.

# The VAPY - (Value APY)

(An Honest Measure of Yield Generation)

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The APY (Annual Percent Yield) is a measure of nominal tokens being earned by the user, but it may not say much about the amount of actual *value* being awarded. We present an alternative formula, called the VAPY, which is meant to measure the amount of economic value that the user actually receives under the pretense of a quoted APY number. This measurement is extremely simple (and fairly obvious), and the only difficulty in application is the availability of certain pertinent information. The document will be divided into 4 sections:

- Section 1: Introduction
- Section 2: What Is APY?
- Section 3: The VAPY
- Section 4: A Case Study

## Section 1: Introduction

The world of DeFi seems to suffer from a lack of a widely accepted and clearly defined taxonomy of its most critical components. This can result in projects adopting terminology and labels which conflate multiple heterogeneous concepts and processes. This is specifically the case as it relates to yield generation mechanisms, where terms from TradFi are adopted liberally for various concepts that may not exactly correspond to their TradFi origin. Presently, we look at the notion of APYs. Within the world of TradFi, the APY represents the rate of return of value earned on an investment (with an emphasis of the word *value*). In the world of DeFi, the APY refers to the return rate of nominal tokens, but with values that can reach into the tens of thousands, it can be doubtful that it represents actual value.

In this article, we will derive a simple mathematical modification to the notion of APY (as it is typically used in DeFi) which will hopefully bring it back down to reality, by attaching it to an actual amount of value earned. We call this the *Value* APY, or VAPY.

## Section 2: What Is APY?

Before we define and derive the VAPY in the next section, we will first use this section to go over some basic fundamental properties of the APY. This may be basic material for some readers, but we include it here for completeness. The phrase *Annual Percent Yield* (APY) suggests that it is a measure of the total percent yield of some asset accumulated after a full year. However, in practice it is more frequently used as a kind of *instantaneous* rate of yield generation, rather than a reflection of an annual aggregate amount. Analogously, one may consider the state of traveling 60 miles per hour. Technically, this means that 60 miles will be traveled over the duration of one hour, but we rarely use it this way in real life. Rather, we use it to describe our instantaneous speed, knowing that, of course, a lot can change during the next hour.

This distinction is simple enough, but how do we quantify it? Let us consider a return bearing instrument, and suppose that we have $n$ return periods over the course of a year. Denote the return rate in the first period by $r_1$, the return rate in the second period is $r_2$, and so on (each expressed as a fraction, not a percent). After the $i^{th}$ return period, the value is increased by a factor of $(1+r_i)$. After $n$ periods, the total value will be

$$\begin{bmatrix}\text{Year End}\\\text{Amount}\end{bmatrix} = \begin{bmatrix}\text{Original}\\\text{Amount}\end{bmatrix}\big(1+r_1\big)\big(1+r_2\big)...\big(1+r_n\big) \quad (1)$$

We can articulate the net effect of these return periods by writing the string of coefficients in expression (1) as '1 plus something', where this 'something' would be the annual percent yield (APY):

$$\begin{bmatrix}\text{Year End}\\\text{Amount}\end{bmatrix} = \begin{bmatrix}\text{Original}\\\text{Amount}\end{bmatrix}\big(1+APY\big) \quad (2)$$

Comparing (1) and (2), we would thus have

$$\big(1+APY\big) = \big(1+r_1\big)\big(1+r_2\big)...\big(1+r_n\big) \quad (3)$$

However, expression (3) is not often what people actually mean when they quote the APY. Instead, they often consider the *current* return rate (let's just call it $r$), and they *extrapolate* what the total annual return *would* be if this $r$ held constant over the $n$ periods in the year:

$$\begin{aligned}\big(1+APY\big) &= \big(1+r\big)\big(1+r\big)...\big(1+r\big)\\ &= \big(1+r\big)^n\end{aligned} \quad (4)$$

Again, in this context the APY is an instantaneous measure of yield that just happens to be reported in a way that is extrapolated to a year's worth of yield, *were* the rate to remain constant at its current value. **This is important because it will inform the way we measure APY when analyzing empirical data in Section 4**. We will return to this in a moment, but first let's establish an important definition.

One may notice from equations (1)-(4) that, rather than considering return rates, it appears that the more relevant quantity is 1+[return rate]. This is indeed an important quantity, and unfortunately, there does not seem to be a consensus on what to call it, even in TradFi (the phrase *gross returns* is sometimes used). Specifically, for a return rate $r$, the quantity $(1+r)$ represents the total factor by which an investors value has been multiplied. For example, if one experiences returns of 15%, then we have $r = 0.15$ and the gross return is 1.15. In other words, their overall value has been multiplied by a factor of 1.15. For the sake of this article, we will call this quantity the *gross returns factor* (GRF):

$$GRF := 1 + \begin{bmatrix}\text{return rate}\\\text{(as a decimal)}\end{bmatrix} \quad (5)$$

Now let's return to the question of how we measure APY in practice. Imagine that we had 30 days of data for a token that generates daily returns with APY values that fluctuate day after day. Let's denote theses daily APY values by $\{APY_1, APY_2, ..., APY_{30}\}$. How do we condense these numbers into a single APY number that describes the returns over the *whole* month? A naive guess would be to take their arithmetic mean,

$$\underset{(month)}{APY} \stackrel{?}{=} \frac{APY_1 + APY_2 + ... + APY_{30}}{30}, \quad (6)$$

but this would be unwise. The correct approach would be to measure the *net* return rate over the whole month (with no regard to the daily returns whatsoever), and plug this into equation (4), with $n = 12$ for the number of months in the year:

$$\Big(1 + \underset{(month)}{APY}\Big) = \left(1 + \begin{bmatrix}\text{net return rate}\\\text{over the month}\end{bmatrix}\right)^{12} \quad (7)$$

Analogously, one could think of measuring a rate of miles per *hour*, but over, say, a *20 minute window*; we need only measure the net distance over the interval.

For an arbitrary time interval, we can generalize the formula in equation (7) as follows:

$$\left(1+\underset{(interval)}{APY}\right) = \left(1+\begin{bmatrix}\text{net returns}\\\text{over interval}\end{bmatrix}\right)^{\left[\frac{|\text{year}|}{|\text{interval}|}\right]} \tag{8}$$

where the exponent |year|/|interval| denotes the length of the year divided by the length of the interval (i.e. the number of intervals in a year). This is the formula that we will use to define our APY value over an arbitrary interval, and it will be critical for our case study in Section 4. As an interesting mathematical aside, it can be shown (below) that the formula in (7) is equivalent to the *geometric* mean of the daily GRFs (roughly), as opposed to our naive guess in (6) of the arithmetic mean (shown in the following mathematical detour):

### Mathematical Detour

Let's briefly return to our previous problem; given a set of daily $APY$ values $\{APY_1, APY_2, ..., APY_{30}\}$, how does one condense these into one net $APY$ figure for the whole month? First, let's relate each daily $APY_i$ to the daily return rate $r_i$ by inverting equation (4):

$$(1 + APY_i)^{1/365} = (1 + r_i) \tag{9}$$

Now, let $\bar{r}$ be the actual net return rate over the month:

$$(1 + \bar{r}) = (1 + r_1)(1 + r_2)...(1 + r_{30}) \tag{10}$$

Then, according to formula (4), the extrapolated $APY$ for the month would be

$$1 + \underset{(month)}{APY} = (1 + \bar{r})^{12} \tag{11}$$

We substitute (10) into (11), and then replace each factor of $(1+r_i)$ with $(1+APY_i)$ using (9) to find the following:

$$\begin{aligned}
1+\underset{(month)}{APY} &= (1 + \bar{r})^{12} \\
&= \Big((1 + r_1)\dots(1 + r_{30})\Big)^{12} \\
&= \Big((1 + APY_1)^{1/365}\dots(1 + APY_{30})^{1/365}\Big)^{12} \\
&= \Big((1 + APY_1)\dots(1 + APY_{30})\Big)^{12/365} \\
&\approx \Big((1 + APY_1)\dots(1 + APY_{30})\Big)^{1/30}
\end{aligned} \tag{12}$$

From (12) we can confirm: the effective APY over the whole month is such that the corresponding GRF is the geometric mean of the GRFs for each day (although one may notice this is only *roughly* true - it would only be *exact* if the interval fit perfectly into one whole year - after all, we had to slightly fudge the last line in (12), when we wrote $12/365 \approx 1/30$).

## Section 3: The VAPY

Now we will actually define the VAPY. Before we ultimately give a mathematical derivation, we first express it conceptually (in terms of GRFs):

$$\left(1+VAPY\right) = \left(1+APY\right)\frac{\left(1+\begin{bmatrix}\text{rate of value}\\\text{capture}\end{bmatrix}\right)}{\left(1+\begin{bmatrix}\text{rate of}\\\text{inflation}\end{bmatrix}\right)} \tag{13}$$

This is actually quite intuitive; the VAPY is given in terms of the APY, but amplified by increases in value capture, and is suppressed by increases in inflation.

As reasonable as this definition may sound, let's give a concrete derivation. First, we must define some variables. For a given token ecosystem, define the following:

- $T$ = the total token supply
- $K$ = the economic value captured in the system

For the $i^{th}$ individual user, define the following:

- $T_i$ = the number of tokens owned by the user
- $K_i$ = the amount of economic value that the user can rightfully be said to own

Clearly the quantities $T$ and $T_i$ can be measured empirically. We also assume the quantity $K$ can be measured or approximated somehow. The quantity $K_i$, however, is not entirely well defined, and so we will define $K_i$ as follows; since an individual user can claim a fraction $T_i/T$ of ownership of the token supply, then they therefore own a similar fraction of the economic value captured. In symbols,

$$K_i := (T_i/T)K \tag{14}$$

Now, fix a return period of $d$ days. Over this interval, we define the following rates:

- $\alpha$ := the nominal APY a user receives
- $\tau$ := the growth of the token supply (inflation rate)
- $\kappa$ := the growth rate of economic value capture
- $\beta$ := the VAPY that we wish to define

Note that in order to make meaningful comparisons, all of these quantities must be measured in a way analogous to formula (8):

$$(1+\alpha) = \left(1+\begin{bmatrix}\text{net returns over}\\\text{entire interval}\end{bmatrix}\right)^{365/d} \tag{15}$$

$$(1+\tau) = \left(1+\begin{bmatrix}\text{fractional change}\\\text{in token supply}\end{bmatrix}\right)^{365/d} \tag{16}$$

$$(1+\kappa) = \left(1+\begin{bmatrix}\text{fractional change}\\\text{in value capture}\end{bmatrix}\right)^{365/d} \tag{17}$$

Now, because we assume that $\{T_i, T, K\}$ can be measured empirically, then the rates $\{\alpha, \tau, \kappa\}$ should be empirically measurable as well. Thus, our goal in this section is to find a formula for the remaining rate $\beta$ in terms of the quantities $\{\alpha, \tau, \kappa\}$. To this end, suppose that $t_1$ and $t_2$ correspond to the beginning and end of the period of $d$ days in which returns are realized. Then, by definition, we can write the following:

$$T_i(t_2) = T_i(t_1)(1+\alpha) \tag{18}$$

$$T(t_2) = T(t_1)(1+\tau) \tag{19}$$

$$K(t_2) = K(t_1)(1+\kappa) \tag{20}$$

A reasonable definition for $\beta$ (the VAPY) would be that it quantifies the rate of returns for the value captured that can be claimed by an individual user (which we denoted by $K_i$):

$$K_i(t_2) = K_i(t_1)(1+\beta) \tag{21}$$

Now, let's rearrange (21):

$$\big(1+\beta\big) = \frac{K_i(t_2)}{K_i(t_1)} \tag{22}$$

Next, we substitute definition (14) into (22), and then simplify:

$$\begin{aligned}\big(1+\beta\big) &= \frac{\Big[\Big(T_i(t_2)/T(t_2)\Big)K(t_2)\Big]}{\Big[\Big(T_i(t_1)/T(t_1)\Big)K(t_1)\Big]} \\ &= \left[\frac{T_i(t_2)}{T_i(t_1)}\right]\left[\frac{K(t_2)}{K(t_1)}\right]\left[\frac{T(t_1)}{T(t_2)}\right] \\ &= \big(1+\alpha\big)\big(1+\kappa\big)\big(1+\tau\big)^{-1} \end{aligned} \tag{23}$$

where the last line follows by using (18)-(20). Thus, we have our formula for $\beta$ (expressed, of course, as a GRF):

$$\big(1+\beta\big) = \frac{\big(1+\alpha\big)\big(1+\kappa\big)}{\big(1+\tau\big)} \tag{24}$$

One can easily check that, according to all stated definitions, equation (24) is indeed equivalent to the initial definition given in (13), as promised.

Notice that, in the 'fairest' scenario, where the inflation rate keeps pace with the rate of value captured (i.e. when $\tau = \kappa$), then equation (24) will reduce to $\beta = \alpha$. In other words:

> The APY and the VAPY are equal precisely when tokens are minted only in response to value that is captured, as we might expect

(As a final comment to this section, we justify the choice of the letter $\beta$ to represent the VAPY. Specifically, the letters $\{\alpha, \tau, \kappa\}$ were chosen to correspond phonetically to $\{APY, token, capture\}$, respectively. For the VAPY, we choose the letter $\beta$ which, in modern Greek, can be used for the english 'v' sound. )

## Section 4: A Case Study

During the year 2021 into 2022, Olympus DAO and its many forks achieved what many might consider to be the pinnacle of the APY arms race, offering APY rates in the tens and even hundreds of thousands. Given these audacious return rates and the spectacular price implosion we witnessed thereafter, one must wonder what the actual rate of economic return was at any point in time. In this section, we will apply the VAPY formulation to various time periods (30 day intervals) over the period from April 2021 to January 2022. Our data can be found here. It must be noted that getting an accurate measure of the economic value captured by the system is an art in and of itself, and in this current work we are simply relying on the data provided (perhaps future work might be done to address the very question of how to measure economic value captured).

In each of the figures below, we plot the total token supply and the total economic value captured, side by side as parallel time series, as in Figure 1:

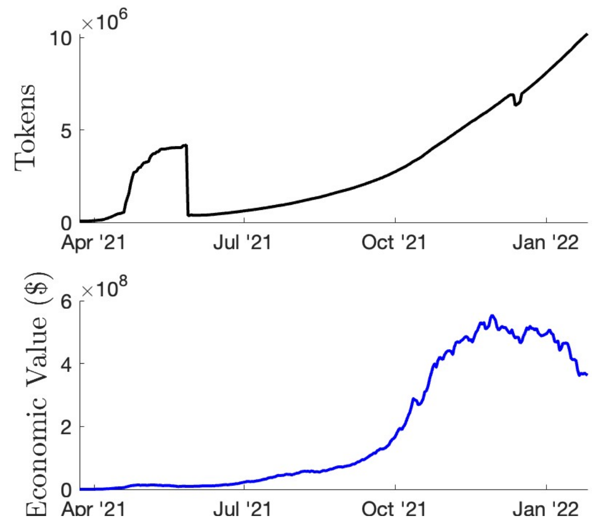


**Figure 1.** Token Supply and Economic Value Capture.

In each figure to follow, we will select a 30 day period, highlighted in red. For each 30 day interval, we use the data provided to compute the values of $\{\alpha, \tau, \kappa\}$ according to their definitions (15)-(17). We then plug these values in to (24) for $\beta$ to get our VAPY value, which can then be compared to $\alpha$, our nominal APY. This comparison, between APY and VAPY, is of course the whole point of the analysis.

The four specific 30-day intervals that we will investigate are the following:

1. 9/Aug/2021 - 8/Sep/2021 (Figure 2)
2. 18/Sep/2021 - 10/Oct/2021 (Figure 3)
3. 27/Nov/2021 - 27/Dec/2021 (Figure 4)
4. 27/Dec/2021 - 26/Jan/2022 (Figure 5)

In Figure 2, we begin in the fall of 2021. At this point the advertised APY had come down to a rate of 7,641% (i.e. $\alpha = 76.41$), which is actually a long way down from previous APY values in the tens of thousands. Never the less, economic value was indeed entering the system due to the incredible enthusiasm ($\kappa = 124.268$). However, the rate of token inflation ($\tau = 243.455$) effectively diluted the real value of the staking returns, being dragged down to a VAPY amount of 3,719% ($\beta = 37.19$). This is a bit less than the 7,641% as promised, but one could argue it's still a good deal.

Moving on to late autumn, we turn to Figure 3. Now we see the rate of economic value begins to dramatically increase ($\kappa = 64934.6$). This has the effect of giving us a VAPY of 15,761%, which is actually *higher* than the quoted APY of 7,641%.

However, passing into the winter (Figure 4), we see that the rate of economic value captured begins to level off ($\kappa = -0.42223$). Users are still being rebased an enormous (albeit lower) amount of tokens with an APY of 4,500%, but the negative $\kappa$ results in a VAPY of 45%. This is literally two orders of magnitude lower.

Lastly, in Figure 5, we turn to early 2022, when the loss of economic value finally outpaces the rebasing rate, and the VAPY turns negative at -99.3%. This occurs as users are receiving a nominally *positive* APY of 900%!

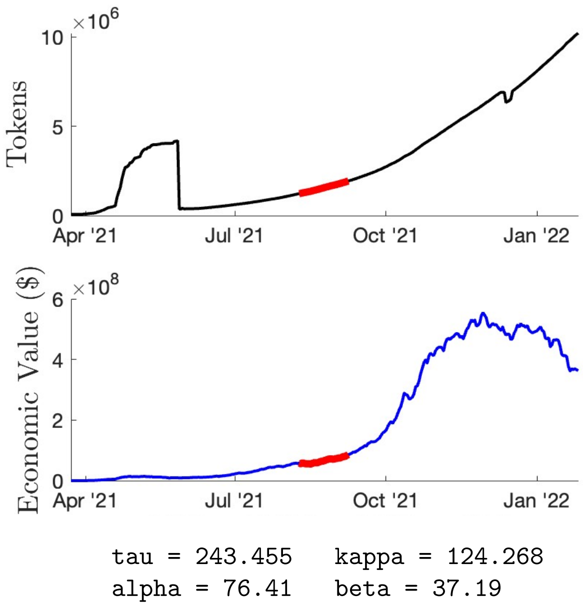


**Figure 2.** 9/Aug/2021 - 8/Sep/2021

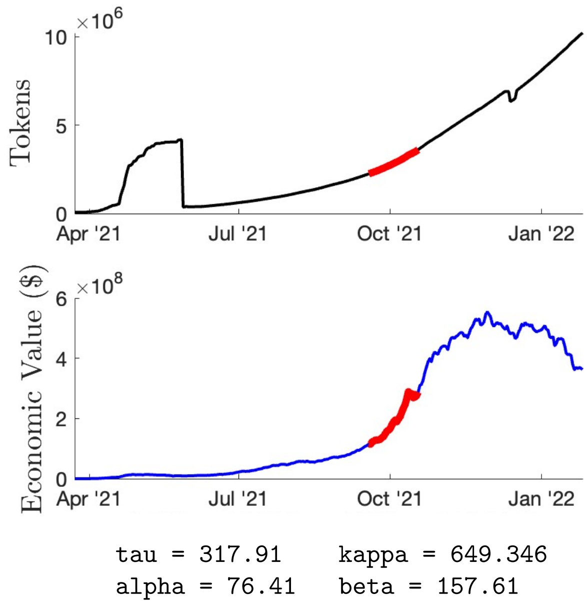


**Figure 3.** 18/Sep/2021 - 10/Oct/2021

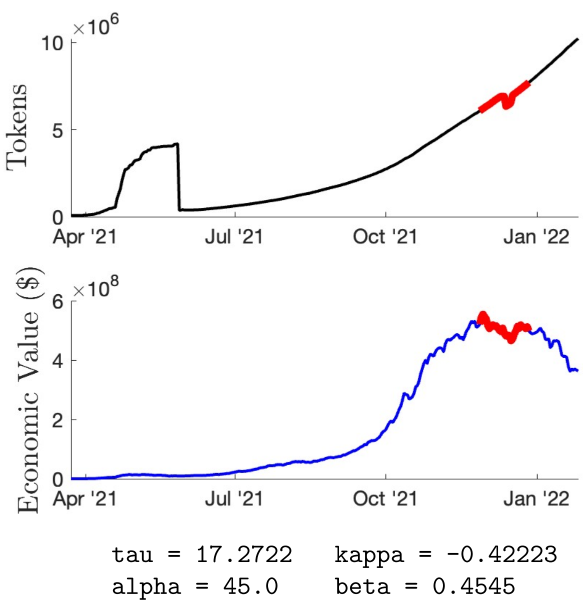


**Figure 4.** 27/Nov/2021 - 27/Dec/2021

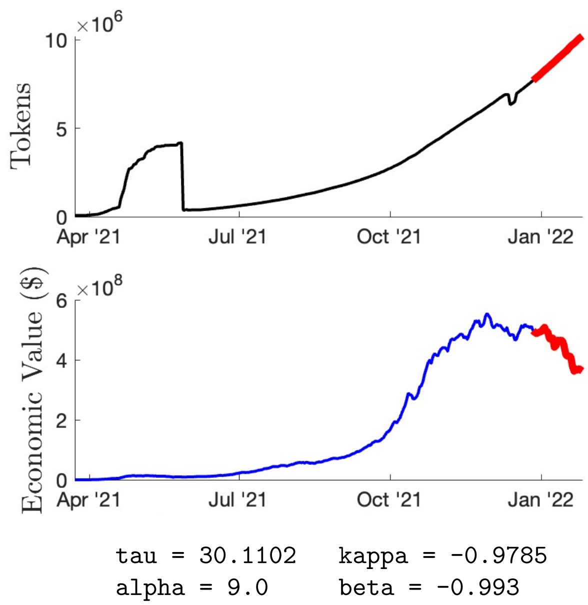


**Figure 5.** 27/Dec/2021 - 26/Jan/2022