Subject BEEE

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UNIT-I

DC & AC Circuits:

Electrical circuit elements (R, L and C):-

Basic definitions:-

Voltage:- The difference in potential Energy of the charges is called the potential difference.

(or)

Potential difference in Electrical terminology is known as "Voltage" (V).

$$\boxed{V = \frac{W}{Q}}$$

where
$W \rightarrow$ Energy (Joule)
$Q \rightarrow$ charge (coulombs)

Current:-

Current is defined as the rate of flow of Electrons in a conductive or semiconductive material.

It is measured by the number of Electrons that flow past a point in unit time.

Expressed mathematically; $\boxed{I = \frac{Q}{t}}$

Units - Amperes (A)

Power and Energy:-

Power is the rate of change of Energy, and is denoted by either 'P' or 'p'.

$$\boxed{P = VI}$$

Energy:- Energy is the capacity for doing work, i.e., energy is nothing but stored work.

→ Energy may exist in many forms such as mechanical, chemical, electrical and so on. $\boxed{W = P.t}$ P → power
t → time

Basic Electrical circuit elements:- (R, L and C)

Resistance:- (R)

→ When a current flows in a material, the free Electrons move through the material and collide with other atoms.

→ These collisions cause the Electrons to lose some of their energy.

→ This loss of energy per unit charge is the drop in potential across the material.

→ The amount of energy lost by the Electrons is related to the physical property of the material.

→ These collisions restrict the movement of electrons.

→ The property of a material to restrict the flow of Electrons is called "resistance"

→ It is denoted by 'R'.

→ The symbol of resistor is "—/\/\/\—"

→ The unit of resistor is "ohm (Ω)."

→ According to ohm's law,

The voltage is directly proportional to the current and total resistance of the circuit at constant temperature

i.e., $\boxed{V = IR}$ where: V – voltage (V)
I – current (A)
R – Resistance (Ω)

→ The power absorbed by the resistor is given by, $P = VI = (IR)I$

$$\boxed{P = I^2R} \text{ watts.}$$

where I → current in the resistor in amps.
V → voltage across the resistor in volts.

→ Energy lost in a resistance in time 't' is given by,

$$W = \int_0^t P.dt = P.t = I^2R.t \text{ (or) } \frac{V^2}{R}.t$$

'W' is in Joules.

Inductance :- (L)

→ If current is made to pass through an inductor, an electromagnetic field is formed.
→ The unit of inductance is 'Henry'. It is denoted by 'H'
→ The symbol for inductance is ' '
→ The current - voltage relation is given by,

$$\boxed{V = L\frac{di}{dt}}$$

where V → is the voltage across inductor in volts.
i → is the current through inductor in amps.

$$V.dt = L\,di$$

$$\frac{V}{L}.dt = di$$

Integrating both sides, $\boxed{i = \frac{1}{L}\int V.dt}$

→ The power absorbed by Inductor is,

$$P = VI$$

$$P = \left[L \frac{di}{dt}\right] . I \quad \text{watts.}$$

→ The Energy stored by the Inductor is,

$$W = \int_0^t P\,dt = \int_0^t \left[L \frac{di}{dt}\right] . I\, dt$$

$$= L \int_0^t I\, di = \frac{1}{2} L I^2$$

$$\boxed{W = \frac{1}{2} L I^2} \quad \text{Jolues}$$

→ Inductor opposes the sudden change in current.

<u>Capacitance:-(C)</u>

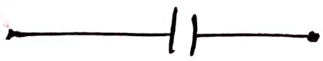
→ Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor.

→ A capacitor stores Energy in the form of "~~Eelectro~~ Electro Static field".

→ It is denoted by 'C'.

→ Units - Farads (F).

→ Symbol -



C

$$\boxed{i = C \frac{dv}{dt}}$$

where: V → is the voltage across capacitor

i → current through the capacitor.

$$i . dt = C\, dV$$

$$\frac{i}{C} . dt = dV$$

Integrating on both sides

$$\boxed{V = \frac{1}{C} \int i\, dt}$$

The power absorbed by the capacitor is given by,

$$P = VI = V C \frac{dV}{dt} \text{ Watts}$$

The energy stored by the capacitor is,

$$W = \int_0^t P\,dt = \int_0^t V C \frac{dV}{dt} \cdot dt$$

$$\boxed{W = \frac{1}{2} C V^2} \text{ Joules.}$$

→ Capacitor opposes the sudden change in Voltage.

Problems:-

1) A 10Ω resistor is connected across a 12 V battery. How much current flows through the resistor?

Sol: Given details:- $R = 10\Omega$

$V = 12V$

$I = ?$

Ohm's law, $V = IR$.

$12 = I(10) \Rightarrow I = \frac{12}{10} = 1.2\,A$

$$\boxed{I = 1.2A}$$

2) Find the inductance of a coil through which flows a current of 0.2A with an energy of 0.15 J.

Sol:- G.D: $I = 0.2A$.

$E = 0.15J$

$L = ?$

We know that $E = \frac{1}{2} L I^2$

$$0.15 = \frac{1}{2} L (0.2)^2$$

$$L = \frac{2 \times 0.15}{(0.2)^2}$$

$$\boxed{L = 7.5H}$$

3) A capacitor having a capacitance $2\mu F$ is charged to a voltage of 1000V. Calculate the stored energy in Joules.

Sol: G.D: $C = 2\mu F$

$V = 1000V$

$E = ?$

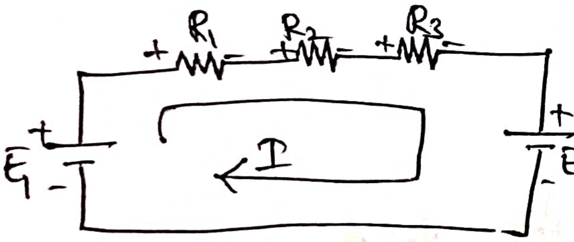
$$E = \frac{1}{2}CV^2 \Rightarrow \frac{1}{2} \times 2 \times 10^{-6} \times (1000)^2$$

$$\boxed{E = 1J}$$

Kirchhoff's laws:-

Kirchhoff's voltage law:-

Statement: The algebraic sum of all voltage drops and voltage raises in a closed loop in a circuit is equal to zero.



→ From Source 'E_1'

→ when current direction flows from '+' to '−' take it as voltage drop.

→ when current flows from '−'ve to '+'ve then it take as 'voltage raise'.

→ law of conservation of Energy.

+ to − → voltage drop = '−ve'
→ I

− to + → voltage raise = +ve
→ I

From the above circuit

$$-V_{R_1} - V_{R_2} - V_{R_3} - E_2 + E_1 = 0$$

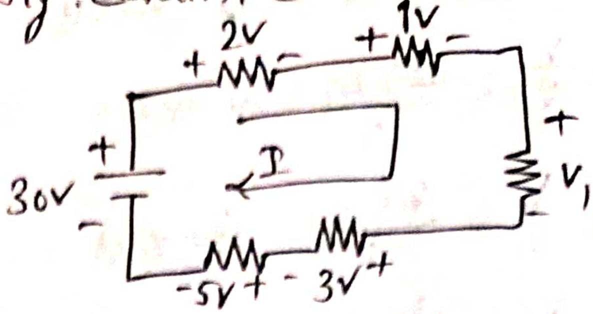
$$-IR_1 - IR_2 - IR_3 - E_2 + E_1 = 0$$

$$-I[R_1 + R_2 + R_3] - E_2 + E_1 = 0$$

$$E_1 - E_2 = I\{R_1 + R_2 + R_3\}$$

$$\boxed{I = \frac{E_1 - E_2}{R_1 + R_2 + R_3}} A$$

Problem:- For the circuit shown in fig. determine the unknown voltage drop V_1.



Sol:- Apply KVL,

$$-2-1-V_1-3-5V+30=0$$

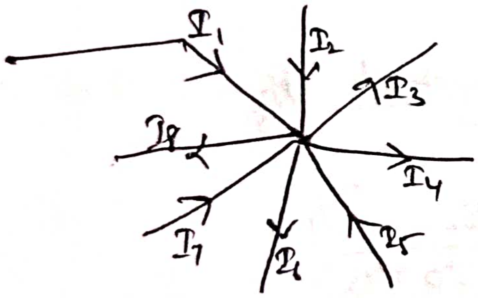
$$-11+30-V_1=0$$

$$19-V_1=0$$

$$\boxed{V_1=19V}$$

Kirchhoff's current law (KCL):-

Statement:- The alzebraic sum of entering current and the alzebraic sum of leaving currents at a node is equal to zero.



→ Entering current at a node is '+ve'

→ leaving currents at a node is '-ve'

Here entering currents = I_1, I_2, I_5, I_7

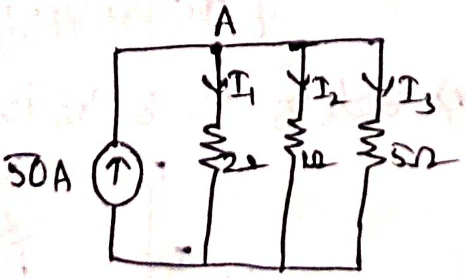
leaving currents = I_3, I_4, I_6, I_8

Apply KCL

$$I_1+I_2+I_5+I_7-I_3-I_4-I_6-I_8=0$$

→ law of conservation of charge

Problem:- Determine the current in all resistors in the circuit shown



Sol:- Apply KCL, at node A. $I=I_1+I_2+I_3$

where $I=50A$

$$50=\frac{V}{2}+\frac{V}{1}+\frac{V}{5}$$

$$\boxed{V=29.41V}$$

Find I_1, I_2, I_3 values.

$$I_1 = \frac{V}{2} = \frac{29.41}{2} = 14.705\,A$$

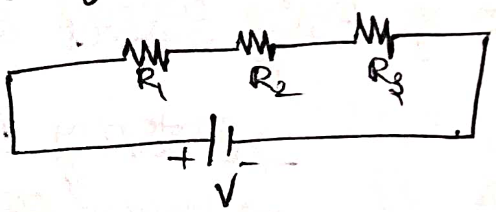
$$I_2 = \frac{V}{1} = \frac{29.41}{1} = 29.41\,A$$

$$I_3 = \frac{V}{5} = \frac{29.41}{5} = 5.882\,A$$

Voltage division Rule:-

→ The series circuit acts as a voltage divider. Since the same current flows through each resistor, the voltage drops are proportional to the values of resistors.

→ Take one circuit:



→ The total voltage is divided by V_{R_1}, V_{R_2} and V_{R_3}.

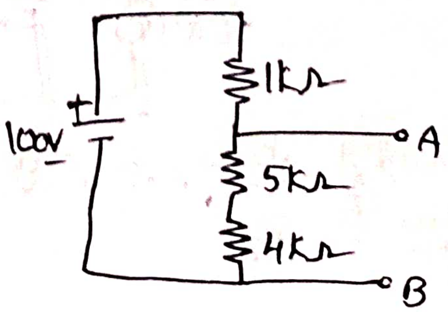
$$V = V_{R_1} + V_{R_2} + V_{R_3}$$

Voltage drop across $R_1 \Rightarrow V_{R_1} = V\left[\frac{R_1}{R_1+R_2+R_3}\right]$

Voltage drop across $R_2 \Rightarrow V_{R_2} = V\left[\frac{R_2}{R_1+R_2+R_3}\right]$

Voltage drop across $R_3 \Rightarrow V_{R_3} = V\left[\frac{R_3}{R_1+R_2+R_3}\right]$

Problem:- Find the voltage b/w A and B in a voltage divider network shown in fig.



Sol:- Apply voltage division Rule,

Between A & B → $9\,k\Omega$ present.

Voltage across AB, $V_{AB} = 100\left[\frac{9}{10}\right]$

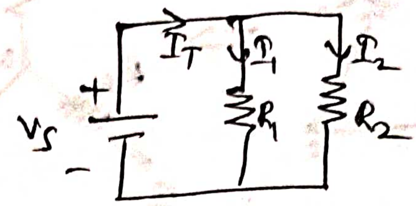
$$\boxed{V_{AB} = 90V}$$

Current division Rule:-

→ The parallel circuit acts as a current division, Since the current is divided in parallel but the voltages are same.

Take one circuit

current division Rule is apply

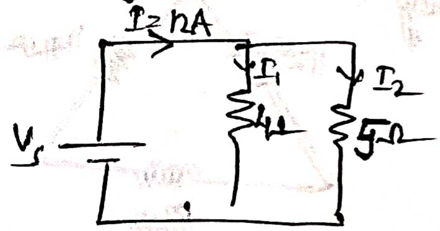


current passing through the $R_1 \Rightarrow I_{R_1} = I\left[\frac{R_2}{R_1+R_2}\right]$

current passing through the $R_2 \Rightarrow I_{R_2} = I\left[\frac{R_1}{R_1+R_2}\right]$

Problem:- Determine the current passing through each resistor in circuit

Sol:- Apply. Current division Rule



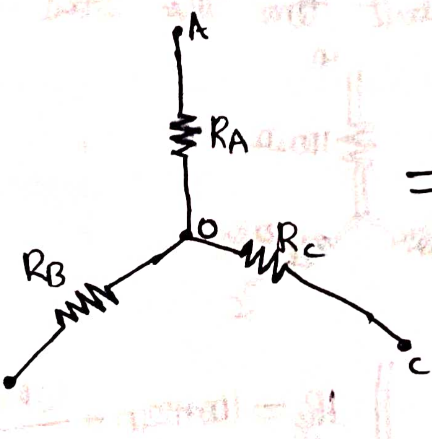
current passing through 4Ω

$$I_{4\Omega} = 12\left[\frac{5}{5+4}\right] = \frac{60}{9}\,A$$

current passing through 5Ω, $I_{5\Omega} = 12\left[\frac{4}{4+5}\right] = \frac{48}{9}\,A$

$$\boxed{I_1 = \frac{60}{9}\,A,\quad I_2 = \frac{48}{9}\,A}$$

Star-Delta Transformation:-



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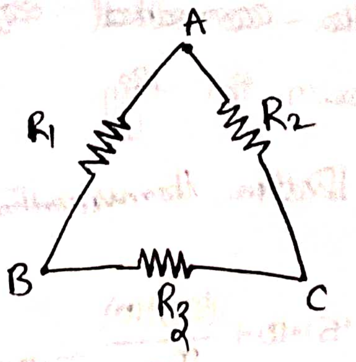


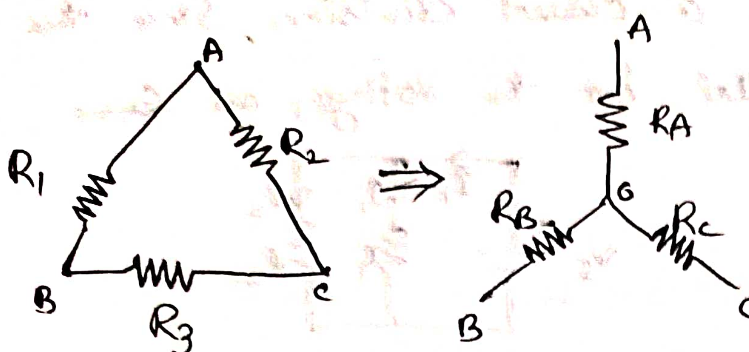
$R_1 = ?,\ R_2 = ?,\ R_3 = ?$

$$R_1 = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_2 = R_A + R_C + \frac{R_A R_C}{R_B}$$

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

Delta - Star Transformation:-



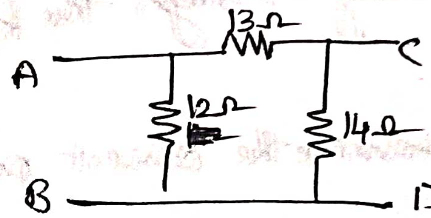
$R_A = ?$, $R_B = ?$, $R_C = ?$

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

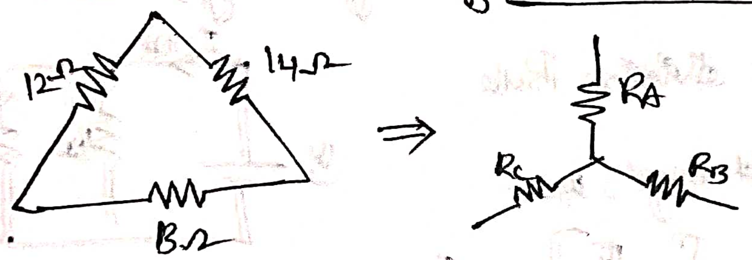
$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Problem:- Obtain the star connected equivalent for the delta connected circuit shown in fig.



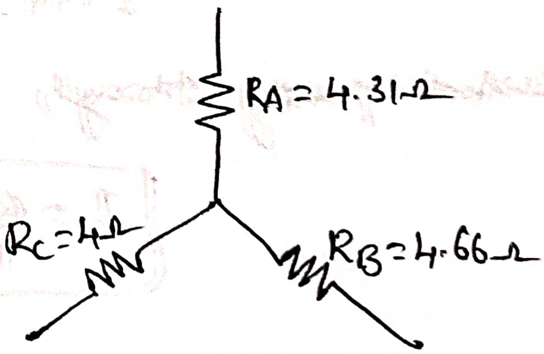
Sol:- The above circuit can be replaced by



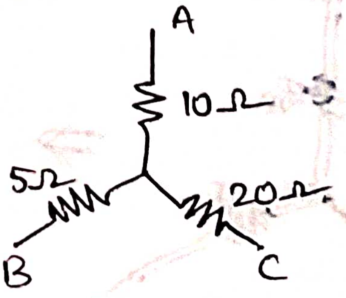
where
$$R_A = \frac{(12)(14)}{12+13+14} = 4.31\,\Omega$$

$$R_B = \frac{(14)(13)}{12+13+14} = 4.66\,\Omega$$

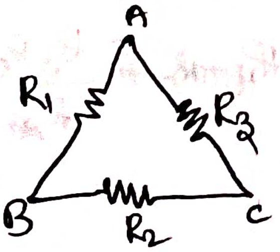
$$R_C = \frac{(12)(13)}{12+13+14} = 4\,\Omega.$$



Problem:- Obtain the delta-connected equivalent for the star-connected circuit shown in fig.



Sol:- Performing star to Delta transformation



$$R_1 = 5 + 10 + \frac{(5)(10)}{20} = 17.5\,\Omega$$

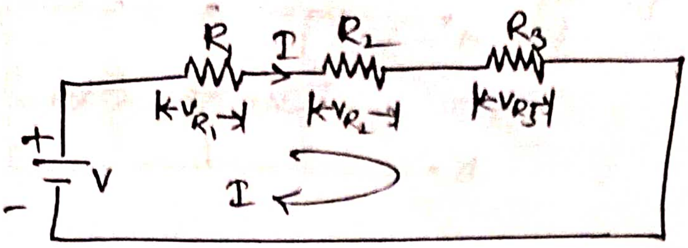
$$R_2 = 5 + 20 + \frac{(5)(20)}{10} = 35\,\Omega$$

$$R_3 = 10 + 20 + \frac{(10)(20)}{5}$$

$$\boxed{R_3 = 70\,\Omega}$$

Series and parallel connection of resistances with DC excitation:

Series:-



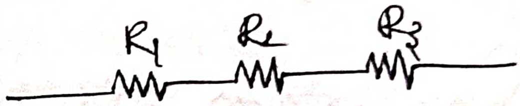
Let the three pure resistances R_1, R_2 and R_3 be connected in Series against a D.C voltage source V. as shown in fig.

Apply KVL: $V = V_{R_1} + V_{R_2} + V_{R_3}$

$$IR = IR_1 + IR_2 + IR_3$$

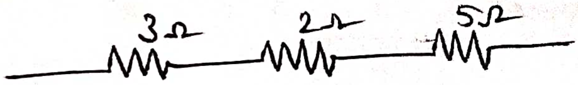
$$\cancel{I}R = \cancel{I}[R_1 + R_2 + R_3]$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$



$R_{eq} = R_1 + R_2 + R_3$

Ex:-



Find $R_{eq} = ?$

$$R_{eq} = 3 + 2 + 5 = 10\Omega$$

In Series connection current is same
Voltage are different.

Parallel:-

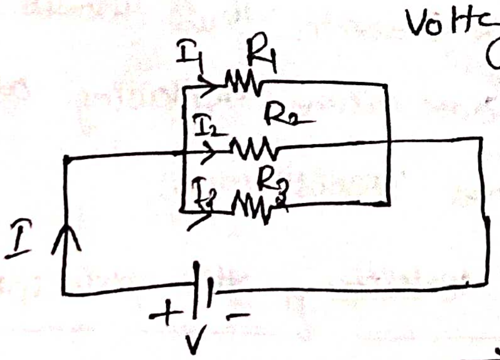


Fig. Shows the Connection of three resistors in parallel across a dc. Voltage Source V.

→ Here current will be devided and voltage will be same.

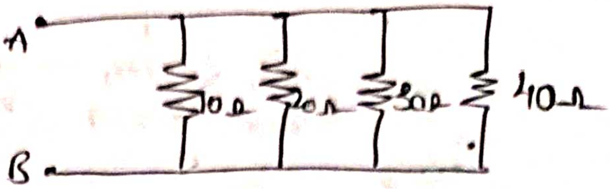
Apply KCL: $I = I_1 + I_2 + I_3$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\cancel{V}\left[\frac{1}{R}\right] = \cancel{V}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\boxed{\frac{1}{R_{eq}} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]}$$

Problem:- Determine the parallel resistance between points A and B of the circuit shown in fig.



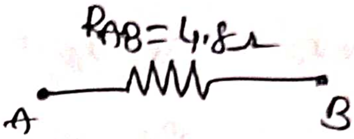
Sol:- $R_{AB} = ?$

Parallel connection,

$$\frac{1}{R_{AB}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$$

$$\frac{1}{R_{AB}} = 0.1 + 0.05 + 0.033 + 0.025$$

$$\frac{1}{R_{AB}} = 0.208 \;, \quad \boxed{R_{AB} = 4.8\,\Omega}$$



Superposition theorem:-

This theorem finds use in solving a network where two or more sources are present.

Statement of Superposition theorem:-

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Steps for solution of a Network utilizing the principle of Superposition:-

Step1:- Take only one independent source of voltage / current and deactivate the other independent voltage / current sources.

a) Remove the voltage source with short circuit their respective circuit terminals.

b) Remove current source with open circuit the

Obtain branch currents.

Step 2: Repeat the above step for each of the independent sources

Step 3: * To determine the net branch current utilizing superposition theorem.

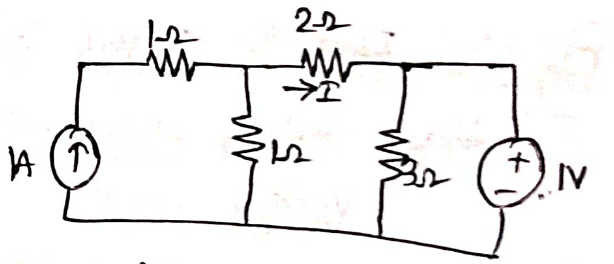
* Just add the currents obtained in step 1 & step 2 for each branch.

* If the currents obtained in step 1 and step 2 are in same direction, just add them.

* If the currents obtained in opposite direction, the direction of the current is clockwise take +ve, the direction of the current is anticlockwise take −ve,

* Find net current in branch obtained.

<u>Problem</u>: Find 'I' in the circuit shown using superposition theorem.

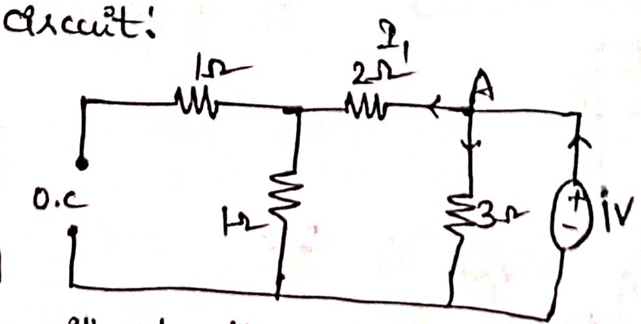


<u>Sol</u>:- <u>Step 1</u>:- To find the current in 2Ω resistance,

First we take voltage source → active and all other sources are set to zero. ie; current source − open circuit

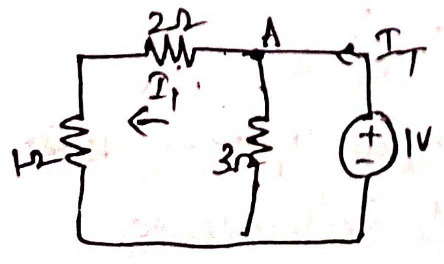
Redraw the circuit:

[Assume current flowing in 2Ω is 'I_1' in step 1]



In 1Ω current will not flows because of O.C connection.

Redraw;



At node 'A' V is 1V because no resistance is connected in b/w node A and voltage source.

Applying nodal Analysis,

Find total current from the voltage source,

$$I_T = \frac{1V}{\{(1+2)\|3\}} = \frac{1}{1.5}$$

$$\boxed{I_T = \frac{1}{1.5}\,A}$$

Current passing through '2Ω' is given by

$$I_1 = I_T\left[\frac{3}{3+3}\right]$$

$$= \frac{1}{1.5}\left[\frac{3}{6}\right] = \frac{1}{3}\,A.$$

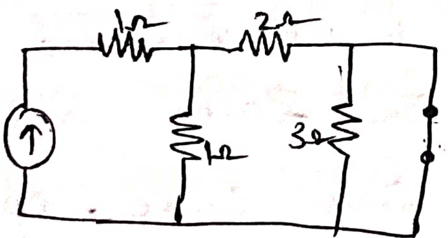
Current passing through '2Ω' is $\boxed{I_{2\Omega} = \frac{1}{3}\,A.}$

Step 2:- Find the current in '2Ω'

Here current source - active

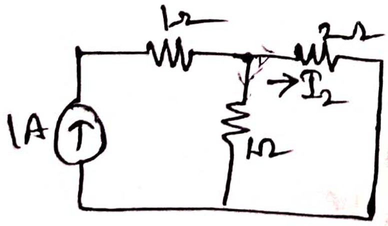
Voltage source - Short circuited.

Redraw:-



Here current will not flows through the '3Ω' because S.C. is present.

Redraw:



Find current flowing through the '2Ω'

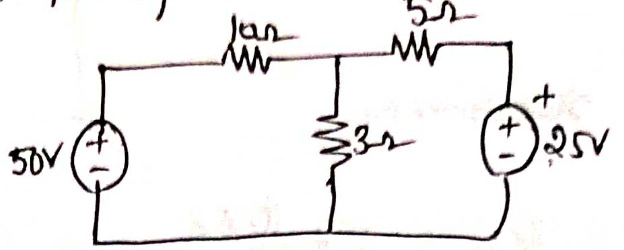
Using current division Rule

$$I_2 = 1\left[\frac{1}{1+2}\right] = \frac{1}{3}\,\Omega$$

Step 3:- Here I_2 values takes +ve and I_1 value takes as -ve because the current direction is anticlockwise.

Total net current passing through the ' ... ' is $= I_1 + I_2 = -\frac{1}{3} + \frac{1}{3} = 0$

Problem Q: For the resistive network shown in fig, Find the current in each resistor, using the superposition principle.

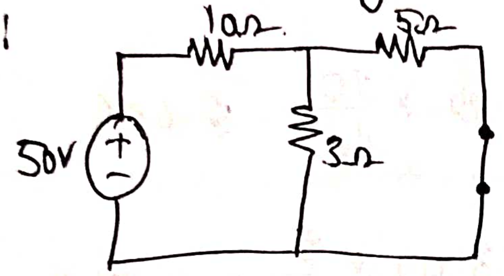


Sol:-

Step1: 50V → voltage source is active
25V → voltage source is short circuited.

Find the currents flowing through all the resistors.

Redraw!



To find out Total current we find the Req.

$$5 \| 3 = \frac{(5)(3)}{5+3} = \frac{15}{8}\Omega$$

→ $\frac{15}{8}\Omega$ is series with the 10Ω, $R_{eq} = 10 + \frac{15}{8} = \frac{80+15}{8}$

$$= \frac{95}{8} = 11.9\Omega.$$

$$\boxed{R_{eq} = 11.9\Omega}$$

current passing through the 10Ω,

$$I_{10} = \frac{50}{11.9} = 4.2A.$$

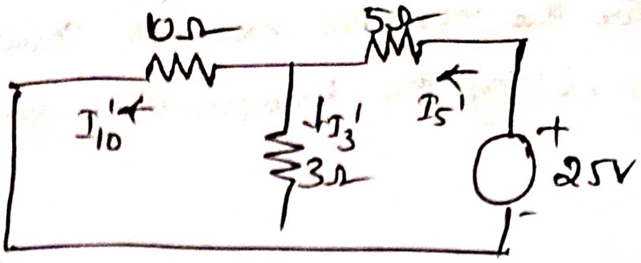
current passing through the 3Ω, $I_3 = 4.2 \times \frac{5}{8} = 2.63A$

current passing through the 5Ω, $I_5 = 4.2 \times \frac{3}{8} = 1.58A$.

Step2:- 25V → voltage source is active.
50V → voltage source is short circuited.

Find the currents flowing through all the resistors.

Redraw the circuit :-



Total resistance,

$$R_T = 5 + \frac{10 \times 3}{13} = 7.31\,\Omega$$

Current in the 5Ω resistor, $I_5' = \frac{25}{7.31} = 3.42\,A$

current in the 3Ω, $I_3' = 3.42 \times \frac{10}{13} = 2.63\,A$

current in the 10Ω, $I_{10}' = 3.42 \times \frac{3}{13} = 0.79\,A$

According to the superposition theorem,

current in the 10Ω $= I_{10} - I_{10}'$

$= 4.2 - 0.79$

$= \underline{3.41A}$

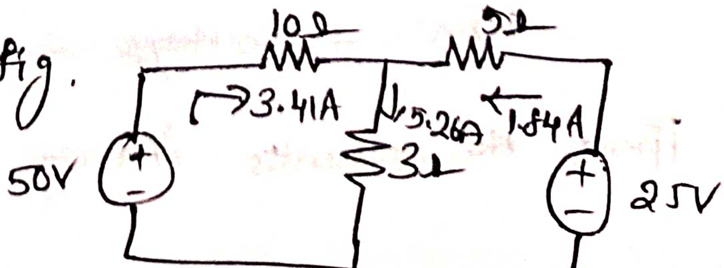
Current in the 3Ω $= I_3 + I_3' = 2.63 + 2.63$

$= 5.26\,A$

Current in the 5Ω $= I_5' - I_5 = 3.42 - 1.58$

$= 1.84\,A.$

When both sources are operative, the directions of the currents are shown in fig.



Representation of Sinusoidal waveforms:-

Sine wave:-

→ The Sinusoidal wave is generally refered to as a Sinewave

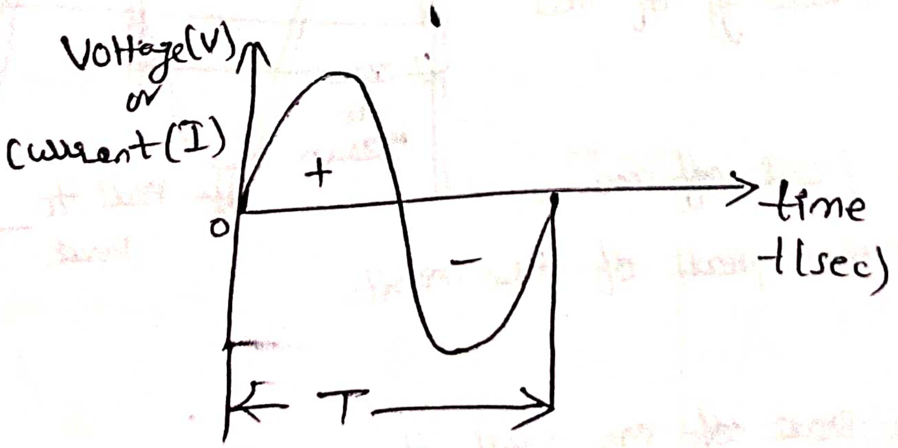
→ Basically an alternating voltage or current waveform is defined as the voltage or current that fluctuates with time periodically, with change in polarity and direction.

→ In general, the wave of sine is more useful than other waveforms, like pulse, sawtooth, square, triangular wave etc.

→ There are a number of reasons for this,

* If we take any second order system, the response of this system is a sinusoid.
* Sinusoidal functions is easy to generate.
* Sinusoidal functions is easy to analyse.
* Its derivatives and integrals are also sinusoids.
* It is more useful in the power industry.

→ The shape of a sinusoidal waveforms is shown in fig.



→ The waveform may be either a current waveforms, or a voltage waveform.

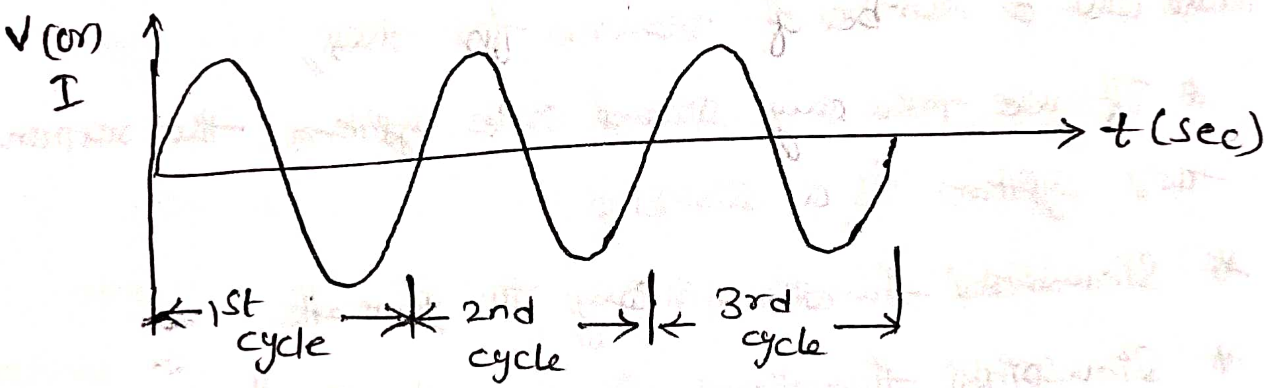
→ As seen from figure, the wave changes its magnitude and direction with time.

→ If we start at time t=0, the wave goes to a maximum value and returns to zero, and then decreases to a negative maximum value and before returning to zero.

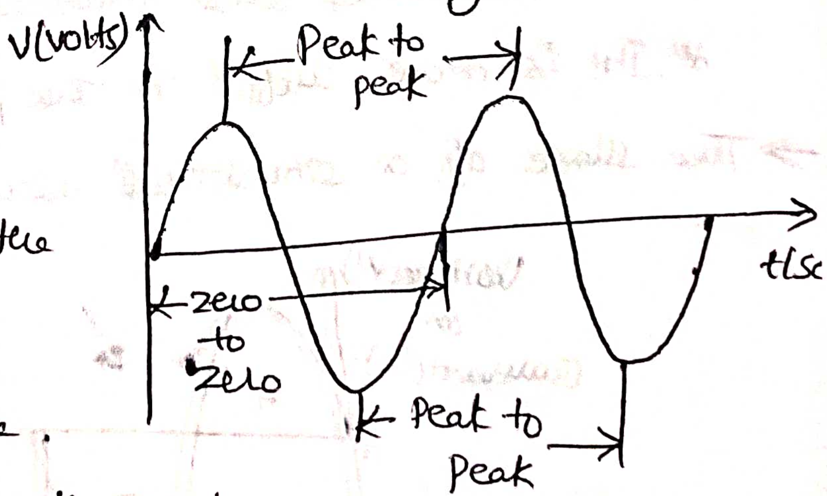
→ The complete positive and negative portion of the wave is one cycle of the sine wave.

→ Time is designated by 't'.

→ The time taken for any wave to complete one full cycle is called the period (T).



The period can be measured in the following different ways,



* From zero crossing of one cycle to zero crossing of the next cycle.
* From positive peak of one cycle to positive peak of the next cycle. and
* From Negative peak of one cycle to negative peak of the next cycle.

Frequency:-
The frequency of a wave is defined as the number of cycles that a wave completes in one second.

Ex:- 60 HZ → 60 cycles per one second

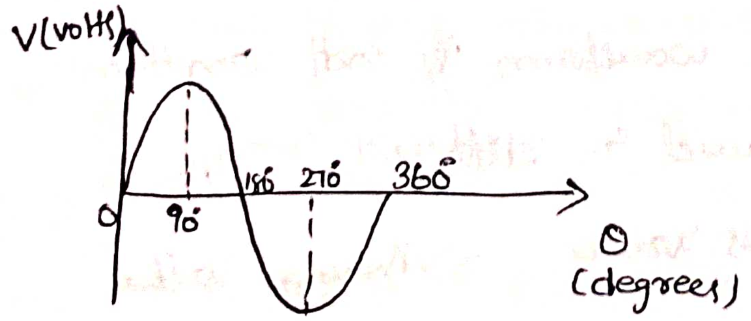
3 HZ → 3 cycles per one seccond.

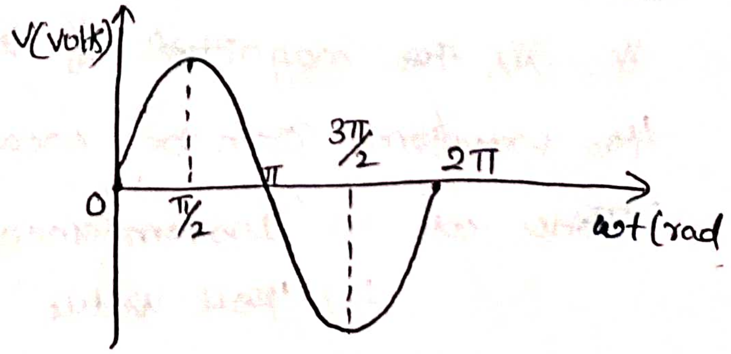
The relation between time period and frequency is given by

$$\boxed{f = \frac{1}{T}}$$

Angular Relation of a sine wave:-

* A sine wave can be measured along the x-axis on a time base which is frequency - dependent.
* A sine wave can also be expressed in terms of an angular measurement.
* This angular measurement is expressed in degrees or radians.
* One radian is equal to 57.3°
* The angular measurement of a sine wave is based on 360° or 2π radians for a complete cycle.
* A sine wave completes a half cycles in 180° or π radian A quarter cycle in 90° or π/2 radians.



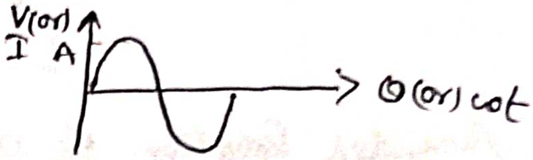


Sine wave equation:-

The amplitude of a sine wave is represented on vertical axis. The angular measurement is represented on horizontal axis. Amplitude A is the max. value of the voltage or current on the y-axis.

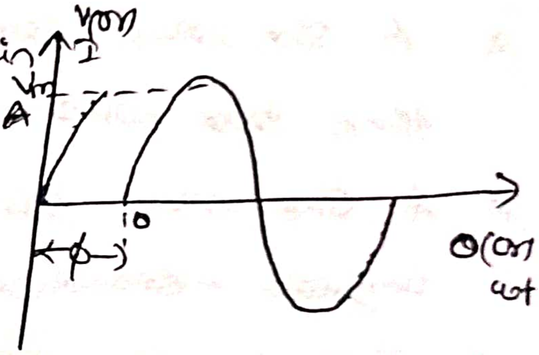
* In general, the sine wave is represented by the equation,

$$V(t) = V_m \sin \omega t$$



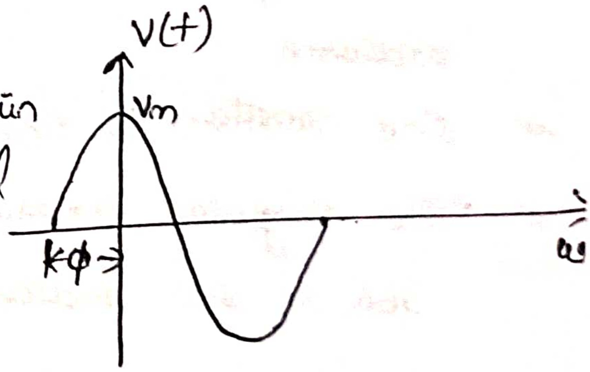
* When a sine wave is shifted to the right of the reference wave by a certain angle ϕ, shown in fig.
The general expression is

$$V(t) = V_m \sin(\omega t - \phi)$$



* When a sine wave is shifted to the left of the reference wave by a certain angle ϕ, as shown in fig., the general expression can be written as

$$V(t) = V_m \sin(\omega t + \phi)$$

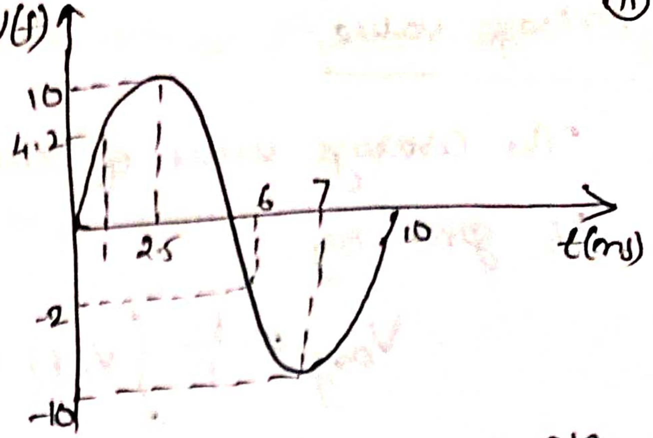


Voltage and current values of a sine wave:-

* As the magnitude of the waveform is not constant, the waveform can be measured in different ways.

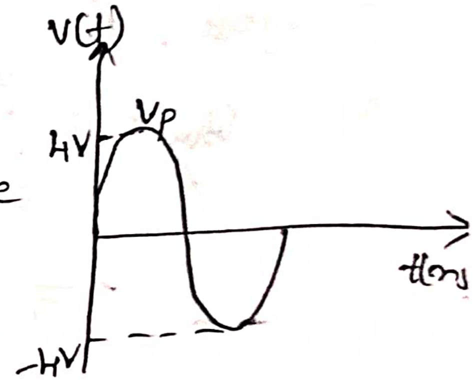
These are
1) Instantaneous value
2) Peak value
3) Peak to peak value
4) Root mean square
5) Average value.
6) Form factor
7) Power factor

Instantaneous value :-



* At any given time, it has some instantaneous value.
* This value is different at different points along the waveform.
* During the positive cycle, the instantaneous values are positive. During the -ve half cycle, the instantaneous values are Negative.
* Ex:- 1ms, the value is 4.2V. ; 6ms, the value is -2V
  2.5ms, the value is 10V ; 7ms, the value is -10V.

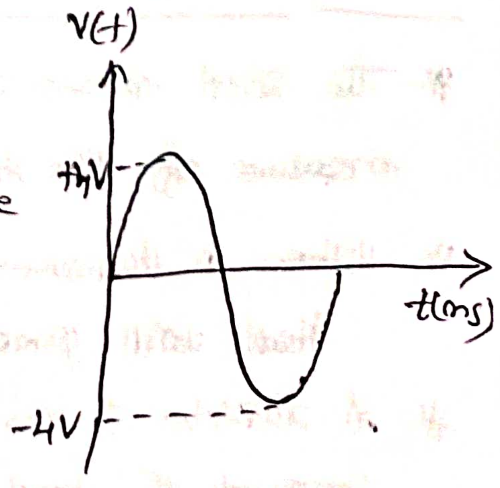
Peak value :-



* The peak value of the sine wave is the maximum value of the wave during positive half cycle, (or)
  max. value of wave during negative half cycle.
* The value of these two are equal in magnitude, a sine wave is characterised by a single peak value.

  Ex:- The peak value of the sine wave is shown. Here the peak value of the sine wave is 4V.

Peak to peak Value :-



* The peak to peak value of a sine wave is the value from the positive to the negative peak as shown in fig.

  Here the peak to peak value is 8V

Average value:-

The average value of any function v(t), with period 'T' is given by,

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\, dt$$

Here $v(t) = V_p \sin \omega t$

* The average value of a sine wave over one complete cycle is always zero.
* So, the avg. value of a sine wave is defined over a half cycle, is

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_p \sin \omega t \; d(\omega t)$$

$$= \frac{1}{\pi}\left[-V_p \cos \omega t\right]_0^{\pi} \Rightarrow \frac{1}{\pi}\left[(-V_p \cos \pi) - (-V_p \cos 0)\right]$$

$$= \frac{1}{\pi}\left[-V_p(-1) - (-V_p(1))\right]$$

$$V_{avg} = \frac{2V_p}{\pi}$$

$$\boxed{V_{avg} = 0.637\, V_p}$$

Root mean square value:-

* The root mean square (RMS) value of a sine wave is a measure of the heating effect of the wave.
* When a Resistance is connected across a dc voltage source, Heat will produced.
* A resistor is connected across an ac voltage source, the same amount of heat is produced.

This value is called "RMS value"

* The rms value of any function with period T has an effective value is given by,

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt} \qquad \text{Here } v(t) = V_p \sin\omega t$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_p \sin\omega t)^2\,d\omega t}$$

$$= \sqrt{\frac{V_p^2}{2\pi}\int_0^{2\pi} (\sin\omega t)^2\,d\omega t} = \sqrt{\frac{V_p^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t}$$

$$= \frac{V_p}{\sqrt{2}} = 0.707\,V_p \qquad = \sqrt{\frac{V_p^2}{4\pi}\left[2\pi - 0\right]}$$

$$\boxed{V_{rms} = 0.707\,V_p} \qquad = \sqrt{\frac{V_p^2}{4\pi} \times 2\pi} = \frac{\sqrt{V_p^2}}{\sqrt{2}}$$

$$\boxed{V_{rms} = \frac{V_p}{\sqrt{2}}}$$

Peak factor:-

* The peak factor of any waveform is defined as the ratio of the peak value of the wave to the rms value of the wave.

$$\text{Peak factor} = \frac{V_p}{V_{rms}}$$

$$\text{Peak factor of the sinusoidal waveform} = \frac{V_p}{V_p/\sqrt{2}} = \sqrt{2}$$

$$\boxed{\text{Peak factor} = 1.414}$$

Form Factor:-

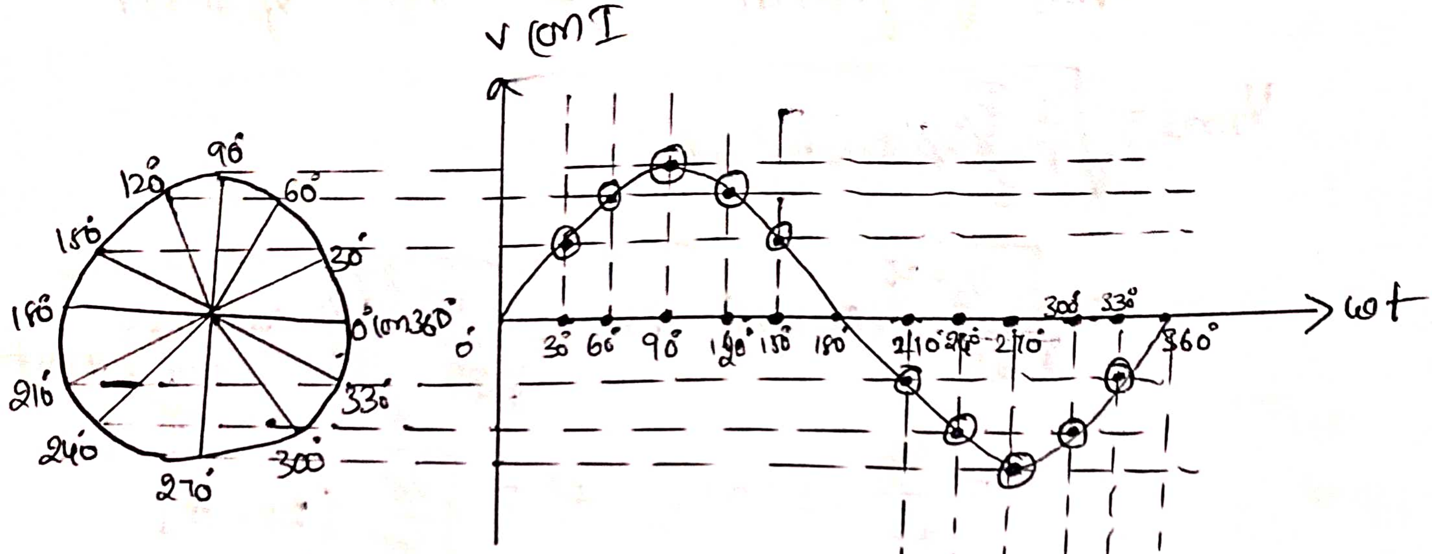
Form Factor of a waveform is defined as the ratio of rms value to the average value of the wave.

$$\text{Form Factor} = \frac{V_{rms}}{V_{avg.}}$$

$$\text{Form factor of the sinusoidal waveform} = \frac{V_p/\sqrt{2}}{0.637\,V_p} = 1.11$$

$$\boxed{\text{Form Factor} = 1.11}$$

Phasor Representation:-



Rotating Phasor

Sinusoidal waveform in the time domain.

Power:-

DC power:- $\boxed{P_{dc} = V_L I_L}$ watts.

AC power:- $\boxed{P_{ac} = V_L I_L \cos\phi}$ watts.

Real power:-

The power which is actually consumed or utilised in an AC circuit is called "Real power".

It is also known as "True power" or "average power" or "Active power".

* It is measured in "watts"

$$\boxed{P_{avg} = VI\cos\phi}$$

Reactive power:-

The power which flows back and forth that means it moves in both the directions in the circuit or reacts upon it self, is called "Reactive power".

* The reactive power is measured in "KVAR"

KVAR → kilo Volt-ampere reactive.

$\boxed{P_{reactive} = VI\sin\phi}$ (or Q)

Apparent power:-

The product of root mean square (RMS) value of voltage and current is known as "Apparent power".

This power is measured in KVA

KVA - kilo volt ampe...

$$\boxed{P_a \text{ (or) } S = VI}$$

Power factor:-

Power factor is the ratio of working power, measured in kilowatts (KW), to apparent power, measured in KVA.

* power factor is an expression of energy efficiency.

$$P.F = \frac{\text{True power}}{\text{apparent power}}$$

$$= \frac{\cancel{VI}\cos\phi}{\cancel{VI}}$$

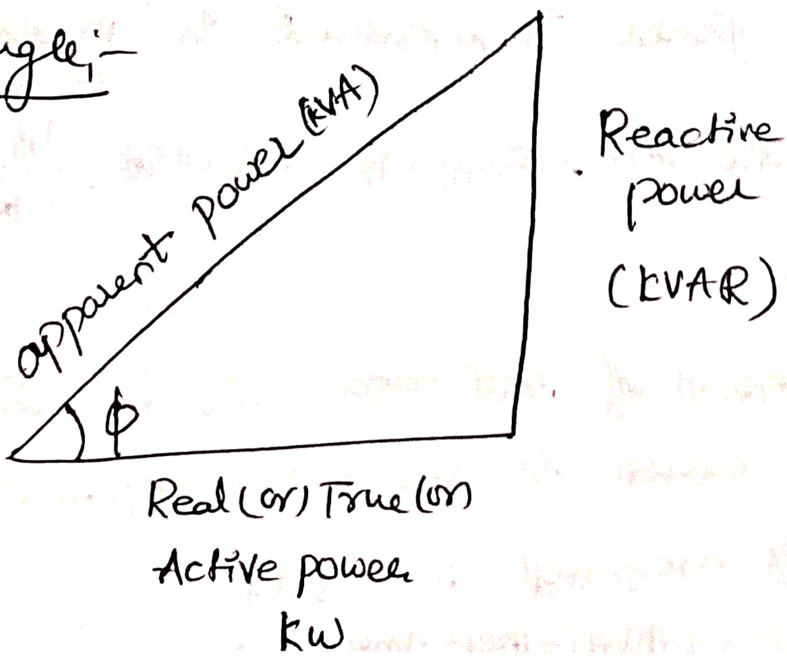
$$\boxed{P.f = \cos\phi}$$

* In a Electrical circuit, Pf is defined as the ratio of 'R' to the Impedance (Z).

$$\boxed{\cos\phi = \frac{R}{Z}}$$

* The cosine angle b/w voltage & current is also called as "power factor".

Power triangle:-



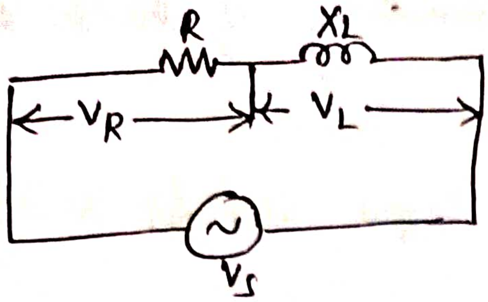
Analyses of single-phase ac circuits consisting of RL-RC-RLC :-

Series circuits:-

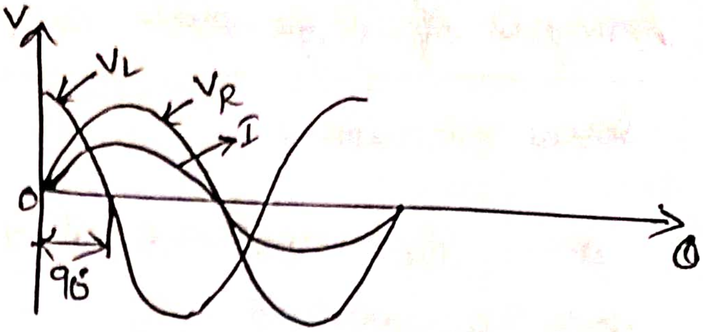
* The impedance diagram is a useful tool for analysing series ac circuit.
* Basically, we can divide the series circuits as RL, RC & RLC circuits.
* In the analysis of series ac circuit, one must draw the impedance diagram.

Series RL circuit:- fig (a)

* If we apply a sinusoidal input to an RL circuit, the current in the circuit and all voltages across the elements are sinusoidal.
* In the analysis of the RL series circuit, we can find the impedance, current, phase angle and voltage drops.
* The resistor voltage (V_R) and current (I) are inphase with each other, but lag behind the voltage source (V_s).
* The inductor voltage (V_L) leads the voltage source (V_s).
* The phase angle b/w current and voltage in pure inductor is always 90°.
* The amplitude of voltages and currents in the circuit are completely dependent on the values of elements (ie., R & X_L).
* The phase angle is in between zero and 90° because of the series combination of RL, which depends on the relative values of R and X_L.
* The phase relation between current and voltages in a series RL circuit. is shown in fig (b)

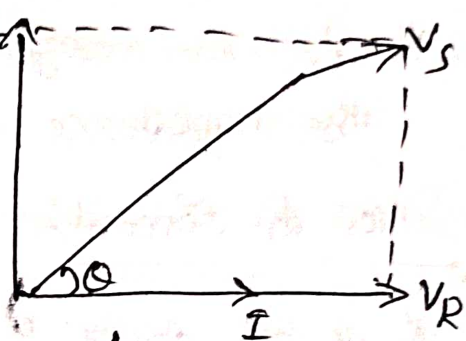


fig(a).



fig(b).

* The phasor diagram for the series RL circuit is shown in fig(c).
* Here V_R & I are inphase.
* current (I) lags inductor voltage (V_L) by 90°.



fig(c).

* From the KVL,

the sum of the voltage drops must equal to the applied voltage.

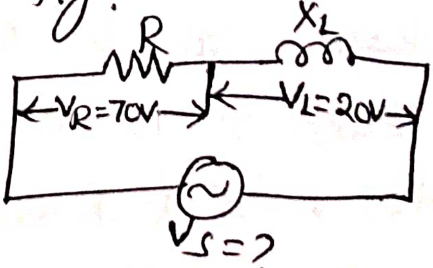
The phasor sum of V_R & V_L is the source voltage (V_s).

$$V_s^2 = V_R^2 + V_L^2$$

$$\boxed{V_s = \sqrt{V_R^2 + V_L^2}}$$

The phase angle between resistor voltage and source voltage (V_s) is $\theta = \tan^{-1}\left[\frac{V_L}{V_R}\right]$

P-1] Determine the source voltage and the phase angle, if $V_R = 70V$, $V_L = 20V$. as shown in fig.



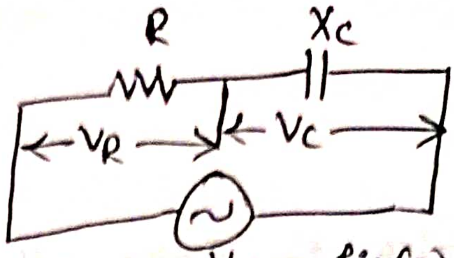
Sol:- The source voltage is given by,

$$V_s = \sqrt{V_R^2 + V_L^2} = \sqrt{(70)^2 + (20)^2}$$

$$V_s = 72.8\,V.$$

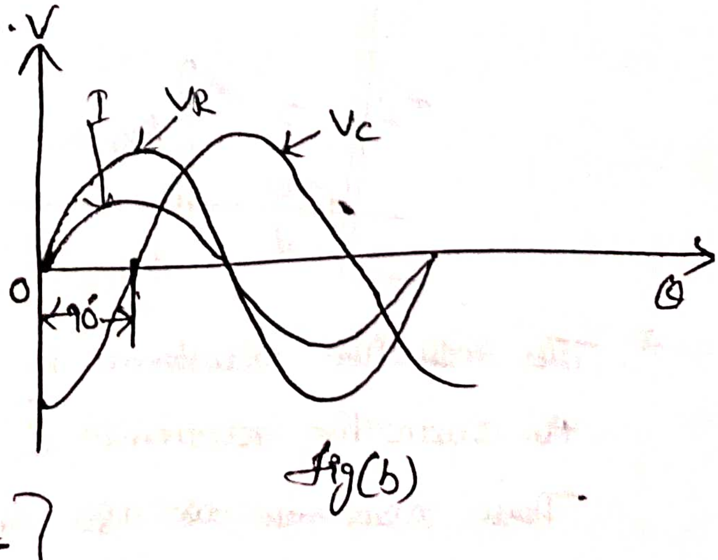
The angle between current and source voltage is

Series RC circuit :-



fig(a).

* when a sinusoidal voltage is applied to an RC series circuit, the current in the circuit and voltages across each of the elements are sinusoidal.
* The series RC circuit is shown in fig(a).
* The resistor voltage (V_R) and current are inphase with eachother
* The capacitor voltage (V_C) lags the current by 90°.
* The amplitude and the phase relations between the voltages & current depend on the ohmic values of the resistance & the capacitive reactance (X_C).
* The phase angle between the applied voltage and the total current is in b/w zero & 90°. depending on the relative values of the resistance and reactance.
* In series RC circuit, the current is the same through the resistor and the capacitor.
* Thus, the voltage of resistance (V_R) is inphase with the current, and the capacitor voltage (V_C) lags the current by 90°. as shown in fig(b).
* The phasor diagram of RC series circuit is shown in fig(c).



fig(b)

$$\boxed{V_S = \sqrt{V_R^2 + V_C^2}}$$

Phase angle is, $\theta = \tan^{-1}\left(\frac{V_C}{V_R}\right)$

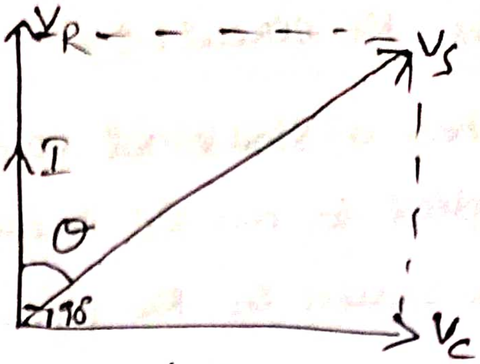
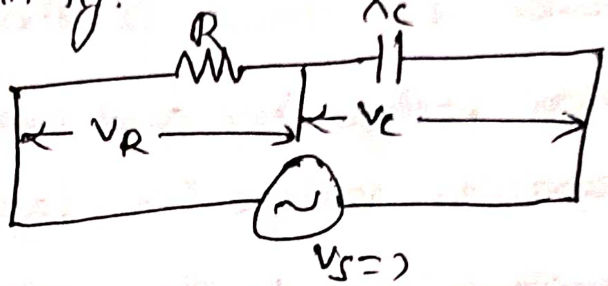


fig (c).

P-2) Determine the source voltage & phase angle when $V_R = 20V$ and the $V_C = 30\,V$ as shown in fig.



Sol:- The source voltage, V_S

$$V_S = \sqrt{V_R^2 + V_C^2}$$

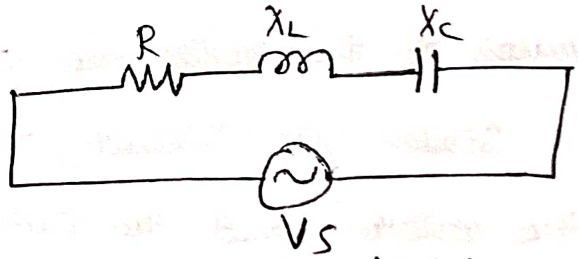
$$= \sqrt{(20)^2 + (30)^2}$$

$$\boxed{V_S = 36V}$$

The angle b/w the current and source voltage is

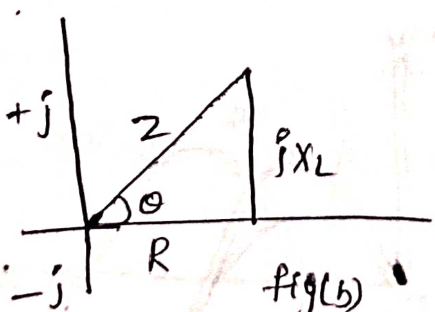
$$\theta = \tan^{-1}\left[\frac{V_C}{V_R}\right] = \tan^{-1}\left[\frac{30}{20}\right] = 56.3^\circ$$

Series RLC circuit:-

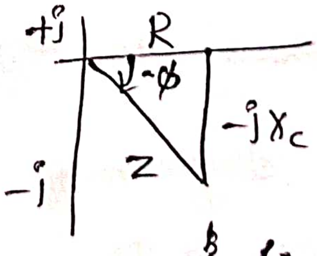


fig(a).

* The series RLC circuit is shown in fig (a).
* The impedance diagram of series RL, and series RC circuit as shown in fig. below. fig(b) & fig(c).



fig(b)



fig(c)

* The inductive reactance X_L, is displayed on the $+j$ axis and the capacitive reactance X_C, is displayed on the $-j$ axis. These reactance are 180° apart and tend to cancel each other.

* The magnitude and type of reactance in a series 'RLC' circuit is the difference of the two reactances.

* The impedance for an RLC series circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

The phase angle for an RLC circuit is,

$$\boxed{\theta = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]}.$$

Resonance:-

1) Series Resonance
2) Parallel Resonance.

1) Series Resonance:-

* Resonance is a very important phenomenon in Electrical circuits.
* The study of resonance is very useful, particularly in the area of communications.
* For example,

The ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequen from other stations is based on the principle of resonance.

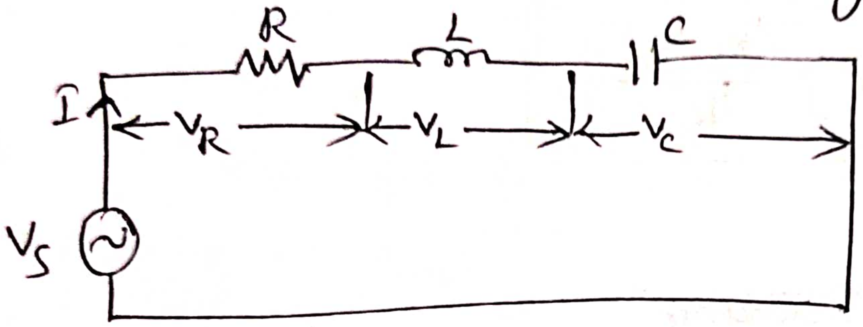
* In a series RLC circuit, the current lags behind, or leads the applied voltage depending up on the values of X_L and X_C.
* 'X_L' causes the total current to lag behind the applied voltage while 'X_C' causes the total current to lead the applied voltage.
* While $X_L > X_C$, the circuit is predominantly inductive, when $X_C > X_L$, the circuit is predominantly capacitive.

* If one of the parameters of the series RLC circuit is varied in a such a way that the current in the circuit is inphase with the applied voltage,

↓

then the circuit is said to be in resonance.

* The series RLC circuit shown in fig.



* The total impedance for the series RLC circuit is,

$$Z = R + j(X_L - X_C)$$

$$= R + j\left[\omega L - \frac{1}{\omega C}\right]$$

* It is clear from the circuit that the current, $I = \frac{V_S}{Z}$.
* In a series RLC circuit, series resonance occurs when $\boxed{X_L = X_C}$.
* The frequency at which the resonance occurs is called the "Resonant frequency".
* At the resonant frequency, f_r, the voltage across capacitance and inductance are equal in magnitude.
* They are 180° out of phase with each other, they cancel each other and, hence zero voltage appears across the LC combination.

At resonance, $X_L = X_C$ ie, $\omega L = \frac{1}{\omega C}$

solving for resonant frequency, we get

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

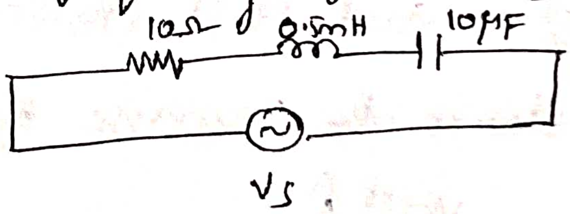
* In a Series RLC circuit, resonance may be produced by varying the frequency, keeping 'L' and 'C' constant.

Otherwise,

resonance may be produced by varying either L or C for a fixed freq.

P1

Determine the resonant frequency for the circuit shown in fig.



Sol:- The resonant frequency

is, $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5\times10^{-3}\times10\times10^{-6}}}$

$$\boxed{f_r = 2.25\ KHz}$$

Impedance of a series resonant circuit:-

The impedance of a series RLC circuit is

$$Z = R + j(X_L - X_C)$$

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

magnitude of Impedance is,

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

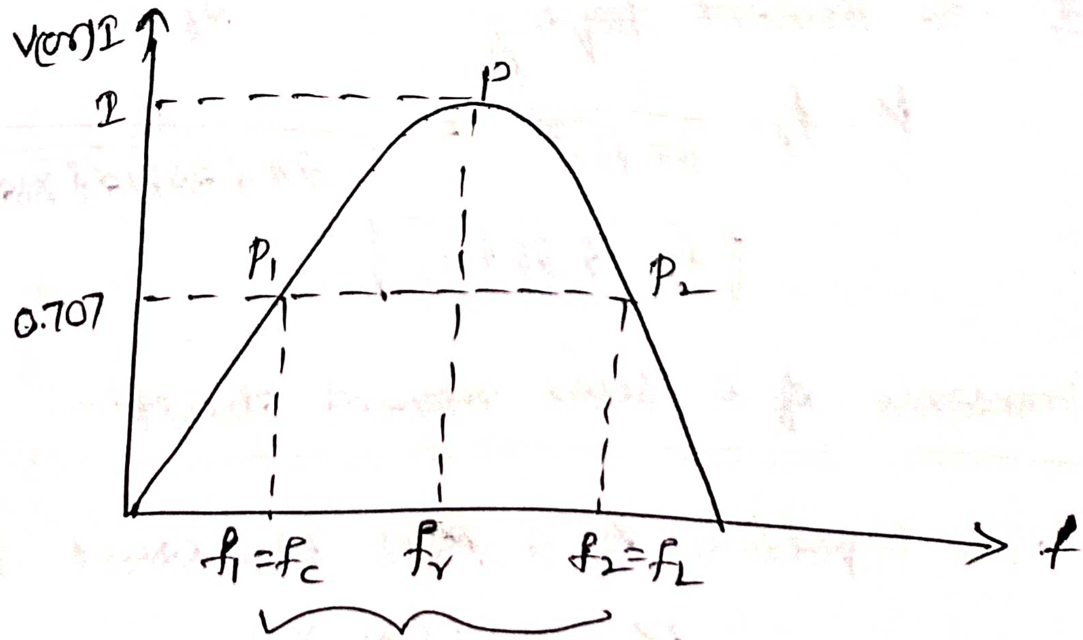
phase angle, $\phi = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$

* Inductance frequency, $f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \frac{R^2 C}{2L}}}$

* capacitance frequency, $f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$

Bandwidth of an RLC circuit:-

* The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by BW.
* figure shows the response of a series RLC circuit.



* $f_1 \rightarrow$ lower cut off frequency
  $f_2 \rightarrow$ upper cut off frequency

* The bandwidth (or) BW, is defined as the frequency difference between f_2 and f_1

$$\boxed{BW = f_2 - f_1}$$

* The unit of BW is hertz (Hz).
* In series circuit, the quality factor, $\boxed{Q = \frac{X_L}{R} \text{ (or) } \frac{X_C}{R}}$

* The relation b/w bandwidth and quality factor,

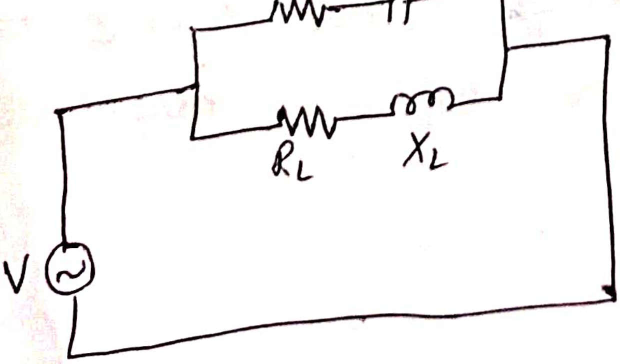
$$\boxed{Q = \frac{f_r}{BW}}$$

magnification:-

The ratio of voltage across either L or C to the voltage applied at resonance can be defined as magnification.

magnification $\Rightarrow \boxed{Q = \frac{V_L}{V} \text{ or } \frac{V_C}{V}}$

Parallel resonance:-



frequency at resonance,

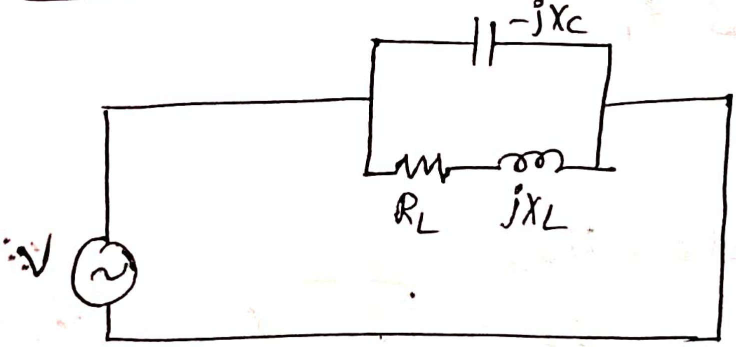
$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

In parallel connection,

Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{\ }}\sqrt{\frac{R_L^2 - (L/C)}{\ }}$

Resonant frequency for a tank circuit :-



The resonant frequency for the tank circuit is,

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}}$$

Quantity factor of parallel resonance :-

* In parallel circuit, the quality factor, $Q = \frac{R}{X_L}$ (or) $\frac{R}{X_C}$

$$\boxed{Q = \frac{R}{\omega L} \text{ (or) } \omega CR}$$

Magnification :-

The ratio of current through either L or C to the current at resonance can be defined as 'magnification'

magnification $\Rightarrow$ $\boxed{Q = \frac{I_L}{I} \text{ or } \frac{I_C}{I}}$