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SS ZG 526: Distributed Computing (CS2)

Logical Time

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- concept of causality between events -> important for design and analysis of parallel and distributed computing and operating systems
- causality is tracked using physical time
- But in distributed systems
 - not possible to have global physical time
 - possible to realize only an approximation of physical time
- way of measuring time
 - asynchronous distributed computations progress in spurts
 - logical time also advances in jumps
 - sufficient to capture the fundamental monotonicity property associated with causality in distributed systems

- ❖ monotonicity property: if event a causally affects event b, then timestamp of a is smaller than timestamp of b
- Ways to represent logical time:
 - * scalar time
 - vector time

- Causality / Causal Precedence Relation
- described among events in a distributed system
- useful in reasoning, analyzing, and drawing inferences about a computation
- helps to solve several problems in distributed systems
 - distributed algorithms design
 - tracking of dependent events
 - knowledge about progress
 - concurrency measure

> Distributed algorithms design

knowledge of the causal precedence relation among events helps to:

- •ensure liveness and fairness in mutual exclusion algorithms
- maintain consistency in replicated databases
- design correct deadlock detection algorithms

> Tracking of dependent events

- helps construct a consistent state for resuming re-execution
- •in failure recovery, helps build a checkpoint
- •in replicated databases, aids in detection of file inconsistencies



- Knowledge about progress knowledge of causal dependency among events
 - •helps measure progress of processes in distributed computation
 - •helps to discard obsolete information, garbage collection, and termination detection
- Concurrency measure knowledge regarding how many events are causally dependent
 - useful in measuring amount of concurrency
 - events not causally related are concurrent
 - analysis of causality tells about concurrency in program

Capturing Causality between Events



- •in distributed systems
 - rate of event occurrence is of high magnitude
 - event execution time is of low magnitude
- physical clocks must be precisely synchronized
- in distributed computation
 - clocks accurate to a few tens of milliseconds are not sufficient
 - progress occurs in spurts
 - interaction between processes occurs in spurts
- logical clocks can accurately capture
 - causality relation between events produced by a program execution
 - •fundamental monotonicity property

Logical Clocks



- •every process has a logical clock
- •logical clock is advanced using a set of rules
- each event is assigned a timestamp
- •causality relation between events can be inferred from their timestamps
- timestamps follow the monotonicity property

Representing Logical Clocks

- 1. Lamport's scalar clocks
 - time is represented by non-negative integers
- 2. Vector clocks (Fidge, Mattern and Schmuck)
 - time is represented using a vector of non-negative integers

Real world applications of Logical clocks

- ❖ Scalar clocks => Amazon S3
- Vector clocks => Voldemort (Linkedin) Amazon Dynamo, Version control systems etc.

Self assessment Question:

Why Lamport's clock cannot be used in blockchain technology?

Definition of Logical Clocks

- system of logical clocks consists of
 - •time domain T
 - •logical clock C
- •elements of T form a partially ordered set over a relation <
- •<:
 - happened before or causal precedence relation
 - analogous to 'earlier than' relation provided by physical time

Definition of Logical Clocks

- •logical clock C
 - •is a function that maps an event *e* in a distributed system to an element in the time domain *T*
 - •denoted as C(e)
 - •C(e): timestamp of e
 - •C is defined as: $C: H \mapsto T$
 - satisfies the following property
 - for any 2 events e_i and e_j , $e_i \rightarrow e_j \Longrightarrow C(e_i) < C(e_j)$.
 - monotonicity property
 - known as clock consistency condition

Definition of Logical Clocks

- •system of clocks is *strongly consistent* if *T* and *C* satisfy the following:
 - •for 2 events e_i and e_j ,

$$e_i \rightarrow e_j \Leftrightarrow \mathsf{C}(e_i) < \mathsf{C}(e_j)$$

- Two issues need to be addressed
 - data structures local to every process to represent logical time
 - a protocol to update the data structures to ensure the consistency condition

Data Structures

- •Each process p_i maintains two data structures
- •A local logical clock
 - •denoted by *lc_i*
 - •helps process p_i measure its own progress
- •A logical global clock
 - •denoted by gci
 - •represents p_i 's local view of the logical global time
 - •allows p_i to assign consistent timestamps to its local events
 - •lc; is a part of gc;



<u>Protocol - ensures consistent management of:</u>

- •a process's logical clock
- process's view of global time
- consists of the following 2 rules:
- R1: governs how the local logical clock is updated by a process when it executes an event (send, receive, or internal)
- R2:
 - •governs how a process updates its global logical clock to update its view of the global time and global progress
 - •dictates what information about logical time is piggybacked in a message
 - •how this information is used by the receiving process to update its view of the global time

- systems of logical clocks differ in
 - representation of logical time
 - the protocol to update the logical clocks
- •all logical clock systems
 - •implement rules **R1** and **R2**
 - •ensure the fundamental monotonicity property associated with causality
 - provide users with some additional properties

Scalar Time - Definition

- •representation was proposed by Lamport in 1978 to totally order events in distributed system
- •in this representation, time domain is the set of non-negative integers
- •*C_i*:
 - integer variable
 - •denotes logical local clock of p_i and its local view of global time

Scalar Time - Definition

Rules for updating clock:

•R1: Before executing an event (send, receive, or internal), process p_i executes $C_i := C_i + d$ (d > 0)

For each execution of R1,

- •d can have different value
- •value of d may be application-dependent

Usually d = 1

- helps to identify the time of each event uniquely at a process
- •keeps rate of increase of d to lowest level

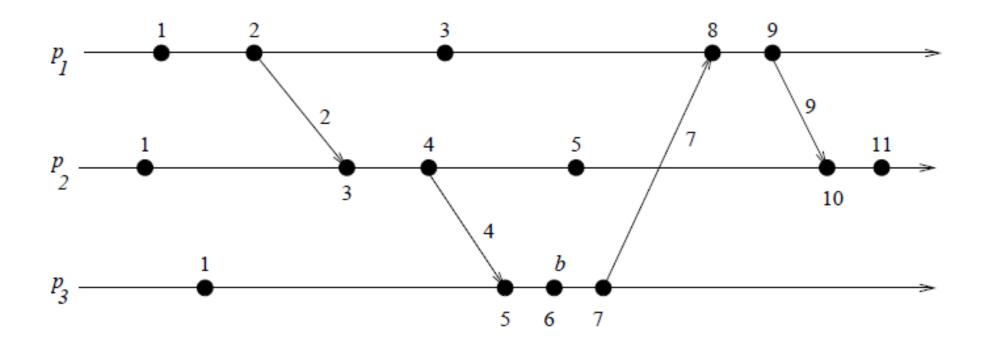
Scalar Time - Definition

Rules for updating clock (contd. from previous slide):

R2: When process p_i receives a message with timestamp C_{msg} , it executes the following actions:

- 1. $C_i = max(C_i, C_{msq});$
- 2. execute **R1**;
- 3. deliver the message.

Evolution of Scalar Time



Space-time diagram of a distributed system

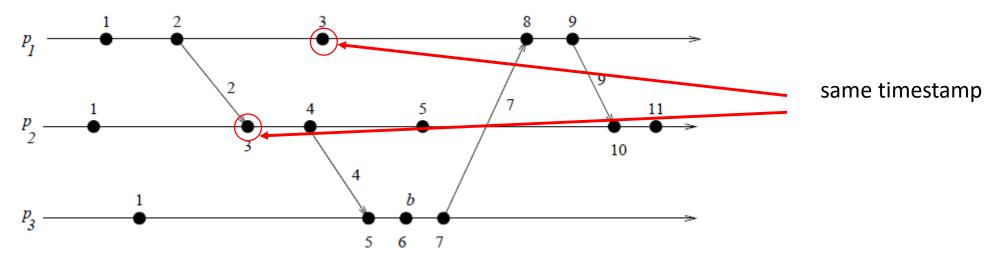
- Consistency
- Total Ordering
- Event Counting
- No Strong Consistency

Consistency Property

- for two events e_i and e_j , $e_i \rightarrow e_j \Longrightarrow C(e_i) < C(e_j)$
- scalar clocks satisfy monotonicity property
- hence, they also satisfy consistency property

Total Ordering

- Scalar clocks are used to totally order events in a distributed system
- Problem in totally ordering events 2 or more events at different processes may have same timestamp for 2 events e_1 and e_2 , $C(e_1) = C(e_2) \Longrightarrow e_1 \parallel e_2$



Total Ordering

- Tie breaking mechanism is needed
- Tie breaking procedure:
 - process identifiers are linearly ordered
 - break tie among events with identical scalar timestamps on the basis of their process identifiers
 - lower process identifier implies higher priority
 - timestamp of an event is denoted by a tuple (t, i)
 - $t \rightarrow$ time of occurrence
 - i → identity of the process where it occurred

Total Ordering

• Total order relation \prec on 2 events x and y with timestamps (h, i) and (k, j) respectively, is defined:

$$x \prec y \Leftrightarrow h < k \text{ or } (h = k \text{ and } i < j)$$

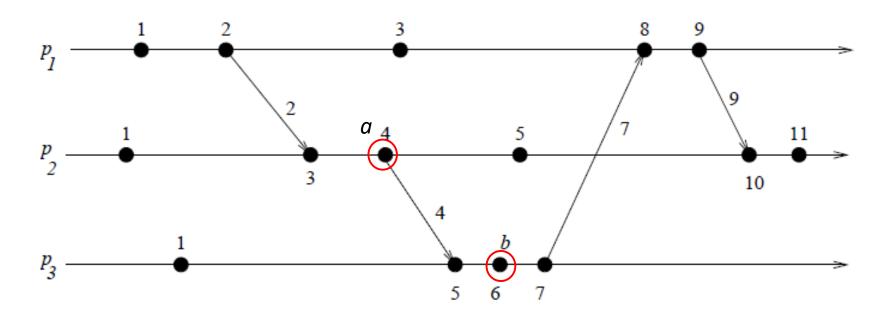
- events that occur at the same logical scalar time are independent (i.e., they are not causally related)
- such events can be ordered using any arbitrary criterion without violating the causality relation ->
- so, total order is consistent with the causality relation " \rightarrow "
- Note:- $x < y \Rightarrow x \rightarrow y \lor x \mid \mid y$

Total Ordering

- Utility
 - total order is generally used to ensure liveness properties in distributed algorithms
 - Liveness property:
 - something good eventually happens
 - system makes progress, no starvation, programs terminate
 - requests are timestamped and served according to the total order based on these timestamps

Event Counting

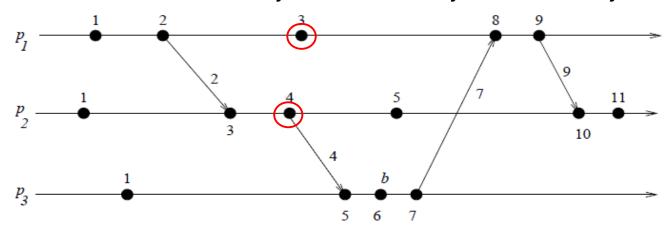
- If the increment value d = 1, scalar time has the following property:
 - if event e has a timestamp h, then h-1
 - is the minimum logical duration required before producing event e
 - counted in units of events
 - called the *height* of event *e*
 - implies that *h* 1 events have been produced sequentially before event *e*, irrespective of the processes that produced these events



- height of event a = 4 1 = 3, i.e., 3 events precede a on the longest causal path ending at a
- height of event b = 6 1 = 5, i.e., 5 events precede b on the longest causal path ending at b

No strong consistency

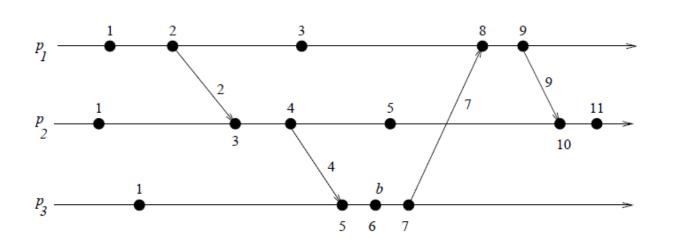
- system of scalar clocks is not strongly consistent
- for 2 events e_i and e_j , $C(e_i) < C(e_j) \not\Rightarrow e_i \rightarrow e_j$



- scalar timestamp of 3rd event of p_1 < scalar timestamp of 3rd event of p_2
- but the former did not happen before the latter

No strong consistency

- scalar clocks are not strongly consistent because logical local clock and logical global clock of a process are combined into one
- results in the loss causal dependency information among events at different processes



- when p_2 receives the 1st message from p_1 , p_2 updates its clock to 3
- forgets that the timestamp of the latest event at p_1 on which it depends is 2

- time domain is represented by a set of *n*-dimensional nonnegative integer vectors
- each process p_i maintains a vector $vt_i[1..n]$
 - vt_i[i] : local logical clock of p_i
 - describes progress of logical time at p_i
 - $vt_i[j]$: represents process p_i 's latest knowledge of process p_j 's local time
 - if $vt_i[j] = x$
 - p_i knows that local time at p_j has progressed till x
- vector vt_i constitutes p_i 's view of the global logical time
- vt_i is used to timestamp events

Rules used by p_i for clock updating

• R1: Before executing an event, process p_i updates its local logical time as follows:

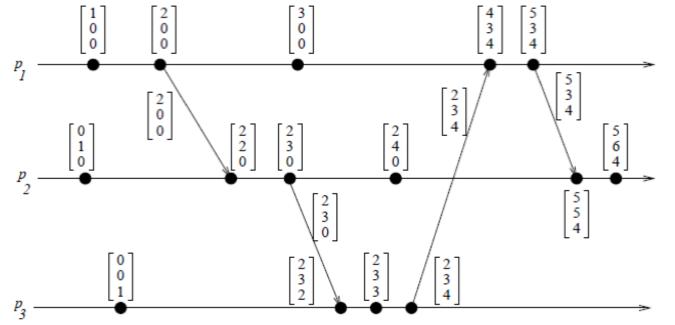
$$vt_i[i] = vt_i[i] + d, d > 0$$

Rules used by p_i for clock updating

- R2:
 - each message m is piggybacked with the vector clock vt of the sender process at sending time
 - when p_i receives message (m, vt), it executes the following sequence of actions:
 - 1. update its global logical time as: $1 \le k \le n$: $vt_i[k] = max(vt_i[k], vt[k])$;
 - 2. execute R1;
 - 3. deliver the message *m*.

Rules used by p_i for clock updating

- R2:
 - timestamp associated with an event is the value of the vector clock of its process when the event is executed
 - initially, a vector clock is [0, 0, 0,, 0]



vector clock's progress with d = 1

Relations for comparing 2 vector timestamps vh and vk

- $vh = vk \Leftrightarrow \forall x : vh[x] = vk[x]$
- $vh \le vk \iff \forall x : vh[x] \le vk[x]$
- $vh < vk \Leftrightarrow vh \leq vk \text{ and } \exists x : vh[x] < vk[x]$
- $vh \mid | vk \Leftrightarrow \neg(vh < vk) \land \neg(vk < vh)$

Isomorphism

Strong Consistency

Event Counting

Isomorphism

- if events in a distributed system are timestamped using a system of vector clocks, following property holds:
 - if 2 events x and y have timestamps vh and vk, respectively, then

$$x \rightarrow y \Leftrightarrow vh < vk$$

 $x \mid \mid y \Leftrightarrow vh \mid \mid vk$

- there is an isomorphism between the set of partially ordered events produced by a distributed computation and their vector timestamps
- very powerful, useful, and interesting property of vector clocks

Isomorphism

- if the process at which an event occurred is known, test for comparing 2 timestamps can be simplified as:
 - if events x and y respectively occurred at processes p_i and p_j and have timestamps vh and vk respectively, then

$$x \rightarrow y \Leftrightarrow vh[i] \leq vk[i]$$

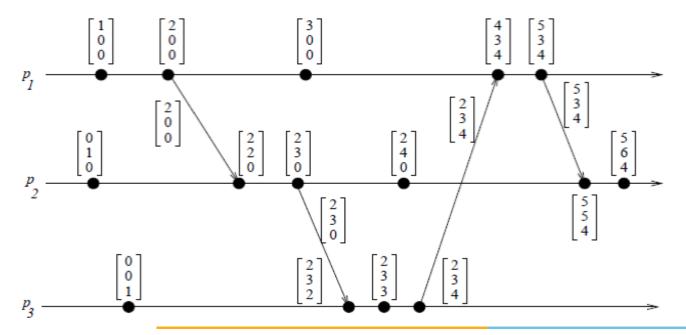
 $x \mid | y \Leftrightarrow vh[i] > vk[i] \land vh[j] < vk[j]$

Strong Consistency

- system of vector clocks is strongly consistent
- examination of vector timestamps of 2 events can determine if the events are causally related
- dimension of vector clocks can't be less than n for this property to hold
 - (n = total no. of processes in the distributed computation)

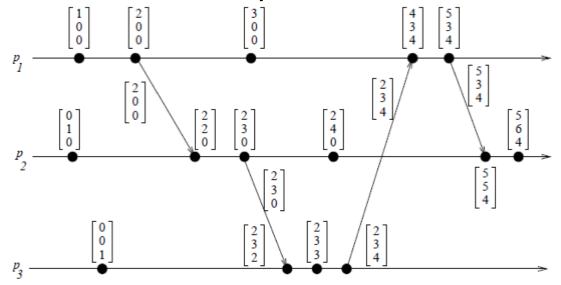
Event counting

- if d is always 1 in rule $\mathbf{R1}$ and $vt_i[i]$ is the i^{th} component of vector clock at process p_i
 - then $vt_i[i]$ = no. of events that have occurred at p_i until that instant



Event counting

- •if an event e has timestamp vh
 - vh[j] = number of events executed by process p_j that causally precede e
 - $\Sigma vh[j] -1$: total no. of events that causally precede e in the distributed computation



Efficient Implementation of Vector Clocks

Why is efficient implementation necessary?

- when no. of processes in a distributed computation is large
 - vector clocks will require piggybacking of huge amount of information in messages
 - messages are required for disseminating time progress and updating clocks
 - message overhead grows linearly with the no. of processes
 - message overhead is not affected by the no. of events occurring at the processors

Singhal–Kshemkalyani's Differential Technique



- based on the observation between successive message sends to the same process, only a few entries of the vector clock at the sender process are likely to change
 this is more likely when the number of processes is large
 - reason: only a few of them will interact frequently by passing messages
- fundamental idea behind the technique
 - when process p_i sends a message to process p_j , it piggybacks only those entries of its vector clock that differ since the last message sent to p_i

Singhal-Kshemkalyani's Differential Technique

- •if entries i_1 , i_2 ,, i_{n_1} of the vector clock at p_i have changed to v_1 , v_2 ,, v_{n_1} , respectively, since the last message sent to p_j , •then p_i piggybacks a compressed timestamp of the form $\{(i_1, v_1), (i_2, v_2),, (i_{n_1}, v_{n_1})\}$ to the next message to p_i
- on receiving this message, p_j updates its vector clock as follows: $vt_j[i_k] = max(vt_j[i_k], v_k)$ for $k = 1, 2, ..., n_1$

> Benefit:

reduces message size, communication bandwidth and buffer (to store messages) requirements

Worst case:

- \triangleright every element of the vector clock has been updated at p_i since the last message sent to p_i
- \triangleright next message from p_i to p_j will need to carry the entire vector timestamp of size n
- Average case: size of the timestamp on a message will be less than
 n

Requirements for implementation:

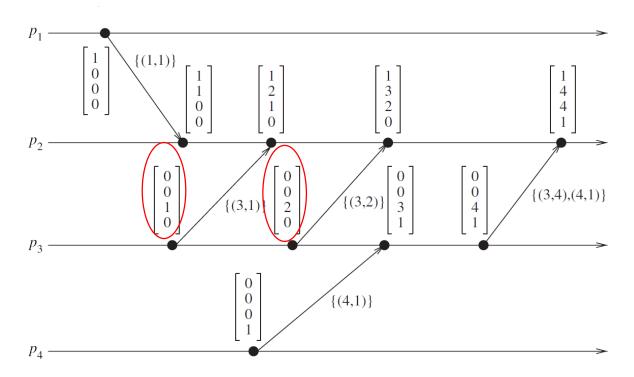
- •each process must remember the vector timestamp in the message last sent to every other process
- •communication channels follow FIFO discipline for delivery of messages

Singhal–Kshemkalyani's Differential Technique



in general, the timestamp is of the form:
 {(p₁, latest_value), (p₂, latest_value),...}

p_i indicates p_ith component of the vector clock has changed



- 2nd message from p₃ to p₂ contains a timestamp {(3, 2)}
- informs p₂ that the 3rd component of the vector clock has been modified and the new value is 2

Benefits:

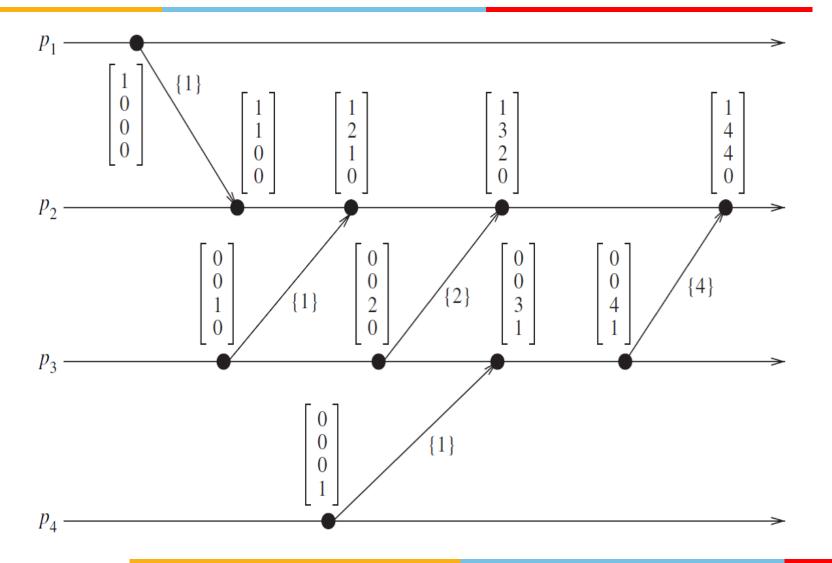
- cost of maintaining vector clocks in large systems can be substantially reduced
- especially if the process interactions exhibit temporal or spatial localities
- useful in applications
 - causal distributed shared memories
 - distributed deadlock detection
 - enforcement of mutual exclusion and localized communications



- reduces the size of messages by transmitting only a scalar value
- no vector clocks are maintained on-the-fly
- process only maintains information regarding direct dependencies on other processes
- vector time for an event represents transitive dependencies on other processes
- vector time for event is constructed off-line from a recursive search of the direct dependency information at processes



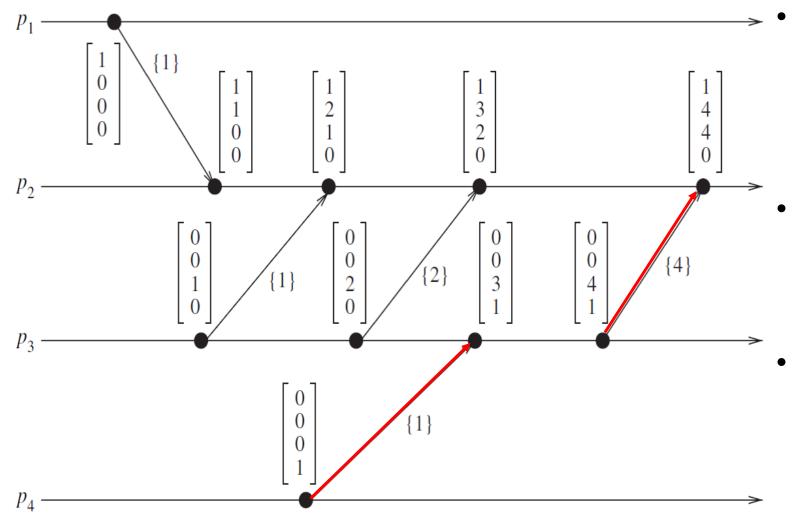
- p_i maintains a dependency vector D_i
- initially, $D_i[j] = 0$ for j = 1,....,n
- D_i is updated using the following rules:
- 1. When an event occurs at p_i , $D_i[i] = D_i[i] + 1$ (i.e., increment the vector component corresponding to its own local time by one)
- 2. When p_i sends a message to p_j , it piggybacks the updated value of $D_i[i]$ in the message
- 3. When p_i receives a message from p_j with piggybacked value d, p_i updates its dependency vector as follows: $D_i[j] = max\{D_i[j], d\}$





- *D_i* reflects only direct dependencies
- at any instant, $D_i[j]$ denotes the sequence no. of the latest event on p_i that **directly** affects the current state
- this event may precede the latest event at p_j that causally affects the current state





- when p_4 sends a message to p_3 , it piggybacks a scalar that indicates the direct dependency of p_3 on p_4 because of this message
- then p₃ sends a message to p₂
 piggybacking a scalar to indicate
 the direct dependency of p₂ on
 p₃ because of this message
- p_2 is indirectly dependent on p_4 since p_3 is dependent on p_4



- transitive (indirect) dependencies are not maintained by this method
- transitive dependencies
 - can be obtained only by recursively tracing the direct dependency vectors of the events off-line
 - involves computational overhead and latencies
- this method is ideal only for those applications
 - that do not require computation of transitive dependencies on the fly
 - eg. applications: causal breakpoints, asynchronous checkpoint recovery



- saves cost considerably
- not suitable for applications that require on-the-fly computation of vector timestamps

n





Physical Clock Synchronization

- no global clock or common memory
- each processor has its own internal clock and its own notion of time
- clocks can drift apart by several seconds per day, accumulating significant errors over time
- clock rates are different, may not remain always synchronized
- for most applications and algorithms that run in a distributed system, need to know time in one or more contexts:
 - time of the day at which an event happened on a specific machine in the network
 - time interval between two events that happened on different machines in network
 - relative ordering of events that happened on different machines in network

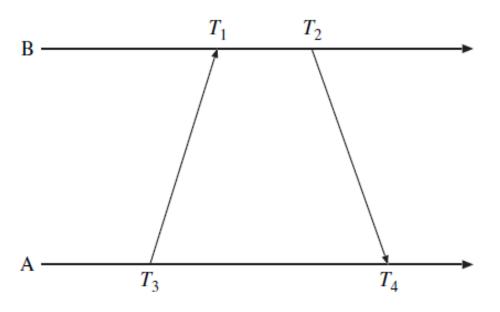
innovate achieve lead

Physical Clock Synchronization

- Clock synchronization
 - ensuring that physically distributed processors have a common notion of time
 - performed to correct clock skew in distributed systems
 - synchronized to an accurate real-time standard like UTC (Universal Coordinated Time)
- Offset
 - difference between the time reported by a clock and the real time
 - offset of the clock C_a is given by C_a(t) -t (C_a(t) time of clock)
 - offset of clock C_a relative to C_b at time $t \ge 0$ is given by $C_a(t) C_b(t)$

- uses offset delay estimation method
- involves a hierarchical tree of time servers
- primary server at the root synchronizes with the UTC
- next level contains secondary servers, which act as a backup to the primary server
- at the lowest level is the synchronization subnet which has the clients





- Each NTP message includes the latest three timestamps T_1 , T_2 , and T_3
- T_4 is determined upon arrival
- Peers A and B can independently calculate delay and offset using a bidirectional message stream

- T_1 , T_2 , T_3 , T_4 values of the 4 most recent timestamps as shown
- assume that clocks A and B are stable and running at the same speed
- $a = T_1 T_3$, $b = T_2 T_4$
 - if the network delay difference from A to B and from B to A, called differential delay, is small, the clock offset θ and roundtrip delay δ of B relative to A at time T₄

$$\theta = (a+b)/2$$

$$\delta = a - b$$



- A pair of servers exchange pairs of timing messages.
- A store of data is then built up about the relationship between the two
- Servers (pairs of offset and delay).
- Each peer maintains pairs (O_i,D_i), where:
 - O_i measure of offset (θ)
 - D_i transmission delay of two messages (δ)
- Offset corresponding to the minimum delay is chosen
- Assume that message m takes time t to transfer and m' takes t' to transfer
- Offset between A's clock and B's clock is O
- If A's local clock time is A(t) and B's local clock time is B(t)
 - A(t) = B(t) + O
- Then, $T_{i-2} = T_{i-3} + t + O$, $T_i = T_{i-1} O + t$



- Assume t = t'
- Offset O_i can be estimated as $O_i = (T_{i-2} T_{i-3} + T_{i-1} T_i)/2$
- Round-trip delay is estimated as $D_i = (T_i T_{i-3}) (T_{i-1} T_{i-2})$
- 8 most recent pairs of (O_i, D_i) are retained
- Value of O_i that corresponds to minimum D_i is chosen to estimate O

Recap Quiz

1. If	f an event e has a t	imestamp h in scalar cloc	k, then the minimum log	ical duration required before producing event e is
(a) h		(b) h+1	(c) h — 1	(d) 1/h
2. If th	ne height of an eve	nt b = 5, then how many	events would have prece	ded b on the longest causal path ending at b if we use a scalar clock?
(a)4		(b) 5	(c) 3	(d) 2
3. In the	he tie breaking me	chanism used in total ord	dering in scalar clocks, a l	ower process identifier implies priority
(a) E	qual	(b) lower	(c) higher	(d) none of the above
4. Whi	ich of the following	g is not a mechanism to r	epresent logical time in d	listributed computing systems?
(a) So	calar clock	(b) vector clock	(c) matrix clock	(d)list clock
5. Con	nsider the event co	ounting in vector clocks.	if an event e has timesta	amp vh, then the umber of events executed by process pj that causally
preced	de e will be			
(a) vl	h[j]	(b)vh[j-1]	(c) vh[j-1]	(d)vh[0]
6. Let	the total number	of processes $n = 8$, in a	distributed environment	. Then the dimension of the vector clocks can't be 8 for strong
consis	stency			
(a) <		(b) >	(c)<=	(d) >=
7. Wh	nich of the followi	ng would ensure the eff	ficient implementation o	of vector clocks if the process interactions exhibit temporal or spatia
localit	ies in a distributed	l system?		
(a)Sing	ghal-Kshemkalyani	i (b)Fowler-Zwaenepoel	(c) physical	clock synchronization (d) NTP
8. Whi	ile implementing N	NTP in distributed compu	ting systems, if A's local o	lock time is A(t) and B's local clock time is B(t), then the Offset O is
(a) A	(t) + B(t)	(b) A(t) * B(t)	(c) A(t)/B(t)	(d) A(t) - B(t)
9. Fow	vler-Zwaenepoel a _l	pproach does not maintai	in dependencies	
(a) C	Commutative	(b) associative	(c) transitive	(d) none of the above
10. In	distributed compu	iting systems, the clock sl	kew can be corrected by _	clock synchronization
(a) Sca	alar	(b) vector	(c) matrix	(d) physical

Recap Quiz - key

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
С	b	С	d	а	а	а	d	С	d



Major References

• Ajay D. Kshemkalyani, and Mukesh Singhal, Chapter 3, "Distributed Computing: Principles, Algorithms, and Systems", Cambridge University Press, 2008 (Reprint 2013).