1a) $\phi(x) = [1,1,0,0,0,0]$ $y=1$

$[0,0,1,1,0,0]$ $y=-1$

$[0,0,1,0,1,0]$ $y=1$

$[1,0,0,0,0,1]$ $y=1$

$[0,0,0,0,0,0]$ $\eta=1$

$\underline{i=1}$ $w\cdot\phi(x)y=0$ $-\phi(x)y=[-1,-1,0,0,0,0]$

$w=[1,1,0,0,0,0]$

$\underline{i=2}$ $w\cdot\phi(x)y=0$ $-\phi(x)y=[0,0,-1,-1,0,0]$

$w=[1,1,1,1,0,0]$

$\underline{i=3 \,\&\, 4}$ $w\cdot\phi(x)y=1$

$w=[1,1,1,1,0,0]$

1b) good 1 $[0,1,0]$

bad -1 $\Rightarrow [0,0,1]$

not good -1 $[1,1,0]$

not bad 1 $[1,0,1]$

Prove $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} > 0$

$$\Rightarrow \begin{bmatrix} y_1 w_2 \\ y_2 w_3 \\ y_3 w_1 + y_3 w_2 \\ y_4 w_1 + y_4 w_3 \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Lets satisfy the constraints in order.

Since we know y, we have

$$\begin{bmatrix} w_2 \\ -w_3 \\ -w_1 - w_2 \\ w_1 + w_3 \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$w_2 > 0$

$-w_3 > 0$ so $w_3 < 0$

$w_2 =$ positive so $-w_2 =$ negative

$-w_1 =$ positive so $w_1 =$ negative

w_1 is negative so in the fourth equation, w_3 must be positive to equal zero but we know w_3 is negative from the second equation. We have reached a contradiction and it is impossible.

It is impossible to linearly separate the data so try a nonlinear approach.

$\phi(\underline{x}) = \phi(x_1, x_2, x_3, x_4)$ $\quad x_4 = x_1^2 + x_2^2 + x_3^2 + x_1 x_3 + x_3$

2b) $\nabla_w \text{Loss}(x, y, w) = 2(\sigma(w \cdot \phi(x)) - y) \cdot \frac{\partial}{\partial w} \sigma(w \cdot \phi(x))$

Note: $\frac{\partial}{\partial w} e^{-(w \cdot \phi(x))} = e^{-(w \cdot \phi(x))} \cdot -\phi(x)$

$$\boxed{\nabla \text{Loss} = 2\left[(1+e^{-w\phi(x)})^{-1} - y\right] * -(1+e^{-w\phi(x)})^{-2} \cdot e^{-w\phi(x)} \cdot -\phi(x)}$$

2a) $\text{Loss}(x, y, w) = (\sigma(w \cdot \phi(x)) - y)^2$

$$= \boxed{\left(\frac{1}{1+e^{-w \cdot \phi(x)}} - y\right)^2}$$

2c) $\frac{\phi(x) e^{-w\phi(x)}}{-2(1+e^{-w\phi(x)})^3}$ $\quad w = \infty \Rightarrow \frac{\phi(x)}{2 \cdot e^{\infty}} = 0$

Note $e^{-\infty} = 1$ $\quad w = \infty$ gives minimum

2d) $w = 0 = \frac{\phi(x)}{2 \cdot 8} = \boxed{\frac{\phi(x)}{16} \text{ gives the maximum}}$

4a)

		0, -1	2, 2
$\phi(x_1)$	1, 0	2	5
$\phi(x_2)$	2, 1	8	$\underline{1}$
$\phi(x_3)$	0, 0	$\underline{1}$	8
$\phi(x_4)$	0, 2	9	4

0, -1	2, 2
$\phi(x_1)$	$\phi(x_2)$
$\phi(x_3)$	$\phi(x_4)$

centers $= \left\{ \frac{1,0}{2} , \frac{2,3}{2} \right\}$

		1/2, 0	1, 3/2
$\phi(x_1)$	1, 0	.25	9/4
$\phi(x_2)$	2, 1	13/4	
$\phi(x_3)$	0, 0		
$\phi(x_4)$	0, 2		

$$(2 - 1/2)^2 + (1-0)^2 = \left(\frac{3}{2}\right)^2 + 1 = \frac{9}{4} + 1 = \frac{13}{4}$$