Ministry of Science and Higher Education of the Russian Federation

National Research University Higher School of Economics

Faculty of Computer Science

School of Data Analysis and Artificial Intelligence

HOMEWORK REPORT

Practical homework №2

Subject: *Ordered Sets for Data Analysis*

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QUESTION 1

Task. Given the following formal context, find all formal concepts, draw the concept lattice, and find all non-trivial implications.

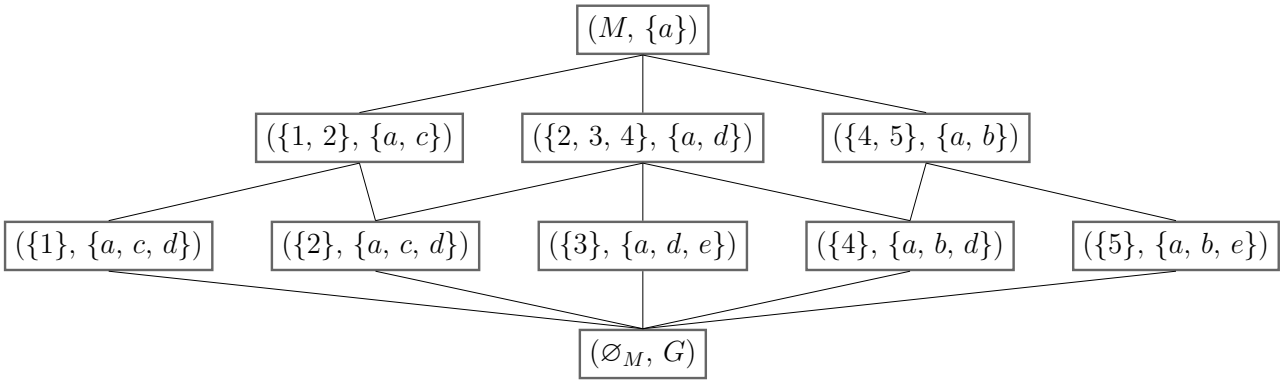
	a	b	c	d	e
1	1		1		1
2	1		1	1	
3	1			1	1
4	1	1		1	
5	1	1			1

Solution. Let the set of all objects be denoted as $G = \{1, 2, 3, 4, 5\}$ and the set of all attributes as $M = \{a, b, c, d, e\}$. We will condense all the data about formal concepts in this context into a table, where $A \subseteq G$ is set of objects defining a concept.

A	A'	A''	Formal concept?
$\{\varnothing_M\}$	G	$\{\varnothing_M\}$	Yes
M	$\{a\}$	M	Yes
$\{1\}$	$\{a, c, e\}$	$\{1\}$	Yes
$\{2\}$	$\{a, c, d\}$	$\{2\}$	Yes
$\{3\}$	$\{a, d, e\}$	$\{3\}$	Yes
$\{4\}$	$\{a, b, d\}$	$\{4\}$	Yes
$\{5\}$	$\{a, b, e\}$	$\{5\}$	Yes
$\{1, 2\}$	$\{a, c\}$	$\{1, 2\}$	Yes
$\{1, 3\}$	$\{a\}$	M	No
$\{1, 4\}$	$\{a\}$	M	No
$\{1, 5\}$	$\{a, e\}$	$\{1, 3, 5\}$	No
$\{2, 3\}$	$\{a, d\}$	$\{2, 3, 4\}$	No
$\{2, 4\}$	$\{a, d\}$	$\{2, 3, 4\}$	No

A	A'	A''	Formal concept?
$\{2, 5\}$	$\{a\}$	M	No
$\{3, 4\}$	$\{a, d\}$	$\{2, 3, 4\}$	No
$\{3, 5\}$	$\{a\}$	M	No
$\{4, 5\}$	$\{a, b\}$	$\{4, 5\}$	Yes
$\{1, 2, 3\}$	$\{a\}$	M	No
$\{1, 2, 4\}$	$\{a\}$	M	No
$\{1, 2, 5\}$	$\{a\}$	M	No
$\{2, 3, 4\}$	$\{a, d\}$	$\{2, 3, 4\}$	Yes
$\{2, 3, 5\}$	$\{a\}$	M	No
$\{3, 4, 5\}$	$\{a\}$	M	No
$\{1, 2, 3, 4\}$	$\{a\}$	M	No
$\{1, 2, 3, 5\}$	$\{a\}$	M	No
$\{1, 2, 4, 5\}$	$\{a\}$	M	No
$\{1, 3, 4, 5\}$	$\{a\}$	M	No
$\{2, 3, 4, 5\}$	$\{a\}$	M	No

Having found all the formal concepts, we are able to construct the concept lattice.



QUESTION 2

Task. Using the diagram find:

1. $f \wedge m$, $a \vee l$, $i \wedge k$;
2. $a \wedge (c \vee d)$, $(i \wedge g) \vee (c \wedge d)$, $\vee(b, c, d)$;
3. $\wedge\varphi$, $\vee\varphi$;

and determine whether the diagram represents an upper semi-lattice, a lower semi-lattice or a lattice.