This is to document the studies for the short volatility momentum project on SPY.

Step 1: Vol Series Study

When the underlying moves, suppose the vol curve is static, calculate ATMVolDiff. We want to identify the trend and take advantage of it. The precomputed data used is under directory: *//pvt-nj04-fs001/tseries/tradedate/volsummary/SPY/*. The vol series data is resampled to a frequency of 10 seconds. Two sets of signals are calculated. The definition is:

$$s_t = sign(y_t - y_{t-n}) \frac{(y_t - y_{t-n})^2}{\Sigma_{i=0}^{n-1} |y_{t-i} - y_{t-i-1}|} \tag{1}$$

where n is 10 or 30, y_t is the ATMVolDiff series. We call the two signals 10-tick vol signal and 30-tick vol signal. The trends that goes up smoothly tend to have a larger vol signal magnitude than the ones that oscillate a lot. Oscillations make the denominator much larger, thus make the vol signal magnitude smaller.
Two examples of the 10-tick/30-tick vol signals from April 22^{nd} and June 26^{th} 2015 are shown in Figures 1 and 2.

Figure 1: ATMVolDiff and computed vol signals on April 22^{nd} 2015

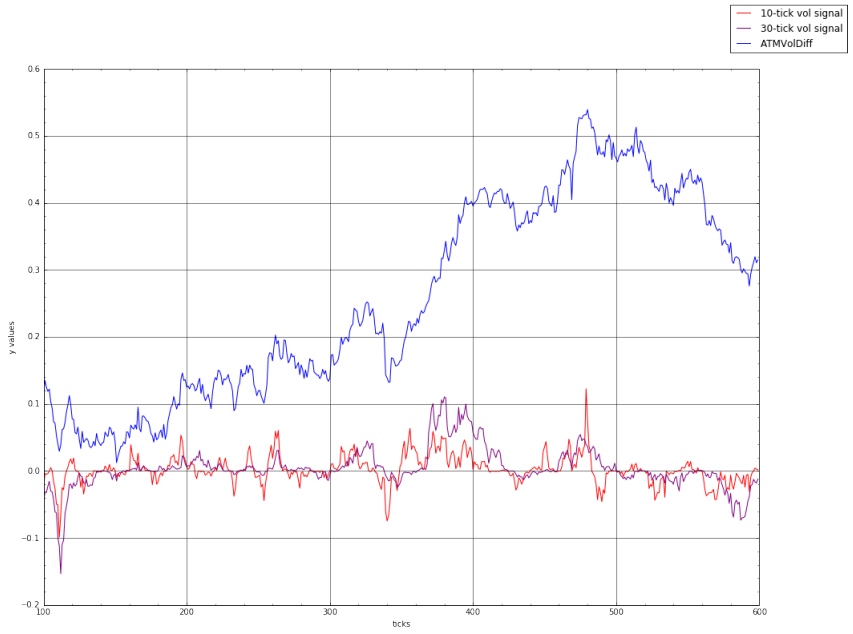

Figure 2: ATMVolDiff and computed vol signals on June 26^{th} 2015

As long as we detect an entry signal, we will enter the trade and then hold the position for a certain time. We computed the statistics or P/L for different

holding period, ranging from 10 ticks to 80 ticks. Ideally we should ride to the end of the trend.
The first thing calculated is the ATMVolDiff change during certain number of ticks w.r.t. computed 10-tick vol signal. Different curves are for different holding periods. Good correlation is observed in Figure 3. The figure tells us that when the 10-tick vol signal is higher, the ATMVolDiff also tend to increase.

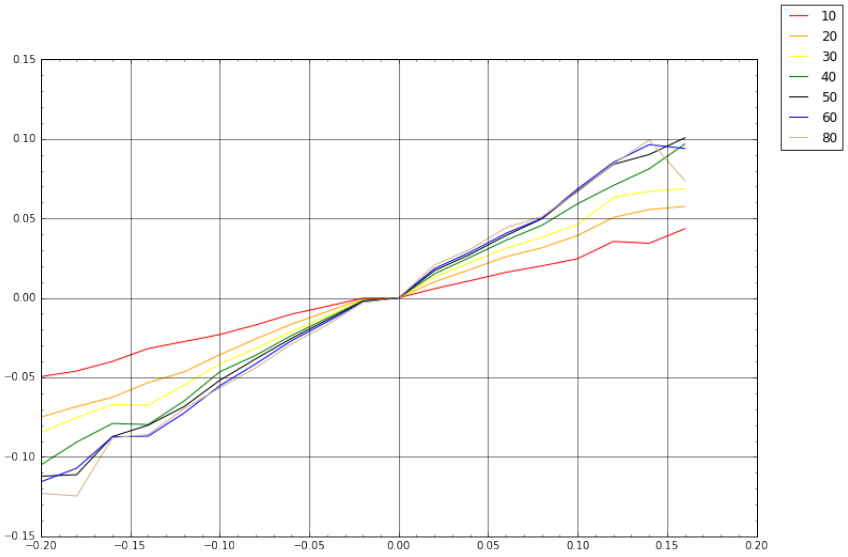

Figure 3: ATMVolDiff change for different ticks w.r.t. short signal strength

We put a threshold on the short/long signal strength. A buy trade is triggered when the threshold is crossed while a sell trade is initiated when it cross negate of the threshold.
The expected P/L are plotted against the threshold for different holding periods in Figures 4 and 5. The X-axis is the vol signal threshold, Y-axis is the expected gain. Different curves are for different holding periods. The expected gains increase first when we increase the thresholds, but they peak at some point and drop as we keep increasing vol signal thresholds.

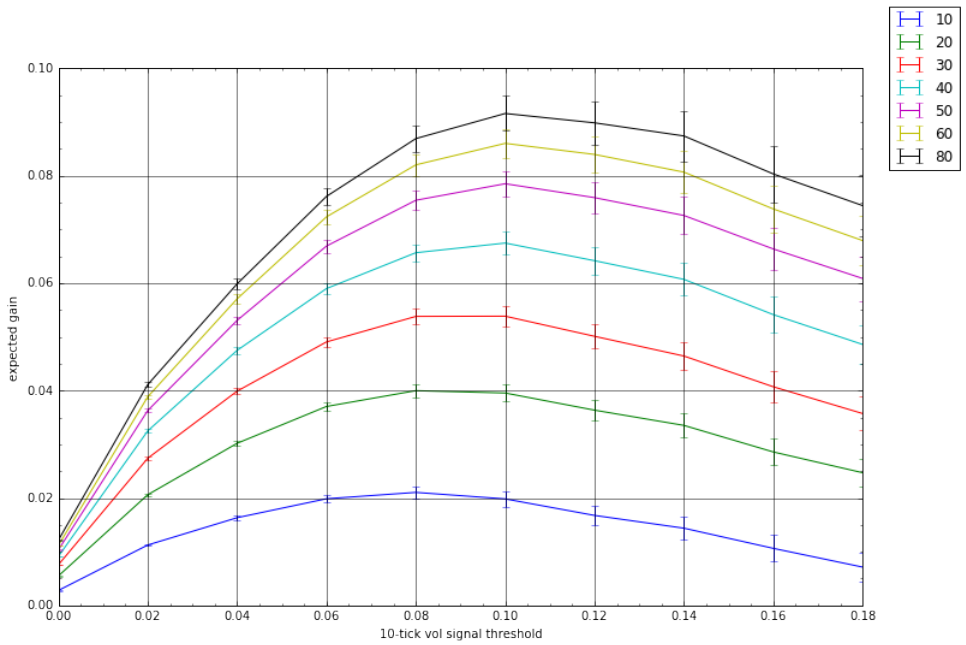

Figure 4: 10-tick vol signal expected gain

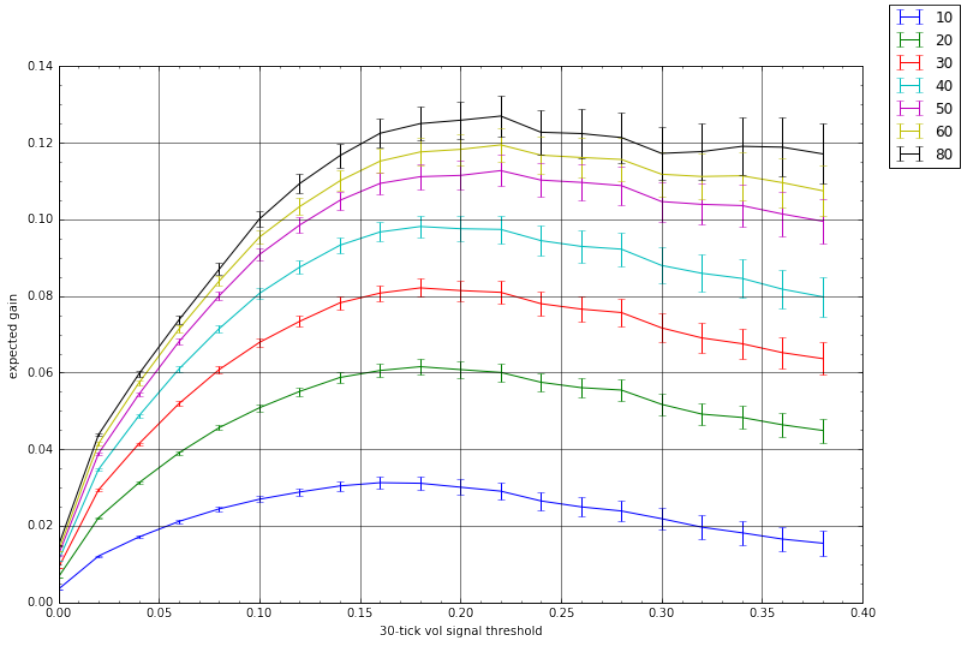

Figure 5: 30-tick vol signal expected gain

In case of outliers, we also computed the expected gains dropping the left

and right 10% tails of the P/L.

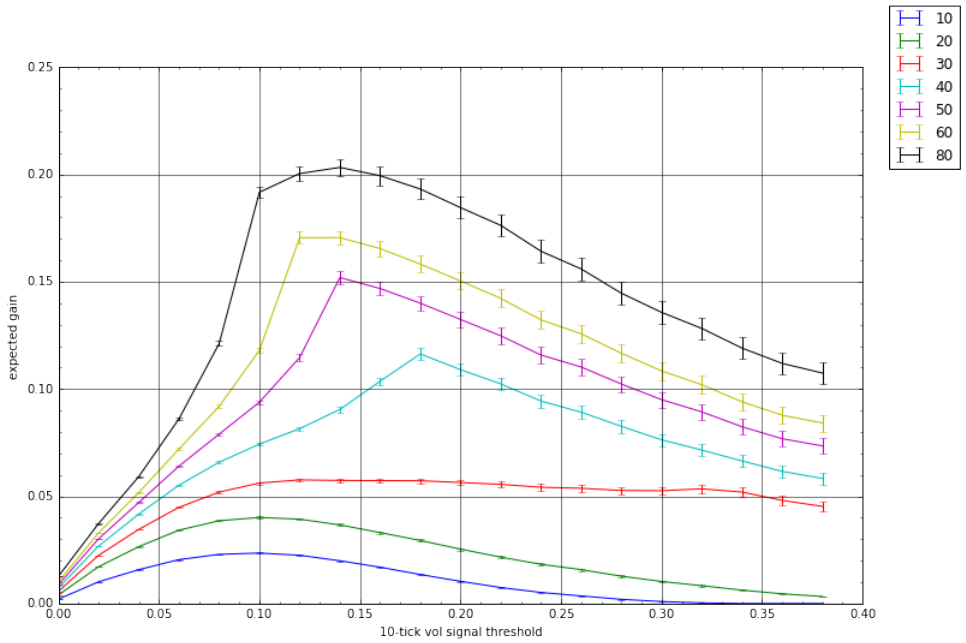

Figure 6: 10-tick vol signal expected gain cutting out left and right 10% tails

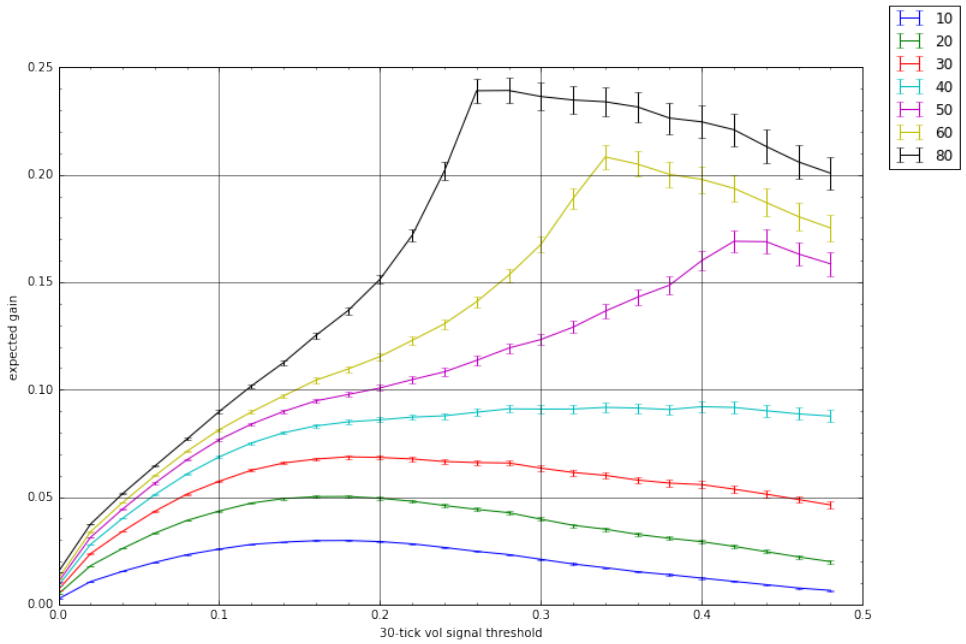

Figure 7: 30-tick vol signal expected gain cutting out left and right 10% tails

The signal duration/survival rate is also studied. When a signal is observed, we will count the ticks until it dies out. We do it for all signals and compute the ratio of survival v.s. the duration. Figures 8 and 9 show the results. After 10 ticks about 5% of 10-tick vol signals are alive, while for 30-tick vol signal, 20% are still alive.

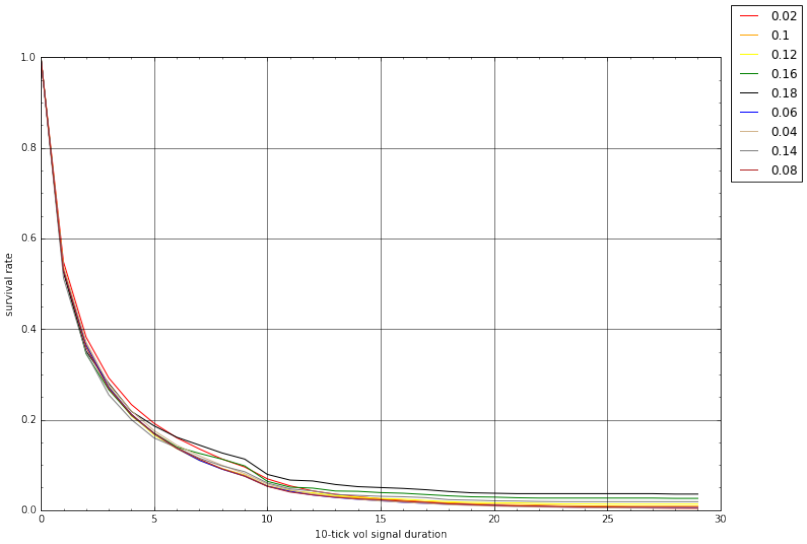

Figure 8: 10-tick vol signal survival ratio v.s. number of ticks

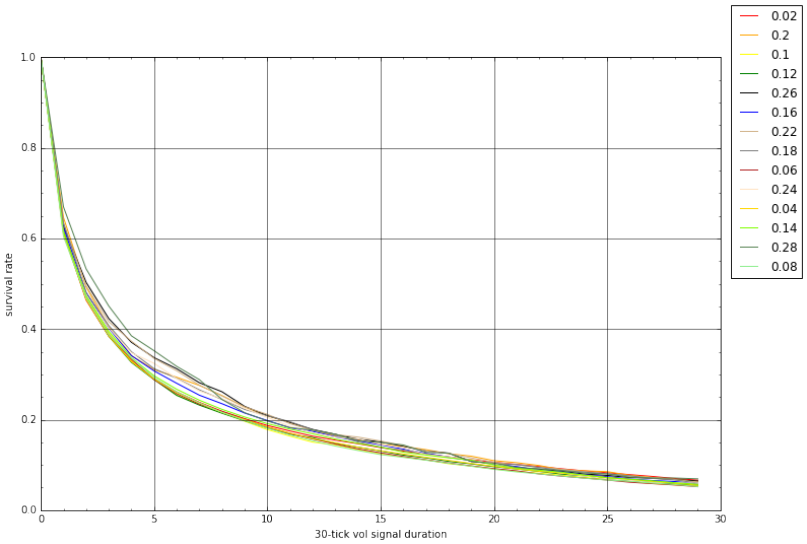

Figure 9: 30-tick vol signal survival ratio v.s. number of ticks

Convex trends are also studied. They are picked up by putting a thresh-

old on the 30-tick vol signal, and also requires the 10-tick vol signals to be strengthening during the most recent 30 ticks. We use a more stable measure for the P/L here. Suppose we hold the position for p = 2n ticks, then the measure is

$$R_p = \frac{1}{n}\Sigma_{k=1}^{n} r_{2k} \tag{2}$$

where r_{2k} is the return after $2k$ ticks.

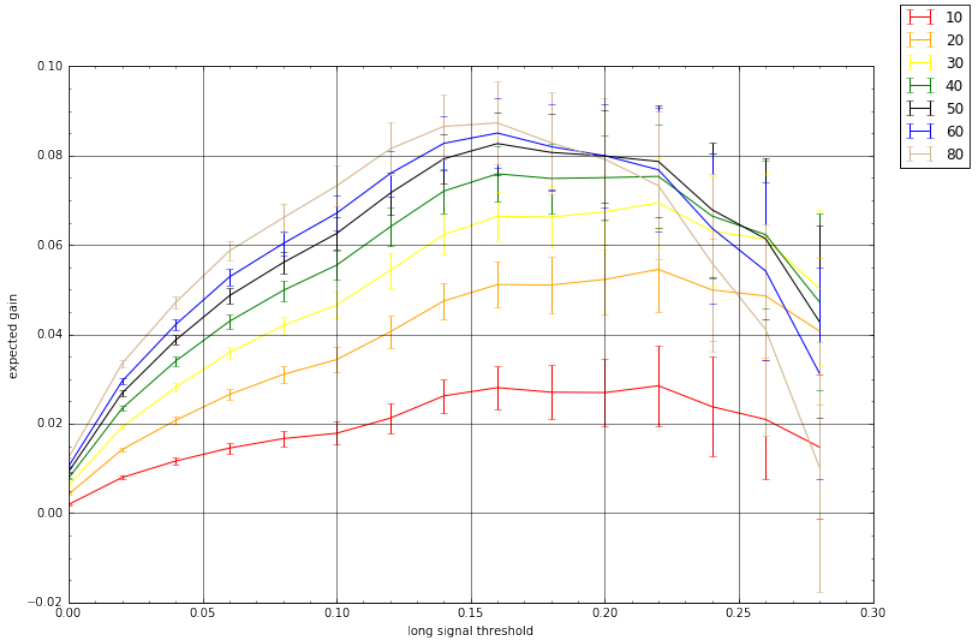

Figure 10: Expected gains (R_p) for convex vol signals

Step 2: Vol Intraday Momentum Strategy Study with Underlying

The underlying price can provide important information for the volatility momentum strategy.
The underlying price movement is much faster, so we resample the data to 3 seconds which is different from 10 seconds per tick for the vol data. The underlying signal is computed as

$$s_t = sign(y_t - y_{t-n})\frac{(y_t - y_{t-n})^2}{\Sigma_{i=0}^{n-1}|y_{t-i} - y_{t-i-1}|} \tag{3}$$

where y_t is the underlying price series. We choose $n = 20$ which means we use the most recent 20 ticks to compute underlying signal. The smooth trends will have larger magnitude while oscillations will have smaller magnitude due to much larger denominators.
An example of the 20-tick underlying signal is plotted in Figure 11.

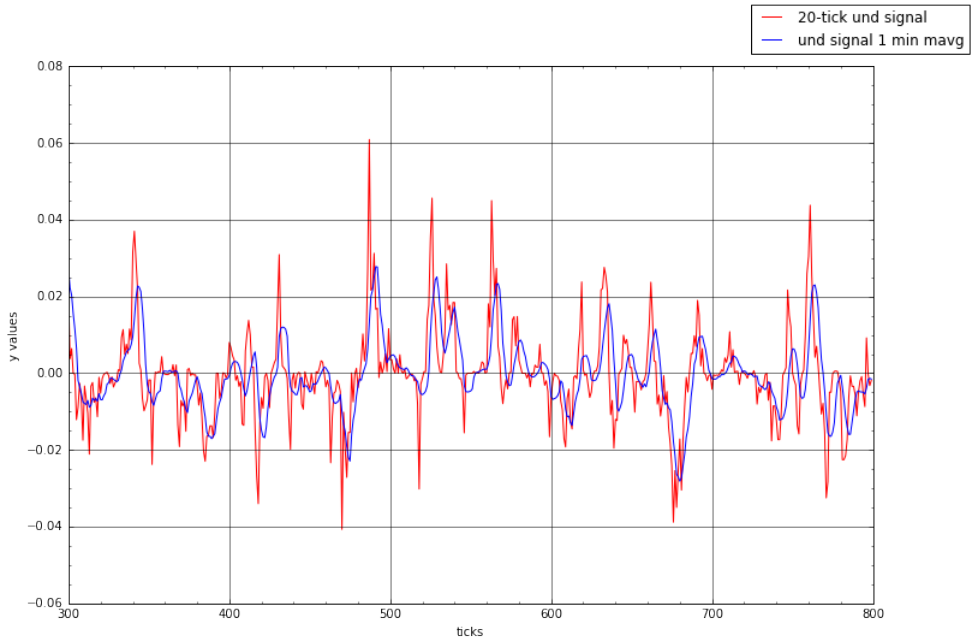

Figure 11: 20-tick underlying signal and its moving average with time window of 1 minute

In this section, we only use the 30-tick vol signal since previous section shows it has better performance than 10-tick vol signal.
Buy signals tend to perform better when the underlying is dropping quickly, i.e. the underlying signal is very small. Sell signals should perform better when the underlying is increasing or the market is very quiet, i.e. when the underlying signals are big and positive or very close to 0. Figures 12 and 13 prove this. These two plots show the ATMVolDiff change for buy/sell trades in different underlying signal regions. No vol signal threshold is applied here. The underlying signals here are the moving average with time window of one minute. From the plots, buy trade gains have negative correlation with underlying signal, while sell gains have positive correlation with underlying signal. This prove our previous points.

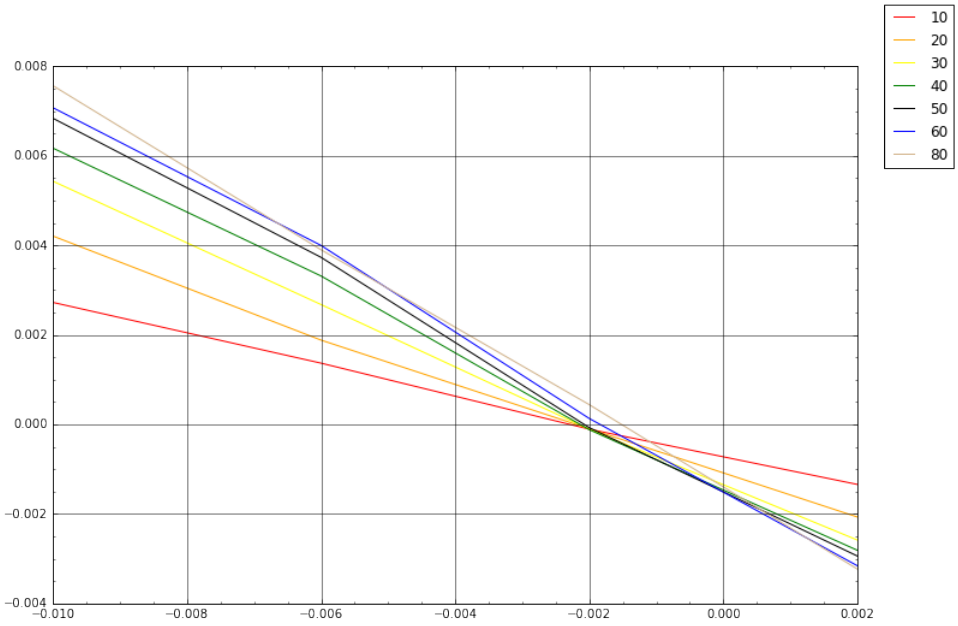

Figure 12: Buy signal performance vs moving average of underlying signal within time window of 1 minute. No vol signal threshold is applied.

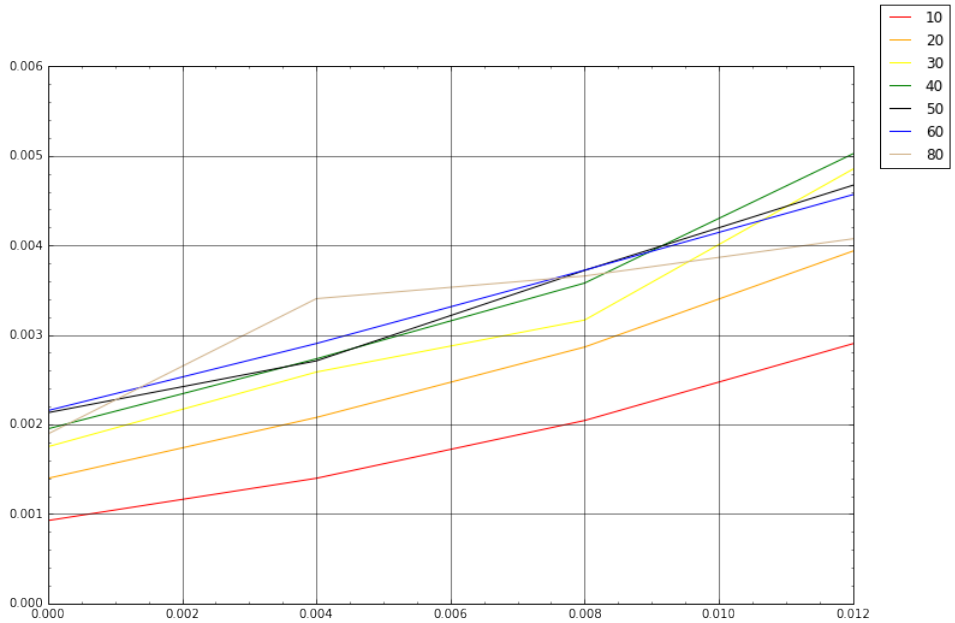

Figure 13: Sell signal performance vs moving average of underlying signal within time window of 1 minute. No vol signal threshold is applied.

To check the expected gain for the strategy with the underlying, we do it separately for buy and sell trades. For buy trades, we require the moving average of the underlying signal in a time window of one minute to be below some thresholds, as big negative underlying signal favors buy trades. For the sell trades, we do the opposite, i.e. require the moving average of the underlying signal to be above the thresholds. Figures 14 and 15 show the results. The threshold for the vol signal is 0.08. The expected gains are higher compared with that from only with the vol series.
The same procedure is repeated with vol signal threshold of 0.04. Figures 16 and 17 show the results.
For buy trades, the expected gains increase as we decrease the thresholds on the underlying signal. The opposite is true for sell trades, since vol drops as the underlying is quiet or moves up which is reflected by a big underlying signal.

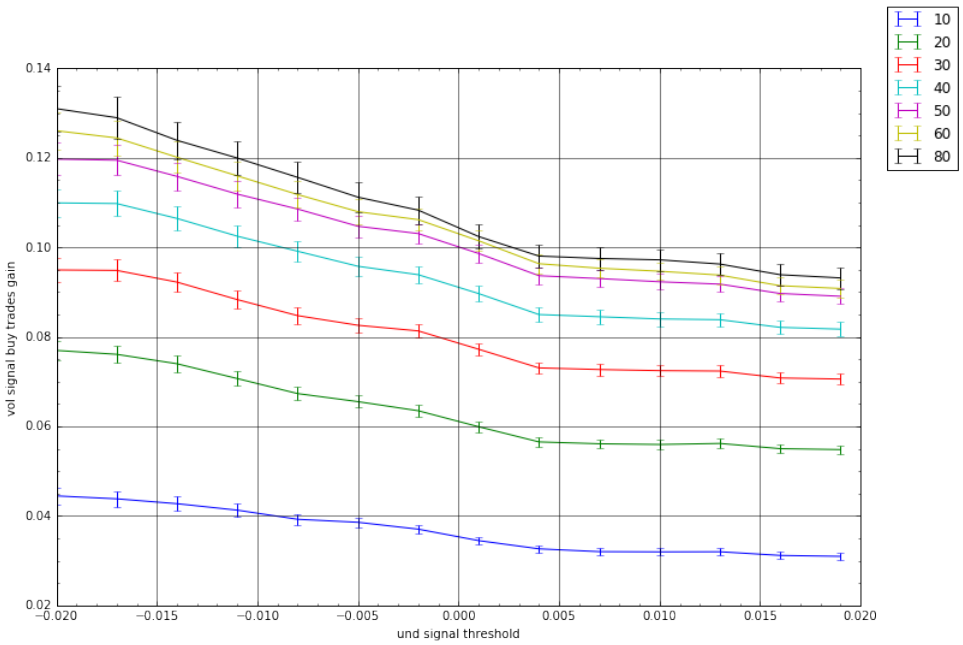

Figure 14: Buy trades expected gain using moving average of underlying signal in one minute. The threshold for the vol signal is 0.08.

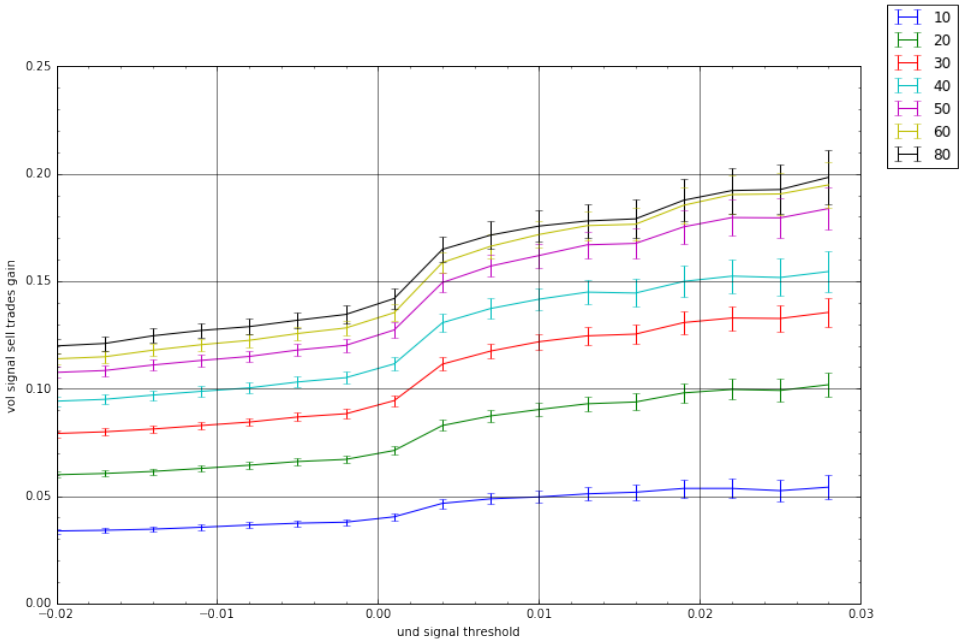

Figure 15: Sell trades expected gain using moving average of underlying signal in one minute. The threshold for the vol signal is 0.08.

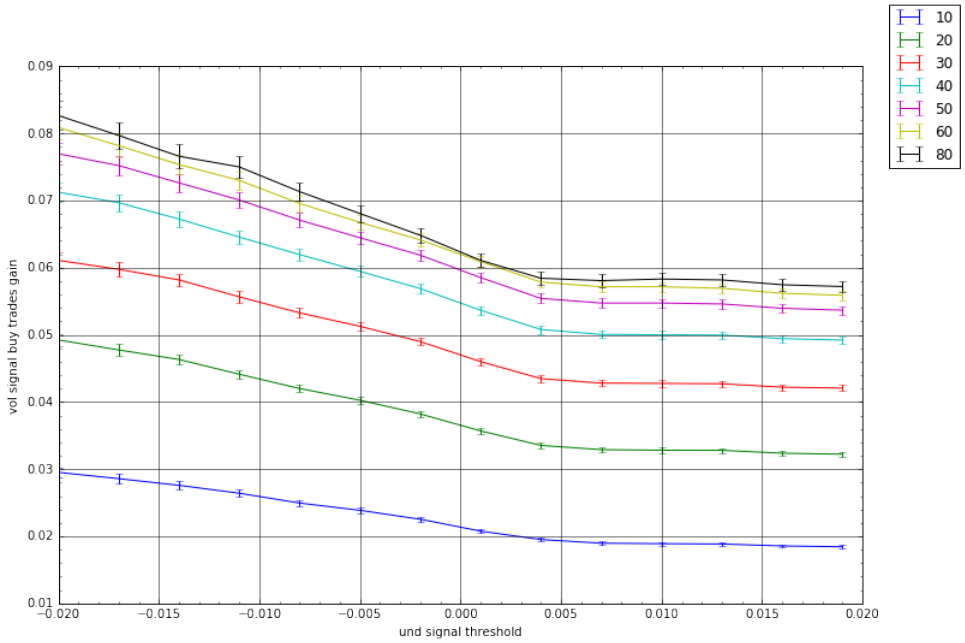

Figure 16: Buy trades expected gain using moving average of underlying signal in one minute. The threshold for the vol signal is 0.04.

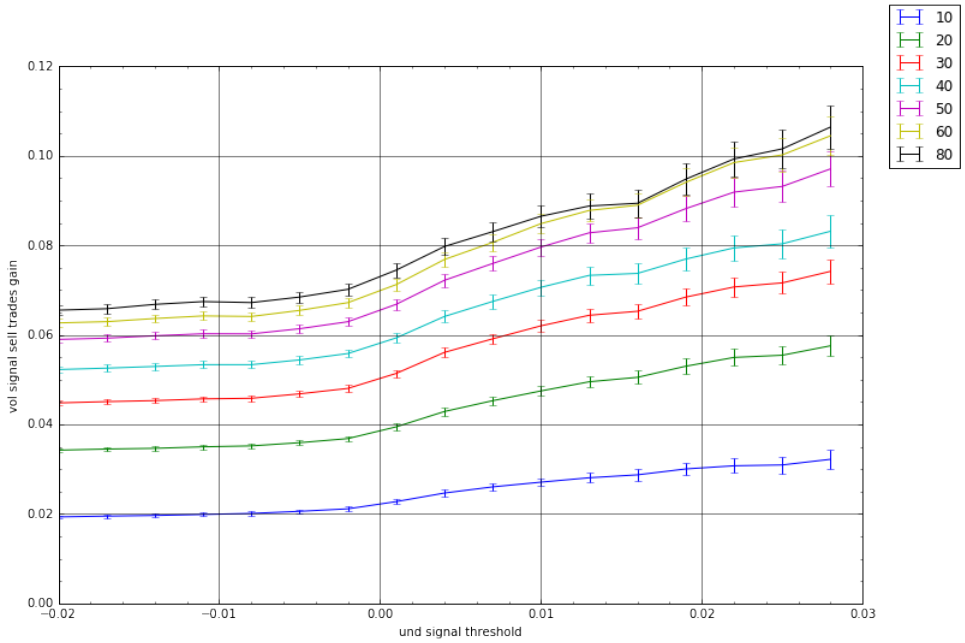

Figure 17: Sell trades expected gain using moving average of underlying signal in one minute. The threshold for the vol signal is 0.04.

Figure 18 examines expected gain for holding 80 ticks with different vol signal thresholds shown for different curves. The X-axis is the underlying signal thresholds. Comparing different curves, the expected gain first increase as we increase the vol signal thresholds. But as we keep increasing the vol signal thresholds, the expected gain drops.

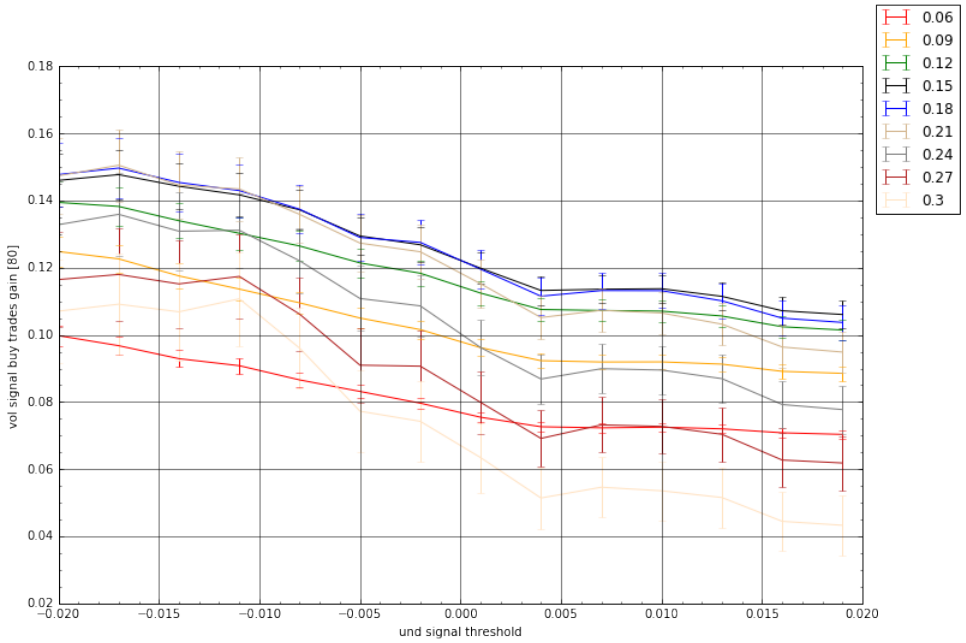

Figure 18: Buy trades expected gain using moving average of underlying signal in one minute. Comparing different curves show that gain first increase as we increase vol signal thresholds, but it peaks and then drops as we keep increasing vol signal thresholds.

Given that we want to trigger buy/sell signals more when the underlying signals are negative/positive, we do a scaling of the vol signals. For vol signals to trigger buy trades, the scaling is:

$$f(x) = \begin{cases} 1, x \leq -0.005 \\ e^{-100x-0.5}, -0.005 \leq x \leq 0.005 \\ 0, x > 0.005. \end{cases} \tag{4}$$

It filter out all buy trades with underlying signal above 0.005 and scale the vol signal by a factor of $\frac{1}{e} = 0.37$ when the underlying signal is at 0.005. In the region of [-0.005, 0.005), it's exponential.

For sell trades, the scale function is

$$g(x) = \begin{cases} e^{100x-1}, x < 0.01 \\ 1, x \geq 0.01. \end{cases} \tag{5}$$

The coefficients are chosen so that it scales the vol signal by a factor of $\frac{1}{e} = 0.37$ when the underlying signal is 0.
Figure 19 shows the function values for different underlying signals.

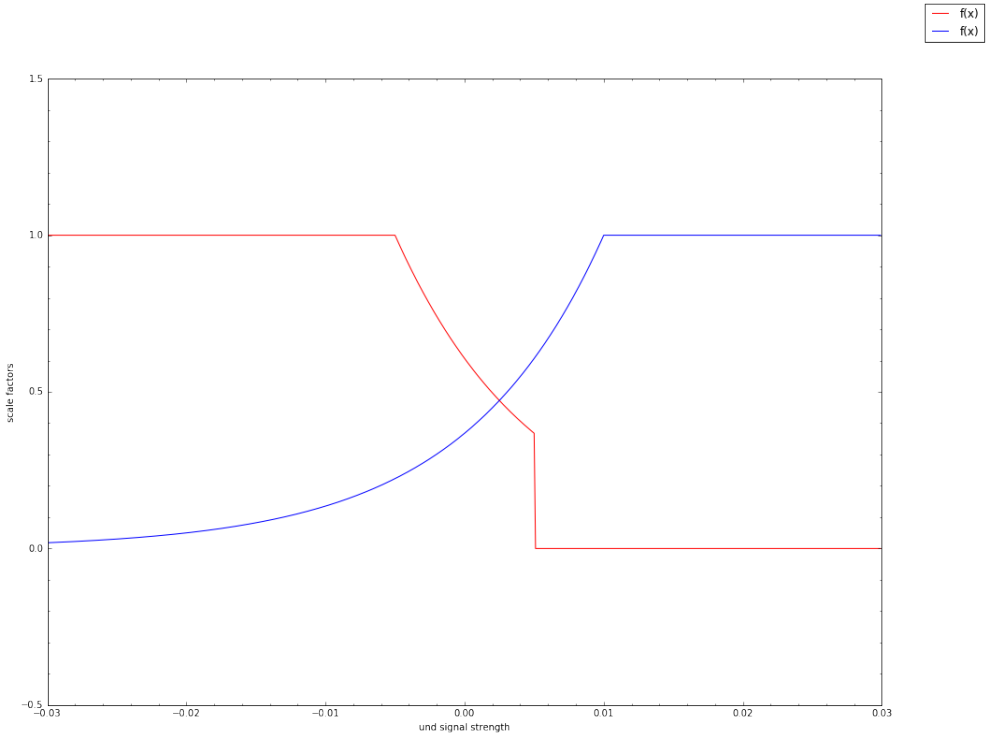

Figure 19: Scaled vol signal for triggering sell trades

Figures 20 and 21 show the scaled buy/sell vol signals.

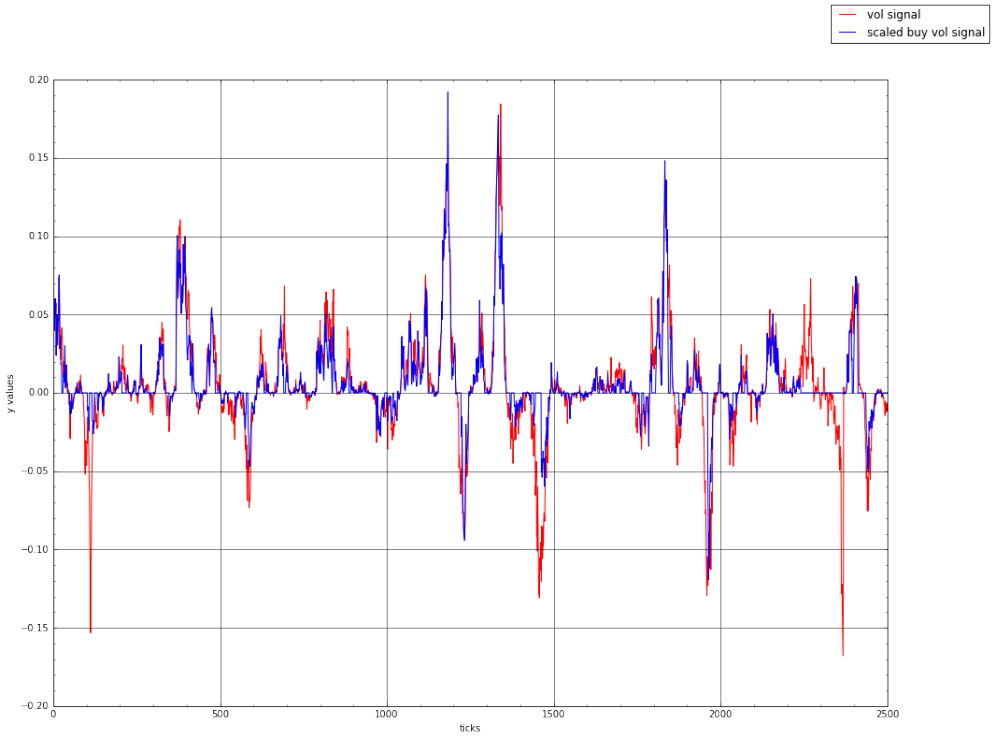

Figure 20: Scaled vol signal for triggering buy trades

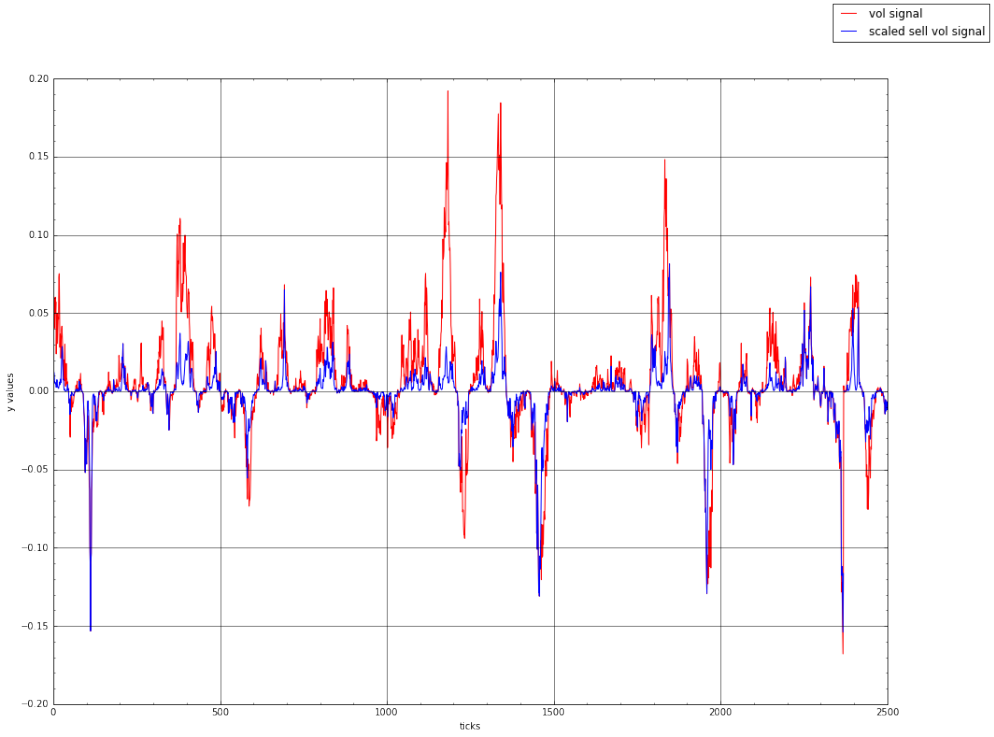

Figure 21: Scaled vol signal for triggering sell trades

The buy/sell expected gains with scaled vol signals are plotted in Figures 22 and 23.

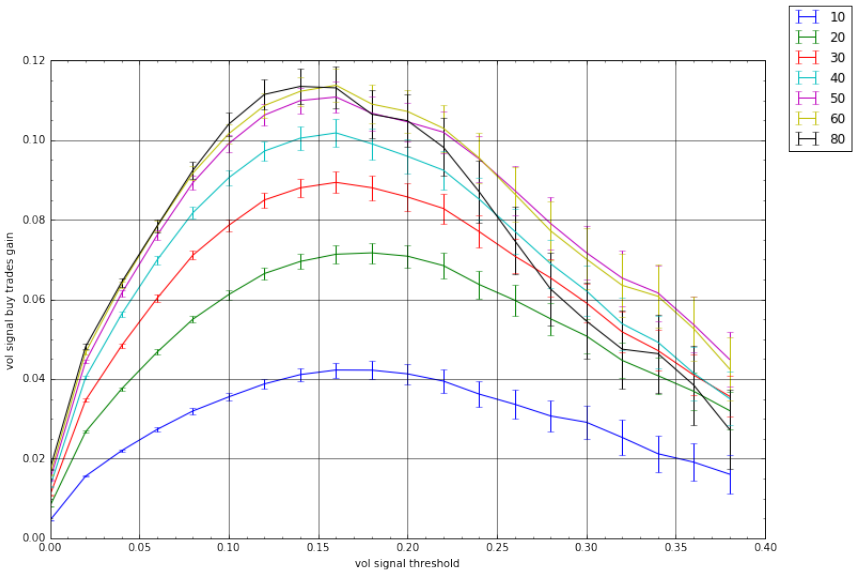

Figure 22: Buy trades expected gain with scaled signal

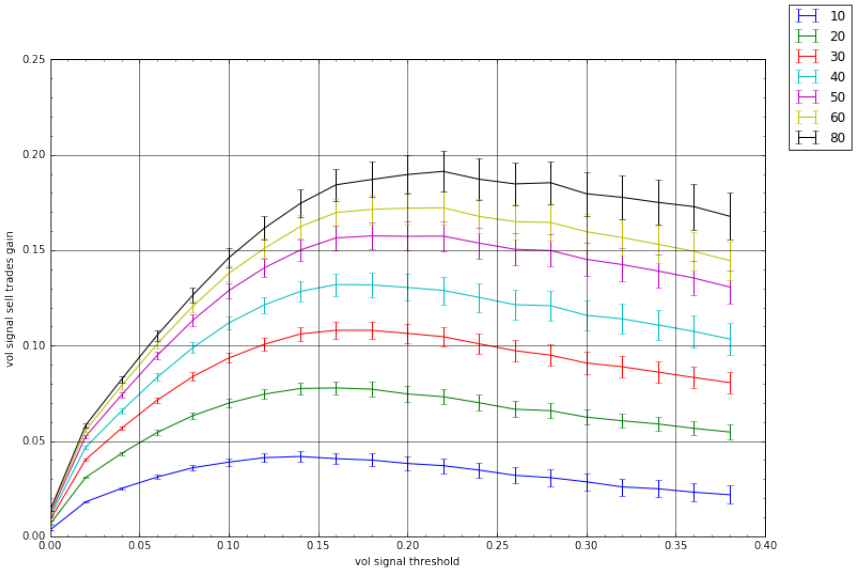

Figure 23: Sell trades expected gain with scaled signal

The gains combining scaled buy and sell signals are shown in Figure 24.

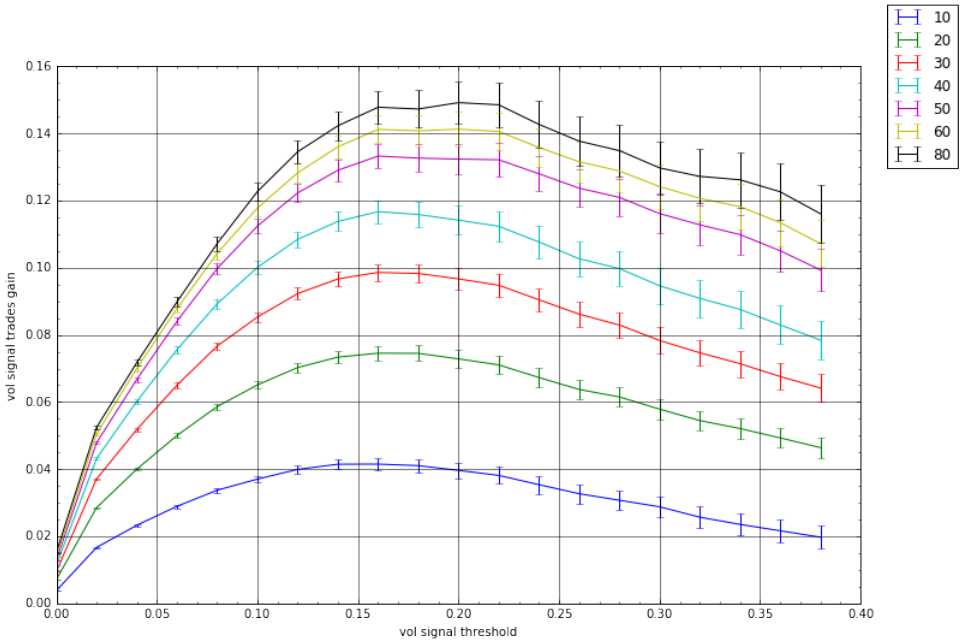

Figure 24: Buy and sell trades expected gain with scaled signal

A same plot is made for vol signal gains with buy/sell trades combined, but without scaling in Figure 25 for comparison. Expected gains are improved by scaling the vol signals.

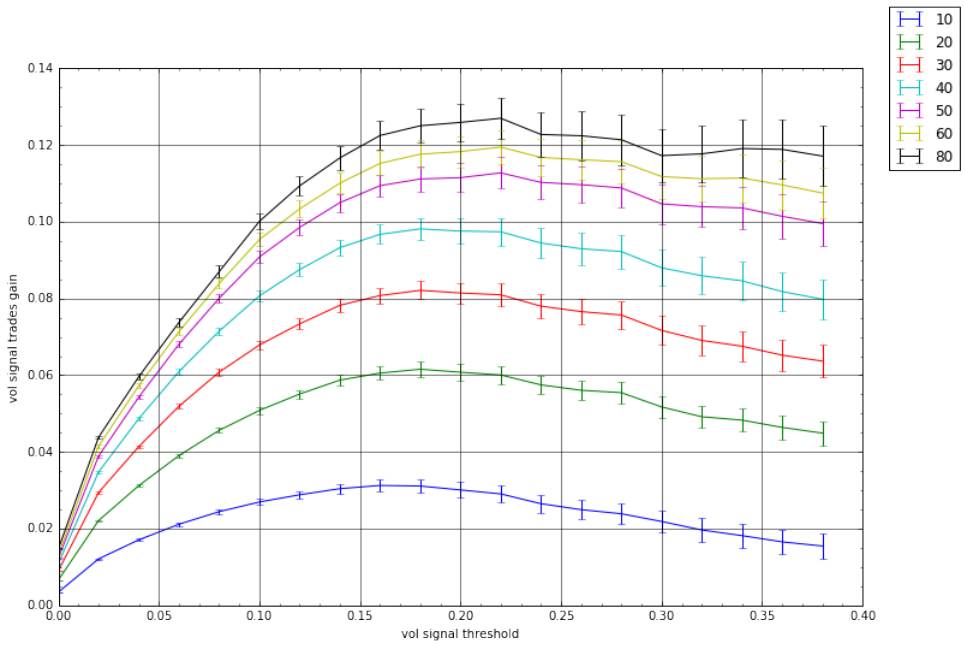

Figure 25: Buy and sell trades expected gain without scaling

Step 3: Trading Volume

We also added the trading volume information to see whether we can learn anything.

Instead of just the current trading volume, we do a moving average with time window of 300 seconds which is the same window with calculating our vol signal. We also tried exponentially weighted moving average of trading volume. It turns out it does not perform better, so we stick to the moving average without exponential reweighting. Trading volumes vary with days to expiration. We bin/bucket the (moving average) trading volume according to the days to expiration, then calculate the median for each bin. We differentiate our trades by whether it has trading volume bigger or less than the median. The expected gain are plotted in Figures 26 and 27. The threshold on the scaled vol signal is 0.08.

From the plots, we can see that they have very different peak expected gains in different trading volume regions. Its calibration will be affected by how much money we can tolerate losing.

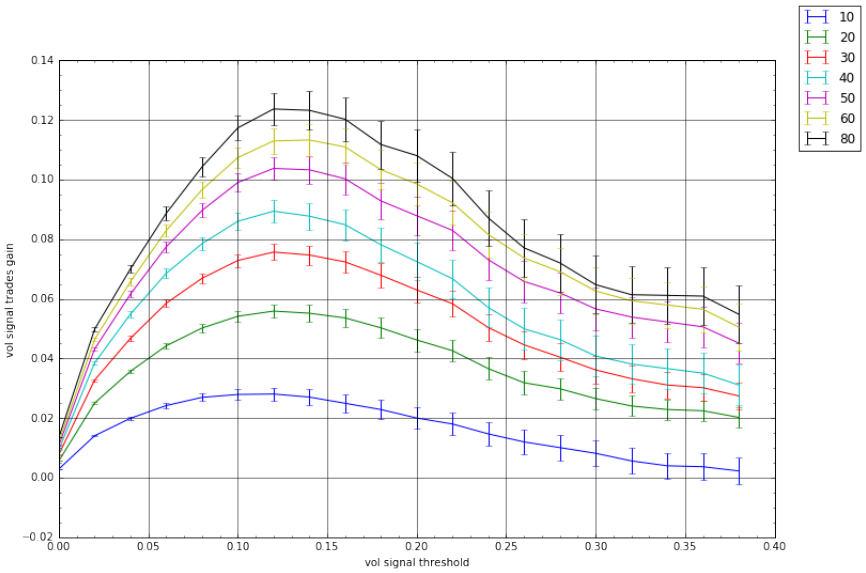

Figure 26: Expected gain with trading volume smaller than the median. Vol signal threshold is 0.08.

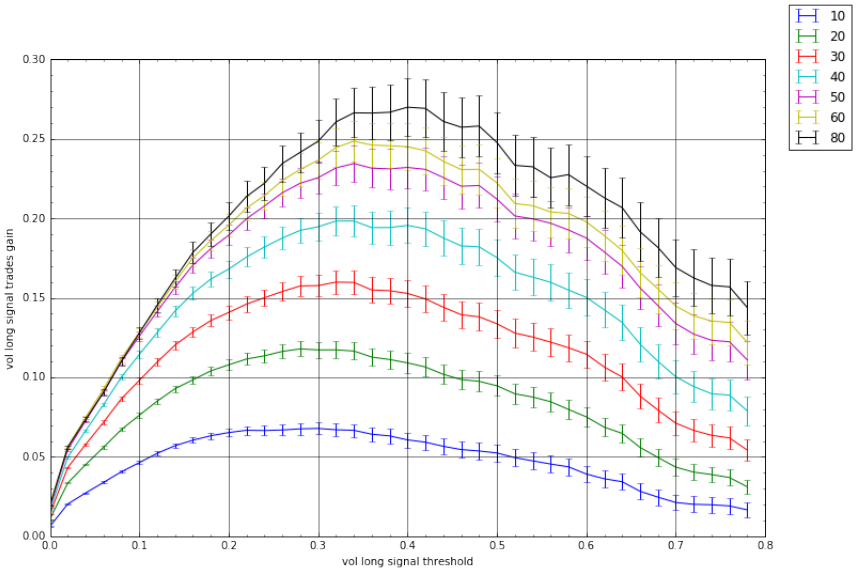

Figure 27: Expected gain with trading volume larger than the median. Vol signal threshold is 0.08.

We calculated the ratio of trading volume over the median. Different thresholds are put on the ratio and the expected gains are plotted in Figure 28.

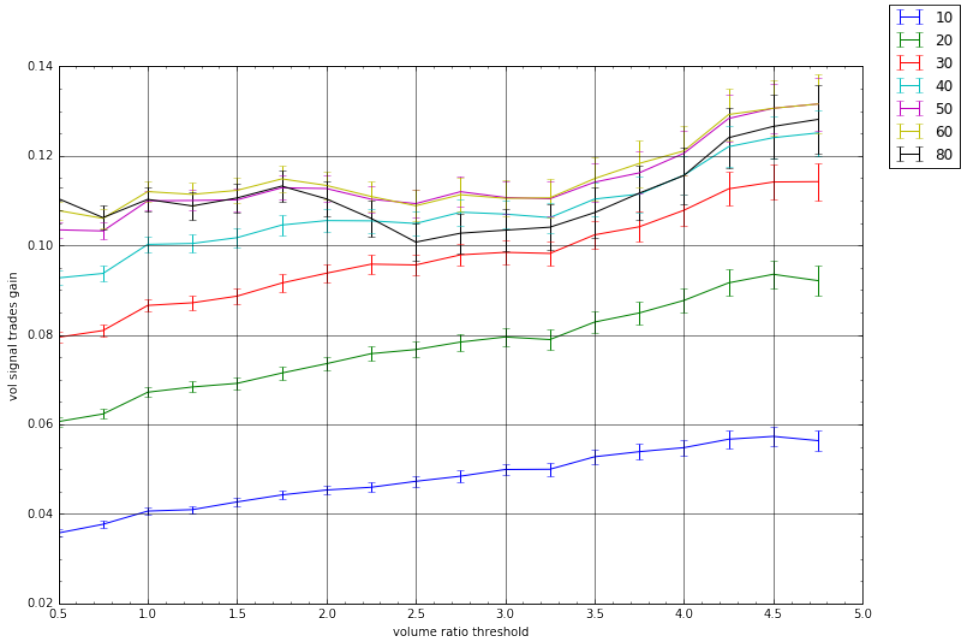

Figure 28: Expected gains with different thresholds on trading volume ratio.