Cubing of Numbers

The cube of a number is simply the square of the number multiplied by the number itself.
The symbol for cubed is ^{3} - To write 4 cubed we would write 4^3
This tells us how many times we need to multiply 4 by itself.
4^3 means 4 x 4 x 4 = 64 ...... or 5^3 means 5 x 5 x 5 = 125
They are called cube numbers because these numbers actually form cubes.
In Vedic maths there are some very simple methods to find the cube of numbers and we will explore two methods in particular: Cubing by the **Nikhilam Method** and; Cubing by the **Anurupyena Method.**

First things first, before exploring the Cubing Methods let us familiarize ourselves with the cubes of the first nine natural numbers, which are: 1, 8, 27, 64, 125, 216, 343, 512 and 729 respectively.

Cubing by the Nikhilam Method

This method is similar to the Yavadunum Method of Squaring *(see Vedic Methods (ii)*) and so numbers near to the base can, with a slight modification, be cubed. Also, taking into consideration the sub-Base : Base RATIO, numbers near a sub-Base can also be cubed.

Cubing of Numbers near the Base

- Tri-furcate the answer space using two forward slashes dividng area into: LHS, MIDDLE and RHS sections
- The LHS is simply the Number, N, plus twice the Deviation, D i.e. **N + 2D**
- The MIDDLE is simply three time the square of the Deviation i.e. $\mathbf{3D^2}$
- The RHS is simply the cube of the Deviation i.e. $\mathbf{D^3}$

Now, the MIDDLE and RHS of the answer will contain the same number of digits as zeros in the Base. As usual, if there are any excess digits in these sections, then they are carried over but if there are deficient digits, these are filled up with zeros, to the left of that answer part.

Example 1: 107^3
Split the answer box into three sections; using Base = 100, the Deviation is 07. RHS = 07^3 = 343
But since only two digits are allowed we carry over the 3 to the MIDDLE.
MIDDLE = 3 x 07^2 = 147. Again, since only two digits are allowed we carry over the 1 to the LHS.
LHS = 107 + 2 x 07 = 121. Thus 107^3 = **1225043**

			LHS	/	MIDDLE	/	RHS
			N + (2 x Deviation)	/	(3 x Deviation^2)	/	(Deviation^3)
eg.(1) 107^3	N = 107	=	107 + (2 x 07)	/	3 x 07^2	/	'07^3
	Base = 100	=	121	/	$_1 47$	/	$_3 43$
	Deviation = 07	=	122	/	50	/	43
1225043							

Example 2: 992^3

Split the answer box into three sections; using Base 1000, the Deviation is $00\bar{8}$. RHS = $00\bar{8}^3 = \overline{512}$
MIDDLE = 3 x $00\bar{8}^{\,2}$ = 192. LHS = 992 + 2 x $00\bar{8}$ = 976. Thus 992^3 = **976191488**

			LHS	/	MIDDLE	/	RHS
			N + (2 x Deviation)	/	(3 x Deviation^2)	/	(Deviation^3)
eg.(2) 992^3	N = 992	=	992 + (2 x $00\bar{8}$)	/	3 x $00\bar{8}^2$	/	$00\bar{8}^3$
	Base = 1000	=	976	/	192	/	$\overline{512}$
	Deviation = $00\bar{8}$	=	976		191		488
976191488							

Example 3: 1008^3

			LHS	/	MIDDLE	/	RHS
			N + (2 x Deviation)	/	(3 x Deviation^2)	/	(Deviation^3)
eg.(3) 1008^3	N = 1008	=	1008 + (2 x 08)	/	3 x 008^2	/	008^3
	Base = 1000	=	1024	/	192	/	512
	Deviation = 08	=	1024		192		512
1024192512							

Example 4: 1011^3

			LHS	/	MIDDLE	/	RHS
			N + (2 x Deviation)	/	(3 x Deviation^2)	/	(Deviation^3)
eg.(4) 1011^3	N = 1011	=	1011 + (2 x 11)	/	3 x 011^2	/	011^3
	Base = 1000	=	1033	/	363	/	$_1331$
	Deviation = 011	=	1033		364		331
1033364331							

Example 5: 9997^3

			LHS	/	MIDDLE	/	RHS
			N + (2 x Deviation)	/	(3 x Deviation^2)	/	(Deviation^3)
eg.(5) 9997^3	N = 9997	=	9997 + (2 x $000\bar{3}$)	/	3 x $000\bar{3}^2$	/	$000\bar{3}^3$
	Base = 10000	=	9991	/	0027	/	$00\overline{27}$
	Deviation = $000\bar{3}$	=	9991		0026		9973
999100269973							

Algebraic explanation of cubing by the Nikhilam Method

Once again, for some number N_1, near to base x, with a deviation of d, the cube N_1^3, is given by:

$(x + d)^3 = (x^2 + 2dx + d^2)(x + d)$
$= (x^3 + dx^2 + 2dx^2 + 2d^2x + d^2x + d^3$
$= x^3 + 3dx^2 + 3d^2x + d^3$
$= x^2(x + 3d) + 3d^2x + d^3$

Writing in term of units, the base and the base squared, using forward slashes to denote the various orders of ten:

$(x + 3d) / 3d^2 / d^3$

$(x + d + 2d) / 3d^2 / d^3$

So, what about cubing a number near to a sub-base?
Let n be the multiple of the base i.e. nx is the sub-Base

$(nx + d)^3 = (n^2x^2 + 2dnx + d^2)(nx + d)$
$= (n^3x^3 + dn^2x^2 + 2n^2dx^2 + 2nd^2x + d^{2n}x + d^3$
$= n^3x^3 + 3dn^2x^2 + 3nd^2x + d^3$
$= x^2(x + 3d) + 3d^2x + d^3$
$= n^2x^2(nx + 3d) + nx(3d^2) + d^3$
$= n^2x^2(nx + d + 2d) + 3nxd^2 + d^3$

Writing in term of units, the base and the base squared, using forward slashes to denote the various orders of ten:

$n^2x^2 (x + 3d) / 3nd^2x / d^3$

$n^2(x + d + 2d) / 3nd^2 / d^3$

So, here we see that there is a great similarity between this result and the previous result, the difference being: the RHS is the same; the MIDDLE term is multiplied by **n** and; the LHS is multiplied by $\mathbf{n^2}$.

Cubing by the Anurupyena Method
This method is similar to the Anurupyena method of squaring. (Please refer to the Squaring of Numbers sub-section in Vedic Maths (ii) for a comparison). It is a particularly useful method for cubing two-digit numbers.

As with with the Anurupyena squaring method, we find the ratio of two digits of a given number. In the answer section, we write the cube of the left-most digit followed by three numbers that are in the same proportion as the ratio. These numbers are in geometric proportions of the cube of the left-most digit depending on the ratio.

Example 1: 14^3
First, We work out the ratio of the two digits i.e. 1 : 4. Maintaining this ratio between adjacent numbers, the four numbers, starting from the cube of the left-most digit are:
1 : 4 : 16 : 64

Next, we now add, in place-wise order, twice the second and third numbers in the row below. These four resulting numbers, written in ratio format, occupy from the right side, the units, tens, hundreds and thousands place respectively. Any numbers that contain more than one digit will be carried over to the next place.

eg(1) 14^3								
		1	:	4	:	16	:	64
				8		32		
	=	1		$_{1}2$		$_{4}8$		$_{6}4$
	=	2		7		4		4
2744								

Or, take another example:

Example 2: 16^3

eg(2) 16^3								
		1	:	6	:	36	:	216
				12		72		
	=	1		$_{1}8$		$_{10}8$		$_{21}6$
	=	4		0		9		6
4096								

Example 3 (part i): 29^3
We can do this calculation in two ways: the first way, without Vinculum and the second way with the Vinculum.

eg(3) 29^3	(without using Vinculum)							
		8	:	36	:	162	:	729
				72		324		
	=	8		$_{10}8$		$_{4}86$		$_{72}9$
	=	24		3		8		9
24389								

Example 3 (part ii): 29^3 (Using Vinculum)

eg(3) 29^3	(using Vinculum)							
$29^3 = 3\bar{1}$		27	:	$\bar{9}$	:	3	:	$\bar{1}$
				$\overline{18}$		6		
	=	27		${}_{\bar{2}}\bar{7}$		9		$\bar{1}$
	=	25		$\bar{7}$		9		$\bar{1}$
	=	24		3		8		9
24389								

Example 4 (part i): 27^3

eg(4) 27^3	(without using Vinculum)							
		8	:	28	:	98	:	343
				56		196		
	=	8		${}_{8}4$		${}_{29}4$		${}_{34}3$
	=	19		6		8		3
19683								

Example 4 (part ii): 27^3

eg(4) 27^3	(using Vinculum)							
$27^3 = 3\bar{3}$		27	:	$\overline{27}$	:	27	:	$\overline{27}$
				$\overline{54}$		54		
	=	27		${}_{\bar{8}}\bar{1}$		${}_{8}1$		${}_{\bar{2}}\bar{7}$
	=	19		7		$\bar{1}$		$\bar{7}$
	=	19		6		8		3
19683								

Example 5 (part i): 38^3 (without using Vinculum)

Eg(5) 38^3	(without using Vinculum)							
		27	:	72	:	192	:	512
				144		384		
	=	27		$_{21}6$		$_{57}6$		$_{51}2$
	=	54		8		7		2
54872								

Example 5 (part ii): 38^3 (using Vinculum)

Eg(5) 38^3 38^3 '= $4\overline{2}$	(using Vinculum)							
		64	:	$\overline{32}$	:	16	:	$\overline{8}$
				$\overline{64}$		32		
	=	64		$\overline{96}$		48		$\overline{8}$
	=	55		$\overline{2}$		8		$\overline{8}$
	=	54		8		7		2
54872								

Algebraic Explanation for Cubing using Anurupyena Method

When squaring a two-digit number using the Anurupyena Method, we used the solution to $(ax + b)^2$. Here, we follow a similar method for $(ax + b)^3$.
When $(ax + b)^3$ is expanded, we get: $a^3x^3 + 3a^2bx^2 + 3ab^2x^1 + b^3x^0$

Consider, for now, a two-digit number with digit 'b' at the units place and digit 'a' at the tens place, with x = 10, i.e. Base 10. We have:

$\mathbf{x^3}$	$\mathbf{x^2}$	$\mathbf{x^1}$	$\mathbf{x^0}$
a^3	$3a^2b$	$3ab^2$	b^3

Rewriting this, we have:

$\mathbf{x^3}$	$\mathbf{x^2}$	$\mathbf{x^1}$	$\mathbf{x^0}$
a^3	a^2b	ab^2	b^3
	$2a^2b$	$2ab^2$	

This is the same expression written in a slightly longer way; we add the $2a^2b$ and the $2ab^2$ in their respective columns to complete the terms in x^2 and x^1. Now, after the adjustment, the first line consists of a^3, a^2b, ab^2 and b^3 and, crucially, the successive ratios a^3: a^2b, a^2b : ab^2 and ab^2 : b^3 all conveniently have the same ratio a : b, which is the initial ratio of the digits in the number being cubed. All we then need to do, as mentioned previously, is to add the x^2 and x^1 terms.