Poisson's equation is a partial differential equation that describes the electrostatic potential ϕ in a region with charge density ρ. In one dimension, Poisson's equation is given by:

$$\frac{d^2\phi}{dx^2} = -\frac{1}{\epsilon_0}\rho \tag{1}$$

To solve this equation numerically, we discretize the domain into a grid of N points with grid spacing dx. We define ϕ_i to be the potential at the ith point, and ρ_i to be the charge density at the ith point.

We can approximate the second derivative of ϕ with respect to x using a finite difference approximation:

$$\frac{d^2\phi}{dx^2} \approx \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{dx^2} \tag{2}$$

Using this approximation, we can discretize Poisson's equation as:

$$\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{dx^2} = -\frac{1}{\epsilon_0}\rho_i \tag{3}$$

Rearranging this equation, we get an expression for ϕ_i in terms of neighboring grid points:

$$\phi_i = \frac{1}{2}(\phi_{i+1} + \phi_{i-1} - \frac{\rho_i}{\epsilon_0 dx^2}) \tag{4}$$

This equation allows us to update the potential at interior points on the grid based on the potential and charge density at neighboring points. We can use this equation to iteratively solve for the potential until it converges to a solution.

In the code, we initialize the potential and charge density arrays to zero, with the exception of the boundary conditions. We then iterate over the grid, updating the potential at interior points using the finite difference equation above. We repeat this process until the potential converges to a solution.

Once we have the potential solution, we can calculate the electric field by taking the gradient of the potential:

$$E = -\frac{d\phi}{dx} \tag{5}$$

This gives us the electric field as a function of position. We can then plot the potential and electric field to visualize the solution.