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ĀRYABHAṬA'S KALIYUGA REVISITED: AN OPTIMISATION PROBLEM

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Āryabhaṭīya of Āryabhaṭa mentions a conjunction of all planets, moon and node at the start of Kaliyuga. Studies have been made earlier to look for such a conjunction and to fix the Kaliyuga epoch. Āryabhaṭa's constants have been checked for their accuracy by comparing it with modern values in earlier studies.

The Kaliyuga epoch is well defined as 17/18 February 3102 BC when all the bodies were in super conjunction. This paper is an attempt to revisit the issue as an optimisation problem. The aim is to look for the conjunction and to derive the Kaliyuga epoch from this using an optimisation scheme. The primary assumption is that Āryabhaṭa's model is a mean model and he had derived this based on accurate measurements during his lifetime. Based on these measurements he had generated the mean model with the planets at zero degree *nirayana* at the start of Kaliyuga.

A mean model on similar lines as that of Āryabhaṭa's has been derived. Mean values for Āryabhaṭa's time (499 AD) have been computed using present day equations. A simplex algorithm is used to minimise the *nirayana* longitudes of planets at Kaliyuga epoch. It is observed that there indeed is a conjunction of the planets at Kaliyuga epoch and Kaliyuga started on 18 February 3102 BC at sunrise at Laṅkā.

Keywords: Āryabhaṭa, Kaliyuga, Mean Motion, Planetary Conjunction

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SYMBOLS AND UNITS USED

LS_{sun}	=	*Sāyana* longitude of Sun w.r.t. vernal equinox point
LS_{mer}	=	*Sāyana* longitude of Mercury (Heliocentric)
LS_{ven}	=	*Sāyana* longitude of Venus (Heliocentic)
LS_{mar}	=	*Sāyana* longitude of Mars (Heliocentric)
LS_{jup}	=	*Sāyana* longitude of Jupiter (Heliocentric)
LS_{sat}	=	*Sāyana* longitude of Saturn (Heliocentric)
LS_{moon}	=	*Sāyana* longitude of Moon (Geocentric)
LS_{node}	=	*Sāyana* longitude of Node (Geocentric)
$LS_{(.)}$	=	*Sāyana* longitude of all above objects
$LN_{(.)}$	=	*Nirayana* longitude of all above objects
$LK_{(.)}$	=	*Nirayana* longitude of all above objects at Kaliyuga epoch
LK_{node}	=	*Nirayana* longitude of Node at Kaliyuga epoch
$NR_{(.)}$	=	Revolution number of objects in 4320000 sidereal years
S_{arya}	=	Sidereal period of Āryabhaṭa (15779175000/4320000)
JD	=	Julian Day number
JD_{ky}	=	Julian Day number of Kaliyuga epoch 588465.5
JD_x	=	Julian Day number of Kaliyuga used as a control variable
Kaliyuga	=	1 Kaliyuga (unit) = 432000 sidereal years
IP_{hel}	=	*Sāyana* longitude of Initial Point (Heliocentric)
IP_{geo}	=	*Sāyana* longitude of Initial Point (Geocentric)

INTRODUCTION

1500 years ago Āryabhaṭa stated in his *Āryabhaṭīya* that the planets, the moon and the moon's ascending node were all at the Fixed Initial Point (IP) *(Meṣādi)* at a point of time 7.5 kaliyugas (7.5 x 432000 sidereal years) before the present Kaliyuga epoch.[1] This is the same as saying that the planets and the moon were all at this very same point at the start of the present Kaliyuga and that the node was 180 degrees away. The start of the

[1] K. S. Shukla, & K. V. Sarma, *Āryabhaṭīya of Āryabhaṭa*, Indian National Science Academy, New Delhi 1976.

present Kaliyuga is assumed to be at midnight or at sunrise 17/18 February 3102 BC (-3101 AD astronomical).

S. K. Chatterjee has taken the VSOP planetary theory and computed the planetary positions at this epoch to arrive at a suitable conclusion.[2] The main result however is that there was no such super conjunction at the epoch! Was Āryabhaṭa then a fraud? This cannot be so as he is hailed by later scholars as a great astronomer. K. Chandra Hari points this out by giving the synodic period of the planets of the *Āryasidhānta* and shows how well they tally with the present value.[3] The astronomy of the ancients was based on computing the mean positions of the bodies and it was heliocentric for the planets.[4] Perhaps the fault then lies in assuming the present day methods for computing planetary positions and checking at the Kaliyuga epoch for a super conjunction.

The planetary positions are computed today accurately using N-body dynamics and integration. Using the data thus generated attempts have been made to fit a curve to this data to simplify the computations. Another method is to fix an epoch in time and to compute the planetary positions at this epoch accurately. The mean longitude of the planet is then computed using the velocity and acceleration terms associated with it.

The actual positions are then computed by solving Kepler's equation.[5] It would not be too erroneous to assume that Āryabhaṭa would have actually measured the planetary positions in his lifetime and also would have devised a method to predict their positions in future. He would have devised a scheme using mean values and fixed an epoch in time very much like the present day methods.

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2. S. K. Chatterjee, "A Note on Kali Era," IJHS, 32 (1997) 69-86.

3. K. Chandra Hari, "On the Origin of Kaliyugādi Synodic Super Conjunction," IJHS, 32 (1998) 195-201.

4. S. N. Sen and K. S. Shukla (ed), *History of Astronomy in India*, Indian National Science Academy, New Delhi 1985, pp. 149-175.

5. Jean Meeus, *Astronomical Formulae for Calculators*, Willmann-Bell, Inc, USA, fourth ed, 1988.

This paper presents an optimised approach to fix the IP and Kaliyuga epoch. It is an attempt to prove that Āryabhaṭa indeed used a mean model assuming that the planets and moon were at sidereal zero degrees at the start of Kaliyuga and the node was 180 degrees away.

ASSUMPTIONS

The following assumptions have been made for the generation of the model for optimisation.

a) Āryabhaṭa had measured the positions of the planets, the moon and the moon's ascending node accurately during his time. He had arrived at the mean values for the objects from the measured data and generated the revolution numbers given in Table 1 below. It is believed that Āryabhaṭa had taken the revolution from some earlier date and used it. The assumption still holds good as he would have surely tested this at his time and perhaps tuned the numbers to match the planetary positions in his time.

b) He had fixed an initial epoch when all these bodies were together at IP. Sun would be at IP every sidereal year. The other bodies would be at various longitudes with respect to this IP (*nirayana* longitude) depending on their respective revolution number.

Table 1 Āryabhaṭa's Revolution numbers for the planets

Object	Revolutions in 4320000 sidereal Years
Sun	4320000
Mercury	17937020
Venus	7022388
Mars	2296824
Jupiter	364224
Saturn	146564
Moon	57753336
Moon's Asc. Node	232226
No of days	1577917500

c) The mean planets would move around the sun along the ecliptic with zero latitude and circular orbit of unit radius. The moon and node would move round the earth with zero latitude and circular orbit of unit radius.

The correction applied to mean longitude of planets to compute the actual position does show that a heliocentric model was assumed for the planets at that time.[6] The latitude corrections and other perturbation corrections had been carried out later to compute the exact position, as done today.

PROBLEM FORMULATION

The start of Kaliyuga, for the problem formulation, is assumed as midnight (UT) 17 February –3101 AD. The Julian Day Number for Kaliyuga is assumed as JD_{ky} = 588465.5. Assuming 499 AD as the time of Āryabhaṭa, the mean positions of the planets (heliocentric), the moon and the moon's ascending node (geocentric) are computed at that time (499 AD). Jean Meeus gives the mean model for the planetary positions as[7]

$$LS_{sun} = 279.6968 + 36000.76892 * T + 0.0003025 * T^2$$

$$LS_{mer} = 178.179078 + 149474.07078 * T + 0.0003011 * T^2$$

$$LS_{ven} = 342.767053 + 58519.21191 * T + 0.0003097 * T^2$$

$$LS_{mar} = 293.737334 + 19141.69551 * T + 0.0003107 * T^2$$

$$LS_{jup} = 238.049257 + 3036.301986 * T + 0.0003347 * T^2 - 0.00000165 * T^3$$

$$LS_{sat} = 266.564377 + 1223.509884 * T + 0.0003245 * T^2 - 0.0000058 * T^3$$

$$LS_{moon} = 270.434164 + 481267.8831 * T - 0.001133 * T^2 + 0.0000019 * T^3$$

$$LS_{node} = 259.183275 - 1934.142 * T + 0.002078 * T^2 + 0.00000222\ T^3 * \qquad \ldots 1$$

6. Sen & Shukla, op. cit.

7. Jean Meeus, op. cit.

where T = (JD-2415020)/36525 and JD is the Julian Day Number for the time at which the mean longitudes are computed (e.g. 499 AD).

These are the *sāyana* longitudes, heliocentric for the planets and geocentric for sun, moon and the node. These have to be converted to *nirayana* longitudes with respect to the IP. It is assumed that at the start of every sidereal year starting from the Kaliyuga epoch JD_{ky} the sun is at the point. For the year 499 AD the start of the sidereal year is 3600 sidereal years after JD_{ky}, i.e.

$$JD = JD_{ky} + 3600^{*}\ S_{arya} \qquad \dots 2$$

where S_{arya} is length of the sidereal year of Āryabhaṭa = 365.2586805555555 days.

The *nirayana* heliocentric longitudes of the planets and *nirayana* geocentric longitudes of moon and node at JD are given as

$$LN_{(.)} = LS_{(.)} - LS_{sun} \qquad \dots 3$$

Subtracting LS_{sun} from the *sāyana* longitudes of moon and node is understandable, as all the parameters are geocentric. This is perfectly valid in case of heliocentric longitudes of the planets also. This is because the longitude of IP is taken as the geocentric longitude of Sun and therefore its heliocentric longitude is also the same. Assuming unit radius vector for the planets and zero latitude the conversion from heliocentric longitude is given by

$$\tan(IP_{geo}-LS_{sun}) = \sin(IP_{hel}-LS_{sun}) / \{\cos(IP_{hel}-LS_{sun}) + 1\} \qquad \dots 4$$

where IP_{hel} is the heliocentric longitude of IP and IP_{geo} is the geocentric longitude of IP. Here $IP_{geo}-LS_{sun} = 0$ and therefore $IP_{hel}-LS_{sun}$ should be zero or 180.

$LN_{(.)}$ the *nirayana* longitudes of the objects having been obtained using the above equations (1-3), their positions at the start of Kaliyuga are computed. This would be similar in nature to how Āryabhaṭa would have done it 1500 years ago. He would have, however, computed the mean *nirayana* positions based on his observations. The *nirayana* longitudes at Kaliyuga are given by

$LK_{(.)} = LN_{(.)} \pm NR_{(.)}$ ...5

The addition in equation (5) is used in case of the ascending node, which moves in the opposite direction compared to the planets. Here $NR_{(.)}$ is the number of revolutions of the particular object for the given number of sidereal year (3600 here) based on Table 1. The number of revolutions are converted to degrees (one revolution is 360 degrees). All longitudes are between 0 to 360 degrees.

$LK_{(.)}$ and $LK_{node} \pm 180$ should be very close to zero!

OPTIMISATION

Having formulated the problem, a simplex algorithm is used to minimise the *nirayana* longitudes $LK_{(.)}$ and $LK_{node} + 180$. The control variable is the Julian Day Number of Kaliyunga epoch JD_x. Starting from the assumed value of Kaliyuga JD_{ky} the variable JD_x is iteratively changed such that the values of $LK_{(.)}$ and $LK_{node} + 180$ is minimised (ideally zero). At every iteration a new value of JD_x is selected. The corresponding JD (at 499 AD) is recomputed along with the $LN_{(.)}$. The $LK_{(.)}$ is recomputed and checked for minimum. This process is repeated till the error between successive values of JD_x is less than 1.0e-6.

It was found during the optimisation process that the minimum varied based on what combination of planets was selected to compute the error. It was suggested by the reviewers that all the combinations should be studied to arrive at the minimum. The problem has been simulated from 400 AD to 600 AD taking all possible combinations of planets, i.e. only one planet at a time, combination of 2 planets at a time and so on. The minimum for each year has been selected as the minimum for the year.

RESULTS

Fig. 1 shows the results of this optimisation. It is found that a combination of Venus and Moon or Venus, Mars and Moon gives the best result. The two curves show the error for the two sets of combinations. It is found that the minimum occurs at 505 AD and 506 AD. This is the year of composition of *Āryabhaṭīya* by Aryabhaṭa. Table 2 summarises the results.

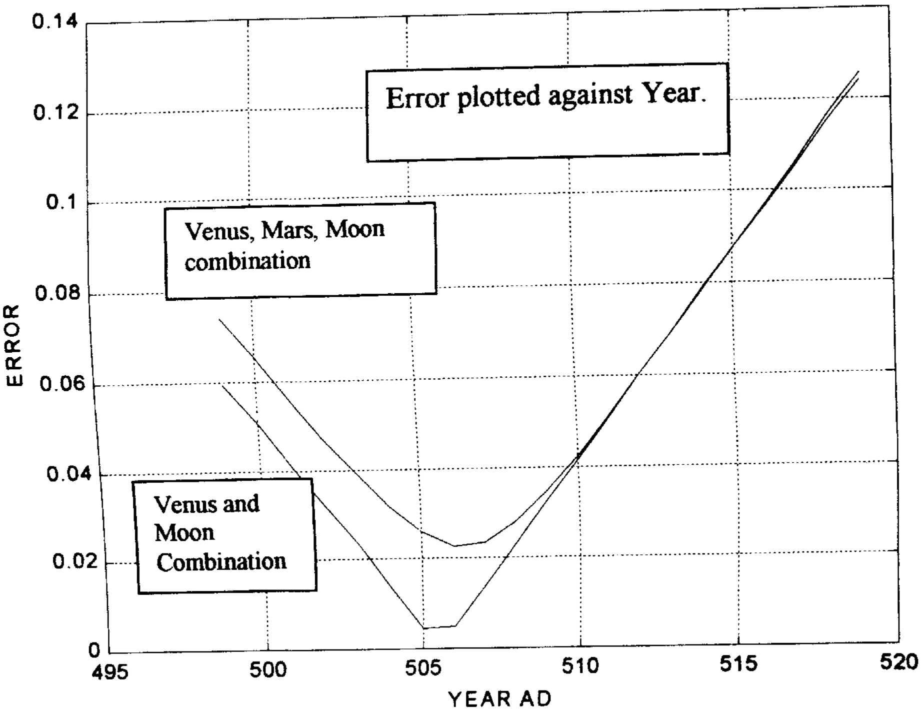

Fig. 1 : Optimisation results

Table 2 Results of tabulation: Venus-Moon combination

Parameter		499 AD	505 AD
Kaliyuga JD		588465.58754588	588465.588811057
Kaliyuga Year		18-2-3102 BC	18-2-3102 BC
Time UT		2h 6m 3.912s	2h 7m 58.4635s
JD for Start of Sidereal Year		1903396.837545288	1908875.719079439
Error (deg)	Mercury	2.3858	2.2569
	Venus	0.0594	0.0781
	Mars	0.0447	0.0036
	Jupiter	0.2345	0.1655
	Saturn	0.5941	0.5138
	Moon	0.0029	0.0039
	Asc. Node	-0.0886	0.0094
Mean Error		0.4618 deg	0.4330 deg
Standard Deviation		0.8775 deg	0.8245 deg

The mean error for 499 AD is 0.4618 degrees and for 505 AD it is 0.4330 degrees. The Kaliyuga Epoch according to Āryabhaṭa was at sunrise of 18 February 3102 BC at Laṅkā at 2h 7m 58.6435s UT. Taking the longitude of Laṅkā as 75 degree East the sunrise at Laṅkā was at 7h 7m 58.4635 & Local Time.

CONCLUSION

Āryabhaṭa used a mean model when he said that the planets were in conjunction at Kaliyuga epoch. The mean model for the planets was heliocentric. He had based this observation from the measurements during his lifetime and proper conversion to mean longitudes. He had back computed the planetary positions and developed a mean model to enable him to compute the future planetary positions.

The Kaliyuga epoch was at sunrise of 18 February 3102 BC at Laṅkā. There indeed was a conjunction at the start of Kaliyuga.