

# Proposal for the Universal Unit System

Takashi Suga, Ph.D.

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## Abstract

This paper presents recent extensions and revisions to the Universal Unit System.<sup>2</sup> This system defines coherent units by employing the duodecimal number system (base 12) and fundamental physical constants, such as the speed of light in a vacuum, the quantum of action, and the Boltzmann constant. The central aim is to unify physical measurements (e.g., length, mass, temperature, and time) with a consistent duodecimal structure while accommodating human, atomic, and cosmic phenomena.

A noteworthy feature is the Harmonic System, which was conceptually designed so that crucial physical constants (including the Bohr radius and the atomic mass unit) and astronomical constants (including the year and day length of the Earth, the age of the solar system, and the universe) can be approximated using multiples or submultiples of integer powers of twelve. The Harmonic System simultaneously embraces duodecimal efficiency, atomic and cosmic relevance, and human convenience.

Compared to the previous official version, this revised document expands the discussion of calendar time units by introducing new definitions and clarifications. Specifically, it replaces the former “clock” unit with “nodus” ( $2^{-7}$  day) and presents “hexon” ( $2^6$  years  $\doteq 12^6$  nodus) and “ternon” ( $12^{-3}$  nodus) as derived calendar intervals.

Additionally, the system identification for units has been shifted from the suffix to the prefix, and the identifier for the Harmonic System has changed from “h” to “±.” The optional proposal to rename specific metric units after mythological figures, first introduced in the previous version, has been further refined in footnote 29 to address improved consistency.

## 1. The Universal Unit System

### 1.1. Before the Universal Unit System

A unit of measure is “a quantity that is used as the basis for expressing a given quantity and is of the same type as the quantity that is to be expressed”. A unit that is used in exchanges between people must be guaranteed to have a constant magnitude within the scope of that exchange. Quantities that can, by consensus, serve as common standards over a broad scope were sought and selected to serve as units. The ultimate example of such a quantity is an entity common to all of the humankind, the Earth itself, which was selected as the foundation for the metric system.

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<sup>1</sup> In this paper, SI units are combined only with the decimal figures (indicated by a period “.” as the radix point), and units of the Universal Unit Systems are combined only with duodecimal figures (indicated by a semicolon “;” as the radix point. ‘X’ expresses ten and ‘E’ expresses eleven). Both notations may use a comma “,” and “\_” as the digit group separator. Non-dimensional quantities are mostly expressed using all figures in duodecimal first, with the decimal given parenthetically. There are also cases where the octal radix point is represented by “@”.

<sup>2</sup> This paper revises the paper currently retrievable at <http://dozenal.com> and [https://github.com/suchowan/a\\_converter](https://github.com/suchowan/a_converter). See also footnote 27. The first Japanese version was released May 1194;(1984.)

## 1.2. The next stage?

Of course, we can consider going beyond the framework of the Earth and defining units with concepts for which agreement can be reached within a broader scope. The quantities that then become available to serve as the standards for defining units include the quantities of the fundamental physical constants, quantities such as the speed of light in a vacuum ( $c_0$ ), the quantum of action ( $\hbar$ ), the Boltzmann constant ( $k_B$ ), and so on. These quantities are believed to have values that remain constant everywhere in the universe. When trying to construct a coherent unit system, however, it is not possible to use all of the fundamental physical constants in the definitions of units. Therefore, would not we expect the fundamental physical constants that were not used in defining units to have fractional magnitudes of unit quantities of the same dimension?

For example, the Rydberg constant ( $R_\infty$ ) is 115,3789;470075/ft (10973731.56816/m)<sup>3</sup> and the Bohr radius ( $a_B$ ) is 0;X8EE6E522  $\times 10^{-9}$ ft (5.29177210544  $\times 10^{-11}$  m). Therefore, the relation between these two constants is:

DOZENAL	DECIMAL	
$R_\infty^{-1} = \text{EE6;06604 } a_B$	$R_\infty^{-1} = 1,722.04515 a_B$	(1)

If one of these two constants is chosen as a unit quantity, the other constant cannot be expressed as a unit quantity.

By surprising coincidences<sup>4</sup> described in Appendix 0 and §2.1 of <http://dozenal.com>, however, if the duodecimal number system is used to express the speed of light in vacuum and the quantum of action as the defining constants such that these constants are strictly multiples of integer powers of twelve of the unit quantities, it is possible to construct a coherent unit system in which not only the constant that was used in the definition but also the Rydberg constant, the Bohr radius, the unified atomic mass unit ( $u$ ), and half the value of the Planck length ( $l_P/2 = \sqrt{36 \cdot \hbar/c_0^3/12}$ ) can be approximated to about or within an error of 2 per gross (1<sup>1/2</sup>%) by a multiple of integer powers of twelve of the unit quantities.

In that case, many other physical constants, including the charge and mass of an electron, the fine structure constant, the molar volume of an ideal gas under standard conditions, the black-body radiation at the ice point, the density and surface tension of water, and others, can be approximated by multiples of integer powers of twelve of the unit quantities. Moreover, by adding the Boltzmann constant and using it in the definition of thermodynamic temperature, the gas constant of an ideal gas can be approximated by a multiple of an integer power of twelve of the unit quantity and the Stefan-Boltzmann constant, and the specific heat of water can be approximated by multiples of integer powers of twelve of the unit quantities with a factor 2 remaining. These conclusions are

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<sup>3</sup> In this paper, plane angle phase factor  $2\pi$  is often treated as a non-dimensional parameter and omitted in order to simplify the explanation. See §A.2 and 3.2.2 of the paper <http://dozenal.com>.

<sup>4</sup> To prevent any misunderstanding, let us emphasize that **these are merely coincidences as far as physical science is concerned.**

shown in Table 5.<sup>5</sup>

**For putting these coincidences to use, the duodecimal number system is the only choice.** It seems that the combination of fundamental physical constants “forces” us to use base twelve.

We define the Universal Unit System as “the unit system that is constructed by using the duodecimal number system and the speed of light in vacuum, the quantum of action, and the Boltzmann constant as the defining constants in such a way that these constants become strict multiples of integer powers of twelve of the unit quantities, and the Rydberg constant, the unified atomic mass unit, the Bohr radius, and half the value of the Planck length can be approximated by multiples of integer powers of twelve of the unit quantities”.

### 1.3. Variation of the Universal Unit Systems

To define three units for time, length, and mass, the Universal Unit System uses the speed of light in vacuum and the quantum of action. Another constant is necessary to define these three units. Therefore, the Universal Unit System has some variations in the constant that the system chooses as the last definition constant.

Universal Unit System with constant  $A$  is the Universal Unit System that uses constant  $A$  as the last definition constant and whose unit quantity of the last dimension is equal to constant  $A$  or its multiples of integer powers of twelve. In particular, the Universal Unit System with the Rydberg constant whose length unit is  $10;^6 / R_\infty$  ( $12.^6 / R_\infty$ ) and velocity unit is  $10;^8 c_0$  ( $12.^8 c_0$ ) is called the Universal System of Units Standard<sup>6</sup> corresponding to the International System of Units Standard (SI). We will use a symbol corresponding to the SI unit symbol prefixed with ‘u’ as a new symbol required by the Universal System of Units Standard; ‘u’ is the ‘universal’ system prefix. The noun form is ‘univer’. For example, the length unit is  ${}_u\text{m}$  and is called the ‘universal meter’ or simply the ‘univer’<sup>7</sup>, and the time unit is  ${}_u\text{s}$  and is called the ‘universal second’<sup>8</sup>. **This unit system is comprised of six quartets.** The units of this system are listed in the 5th column of Table 4, and physical, material, and astronomical constants expressed using this system are presented in the 3rd column of Table 5. The ratio of the time unit  $s_u$  and the SI second is 0;4824707(0.3902675).

The Universal Unit System with the Bohr radius whose length unit is  $10;^9 a_B$  ( $12.^9 a_B$ ) and velocity unit is  $10;^8 c_0$  ( $12.^8 c_0$ ) can be defined in the same way. Its time unit is 1;005E857  ${}_u\text{s}$

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<sup>5</sup> A more detailed table is retrievable at <http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/condensed.xlsx>.

<sup>6</sup> The Universal System of Units Standard is strictly defined in §3 of the paper <http://dozenal.com>.

<sup>7</sup> The noun form of the system prefix is considered to be the abbreviation of the length unit like the length unit of the metric system is ‘meter’.

<sup>8</sup> Note that there is no vagueness at all even if the prefix is omitted if the notation of footnote 1 is adopted. Prefixes are necessary to identify plural Universal Unit Systems mutually.  $4 \times 10.^7 \text{ m}$ ,  $4 \times 10.^7 m_u$ , and the Earth’s meridian length are nearly equal.  $10.^5 \text{ s}$ ,  $10;^5 {}_u\text{s}$ , and  $1\frac{1}{8} \text{ days}$  are nearly equal.

(0.3916171 s). Roughly speaking, **if a time constant or its multiples of integer powers of twelve falls within the range between 1;<sub>us</sub> (0.3902675 s) and 1;005E857<sub>us</sub> (0.3916171 s), we can construct the Universal Unit System using the constant as a time unit.**

If half the value of the Planck length is used instead of the Bohr radius, the time constant becomes 1;0223<sub>us</sub> (0.3962 s). This is out of range. To keep the time constant within the range, we should use  $\sqrt{35.Gh/c_0^3}$  instead of  $\sqrt{36.Gh/c_0^3}$ . Then, the time constant becomes 1;00186<sub>us</sub> (0.39065 s). We call this system the Gravitic System. The Gravitic System can be interpreted as a unit system that 35.  $G$ ,  $\alpha$ ,  $\hbar$ ,  $K_B$  and  $Z_P$  are strict multiples of integer powers<sup>9</sup> of twelve of the unit quantities. However, it is not practical because  $G$ 's measurement accuracy is insufficient.

## 2. The GCD Unit System

### 2.1. Basic Concept

The length of the tropical year is strictly 265;2XX6 days in a certain year at the end of the 20th century. For human activities on Earth, year and day cannot be ignored as calendar time units. However, the ratio of year and day is not simple. **Therefore, any calendar time unit system must be a mixed radix system.** The ratio of one tropical year and one day is:

$$\begin{array}{c} \text{DOZENAL} \\ \frac{\text{year}}{\text{day}} = 265; + \frac{27;}{X8;} = 1;003628 \times 264;6 = 1;003628 \times \frac{3^6}{2} \\ \text{DECIMAL} \end{array} \quad (2)$$

$$\frac{\text{year}}{\text{day}} = 365. + \frac{31.}{128.} = 1.002036.. \times 364.5 = 1.002036.. \times \frac{3^6}{2}$$

Because one year consists of twelve months, it is reasonable to adopt twelve as one of the radixes. Though this ratio contains the extra factor 3 six times,<sup>10</sup> by multiplying by factor  $2^6 (= 8 \times 8)$  twice, we can cancel factor 3 and obtain powers of twelve ( $= 3 \times 2 \times 2$ ):

$$\begin{array}{c} \text{DOZENAL} \\ \frac{2^6 \text{year}}{2^{-6} \text{half day}} = 2^6 \times 1;003628 \times \frac{3^6}{2} \times 2^7 = 1;003628 \times 10;^6 \\ \text{DECIMAL} \end{array} \quad (3)$$

$$\frac{2^6 \text{year}}{2^{-6} \text{half day}} = 2^6 \times 1.002036.. \times \frac{3^6}{2} \times 2^7 = 1.002036.. \times 12.^6$$

<sup>9</sup> If these integers are all set to 0, various constants are approximately expressed as follows: [https://en.wikipedia.org/wiki/Talk:Planck\\_units/Archive\\_3#Other\\_possible\\_normalizations](https://en.wikipedia.org/wiki/Talk:Planck_units/Archive_3#Other_possible_normalizations).

<sup>10</sup> Yang Xiong (45;(53.) BCE–16;(18.)) used the ratio  $3^6/2$  in his Tai Xuan calendar (太玄曆).

See Hideki Kawahara “Chinese Scientific Thought (中国の科学思想)” 11X4;(1996.) [ ISBN: 978-4423194126] and Wikipedia [http://en.wikipedia.org/wiki/Yang\\_Xiong\\_\(author\)](http://en.wikipedia.org/wiki/Yang_Xiong_(author)).

If we define ‘hexon’ (=octal century, symbol: ☿) as  $2^6$  years, and define ‘nodus’ (symbol: ☆) as  $2^{-6}$  of a half day<sup>11</sup>, the relation between these two units is:

$$\text{hexon} = 1;003628 \times 10;^6 \text{ nodus} \quad (4)$$

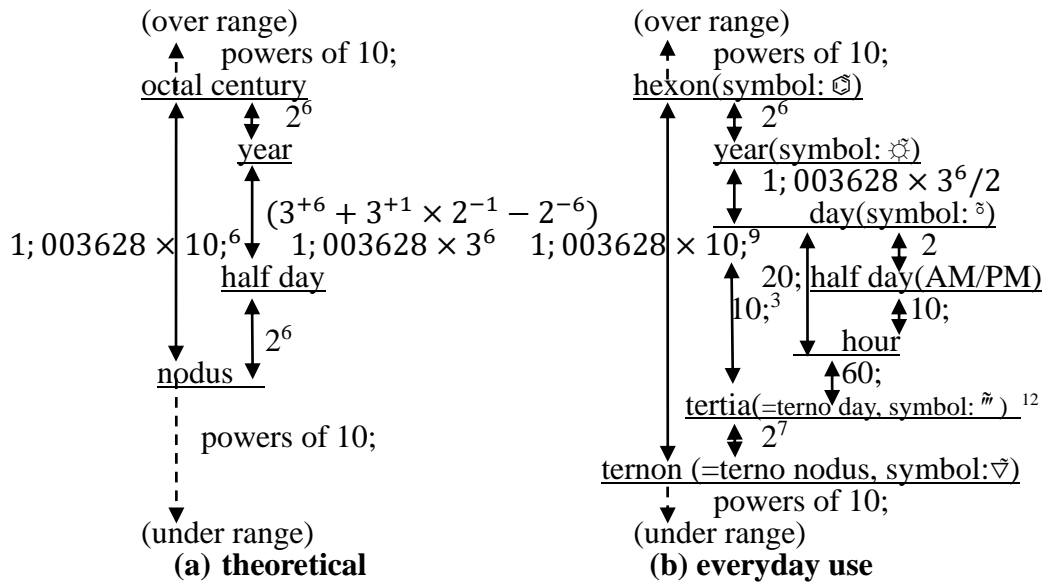
There are some interesting coincidences. Also, please see Table 1.

- One nodus is the difference between a tropical year and a Julian year.
- One nodus is the greatest common divisor (GCD) of the length of a day and a tropical year.
- Two octal centuries are the least common multiple (LCM) of the length of a day and a tropical year, and so leap year rules become simpler than those of the Gregorian calendar.
- The geometric mean of one octal century and one nodus is approximately one fortnight.

**Table 1 Coincidences of rotation and revolution of Earth and other planets**

Quantity A	Quantity B	Type	Common Quantity
day	Julian year	GCD	1/4 day
		LCM	4 years
	tropical year	GCD	$1/X8;(2^7)$ day
		LCM	$X8;(2^7)$ years
year	rotation of Venus	LCM	2 years and -3 rotations
	revolution of Venus		8 years and 11;(13.) revolutions
	revolution of Mars		28;(2 <sup>5</sup> ) years and 15;(17.) revolutions

It seems that the combination of the rotation and revolution of the Earth “forces” us to use base two in the middle of the calendar time range. People have a proclivity for using large units for large quantities and using small units for small quantities, so we can cover a wide calendar time range by a duodecimal number system using the following hierarchy shown in Figure 1 (a).



**Figure 1 The GCD Unit hierarchy**

<sup>11</sup> The Latin word ‘nodus’ signifies a knot or node, a point where multiple lines or elements intersect. See the Natural Time Scale Chart in Appendix E. The neologisms ‘hexon’ (nearly  $10;^{+6}$  nodus) and ‘ternon’ ( $10;^{-3}$  nodus) were coined by abbreviating hexa[6]on[+]nodus → hex(a)on n(odus) → hexonn → hexon, and ter[3]no[-]nodus → terno n(odus) → ternon, respectively. See Appendix C and blog post: [https://suchowan.seesaa.net/article/202501article\\_17\\_3.html](https://suchowan.seesaa.net/article/202501article_17_3.html).

<sup>12</sup> A terno day(‘tertia’) is equal to H. C. Churchman's ‘moment’. Please see <http://www.dozenal.org/archive/DuodecimalBulletinIssue112-web.pdf>.

We define ‘the GCD Unit’ as “the GCD of the length of a day and a tropical year, or its multiples of integer powers of twelve”, and ‘the GCD Unit System’ as “the 2:10;(2:12.) mixed-radix calendar time system using a day, a tropical year, and the GCD Unit”. **It is very natural to adopt a unit of calendar time that is an integer division of both a year and a day.**

## 2.2. Everyday use

Calendar time units are the most conservative units. Considering the easiness of the shift from the present 10;:50;(12.:60.) mixed-radix system to the future 2:10;(2:12.) mixed-radix system, there are some variations of connection points of the binary number system and duodecimal number system<sup>13</sup>. A variation designed for everyday use is shown in Figure 1 (b) and Table 8.

$$1 \text{ day} = 10;^3 \text{ tertias} = 10;^3 \times 2^7 \text{ ternons} \quad (5)$$

This is a variation that is designed to maximize the range expressed by multiples of integer powers of twelve of a day. This permits ‘half day(AM/PM)’ and ‘hour’ to be used for clock notations. That is, the following four clock notations are all available. The notation ‘hour’ may fade out.

**Table 2 Clock Notations**

clock notation	format	2:10;(2:12.) system	10;:50;(12.:60.) system
10;-hour notation	H:TT <sup>m</sup> NN <sup>v</sup> nn...(AM/PM)	3:16 <sup>m</sup> 28 <sup>v</sup> 0 PM	3:15:12.5 PM
20;-hour notation	HH:TT <sup>m</sup> NN <sup>v</sup> nn...	13:16 <sup>m</sup> 28 <sup>v</sup> 0	15:15:12.5
1000;-tertia notation	UTT <sup>m</sup> NN <sup>v</sup> nn...	776 <sup>m</sup> 28 <sup>v</sup> 0	Not used
10;-unitia notation	TT(to[-]/after[+])U	46; tertias to the 8 <sup>th</sup> unitia	15 minutes after 3 PM

Following the Titius-Bode law, the orbital semi-major axis of planets can be approximated by  $(3 \times 2^N + 4)$  light solar tertias (see also Table 8), where  $N = -\infty, 0, 1, 2, 4, 5, 6$  (Mercury, ..., and Uranus). Ratio 2<sup>7</sup>:1 is the same as the ratio of the U.S. liquid gallon and fl oz. **The Harmonic Ratio 2<sup>7</sup>:10;<sup>2</sup> (2<sup>3</sup>:3<sup>2</sup>)** corresponds to the major tone of the just intonation.

## 3. The Harmonic System

The GCD Unit is derived from the combination of the rotation and revolution of the Earth without using the Universal Unit System concept. However, we encounter the final surprising coincidences. 1; ternon is equal to a day/(10;<sup>3</sup>×2<sup>7</sup>). It is equivalent to 1;0016EE1<sub>us</sub> (0.3906250 s). Therefore the equivalent value of 1; ternon falls within the range between 1;<sub>us</sub> (0.3902675 s) and 1;005E857<sub>us</sub> (0.3916171 s). This value also almost agrees with the time constant of the Gravitic System 1;0018<sub>us</sub> (0.3906 s). We can construct the Universal Unit System with the GCD Unit! <sup>14</sup> The Dozenal Society of America seems to recommend ‘dour’(day/10;) and ‘moment’(day/10;<sup>3</sup>). Of course, these units are all available, but because the calendar time unit system includes ‘year’ by all

<sup>13</sup> See detailed discussion in <https://www.tapatalk.com/groups/dozenonline/the-universal-unit-system-f34/>.

<sup>14</sup> See the sheet ‘Clock’ in <http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/condensed.xlsx>.

means, it does not become a pure duodecimal system even if we choose  $1/10$ ;<sup>2</sup> of ‘moment’ as a unit. **We can enjoy all the advantages of Table 5 if we choose not  $1/10$ ;<sup>2</sup> but  $1/2$ .**<sup>7</sup>

Because of tidal friction, the physical time length of a day is not constant and becomes longer little by little.<sup>15</sup> A length unit can be redefined exactly as  $100,1700/R_{\infty}$  so that a physical time unit becomes exactly  $1;001700_{\text{us}}$  (0.3906251 s) and approximately  $1;0016EE1_{\text{us}}$  (0.3906250 s) similar to the redefinition of SI units to provide better stability and reproducibility. The calendar time unit length will strictly correspond to the physical time length of a day about 20; octal centuries (1400. years) later. We call this redefined system ‘the Harmonic Universal Unit System with the GCD Unit’, or simply ‘the Harmonic System’ and will use the prefix ‘ $\pm$ ’ for units of this system<sup>16</sup>. The length unit is  $\pm h$  and is called ‘harmonic universal meter’, ‘harmonic meter’, or simply ‘harmon’. We specifically granted dedicated aliases for the Harmonic System’s length, time, mass and impedance unit. Please see footnote 26 on ‘harmon’( $\pm h$ ), ‘nic’( $\pm n$ ), ‘looloh’( $\pm l$ ) and ‘nohm’( $\pm \Omega$ ).

We notice that the Harmonic System is also a unit system that redefines the Gravitic System which is not practical because G’s measurement accuracy is insufficient. In other words, the Harmonic System can also be said to have given a strict and practical definition to the conceptual Gravitic System.

The units of this system are listed in the 7th column of Table 4 and physical, material, and astronomical constants expressed using this system are presented in the 4th column of Table 5. Please see also Table 6 and Table 7 for details. The approximations shown in Table 5 are remarkable. For putting coincidences described in this paper to use, the duodecimal number system is indispensable. **We hope that the Harmonic System is acceptable for humans on Earth.**

## A. The Earth local extension

The Earth local extension, which consists of four unit series, and three supplementary constants, is designed for local use on Earth. Please see Table 8.

In this scheme, the CGD unit system is treated as part of the Earth local extension. **To distinguish calendar time units from physical time units, we regard the dimension of calendar time units as the plane angle.**<sup>17</sup>

A new temperature unit,  $^{\circ}H$  (degree H), is introduced under the “Earth local extension” category to accommodate everyday use within the universal system. We define  $0^{\circ}H$  at  $T_E$  ( $=118,2354; \pm K$  (approximately  $-74.36^{\circ}C$ ,  $-101.85^{\circ}F$ )), and set  $100;^{\circ}H$  to match the boiling point of

<sup>15</sup> See Stephenson, F. R.; Morrison, L. V. (April 11X3;(1995.)) “Long-term fluctuations in the Earth’s rotation: 700 BC to AD 1990.” retrievable at <http://adsabs.harvard.edu/abs/1995RSPTA.351..165S> .

<sup>16</sup> In principle, the prefix ‘ $\pm$ ’ is pronounced as ‘harmonic.’ However, if the unit is the same amount in both the Universal and Harmonic systems, it is pronounced as ‘universal.’ For example, ‘ $\pm A$ ’ is ‘harmonic Ampere,’ but ‘ $\pm C$ ’ is ‘universal Coulomb.’ Also, if the context tells you it is a Harmonic System unit, you do not need to pronounce ‘ $\pm$ .’ For example, in the voltage unit ‘ $\pm \Omega A$ ,’ when you read ‘ $\pm \Omega$ ’ as ‘nohm,’ you know that the next unit is not an SI but a Harmonic System unit so that you can pronounce it as ‘nohm Ampere’ instead of ‘nohm harmonic Ampere.’

<sup>17</sup> See Seaman, Rob (April 11XB;(2003.)). “A Proposal to Upgrade UTC” retrievable at <https://web.archive.org/web/20150419125423/http://iraf.noao.edu/~seaman/leap/>. It seems that the dimension of the quantity of a day (=calendar time) should be a plane angle rather than physical time. The calendar time is, in a word, the rotation angle of the Earth derived by using the direction of the sun as a coordinate origin.

water (99.9839 °C). Consequently, 1 °H interval corresponds exactly to 1 **H**yper Kelvin (=1,0000; ± K ÷ 1.210724 K). This arrangement ensures that typical ambient temperatures on Earth remain in a convenient positive range, avoiding negative values for everyday measurements. While some may question the choice of offset and the use of 10;<sup>2</sup> as the boiling point reference, this design balances the duodecimal scaling of the universal system with practical concerns for daily life. H is an initial letter for both the Hyper and Human scale.

The supplementary constant  $g_E$  is used to represent any force quantity as a corresponding mass quantity, like  $1 \pm 1 g_E$  (one looloh gee). In the same way, the supplementary constants  $s_E$  and  $m_E$  are used to represent any physical time and length quantity as a corresponding plane angle quantity.

## B. Gravitational constant and gravity field equations

The equations of some categories<sup>18</sup> can be used efficiently if we introduce new constants. At this time, the total solid angle of a sphere<sup>19</sup>,  $\Omega_2$ , and the speed of light in vacuum,  $c_0$ , appear in the equations. This is the reason that the speed of light in a vacuum should strictly be multiples of integer powers of the base number of the unit quantities.

**When representing the mass of a celestial body using the Universal Unit System, the gravitational radius (half the Schwarzschild radius) is used rather than using mass directly.** Because the accuracy of measuring the Newtonian constant of gravitation is poor, representing the mass of a celestial body directly in terms of mass results in poor accuracy, but the gravitational radius can be measured to an accuracy of around ten digits.

If we define a new constant that has the dimension ‘force’ as ‘the Planck force’, there is a good chance that the geometrical parts can be separated from the coefficients in the formula<sup>20</sup>. Make the Planck force,  $F_P = c_0^4 G^{-1} = \hbar c_0 / l_P^2 = 35. \hbar c_0 / m_G^2$ , then:

$$\text{gravitational radius, } r_m = \frac{Gm}{c_0^2} = \frac{mc_0^2}{F_P} \text{ (half the Schwarzschild radius)} \quad (6)$$

$$\text{gravitational force, } f = F_P \frac{r_m r_{m'}}{r^2} = c_0^2 \frac{r_m m'}{r^2} \quad (7)$$

$$\text{gravitational acceleration, } g = c_0^2 \frac{r_m}{r^2} = \frac{r_m}{(r/c_0)^2} \quad (8)$$

$$\text{gravity field equation, } \frac{T_{ik}}{F_P} = \frac{1}{2\Omega_2} \left( R_{ik} - \frac{1}{2} R g_{ik} + \Lambda g_{ik} \right) \quad (9)$$

<sup>18</sup> For a case of electromagnetism, see §B of the paper <http://dozenal.com>. In the electromagnetic field, the natural unit of impedance,  $\imath\Omega$  (=Z<sub>P</sub>: the Planck impedance), plays the role of the Planck force in the gravitational field.  $\imath\Omega$ ,  $e$ ,  $\theta_W$ , and  $F_P$  are constants for the four fundamental forces.

<sup>19</sup> See electromagnetic units in Appendix E and §3.2.2 of the paper <http://dozenal.com>, and <http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/electromagnetism.pdf>.

<sup>20</sup> Please note that Eq. (8) and (9) are geometrical and have no mass dimension.



## C. Number Counting

Many of the constants introduced in footnote 9 often have orders  $8n - 1$ .<sup>21</sup> Therefore, it is convenient to use the factor  $U(=10;^8(12.^8))$  to make the units of the Universal Unit System into human scale. The factor  $U$  can be regarded as a conversion factor between atomic scale, human scale, and cosmic scale. Since power  $8(=2^3)$  is a power of 2, the decimal myriad system<sup>22</sup> has affinity for our system. So, we propose a duodecimal myriad system in Table 3<sup>23</sup> replacing ten/hundred with dozen/gross. Larger numbers consist of uni(1), di(2), ter(3), tetra(4), penta(5), hexa(6), hepta(7), lli(0), on(+), and reciprocals are expressed by replacing on(+, positive power) with no(-, negative power)<sup>11</sup>.

**Table 3 Duodecimal myriad system**

decimal	dozenal		read as 'one-'	origin of prefix part	decimal	dozenal		read as 'one-'	origin of prefix part
$12.^1$	10;		dozen	Old Norse	$12.^{-1}$	$U^{-@1}$	1;'	unino	Latin
$12.^2$	100;		gross	Old French	$12.^{-2}$	$U^{-@2}$	1;''	dino	Greek
$12.^3$	1000;		doz gross		$12.^{-3}$	$U^{-@3}$	1;'''	terno	Latin
$12.^4$	1,0000;	1,;	myriad	Greek	$12.^{-4}$	$U^{-@4}$	1;,,	tetrano	Greek
$12.^5$	10,0000;	10,;	dozen myriad		$12.^{-5}$	$U^{-@5}$	1;,'	pentano	Greek
$12.^6$	100,0000;	100,;	gross myriad		$12.^{-6}$	$U^{-@6}$	1;''''	hexano	Greek
$12.^7$	1000,0000;	1000,;	doz gross myriad		$12.^{-7}$	$U^{-@7}$	1;'''''	heptano	Greek
$12.^8$	U	1_;	unillion		$12.^{-8}$	$U^{-1}$	1;_	unillino	
$12.^{16}$	$U^2$	1_.; or 1__;	dillion		$12.^{-16}$	$U^{-2}$	1;_or 1;__	dillino	
$12.^{24}$	$U^3$		terllion		$12.^{-24}$	$U^{-3}$		terllino	
$12.^{32}$	$U^4$		tetrallion		$12.^{-32}$	$U^{-4}$		tetrallino	
$12.^{40}$	$U^5$		pentallion		$12.^{-40}$	$U^{-5}$		pentallino	
$12.^{48}$	$U^6$		hexallion		$12.^{-48}$	$U^{-6}$		hexallino	
$12.^{56}$	$U^7$		heptallion		$12.^{-56}$	$U^{-7}$		heptallino	
$12.^{64}$	$U^{10@}$		unillillion		$12.^{-64}$	$U^{-10@}$		unillillino	
$12.^{128}$	$U^{20@}$		dillillion		$12.^{-128}$	$U^{-20@}$		dillillino	
...	...		...		...	...		...	

For example,  $1\_2345,6789$ ; is read as 'one unillion two doz three gross four dozen five myriad six doz seven gross eight dozen nine'. The first 'one' should not be omitted. The term 'dozen' is abbreviated as 'doz' when it appears in the  $10;^{4n-1}$  position. The expression 'Unillion to the power of *octal number*' is also used as exponential expression<sup>24</sup>. For example,  $1;2 \times U^{3@4}$  is read as 'one point two times unillion to the power of three point four'. '@4' can also be written 'H', in which case 'H' is read as 'and half'. The characters "' ", '" ', "' ", ", ", and (diacritical mark) " \_ " following a radix point or digit group separator<sup>1</sup> shift them by 1, 2, 3, 4, and 8 places, respectively.<sup>36</sup>

<sup>21</sup> See also footnote 34.

<sup>22</sup> <http://en.wikipedia.org/wiki/-yllion> .

<sup>23</sup> "@" is the octal radix point.

<sup>24</sup> Dr. Isaac Asimov made a similar proposal called 'T-formation'. See <http://www.isfdb.org/cgi-bin/title.cgi?62431> and <http://www.arvindguptatoys.com/arvindgupta/asimov-on-numbers.pdf> .

## D. Tables

**Table 4 Units with special names and symbols<sup>25</sup>**

ALL VALUES DECIMAL

Unit Category		Dimension	The Universal Unit Systems			
			with the Rydberg constant(u)		Harmonic System ( $\pm$ )	
Coherent	base units that are not natural units	length	${}_u\text{m}$	272.102883 mm	$\pm\text{h}$ <sup>26</sup>	272.352206 mm
		time	${}_u\text{s}$	390.267520 ms	$\pm\text{n}$	390.625115 ms
		energy	${}_u\text{J}$	64.143275 mJ	$\pm\text{J}$	64.084556 mJ
		temperature <sup>27</sup>	${}_u\text{K}$	58.441041 $\mu\text{K}$	$\pm\text{K}$	58.387542 $\mu\text{K}$
	base units that are natural units	plane angle	rad	$(2/\pi)$ arc sin(1)		
		logarithm	neper	log(e)		
		amount of substance	${}_s\text{mol}$ or $N_A^{-1}$	mol / $6.02214076 \times 10^{23}$ . In this context ‘ ${}_s$ ’ is equivalent to ‘3-’ and ${}_s\text{mol}$ is called ‘natural mol.’		
		impedance	${}_s\Omega$ or $Z_P$	$29.979245796 \Omega (=1\text{sr}/(\epsilon_0 c_0))$ <sup>28</sup> ${}_s\Omega$ is called ‘natural ohm’ or more simply ‘nohm.’		
	derived units of electromagnetic quantities	charge	$\pm\text{C}$	28.896578 mC (is called ‘universal Coulomb’ (or ‘Clio’ <sup>29</sup> ))		
		electric current	${}_u\text{A}$	74.043001 mA	$\pm\text{A}$	73.975219 mA
		field strength	${}_u\text{E}$ <sup>19,29</sup>	272.113988 mA/m	$\pm\text{E}$	271.616007 mA/m
		flux density	${}_u\text{T}$	390.283447 mC/m <sup>2</sup>	$\pm\text{T}$	389.569211 mC/m <sup>2</sup>
	derived units of dynamical quantities	mass	${}_u\text{g}$	131.950082 g	$\pm\text{l}$ ( $\times 006\text{C}$ )	131.829289 g
		power	${}_u\text{W}$	164.357196 mW	$\pm\text{W}$	164.056415 mW
		force	${}_u\text{N}$	235.731701 mN	$\pm\text{N}$	235.300301 mN
		Pressure	${}_u\text{P}$	3.183843 Pa	$\pm\text{P}$	3.172201 Pa

<sup>25</sup> Please see also <http://www.asahi-net.or.jp/~dd6t-sg/univunit-e/units.pdf> for details. A web-based unit converter is available at <http://hosi.org/cgi-bin/conv.cgi>. This converter also teaches us the representation of units that belong to various unit systems.

<sup>26</sup> ‘harmon’( $\pm\text{h}$ ), ‘nic’( $\pm\text{n}$ ), ‘looloh’<sup>31</sup>( $\pm\text{l}$ ), ‘l’ can also be a cursive ‘ $\ell$ ’ ( $\times 2113$ )), and ‘nohm’( ${}_s\Omega$ ) constitutes a quartet. These are the alias for common use.

<sup>27</sup> The unit of thermodynamic temperature has been changed. The new unit is one-1,0000;<sup>th</sup> of the old unit in the paper <http://dozenal.com> along with the introduction of the Earth local extension.

<sup>28</sup> If we adopt the elementary charge as one of the definition constants,  $\pm\Omega$  is used in substitution for  ${}_s\Omega$ .

<sup>29</sup> The unit symbol E (Ørsted) is associated with the CGS system. In this paper, we adopt metric unit names based on the scientists’ names as is.

However, under the Harmonic System, an alternative proposal suggests replacing these units with the names of Muses bearing the same initials — namely, Newton→**Nete**, Pascal→(Polymnia→)**Polym**, Coulomb→**Clio**, Ampere→**Aoide**, Ørsted→**Erato**, Tesla→**Thalia**, and Kelvin→**Kalliope**. This proposal has two advantages: (1) it does not honor any individual, and (2) it allows the omission of redundant ‘harmonic’ terms<sup>16</sup>. The unit converter for this proposal is available at [http://hosi.org/cgi-bin/conv\\_muse.cgi](http://hosi.org/cgi-bin/conv_muse.cgi).

This proposal also renames units for which no corresponding Muse is found, such as Joule→**Juno**, Watt→(Walküre→)**Walku**, naper→(Nephelē→)**nephe**, dirac→**diana**, and  $\Omega\text{hm}$ → **$\Omega$ (Omega)**. Since no suitable Muse exists for Joule, Watt, or naper, the proposal instead borrows names from Roman, Norse, and Greek mythology. Moreover, because of the electromagnetic symmetry required to pair  ${}_s\Omega$  and  $\Omega_2$  (see the 3<sup>rd</sup> part of p.14), ‘ $\Omega$  (Omega)’ is adopted without a Muse equivalent.

Non-coherent	defining constants	wave number	$R_\infty$	10,973,731.568157/m ( is called ‘Rydberg’)
		velocity	$c_0$ or $\frac{1}{4} \gamma$	299,792,458 m/s (defined, and is called ‘light’)
		action	$\hbar$	$6.62607015 \times 10^{-34} \text{ J s} / 2\pi$ ( is called ‘quantum’)
		heat capacity	$k_B$	$1.380649 \times 10^{-23} \text{ J/K}$ ( is called ‘Boltzmann’)
	supplementary constants	the total solid angle of a hypersphere	$\Omega_k$	$\frac{2\pi^{\frac{k+1}{2}}}{\Gamma(\frac{k+1}{2})} \text{ rad}^k$ $k=0,1,2$ $\Omega_0=2$ $\Omega_1=2\pi \text{ rad}$ ( is called ‘cycle’) $\Omega_2=4\pi \text{ sr}$ ( is called ‘turn’)
		logarithm of an integer	$f_k$	$\log(2^k)$ $k=1(\text{bit}), d(\text{figure}), 4(\text{nibble}), 8(\text{byte}), \dots$ $d=\log_2(12.)$
		amount of substance	$\pm \text{mol}$	$132.007620 \text{ mol}$ $(=12.24 / N_A)$ $(\pm \text{mol}$ is called ‘universal mol’)
		elementary charge	$e$	$1.6021766340 \times 10^{-19} \text{ C}$ $(e$ is called ‘electron’) $(=\sqrt{\frac{\alpha \hbar}{\Omega_n}})$

**Table 5 Physical, material and astronomical constants<sup>30</sup>**

ALL VALUES DOZENAL

Constant Symbols and Name (UNDERLINE INDICATES CONSTANT MAINTAINS SAME VALUE BETWEEN SYSTEMS u AND h)		Constant Value expressed by the Universal Unit Systems		Exponent $N$ of $10^N$	Unit Symbol (u and h prefixes omitted)
		with the Rydberg constant (u)	Harmonic System (h)		
$R_\infty$	Rydberg constant	1	1;00170000	6;	$\Omega_1/\text{m}$
$c_0$	<u>the speed of light in vacuum</u>	1		8;	m/s
$\hbar$	<u>quantum of action</u>	1		-26;	J s
$k_B$	<u>Boltzmann constant</u>	1		-20;	J/K
$N_A$	<u>Avogadro constant</u>	1		20;	$\text{mol}^{-1}$
$R$	<u>gas constant</u>	1		0;	J/(mol K)
$u$	unified atomic mass unit	1;00090610	1;00240733	-20;	$\text{g}^{31}$
$a_B$	Bohr Radius	1;005E85684	1;00447X74	-9;	m
$\alpha$	<u>fine structure constant</u>	1;0739940472		-2;	-
$e$	<u>elementary charge</u>	1;0374439E14		-14;	C
$m_e$	electron mass	0;E4692217E0	0;E48324X245	-23;	g
$\sigma$	<u>Stefan-Boltzmann constant</u>	1;E82E28		-1E;	W/(m <sup>2</sup> K <sup>4</sup> )

<sup>30</sup> If CODATA (2022) values are required, see <http://physics.nist.gov/cuu/Constants/index.html> .

<sup>31</sup> Because  $u_g$  is approximately  $100^{10}$ ;  $u$ , I add alias name ‘looloh’ ( $\text{lú:lou/əu}$ ,  $\pm 1$ ) to mass unit of the Harmonic System.

$m_G$	gravitic meter $(\sqrt{2E}; l_P)$	1;00186	1;00016	-27;	m
$l_P$	Planck length	2;0445E	2;04134	-28;	m
$F_P$	Planck force $(\hbar c_0 / l_P^2)$	2;XE206	2;XEE32( $\div 2;E$ ) <sup>32</sup>	35;	N
$G$	Newtonian constant of gravitation $(c_0^4 / F_P)$	4;15768	4;14663	-X;	$(m^4/s^4)/N$
$\theta_W$	<u>weak mixing angle</u>	E;304		-2;	$\Omega_1$
$V_m$	molar volume of an ideal gas under standard conditions	1;02X469	1;025665	2;	$m^3/mol$
	black-body radiation at the ice point	0;EX2466	0;EX8784	2;	$W/m^2$
	maximum density of water	1;088183	1;092X47 ( $\div 15;14;$ )	2;	$g/m^3$
	density of ice at the ice point	0;E7E9	0;E85E	2;	$g/m^3$
	specific heat of water <sup>33</sup>	0;6052	0;6045 ( $\div 1/2$ )	0;	$J/(g\ K)$
	surface tension of water at 25°C	0;EE68	0;EEE4	-1;	$N/m$
atm	standard atmosphere	1;65008E	1;659967 ( $\div 1;66$ )	4;	P
$g_n$	standard gravitational acceleration	5;5X54XE9	5;5E21264 ( $\div E;2$ )	0;	$m/s^2$
$r_E$	gravitational radius of Earth	2;41E8982X0X	2;418030652	-2;	m
au	astronomical unit	8;X67575535	8;X55509X31	X;	m
	<u>astronomical unit</u>	9;E91731X53		-3;	$c_0\ s_E\ day$

**Table 6 Power prefixes**

name	symbol	T <sub>EX</sub> text	value	name	symbol	T <sub>EX</sub> text	value
dirac <sup>34</sup>	$\mathcal{V}\#$	dirac	10;				
<b>hyper</b>	$\#$ (x266F)	hyper	<b>10;<sup>4</sup></b>	<b>sub</b>	$\mathfrak{b}$ (x266D)	sub	<b>10;<sup>-4</sup></b>
<b>cosmic</b>	+	$\_+$	<b>10;<sup>8</sup>(=U)</b>	<b>atomic</b>	-	$\_-$	<b>U<sup>-1</sup></b>
di-cosmic	2+	$\_{2+}$	U <sup>2</sup>	di-atomic	2-	$\_{2-}$	U <sup>-2</sup>
ter-cosmic	3+	$\_{3+}$	U <sup>3</sup>	ter-atomic	3-	$\_{3-}$	U <sup>-3</sup>
tetra-cosmic	4+	$\_{4+}$	U <sup>4</sup>	tetra-atomic	4-	$\_{4-}$	U <sup>-4</sup>
penta-cosmic	5+	$\_{5+}$	U <sup>5</sup>	penta-atomic	5-	$\_{5-}$	U <sup>-5</sup>
hexa-cosmic	6+	$\_{6+}$	U <sup>6</sup>	hexa-atomic	6-	$\_{6-}$	U <sup>-6</sup>
hepta-cosmic	7+	$\_{7+}$	U <sup>7</sup>	hepta-atomic	7-	$\_{7-}$	U <sup>-7</sup>

<sup>32</sup> If this is expressed as 2;E, the error from CODATA (2018) becomes -6;61(-6.51) times standard deviation.

<sup>33</sup> This corresponds to the definition of the thermodynamic calorie.

<sup>34</sup> ‘dirac’ is only used when expressing the unit of the Gravitic System with the Harmonic System. (i.e., gravitic meter = tetra-atomic dirac harmon, gravitic second = penta-atomic dirac nic, gravitic gram = atomic dirac looloh)

**Table 7 Examples of natural scale quantity representation <sup>35</sup>**

quantity	symbol	value	refer to
2E; penta-cosmic Newton	2E; <sub>5</sub> N	2E; $\times$ U <sup>5</sup> [harmonic] Newton	the Planck force
6;di-cosmic nic	6; <sub>2</sub> n	6; $\times$ U <sup>2</sup> [harmonic]nic[second]	the age of the universe
cosmic hyper bit [Boltzmann]	+ <sub>#</sub> f <sub>1</sub> [k <sub>B</sub> ]	U <sup>1@4</sup> log2 <sup>1</sup> [Boltzmann]	1.01 Tera Byte(=2 <sup>43</sup> .bit)
cosmic harmon	+h	U <sup>1</sup> harmon[ic meter]	the speed of light in vacuum
ato[mic ][light]	- $\gamma$	harmon[ic meter]/ [harmonic]nic[second]	U <sup>-1</sup> light( $\div$ 2.51 km / hour)
atomic uninol h[armon]	1; '[0].h <sup>36</sup>	U <sup>-1@1</sup> harmon[ic meter]	the Bohr radius
di-atomic Coulomb	<sub>2</sub> .C	U <sup>-2</sup> [universal] Coulomb	the elementary charge
di-atomic effective Watt <sup>37</sup>	<sub>2</sub> . $\bar{W}$	U <sup>-2</sup> [harmonic]effective Watt	a photon power (540.THz)
ter-atomic looloh	<sub>3</sub> .l	U <sup>-3</sup> looloh	the unified atomic mass unit
2; tetra-atomic harmon	2; <sub>4</sub> .h	2; $\times$ U <sup>-4</sup> harmon[ic meter]	the Planck length

**Table 8 The Earth local extension for the Harmonic Universal Unit System**

category		name / description	symbol	plain text	value
Non-coherent calendar time	units	year	☼ <sub>(x263C)</sub>	year	☼ = 365.days 31.nodus (265; <sup>27</sup> ☼)
		month	☾ <sub>(x263D)</sub>	month	☾ = 10; <sup>-1</sup> ☼
		day	<sup>27</sup> ☼ <sub>(x00B0)</sub>	day	Ω <sub>1</sub> = 1 <sup>27</sup> = 10; <sup>7</sup> = 100; <sup>27</sup> = 1000; <sup>36</sup>
		unino day dino day terno day (tertiary 12 divisions of one day)	<sup>27</sup> ☼ <sub>(x2032)</sub> <sup>27</sup> ☼ <sub>(x2033)</sub> <sup>27</sup> ☼ <sub>(x2034)</sub>	unitia ditia tertia	‘day’ corresponds to 86,400. s at the beginning of year 1900. <b>Each calendar time unit symbol is distinguished from existing systems by adding a tilde (“~”, x0303) or by superscripting the symbol itself.</b>
Non-coherent unit and constants		nodus	☼ <sub>(x2606)</sub>	nodus	<sup>27</sup> = 2 <sup>+7</sup> ☼
		terno nodus → terno n(odus) → ternon	☼ <sub>(x25BD)</sub>	ternon	☼ <sup>27</sup> = 10; <sup>-3</sup> ☼
		hexaon nodus → hex(a)o(n) n(odus) → hexon	☼ <sub>(x232C)</sub>	hexon	☼ <sup>27</sup> = 2 <sup>+6</sup> ☼ = 1; 003628 × 10; <sup>+6</sup> ☼
		difference between thermodynamic temperature and <b>T<sub>E</sub></b> (=118,2354; ±K (-74.36°C, -101.85°F))	°H	deg H	1,0000; ±K (÷1.210724 K ÷ 23./19. K)
		approximate formula			
		°C = $\frac{1E;}{17;}$ °H - 62;4    °H = $\frac{17;}{1E;}$ °C + 51;5			
supple- mentary constants		the gravitational acceleration of the Earth (is called ‘gee [of Earth] ’)	<b>g<sub>E</sub></b>	g_ E or gee	5;611X615 harmon/nic <sup>2</sup> g <sub>E</sub> is defined as c <sub>0</sub> <sup>2</sup> r <sub>E</sub> (m <sub>E</sub> rad) <sup>-2</sup>
		the rotation period of the Earth (is called ‘[Earth] solar’) at the beginning of year 1900.	<b>s<sub>E</sub></b>	s_ E or solar	0;EEEEEE15336X nic/ ternon (This should be ‘coordinated’ . <sup>17</sup> )
		the meridian length of the Earth (is called ‘[Earth] meridian’)	<b>m<sub>E</sub></b>	m_ E or meridian	4124,216E; harmon/Ω <sub>1</sub>

<sup>35</sup> The part enclosed with '[']' can be omitted in Table 7 and Table 8.

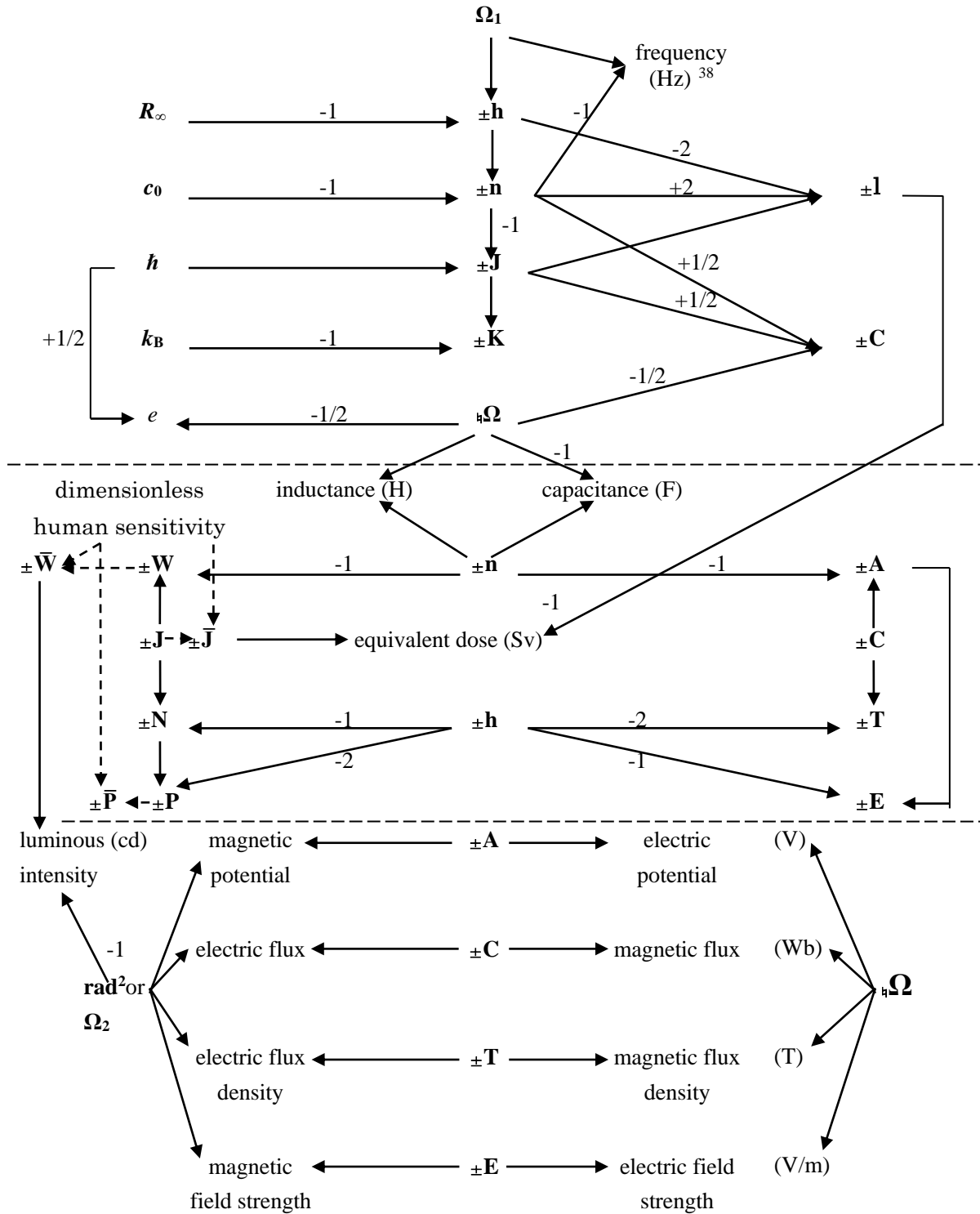
<sup>36</sup> This is the notation explained at the end of Appendix C.

$\bar{W}$  corresponds to 1;di-cosmic photon energy(540.THz) / nic and 115.667212 lumen.

<sup>37</sup> Units for quantity weighted by dimensionless human sensitivity are indicated by 'effective' and symbolled by overline.

$\bar{W}$  corresponds to 1;di-cosmic photon energy(540.THz) / nic and 115.667212 lumen.

## E. Relation of Units and Dimensions



<sup>38</sup> The units enclosed with '()' are units of SI.

Force between electrical quantities

$$f = \frac{1}{\epsilon_0} \frac{\Omega_2 Q}{4\pi r^2} Q' = \Omega_n c_0 \frac{Q Q'}{r^2}$$

Force between electrical currents

$$df = \mu_0 \frac{\Omega_2 I}{2\pi r} I' = \frac{2\Omega_n}{c_0} \frac{I I'}{r}$$

Lorentz force

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy density of an electromagnetic field

$$u = \frac{1}{2\Omega_2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B})$$

Poynting vector

$$\mathbf{S} = \frac{1}{\Omega_2} \mathbf{E} \times \mathbf{H}$$

Electromagnetic induction law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Gauss' theorem (differential form)

$$\nabla \times \mathbf{H} = +\frac{\partial \mathbf{D}}{\partial t} + \Omega_2 \mathbf{J}$$

Charge conservation law

$$\begin{cases} \nabla \cdot \mathbf{D} = \Omega_2 \rho \\ \nabla \cdot \mathbf{B} = 0 \end{cases}$$

Scalar potential

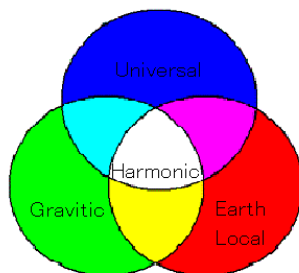
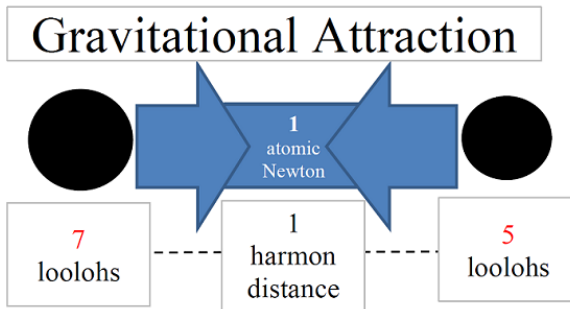
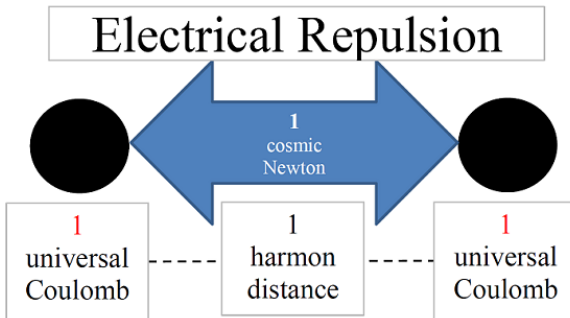
$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

Vector potential

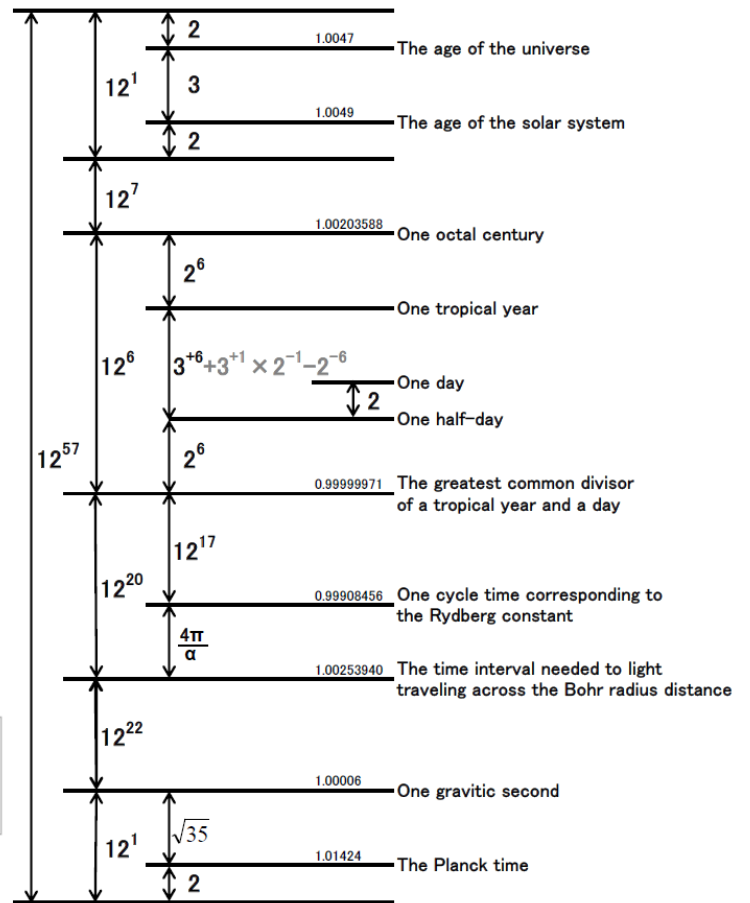
$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

Equation that satisfies the potential

$$\begin{cases} \Delta \phi - \epsilon_0 \mu_0 \frac{\partial^2 \phi}{\partial t^2} = -\Omega_2 \frac{\rho}{\epsilon_0} \\ \Delta \mathbf{A} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\Omega_2 \mu_0 \mathbf{J} \end{cases}$$



## Natural Time Scale



1 hexon = 2 <sup>6</sup> years ≐ 10;(12.) <sup>6</sup> nodus = 10;(12.) <sup>9</sup> ternons			
0 <sup>th</sup> year	1 <sup>st</sup> year	...	53;(63.) <sup>rd</sup> year
1 year = 10;(12.)months			
0 <sup>th</sup> month	1 <sup>st</sup> month	...	E;(11.) <sup>th</sup> month
1 month = 26;(30.)days or 27;(31.)days			
0 <sup>th</sup> day	1 <sup>st</sup> day	...	last day
1 day = 10; <sup>3</sup> (12. <sup>3</sup> ) tertias			
0 <sup>th</sup> tertia	1 <sup>st</sup> tertia	...	EEE;(1727.) <sup>th</sup> tertia
1 tertia = 2 <sup>7</sup> ternons			
0 <sup>th</sup> ternon	1 <sup>st</sup> ternon	...	X7;(127.) <sup>th</sup> ternon

## F. Ratios of fundamental physical constants

### F.1. The fine structure constant and the elementary charge

The fine structure constant,  $\alpha$ , a dimensionless quantity, was originally introduced for the purpose to explain the fine structure spectral emission lines.

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c_0 \hbar} \quad (\text{X})$$

By multiplying both sides of Eq. (X) by  $\frac{c_0 \hbar}{r^2}$ , we get

$$\alpha \frac{c_0 \hbar}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad (\text{E})$$

The right side of Eq. (E) expresses the Coulomb force acting between two elementary charges (i.e., the electrical charge of an electron) separated by a distance of  $r$ . The left side indicates that this force is proportional to  $\frac{c_0 \hbar}{r^2}$  by a factor of  $\alpha$ . For this reason, the fine structure constant,  $\alpha$ , can be interpreted as a dimensionless quantity that represents the strength of electromagnetic interaction.

The value of the fine structure constant,  $\alpha$ , is close to  $10^{-2}$  (12<sup>-2</sup>).

DOZENAL

$$\alpha = \frac{1}{\text{E5;052258E7}} = 1;073994047 \times 10^{-2}$$

DECIMAL

(10;)

$$\alpha = \frac{1}{137.03599918} = 1.050818769 \times 12^{-2}$$

Therefore, the ratio of the elementary charge,  $e$ , and “the dimensioned quantity of charge, which is derived from the speed of light in vacuum,  $c_0$ , and the quantum of action,  $\hbar$ ” is:

DOZENAL

$$\alpha^{\frac{1}{2}} = \frac{e}{\sqrt{4\pi\epsilon_0 c_0 \hbar}} = 1;0374439\text{E1} \times 10^{-1}$$

DECIMAL

(11;)

$$\alpha^{\frac{1}{2}} = \frac{e}{\sqrt{4\pi\epsilon_0 c_0 \hbar}} = 1.025094517 \times 12^{-1}$$

### F.2. The Rydberg constant and the Bohr radius

The deviation of the fine structure constant,  $\alpha$ , from an integer power of twelve is nearly the same as the deviation of  $4\pi$  from twelve.

DOZENAL

$$4\pi = 1;069683171 \times 10^{-1} = \frac{1}{\text{E5;6150822}} \times 10^{-3}$$

DECIMAL

(12;)

$$4\pi = 1.047197551 \times 12^1 = \frac{1}{137.5098708} \times 12^3$$



The ratio of the Bohr radius,  $a_B$ , and “the dimensioned quantity of length,  $L=R_\infty^{-1}$ , where  $R_\infty$  is the Rydberg constant” is:

DOZENAL

$$\frac{a_B}{L} = \frac{\alpha}{4\pi} (\text{strict}) = 1;005\text{E}85684 \times 10;^{-3}$$

DECIMAL

$$\frac{a_B}{L} = \frac{\alpha}{4\pi} (\text{strict}) = 1.003458009 \times 12^{-3}$$

(13;)

### F.3. The electron mass and the unified atomic mass unit

The ratio of the mass of an electron,  $m_e$ , and “the dimensioned quantity of mass,  $M$ , which is derived from  $L$ , the speed of light in vacuum,  $c_0$ , and the quantum of action  $\hbar$ ,”

$$M = \frac{\hbar}{c_0 L}, \quad (14;)$$

is:

DOZENAL

$$\frac{m_e}{M} = \frac{4\pi}{\alpha^2} (\text{strict}) = 0;\text{E}4692218 \times 10;^5$$

DECIMAL

$$\frac{m_e}{M} = \frac{4\pi}{\alpha^2} (\text{strict}) = 0.948359448 \times 12^5$$

(15;)

The ratio of the mass of an electron,  $m_e$  and the unified atomic mass unit,  $u$ , is:

DOZENAL

$$\frac{m_e}{u} = \frac{1}{107\text{X};\text{X}7\text{E}4} = \frac{4\pi}{\alpha^2} \times 0.\text{EEE}2\text{E}66 \times 10;^{-8}$$

DECIMAL

$$\frac{m_e}{u} = \frac{1}{1822.8885} = \frac{4\pi}{\alpha^2} \times 0.9995641 \times 12^{-8}$$

(16;)

This ratio corresponds to the ratio of typical nuclear energy and chemical energy. The deviations of ratio Eq. (15;) and ratio Eq. (16;) from multiples of an integer power of twelve are near to the same magnitude. Therefore:

DOZENAL

$$\frac{u}{M} = 1;0009060\text{E} \times 10;^8$$

DECIMAL

$$\frac{u}{M} = 1.00043606 \times 12^8$$

(17;)

### F.4. The Planck length

The ratio of the general expression of the Planck length,  $\sqrt{\frac{G\hbar}{c_0^3}}$ , and  $L$  is close to 2 when factors of multiples of an integer power of twelve are eliminated.

DOZENAL

$$\sqrt{\frac{G\hbar}{c_0^3}}/L = 2 \times 1;0222E \times 10;^{-22}$$

DECIMAL

(18;)

$$\sqrt{\frac{G\hbar}{c_0^3}}/L = 2 \times 1.01519 \times 12^{-26}$$

Taking the expression  $\sqrt{\frac{G\hbar}{c_0^3\alpha}}$ , which has been adjusted<sup>39</sup> by the fine structure constant,  $\alpha$ , in order to express the tensile force in a superstring in terms of the Planck length, the ratio of the adjusted Planck length and  $L$  then becomes:

DOZENAL

$$\sqrt{\frac{G\hbar}{c_0^3\alpha}}/L = 2 \times 0;EX737 \times 10;^{-21}$$

DECIMAL

(19;)

$$\sqrt{\frac{G\hbar}{c_0^3\alpha}}/L = 2 \times 0.99034 \times 12^{-25}$$

The Gravitic Universal Unit System uses 140.0 to approximate  $\alpha^{-1}(=137.03599918)$ . Because 1,0017;(20,755.=35.  $\times$  593.) is divisible with 2E;(35.), we can approximate  $G$  by Eq.(1X;) about 6.51 times standard deviation error of CODATA(2018):

$$G \doteq \frac{(5 \times 7)^2 \times 415;^2 c_0^3}{5 \times 7 \times \hbar R_{\odot}^2} \times 10;^{-4X} = \frac{5 \times 7 \times 415;^2 c_0^3}{\hbar R_{\odot}^2} \times 10;^{-4X} \quad (1X;)$$

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<sup>39</sup> See E. Witten, 'Reflections on the fate of spacetime' p. 24. in April 11X4;(1996.) *Physics Today*.