IS-ZC444: Artificial Intelligence

Lecture-08: SA, GA, ALPHA-BETA Pruning

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Simulated Annealing (Metallurgy approach)

- Simulated annealing is hill climbing combined with **random walk**
- Step size is gradually reduced
- First Applied around 1980, for VLSI layout problem

Algorithm 1: Simulated-Annealing(problem, schedule)

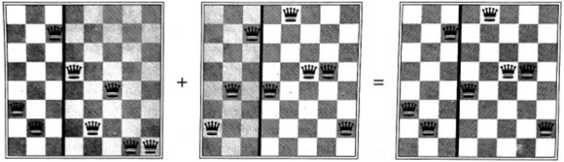
current $\leftarrow$ Make-Node (*problem*.Initial-State)
for $t = 1$ *to* ∞ **do**
  T $\leftarrow$ *schedule(t)*
  if $T = 0$ **then return** current
  next $\leftarrow$ a randomly selected successor of current
  $\Delta E \leftarrow$ next.Value - current.Value
  if $\Delta E > 0$ **then** current $\leftarrow$ next
  else current $\leftarrow$ next with probability $e^{\Delta E / T}$

Genetic Algorithms

Evolution

- Recall Darwin's theory of evolution: "*Survival of the fittest*" [1]

Is it true? Let's formulate and try..



[1] Images taken from various sources on Internet

Genetic Algorithms

Learning approach of Genetic algorithms is based on simulated evolution (appeared in 1975)

- State are represented using fixed length bit strings (chromosome)
- Search for a goal state begins with a population of initial states
  - Members of the current population give rise to the next generation population using random **mutation** and **crossover**
  - States are evaluated using some **fitness** measure
  - Most fit state act as a seeds for producing next generation
- Applied a variety of learning tasks and optimization problems (like robot control and learning parameters for ANN)
- Search can move abruptly. Crowding can happen

Without **guarantee**, GA often finds an object of high fitness

Genetic Algorithms

Algorithm 2: GA ($\texttt{Fitness}$, F_{th}, p, r, m)

P $\leftarrow$ generate p states at random

while $max(\texttt{Fitness}(h_1), \texttt{Fitness}(h_2), ..., \texttt{Fitness}(h_p)) < F_{th}$ **do**

    Select: $(1 - r)p$ members of P

    Crossover: on $(r \times p)/2$ pairs to produce two offspring

    Mutation: randomly invert a bit of m percentage of population

return state h_i having maximum $\texttt{Fitness}(h_i)$

- Fitness function is typically a heuristic
- The fitness function is a criterion for ranking states to select states probabilistically for inclusion in the next generation population

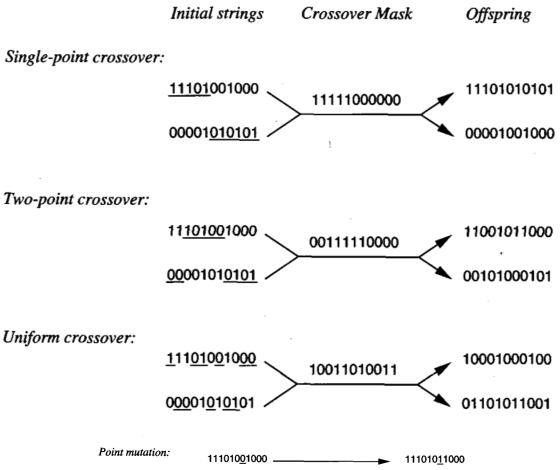
Selection

- **Fitness proportionate.** Probability of selecting a state in next generation is

$$Pr(h_i) = \frac{\texttt{Fitness}(h_i)}{\sum_{j=1}^{p} \texttt{Fitness}(h_j)}$$

- **Tournament selection.** randomly pick two states and then with some predefined probability p the more fit of these two is then selected, and with probability $(1 - p)$ the less fit state is selected
- **Rank selection.** states are sorted by fitness and the probability of selection of a state is proportional to its rank in this sorted list, rather than its fitness
- **Elitist Model.** select a small proportion of the fittest candidates in current population intact into the next generation

Genetic Operators (crossover and mutation)



Does GA works?

- Can we mathematically characterize the evolution
- **Schema:** string of 0, 1 or $*$ like $0 * 1$ denoting set $\{001, 011\}$
- String 1011 is **representative** of 2^4 schema
- Let $m(s, t)$ be number of instances of schema s in generation t
- Consider *selection*, let fitness of individual h be $f(h)$ and average fitness of whole n size population at time t be $\bar{f}(t)$
- $h \in s \cup p_t$ means 1) h is representative of s and 2) it is present in the population at time t
- Let $\hat{u}(s, t)$ be average fitness of instances of s at time t

$$\hat{u}(s, t) = \frac{\sum_{h \in s \cup p_t} f(h)}{m(s, t)}$$

- We know $Pr(h) = f(h)/(\sum f(h)) = f(h)/(n\bar{f}(t))$

Does GA works? (contd..)

- Probability that we will select a representative of schema s is

$$
\begin{aligned}
Pr(h \in s) &= \sum_{h \in s \cup p_t} Pr(h) = \sum_{h \in s \cup p_t} f(h)/(n\bar{f}(t)) \\
&= \frac{\hat{u}(s,t)}{n\bar{f}(t)} m(s,t)
\end{aligned}
$$

- Expected number of instances of s resulting from the n independent selection steps that create the entire new generation is just n times this probability. Therefore,

$$
E[m(s,t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)
$$

Does GA works? (contd..)

$$E[m(s, t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)$$

Expected number of representative instances of a schema s in the generation at time $t+1$ is

1. Proportional to the average fitness $\hat{u}(s,t)$ of instances of this schema at time t , and
2. Inversely proportional to the average fitness $\bar{f}(t)$ of all members of the population at time t

Thus, we can expect schema with above average fitness to be represented with increasing frequency on successive generations

Does GA works? (contd..)

Also consider negative effects of single point crossover and mutation

- Let p_c represents the probability of **crossover** on an individual. $d(s)$ be the distance between left most and right most defined bit of s and l be the length of individual bit string
- Let p_m represents the probability of **mutation** on an individual and $o(s)$ be number of defined bits in s

Full schema theorem thus provides a lower bound on the expected frequency of schema s, as follows

$$E[m(s,t+1)] \geq \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)\left(1 - p_c\frac{d(s)}{l-1}\right)(1-p_m)^{o(s)}$$

Similar expression. More fit schemas will tend to grow in influence.

Local Search in Continuous State

Issue: Number of next states (branching factor) becomes infinite

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Example: Induct three new airports in Romania

- Let at (x_1, y_1), (x_2, y_2) and (x_3, y_3) on the map
- Minimize sum of distances of all the cities from its nearest airport

$$f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} ((x_i - x_c)^2 + (y_i - y_c)^2)$$

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- Given locally correct values of $\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_i - x_c)$ one can perform steepest-ascent using $x \leftarrow x + \alpha \bigtriangledown f$

Search with Non-Deterministic Actions

Non-Deterministic: not sure what would be the next state[2]

[2] Percepts would tell where have we reached.

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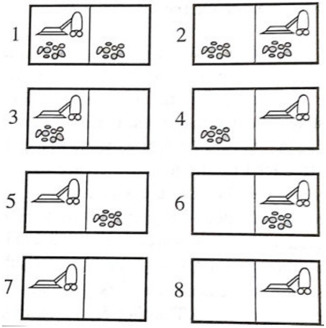

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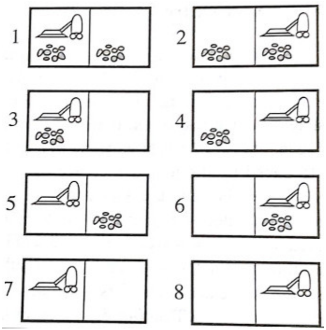

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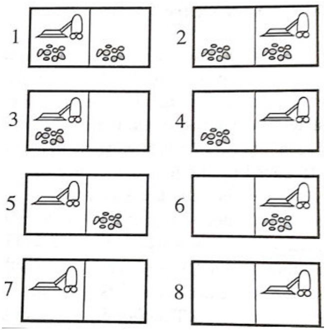

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[*suck*, **if** state=5 **then** [*right,suck* **else** []]

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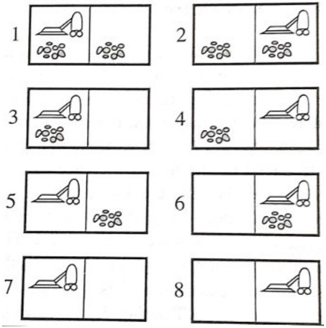

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- Search tree would contain some OR nodes and some AND nodes

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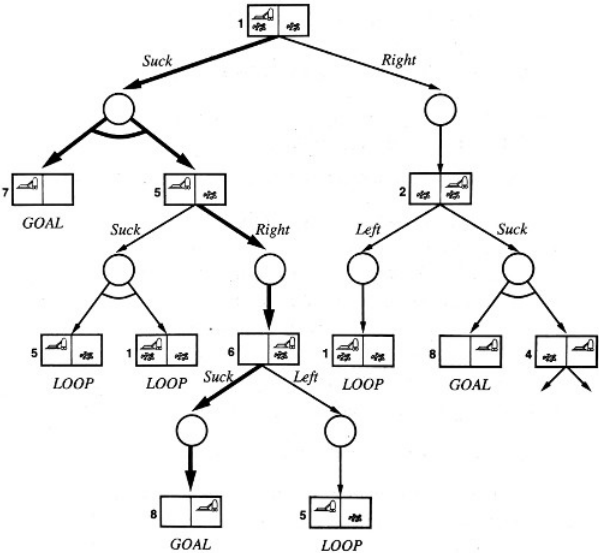
AND-OR Search Tree



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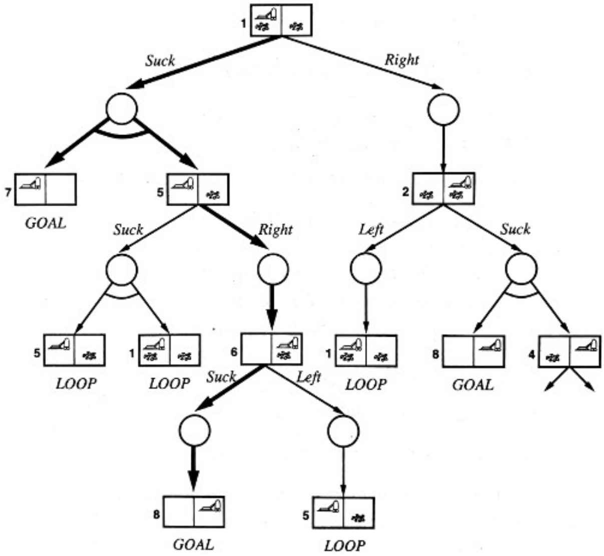

Solution

1. has goal node at every leaf
2. takes one action at each OR node
3. includes every outcome branch at each AND node

Searching with Partial Observations

When percepts do not suffice to pin down the exact state

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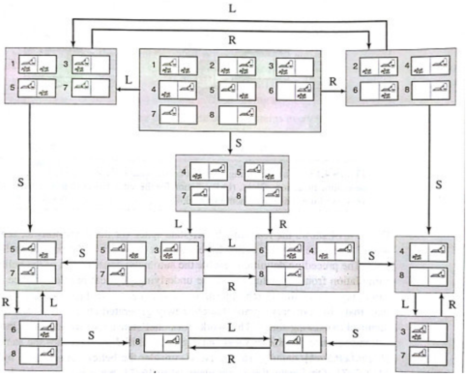
When percepts do not suffice to pin down the exact state

- **Sensor less.** consider [*right,suck,left,suck*] guarantees to reach in state 7 that is a goal state (traverses through belief states)
- All possible belief states may not be reachable (only 12 out of 2^8)



Online Search and Unknown Environment

Agent interleaves computation and action

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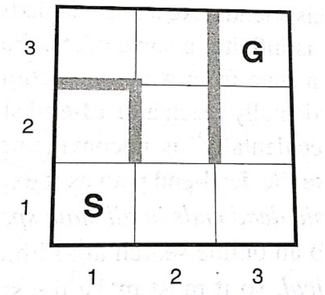

1. Consider following maze problem
2. A robot need to go from S to G
3. Knows nothing about the environment

Random-walk?

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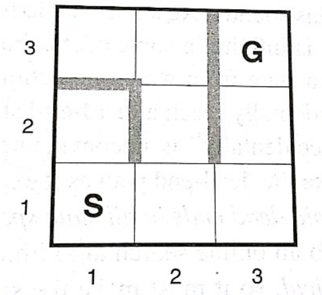

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No algorithm can avoid dead-end in all state space

Adversarial Search (game)

Agents having conflicting goals in competitive multiagent environment

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- Chess has roughly branching factor 35, moves 50 so tree search space is $35^{100} = 10^{154}$ however, graph has 10^{40} nodes
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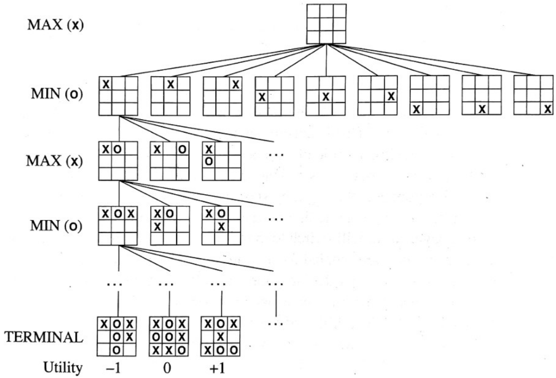
Game is between MAX and MIN (MAX moves first)

- S_0: the initial state
- PLAYER(s): defines which player has move to start
- ACTIONS(s): returns set of legal moves in a state
- RESULT(s, a):termination model defining result of a move
- TERMINAL_TEST(s): is true when game is over
- UTILITY(s, p): utility function defining reward (for chess +1,0,1/2)

Game Tree for tic-tac-toe



The search tree of the game has less than $9! = 362880$ nodes.

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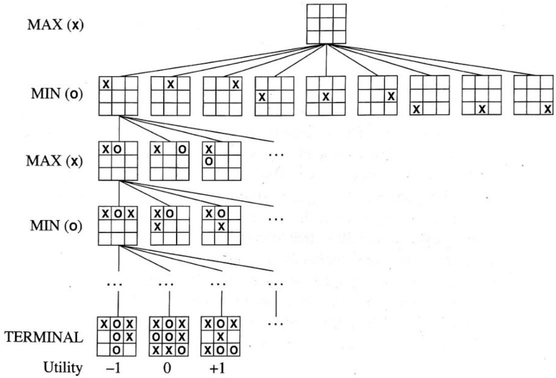
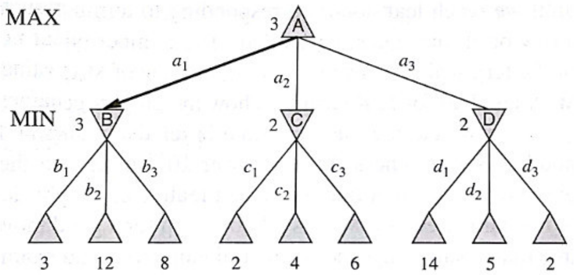


The search tree of the game has less than $9! = 362880$ nodes.

MAX must find a contingent **strategy**.

Analogous to AND-OR search (MAX plays OR and MIN plays AND)

Two half moves is one **ply**



[3] utility value for MAX of being in corresponding state (assuming then onwards both player play optimally)

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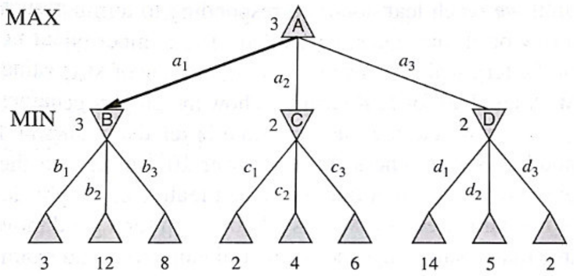

Given the game tree, optimal strategy can be determined from **minimax value** of each node.

$$MINIMAX(s) = \begin{cases} UTILITY(s) & \text{if } TERMINAL\_TEST(s) \\ argmax_{a \in Actions(s)} MINIMAX(RESULT(s, a)) & \text{if } PLAYER(s) = MAX \\ argmin_{a \in Actions(s)} MINIMAX(RESULT(s, a)) & \text{if } PLAYER(s) = MIN \end{cases}$$

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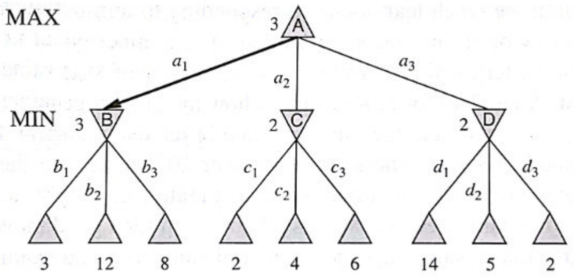

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Action a_1 is the optimal choice [3]

(essentially optimizing worst-case outcome for MAX)

[3] utility value for MAX of being in corresponding state (assuming then onwards both player play optimally)

MINIMAX Algorithm

Returns the action corresponding to best move

function MINIMAX-DECISION(*state*) **returns** *an action*
  return $\arg\max_{a \in \text{ACTIONS}(s)}$ MIN-VALUE(RESULT(*state*, *a*))

---

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*s*, *a*)))
  return v

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Recursion proceeds all the way down to the leaves.

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Recursion proceeds all the way down to the leaves.Time complexity $O(b^m)$ that is impractical but provides a basis of solution.

ALPHA-BETA Pruning

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- Sometime we can make it $O(b^{m/2})$ using **alpha-beta pruning**

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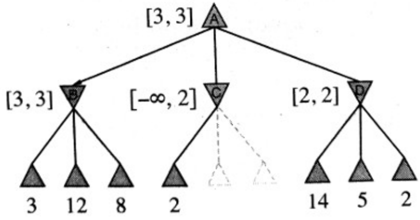

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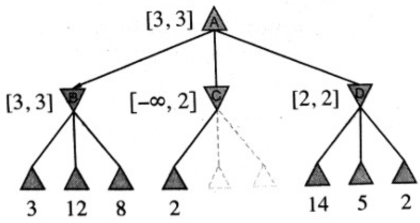

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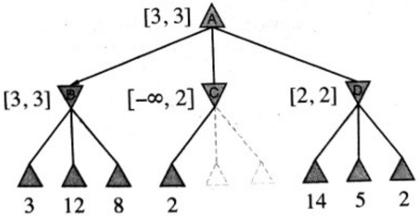

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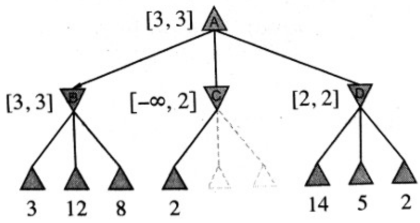

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= max(3, min(2,x,y), 2)
= max(3, z, 2) where z=min(2,x,y)$\leq$ 2

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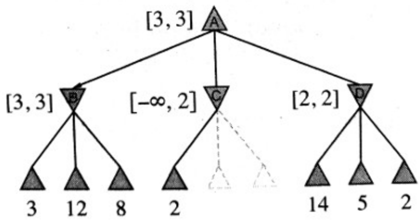
- Sometime we can make it $O(b^{m/2})$ using **alpha-beta pruning**
- When applied to standard minimax tree, it returns the same move as minimax but, prunes away branches that cannot possibly influence the decision.

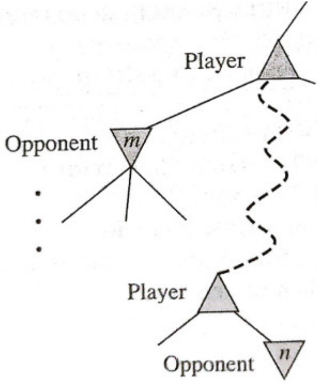


Consider two unevaluated successor of node C have value x and y

MINIMAX(root)
= max(min(3,12,8), min(2,x,y), min(14,5,2))
= max(3, min(2,x,y), 2)
= max(3, z, 2) where z=min(2,x,y)$\leq$ 2
= 3

ALPHA-BETA Pruning

- Alpha-beta pruning can be applied to trees of any depth, and it is often possible to prune entire subtree rather than just leaves.



If *m* is better than *n* for player then we would never go to *n* in play

α	$=$	value of best choice (highest) found so far for MAX
β	$=$	value of best choice (lowest) found so far for MIN

ALPHA-BETA Pruning

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in ACTIONS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

---

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

ALPHA-BETA Pruning

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v

function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v

function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Order matters. So, examine likely to be best successor first.

ALPHA-BETA Pruning

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in ACTIONS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
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  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

---

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Order matters. So, examine likely to be best successor first.

Is it possible?

ALPHA-BETA Pruning

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE($state, -\infty, +\infty$)
  return the *action* in ACTIONS(*state*) with value v

---

function MAX-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

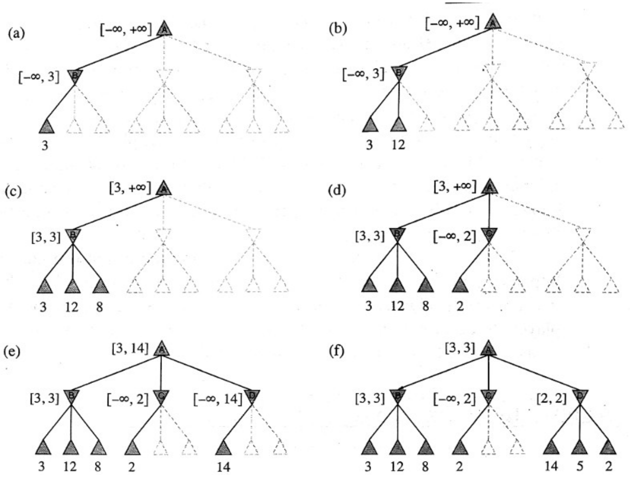
---

function MIN-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a), α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Order matters. So, examine likely to be best successor first.

Is it possible?
No

In-action: ALPHA-BETA Pruning



Thank You!

Thank you very much for your attention!

Queries ?

(Reference[4])

[4] 1) Book - *AIMA*, ch-03+04+05, Russell and Norvig.