BITS Pilani
Pilani Campus

Hypothesis Testing

Akanksha Bharadwaj
Asst. Professor, CS/IS Department

Forms Of Statistical Inference

Three forms of statistical inference

- Point estimation
- Interval estimation
- Hypothesis testing

Estimating The Population Mean Using The Z Statistic

Point Estimate

- A **point estimate** is a statistic taken from a sample that is used to estimate a population parameter.

- A point estimate is only as good as the representativeness of its sample.

- If other random samples are taken from the population, the point estimates derived from those samples are likely to vary.

Interval Estimate

- Because of variation in sample statistics, estimating a population parameter with an interval estimate is often preferable to using a point estimate.

- An interval estimate (confidence interval) is a range of values within which the analyst can declare, with some confidence, the population parameter lies.

Central Limit Theorem

- z formula for sample means can be used if the population standard deviation is known when sample sizes are large, regardless of the shape of the population distribution, or for smaller sizes if the population is normally distributed. $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

- Rearranging this formula algebraically to solve for μ gives $\mu = \bar{x} - z\dfrac{\sigma}{\sqrt{n}}$

- Because a sample mean can be greater than or less than the population mean, z can be positive or negative.

- Thus, μ can be $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$ (**confidence interval formula**)

Confidence Interval to Estimate μ

$100(1-\alpha)\%$ CONFIDENCE INTERVAL TO ESTIMATE μ: σ KNOWN (8.1)

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

or

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

where

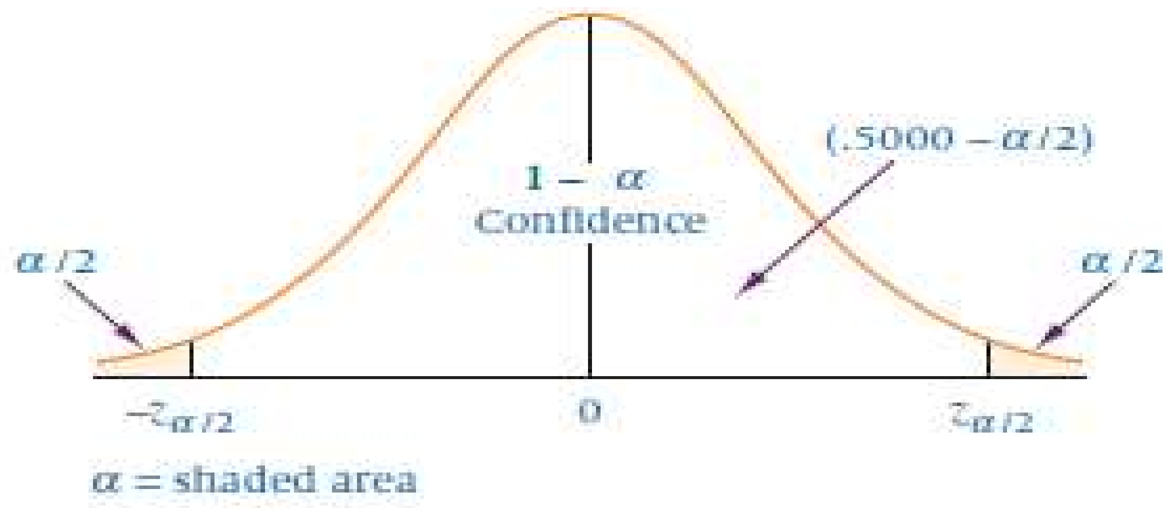
α = the area under the normal curve outside the confidence interval area

$\alpha/2$ = the area in one end (tail) of the distribution outside the confidence interval

- Alpha is the area under the normal curve in the tails of the distribution outside the area defined by the confidence interval

- The confidence interval formula (8.1) yields a range (interval) within which we feel with some confidence that the population mean is located.

Z Scores for Confidence Intervals in Relation to alpha



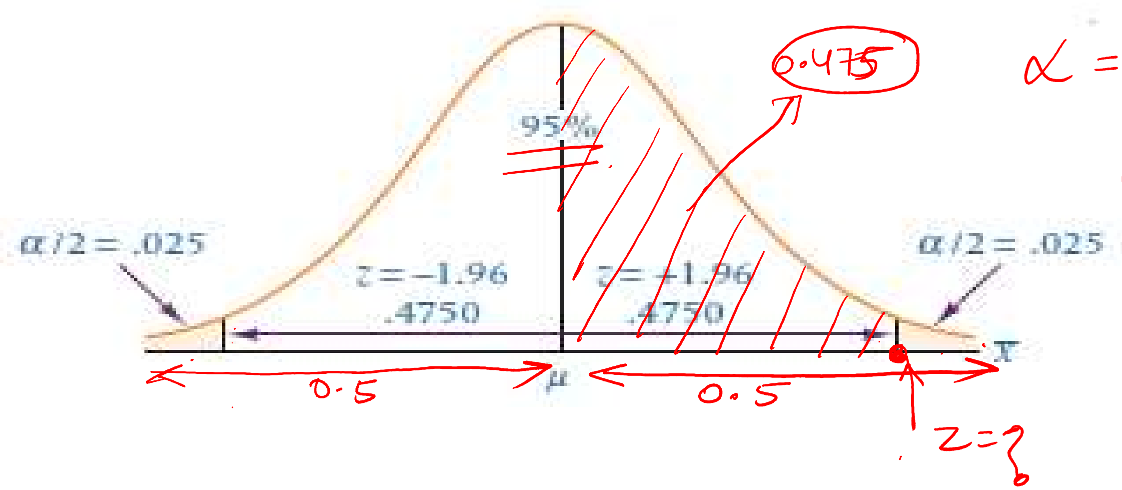
- Here we use α to locate the *z* value in constructing the confidence interval
- Because the standard normal table is based on areas between a z of 0 and $z_{\alpha/2}$,
- the table z value is found by locating the area of .5000 , which is the part of the normal curve between the middle of the curve and one of the tails.

Distribution of Sample Means for 95% Confidence

$1 = 0.95 + \alpha$

$\alpha = 0.05$

$\alpha/2 = 0.025$



- For 95% confidence, α = .05 and α/2 = .025.
- The value of $z_{\alpha/2}$ or $z_{.025}$ is found by looking in the standard normal table under .5000 - .0250 = **.4750**.
- This area in the table is associated with a *z* value of **1.96**.

Example

- In the cellular telephone company problem of estimating the population mean number of minutes called per residential user per month, from the sample of 85 bills it was determined that the sample mean is 510 minutes. Suppose past history and similar studies indicate that the population standard deviation is 46 minutes. Determine a 95% confidence interval.

$n = 85$

$\bar{x} = 510$ min.

$\sigma = 46$ min

for area $= 0.5 - 0.025$
$= 0.475$

value of $z = \pm 1.96$

$1 - \alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

Solution

$$\bar{x} - \frac{Z\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{Z\sigma}{\sqrt{n}}$$

$$510 - 1.96 \times \frac{46}{\sqrt{85}} \leq \mu \leq 510 + 1.96 \times \frac{46}{\sqrt{85}}$$

$$500.22 \leq \mu \leq 519.78$$

Finite correction factor

Confidence Level	z Value
90%	1.645
95%	1.96
98%	2.33
99%	2.575

Values of z for Common Levels of Confidence

CONFIDENCE INTERVAL TO ESTIMATE μ USING THE FINITE CORRECTION FACTOR (8.2)

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

Exercise

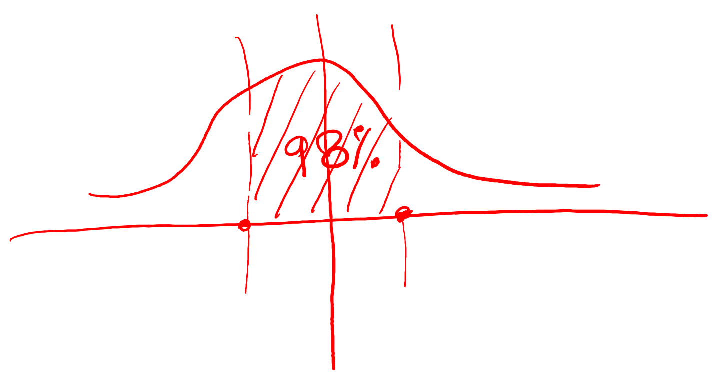
A study is conducted in a company that employs 800 engineers. A random sample of 50 engineers reveals that the average sample age is 34.3 years. Historically, the population standard deviation of the age of the company's engineers is approximately 8 years. Construct a 98% confidence interval to estimate the average age of all the engineers in this company.

$N = 800$

$n = 50$

$\bar{x} = 34.3$ yrs

$\sigma = 8$ yrs.



Solution

$1 - \alpha = 0.98$

$\alpha = 0.02$, $\alpha/2 = 0.01$

area under curve from mean to z value $= 0.5 - 0.01 = 0.49$

for $\underline{\underline{0.49}}$ value of z will be 2.33

$$34.3 - 2.33 \times \frac{8}{\sqrt{50}} \sqrt{\frac{800-50}{800-1}} \leq \mu \leq 34.3 + 2.33 \times \frac{8}{\sqrt{50}} \sqrt{\frac{750}{799}}$$

$$31.75 \leq \mu \leq 36.85$$

Estimating The Population Proportion

- Methods similar to those used earlier can be used to estimate the population proportion.
- The central limit theorem for sample proportions led to the following formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

- where q = 1 - p. Recall that this formula can be applied only when n*p and n*q are greater than 5.

- **for confidence interval purposes only and for large sample sizes**— is substituted for p in the denominator, yielding

$$z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}}$$

Confidence Interval To Estimate P

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$$

where

$\hat{p}$ = sample proportion
$\hat{q} = 1 - \hat{p}$
p = population proportion
n = sample size

In this formula, $\hat{p}$ is the point estimate and $\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$ is the error of the estimation.

Example

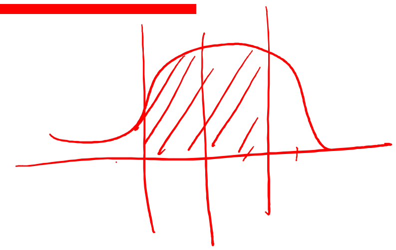
- A study of 87 randomly selected companies with a telemarketing operation revealed that 39% of the sampled companies used telemarketing to assist them in order processing. Using this information, how could a researcher estimate the *population* proportion of telemarketing companies that use their telemarketing operation to assist them in order processing?
- Use 95% confidence interval

$n = 87$

$\hat{p} = 0.39$ $\quad \hat{q} = 1 - 0.39 = 0.61$

Solution

95% confidence interval



$$z = ?$$

$$z = \pm 1.96$$

$$0.39 - 1.96\sqrt{\frac{0.39 \times 0.61}{87}} \leq p \leq 0.39 + 1.96\sqrt{\frac{0.39 \times 0.61}{87}}$$

$$0.2875 \leq p \leq 0.4925$$

Exercise (HW)

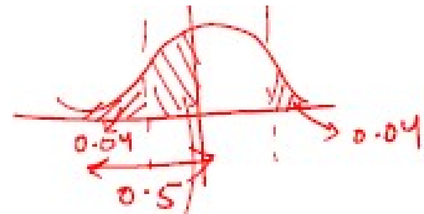
- Coopers & Lybrand surveyed 210 chief executives of fast-growing small companies. Only 51% of these executives had a management succession plan in place. A spokesperson for Cooper & Lybrand said that many companies do not worry about management succession unless it is an immediate problem. However, the unexpected exit of a corporate leader can disrupt and unfocus a company for long enough to cause it to lose its momentum. Use the data given to compute a 92% confidence interval to estimate the propor

Solution

$1-\alpha = 0.92$

$\therefore \alpha = 0.08$

$\alpha/2 = 0.04$



$\hat{p} = 0.51$

$n = 210$

$0.5 - 0.04$

$= 0.46$

for 0.46 my $z = \pm 1.75$

$$0.51 - 1.75\sqrt{\frac{0.51 \times 0.49}{210}} \le p \le 0.51 + 1.75\sqrt{\frac{0.51 \times 0.49}{210}}$$

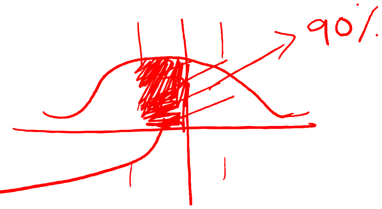
$$\underline{\underline{0.45 \le p \le 0.57}}$$

Exercise

- A clothing company produces men's jeans. The jeans are made and sold with either a regular cut or a boot cut. In an effort to estimate the proportion of their men's jeans market in Oklahoma City that prefers boot-cut jeans, the analyst takes a random sample of 212 jeans sales from the company's two Oklahoma City retail outlets. Only 34 of the sales were for boot-cut jeans. Construct a 90% confidence interval to estimate the proportion of the population in Oklahoma City who prefer boot-cut jeans.

$n = 212$ $\hat{p} = \frac{34}{212} = 0.16$

Solution



$\hat{q} = 1-\hat{p}$
$= 1-0.16$
$= 0.84$

$1 = 0.9+\alpha$

$\alpha = 0.1$

$\alpha/2 = 0.05$

area under curve $= 0.5 - 0.05 = 0.45$

for 0.45 value of $z = \pm 1.645$

$$0.16 - 1.645\sqrt{\frac{0.16\times 0.84}{212}} \le p \le 0.16 + 1.645\sqrt{\frac{0.16\times 0.84}{212}}$$

$$0.119 \le p \le 0.201$$

BITS Pilani
Pilani Campus

SS ZG536, ADV STAT TECHNIQUES FOR ANALYTICS
Contact Session 6

Need for testing of hypothesis

- Often the decisions are made based on samples estimates to generalize on population parameter (as described in sampling and estimation).

- In this process, there may be a difference between the estimate and the parameter

Need for testing of hypothesis

The following possibilities might arise due to sampling

$$\left|\text{Estimate} - \text{Parameter}\right| = \begin{cases} 0 \\ \text{Small} \\ \text{Large} \end{cases}$$

Case(i): If the difference is zero, it is called unbiased

Need for testing of hypothesis

Case(ii): If the difference is small, it may be due to chance or sampling error (improper sampling technique used leads to sampling error)

Case(iii): If the difference is large, it may be a real one or due to sampling error (improper sampling technique used leads to sampling error)

Hence, there is a need to test what type of difference is between estimate and parameter.

Statistical Hypothesis

A statement which is yet to be proved/ established or a statement on the parameter(s) of the Probability distribution to be tested

Null Hypothesis

there is nothing new happening, the old theory is still true, the old standard is correct, and the system is in control.

Alternative Hypothesis

states that the new theory is true, there are new standards, the system is out of control, and/or something is happening.

Hypothesis testing (Non-statistical)

Criminal Trial

A suspected criminal is produced before jury. The Jury has to decide whether the defendant is innocent or guilty.

Jury

Jury must decide between two hypotheses

The null hypothesis → **H_0**: The defendant may be innocent

The alternative hypothesis → **H_1**: The defendant may be guilty

or H_a

Hypothesis - Formulation

Judge 1

Judge 2

Suppose based on evidences, if we are interested in finding proportion of false positivity in the judgment of two Judges

Formulate the hypotheses ???

Hypothesis - Formulation

H_0 The proportion of false positive judgement between Judges may be same

$$H_0 : P_1 = P_2$$

H_1 The proportion of false positive judgement by Judge 1 may be **lower than** proportion of false positive judgement by Judge 2

$$H_1 : P_1 < P_2$$

Hypothesis - Formulation

H_1

The proportion of false positive judgement by Judge 1 may be **more than** proportion of false positive judgement by Judge 2

$$H_1 : P_1 > P_2$$

H_1

The proportion of false positive judgement between both Judges may be different

$$H_1 : P_1 \neq P_2$$

Example

- Suppose flour packaged by a manufacturer is sold by weight; and a particular size of package is supposed to average 40 ounces. Suppose the manufacturer wants to test to determine whether their packaging process is out of control as determined by the weight of the flour packages.

- The null hypothesis for this experiment is that the average weight of the flour packages is 40 ounces (no problem).

- The alternative hypothesis is that the average is not 40 ounces (process is out of control).

$$H_0: \mu = 40 \text{ oz.}$$
$$H_a: \mu \neq 40 \text{ oz.}$$

Exercise

- According to the Cancer Control and Prevention committee, the proportion of Indian adults age 25 or older who smoke is 0.12. A researcher suspects that the rate is lower among Indian adults 25 or older who have a bachelor's degree or higher education level.

- **What is the null hypothesis in this case?**
- The Proportion of smokers among Indian adults 25 or older who have a bachelor degree or higher is 0.12
- **What is the alternative hypothesis in this case?**
- The Proportion of smokers among Indian adults 25 or older who have a bachelor degree or higher is less than 0.12

Rejection and Non-rejection Regions

The possible statistical outcomes of a study can be divided into two groups:

1. Those that cause the rejection of the null hypothesis

2. Those that do not cause the rejection of the null hypothesis.

Example

- Consider the flour-packaging manufacturing example. The null hypothesis is that the average fill for the population of packages is 40 ounces.

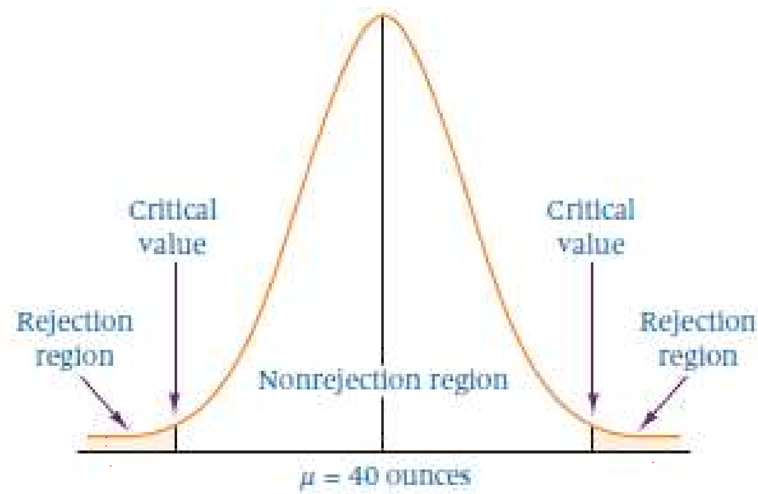
- Suppose a sample of 100 such packages is randomly selected, and a sample mean of 50 ounces is obtained

- This sample mean may be so far from what is reasonable to expect for a population with a mean of 40 ounces that the decision is made to reject the null hypothesis.

- **This promps the question: when is the sample mean so far away from the population mean that the null hypothesis is rejected**?

Critical values

- In each direction beyond the critical values lie the **rejection** regions.



- Any sample mean that falls in that region will lead the business researcher to reject the null hypothesis.

- Sample means that fall between the two critical values are close enough to the population mean that the business researcher will decide not to reject the null hypothesis. These means are in the **non-rejection** region.

$$\text{Test} = \begin{cases} \mu_1 < \mu_2 \Rightarrow \text{One} - \text{tailed test} \\ \mu_1 > \mu_2 \Rightarrow \text{One} - \text{tailed test} \\ \mu_1 \neq \mu_2 \Rightarrow \text{Two} - \text{tailed test} \end{cases}$$

Type I Errors

- **The null hypothesis is true, but the business researcher decides that it is not.**
- As an example, suppose the flour-packaging process actually is "in control" and is averaging 40 ounces of flour per package. Suppose also that a business researcher randomly selects 100 packages, weighs the contents of each, and computes a sample mean.
- It is possible, by chance, to randomly select 100 of the more extreme packages (mostly heavy weighted or mostly light weighted) resulting in a mean that falls in the rejection region.
- The decision is to reject the null hypothesis even though the population mean is actually 40 ounces. In this case, the business researcher has committed a Type I error.

Alpha

- Means that fall beyond the critical values will be considered so extreme that the business researcher chooses to reject the null hypothesis.
- However, if the null hypothesis is true, any mean that falls in a rejection region will result in a decision that produces a Type I error.
- The *probability of committing a Type I error* is called **alpha ( )** or **level of significance**.
- **Alpha equals the area under the curve that is in the rejection region beyond the critical value(s).**

Type II error

- It is committed when a business researcher ***fails to reject a false null hypothesis.***
- In this case, the null hypothesis is false, but a decision is made to not reject it.
- Suppose in the case of the flour problem that the packaging process is actually producing a population mean of 41 ounces even though the null hypothesis is 40 ounces.
- A sample of 100 packages yields a sample mean of 40.2 ounces, which falls in the non-rejection region. The business decision maker decides not to reject the null hypothesis.
- A Type II error has been committed.

Beta

- The probability of committing a Type II error is **beta** ( ).
- Beta occurs only when the null hypothesis is not true, the computation of beta varies with the many possible alternative parameters that might occur.
- For example, in the flour-packaging problem, if the population mean is not 40 ounces, then what is it? It could be 41, 38, or 42 ounces. A value of beta is associated with each of these alternative means

Relation between alpha and beta

- Power, which is equal to 1 - β, is the probability of a statistical test rejecting the null hypothesis when the null hypothesis is false.

	Reality	
Decision	H_0 is true	H_0 is false
Reject H_0, (conclude H_a)	Type I error	☺ Correct decision
Fail to reject H_0	☺ Correct decision	Type II error

- A business researcher cannot commit both a Type I error and a Type II error at the same time on the same hypothesis test.

- Generally, alpha and beta are inversely related.

- If alpha is reduced, then beta is increased, and vice versa.

Decision on α –error and β - error

Conventionally

- Fix α at 5%
- Minimize β
  - Maximize 1-β

power

Steps involved in Testing of Hypothesis

Typically, statisticians and researchers present the hypothesis testing process in terms of an eight-step approach:

- Step 1. Establish a null and alternative hypothesis.
- Step 2. Determine the appropriate statistical test.
- Step 3. Set the value of alpha, the Type I error rate.
- Step 4. Establish the decision rule.
- Step 5. Gather sample data.
- Step 6. Analyze the data.
- Step 7. Reach a statistical conclusion.
- Step 8. Make a business decision.

innovate achieve lead

BITS Pilani

Testing hypotheses about a population mean using the Z statistic

Z Test For A Single Mean

- Below formula can be used to test hypotheses about a single population mean when σ is known if the sample size is large (n >= 30) for any population and for small samples (n < 30) if x is known to be normally distributed in the population.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example

- A survey of CPAs across the United States found that the average net income for sole proprietor CPAs is $74,914. Because this survey is now more than ten years old, an accounting researcher wants to test this figure by taking a random sample of 112 sole proprietor accountants in the United States to determine whether the net income figure changed. The researcher could use the eight steps of hypothesis testing to do so. Assume the population standard deviation of net incomes for sole proprietor CPAs is $14,530.

Solution

- At step 1, the hypotheses must be established. Because the researcher is testing to determine whether the figure has changed, the alternative hypothesis is that the mean net income is not $74,914.

- The null hypothesis is that the mean still equals $74,914. These hypotheses follow.

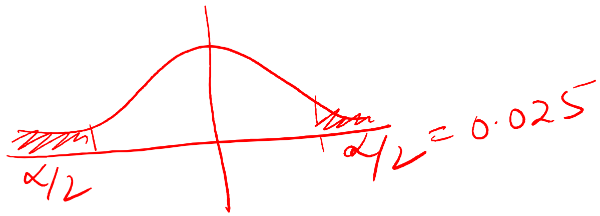
$$H_0: \mu = \$74{,}914$$

$$H_1 / H_a: \mu \neq \$74{,}914$$

TEST:

- Step 2 is to determine the appropriate statistical test and sampling distribution. Because the population standard deviation is known ($14,530) and the researcher is using the sample mean as the statistic, **the z test for a single mean is the appropriate test statistic.**

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



- Step 3 is to specify the Type I error rate, or alpha, which is .05 in this problem.

- Step 4 is to state the decision rule. Because the test is two tailed and alpha is .05, there is 2 or .025 area in each of the tails of the distribution. Thus, the rejection region is in the two ends of the distribution with 2.5% of the area in each.

- There is a .4750 area between the mean and each of the critical values that separate the tails of the distribution (the rejection region) from the non-rejection region.

- By using this .4750 area and Z Table, the critical *z* value can be obtained.

$$z_{\alpha/2} = \pm 1.96 \longrightarrow \text{critical } z \text{ value}$$

Step 5 is to gather the data. Suppose the 112 CPAs who respond produce a sample mean of \$78,695. At step 6, the value of the test statistic is calculated by using $\bar{x}$ = \$78,695, $n = 112$, σ = \$14,530, and a hypothesized μ = \$74,914:

observed z value $= \dfrac{78,695 - 74,914}{\dfrac{14530}{\sqrt{112}}}$

$= \underline{\underline{2.75}}$

ACTION:

- Because this test statistic, $z = 2.75$, is greater than the critical value of z in the upper tail of the distribution, $z = +1.96$,

- the statistical conclusion reached at step 7 of the hypothesis- testing process is to reject the null hypothesis.

- *The calculated test statistic* is often referred to as the **observed value**. Thus, the observed value of z for this problem is 2.75 and the critical value of z for this problem is 1.96.

Reject NULL hypothesis

Testing the Mean with a Finite Population

- Remember that if the sample size is less than 5% of the population, the finite correction factor does not significantly alter the solution.

FORMULA TO TEST HYPOTHESES ABOUT μ WITH A FINITE POPULATION (9.2)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}$$

- In the CPA net income example, suppose only 600 sole proprietor CPAs practice in the United States.
- Z= ?

$$Z = \frac{78,695 - 74,914}{\frac{14530}{\sqrt{112}}\sqrt{\frac{600-112}{600-1}}} = \frac{3781}{1239.2}$$

$$= 3.05$$

Reject NULL hypothesis

Using the p-Value to Test Hypotheses

- Another way to reach a statistical conclusion in hypothesis testing problems is by using the ***p*-value**, sometimes referred to as **observed significance level**

- The p-value defines the smallest value of alpha for which the null hypothesis can be rejected.

- For example, if the p-value of a test is .038, the null hypothesis cannot be rejected at α = .01 because .038 is the smallest value of alpha for which the null hypothesis can be rejected. However, the null hypothesis can be rejected for α = .05.

Rejecting the Null Hypothesis Using p-Values

Range of p-Values	Rejection Range
p-value $> .10$	Cannot reject the null hypothesis for commonly accepted values of alpha
$.05 < p$-value $\leq .10$	Reject the null hypothesis for $\alpha = .10$
$.01 < p$-value $\leq .05$	Reject the null hypothesis for $\alpha = .05$
$.001 < p$-value $\leq .01$	Reject the null hypothesis for $\alpha = .01$
$.0001 < p$-value $\leq .001$	Reject the null hypothesis for $\alpha = .001$

Critical Value Method to Test Hypotheses

- The critical value method determines the **critical mean value** required for z to be in the rejection region and uses it to test the hypotheses.

$\alpha = 0.05$, $Z_c = \pm 1.96$

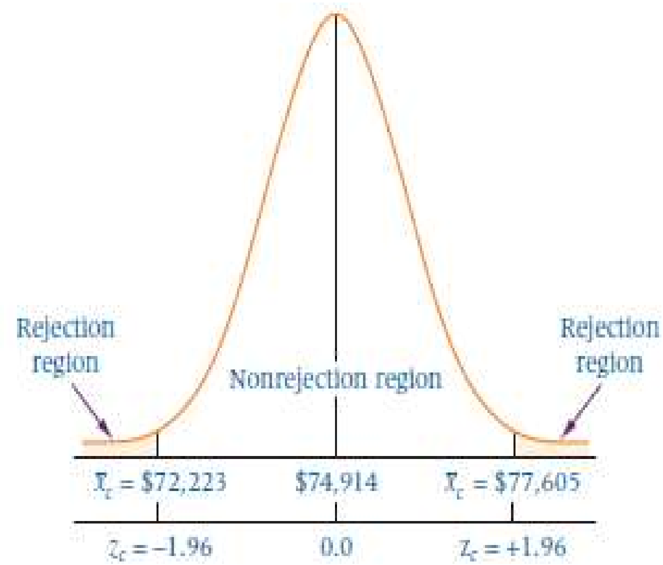
$$\pm 1.96 = \frac{\bar{x}_c - 74914}{\frac{14530}{\sqrt{112}}}$$

$$\bar{x}_c = 74914 \pm 2629$$

$$72,223 \leq \bar{x}_c \leq 77,605$$

Sample $\bar{x} = 78,695$

Reject NULL hypothesis



Exercise

- In an attempt to determine why customer service is important to managers in the United Kingdom, researchers surveyed managing directors of manufacturing plants in Scotland. One of the reasons proposed was that customer service is a means of retaining customers. On a scale from 1 to 5, with 1 being low and 5 being high, the survey respondents rated this reason more highly than any of the others, with a mean response of **4.30**. Suppose U.S. researchers believe American manufacturing managers would not rate this reason as highly and conduct a hypothesis test to prove their theory. Alpha is set at .**05**. Data are gathered and the following results are obtained. Use these data and the eight steps of hypothesis testing to determine whether U.S. managers rate this reason significantly lower than the **4.30** mean ascertained in the United Kingdom. Assume from previous studies that the population standard deviation is **0.574**.

3 4 5 5 4 5 5 4 4 4 4

4 4 4 4 5 4 4 4 3 4 4

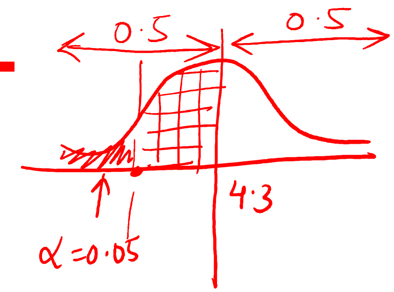
4 3 5 4 4 5 4 4 4 5

sample data

$\bar{x} \approx 4.156$

Solution

① $H_0: \mu = 4.3$, $H_a: \mu < 4.3$



② σ is known z test & $n = 32$.
one tail test

③ $\alpha = 0.05$

④ Shaded area = $0.5 - 0.05 = 0.45$

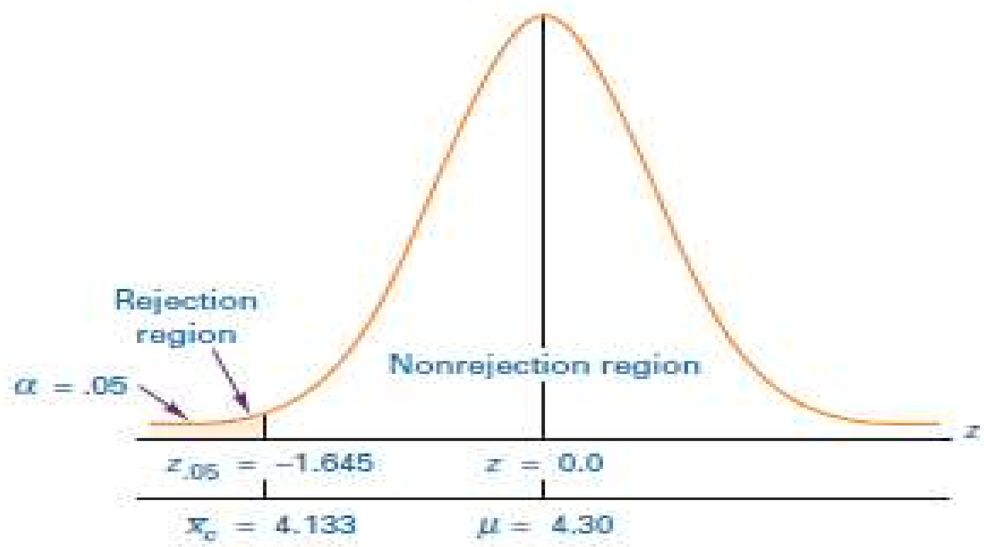
z value for 0.45 -1.645

$Z_c = -1.645$

⑤ observed value of z = $\dfrac{4.156 - 4.3}{0.574/\sqrt{32}} \simeq -1.42$

∴ -1.42 is not in rejection area
we will accept the
NULL hypothesis.
or fail to reject H_0

Solution



the probability of getting a z value at least this extreme when the null hypothesis is true is $.5000 - .4222 = .0778$. Hence, the null hypothesis cannot be rejected at $\alpha = .05$ because the smallest value of alpha for which the null hypothesis can be rejected is .0778. Had $\alpha = .10$, the decision would have been to reject the null hypothesis.

Using the critical value method: For what sample mean (or more extreme) value would the null hypothesis be rejected? This critical sample mean can be determined by using the critical z value associated with alpha, $z_{.05} = -1.645$.

$$z_c = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$-1.645 = \frac{\bar{x}_c - 4.30}{\frac{.574}{\sqrt{32}}}$$

$$\bar{x}_c = 4.133$$

The decision rule is that a sample mean less than 4.133 would be necessary to reject the null hypothesis. Because the mean obtained from the sample data is 4.156, the researchers fail to reject the null hypothesis. The preceding diagram includes a scale with the critical sample mean and the rejection region for the critical value method.

Testing hypotheses about a population mean using the t Statistic

Introduction

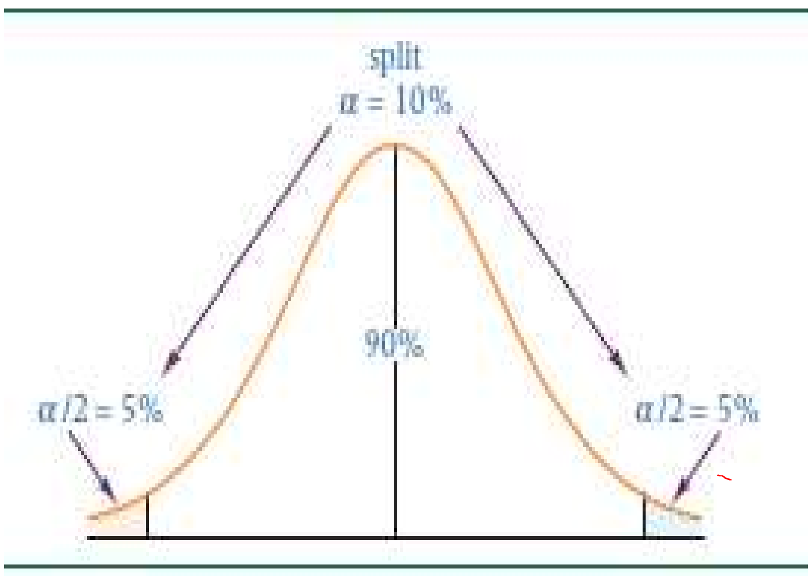
- Very often when a business researcher is gathering data to test hypotheses about a single population mean, the value of the population standard deviation is unknown and the researcher must use the sample standard deviation as an estimate of it. In such cases, the *z* test cannot be used.

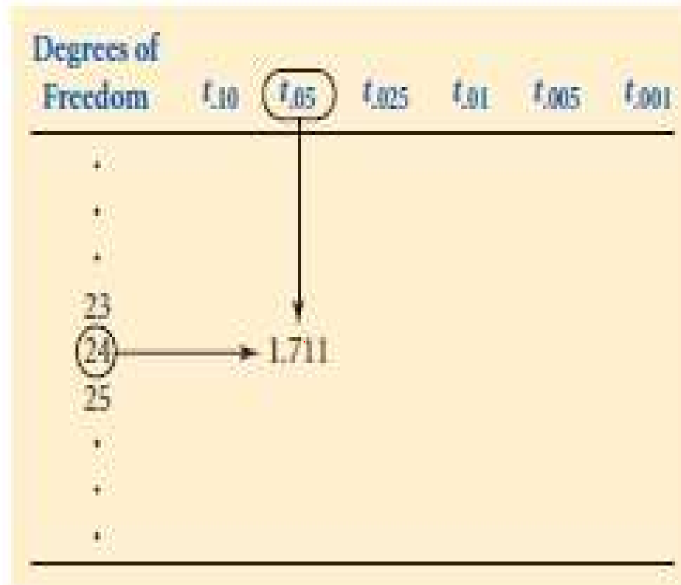
- Gosset developed the **t distribution**, which is used instead of the z distribution for doing inferential statistics on the population mean when the population standard deviation is unknown and the population is normally distributed. The formula for the t statistic is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Reading the t Distribution Table

- For example, if a 90% confidence interval is being computed the total area in the two tails is 10%. Thus, α is.10 and α/2 is .05, as indicated. n=25





Reading the t Distribution Table

- To find a value in the *t* distribution table requires knowing the degrees of freedom; each different value of degrees of freedom is associated with a different *t* distribution.
- The degrees of freedom for the t statistic presented in this section are computed by **n - 1.**
- The emphasis in the t table is on α, and each tail of the distribution contains of the area under the curve when confidence intervals are constructed.
- For confidence intervals, **the table t value is found in the column under the value of α/2 and in the row of the degrees of freedom (df) value**

Example

The U.S. Farmers' Production Company builds large harvesters. For a harvester to be properly balanced when operating, a 25-pound plate is installed on its side. The machine that produces these plates is set to yield plates that average 25 pounds. The distribution of plates produced from the machine is normal. However, the shop supervisor is worried that the machine is out of adjustment and is producing plates that do not average 25 pounds. To test this concern, he randomly selects 20 of the plates produced the day before and weighs them. Table 9.1 shows the weights obtained, along with the computed sample mean and sample standard deviation.

1 $H_0: \mu = 25$ pounds

$H_a: \mu \neq 25$ pounds

TABLE 9.1

Weights in Pounds of a Sample of 20 Plates

22.6	22.2	23.2	27.4	24.5
27.0	26.6	28.1	26.9	24.9
26.2	25.3	23.1	24.2	26.1
25.8	30.4	28.6	23.5	23.6

$\bar{x} = 25.51$, $s = 2.1933$, $n = 20$

Solution

(2) $\because \sigma$ is not known, we will use the t-statistic, 2-tailed test

(3) $\alpha = 0.05$

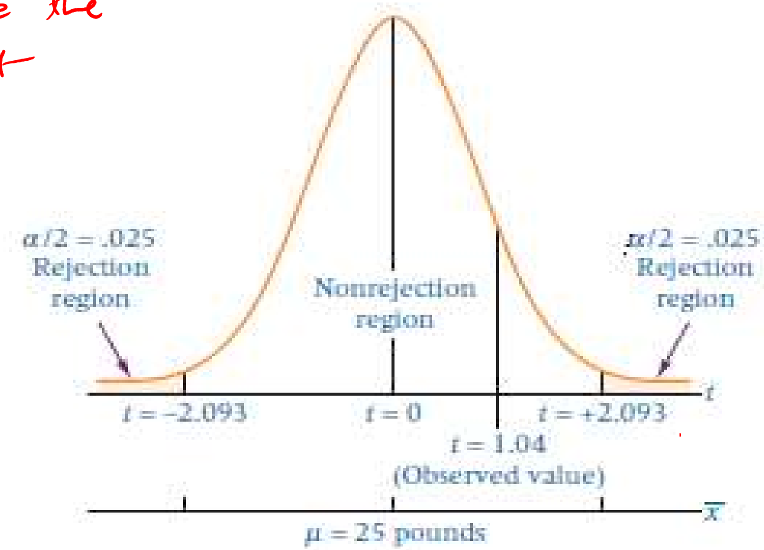
(4) $\alpha = 0.05$ ($\because$ 2 tail test)

$\alpha/2 = 0.025$

$n = 20$

$df = 20 - 1 = 19$

$t_{c\,0.025,19} = \pm 2.093$



(5) observed value of $t = \dfrac{25.51 - 25}{\frac{2.1933}{\sqrt{20}}} \approx \underline{\underline{1.04}}$

Fail to reject / Accept Null hypo.

Exercise (HW)

- Figures released by the U.S. Department of Agriculture show that the average size of farms has increased since 1940. In 1940, the mean size of a farm was 174 acres; by 1997, the average size was 471 acres. Between those years, the number of farms decreased but the amount of tillable land remained relatively constant, so now farms are bigger. This trend might be explained, in part, by the inability of small farms to compete with the prices and costs of large-scale operations and to produce a level of income necessary to support the farmers' desired standard of living. Suppose an agribusiness researcher believes the average size of farms has now increased from the 1997 mean figure of 471 acres. To test this notion, she randomly sampled 23 farms across the United States and ascertained the size of each farm from county records. The data she gathered follow. Use a 5% level of significance to test her hypothesis. Assume that number of acres per farm is normally distributed in the population.

445 489 474 505 553 477 454 463 466
557 502 449 438 500 466 477 557 433
545 511 590 561 560

Solution

(1) $H_0: \mu = 471$ $H_1: \mu > 471$

(2) $\because \sigma$ is not known t test

(3) $\alpha = 0.05$

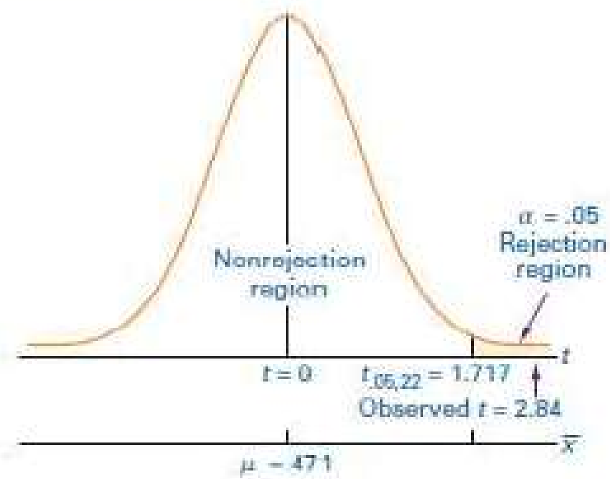
(4) one tail test $df = 22$

$t_{0.05, 22} = 1.717$

(5) observed t value

$\bar{x} = 498.78$, $s = 46.94$

$$t = \frac{498.78 - 471}{46.94/\sqrt{23}} = 2.84$$



$\because$ observed value > than the critical value

Reject NULL hypothesis