Probability Practice Questions

Ques1. The client company data from the Decision Dilemma reveal that 155 employees worked one of four types of positions. Shown here again is the raw values matrix (also called a contingency table) with the frequency counts for each category and for subtotals and totals containing a breakdown of these employees by type of position and by sex. If an employee of the company is selected randomly, what is the probability that the employee is female or a professional worker?

COMPANY HUMAN RESOURCE DATA

		Sex		
		Male	Female	
Type of Position	Managerial	8	3	11
	Professional	31	13	44
	Technical	52	17	69
	Clerical	9	22	31
		100	55	155

Ans.

Let F denote the event of female and P denote the event of professional worker. The question is P (F U P) = ?

By the general law of addition,

P (F U P) = *P* (F) + *P* (P) - *P* (F ∏ P)

Of the 155 employees, 55 are women. Therefore, P (F) = 55 155 = .355. The 155 employees include 44 professionals. Therefore, P (P) = 44 155 = .284. Because 13 employees are both female and professional, P (F P) = 13 155 = .084. The union probability is solved as

P (F U P) = .355 + .284 - .084 = .555.

Ques2. Shown here are the raw values matrix and corresponding probability matrix for the results of a national survey of 200 executives who were asked to identify the geographic locale of their company and their company's industry type. The executives were only allowed to select one locale and one industry type.

RAW VALUES MATRIX

		Geographic Location				
		Northeast D	Southeast E	Midwest F	West G	
Industry Type	Finance A	24	10	8	14	56
	Manufacturing B	30	6	22	12	70
	Communications C	28	18	12	16	74
		82	34	42	42	200

Suppose a respondent is selected randomly from these data.

a. What is the probability that the respondent is from the Midwest (F)?

b. What is the probability that the respondent is from the communications industry (C) or from the Northeast (D)?

c. What is the probability that the respondent is from the Southeast (E) or from the finance industry (A)?

Ans.

Ref: Ken Black

PROBABILITY MATRIX

Industry Type	Geographic Location: Northeast D	Southeast E	Midwest F	West G	
Finance A	.12	.05	.04	.07	.28
Manufacturing B	.15	.03	.11	.06	.35
Communications C	.14	.09	.06	.08	.37
	.41	.17	.21	.21	1.00

a. P (Midwest) = P (F) = .21
b. P (C D) = P (C) + P (D) - P (C D) = .37 + .41 - .14 = .64
c. P (E ´ A) = P (E) + P (A) - P (E ¨ A) = .17 + .28 - .05 = .40

Ques3. If a worker is randomly selected from the company described in Problem 1, what is the probability that the worker is either technical or clerical? What is the probability that the worker is either a professional or a clerical?

Ans.

Let T denote technical, C denote clerical, and P denote professional. The probability that a worker is either technical or clerical is

P (T U C) = *P* (T) + *P* (C) = 69/155 + 31/155 = 100/155 = .645

The probability that a worker is either professional or clerical is

P(P U C) = P (P) + P (C) = 44/155 + 31/155 = 75/155 = .484

Ques4. Use the data from the matrices in Problem 2. What is the probability that a randomly selected respondent is from the Southeast or the West?

P (E U G) = ?

Ans.

Because geographic location is mutually exclusive (the work location is either in the Southeast or in the West but not in both),

P (E U G) = P (E) + P (G) = .17 + .21 = .38

Ques5. A company has 140 employees, of which 30 are supervisors. Eighty of the employees are married, and 20% of the married employees are supervisors. If a company employee is randomly selected, what is the probability that the employee is married and is a supervisor?

Ans.

Let M denote married and S denote supervisor. The question is:

P (M ∏ S) = ?

First, calculate the marginal probability.

P (M) = 80/140 = .5714

Then, note that 20% of the married employees are supervisors, which is the conditional probability, *P* (S|M) = .20. Finally, applying the general law of multiplication gives

(M ∏ S) = *P* (M) * *P* (S|M) = (.5714)(.20) = .1143

Hence, 11.43% of the 140 employees are married and are supervisors.

Ref: Ken Black

Ques6. From the data obtained from the interviews of 200 executives in Problem 2, find:
a. P (B ∏ E)
b. P (G ∏ A)
c. P (B ∏ C)
Ans.

PROBABILITY MATRIX

Industry Type	Geographic Location: Northeast D	Southeast E	Midwest F	West G	
Finance A	.12	.05	.04	.07	.28
Manufacturing B	.15	.03	.11	.06	.35
Communications C	.14	.09	.06	.08	.37
	.41	.17	.21	.21	1.00

a. From the cell of the probability matrix, P (B ∏ E) = .03. To solve by the formula, P (B ∏ E) = P (B) * P (E|B), first find P (B):
P (B) = .35
The probability of E occurring given that B has occurred, P (E|B), can be determined from the probability matrix as P (E|B) = .03/.35.
Therefore, P (B ∏ E) = P (B) * P (E|B) = (.35) (.03/.35) = .03

b. (G ∏ A) = P (G)*P (A|G) = (.21) (.07/.21) = .07
or
P (G ∏ A) = P (A)*P (G |A) = (.28) (.07/.28) = .07

c. The probability P (B ∏ C) means that one respondent would have to work both in the manufacturing industry and the communications industry. The survey used to gather data from the 200 executives, however, requested that each respondent specify only one industry type for his or her company. The matrix shows no intersection for these two events. Thus B and C are mutually exclusive. None of the respondents is in both manufacturing and communications. Hence,
P (B ∏ C) = .0

Ques7. A manufacturing firm produces pads of bound paper. Three percent of all paper pads produced are improperly bound. An inspector randomly samples two pads of paper, one at a time. Because a large number of pads are being produced during the inspection, the sampling being done, in essence, is with replacement. What is the probability that the two pads selected are both improperly bound?
Ans.
Let I denote improperly bound. The problem is to determine
$P(I_1 \cap I_2) = ?$
The probability of I = .03, or 3% are improperly bound. Because the sampling is done with replacement, the two events are independent. Hence,

P (I1 ∏ I2) = P (I1) * P (I2) = (.03)(.03) = .0009

Ques8. Use the data from Table below and the special law of multiplication to find P (B ∏ D).

Contingency Table of Data from Independent Events

	D	E	
A	8	12	20
B	20	30	50
C	6	9	15
	34	51	85

Ans.
P (B ∏ D) = P (B) * P (D) = (50/85) * (34/85) = .2353

Ques9. The data from the executive interviews given in Problem 2 are repeated here. Use these data to find:
a. P (B|F)
b. P (G|C)
c. P (D|F)
Ans.

PROBABILITY MATRIX

Industry Type	Geographic Location: Northeast D	Southeast E	Midwest F	West G	
Finance A	.12	.05	.04	.07	.28
Manufacturing B	.15	.03	.11	.06	.35
Communications C	.14	.09	.06	.08	.37
	.41	.17	.21	.21	1.00

a. P (B|F) = P (B ∏ F)/P (F) = .11/.21 = .524

b. P (G|C) = P (G ∏ C)/P (C) = .08/.37 = .216

c. P (D|F) = P (D ∏ F)/P (F) = .00/.21 = .00
Because D and F are mutually exclusive, P (D ∏ F) is zero

Ques10. Test the matrix for the 200 executive responses to determine whether industry type is independent of geographic location.

RAW VALUES MATRIX

Industry Type	Geographic Location: Northeast D	Southeast E	Midwest F	West G	
Finance A	24	10	8	14	56
Manufacturing B	30	6	22	12	70
Communications C	28	18	12	16	74
	82	34	42	42	200

Ans.
Select one industry and one geographic location (say, A-Finance and G-West). Does P(A|G) = P(A)?
P (A|G) = 14/42 and P (A) = 56/200

Does 14/42 = 56/200? No, .33 .28. Industry and geographic location are not independent because at least one exception to the test is present

Ques11. Determine whether the contingency table shown below contains independent events.

	D	E	
A	8	12	20
B	20	30	50
C	6	9	15
	34	51	85

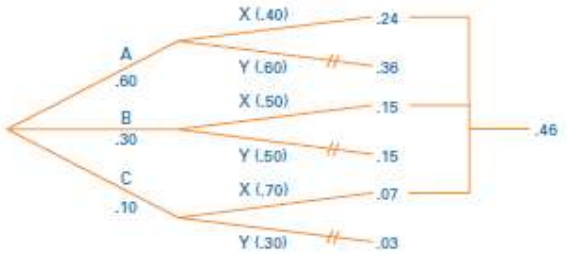
Ans.
Check the first cell in the matrix to find whether P (A|D) = P (A).
P(A|D) = 8/34 = .2353
P (A) = 20/85 = .2353
The checking process must continue until all the events are determined to be independent.

Ques12. Machines A, B, and C all produce the same two parts, X and Y. Of all the parts produced, machine A produces 60%, machine B produces 30%, and machine C produces 10%. In addition,
40% of the parts made by machine A are part X.
50% of the parts made by machine B are part X.
70% of the parts made by machine C are part X.
A part produced by this company is randomly sampled and is determined to be an X part. With the knowledge that it is an X part, revise the probabilities that the part came from machine A, B, or C.

Ans.

Event	Prior $P(E_i)$	Conditional $P(X \mid E_i)$	Joint $P(X \cap E_i)$	Posterior
A	.60	.40	(.60)(.40) = .24	$\frac{.24}{.46} = .52$
B	.30	.50	.15	$\frac{.15}{.46} = .33$
C	.10	.70	.07	$\frac{.07}{.46} = .15$
			$P(X) = .46$	

Probability Practice Questions



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