DEVELOPMENT OF SURFACES

13.1 INTRODUCTION

The complete surface of an object, laid-out on a plane is called the development of the surface or flat pattern of the object. The development of geometrical surfaces is important in the fabrication of not only small, simple shapes made of thin sheet metal, but also, sophisticated pieces of hardware such as space capsules. In actual practice, bend allowances have to be made in the layout of the pattern to ensure proper fabrication. These allowances depend upon the degree of bend, thickness of the metal and type of metal being used. However, these allowances are not considered in presenting the subject here, as it falls outside the scope of the book.

In making the development of a geometrical surface, the opening should be determined first. Every line used in making the development must represent the true length of that line on the actual surface.

Developments are made possible, with the application of basic graphic and geometric principles, in conjunction with mathematics. Since, different shapes must be joined together in many instances, principles of intersections are closely related to developments. If a development problem is resolved into basic geometric elements, the solution will be simpler.

13.2 CLASSIFICATION OF OBJECTS

In general, objects are bounded by geometric surfaces. These may be classified as:

1. Solids bounded by plane surfaces - cube, prism, pyramid, etc.
2. Solids bounded by single curved surfaces - cylinder, cone, etc.
3. Solids bounded by double curved surfaces - sphere, paraboloid, etc.
4. Solids bounded by warped surfaces - ellipsoid, hyperboloid, etc.

The first two types of solids can be developed accurately, whereas the last two can only be developed approximately, by dividing them into a number of parts.

13.3 METHODS OF DEVELOPMENT

Solids bounded by plane surfaces and single curved surfaces can be developed by: (i) Parallel line development method, based on stretch-out line principle; used for prisms and cylinders, (ii) radial line development method, making use of true length of slant edge or generator; used for pyramids and cones, (iii) triangulation method normally used for developing the transition pieces - connecting ducts, pipes, openings and similar objects with various sizes and shapes and (iv) approximate method, used to develop the objects with double curved or warped surfaces such as sphere, paraboloid, ellipsoid, hyperboloid, etc.

Only the lateral surfaces are generally developed and shown as presented here, omitting the bases or ends of solids.

13.3.1 Parallel line development

The surfaces of right prisms, cylinders and also oblique prisms and cylinders may be developed by this method.

Problem 1 *A square prism of side of base 40 and axis 80 long, is resting on its base on H.P such that, a rectangular face of it is parallel to V.P. Draw the development of the prism.*

Construction (Fig. 13.1)

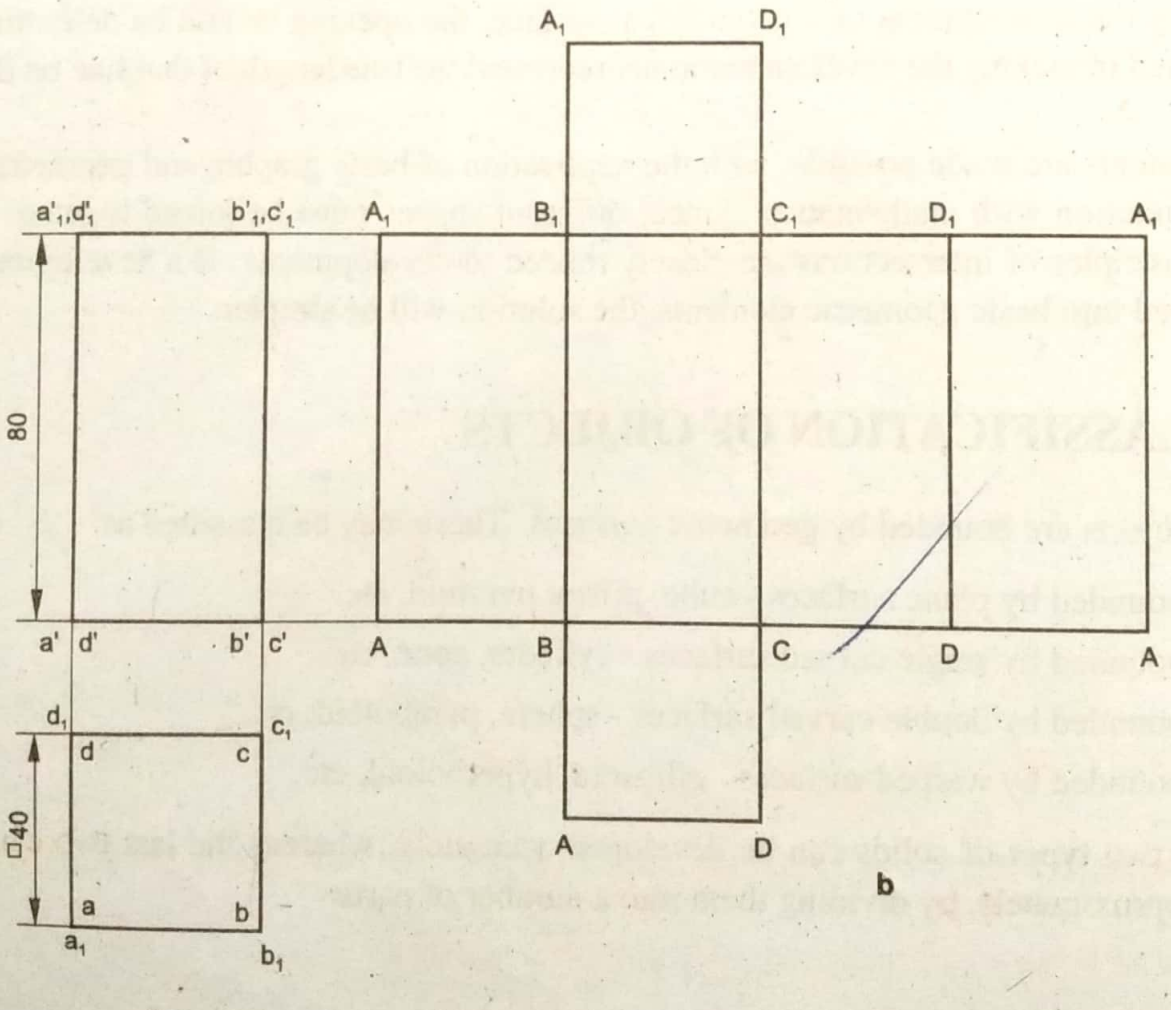


Fig.13.1

1. Draw the projections of the prism.
2. Draw the stretch-out line AA and mark - off the sides of the base along this line in succession, i.e., AB, BC, CD and DA.
3. Erect perpendiculars through these points and mark the edges AA_1, BB_1, etc.
4. Add the bases ABCD and $A_1B_1C_1D_1$ suitably.

NOTE (i) Stretch-out line is drawn in-line with the base in the front view, to complete the development quickly.

(ii) All the lines on the development should represent the true lengths.

Problem 2 *A hexagonal prism of side of base 30 and axis 75 long, is resting on its base on H.P such that, a rectangular face is parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at 30° to H.P. The section plane is passing through the top end of an extreme lateral edge of the prism. Draw the development of the lateral surface of the cut prism.*

Construction (Fig. 13.2)

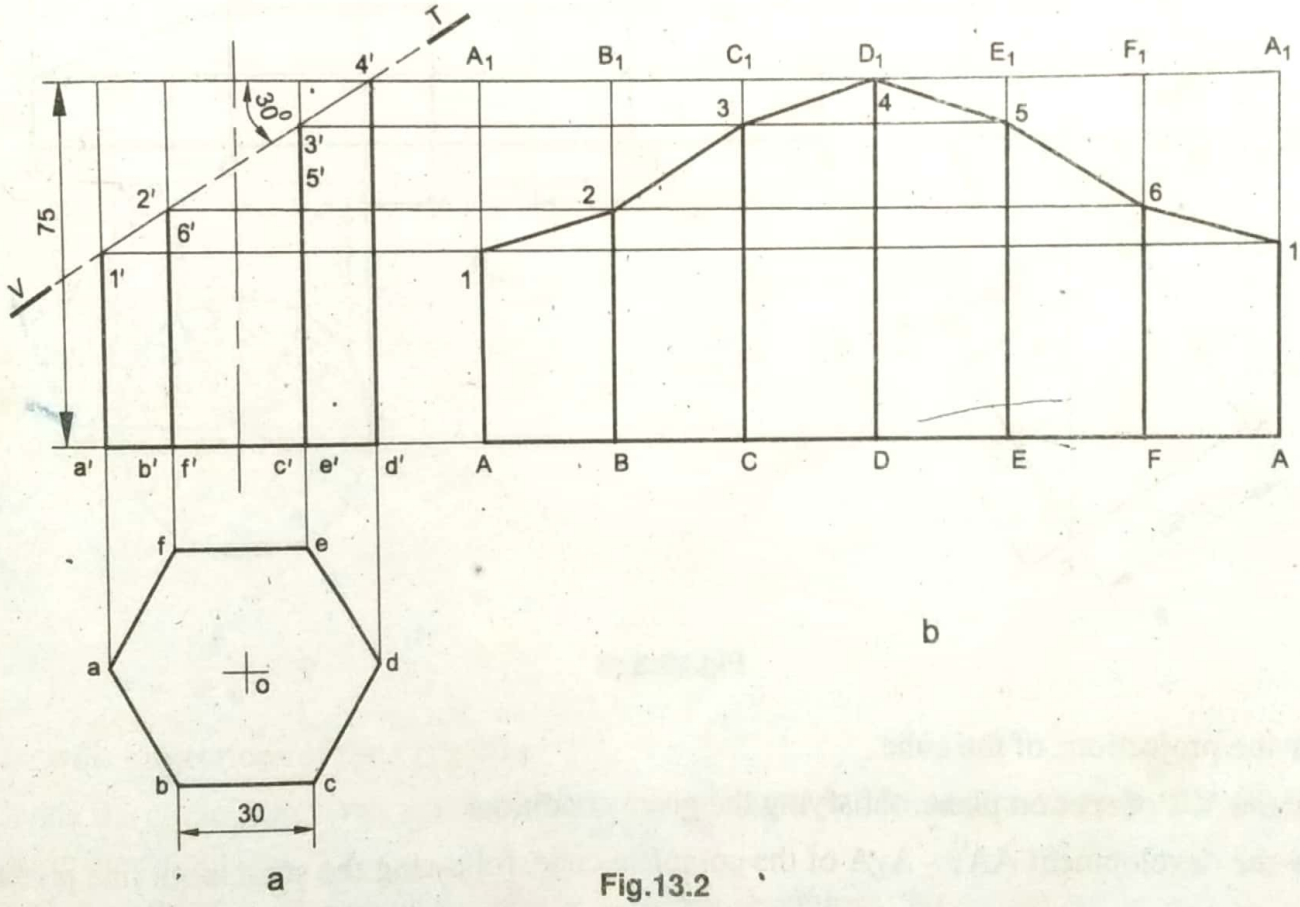


Fig.13.2

1. Draw the projections of the prism.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA_1 - A_1A of the complete prism, following the stretch-out line principle.
4. Locate the points of intersection 1′, 2′, etc., between the V.T and edges of the prism.

5. Draw horizontal lines through 1′, 2′, etc., and obtain 1, 2, etc., on the corresponding edges in the development.
6. Join the points 1, 2, etc., by straight lines and darken the sides, corresponding to the retained portion of the solid.

NOTE It is the usual practice to cut open (for the development) the surface of the solid at the shortest edge/length.

Problem 3 *A cube of 50 edge, is resting on a face on H.P such that, a vertical face is inclined at 30° to V.P. It is cut by a section plane perpendicular to V.P and inclined to H.P at 30° and passing through a point at 12 from the top end of the axis. Develop the lateral surface of the lower portion of the cube.*

Construction (Fig. 13.3)

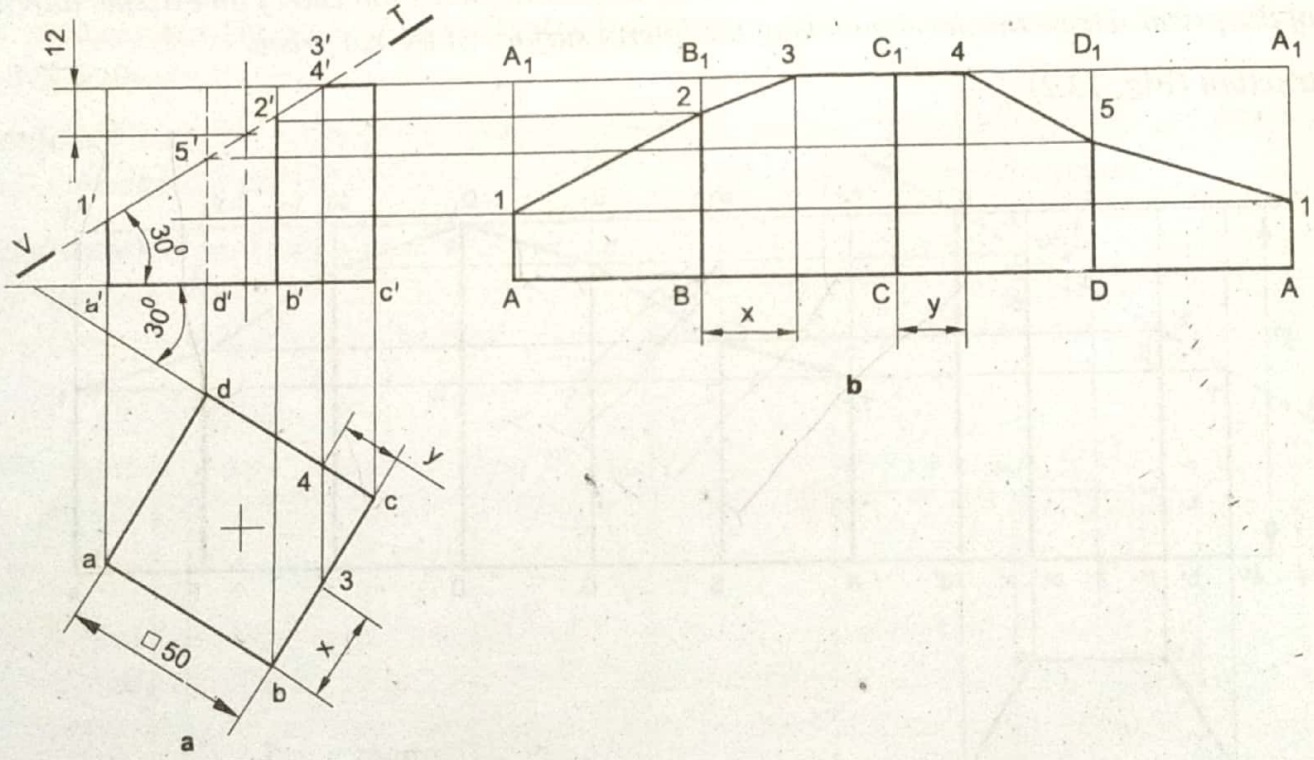


Fig.13.3

1. Draw the projections of the cube.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA_1 - A_1A of the complete cube, following the stretch-out line principle.
4. Repeat steps 4 to 6 of Construction: Fig. 13.2 and obtain the development of the cut solid.

NOTE 1. The points 3′ and 4′ are on the top surface of the cube. To locate the corresponding points in the development:

(i) Draw a projector through 3′ (4′), meeting bc at 3 and cd at 4 in the top view.

(ii) Mark the points 3 and 4 in the development such that, $B_1 3 = b3 = x$ and $C_1 4 = c4 = y$.

2. The sectioned portion in the top view is not cross-hatched as it is made use of only for locating certain points in the development.

Problem 4 *A cylinder of diameter of base 40 and axis 55 long, is resting on its base on H.P. It is cut by a section plane, perpendicular to V.P and inclined at 45° to H.P. The section plane is passing through the top end of an extreme generator of the cylinder. Draw the development of the lateral surface of the cut cylinder.*

Construction (Fig. 13.4)

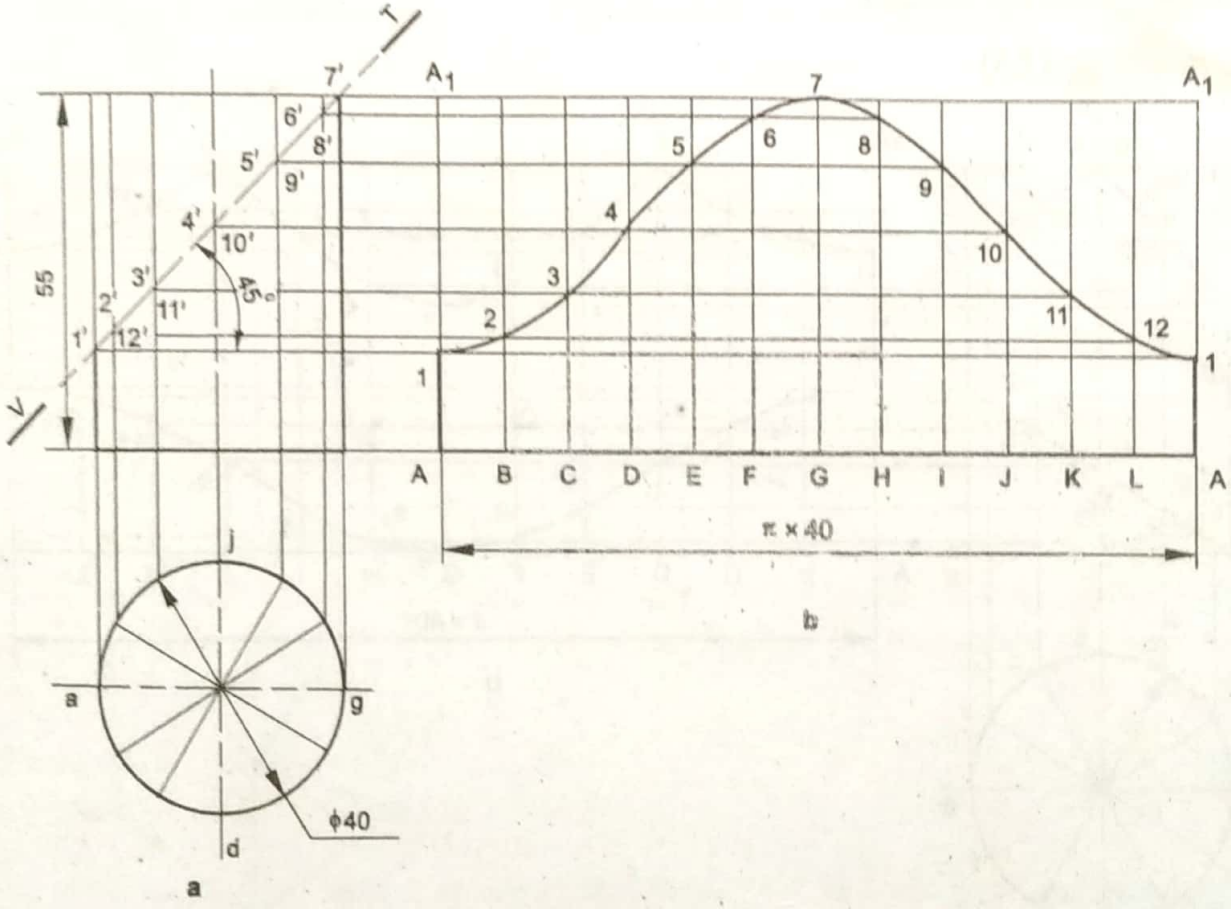


Fig.13.4

1. Draw the projections of the cylinder.
2. Divide the circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the V.T of section plane, satisfying the given conditions.
4. Draw the stretch-out line AA, equal to the circumference of the base of the cylinder.
5. Divide the stretch-out line AA, into the same number of equal parts as that of the base circle/ set-off chord lengths by a divider and locate the generators through the division points B, C, D, etc.
6. Locate the points of intersection 1′, 2′, etc., between the V.T and generators.

13.3.2 Radial line development

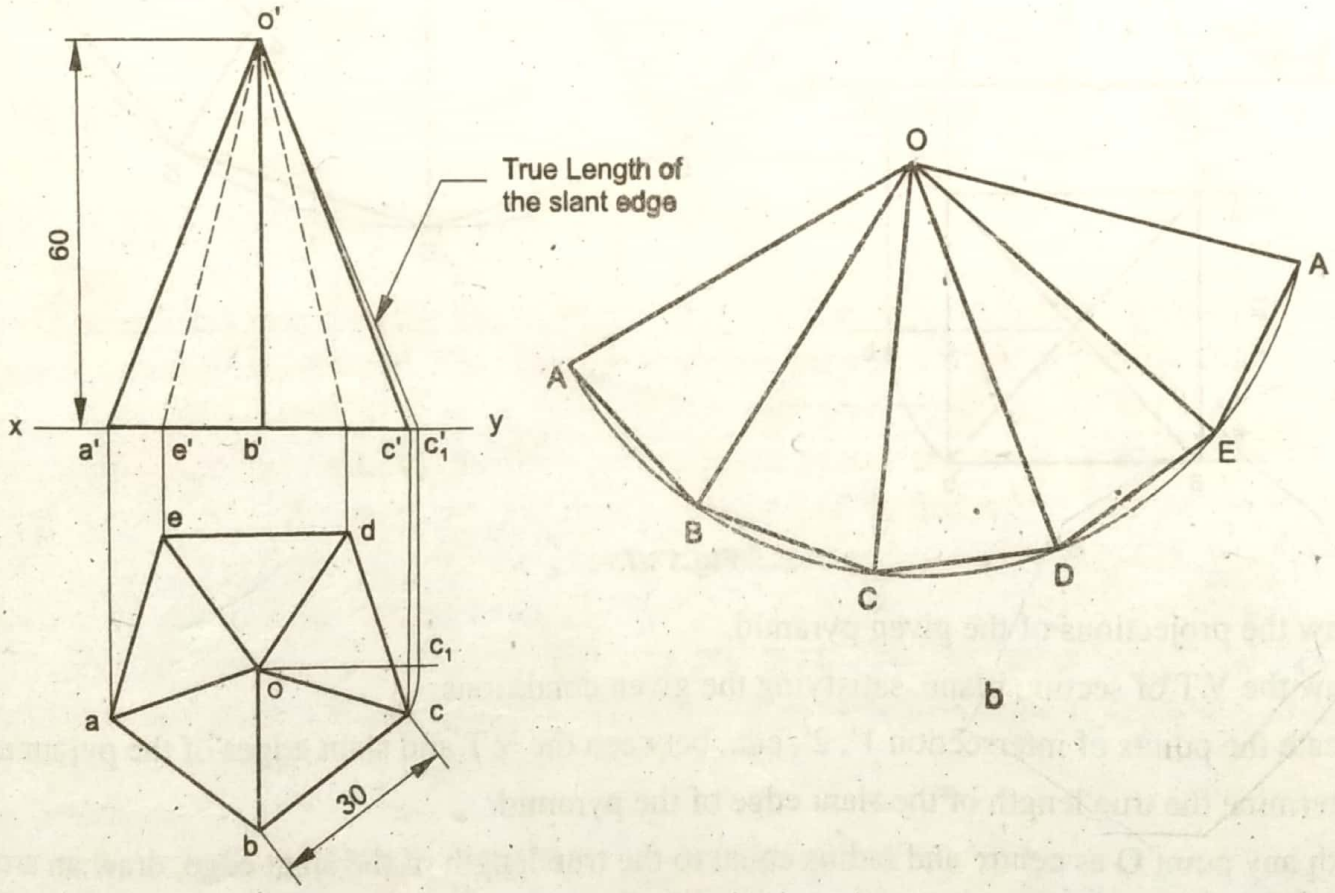
The lateral surfaces of right pyramids and cones may be developed by this method.

Problem 6 *A pentagonal pyramid of side of base 30 and axis 60 long, is resting on its base on H.P, with an edge of the base parallel to V.P. Draw the development of the lateral surface of the pyramid.*

HINT (i) The development of a pyramid consists of a number of equal isosceles triangles. The base of the triangle is equal to the edge of the base and the sides equal to the slant height of the pyramid respectively.

(ii) The true length of the slant edge may be measured from the front view, by making its top view parallel to xy.

Construction (Fig. 13.6)



a

Fig.13.6

1. Draw the projections of the pentagonal pyramid.
2. Determine the true length of the slant edge OC of the pyramid as shown.
3. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
4. With radius equal to the side of the base, step-off five divisions on the above arc.
5. Join the above division points in the order and also with the centre of the arc.

The figure thus formed is the development of the lateral surface of the pyramid.

Problem 7 *A square pyramid, with side of base 30 and axis 50 long, is resting on its base on H.P, with an edge of the base parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at 45° to H.P. The section plane is passing through the mid-point of the axis. Draw the development of the surface of the cut pyramid.*

Construction (Fig. 13.7)

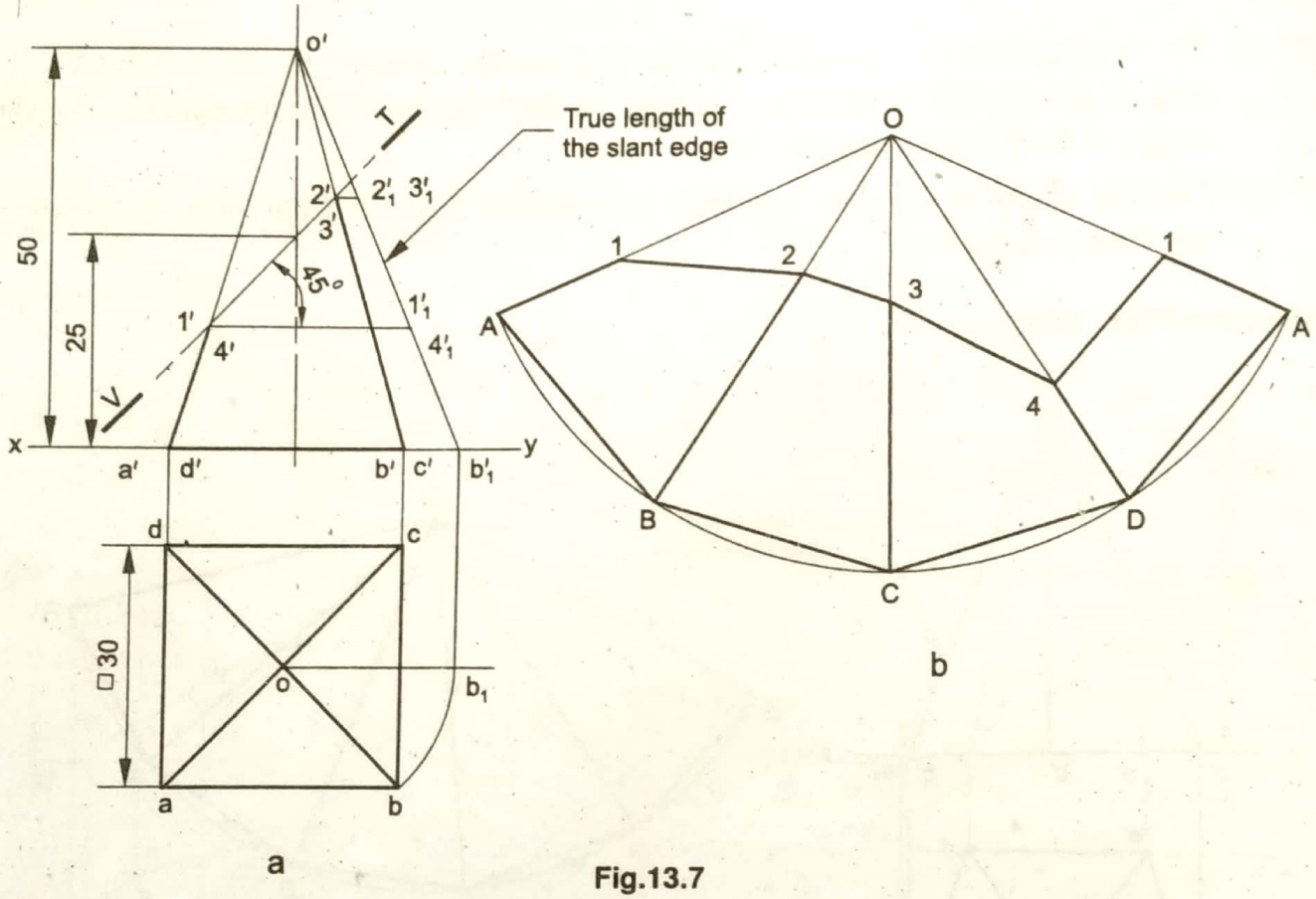


Fig.13.7

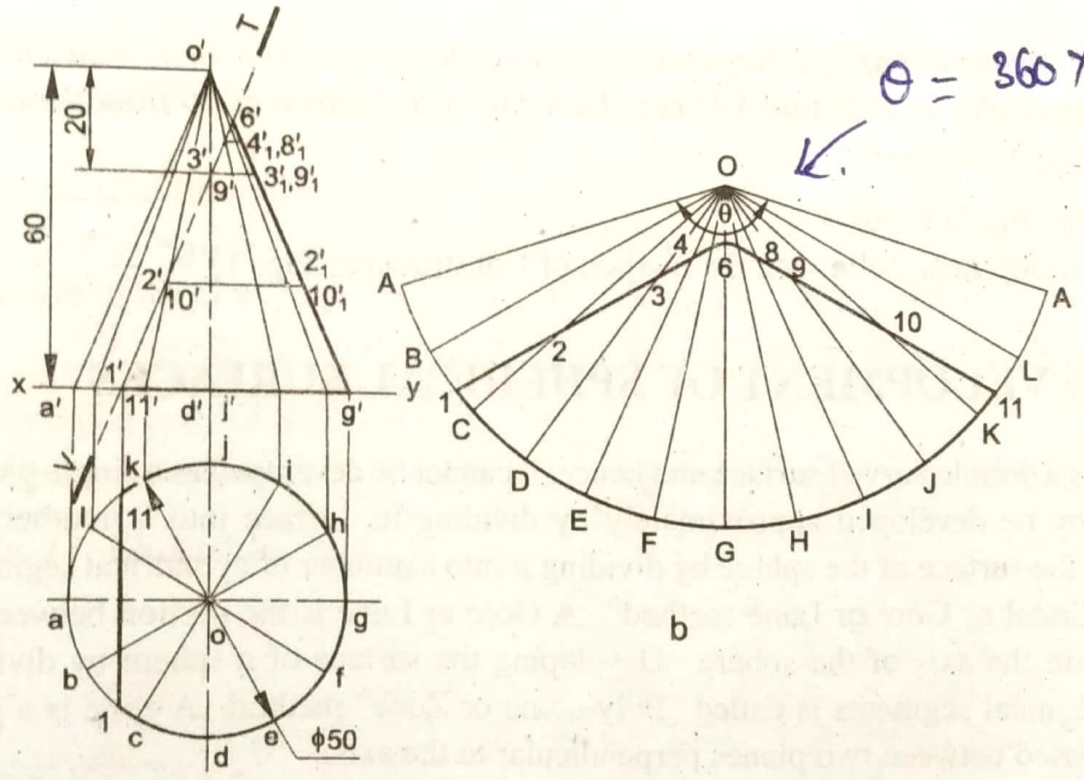
1. Draw the projections of the given pyramid.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and slant edges of the pyramid.
4. Determine the true length of the slant edge of the pyramid.
5. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of the circle.
6. Follow steps 4 and 5 of Construction: Fig.13.6 and obtain the full development of the pyramid.
7. Obtain the true lengths of the slant edges of the cut pyramid, by projecting on to the true length line.
8. Transfer the above true lengths to the corresponding edges in the development.
9. Join the points 1, 2, etc., by straight lines and obtain the development of the cut pyramid.

Problem 9 *A cone of base 50 diameter and axis 60 long, is resting on its base on H.P. It is cut by a section plane perpendicular to V.P and parallel to an extreme generator and passing through a point on the axis at a distance of 20 from the apex. Draw the development of the retained solid.*

Construction (Fig. 13.9)

1. Draw the projections of the cone.
2. Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in the front view.
3. Draw the development of the complete cone, following Construction: Fig. 13. 8.
4. Draw the V.T of section plane, satisfying the given conditions.
5. Locate the points of intersection between the V.T and generators and base of the cone.
6. Determine the true lengths of o′2′, o′3′, etc., by drawing horizontal lines to the extreme generator.
7. Transfer these true lengths to the development.
8. Join the points 1, 2, etc., by a smooth curve and obtain the required development.

$$\theta = 360 \times \frac{\text{radius of Base circle}}{\text{Slant length of the cone}}$$



a b

Fig. 13.9

13.5 EXAMPLES

Problem 12 *Figure13.13a shows the projections of a hexagonal prism, cut by two section planes as indicated. Draw the development of the retained portion of the prism.*

Construction (Fig. 13.13b)

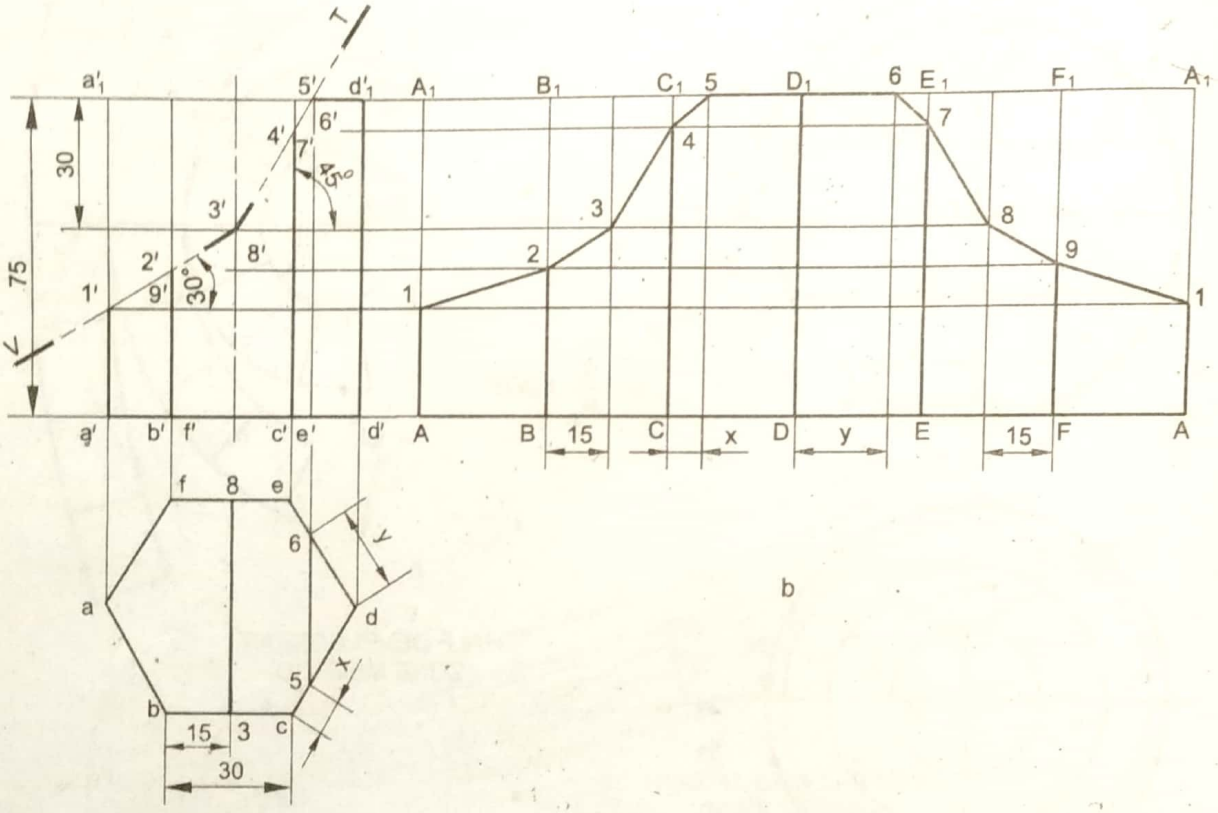
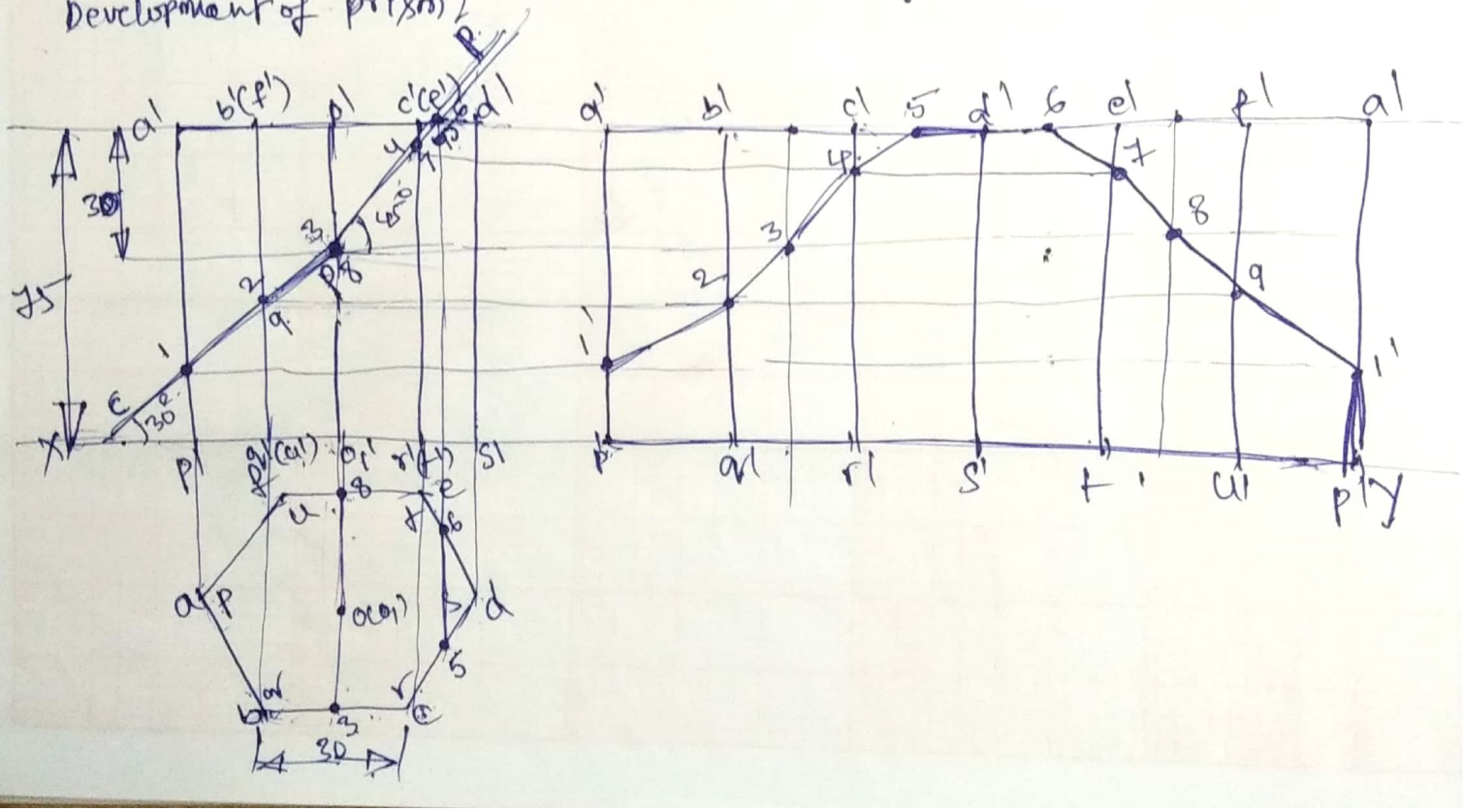


Fig. 13.13

1. Draw the development AA_1-A_1A of the complete prism, following the stretch-out line princi-
2. Locate the points of intersection 1′, 2′, etc., between the V.T of cutting plane and edges of the prism.

* A Hexagonal ~~Pyramid~~ Prism stands on its Base on 'Hp', one of the Rectangular Face is parallel to 'VP'. A solid is cut by two section planes- one Cutting plane Cuts solid, at 30° to HP at point 30 mm. from Top on axis. and another cutting plane Having 45° to 'Hp' from same point. Take one Side of prism, 30mm, and axis, 75 long. Draw the lateral Development of prism?



3. Transfer the intersection points to the development by horizontal projectors, except 5′ and 6′, to the corresponding edges in the development.
4. Locate the points 5 and 6 such that $C_1\ 5 = c5 = x$ and $D_1\ 6 = d6 = y$.
5. Join these points by straight lines and obtain the development of the cut prism, by darkening the edges.

Problem 13 *A cube of 50 edge, stands on one of its fo s on H.P, with the vertical faces equally inclined to V.P. A hole of 35 diameter is drilled cen ully through the cube such that, the axis of the hole is perpendicular to V.P. Draw the development of the cube.*

Construction (Fig. 13.14)

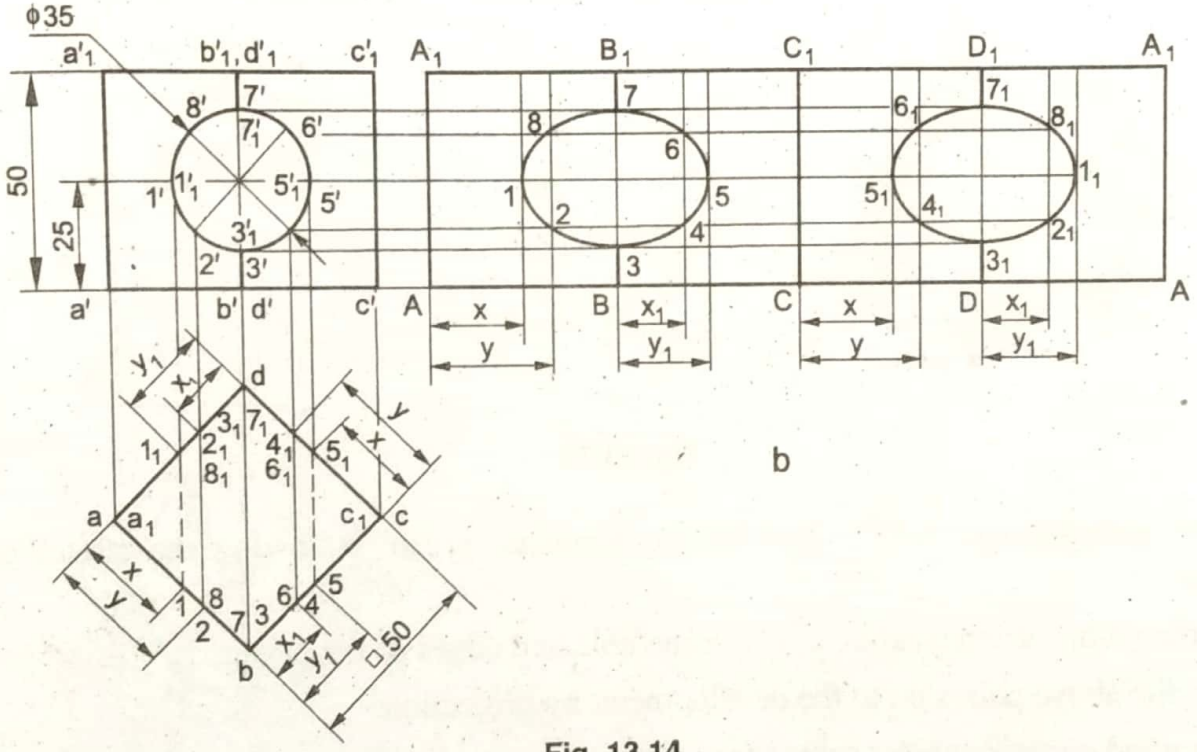


Fig. 13.14

1. Draw the projections of the cube.
2. Draw the development AA_1 - A_1A of the cube, following the stretch-out line principle.
3. Divide the circle in the front view, into equal number of parts, say 8 and obtain the corresponding generators in the top view.
4. Locate the points of intersection between the above generators and edges of the cube in the top view.
5. Draw horizontal projectors from 1′ (1_1′), 2′ (2_1′), etc., and locate the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., on the corresponding generators in the development.
6. Join the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., by smooth curves; representing the two openings of the hole in the development. Darken the edges corresponding to the retained portion of the cube.

Problem 15 *A hexagonal prism of 20 side of base and 50 height, rests on a base on H.P, with a vertical face parallel to V.P. A circular hole of 35 diameter, is drilled through the prism such that, the axis of the hole bisects the axis of the prism and is perpendicular to V.P. Draw the development of the prism.*

Construction (Fig.13.16)

1. Draw the projections of the prism with the hole.
2. Draw the development AA_1-A_1A of the complete prism, following the stretch-out line principle.

3. Divide the circle in the front view into a number of parts such that, certain points lie on the longer-edges of the prism. Also, locate the transition points $1'$, $5'$ and $1_1'$, $5_1'$, and obtain the corresponding points in the top view.
4. Draw horizontal lines from $1'$ ($1_1'$), $2'$ ($2_1'$), etc., and locate the points 1,2,3, etc., and 1_1,2_1,3_1, etc., on the corresponding edges (lines) in the development.
5. Join the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., by smooth curves, representing the two openings of the hole in the development.
6. Darken the edges corresponding to the retained portion of the prism and complete the development.

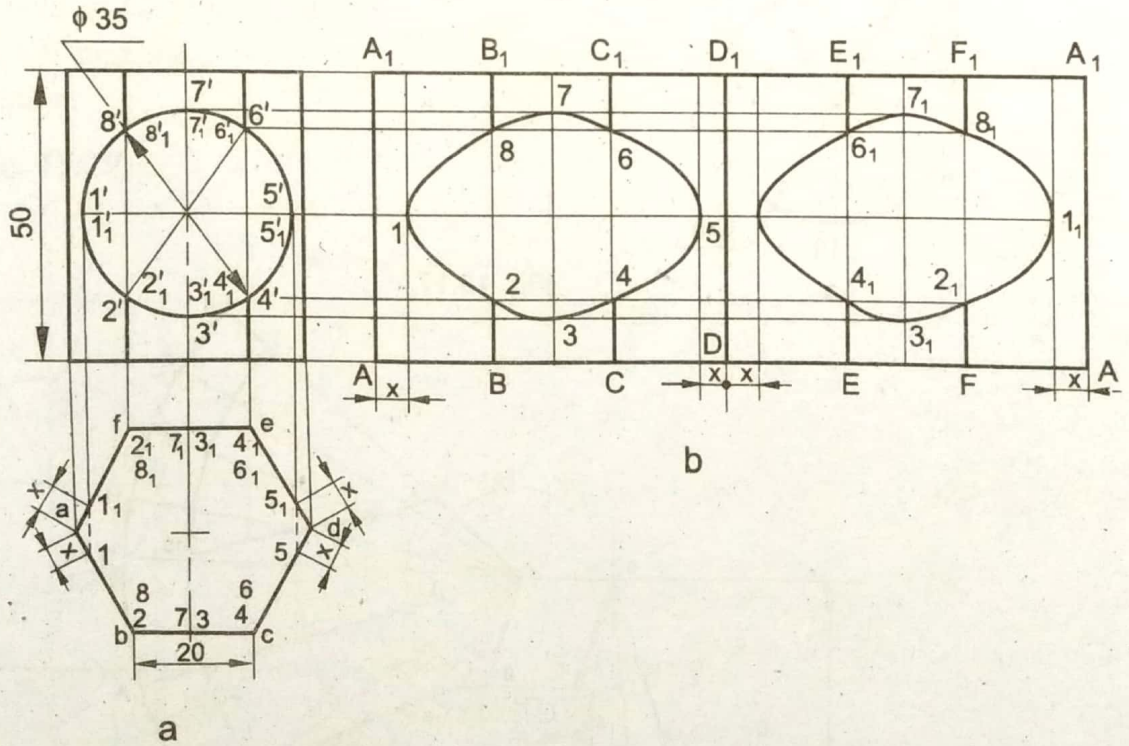


Fig. 13.16

Problem 18 *A cylinder of base 80 diameter and axis 110 long, is resting on its base on H.P. It has a circular hole of 60 diameter, drilled through centrally such that, the axis of the hole is perpendicular to V.P. and bisects the axis of the cylinder at right angles. Develop the lateral surface of the cylinder.*

Construction (Fig. 13.19)

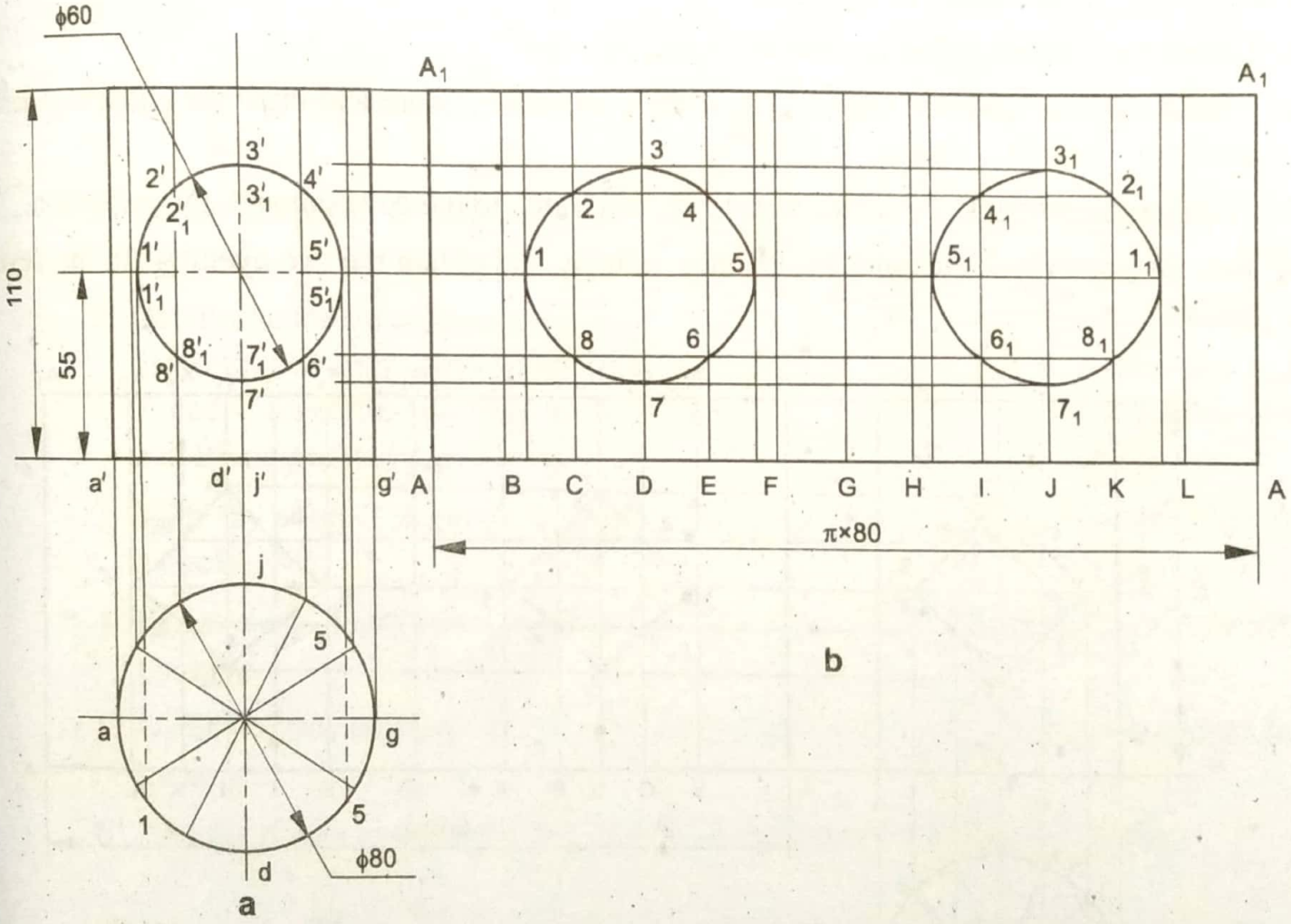


Fig. 13.19

1. Draw the projections of the cylinder, with the hole through it.
2. Divide the circle (top view of the cylinder) into a number of equal parts, say 12 and draw the corresponding generators in the front view.
3. Obtain the complete development AA_1-A_1A of the cylinder and draw the generators on it.

4. Determine the points of intersection $1'$, $2'$, etc., and $1_1'$, $2_1'$, etc., between the hole and generators in the front view.
5. Transfer these points on to the development by projection, including the transition points $1'$ ($1_1'$) and $5'$ ($5_1'$).
6. Join the points 1, 2, etc., and 1_1, 2_1, etc., by smooth curves and obtain the two openings in the development.

Problem 19 *Draw the development of a cylinder of 50 diameter and 75 height, containing a square hole of 25 side. The sides of the hole are equally inclined to the base and the axis of the hole bisects the axis of the cylinder.*

Construction (Fig.13.20)

1. Draw the projections of the cylinder, with the hole.
2. Draw number of generators in the front view, passing through the chosen points $1'$ ($1_1'$), $2'$ ($2_1'$), etc., on the edges of the hole.
3. Locate the corresponding generators in the top view.
4. Obtain the development AA_1 - A_1A of the complete cylinder and draw the chosen generators on it.
5. Transfer the points $1'$,$2'$,$3'$, etc., and $1_1'$ $2_1'$, $3_1'$, etc., to the development, by projection.
6. Join the points 1, 2, etc., and 1_1, 2_1, etc., suitably and obtain the two openings in the development.

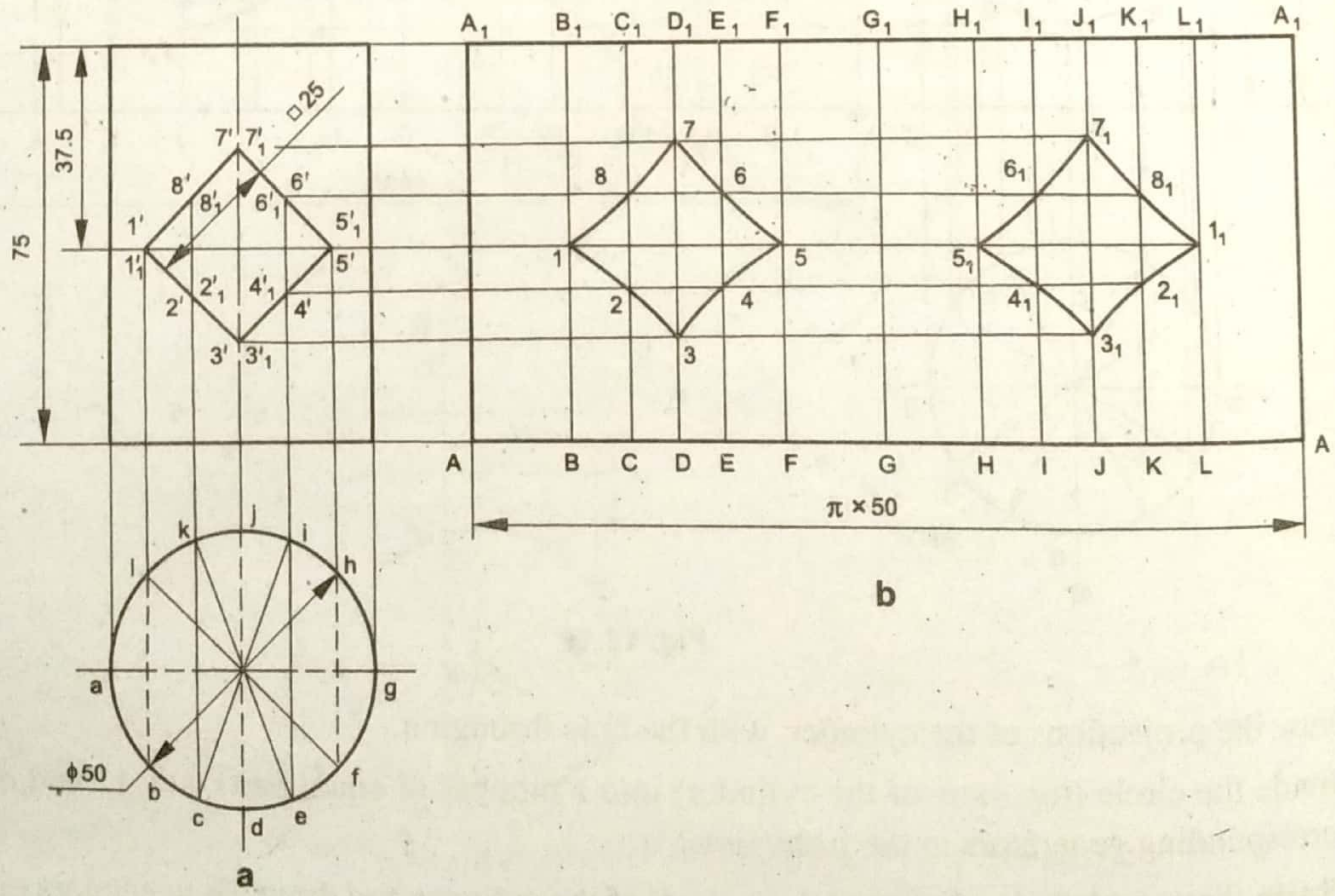


Fig. 13.20

Problem 25 *A cone of base diameter 80 and axis height 80, rests on H.P on its base. A square hole of side 40 is cut horizontally through the cone such that, the axis of the hole and the cone intersect at a height of 16 from the base. If the sides of the hole are equally inclined to H.P, draw the development of the lateral surface of the cone.*

Construction (Fig. 13.26)

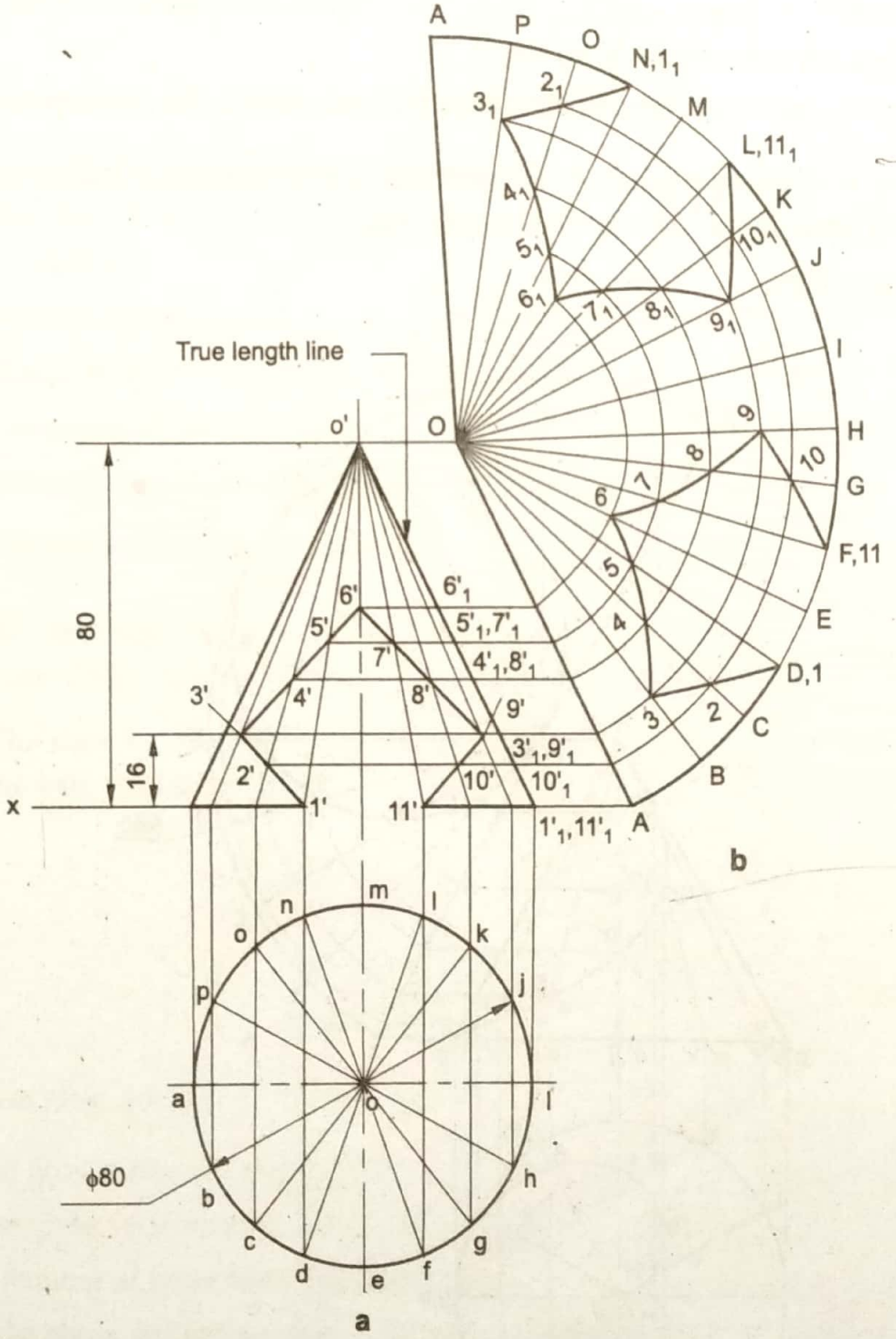


Fig. 13.26

1. Draw the projections of the cone, with the square hole through it.
2. Locate a number of points 1′, 2′, 3′, etc., on the edges of the hole in the front view and draw generators through them.

Problem 31 *Figures 13.32a and 13.33a show the orthographic views of rectangular scoop, and round scoop respectively. Figures 13.32b and 13.33b show the developments of the same; the constructions of which are self explanatory.*

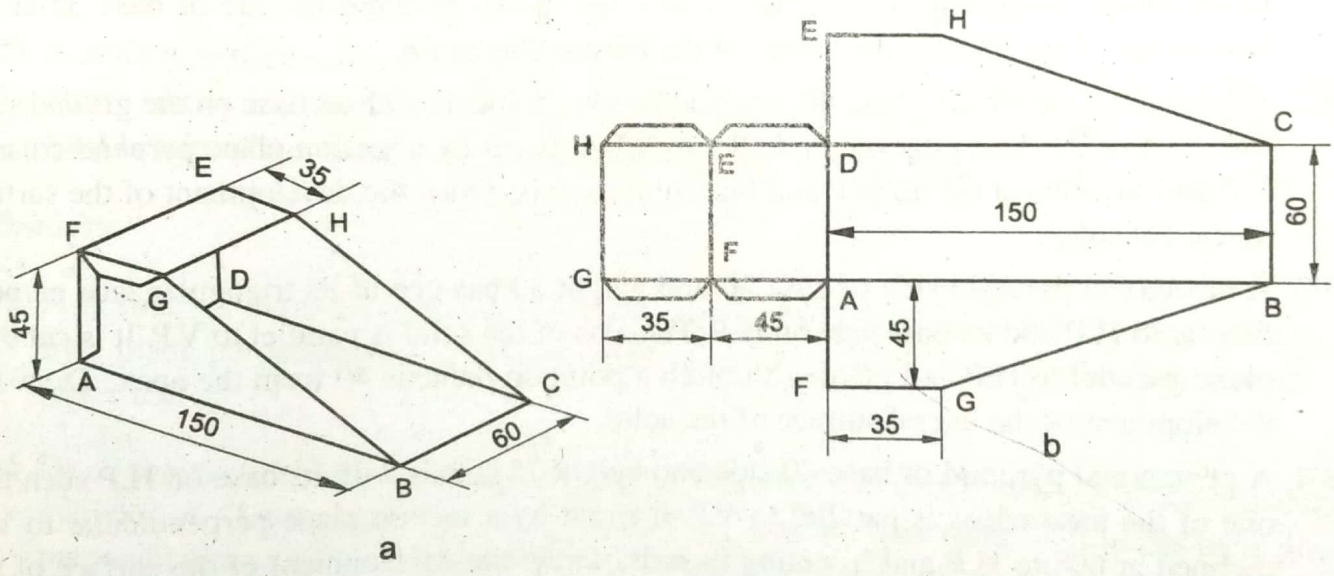


Fig. 13.32