p.v.s.sucharitha
192311103
CSA0669.

Analytical Assignment:-3

(1) calculate the number of ways to acheive a sum of 15 when rolling four six-sided dice provide a detailed step-by-step solution

Let us assume

$x_1+x_2+x_3+x_4=15 \quad (1<x_i<6)$

consider that

$x_i=y+1$

$(y_1+1)+(y_2+1)+(y_3+1)+(y_4+1)=15$

$y_1+y_2+y_3+y_4=11$

By using Stars and Bar theorm

$\binom{n+k-1}{n-1} = \binom{11+4-1}{4-1} - \binom{14}{3} = \frac{14\times13\times\cancel{12}^{2}}{1\times\cancel{2}\times\cancel{3}} = 364$

Assume that,

$y_1=6 \, (i\leq 6)$

$y_1+y_2+y_3+y_4=5$

$\binom{5+4-1}{4-1} = \binom{5+3}{3} = \binom{8}{3} = \frac{8\times7\times\cancel{6}^{2}}{1\times\cancel{2}\times\cancel{3}} = 56$

Since any of four variables $=4\times56=224$

$\Rightarrow y_2=6 \quad (i\leq6)$

$y_1+y_2+y_3+y_4=-1$

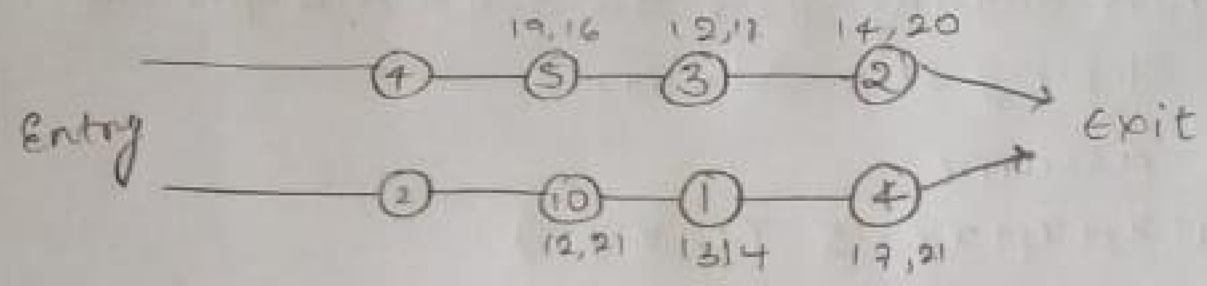
As only non negative values are considered

∴ The No.of ways

$=364-224$

$=140$

(2) Two assembly lines have station times as follows line 1: [4,5,3,2] ;Line 2: [2,10,1,4) Transfer times b/w lines are from Line1 to line 2: [7,4,5], from Line2 to Line 1: [9,2,8] Calculate the Min time to assemble a Product.



F1[j]	4	9	12	14
F2[j]	2	12	13	17

l1[j]	1	1←	1←	1←
l2[j]	2	2	2	2

(3) Given keys {10,20,30,40} with access probabilites {0.1,0.2,0.4,0.3} construct the optimal binary search tree calculate total cost of tree.

(A) Given that,

K= {10, 20,30,40}

P= {0·1, 0·2, 0·4, 0·3}

	0	1	2	3	4
0	0	0.1	0.4^[3]	1.1^[2]	1.4^[3]
1		0	0.2	0.8^[3]	1.0^[2]
2			0	0.4	1^[3]
3				0	0.3
4					0

1> j-i =0

0-0=0[0,0)

1-1 = 0 [1,1)

2-2 = 0 [2,2]

3-3 = 0 [3,3]

4-4 = 0 [4,4]

j-i = 1

1-0 = [0,1)

2-1 = [1,2]

3-2 = [2,3]

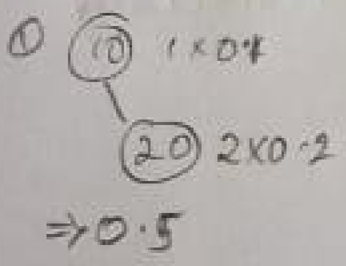
4-3 = [3,4]

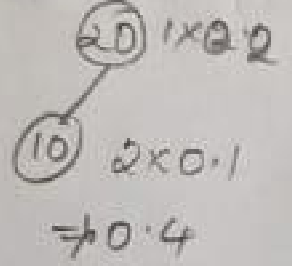
j-i = 2.

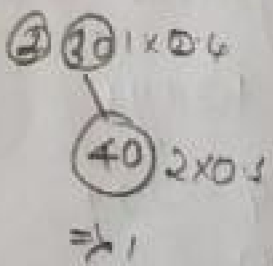
2-0 = [0,2]

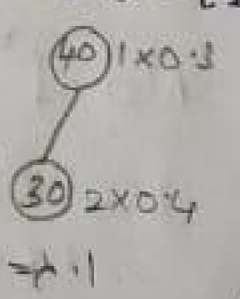
3-1 = [1,3]

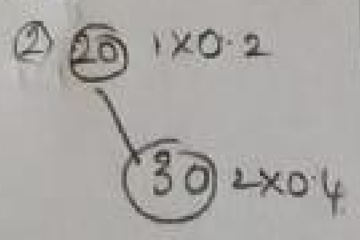
[4,2] = [2,4]

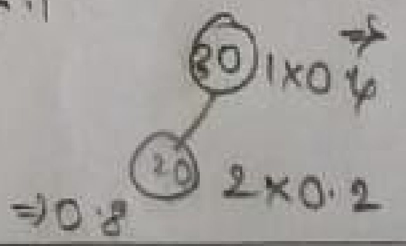












(4) $j - i = 3$

$3-0 = [0,3]$ $\quad$ $cost(i,j) = Min\{(i,a,k-1) + cost(k,j)\} + w_i$

$4-1 = [1,4]$

$$cost(0,3) = \underset{k=1,2,3}{Min} \begin{Bmatrix} cos(0,1-1) + cost(1,3) \\ cost(0,2-1) + cost(2,3) \\ cost(0,3-1) + cost(3,3) \end{Bmatrix} + 0.7$$

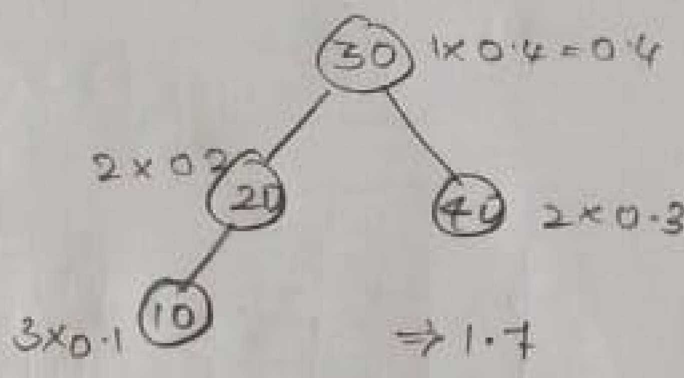
$$= Min \begin{Bmatrix} 0+0.8 \\ 0.1+0.4 \\ 0.4+0 \end{Bmatrix} + 0.7$$

$$Min \begin{Bmatrix} 1.5 \\ 1.2 \\ \boxed{1.1} \end{Bmatrix}.$$

⑤ $j - i = 4$

$4-0 = [0,4]$

$$= Min \begin{Bmatrix} 2.0 \\ 2.1 \\ 1.7 \\ 2.1 \end{Bmatrix} \Rightarrow 1.7$$

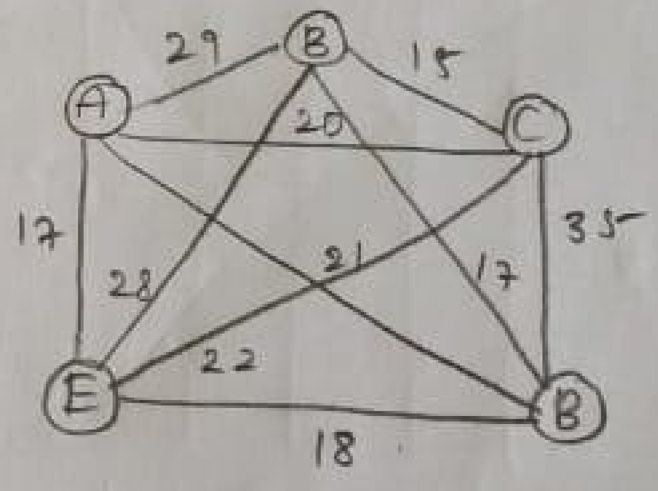


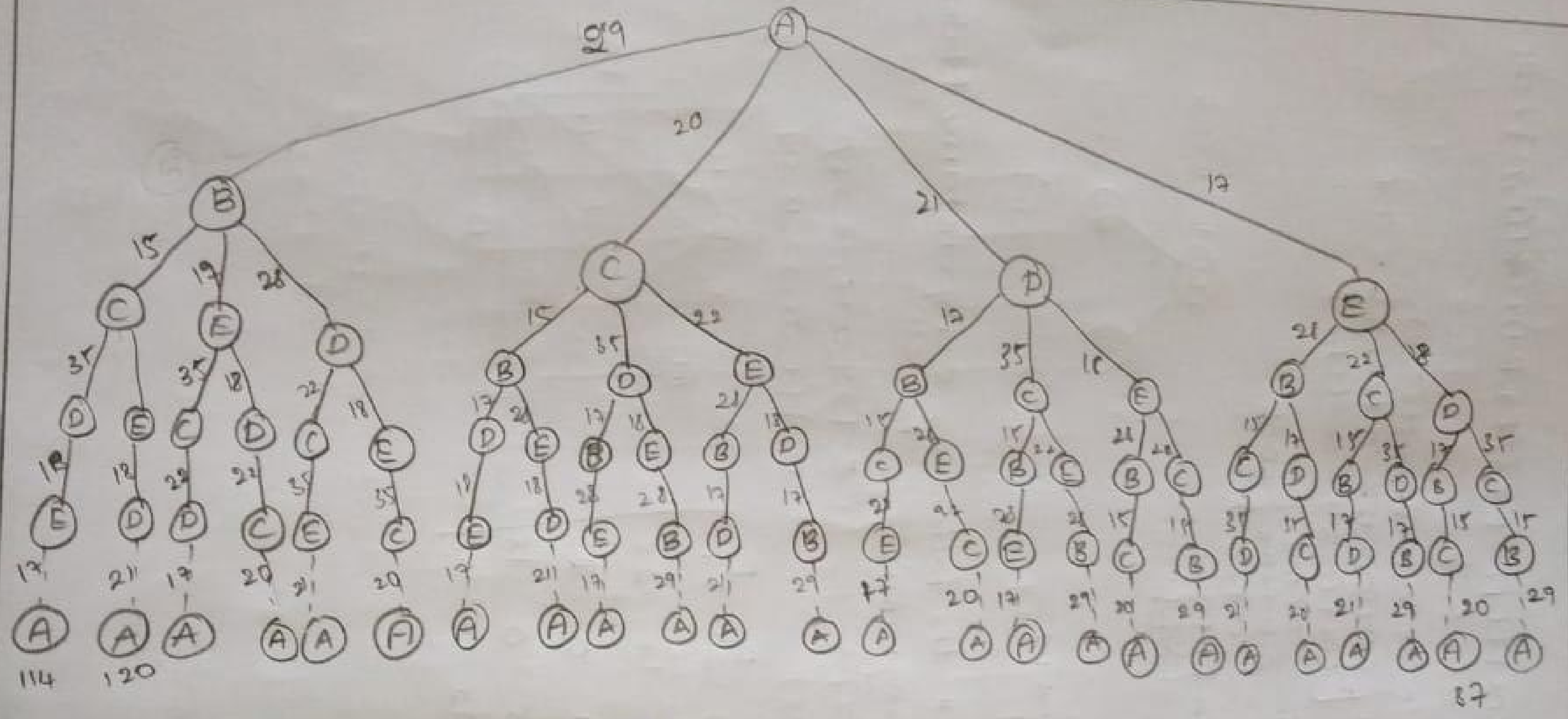
(4) Solve the TSP for the following 5-city distance Matrix using dynamic programming

A: [0, 29, 20, 21, 17] $\quad$ B: [29, 0, 15, 17, 28]

C: [20, 19, 35, 0, 18) $\quad$ E: [17, 28, 22, 18, 0]

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ A & 0 & 29 & 20 & 21 & 17 \\ B & 29 & 0 & 15 & 17 & 28 \\ C & 20 & 15 & 0 & 35 & 22 \\ D & 21 & 17 & 35 & 0 & 18 \\ E & 17 & 28 & 22 & 18 & 0 \end{array}$$





∴ The Minimum cost = 87

A — E — D — B — C — A

(5) You have a knapsack with a capacity of 50 units. There are 4 items with following weights and values:

$I_1 \div W_1 = 10;\ V = 60$ $\qquad I_2 \div 1$

$J_2 \div W_2 = 20,\ V = 10$

$J_3 \div W_3 = 30,\ V = 120$

$I_4 : W_4 = 40,\ V = 200$ Determine using 0/1 knapsack

I	W	V
1	10	60
2	20	100
3	30	120
4	40	200

W/V	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

Formula:

$V[i, W] = \max\{V[i-1, W], V[i-1, W-W[i]] + value[i]\}$

$V[4, 50] = \max\{V[3, 50], V[3, 50-200] + value[4]\}$

$= \max\{220, 260\} = 260$

(6) Given the following directed graph with vertices A, B, C, DA, B, C, DA, B, C, D and edges with weights:

A → BA \ ग.a BA → B with wt 1

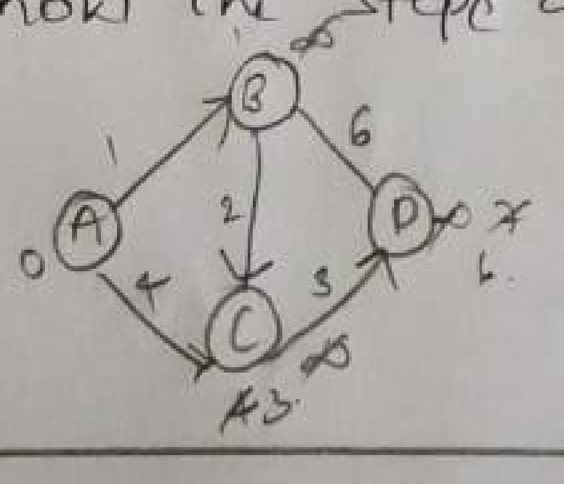
A → CA \ ग.a CA → C wt 4

B → CB \ r.a CB → D wt 2

B → DB \ r.a DB → D wt 2

C → DC \ r.a DC → D wt 3

Use the Bellman-Ford algorithm to find the shortest path from vertex AAA to all other vertices. Show the steps and the final distances.



A → B	1
A → C	4
B → C	2
B → D	6
C → D	3

Initialize

v	A	B	C	D
d	0	∞	∞	∞
p	-	-	-	-

①

v	A	B	C	D
d	0	1	4	∞
p	-	A	A	B

②

v	A	B	C	D
d	0	1	3	7
p	-	A	B	B

③

v	A	B	C	D
d	0	1	3	6
p	-	A	B	C

Path	shortest dis	shortest Path
A-B	1	A-B
A-C	3	A-B-C
A-D	6	A-B-C-D

O/P → A → B → C → D.

7) Determine the probability of rolling five dice such that the Sum is Exactly 20. Include a combinatial approach to arrive at the Sol^n.

(A) $6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $1 \le x_i < 6$

$y_i = x_j - 1$ for $i = 1, 2, 3, 4, 5$

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) + (y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

By "stars and bars"

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19\times18\times17\times16}{4\times3\times2\times1} = 3876$$

If $y_1 \ge 6$, let $y_i = y_1 - 6$

$y_i + y_2 + y_3 + y_4 + y_5 = 9$.

$$\binom{9+5-}{5-1} = \binom{13}{14}.$$

$$\binom{13}{4} = \frac{13\times12\times11\times10}{4\times3\times2\times1} = 715$$

there are 5 such variables.

$5\times715 = 3575$

If two variables $y_i - y_j \geq 6$, let $y_1 = y_1 - 6$ and $y_2' = y_2 - 6$

$$y_1' + y_2' + y_3 + y_4 + y_5 = 3$$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

$$\binom{7}{4} = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$$

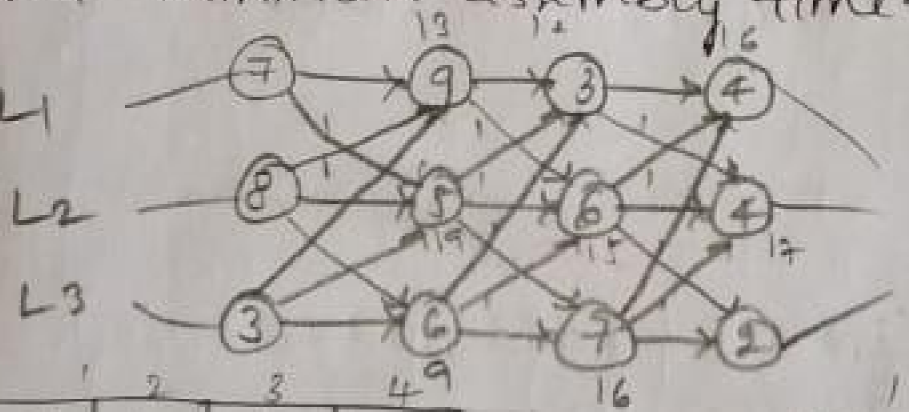
$$\binom{5}{2} \times 35 = 10\times35 = 350$$

Using the Inclusion - Exclusion principle:

$3876 - 3575 + 350 = 651$

$$\frac{651}{7776} = \frac{651}{7776} = 0.0837$$

⑧ For three assembly lines with stations times: Line: [7,9,3,4], Line 2 · [8,5,6,4), line 3: [3,6,7,2], & transfer times b/w lines given, determine the optimal scheduling & the total minimum assembly time.



	1	2	3	4
$f_1[j]$	7	13	12	16
$f_2[j]$	8	9	15	17
$f_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1	1
$L_2[j]$	2	3	2	1
$L_3[j]$	3	3	3	1

$$F_i[j] : \text{Min}\{f_1[j-1] + a_{1j}), (f_2(j-1) + (t_2, j-1) + a_1(j)), (f_3(j-1) + (t_3, j-1) + a_{1j}\}$$

$$= \text{Min}\{(7+9), (8+1+9), (3+1+9)\} = 13$$

⑨ Consider keys {15, 25, 35, 45, 55} | {15, 25, 35, 45, 55} | {15, 25, 35, 45, 55} with access Probabilities {0.05, 0.5, 0.4, 0.25, 0.15} Determine the structure of the optimal binary search tree and compute the Expected cost

Given that,

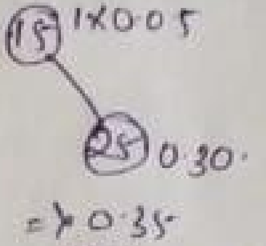
$\{\overset{1}{15}, \overset{2}{25}, \overset{3}{35}, \overset{4}{45}, \overset{5}{55}\}$

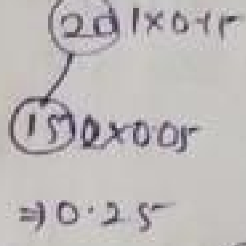
{0.05, 0.15, 0.4, 0.25, 0.15}

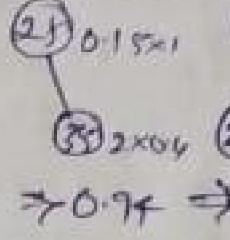
	0	1	2	3	4	5
0	0	0.05	0.25 [2]	0.85 [2]	1.25 [2]	1.80 [2]
1		0	0.15	0.70 [2]	1.20 [3]	1.80 [4]
2			0	0.80	0.98 [3]	1.35 [3,4]
3				0	0.25	0.55 [4]
4					0	0.15
5						0

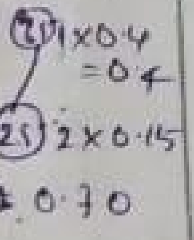
j-i=1
1-0=1 (0,1) (1,1)
2-1=1 (1,2) (2,2)
3-2=1 (2,3) (3,3)
4-3=1 (3,4) (4,4)
5-4=1 (4,5) (5,5)

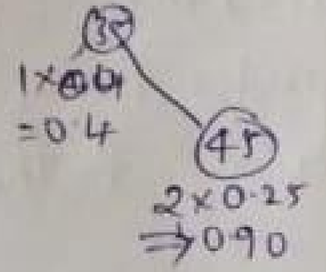
j-i=2
2-0=2 (0,2) (1,2)
3-1=2 (1,3) (2,3)
4-2=2 (2,4) (3,4)
5-3=2 (3,5) (4,5).

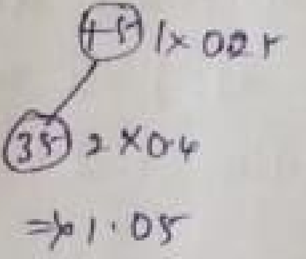


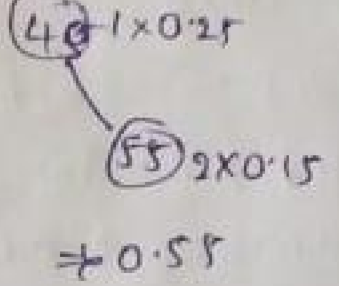


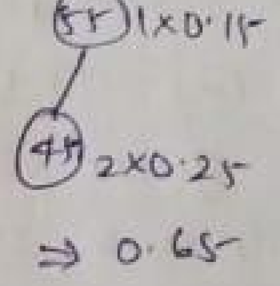












j-i=3
3-0=3 (0,3) (1,3)
4-1=3 (1,4) (2,4)
5-2=3 (2,5) (3,5)

cost (i,j) = min {cost (i, k-1) + cost (k, j) + w}

$$\text{cost } (0,3) = \min \begin{Bmatrix} 0+0.70 \\ 0.05+0.4 \\ 0.25+0 \end{Bmatrix} + 0.6$$

$$\min \begin{Bmatrix} 1.30 \\ 1.05 \\ 0.85 \end{Bmatrix} = 0.85$$

$$\text{cost } (1,4) = \min \begin{Bmatrix} 0+0.90 \\ 0.15+0.25 \\ 0.70+0 \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 1.70 \\ 1.20 \\ 1.30 \end{Bmatrix} = 1.20$$

$$\text{cost}(2,5) = \min \begin{Bmatrix} 0 + 0.55 \\ 0.4 + 0.15 \\ 0.90 + 0 \end{Bmatrix} + 0.8$$

$$\min \begin{Bmatrix} 1.35 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$$

$j - i = 4.$

$4 - 0 = 4 \quad (0,4)\ (1,4)$

$5 - 1 = 4 \quad (1,5) \quad (2,5)$

$$\text{cost}(0,4) = \min \begin{Bmatrix} 0 + 1.20 \\ 0.05 + 0.90 \\ 0.25 + 0.25 \\ 0.85 + 0 \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.\text{[illegible]} \end{Bmatrix} \Rightarrow 1.35$$

$$\text{cost}(1,5) = \min_{k=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 0 + 1.35 \\ 0.15 + 1.35 \\ 0.70 + 0.15 \\ 1.20 + 0 \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80.$$

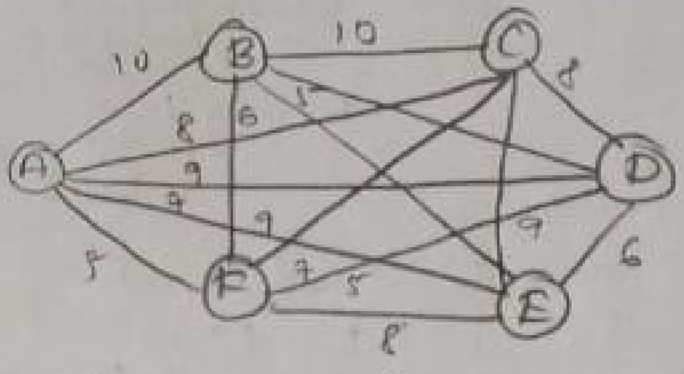
$$\text{cost}(0,5) = \min_{k=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ c(0,1) + c(2,5) \\ c(0,2) + c(3,5) \\ c(0,3) + c(4,5) \\ c(0,4) + c(5,5) \end{Bmatrix} + 1$$

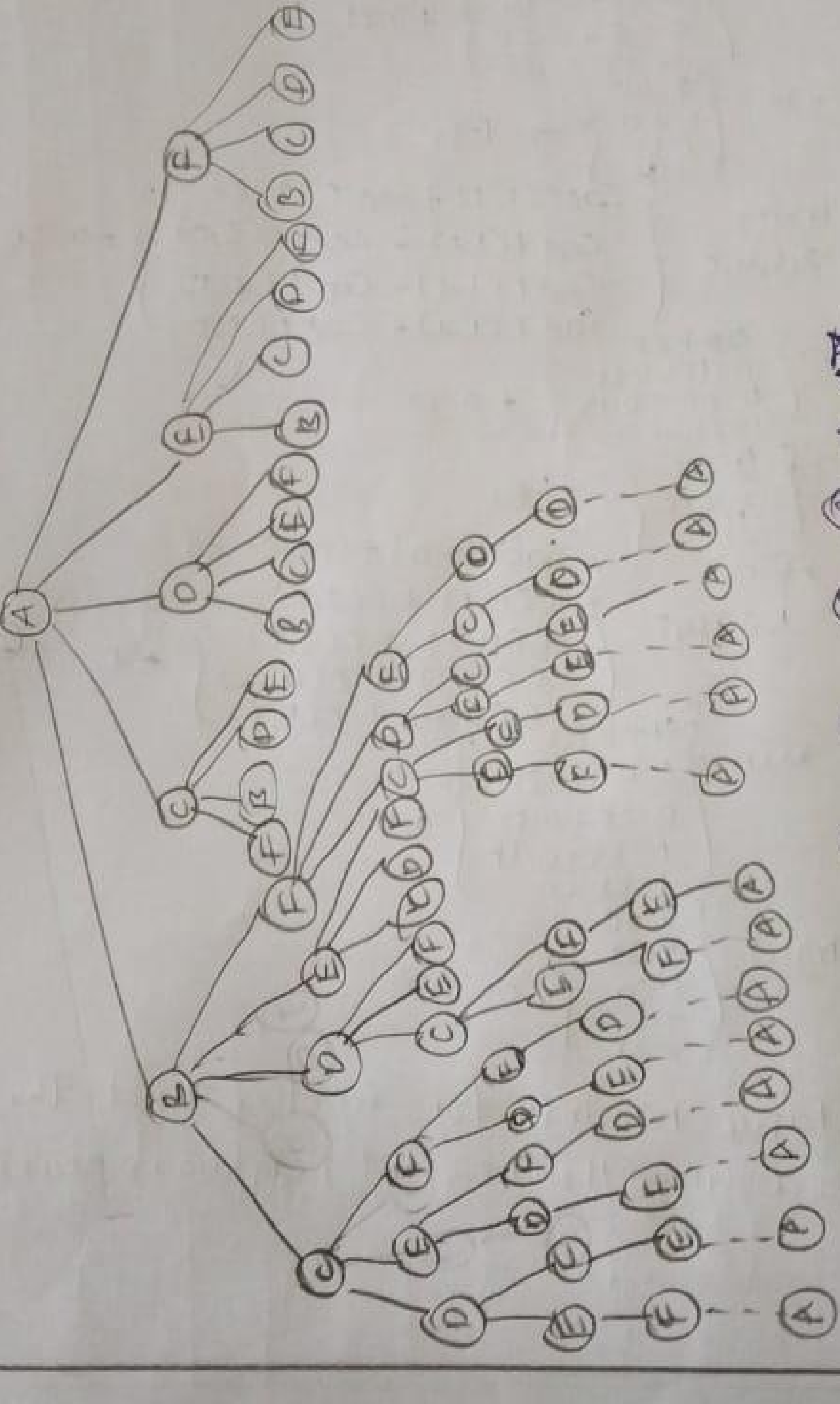
$$\Rightarrow \min \begin{Bmatrix} 0 + 1.80 \\ 0.05 + 1.35 \\ 0.25 + 0.55 \\ 0.85 + 0.15 \\ 1.35 + 0 \end{Bmatrix} + 1$$

$$\Rightarrow \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} \Rightarrow 1.80$$

10) Given a distance matrix for 6 cities, find the Shortest Path using the nearest neighbour heuristic

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline A & 0, & 10, & 8, & 9, & 7 & 5 \\ B & 10 & 0 & 10 & 5 & 6 & 9 \\ C & 8 & 10 & 0 & 8 & 9 & 5 \\ D & 9 & 5 & 8 & 0 & 6 & 5 \\ E & 7 & 6 & 9 & 6 & 0 & 8 \\ F & 5 & 9 & 7 & 5 & 8 & 0 \end{array}$$





A — F — E — D — C — B — A = 47

11) Solve the fractional knapsack problem for a knapsack with a capacity of 60 units and the following items:

Item1: W=20, V=100

Item2: W=30, V=120

Item3: W=10, V=60

~~Item~~ calculate the max value that can be achieved and describe the fractions of items taken

(A)

Item	weight	value(P)
1,	20	100
2.	30	120
3.	10	60

W=60 units

v\w	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Formula :-

$V[i, W] = \max\{V[i-1, W], V[i-1, w-w[i]] + value[j]\}$

12) consider a directed graph with 5 vertices $V_1, V_2, V_3, V_4, V_5 V_1, V_2, V_3, V_4, V_5 V_1, V_2, V_3, V_4, V_5$ and the following Edges with weights.

$V_1 \rightarrow V_2 V_1$ \rightarrow $V_2 V_1 \rightarrow V_2$ with W-3

$V_1 \rightarrow V_3 V_1$ W-8

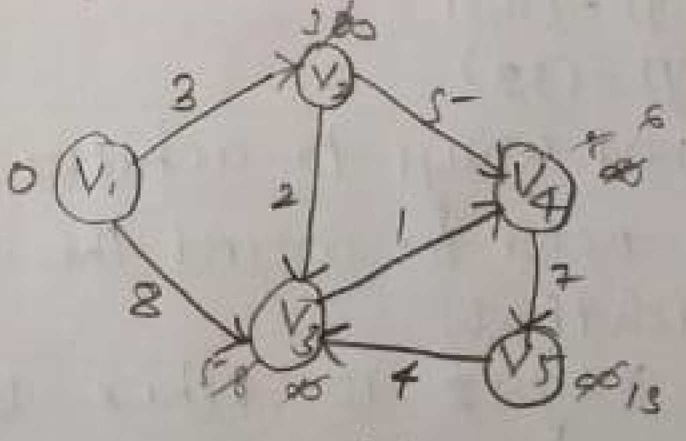
$V_2 \rightarrow V_3 V_2$ W-2

$V_2 \rightarrow V_4 V_2$ W-5

$V_3 \rightarrow V_4 V_3$ W-1

$V_4 \rightarrow V_5 V_4$ W-7

$V_5 \rightarrow V_3 V_5$ W-4



Apply Bellman ford algorithm.

Initialize

V	V_1	V_2	V_3	V_4	V_5
d	0	∞	∞	∞	∞
P	-	-	-	-	-

①

V	V_1	V_2	V_3	V_4	V_5
d	0	3	8	∞	∞
P	-	V_1	V_1	V_2	V_4

$V_1 \rightarrow V_2$ -3

$V_1 \rightarrow V_3$ -8

$V_2 \rightarrow V_3$ -2

$V_2 \rightarrow V_4$ -5

$V_3 \rightarrow V_4$ 1

$V_4 \rightarrow V_5$ 7

$V_5 \rightarrow V_3$ 4

②

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	8	∞
p	-	V_1	V_2	V_2	V_4

③

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	15
p	-	V_1	V_2	V_3	V_4

④

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
P	-	V_1	V_2	V_3	V_4

Path	Shortest distance	Shortest Path
V_1-V_2	3	V_1-V_2
V_1-V_3	5	$V_1-V_2-V_3$
V_1-V_4	6	$V_1-V_2-V_3-V_4$
V_1-V_5	13	$V_1-V_2-V_3-V_4-V_5$

13) Given two eight-sided dice, compute the num of ways to achieve a sum of 10. Then, extend this to three dice and find the new no.of ways to get the same sum.

(A) We need to count the pairs (x,y) such that $x+y=10$ where $1 \leq x, y \leq 8$.

Possible pairs -

$(x,y) = (2,8)$
$(x,y) = (3,7)$
$(x,y) = (4,6)$
$(x,y) = (5,5)$
$(x,y) = (6,4)$
$(x,y) = (7,3)$
$(x,y) = (3,2)$

No.of ways to achieve a sum of 10 is : 7.

We need to count the no.of triples (x,y,z). Such that,

$$x+y+z=10 \text{ where } 1 \leq x, y, z \leq 8$$

1. $x=1$

$y+z=9$

$(1,1,8)$, $(1,2,7)$, $(1,3,6)$ $(1,4,5)$, $(1,5,4)$, $(1,6,3)$, $(1,7,2)$, $(1,8,1)$

2. $x=2$:

$y+z=8$

$(2,1,7)$, $(2,2,6)$, $(2,3,5)$, $(2,4,4)$, $(2,5,3)$, $(2,6,2)$, $(2,7,1)$

3. x=3:

y+z=7:

(3,1,6), (3,2,5), (3,3,4), (3,4,3), (3,5,2), (3,6,1)

x=4:

4. y+z=6:

(4,1,5), (4,2,4), (4,3,3), (4,4,2), (4,5,1)

x=5:

5. y+z=5:

(5,1,4), (5,2,3), (5,3,2), (5,4,1)

x=6:

6. y+z=4:

(6,1,3), (6,2,2), (6,3,1)

7. x=7:

y+z=3

(7,1,2), (7,2,1)

8. x=8

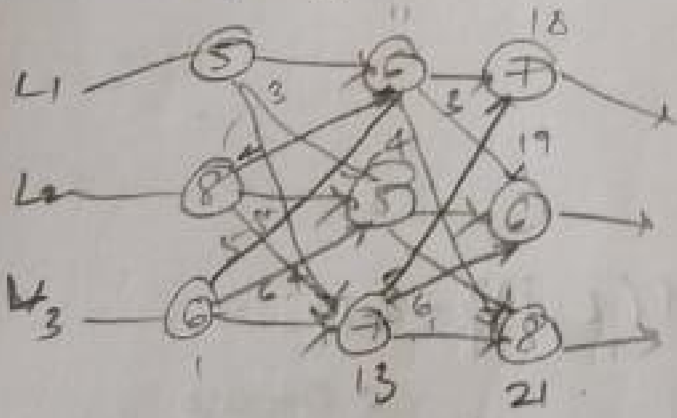
y+z=2 (8,1,1)

Sum = 8+7+6+5+4+3+2+1=36

So, the no of ways to a sum of 10 = 36.

14) Given station time for Line 1: [5,6,9], Line 2: [3,5,6], and Line 3: [6,7,8] and transfer times b/w lines [3,4], [4,5], & [5,6].

(A) Given that,



$F_1[j]$	5	11	18
$F_2[j]$	8	13	19
$F_3[j]$	6	13	21

$L_1[j]$	1←	1←	1←
$L_2[j]$	2	2	2
$L_3[j]$	3	3	3

$$F_1[j] = \text{Min}\{(f_1(j-1)+a_{1}j), (f_2(j-1)+(t_{2},j-1)+a_{1}j), f_3(j-1)+(t_3, j-1)+a_{1}j)\}$$

$$= \text{Min}\{11, 18, 17\}$$

(15) Given key {5, 15, 25, 35, 45, 55} with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1}; use dp to find the OBST. Show the steps of your calculation & the resulting cost

Given that

$\{\overset{1}{5}, \overset{2}{15}, \overset{3}{25}, \overset{4}{35}, \overset{5}{45}, \overset{6}{55}\}$

{0.1, 0.05, 0.2, 0.25, 0.3, 0.1}.

	0	1	2	3	4	5	6
0	0	0.1	0.2 (1)	0.55 [1]	1.05 [3]	1.75 (4)	2.05 (4)
1		0	0.05	0.3 [3]	0.8 [34]	1.4 [4]	1.7 (4)
2			0	0.2	0.65 [4]	1.25 (4)	1.55 (4)
3				0	0.25	0.8 [5]	1.35 (5)
4					0	0.3	0.5 [5]
5						0	0.1
6							0

j-1=1 t

1-0=1 (0,1)

2-1=1 (1,2)

3-2=1 (2,3)

4-3=1 (3,4)

5-4=1 (4,5)

6-5=1 (5,6)

j-1=2

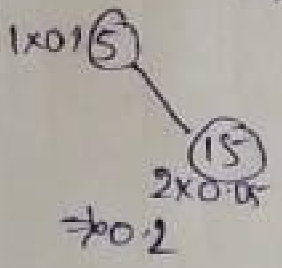
2-0=2 (0,2) (1,2)

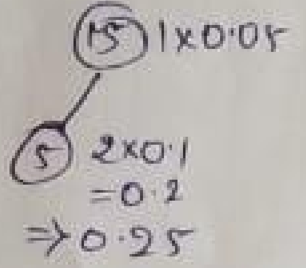
3-1=2 (1,3) (2,3)

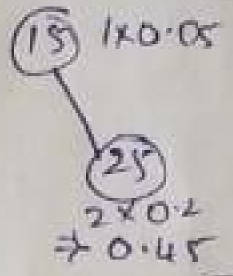
4-2=2 (2,4) (3,4)

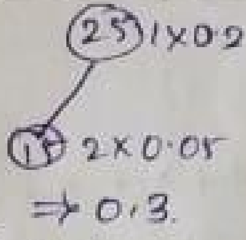
5-3=2 (3,4) (4,5)

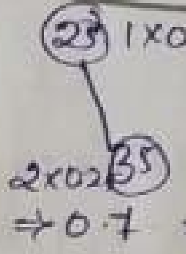
6-4=2 (4,6) (5,6)

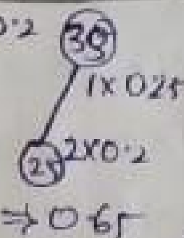


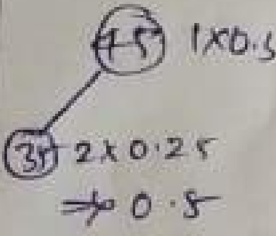


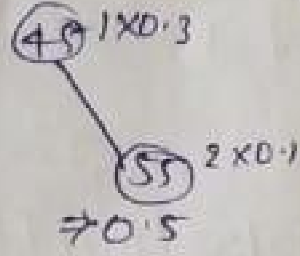


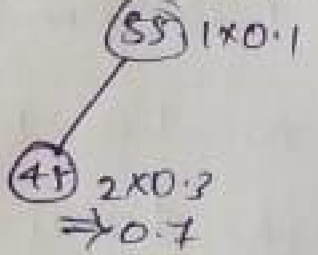












j-i=3

3-0=3 (0,3) (1,3)

4-1=3 (1,4) (2,4)

5-2=3 (2,5) (3,5)

6-3=3 (3,6) (4,6)

cost(i,j) = min {cost(i,k-1) + cost(k,j)} + wi

$$\text{cost}(0,3) = \min_{k=1,2,3} \left\{ \begin{array}{l} C(0,0)+C(1,3) \\ C(0,1)+C(2,3) \\ C(0,3)+C(3,3 \end{array} \right\} + 0.35 \Rightarrow \left\{ \begin{array}{c} 0+0.3 \\ 0.1+0.65 \\ 0.2+0 \end{array} \right\} + 0.35$$

$$\rightarrow \left\{ \begin{array}{c} 0.65 \\ 1.1 \\ 0.55 \end{array} \right\} \Rightarrow 0.55$$

$$\text{cost}(1,4) = \min_{k=2,3,4} \left\{ \begin{array}{l} \text{cost}(1,1)+C(2,4) \\ C(1,2)+C(3,4) \\ C(1,3)+C(4,4 \end{array} \right\} + 0.5 \Rightarrow \left\{ \begin{array}{c} 0+0.65 \\ 0.05+0.25 \\ 0.3+0 \end{array} \right\} + 0.5 \Rightarrow \left\{ \begin{array}{c} 1.15 \\ 0.8 \\ 0.[illegible] \end{array} \right\}$$

$\Rightarrow 0.8$

$$\text{Cost}(2,5) = \min_{k=3,4,5} \begin{cases} C(2,2)+\text{Co}(3,5) \\ C(2,3)+C(4,5) \\ C(2,4)+C(5,5 \end{cases} + 0.75 \Rightarrow \min \begin{cases} 0.08 \\ 0.2+0.3 \\ 0.65+0 \end{cases} + 0.7$$

$$\Rightarrow 1.25$$

$$\text{Cost}(3,6) = \min_{k=4,5,6} \begin{cases} 1.15 \\ 1 \\ 1.45 \end{cases} \Rightarrow 1$$

$j-i = 4$

$4-0=4 \quad (0,4)\ (1,4)$

$5-1=4 \quad (1,5)\ (2,5)$

$6-2=4 \quad (2,6)\ (3,6)$

$$\text{Cost}(0,4) = \min_{k=1,2,3,4} \begin{cases} C(0,0)+C(1,4) \\ C(0,1)+C(2,4) \\ C(0,2)+C(3,4) \\ C(0,3)+C(4,4) \end{cases} + 0.6$$

$$\Rightarrow \begin{cases} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{cases} \Rightarrow 1.05$$

$$\text{Cost}(1,5) = \min_{k=2,3,4,5} \begin{cases} C(1,1)+C(2,5) \\ C(1,2)+C(3,5) \\ C(1,3)+C(4,5) \\ C(1,4)+C(5,5 \end{cases} + 0.8 \Rightarrow \begin{cases} 2.02 \\ 1.65 \\ 1.4 \\ 1.6 \end{cases} \Rightarrow 1.4.$$

$$\text{Cos}(2,6) = \min_{k=3,4,5,6} \begin{cases} C(2,2)+C(3,6) \\ C(2,3)+C(4,6) \\ C(2,4)+C(5,6) \\ C(2,5)+C(6,6) \end{cases} + 0.85 \Rightarrow \begin{cases} 1.85 \\ 1.55 \\ 1.6 \\ 2.1 \end{cases} \Rightarrow 1.55$$

$j-1 = 5$

$5-0=5\ (0,5)\ (1,5)$

$6-1=5\ (1,6)\ (2,6)$

$$\text{Cos}(0,5) = \min_{k=1,2,3,4,5} \begin{cases} C(0,0)+C(1,5) \\ C(0,1)+C(2,5) \\ C(0,2)+C(3,5) \\ C(0,3)+C(4,5) \\ C(0,4)+C(5,5 \end{cases} + 0.9 \Rightarrow \begin{cases} 2.45 \\ 1.95 \\ 1.8 \\ 1.75 \end{cases} + 0$$

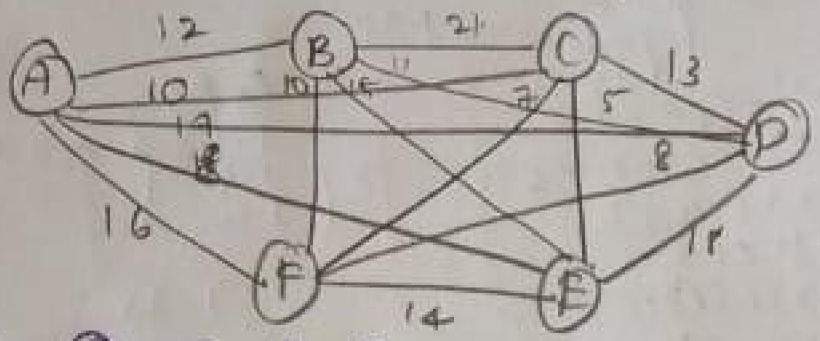
$$\text{Cost}(1,6) = \min_{k=2,3,4,5,6} \begin{cases} C(1,1)+C(2,6) \\ C(1,2)+C(3,6) \\ C(1,3)+C(4,6) \\ C(1,4)+C(5,6) \\ C(1,5)+C(6,6) \end{cases} + 0.9 \Rightarrow \begin{cases} 2.45 \\ 2 \\ 1.7 \\ 1.8 \\ 2.3 \end{cases} \Rightarrow 1.7$$

$$\text{Cost}(0,6) = \min_{k=1,2,3,4,5,6} \begin{cases} C(0,0)+C(1,6) \\ C(0,1)+C(2,6) \\ C(0,2)+C(3,6) \\ C(0,3)+C(4,6) \\ C(0,4)+C(5,6) \\ C(0,5)+C(6,6) \end{cases} + 1$$

$$\Rightarrow \begin{cases} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{cases} \Rightarrow 2.05$$

(16) Extend the following distance Matrix to 7 cities & Solve the TSP

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ A & 0 & 12 & 10 & 19 & 8 & 16 \\ B & 12 & 0 & 21 & 11 & 15 & 10 \\ C & 10 & 21 & 0 & 13 & 5 & 7 \\ D & 19 & 11 & 13 & 0 & 18 & 8 \\ E & 8 & 15 & 5 & 18 & 0 & 14 \\ F & 16 & 10 & 7 & 8 & 14 & 0 \end{array}$$



O/P → Ⓐ → Ⓑ – Ⓒ – Ⓓ – Ⓔ – Ⓕ ⇒ 83%.

(17) Given a knapsack capacity of 70 units and the following items:

I₁ : w=25, v=80
2 35, 90
3 45, 120
4 30, 70.

Use dp to solve 0/1 knapsack problem.

v\w	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

For a graph
A → BA, W-1
A → CA, W-4
B → CB, W-3
B → DB, W-2
B → EB, W-2
D → BD, W-1
D → CD, W-5.
E → DE, W-3
Use Bellman-Ford and solve it

A→B=1
A→C=4
B→C=3
B→D-2
B→E=2
D→B-1
D→C=5
E→D—-3

V	A	B	C	D	E
d	0	∞	∞	∞	∞
p	-	-	-	-	-

V	A	B	C	D	E
d	0	1	4	∞	∞
p	-	A	A	-	-

V	A	B	C	D	E
d	0	-1	4	∞	1
p	-	A	A	-	B

V	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

V	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

Path	distance	Shortest Path
A	0	A
B	-1	A→B
C	4	A→C
D	3	A→E→D
E	1	A→B→C

9. Find the Expected value of the Sum of outcomes when rolling 3 four-sided dice. show your calculation & reasoning

Sum=3(1+1+1)

Sum

$4 = \frac{3}{64}(1+1+2,\ 1+2+1,\ 2+1+1)$

$5 = \frac{6}{64}(1+1+3,\ 1+2+2,\ 1+3+1,\ 2+1+2,\ 2+2+1,\ 3+1+1)$

$6 = \frac{10}{64}\left(\begin{array}{l}1+2+3,\ 1+3+2,\ 2+1+3,\ 2+2+2,\ 2+3+1,\ 3+1+2, \\ 3+2+1,\ 1+4+1,\ 2+2+2,\ 2+3+1\end{array}\right)$

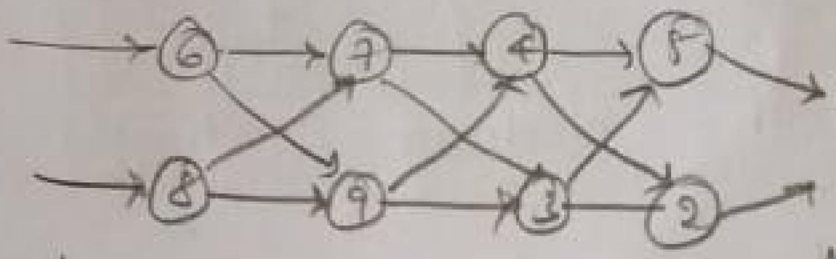
8 = 12/64
9 - 10/64
10 = 6/64
11 = 3/64
12 = 1/64

Σ (sum * probability)

$$\Rightarrow \left(2\times\frac{1}{64}\right)+\left(4\times\frac{3}{64}\right)+\left(6\times\frac{6}{64}\right)+\left(6\times\frac{10}{64}\right)+\left(7\times\frac{12}{64}\right)+\left(8\times\frac{12}{64}\right)$$
$$+\left(9\times\frac{10}{64}\right)+\left(10\times\frac{6}{64}\right)+\left(11\times\frac{3}{64}\right)+\left(12\times\frac{1}{64}\right)$$

$$\Rightarrow \frac{480}{64} = 7.5\%.$$

(20) Calculate the min time for Line 1: [6,7,4,5], Line 2: [8,9,3,2] With transfer lines [4,5,6] 1 to 2 and [6,5,4] 2 to 1



	1	2	3	4
F1[j]	6	13	17	12
F2[j]	8	17	20	22

	1	2	3	4
L1[j]	1	1	1	1
L2[j]	2	2	2	2

(21) Keys {10,20,30} have probabilities {0.2, 0.5, 0.3}
alo OBST

k = {10,20,30}
v = {0.2, 0.5, 0.3}

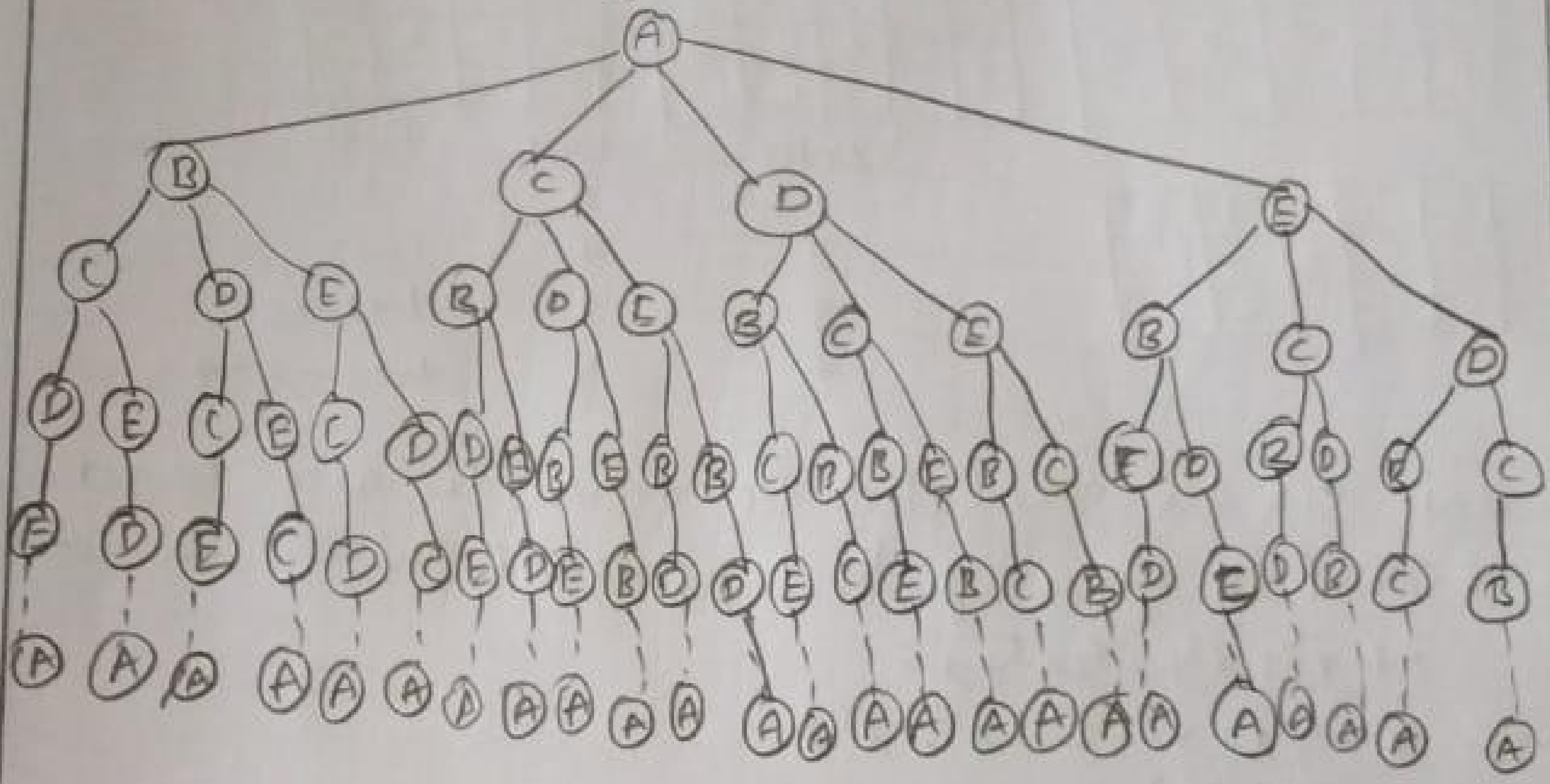
j-1 = 3
3-0 = (9.3)

$$\text{cost}(0,3) = \min\begin{Bmatrix}2.1\\1.5\\1.1\end{Bmatrix}$$

	0	1	2	3
0	0	0.2	0.7	1.1
1		0	0.5	1.1
2			0	0.3
3				0

(22) Using 5 cities

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ A & 0 & 14 & 4 & 10 & 20 \\ B & 14 & 0 & 7 & 8 & 7 \\ C & 4 & 7 & 0 & 12 & 6 \\ D & 10 & 8 & 12 & 0 & 15 \\ E & 20 & 7 & 6 & 15 & 0 \end{array}$$



(23) knapsack 0/150 units

I_1 ⇒ W=10, V=50
2 → 20 70
3 → 30 90
4 → 25 60
5 → 15 40

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	90	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

(24) Bellman-ford

1→21, ω=4
1→31, ω=5
2→32, ω=-2
3→43, ω=3
4→21, ω=-10

1→2-4
1→3→5
2→3 - 3
4→2 =-10

V	1	2	3	4
d	0	∞	∞	∞
p	–	–	–	–

V	1	2	2	4
d	0	4	5	∞
p	–	1	1	–

V	1	2	3	4
d	0	4	2	∞
p	–	1	2	–

V	1	2	3	4
d	0	4	2	5
p	–	1	2	3

Vertex	Dist	Path
1	0	1
2	4	1→2
3	2	1→2→3
4	5	1→2→3→4

(25) Roll six six-sided dice Determine the no of to get a sum of 18, Ensuring that at least one die shows a

$x+x^2+x^3+x^4+x^5+x^6$

$\Rightarrow x(1+x+x^2+x^3+x^4+x^5)$

$\Rightarrow \frac{x(1+x^6)}{1-x}$

For six dice

$\left(\frac{x(1-x^6)}{1-x}\right)^6 \Rightarrow = x^6(1-x^6)^6 \cdot (1-x)^{-6} \Rightarrow (x^{18})$

$= 340$ //

(26) Given line 1 : [10, 12, 11] Line 2 : [9, 14, 13], Transfer lines [6, 5], by 2 units

Before reduction

4	6	5
L2	30	30

After Reduction

4	4	5
L2	28	27

(27) For keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1} Determine OBST using dp

(A) (8, 12, 16, 20, 24)

(0.2, 0.05, 0.4, 0.25, 0.1)

j-i=0
j-1=1
j-i=2
2-0=(0,2)
3-1=(1,3)
4-2=(2,4)
5-3=(3,5)

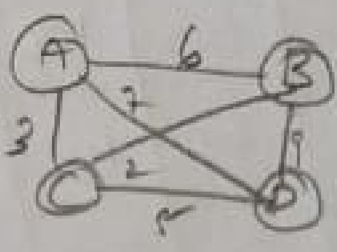
	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

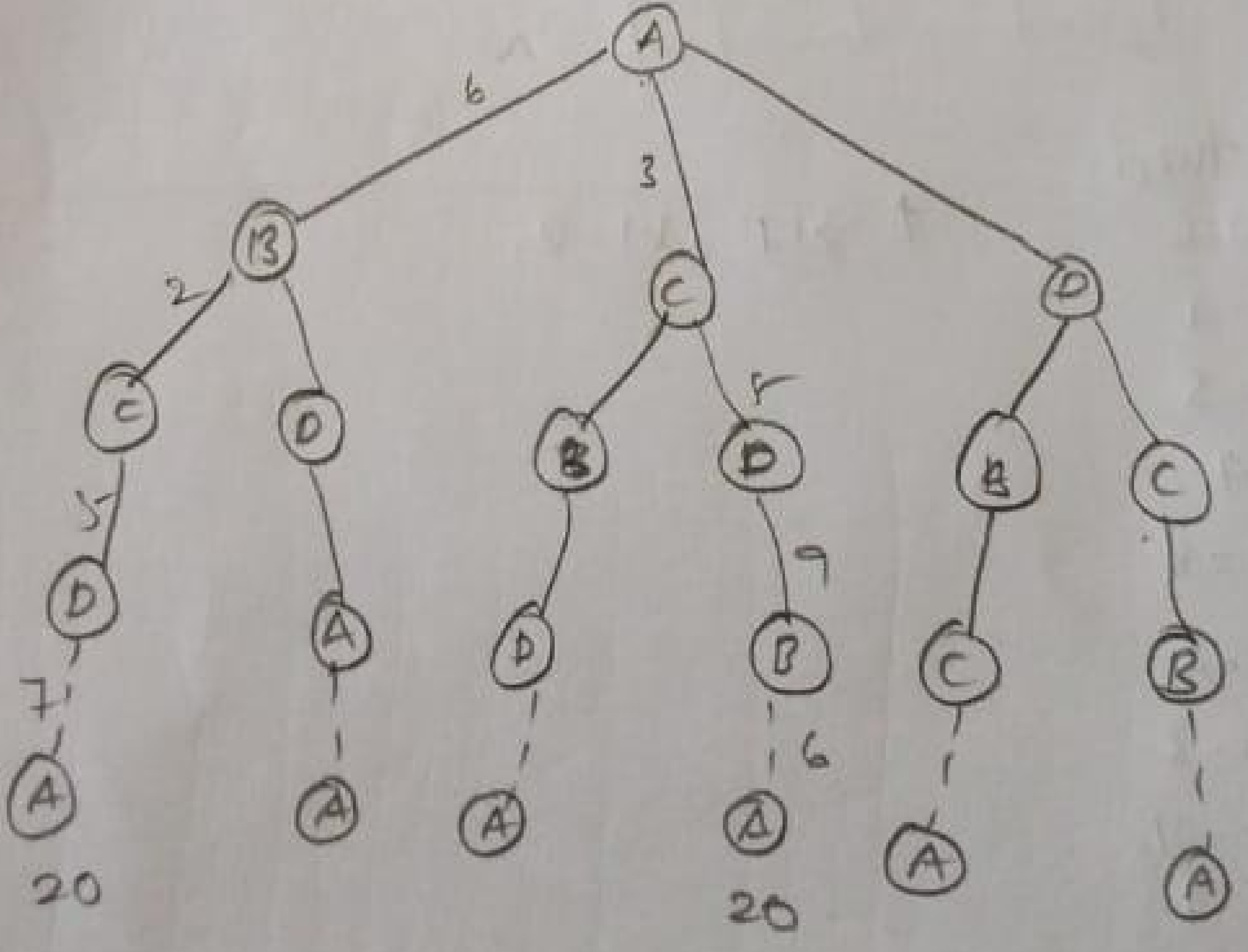
$$\text{Cost } C(1,5) = \min\begin{cases} C(1,1)+C(2,5) \\ C(1,2)+C(3,5) \\ C(1,3)+C(4,5) \\ C(1,4)+C(5,5) \end{cases} + 0.8 \Rightarrow \min\begin{cases} 0+1.2 \\ 0.05+0.45 \\ 0.5+0.1 \\ 1+0 \end{cases} + 0.8$$

$$\Rightarrow \min\begin{cases} 2 \\ 1.3 \\ 1.4 \\ 1.8 \end{cases} \Rightarrow 13$$

(28) Solve TSP 4 cities

A : [0, 6, 3, 7]   C : [3, 2, 0, 5]
B : (6, 0, 2, 9)   D : [7, 9, 5, 0]





Ⓐ—Ⓑ—Ⓒ—Ⓓ—Ⓐ = 20
Ⓐ→Ⓓ—Ⓒ—Ⓑ—Ⓐ→ 20 } Min optimal path

29) Knapsack 0/1 50 units

I_1 $W_1=10$ $V_1=60$

I_2 $W_2=20$ $V_2=100$

I_3 $W_3=30$ $V_3=120$

I_4 $W_4=40$ $V_4=200$

v/w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

30) Bell-man ford.

A → BA, W=6
A → DA, W=7
B → CB, W=5
B → EB, W=4
B → DB, W=8
B → BD, W=-2
D → CD, W=-3
D → ED, W=-9.
E → FE, W=7.
F → CF, W=2.

Intialize

v	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
p	–	–	–	–	-	–

①

v	A	B	C	D	E	F
d	0	6	4	7	2	9
p	–	A	D	A	B	E

②

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

③

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

④

v	A	B	C	D	E	F
d						
p						

⑤

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

vertex	Dist	Path
A	0	A
B	2	A–D–C–B
C	4	A–D–C
D	7	A–D
E	2	A–D–C–B–E
F	9	A–D–C–B–E–F