



**BITS Pilani**  
Hyderabad Campus

Distributed Computing (CS 5 – M5)

Distributed Mutual Exclusion

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# What is mutual exclusion?



- ❖ Mutual Exclusion also known as **Mutex** was first identified by Dijkstra
- ❖ When a process is accessing a shared variable, it is said to be in a critical section (code segment)
- ❖ When no two processes can be in Critical Section at the same time, this state is known as **Mutual Exclusion** which is a property of concurrency control and is used to prevent **race condition** (happens when the result of multiple thread execution in critical section differs according to the order in which the threads execute)

Mutual Exclusion Devices:-

Locks, recursive locks, semaphores, monitor, message passing etc.,

# Mutual exclusion in synchronization



- ❖ **Mutual exclusion** is a property of process synchronization which states that “no two processes can exist in the critical section at any given point of time”
- ❖ Any process synchronization technique being used must satisfy the property of mutual exclusion, without which it would not be possible to get rid of a race condition
- ❖ During concurrent execution of processes, processes need to enter the critical section (or the section of the program shared across processes) at times for execution
- ❖ It might so happen that because of the execution of multiple processes at once, the values stored in the critical section become inconsistent; in other words, the values depend on the sequence of execution of instructions – also known as a **race condition**
- ❖ The primary task of process synchronization is to get rid of race conditions while executing the critical section

# Mutual Exclusion in Distributed Computing



Mutual exclusion is a concurrency control property which is introduced to prevent race conditions; the requirement that a process can not enter its critical section while another concurrent process is currently present or executing in its critical section i.e., only one process is allowed to execute the critical section at any given instance of time

In single computer system, memory and other resources are shared between different processes. The status of shared resources and the status of users is easily available in the shared memory so with the help of shared variable (For example: Semaphores) mutual exclusion problem can be easily solved

In Distributed systems, we neither have shared memory nor a common physical clock and therefore we can not solve mutual exclusion problem using shared variables. To solve the mutual exclusion problem in distributed systems, message passing is used

A site in distributed system does not have the complete information of the state of the system due to lack of shared memory and a common physical clock

# Mutual Exclusion

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- ❖ for distributed systems –
  - ❖ decision as to which process is allowed access to the CS next is made by **message passing**
- ❖ must deal with **unpredictable message delays and incomplete knowledge of the system state**

# Mutual Exclusion

- ❑ ensures that concurrent access of processes to a shared resource or data is **serialized**
- ❑ executed in a mutually exclusive manner
- ❑ for distributed system –
  - ❑ only one process is allowed to execute the critical section (CS) at any given time
  - ❑ semaphores or a local kernel cannot be used to implement mutual exclusion

# Three approaches for mutual exclusion in distributed computing systems

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- ☐ Token based
- ☐ Non-token based
- ☐ Quorum based

# Token-Based Approach

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- ❖ a unique token is shared among the sites
- ❖ token is also called **PRIVILEGE** message
- ❖ a site is allowed to enter its CS if
  - ❖ it possesses the **token**
  - ❖ it continues to hold the token until the execution of the CS is over
- ❖ mutual exclusion is ensured because the token is unique



# Non-Token-Based Approach

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- two or more successive rounds of **messages are exchanged** among the sites to determine which site will enter the CS next
- a site enters the CS when an **assertion** becomes true
- **mutual exclusion is enforced because the assertion becomes true only at one site at any given time**

# Quorum-Based Approach

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- ❑ each site requests permission to execute the CS from a subset of sites
- ❑ subset of sites is called quorum
- ❑ quorums are formed in such a way that when two sites concurrently request access to the CS
  - ❑ at least one site receives both the requests
  - ❑ this site is responsible to make sure that only one request executes the CS at any time

# System Model for Mutual Exclusion

- ✓ system consists of **N sites**,  $S_1, S_2, \dots, S_N$
- ✓ without loss of generality, assume that a single process is running on each site
- ✓ **process at site  $S_i$  is  $p_i$**
- ✓ processes communicate **asynchronously** over an underlying communication network
- ✓ any process wishing to enter the CS
  - ✓ requests all other or a subset of processes by sending **REQUEST** messages
  - ✓ waits for appropriate **replies** before entering the CS
- ✓ while waiting the process is not allowed to make further requests to enter the CS

# System Model for Mutual Exclusion contd..



- ❑ site can be in one of the following 3 states:
  - ❑ requesting the CS
  - ❑ executing the CS
  - ❑ neither of the 2
- ❑ “requesting the CS” state - site is blocked and cannot make further requests for the CS
- ❑ “idle” state - site is executing outside the CS
- ❑ for token-based algorithms
  - ❑ a site can also be in a state where a site holding the token is executing outside the CS
  - ❑ such state is called *idle token state*

# System Model for mutual exclusion contd..



- ❖ at any instant, a site may have **several pending requests** for CS
- ❖ a site **queues** up these requests and serves them one at a time
- ❖ **nature of channels (FIFO or not) is algorithm specific**
- ❖ assume that:
  - ❖ channels reliably deliver all messages
  - ❖ sites do not crash
  - ❖ network does not get partitioned
- ❖ **timestamps are used to decide the priority of requests in case of a conflict**
- ❖ ***general rule - smaller the timestamp of a request, the higher its priority to execute the CS***

# System Model contd..

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## Notations:

- ❑ N - number of processes or sites involved in invoking the critical section
- ❑ T - average message delay
- ❑ E - average critical section execution time

# Requirements of mutual exclusion algorithms



## ❑ No Deadlock:

Two or more sites should not endlessly wait for any message that will never arrive

## ❑ No Starvation:

Every site which wants to execute critical section should get an opportunity to execute it in finite time. Any site should not wait indefinitely to execute critical section while other sites are repeatedly executing the critical section

## ❑ Fairness:

Each site should get a fair chance to execute the critical section. Any request to execute the critical section must be executed in the order they are made i.e Critical section execution requests should be executed in the order of their arrival in the system

## ❑ Fault Tolerance:

In case of failure, it should be able to recognize it by itself in order to continue functioning without any disruption

# Properties of Mutual Exclusion Algorithms

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A mutual exclusion algorithm should satisfy:

- ❖ Safety Property – absolutely necessary
- ❖ Liveness Property – important
- ❖ Fairness – important



# Requirements of Mutual Exclusion Algorithms



## ☐ **Safety property**

- ☐ at any instant, only one process can execute the critical section
- ☐ absolutely necessary property

## ☐ **Liveness property**

- ☐ absence of deadlock and starvation
- ☐ a site must not wait indefinitely to execute the CS while other sites are repeatedly executing the CS
- ☐ every requesting site should get an opportunity to execute the CS in finite time

# Mutual Exclusion Algorithms

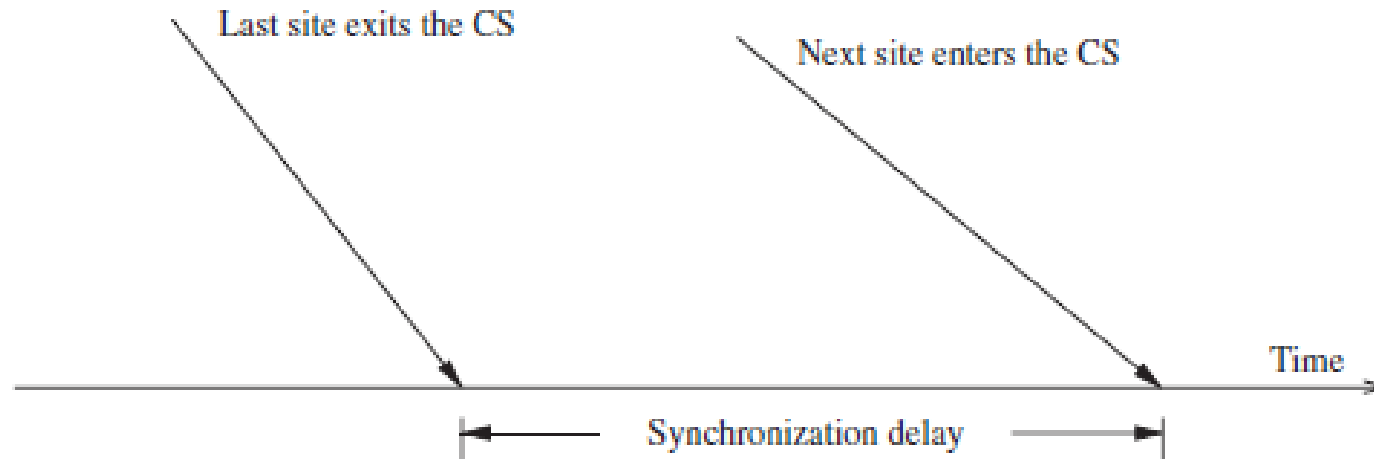


## ❑ Fairness

- ❑ each process gets a **fair chance** to execute the CS
- ❑ CS execution requests are executed in **order of their arrival** in the system
- ❑ time is determined by a **logical clock**

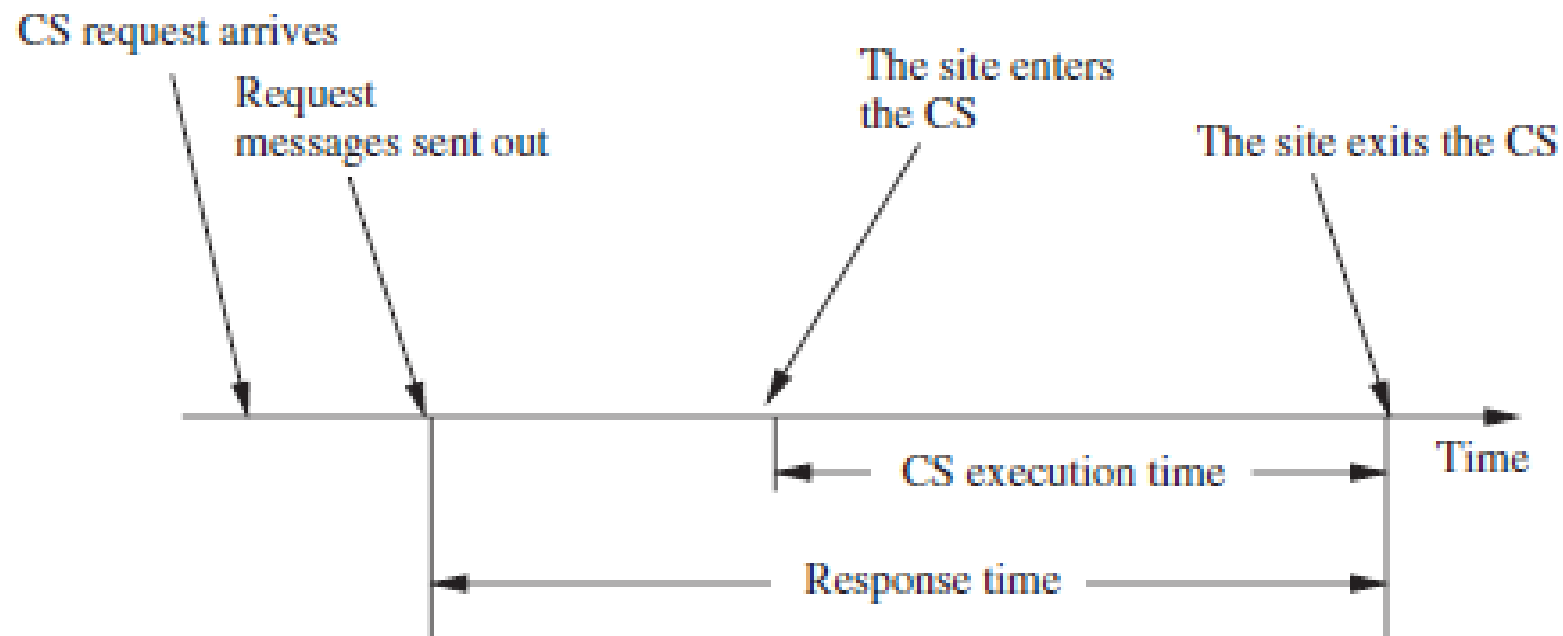
# Performance Metrics

- ❑ **Message complexity** - number of messages that are required per CS execution by a site
- ❑ **Synchronization delay** –
  - ❑ after a site leaves the CS, the time required before the next site enters the CS
  - ❑ one or more sequential message exchanges may be required after a site exits the CS and before the next site can enter the CS



# Performance Metrics

- ❑ **Response time** – time interval a request waits for its CS execution to be over after its request messages have been sent out
- ❑ **System throughput** -
  - ❑ rate at which the system executes requests for the CS
  - ❑ SD is Synchronization Delay
  - ❑ E is average critical section execution time
  - ❑ **System throughput =  $1/(SD+E)$**



# Performance Metrics

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## Low and high load performance –

- ☐ “low load” – not more than one request for the critical section present in the system simultaneously
- ☐ “high load” -
  - ☐ there is always a pending request for critical section at a site
  - ☐ after having executed a request, a site immediately initiates activities to execute its next CS request
  - ☐ a site is seldom in the idle state

# Lamport's Algorithm

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- ❑ time is determined by logical clocks
- ❑ when a site processes a request for the CS, it
  - ❑ updates its local clock
  - ❑ assigns the request a timestamp
- ❑ algorithm is fair - executes CS requests in the increasing order of timestamps
- ❑ every site  $S_i$  keeps a queue, `request_queuei`
- ❑ `request_queuei` contains mutual exclusion requests ordered by their timestamps

# Lamport's Algorithm

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- ❖ algorithm requires communication channels to deliver messages in FIFO order
- ❖ when a site removes a request from its request queue
  - ❖ its own request may come at the top of the queue
  - ❖ enables it to enter the CS
- ❖ when a site receives a REQUEST, REPLY, or RELEASE message
  - ❖ it updates its clock using the timestamp in the message

# Lamport's Algorithm

- ❖ **Requesting the critical section:**
- ❖ When a site  $S_i$  wants to enter the CS, it broadcasts a  $\text{REQUEST}(ts_i, i)$  message to all other sites and places the request on  $\text{request\_queue}_i$ .  $((ts_i, i))$  denotes the timestamp of the request)
- ❖ When a site  $S_j$  receives the  $\text{REQUEST}(ts_i, i)$  message from site  $S_i$ , it places site  $S_i$ 's request on  $\text{request\_queue}_j$  and returns a timestamped **REPLY** message to  $S_i$



# Lamport's Algorithm



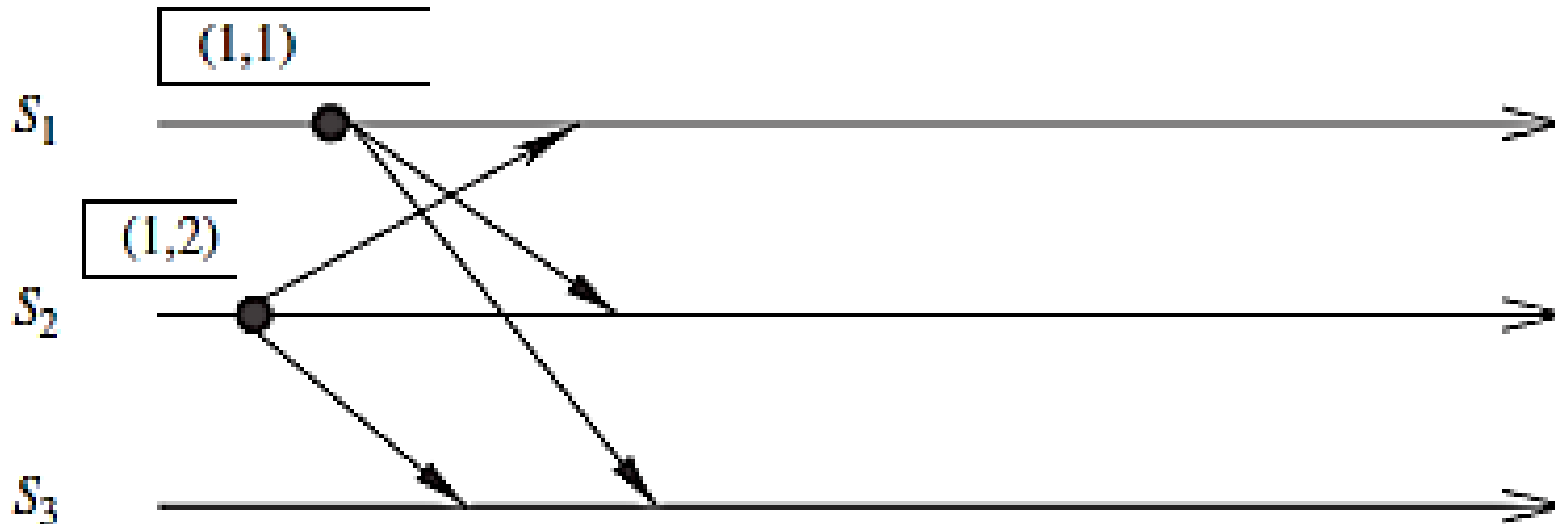
## ❑ Executing the critical section:

- ❑ Site  $S_i$  enters the CS when the following two conditions hold:
  - ❑ **L1:**  $S_i$  has received a message with timestamp larger than  $(ts_i, i)$  from all other sites
  - ❑ **L2:**  $S_i$ 's request is at the top of the request\_queue $_i$

## ❑ Releasing the critical section:

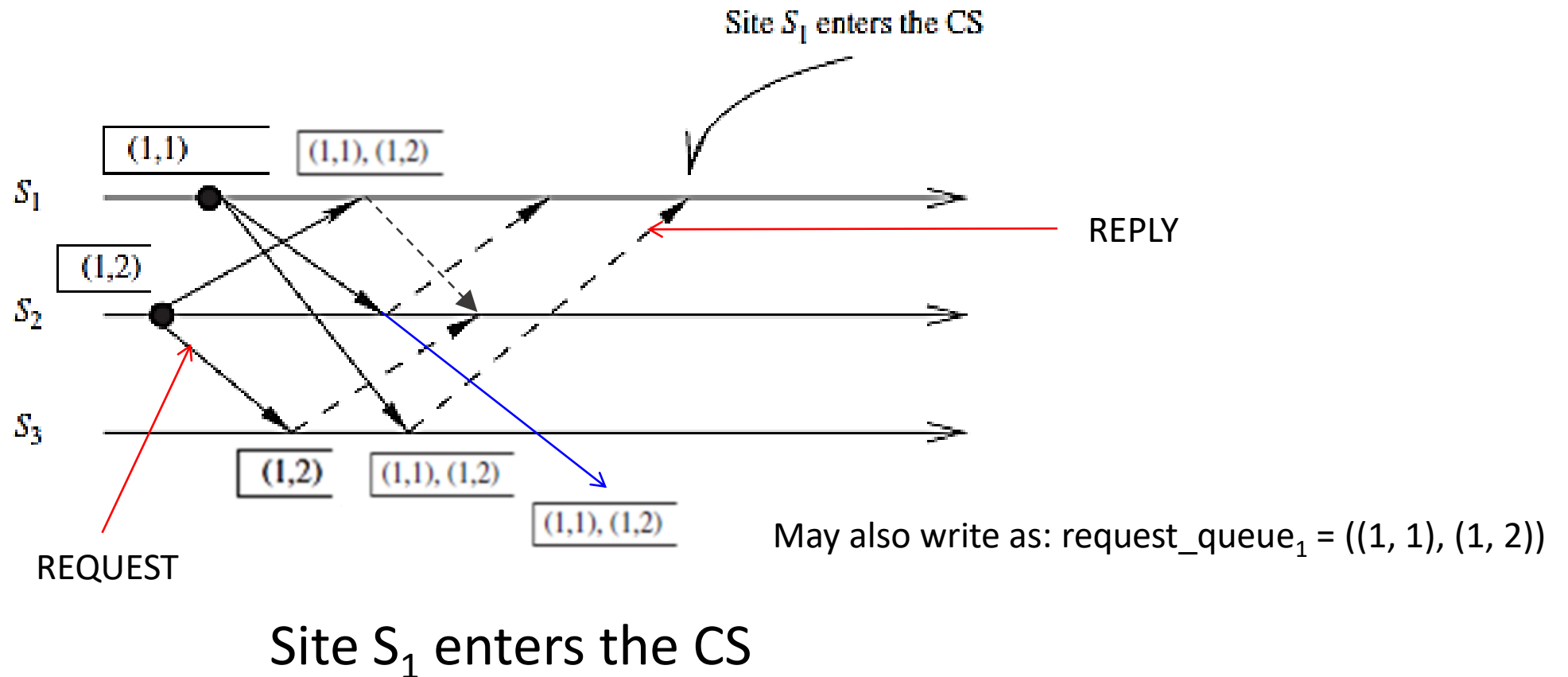
- ❑ • Site  $S_i$ , upon exiting the CS, removes its request from the top of its request queue and broadcasts a timestamped RELEASE message to all other sites
- ❑ • When a site  $S_j$  receives a RELEASE message from site  $S_i$ , it removes  $S_i$ 's request from its request queue

# Lamport's Algorithm

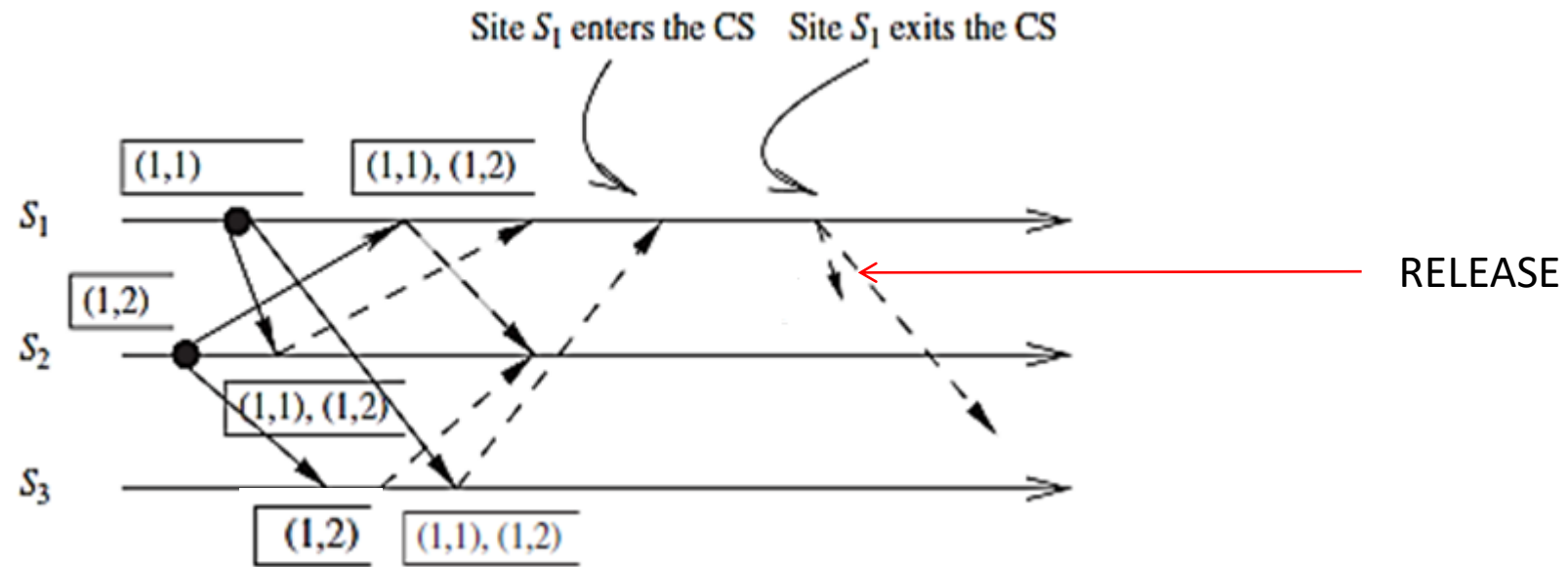


Sites  $S_1$  and  $S_2$  make requests for the CS

# Lamport's Algorithm

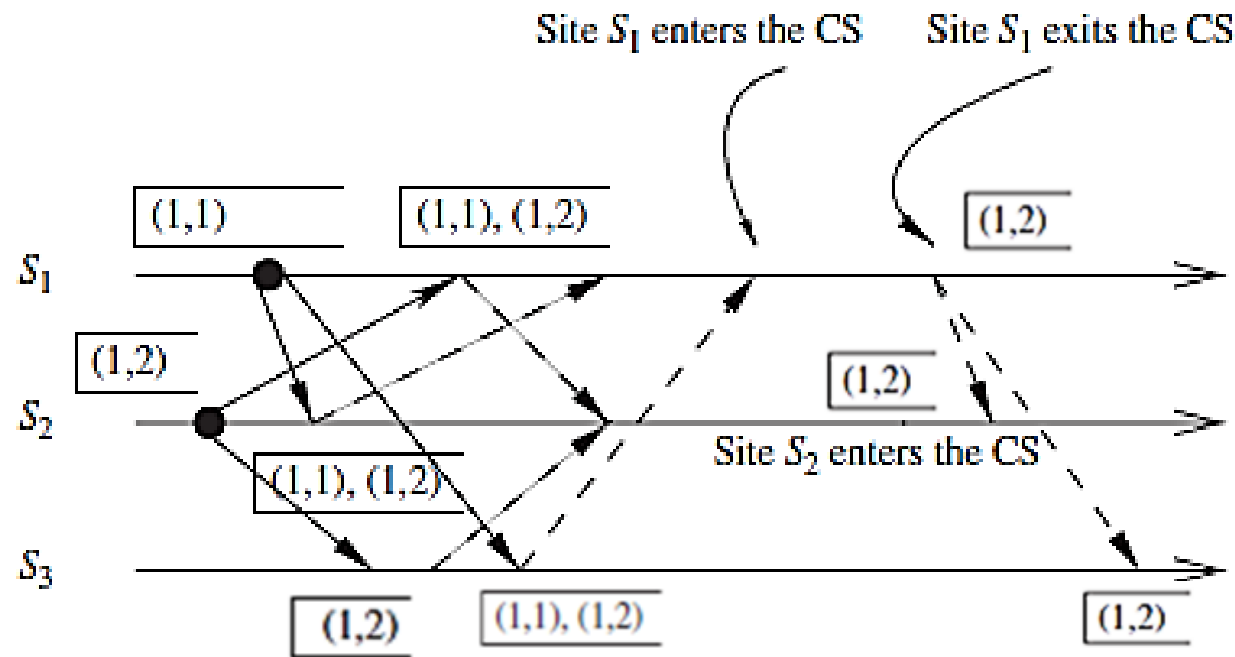


# Lamport's Algorithm



Site  $S_1$  exits the CS and sends RELEASE messages

# Lamport's Algorithm



Site  $S_2$  enters the CS

# Lamport's Algorithm

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## ☐ **Correctness**

- ☐ *Lamport's algorithm achieves mutual exclusion*
- ☐ *Lamport's algorithm is fair - requests for CS are executed in the order of their timestamps*

## ☐ **Performance**

- ☐ For each CS execution, requires  $(N - 1)$  REQUEST messages,  $(N - 1)$  REPLY messages, and  $(N - 1)$  RELEASE messages
- ☐ Requires  $3(N - 1)$  messages per CS invocation
- ☐ Synchronization delay in the algorithm is  $T$  ( $T$  = avg. message delay)

# Ricart–Agrawala Algorithm

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- ❖ communication channels are not required to be FIFO
- ❖ uses two types of messages:
  - ❖ REQUEST
  - ❖ REPLY
- ❖ a process sends a REQUEST message to all other processes to request their permission to enter the critical section
- ❖ a process sends a REPLY message to a process to give its permission to that process
- ❖ processes use Lamport-style logical clocks to assign a timestamp to critical section requests

# Ricart–Agrawala Algorithm

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- timestamps are used to decide the priority of requests in case of conflict –
  - if a process  $p_i$  that is waiting to execute the CS receives a REQUEST message from process  $p_j$ , then if the priority of  $p_j$ 's request is lower,  $p_i$  defers the REPLY to  $p_j$  and sends a REPLY message to  $p_j$  only after executing the CS for its pending request
  - otherwise,  $p_i$  sends a REPLY message to  $p_j$  immediately, provided it is currently not executing the CS
- if several processes are requesting execution of the CS, the highest priority request succeeds in collecting all the needed REPLY messages and gets to execute the CS



# Ricart–Agrawala Algorithm

- ❖ each process  $p_i$  maintains the request-deferred array,  $RD_i$
- ❖ size of  $RD_i$  = no. of processes in the system
- ❖ initially,  $\forall i \forall j: RD_i[j] = 0$
- ❖ whenever  $p_i$  defers the request sent by  $p_j$ , it sets  $RD_i[j] = 1$ ,
- ❖ after it has sent a REPLY message to  $p_j$ , it sets  $RD_i[j] = 0$
- ❖ when a site receives a message, it updates its clock using the timestamp in the message
- ❖ when a site takes up a request for the CS for processing, it
  - ❖ updates its local clock
  - ❖ assigns a timestamp to the request
- ❖ execution of the CS requests is always in the order of their timestamps

# Ricart–Agrawala Algorithm

## Requesting the critical section:

- (a) When a site  $S_i$  wants to enter the CS, it broadcasts a timestamped REQUEST message to all other sites
- (b) When site  $S_j$  receives a REQUEST message from site  $S_i$ , it sends a REPLY message to site  $S_i$  if site  $S_j$  is neither requesting nor executing the CS, or if the site  $S_j$  is requesting and  $S_i$ 's request's timestamp is smaller than site  $S_j$ 's own request's timestamp. Otherwise, the reply is deferred and  $S_j$  sets  $RD_j[i] = 1$

# Ricart–Agrawala Algorithm

## Executing the critical section:

(c) Site  $S_i$  enters the CS after it has received a REPLY message from every site it sent a REQUEST message to

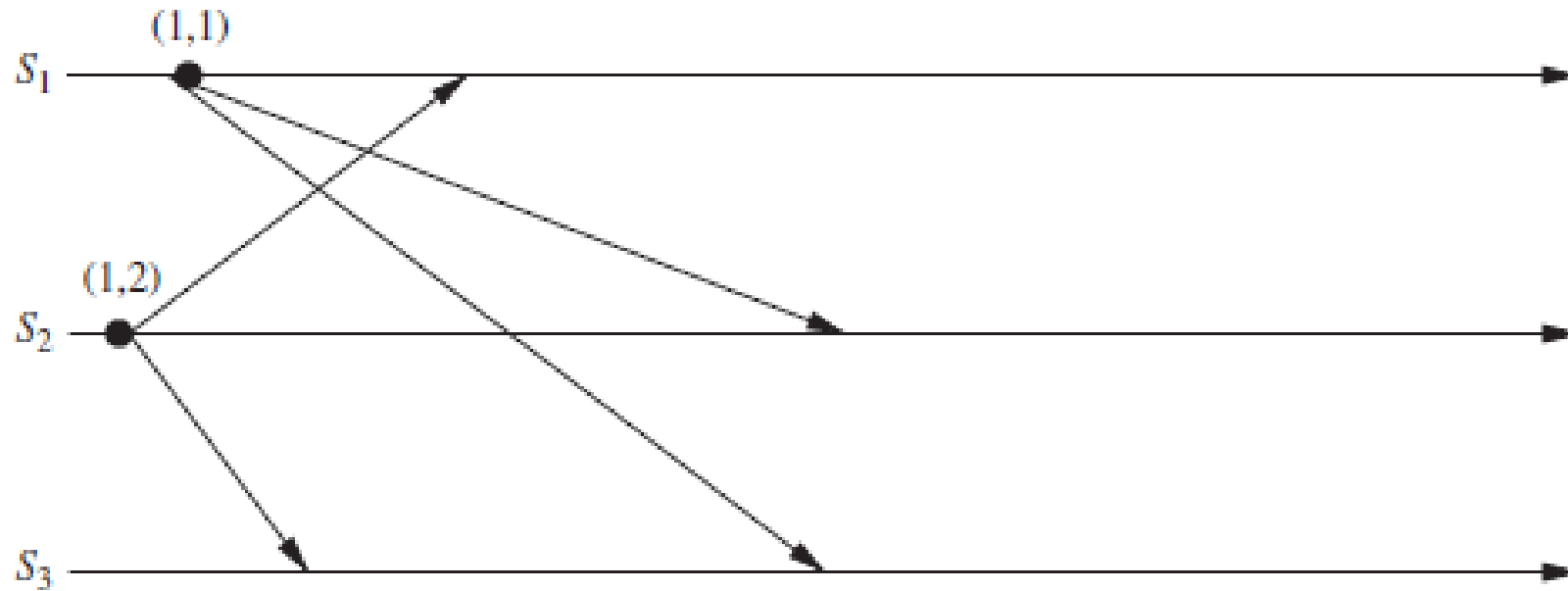
## Releasing the critical section:

(d) When site  $S_i$  exits the CS, it sends all the deferred REPLY messages:  
 $\forall j$  if  $RD_i[j] = 1$ , then  $S_i$  sends a REPLY message to  $S_j$  and sets  $RD_i[j] = 0$

## Correctness

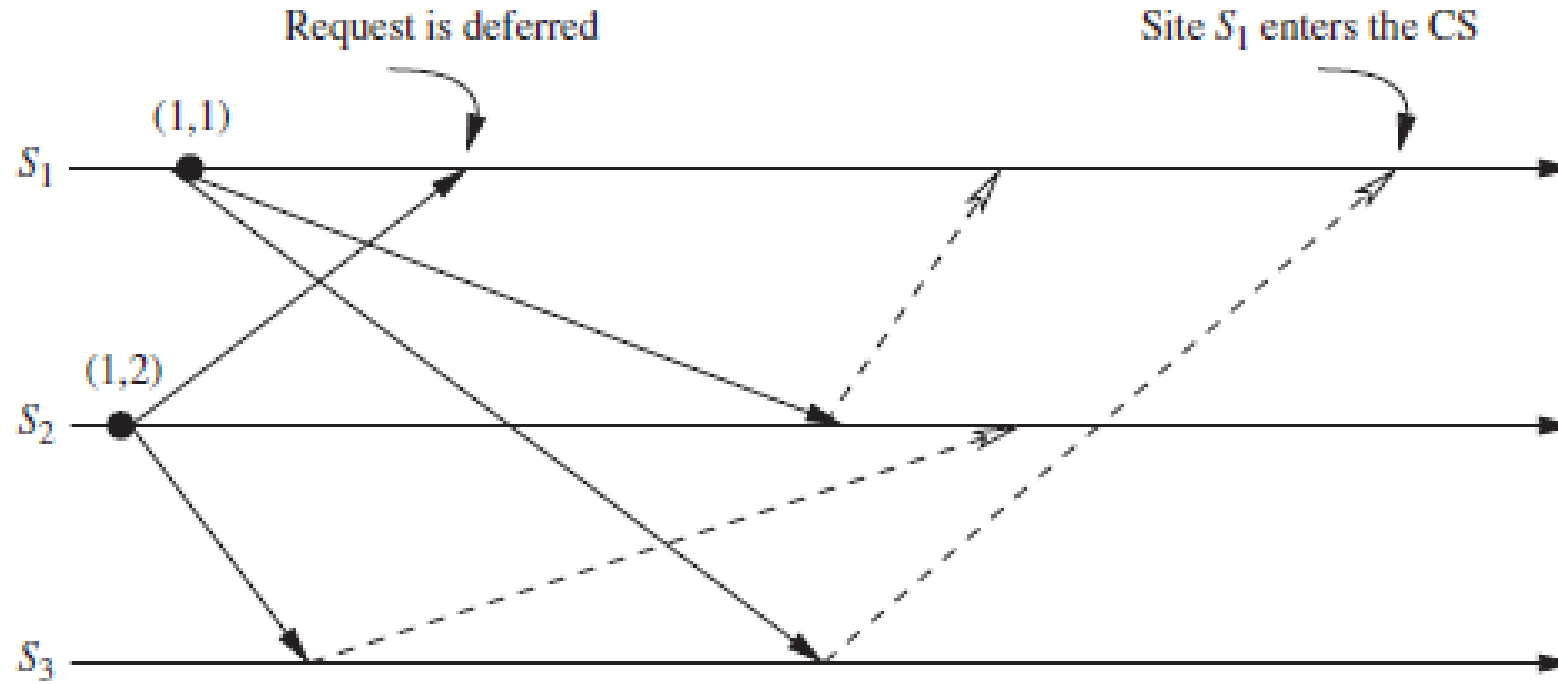
- *Ricart–Agrawala algorithm achieves mutual exclusion*

# Ricart–Agrawala Algorithm



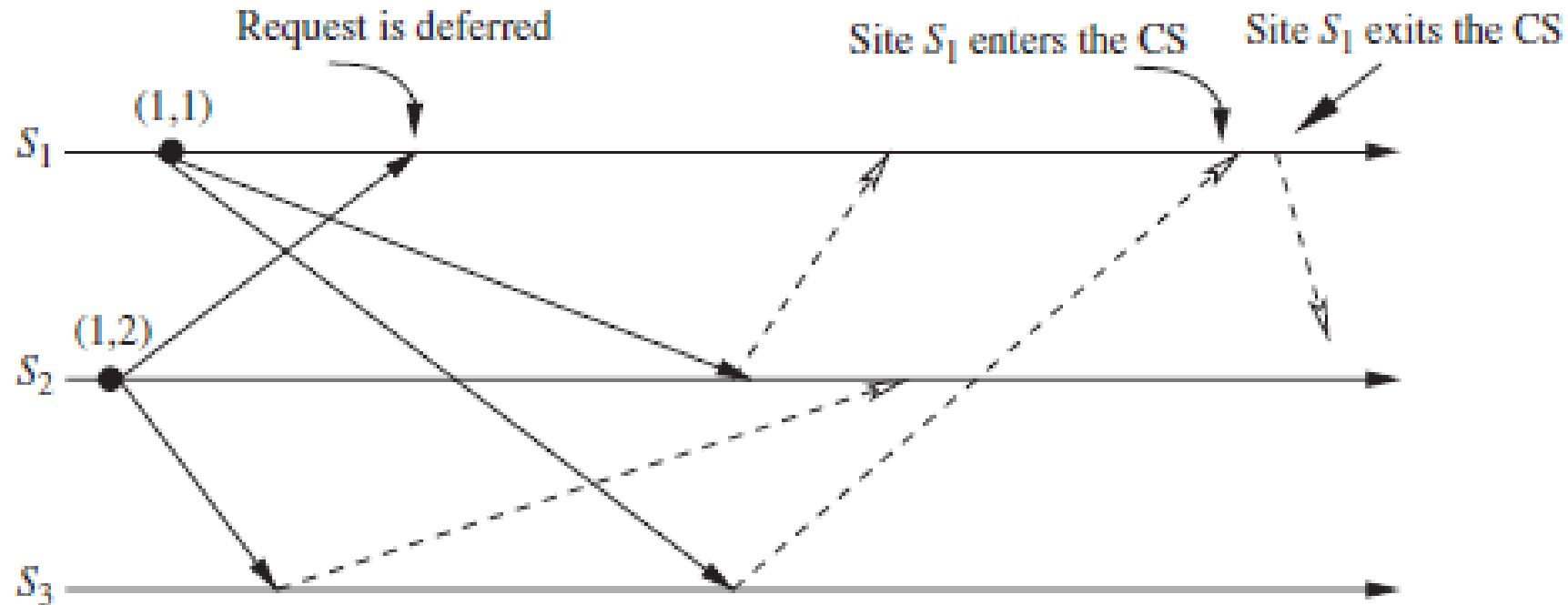
Sites  $S_1$  and  $S_2$  each make a request for the CS

# Ricart–Agrawala Algorithm



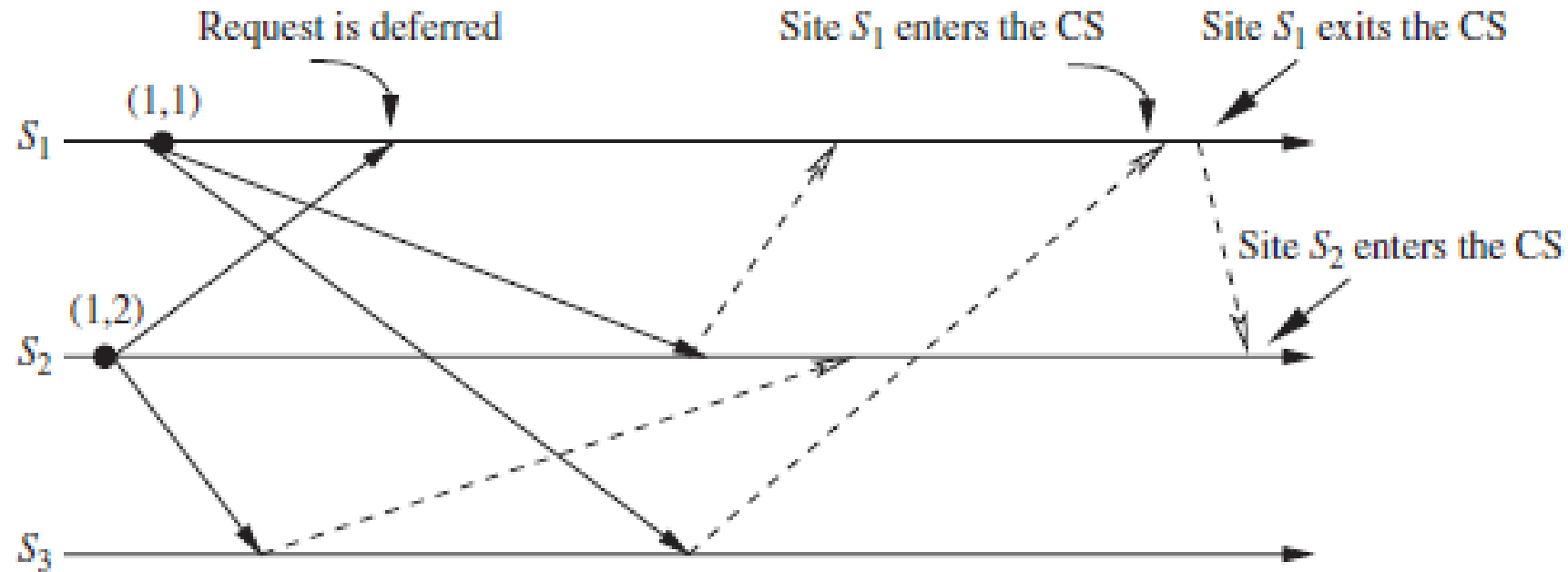
Site  $S_1$  enters the CS

# Ricart–Agrawala Algorithm



Site  $S_1$  exits the CS and sends a REPLY message to  $S_2$ 's deferred request

# Ricart–Agrawala Algorithm



Site  $S_2$  enters the CS

# Ricart–Agrawala Algorithm



## Performance

- For each CS execution, requires  $(N - 1)$  REQUEST messages and  $(N - 1)$  REPLY messages
- requires  $2(N - 1)$  messages per CS execution
- synchronization delay in the algorithm is  $T$



# Maekawa's Algorithm



- quorum-based mutual exclusion algorithm
- request sets for sites (i.e., quorums) are constructed to satisfy the following conditions:
  - M1:  $(\forall i \forall j : i \neq j, 1 \leq i, j \leq N :: R_i \cap R_j \neq \phi)$
  - M2:  $(\forall i : 1 \leq i \leq N :: S_i \in R_i)$
  - M3:  $(\forall i : 1 \leq i \leq N :: |R_i| = K \text{ for some } K)$
  - M4: Any site  $S_j$  is contained in  $K$  number of  $R_i$ 's,  $1 \leq i, j \leq N$
- Maekawa showed that  $N = K(K - 1) + 1$
- This relation gives  $|R_i| = K = \sqrt{N}$  (square root of  $N$ )

# Maekawa's Algorithm



- ❑ there is at least one common site between the request sets of any two sites (condition M1)
- ❑ every pair of sites has a common site which mediates conflicts between the pair
- ❑ a site can have only one outstanding REPLY message at any time; that is,
  - ❑ it grants permission to an incoming request if it has not granted permission to some other site
  - ❑ mutual exclusion is guaranteed
- ❑ requires delivery of messages to be in the order they are sent between every pair of sites

# Maekawa's Algorithm



- ❑ conditions M1 and M2 are necessary for correctness
- ❑ M3 ( $\forall i : 1 \leq i \leq N :: |R_i| = K$  for some  $K$ ) states that the size of the requests sets of all sites must be equal
  - ❑ equal amount of work to invoke mutual exclusion
- ❑ M4 (Any site  $S_j$  is contained in  $K$  number of  $R_i$ 's,  $1 \leq i, j \leq N$ ) enforces that exactly the same number of sites should request permission from any site
  - ❑ equal responsibility

# Maekawa's Algorithm



## Requesting the critical section:

- (a) A site  $S_i$  requests access to the CS by sending REQUEST( $i$ ) messages to all sites in its request set  $R_i$ .
- (b) When a site  $S_j$  receives the REQUEST( $i$ ) message, it sends a REPLY( $j$ ) message to  $S_i$  provided it hasn't sent a REPLY message to a site since its receipt of the last RELEASE message. Otherwise, it queues up the REQUEST( $i$ ) for later consideration.

## Executing the critical section:

- (c) Site  $S_i$  executes the CS only after it has received a REPLY message from every site in  $R_i$ .

# Maekawa's Algorithm



## Releasing the critical section:

(d) After the execution of the CS is over, site  $S_i$  sends a RELEASE( $i$ ) message to every site in  $R_i$ .

(e) When a site  $S_j$  receives a RELEASE( $i$ ) message from site  $S_i$ , it sends a REPLY message to the next site waiting in the queue and deletes that entry from the queue. If the queue is empty, then the site updates its state to reflect that it has not sent out any REPLY message since the receipt of the last RELEASE message.

# Maekawa's Algorithm



**Correctness:** *Maekawa's algorithm achieves mutual exclusion*

**Performance:**

- size of a request set is  $\sqrt{N}$
- an execution of the CS requires  $\sqrt{N}$  REQUEST,  $\sqrt{N}$  REPLY, and  $\sqrt{N}$  RELEASE messages, resulting in  $3\sqrt{N}$  messages per CS execution
- synchronization delay  $2T$

# Reference



- ❖ Ajay D. Kshemkalyani, and Mukesh Singhal, Chapter 9, “Distributed Computing: Principles, Algorithms, and Systems”, Cambridge University Press, 2008 (reprint: 2013).