

**ADVANCED
ENGINEERING MATHEMATICS**

ADVANCED ENGINEERING MATHEMATICS

[For the Students of M.E., B.E. and other Engineering Examinations]

H.K. DASS

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Diploma in Specialist Studies (Maths.)

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PREFACE TO THE TWENTYFIRST REVISED EDITION

I am happy to be able to bring out this revised edition.

Misprints and errors which came to my notice have been corrected.

Suggestions and healthy criticism from students and teachers to improve the book shall be personally acknowledged and deeply appreciated to help me to make it an ideal book for all.

We are thankful to the Management Team and the Editorial Department of S. Chand & Company Pvt. Ltd. for all help and support in the publication of this book.

D-1/87, Janakpuri
New Delhi-110 058
Tel. 28525078, 28521776
Mob. 9350055078
hk_dass@yahoo.com

H.K. DASS

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Further, the appearance of the personal name, location, place and incidence, if any; in the illustrations used herein is purely coincidental and work of imagination. Thus the same should in no manner be termed as defamatory to any individual.

PREFACE TO THE FIRST EDITION

It gives me great pleasure to present this textbook of Mathematics to the students pursuing I.E.T.E and various engineering courses.

This book has been written according to the new revised syllabus of Mathematics of I.E.T.E. and includes topics from the syllabi of the other engineering courses. There is not a single textbook which entirely covers the syllabus of I.E.T.E. and the students have all along been facing great difficulties. Endeavour has been made to cover the syllabus exhaustively and present the subject matter in a systematic and lucid style. More than 550 solved examples on various topics have been incorporated in the textbook for the better understanding of the students. Most of the examples have been taken from previous question papers of I.E.T.E. which should make the students familiar with the standard and trend of questions set in the examinations. Care has been taken to systematically grade these examples.

The author possesses very long and rich experience of teaching Mathematics to the students preparing for I.E.T.E. and other examinations of engineering and has first hand experience of the problems and difficulties that they generally face.

This book should satisfy both average and brilliant students. It would help the students to get through their examination and at the same time would arouse greater intellectual curiosity in them.

I am really thankful to my Publishers, Padamshree Lala Shyam Lal Gupta, Shri Ravindra Kumar Gupta for showing personal interest and his General Manager, Shri P.S. Bhatti and Km. Shashi Kanta for their co-operations. I am also thankful to the Production Manager, Shri Ravi Gupta for bringing out the book in a short period.

Suggestions for the improvement of the book will be gratefully acknowledged.

D-1/87, Janakpuri
New Delhi-110 058

H.K. DASS

FOREWORD

On my recent visit to India, I happened to meet Prof. H.K. Dass, who has written quite a number of successful books on Mathematics for students at various levels.

During my meeting, Prof. H.K. Dass presented me with the book entitled “Advanced Engineering Mathematics” I am delighted to write this Foreword, as I am highly impressed on seeing the wide variety of its contents. The contents includes many key topics, for examples, advanced calculus, vector analysis, tensor analysis, fuzzy sets, various transforms and special functions, probability (curiously some tests of significance are given under that chapter), numerical methods; matrix algebra and transforms. In spite of this breadth , the development of the material is very lucid, simple and in plain English.

I know of quite a number of other textbooks on Engineering Mathematics but the material that has been included in this textbook is so comprehensive that the students of all the engineering streams will find this textbook useful. It contains problems, questions and their solutions which are useful both to the teachers and students, and I am not surprised that it has gone through various editions. The style reminds me of the popular books of Schaum’s Series. I believe that this book will be also helpful to non-engineering students as a quick reference guide.

This book is a work of dedicated scholarship and vast learning of Mr. Dass, and I have no hesitation in recommending this book to the students for any Engineering degree world-wide.

Prof. K.V. Mardia
M.Sc. (Bombay), M.Sc.(Pune)
Ph.D. (Raj.), Ph.D. (N'cle),D.Sc.(N'cle)
Senior Research Professor
University of Leeds,
LEEDS (England)

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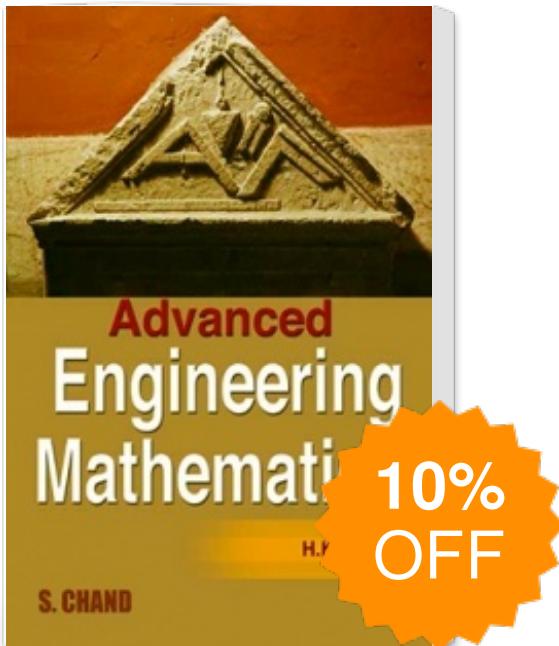
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