Effectiveness of Regularization in Neuro-Fuzzy Computerized Tomography

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Key words : *computerized tomography, Neuro-Fuzzy, RBF, regularization*

Abstract

A Neuro-Fuzzy approach to computerized tomography (CT) was proposed for the reconstruction of smooth distribution of some material parameters. Unfortunately, detailed pictures of the spatial distribution is hard to reconstruct from very small number of projection data. In this paper we discuss the effectiveness of regularization conditions in Neuro-Fuzzy computerized tomography.

1. INTRODUCTION

Computerized tomography (CT) using Radon and Fourier Transform has revolutionized medical X-ray imaging and non-destructive examination. A method of computerized tomography using Neuro-Fuzzy was proposed for the reconstruction from small number of projection data [2]. The method is based on an iterative learning algorithm using fuzzy models consist of Gaussian membership functions. Unfortunately, detailed pictures of the spatial distribution is hard to reconstruct from very small number of projection data.

In this paper we discuss the effectiveness of regularization conditions in Neuro-Fuzzy CT. Furthermore, by numerical examples we show the method is effective even when projection data contains observational errors.

2. NEURO-FUZZY CT WITH REGULARIZATION CONDITIONS

Let A_{ik} denote the membership function of the k th fuzzy rule in the domain of the i th input variable x_i. Using the Gaussian membership function ,A_{ik} is defined as :

$$A_{ik}(x_i) = \exp\left(-\frac{(x_i - a_{ik})^2}{b_{ik}}\right) \tag{1}$$

where the parameters a_{ik} and $b_{ik}(i = 1, ..., N)$ are given for each k and are changed in the training procedure. The final output y is written as :

$$y = f(x_1, x_2) = \sum_{k=1}^{K} \mu_k(x_1, x_2) \cdot w_k \tag{2}$$

where $\mu_k(x_1, x_2)$ is the compatibility degree of the premise part of the k th fuzzy rule, which is defined as :

$$\mu_k(x_1, x_2) = A_{1k}(x_1) \times A_{2k}(x_2) \tag{3}$$

This simplified fuzzy model is equivalent to the networks of Gaussian RBFs (e.g. by Moody and Darken[3]). In Fig.1 the line from A $(0,x_2^L)$ to B $(1,x_2^L)$ can be written as $x_2 = \alpha x_1 + x_2^L$ where $\alpha = x_2^R - x_2^L$. And the line integral of $f(x_1, x_2)$ along the path AB can be written as :

$$\int_{-\infty}^{\infty} f(z)dz = \sum_{k=1}^{K} \zeta_k \sqrt{\pi b_k} \cdot w_k \tag{4}$$

where z is a parameter denoting the length from A to the point (x_1, x_2). In a similar manner the line integral of $f(x_1, x_2)$ along the path $A'B'$ can be calculated.

Let the line segment AB and $A'B'$ be propagation paths L_p, L'_p. We express the distribution of some material parameter such as the attenuation rate by the Neuro-Fuzzy model $f(x_1, x_2)$. Let the experimental data be I_p^* along L_p and $I_p'^*$ along L'_p respectively. Let a cost function for the evaluation of the approximation be

$$E_1 = \frac{1}{2}\left\{\sum_{p=1}^{P}(I_p - I_p^*)^2 + \sum_{p=1}^{P}(I'_p - I_p'^*)^2\right\} \tag{5}$$

and a cost function for the regularization be

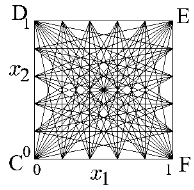

Figure 1: A region of probing

$$E_2 = \frac{1}{2} \sum_{(x_1,x_2)\in W} \left\{ \left(\frac{\partial^2 f(x_1,x_2)}{\partial {x_1}^2} \right)^2 + \left(\frac{\partial^2 f(x_1,x_2)}{\partial {x_2}^2} \right)^2 \right\} \quad (6)$$

where W is the set of data points in the region. Let a total cost function be $E = E_1 + \eta E_2$, and formulate a nonlinear mathematical programming problem to find the values of parameters a_{ik} , b_k and w_k in the Neuro-Fuzzy model, which minimize the total cost function. The learning rules of a_{ik} , b_k and w_k are based on the gradient descent method.

3. COMPUTER SIMULATION

Fig.2 and Fig.3 show the computational results with the regularization conditions in Neuro-Fuzzy computerized tomography. In Fig.2 and Fig.3, (a) the simulated test distribution of a material parameter, (b) the output of $f(x_1, x_2)$ after learning without regularization conditions and (c) the output after learning with regularization conditons are shown. Fig.4 shows the output of $f(x_1, x_2)$ after learning from projection data with Gaussian noise $N(0, \sigma^2)$, (a)$\sigma = 0.01$, (b)$\sigma = 0.05$ and (c)$\sigma = 0.1$. The detailed pictures of the spatial distribution from projection data with relatively small noise can be reconstructed.

4. CONCLUSION

In this paper we have proposed the method of computerized tomography with regularization conditions, and the effect of regularization is clearly shown by the computer simulations. Furthermore, the spatial distribution is recovered from projection data with noise.

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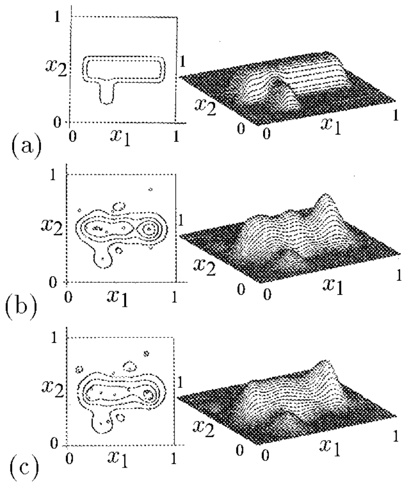

Figure 2: Effect of regularization conditions (Case I : the number of propagation paths is 32)

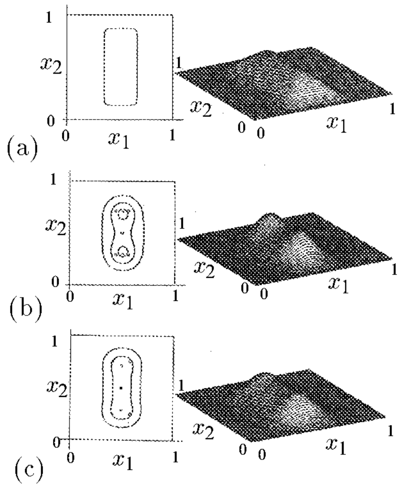

Figure 3: Effect of regularization conditions (Case II : the number of propagation paths is 16)

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Figure 4: Reconstruction from the data with noise $\varepsilon \sim N(0, \sigma^2)$.