Sauer-Lemma-Verification

Verifying Sauer's Lemma for a Convex hypothesis set

Convex Sets

The hypothesis set considered here is a convex set. This set contains all hypothesis in 2D space h:R^2 -> {-1, +1}, that are positive inside some convex set and negative elsew here.

To create a convex set N points are randomly chosen along the perimeter of a random circle. We consider any dichotomy of these points, assigning arbitrary pattern of +/-1's.

Theory

For a convex set the grow th function, i.e. m(N) = 2^N. Grow th function only has to do w ith the upper bound of m(N) for an hypothesis set, for a convex set w e can easily create this upper bound by taking the N points along the perimeter of a circle.

Since the m(N) = 2^N, this means that any set of N points can be shattered by our hypothesis set H(convex set). This means that the breakpoint = k = infinity and the VC dimension = d = infinity.

If B(N,k) is the maximum number of dichotomies on N points such that no subset of size k of the N points can be shattered by these dichotomies, then,

$$m_{\mathcal{H}}(N) \leq B(N,k)$$

Now, according to Sauer's lemma,

$$B(N,k) \leq \sum_{i=0}^{k-1} \binom{N}{i}$$

Thus,

$$m_{\mathcal{H}}(N) \leq \sum_{i=0}^{k-1} \binom{N}{i}$$

For the case of convex set, since k -> infinity, w e can take the upper limit of the summation to be N instead of k - 1

Running the code

- Give the follow ing command in the MATLAB command terminal

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sauermain.m
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- Enter the value of N(number of points to be taken)

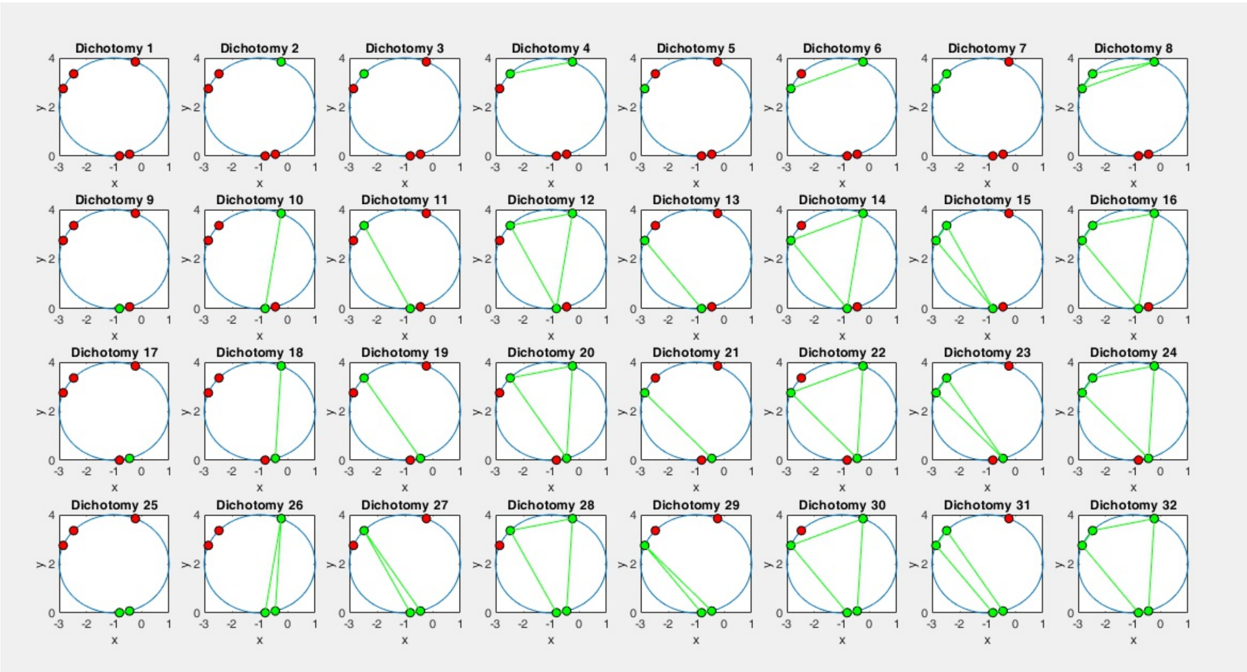
Observation

If we connect the +1 points with a polygon, the hypothesis made up of the closed interior of the polygon(which will always be convex) agrees with the dichotomy of all N points.

- RHS is computed and displayed using,

$$B(N,k) \leq \sum_{i=0}^{k-1} \binom{N}{i}$$

- All possible dichotomies plotted as shown below,



Result

We can see that the total number of creatable dichotomies for any N equals 2^N, i.e. the N points are shattered no matter what N we take.

The RHS of Sauer's lemma when computed for this case, keeping the upper limit to be N(since k = infinity for convex set) comes out to be equal to 2^N for all cases.

Thus LHS <= RHS(LHS = RHS in this case) is true, hence Sauer's lemma is verified for the convex hypothesis set.