

# Answers for Stochastic Calculus for Finance I; Steven Shreve VJul 15 2009

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July 25, 2009

## Chapter 1

→ 1.1: Since  $S_1(H) = uS_0$ ,  $S_1(T) = dS_0$ ,  $X_1(H) = \Delta_0 S_0(u - (1 + r))$  and  $X_1(T) = \Delta_0 S_0(d - (1 + r))$ . Therefore,  $X_1(H)$  positive implies  $X_1(T)$  negative and vice-versa.

→ 1.2: Check that  $X_1(H) = 3\Delta_0 - 3/2\Gamma_0 = -X_1(T)$ . Therefore if  $X_1(H)$  is positive,  $X_1(T)$  is negative and vice-versa.

→ 1.3: By (1.1.6),  $V_0 = S_0$ .

→ 1.4: mutatis-mutandis the proof in Theorem 1.2.2 replacing  $u$  by  $d$  and  $H$  by  $T$ .

→ 1.5: many computations.

→ 1.6: We have  $1.5 - V_1 = \Delta_0 S_1 + (-\Delta_0 S_0)(1 + r)$ . We determine  $\Delta_0 = -1/2$ . So we should sell short  $1/2$  stocks.

→ 1.7: see last exercise.

→ 1.8:

- (i)  $v_n(s, y) = 2/5[v_{n+1}(2s, y + 2s) + v_{n+1}(s/2, y + s/2)]$
- (ii)  $v_0(4, 4) = 2.375$ . One can check that  $v_2(16, 28) = 8$ ,  $v_2(4, 16) = 1.25$ ,  $v_2(4, 10) = 0.25$  and  $v_2(1, 7) = 0$ .
- (iii)  $\delta(s, y) = [v_{n+1}(2s, y + 2s) - v_{n+1}(s/2, y + s/2)]/[2s - s/2]$ .

→ 1.9:

- (i)  $V_n(w) = 1/(1 + r_n(w))[\tilde{p}_n(w)V_{n+1}(wH) + \tilde{q}_n(w)V_{n+1}(wT)]$  where  $\tilde{p}_n(w) = (1 + r_n(w) - d_n(w))/(u_n(w) - d_n(w))$  and  $\tilde{q}_n(w) = 1 - \tilde{p}_n(w)$ .
- (ii)  $\Delta_n(w) = [V_{n+1}(wH) - V_{n+1}(wT)]/[S_{n+1}(wH) - S_{n+1}(wT)]$
- (iii)  $\tilde{p} = \tilde{q} = 1/2$ .  $V_0 = 9.375$ . One can check that  $V_2(HH) = 21.25$ ,  $V_2(HT) = V_2(TH) = 7.5$  and  $V_2(TT) = 1.25$ .

## Chapter 2

→ 2.1:

(i)  $A$  and  $A^c$  are disjoint and their union is the whole space. Therefore  $P(A \cup A^c [= \text{whole space}]) = 1 = P(A) + P(A^c)$  [they are disjoint].

(ii) it is enough to show it for two events  $A$  and  $B$ . By induction it follows for any finite set of events. Since  $A \cup B = A \cup (B/A)$ , and this is a disjoint union,  $P(A \cup B) = P(A) + P(B/A)$ . Since  $B/A$  is a subset of  $B$ ,  $P(B/A) \leq P(B)$  by definition (2.1.5).

→ 2.2:

(i)  $\tilde{P}(S_3 = 32) = \tilde{P}(S_3 = 0.5) = (1/2)^3 = 1/8$ ;  $\tilde{P}(S_3 = 2) = \tilde{P}(S_3 = 8) = 3(1/2)^3 = 3/8$ .

(ii)  $\tilde{E}S_1 = (1+r)S_0 = 4(1+r)$ ,  $\tilde{E}S_2 = (1+r)^2S_0 = 4(1+r)^2$ ,  $\tilde{E}S_3 = (1+r)^3S_0 = 4(1+r)^3$ , Average rate of growth is  $1+r$  (see p.40, second paragraph).

(iii)  $P(S_3 = 32) = (2/3)^3 = 8/27$ ,  $P(S_3 = 8) = 4/9$ ,  $P(S_3 = 2) = 2/9$ ,  $P(S_3 = 0.5) = (1/3)^3 = 1/27$

We can compute  $ES_1$  directly or use p.38, second paragraph:  $E_n[S_{n+1}] = (3/2)S_n$ . Therefore,  $E_0[S_1] = E[S_1] = (3/2)S_0$ ,  $E_1[S_2] = (3/2)S_1$ . Applying  $E$  to both sides,  $E[E_1[S_2]] = E[S_2] = (3/2)E[S_1] = (3/2)^2S_0$ . Following the same reasoning,  $E[S_3] = (3/2)^3S_0$ . Therefore, the average rate of growth is  $3/2$ .

→ 2.3: Use Jensen's inequality and the martingale property:  $\phi(M_n) = \phi(E_n[M_{n+1}]) \leq E_n[\phi(M_{n+1})]$ .

→ 2.4:

(i)  $E_n(M_n + 1) = \sum_{j=1}^{n+1} E_n[X_j] = (\text{taking out what is known}) = E_n[X_{n+1}] + \sum_{j=1}^n X_j = E_n[X_{n+1}] + M_n$ . Since  $X_j$  assumes 1 or  $-1$  with equal probability, and depends only on the  $n+1$  coin toss (independence),  $E_n[X_{n+1}] = E[X_{n+1}] = 0$ . Therefore,  $E_n(M_{n+1}) = M_n$ .

(ii) Since  $X_{n+1}$  depends only on the  $n+1$  coin toss (independence),  $E_n(e^{\sigma X_{n+1}}) = E(e^{\sigma X_{n+1}}) = (e^{\sigma} + e^{-\sigma})/2$ .  $E_n[e^{\sigma M_{n+1}}] = (\text{taking out what is known}) = e^{\sigma M_n} E_n[e^{\sigma X_{n+1}}] = e^{\sigma M_n} (e^{\sigma} + e^{-\sigma})/2$ .

→ 2.5:

(i) Hint:  $M_{n+1} = M_n + X_{n+1}$  (why?) and  $(X_j)^2 = 1$  and therefore,  $M_{n+1}^2 = M_n^2 + 2M_nX_{n+1} + 1$ . Also  $I_{n+1} = M_n(M_{n+1} - M_n) + I_n = M_nX_{n+1} + I_n$ . One can prove by induction on  $n$  since, by induction hypotheses,  $I_n = 1/2(M_n^2 - n)$  and therefore,  $I_{n+1} = 1/2(M_n^2 + 2M_nX_{n+1} - n) = 1/2(M_{n+1}^2 - 1 - n)$ .

(ii) from (i),  $I_{n+1} = 1/2(M_n + X_{n+1})^2 - (n+1)/2$ . Since  $X_{n+1} = 1$  or  $-1$  with same probability,  $E_n(f(I_{n+1})) = j(M_n)$ , where  $j(m) = E[f(1/2(m + X_{n+1})^2 - (n+1)/2)] = 1/2f(1/2(m+1)^2 - (n+1)/2) + f(1/2(m-1)^2 - (n+1)/2) = 1/2f(1/2(m^2 - n) + m) + f(1/2(m^2 - n) - m)$

Since  $I_n = 1/2(M_n^2 - n)$ ,  $E_n(f(I_{n+1})) = j(M_n) = 1/2f(I_n + M_n) + f(I_n - M_n)$ . Now we need to make the rhs to depend on  $I_n$  only. Since  $I_n = 1/2(M_n^2 - n)$ ,  $M_n^2 = \pm\sqrt{2I_n + n}$ .

So  $E_n(f(I_{n+1})) = g(I_n)$  where  $g(i) = 1/2f(i + \sqrt{2i + n}) + f(i - \sqrt{2i + n})$ .

→ 2.6: It is easy to show that  $I_n$  is an adapted process.  $E_n[I_{n+1}] = (\text{taking out what is known}) = \Delta_n(E_n(M_{n+1}) - M_n) + I_n$ . Since  $M_n$  is a martingale,  $E_n(M_{n+1}) = M_n$  and the first term is zero.

→ 2.7

→ 2.8

(i)  $M'_{N-1} = E(M'_{N-1}) = E_{N-1}(M_N) = M_{N-1}$ . Therefore  $M'_{N-1} = M_{N-1}$ . Proceed by induction.

(ii)  $E_n[V_{n+1}](w) = pV_{n+1}(wH) + qV_{n+1}(wT) = (1+r)V_n(w)$  from algorithm 1.2.16. Now is is easy to prove.

(iii) this is a consequence of the fact that  $E_n[Z]$ , for any  $Z$ , is a martingale.

→ 2.9 see p. 46 2.4.1: “models with random interest rates, it would ...”

(i)  $\tilde{P}(HH) = 1/4$ ,  $\tilde{P}(HT) = 1/4$ ,  $\tilde{P}(TH) = 1/12$ ,  $\tilde{P}(TT) = 5/12$ .

(ii)  $V_2(HH) = 5$ ,  $V_2(HT) = 1$ ,  $V_2(TH) = 1$ ,  $V_2(TT) = 0$ ,  $V_1(H) = 1/9$ ,  $V_1(T) = 12/5$ ,  $V_0 = 226/225$ .

(iii)  $\Delta_0 = 103/270$ .

(iv)  $\Delta_1(H) = 1$ .

→ 2.10

(i) Follow the proof of 2.4.5 (p. 40)

(ii) easy

(iii) easy

→ 2.11

(i) easy since  $C_N = (S_N - K)^+$ ,  $F_N = S_N - K$  and  $P_N = (K - S_N)^+$ .

(ii) trivial

(iii)  $F_0 = 1/(1+r)^N \tilde{E}(F_N) = 1/(1+r)^N \tilde{E}(S_N - K)$ . Since  $S_N = S_0(1+r)^N$ ,  $F_0 = S_0 - K/(1+r)^N$ .

(vi) Yes,  $C_n = P_n$ .

→ 2.12

→ 2.13

(i) we write  $(S_{n+1}, Y_{n+1}) = (\frac{S_{n+1}}{S_n} S_n, Y_n + \frac{S_{n+1}}{S_n} S_n)$ . Now we apply the independence lemma to variables:  $S_{n+1}/S_n$ ,  $S_n$  and  $Y_n$ .

We obtain:

$E_n(f(S_{n+1}, Y_{n+1})) = pf(uS_n, Y_n + uS_n) + qf(dS_n, Y_n + dS_n)$ . Therefore,  $g(s, y) = pf(us, y + us) + qf(ds, y + ds)$ .

(ii)  $v_N(s, y) = f(y/(N+1))$ .  $V_n = v_n(S_n, Y_n) = \frac{1}{r} \tilde{E}_n(V_{n+1}) = \frac{1}{r} \tilde{E}_n(v_{n+1}(S_{n+1}, Y_{n+1})) = \frac{1}{r} (\tilde{p}v_{n+1}(uS_n, Y_n + uS_n) + \tilde{q}v_{n+1}(dS_n, Y_n + dS_n))$ .

So,  $v_n(s, y) = \frac{1}{r} [\tilde{p}v_{n+1}(us, y + us) + \tilde{q}v_{n+1}(ds, y + ds)]$ .

→ 2.14

(i) see 2.13 (i)

(ii)  $v_N(s, y) = f(\frac{y}{N-M})$ . For  $0 \leq n < M$ ,  $v_n(s) = \frac{1}{r} (\tilde{p}v_{n+1}(us) + \tilde{q}v_{n+1}(ds))$

For  $n > M$ ,  $v_n(s, y) = \frac{1}{r} [\tilde{p}v_{n+1}(us, y + us) + \tilde{q}v_{n+1}(ds, y + ds)]$ .

For  $n = M$ ,  $v_M(s) = \frac{1}{r} [\tilde{p}v_{n+1}(us, us) + \tilde{q}v_{n+1}(ds, ds)]$

## Chapter 3

→ 3.1

- (i) Since  $\tilde{P} > 0$ ,  $Z > 0$ . Therefore  $\frac{1}{Z(w)} > 0$  for every  $w$ .
- (ii)  $\tilde{E}\frac{1}{Z} = \sum \frac{1}{Z(w)}\tilde{P}(w) = \sum P(w) = 1$  replacing  $Z$  by its definition.
- (iii)  $EY = \sum YP = \sum Y(w)\tilde{P}(w)/Z(w) = \tilde{E}(\frac{1}{Z}Y)$

→ 3.2

- (i)  $\tilde{P}(\Omega) = \sum Z(w)P(w) = EZ = 1$ .
- (ii)  $\tilde{E}Y = \sum Y\tilde{P} = \sum YZP = E(YZ)$
- (iii) Since  $P(A) = 0$ ,  $P(w) = 0$  for every  $w \in A$ . Now  $\tilde{P}(A) = \sum_{w \in A} \tilde{P}(w) = \sum_{w \in A} Z(w)P(w) = 0$  since  $P(w) = 0$  for every  $w \in A$ .
- (iv) If  $\tilde{P}(A) = \sum_{w \in A} \tilde{P}(w) = \sum_{w \in A} Z(w)P(w) = 0$ . Since  $P(Z > 0) = 1$ ,  $Z(w) > 0$  for every  $w$ . Therefore the sum  $\sum_{w \in A} Z(w)P(w)$  can be zero <sup>1</sup> iff  $P(w) = 0$  for every  $w \in A$ . Therefore,  $P(A)=0$ .
- (v)  $P(A) = 1$  iff  $P(A^c) = 1 - P(A) = 0$  iff  $\tilde{P}(A^c) = 0 = 1 - \tilde{P}(A)$  iff  $\tilde{P}(A) = 1$ .
- (vi) Let  $\Omega = \{a, b\}$ ,  $Z(a) = 2$ ,  $Z(b) = 0$ . Let  $P(a) = P(b) = 1/2$ . Now  $\tilde{P}(a) = 1$ ,  $\tilde{P}(b) = 0$ .

→ 3.3  $M_0 = 13.5$ ,  $M_1(H) = 18$ ,  $M_1(T) = 4.5$ ,  $M_2(HH) = 24$ ,  $M_2(HT) = 6$ ,  $M_2(TH) = 6$ ,  $M_2(TT) = 1.5$

Now,  $E_n[M_{n+1}] = E_n[E_{n+1}[M_n]]$ . From the properties of conditional expectation, this is equal to  $E_n[M_n] = M_n$ .

→ 3.5

- (i)  $Z(HH) = 9/16$ ,  $Z(HT) = 9/8$ ,  $Z(TH) = 9/24$ ,  $Z(TT) = 45/12$ .
- (ii)  $Z_1(H) = 3/4$ ,  $Z_1(T) = 3/2$ ,  $Z_0 = 1$ .

## Chapter 4

→ 4.1

- (ii)  $V_0^C = 320/125 = 2.56$ ,  $V_2(HH) = 64/5 = 12.8$ ,  $V_2(HT) = V_2(TH) = 8/5 = 1.6$ ,  $V_2(TT) = 0$ ,  $V_1(H) = 144/25 = 5.76$ ,  $V_1(T) = 16/25 = 0.64$ .

→ 4.2  $\Delta_0 = (0.4 - 3)/6$ ,  $\Delta_1(H) = -1/12$ ,  $\Delta_1(T) = -1$ ,  $C_0 = 0$ ,  $C_1(H) = 0$ ,  $C_1(T) = 1$ .

→ 4.3 The time-zero price is  $V_0 = 0.4$  and the optimal stopping time is  $\tau^*(H \dots) = +\infty$  and  $\tau^*(T \dots) = 1$ .

Moreover,  $V_1(H) = V_2(HH) = V_2(HT) = 0$ ,  $V_1(T) = 1 = \max(1, 14/15)$ ,  $V_2(TH) = 2/3 = \max(2/3, 4/10)$ ,  $V_2(TT) = 5/3 = \max(5/3, 31/20)$ ,  $V_3(TTH) = 7/4 = 1.75$ ,  $V_3(TTT) = 8.5/4 = 2.125$

→ 4.4 No. With 1.36 we can make a hedge such that we can pay Y for any outcome and we will have some spare cash. In this case he will not exercise at optimal times. If we have HH or HT we will keep 0.36 at time one with probability 1/2. If we have TT we get nothing and if we have TH we keep 1 at time 1 with probability 1/4.

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<sup>1</sup> iff means if, and only if.

Therefore at time zero we would have  $0.35(1/2) + 4/5(1/4)1 = 0.38 = 1.74 - 1.36$ .

→ 4.5 List of 11 stopping times that never exercises when the option is out of money:

t(HH)	t(HT)	t(TH)	t(TT)	value
0	0	0	0	1
inf	2	1	1	1.36
inf	2	2	inf	0.32
inf	2	inf	2	0.8
inf	2	2	2	0.96
inf	2	inf	inf	0.16
inf	inf	1	1	1.2
inf	inf	2	inf	0.16
inf	inf	inf	2	0.64
inf	inf	2	2	0.8
inf	inf	inf	inf	0
t(HH)	t(HT)	t(TH)	t(TT)	value

→ 4.6

(i) Since  $S_n/(1+r)^n$  is a martingale,  $S_{n \wedge \tau}/(1+r)^{n \wedge \tau}$  is a martingale by theorem 4.3.2.

Therefore,  $\tilde{E}[S_{0 \wedge \tau}/(1+r)^{0 \wedge \tau}] = \tilde{E}[S_{N \wedge \tau}/(1+r)^{N \wedge \tau}]$ . Now the first term is equal to  $\tilde{E}[S_0/(1+r)^0] = S_0$ . The last term is equal to  $\tilde{E}[S_\tau/(1+r)^\tau]$  since  $\tau \leq N$ .

Therefore, for any  $\tau$ ,  $\tilde{E}[G_\tau/(1+r)^\tau] = \tilde{E}[(K - S_\tau)/(1+r)^\tau] = \tilde{E}[K/(1+r)^\tau] - \tilde{E}[S_\tau/(1+r)^\tau]$ . Since the discounted price is a martingale, this is equal to  $\tilde{E}[K/(1+r)^\tau] - S_0$ . Since  $(1+r) \geq 1$ , the maximum for this term is when  $\tau(w) = 0$  for every  $w$ . In this case, the first term will be  $K$ . Therefore the value is  $K - S_0$ .

→ 4.7 Using an argument similar to 4.6 (i), taking  $\tau(w) = N$  for every  $w$ , the value will be  $\frac{S_0 - K}{(1+r)^N}$ .

## Chapter 5

→ 5.1

(i) We need the following property: If  $X$  and  $Y$  are independent r.v. then  $E(XY) = E(X)E(Y)$ .

Since  $\tau_2 - \tau_1$  and  $\tau_1$  are independent,

$$E(\alpha^{\tau_2}) = E(\alpha^{\tau_2 - \tau_1 + \tau_1}) = E(\alpha^{\tau_2 - \tau_1} \alpha^{\tau_1}) = E(\alpha^{\tau_2 - \tau_1})E(\alpha^{\tau_1}) = (E(\alpha^{\tau_1}))^2$$

(ii) Write  $\tau_m = (\tau_m - \tau_{m-1}) + (\tau_{m-1} - \tau_{m-2}) + \dots + (\tau_2 - \tau_1) + \tau_1$ . Using the same argument as in (i) we obtain the result.

(iii) Yes since the probability of rising from level 0 to level 1 is the same as rising from level 1 to level 2. This is different though from the probability to go to level -1.

→ 5.2

(i) We can write  $f(\tau) = p(e^\tau - e^{-\tau}) + e^{-\tau}$ . Since  $p > 1/2$  and  $(e^\tau - e^{-\tau}) > 0$ ,  $f(\tau) > 1/2(e^\tau - e^{-\tau}) + e^{-\tau} = \cosh(\tau) > 1$

Another proof is determining  $\tau_0$  such that  $f'(\tau_0) = 0$ . This have only one solution  $\tau_0 = \log(q/p)/2$ . Since  $q/p < 1$ ,  $\tau_0 < 0$ . Also  $f'(0) = p - q > 0$ . Therefore  $f$  is strictly increasing for  $\tau \geq 0$ . Since  $f(0) = 1$ ,  $f(\tau) > 1$  for every  $\tau > 0$ .

(ii)  $E_n[S_{n+1}] = S_n E_n[e^{\sigma X_{n+1}}]/f(\sigma) = S_n E[e^{\sigma X_{n+1}}]/f(\sigma) = S_n f(\sigma)/f(\sigma) = S_n$

(iii) Follow the same argument as in pages 121 and 122.

(iv) (see p.123) Given  $\alpha \in (0, 1)$  we need to solve for  $\sigma > 0$  which satisfies  $\alpha = 1/f(\sigma)$ . This is the same as to solve  $\alpha p e^\sigma + \alpha(1-p)e^{-\sigma} = 1$ . This is a quadratic equation for  $e^{-\sigma}$  and we obtain that  $e^{-\sigma} = \frac{1 - \sqrt{1 - 4\alpha^2 p(1-p)}}{2\alpha(1-p)}$ . Since  $1 - \sqrt{\dots} < 2\alpha(1-p)$  **iff**  $1 - 2\alpha(1-p) < \sqrt{\dots}$  **iff**  $\alpha < 1$ , we have that  $e^{-\sigma} < 1$ . We conclude that  $\sigma > 0$ .

Therefore,  $E\alpha^{\tau_1} = \frac{1 - \sqrt{1 - 4\alpha^2 p(1-p)}}{2\alpha(1-p)}$

(v) Follow corollary 5.2 (p.124): Let  $\theta = \sqrt{1 - 4\alpha^2 p(1-p)}$   $E[\tau_1 \alpha^{\tau_1}] = \frac{\partial}{\partial \alpha} \frac{1-\theta}{2\alpha(1-p)} = \frac{1}{2\alpha^2} \frac{1-\theta}{(1-p)\theta}$  (I used a CAS=computer algebra system: Maxima) Now if we let  $\alpha$  goes to 1 then  $\theta$  goes to  $2p-1$  (since  $\sqrt{1 - 4p(1-p)} = \sqrt{(2p-1)^2} = |2p-1|$ , since  $p > 1/2$ , this is equal to  $2p-1$ ). Therefore we obtain that  $E\tau_1 = \frac{1}{2p-1}$ .

→ 5.3

(i)  $\sigma_0 = \log(q/p)$  which is greater that 0 since  $q > p$ .

(ii) Follow the steps in p.121 and p.122 but replacing  $2/(e^\sigma + e^{-\sigma})$  by  $1/f(\sigma)$ .

Now the equation 5.2.11 from p.122 (with the replacement above) is true for  $\sigma > \sigma_0$ . Here we cannot let  $\sigma$  goes to zero. Instead we let  $\sigma$  goes to  $\sigma_0$ . Since from (i)  $e^{\sigma_0} = q/p$ , we obtain the answer:  $p/q$ .

(iii) We can write  $E\alpha^{\tau_1} = E(I_{\{\tau_1=\infty\}}\alpha^{\tau_1}) + E(I_{\{\tau_1<\infty\}}\alpha^{\tau_1})$ . Since  $0 < \alpha < 1$ ,  $\alpha^{\tau_1} = 0$  for the first term. Therefore,  $E\alpha^{\tau_1} = E(I_{\{\tau_1<\infty\}}\alpha^{\tau_1})$ .

Following exercise 5.2 (iv):  $E\alpha^{\tau_1} = \frac{1 - \sqrt{1 - 4\alpha^2 p(1-p)}}{2\alpha(1-p)}$  (note that  $E\alpha^{\tau_1} = E(I_{\{\tau_1<\infty\}}\alpha^{\tau_1}) + E(I_{\{\tau_1=\infty\}}\alpha^{\tau_1})$ ).

(iv) Follow exercise 5.2 (v): Let  $\theta = \sqrt{1 - 4\alpha^2 p(1-p)}$   $E[I_{\{\tau_1<\infty\}}\tau_1 \alpha^{\tau_1}] = \frac{\partial}{\partial \alpha} \frac{1-\theta}{2\alpha(1-p)} = \frac{1}{2\alpha^2} \frac{1-\theta}{(1-p)\theta}$  (I used a CAS=computer algebra system: Maxima) Now if we let  $\alpha$  goes to 1 then  $\theta$  goes to  $1-2p$  (since  $\sqrt{1 - 4p(1-p)} = \sqrt{(2p-1)^2} = |2p-1|$ , since  $p < 1/2$ , this is equal to  $1-2p$ ). Therefore we obtain that  $E[I_{\{\tau_1<\infty\}}\tau_1] = \frac{p}{(1-p)(1-2p)}$ .

→ 5.4

(ii)  $P(\tau_2 = 2k) = P(\tau_2 \leq 2k) - P(\tau_2 \leq 2k-2)$

By the reflection principle,  $P(\tau_2 \leq 2k) = P(M_{2k} = 2) + 2P(M_{2k} \geq 4)$ . Since the random walk is symmetric,  $2P(M_{2k} \geq 4) = P(M_{2k} \geq 4) + P(M_{2k} \leq -4)$ .

Therefore,  $P(\tau_2 \leq 2k) = P(M_{2k} = 2) + P(M_{2k} \geq 4) + P(M_{2k} \leq -4)$ . This is equal to  $P(\tau_2 \leq 2k) = 1 - P(M_{2k} = 0) - P(M_{2k} = -2)$ .

Replacing  $k$  by  $k-1$ ,  $P(\tau_2 \leq 2k-2) = 1 - P(M_{2k-2} = 0) - P(M_{2k-2} = -2)$ .

Therefore,

$P(\tau_2 = 2k) = P(M_{2k-2} = 0) + P(M_{2k-2} = -2) - P(M_{2k} = 0) - P(M_{2k} = -2)$ .

The complete formula can be written since  $P(M_m = 0) = \frac{m!}{((m/2)!)^2} \left(\frac{1}{2}\right)^m$   
and  $P(M_m = -2) = \frac{m!}{(m/2-1)!(m/2+1)!} \left(\frac{1}{2}\right)^m$  and  
 $\longrightarrow$  5.5  
(ii) replace  $\left(\frac{1}{2}\right)^n$  in (i) by  $p^{\frac{n-b}{2}+m} q^{\frac{n+b}{2}-m}$ .