UNIT-3
Distributions

- Distribution
  - Discrete
    - Binomial Distribution
    - Poison Distribution
  - Continuous
    - Normal distribution.

Binomial Distribution:- A discrete Random variable 'x' is said to fallow binomial distribution function. If it assumes only non negetive values and its probability mass function is given by $P(X=x) = P(x) = n_{c_x}\, p^{x} q^{n-x}$; $x = 0, 1, 2, \ldots n$, $p+q=1$

where "n" is no. of trials
"p" is probability of success in a trial
"q" is probability of failure in a trial

Here n & p are the parameters of binomial distribution.

Note :— (1) Let x be a binomial variable with parameters n & p we write $x \sim B(n, p)$

(2) The assignment of probabilities in Binomial distribution is Permissible (finite) because

$[\because (x+y)^n = n_{c_0} x^0 y^n + n_{c_1} x^1 y^{n-1} + \ldots]$

$$\sum_{x=0}^{n} P(x) \Rightarrow \sum_{x=0}^{n} n_{c_x}\, p^x q^{n-x}$$

$$\Rightarrow n_{c_0}\, p^0 q^{n-0} + n_{c_1}\, p^1 q^{n-1} + \ldots + n_{c_n}\, p^n q^{n-n}$$

$$\Rightarrow (q+p)^n$$

$$\Rightarrow 1$$

Constants of binomial distribution :—

(i) **mean** :— By def, mean $E(x) = \sum_{x=0}^{n} x\, P(x)$

$$= \sum_{x=0}^{n} x \; {}^{n}C_{x} \, p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!\,(n-x)!} p^{x} q^{n-x}$$

$$= 0 \frac{n!}{0!\,(n-0)!} p^{0} q^{n-0} + \sum_{x=1}^{n} x \frac{n!}{x!\,(n-x)!} p^{x} q^{n-x}$$

$$= 0 + \sum_{x=1}^{n} x \frac{n(n-1)!}{x(x-1)!\,(n-x)!} p^{x-1+1} q^{n-x}$$

$$= \sum_{x=1}^{n} n \frac{(n-1)!}{(x-1)!\,((n-1)-(x-1))!} p^{x-1} p \, q^{(n-1)-(x-1)}$$

$$= \sum_{x=1}^{n} np \frac{(n-1)!}{(x-1)!\,((n-1)-(x-1))!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} {}^{n-1}C_{x-1} \, p^{x-1} q^{(n-1)-(x-1)}$$

$$= np\,(q+p)^{n-1} \qquad [\because p+q=1]$$

$$= np\,(1)^{n-1}$$

$$= np(1)$$

Mean $= E(x) = np$

Variance :—

By def, variance $= E(x^2) - [E(x)]^2$

Now, $E(x^2) = \sum_{x=0}^{n} x^2 \, p(x)$

$$= \sum_{x=0}^{n} (x(x-1)+x) \, p(x)$$

$$= \sum_{x=0}^{n} x(x-1) p(x) + \sum_{x=0}^{n} x \, p(x)$$

$$= \sum_{x=0}^{n} x(x-1) \, {}^{n}C_{x} \, p^{x} q^{n-x} + \sum x \, p(x)$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^{x} q^{n-x} + np$$

$$= 0 + 0 + \sum_{x=2}^{n} \cancel{x(x-1)} \frac{n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!\,(n-x)!} p^{x-2} \cdot p^{2} q^{n-x} + np$$

$$= n(n-1)\,p^{2} \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,((n-2)-(x-2))!} p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{n-2}c_{x-2}\ p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 (p+q)^{n-2} + np$$

$$= n(n-1)p^2 (1)^{n-2} + np \qquad [\because p+q=1]$$

$$E(x^2) = n(n-1)p^2 + np$$

Now variance $v(x) = \sigma_x^2 = E(x^2) - (E(x))^2$

$$\sigma_x^2 = n(n-1)p^2 + np - (np)^2$$

$$= n[(n-1)p^2 + p] - n^2p^2$$

$$= n[(n-1)p^2 + p] - n[np^2]$$

$$= n[(n-1)p^2 + p - np^2]$$

$$= n[\cancel{np^2} - p^2 + p - \cancel{np^2}]$$

$$= n[p - p^2]$$

$$= n[p(1-p)]$$

$$= np(1-p) \qquad [\because q = 1-p]$$

variance $\sigma_x^2 = v(x) = npq$

Recurance relation for Binomial distribution :-

St:- Let 'x' be a binomial variate with p.m.f is denoted by $P(x) = n_{c_x} p^x q^{n-x}$ and

$$P(x+1) = n_{c_{x+1}} p^{x+1} q^{n-(x+1)}$$

Now, $$\frac{P(x+1)}{P(x)} = \frac{n_{c_{x+1}} p^{x+1} q^{n-(x+1)}}{n_{c_x} p^x q^{n-x}}$$

$$= \frac{\frac{n!}{(x+1)!(n-(x+1))!} p^{x+1} q^{n-(x+1)}}{\frac{n!}{x!(n-x)!} p^x q^{n-x}}$$

$$= \frac{\frac{n!}{(x+1)!\,(n-x-1)!}\, p^{x+1}\, q^{n-(x+1)}}{\frac{n!}{x!\,(n-x)!}\, p^{x}\, q^{n-x}}$$

$$= \frac{\frac{n!}{(x+1)!(n-x-1)!}\, \cancel{p^{x}}\cdot p\ \ \cancel{q^{n-x}}\cdot q^{-1}}{\frac{n!}{x!\,(n-x)!}\, \cancel{p^{x}}\ \cancel{q^{n-x}}}$$

$$= \frac{\frac{\cancel{n!}}{(x+1)!\,(n-x-1)!}\ p}{\frac{\cancel{n!}}{x!\,(n-x)!}\ q}$$

$\frac{p}{\frac{1}{q}} \quad p \cdot \frac{1}{q}$

$$= \frac{p}{(x+1)!\,(n-x-1)!} \cdot \frac{x!\,(n-x)!}{q}$$

$$= \frac{x!\,(n-x)!}{(x+1)!\,(n-x-1)!}\ \frac{p}{q}$$

$$= \frac{\cancel{x!}\,(n-x)\,\cancel{(n-x-1)!}}{(x+1)\,\cancel{x!}\,\cancel{(n-x-1)!}}\ \frac{p}{q}$$

$$\frac{P(x+1)}{P(x)} = \frac{n-x}{x+1}\cdot\frac{p}{q}$$

$$P(x+1) = \frac{n-x}{x+1}\,\frac{p}{q}\cdot P(x)$$ which is the recurrance relation of Binomial Distribution

Q:- Fit a Binomial distribution with a following data

x	0	1	2	3	4	5
$f(x)$	2	14	20	34	22	8

and calculate expected frequencies.

Sol:- from the given data mean $= \frac{\Sigma x f(x)}{N} = \frac{\Sigma x f(x)}{\Sigma f}$

x	0	1	2	3	4	5	
$f(x)$	2	14	20	34	22	8	$= 100 = N = \Sigma f$
$x f(x)$	0	14	40	102	88	40	$= 284 = \Sigma x f(x)$

$$\text{mean} = \frac{\sum x f(x)}{N} = \frac{284}{100} = 2.84$$

In a binomial distribution mean $= np$

$$p = \frac{\text{mean}}{n}$$

$$= \frac{2.84}{5}, \text{ Here } n = 5$$

$$= 0.568$$

and $q = 1 - p$ $\quad [\because p+q=1]$

$$= 1 - 0.568$$

$$= 0.432$$

Hence the fitting of Binomial distribution is

$$B(n,p) = P(x) = n_{c_x} \, p^x \, q^{n-x}$$

$$= 5_{c_x} (0.568)^x (0.432)^{5-x}, \quad x = 0,1,2,\ldots 5$$

The expected frequencies are

Expected frequency $f(x) = N * P(x)$ $\quad f(3) = 3\ldots5$

(or) $\quad = 22.48$

$E(x) = N * P(x)$ $\quad = 5.912$

$$f(x) = 100 \times 5_{c_x} (0.568)^x (0.432)^{5-x}$$

If $x = 0 \Rightarrow f(0) = 100 \times 5_{c_0} (0.568)^0 (0.432)^{5-0}$

$$= 100 \times 1 \times (1)(0.432)^5 = 1.505 \simeq 2$$

If $x = 1 \Rightarrow f(1) = 100 \times 5_{c_1} (0.568)^1 (0.432)^{5-1}$

$$= 100 \times 5 \times (0.568)(0.432)^4$$

$$= 100 \times 2.840(0.034)$$

$$= 9.8911 \simeq 1.$$

If $x = 2 \Rightarrow f(2) = 100 \times 5_{c_2} (0.568)^2 (0.432)^3$

$$= 100 \times 10 \times 0.322 \times 0.080$$

$$= 26.041 \simeq 26$$

$f(3) = 100 \times 5_{c_3} (0.568)^3 (0.432)^2$

$= 100 \times 10 \times 0.1832 \times 0.1866 = 34.15 \simeq 34$

$f(4) = 100 \times 5_{c_4} (0.568)^4 (0.432)^1$

$= 100 \times 5 \times 0.104 \times 0.432 = 22.48 \simeq 22$

$f(5) = 100 \times 5_{c_5} (0.568)^5 (0.432)^0$

$= 100 \times 1 \times 0.0591 \times 1 = 5.910 \simeq 6$

(2) Fit a B.D to the following data and find the expected frequencies

x	0	1	2	3	4	5
$f(x)$	1	10	24	35	18	8

Sol:- From the given data mean $= \frac{\Sigma x f(x)}{N}$ (or) $\frac{\Sigma x f(x)}{\Sigma f}$

x	0	1	2	3	4	5	
$f(x)$	1	10	24	35	18	8	$= 96 = N = \Sigma f$
$xf(x)$	0	10	48	105	72	40	$= 275 = \Sigma x f(x)$

mean $= \frac{\Sigma x f(x)}{N} = \frac{275}{96} = 2.86$

In a Binomial distribution mean $= np$

$p = \frac{\text{mean}}{n}$, Here $n = 5$

$= \frac{2.86}{5} = 0.572$

and $q = 1 - p = 1 - 0.572 = 0.428$

Hence the fitting of B.D is, $P(x) = n_{c_x} p^x q^{n-x}$

$B(n,p) = P(x) = 5_{c_x} (0.572)^x (0.428)^{5-x}, x = 0, 1, 2, \ldots 5$

The Expected frequencies is,

Expected frequency $f(x) = N * P(x)$

$f(x) = 96 * 5_{c_x} (0.572)^x (0.428)^{5-x}$

If $x = 0 \Rightarrow f(0) = 96 \times 5_{c_0} (0.572)^0 (0.428)^{5-0}$

$= 96 \times 1 \times 1 \times (0.428)^5 = 6.893 \cong 7$

If $x = 1 \Rightarrow f(1) = 96 \times 5_{c_1} (0.572)^1 (0.428)^{5-1}$

$= 96 \times 5 \times 0.572 \times 0.033 = 9.060 \cong 9$

If $x=2 \Rightarrow f(2) = 96 \times 5_{c_2} \times (0.572)^2 (0.428)^{5-2}$

$= 96 \times 5_{c_2} \times 0.327 \times 0.078$

$= 24.48 \cong 24$

If $x=3 \Rightarrow f(3) = 96 \times 5_{c_3} (0.572)^3 (0.428)^{5-3}$

$= 96 \times 10 \times 0.187 \times 0.183$

$= 32.85 \cong 33$

If $x=4 \Rightarrow f(4) = 96 \times 5_{c_4} (0.572)^4 (0.428)^{5-4}$

$= 96 \times 5 \times 0.107 \times 0.428$

$= 21.98 \cong 22$

If $x=5 \Rightarrow f(5) = 96 \times 5_{c_5} (0.572)^5 (0.428)^{5-5}$

$= 96 \times 1 \times 0.061 \times 1$

$= 5.856 \cong 6$ //

(3) 4 coins are tossed 160 times, the no. of times x, heads occur ($x = 0, 1, 2, 3, 4$) is given below..

x	0	1	2	3	4
No. of times	8	34	69	43	6

Fit a binomial distribution to this data on the hypothesis that coins are unbiased and find expected frequencies.

Sol:- Here n = no. of times = 4 and the hyposis that the coins are unbaised

i.e, $p = 1/2$ and $Q = 1/2$

$\therefore$ The fitting of B.D is $P(x) = 4_{c_x}\, p^x\, q^{4-x}$

Expected frequency $E(x) = f(x) = N \times P(x) = 160 \times 4_{c_x} (1/2)^x (1/2)^{4-x}$

If $x=0 \Rightarrow f(0) = 160 \times 4_{c_0} (1/2)^0 (1/2)^{4-0} = 10$

If $x=1 \Rightarrow f(1) = 160 \times 4_{c_1} (1/2)^1 (1/2)^3 = 40$

If $x=2 \Rightarrow f(2) = 160 \times 4_{c_2} (1/2)^2 (1/2)^2 = 60$

If $x=3 \Rightarrow f(3) = 160 \times 4_{c_3} (1/2)^3 (1/2)^1 = 40$

If $x=4 \Rightarrow f(4) = 160 \times 4_{c_4} (1/2)^4 (1/2)^0 = 10$

Expected frequency 10, 40, 60, 40, 10

4. The mean and varience of a B.d are 4 & 4/3 respectively
i, find parameter ii, Distribution of x iii, $P(x \geq 1)$

Sol: Given mean = 4 and varience = 4/3

In B.D mean = $np = 4$
varience = $npq = 4/3$

Now, $\frac{\text{varience}}{\text{mean}} \Rightarrow \frac{npq}{np} = \frac{4/3}{4} = \frac{\cancel{4}}{3} \times \frac{1}{\cancel{4}} = \frac{1}{3}$

$q = 1/3$

i, Parameter $p = 1 - q$, $np = 4$
$= 1 - 1/3$ $n \times \frac{2}{3} = 4^2$
$= 2/3$ $n = 6$

ii, Hence the B.D is,

$B(n,p) = P(x) = 6_{C_x} \left(\frac{2}{3}\right)^x (1/3)^{6-x}$

iii, $P(x \geq 1) = 1 - P(x < 1)$
$= 1 - P(x=0)$
$= 1 - 6_{C_0} \left(\frac{2}{3}\right)^0 (1/3)^{6-0}$
$= 1 - (1/3)^6$
$= 1 - 0.0014 = 0.9986$

5, If X is a Binomial random variable
$E(x) = 2$ and $V(x) = 4/3$ then find the distribution of x

Sol: Given mean $E(x) = 2$
and variance $V(x) = 4/3$

In B.D mean = $np = 2$
varience = $npq = 4/3$

Now, $\frac{\text{Variance}}{\text{mean}} = \frac{npq}{np} = \frac{4/3}{2} = \frac{\cancel{4}^2}{3} \times \frac{1}{\cancel{2}} = 2/3$
$q = 2/3$

Parameter $p = 1 - q$, $np = 2$

$= 1 - 2/3$ $\qquad n \cdot \frac{1}{3} = 2$

$= 1/3$ $\qquad n = 6$

Hence the B.D is $p(x=x) = p(x) = {}^nc_x\, p^x q^{n-x}$

$$p(x) = {}^6c_x\, (1/3)^x\, (2/3)^{6-x}$$

(6) The mean and variance of a B. variable x with parameters n & p are 16 & 8. find $p(x \geq 2)$

Sol:- Given mean $E(x) = 16$

and variance $v(x) = 8$

Now, $\quad = \dfrac{\text{variance}}{\text{mean}} = \dfrac{8}{16} = \dfrac{1}{2}$

$q = 1/2$

parameter $p = 1 - q$ $\qquad , \; np = 16$

$= 1 - 1/2 = 1/2$ $\qquad n(1/2) = 16$

$n = 32$

Hence the B.D is $p(x) = {}^nc_x\, p^x q^{n-x}$

$$p(x) = {}^{32}c_x\, (1/2)^x\, (1/2)^{32-x}$$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[{}^{32}c_0 (1/2)^0 (1/2)^{32-0} + {}^{32}c_1 (1/2)^1 (1/2)^{32-1}\right]$

$= 1 - [1 \times 1 \times 0 + 32 \times \frac{1}{2} \times 0]$

$= 1$

(7) The mean of B.D is 3 and the varience is $9/4$ find (i) The value of n (ii) Probability that $(x \geq 7)$ (iii) probability that $(1 \leq x < 6)$

Sol:- Given mean $= 3$ and varienc $= 9/4$

Now $\dfrac{\text{varience}}{\text{mean}} = \dfrac{npq}{np} = \dfrac{9/4}{3} = \dfrac{9}{4} \times \dfrac{1}{3} = 3/4$

(i) Parameter $p = 1 - q = 1 - 3/4 = 1/4$, $np = 3 \Rightarrow n \cdot \frac{1}{4} = 3$

$n = 12$

ii, $P(x \geq 7) = 1 - P(x < 7)$ Hence the B.D is $P(x) = {}^{12}C_x (1/4)^x (3/4)^{12-x}$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)]$$

$$= 1 - [{}^{12}C_0 (1/4)^0 (3/4)^{12-0} + {}^{12}C_1 (1/4)^1 (3/4)^{11} + {}^{12}C_2 (1/4)^2 (3/4)^{10} + {}^{12}C_3 (1/4)^3 (3/4)^9 + {}^{12}C_4 (1/4)^4 (3/4)^8 + {}^{12}C_5 (1/4)^5 (3/4)^7 + {}^{12}C_6 (1/4)^6 (3/4)^6]$$

$$= 1 - [1 \times \tfrac{1}{4} \times (\tfrac{3}{4})^{12} + 12 \times \tfrac{1}{4} \times (\tfrac{3}{4})^{11} + 66 \times \tfrac{1}{16} \times (\tfrac{3}{4})^{10} + 220 \times \tfrac{1}{64} \times (\tfrac{3}{4})^9 + 495 \times \tfrac{1}{256} \times (\tfrac{3}{4})^8 + 792 \times \tfrac{1}{1024} \times (\tfrac{3}{4})^7 + 924 \times \tfrac{1}{4096} \times (\tfrac{3}{4})^6]$$

$$= 1 - [0.007 + 0.126 + 0.232 + 0.257 + 0.193 + 0.103 + 0.040]$$

$$= 1 - 0.958 = 0.042$$

iii, $P(1 \leq x < 6) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$$= {}^{12}C_1 (1/4)^1 (3/4)^{11} + {}^{12}C_2 (1/4)^2 (3/4)^{10} + {}^{12}C_3 (1/4)^3 (3/4)^9 + {}^{12}C_4 (1/4)^4 (3/4)^8 + {}^{12}C_5 (1/4)^5 (3/4)^7$$

$$= 12 \times \tfrac{1}{4} \times (\tfrac{3}{4})^{11} + 66 \times \tfrac{1}{16} \times (\tfrac{3}{4})^{10} + 200 \times \tfrac{1}{64} \times (\tfrac{3}{4})^9 + 495 \times \tfrac{1}{256} \times (\tfrac{3}{4})^8 + 792 \times \tfrac{1}{1024} \times (\tfrac{3}{4})^7$$

$$= 0.126 + 0.232 + 0.257 + 0.193 + 0.103$$

$$= 0.911$$

(8) In a family of 5 children find the probability that there are i, Two boys ii, atleast one boy iii, all are boys iv, no boys.

Sol: Let x is the no. of boys

Here no. of children $n = 5$

and p = probability of getting a boy $= 1/2$

$q = 1/2$

i, Two boys :-

Hence the B.D is, $P(x) = {}^5C_x (1/2)^x (1/2)^{5-x}$

The probability of two boys in a family

$$P(x = 2) = {}^5C_2 (1/2)^2 (1/2)^3$$

$$= 10 \times 0.25 \times 0.125$$

$$= 0.3125$$

(ii) The probability of atleast one boy in a family

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 5_{C_0} (1/2)^0 (1/2)^{5-0}$

$= 1 - 1 \times 1 \times 0.0313$

$= 1 - 0.0313$

$= 0.968$

(iii) The probability of all are boys ie 5 boys in a family

$P(x=5) = 5_{C_5} (1/2)^5 (1/2)^{5-5}$

$= 1 \times 0.0313 \times 1$

$= 0.0313$

(iv) The probability of no boys ie 0 boys in a family.

$P(x=0) = 5_{C_0} (1/2)^0 (1/2)^{5-0}$

$= 1 \times 1 \times 0.0313 \simeq 0.0313$

(9) 10 coins are thrown simultaneously. find the probability of getting atleast 7 heads

Sol: Let Random vaiable x is the no. of heads

Here $n = 10$

p = probability of getting one head = $1/2$

q = probability of not getting one head = $1/2$

Hence B.D is, $P(x) = 10_{C_x} (1/2)^x (1/2)^{10-x}$

∴ probability of getting atleast 7 heads in 10 throws

is, $P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= 10_{C_7} (1/2)^7 (1/2)^{10-7} + 10_{C_8} (1/2)^8 (1/2)^{10-8} + 10_{C_9} (1/2)^9 (1/2)^{10-9} + 10_{C_{10}} (1/2)^{10} (1/2)^{10-10}$

$= 120 \times 0.0078 \times 0.125 + 45 \times 0.0039 \times 0.25 + 10 \times 0.0019 \times 0.5 + 1 \times 0.0009 \times 1$

$= 0.1170 + 0.0438 + 0.0095 + 0.0009$

$= 0.1712$

10. A die is thrown 8 times if getting a two or 4 is a success. Find the probability of

i. 4 success ii. $x < 3$ success

Sol: Here $n = 8$

p = probability of getting 2 or 4 success $= 1/6 + 1/6 = 2/6 = 1/3$

$q = 1 - p$

$= 1 - 1/3 = 2/3$

Hence the B.D is, $P(x) = 8_{c_x} (1/3)^x (2/3)^{8-x}$

(i) $p(x=4) = 8_{c_4} (1/3)^4 (2/3)^{8-4}$

$= 70 \times 0.0122 \times 0.1974$

$= 0.164$

ii. $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= 8_{c_0} (1/3)^0 (2/3)^{8-0} + 8_{c_1} (1/3)^1 (2/3)^{8-1} + 8_{c_2} (1/3)^2 (2/3)^{8-2}$

$= 1 \times 1 \times 0.0387 + 8 \times 0.333 \times 0.0581 + 28 \times 0.110 \times 0.0872$

$= 0.0387 + 0.15477 + 0.2685$

$= 0.4619$

11. Among the items produced by, in a factory 5% are defective. Find the probability that a sample of 8 coins

i. Exactly 2 defective ii. atleast one defective item.

Sol: Here $n = 8$

p = probability of defective items $= \frac{5}{100} = 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

Hence the B.D is, $P(x) = 8_{c_x} (0.05)^x (0.95)^{8-x}$

i) The probability of getting exactly two defective items

$P(x=2) = 8_{c_2} (0.05)^2 (0.95)^{8-2} = 28 \times 0.0025 \times 0.7350 = 0.0515$

(ii) The probability of getting atleast one defective item

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 8_{C_0} (0.05)^0 (0.95)^{8-0}$

$= 1 - [1 \times 1 \times 0.6634] = 1 - 0.6634$

$= 0.3366$

12. The probability that John hits a target is 1/2, he fires 6 times find the probability that he hits the target.

i) Exactly 2 times ii) atleast once.

Sol: Here $n = 6$

$p = 1/2 \quad , \quad q = 1/2$

Hence the B.D is, $P(x) = 6_{C_x} (1/2)^x (1/2)^{6-x}$

(i) The probability of John hits the target exactly 2 times

$P(x=2) = 6_{C_2} (1/2)^2 (1/2)^{6-2}$

$= 6_{C_2} (1/2)^2 (1/2)^4$

$= 15 \times 0.25 \times 0.0625$

$P(x=2) = 0.234$

(ii) probability of John hits the target atleast once

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 6_{C_0} (1/2)^0 (1/2)^{6-0}$

$= 1 - [1 \times 1 \times 0.0156]$

$= 1 - 0.0156$

$P(x \geq 1) = 0.9844$ //

13, If 3 of 20 tyres are defective and 4 of them are randomly choosen for inspection, what is the probability that only one of the defective tyre will be included

Sol: Here $n = 4$

p = probability of getting one defective tyre $= \frac{^3C_1}{^{20}C_1} = \frac{3}{20} = 0.150$

$q = 1 - p = 1 - \frac{3}{20} = \frac{17}{20}$

Hence the B.D is, $P(x) = {}^4C_x \, (3/20)^x \, (17/20)^{4-x}$

The probability of getting one defective tyre is

$P(x = 1) = {}^4C_1 \, (3/20)^1 \, (17/20)^{4-1}$

$= 4 \times 0.150 \times 0.614$

$= 0.368$ //

14. 6 dies are thrown 729 times how many times do you Expected atleast 3 dies to show a 5 or 6.

Sol: Here $n = 6$

p = probability of getting a 5 or 6 = $\frac{1}{6}+\frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$q = 1 - \frac{1}{3} = \frac{2}{3}$

Hence the B.D is, $P(x) = 6_{c_x} (\frac{1}{3})^x (\frac{2}{3})^{6-x}$

The Expected No. of times atleast 3 dies to showing 5 or 6 is $N \times P(x \geq 3)$

$= 729 \times [P(x=3) + P(x=4) + P(x=5) + P(x=6)]$

$= 729 \times [6_{c_3} (\frac{1}{3})^3 (\frac{2}{3})^{6-3} + 6_{c_4} (\frac{1}{3})^4 (\frac{2}{3})^{6-4} + 6_{c_5} (\frac{1}{3})^5 (\frac{2}{3})^{6-5} + 6_{c_6} (\frac{1}{3})^6 (\frac{2}{3})^{6-6}]$

$= 729 \times [20 \times 0.0370 \times 0.2962 + 15 \times 0.0122 \times 0.444 + 6 \times 0.0041 \times 0.6666 + 1 \times 0.0013 \times 1]$

$= 729 \times [0.2191 + 0.08125 + 0.01639 + 0.0013]$

$= 729 \times [0.3197]$

$= 233.62 \cong 234$

15. out of 800 families with 5 children in each how many would you Expect to have i. 3 boys ii. Either 2 or 3 boys Assume Equal probabilities for boys & girls.

Sol: Here $n = 5$ $N = 800$

$p = \frac{1}{2}$, $q = \frac{1}{2}$

Hence the B.D is, $P(x) = 5_{c_x} (\frac{1}{2})^x (\frac{1}{2})^{5-x}$

(i) probability of family with 3 boys = $N \times P(x)$

$P(x=3) = 5_{c_3} (\frac{1}{2})^3 (\frac{1}{2})^{5-3}$

$= 10 \times 0.125 \times 0.25 = 0.3125$

Expected no. of families with 3 boys is

$N \times P(x=3) = 800 \times 0.3125$

$= 250$

probability of family with 2 (or) 3 boys

ie, $P(2 \text{ or } 3) = [P(x=2) + P(x=3)]$

$= [5_{C_2} (1/2)^2 (1/2)^{5-2} + 5_{C_3} (1/2)^3 (1/2)^{5-3}]$

$= [10 \times 0.25 \times 0.125 + 10 \times 0.125 \times 0.25]$

$P(2 \text{ or } 3) = [0.3125 + 0.3125] = 0.625$

The expected no. of families with 2 (or) 3 boys is

$N \times P(2 \text{ or } 3) = 800 \times 0.625$

$= 500$

16, with the equal notations find p for a Binomial varience 'x'. If $n=6$ and $9P(x=4) = P(x=2)$

Sol: Here $n=6$

The B.D is $P(x) = 6_{C_x} p^x q^{6-x}$

Now, $P(x=4) = 6_{C_4} p^4 q^{6-4}$

$= 15 p^4 q^2$

$P(x=2) = 6_{C_2} p^2 q^{6-2}$

$= 15 p^2 q^4$

Given

$9P(x=4) = P(x=2)$

$9 \times \cancel{15} p^{\cancel{4}\,2} \cancel{q^2} = \cancel{15} \cancel{p^2} q^{\cancel{4}\,2}$

$9p^2 = q^2$

$\frac{p^2}{q^2} = \frac{1}{9} \Rightarrow \frac{p}{q} = \pm\frac{1}{3}$

$\Rightarrow \frac{1-q}{q} = \pm 1/3$

$\Rightarrow \frac{1}{q} - 1 = \pm 1/3$

$\Rightarrow \frac{1}{q} = 1 + \frac{1}{3}$ (or), $\frac{1}{q} = 1 - \frac{1}{3}$ $(q>0)$

$\Rightarrow \frac{1}{q} = 4/3$

$\Rightarrow q = 3/4$

$\therefore p = 1 - q$

$= 1 - 3/4 = 1/4$

Hence the B.D is $p(x) = 6_{c_x} (1/4)^x (3/4)^{6-x}$

17) with the usual notations find 'p' for a Binomial variable 'x' if $n=8$ and $4P(x=5) = P(x=3)$

Sol. Here $n=8$

The B.D is $P(x) = 8_{c_x} p^x q^{8-x}$

$P(x=5) = 8_{c_5} p^5 q^{8-5}$, $P(x=3) = 8_{c_3} p^3 q^{8-3}$

$= 56 p^5 q^3$ $= 56 p^3 q^5$

Given

$\because 4P(x=5) = P(x=3)$

$4 \times \cancel{56} \cancel{p^5}^{2} \cancel{q^3} = \cancel{56} \cancel{p^3} \cancel{q^5}^{2}$

$4p^2 = q^2$

$\frac{p^2}{q^2} = \frac{1}{4} \Rightarrow \frac{p}{q} = \pm\frac{1}{2}$

$\Rightarrow \frac{1-q}{q} = \pm\frac{1}{2}$

$\Rightarrow \frac{1}{q} - 1 = \pm 1/2$ $(\because q > 0)$

$\Rightarrow \frac{1}{q} = 1 + 1/2$

$\Rightarrow \frac{1}{q} = 3/2 \Rightarrow q = 2/3$

$\Rightarrow p = 1 - q$

$= 1 - 2/3 = 1/3$

Hence the B.D is $P(x) = 8_{c_x} (1/3)^x (2/3)^{8-x}$

18) In a B.D consisting of 5 independent trials, probability of 1 & 2 success are 0.4096 & 0.2048 respectively. Find the parameter 'p' of the distribution and the B.D.

Sol. Here n = no. of trails = 5

Now, $P(x=1) = 0.4096$

$P(x=2) = 0.2048$

by Def of Binomial distribution $p(x) = {}^nC_x p^x q^{n-x}$

Now

$$\frac{P(x=1)}{P(x=2)} = \frac{0.4096}{0.2048}$$

$$\frac{5_{c_1} p^1 q^{5-1}}{5_{c_2} p^2 q^{5-2}} = \frac{0.4096}{0.2048}$$

$$\frac{\cancel{5} \cancel{p} q^4}{\cancel{10}_2 p^{\cancel{2}} q^3} = 2$$

$$= \frac{q}{2p} = 2$$

$$\frac{q}{p} = 4$$

$$\frac{p}{q} = \frac{1}{4}$$

$$\frac{1-q}{q} = \frac{1}{4}$$

$$\frac{1}{q} - 1 = 1/4$$

$$\frac{1}{q} = 1 + 1/4$$

$$\frac{1}{q} = 5/4$$

$$q = 4/5$$

$$\therefore p = 1 - q$$

$$= 1 - 4/5$$

$$= 1/5$$

Hence B.D is $P(x) = n_{c_x} p^x q^{n-x}$

$$P(x) = 5_{c_x} (1/5)^x (4/5)^{5-x}$$

Poisson distribution :— A discrete random variable 'x' is said to follow poisson distribution if it assumes only non-negative values and its probability mass function is given by

$$P(X=x) = p(x) = e^{-\lambda} \frac{\lambda^x}{x!} \quad : x = 0, 1, 2 \ldots \text{ and } \lambda > 0$$

$$= 0 \quad : \text{otherwise.}$$

Here 'λ' is the parameter of poisson distribution.

NOTE :— The Assignment of probabilities is permissable because

$$\sum_{x=0}^{\infty} p(x) = \sum_{x=0}^{\infty} e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \infty\right]$$

$= e^{-\lambda} e^{\lambda}$

$= 1$

constants of poisson distribution :—

(1) mean of poison distribution :—

$E(x) = \sum_{x=0}^{\infty} x\, p(x)$

$= \sum_{x=0}^{\infty} x\, \frac{e^{-\lambda} \lambda^x}{x!}$

$= e^{-\lambda} \sum_{x=0}^{\infty} x \frac{\lambda^x}{x!} = e^{-\lambda} \left[0 \frac{\lambda^0}{0!} + \sum_{x=1}^{\infty} x \frac{\lambda^x}{x!} \right]$

$= e^{-\lambda} \left[0 + \sum_{x=1}^{\infty} \cancel{x} \frac{\lambda \lambda^{x-1}}{\cancel{x}(x-1)!} \right.$

$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$

$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \infty \right]$

$= \lambda e^{-\lambda} (e^{\lambda}) = \lambda e^{-\lambda+\lambda}$ $\qquad [\because e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$= \lambda e^0$

$\boxed{\therefore \text{mean} = \lambda}$

(2) variance of P.D :—

$\text{varience} = E(x^2) - (\text{mean})^2 = E(x^2) - (E(x))^2$

Now, $E(x^2) = \sum_{x=0}^{\infty} x^2 p(x)$

$\Rightarrow \sum_{x=0}^{\infty} (x(x-1) + x)\, p(x)$

$= \sum_{x=0}^{\infty} x(x-1)\, p(x) + \sum_{x=0}^{\infty} x\, p(x)$

$[\because E(x) = \sum x\, p(x) = \lambda]$ $\qquad = \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^x}{x!} + \lambda$

$= 0(0-1) \frac{e^{-\lambda} \lambda^0}{0!} + \frac{1(1-1) e^{-\lambda} \lambda^1}{1!} + \sum_{x=2}^{\infty} \frac{x(x-1) e^{-\lambda} \lambda^x}{x!} + \lambda$

$= 0 + 0 + \sum_{x=2}^{\infty} \frac{x(x-1)\, e^{-\lambda} \cdot \lambda^{x-2+2}}{x(x-1)(x-2)!} + \lambda$

$= \lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$

$= \lambda^2 e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \infty\right] + \lambda$

$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$ $\quad \left[\because e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots\right]$

$= \lambda^2 e^0 + \lambda$

$E(x^2) = \lambda^2 + \lambda$

variance $V(x) = E(x^2) - (E(x))^2$

$\therefore$ variance $= \lambda^2 + \lambda - (\lambda)^2 = \lambda$

Note :- $\therefore$ Mean and variance are equal in poisson distribution.

Recurrence Relation of P.D :— Let 'x' be a poisson variable with P.M.F is given by $P(x) = e^{-\lambda}\frac{\lambda^x}{x!}$ and $P(x+1) = e^{-\lambda}\frac{\lambda^{x+1}}{(x+1)!}$

Now, $\frac{P(x+1)}{P(x)} = \frac{\cancel{e^{-\lambda}} \frac{\lambda^{x+1}}{(x+1)!}}{\cancel{e^{-\lambda}} \frac{\lambda^x}{x!}}$

$= \frac{\lambda^{x+1}}{(x+1)\cancel{x!}} \times \frac{\cancel{x!}}{\lambda^x}$

$= \frac{\cancel{\lambda^x} \cdot \lambda}{x+1} \times \frac{1}{\cancel{\lambda^x}}$

$\frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1}$

$P(x+1) = \frac{\lambda}{x+1} \cdot P(x)$

Poisson distribution as Integrates of Binomial distribution :—

Proof :— poisson distribution as integrates of B.D under the conditions that

1. n is very large i.e, $n \to \infty$
2. p is very small ie, $p \to 0$
3. $np = \lambda$ is finite.

Let 'x' be a binomial variable with p.m.f is given by

$P(x) = n_{C_x} p^x q^{n-x} \quad ; \quad x = 0, 1, 2, \cdots n$

$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$

$$= \frac{n(n-1)(n-2)\cdots(n-x+1)\cancel{(n-x)!}}{x!\,\cancel{(n-x)!}} p^x q^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} p^x q^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} p^x q^{n-x} \times \frac{n^x}{n^x}$$

$$= \frac{(np)^x}{x!} \frac{n(n-1)(n-2)\cdots(n-x+1)}{n \cdot n \cdots n(x\text{ times})} (1-p)^{n-x} \quad [\because p+q=1$$

$$P(x) = \frac{\lambda^x}{x!} (\cancel{n}/\cancel{n})(1-1/n)(1-2/n)\cdots\times(1-\lambda/n)^n(1-\lambda/n)^{-x} \quad \left[\because np=\lambda,\ p=\frac{\lambda}{n}\right]$$

as $n \to \infty$

$$= \frac{\lambda^x}{x!} \times 1 \times 1 \times 1 \times \cdots 1 \times \lim_{n\to\infty}(1-\lambda/n)^n \lim_{n\to\infty}(1-\lambda/n)^{-x}$$

$$P(x) = \frac{\lambda^x}{x!} \times e^{-\lambda} \times 1 \qquad \left[\because \lim_{n\to\infty}(1-1/n)^n = e^{-\lambda}\right]$$

$$B(n,p) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0,1,2,\cdots\infty \qquad [\because n\to\infty$$

1) Fit a poison distribution for the following data and calculate Expected frequencies.

x	0	1	2	3	4	5
freq	125	95	49	20	8	3

Sol:- from the given data mean $= \frac{\sum x f(x)}{N}$ (or) $\frac{\sum x f(x)}{\sum f}$

$$= \frac{0\times125+1\times95+2\times49+3\times20+4\times8+5\times3}{300}$$

$$= \frac{0+95+98+60+32+15}{300}$$

$$\text{mean} = \frac{300}{300} = \frac{3}{3} = 1$$

In a poisson distribution mean $= \lambda$

$$\therefore \lambda = 1$$

Hence the poisson distribution is $P(x) = e^{-\lambda}\frac{\lambda^x}{x!}$

$$= e^{-1}\frac{1^x}{x!} = \frac{e^{-1}}{x!}, \quad x = 0,1,2\ldots5$$

Calculate Expected frequices as given below:

$$f(x) = NP(x)$$

$$= N \times \frac{e^{-1}}{x!} \quad , x = 0,1,2, \cdots 5$$

take : $x = 0, 1, \cdots 5$

$f(0) \Rightarrow x = 0 \Rightarrow 300 \times \frac{e^{-1}}{0!} = 110.1 \cong 110$

$x = 1 \Rightarrow f(1) \Rightarrow 300 \times \frac{e^{-1}}{1!} = 110.36 \cong 110$

$x = 2 \Rightarrow f(2) \Rightarrow 300 \times \frac{e^{-1}}{2!} = 55.181 \cong 55$

$x = 3 \Rightarrow f(3) \Rightarrow 300 \times \frac{e^{-1}}{3!} = 18.39 \cong 18$

$x = 4 \Rightarrow f(4) \Rightarrow 300 \times \frac{e^{-1}}{4!} = 4.59 \cong 5$

$x = 5 \Rightarrow f(5) \Rightarrow 300 \times \frac{e^{-1}}{5!} = 0.91 \cong 1$

$\therefore$ Expected frequencies are 110, 110, 55, 18, 5, 1

(2) fit a poisson distribution to the following data and find the Expected frequencies

x	0	1	2	3	4	5
$f(x)$	142	156	69	27	5	1

Sol:- from the given data mean $= \frac{\sum x f(x)}{N}$ (or) $\frac{\sum x f(x)}{\sum f}$

$$= \frac{0 \times 142 + 1 \times 156 + 2 \times 69 + 3 \times 27 + 4 \times 5 + 5 \times 1}{400}$$

$$= \frac{0 + 156 + 138 + 71 + 20 + 5}{400}$$

$$= \frac{390}{400}$$

$$\text{mean} = \frac{390}{400} = 0.975$$

In a poisson distribution mean $= \lambda \therefore \lambda = 0.975$

Hence the poisson distribution is $p(x) = e^{-0.975} \cdot \frac{(0.975)^x}{x!}$, $x = 0, 1, \cdots 5$

calculate the Expected frequencies as given below.

$$f(x) = N \times P(x)$$

$$= N \times e^{-0.975} \cdot \frac{(0.975)^x}{x!} \quad , x = 0, 1, \cdots 5$$

take $x = 0, 1, \cdots 5$

$x = 0 \Rightarrow f(0) = 400 \times \frac{e^{-0.975}(0.975)^0}{0!} = 400 \times 0.3771 \times 1 = 150.87 \cong 151$

$x=1 \Rightarrow$

$$f(1) = 400 \times e^{-0.975} \frac{(0.975)^1}{1!} = 400 \times 0.3771 \times 0.975 = 147.06 \cong 147$$

$x=2 \Rightarrow$

$$f(2) = 400 \times e^{-0.975} \frac{(0.975)^2}{2!} = 400 \times 0.3771 \times \frac{0.9506}{2}$$

$$= 400 \times 0.3771 \times 0.4753$$

$$f(2) = 71.69 \cong 72$$

$x=3 \Rightarrow$

$$f(3) = 400 \times e^{-0.975} \frac{(0.975)^3}{3!} = 400 \times 0.3771 \times \frac{0.9268}{6}$$

$$= 400 \times 0.3771 \times 0.1544$$

$$f(3) = 23.28 \cong 23$$

$x=4 \Rightarrow$

$$f(4) = 400 \times e^{-0.975} \frac{(0.975)^4}{4!} = 400 \times 0.3771 \times \frac{0.9036}{24}$$

$$= 400 \times 0.3771 \times 0.0376$$

$$= 5.671 \cong 6$$

$x=5 \Rightarrow$

$$f(5) = 400 \times e^{-0.975} \frac{(0.975)^5}{5!} = 400 \times 0.3771 \times \frac{0.8810}{120}$$

$$= 400 \times 0.3771 \times 0.0073$$

$$= 1.10 \cong 1$$

Q

3, Fit a poisson distribution to the following data and find the Expected frequencies

x	0	1	2	3	4
f(x)	109	65	22	3	1

Sol:- From the given data mean $= \frac{\Sigma x f(x)}{N}$ (or) $\frac{\Sigma x f(x)}{\Sigma f}$

$$= \frac{0 \times 109 + 1 \times 65 + 2 \times 22 + 3 \times 3 + 4 \times 1}{200}$$

$$= \frac{0 + 65 + 44 + 9 + 4}{200}$$

$$\text{mean} = \frac{122}{200} = 0.61$$

In a poisson distribution mean $= \lambda$; $\therefore \lambda = 0.61$

Hence the poisson distribution is $p(x) = \frac{e^{-\lambda}}{x!} \lambda^x$

$$= e^{-0.61} \frac{(0.61)^x}{x!}, \quad x = 0, 1, 2, \dots 4$$

$x=0 \Rightarrow$
$f(0) = 200 \times e^{-0.61} \times \frac{(0.61)^0}{0!} = 200 \times 0.543 \times 1 = 108.67 = \simeq 109$

$x=1 \Rightarrow$
$f(1) = 200 \times e^{-0.61} \times \frac{(0.61)^1}{1!} = 200 \times 0.543 \times 0.61 = 66.24 \cong 66$

$x=2 \Rightarrow$
$f(2) = 200 \times e^{-0.61} \times \frac{(0.61)^2}{2!} = 200 \times 0.543 \times 0.186 = 20.19 \cong 20$

$x=3 \Rightarrow$
$f(3) = 200 \times e^{-0.61} \times \frac{(0.61)^3}{3!} = 200 \times 0.543 \times 0.037 = 4.01 \cong 4$

$x=4 \Rightarrow$
$f(4) = 200 \times e^{-0.61} \times \frac{(0.61)^4}{4!} = 200 \times 0.543 \times 0.0057 = 0.61 \cong 1$

The Expected frequencies are 109, 66, 20, 4, 1

(4) using Recurrance formula find the probabilities when $x=0,1,2,\ldots 5$ if the mean of poisson distribution is 3.

Sol:- The given mean $\lambda = 3$

The poisson distribution is $P(x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-3} 3^x}{x!}$

Take $x=0$, $P(0) = e^{-3} \frac{(3)^0}{0!} = e^{-3} = 0.0497$

$\therefore$ The Recurrance formula for poisson distribution is

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

$$P(x+1) = \frac{3}{x+1} P(x)$$

$x=0 \Rightarrow P(1) = \frac{3}{1} P(0)$

$= 3 \times 0.0497 = 0.1494$

$x=1 \Rightarrow P(2) = \frac{3}{2} P(1)$

$= \frac{3}{2} e^{-3} \times \frac{(3)^1}{1!}$

$= \frac{3}{2} \times 0.0497 \times 3 = 0.2241$

$x=2 \Rightarrow P(3) = \frac{3}{3} P(2)$

$= e^{-3} \frac{(3)^2}{2!} = 0.0497 \times \frac{\cancel{6}3}{\cancel{2}} = 0.2241$

$x=3 \Rightarrow P(4) = \frac{3}{4} P(3)$

$= \frac{3}{4} \times e^{-3} \frac{(3)^3}{3!} = \frac{\cancel{3}}{4} \times 0.0497 \times \frac{27}{\cancel{6}2} = 0.1680$

$x=4 \Rightarrow P(5) = \frac{3}{5} P(4)$

$= \frac{3}{5} \times e^{-3} \frac{(3)^4}{4!} = \frac{3}{5} \times 0.0497 \times 3.375 = 0.1008$

(5) Suppose that 'x' has a poison varient if $P(x=1) = \frac{2}{3}P(x=3)$ find
(1) mean of the distribution. (2) $P(x \geq 1)$

Sol:- let 'x' be a poison varient if p.m.f given by

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Now, $P(x=1) = \frac{2}{3} P(x=3)$

$$\cancel{e^{-\lambda}} \frac{\lambda^1}{1!} = \frac{2}{3} \cancel{e^{-\lambda}} \frac{\lambda^{3^2}}{3!}$$

$$\frac{3}{2} = \frac{\lambda^2}{3!}$$

$$\frac{3}{\cancel{2}} \times \cancel{6}^3 = \lambda^2 \Rightarrow \lambda^2 = 9 \Rightarrow \lambda = \pm 3$$

$$\therefore \lambda = 3 \; (\because \lambda > 0)$$

(1) mean of the distribution is λ $\therefore \lambda = 3$

(2) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - e^{-3} \frac{(3)^0}{0!}$

$= 1 - 0.0497$

$= 0.951$

(6) If x is a poison distribution variant such that $P(1) = P(2)$
find (1) distribution of 'x'
(2) $P(1 \leq x < 3)$

Sol:- let 'x' be a poisson varient p.m.f is given by

$$P(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

Now, $P(1) = P(2)$

$$\cancel{e^{-\lambda}} \frac{\lambda^1}{1!} = \cancel{e^{-\lambda}} \frac{\lambda^{2^1}}{2!}$$

$$2 = \lambda \Rightarrow \lambda = 2$$

(1) distribution of 'x' is $P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

hence the p.p is $P(x) = e^{-2} \frac{(2)^x}{x!}$

(2) $P(1 \leq x < 3) = P(x=1) + P(x=2)$

$$= e^{-2}\frac{(2)^1}{1!} + e^{-2}\frac{(2)^2}{2!}$$

$$= 0.27 + 0.135 \times \frac{4}{2}$$

$$= 0.27 + 0.27$$

$$= 0.54.$$

7) If 'x' is a poisson variant such that $3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$ find (1) distribution of 'x' and also (2) standard deviation.

Sol:- Let 'x' be a poisson variant p.m.f given by

$$P(x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

Now, $3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$

$$3e^{-\lambda}\frac{(\lambda)^4}{4!} = \frac{1}{2}\left[e^{-\lambda}\frac{\lambda^2}{2!}\right] + \left[e^{-\lambda}\frac{\lambda^0}{0!}\right]$$

$$3e^{-\lambda}\frac{\lambda^4}{4!} = \frac{1}{4}e^{-\lambda}\lambda^2 + e^{-\lambda}$$

$$3\cancel{e^{-\lambda}}\frac{\lambda^4}{4!} = \cancel{e^{-\lambda}}\left[\frac{\lambda^2}{4} + 1\right]$$

$$3\frac{\lambda^4}{4!} = \frac{\lambda^2+4}{4}$$

$$\frac{3\lambda^4}{\cancel{24}_6} = \frac{\lambda^2+4}{\cancel{4}}$$

$$3\lambda^4 = 6\lambda^2 + 24$$

$$3\lambda^4 - 6\lambda^2 - 24 = 0$$

$$\lambda^4 - 2\lambda^2 - 8 = 0$$

$$\lambda^4 - 4\lambda^2 + 2\lambda^2 - 8 = 0$$

$$\lambda^2(\lambda^2-4) + 2(\lambda^2-4) = 0$$

$$(\lambda^2-4)(\lambda^2+2) = 0$$

$$\lambda^2 = 4, \quad \lambda^2 = -2$$

$\lambda = \pm 2$ Here we consider $\lambda = 2$ [$\because$ mean is +ve]

(1). Hence the P.D is $p(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2}2^x}{x!}$

(2). S.D = $\sqrt{\text{variance}} = \sqrt{\lambda} = \sqrt{2}$

(8) If 'x' is poisson varient such that $p(x=2) = 9\,p(x=4) + 90\,p(x=6)$ find (1) λ (2) distribution of 'x'.

Sol:- Let 'x' be a poisson variable $p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

Now, $P(x=2) = 9\,P(x=4) + 90\,P(x=6)$

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = 9\frac{e^{-\lambda}\lambda^{4}}{4!} + 90\frac{e^{-\lambda}\lambda^{6}}{6!}$$

$$\cancel{e^{-\lambda}}\frac{\cancel{\lambda^{2}}}{2!} = \cancel{e^{-\lambda}}\cancel{\lambda^{2}}\left[9\frac{\lambda^{2}}{4!} + 90\frac{\lambda^{4}}{6!}\right]$$

$$\frac{1}{2} = \frac{9\lambda^{2}}{24} + \frac{\cancel{90}\lambda^{4}}{\cancel{720}\,8}$$

$$\frac{1}{2} = \frac{3\lambda^{2}}{8} + \frac{\lambda^{4}}{8}$$

$$\frac{1}{\cancel{2}} = \frac{3\lambda^{2}+\lambda^{4}}{\cancel{8}\,4}$$

$$4 = 3\lambda^{2}+\lambda^{4}$$

$$\lambda^{4}+3\lambda^{2}-4 = 0$$

$$\lambda^{4}+4\lambda^{2}-\lambda^{2}-4 = 0$$

$$\lambda^{2}(\lambda^{2}+4) - (\lambda^{2}+4) = 0$$

$$(\lambda^{2}+4)(\lambda^{2}-1) = 0$$

$\lambda^{2} = -4$ $\quad\quad \lambda^{2} = 1$

$\lambda = \pm 1$

$\lambda = 1$ or $\lambda = -1$, Here we consider $\lambda = 1$ $[\because \lambda > 0$

(2) distribution of x is $p(x=x) = p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$$p(x) = \frac{e^{-1}(1)^{x}}{x!} \quad [\because \lambda = 1$$

(9) Average number of accidents on any date on a national highway is 1.8 determine the probability that the number of accidents are (1) atleast one (2) atmost one.

Sol:- Let 'x' be the no. of accidents on any date on a national highway.

mean, $\lambda = 1.8$

Hence the P.D is $P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$= \frac{e^{-1.8} (1.8)^x}{x!}$

i, P(atleast one) $= P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \frac{e^{-1.8} (1.8)^0}{0!}$

$= 1 - 0.1653 = 0.8347$

ii, $P(x \leq 1) = P(x=0) + P(x=1)$

$= \frac{e^{-1.8} (1.8)^0}{0!} + \frac{e^{-1.8} (1.8)^1}{1!}$

$= 0.1653 + 0.1652 \times 1.8$

$= 0.1653 + 0.2973 = 0.4628.$

10, The Average no. of phone calls/minute coming into a switch board between 2pm and 4 pm is 2.5. Determine the probability that during 1 perticular minute their will be i, 4 or Fewer ii, more than 6 calls.

Sol:- Let 'x' be the no. of phone calls / minute coming into a switch board b/w 2pm and 4 PM

Given mean $\lambda = 2.5$, P.D is $P(x=x) = \frac{e^{-2.5} (2.5)^x}{x!}$

i, $P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$$= \frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!} + \frac{e^{-2.5}(2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!} + \frac{e^{-2.5}(2.5)^4}{4!}$$

$$= e^{-2.5}\left[1 + 2.5 + \frac{6.25}{2} + \frac{15.625}{6} + \frac{39.0625}{24}\right]$$

$$= e^{-2.5}\left[1 + 2.5 + 3.125 + 2.6042 + 1.6276\right]$$

$$= 0.0821 \times 10.8568$$

$P(x \le 4) = 0.8913$

(probability of more than 6 calls)

$$P(x > 6) = 1 - P(x \le 6)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)]$$

$$= 1 - [P(x \le 4) + P(x=5) + P(x=6)]$$

$$= 1 - \left[0.8913 + \frac{e^{-2.5}(2.5)^5}{5!} + \frac{e^{-2.5}(2.5)^6}{6!}\right]$$

$$= 1 - [0.8913 + 0.0668 + 0.0278]$$

$P(x \geq 6) = 0.0141$

11) Suppose 2% of the people on the average of left handed. find

i) The probability of finding none or one left handed

ii) The probability of finding 3 or more left handed.

Sol:- Let 'x' be the no. of people left handed.

Given mean $\lambda = 2\% = \frac{2}{100} = 0.02$

Hence the P.D is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$

i) $P(x \le 1) = P(x=0) + P(x=1)$

$$= \frac{e^{-0.02}(0.02)^0}{0!} + \frac{e^{-0.02}(0.02)^1}{1!}$$

$$= 0.980 \times 1 + 0.980 \times 0.02$$

$$= 0.980 + 0.0196 = 0.9998$$

ii) $P(x \geq 3) = 1 - P(x < 3)$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[0.980 + 0.0196 + \frac{0.0004}{2}\right] = 1 - [0.9998 + 0.0002]$$

$$= 1 - 1 = 0$$

12. 2% of the items of a factory are defective. The items are packed in boxes what is the probability that their will be two defectives in a box of 100 items.

Sol:- Here $n = 100$ and p = probability of defective items

$$p = \frac{2}{100} = 0.2$$

Here mean, $\lambda = np$

$$= 100 \times \frac{2}{100} = 2$$

Hence the p.D is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2}2^x}{x!}$

$$P(x=2) = \frac{e^{-2}(2)^2}{2!} = 0.135 \times \frac{2^2}{2} = 0.2707$$

NOTE :— using Binomial distribution probability

B.D is $P(x) = 100_{C_x} (0.02)^x (0.98)^{100-x}$

$$P(x=2) = 100_{C_2} (0.02)^2 (0.98)^{98}$$

$$= 4950 \times 0.0004 \times 0.1380$$

$$= 0.2734$$

13. A car hire firm has two cars which it hires from day to day. The no. of demands of a car on each day is distributed has a poisson variable with mean 1.5. Calculate the proportion of days on which i, neither car is used ii, some demand is refused

Sol Let 'x' be the no. of demands for a car on each day

Given mean of poison distribution $\lambda = 1.5$

The poison distribution is $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.5}(1.5)^x}{x!}$

(i) P(neither car is used, that means no demand) $= P(X=0)$

$$= \frac{e^{-1.5}(1.5)^0}{0!}$$

$$= 0.2231 \times 1 = 0.2231$$

The number of days in a year there is no demand of cars $= N \times P(x=0)$

$= 365 \times P(x=0)$

$= 365 \times 0.2231$

$= 81.4315 \cong 81$ days

(ii) P(some demand is refused that means no. of demands of car more than two) is

$$P(x>2) = 1-P(x\leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!} \right]$$

$$= 1 - \left[\frac{e^{-1.5}(1)}{1} + \frac{e^{-1.5}(1.5)}{1} + \frac{e^{-1.5}(2.25)}{2} \right]$$

$$= 1 - [0.2231 + 0.3347 + 0.2510]$$

$$P(x>2) = 0.1912$$

The number of days in a year when some demands are refused $= N \times P(x>2)$

$= 365 \times 0.1912$

$= 69.82$

$\cong 70$ days

Normal distribution :— A continuous random variable 'x' is said to have a normal distribution with parameter μ (called mean) and σ^2 (called variance). If its probability density function is given by $f(x, \mu, \sigma) = f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\sigma^2 > 0$

Here μ and σ^2 are the two parameters of N.D

NOTE :— 1) A random variable 'x' with mean 'μ' and variance σ^2 the above function is expressed by $X \sim N(\mu, \sigma^2)$.

2) If $X \sim N(\mu, \sigma^2)$ then $z = \frac{x-\mu}{\sigma}$ is a standard Normal variant with mean "zero" and variance '1'

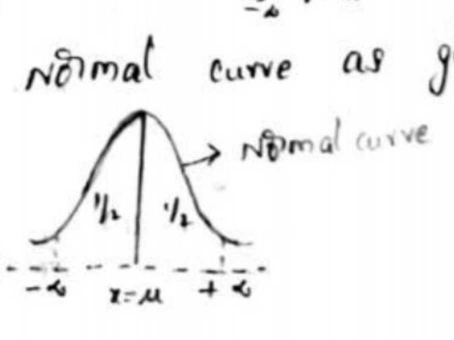
3) The P.D.F of standard N.D in 'z' is given by

$$f(z) \text{ (or) } \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < +\infty$$

(4) The area under the normal curve is unity.

i.e, $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} f(z)\,dz = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz = 1$

(5) The graph of Normal curve as given below.



Mean and variance of N.D :—

Proof:- we know that $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Mean :-

$$\text{mean} = E(x) = \int_{x=-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{x=-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{x=-\infty}^{\infty} x\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + \sigma z$

$\frac{dz}{dx} = \frac{1}{\sigma}$ L.L : $x = -\infty \Rightarrow z = \frac{-\infty-\mu}{\sigma} = -\infty$

$dx = \sigma\, dz$ U.L : $x = \infty \Rightarrow z = \frac{\infty-\mu}{\sigma} = \infty$

substituting these in above integral

$$\text{mean} = \frac{1}{\sqrt{2\pi}\cdot\cancel{\sigma}} \int_{z=-\infty}^{\infty} (\mu+\sigma z)\, e^{-z^2/2} \cdot \cancel{\sigma}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} \mu\, e^{-z^2/2} dz + \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} \sigma z\, e^{-z^2/2} dz$$

$$= \mu \int_{z=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + \sigma \frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} z\, e^{-z^2/2} dz$$

$= \mu \times 1 + \sigma \times 0$ [Since Integrand $z e^{-z^2/2}$ is an odd function]

$$\boxed{\text{mean} = \mu}$$

variance :-

We know that varience $V(x) = E[x - E(x)]^2$

$$= E(x-\mu)^2$$

$$= \int_{x=-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_{x=-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $z = \frac{x-\mu}{\sigma} \Rightarrow x-\mu = \sigma z$

$\frac{dz}{dx} = \frac{1}{\sigma}$

$\underline{L.L}: x = -\infty \Rightarrow z = \frac{-\infty-\mu}{\sigma} = -\infty$

$\underline{U.L}: x = \infty \Rightarrow z = \frac{\infty-\mu}{\sigma} = \infty$

substituting these in above integral

$$V(x) = \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{z=-\infty}^{\infty} (\sigma z)^2 \, e^{-z^2/2} \cdot \cancel{\sigma}\, dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} z^2 e^{-z^2/2}\, dz$$

$$V(x) = \frac{2\sigma^2}{\sqrt{2\pi}} \int_{z=0}^{\infty} z^2 e^{-z^2/2}\, dz \qquad [\because z^2 e^{-z^2/2} \text{ is even function}]$$

let $t = z^2/2 \Rightarrow z^2 = 2t \Rightarrow z = \sqrt{2}\, t^{1/2}$

$\frac{dt}{dz} = z$

$dz = \frac{dt}{z} = \frac{dt}{\sqrt{2}\, t^{1/2}}$

$\underline{L.L}\; z = 0 \Rightarrow t = 0$

$\underline{U.L}\; z = \infty \Rightarrow t = \frac{(\infty)^2}{2} = \infty$

substituting these in above integral

$$V(x) = \frac{2\sigma^2}{\sqrt{2\pi}} \int_{t=0}^{\infty} \cancel{2} t\, e^{-t} \frac{dt}{\sqrt{2}\, t^{1/2}}$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{t=0}^{\infty} \cancel{\sqrt{2}}\, e^{-t} t^{1/2}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{t=0}^{\infty} e^{-t}\, t^{(1/2+1)-1}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{t=0}^{\infty} e^{-t}\, t^{3/2-1}\, dt$$

$[\because \Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$

$\Gamma(n) = (n-1)!$

$\Gamma(n) = (n-1)\Gamma(n-1)$

$\Gamma(1/2) = \sqrt{\pi}\;]$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times \Gamma(3/2)$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times \left(\frac{3}{2}-1\right) \Gamma\left(\frac{3}{2}-1\right)$$

$$= \frac{\cancel{2}\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{\cancel{2}} \times \Gamma(1/2)$$

$$= \frac{\sigma^2}{\cancel{\sqrt{\pi}}} \cdot \cancel{\sqrt{\pi}}$$

$\boxed{\text{variance } V(x) = \sigma^2}$

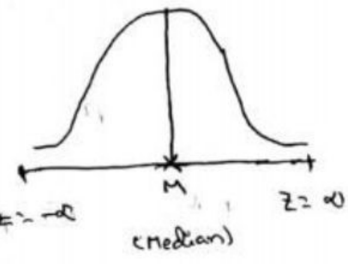
CamScanner

Median of N.D :—

Proof :- we know that $f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

If 'm' is the median of N.D, we have

$$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{+\infty} f(x)\,dx = \frac{1}{2}$$



$$\Rightarrow \int_{-\infty}^{M} f(x)\,dx = \frac{1}{2}$$

$$\Rightarrow \int_{-\infty}^{M} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

$$\Rightarrow \int_{-\infty}^{\mu} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx + \int_{\mu}^{M} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

$$\Rightarrow P(-\infty < X < \mu) + \int_{\mu}^{M} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dn = \frac{1}{2}$$

we have suppose consider median M is μ [$\because$ from the graph of normal curve]

$$\Rightarrow \cancel{\tfrac{1}{2}} + \int_{\mu}^{M} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \cancel{\tfrac{1}{2}}$$

$$\Rightarrow \int_{\mu}^{M} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 0 \qquad [\because \text{median} = \text{mean}]$$

$\boxed{\text{this is possible when } M = \mu}$

Hence for the N.D median = mean = μ

mode of N.D :—

Proof :- _ have $f(\,) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

mode, the value of x for which $f(x)$ is maximum that is mode is the solution of $f'(x)=0$ and $f''(x)<0$

we know that $f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Taking logarithm on B.S

$$\log_e f(x) = \log c - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 \log_e e$$

$$\log f(x) = \log c - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 \quad (1) \qquad [\because \log_e^e = 1$$

Differentiate with respect to 'x'

$$\frac{1}{f(x)} f'(x) = 0 - \frac{1}{2} \times 2\left(\frac{x-\mu}{\sigma}\right) \cdot \frac{1}{\sigma}$$

$$\frac{f'(x)}{f(x)} = -\left(\frac{x-\mu}{\sigma^2}\right)$$

$$f'(x) = -f(x)\left(\frac{x-\mu}{\sigma^2}\right) \quad \text{— } (1)$$

we know that the function $f(x)$ has maximum value when $f'(x)=0$

Now $f'(x) = 0$

$$\Rightarrow -f(x)\left(\frac{x-\mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \frac{x-\mu}{\sigma^2} = 0 \qquad [\because f(x) \neq 0]$$

$$\Rightarrow x - \mu = 0$$

$$\Rightarrow \boxed{x = \mu}$$

Again. Diff (1) with respect to 'x'

$$f''(x) = -\left\{f'(x)\left(\frac{x-\mu}{\sigma^2}\right) + f(x)\left(\frac{1}{\sigma^2}\right)\right\}$$

$$= -\left\{-f(x)\left(\frac{x-\mu}{\sigma^2}\right)\left(\frac{x-\mu}{\sigma^2}\right) + \frac{f(x)}{\sigma^2}\right\}$$

$$f''(x) = -\frac{f(x)}{\sigma^2}\left[1 - \left(\frac{x-\mu}{\sigma}\right)^2\right]$$

At the point $x = \mu$

$$\left[f''(x)\right]_{x=\mu} = \frac{-f(\mu)}{\sigma^2} < 0$$

Here at $x=\mu$ we have $f''(x)<0$ so the function $f(x)$ has maximum value at $x=\mu$

Hence the mode of normal distribution is μ.

Hence the normal distribution mean, median & mode coincide i.e, mean = mode = median

$\therefore$ The N.D is symmetrical.

* <u>mean deviation from mean for N.D</u> :—

By def mean deviation from mean

$$M.D(\text{from mean}) = E|x - \text{mean}|$$

$$= E|x-\mu|$$

$$= \int_{x=-\infty}^{\infty} |x-\mu| \, f(x)\, dx \qquad [\because x \text{ is continuous}$$

$$= \int_{x=-\infty}^{\infty} |x-\mu| \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $\frac{x-\mu}{\sigma} = z$ $\qquad$ L.L $x=-\infty \Rightarrow z = \frac{-\infty-\mu}{\sigma} = -\infty$

$\Rightarrow \frac{dz}{dx} = \frac{1}{\sigma}$ $\qquad$ U.L $x=\infty \Rightarrow z = \frac{\infty-\mu}{\sigma} = \infty$

substituting these in above integral

$$\Rightarrow \int_{z=-\infty}^{\infty} |\sigma z| \frac{1}{\sqrt{2\pi}\cdot\cancel{\sigma}} e^{-z^2/2} \sigma\, dz$$

$$\Rightarrow \frac{\sigma}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} |z|\, e^{-z^2/2}\, dz$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_{z=0}^{\infty} z\, e^{-z^2/2}\, dz \qquad [\because |z| e^{-z^2/2} \text{ is even function}]$$

$$= \sqrt{\frac{2}{\pi}}\cdot\sigma \int_{z=0}^{\infty} z\, e^{-z^2/2}\, dz$$

let $z^2/2 = t \Rightarrow \frac{dt}{dz} = z$ $\qquad$ L.L $z=0 \Rightarrow t = \frac{(0)^2}{2} = 0$

substituting these in above integral $\qquad$ U.L $z=\infty \Rightarrow t = \frac{(\infty)^2}{2} = \infty$

$$= \sqrt{\frac{2}{\pi}}\cdot\sigma \int_{t=0}^{\infty} \cancel{z}\, e^{-t} \frac{dt}{\cancel{z}}$$

$$= \sqrt{\frac{2}{\frac{22}{7}}}\cdot\sigma \int_{0}^{\infty} e^{-t}\, dt$$

$$\Big|\; e^{-\infty} = 0, \quad e^{\infty} = \infty$$

$$= \sqrt{\frac{14}{22}}\cdot\sigma \left[\frac{e^{-t}}{-1}\right]_0^{\infty}$$

$$= \sqrt{\frac{14}{22}}\cdot\sigma \left[\frac{0-1}{-1}\right]$$

$$= 0.79\sigma$$

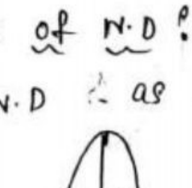
$$\simeq 0.8\sigma \simeq \frac{4}{5}\sigma$$

Mean deviation from mean $\simeq \frac{4}{5}\sigma$

Hence mean deviation from mean of normal distribution is equal to $\frac{4}{5}$ times of standard deviations.

Chief characteristics of N.D :—

i, The graph of N.D is as shown below



2) The curve is bell shaped symmetrical about the line $x = \mu$ i.e $z = 0$

3) The area under the normal curve is unity.

4) Mean, median and mode of the N.D coincide. So normal curve is uni-modal.

5) x-axis assymptote to the curve.

6) Linear combination of independent normal variates is also a normal variate.

7) The points of inflection of the curve are at $x = \mu \pm \sigma$

8) In a N.D, Q.D : M.D : S.D : : 10 : 12 : 15.

NOTE :-

1. $P(0 \le x \le a) = P(a \le x \le 0) = A(a)$

2. $P(a \le x \le b) = A(b) - A(a)$

3. $A(-a) = A(a)$

standard normal distribution :- (S.N.D)

The normal distribution for mean $\mu = 0$ and standard deviation $\sigma = 1$ is known as standard normal Distribution.

NOTE :- Areas under standard normal Distribution

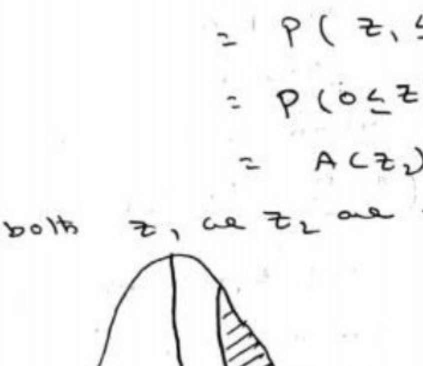
1. $P(x_1 \le x \le x_2) = P\left(\frac{x_1-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{x_2-\mu}{\sigma}\right)$

$$= P(z_1 \le z \le z_2)$$

$$= P(0 \le z \le z_2) - P(0 \le z \le z_1)$$

$$= A(z_2) - A(z_1)$$

Here both z_1 & z_2 are +ve (or) -ve



2. $P(x \geq x_1) = P\left(\frac{x-\mu}{\sigma} \geq \frac{x_1-\mu}{\sigma}\right)$

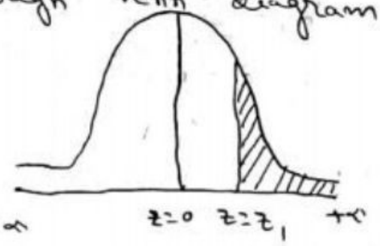
$= P(z \geq z_1)$ where $z = \frac{x-\mu}{\sigma}$

(i) If z_1 is positive then

$P(z \geq z_1) = \frac{1}{2} - P(0 \leq z \leq z_1)$

$= 0.5 - A(z_1)$

Area through venn Diagram



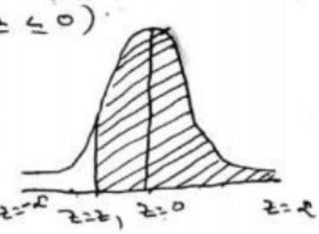
ii If z_1 is negative then

$P(z \geq z_1) = \frac{1}{2} + P(-z_1 \leq z \leq 0)$

$= 0.5 + A(-z_1)$

$= 0.5 + A(z_1)$

Area through venn Diagram



3. $P(x \leq x_1) = P\left(\frac{x-\mu}{\sigma} \leq \frac{x_1-\mu}{\sigma}\right)$

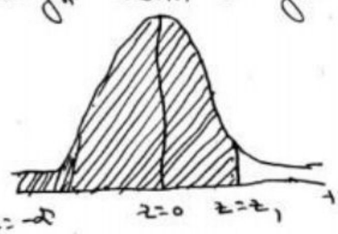
$= P(z \leq z_1)$ where $z = \frac{x-\mu}{\sigma}$

(i) If z_1 is positive then

$P(z \leq z_1) = \frac{1}{2} + P(0 \leq z \leq z_1)$

$= 0.5 + A(z_1)$

Area through venn Diagram



(ii) If z_1 is negative then

$P(z \leq z_1) = \frac{1}{2} - P(z_1 \leq z \leq 0)$

$= \frac{1}{2} - A(z_1)$

Area through venn Diagram

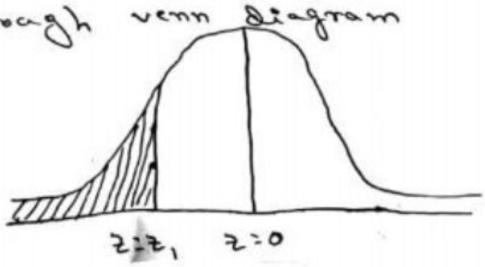
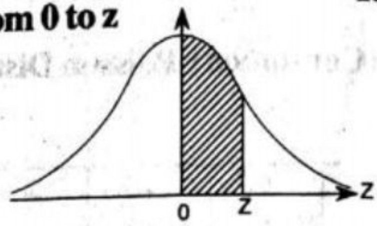


Table – 3

Areas under the Standard Normal Curve from 0 to z

$$Z = \frac{x - \mu}{\sigma}$$



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0938	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1256	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1916	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2649
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4654	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4979	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Examples :-

Imp 1) The mean and standard deviation of a normal variable are 8 & 4 respectively. find

i) $P(5 \le x \le 10)$ ii) $P(x \le 15)$ iii) $P(x \ge 5)$

Sol:- Given mean $\mu = 8$ and standard deviation $(\sigma) = 4$

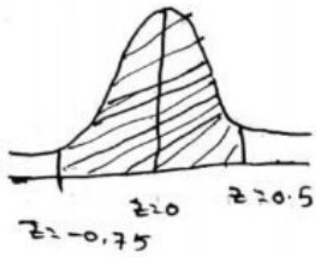
we know that, standard normal variable (SNV)

$$z = \frac{x-\mu}{\sigma} = \frac{x-8}{4}$$

i) $P(5 \le x \le 10)$

when $x=5$, from $z = \frac{5-8}{4} = -\frac{3}{4} = -0.75$ $\left[\because z = \frac{x-\mu}{\sigma}\right.$

when $x=10$, from $z = \frac{10-8}{4} = \frac{2}{4} = 0.5$



$$P(5 \le x \le 10) = P(-0.75 \le z \le 0.5)$$

$= A(0.5) + A(-0.75)$

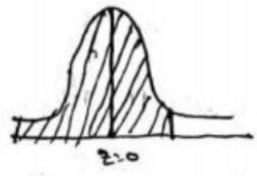
$= A(0.5) + A(0.75)$ [$\because$ by symmetric property

$= 0.1916 + 0.2734$ [from normal distribution Table

$= 0.4650$

ii, $P(x \le 15)$

when $x = 15$, from $z = \frac{15-8}{4} = \frac{7}{4} = 1.75$



$P(x \le 15) = P(z \le 1.75)$

$= 0.5 + A(1.75)$ [∵ from graph]

$= 0.5 + 0.4599$

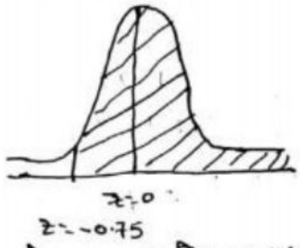
$= 0.9599$ [∵ normal distribution table

iii, $P(x \geq 5)$

when $x = 5$, from $z = \frac{5-8}{4} = \frac{-3}{4} = -0.75$

$P(x \geq 5) = P(z \geq -0.75)$

$= \cancel{?} + P(-0.75 \le z \le 0)$



$= 0.5 + A(-0.75)$

$= 0.5 + A(0.75)$ [∵ from normal distribution table

$= 0.5 + 0.2734$

$= 0.7734$

Imp

4, If x is a normal variate is mean 30 and S.D is 5 find the probabilities that

i, $P(26 \le x \le 40)$

ii, $x \geq 45$ iii, $|x-30| > 5$

Sol:- Given mean $\mu = 30$

S.D $\sigma = 5$

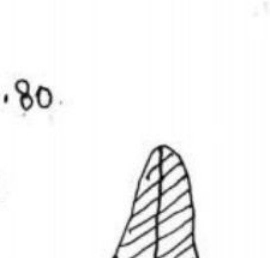
We know that, S.N.V, $z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$

i, $P(26 \le x \le 40)$

when $x = 26$, from $z = \frac{26-30}{5} = \frac{-4}{5} = -0.80$

when $x = 40$, from $z = \frac{40-30}{5} = \frac{10}{5} = 2$

$P(26 \le x \le 40) = P(-0.80 \le z \le 2)$



$= A(2) + A(-0.80)$

$= A(2) + A(0.8)$ [∵ by symmetry]

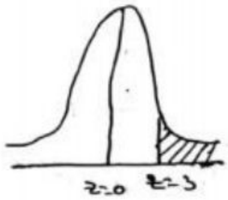
$= 0.4772 + 0.2881$ [∵ from N.D table

$= 0.7653$

(ii) $P(x \geq 45)$

when $x = 45$, from $z = \frac{45-30}{5} = \frac{15}{5} = 3$

$P(x \geq 45) = P(z \geq 3)$



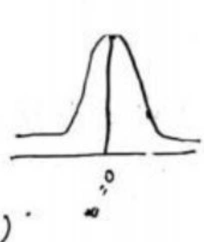
$= \frac{1}{2} - P(0 \leq z \leq 3)$

$= 0.5 - A(3)$

$= 0.5 - 0.4987$ [∵ normal distribution table]

$= 0.0013$

(iii) $P(|x-30| > 5) = 1 - P(|x-30| \leq 5)$



$= 1 - P(-5 \leq x-30 \leq 5)$

$= 1 - P(-5+30 \leq x - \cancel{30} + \cancel{30} \leq 5+30)$

$= 1 - P(25 \leq x \leq 35)$

Now, $P(25 \leq x \leq 35)$

when $x = 25$, from $z = \frac{25-30}{5} = \frac{-5}{5} = -1$

when $x = 35$, from $z = \frac{35-30}{5} = \frac{5}{5} = 1$

$P(25 \leq x \leq 35) = P(-1 \leq z \leq 1)$

$= A(1) + A(-1)$

$= A(1) + A(1)$ [∵ N.D table

$= 0.3413 + 0.3413$

$= 0.6826$

Now

$P(|x-30| > 5) = 1 - P(25 \leq x \leq 35)$

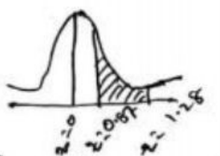
$= 1 - 0.6826$

$= 0.3174$

3. Find the probability that a random variable having the standard N.D will take on a value between 0.87 & 1.28

Sol:- Given $z_1 = 0.87$ and $z_2 = 1.28$

$\therefore P(0.87 \le z \le 1.28) = A(1.28) - A(0.87)$



$= 0.3997 - 0.3078$ [∵ from N.D. table

$= 0.0919.$

(4) If x is a normal variate find the area 1) to the left of $z = -1.78$ 2) to the right of $z = -1.45$

3) corresponding to $-0.8 \le z \le 1.53$.

4) to the left of $z = -2.52$ and to the right of $z = 1.83$.

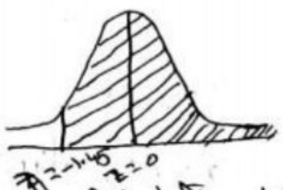
Sol:-

(i) To the left of $z = -1.78$

$P(z \le -1.78) = \frac{1}{2} - P(-1.78 \le z \le 0)$
$= \frac{1}{2} - A(-1.78) = \frac{1}{2} - A(1.78)$
$= 0.0375$

(ii) To the right of $z = -1.45$

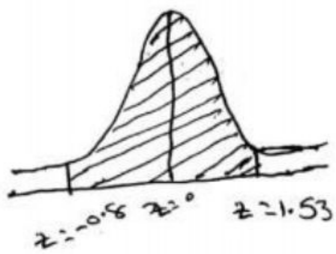
$P(z \ge -1.45) = \frac{1}{2} + P(-1.45 \le z \le 0)$
$= \frac{1}{2} + A(-1.45)$
$= \frac{1}{2} + A(1.45)$



$= 0.5 + 0.4265$ [∵ Normal distribution table

$= 0.9265$

(iii) $P(-0.8 \le z \le 1.53) = A(1.53) + A(-0.8)$
$= A(1.53) + A(0.8)$
$= 0.4370 + 0.2881$
$= 0.7251$

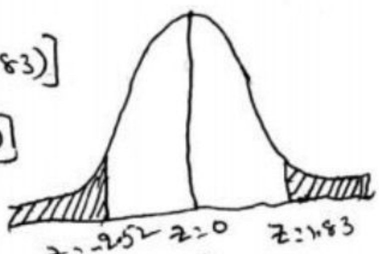


$P(z \le -2.52) + P(z \ge 1.83)$

$= [\frac{1}{2} - P(-2.52 \le z \le 0)] + [\frac{1}{2} - P(0 \le z \le 1.83)]$

$= [0.5 - A(-2.52)] + [0.5 - A(1.83)]$

$= [0.5 - A(2.52)] + [0.5 - A(1.83)]$



$= [0.5 - 0.4941] + [0.5 - 0.4664]$ [from N.D table]

$= 0.0059 + 0.0336$

5, If the masses of 300 students are normally distributed with mean 68 kgs and S.D 3 kgs How many students have masses i, ≤ 64 kg ii, > 72 kgs iii, b/w 65 & 71 kgs inclusive.

Sol:- Given, let x denotes the masses of students

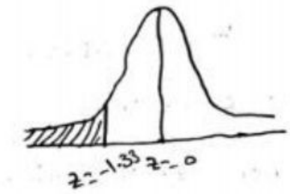
$\therefore$ mean, $\mu = 68$ kgs

$\sigma = 3$ kgs

W.K.T S.D, $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-68}{3}$

i, $P(x \le 64)$ $\quad x = \frac{64-68}{3} = \frac{-4}{3} = -1.33$



$P(x \le 64) = P(z \le -1.33) = \frac{1}{2} - A(-1.33) = \frac{1}{2} - A(1.33) = 0.5 - 0.4082$

$= 0.0918$

No. of students with < 64 kgs, $= 300 \times$ Prob.

$= 300 \times 0.0918$

$= 27.36$

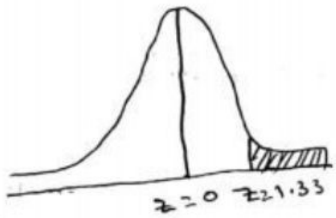
$\cong 27$

ii, $P(x > 72)$

where $z = \frac{72-68}{3} = \frac{4}{3} = 1.33$

$P(x > 72) = P(z > 1.33) = \frac{1}{2} - P(0 < z < 1.33)$

$= \frac{1}{2} - A(1.33) = 0.0918$



No. of students with more than 72, $= 300 \times$ prob

$= 300 \times 0.0918$

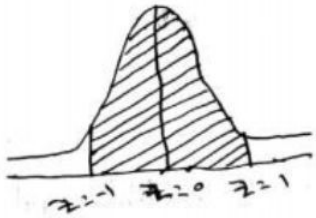
$= 27.36$

$\cong 27$

iii $P(65 \le x \le 71)$

where $x = 65$, then $z = \frac{65-68}{3} = \frac{-3}{3} = -1$

$x = 71$, then $z = \frac{71-68}{3} = \frac{3}{3} = 1$



$P(65 \le x \le 71) = P(-1 \le z \le 1)$

$= A(1) + A(-1)$

$= A(1) + A(1)$

$= 0.3413 + 0.3413$

$= 0.6826$

$\therefore$ No. of students b/w 65 & 71 kgs, $= 300 \times 0.6826$

$= 204.78$

$\cong 205$

Q62) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and S.D 11%. Determine

i, How many students marks b/w 60% and 70%.

ii, How many students got marks above 90%.

Sol: let the variable 'x' denotes the student marks,

Given, $\mu = 78\%$

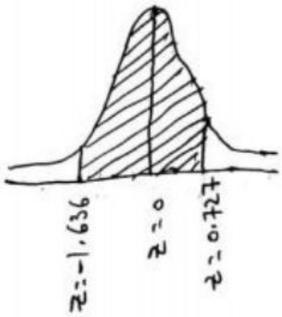
$\sigma = 11\%$

w.k.T S.D.V, $z = \frac{x-78}{11}$

i, $P(60 \le x \le 70)$

where $x = 60$, then $z = \frac{60-78}{11} = -\frac{18}{11} = -1.636$

$x = 70$, then $z = \frac{70-78}{11} = \frac{-8}{11} = 0.727$



$P(-1.636 \le z \le 0.727) = A(0.727) + A(-1.636)$

$= A(0.727) + A(1.636)$

$= 0.2671 + 0.4495$

$= 0.7166$

No. of students whoes marks lies b/w 60 & 70% is,

$= 1000 \times prob$

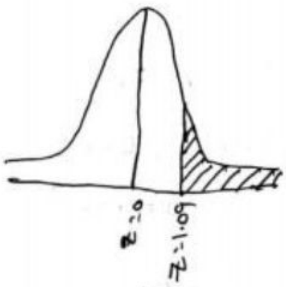
$= 1000 \times 0.7166$

$= 716.6$

$\cong 717$

ii, $P(x > 90)$

where $x > 90$, then $z = \frac{90-78}{11} = \frac{12}{11} = 1.09$



$P(z > 1.09) = \frac{1}{2} - A(1.09)$

$= \frac{1}{2} - 0.3621$

$= 0.1379$

No. of students got marks above 90% is, $= 1000 \times 0.1879$

$= 137.9$

$\approx 138\%$

(7) Given that mean heights of students in a class 158cm with a S.D 20cm. find how many students heights lies b/w 150cm and 170cm, if there are 100 students in the

class.

Sol: let 'x' denotes the heights of students

Given mean $\mu = 158$ cm

and S.D $\sigma = 20$ cm

W.K.T $z = \frac{x-158}{20}$

$P(150 < x < 170)$

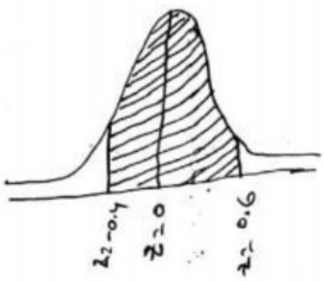
when $x = 150$, then $z = \frac{150-158}{20} = \frac{-8}{20} = -0.4$

$x = 170$, then $z = \frac{170-158}{20} = \frac{12}{20} = 0.6$

$P(150 < x < 170) = P(-0.4 < z < 0.6)$

$= A(0.6) + A(-0.4)$
$= A(0.6) + A(0.4)$
$= 0.2258 + 0.1554$

$= 0.3812$



No. of students in the class whose heights lies b/w 150 & 170 cm $= 100 \times 0.3812$
$= 38.12$
$\simeq 38$

In the N.D 7% of the items are under 35 and 89% are under 63. Find the mean and S.D of the distribution.

Sol:- let μ and σ be the mean and S.D

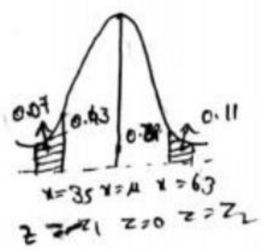
Given, $P(x < 35) = 0.07$ and
$P(x < 63) = 0.89$

$P(x > 63) = 1 - P(x < 63)$

$= 1 - 0.89 = 0.11$

The points $x = 35$ and $x = 63$ are shown in the following figure,

$0.07 - 0.5 = -0.43$
$0.11 - 0.5 = 0.39$



when $x=35$, from $z=\frac{35-\mu}{\sigma}=-z_1$ (say) — (1)

$x=63$, from $z=\frac{63-\mu}{\sigma}=+z_2$ (say) — (2)

from the figure, we have.

$P(-z_1<z<0)=0.43$ and

$P(0<z<z_2)=0.39$

From normal tables,

where area 0.43 then $z_1=-1.48$ and

when area 0.39 then $z_2=1.23$.

from (1), we have, $\frac{35-\mu}{\sigma}=-1.48$

$35-\mu=-1.48\sigma$

$35=\mu-1.48\sigma$ — (3)

from (2), we have $\frac{63-\mu}{\sigma}=1.23$

$63-\mu=1.23\sigma$

$63=\mu+1.23\sigma$ — (4)

Solving (3) & (4) we get

$$\mu-1.48\sigma=35$$
$$\underline{\mu+1.23\sigma=63}$$
$$-2.71\sigma=-28$$

$2.71\sigma=28$

$\sigma=\frac{28}{2.71}=10.332$

from (3) $\Rightarrow 35=\mu-(1.48)(10.332)$

$=\mu-15.2913$

$\mu=35+15.2913$

$\mu=50.29$

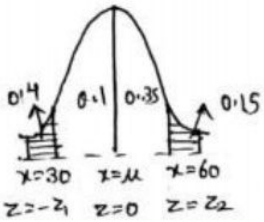
(9) The marks obtained in statistics in a certain Examination found to be N.D if 15% of the students ≥ 60 marks & 40% <30 marks find mean and S.D

Sol:- let μ be the mean and σ be the S.D

Given $P(x\geq 60)=15\%=0.15$ and

$P(x<30)=0.4$

The points $x=60$ and $x=30$ are shown in the following figure.



when $x=60$, then $z = \frac{60-\mu}{\sigma} = +z_2$ (say) — (1)

$x=30$, then $z = \frac{30-\mu}{\sigma} = -z_1$ (say) — (2)

From the figure we have $P(-z_1 < z < 0) = 0.1$ and $P(0 < z < z_2) = 0.35$

From the normal tables, when area 0.1 then $z_1 = -0.26$ and when area 0.35 then $z_2 = 1.04$

from (1), we have $\frac{60-\mu}{\sigma} = 1.04$

$$60-\mu = 1.04\sigma$$

$$60 = \mu + 1.04\sigma \quad — (3)$$

from (2), we have $\frac{30-\mu}{\sigma} = -0.26$

$$30-\mu = -0.26\sigma$$

$$30 = \mu - 0.26\sigma \quad — (4)$$

Solving (3) & (4) we get

$$60 = \mu + 1.04\sigma$$
$$30 = \mu - 0.26\sigma$$
$$30 = 1.30\sigma$$
$$\sigma = \frac{30}{1.3} = 23.07$$

(3) $\Rightarrow 60 = \mu + 1.04(23.07)$

$$60 = \mu + 23.99$$
$$\mu = 60 - 23.99$$
$$\mu = 35.99$$

$\therefore \sigma = 23.07$, $\mu = 35.99$

10, calculate the mean and S.D of a normal distribution in which 31% of the items are under 45 and 8% are over 64

Sol:- let μ be the mean and S.D be the σ,

Given, $P(x<45) = 0.31$ and $P(x>64) = 0.08$

The points $x=45$ and $x=64$ are shown in the following figure.

when $x=45$, then $z = \frac{45-\mu}{\sigma} = z_2$ (say) — (1)

when $x=64$, then $z = \frac{64-\mu}{\sigma} = -z_1$ (say) — (2)

From the figure we have $P(z_1 < z < 0) = 0.42$ and

$P(0 < z < z_2) = 0.19$

from the Normal tables, when area 0.42 then $z_1 = -1.41$ and when area 0.19 then $z_2 = 0.05$ 0.50

from (1), we have $\Rightarrow \frac{45-\mu}{\sigma} = 0.05$

$$45-\mu = 0.05\sigma$$

$$45 = \mu + 0.05\sigma \quad \text{--- (3)}$$

from (2), we have $\Rightarrow \frac{64-\mu}{\sigma} = -1.41$

$$64-\mu = -1.41\sigma$$

$$64 = \mu - 1.41\sigma \quad \text{--- (4)}$$

Solving (3) & (4) we get

$$45 = \mu + 0.05\sigma$$

$$\underline{\begin{matrix} 64 = \mu - 1.41\sigma \\ - \quad - \quad + \end{matrix}}$$

$$-19 = 1.46\sigma$$

$$\sigma = \frac{-19}{1.46} = 13.01$$

(3) $\Rightarrow 45 = \mu + (0.05)(-13.01)$

$$45 = \mu - 0.65$$

$$\mu = 45 + 0.65$$

$$\mu = 45.65$$

$$\therefore \sigma = 13.01, \quad \mu = 45.65$$

The Normal appropriation to the B.D :— Let x be a Binomial random variable with np and S.D $\sqrt{npq}$ Then the limiting form of the Standard N.D of $z = \frac{x-np}{\sqrt{npq}}$ as $n \to \infty$

Suppose the No. of sucess 'x' ranges from x_1 to x_2. Let z_1 & z_2 be the values of z, corresponding to x_1 & x_2 of X respectively. Then $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$ and this can be determine by using normal tables. where $z_1 = \frac{(x_1 - \frac{1}{2}) - np}{\sqrt{npq}}$

$$z_2 = \frac{(x_2 + \frac{1}{2}) - np}{\sqrt{npq}}$$

1, Find the probability of getting 1 or 4 or 5 or 6 in throwing a die 5 to 7 times among 9 trials using
i, Binomial distribution ii, N.D

Sol:- Given $n = 9$ (no. of trials)

$p = P$ (getting a 1 or 4 or 5 or 6 in throwing a die)

$$P = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\therefore P = \frac{4}{6} = \frac{2}{3} \text{ and } q = 1 - P = 1 - \frac{2}{3} = 1/3$$

Hence the B.D is , $P(x) = n_{c_x} p^x q^{n-x}$

$$= 9_{c_x} (2/3)^x (1/3)^{9-x}$$

i, using B.D, $P(5 \le x \le 7) = P(x=5) + P(x=6) + P(x=7)$

$$= \left[9_{c_5} (2/3)^5 (1/3)^4 + 9_{c_6} (2/3)^6 (1/3)^3 + 9_{c_7} (2/3)^7 (1/3)^2 \right]$$

$$= 126 \times 0.1317 \times 0.0123 + 84 \times 0.0878 \times 0.0370 + 36 \times 0.0585 \times 0.1111$$

$$= 0.2041 + 0.2729 + 0.2340$$

$$= 0.7110$$

ii, using N.D, $n=9$, $p = 2/3$, $q = 1/3$, $x_1 = 5$, $x_2 = 7$

$P(5 \le x \le 7)$

W.K.T $z_1 = \dfrac{(5 - 1/2) - \overset{3}{\cancel{9}} \times 2/\cancel{3}}{\sqrt{9 \times 2/3 \times 1/3}}$

$$= \frac{\left(\frac{10-1}{2}\right) - 6}{\sqrt{2}}$$

$$= \frac{\frac{9}{2} - 6}{\sqrt{2}} = \frac{9-12}{2\sqrt{2}} = \frac{-3}{2\sqrt{2}} = \frac{-3}{2.8284}$$

$$= -1.0607$$

$z_2 = \dfrac{(7 + 1/2) - \overset{3}{\cancel{9}} \times 2/\cancel{3}}{\sqrt{9 \times 2/3 \times 1/3}} = \dfrac{\frac{14+1}{2} - 6}{\sqrt{2}}$

$$= \frac{\frac{15}{2} - 6}{\sqrt{2}} = \frac{15-12}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} = 1.0607$$

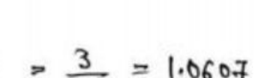
Now, $P(5 < x < 7) = P(z_1 < z < z_2)$

$$= P(-1.0607 < z < 1.0607)$$

$$= A(1.06) + A(-1.06)$$

$$= A(1.06) + A(1.06)$$

$$= 0.3554 + 0.3554 = 0.7108$$



2, 8 coins are tossed together find the probability of getting 1 to 4 heads in a single toss.

Sol:- Here, n = no. of coins = 8

p = P(getting 1 to 4 heads) = $1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

We have, $z_1 = \dfrac{(1 - 1/2) - 8 \times 1/2}{\sqrt{8 \times 1/2 \times 1/2}} = \dfrac{\frac{2-1}{2} - 4}{\sqrt{2}}$

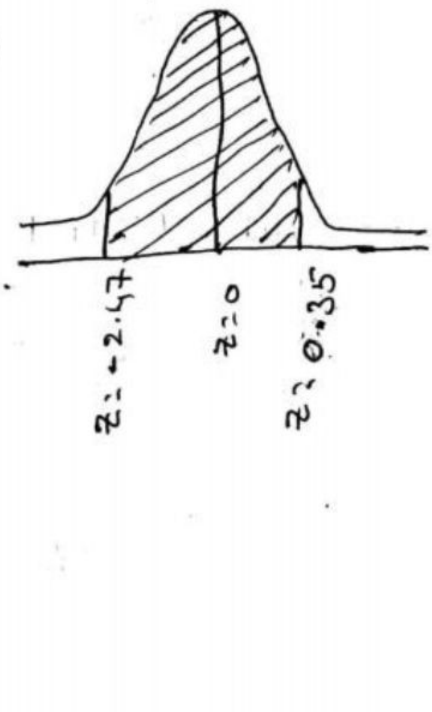
$$= \frac{\frac{1}{2} - 4}{\sqrt{2}} = \frac{1-8}{2\sqrt{2}} = \frac{-7}{2.8284} = -2.4749$$

$$z_2 = \frac{(4+1/2) - 8 \times 1/2}{\sqrt{8 \times 1/2 \times 1/2}}$$

$$= \frac{\frac{8+1}{2} - 4}{\sqrt{2}}$$

$$= \frac{\frac{9}{2} - 4}{\sqrt{2}} = \frac{9-8}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} = 0.3536$$

$$P(z_1 < z < z_2) = P(-2.4749 < z < 0.3536)$$



$$= A(0.3536) + A(-2.4749)$$

$$= A(0.3536) + A(2.4749)$$

$$= 0.1368 + 0.4932$$

$$= 0.6300 \quad //$$