1. Combinations : ${}^nC_x = n!/x!(n-x)!$: No of distinct combinations of n objects taking x at a time. Order is not important.

2. Permutations: ${}^nP_x = n!/(n-x)!$: No. of ways to arrange n objects taking x at a time. Order of the choice is important.

3. Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$; $P(B) > 0$.

4. Total Probability: $P(A) = \sum P(B_i) \cdot P(A|B_i) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A|B_n)$

5. Bayes Theorem: $\underbrace{P(A|B)}_{\text{Posterior}} = \dfrac{\overbrace{P(B|A)}^{\text{Likelihood}} \cdot \overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{Marginalization}}}$

6. Independence : A & B are independent if $P(A \cap B) = P(A) \cdot P(B)$. ie; $P(A|B) = P(A)$

7. Expected Value : $E(x) = \sum_{x \in D} x \cdot p(x)$ | $\mu_x = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$.

Discrete RV | Continous RV

8. Expectation of a function: $E(h(x)) = \sum h(x) \cdot p(x)$ | $E[h(x)] = \int_{-\infty}^{\infty} h(x) \cdot f(x)\, dx$

9. Variance: $V(x) = \sigma^2 = \sum (x-\mu)^2\, p(x)$
$= E((x-\mu)^2) = E(x^2) - [E(x)]^2$ | $V(x) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$
$= E[(x-\mu)^2] = E(x^2) - [E(x)]^2$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$.

10. Binomial Random variable : X = no. of S (success) among n trials.

11. Pmf of binomial RV: $b(x, n, p) \begin{cases} = {}^nC_x \cdot p^x (1-p)^{n-x} & ; \text{ if } x = 0, 1, 2, \dots, n \\ = 0 & ; \text{ otherwise} \end{cases}$

12. Binomial Mean & Variance: $\mu_x = E(x) = np$
$\sigma^2 = Var(x) = npq \Rightarrow SD(x) = \sqrt{npq}$ | $q = 1-p$

13. Bernoulli's Trial : Special case of binomial with $n = 1$. $E(x) = p$ | $Var(x) = p \cdot (1-p)$

14. Poisson Distribution : Events happen at a constant rate over time.

$\boxed{\mu = \sigma^2 = \lambda}$; λ = Expected no. of hits in a given period = rate × time.

15. Poisson PMF: $p(x, \mu) = \frac{e^{-\mu}\mu^x}{x!}$; $x = 0, 1, 2, 3 \ldots$

16. Binomial → Poisson: Any binomial experiment with large n and small p ≈ Poisson process.

$\mu = np$; if $n > 50$ & $np < 5$.

17. Normal Distribution:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \; ; \; -\infty < x < \infty$$

$1\sigma = 68.26\%$
$2\sigma = 95.46\%$
$3\sigma = 99\%$

18. Standard Normal: $\mu = 0$; $\sigma = 1$ | $f(z, 0, 1) = \frac{1}{2\pi} e^{-z^2/2}$

19. Standardization: $Z = \frac{x - \mu}{\sigma}$; $\sigma \neq 0$

20. Z-score: No. of std deviations a value of x is above/below the mean.

21. Binomial → Normal: As sample size become larger, binomial distribution → Normal.

Occurs faster for smaller values for n; when p is near 0.5.

$P(X \leq x) = B(x, n, p) = \phi\left(\frac{x + 0.5 - np}{\sqrt{npq}}\right)$; if $np > 10$ & $nq > 10$

$\mu = np$ & $\sigma = \sqrt{npq}$

22. Random sampling: Every unit of the population has same probability of being selected.

23. Central Limit Theorem: Sample means are normally distributed for sufficiently large ($n \geq 30$); regardless of the shape of the population.

$\boxed{\mu_{\bar{x}} = \mu}$ $\boxed{\sigma_{\bar{x}} = \sigma/\sqrt{n}}$; $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \Rightarrow \boxed{Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$

24. Finite Correction Factor: $\sqrt{\frac{N-n}{N-1}}$; if $\frac{n}{N} > 0.05 \Rightarrow Z = \frac{x - \bar{\mu}}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}$

25. Sample Proportion: $\hat{p} = x/n$ | $Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$

p = population proportion ; $q = 1 - p$

$\hat{p}$ = Sample proportion

$np > 5$ & $nq > 5$ SD of Sample Prop $= \sqrt{pq/n}$

26. Confidence Interval: Range of values within which we declare, with some confidence, the population parameter lies.

27. **CI for μ:** $\mu = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$; $\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

α is the area outside CI area in curve.

28. **CI for proportions:** $\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} \le p \le \hat{p} + Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$; $\hat{q} = 1 - \hat{p}$

29. **Hypothesis Testing:** Decisions are based on sample estimates to generalize on population.

$$|\text{Estimate} - \text{Parameter}| = \begin{cases} 0 \\ \text{Small} \\ \text{Large} \end{cases}$$

30. **Critical Value:** Beyond critical value lies rejection region. $\mu_1 \neq \mu_2$: Two tailed

$\mu_1 < \mu_2$ or $\mu_1 > \mu_2$: One tailed

31. **Type 1 Error:** Wrongly rejecting null hypothesis. Probability for commiting type-1 error is called level of significance (α).

32. **Type 2 Error:** Failure to reject false null hypothesis. Probability of T_2 error is beta.

33. **Z-test (Single Mean):** $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$; $n \geq 30$ | $Z = \frac{x - \mu}{\sigma/\sqrt{n} \sqrt{\frac{N-n}{N-1}}}$; if $\frac{n}{N} > 0.05$

34. **t-test (Single Mean):** $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$; when population SD is not known, use the sample SD

35. **CLT (2-mean):** Difference of two sample means are normally distributed.

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 \; ; \; \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

36. **Z-test for 2-mean:** $\boxed{Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$.

37. **CI for 2-mean:** $(\bar{x}_1 - \bar{x}_2) - Z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + Z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

38. **Z-test (Proportions):** $Z = \frac{\hat{p} - p}{\sqrt{p \cdot q/n}}$; $np \geq 5$; $nq \geq 5$

$q = 1 - p$.

39. **Matched Pairs test:** Analyze dependent samples or related samples. Also called as correlated t-test.

40. t-test (Matched pairs):

$$\boxed{t = \frac{\bar{d} - D}{\frac{s_d}{\sqrt{n}}}}$$

$\bar{d}$: mean sample diff.

D: mean population diff.

$$\bar{d} = \frac{\sum d}{n} \quad ; \quad s_d = \sqrt{\frac{\sum (d-\bar{d})^2}{n-1}}$$

41. CI for matched pairs: $\bar{d} - \frac{t \cdot s_d}{\sqrt{n}} \leq D \leq \bar{d} + \frac{t \cdot s_d}{\sqrt{n}}$

42. Z-test for two proportions:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1 \cdot q_1}{n_1} + \frac{p_2 \, q_2}{n_2}}}$$

On large populations:

$$\boxed{Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p} \cdot \bar{q} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}}$$

; $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$

$\bar{q} = 1 - \bar{p}$