Unit-1. Wave Optics

1.0 Interference

One of the best phenomena that provide direct evidence for the wave nature of light is interference. Thomas young demonstrated the concept of interference of light in 1802 itself. Examples are the multiple colours on soap bubbles as well as on floating oil film when viewed under sun light. Interference is based on the principle of **superposition of waves.**

Interference can be defined as the superposition of two or more coherent waves with each other such that the intensity modifies maxima and minima which results in a new wave pattern.

1.01. Constructive interference

If two waves are in phase with the amplitudes A_1 and A_2 and superpose with each other then the crest of one wave will coincide with the crest of the other wave and the amplitude reaches to a maximum. The resultant wave will have amplitude is $A = A_1+A_2$.

The condition for constructive interference is the path difference should be equal to $n\lambda$.

i.e., $\Delta = n\lambda$; $n = 0,\pm1,\pm2,\ldots\ldots$

1.02. Destructive interference

If two waves are out of phase with the amplitudes A_1 and A_2 and superpose with each other then the crest of one wave will coincide with the trough of the other wave and so will tend to cancel out. The resultant wave will have amplitude is $A = |A_1-A_2|$.

The condition for destructive interference is the path difference should be equal to odd multiples of $\lambda/2$.

i.e., $\Delta = (2n+1)\lambda/2$; $n = 0,\pm1,\pm2,\ldots\ldots$

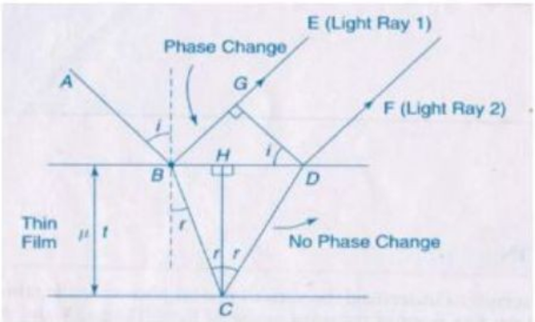
1.03. Conditions for Interference

1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelength and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be large.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow.
8. The sources should be monochromatic.

Unit-1. Wave Optics

1.1. Interference in thin film by reflection

When light is incident on a plane parallel thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the lower surface and emerges through the upper surface. These reflected light beams superimpose with each other, producing interference and forming interference patterns.



Consider a transparent plane parallel thin film of thickness 't' with refractive index μ. Let a monochromatic light ray AB be incident at an angle of incidence of 'i' on the upper surface of the film. BE and BC are the reflected and transmitted light rays. Let the angle of refraction is 'r'. The ray BC will be reflected into the film and emerge through the film in the form of the light ray DF. These two light rays superimpose and depending upon path difference between them, they produce interference patterns.

To know the path difference, let us draw the normal DG and BE. From the points D and G onwards, the light rays travel equal distances. By the time the light ray travels from B to G, the transmitted light ray has to travel from B to C and C to D.

The path difference between light rays (1) & (2) is

Path difference = μ(BC+CD) in film – BG in air -----------→ (1)

Consider the ΔBCH, $\cos r = \frac{\text{HC}}{\text{BC}}$

$$\text{BC} = \frac{\text{HC}}{\cos r} = \frac{\text{t}}{\cos r}$$

Similarly, from ΔDCH, $\text{CD} = \frac{\text{t}}{\cos r}$

To calculate BG, first BD which is equal to (BH+HD) has to be determined.

From ΔBHC, $\tan r = \frac{\text{BH}}{\text{CH}} = \frac{\text{BH}}{\text{t}}$

$$\text{BH} = \text{t} \tan r$$

Similarly, $\text{HD} = \text{t} \tan r$

$$\therefore \text{BD} = \text{BH} + \text{HD} = 2\text{t} \tan r$$

From ΔBGD, $\sin i = \frac{BG}{BD}$

$$BG = BD \sin i$$

$$= 2t \tan r \sin i$$

From Snell's law, $\sin i = \mu \sin r$

$$\therefore BG = 2\mu t \tan r \sin r$$

∴ From eqn(1), we have

$$\text{Path difference} = \frac{2\mu t}{\cos r} - 2\mu t \tan r \sin r$$

$$= 2\mu t \cos r$$

At the point B, reflection occurs from the upper surface of the thin film. Light ray(1) undergoes an additional phase change of π or an additional path difference of $\lambda/2$.

$$\text{Total path difference} = 2\mu t \cos r + \lambda/2$$

When the path difference is equal to integral multiples of λ then the rays (1) & (2) meet in phase and undergoes constructive interference. The condition for bright fringe is $2\mu t \cos r = (2n-1)\lambda/2$ where n = 0,1,2,3,………..

When the path difference is equal to half integral multiples of λ then the rays (1) & (2) meet out of phase and undergoes destructive interference. The condition for dark fringe is $2\mu t \cos r = n\lambda$ where n = 0,1,2,3,………..

1.2. Newton's ring

Newton's rings are one of the best examples for the interference in a non uniform thin film. When a planoconvex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and the glass plate are seen. These circular rings were discovered by Newton and are called Newton's ring.

Experimental arrangement

The planoconvex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate (P). The lens makes the contact with the plate at 'O'. The monochromatic light falls on a glass plate G held at an angle of 45° with the vertical. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P. A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen near the upper surface of the air film through the microscope.

Unit-1. Wave Optics

Explanation of Newton's ring

A part of the incident monochromatic light AB is reflected at B in the form of the ray (1) with an additional phase (or path) change. The other part of the light is refracted through BC. Then at C, it is again reflected in the form of the ray (2) with additional phase change of π or path change of $\lambda/2$.

As the rings are observed in the reflected light, the path difference between them is $2\mu t \cos r+\lambda/2$. For air film $\mu=1$ and for normal incidence $r=0$, path difference is $2t+ \lambda/2$.

At the point of contact $t=0$, path difference is $\lambda/2$. Then the incident and reflected lights are out of phase and interface destructively. Hence the central spot is dark.

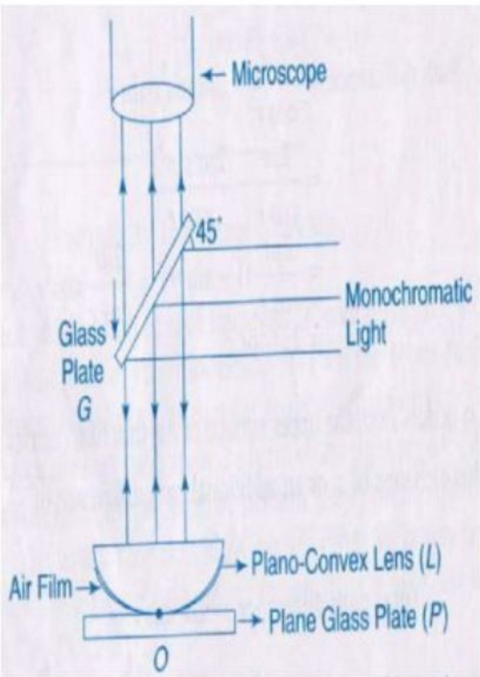
The condition for bright ring is $2t+ \lambda/2 = n\lambda$

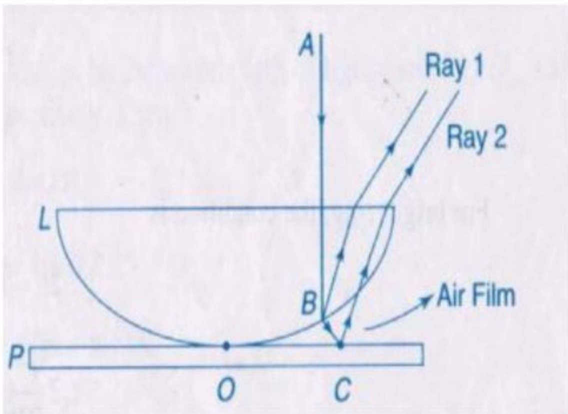
$$2t = (2n-1)\lambda/2 \;;\; n = 1,2,3,\ldots$$

The condition for dark ring is $2t+ \lambda/2 = (2n+1)\lambda/2$

$$2t = n\lambda \;;\; n = 0,1,2,3,\ldots$$

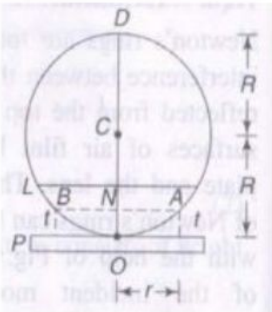
For monochromatic light, the bright and dark rings depends on thickness of the air film. For a Newton's rings system, the focus of points having same thickness lie on a circle having its centre at the point of contact. Thus, we get bright and dark circular rings with the point of contact as the centre.

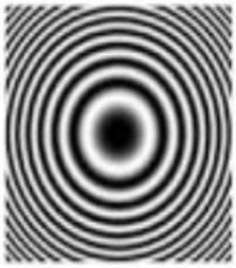




Theory of Newton's ring

To find the diameters of dark and bright rings, let 'L' be a lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness 't'.





From the property of the circle, NA x NB = NO x ND

Substituting the value, r x r = t x (2R-t)

$$r^2 = 2Rt - t^2$$

As t is small, t^2 will be negligible

$$r^2 = 2Rt$$

$$t = \frac{r2}{2R}$$

For bright ring, the condition is

$$2t = (2n-1)\lambda/2$$

$$2\left(\frac{r2}{2R}\right) = (2n-1)\lambda/2$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by D/2, the diameter of n^{th} bright ring will by

$$\frac{D^2}{4} = \frac{(2n-1)}{2}\lambda R$$

$$D = \sqrt{2n-1}\sqrt{2\lambda R}$$

$$D \propto \sqrt{2n-1}$$

$$D \propto \sqrt{\text{odd natural numbers}}$$

For dark ring, the condition is

$$2t = n\lambda$$

$$2\frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{\text{natural number}}$$

With increase in the order (n), the rings get closer and the fringe width decreases and is shown in fig.

Determination of wavelength of a light source

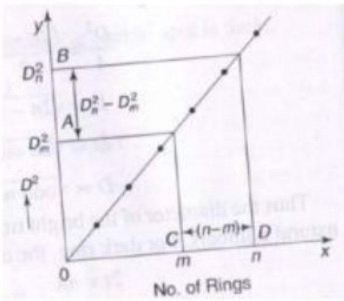
Let R be the radius of curvature of a planoconvex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of m^{th} and n^{th} dark rings respectively. Then

$$D_m^2 = 4m\lambda R \text{ and } D_n^2 = 4n\lambda R$$

$$D_n^2 - D_m^2 = 4(n-m)\,\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

Newton's rings are formed with suitable experimental setup. With the help of a travelling microscope, the readings for different orders of dark rings were noted from one edge of the rings to the other edge. The diameters of different orders of the rings can be known. A plot between D^2 and the no.of rings gives a straight line as shown in fig.



From the graph,

$$\frac{D_n^2 - D_m^2}{(n-m)} = \frac{AB}{CD}$$

The radius R of the planoconvex lens can be obtained with the help of a spherometer. Using these values in the formula, λ can be calculated.

2.0. Diffraction

The phenomenon of bending of light round the corners of obstacles and spreading of light waves into the geometrical shadow region of an obstacle placed in the path of light is called diffraction. The phenomenon of diffraction can be divided into two types.

1) Fraunhoffer diffraction
2) Fresnel diffraction.

Unit-1. Wave Optics

2.01. Fraunhoffer diffraction

In this diffraction, the source and screen are placed at infinite distances from the obstacle. Due to the above fact, we need lenses to study the diffraction. This diffraction can be studied in any direction. In this case, the incident wave front is plane.

2.02. Fresnel diffraction

To study diffraction, there should be a light source, obstacle & screen. In this class of diffraction, the source & the screen are placed at finite distances from the obstacle. To study this diffraction, lenses are not necessary as the source &screen are at a finite distance. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

2.03. Comparison of Fresnel diffraction and Fraunhoffer diffraction

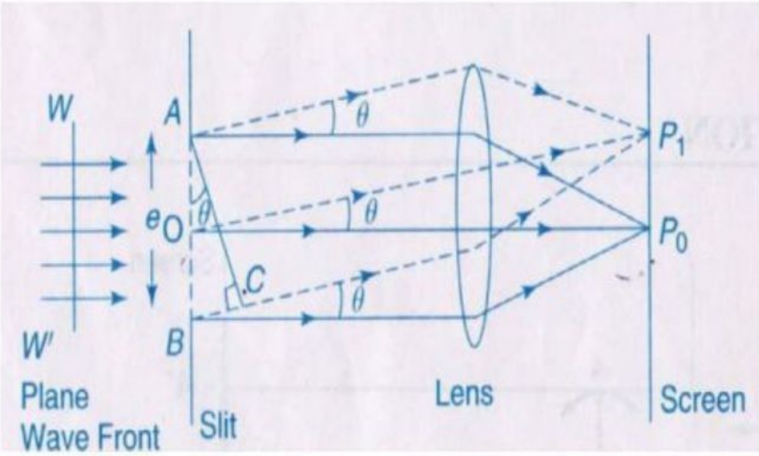
Fresnel Diffraction	Fraunhoffer Diffraction
1) Either a point source or an illuminated narrow slit is used 2) The wavefront undergoing diffraction is either spherical or cylindrical. 3) The source and the screen are finite distances from the obstacle producing diffraction. 4) No lens is used to focus the rays.	1) Extended source at infinite distance is used. 2) The wavefront undergoing diffraction is a plane wavefront. 3) The source and the screen are at infinite distances from the obstacle producing diffraction. 4) Converging lens is used to focus parallel fringes.

2.04. Difference between interference and diffraction

Interference	Diffraction
1) Superposition is due to two separate wavefronts originating from two coherent waves. 2) The fringes normally have equal widths. 3) All the bright fringes have the same intensity. 4) All the dark fringes have zero intensity.	1) Superposition is due to secondary wavelets originating from different parts of the same wavefront. 2) The width between the fringes is never equal. 3) The intensity of bright fringes usually decreases with increase of order. 4) The intensity of dark fringes is not zero.

3.1 Fraunhoffer diffraction at single slit

Consider a slit AB of width 'e'. Let a plane wavefront WW^1 of monochromatic light of wavelength λ propagating normally towards the slit is incident on it. The diffracted light through the slit is focused by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygens-Fresnel, every point on the wavefront in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit, i.e., along the direction OP_o are brought to focus at P_o by the lens. Thus, P_o is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1 on the screen. Depending on path difference, the point P_1 may have maximum or minimum intensities. To find intensity at P_1, let us draw the normal AC from A to the light ray at B.



The path difference between the wavelets from A & B in the direction θ is given by

$$\text{Path difference} = BC = AB \sin\theta = e\sin\theta$$

$$\text{Corresponding phase difference} = \frac{2\pi}{\lambda} \text{ x path difference}$$

$$= \frac{2\pi}{\lambda} e\sin\theta$$

Let the width of the slit be divided into n equal parts and the amplitude of the wave from each part is ‘a’. The phase difference between any two successive waves from these parts would be

$$\frac{1}{n}[\text{total phase}] = \frac{1}{n}[\frac{2\pi}{\lambda} e\sin\theta] = d$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin d/2}{\sin d/2}$$

$$= a\frac{\sin(\pi e \sin\theta/\lambda)}{\sin(\pi e \sin\theta/n\lambda)}$$

$$= a\frac{\sin\alpha}{\sin\alpha/n} \text{ where } \alpha = \pi e \sin\theta/\lambda$$

$$= a\frac{\sin\alpha}{\alpha/n} \ \text{< since } \alpha/n \text{ is very small>}$$

$$= na\frac{\sin\alpha}{\alpha}$$

$$= A\frac{\sin\alpha}{\alpha}$$

$$\text{Intensity} = I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \text{--------------} \rightarrow (1)$$

Principal maximum

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\dots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\dots\right]$$

I will be maximum, when the value of R is maximum. For maximum value of R, the negative terms must vanish i.e., $\propto = 0$

$$\pi e \sin\theta / \lambda = 0$$

$$\sin\theta = 0$$

$$\theta = 0$$

Then R = A

$$\therefore I_{max} = R^2 = A^2$$

The condition $\theta = 0$ means that the maximum intensity is formed at P_o and is known as principal maximum.

Minimum intensity position

I will be minimum, when $\sin\alpha = 0$

$$\alpha = \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi, \dots\dots\dots$$

$$\alpha = \pm m\pi$$

$$\pi e \sin\theta / \lambda = \pm m\pi$$

$$e\sin\theta = \pm m\lambda \text{ where } m = 1,2,3,\dots\dots\dots$$

Thus, we obtain the points of minimum intensity on either side of the principal maximum. For m = 0, $\sin\theta = 0$, which corresponds to principal maximum.

Secondary maxima

In between these minima, we get secondary maxima. The positions can be obtained by differentiating the expression of I wrt α and equating to zero. We get

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

Unit-1. Wave Optics

$$A^2\, 2\, \frac{\sin\alpha}{\alpha} \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

Either $\sin\alpha = 0$ or $\alpha\cos\alpha - \sin\alpha = 0$

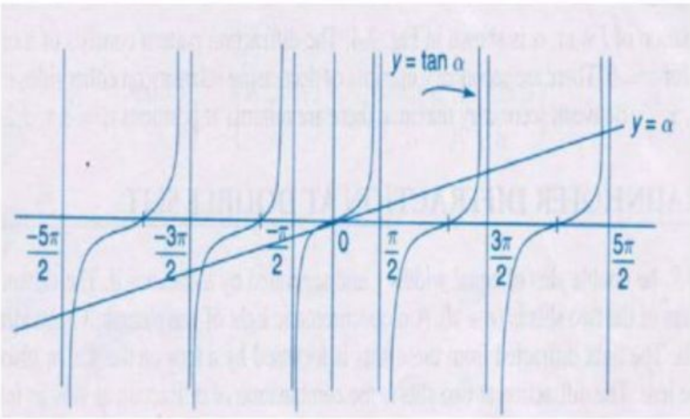
$\sin\alpha = 0$ gives position of minima

Hence the position of secondary maxima are given by

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha$$

The values of α satisfying the above eqn are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ on the same graph. The points of intersection of the two curves gives the values of α which satisfy the above equation. The plots of $y = \alpha$ and $y = \tan\alpha$ are shown in fig.



The points of intersection are

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots$$

Using the above values in eqn (1), we get the intensities in various maxima.

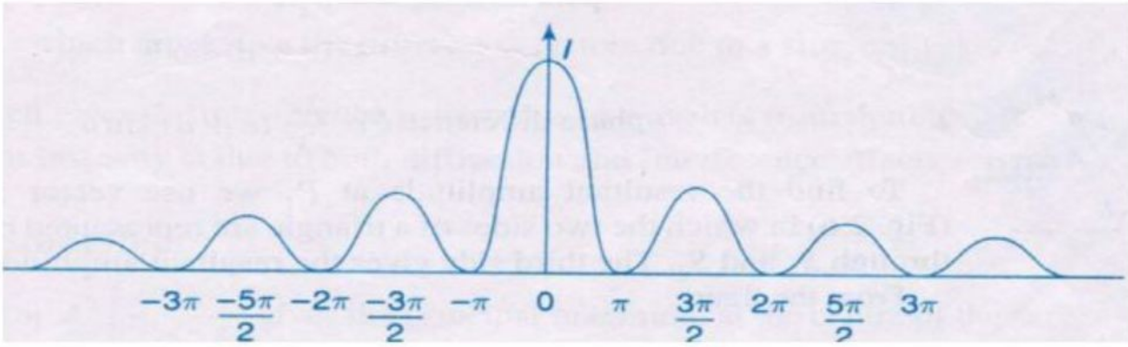
$\alpha = 0,\ I_o = A^2$ (principal maximum)

$\alpha = \frac{3\pi}{2},\ I_1 = A^2 \left[\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2 \cong \frac{A^2}{22}$ (1st secondary maximum)

$\alpha = \frac{5\pi}{2},\ I_2 = A^2 \left[\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2 \cong \frac{A^2}{62}$ (2nd secondary maximum)

and so on

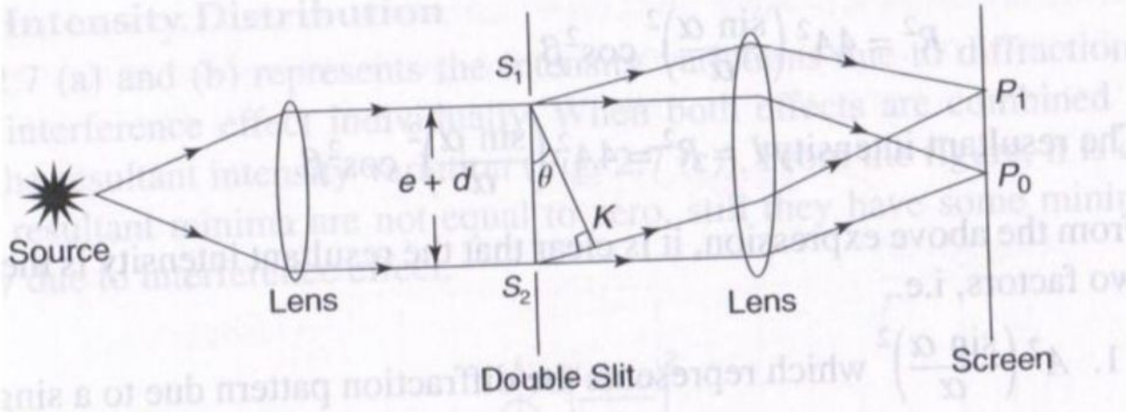
Intensity distribution



The variation of I wrt α is shown in fig. the diffraction pattern consists of a central principal maximum for $\alpha = 0$. There are secondary maxima of decreasing intensity on either of it at positions $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$,....... Between secondary maxima, there are minima at positions $\alpha = \pm\pi, \pm 2\pi$,............

2.2. Fraunhoffer diffraction at a double slits

Let S_1 & S_2 be double slits of equal widths 'e' and separated by a distance d. the distance between the middle points of the two slits is (e+d). A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focused by a lens on the screen placed in the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference, the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superimposed. When a plane wavefront is incident normally on two slits, the secondary wavelets from the slits travel uniformly in all directions. The wavelets travelling in the direction of incident light come to a focus at P_0 while the wavelets travelling in a direction making an angle θ, come to focus at P_1.



From the study of diffraction due to single slit, the resultant amplitude = $A\frac{\sin\alpha}{\alpha}$ where

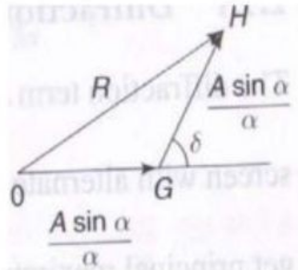
$\alpha = \frac{\pi e \sin\theta}{\lambda}$. Since we use double slit, from each slit we get a wavelet of amplitude $A\frac{\sin\alpha}{\alpha}$ in a direction θ. These two wavelets interfere and meet at a point P_1 on the screen. To calculate the path difference between the wavelets, let us draw a normal S_1K to the wavelets through S_2.

$$\text{Path diff} = S_2K$$

$$= (e+d)\sin\theta$$

$$\text{Phase diff} = \frac{2\pi}{\lambda}(e+d)\sin\theta = \delta$$

To find the resultant amplitude at P_1 we use vector addition method is which the two sides of a triangle are represented by the amplitudes through S_1 & S_2. The third side gives the resultant amplitude. From the fig,



$$(OH)^2 = (OG)^2 + (GH)^2 + 2(OG)(GH)\cos\delta$$

$$R^2 = (A\frac{\sin\alpha}{\alpha})^2 + (A\frac{\sin\alpha}{\alpha})^2 + 2(A\frac{\sin\alpha}{\alpha})(A\frac{\sin\alpha}{\alpha})\cos\delta$$

$$= 2A^2(\frac{\sin\alpha}{\alpha})^2[1+\cos\delta]$$

$$= 2(A\frac{\sin\alpha}{\alpha})^2(1 + 2\cos^2\delta/2 - 1)$$

$$= 4A^2(\frac{\sin\alpha}{\alpha})^2\cos^2(\frac{\pi(e+d)\sin\theta}{\lambda})$$

$$\text{Let } \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

$$R^2 = 4A^2(\frac{\sin\alpha}{\alpha})^2\cos^2\beta$$

The resultant intensity $I = R^2 = 4A^2(\frac{\sin\alpha}{\alpha})^2\cos^2\beta$

From the above expression, it is clear that the resultant intensity is the product of two factors, i.e.,

1) $A^2(\frac{\sin\alpha}{\alpha})^2$ which represents the diffraction pattern due to a single slit.
2) $\cos^2\beta$ which gives the interference pattern due to wavelets from double slits. The resultant intensity is due to both diffraction and interference effects.

Diffraction effect

The diffraction term $A^2(\frac{\sin\alpha}{\alpha})^2$ gives the principal maximum at the centre of the screen with alternate minima and secondary maxima of decreasing intensity. We get principal maxima for $\theta = 0$. We get minima for $\sin\alpha = 0$

Unit-1. Wave Optics

$$\alpha = \pm m\pi$$

$$\pi e \sin\theta/\lambda = \pm m\pi$$

$$e \sin\theta = \pm m\lambda \text{ where } m = 1,2,3,\ldots\ldots\ldots$$

The position of secondary maxima occurs for

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots.$$

Interference effect

The interference term $cos^2\beta$ gives the equidistant bright and dark fringes. The maxima will occur for $cos^2\beta = 1$

$$\beta = \pm n\pi$$

$$\pi(e + d)\sin\theta/\lambda = \pm n\pi$$

$$(e + d)\sin\theta = \pm n\lambda \text{ where } n = 0,1,2,3,\ldots\ldots\ldots$$

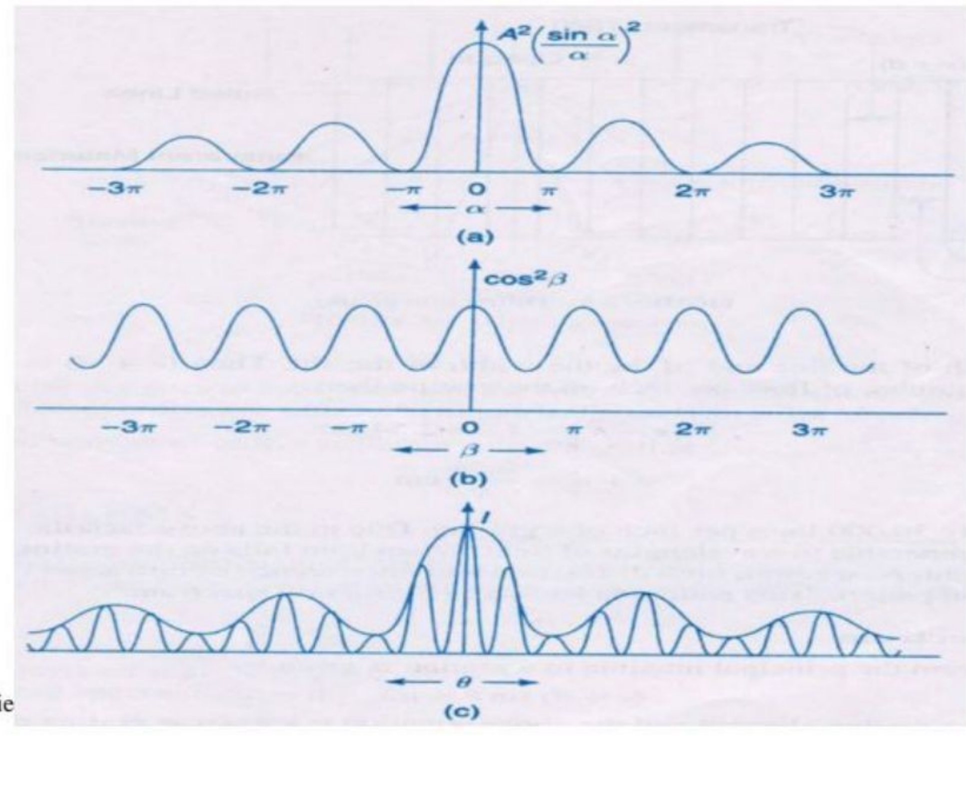
The minima will occur for $cos^2\beta = 0$

$$\beta = \pm(2n+1)\pi/2 \text{ where } n = 0,1,2,\ldots.$$

$$(e + d)\sin\theta = \pm(2n+1)\lambda/2$$

Intensity distribution

When both effects are combined then we get the resultant intensity variation. From fig, it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference effect.



Unit-1. Wave Optics

Diffraction grating

2.7 Diffraction Grating

A diffraction grating is an optical device consisting of a large number of equidistant narrow parallel, rectangular slits of equal width and separated by opaque portions. This was first constructed by the German physicist Joseph von Fraunhofer in 1821.

Construction

➢A diffraction grating is prepared by ruling a large number of equidistant narrow, parallel lines on an optically plane glass plate with a fine diamond point.

➢The ruled lines are opaque to light called opaque portions while the space between any two successive ruled lines is transparent to light called transparent portions and act as slits.

➢The distance between any two successive slits (or) ruled lines is called grating element as shown in Fig.

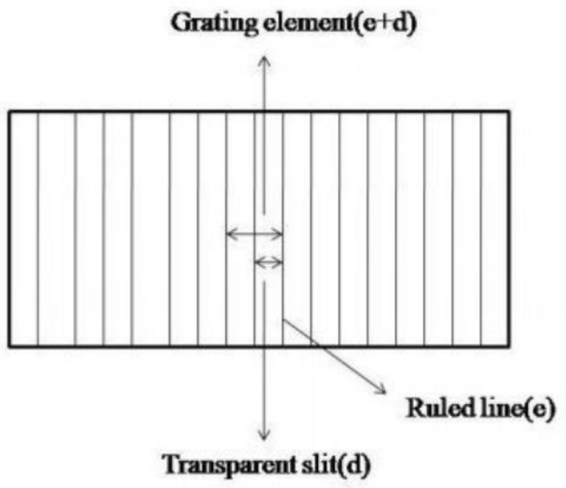


Fig: Construction of diffraction grating

- The Commercial gratings are made up a thin layer of colloidal solution (ex: Solution of cellulose Acetate) is poured on the ruled surface and allowed to dry to form a thin film.
- The colloidal thin film is peeled carefully from the ruled surface. The film retains impression of the ruling of ruled surface.
- The ruled lines acts as opaque portions where as the space between them act as transparent portions which transmit incident light. The film is mounted between two glass plates called grating.

Unit-1. Wave Optics

- When the light is incident on the grating surface then light is transmitted through transmission portions or slits and obstructed by the opaque portions or ruled lines. Such a grating is known as transmission grating.

Theory

Let' e' be the width of each ruled line (or) opaque portion and 'd' be the width of transparent portion (or) slit then (e+ d) represents the grating element. The relation between grating element and the angles of diffracted beams of light is known as grating equation. It is given by

$$(e+d)\sin\theta=n\lambda, \text{ Where } n = 0, 1, 2, 3\ldots.$$

This expression is known as grating equation.

$$\sin\theta= n\lambda /(e+d)$$

$$\sin\theta= N\, n\lambda$$

Where $1/(e+d) = N$ is the number of grating elements or lines per unit width of the grating.

$$N\,(e+d) = 1\text{inch} = 2.54 \text{ cm}$$

$$e+d=2.54/N \text{ cm}$$

Grating Spectrum

The diffraction pattern formed with a grating is known as grating spectrum.

The positions of principle maxima in grating are given by

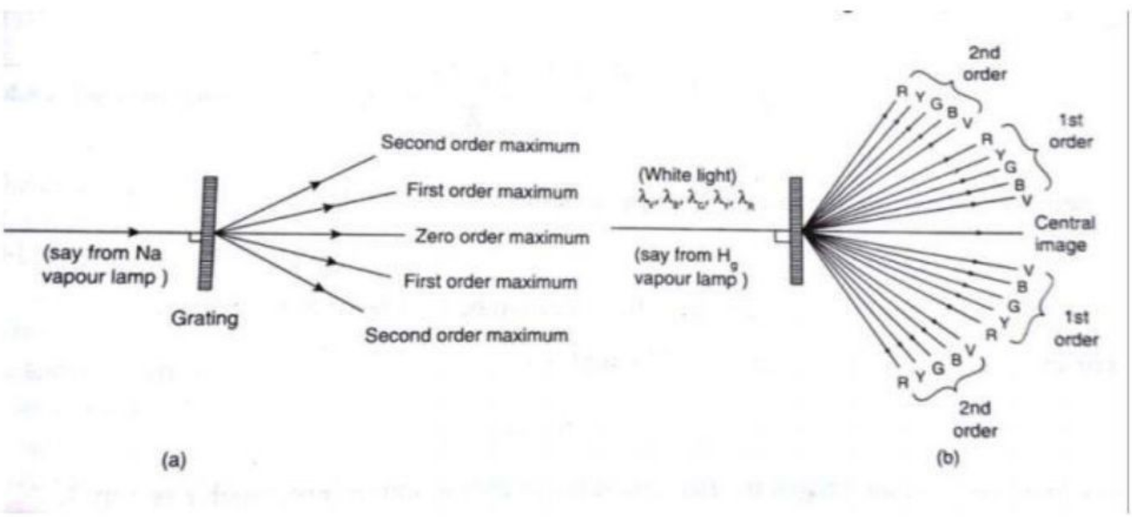
$(e+d)\sin\theta=n\lambda$, Where n = 0, 1, 2, 3….

This expression is known as grating equation.

For a particular wavelength λ , the diffraction angle is different for different for principal maxima of different orders.

The first order maxima is obtained for n=1, then $(e+d)\ \sin\theta_1=\lambda$

The second order maxima is obtained for n=2, then $(e+d)\ \sin\theta_2=2\lambda$ The different orders of principal maxima are obtained on both sides of zero order maxima as shown Fig.



Unit-1. Wave Optics

Determination of Wavelength (λ) of given source of light

The positions of principle maxima in grating are given by

$(e+d)sin\theta=n\lambda$, Where n = 0, 1, 2, 3…. $sin\theta= n\lambda /(e+d)$

$sin\theta=$ N $n\lambda$

$\lambda=sin\theta$N n

Where $1/e+d$ = N is the number of grating elements or lines per unit width of the grating.

By knowing the values of angle of diffraction, order of the principal maxima and grating element, the wavelength can be determined.

Applications of Diffraction

The phenomenon of diffraction can be used in various engineering applications; some of the applications are given below.

1. It is used in x-ray diffraction studies of crystals.

2. It is also used in holography for reconstructing 3D images of objects using laser light.

3. Diffraction gratings are used in spectroscopes to separate a light source into its component wavelengths.

4. Diffraction grating can be chosen to specifically analyze a wavelength of light emitted by molecules in diseased cells in a biopsy sample.

5. Diffraction gratings are used in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths.

3. Polarization of Light

3.1 Introduction

The phenomena of interference and diffraction tells us that light is a form of wave. But they do not tell us whether light is transverse or longitudinal waves in nature. The phenomenon **polarization** confirms that light waves are transverse in nature.

In general, light waves are electromagnetic waves and contain vibrations of electric and magnetic field vectors (*E*&*B*) which are perpendicular to each other and also perpendicular to the direction of propagation of light waves .i.e., transverse waves. Therefore, light is a transverse wave motion. It has been found experimentally that the electric field vector (E) component of light is mainly responsible for polarization and other optical effects but not due to the magnetic field vector. Therefore, the electric field vector E is also called the light vector and vibration of light means vibrations of electric field vector of light.

The word polarization comes from the Greek word *"polos"* which means orientation. Therefore the phenomenon of restricting or orienting the vibrations of the electric field vector in a particular direction is called polarization of light.

3.11 Un-Polarized light

If the light vector (electric field vector *E*) vibrates in all possible directions which are perpendicular to the direction of propagation then the light is said to be un-polarized light.

Let us consider an ordinary light ray passing perpendicular to the plane of the paper and into the paper. The electric field vectors are perpendicular to the ray propagating with equal amplitude in all possible directions as shown in Fig. 3 . 1 . This is the nature of un-polarized light. A double headed arrows show to and fro vibrations of the electric vector.

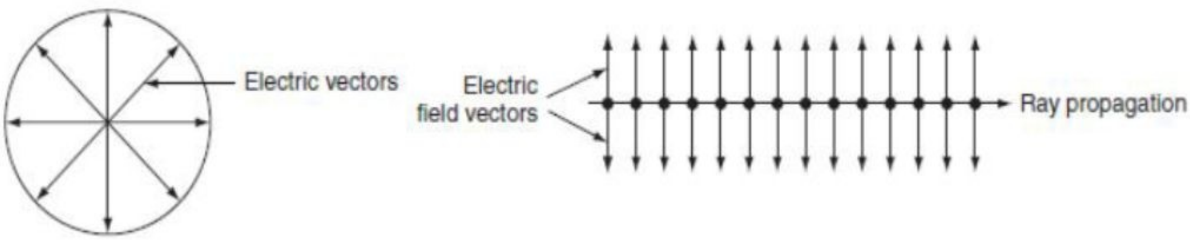


Fig: 3. 1 Representation of un-polarized light

Unit-1. Wave Optics

2.12 Polarized light

The phenomenon of restricting the vibrations of the light vector or electric field vector (E) in a particular direction is called polarization of light. The polarized light can be classified into three types. They are

i. Linearly (or) Plane polarized light
ii. Circularly polarized light and
iii. Elliptically polarized light

Linearly (or) Plane polarized light

If the vibrations of light vector (electric field vector E) restricting along straight line only in a plane perpendicular to the direction of propagation of light, it is said to be plane polarized light.

The linearly polarized light is shown in 3.11. The arrowed lines represent in the plane of paper and the dots, the vibrations at right angles (perpendicular) to the paper. In Fig. 3.11(a), the vibrations of electric field vectors lie in the plane of the paper and in Fig. 3.11(b) the vibrations of electric field vectors are perpendicular to the plane of the paper.

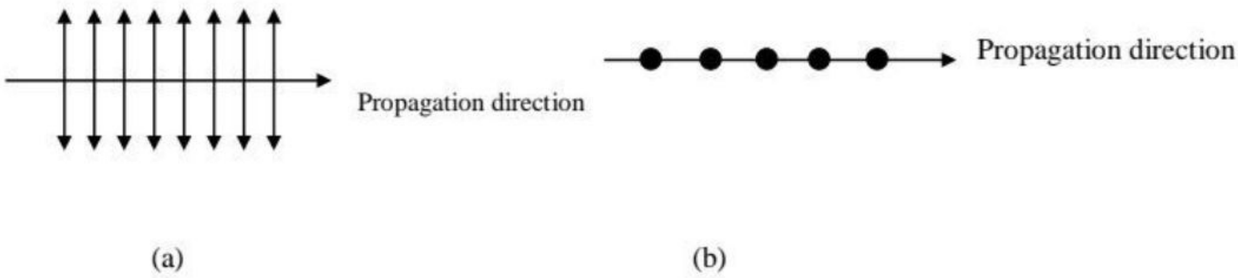


Fig 3.11. Representation of plane polarized light

Circularly polarized light and elliptically polarized light

When two plane polarized light waves are superimposed each other then under certain conditions the resultant light vector may rotates with a constant magnitude in a plane perpendicular to the direction of propagation of light.

If the magnitude of the resultant light vector remains constant while its orientation varies regularly, the tip of the vector traces a circle. Thus the light is said to be circularly polarized.

If however, both magnitude and orientation of light vector vary, the tip of the vector traces an ellipse. Thus the light is said to be elliptically polarized.

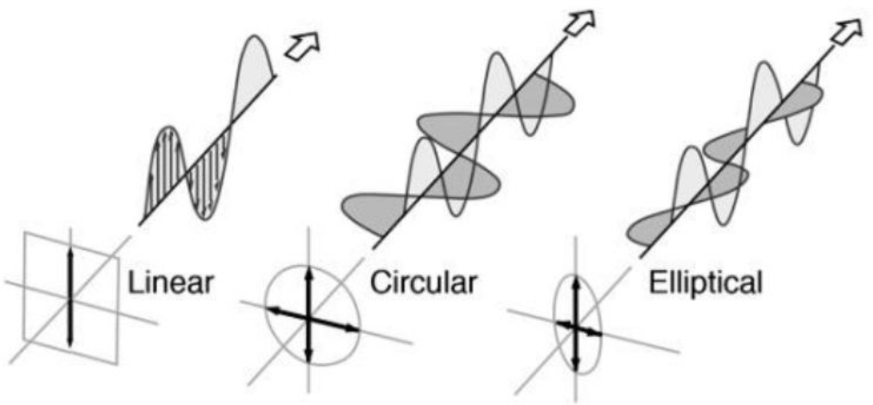


Fig 3.12. Representation of plane, circular and elliptically polarized light

2.13 Methods of producing polarized light

Polarized light can be produced by any one of the following.

i i. Polarization by reflection

ii ii. Polarization by refraction

iii iii. Polarization by double refraction

iv iv. Polarization by scattering

v v. Polarization by selective absorption

Present section we only discuss only polarization by double refraction.

2.14 Polarization by double refraction

When a light ray is passed through a glass plate, we get only one refracted ray. But when a light ray is passed through a certain crystals like calcite or quartz, we get two refracted rays, one refracted light ray which obeys the laws of refraction is called ordinary or *o*-ray. The other ray does not obey the laws of refraction and is extraordinary ray or *e*-ray. This phenomenon is called double refraction or birefringence. This was first discovered by Erasmus Bartholinus in 1869.

When an ink dot is marked on a white paper and is seen through a calcite crystal, then two images (dots) are observed. If the crystal is rotated slowly with incident ray as vertical axis, then it is observed that one image remains fixed and the other image rotates with the rotation of the calcite crystal. The fixed image is called ordinary image and its refracted ray is called *o*-ray which has vibrations perpendicular to plane of the white paper. The other image is called extraordinary image and its refracted ray is called *e*-ray which has vibrations in the plane of the paper as shown in Fig .1.23.

Inside the crystal the *o*-ray travels with the same velocity in all direction. But the *e*-ray has different velocities in different directions. If the incident ray strikes the crystal along a certain direction called the **optical axis** of the crystal, there will be no double refraction. Hence optical axis of the crystal is a direction along which both *o*-ray and *e*-ray travel in the same direction with the same velocity.

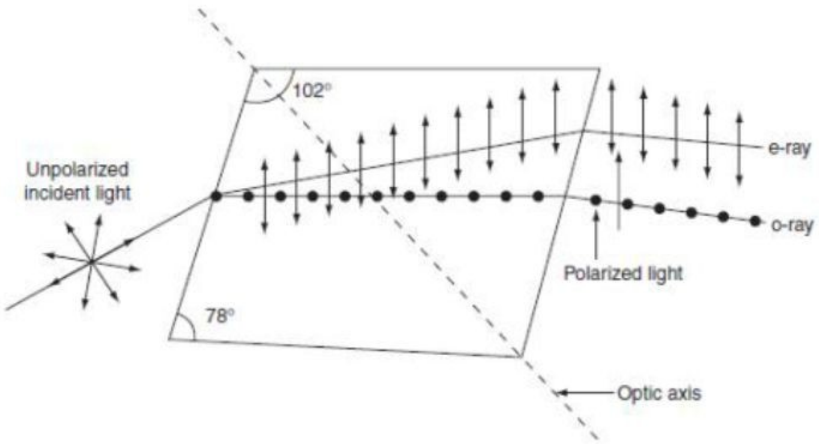


Fig.1.23. Double refraction in calcite crystal

2.15 Double refracting crystals

The crystals which exhibit double refraction are called double refracting crystals.

There are two types of double refracting crystals

i. uni-axial crystals
ii. biaxial crystals

Uni-axial crystals

If only one optic axis is present in the crystals, then they are called uni-axial crystals. *Examples:* Calcite, quartz and tourmaline etc.

These crystals further dived into two crystals

i. Uni-axial negative crystals
ii. Uni-axial positive crystals

Uni-axial negative crystals

If the velocity of *e*-ray is more than that of *o*-ray in the crystals, then they are called negative crystals.

i.e., $v_e > v_o$ and $\mu_e < \mu_o$

Examples: Calcite, tourmaline, ruby and emerald etc

Uni-axial positive crystals

If the velocity of *e*-ray is less than that of *o*-ray in the crystals, then they are called negative crystals.

i.e., $v_e < v_o$ and $\mu_e > \mu_o$

Examples: Quartz and iron oxide.

Biaxial crystals

If two optic axes are present in the crystals, then they are called biaxial crystals.

Examples: Topaz and aragonite.

Note:

Optical axis

The direction in which the ray of transmitted light does not suffer double refraction inside the crystal is known as the optic axis. Along this axis both the velocities of *o*-ray and *e*-ray are the same and also refractive indices are same. i.e., $v_e = v_o$ and $\mu_e = \mu_0$. So, there no double refraction.Optical axis is a direction but not line.

2.16 Wave Plates or Retardation Plates

Wave plates are the doubly refracting uniaxial (calcite) crystals whose refracting faces are cut parallel to the direction of the optic axis and are used to produce a phase difference between ordinary and extraordinary rays when they emerging from the doubly refracting crystals.

These are of mainly two types

i. Quater-wave plate
ii. Half-wave plate

2.16.1Quarter-wave plate

A quater-wave plate is a thin double refracting crystal (calcite) having a thickness 't', cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/4$ or phase difference of $\pi/2$ between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

Consider a calcite crystal of thickness of 't'. When a plane polarized light is incident normally on the surface of calcite crystal, then the light will split up into o-ray and e-ray. These rays travel with different velocities in the crystal. As a result, when *o*-ray and *e*-ray emerging from the crystal, they have a phase or path difference between them due to varaition in their velocities.

Let $\mu_o$and μ_e are the refractive indices of *o*-ray and *e*-ray respectively. Let 't' be the thickness of the crystal. Hence the path difference between the two rays is

Δ = Optical path for *o*-ray − Optical path for *e*-ray

$$= \mu_o t - \mu_e t$$

$$= (\mu_o - \mu_e)t \qquad \rightarrow(1.57)$$

As the crystal is a quater-wave plate, it introduces a path difference of $\lambda/4$ between o-ray and e-ray.

$$\Delta = \lambda/4 \qquad \rightarrow(1.58)$$

From Eqs.(1.57) and (1.58),we get

$$(\mu_o - \mu_e)\text{t} = \frac{\lambda}{4}$$

Therefore $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$

For positive crystal $\mu_e > \mu_o$, then $t = \dfrac{\lambda}{4(\mu_e - \mu_o)}$

For negative crystal $\mu_o > \mu_e$, then $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$

Applications

1. A quarter wave plate is used to produce circularly and elliptically polarized light.
2. Quarter wave plate converts plane-polarized light into elliptically or circularly polarized light depending upon the angle that the incident light vector makes with the optic axis of the quarter wave plate.

2.16.2 Half-wave plate

A half-wave plate is a thin double refracting crystal (calcite) having a thickness 't', cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/2$ or phase difference of π between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

As the crystal is a haf-wave plate, it introduces a path difference of $\lambda/2$ between *o*-ray and *e*-ray.

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

$$\text{Therefore } t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$\text{For positive crystal } \mu_e > \mu_o, \text{ then } t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$\text{For negative crystal } \mu_o > \mu_e, \text{ then } t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

Applications

1. A half wave plate is used to produce plane polarized light.
2. It produces a phase difference of π between the ordinary and extraordinary ray.

2.17 Nicol's Prism

Nicol prism is an optical device used to produce and analyze plane polarized light. This was invented by William Nicol in the year 1828 and is known as Nicol prism. It is made from a double refracting calcite crystal.

Principle

It is based on the phenomenon of double refraction. When a light ray is passed through a calcite crystal, it splits up into two refracted rays such as ordinary (*O*-ray) and extraordinary ray (*E*-ray). Nicol prism transmits the extraordinary rays and eliminates ordinary rays with the help of the phenomenon of total internal reflection.

Construction

- It is constructed from the calcite crystal ABCD having length three times of its width.
- The end faces AB and CD are cut down such that the angles of principal section are 68^0 and 112^0 instead of 71^0 and 109^0.
- The crystal is then cut diagonally into two parts. The surfaces of these parts are ground to make optically flat and then these are polished.

- These polished surfaces are connecting together with special cement known as Canada balsam which is a transparent liquid material.
- Canada balsam is optically (refractive index) more dense than e-ray and less dense for o-ray ($\mu_o = 1.6584, \mu_{ca} = 1.55$ and $\mu_e = 1.4864$).

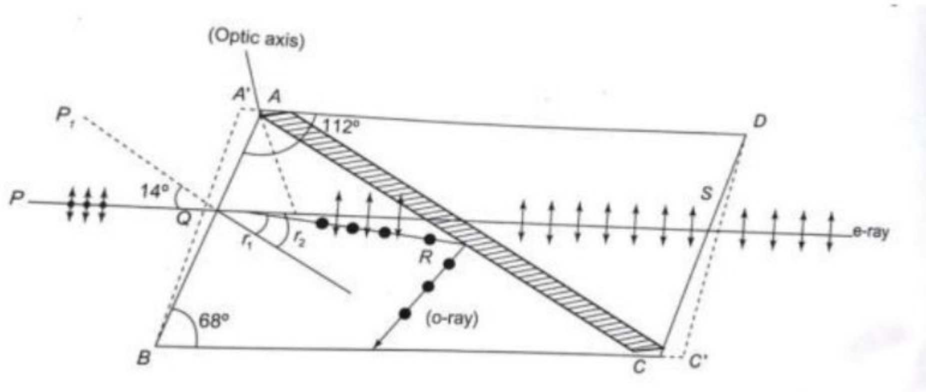


Fig.1.24.Nicol prism

Action or Working

- When a beam of unpolarized light PQ is incident on the face of AB, it splits into two refracted rays o-ray (QR) and e-ray (QS) due to double refraction of calcite crystal.
- These two rays are plane polarized.
- From the refractive index values, we know that the Canada balsam acts as a rarer medium for the ordinary ray and it acts as a denser medium for extraordinary ray.
- When o-ray of light travels in the calcite crystal and enters the Canada balsam cement, it passes from denser to rarer medium. When the angle of incidence for ordinary ray on the Canada balsam is greater than the critical angle then the incident ordinary ray is totally internally reflected from the crystal and only e-ray is transmitted through the prism and emerges out of Nicol prism.
- In this way, plane polarized light is produced.

2.18 Nicol prism as polarizer and analyzer

- Nicol prism can be used as polarizer and analyzer.
- In order to produce and analyse the plane polarized light, two Nicol prisms are arranged adjacently as shown in Fig.1.25.
- The first Nicol prism is used to produce plane polarized light and is called polarizer. The second Nicol prism is used to test the emerging light and is called analyzer.

➢ In the parallel positions, the extraordinary ray passes through both the prisms as shown in Fig.1.29 (a). In this case, the intensity of emergent extraordinary light is maximum.

➢ If the second prism is slowly rotated, then the intensity of the extraordinary ray decreases. When they are perpendicular to each other, no light come out of the second prism because the e-ray that comes out from first prism will enter into the second prism and act as an ordinary ray. So, this light is reflected in the second prism as shown in Fig1.29(b).

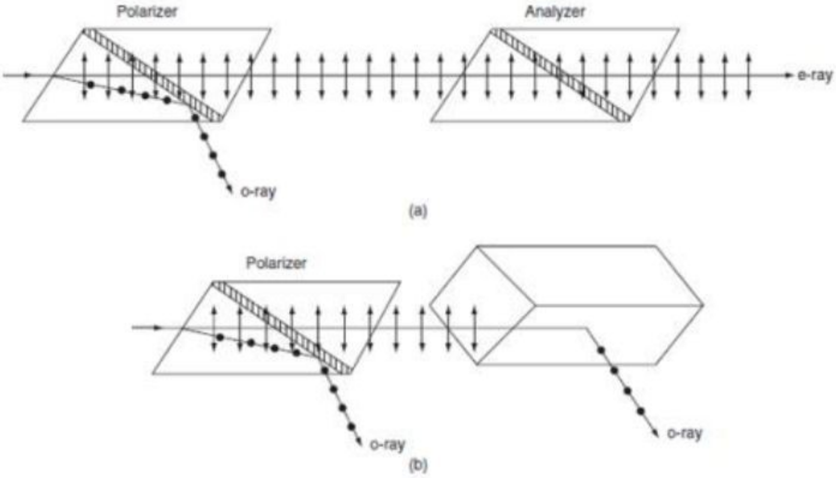


Fig.1.26. Nicol prism as polarizer and analyzer

2.19 Engineering applications of Polarization

The phenomenon of Polarization can be used in various engineering applications; some of the applications are given below.

1. Polaroid glasses are used to reduce the amount of light that is approachable to eye.
2. Polarization is useful in receiving and transmitting wave signals.
3. Laser is an outcome of polarization of waves.
4. 3D movies (or stereoscopic movies) are possible because of polarization of light.
5. Photographic filters.
6. Photo elasticity – To study the objects with irregular boundaries and stress analysis.
7. LCD's.