

a)

The given inputs

$$x_1 = 0.3$$

$$x_2 = 0.6$$

The given weights

$$w_{3,1} = -0.1, \quad w_{4,1} = 0.2.$$

$$w_{3,2} = -0.2, \quad w_{4,2} = 0.3$$

$$w_{3,0} = 0.1, \quad w_{4,0} = 0.1$$

$$w_{5,3} = -0.1, \quad w_{6,3} = 0.1$$

$$w_{5,4} = 0.2, \quad w_{6,4} = -0.2$$

$$w_{5,0} = 0.1 \quad w_{6,0} = 0.1$$

For neuron 3

$$\begin{aligned} z_3 &= (x_1 \times w_{3,1}) + (x_2 \times w_{3,2}) + w_{3,0} \\ &= (0.3 \times -0.1) + (0.6 \times -0.2) + 0.1 \\ &= -0.03 - 0.12 + 0.1 \\ &= -0.05 \end{aligned}$$

Since z_3 is negative the ReLU activation function will output zero.

$$a_3 = \max(0, z_3) = 0.$$

for neuron 4:

$$z_4 = (x_1 \times w_{41}) + (x_2 \times w_{42}) + w_{40}$$

$$z_4 = (0.3 \times 0.2) + (0.6 \times 0.3) + 0.1$$

$$\begin{aligned} z_4 &= 0.06 + 0.18 + 0.1 \\ &= 0.34 \end{aligned}$$

$$a_4 = \max(0, z_4) = 0.34$$

for neuron 5:

$$\begin{aligned} z_5 &= (x_3 \times w_{53}) + (x_4 \times w_{54}) + w_{50} \\ &= (0 \times -0.1) + (0.34 \times 0.2) + 0.1 \\ &= 0 + 0.068 + 0.1 \\ &= 0.168 \end{aligned}$$

$$a_5 = \max(0, z_5) = 0.168$$

for neuron 6:

$$\begin{aligned} z_6 &= (x_3 + w_{63}) + (x_4 \times w_{64}) \\ &\quad + w_{60} \\ &= (0 \times 0.1) + (0.34 \times -0.2) + 0.1 \\ &= 0 - 0.068 + 0.1 \end{aligned}$$

$$z_6 = 0.032$$

$$a_6 = \max(0, z_6) = 0.032$$

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The values for the output
Neurons

$$\text{Neuron 5 : } z_5 = 0.168 \quad f_{a5} = 0.168$$

$$\text{Neuron 6 : } z_6 = 0.032 \quad f_{a6} = 0.032$$



b) The desired outputs are:

$$\text{Neuron 5} : 0.7$$

$$\text{Neuron 6} : 0.4$$

The actual outputs are

$$\text{Neuron 5} : 0.168$$

$$\text{Neuron 6} : 0.032$$

$$\begin{aligned}
 SSE &= \frac{1}{2} (d_5 - a_5)^2 + (d_6 - a_6)^2 \\
 &= \frac{1}{2} (0.7 - 0.168)^2 + (0.4 - 0.032)^2 \\
 &= \frac{1}{2} (0.532)^2 + (0.368)^2 \\
 &= \frac{1}{2} (0.2830 + 0.1354) \\
 &\approx \frac{1}{2} (0.4184) = 0.2092
 \end{aligned}$$

d_5, d_6 - desired outputs

a_5, a_6 - actual outputs

$$\begin{aligned}
 c) \quad \delta_5 &= (d_5 - a_5) \times \text{ReLU}(a_5) \\
 &= (0.7 - 0.168) \times 1 \\
 &= 0.532.
 \end{aligned}$$

The above is the error delta for neuron 5 (δ_5)

$$\begin{aligned}
 \delta_6 &= (d_6 - a_6) \times \text{ReLU}(a_6) \\
 &= (0.4 - 0.032) \times 1 \\
 &= 0.368
 \end{aligned}$$

For the Hidden neurons. (3 & 4)

For neuron 4.

$$\begin{aligned}
 \delta_4 &= (\delta_5 \times w_{54} + \delta_6 \times w_{64}) \\
 &\quad \times \text{ReLU}(a_4) \\
 &= (0.532 \times 0.2 + 0.368 \times -0.2) \times 1 \\
 &= (0.1064 - 0.0736) \\
 &= 0.0328
 \end{aligned}$$

$$\delta_5 = 0.532.$$

$$\delta_6 = 0.368$$

$\delta_3 = 0$ (since neuron 3's output is 0 its contribution to the error is 0)

$$\delta_4 = 0.0328$$

$$\text{d) } \frac{de}{dw_{6,4}} = \varphi_6 \cdot a_4 = 0.368 \times 0.34 \\ \cong 0.12512$$

$$\frac{de}{dw_{6,3}} = \varphi_6 \cdot a_3 = 0.368 \times 0 \\ = 0.$$

$$\frac{de}{dw_{6,0}} = \varphi_6 = 0.368$$

$$\frac{de}{dw_{5,4}} = \varphi_5 \times a_4 = 0.532 \times 0.34 \\ \cong 0.18088$$

$$\frac{dE}{dw_{5,3}} = \delta_5 \times a_3 = 0.532 \times 0 \\ = 0.$$

$$\frac{dE}{dw_{5,0}} = \delta_5 = 0.532.$$

$$\frac{dE}{dw_{4,2}} = \delta_4 \times a_2 = 0.0328 \times 0.6 \\ = 0.01968.$$

$$\frac{dt}{dw_{4,1}} = \delta_4 \times a_1 = 0.0328 \times 0.3 \\ = 0.00984$$

$$\frac{dt}{dw_{4,0}} = \delta_4 = 0.0328$$

$$\frac{dt}{dw_{3,2}} = \delta_3 \times a_2 = 0 \times 0.6 = 0$$

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$$\frac{d\epsilon}{d\omega_{3,1}} = \alpha_3 \times a_1 = 0 \times 0.3$$

$$= 0.$$

~~$$\frac{d\epsilon}{d\omega_{3,0}} = \alpha_3 = 0.$$~~

e. Learning rate $\alpha = 0.1$

Weights.

$$w_{\text{new}} = w_{\text{old}} - \alpha \times \frac{d e}{d w}$$

bias.

$$b_{\text{new}} = b_{\text{old}} - \alpha \times \delta.$$

~~w_{6,4} new~~

$$w_{6,4 \text{ new}} = w_{6,4} - \frac{\alpha d e}{d w_{6,4}}$$

$$\Rightarrow \cancel{0.2 \times 0.12512}$$

$$= -0.2 - 0.1 \times 0.12512.$$

$$\approx -0.212512.$$

$$w_{6,3 \text{ new}} = \cancel{w_{6,3} \times d}$$

$$= w_{6,3} - \frac{\alpha d e}{d w_{6,3}}$$

$$= 0.1 - 0.1 \times 0.1 = 0.1$$

e. Learning rate $\alpha = 0.1$

Weights.

$$w_{\text{new}} = w_{\text{old}} - \alpha \frac{\partial E}{\partial w}$$

bias.

$$b_{\text{new}} = b_{\text{old}} - \alpha \times \delta_b$$

~~w_{6,4}, new~~

$$\begin{aligned} w_{6,4 \text{ new}} &= w_{6,4} - \frac{\partial E}{\partial w_{6,4}} \\ &= -0.2 - 0.1 \times 0.12512 \\ &\approx -0.212512. \end{aligned}$$

$$\begin{aligned} w_{6,3 \text{ new}} &= w_{6,3} - \frac{\partial E}{\partial w_{6,3}} \\ &= 0.1 - 0.1 \times 0.1 = 0.1 \end{aligned}$$

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$$w_{6,0 \text{ new}} = w_{6,0} - \alpha \frac{de}{dw_{6,0}}$$

$$\begin{aligned} &= 0.1 - 0.1 \times (0.368) \\ &= 0.1 - 0.0368 \\ &= 0.0632 \end{aligned}$$

$$\begin{aligned} w_{5,4 \text{ new}} &= w_{5,4} - \alpha \frac{de}{dw_{5,4}} \\ &= 0.2 - 0.1 \times (0.18088) \\ &= 0.2 - 0.018088 \\ &= 0.181912 \end{aligned}$$

$$\begin{aligned} w_{5,3 \text{ new}} &= w_{5,3} - \alpha \frac{de}{dw_{5,3}} \\ &= -0.1 - 0.1 \times (0) \\ &= -0.1 \end{aligned}$$

$$w_{5,0} \text{ new} = w_{5,0} - \alpha \frac{de}{dw_{5,0}}$$

$$\begin{aligned} &= 0.1 - 0.1 \times (0.532) \\ &= 0.1 - 0.0532 \\ &= 0.0468 \end{aligned}$$

$$w_{4,2} \text{ new} = w_{4,2} - \alpha \frac{de}{dw_{4,2}}$$

$$\begin{aligned} &= 0.3 - 0.1 \times (0.01968) \\ &= 0.3 - 0.001968 \\ &= 0.298032 \end{aligned}$$

$$w_{4,1} \text{ new} = w_{4,1} - \alpha \frac{de}{dw_{4,1}}$$

$$= 0.2 - 0.1 (0.00984)$$

$$\begin{aligned} &= 0.2 - 0.000984 \\ &= 0.199016 \end{aligned}$$

$$w_{4,0} \text{ new} = w_{4,0} - \alpha \frac{\partial E}{\partial w_{4,0}}$$

$$= 0.1 - 0.1(0.0328)$$

$$= 0.1 - 0.00328$$

$$= 0.09672$$

$$w_{3,2} \text{ new} = w_{3,2} - \alpha \frac{\partial E}{\partial w_{3,2}}$$

$$= -0.2 - 0.1(0)$$

$$= -0.2$$

$$w_{3,1} \text{ new} = w_{3,1} - \alpha \frac{\partial E}{\partial w_{3,1}}$$

$$= -0.1 - 0.1(0)$$

$$= -0.1$$

$$w_{3,0} \text{ new} = w_{3,0} - \alpha \frac{\partial e}{\partial w_{3,0}}$$

$$= 0.1 - 0.1 \times (0)$$

$$= 0.1 - 0 = 0.1$$

f. For Neuron 3.

Since there is no update
in weights of w_{31} & w_{32} .

$$z_3 = -0.05$$

$$a_3 = \max(0, -0.05) = 0$$

For neuron 4.

$$z_4 = (x_1 \times w_{41\text{new}}) + (x_2 \times w_{42\text{new}})$$

$$z_4 = (0.3 \times 0.0199016) + (0.6 \times 0.298032)$$

$$+ 0.1$$

$$= 0.057048 + 0.1788192 + 0.1$$

$$= 0.338524$$

$$a_4 = \max(0, 0.338524) =$$

$$= 0.338524$$

For neuron 5:

$$\begin{aligned}
 z_5 &= (x_3 \times w_{53}) + (x_4 \times w_{54}) + w_{510} \\
 &= (0 \times -0.1) + (0.338524 \times \\
 &\quad 0.181912) \\
 &\quad + 0.0468
 \end{aligned}$$

$$\begin{aligned}
 z_5 &= 0 + 0.061572 + 0.0468 \\
 &\approx 0.108372.
 \end{aligned}$$

$$\begin{aligned}
 a_5 &= \max(0, 0.108372) \\
 &= 0.108372
 \end{aligned}$$

For neuron 6.

Since $a_3 = 0$, it does not contribute to z_6

$$\begin{aligned}
 z_6 &= (0 \times 0.1) + (0.338524 \times \\
 &\quad -0.212512) \\
 &\quad + 0.0632.
 \end{aligned}$$

$$\begin{aligned}
 z_6 &= 0 - 0.071928 + 0.0632 \\
 &\approx -0.008728
 \end{aligned}$$

$$a_6 = \max(0, -0.008728) = 0$$

g. The new SSE

$$\text{SSE} = \frac{1}{2} (d_5 - a_5)^2 + (d_6 - a_6)^2$$

$$= \frac{1}{2} (0.7 - 0.108372)^2 + (0.4 - 0)^2$$

$$= \frac{1}{2} [(0.59161)^2 + (0.4)^2]$$

$$= \frac{1}{2} [0.35001 + 0.16]$$

$$= \frac{1}{2} [0.5100]$$

$$= 0.255$$

w. Original SSE = 0.2092

New SSE = 0.255

$$\text{Error} = 0.255 - 0.2092 \\ = -0.0458$$

Therefore there is an increase
in error of 0.0458