Square Root of Algebraic Expressions

Once again, to find the square root of an algebraic expression using the Duplex Method, a similar procedure as in the Numerical Method is carried out. If the square root of an algebraic expression happens to be a perfect square then we get a definite number of terms, where the number of terms in the square root will depend on the number of terms in the square. For instance: a one or two term square will give one term in the square root; three or four terms in the square will give two terms in the square root; five or six terms in the square will give three terms in the square root and so on. If the square root is not a perfect square we will get an infinite series just as in the case of numerals where we get a decimal part. With numerals, the excess digits remaining after dividing by the divisor are carried over to the next digit. However, when taking square root of algebraic expressions there are no carry over of terms and this situation will sometimes lead to the production of fractional coefficients of the terms. Aside from this, the rest of the procedure is the same as the that followed to obtain the square root of numerals using the Duplex Method.

Example 1: What is the square root of: $16x^4 - 40x^3 + 81x^2 - 70x + 49$ i.e. What is $\sqrt{16x^4-40x^3+81x^2-70x+49}$?

This expression has 5 terms, so it will have only three terms in its square root if it is a perfect square. Let's see!

Step 1: Set up the sum as in the case of numerical square roots. That is write down the Dividend, leaving a blank line immediately underneath and draw an answer box/Divisor area (to the left) and then put a forward slash in the answer box between first and second terms as shown below. .

Step 1

$$\begin{array}{r|l} & 16x^4 - 40x^3 + 81x^2 - 70x + 49 \\ \hline & \quad\quad / \end{array}$$

Step 2: Then take the square root of the first term and write it in the answer box directly underneath the first term; the Divisor is simply twice this term i.e. 2 times **$4x^2$** or **$\underline{8x^2}$**

Step 2

Divisor $\rightarrow$

$$\begin{array}{r|l} 8x^2 & 16x^4 - 40x^3 + 81x^2 - 70x + 49 \\ \hline & 4x^2 / \end{array}$$

Step 3: Next, we divide the next term of the Dividend, in this case $-40x^3$, by the Divisor ($8x^2$), giving **$-5x$.** This is the second term in the square root.

Step 3

Divisor $\rightarrow$ $8x^2$ | $16x^4 - 40x^3 + 81x^2 - 70x + 49$

$4x^2$ / $-5x$

Step 4: Next, we subtract the Duplex of the second term, from the third term of the Dividend and then divide the result by the Divisor; this gives the third term of the square root.
i.e. $(81x^2 - D(-5x)) \div 8x^2$ = **7**

Step 4

Divisor $\rightarrow$ $8x^2$ | $16x^4 - 40x^3 + 81x^2 - 70x + 49$

$4x^2$ / $-5x + 7$

If the square is perfect then the answer would be an exact square root; the calculation would be complete having a three term answer. But, just to verify that this is the case let us continue......

Step 5: Next, we subtract the Duplex of the second and third terms, from the fourth term of the Dividend and then divide the result by the Divisor; this gives the fourth term of the square root.
i.e. $(-70x - D(-5x+7)) \div 8x^2$ = -70x + 70x = **0**

Step 5

Divisor $\rightarrow$ $8x^2$ | $16x^4 - 40x^3 + 81x^2 - 70x + 49$

$4x^2$ / $-5x + 7 + 0$

Step 6: Next, we subtract the Duplex of the second, third and fourth terms, from the fifth term of the Dividend and then divide the result by the Divisor; this gives the fifth term of the square root.
i.e. $(49 - D(-5x+7+0)) \div 8x^2$ = 49 – 49 = **0**

Step 6

$8x^2$ | $16x^4 - 40x^3 + 81x^2 - 70x + 49$

$4x^2$ / $-5x + 7 + 0 + 0$

$4x^2 -5x + 7$

$$\sqrt{16x^4 - 40x^3 + 81x^2 - 70x + 49} = 4x^2 - 5x + 7$$

So, as we can see, as the expression was a perfect square of five terms, its square root is a trinomial expression.

Example 2: What is $\sqrt{25g^2+40g+16}$?
Step 1: As usual, set up the sum with the answer box and Divisor region to the left

Step 1

$$\begin{array}{l|l} & 25g^2 + 40g + 16 \\ \hline & \quad / \end{array}$$

Step 2: Take the square root of the first term, $25g^2$, giving **5g**....the first term of the square root. Double this term to the Divisor of **<u>10g.</u>**

Step 2

Divisor ➡

$$\begin{array}{r|l} & 25g^2 + 40g + 16 \\ 10g & \\ \hline & 5g \ / \end{array}$$

Step 3: Dividing the second term of the expression, 40g, by the Divisor, 10g, gives: **4.** This is the second term of the square root.

Step 3

Divisor ➡

$$\begin{array}{r|l} & 25g^2 + 40g + 16 \\ 10g & \\ \hline & 5g \ / \ 4 \end{array}$$

Step 4: Subtracting the Duplex of the second term of the square root from the second term in the expression and then dividing the result by the Divisor, 10g, gives **0**, third term of the square root. i.e.

$$(16-D(4))\div 10g \quad =0$$

Step 4

Divisor ➡

$$\begin{array}{r|l} & 25g^2 + 40g + 16 \\ 10g & \\ \hline & 5g \ / \ 4+0 \end{array}$$

5g + 4

So, $\sqrt{25g^2+40g+16} = 5g + 4$

Example 3: What is $\sqrt{64j^2+96jk+36k^2}$?

Step 1: Setup..... as usual

Step 1

Divisor ➡ | $64j^2 + 96jk + 36k^2$

/

Step 2: Take square root of first term gives **8j**; then we double to get Divisor, 16j.

Step 2

Divisor ➡ 16j | $64j^2 + 96jk + 36k^2$

8j /

Step 3: Second term of expression, 96jk, divided by Divisor, 16j, gives **6k**

Step 3

Divisor ➡ 16j | $64j^2 + 96jk + 36k^2$

8j / 6k

Step 4: Third term of expression, $36k^2$ minus Duplex of second term in answer, all divided by Divisor gives **0.**

Step 4

Divisor ➡ 16j | $64j^2 + 96jk + 36k^2$

8j / 6k + 0

8j + 6k

So, $\sqrt{64j^2+96jk+36k^2}$ = 8j + 6k

Example 4: What is $\sqrt{9t^4-42t^3+115t^2-154t+121}$?

Step 1: Setup answer box and Divisor area as usual

Step 1

$$\begin{array}{r|l} & 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \hline & \qquad / \end{array}$$

Step 2: Take the square root of the first term of the expression, $9t^4$, to give $\mathbf{3t^2}$. Double the term to get the all important Divisor of $\underline{\mathbf{6t^2}}$.

Step 2

$$\begin{array}{r|l} & 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \text{Divisor} \quad \rightarrow \quad 6t^2 & \\ \hline & 3t^2 \;\; / \end{array}$$

Step 3: Divide second term of expression, $-42t^3$, by Divisor to give **7t**.

Step 3

$$\begin{array}{r|l} & 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \text{Divisor} \quad \rightarrow \quad 6t^2 & \\ \hline & 3t^2 \;\; / \; 7t \end{array}$$

Step 4: Subtract the Duplex of second term of answer from third term of the expression and divide the result by the Divisor, giving: **11**

Now, if the expression was a perfect square we could finish the calculation there but just to verify, let us continue...

Step 4

$$\text{Divisor} \rightarrow \quad 6t^2 \;\Big|\; \begin{array}{l} 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \hline 3t^2 \;/\; 7t + 11 \end{array}$$

Step 5: Subtract the Duplex of the second and third terms of the answer from the fourth term of the expression and divide the result by the Divisor i.e. $(-154t - D(7t+11)) \div 6t^2$, giving: **0**

Step 5

$$\text{Divisor} \rightarrow \quad 6t^2 \;\Big|\; \begin{array}{l} 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \hline 3t^2 \;/\; 7t + 11 + 0 \end{array}$$

Step 6: Subtract the Duplex of the second, third and fourth terms of the answer from the fifth term of the expression and divide the result by the Divisor i.e. $(121 - D(7t+11+0)) \div 6t^2$ = $121 - 121 \div 6t^2$ = **0**

Step 6

$$\text{Divisor} \rightarrow \quad 6t^2 \;\Big|\; \begin{array}{l} 9t^4 - 42t^3 + 115t^2 - 154t + 121 \\ \hline 3t^2 \;/\; 7t + 11 + 0 + 0 \end{array}$$

$3t^2 + 7t + 11$

So, $\sqrt{9t^4 - 42t^3 + 115t^2 - 154t + 121} = 3t^2 + 7t + 11$

Example 5: What is $\sqrt{4j^4 + 12j^3 + 13j^2 + 8j + 4}$ in **descending** and **ascending** powers of j?

Square Root in descending order of x

Step 1: Setup answer box and Divisor area as usual

Step 1

$$\begin{array}{l|l} & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ \hline & \quad / \end{array}$$

Step 2: Take the square root of the first term of the expression, $4j^4$, to give $\mathbf{2j^2}$. Double the term to get the all important Divisor of $\underline{\mathbf{4j^2}}$.

Step 2

$$\text{Divisor} \Rightarrow \begin{array}{l|l} 4j^2 & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ \hline & 2j^2\ / \end{array}$$

Step 3: Divide second term of expression, $12j^3$, by Divisor $4j^2$ i.e. $12j^3 \div 4j^2 = \mathbf{3j}$

Step 3

$$\text{Divisor} \Rightarrow \begin{array}{l|l} 4j^2 & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ \hline & 2j^2\ /\ 3j \end{array}$$

Step 4: Subtract the Duplex of second term of answer from third term of the expression and divide the result by the Divisor, $(13j^2 - D(3j)) \div 4j^2$ giving: **1**

Step 4

$$\text{Divisor} \Rightarrow \begin{array}{l|l} 4j^2 & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ \hline & 2j^2\ /\ 3j + 1 \end{array}$$

Step 5: Subtract the Duplex of the second and third terms of the answer from the fourth term of the expression and divide the result by the Divisor i.e. , $(8j-D(3j+1))\div 4j^2$ giving: $\frac{1}{2j}$

Step 5

Divisor ➡ $\begin{array}{c|l} & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ 4j^2 & \\ \hline & 2j^2 \ / \ 3j + 1 + \frac{1}{2j} \end{array}$

Step 6: Subtract the Duplex of the second, third and fourth terms of the answer from the fifth term of the expression and divide the result by the Divisor i.e. $(4-D(3j+1+\frac{1}{2j}))\div 4j^2$ =

$(4-7)\div 4j^2$ = $\frac{-3}{4j^2}$

Step 6

Divisor ➡ $\begin{array}{c|l} & 4j^4 + 12j^3 + 13j^2 + 8j + 4 \\ 4j^2 & \\ \hline & 2j^2 \ / \ 3j + 1 + \frac{1}{2j} \quad \frac{-3}{4j^2} \end{array}$

Step 7: Subtract the Duplex of the second, third, fourth and fifth terms of the answer from the sixth term of the expression, which actually happens to be zero, and divide the result by the Divisor, i.e.

$(0-D(3x+1+\frac{1}{2j}-\frac{3}{4j^2}))\div 4j^2$ = $\frac{-7}{8j^3}$

Step 7

Divisor ➡ $\begin{array}{c|l} & 4j^4 + 12j^3 + 13j^2 + 8j + 4 + 0 \\ 4j^2 & \\ \hline & 2j^2 \ / \ 3j + 1 + \frac{1}{2j} \quad \frac{-3}{4j^2} \quad \frac{-7}{8j^3} \end{array}$

Step 8: Subtract the Duplex of the second, third, fourth, fifth and sixth terms of the answer from the seventh term of the expression, which also actually again happens to be zero, and divide the result by the Divisor, i.e. $(0-D(3j+1+\frac{1}{2j}-\frac{3}{4j^2}-\frac{7}{8j^3}))\div 4j^3$ giving $\frac{-13}{16j^4}$

Step 8

$$\text{Divisor} \rightarrow \quad 4j^2 \quad \left| \begin{array}{l} 4j^4 + 12j^3 + 13j^2 + 8j + 4 + 0 + 0 \\ \hline 2j^2 \;/\; 3j + 1 + \frac{1}{2j} \quad \frac{-3}{4j^2} \quad \frac{-7}{8j^3} \quad \frac{-13}{8j^4} \end{array} \right.$$

So, in descending powers of j

$$\sqrt{4j^4+12j^3+13j^2+8j+4} = 2j^2+3j+1+\frac{1}{2j}-\frac{3}{4j^2}-\frac{7}{8j^3}-\frac{13}{8j^4}+\ldots\ldots\ldots$$

Now, let's do the same sum in **ascending** powers of j

Step 1: As usual, set up the answer box and Divisor area but this time with the expression in ascending powers of j

Step 1

$$\left| \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline \quad / \end{array} \right.$$

Step 2: $\sqrt{4}=2$ and so the Divisor = 4

Step 2

$$\text{Divisor} \rightarrow \quad 4 \quad \left| \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline 2 \;/ \end{array} \right.$$

Step 3: $8j \div 4 = 2j$

Step 3

$$\text{Divisor} \rightarrow \quad 4 \quad \left| \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline 2 \;/\; 2j \end{array} \right.$$

Step 4: $(13j^2 - D(2j)) \div 4 = \frac{9}{4}j^2$

Step 4

$$\text{Divisor} \rightarrow \quad 4 \;\Big|\; \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline 2 \;/\; 2j + \frac{9}{4}j^2 \end{array}$$

Step 5: $(12j^3 - D(2j + \frac{9}{4}j^2)) \div 4 = \frac{3}{4}j^3$

Step 5

$$\text{Divisor} \rightarrow \quad 4 \;\Big|\; \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline 2 \;/\; 2j + \frac{9}{4}j^2 + \frac{3}{4}j^3 \end{array}$$

Step 6: $(4j^4 - D(2j + \frac{9}{4}j^2 + \frac{3}{4}j^3)) \div 4 = -(\frac{65}{64})j^3$

Step 6

$$\text{Divisor} \rightarrow \quad 4 \;\Big|\; \begin{array}{l} 4 + 8j + 13j^2 + 12j^3 + 4j^4 \\ \hline 2 \;/\; 2j \; +(\frac{9}{4})j^2 + (\frac{3}{4})j^3 - (\frac{65}{64})j^4 \end{array}$$

We can do as many steps as required depending on the level of accuracy required.
So, we have, in ascending powers of j:

$$\sqrt{4j^4 + 12j^3 + 13j^2 + 8j + 4} = 2 + 2j + (\frac{9}{4})j^2 + (\frac{3}{4}j^3) - (\frac{65}{64}j^4) + \ldots\ldots\ldots$$