A Neuro-Fuzzy Approach to Variational Problems by Using Gaussian Membership Functions

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ABSTRACT

In this paper we propose a neuro-fuzzy direct solution method of variational problems in which the cost function of an integral form is minimized. We deal with two nonlinear systems, one is a direct drive(DD) manipulator system and the other is a trailer truck system. The DD manipulator system is described by a continuous-time dynamical model and the trailer truck system is described by a discrete-time dynamical model. The problem is to find trajectories which minimize the cost function of an integral form. The trajectories of state variables and input variables are represented by fuzzy models that consist of Gaussian membership functions. The networks of Gaussian functions are trained by the steepest descent method to minimize the cost function .

The proposed neuro-fuzzy approach provides a direct solution method of the variational problems by using Gaussian functions. The function is regarded as a simplified fuzzy reasoning model and called neuro-fuzzy.

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1. INTRODUCTION

J.Moody and C.Darken [1, 2] have proposed radial basis function(RBF) networks, a technique for interpolating in a high dimensional space, and reported that the training of RBF networks are potentially 1000 times faster than sigmoidal basis function networks with backpropagation for comparable error rates. The RBF network can be regarded as a three layered neural network[1–4] and a simplified fuzzy reasoning model[5–10]. In this paper we propose an optimal control scheme for nonlinear systems by using RBF networks, which we call a "neuro-fuzzy approach". In the proposed method the cost function of an integral form is minimized by the steepest descent method. The methodology shows promise for application in control problems that are so complex that analytical design techniques are not suitable. It is shown that the RBF networks can be used to solve highly nonlinear control problems.

In this paper we deal with a direct drive (DD) manipulator system and a trailer truck system as severely nonlinear systems. The DD manipulator system is described by a continuous-time dynamical model, and the trailer truck system is described by a discrete-time dynamical model.

For the multijoint arm movement, there exist complicated control problems because of the presence of interactional forces such as colioris forces and reaction forces. When the hand of the multijoint arm is moved from one position to another, there are an infinitely number of possible paths. Though the minimum jerk model proposed by Flash and Hogan [11] takes into account the kinematics of movement, it is independent of the dynamics of the musculoskeltal system. Uno *et al.* [12] have proposed a performance index i.e. the sum of square of the torque change rate integrated over the entire movement period. The model is called a "minimum torque-change model". Uno *et al.* have shown that the hand trajectories yielded by the minimum torque-change model were in better agreement with human arm movement compared with the minimum jerk model. The iterative scheme for the minimum torque-change model uses a method of variational calculus and dynamic optimization theory. Hence, it seems to be a control theoretic method rather than a neuro scientific one. In this paper, we propose a direct solution method of this variational problem using Gaussian radial basis functions (RBF) which can be reinterpreted as a simplified fuzzy reasoning model. The RBF networks are attractive since such networks are potentially faster than the conventional backpropagation networks[13] for comparable error rates in supervised learning [1] .

Control of a trailer truck backing to a loading dock[14, 15] is a difficult problem, for the system is non-linear and unstable. The neural network truck backer-upper control was developed by Nguyen and Widrow[16]. In their approach an emulator, a multilayered neural network[13], learns to identify the system's dynamics characteristics. A controller, another multilayered neural network, then is trained to minimize the final state error and control energy. The advantage of this approach is to realize an optimal feedback control based on a cost function of some state and manipulated variables. However, unfortunately a trained emulator is needed for this approach, and thousands of backups are required. Therefore, training the network using an actual trailer truck is not a realistic approach. Hence, the training is carried out by a computer simulation using a mathematical model of the trailer truck dynamics.

We apply the proposed neuro-fuzzy scheme which is a direct solution method to the variational problem, and the trajectories of state and input variables of nonlinear systems are represented by Gaussian functions.

In Section 2 we describe a "neuro-fuzzy optimal control scheme", and in Section 3 we apply the proposed method to a first order lag system which has an input variable and a state variable. Section 4 and 5 are devoted to describing applications to a DD manipulator system and a trailer truck system respectively.

2. NEURO-FUZZY OPTIMAL CONTROL

Let $\boldsymbol{x} = (x_1, x_2, \cdots, x_Q)'$ be a vector of state variables in an optimal control problem, where $'$ denotes transpose. Let $\boldsymbol{u} = (u_1, u_2, \cdots, u_R)'$ be an input vector of manupulated variables. Then, the state equation is written as :

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{1}$$

Let $\boldsymbol{x}(0)$ be an initial state. T is an appropriately chosen time to terminate control. $\boldsymbol{x}(T)$ is a terminal state. The cost function is the following:

$$J = \int_0^T F(\boldsymbol{x}(t), \boldsymbol{u}(t))dt + G(\boldsymbol{x}(0), \boldsymbol{x}(T)) \tag{2}$$

We seek an optimal control by which the integral of F with respect to t from 0 to T and the initial and final state errors represented by G are minimized. Hence, it is a variational problem to find the functions $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$ which minimize the cost function J.

First, M independent variables are chosen from the state variables (x_1, $x_2, \cdots, x_Q$) and the manipulated variables $(u_1, u_2, \cdots, u_R)$. Each independent variable is represented by Gaussian functions with one input variable t (time). The fuzzy reasoning *if -then* rules are written as:

if t is μ_{mk} then y_m is w_{mk} $\quad (k = 1, \cdots, K)$

where K is the number of fuzzy rules used for representing $y_m(t)$. The membership function of the premise part of each fuzzy rule for the independent variable $y_m(t)$ is defined by a Gaussian function (i.e. a bell shape membership function) as :

$$\mu_{mk}(t) = \exp\left(-\frac{(t - a_{mk})^2}{b_{mk}}\right), \qquad (k = 1, \cdots, K) \tag{3}$$

The mth independent variable can be written as the fuzzy model

$$y_m(t) = \sum_{k=1}^{K} \mu_{mk}(t) \cdot w_{mk}, \qquad (m = 1, 2, \cdots, M) \tag{4}$$

y_m is equivalent to Gaussian radial basis functions [1–4] . Let $\boldsymbol{y} = (y_1, y_2, \cdots, y_M)'$ be a vector of these independent variables. Then, the other state and manipulated variables can be represented by the independent variables $y_1, y_2, \cdots$ and y_M. Substituting $y_1, y_2, \cdots$ and y_M to the state and constraints equations, we have

$$\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}) \tag{5}$$

$$\boldsymbol{g}(\boldsymbol{y}, \dot{\boldsymbol{y}}) = \boldsymbol{O} \tag{6}$$

The cost function of the neuro-fuzzy optimal control can be written as :

$$\begin{aligned} J \;=\; & \int_0^T F(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))dt \\ & + \alpha \int_0^T ||\dot{\boldsymbol{y}}(t) - \boldsymbol{f}(\boldsymbol{y}(t))||^2 dt \\ & + \boldsymbol{G}(\boldsymbol{y}(0), \boldsymbol{y}(T), \dot{\boldsymbol{y}}(0), \dot{\boldsymbol{y}}(T)) \end{aligned} \tag{7}$$

where α is a positive constant. For numerical integration of the cost function, Sympson's formula is adopted. The learning rules based on the gradient descent method are

$$w_{mk}^{NEW} = w_{mk}^{OLD} - \tau \frac{\partial J}{\partial w_{mk}} \tag{8}$$

$$a_{mk}^{NEW} = a_{mk}^{OLD} - \tau \frac{\partial J}{\partial a_{mk}} \tag{9}$$

$$b_{mk}^{NEW} = b_{mk}^{OLD} - \tau \frac{\partial J}{\partial b_{mk}} \tag{10}$$

where τ is the positive learning rate.

3. APPLICATION TO A FIRST ORDER LAG SYSTEM

We apply the neuro-fuzzy optimal control to a simple first order lag system and compare the result with the theoretical solution.

The state equation and the boundary conditions are given as :

$$\dot{x}(t) = -cx(t) + u(t), \qquad x(0) = x_0, \quad x(T) = 0 \tag{11}$$

The problem is to find an optimal control by which the constraints in Eq. (11) are satisfied and the cost function

$$\begin{aligned} J(x(t)) &= \int_0^T \{x^2(t) + u^2(t)\}dt \\ &= \int_0^T \{x^2(t) + (\dot{x}(t) + cx(t))^2\}dt \end{aligned} \tag{12}$$

is minimized. Since the system is linear and the cost function is quadratic, (i.e. the LQ problem), we have a theoretical solution. By our proposed method,

$$x(t) = \sum_{k=1}^{K} \mu_k(t) w_k \tag{13}$$

$$\mu_k(t) = \exp\left(-\frac{(t-a_k)^2}{b_k}\right) \tag{14}$$

$$\begin{aligned} J &= \int_0^T \{x^2(t) + (\dot{x}(t) + cx(t))^2\}dt \\ &+ s_0(x(0) - x_0)^2 + s_T x^2(T) \end{aligned} \tag{15}$$

(a) Optimal trajectory (x) (b) Optimal control input (u)

Figure 1. An approximately optimal solution by the neuro-fuzzy and the theoretical optimal solution.($c = 1$,$x_0 = 10$,$T = 4$)

where s_0 and s_T are the positive constants for evaluating the errors to the initial condition and the terminal condition respectively. From Eq.(13) we have

$$\dot{x}(t) = \sum_{k=1}^{K} \left(-\frac{2(t-a_k)}{b_k} \right) \mu_k(t) w_k \tag{16}$$

The learning rules are as in Eqs.(8)-(10) and $c = 1$, $x_0 = 10$, $T = 4$, $K = 10$, $s_0 = 100$, $s_T = 100$. We use Simpson's formula of numerical integration. In Figure 1-(a) the dotted line represents the computational result of an approximately optimal solution by the neuro-fuzzy approach. The solid line (theoretical solution) and the dotted line almost overlap each other.

The manipulated variable $u(t)$ can be obtained by the relation $u(t) = \dot{x}(t) + cx(t)$ and is shown in Figure 1-(b). Both $x(t)$ and $u(t)$ (shown by the dotted lines) are similar to the theoretical solutions $x^*(t)$ and $u^*(t)$ (shown by the solid lines) respectively.

Figure 2. A two-joint manipulator which moves within a horizontal plane.

4. OPTIMAL CONTROL OF A DD MANIPULATOR

4.1. Neuro-Fuzzy Minimum Torque Change Model for a DD Manipulator

We consider a two joint direct drive manipulator as shown in Figure 2, which moves within a horizontal plane. The manipulator dynamics is given as:

$$\begin{aligned}
&(I_1 + I_2 + 2M_2L_1S_2\cos\theta_2 + M_2(L_1)^2 + J_1)\ddot{\theta}_1 \\
&+(I_2 + M_2L_1S_2\cos\theta_2)\ddot{\theta}_2 \\
&-M_2L_1S_2(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2\sin\theta_2 + r_1\dot{\theta}_1 = k_1v_1
\end{aligned} \tag{17}$$

$$\begin{aligned}
&(I_2 + M_2L_1S_2\cos\theta_2)\ddot{\theta}_1 + (I_2 + J_2)\ddot{\theta}_2 \\
&+M_2L_1S_2(\dot{\theta}_2)^2\sin\theta_2 + r_2\dot{\theta}_2 = k_2v_2
\end{aligned} \tag{18}$$

where M_i, L_i and S_i represent the mass, the length and the distance from the center of mass to joint respectively, and I_i represents the rotary inertia of the link i around the joint. r_i is the viscosity coefficients. The cost function is given as:

$$\begin{aligned}
J(\tau_1(t), \tau_2(t)) \;=\; & \frac{1}{2}\Big(\int_0^T C_1(\dot{\tau}_1)^2 + C_2(\dot{\tau}_2)^2 dt \\
& +C_3(\theta_1^0 - \theta_1(0))^2 + C_4(\theta_1^T - \theta_1(T))^2 \\
& +C_5(\theta_2^0 - \theta_2(0))^2 + C_6(\theta_2^T - \theta_2(T))^2 \\
& +C_7(\dot{\theta}_1(0))^2 + C_8(\dot{\theta}_2(0))^2 + C_9(\dot{\theta}_1(T))^2 \\
& +C_{10}(\dot{\theta}_2(T))^2 + C_{11}(\ddot{\theta}_1(0))^2 + C_{12}(\ddot{\theta}_2(0))^2 \\
& +C_{13}(\ddot{\theta}_1(T))^2 + C_{14}(\ddot{\theta}_2(T))^2\Big)
\end{aligned} \tag{19}$$

where θ_i^0 represents the initial angle of the ith link and θ_i^T is the final angle of the ith link. T represents a given time for movement. The right-hand-sides of the Eqs. (17) and (18) correspond to torque $\tau_i, (i = 1, 2)$ respectively. The derivatives of τ_i are

$$\begin{aligned}\dot{\tau}_1(t) &= M_2L_1S_2\{-(2\dot{\theta}_1\ddot{\theta}_2 + 4\ddot{\theta}_1\dot{\theta}_2 + 3\dot{\theta}_2\ddot{\theta}_2)\sin\theta_2 \\ &\quad +(2\dddot{\theta}_1 + \dddot{\theta}_2 - 2\dot{\theta}_1(\dot{\theta}_2)^2 - (\dot{\theta}_2)^3)\cos\theta_2\} \\ &\quad +(I_1 + I_2 + M_2(L_1)^2 + J_1)\dddot{\theta}_1 + I_2\dddot{\theta}_2 + r_1\ddot{\theta}_1 \end{aligned} \tag{20}$$

and

$$\begin{aligned}\dot{\tau}_2(t) &= M_2L_1S_2\{(2\dot{\theta}_1\ddot{\theta}_1 - \ddot{\theta}_1\dot{\theta}_2)\sin\theta_2 \\ &\quad +((\dot{\theta}_1)^2\dot{\theta}_2 + \dddot{\theta}_1)\cos\theta_2\} \\ &\quad +J_2\dddot{\theta}_2 + I_2(\dddot{\theta}_1 + \dddot{\theta}_2) + r_2\ddot{\theta}_2 \end{aligned} \tag{21}$$

We define joint angle $\theta_i(t)$ as:

$$\theta_i(t) = \sum_{k=1}^{K} \mu_{ik}(t) \cdot w_{ik} \qquad (i = 1, 2) \tag{22}$$

$$\mu_{ik}(t) = \exp\left(-\frac{(t - a_{ik})^2}{b_{ik}}\right) \tag{23}$$

The initial values of the parameters in (16) are set as:

$$a_{ik} = \frac{T}{K-3}(k-2) \tag{24}$$

$$b_{ik} = \frac{T}{2(K-3)} \tag{25}$$

$$w_{ik} = 0.0 \tag{26}$$

where T represents a given time for movement. K is the number of fuzzy rules (Gaussian functions).

4.2. Trajectory Formation by the Gradient Descent Method

The physical parameters of the manipulator are given as in Table 1. The movement from $\theta_1 = \theta_2 = 0.0$ (rad) to $\theta_1 = \theta_2 = 1.0$ (rad) with the duration of one second is assumed. The weight parameters $C_i (i = 1 \sim 14)$ of the cost function are given as in Table 2.

Table 1. Values of the physical parameters of the manipulator shown in Figure 2.

Parameter	Link1	Link2
M_i(kg)	3.0	2.0
L_i(m)	0.50	0.35
S_i(m)	0.21	0.15
I_i(kg $\cdot m^2$)	0.27	0.10
J_i(kg $\cdot m^2$)	0.0005	0.0003
r_i(kg $\cdot m^2/s$)	0.20	0.15
k_i(N $\cdot m/V$)	0.30	0.10

Table 2. Values of the coefficients of the cost function.

η	0.000001	(learning rate)	
C_1	0.01	C_2	0.01
C_3	50000	C_4	50000
C_5	50000	C_6	50000
C_7	10000	C_8	10000
C_9	10000	C_{10}	10000
C_{11}	100	C_{12}	100
C_{13}	100	C_{14}	100

Table 3. Errors at the initial and terminal positions(absolute values).

$\theta_1(0.0)$	0.000370	$\theta_2(0.0)$	0.000112
$\dot{\theta}_1(0.0)$	0.000430	$\dot{\theta}_2(0.0)$	0.000107
$\ddot{\theta}_1(0.0)$	0.015510	$\ddot{\theta}_2(0.0)$	0.003999
$\theta_1(1.0)$	0.000430	$\theta_2(1.0)$	0.000036
$\dot{\theta}_1(1.0)$	0.000622	$\dot{\theta}_2(1.0)$	0.000112
$\ddot{\theta}_1(1.0)$	0.022390	$\ddot{\theta}_2(1.0)$	0.000519

The computational results for the 1st link are shown in Figure 3. The trajectories are depicted by solid lines and the numbers of learning iterations are also shown in the figures. Figure 4 shows the change of the value of the cost function as learning proceeds. Table 3 shows errors at the initial and terminal positions. Figure 5 shows the trajectory passing through a via-point.

angle θ [rad]

time t[s]

angular velocity $\dot{\theta}$ [rad/sec]

time t[s]

torque [N $\cdot$ m]

time t[s]

Figure 3. Trajectories of the 1st link.

value of cost function

iteration number

Figure 4. Changes in the cost function value as learning proceeds.

angle θ [rad]

time t[s]

angular velocity $\dot{\theta}$ [rad/sec]

time t[s]

torque [$\mathrm{N} \cdot \mathrm{m}$]

time t[s]

Figure 5. Obtained trajectories of the 1st link passing through a via-point.

5. TRAILER TRUCK BACKER-UPPER

Figure 6 shows a diagram of a trailer and truck system. The definition of state variables (ϕ,ψ,θ,η and ζ) and the manipulated variable (δ) are also illustrated in Figure 6. The problem is to control the steering of a trailer truck while backing up to a loading dock from an initial position. Only backing up is allowed. It is assumed that the truck moves very slow. Let time step Δt be small, then the dynamics equations of the trailer truck system can be written geometrically as :

$$\psi[i+1] \quad = \quad \psi[i] + \frac{v\Delta t \cdot \tan\delta[i]}{l} \tag{27}$$

$$\theta[i+1] \quad = \quad \theta[i] + \frac{v\Delta t \cdot \sin\phi[i]}{L} \tag{28}$$

$$\zeta[i+1] \quad = \quad \zeta[i] + v\Delta t \cdot \cos\phi[i] \cdot \cos\frac{\theta[i+1]+\theta[i]}{2} \tag{29}$$

$$\eta[i+1] \quad = \quad \eta[i] + v\Delta t \cdot \cos\phi[i] \cdot \sin\frac{\theta[i+1]+\theta[i]}{2} \tag{30}$$

$$\phi[i] \quad = \quad \psi[i] - \theta[i] \tag{31}$$

From above difference equations (27)-(31), the four variables ψ, θ, η and δ can be written only by ϕ as :

$$\theta[i] = \theta_0 + \sum_{n=0}^{i-1} \frac{v\Delta t \cdot \sin\phi[i-1-n]}{L} \tag{32}$$

$$\eta[i] = \eta_0 + \sum_{n=0}^{i-1} \left(v\Delta t \cdot \cos\phi[i-1-n] \cdot \sin\frac{\theta[i-n]+\theta[i-1-n]}{2} \right) \tag{33}$$

$$\delta[i] = \tan^{-1}\left(\frac{l \cdot (\phi[i+1] - \phi[i] + \theta[i+1] - \theta[i])}{v\Delta t} \right) \tag{34}$$

ψ : angle of the truck with horizontal,
θ : angle of the trailer with horizontal,
ϕ : relative angle of the truck with trailer,
δ : steering angle
(η, ζ) : Cartesian coordinate of the robot

Figure 6. Diagram of a truck and trailer.

where θ_0 and η_0 are the initial values of θ and η respectively. $\theta[i]$ in Eqs.(33) and (34) can be substituted by Eq. (32). Hence, only the relative angle ϕ of the truck and the trailer is represented as a neuro-fuzzy model :

$$\phi[i] = \sum_{k=1}^{K} \mu_k[i] \cdot w_k \tag{35}$$

$$\mu_k[i] = \exp(-\frac{(i\Delta t - a_k)^2}{b_k}) \tag{36}$$

The goal is to make the back of the trailer to be parallel to the loading dock and to have θ, ψ, and η equal zero with as little steering as possible. By substituting Eqs.(35) and (36) to Eqs.(32)-(34), we have a cost function of quadratic form as :

$$\begin{aligned} J &= \sum_{i=0}^{N-1} (q_1\phi^2[i] + q_2\theta^2[i] + q_3\eta^2[i] + r\delta^2[i]) \\ &\quad + s_0(\phi[0] - \phi_0)^2 + s_1\phi^2[N] + s_2\theta^2[N] + s_3\eta^2[N] \end{aligned} \tag{37}$$

where ϕ_0 is the given initial value of ϕ. It should be noted that the unknown parameters in Eq.(37) are w_k, a_k and $b_k(k = 1, \cdots, K)$. q_1, q_2, q_3 and r are the positive weights for $\phi[i]$, $\theta[i]$,$\eta[i]$ and $\delta[i]$ respectively. s_0 and $s_i(i = 1, 2, 3)$ are the positive weights for the initial condition and the terminal condition respectively. The learning rules are Eqs.(8)-(10) and the parameters are given as in Table 4. We set $t_1 = 0$ and $t_2 = 1$. Figure 7 shows the computational result. Figure 8 shows the simulation result of the follow-up control. Figure 9 shows another simulation result where the weights in the cost function were changed to $q_1 = r = 0.001$, $q_2 = 0.01$, $q_3 = 0.0001$, $s_0 = s_1 = s_2 = 1000.0$ and $s_3 = 1.0$.

Table 4. The parameters values.

τ	0.000001	K	15	
q_1	0.01	l	2.8	〔m〕
q_2	0.1	L	5.5	〔m〕
q_3	0.0001	v	-1.0	〔m/s〕
r	0.01	Δt	2.0	〔s〕
s_0	1000.0	N	50	
s_1	100.0	ϕ_0	0.0	〔°〕
s_2	100.0	θ_0	-135.0	〔°〕
s_3	0.01	η_0	10.0	〔m〕

6. CONCLUDING REMARKS

We have proposed a direct solution method of variational problems. The trajectories of state and manipulated variables are represented by networks of Gaussian functions which can be reinterpreted as simplified fuzzy reasoning rules. In conventional fuzzy control, the parameter tuning of fuzzy rules is a troublesome problem, since it is a time consuming task for engineers. The proposed method which is based on the mathematical models of a control object may present a convenient way for this optimizing procedure and provides an easy to use techique for engineers.

Figure 7. An approximately optimal control of backing up a trailer truck by the neuro-fuzzy approach.

Figure 8. A locus of the trailer truck following-up to the optimal trajectory.

Figure 9. Simulation result with $q_1 = r = 0.001$, $q_2 = 0.01$, $q_3 = 0.0001$, $s_0 = s_1 = s_2 = 1000.0$ and $s_3 = 1.0$

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List of Tables

List of Figures