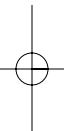
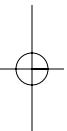
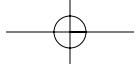
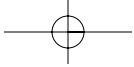


# Advanced Engineering Mathematics







9<sup>TH EDITION</sup>

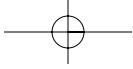
# Advanced Engineering Mathematics

ERWIN KREYSZIG

Professor of Mathematics  
Ohio State University  
Columbus, Ohio



WILEY JOHN WILEY & SONS, INC.



*Vice President and Publisher:* Laurie Rosatone  
*Editorial Assistant:* Daniel Grace  
*Associate Production Director:* Lucille Buonocore  
*Senior Production Editor:* Ken Santor  
*Media Editor:* Stefanie Liebman  
*Cover Designer:* Madelyn Lesure  
*Cover Photo:* © John Sohm/Chromosohm/Photo Researchers

This book was set in Times Roman by GGS Information Services  
and printed and bound by Von Hoffmann, Inc.  
The cover was printed by Von Hoffmann, Inc.

This book is printed on acid-free paper. 

Copyright © 2006 John Wiley & Sons, Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (508) 750-8400, fax (508) 750-4470. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, E-Mail: PERMREQ@WILEY.COM.

Kreyszig, Erwin.

Advanced engineering mathematics / Erwin Kreyszig.—9th ed.

p. cm.

Accompanied by instructor's manual.

Includes bibliographical references and index.

ISBN 0-471-48885-2 (cloth : acid-free paper)

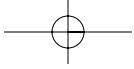
1. Mathematical physics. 2. Engineering mathematics. 1. Title.

ISBN-13: 978-0-471-48885-9

ISBN-10: 0-471-48885-2

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1



## P R E F A C E

See also <http://www.wiley.com/college/kreyszig/>

### Goal of the Book. Arrangement of Material

This new edition continues the tradition of providing instructors and students with a comprehensive and up-to-date resource for teaching and learning *engineering mathematics*, that is, *applied mathematics* for engineers and physicists, mathematicians and computer scientists, as well as members of other disciplines. A course in elementary calculus is the sole *prerequisite*.

The subject matter is arranged into seven parts A–G:

- A Ordinary Differential Equations (ODEs) (Chaps. 1–6)
- B Linear Algebra. Vector Calculus (Chaps. 7–9)
- C Fourier Analysis. Partial Differential Equations (PDEs) (Chaps. 11–12)
- D Complex Analysis (Chaps. 13–18)
- E Numeric Analysis (Chaps. 19–21)
- F Optimization, Graphs (Chaps. 22–23)
- G Probability, Statistics (Chaps. 24–25).

This is followed by five appendices:

- App. 1 References (ordered by parts)
- App. 2 Answers to Odd-Numbered Problems
- App. 3 Auxiliary Material (see also inside covers)
- App. 4 Additional Proofs
- App. 5 Tables of Functions.

This book has helped to pave the way for the present development of engineering mathematics. By a modern approach to those areas A–G, this new edition will prepare the student for the tasks of the present and of the future. The latter can be predicted to some extent by a judicious look at the present trend. Among other features, this trend shows the appearance of more complex production processes, more extreme physical conditions (in space travel, high-speed communication, etc.), and new tasks in robotics and communication systems (e.g., fiber optics and scan statistics on random graphs) and elsewhere. This requires the refinement of existing methods and the creation of new ones.

It follows that students need solid knowledge of basic principles, methods, and results, and a clear view of what engineering mathematics is all about, and that it requires proficiency in all three phases of problem solving:

- **Modeling**, that is, translating a physical or other problem into a mathematical form, into a mathematical *model*; this can be an algebraic equation, a differential equation, a graph, or some other mathematical expression.
- **Solving** the model by selecting and applying a suitable mathematical method, often requiring numeric work on a computer.
- **Interpreting** the mathematical result in physical or other terms to see what it practically means and implies.

It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead they should recognize that mathematics rests on relatively few basic concepts and involves powerful unifying principles. This should give them a firm grasp on the *interrelations among theory, computing, and* (physical or other) *experimentation*.

## PARTS AND CHAPTERS OF THE BOOK

### PART A

Chaps. 1–6

Ordinary Differential Equations (ODEs)

Chaps. 1–4

Basic Material

↓  
Chap. 5  
Series Solutions

Chap. 6  
Laplace Transforms

### PART B

Chaps. 7–10

Linear Algebra. Vector Calculus

Chap. 7  
Matrices,  
Linear Systems

Chap. 9  
Vector Differential  
Calculus

↓  
Chap. 8  
Eigenvalue Problems

↓  
Chap. 10  
Vector Integral Calculus

### PART C

Chaps. 11–12

Fourier Analysis. Partial Differential  
Equations (PDEs)

Chap. 11  
Fourier Analysis

↓  
Chap. 12  
Partial Differential Equations

### PART D

Chaps. 13–18

Complex Analysis,  
Potential Theory

Chaps. 13–17  
Basic Material

↓  
Chap. 18  
Potential Theory

### PART E

Chaps. 19–21

Numeric Analysis

Chap. 19  
Numerics in  
General

Chap. 20  
Numeric  
Linear Algebra

Chap. 21  
Numerics for  
ODEs and PDEs

### PART F

Chaps. 22–23

Optimization, Graphs

Chap. 22  
Linear Programming

Chap. 23  
Graphs, Optimization

### PART G

Chaps. 24–25

Probability, Statistics

Chap. 24  
Data Analysis. Probability Theory

↓  
Chap. 25  
Mathematical Statistics

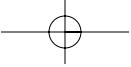
### GUIDES AND MANUALS

Maple Computer Guide

Mathematica Computer Guide

Student Solutions Manual

Instructor's Manual



## General Features of the Book Include:

- **Simplicity of examples**, to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?
- **Independence of chapters**, to provide *flexibility* in tailoring courses to special needs.
- **Self-contained presentation**, except for a few clearly marked places where a proof would exceed the level of the book and a reference is given instead.
- **Modern standard notation**, to help students with other courses, *modern* books, and mathematical and engineering journals.

Many sections were rewritten in a more detailed fashion, to make it a *simpler book*. This also resulted in a *better balance between theory and applications*.

## Use of Computers

The presentation is *adaptable to various levels of technology and use of a computer or graphing calculator*: very little or no use, medium use, or intensive use of a graphing calculator or of an unspecified **CAS** (Computer Algebra System, **Maple**, **Mathematica**, or **Matlab** being popular examples). In either case texts and problem sets form an entity without gaps or jumps. And many problems can be solved by hand or with a computer or both ways. (For **software**, see the beginnings of *Part E* on Numeric Analysis and *Part G* on Probability and Statistics.)

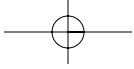
More specifically, this new edition on the one hand gives more prominence to tasks the computer *cannot* do, notably, modeling and interpreting results. On the other hand, it includes **CAS projects**, **CAS problems**, and **CAS experiments**, which *do require* a computer and show its power in solving problems that are difficult or impossible to access otherwise. Here our goal is the combination of *intelligent* computer use with high-quality mathematics. This has resulted in a change from a formula-centered teaching and learning of engineering mathematics to a more quantitative, project-oriented, and visual approach. CAS experiments also exhibit the computer as an instrument for observations and experimentations that may become the beginnings of new research, for “proving” or disproving conjectures, or for formalizing empirical relationships that are often quite useful to the engineer as working guidelines. These changes will also help the student in discovering the *experimental aspect of modern applied mathematics*.

Some **routine and drill work** is retained as a necessity for keeping firm contact with the subject matter. In some of it the computer can (but must not) give the student a hand, but there are plenty of problems that are more suitable for pencil-and-paper work.

## Major Changes

**1. New Problem Sets.** Modern engineering mathematics is mostly **teamwork**. It usually combines analytic work in the process of modeling and the use of computer algebra and numerics in the process of solution, followed by critical evaluation of results. Our problems—some straightforward, some more challenging, some “thinking problems” not accessible by a CAS, some open-ended—reflect this modern situation with its increased emphasis on qualitative methods and applications, and the problem sets take care of this novel situation by including team projects, CAS projects, and writing projects. The latter will also help the student in writing general reports, as they are required in engineering work quite frequently.

**2. Computer Experiments**, using the computer as an instrument of “**experimental mathematics**” for exploration and research (see also above). These are mostly open-ended



experiments, demonstrating the use of computers in experimentally finding results, which may be provable afterward or may be valuable heuristic qualitative guidelines to the engineer, in particular in complicated problems.

**3. More on modeling and selecting methods,** tasks that usually cannot be automated.

**4. Student Solutions Manual and Study Guide enlarged,** upon explicit requests of the users. This Manual contains worked-out solutions to carefully selected odd-numbered problems (to which App. 1 gives only the final answers) as well as general comments and hints on studying the text and working further problems, including explanations on the significance and character of concepts and methods in the various sections of the book.

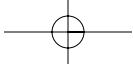
## Further Changes, New Features

- Electric circuits moved entirely to Chap. 2, to avoid duplication and repetition
- Second-order ODEs and Higher Order ODEs placed into two separate chapters (2 and 3)
- In Chap. 2, applications presented before variation of parameters
- Series solutions somewhat shortened, without changing the order of sections
- Material on Laplace transforms brought into a better logical order: partial fractions used earlier in a more practical approach, unit step and Dirac's delta put into separate subsequent sections, differentiation and integration of transforms (not of functions!) moved to a later section in favor of practically more important topics
- Second- and third-order determinants made into a separate section for reference throughout the book
- Complex matrices made optional
- Three sections on curves and their application in mechanics combined in a single section
- First two sections on Fourier series combined to provide a better, more direct start
- Discrete and Fast Fourier Transforms included
- Conformal mapping presented in a separate chapter and enlarged
- Numeric analysis updated
- Backward Euler method included
- Stiffness of ODEs and systems discussed
- List of software (in Part E) updated; another list for statistics software added (in Part G)
- References updated, now including about 75 books published or reprinted after 1990

## Suggestions for Courses: A Four-Semester Sequence

The material, when taken in sequence, is suitable for four consecutive semester courses, meeting 3–4 hours a week:

- |               |  |
|---------------|--|
| 1st Semester. | <i>ODEs</i> (Chaps. 1–5 or 6)                        |
| 2nd Semester. | <i>Linear Algebra. Vector Analysis</i> (Chaps. 7–10) |
| 3rd Semester. | <i>Complex Analysis</i> (Chaps. 13–18)               |
| 4th Semester. | <i>Numeric Methods</i> (Chaps. 19–21)                |



## Suggestions for Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week. For instance:

- Introduction to ODEs (Chaps. 1–2, Sec. 21.1)
- Laplace Transforms (Chap. 6)
- Matrices and Linear Systems (Chaps. 7–8)
- Vector Algebra and Calculus (Chaps. 9–10)
- Fourier Series and PDEs (Chaps. 11–12, Secs. 21.4–21.7)
- Introduction to Complex Analysis (Chaps. 13–17)
- Numeric Analysis (Chaps. 19, 21)
- Numeric Linear Algebra (Chap. 20)
- Optimization (Chaps. 22–23)
- Graphs and Combinatorial Optimization (Chap. 23)
- Probability and Statistics (Chaps. 24–25)

## Acknowledgments

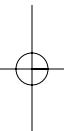
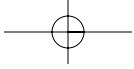
I am indebted to many of my former teachers, colleagues, and students who helped me directly or indirectly in preparing this book, in particular, the present edition. I profited greatly from discussions with engineers, physicists, mathematicians, and computer scientists, and from their written comments. I want to mention particularly Y. Antipov, D. N. Buechler, S. L. Campbell, R. Carr, P. L. Chambre, V. F. Connolly, Z. Davis, J. Delany, J. W. Dettman, D. Dicker, L. D. Drager, D. Ellis, W. Fox, A. Goriely, R. B. Guenther, J. B. Handley, N. Harbertson, A. Hassen, V. W. Howe, H. Kuhn, G. Lamb, M. T. Lusk, H. B. Mann, I. Marx, K. Millet, J. D. Moore, W. D. Munroe, A. Nadim, B. S. Ng, J. N. Ong, Jr., D. Panagiotis, A. Plotkin, P. J. Pritchard, W. O. Ray, J. T. Scheick, L. F. Shampine, H. A. Smith, J. Todd, H. Unz, A. L. Villone, H. J. Weiss, A. Wilansky, C. H. Wilcox, H. Ya Fan, and A. D. Ziebur, all from the United States, Professors E. J. Norminton and R. Vaillancourt from Canada, and Professors H. Florian and H. Unger from Europe. I can offer here only an inadequate acknowledgment of my gratitude and appreciation.

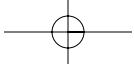
Special cordial thanks go to Privatdozent Dr. M. Kracht and to Mr. Herbert Kreyszig, MBA, the coauthor of the Student Solutions Manual, who both checked the manuscript in all details and made numerous suggestions for improvements and helped me proofread the galley and page proofs.

Furthermore, I wish to thank John Wiley and Sons (see the list on p. iv) as well as GGS Information Services, in particular Mr. K. Bradley and Mr. J. Nystrom, for their effective cooperation and great care in preparing this new edition.

*Suggestions of many readers worldwide were evaluated in preparing this edition. Further comments and suggestions for improving the book will be gratefully received.*

ERWIN KREYSZIG





# CONTENTS

## PART A Ordinary Differential Equations (ODEs) 1

### CHAPTER 1 First-Order ODEs 2

- 1.1 Basic Concepts. Modeling 2
- 1.2 Geometric Meaning of  $y' = f(x, y)$ . Direction Fields 9
- 1.3 Separable ODEs. Modeling 12
- 1.4 Exact ODEs. Integrating Factors 19
- 1.5 Linear ODEs. Bernoulli Equation. Population Dynamics 26
- 1.6 Orthogonal Trajectories. *Optional* 35
- 1.7 Existence and Uniqueness of Solutions 37
- Chapter 1 Review Questions and Problems 42
- Summary of Chapter 1 43

### CHAPTER 2 Second-Order Linear ODEs 45

- 2.1 Homogeneous Linear ODEs of Second Order 45
- 2.2 Homogeneous Linear ODEs with Constant Coefficients 53
- 2.3 Differential Operators. *Optional* 59
- 2.4 Modeling: Free Oscillations. (Mass–Spring System) 61
- 2.5 Euler–Cauchy Equations 69
- 2.6 Existence and Uniqueness of Solutions. Wronskian 73
- 2.7 Nonhomogeneous ODEs 78
- 2.8 Modeling: Forced Oscillations. Resonance 84
- 2.9 Modeling: Electric Circuits 91
- 2.10 Solution by Variation of Parameters 98
- Chapter 2 Review Questions and Problems 102
- Summary of Chapter 2 103

### CHAPTER 3 Higher Order Linear ODEs 105

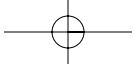
- 3.1 Homogeneous Linear ODEs 105
- 3.2 Homogeneous Linear ODEs with Constant Coefficients 111
- 3.3 Nonhomogeneous Linear ODEs 116
- Chapter 3 Review Questions and Problems 122
- Summary of Chapter 3 123

### CHAPTER 4 Systems of ODEs. Phase Plane. Qualitative Methods 124

- 4.0 Basics of Matrices and Vectors 124
- 4.1 Systems of ODEs as Models 130
- 4.2 Basic Theory of Systems of ODEs 136
- 4.3 Constant-Coefficient Systems. Phase Plane Method 139
- 4.4 Criteria for Critical Points. Stability 147
- 4.5 Qualitative Methods for Nonlinear Systems 151
- 4.6 Nonhomogeneous Linear Systems of ODEs 159
- Chapter 4 Review Questions and Problems 163
- Summary of Chapter 4 164

### CHAPTER 5 Series Solutions of ODEs. Special Functions 166

- 5.1 Power Series Method 167
- 5.2 Theory of the Power Series Method 170



5.3	Legendre's Equation. Legendre Polynomials $P_n(x)$	177
5.4	Frobenius Method	182
5.5	Bessel's Equation. Bessel Functions $J_\nu(x)$	189
5.6	Bessel Functions of the Second Kind $Y_\nu(x)$	198
5.7	Sturm–Liouville Problems. Orthogonal Functions	203
5.8	Orthogonal Eigenfunction Expansions	210
	Chapter 5 Review Questions and Problems	217
	Summary of Chapter 5	218

## CHAPTER 6 Laplace Transforms 220

6.1	Laplace Transform. Inverse Transform. Linearity. $s$ -Shifting	221
6.2	Transforms of Derivatives and Integrals. ODEs	227
6.3	Unit Step Function. $t$ -Shifting	233
6.4	Short Impulses. Dirac's Delta Function. Partial Fractions	241
6.5	Convolution. Integral Equations	248
6.6	Differentiation and Integration of Transforms.	254
6.7	Systems of ODEs	258
6.8	Laplace Transform: General Formulas	264
6.9	Table of Laplace Transforms	265
	Chapter 6 Review Questions and Problems	267
	Summary of Chapter 6	269

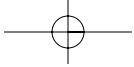
## PART B Linear Algebra. Vector Calculus 271

### CHAPTER 7 Linear Algebra: Matrices, Vectors, Determinants. Linear Systems 272

7.1	Matrices, Vectors: Addition and Scalar Multiplication	272
7.2	Matrix Multiplication	278
7.3	Linear Systems of Equations. Gauss Elimination	287
7.4	Linear Independence. Rank of a Matrix. Vector Space	296
7.5	Solutions of Linear Systems: Existence, Uniqueness	302
7.6	For Reference: Second- and Third-Order Determinants	306
7.7	Determinants. Cramer's Rule	308
7.8	Inverse of a Matrix. Gauss–Jordan Elimination	315
7.9	Vector Spaces, Inner Product Spaces. Linear Transformations. <i>Optional</i>	323
	Chapter 7 Review Questions and Problems	330
	Summary of Chapter 7	331

### CHAPTER 8 Linear Algebra: Matrix Eigenvalue Problems 333

8.1	Eigenvalues, Eigenvectors	334
8.2	Some Applications of Eigenvalue Problems	340
8.3	Symmetric, Skew-Symmetric, and Orthogonal Matrices	345
8.4	Eigenbases. Diagonalization. Quadratic Forms	349
8.5	Complex Matrices and Forms. <i>Optional</i>	356
	Chapter 8 Review Questions and Problems	362
	Summary of Chapter 8	363

**CHAPTER 9 Vector Differential Calculus. Grad, Div, Curl 364**

- 9.1 Vectors in 2-Space and 3-Space 364
- 9.2 Inner Product (Dot Product) 371
- 9.3 Vector Product (Cross Product) 377
- 9.4 Vector and Scalar Functions and Fields. Derivatives 384
- 9.5 Curves. Arc Length. Curvature. Torsion 389
- 9.6 Calculus Review: Functions of Several Variables. *Optional* 400
- 9.7 Gradient of a Scalar Field. Directional Derivative 403
- 9.8 Divergence of a Vector Field 410
- 9.9 Curl of a Vector Field 414
- Chapter 9 Review Questions and Problems 416
- Summary of Chapter 9 417

**CHAPTER 10 Vector Integral Calculus. Integral Theorems 420**

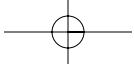
- 10.1 Line Integrals 420
- 10.2 Path Independence of Line Integrals 426
- 10.3 Calculus Review: Double Integrals. *Optional* 433
- 10.4 Green's Theorem in the Plane 439
- 10.5 Surfaces for Surface Integrals 445
- 10.6 Surface Integrals 449
- 10.7 Triple Integrals. Divergence Theorem of Gauss 458
- 10.8 Further Applications of the Divergence Theorem 463
- 10.9 Stokes's Theorem 468
- Chapter 10 Review Questions and Problems 473
- Summary of Chapter 10 474

**PART C Fourier Analysis. Partial Differential Equations (PDEs) 477****CHAPTER 11 Fourier Series, Integrals, and Transforms 478**

- 11.1 Fourier Series 478
- 11.2 Functions of Any Period  $p = 2L$  487
- 11.3 Even and Odd Functions. Half-Range Expansions 490
- 11.4 Complex Fourier Series. *Optional* 496
- 11.5 Forced Oscillations 499
- 11.6 Approximation by Trigonometric Polynomials 502
- 11.7 Fourier Integral 506
- 11.8 Fourier Cosine and Sine Transforms 513
- 11.9 Fourier Transform. Discrete and Fast Fourier Transforms 518
- 11.10 Tables of Transforms 529
- Chapter 11 Review Questions and Problems 532
- Summary of Chapter 11 533

**CHAPTER 12 Partial Differential Equations (PDEs) 535**

- 12.1 Basic Concepts 535
- 12.2 Modeling: Vibrating String, Wave Equation 538
- 12.3 Solution by Separating Variables. Use of Fourier Series 540
- 12.4 D'Alembert's Solution of the Wave Equation. Characteristics 548
- 12.5 Heat Equation: Solution by Fourier Series 552



12.6 Heat Equation: Solution by Fourier Integrals and Transforms	562
12.7 Modeling: Membrane, Two-Dimensional Wave Equation	569
12.8 Rectangular Membrane. Double Fourier Series	571
12.9 Laplacian in Polar Coordinates. Circular Membrane. Fourier–Bessel Series	579
12.10 Laplace’s Equation in Cylindrical and Spherical Coordinates. Potential	587
12.11 Solution of PDEs by Laplace Transforms	594
Chapter 12 Review Questions and Problems	597
Summary of Chapter 12	598

## PART D Complex Analysis 601

### CHAPTER 13 Complex Numbers and Functions 602

13.1 Complex Numbers. Complex Plane	602
13.2 Polar Form of Complex Numbers. Powers and Roots	607
13.3 Derivative. Analytic Function	612
13.4 Cauchy–Riemann Equations. Laplace’s Equation	618
13.5 Exponential Function	623
13.6 Trigonometric and Hyperbolic Functions	626
13.7 Logarithm. General Power	630
Chapter 13 Review Questions and Problems	634
Summary of Chapter 13	635

### CHAPTER 14 Complex Integration 637

14.1 Line Integral in the Complex Plane	637
14.2 Cauchy’s Integral Theorem	646
14.3 Cauchy’s Integral Formula	654
14.4 Derivatives of Analytic Functions	658
Chapter 14 Review Questions and Problems	662
Summary of Chapter 14	663

### CHAPTER 15 Power Series, Taylor Series 664

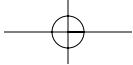
15.1 Sequences, Series, Convergence Tests	664
15.2 Power Series	673
15.3 Functions Given by Power Series	678
15.4 Taylor and Maclaurin Series	683
15.5 Uniform Convergence. <i>Optional</i>	691
Chapter 15 Review Questions and Problems	698
Summary of Chapter 15	699

### CHAPTER 16 Laurent Series. Residue Integration 701

16.1 Laurent Series	701
16.2 Singularities and Zeros. Infinity	707
16.3 Residue Integration Method	712
16.4 Residue Integration of Real Integrals	718
Chapter 16 Review Questions and Problems	726
Summary of Chapter 16	727

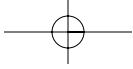
### CHAPTER 17 Conformal Mapping 728

17.1 Geometry of Analytic Functions: Conformal Mapping	729
17.2 Linear Fractional Transformations	734
17.3 Special Linear Fractional Transformations	737



17.4	Conformal Mapping by Other Functions	742
17.5	Riemann Surfaces. <i>Optional</i>	746
Chapter 17 Review Questions and Problems		747
Summary of Chapter 17		748
<b>CHAPTER 18 Complex Analysis and Potential Theory</b>		749
18.1	Electrostatic Fields	750
18.2	Use of Conformal Mapping. Modeling	754
18.3	Heat Problems	757
18.4	Fluid Flow	761
18.5	Poisson's Integral Formula for Potentials	768
18.6	General Properties of Harmonic Functions	771
Chapter 18 Review Questions and Problems		775
Summary of Chapter 18		776
<b>PART E Numeric Analysis</b>		777
<b>Software</b>		778
<b>CHAPTER 19 Numerics in General</b>		780
19.1	Introduction	780
19.2	Solution of Equations by Iteration	787
19.3	Interpolation	797
19.4	Spline Interpolation	810
19.5	Numeric Integration and Differentiation	817
Chapter 19 Review Questions and Problems		830
Summary of Chapter 19		831
<b>CHAPTER 20 Numeric Linear Algebra</b>		833
20.1	Linear Systems: Gauss Elimination	833
20.2	Linear Systems: LU-Factorization, Matrix Inversion	840
20.3	Linear Systems: Solution by Iteration	845
20.4	Linear Systems: Ill-Conditioning, Norms	851
20.5	Least Squares Method	859
20.6	Matrix Eigenvalue Problems: Introduction	863
20.7	Inclusion of Matrix Eigenvalues	866
20.8	Power Method for Eigenvalues	872
20.9	Tridiagonalization and QR-Factorization	875
Chapter 20 Review Questions and Problems		883
Summary of Chapter 20		884
<b>CHAPTER 21 Numerics for ODEs and PDEs</b>		886
21.1	Methods for First-Order ODEs	886
21.2	Multistep Methods	898
21.3	Methods for Systems and Higher Order ODEs	902
21.4	Methods for Elliptic PDEs	909
21.5	Neumann and Mixed Problems. Irregular Boundary	917
21.6	Methods for Parabolic PDEs	922
21.7	Method for Hyperbolic PDEs	928
Chapter 21 Review Questions and Problems		930
Summary of Chapter 21		932



**PART F Optimization, Graphs 935****CHAPTER 22 Unconstrained Optimization. Linear Programming 936**

- 22.1 Basic Concepts. Unconstrained Optimization 936
- 22.2 Linear Programming 939
- 22.3 Simplex Method 944
- 22.4 Simplex Method: Difficulties 947
- Chapter 22 Review Questions and Problems 952
- Summary of Chapter 22 953

**CHAPTER 23 Graphs. Combinatorial Optimization 954**

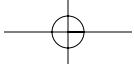
- 23.1 Graphs and Digraphs 954
- 23.2 Shortest Path Problems. Complexity 959
- 23.3 Bellman's Principle. Dijkstra's Algorithm 963
- 23.4 Shortest Spanning Trees: Greedy Algorithm 966
- 23.5 Shortest Spanning Trees: Prim's Algorithm 970
- 23.6 Flows in Networks 973
- 23.7 Maximum Flow: Ford-Fulkerson Algorithm 979
- 23.8 Bipartite Graphs. Assignment Problems 982
- Chapter 23 Review Questions and Problems 987
- Summary of Chapter 23 989

**PART G Probability, Statistics 991****CHAPTER 24 Data Analysis. Probability Theory 993**

- 24.1 Data Representation. Average. Spread 993
- 24.2 Experiments, Outcomes, Events 997
- 24.3 Probability 1000
- 24.4 Permutations and Combinations 1006
- 24.5 Random Variables. Probability Distributions 1010
- 24.6 Mean and Variance of a Distribution 1016
- 24.7 Binomial, Poisson, and Hypergeometric Distributions 1020
- 24.8 Normal Distribution 1026
- 24.9 Distributions of Several Random Variables 1032
- Chapter 24 Review Questions and Problems 1041
- Summary of Chapter 24 1042

**CHAPTER 25 Mathematical Statistics 1044**

- 25.1 Introduction. Random Sampling 1044
- 25.2 Point Estimation of Parameters 1046
- 25.3 Confidence Intervals 1049
- 25.4 Testing Hypotheses. Decisions 1058
- 25.5 Quality Control 1068
- 25.6 Acceptance Sampling 1073
- 25.7 Goodness of Fit.  $\chi^2$ -Test 1076
- 25.8 Nonparametric Tests 1080
- 25.9 Regression. Fitting Straight Lines. Correlation 1083
- Chapter 25 Review Questions and Problems 1092
- Summary of Chapter 25 1093



## Contents

xvii

APPENDIX 1	References	A1
APPENDIX 2	Answers to Odd-Numbered Problems	A4
APPENDIX 3	Auxiliary Material	A60
A3.1	Formulas for Special Functions	A60
A3.2	Partial Derivatives	A66
A3.3	Sequences and Series	A69
A3.4	Grad, Div, Curl, $\nabla^2$ in Curvilinear Coordinates	A71
APPENDIX 4	Additional Proofs	A74
APPENDIX 5	Tables	A94
PHOTO CREDITS		P1
INDEX		I1

