IS-ZC444: Artificial Intelligence

Lecture-09: Adversarial Search, Constraint Satisfaction

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Recap: Local Search in Continuous State

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Example: Induct three new airports in Romania

- Let at (x_1, y_1), (x_2, y_2) and (x_3, y_3) on the map
- Minimize sum of distances of all the cities from its nearest airport

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- Given locally correct values of $\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_i - x_c)$ one can perform steepest-ascent using $x \leftarrow x + \alpha \bigtriangledown f$

Recap: Search with Non-Deterministic Actions

Non-Deterministic: not sure what would be the next state[1]

[1] Percepts would tell where have we reached.

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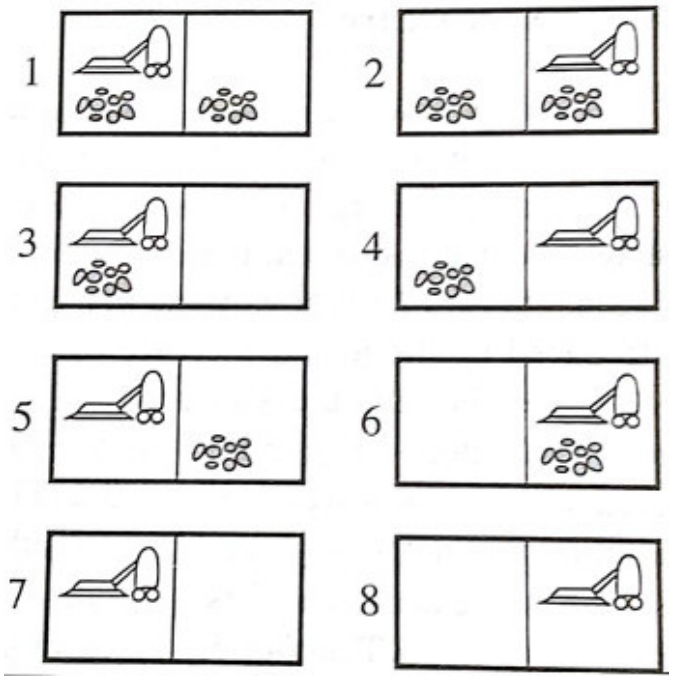

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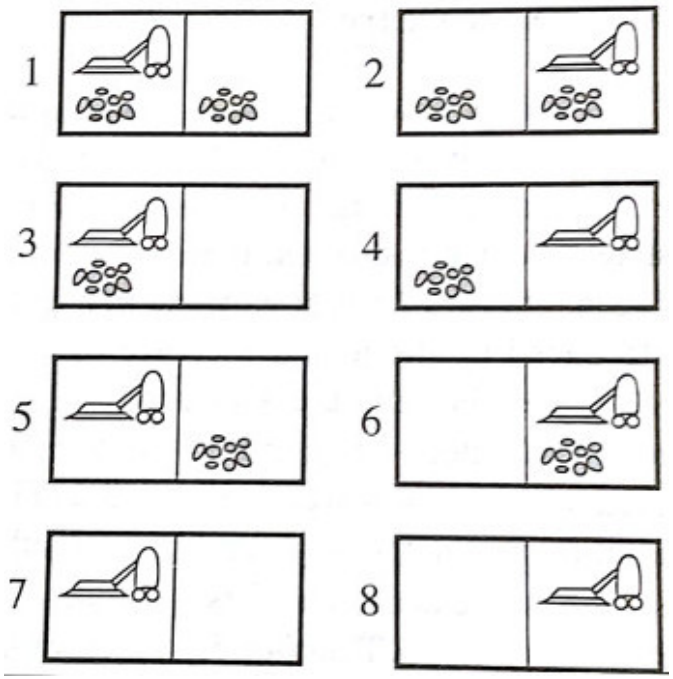

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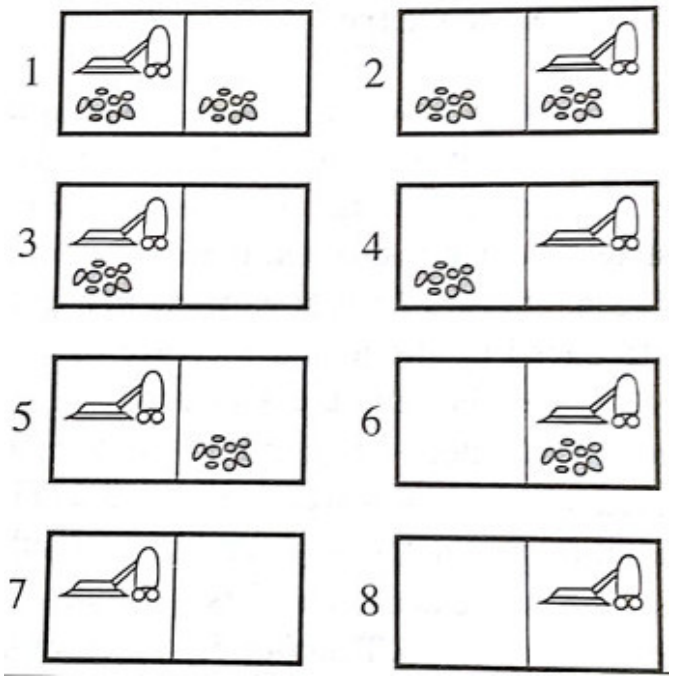

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[*suck*, **if** state=5 **then** [*right,suck* **else** []]

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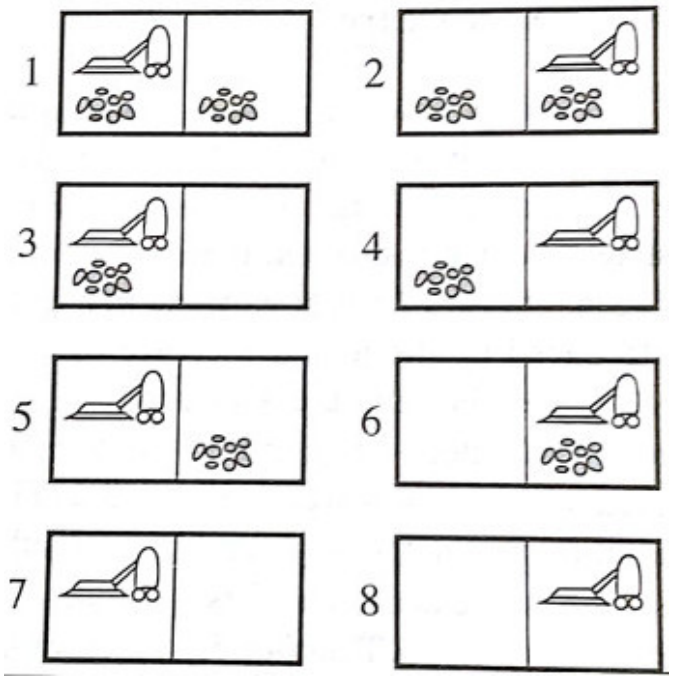

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- Search tree would contain some OR nodes and some AND nodes

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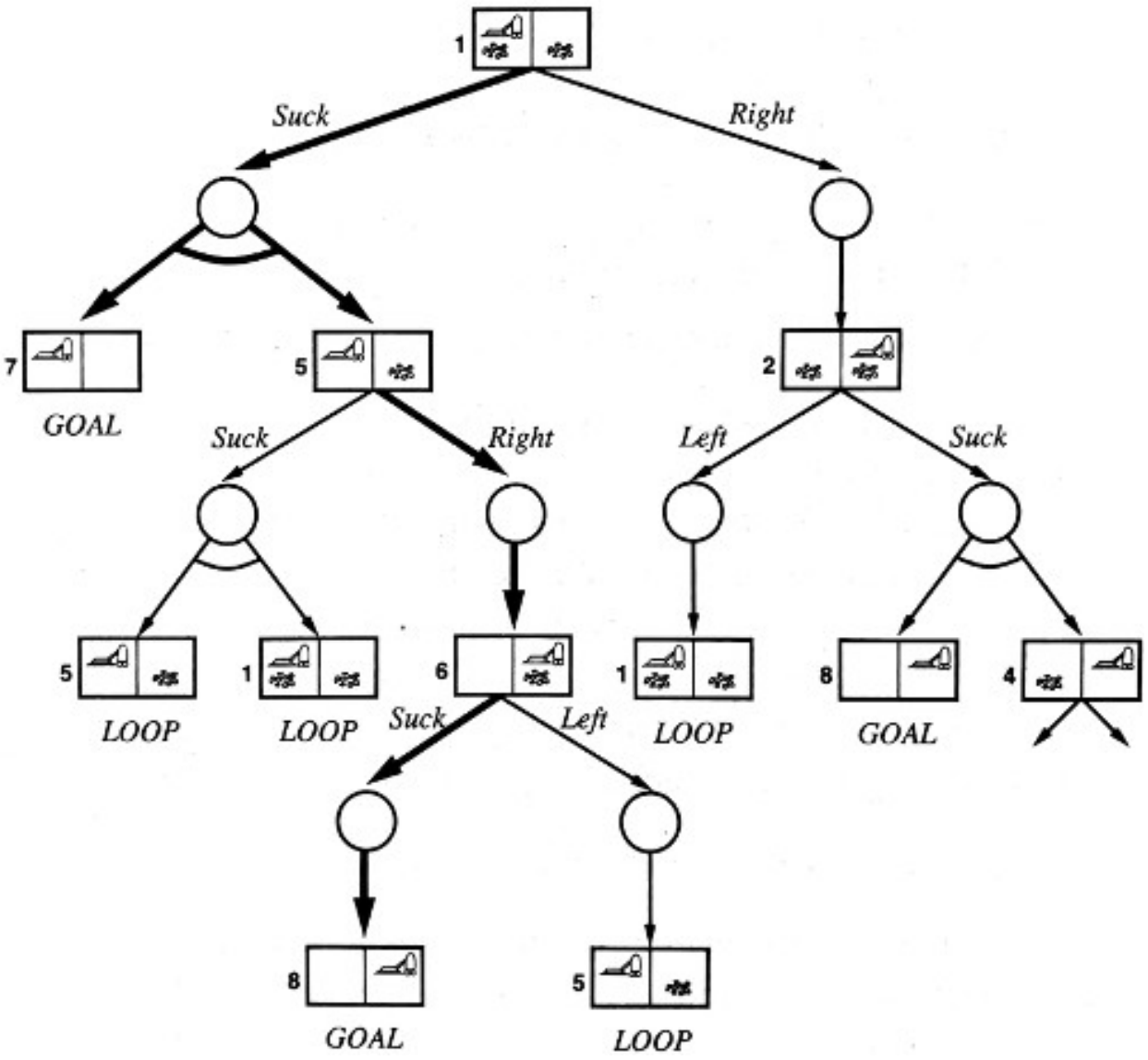
AND-OR Search Tree



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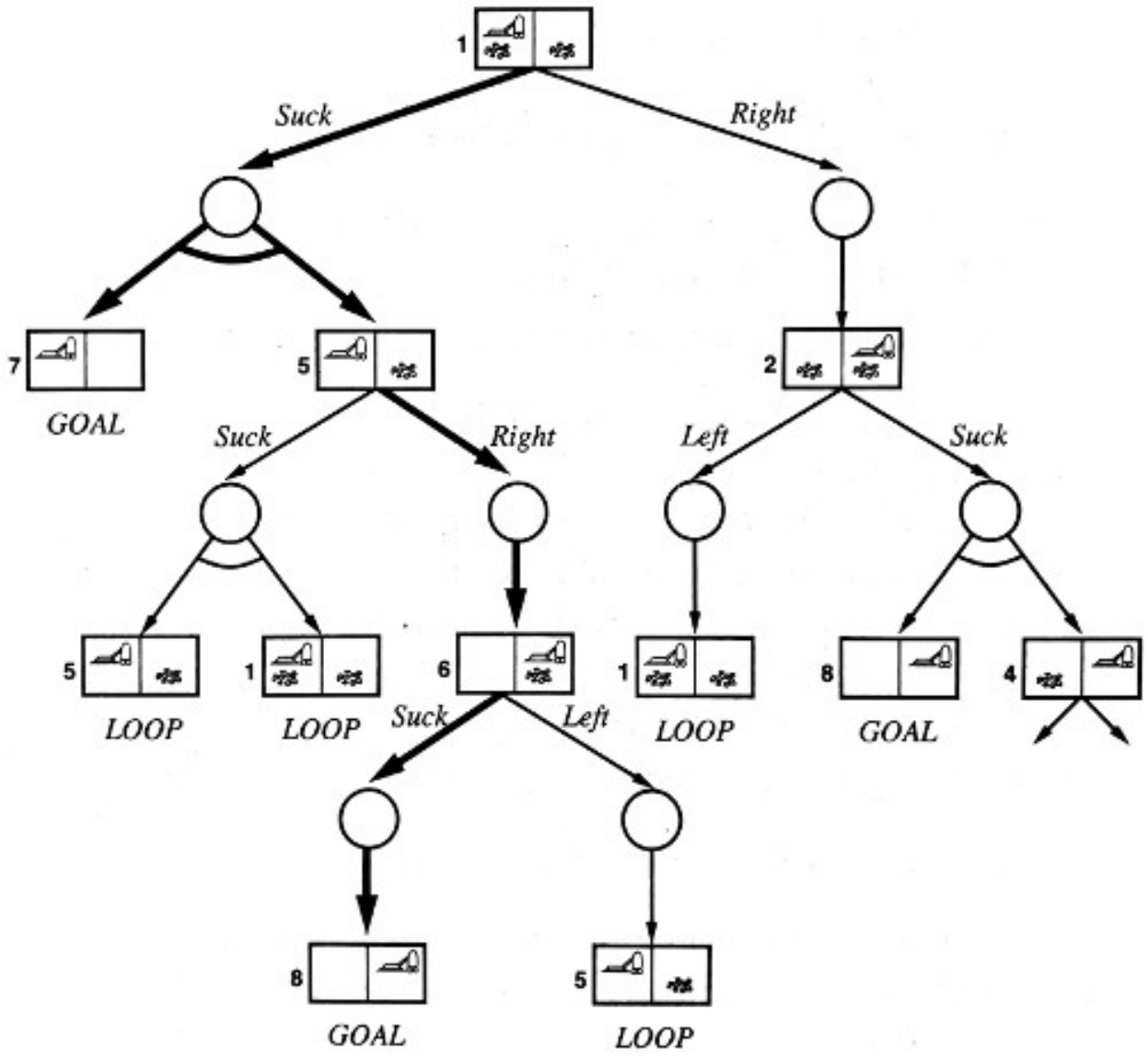

Solution

1. has goal node at every leaf
2. takes one action at each OR node
3. includes every outcome branch at each AND node

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When percepts do not suffice to pin down the exact state

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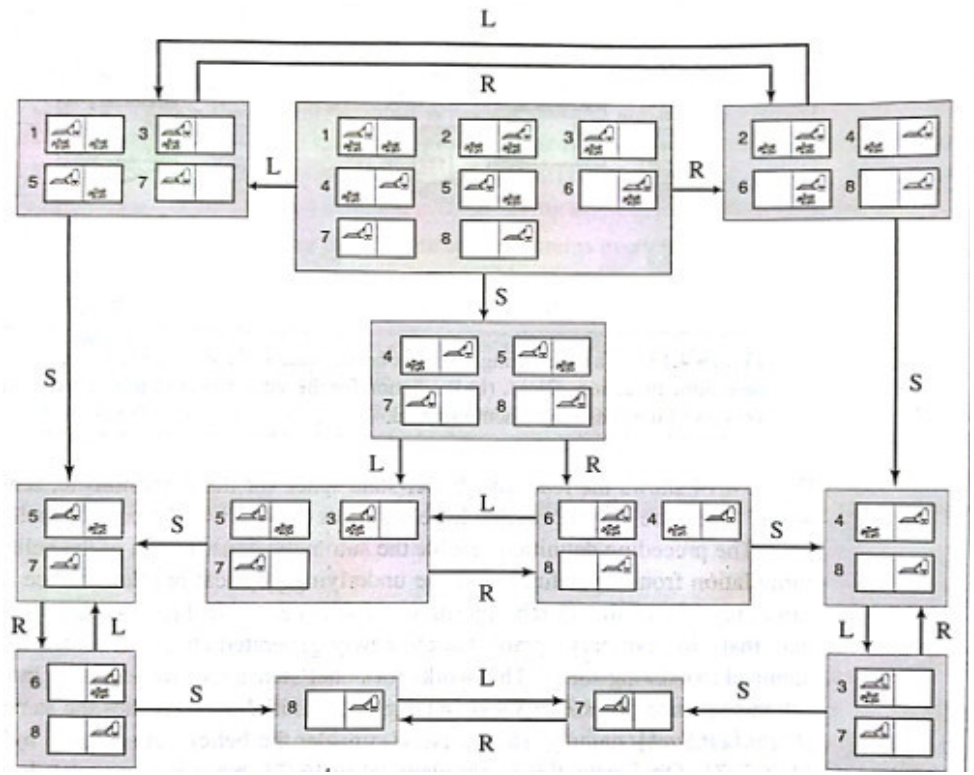
- **Sensor less**. consider [*right,suck,left,suck*] guarantees to reach in state 7 that is a goal state (traverses through belief states)
- All possible belief states may not be reachable (only 12 out of 2^8)



Online Search and Unknown Environment

Agent interleaves computation and action

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Take action $\rightarrow$ observe environment $\rightarrow$ compute next action

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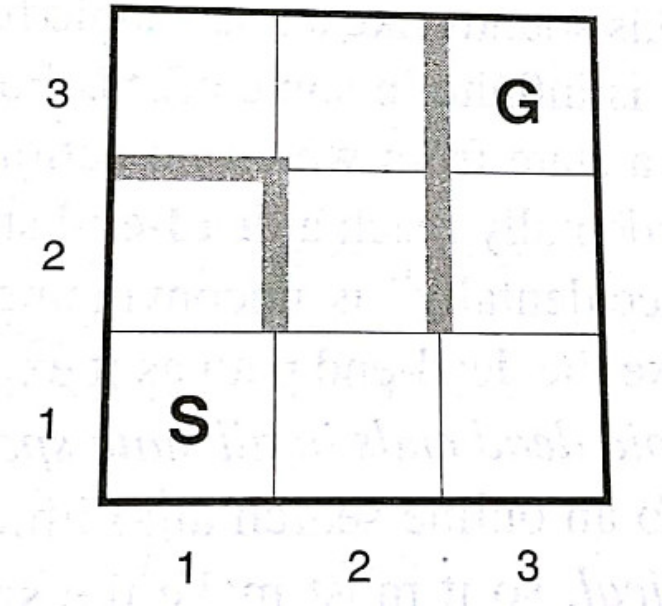

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2. A robot need to go from S to G
3. Knows nothing about the environment

Random-walk?

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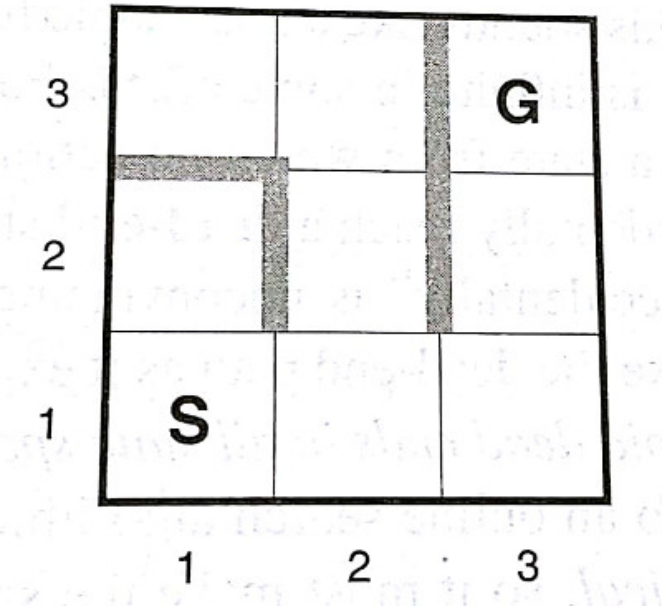

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Random-walk?

No algorithm can avoid dead-end in all state space

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Agents having conflicting goals in competitive multiagent environment

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- Chess has roughly branching factor 35, moves 50 so tree search space is $35^{100} = 10^{154}$ however, graph has 10^{40} nodes
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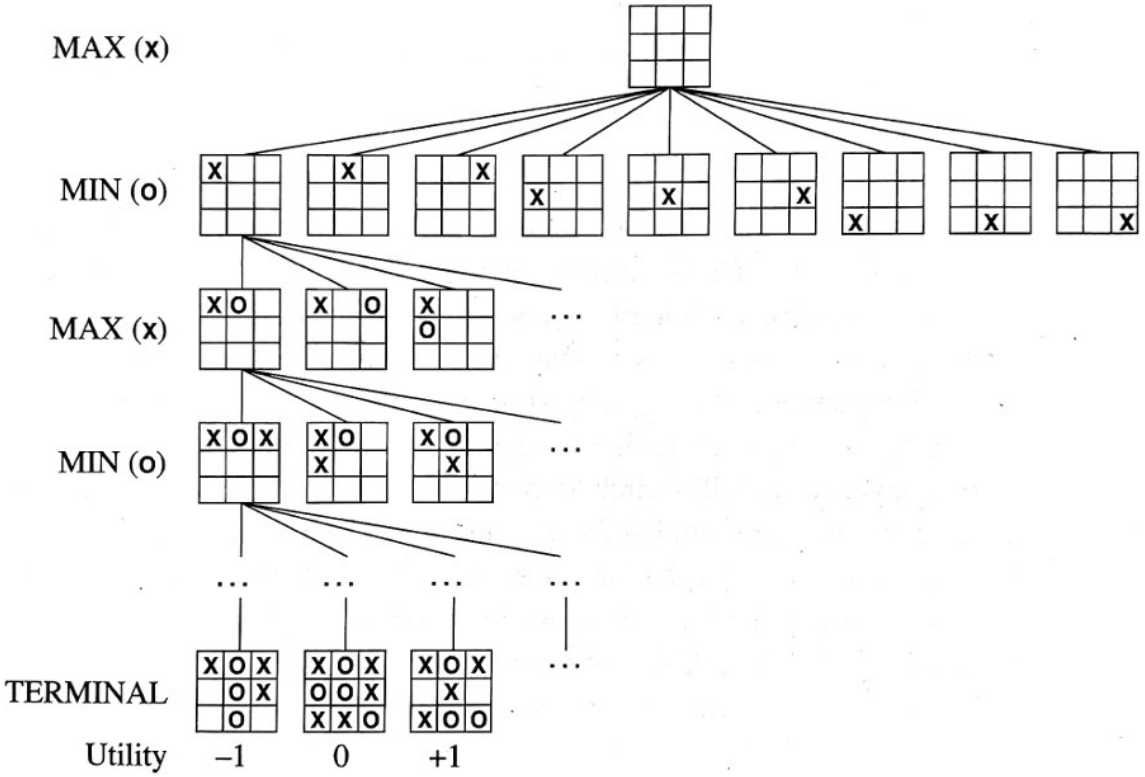
Game is between MAX and MIN (MAX moves first)

- S_0: the initial state
- PLAYER(s): defines which player has move to start
- ACTIONS(s): returns set of legal moves in a state
- RESULT(s, a):termination model defining result of a move
- TERMINAL_TEST(s): is true when game is over
- UTILITY(s, p): utility function defining reward (for chess +1,0,1/2)

Game Tree for tic-tac-toe



The search tree of the game has less than $9! = 362880$ nodes.

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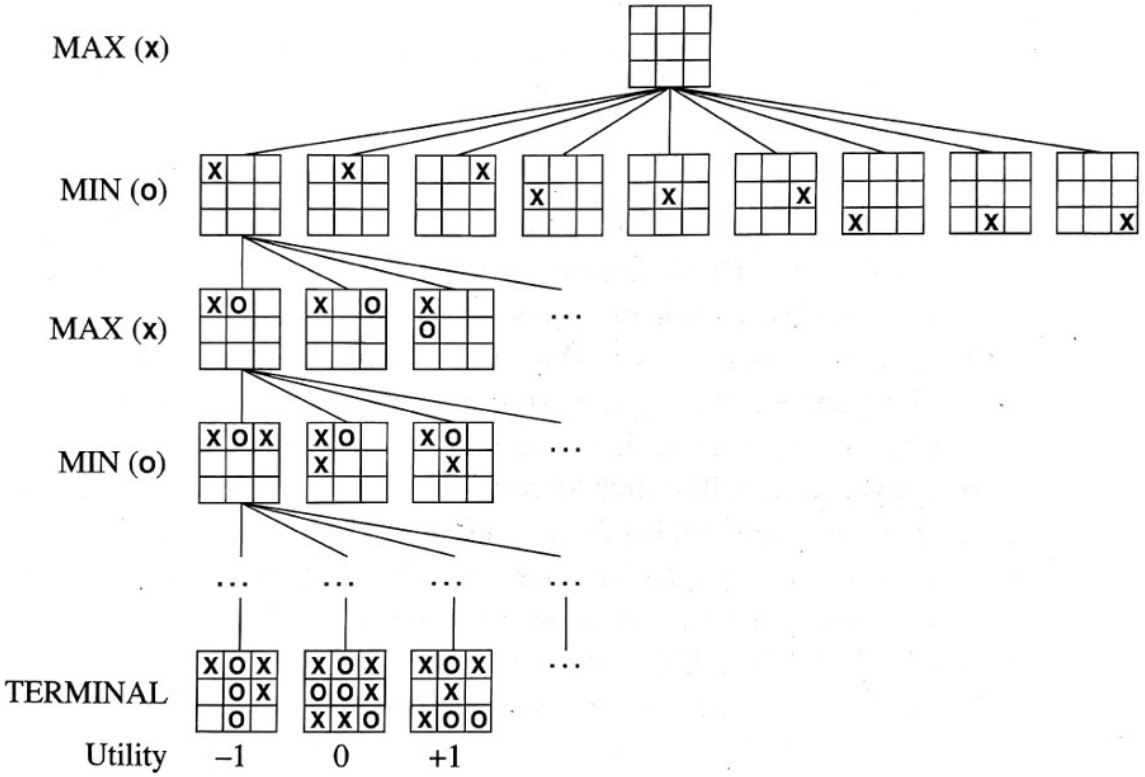

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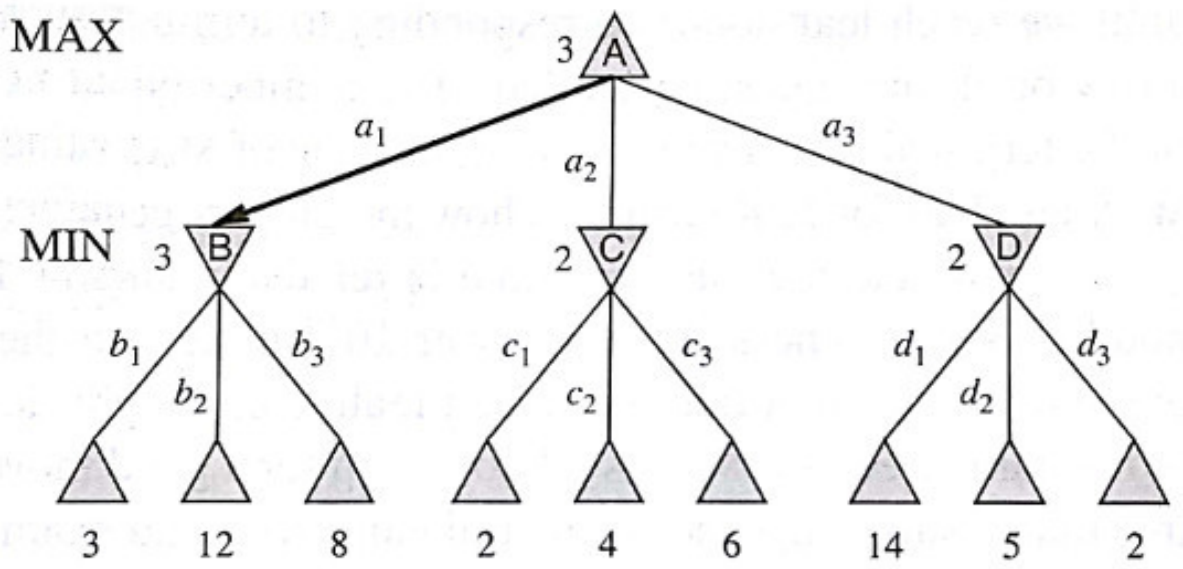
MAX must find a contingent **strategy**.

Analogous to AND-OR search (MAX plays OR and MIN plays AND)

Two half moves is one **ply**



[2]utility value for MAX of being in corresponding state (assuming then onwards both player play optimally)

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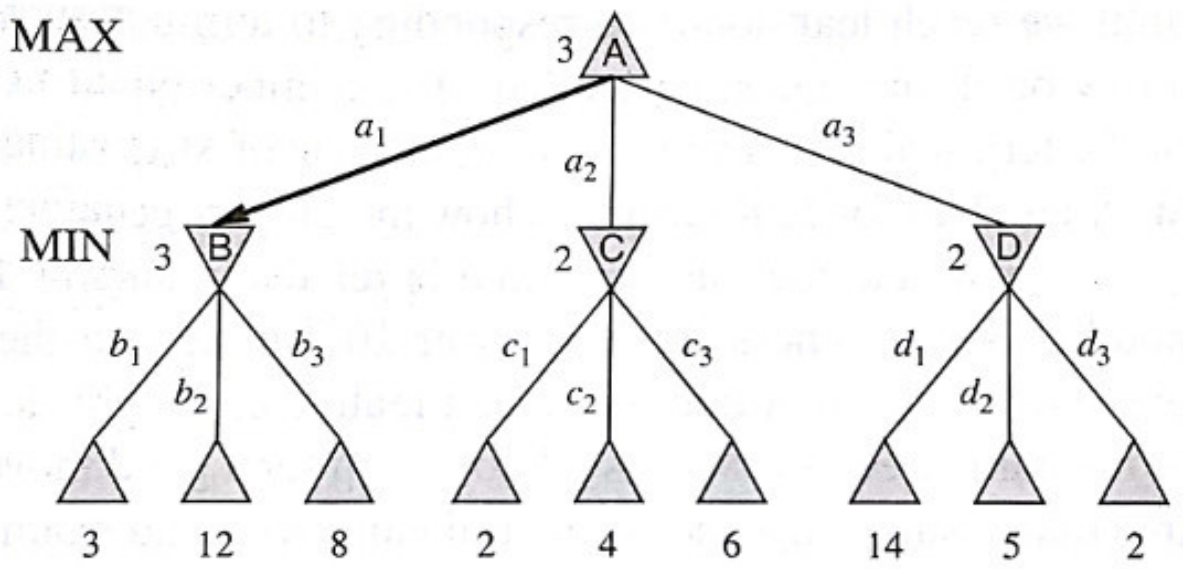

Given the game tree, optimal strategy can be determined from **minimax value** of each node.

$$MINIMAX(s) = \begin{cases} UTILITY(s) & \text{if } TERMINAL\_TEST(s) \\ argmax_{a \in Actions(s)} MINIMAX(RESULT(s,a)) & \text{if } PLAYER(s) = MAX \\ argmin_{a \in Actions(s)} MINIMAX(RESULT(s,a)) & \text{if } PLAYER(s) = MIN \end{cases}$$

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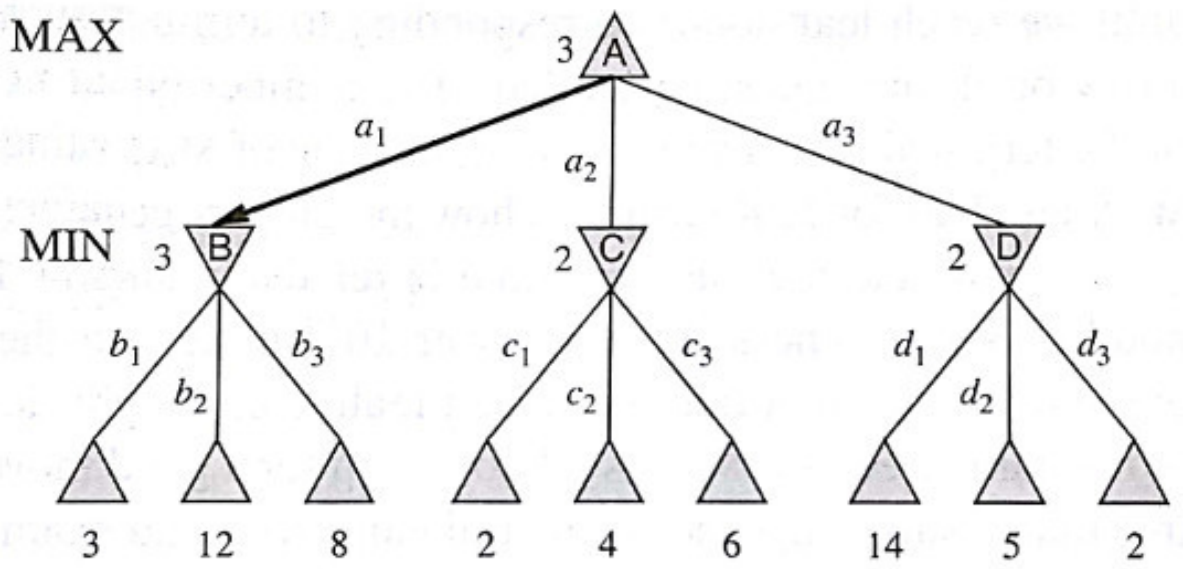

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Action a_1 is the optimal choice [2]
(essentially optimizing worst-case outcome for MAX)

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MINIMAX Algorithm

Returns the action corresponding to best move

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function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
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function MAX-VALUE(state) returns a utility value
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Recursion proceeds all the way down to the leaves.Time complexity $O(b^m)$ that is impractical but provides a basis of solution.

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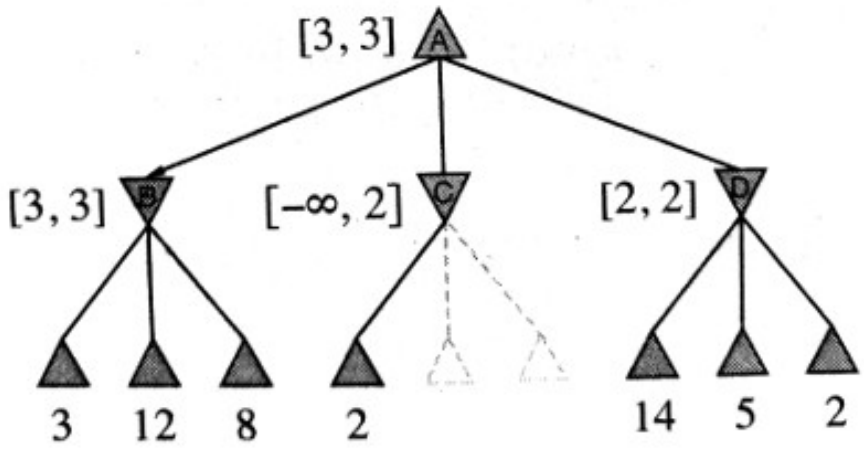

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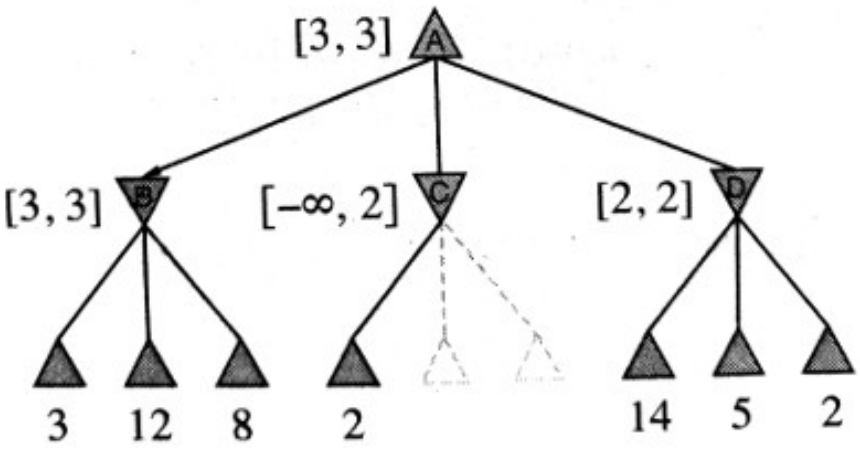

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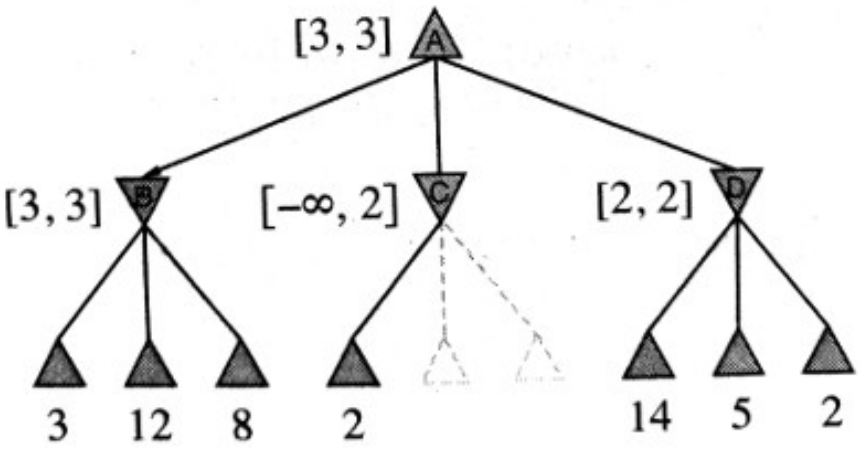

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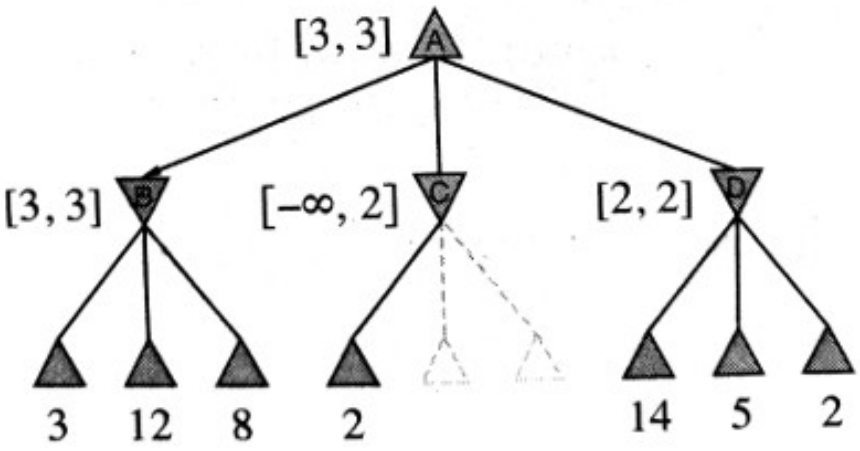

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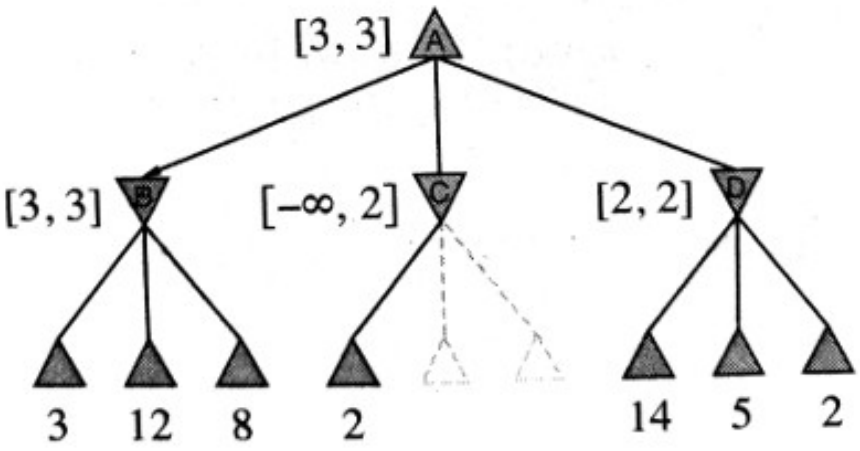

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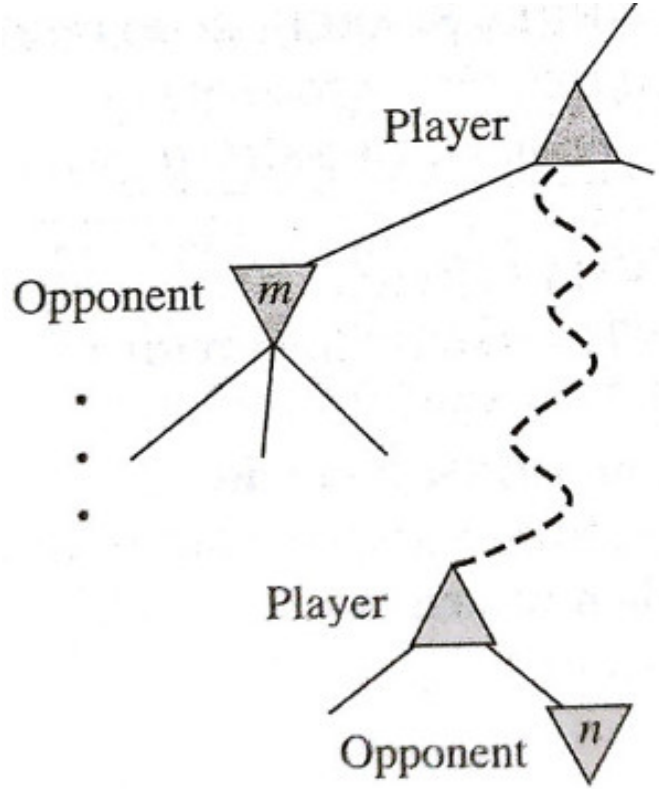
ALPHA-BETA Pruning

- Alpha-beta pruning can be applied to trees of any depth, and it is often possible to prune entire subtree rather than just leaves.



If *m* is better than *n* for player then we would never go to *n* in play

α	$=$	value of best choice (highest) found so far for MAX
β	$=$	value of best choice (lowest) found so far for MIN

ALPHA-BETA Pruning

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function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
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Is it possible?

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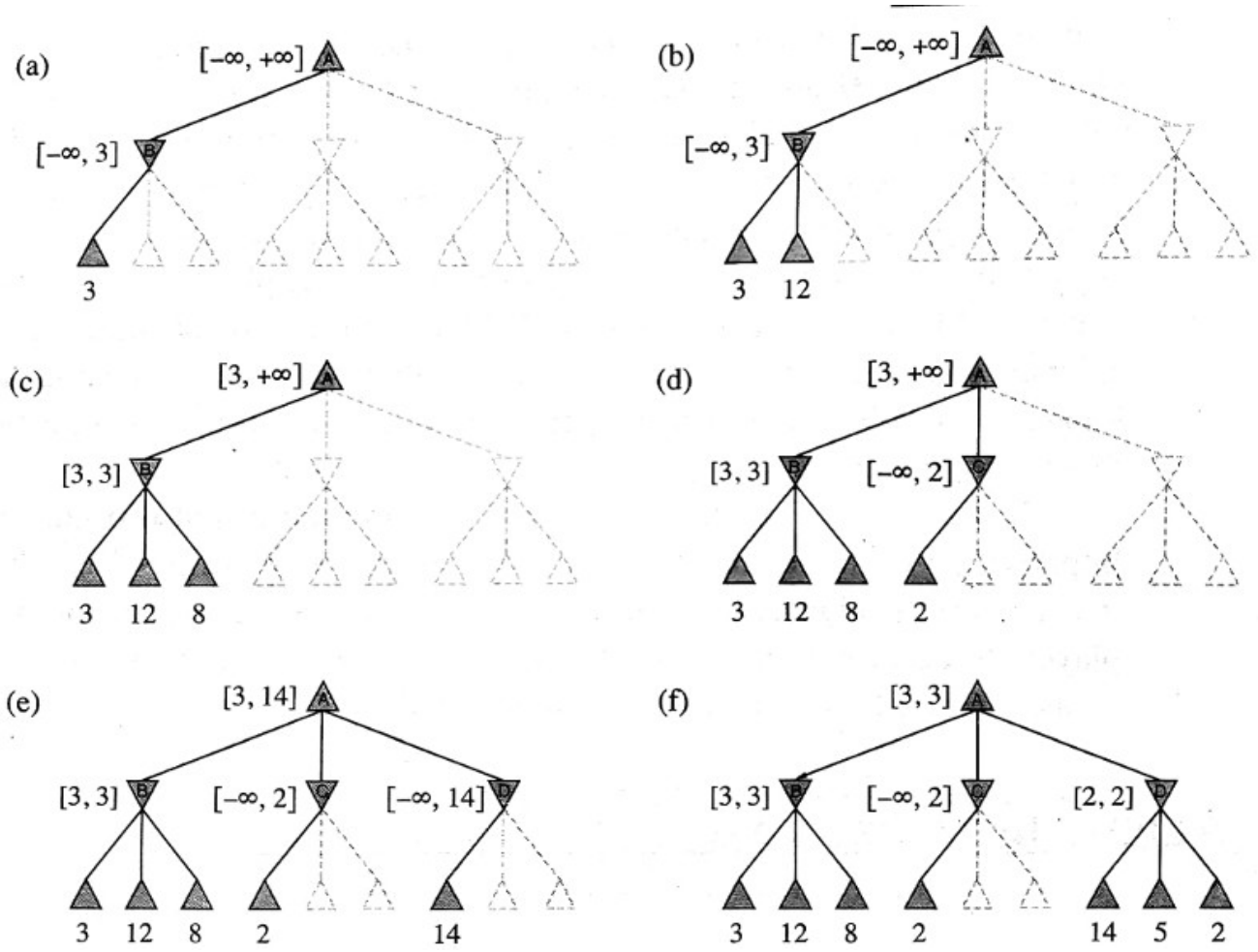
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Order matters. So, examine likely to be best successor first.

Is it possible?
No

In-action: ALPHA-BETA Pruning



Move Ordering

- With perfect ordering we need to examine only $O(b^{m/2})$ nodes
- When successors are examined in random order it needs to examine $O(b^{3m/4})$ nodes
- In chess, a strategy that capture $\rightarrow$ threat $\rightarrow$ forward $\rightarrow$ backward, gets you to within about a factor of 2 of the best case $O(b^{m/2})$
- Try first the move that were found useful in past (killer move)
- Iterative deepening could help (adds constant fraction time)
- Hash table of previously seen positions (**transposition table**) can restrict re-computation of states

Imperfect Real-time Decision

- Generating entire search space is overkill
- Covering till large depth is not possible
- Idea is to cutoff the search earlier

$$H\text{-}MINIMAX(s, d) = \begin{cases} Eval(s) & \text{if } CUTOFF\_TEST(s, d) \\ argmax_{a \in Actions(s)} H\text{-}MINIMAX(RESULT(s, a), d + 1) & \text{if } PLAYER(s) = MAX \\ argmin_{a \in Actions(s)} H\text{-}MINIMAX(RESULT(s, a), d + 1) & \text{if } PLAYER(s) = MIN \end{cases}$$

Evaluation Function

- Returns the estimate of expected utility
- Performance would deeply depend on it
- Eval(win) $\geq$ Eval(draw) $\geq$ Eval(lose)
- Computation should be quick
- Value should relate to actual chance of winning
- Can use features (Number of pawns, Number of queens, ...)
- Can have various categories (all pawn vs one pawn, etc) Suppose experience suggests 72% of states encountered in the two-pawn vs. one pawn category lead to win, 20% lose, and 8% in draw. then eval could be

$$0.72 \times 1 + 0.20 \times 0 + 0.08 \times 0.5 = 0.76$$

- **Weighted linear function** are also possible

$$Eval(s) = w_1 \times f_1(s) + w_2 \times f_2(s) + w_3 \times f_3(s) + ...$$

Important Considerations

- **Cutting off search**: having fixed depth may not be a good idea. **Quiescent** states are unlikely to have wild swing so expand till it. **Horizon effect** is difficult to eliminate so **singular extension**[3] could be used
- **Forward Pruning**: cutting off the some search without further consideration. (we do not evaluate all possibilities **beam search**, we only use some in our mind) This may be fatal.
- **Search Vs Lookup** We have some policy on how to start and finish well. Use them as lookup

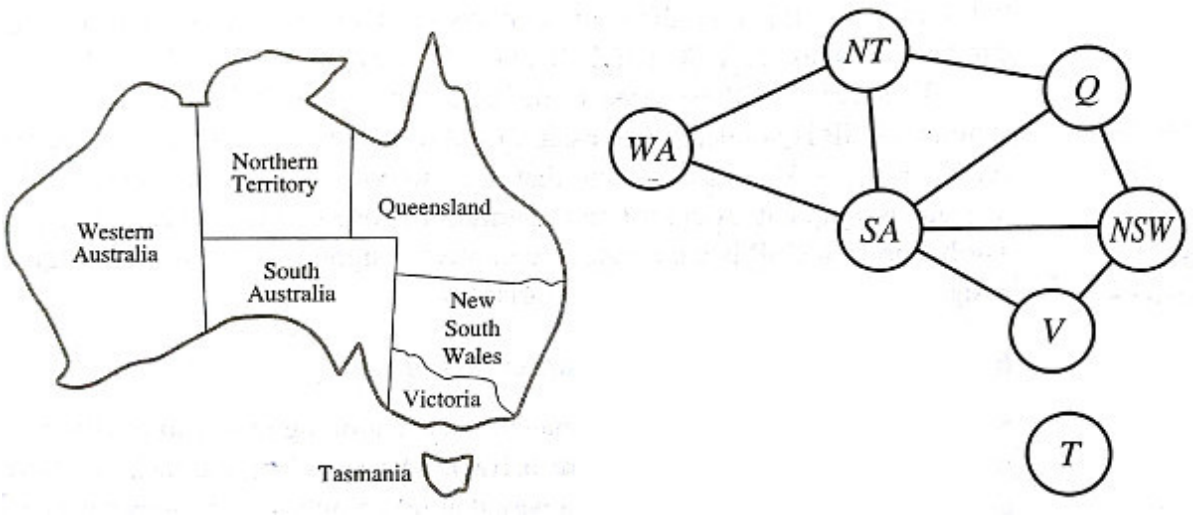
[3] can I take a previous good move

Constraint Satisfaction

- A Constraint Satisfaction Problem (CSP) has three components
  1. X as a set of variables $\{X_1, X_2, X_3, ..., X_n\}$
  2. D as a set of domains $\{D_1, D_2, D_3, ..., D_n\}$
  3. C set of constraints that specify allowable combinations of values
- Each D_i contains allowable set of values $\{v_1, v_2, v_3, ..., v_k\}$ for X_i
- Each C_i contains a pair <scope, relation>
  such as $<(X_1, X_3), X_1 \neq X_3>$
- To solve CSP it needs state space and a notion of solution
- An assignment of variables such as $(X_1 = v_1, X_2 = v_2, ..., X_n = v_n)$ that does not violates any constraints is called **consistent** or legal solution.
- Our target to have complete assignment that is consistent.

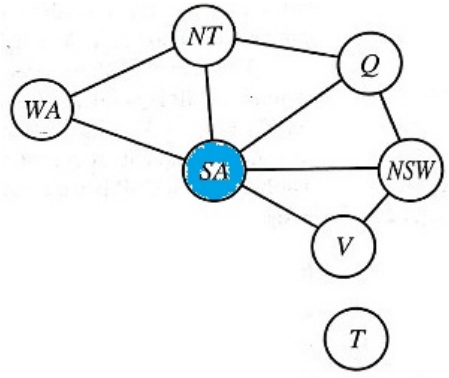
Example: Map Coloring

Given three colors {red,green,blue}, can you color following map such that no two neighboring region have same color.



- X = {WA, NT, Q, NSW, V, SA, T}
- C = {SA≠WA, SA≠NT, SA≠Q, SA≠NSW, SA≠V, WA≠NT, NT≠Q, Q≠NSW, NSW≠V}
- There are many solution to the problem
- It is helpful to visualize as a **constraint graph** (node: variable, edge: participate in constraint)

Constraint Propagation

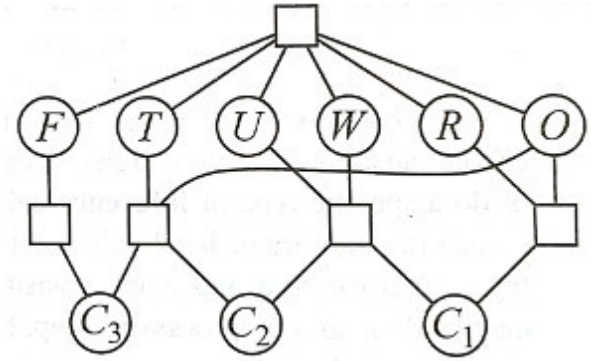


- Once you choose (SA=blue)
- you can conclude that none of its five members could take the color blue
- Without taking advantage of constraint propagation, search needs to consider $3^5 = 243$ assignments for the five neighbors.
- With constraint propagation, it needs $2^5 = 32$ only
- 87% reduction

Example: Cryptarithmethc Puzzle

Consider following addition (we have to find digits)

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$



- Each letter represents a different digit
- We need Alldiff{F,T,U,W,R,O}
- Additional constraints are

$$\begin{aligned} O + O &= R + 10 \times C_{10} \\ C_{10} + W + W &= U + 10 \times C_{100} \\ C_{100} + T + T &= O + 10 \times C_{1000} \\ C_{1000} &= F \end{aligned} \tag{1}$$

Thank You!

Thank you very much for your attention!

Queries ?

(Reference[4])

[4] 1) Book - *AIMA*, ch-04+05, Russell and Norvig.