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Computed Tomography by Neuro-Fuzzy Inversion

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Abstract

Moody and Darken (1989) proposed a network architecture which uses a single internal layer of locally-tuned processing units to learn real-valued function approximaions. The network can be reinterpreted as both neural networks and fuzzy rules. Hence,we call it neuro-fuzzy and propose a method of computerized tomograpy. The line integrals of Gaussian Radial Basis Functions can be obtained in a simple manner and the spatial distribution is calculated from the line integrals along rays in a plane. With this method, detailed pictures of the spatial distribution of attenuation or propagation velocity can be reconstructed from a small number of measured data.

1 Introduction

Supervised learning in artificial neural networks provides a convenient method to find an approximate function which is composed of some kind of basis functions (e.g., sigmoid function). In this paper a neuro-fuzzy model is obtained in the form of Gaussian Radial Basis Functions(RBFs) which can be regarded as a three-layered neural network[3,4] and fuzzy reasoning rules[2]. A simple formula of the integration of Gaussian RBFs is applied to the computerized tomography with a relatively small number of propagation paths. A straight-line ray optics model is assumed for the propagation mechanism. A numerical example of computed tomography by the direct method of solution using the gradient descent method is presented.

2 Gaussian RBFs as a Neuro-Fuzzy Model

Let A_{ik} denote the membership function of the k-th fuzzy rule in the domain of the i-th input variable x_i. The k-th rule is written as "If x_1 is A_{1k} and x_2 is A_{2k}, then y is w_k." The conclusion part of the fuzzy reasoning rule which infers the output y is simplified as real number w_k. In the case of Gaussian membership function ,A_{ik} is defined as :

$$A_{ik}(x_i) = \exp\left(-\frac{(x_i - a_{ik})^2}{b_{ik}}\right)$$

where the parameters a_{ik} and $b_{ik} (i = 1, ..., N)$ are given for each k and are changed in the training procedure. The final output y is written as :

$$y = f(x_1, x_2) = \sum_{k=1}^{K} \mu_k(x_1, x_2) \cdot w_k$$

where $\mu_k(x_1,x_2)$ is the compatibility degree of the premise part of the k-th fuzzy rule, which is defined as :

$$\mu_k(x_1,x_2) = A_{1k}(x_1) \times A_{2k}(x_2)$$

This simplified fuzzy model is equevalent to the networks of Gaussian RBFs first proposed by Moody and Darken[3]. In Figure 1 the line from A $(0,x_2^L)$ to B $(1,x_2^L)$ can be written as $x_2 = \alpha x_1 + x_2^L$ where $\alpha = x_2^R - x_2^L$. Let (x_1,x_2) be coordinate of a point on the line AB. Then

$$x_1 = \frac{z}{\sqrt{1+\alpha^2}}$$

$$x_2 = \frac{\alpha z}{\sqrt{1+\alpha^2}} + x_2^L$$

where z is a parameter denoting the length from A to the point (x_1,x_2) . Let a Gaussian function be

$$\mu_k(x_1,x_2) = \exp\left(-\frac{(x_1-a_{1k})^2+(x_2-a_{2k})^2}{b_k}\right)$$

where it is assumed that b_{1k} equals b_{2k} and is denoted by b_k for simplicity. The circles in Figure 1 represent contour curves of a Gaussian function with two variables. Using parameter z, we have

$$\mu_k(z) = \zeta_k \exp\left(-\frac{(z-a_{3k})^2}{b_k}\right)$$

where

$$\zeta_k = \exp\left(-\frac{(\alpha a_{1k}+x_2^L-a_{2k})^2}{(1+\alpha^2)b_k}\right)$$

$$a_{3k} = \frac{\alpha(x_2^L-a_{2k})-a_{1k}}{\sqrt{1+\alpha^2}}$$

And the line integral of $f(x_1,x_2)$ along the path AB can be written as :

$$\int_{-\infty}^{\infty} f(z)dz = \sum_{k=1}^{K} \zeta_k \sqrt{\pi b_k} \cdot w_k$$

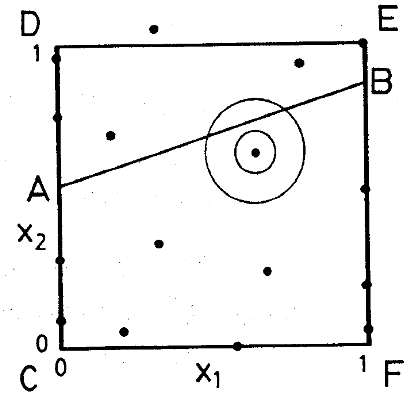

Figure 1: A region of probing

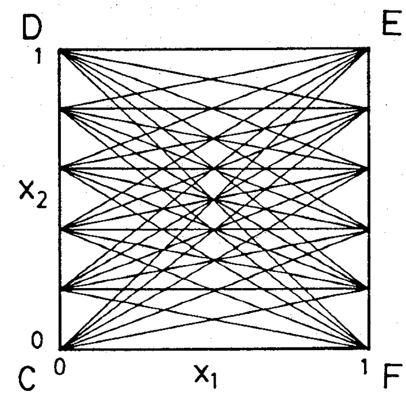

Figure 2: Propagation paths

3 Neuro-Fuzzy Computed Tomography

We here consider an application such as the geophysical tomography[1]. In Figure 1, a typical situation, the data collection system scans the rectangular region (CDEF) with a signal source located in one borehole (CD) and a receiver in the other (EF). A multitude of source and receiver locations are selected to sample the region with a relatively small number of orientations for the transmission paths. The receiver signals carry time-of-flight information for velocity measurements and amplitude information for attenuation measurements. In this section we describe a neuro-fuzzy approach to computed tomography and test its performance on computer generated data. A region of probing (cross section) is shown in Figure 1, where a typical ray path (AB) is indicated for signal propagation. In the region of probing the distribution of a material parameter, such as velocity or attenuation rate, is to be calculated. We assume the straight line ray-optic model of the propagation mechanism.

The line segment AB is a propagation path L_p. We express the distribution of some material parameter such as attenuation rate by the neuro-fuzzy model $f(x_1, x_2)$. Attenuation rate is a measure of how rapidly plane wave energy is dissipated with distance in the medium. If the refractive-index changes are very small, we can assume the straight line ray-optic model. And, the projection along a path L_p can be written as :

$$I_p = \int_{L_p} f(x_1, x_2)dz, (p = 1, 2, \cdots, P)$$

This equation holds regardless of whether we are dealing with attenuation or velocity. A transmitter is located on CD ($x_1 = 0$) and a receiver is on EF ($x_1 = 1$) as shown in Figure 2. Synthetic data I_p^* corresponding to Q equally spaced transmitter locations and R receiver locations can be generated. We can have I_p^* for each of $P(= Q \times R)$ propagation paths. Let a cost function be

$$E = \frac{1}{2}\sum_{p=1}^{P}(I_p - I_p^*)^2$$

and formulate a nonlinear mathematical programming problem to find the values of parameters a_{ik} , b_k and w_k in the neuro-fuzzy model, which minimize the cost function. The line integral I_p of $f(x_1, x_2)$ along the path L_p can be approximated as :

$$I_p = \sum_{k=1}^{K} \zeta_{kp}\sqrt{\pi b_k}w_k + \frac{1}{2}\sum_{k=K+1}^{K+J} \zeta_{kp}\sqrt{\pi b_k}w_k$$

where K and J Gaussian functions are allocated inside the region and on the boundaries of the region (CD and EF in Figure 2) respectively. The learning rule is based on the gradient decent method such as :

$$\begin{aligned} w_k^{NEW} &= w_k^{OLD} - \tau\frac{\partial E}{\partial w_k} \\ a_{ik}^{NEW} &= a_{ik}^{OLD} - \tau\frac{\partial E}{\partial a_{ik}} \\ b_k^{NEW} &= b_k^{OLD} - \tau\frac{\partial E}{\partial b_k} \end{aligned}$$

4 Computer Simulation

36 propagation paths are chosen, as in Figure 2, from 6 equally spaced transmitter and receiver locations. In Figures 3-5, (a) the simulated test distribution of a material parameter and (b) the output of $f(x_1, x_2)$ after learning are shown respectively. Both 3-D graphic and contour curves (an image of the cross section) are shown. In the figures, mark "o" represents a center of Gaussian function after learning.

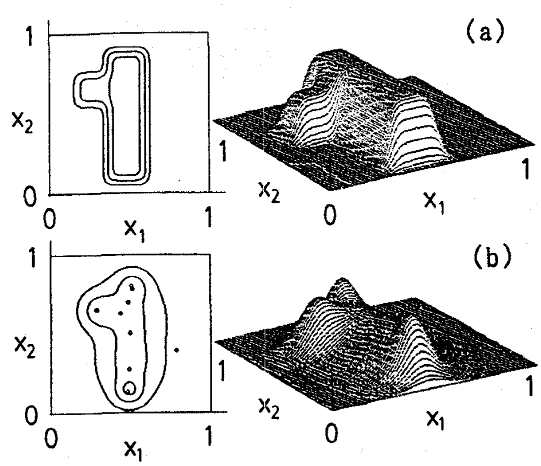

Figure 3: Simulation Results(1)
9 Gaussian functions are used.

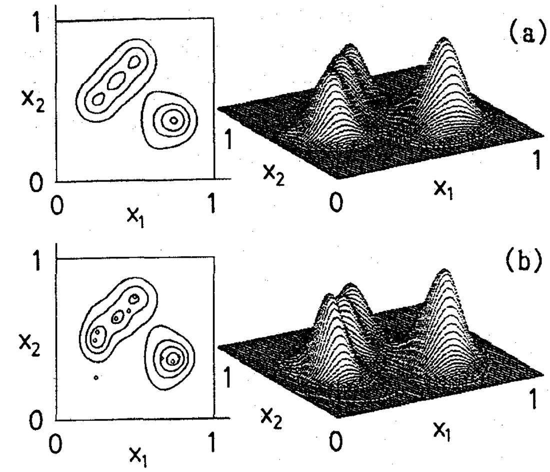

Figure 4: Simulation Results(2)
9 Gaussian functions are used.

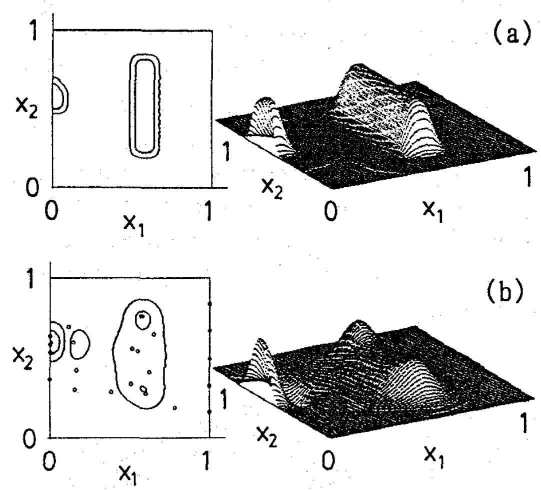

Figure 5: Simulation Results(3)
16 Gaussian functions in the region and 10 Gaussian functions on the boundary are used.

5 Conclusion and Further Work

In this paper we have proposed computed tomography with RBFs networks, assuming straight-ray propagation. Though the number of propagation paths is small, the low-contrast picture of a cross section can be reconstructed. The solution method of the inverse problem is based on the gradient descent method.

Future work is planned to incoroporate ray-bending corrections required to obtain accurate reconstructions for the cases as seismic data.

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