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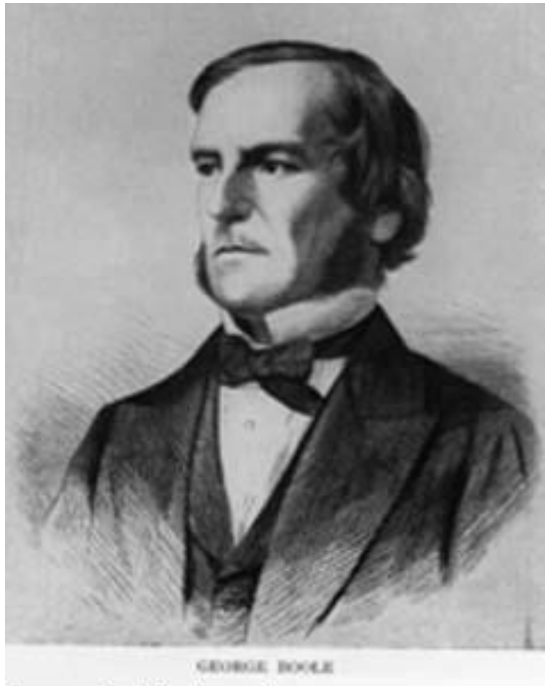
SS ZC416 Mathematical Foundations for Data Science
Brief Introduction to Boolean Functions

Objectives

- Introduction to Graphs
  - Undirected & Directed Graphs
  - Simple Graphs & Multigraphs
  - Special Graphs
  - Adjacency Matrix & Incidence Matrix
  - Connectedness
  - Hamiltonian Paths & Circuits

George Boole, 1815 - 1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland.
- Wrote *An Investigation of the Laws of Thought* (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT.



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AND, OR, Not & Truth Table

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AND $X \bullet Y$ XY

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

OR   X+Y

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

NOT $\bar{X}$ X'

X	Y
0	1
1	0

Boolean Algebra

Boolean algebra provides the operations and the rules for working with the set **{0, 1}.**

We are going to focus on three operations:

- Boolean complementation,
- Boolean sum, and
- Boolean product

Boolean Operations

The **complement** is denoted by a bar or a ‘. It is defined by

$0' = 1$ and $1' = 0$.

The **Boolean sum**, denoted by + or by OR, has the following values:

$1 + 1 = 1, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 0 + 0 = 0$

The **Boolean product**, denoted by · or by AND, has the following values:

$1 \cdot 1 = 1, \quad 1 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 0 \cdot 0 = 0$

Boolean Functions and Expressions

Definition: Let B = {0, 1}. The variable x is called a **Boolean variable** if it assumes values only from B.

A function from B^n, the set $\{(x_1, x_2, \ldots, x_n) \mid x_i \in B, 1 \leq i \leq n\}$, to B is called a **Boolean function of degree n**.

Boolean functions can be represented using expressions made up from the variables and Boolean operations.

Example

Construct the Truth Table for $F(x, y) = (x').y$

x	y	F(x, y)
0	0	0
0	1	1
1	0	0
1	1	0

Truth Table

Construct the Truth Table for F(x, y, z) = (xz + y)’

x	y	z	xz	xz+y	(xz + y)’
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	0	0	1
1	0	1	1	1	0
1	1	0	0	1	0
1	1	1	1	1	0

Equivalence

Definition: The Boolean functions F and G of n variables are **equal** if and only if $F(b_1, b_2, \ldots, b_n) = G(b_1, b_2, \ldots, b_n)$ whenever $b_1, b_2, \ldots, b_n$ belong to B.

Two different Boolean expressions that represent the same function are called **equivalent**.

- For example, the Boolean expressions xy, $xy + 0$, and $xy \cdot 1$ are equivalent.

Complement of a Function

The **complement** of the Boolean function F is the function F’, where
$F'(b_1, b_2, \ldots, b_n) =$
$(F(b_1, b_2, \ldots, b_n))'$

Let F and G be Boolean functions of degree n. The **Boolean sum F+G** and **Boolean product FG** are then defined by

$(F + G)(b_1, b_2, \ldots, b_n) = F(b_1, b_2, \ldots, b_n) + G(b_1, b_2, \ldots, b_n)$

$(FG)(b_1, b_2, \ldots, b_n) = F(b_1, b_2, \ldots, b_n)\, G(b_1, b_2, \ldots, b_n)$

Boolean algebra

A Boolean algebra comprises...

– A set of elements B

– Binary operators {+ , •} Boolean sum and product

– A unary operation { ' } (or { $\bar{\ }$ }) example: A' or $\bar{A}$

…and the following axioms

1. The set B contains at least two elements {a b} with a ≠ b

2. Closure:	a+b is in B	a•b is in B
3. Commutative:	a+b = b+a	a•b = b•a
4. Associative:	a+(b+c) = (a+b)+c	a•(b•c) = (a•b)•c
5. Identity:	a+0 = a	a•1 = a
6. Distributive:	a+(b•c)=(a+b)•(a+c)	a•(b+c)=(a•b)+(a•c)
7. Complementarity:	a+a' = 1	a•a' = 0

Priority

$$\overline{A} \bullet B + C = ((\overline{A}) \bullet B) + C$$

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Thank you!!