Proposal for the Universal Unit System

Takashi Suga, Ph.D.

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Abstract

We define the Universal Unit System based on the combination of fundamental physical constants and propose the Harmonic System as its practical implementation.[2]

1. The Universal Unit System

1.1. Before the Universal Unit System

A unit of measure is "a quantity that is used as the basis for expressing a given quantity and is of the same type as the quantity that is to be expressed". A unit that is used in exchanges between people must be guaranteed to have a constant magnitude within the scope of that exchange. Quantities that can, by consensus, serve as common standards over a broad scope were sought and selected to serve as units. The ultimate example of such a quantity is an entity common to all of the humankind, the Earth itself, which was selected as the foundation for the metric system.

1.2. The next stage?

Of course, we can consider going beyond the framework of the Earth and defining units with concepts for which agreement can be reached within a broader scope. The quantities that then become available to serve as the standards for defining units include the quantities of the fundamental physical constants, quantities such as the speed of light in a vacuum (c_0), the quantum of action ($\hbar$), the Boltzmann constant (k_B), and so on. These quantities are believed to have values that remain constant everywhere in the universe. When trying to construct a coherent unit system, however, it is not possible to use all of the fundamental physical constants in the definitions of units. Therefore, would not we expect the fundamental physical constants that were not used in defining

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[1] In this paper, SI units are combined only with the decimal figures (indicated by a period "." as the radix point), and units of the Universal Unit Systems are combined only with duodecimal figures (indicated by a semicolon";" as the radix point. 'X' expresses ten and 'E' expresses eleven). Both notations may use a comma "," and "_" as the digit group separator. Non-dimensional quantities are mostly expressed using all figures in duodecimal first, with the decimal given parenthetically. There are also cases where the octal radix point is represented by "@".

[2] This paper revises the paper currently retrievable at http://dozenal.com. See also footnote 26. The first Japanese version was released May 1194;(1984.)

units to have fractional magnitudes of unit quantities of the same dimension?

For example, the Rydberg constant (R_∞) is 115,3789;4702X0/ft (10973731.568508/m) [3] and the Bohr radius (a_B) is 0;X8EE6E523×10;$^{-9}$ft (5.2917721067×10.$^{-11.}$ m). Therefore, the relation between these two constants is:

DOZENAL	DECIMAL	
R_∞^{-1} = EE6;06603 a_B	R_∞^{-1} = 1,722.04515 a_B	(1)

If one of these two constants is chosen as a unit quantity, the other constant cannot be expressed as a unit quantity.

By surprising coincidences [4] described in Appendix F and §2.1 of http://dozenal.com, however, if the duodecimal number system is used to express the speed of light in vacuum and the quantum of action as the defining constants such that these constants are strictly multiples of integer powers of twelve of the unit quantities, it is possible to construct a coherent unit system in which not only the constant that was used in the definition but also the Rydberg constant, the Bohr radius, the unified atomic mass unit (u), and half the value of the Planck length ($l_P/2=\sqrt{36.G\hbar/c_0^3}/12.$) can be approximated to about or within an error of 2 per gross ($1^1/_2$%) by a multiple of integer powers of twelve of the unit quantities.

In that case, many other physical constants, including the charge and mass of an electron, the fine structure constant, the molar volume of an ideal gas under standard conditions, the black-body radiation at the ice point, the density and surface tension of water, and others, can be approximated by multiples of integer powers of twelve of the unit quantities. Moreover, by adding the Boltzmann constant and using it in the definition of thermodynamic temperature, the gas constant of an ideal gas can be approximated by a multiple of an integer power of twelve of the unit quantity and the Stefan-Boltzmann constant, and the specific heat of water can be approximated by multiples of integer powers of twelve of the unit quantities with a factor 2 remaining. These conclusions are shown in Table 5.[5]

For putting these coincidences to use, the duodecimal number system is the only choice. It seems that the combination of fundamental physical constants "forces" us to use base twelve.

We define the Universal Unit System as "the unit system that is constructed by using the duodecimal number system and the speed of light in vacuum, the quantum of action, and the

[3] In this paper, plane angle phase factor 2π is often treated as a non-dimensional parameter and omitted in order to simplify the explanation. See §A.2 and 3.2.2 of the paper http://dozenal.com .

[4] To prevent any misunderstanding, let us emphasize that **these are merely coincidences as far as physical science is concerned**.

[5] A more detailed table is retrievable at http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/condensed.xlsx.

Boltzmann constant as the defining constants in such a way that these constants become strict multiples of integer powers of twelve of the unit quantities, and the Rydberg constant, the unified atomic mass unit, the Bohr radius, and half the value of the Planck length can be approximated by multiples of integer powers of twelve of the unit quantities".

1.3. Variation of the Universal Unit Systems

To define three units for time, length, and mass, the Universal Unit System uses the speed of light in vacuum and the quantum of action. Another constant is necessary to define these three units. Therefore, the Universal Unit System has some variations in the constant that the system chooses as the last definition constant.

Universal Unit System with constant *A* is the Universal Unit System that uses constant *A* as the last definition constant and whose unit quantity of the last dimension is equal to constant *A* or its multiples of integer powers of twelve. In particular, the Universal Unit System with the Rydberg constant whose length unit is 10;^{6} / R_∞ (12.^{6} / R_∞) and velocity unit is 10;$^{-8}$ c_0 (12.$^{-8}$ c_0) is called the Universal System of Units Standard[6] corresponding to the International System of Units Standard (SI). We will use a symbol corresponding to the SI unit symbol suffixed with 'u' as a new symbol required by the Universal System of Units Standard; 'u' is the '**u**niversal' system suffix. The noun form is 'univer'. For example, the length unit is m_u and is called the 'universal meter' or simply the 'univer'[7], and the time unit is s_u and is called the 'universal second'[8]. **This unit system is comprised of six quartets.** The units of this system are listed in the 5th column of Table 4, and physical, material, and astronomical constants expressed using this system are presented in the 3rd column of Table 5. The ratio of the time unit s_u and the SI second is 0;4824707(0.3902675).

The Universal Unit System with the Bohr radius whose length unit is 10;^{9} a_B (12.^{9} a_B) and velocity unit is 10;$^{-8}$ c_0 (12.$^{-8}$ c_0) can be defined in the same way. Its time unit is 1;005E857 s_u(0.3916171 s). Roughly speaking, **if a time constant or its multiples of integer powers of twelve falls within the range between 1; s_u (0.3902675 s) and 1;005E857 s_u (0.3916171 s), we can construct the Universal Unit System using the constant as a time unit.**

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6 The Universal System of Units Standard is strictly defined in §3 of the paper http://dozenal.com.

7 The noun form of the system suffix is considered to be the abbreviation of the length unit like the length unit of the metric system is 'meter'.

8 Note that there is no vagueness at all even if the suffix is omitted if the notation of footnote 1 is adopted. Suffixes are necessary to identify plural Universal Unit Systems mutually. 4×10.^{7} m, 4×10;^{7} m_u, and the Earth's meridian length are nearly equal. 10.^{5}s, 10;^{5} s_u, and 1⅛ days are nearly equal.

If half the value of the Planck length is used instead of the Bohr radius, the time constant becomes 1;0223 s_u (0.3962 s). This is out of range. To keep the time constant within the range, we should use $\sqrt{\mathbf{35.}G\hbar/c_0^3}$ instead of $\sqrt{\mathbf{36.}G\hbar/c_0^3}$. Then, the time constant becomes 1;0018 s_u (0.3906 s). We call this system the Gravitic System. The Gravitic System can be interpreted as a unit system that 35.G, c_0, $\hbar$, K_B and Z_P are strict multiples of integer powers[9] of twelve of the unit quantities. However, it is not practical because G's measurement accuracy is insufficient.

2. The GCD Unit System

2.1. Basic Concept

The length of the tropical year is strictly 265;2XX6 days in a certain year at the end of the 20th century. For human activities on Earth, year and day cannot be ignored as calendar time units. However, the ratio of year and day is not simple. **Therefore, any calendar time unit system must be a mixed radix system.** The ratio of one tropical year and one day is:

DOZENAL

$$\frac{\text{year}}{\text{day}} = 265;+\frac{27;}{X8;} = 1;003628 \times 264;6 = 1;003628 \times \frac{3^6}{2}$$

DECIMAL (2)

$$\frac{\text{year}}{\text{day}} = 365.+\frac{31.}{128.} = 1.002036..\times 364.5 = 1.002036..\times \frac{3^6}{2}$$

Because one year consists of twelve months, it is reasonable to adopt twelve as one of the radixes. Though this ratio contains the extra factor 3 six times,[10] by multiplying by factor $2^6(= 8 \times 8)$twice, we can cancel factor 3 and obtain powers of twelve$(= 3 \times 2 \times 2)$:

DOZENAL

$$\frac{2^6\text{year}}{2^{-6}\text{half day}} = 2^6 \times 1;003628 \times \frac{3^6}{2} \times 2^7 = 1;003628 \times 10;^6$$

DECIMAL (3)

$$\frac{2^6\text{year}}{2^{-6}\text{half day}} = 2^6 \times 1.002036..\times \frac{3^6}{2} \times 2^7 = 1.002036..\times 12.^6$$

If we define 'octal century' as 2^6 years, and define 'clock' as 2^{-6} of a half day, the relation between these two units is:

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[9] If these integers are all set to 0, various constants are approximately expressed as follows: https://en.wikipedia.org/wiki/Talk:Planck_units/Archive_3#Other_possible_normalizations .

[10] Yang Xiong (45;(53.) BCE–16;(18.)) used the ratio $3^6/2$ in his Tai Xuan calendar (太玄暦).

See Hideki Kawahara "Chinese Scientific Thought (中国の科学思想)" 11X4;(1996.) [ISBN: 978-4423194126] and Wikipedia http://en.wikipedia.org/wiki/Yang_Xiong_(author).

octal century = 1;003628×10;^{6} clocks (4)

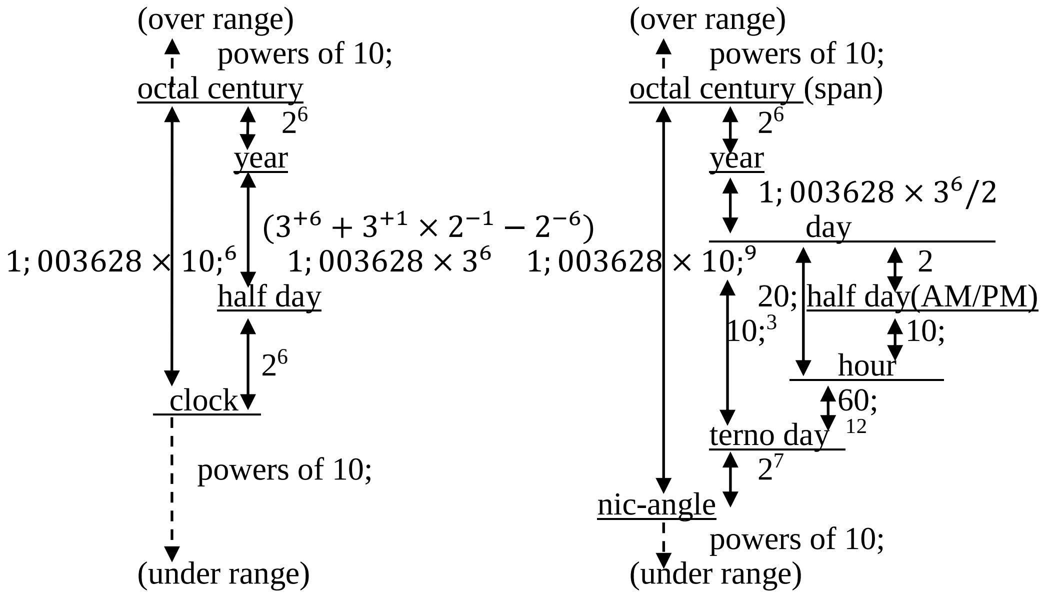
There are some interesting coincidences. Also, please see Table 1.

- One clock is the difference between a tropical year and a Julian year.
- One clock is the greatest common divisor (GCD) of the length of a day and a tropical year.
- Two octal centuries are the least common multiple (LCM) of the length of a day and a tropical year, and so leap year rules become simpler than those of the Gregorian calendar.
- The geometric mean of one octal century and one clock is approximately one fortnight.

Table 1 Coincidences of rotation and revolution of Earth and other planets

Quantity A	Quantity B	Type	Common Quantity
day	Julian year	GCD	1/4 day
		LCM	4 years
	tropical year	GCD	1/X8;(2^{-7}) day
		LCM	X8;(2^{7}) years
year	rotation of Venus	LCM	2 years and -3 rotations
	revolution of Venus		8 years and 11;(13.) revolutions
	revolution of Mars		28;(2^{5}) years and 15;(17.) revolutions

It seems that the combination of the rotation and revolution of the Earth "forces" us to use base two in the middle of the calendar time range. People have a proclivity for using large units for large quantities and using small units for small quantities, so we can cover a wide calendar time range by a duodecimal number system using the following hierarchy shown in Figure 1 (a) [11].



(a) theoretical **(b) everyday use**

Figure 1 The GCD Unit hierarchy

[11] A more detailed figure is retrievable at http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/TimeScale.pdf.

[12] See Appendix C for 'terno'. A terno day is equal to H. C. Churchman's 'moment'. Please see http://www.dozenal.org/archive/DuodecimalBulletinIssue112-web.pdf .

We define 'the GCD Unit' as "the GCD of the length of a day and a tropical year, or its multiples of integer powers of twelve", and 'the GCD Unit System' as "the 2:10;(2:12.) mixed-radix calendar time system using a day, a tropical year, and the GCD Unit". **It is very natural to adopt a unit of calendar time that is an integer division of both a year and a day.**

2.2. Everyday use

Calendar time units are the most conservative units. Considering the easiness of the shift from the present 10;:50;(12.:60.) mixed-radix system to the future 2:10;(2:12.) mixed-radix system, there are some variations of connection points of the binary number system and duodecimal number system[13]. A variation designed for everyday use is shown in Figure 1 (b) and Table 8.

$$1 \text{ day} = 10;^3 \text{ terno days} = 10;^3\times 2^7 \text{ nic-angles} \tag{5}$$

This is a variation that is designed to maximize the range expressed by multiples of integer powers of twelve of a day. This permits 'half day(AM/PM)' and 'hour' to be used for clock notations. That is, the following four clock notations are all available. The notation 'hour' may fade out.

Table 2 Clock Notations

clock notation	format	2:10;(2:12.) system	10;:50;(12.:60.) system
10;-hour clock notation	H:TT'NN;nn…(AM/PM)	3:16'28;0 PM	3:15:12.5 PM
20;-hour clock notation	HH:TT'NN;nn…	13:16'28;0	15:15:12.5
1000;-terno clock notation	UTT'NN;nn…	776'28;0	Not used
10;-unino clock notation	TT(to[-]/after[+])U	46; terno days to the 8th unino day	15 minutes after 3 PM

Following the Titius-Bode law, the orbital semi-major axis of planets can be approximated by $(3\times 2^N+4)$ light solar terno days (see also Table 8), where N = -∞,0,1,2,4,5,6 (Mercury, .., Uranus). Ratio 2^7:1 is the same as the ratio of the U.S. liquid gallon and fl oz. The ratio 2^7:$10;^2$ (8:9) corresponds to the major tone of the just intonation.

3. The Harmonic System

The GCD Unit is derived from the combination of the rotation and revolution of the Earth without using the Universal Unit System concept. However, we encounter the final surprising coincidences. 1; nic-angle is equal to a day/($10;^3\times 2^7$). It is equivalent to 1;0016EE1 s_u (0.3906250 s). Therefore the equivalent value of 1; nic-angle falls within the range between 1; s_u (0.3902675 s) and 1;005E857 s_u (0.3916171 s). This value also almost agrees with the time constant of the Gravitic System 1;0018 s_u (0.3906 s). We can construct the Universal Unit System with the GCD Unit! [14] The Dozenal Society of America seems to recommend 'dour'(day/10;) and 'moment'(day/$10;^3$). Of

[13] See detailed discussion in http://z13.invisionfree.com/DozensOnline/index.php?showtopic=371.

[14] See the sheet 'Clock' in http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/condensed.xlsx.

course, these units are all available, but because the calendar time unit system includes ‘year’ by all means, it does not become a pure duodecimal system even if we choose 1/10; [2] of ‘moment’ as a unit. **We can enjoy all the advantages of Table 5 if we choose not 1/10; [2] but $1/2^7$.**

Because of tidal friction, the physical time length of a day is not constant and becomes longer little by little.[15] A length unit can be redefined exactly as 100,1700;/R_∞ so that a physical time unit becomes exactly 1;001700 s_u (0.3906251 s) and approximately 1;0016EE1 s_u (0.3906250 s) similar to the redefinition of SI units to provide better stability and reproducibility. The calendar time unit length will strictly correspond to the physical time length of a day about 20; octal centuries (1400. years) later. We call this redefined system ‘the Harmonic Universal Unit System with the GCD Unit’, ‘the Harmonic Universal Unit System’, or simply ‘the Harmonic System’ and will use the suffix ‘h’ for units of this system. ‘h’ is for ‘**h**uman’ or ‘**h**armonic universal’ system suffix. The noun form is ‘harmon’. For example, the length unit is m_h and is called ‘harmonic universal meter’, ‘harmonic meter’, or simply ‘harmon’. We specifically granted dedicated aliases for the Harmonic System’s length, time, mass and impedance unit. Please see footnote 25 on ‘harmon’, ‘nic’, ‘looloh’ and ‘nohm’.

We notice that the Harmonic System is also a unit system that redefines the Gravitic System which is not practical because G’s measurement accuracy is insufficient. In other words, the Harmonic System can also be said to have given a strict and practical definition to the conceptual Gravitic System.

The units of this system are listed in the 7th column of Table 4 and physical, material, and astronomical constants expressed using this system are presented in the 4th column of Table 5. Please see also Table 6 and Table **7** for details. The approximations shown in Table 5 are remarkable. For putting coincidences described in this paper to use, the duodecimal number system is indispensable. **We hope that the Harmonic System is acceptable for humans on Earth.**

A. The Earth local extension

The Earth local extension, which consists of five units, and three supplementary constants, is designed for local use on Earth. Please see Table 8.

In this scheme, the CGD unit system is treated as part of the Earth local extension. **To distinguish calendar time units from physical time units, we regard the dimension of calendar time units as the plane angle.**[16]

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[15] See Stephenson, F. R.; Morrison, L. V. (April 11X3;(1995.)) “Long-term fluctuations in the Earth's rotation: 700 BC to AD 1990.” retrievable at http://adsabs.harvard.edu/abs/1995RSPTA.351..165S .

[16] See Seaman, Rob (April 11XB;(2003.)). “A Proposal to Upgrade UTC” retrievable at https://web.archive.org/web/20150419125423/http://iraf.noao.edu/~seaman/leap/. It seems that the dimension of

The newly introduced temperature unit °S (degree S) is designed to meet the relationship that 100; °S corresponds to the boiling point of water (99.9839 °C). The quantity expressed by the unit °S is the difference between thermodynamic temperature and 118,2354; K_h (approximately -74.36°C). The interval of unit °S is one **S**uper Kelvin (=1,0000;K_h≑1.210724 K).

The supplementary constant g_E is used to represent any force quantity as a corresponding mass quantity, like 1 ℓℓ g_E (one looloh gee). In the same way, the supplementary constants s_E and m_E are used to represent any physical time and length quantity as a corresponding plane angle quantity.

B. Gravitational constant and gravity field equations

The equations of some categories[17] can be used efficiently if we introduce new constants. At this time, the total solid angle of a sphere[18], Ω_2, and the speed of light in vacuum, c_0, appear in the equations. This is the reason that the speed of light in a vacuum should strictly be multiples of integer powers of the base number of the unit quantities.

When representing the mass of a celestial body using the Universal Unit System, the gravitational radius (half the Schwarzschild radius) is used rather than using mass directly. Because the accuracy of measuring the Newtonian constant of gravitation is poor, representing the mass of a celestial body directly in terms of mass results in poor accuracy, but the gravitational radius can be measured to an accuracy of around ten digits.

If we define a new constant that has the dimension 'force' as 'the Planck force', there is a good chance that the geometrical parts can be separated from the coefficients in the formula[19]. Make the Planck force, $F_P = c_0^4 G^{-1} = \hbar c_0 / l_P^{\,2} = 35.\hbar c_0 / m_G^{\,2}$, then:

gravitational radius, $r_m = \frac{Gm}{c_0^2} = \frac{mc_0^2}{F_P}$ (half the Schwarzschild radius) (6)

gravitational force, $f = F_P \frac{r_m r_{m'}}{r^2} = c_0^2 \frac{r_m m'}{r^2}$ (7)

gravitational acceleration, $g = c_0^2 \frac{r_m}{r^2} = \frac{r_m}{(r/c_0)^2}$ (8)

gravity field equation, $\frac{T_{ik}}{F_P} = \frac{1}{2\Omega_2}\left(R_{ik} - \frac{1}{2} R g_{ik} - \Lambda g_{ik}\right)$ (9)

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the quantity of a day (=calendar time) should be a plane angle rather than physical time. The calendar time is, in a word, the rotation angle of the Earth derived by using the direction of the sun as a coordinate origin.

17 For a case of electromagnetism, see §B of the paper http://dozenal.com. In the electromagnetic field, the natural unit of impedance, Ω_n (=Z_P: the Planck impedance), plays the role of the Planck force in the gravitational field. Ω_n, e, θ_W, and F_P are constants for the four fundamental forces.

18 See electromagnetic units in Appendix E and §3.2.2 of the paper http://dozenal.com.

19 Please note that Eq. (8) and (9) are geometrical and have no mass dimension.

C. Number Counting

Many of the constants introduced in footnote 9 often have orders $8n-1$.[20] Therefore, it is convenient to use the factor U(=$10;^{8}$($12.^{8}$)) to make the units of the Universal Unit System into human scale. The factor U can be regarded as a conversion factor between atomic scale, human scale, and cosmic scale. Since power 8(=2^{3}) is a power of 2, the decimal myriad system[21] has affinity for our system. So, we propose a duodecimal myriad system in Table 3 [22] replacing ten/hundred with dozen/gross. Larger numbers consist of uni(1),di(2),ter(3),tetra(4),penta(5),hexa(6),hepta(7), lli(0),on(+), and reciprocals are expressed by replacing on(+) with no(-).

Table 3 Duodecimal myriad system

decimal	dozenal	read as 'one-'	origin of prefix part	decimal	dozenal	read as 'one-'	origin of prefix part
$12.^{1.}$	10;	dozen	Old Norse	$12.^{-1.}$	$U^{-@1}$	unino	Latin
$12.^{2.}$	100;	gross	Old French	$12.^{-2.}$	$U^{-@2}$	dino	Greek
$12.^{3.}$	1000;	doz gross		$12.^{-3.}$	$U^{-@3}$	terno	Latin
$12.^{4.}$	1,0000;	myriad	Greek	$12.^{-4.}$	$U^{-@4}$	tetrano	Greek
$12.^{5.}$	10,0000;	dozen myriad		$12.^{-5.}$	$U^{-@5}$	pentano	Greek
$12.^{6.}$	100,0000;	gross myriad		$12.^{-6.}$	$U^{-@6}$	hexano	Greek
$12.^{7.}$	1000,0000;	doz gross myriad		$12.^{-7.}$	$U^{-@7}$	heptano	Greek
$12.^{8.}$	U	unillion		$12.^{-8.}$	U^{-1}	unillino	
$12.^{16.}$	U^{2}	dillion		$12.^{-16.}$	U^{-2}	dillino	
$12.^{24.}$	U^{3}	terllion		$12.^{-24.}$	U^{-3}	terllino	
$12.^{32.}$	U^{4}	tetrallion		$12.^{-32.}$	U^{-4}	tetrallino	
$12^{40.}$	U^{5}	pentallion		$12^{-40.}$	U^{-5}	pentallino	
$12.^{48.}$	U^{6}	hexallion		$12.^{-48.}$	U^{-6}	hexallino	
$12.^{56.}$	U^{7}	heptallion		$12.^{-56.}$	U^{-7}	heptallino	
$12.^{64.}$	$U^{10@}$	unillillion		$12.^{-64.}$	$U^{-10@}$	unillillino	
$12.^{128.}$	$U^{20@}$	dillillion		$12.^{-128.}$	$U^{-20@}$	dillillino	
…	…	…		…	…	…	

For example, 1_2345,6789; is read as 'one unillion two doz three gross four doz five myriad six doz seven gross eight doz nine'. The first 'one' should not be omitted. 'dozen' becomes abbreviated 'doz' before 'one' to 'eleven' or 'gross'. The expression 'Unillion to the power of *octal number*' is also used as exponential expression[23]. For example, $1;2\times U^{3@4}$ is read as 'one point two times unillion to the power of three point four'.

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20 See also footnote 33.

21 http://en.wikipedia.org/wiki/-yllion .

22 "@"is the octal radix point.

23 Dr. Issac Asimov made a similar proposal called 'T-formation'. See http://www.isfdb.org/cgi-bin/title.cgi?62431 .

D. Tables

Table 4 Units with special names and symbols[24]

ALL VALUES DECIMAL

	Unit Category	Dimension	The Universal Unit Systems: with the Rydberg constant(u)			Harmonic System (h)		
Coherent	base units that are not natural units	length	m_u	272.102883	mm	m_h or hm[25]	272.352206	mm
		time	s_u	390.267520	ms	s_h or nc	390.625115	ms
		energy	J_u	64.143274	mJ	J_h	64.084555	mJ
		temperature [26]	K_u	58.441061	μK	K_h	58.387561	μK
	base units that are natural units	plane angle	rad	(2/π) arc sin(1)				
		logarithm	neper	log(e)				
		amount of substance	mol_n or N_A^{-1}	mol / 6.022140857×10.[23.]				
		impedance	Ω_n, Z_P or nh	29.9792458 Ω (=1sr/($\varepsilon_0 c_0$) strict [27], is called ‘nohm’)				
	derived units of electromagnetic quantities	charge	C_u	28.896578	mC			
		electric current	A_u	74.043000	mA	A_h	73.975218	mA
		field strength	E_u [18,28]	272.113986	mA/m	E_h	271.616004	mA/m
		flux density	G_u [28]	390.283444	mC/m^2	G_h	389.569207	mC/m^2
	derived units of dynamical quantities	mass	g_u	131.950080	g	g_h or ℓℓ	131.829287	g
		power	W_u	164.357194	mW	W_h	164.056412	mW
		force	N_u	235.731697	mN	N_h	235.300297	mN
		Pressure	P_u	3.183843	Pa	P_h	3.172201	Pa

[24] Please see also http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/units.pdf and http://z13.invisionfree.com/DozensOnline/index.php?showtopic=371&st=6 for details. A web-based unit converter is available at http://hosi.org:8080/cgi-bin/conv.cgi .

[25] ‘harmon(hm)’, ‘nic(nc)’, ‘looloh(ℓℓ)’, and ‘nohm(nh)’ constitutes a quartet. These are the alias for common use.

[26] The unit of thermodynamic temperature has been changed. The new unit is one-1,0000;th of the old unit in the paper http://dozenal.com along with the introduction of the Earth local extension.

[27] If we adopt the elementary charge as one of the definition constants, Ω_u is used in substitution for Ω_n.

[28] The unit symbol E(Ørsted) and G(Gauß) are associated with the units of CGS unit system. In this paper, we adopt the metric unit names named after the scientists' name as it is. However, an alternative proposal to replace them with the names of the goddesses with the same initials have (i.e., Joule→Juno, Watt→Walküre, Newton→Nereide, Pascal→Polymnia, Coulomb→Clio, Ampere→Aoide, Ørsted→Erato, Gauß→Gaea, Kelvin→Korē) also been proposed.

Non-coherent	defining constants	wave number	R_∞	10,973,731.568508/m (is called 'Rydberg')
		velocity	c_0	299,792,458 m/s (defined, and is called 'light')
		action	$\hbar$	$1.054571800 \times 10.^{-34.}$Js(is called 'quantum')
		heat capacity	k_B	$1.38064852 \times 10.^{-23.}$J/K (is called 'Boltzmann')
Non-coherent	supplementary constants	the total solid angle of a hypershere	Ω_k	$\dfrac{2\pi^{\frac{k+1}{2}}}{\Gamma(\frac{k+1}{2})}$ rad^k k=0,1, 2 Ω_0=2 Ω_1=2π rad (circle, cycle) Ω_2=4π sr (sphere, turn)
		logalithm of an integer	f_k	log(2^k) k=1(bit), d(figure), 4(nibble), 8(byte), .. d=$\log_2$(12.)
		amount of substance	mol_u	132.007618 mol (=$12.^{24.}/N_A$)
		elementary charge	e	$1.6021766208 \times 10.^{-19.}$C (=$\sqrt{\frac{\alpha\hbar}{\Omega_n}}$)

Table 5 Physical, material and astronomical constants[29]

ALL VALUES DOZENAL

Constant Symbols and Name (UNDERLINE INDICATES CONSTANT MAINTAINS SAME VALUE BETWEEN SYSTEMS u AND h)		Constant Value expressed by the Universal Unit Systems: with the Rydberg constant (u)	Constant Value expressed by the Universal Unit Systems: Harmonic System (h)	Exponent N of× $10;^N$	Unit Symbol (u and h suffixes omitted)
R_∞	Rydberg constant	1	1;00170000	6;	Ω_1/m
c_0	<u>the speed of light in vacuum</u>	1		8;	m/s
$\hbar$	<u>quantum of action</u>	1		-26;	J s
k_B	<u>Boltzmann constant</u>	1		-20;	J/K
N_A	<u>Avogadro constant</u>	1		20;	mol^{-1}
R	<u>gas constant</u>	1		0;	J/(mol K)
u	unified atomic mass unit	1;0009061	1;0024073	-20;	g [30]
a_B	Bohr Radius	1;005E85686	1;00447X740	-9;	m
α	<u>fine structure constant</u>	1;07399405		-2;	-
e	<u>elementary charge</u>	1;0374439E		-14;	C
m_e	electron mass	0;E469222	0;E48324X	-23;	g
σ	<u>Stefan-Boltzmann constant</u>	1;E82E28		-1E;	W/(m^{2}K^4)

[29] If CODATA (2014) values are required, see http://physics.nist.gov/cuu/Constants/index.html .

[30] Because g_u is approximately $100;^{10;}$ u, I add alias name 'looloh'(lúːloʊ/əʊ) to g_h.

m_G	gravitic meter ($\sqrt{2E;}$ l_P)	1;0018	1;0001	-27;	m
l_P	Planck length	2;0445	2;0413	-28;	m
F_P	Planck force ($\hbar c_0 / l_P^2$)	2;XE23	2;XEE5(≑ 2;E)[31]	35;	N
G	Newtonian constant of gravitation (c_0^4/F_P)	4;1574	4;1463	-X;	$(m^4/s^4)/N$
θ_W	weak mixing angle	E;304		-2;	Ω_1
V_m	molar volume of an ideal gas under standard conditions	1;02X468	1;025664	2;	m^3/mol
	black-body radiation at the ice point	0;EX2462	0;EX8780	2;	W/m^2
	maximum density of water	1;088184	1;092X47 (≑ 15;/14;)	2;	g/m^3
	density of ice at the ice point	0;E7E9	0;E85E	2;	g/m^3
	specific heat of water[32]	0;6052	0;6045 (≑ 1/2)	0;	J/(g K)
	surface tension of water at 25°C	0;EE68	0;EEE4	-1;	N/m
atm	standard atmosphere	1;65008E	1;659967 (≑ 1;66)	4;	P
g_n	standard gravitational acceleration	5;5X54XE9	5;5E21264 (≑ E;/2)	0;	m/s^2
r_E	gravitational radius of Earth	2;41E8982X13	2;4180306534	-2;	m
au	astronomical unit	8;X67575537	8;X55509X33	X;	m
	astronomical unit	9;E91731X53		-3;	c_0 s_E day

Table 6 Power prefixes

name	symbol	Plain text	value	name	symbol	Plain text	value
dirac [33]	D		10;				
super	**S**		**10;^{4}**	**sub**	**s**		**10;$^{-4}$**
cosmic	$_{+}$	**_+**	**10;^{8}(=U)**	**atomic**	$_{-}$	**_-**	**U^{-1}**
di-cosmic	$_{2+}$	_2+	U^2	di-atomic	$_{2-}$	_2-	U^{-2}
ter-cosmic	$_{3+}$	_3+	U^3	ter-atomic	$_{3-}$	_3-	U^{-3}
tetra-cosmic	$_{4+}$	_4+	U^4	tetra-atomic	$_{4-}$	_4-	U^{-4}
penta-cosmic	$_{5+}$	_5+	U^5	penta-atomic	$_{5-}$	_5-	U^{-5}
hexa-cosmic	$_{6+}$	_6+	U^6	hexa-atomic	$_{6-}$	_6-	U^{-6}
hepta-cosmic	$_{7+}$	_7+	U^7	hepta-atomic	$_{7-}$	_7-	U^{-7}

[31] If this is expressed as 2;E, the error from CODATA (2014) becomes -2;53(-2.44) times standard deviation.

[32] This corresponds to the definition of the thermodynamic calorie.

[33] 'dirac' is only used when expressing the unit of the Gravitic System with the Harmonic System. (i.e., gravitic meter = tetra-atomic dirac harmon, gravitic second = penta-atomic dirac nic, gravitic gram = atomic dirac looloh)

Table 7 Examples of natural scale quantity representation [34]

quantity	symbol	value	refer to
2E; penta-cosmic Newton	2E;$_{5+}$N	2E;×U^{5} [harmonic] Newton	the Planck force
6;di-cosmic nic	6; $_{2+}$nc	6;×U^{2} [harmo]nic[second]	the age of the universe
cosmic super bit [Boltzmann]	$_{+}$Sf$_{1}$[k_{B}]	U$^{1@4}$log2^{1} [Boltzmann]	1.01 Tera Byte(=2$^{43.}$bit)
cosmic harmon	$_{+}$hm	U^{1} harmon[ic meter]	the speed of light in vacuum
unino atomic harmon	0;1$_{-}$hm	U$^{-1@1}$ harmon[ic meter]	the Bohr radius
di-atomic Coulomb	$_{2-}$C	U^{-2} [universal] Coulomb	the elementary charge
di-atomic effective Watt [35]	$_{2-}$W$_{e}$	U^{-2}[harmonic]effective Watt	a photon power (540.THz)
ter-atomic looloh	$_{3-}\ell\ell$	U^{-3} looloh	the unified atomic mass unit
2; tetra-atomic harmon	2; $_{4-}$hm	2;×U^{-4} harmon[ic meter]	the Planck length

Table 8 The Earth local extension for the Harmonic Universal Unit System

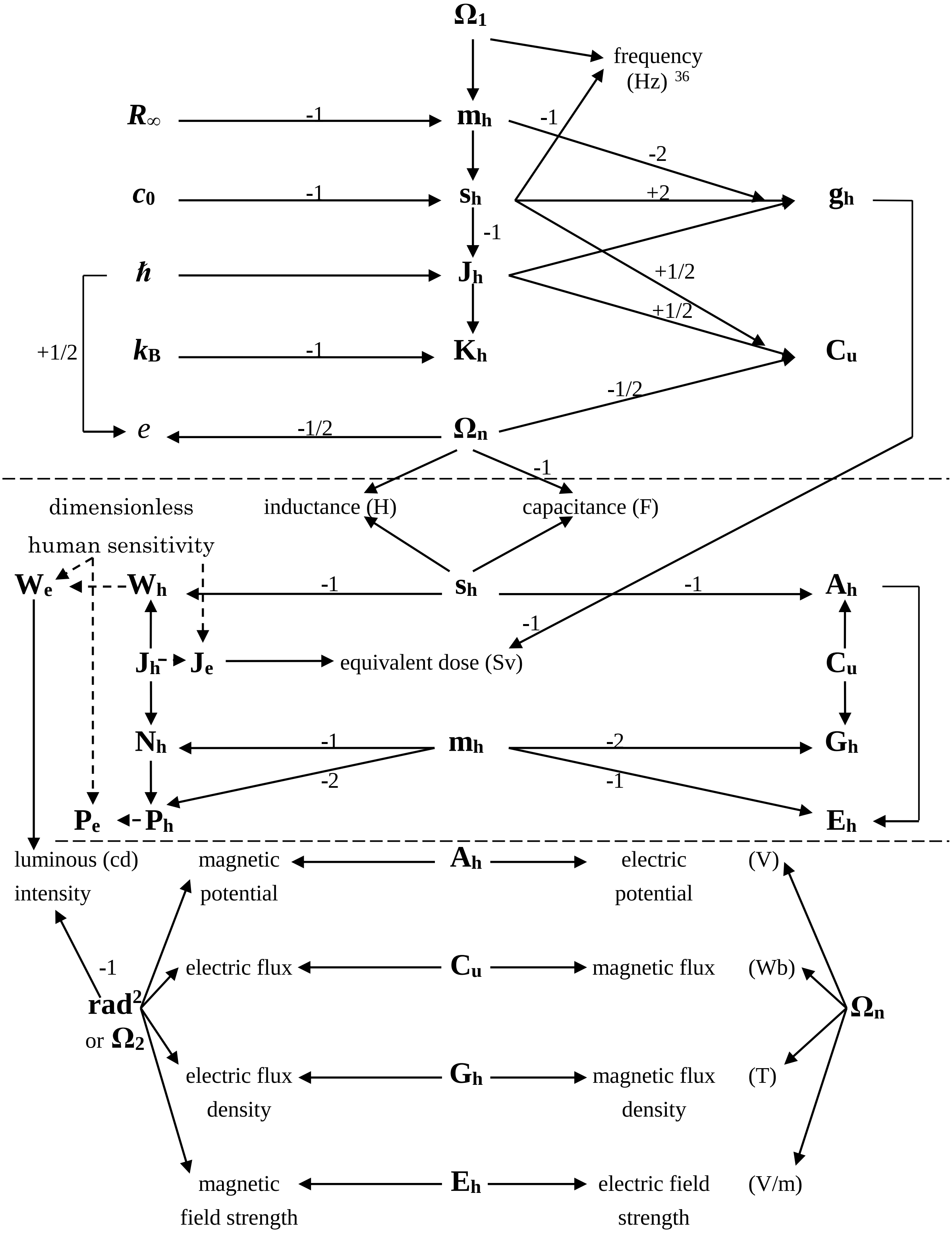
<table>
<tr><th colspan="2">category</th><th>name / description</th><th>symbol</th><th>plain text</th><th>value</th></tr>
<tr><td rowspan="4">Non-coherent calendar time</td><td rowspan="5">units</td><td>nic-angle (or octal second)</td><td colspan="2">na</td><td>10;$^{-3}$ × 2^{-7} day</td></tr>
<tr><td>day</td><td colspan="2">day</td><td>1 Ω_1
'day' corresponds to 86,400. s at the beginning of year 1900.</td></tr>
<tr><td>year</td><td colspan="2">y or a</td><td>(365.+ 31./128.) days</td></tr>
<tr><td>span (or octal century)</td><td colspan="2">span or "`"</td><td>64. years</td></tr>
<tr><td rowspan="4">Non-coherent unit and constants</td><td>difference between thermodynamic temperature and 118,2354; K$_{h}$ (≒ -74.36°C)
<table><tr><th colspan="2">approximate formula</th></tr><tr><td>$℃ = \frac{1E;}{17;}$°S-62;4</td><td>$°S = \frac{17;}{1E;}℃ + 51;5$</td></tr></table></td><td>°S</td><td>deg S</td><td>1,0000; K$_{h}$ (≒1.210724 K ≒ 23./19. K)
<table><tr><td>100; 0000°S</td><td>is</td><td>99.9839 °C</td></tr><tr><td>78;0000°S</td><td>is</td><td>37.0262°C</td></tr><tr><td>61;0000°S</td><td>is</td><td>14.0224°C</td></tr><tr><td>51;5026°S</td><td>is</td><td>0.0000°C</td></tr></table>99.9839 °C is the boiling point of water at the standard atmosphere.</td></tr>
<tr><td rowspan="3">supple-mentary constants</td><td>the gravitational acceleration of the Earth (is called 'gee [of Earth] ')</td><td>g_{E}</td><td>g_E or gee</td><td>5;611X615 harmon/nic^{2}
g_{E} is defined as $c_0^{2} r_{\mathrm{E}} (m_{\mathrm{E}}\ \mathrm{rad})^{-2}$</td></tr>
<tr><td>the rotation period of the Earth (is called '[Earth] solar')
at the beginning of year 1900.</td><td>s_{E}</td><td>s_E or solar</td><td>0;EEEEEE153565 nic/nic-angle
(This should be 'coordinated'.[16])</td></tr>
<tr><td>the meridian length of the Earth (is called '[Earth] meridian')</td><td>m_{E}</td><td>m_E or meridian</td><td>4124,216E; harmon/Ω_1</td></tr>
</table>

[34] The part enclosed with'[]' can be omitted in Table 7 and Table 8.

[35] Units for quantity weighted by dimensionless human sensitivity are indicated by 'effective'.

W$_{e}$ corresponds to 1;di-cosmic photon energy(540.THz) / nic and 115.667210 lumen.

E. Relation of Units and Dimensions



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36 The units enclosed with‘()’ are units of SI.

F. Ratios of fundamental physical constants

F.1. The fine structure constant and the elementary charge

The fine structure constant, α, a dimensionless quantity, was originally introduced for the purpose to explain the fine structure spectral emission lines.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 c_0 \hbar} \qquad \text{(X)}$$

By multiplying both sides of Eq. (X) by $\frac{c_0 \hbar}{r^2}$, we get

$$\alpha \frac{c_0 \hbar}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \qquad \text{(E)}$$

The right side of Eq. (E) expresses the Coulomb force acting between two elementary charges (i.e., the electrical charge of an electron) separated by a distance of r. The left side indicates that this force is proportional to $\frac{c_0 \hbar}{r^2}$ by a factor of α. For this reason, the fine structure constant, α, can be interpreted as a dimensionless quantity that represents the strength of electromagnetic interaction.

The value of the fine structure constant, α, is close to $10;^{-2}(12.^{-2})$.

DOZENAL

$$\alpha = \frac{1}{\text{E5;052258X}} = 1;073994049 \times 10;^{-2}$$

DECIMAL (10;)

$$\alpha = \frac{1}{137.03599914} = 1.050818770 \times 12^{-2}$$

Therefore, the ratio of the elementary charge, e, and "the dimensioned quantity of charge, which is derived from the speed of light in vacuum, c_0, and the quantum of action, $\hbar$" is:

DOZENAL

$$\alpha^{\frac{1}{2}} = \frac{e}{\sqrt{4\pi\varepsilon_0 c_0 \hbar}} = 1;0374439\text{E}2 \times 10;^{-1}$$

DECIMAL (11;)

$$\alpha^{\frac{1}{2}} = \frac{e}{\sqrt{4\pi\varepsilon_0 c_0 \hbar}} = 1.025094517 \times 12^{-1}$$

F.2. The Rydberg constant and the Bohr radius

The deviation of the fine structure constant, α, from an integer power of twelve is nearly the same as the deviation of 4π from twelve.

DOZENAL

$$4\pi = 1;069683171 \times 10;^{1} = \frac{1}{\text{E5;6150822}} \times 10;^{3}$$

DECIMAL (12;)

$$4\pi = 1.047197551 \times 12^1 = \frac{1}{137.5098708} \times 12^3$$

The ratio of the Bohr radius, a_B, and "the dimensioned quantity of length, $L=R_\infty^{-1}$, where R_∞ is the Rydberg constant" is:

DOZENAL

$$\frac{a_B}{L} = \frac{\alpha}{4\pi}(\text{strict}) = 1;005E85686 \times 10;^{-3}$$

DECIMAL (13;)

$$\frac{a_B}{L} = \frac{\alpha}{4\pi}(\text{strict}) = 1.003458009 \times 12^{-3}$$

F.3. The electron mass and the unified atomic mass unit

The ratio of the mass of an electron, m_e, and "the dimensioned quantity of mass, M, which is derived from L, the speed of light in vacuum, c_0, and the quantum of action$h$,"

$$M = \frac{h}{c_0 L}, \qquad (14;)$$

is:

DOZENAL

$$\frac{m_e}{M} = \frac{4\pi}{\alpha^2}(\text{strict}) = 0;E4692218 \times 10;^5$$

DECIMAL (15;)

$$\frac{m_e}{M} = \frac{4\pi}{\alpha^2}(\text{strict}) = 0.948359448 \times 12^5$$

The ratio of the mass of an electron, m_e and the unified atomic mass unit, u, is:

DOZENAL

$$\frac{m_e}{u} = \frac{1}{107X;X7E4} = \frac{4\pi}{\alpha^2} \times 0.EEE2E66 \times 10;^{-8}$$

DECIMAL (16;)

$$\frac{m_e}{u} = \frac{1}{1822.8885} = \frac{4\pi}{\alpha^2} \times 0.9995641 \times 12^{-8}$$

This ratio corresponds to the ratio of typical nuclear energy and chemical energy. The deviations of ratio Eq. (15;) and ratio Eq. (16;) from multiples of an integer power of twelve are near to the same magnitude. Therefore:

DOZENAL DECIMAL

$$\frac{u}{M} = 1;0009060E \times 10;^8 \qquad \frac{u}{M} = 1.00043606 \times 12^8 \qquad (17;)$$

F.4. The Planck length

The ratio of the general expression of the Planck length, $\sqrt{\frac{G\hbar}{c_0^3}}$, and L is close to 2 when factors of multiples of an integer power of twelve are eliminated.

DOZENAL

$$\sqrt{\frac{G\hbar}{c_0^3}}/L = 2 \times 1;02227 \times 10;^{-22}$$

DECIMAL (18;)

$$\sqrt{\frac{G\hbar}{c_0^3}}/L = 2 \times 1.01517 \times 12^{-26}$$

Taking the expression $\sqrt{\frac{G\hbar}{c_0^3\alpha}}$, which has been adjusted[37] by the fine structure constant, α, in order to express the tensile force in a superstring in terms of the Planck length, the ratio of the adjusted Planck length and L then becomes:

DOZENAL

$$\sqrt{\frac{G\hbar}{c_0^3\alpha}}/L = 2 \times 0;\text{EX}733 \times 10;^{-21}$$

DECIMAL (19;)

$$\sqrt{\frac{G\hbar}{c_0^3\alpha}}/L = 2 \times 0.99032 \times 12^{-25}$$

The Gravitic Universal Unit System uses 140.0 to approximate α^{-1}(=137.03599914). Because 1,0017;(20,755.=35.×593.) is divisible with 2E;(35.), we can approximate G by Eq.(1X;) about 2.44 times standard deviation error of CODATA(2014):

$$G \fallingdotseq \frac{(5\times7)^2 \cancel{\times}415;^2 c_0^3}{\cancel{5\times7\times\hbar R_\infty^2}} \times 10;^{-4X} = \frac{5\times7\times415;^2 c_0^3}{\hbar R_\infty^2} \times 10;^{-4X} \qquad (1X;)$$

[37] See E. Witten, ‘Reflections on the fate of spacetime’ p. 24. in April 11X4;(1996.) *Physics Today*.