# CHAPTER - 11

# PROJECTIONS OF SOLIDS

## 11.1 INTRODUCTION

A solid is a three dimensional object having length, breadth and thickness. In engineering practice, one often comes across solids bounded by simple or complex geometric surfaces. To represent a solid in orthographic projections, the number and types of views necessary will depend upon the type of solid and its orientation with respect to the principal planes of projection. The applications of auxiliary planes are also considered here.

## 11.2 TYPES OF SOLIDS

Solids may be classified as (i) polyhedra and (ii) solids of revolution.

### 11.2.1 Polyhedra

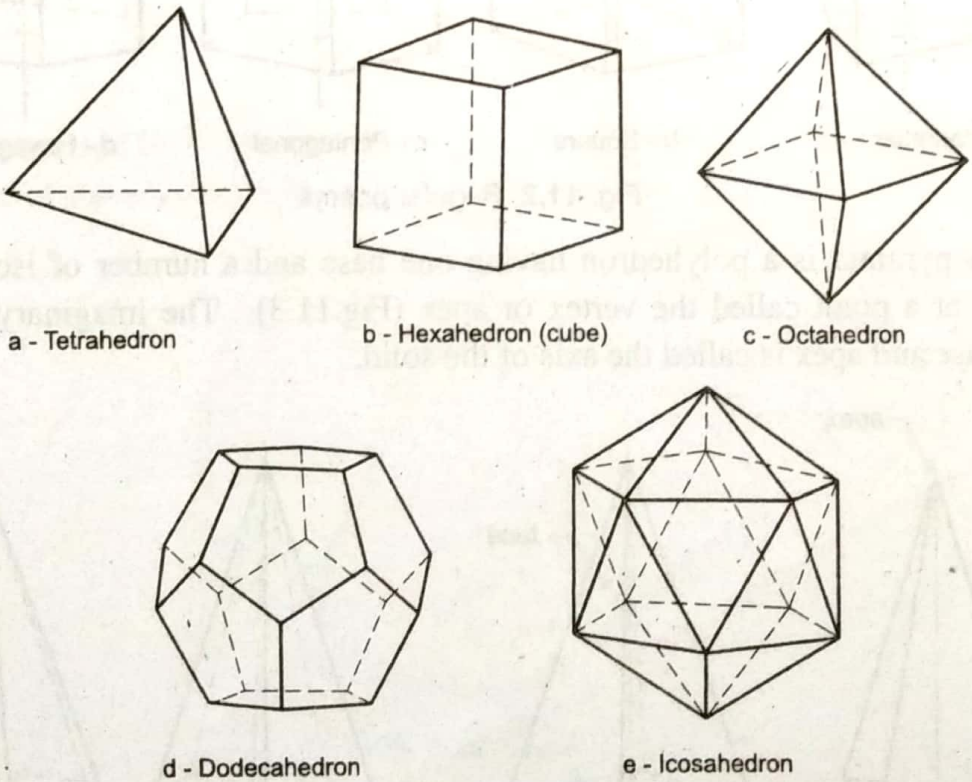



**Fig. 11.1** Regular polyhedra

A polyhedron is defined as a solid bounded by plane surfaces called faces. There are two types of polyhedra: (i) Regular and (ii) irregular or oblique polyhedra.

### 11. 2. 1. 1 *Regular polyhedra*

A regular polyhedron is a solid bounded by plane surfaces, which are equal and regular. The examples are: (i) Tetrahedron, (ii) hexahedron, (iii) octahedron, (iv) dodecahedron and (v) icosahedron.

*Tetrahedron* It has four equal faces, each an equilateral triangle (Fig. 11.1a).

*Hexahedron or cube* It has six equal faces, each a square (Fig. 11.1b).

*Octahedron* It has eight equal faces, each an equilateral triangle (Fig. 11.1c).

*Dodecahedron* It has twelve equal faces, each a regular pentagon (Fig. 11.1d).

*Icosahedron* It has twenty equal faces, each an equilateral triangle (Fig. 11.1e).

There are two more categories of polyhedra, namely (i) prisms and (ii) pyramids.

***Prism*** A prism is a polyhedron having two equal ends or bases, parallel to each other. The two bases are joined by faces, which are rectangles (Fig.11.2). The imaginary line joining the centres of the bases is called the axis of the solid.

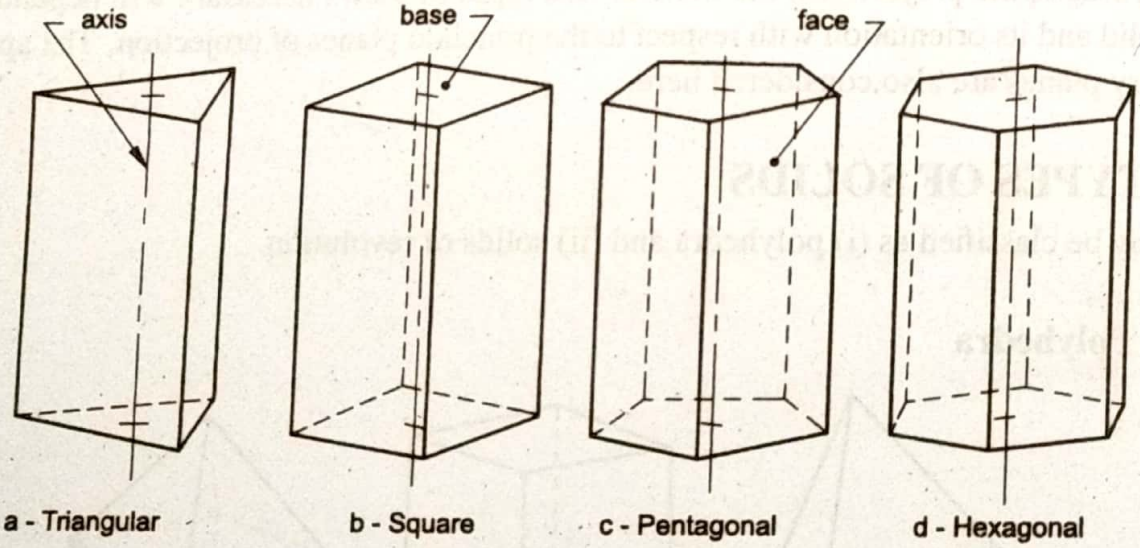



**Fig. 11.2** Regular prisms

***Pyramid*** A pyramid is a polyhedron having one base and a number of isosceles triangular faces, meeting at a point called the vertex or apex (Fig.11.3). The imaginary line joining the centre of the base and apex is called the axis of the solid.

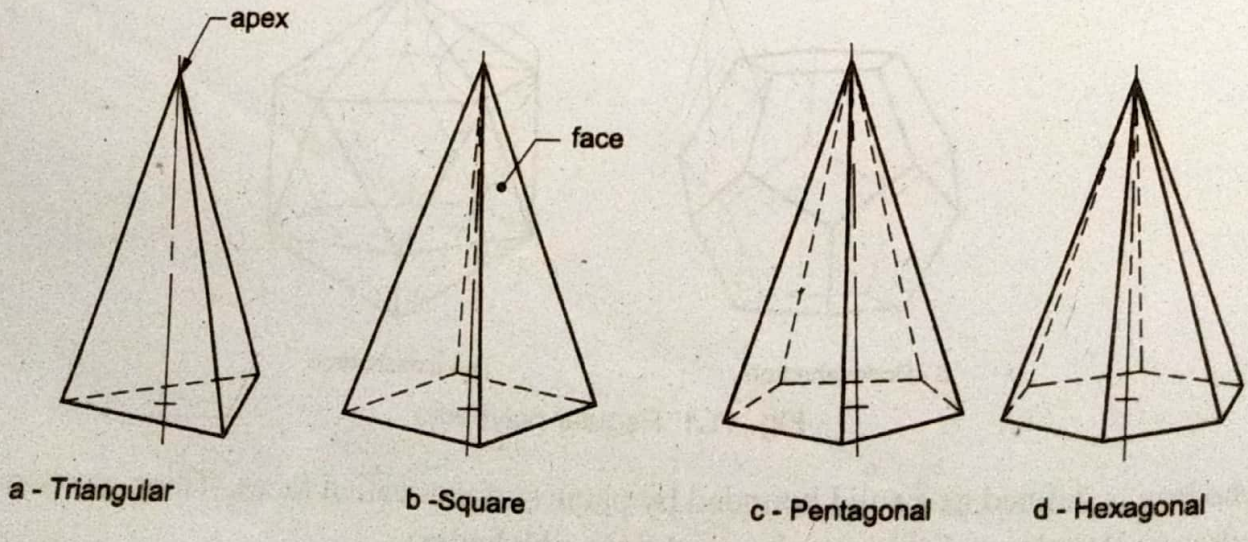



**Fig. 11.3** Regular pyramids

A pyramid or prism is said to be regular when the axis is perpendicular to the base. Both pyramids and prisms are named according to the shape of the base, viz., triangular pyramid/prism, pentagonal pyramid/prism and so on.

### 11.2.2 Solids of revolution

Solids of revolution are obtained or generated by rotating a plane figure about one of its edges. Examples are: (i) cylinder, (ii) cone and (iii) sphere. Solids of revolution, viz., cylinders and cones, may also be classified as: (i) regular and (ii) oblique.

#### 11. 2. 2. 1 *Regular solids of revolution*

***Cylinder*** A cylinder is generated by rotating a rectangle about one of its edges. The lateral surface is connected at its either end by two circular bases (Fig. 11.4).

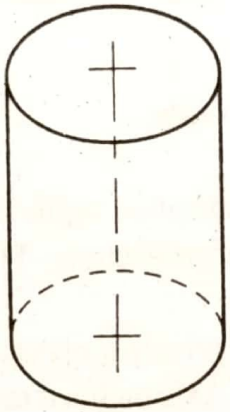


**Fig. 11.4** Cylinder

***Cone*** A cone is generated by rotating a right angled triangle about one of its perpendicular sides. The lateral surface of the cone is connected by a circular base (Fig.11.5).

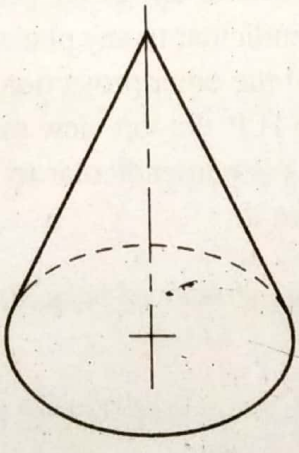


**Fig. 11.5** Cone

A cylinder or a cone is said to be regular when the axis is perpendicular to the base. The lines drawn on the surface of a cylinder and parallel to the axis, are known as generators. The length of a generator is equal to the height of the cylinder. A line drawn from the vertex to any point on the base of a cone is also known as a generator, whose length is equal to the slant height of the cone.

***Sphere*** A sphere is also a solid of revolution generated by rotating a semi-circle about its diameter (Fig. 11.6). The mid-point of the diameter is the centre of the sphere. All points on the surface of a sphere are equidistant from the centre.

Cylinders and cones may also be termed as solids with single curved surfaces, whereas a sphere has a double curved surface.

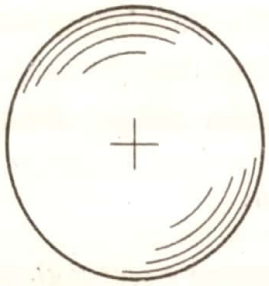


**Fig. 11.6** Sphere

## 11.3 TWO-VIEW DRAWINGS

The position of a solid in space may be specified by the location of either the axis, edges, diagonals or surfaces, with the principal planes of projection. The following are some of the positions of solids:

(i) Axis perpendicular to one of the principal planes, (ii) Axis inclined to one of the principal planes and parallel to the other and (iii) Axis inclined to both the principal planes.

### 11.3.1 Axis perpendicular to one of the principal planes

When the axis of a solid is perpendicular to one of the principal planes, it is parallel to the other. Also, when the axis of a solid is perpendicular to any plane, the projection on that plane, will show the true shape and size of its base and the other projection will reveal the true length of the solid. So, when the axis is perpendicular to H.P, the top view must be drawn first and the front view is then projected from it. When the axis is perpendicular to V.P, the front view must be drawn first and then the top view is projected from it.

**Problem 1** *Draw the projections of a cylinder of base 30 diameter and axis 50 long, when it is resting on H.P on one of its bases.*

(Aug/Sep 2008, JNTU)

***Construction (Fig. 11.7)***

1. Draw a circle of 30 diameter, representing the top view of the cylinder.
2. Project the front view, which is a rectangle of height 50.

The bottom base in the front view, coincides with xy, as the solid is resting on H.P on one of its bases.

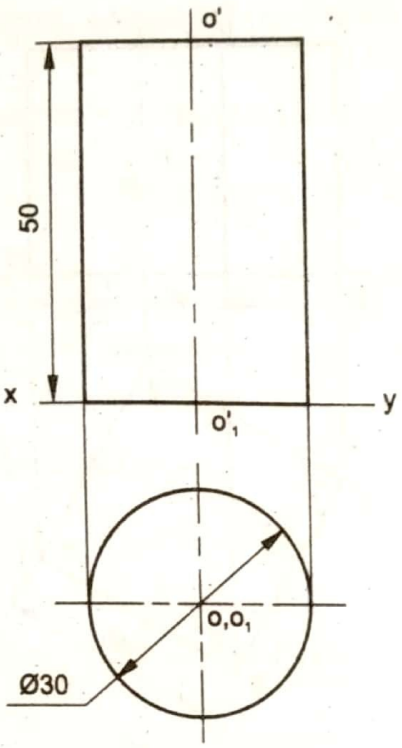



Fig. 11.7

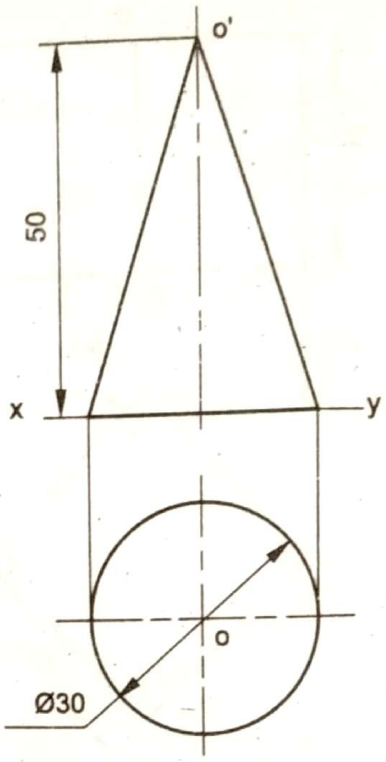



Fig. 11.8

**Problem 2** *Draw the projections of a cone of base 30 diameter and axis 50 long, when it is resting on H.P on its base.* (Aug/Sep 2008, 2010, JNTU)

***Construction (Fig. 11.8)***

1. With centre o, draw a circle of 30 diameter; representing the top view of the cone.
2. Through o, draw a projector and locate the apex o′, at 50 above xy.
3. Obtain the front view which is a triangle, making the base coinciding with xy.

**Problem 3** *A cube of 40 side is resting with a face on H.P such that, the vertical faces are equally inclined to V.P. Draw, its projections.* (May/June 2008, 2010, JNTU)

***Construction (Fig. 11.9a)***

1. Draw a square abcd of side 40 and with its sides making 45° with xy. This represents the top view of the cube.
2. Project the front view, keeping the height equal to 40.

It may be noted from the front view that the front view d′4′ of the invisible edge D4 coincides with the front view b′2′ of the visible edge B2.

Figure 11.9b shows the projections of the cube, when one of its vertical faces is inclined at 30° to V.P. It may be noted that the edge D4 in the front view (d′4′) is invisible and hence, represented by dotted line.

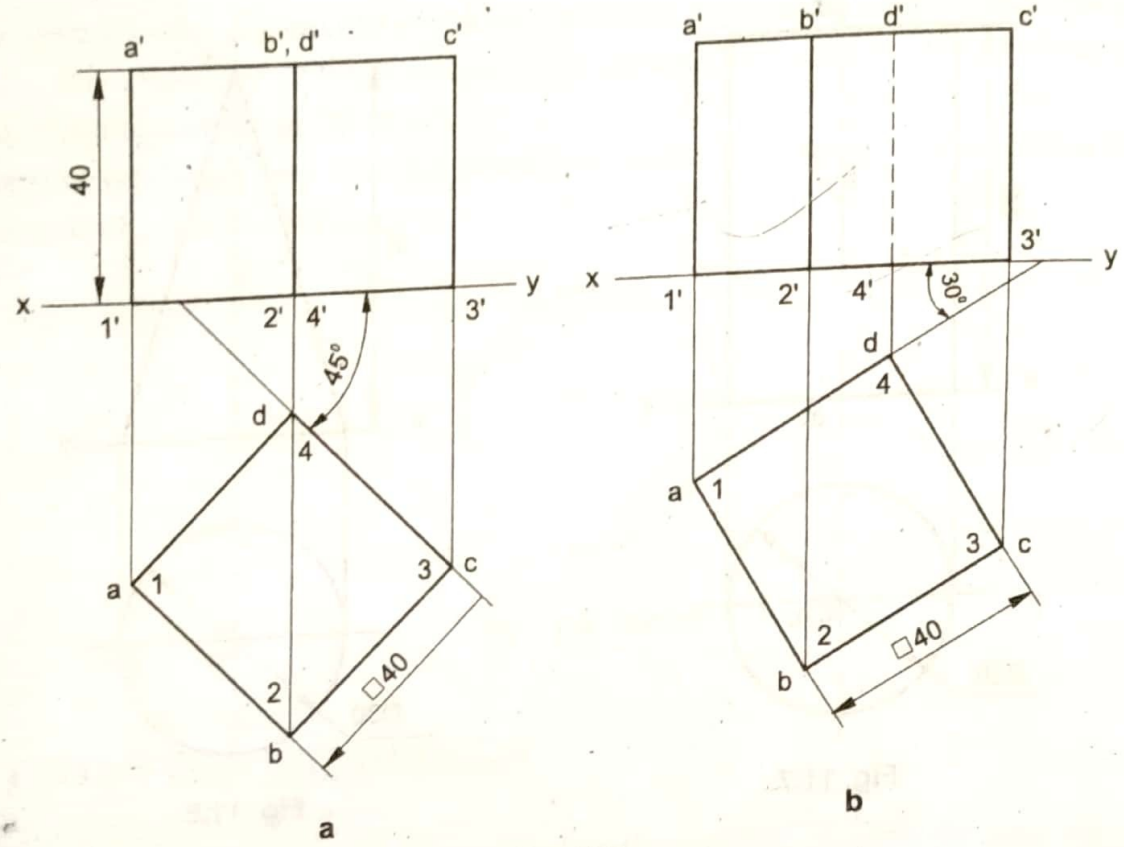



Fig. 11.9

**Problem 4** *A triangular prism of base 30 side and axis 50 long, is resting on H.P on one of its bases, with a face perpendicular to V.P. Draw the projections of the solid.*

(May/June 2008, JNTU)

***Construction (Fig. 11.10a)***

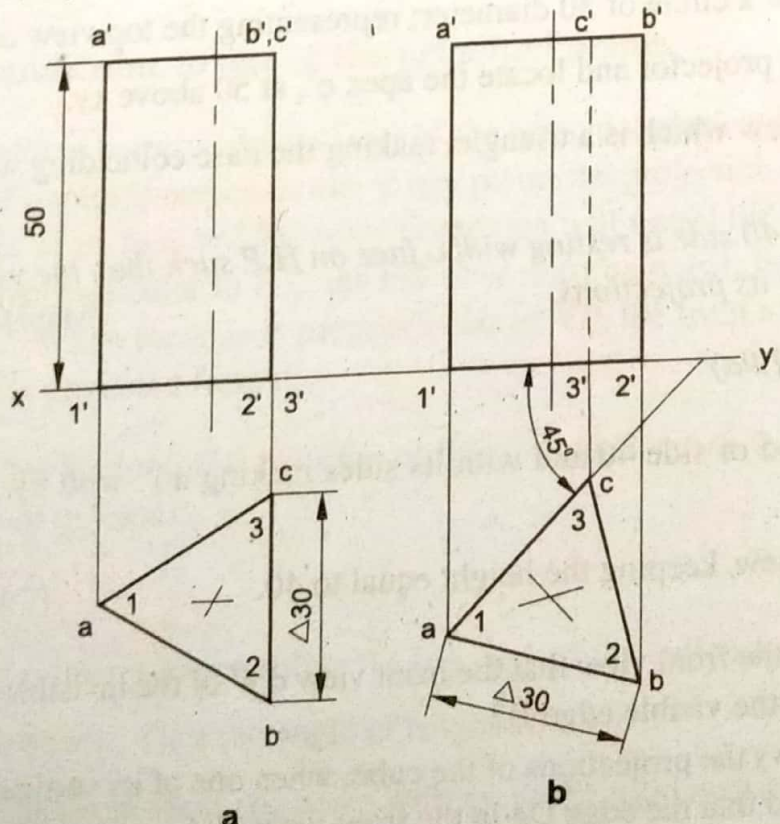



Fig. 11.10

1. Draw the top view, an equilateral triangle of 30 side such that, one edge of it is perpendicular to xy.
2. Project the front view, keeping its height equal to 50 and the bottom edge coinciding with xy.

Figure 11.10b shows the projections of the above solid, when one of its faces is inclined at 45° with V.P.

**Problem 5** *A pentagonal pyramid of base 25 side and axis 60 long, is resting on an edge of the base on H.P. Draw the projections of the pyramid, when its axis is perpendicular to V.P and the base is at 15 from V.P.*

***Construction (Fig. 11.11a)***

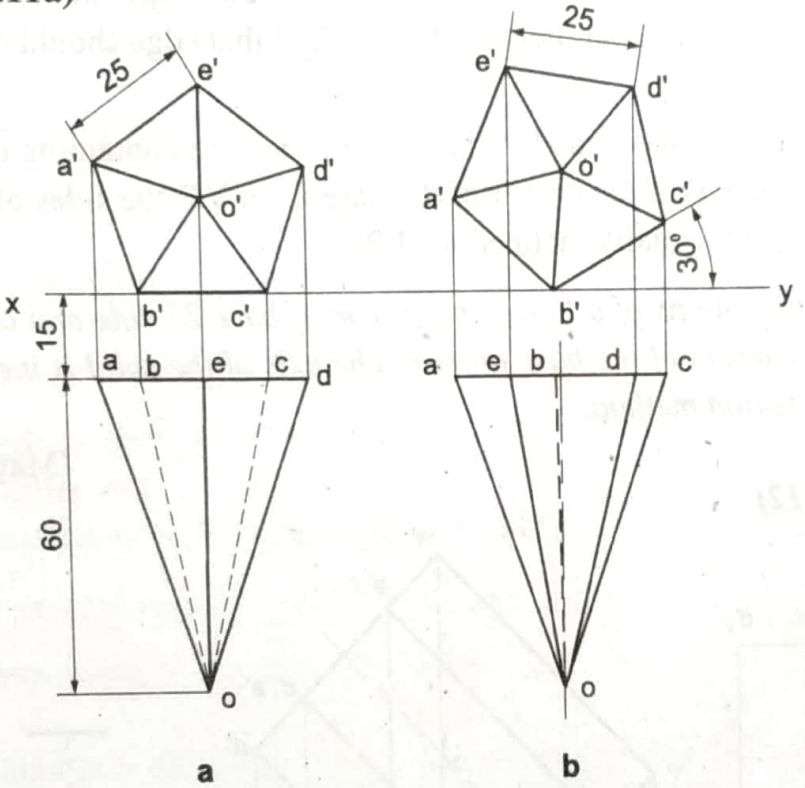



**Fig. 11.11**

1. Draw the front view of the base a′b′c′d′e′, a pentagon of side 25, with the side b′c′ coinciding with xy.
2. Project the top view such that, the top view of the base abcde is at 15 from xy and the apex o is at 60 from abcde.

**NOTE** For the given position of the solid, when it is seen from above, the slant edges OB and OC are invisible. Hence, the lines, ob and oc are represented by dotted lines in the top view.

Figure 11.11b shows the projections of the pentagonal pyramid, when it is resting on H.P on a base corner, with an edge of the base containing that corner, making 30° with H.P.

### 11.3.2 Axis inclined to one of the principal planes and parallel to the other

When the axis of the solid is inclined to any principal plane, the final projections are drawn in two stages. In the first stage, the axis of the solid is assumed to be perpendicular to the principal plane,

to which it is actually inclined and the views are drawn. In the second stage, one of the views is redrawn to suit the given condition and the other view is projected from it. This method is known as change of position method.

In the second stage, instead of reconstructing one of the views as mentioned above, an auxiliary plane is imagined satisfying the given condition and the other view is projected on it. This method is known as change of reference line method. This is advantageous compared to the former one, as this avoids redrawing one of the views in the second stage. This advantage may be appreciated with respect to the solids having curved surfaces or too many edges.

While drawing the projections of solids, the following must be observed:

1. If a solid has an edge of its base parallel to H.P or on H.P, that edge should be kept perpendicular to V.P. If the edge of the base is parallel to V.P or on V.P, that edge should be kept perpendicular to H.P.
2. If a solid has a corner of its base on H.P, the sides of the base containing that corner should be kept equally inclined to V.P; if the corner of the base is on V.P, the sides of the base containing that corner should be kept equally inclined to H.P.

**Problem 6** *Draw the projections of a hexagonal prism of base 25 side and axis 60 long, when it is resting on one of its corners of the base on H.P. The axis of the solid is inclined at 45° to H.P. Follow the change of position method.*

(May/June 2008, JNTU)

***Construction (Fig. 11.12)***

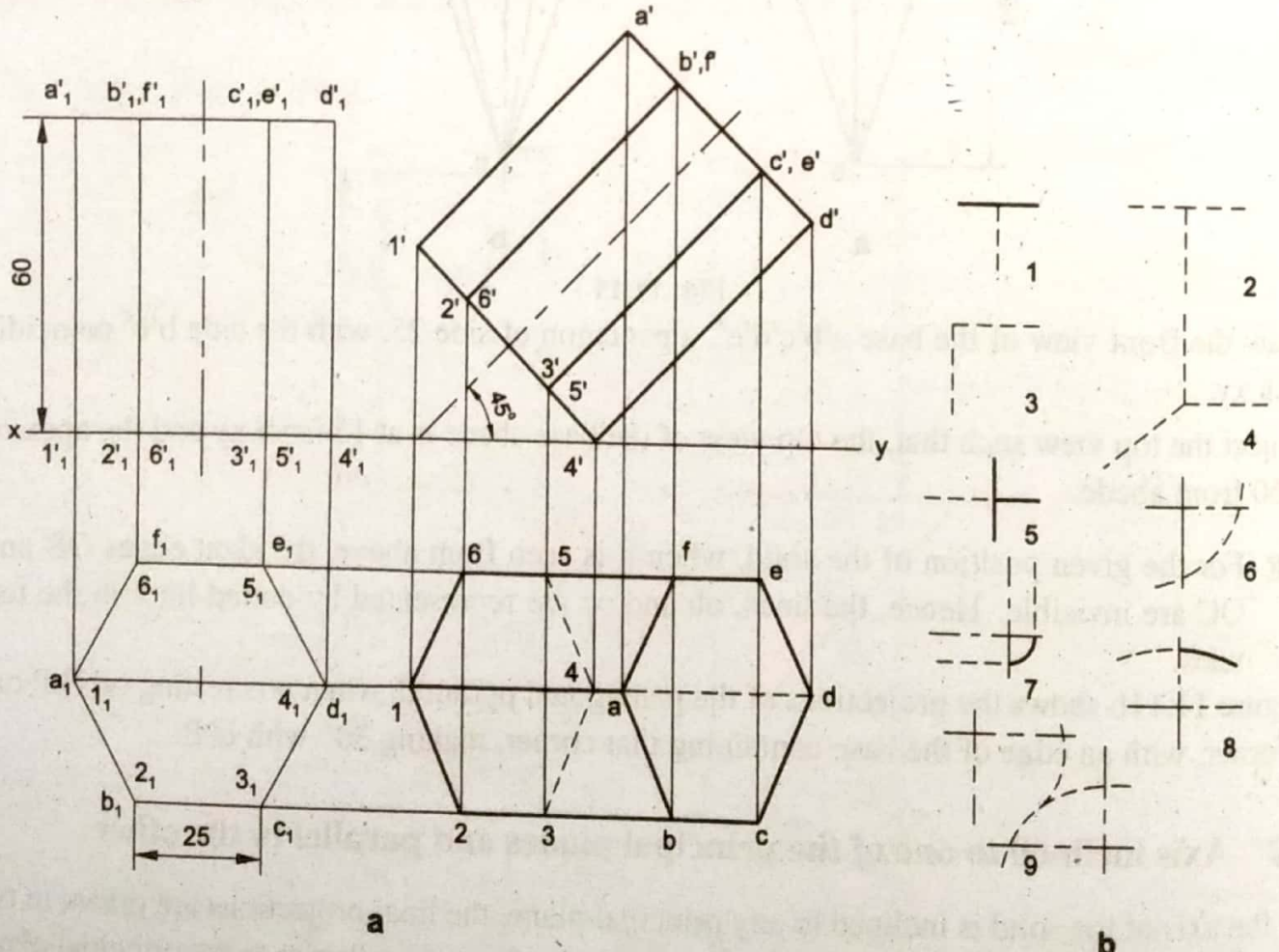



**Fig. 11.12**

**Problem 8** *Draw the projections of a pentagonal prism of base 25 side and axis 50 long, when it is resting on one of its rectangular faces on H.P. The axis of the solid is inclined at 45° to V.P. Follow the change of position method.* (May/June 2008, JNTU)

***Construction (Fig. 11.14)***

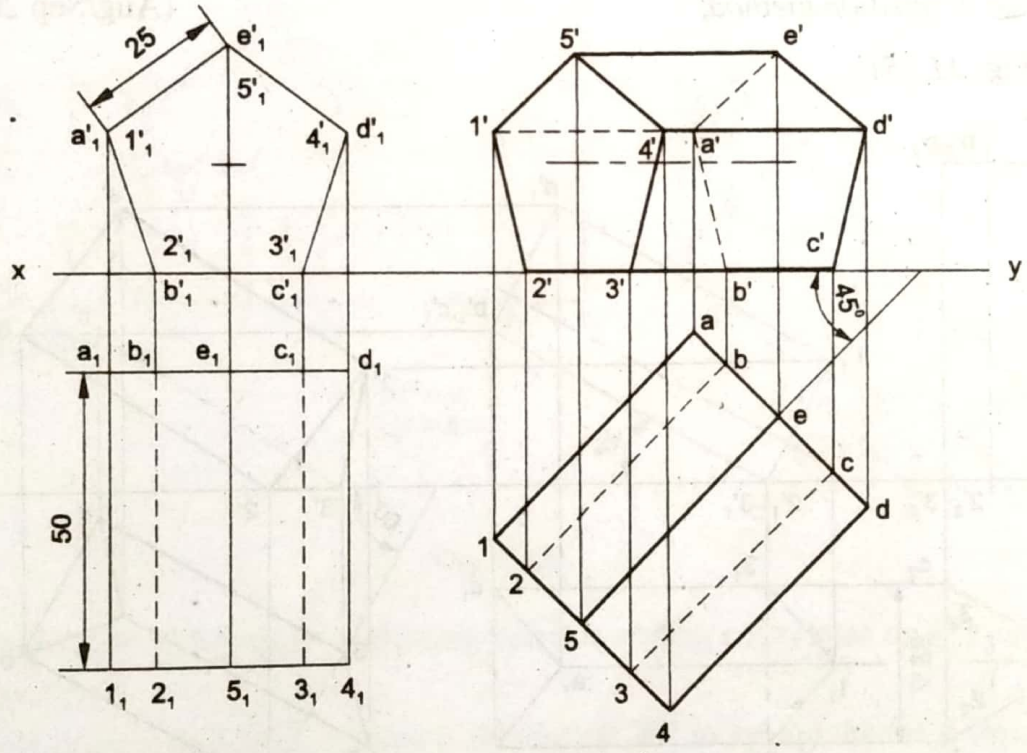



**Fig.11.14**

***Stage I***

1. Draw the projections of the solid, assuming that it is resting on one of its faces on H.P, with its axis perpendicular to V.P.

***Stage II***

2. Redraw the top view such that, the axis makes 45° with xy; forming the final top view.
3. Obtain the final front view, by projection.

### 11.3.3 Axis inclined to both the principal planes

A solid is said to be inclined to both the planes when, (i) the axis is inclined to both the planes or (ii) the axis is inclined to one plane and an edge of the base is inclined to the other. In all such cases, the final projections are obtained in three stages.

***Stage I*** *Assume that the axis is perpendicular to one of the planes and draw the projections.*

***Stage II*** *Redraw one of the views, by making the axis inclined to one of the planes and project the other view from it.*

***Stage III*** *Redraw one of the views obtained in stage II, satisfying the remaining condition and project the other view from it.*

Stages II and III may also be drawn by the use of auxiliary plane method.

**Problem 9** *An equilateral triangular prism of side of base 25 and axis 50 long, is resting on an edge of its base on H.P. The face containing that edge is inclined at 30° to H.P. Draw the projections of the prism, when the edge on which the prism rests, is inclined at 60° with V.P. Follow the change of position method.* (Aug/Sep 2008, JNTU)

***Construction (Fig. 11.15)***

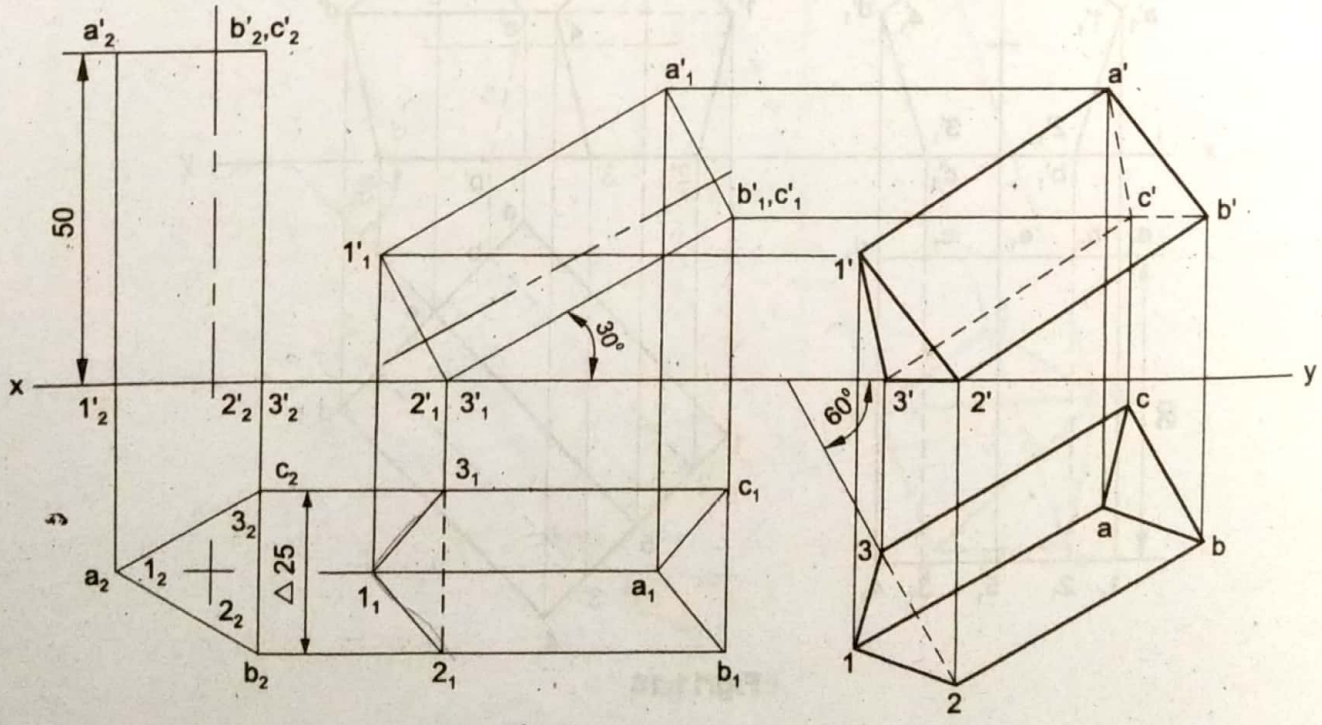



**Fig.11.15**

*Stage I Assume that the solid is resting on its base on H.P, with one edge perpendicular to V.P.*

1. Draw the projections of the solid.

*Stage II Tilt the solid about the edge, which is perpendicular to V.P such that, the face containing the edge makes 30° to H.P.*

2. Redraw the front view such that, the front view of the face 23BC is inclined at 30° to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.

*Stage III Rotate the solid, till the edge on which it rests is inclined at 60° to V.P.*

4. Redraw the above top view such that, the top view of the edge 2-3 is inclined at 60° to xy. This forms the final top view.
5. Obtain the final front view, by projection.

**Problem 10** *Draw the projections of a square prism, side of base 30 and axis 60 long, resting with one of the edges of its base on H.P. Its axis is inclined at 30° to H.P and the top view of the axis at 45° to xy line.*

***Construction (Fig. 11.16)***

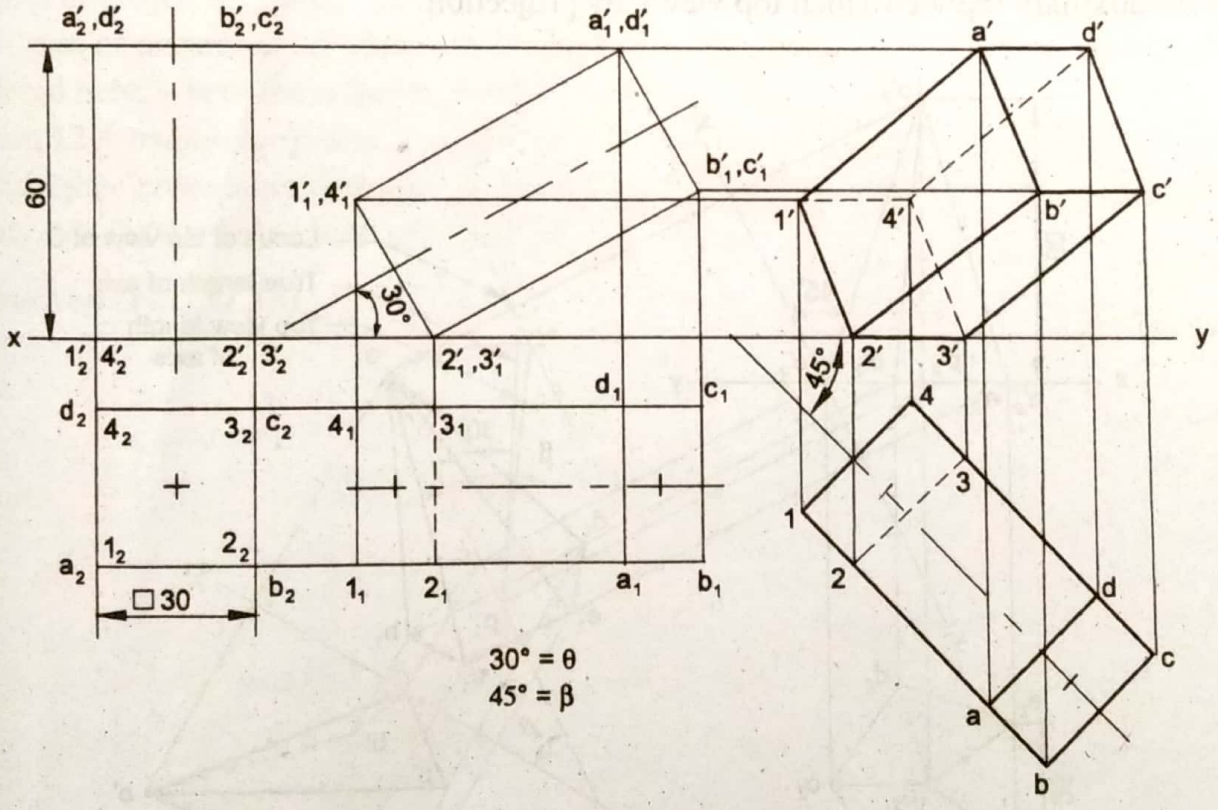



**Fig.11.16**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with one edge perpendicular to V.P.
2. Redraw the front view such that, the axis is inclined at 30° to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.

**Problem 17** *A triangular pyramid of base 30 side and axis 50 long, is resting on H.P on its base, with a face perpendicular to V.P. Draw the projections of the pyramid.*

***Construction (Fig. 11.24a)***

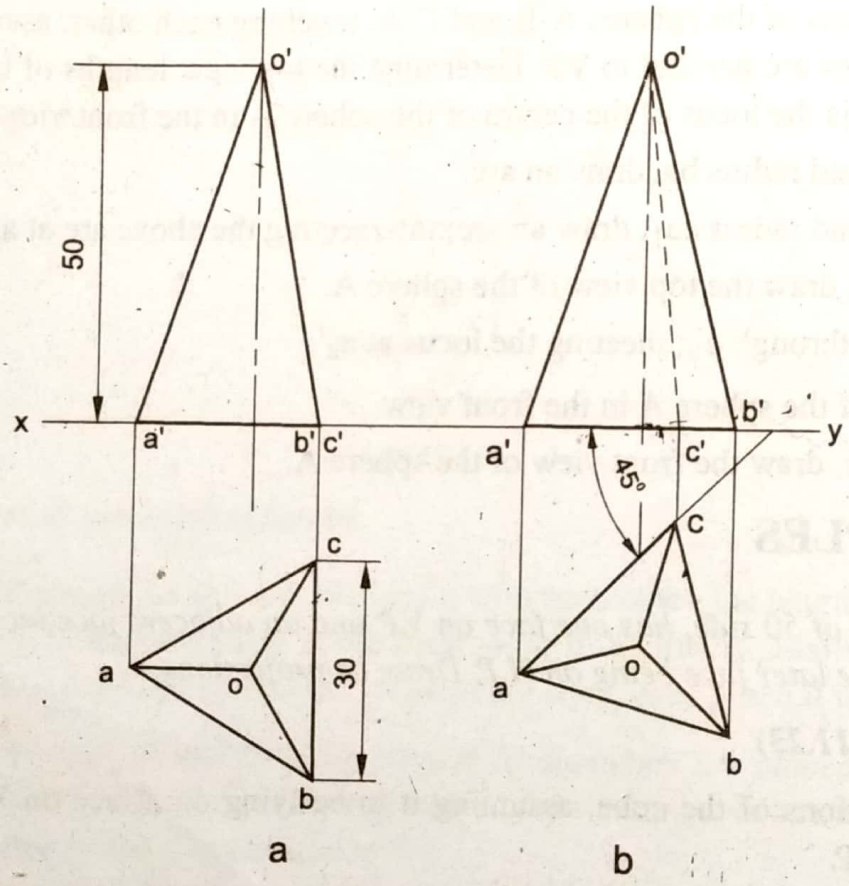



Fig.11.24

**Problem 22** *Draw the projections of a cylinder of 40 diameter and axis 60 long, when it is lying on H.P, with its axis inclined at 45° to H.P and parallel to V.P. Follow the change of position method.*

(May/June 2010, May 2012 JNTU)

***Construction (Fig. 11.29)***

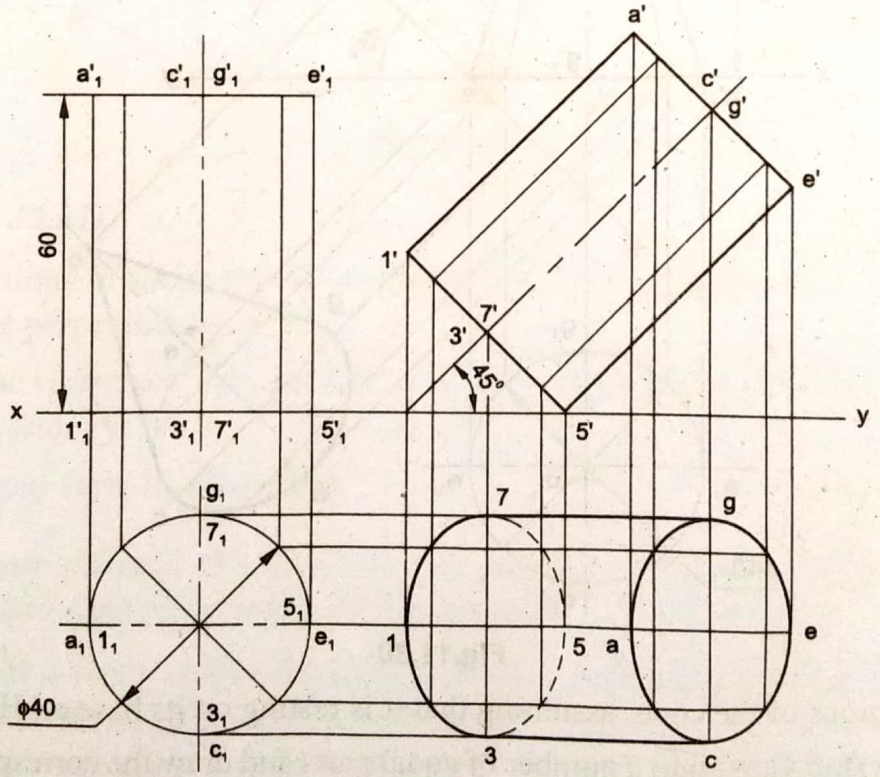



**Fig.11.29**

**Problem 24** *Draw the projections of a hexagonal pyramid, with side of base 30 and axis 70 long, which is resting with a slant face on H.P such that, the axis is parallel to V.P. Follow the change of position method.*

(May/June 2010, JNTU)

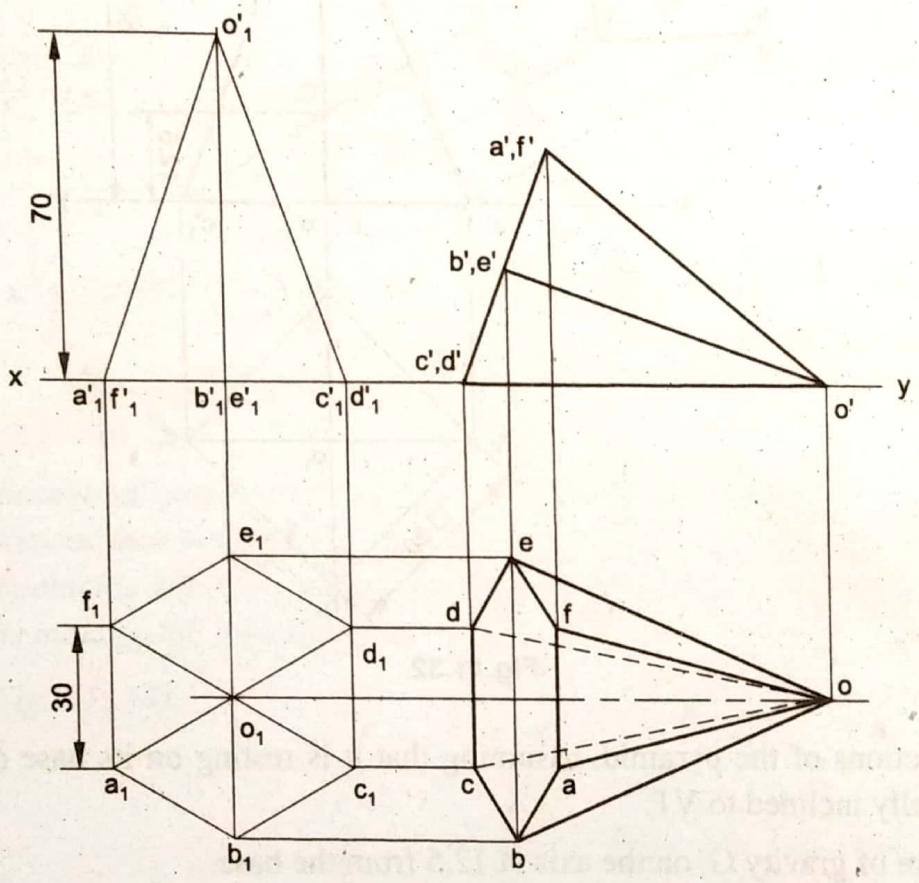



**Fig.11.31**

***Construction (Fig. 11.31)***

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P, with two edges of the base perpendicular to V.P.
2. Redraw the front view such that, the line o′ c′ (d′), representing front view of the slant face OCD, coincides with xy.
3. Obtain the final top view, by projection.

**Problem 26** *A hexagonal prism of side of base 25 and axis 60 long is freely suspended from a corner of the base. Draw the projections, by the change of position method.*

(May/June 2010, JNTU)

***Construction (Fig. 11.33)***

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with an edge of the base parallel to V.P.
2. Locate the centre of gravity G, at the mid-point of the axis.
3. Redraw the front view such that, the line passing through, say $d_1'$ G is perpendicular to xy.
4. Obtain the final top view, by projection.

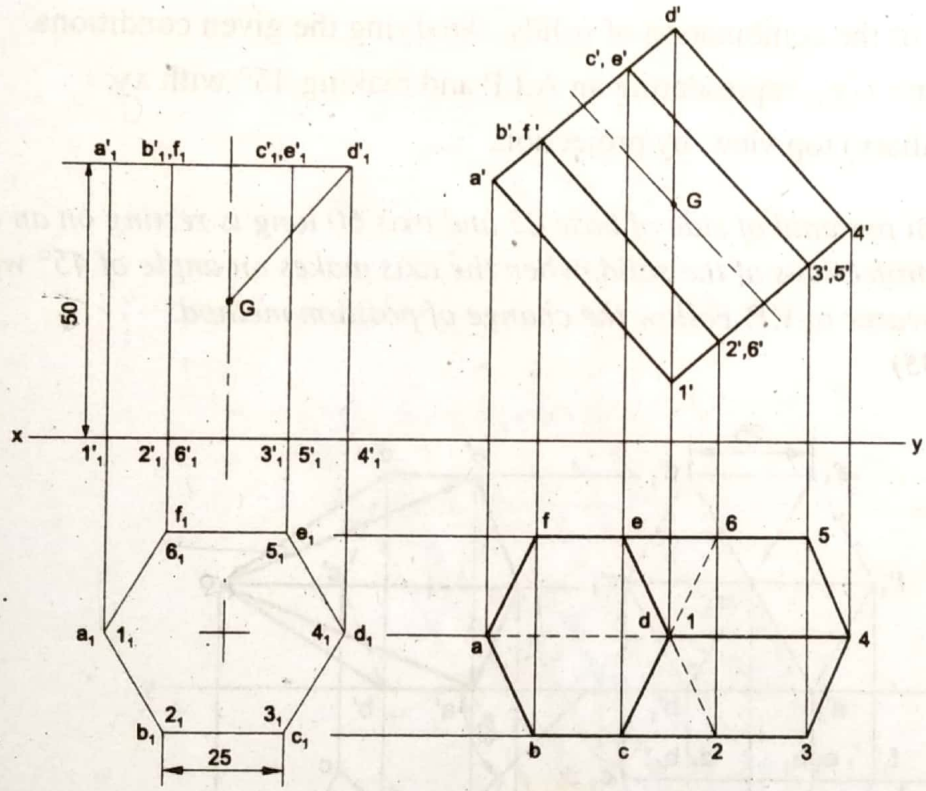

d'
c', e'
b', f'
a'
G
4'
3',5'
2',6'
1'
a'1 b'1,f1 c'1,e'1 d'1
50
G
x
y
1'1 2'1 6'1 3'1 5'1 4'1
f1
e1
61 51
a1 11 41 d1
21 31
b1 c1
25
f
e
6
5
d
1
a
4
b
c
2
3

Fig.11.33

**Problem 28** *A hexagonal pyramid of side of base 25 and axis 60 long is resting on an edge of the base on H.P. Draw the projections of the solid, when the axis makes an angle of 45° with V.P and the base of the solid is nearer to V.P. Follow the change of position method.*

***Construction (Fig. 11.35)***

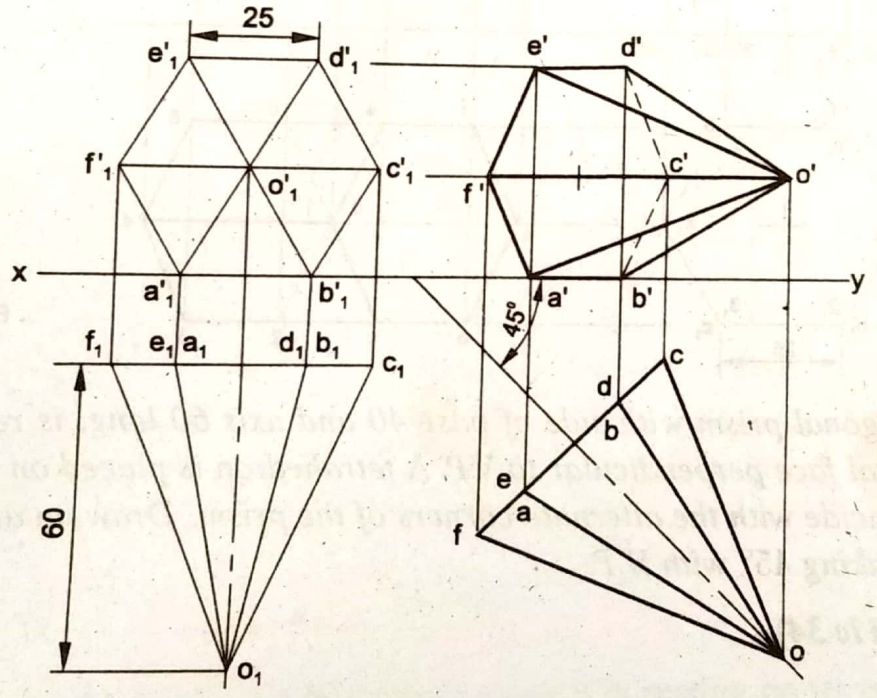



**Fig.11.35**

1. Draw the projections of the solid, assuming that its base is parallel to V.P and an edge of the base on H.P.
2. Redraw the top view such that, the axis makes an angle of 45° with xy.
3. Obtain the final front view, by projection

*... hexagonal hole of 25 side, cut*

**Problem 30** *Draw the projections of a cube of side 25, resting on H.P on one of its corners, with a solid diagonal perpendicular to V.P. Follow the change of position method.*

***Construction (Fig. 11.37)***

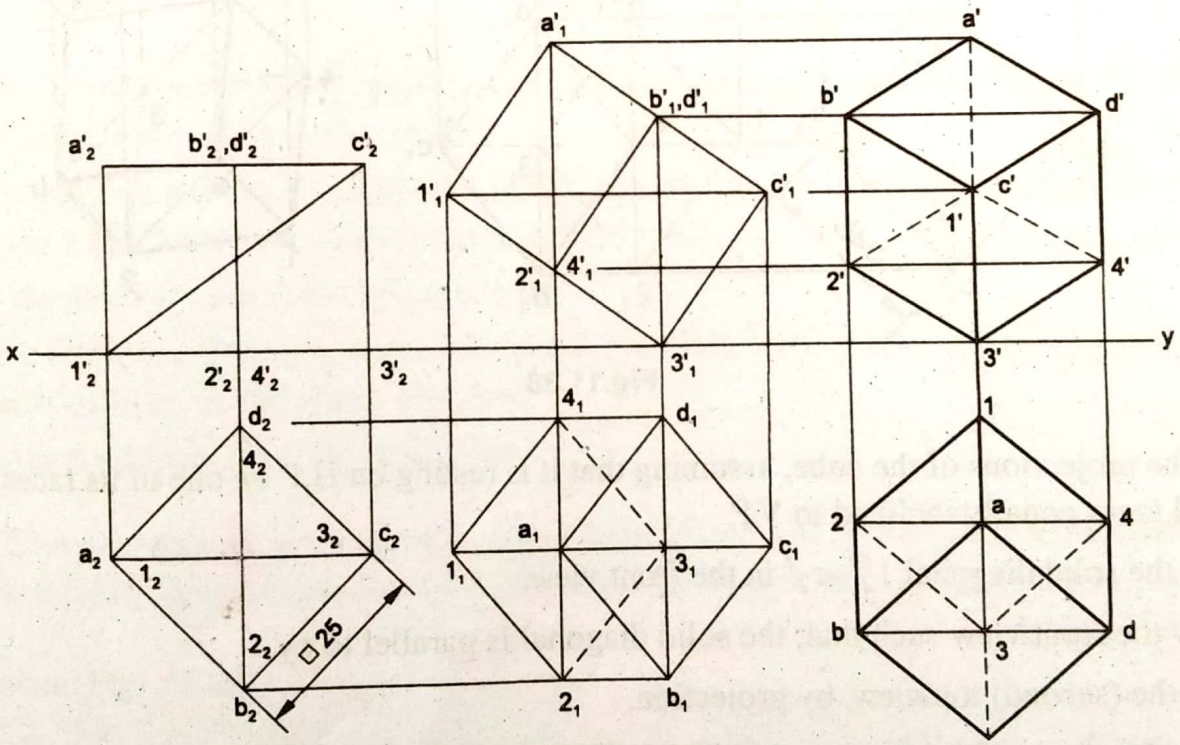



**Fig.11.37**

**Problem 34** *A hexagonal pyramid with side of base 30 long and height 80, has one of its triangular faces perpendicular to H.P and inclined at 45° to V.P. The base-side of this triangular face is on H.P. Draw its projections.* (Aug/Sep 2008, JNTU)

***Construction (Fig. 11.41)***

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P with an edge of the base, perpendicular to xy.
2. Redraw the front view such that, the front view ($o_1'd_1'e_1'$) of the triangular face ODE is perpendicular to xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the top view (ode) of the triangular face ODE makes 45° with xy.
5. Obtain the final front view, by projection.

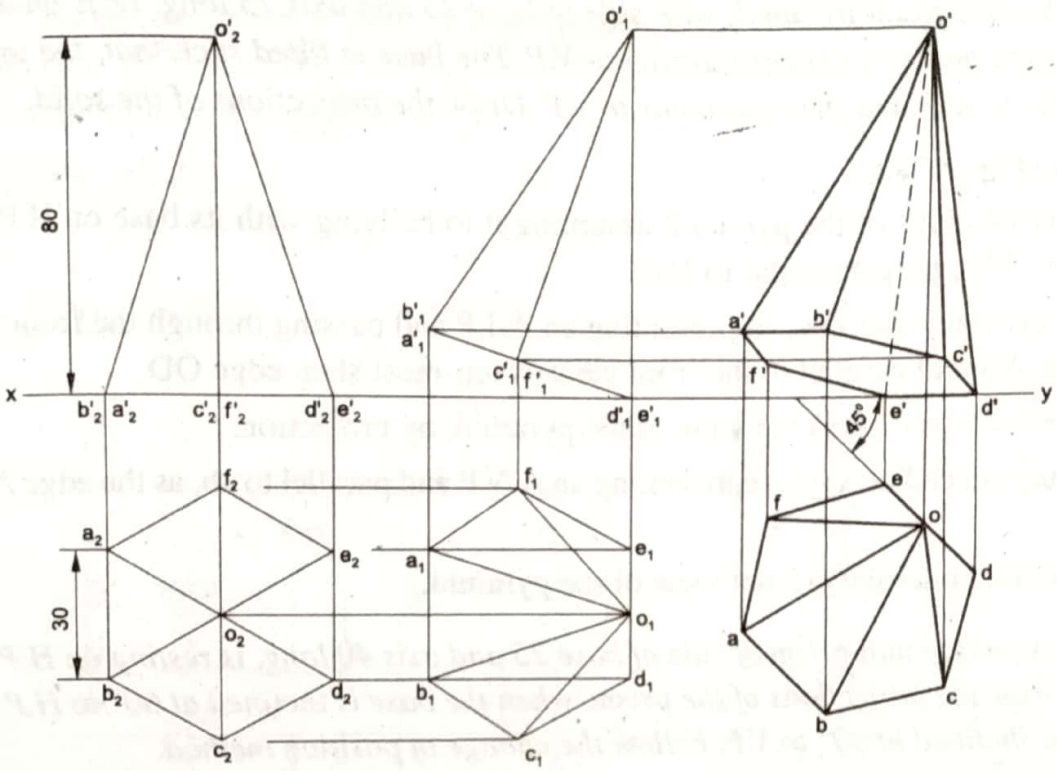

80
30
x
y
45°
o'2
o'1
o'
b'2 a'2 c'2 f'2 d'2 e'2
b'1 a'1 c'1 f'1 d'1 e'1
a' b' c' f' d' e'
f2 a2 e2 o2 b2 d2 c2
f1 a1 e1 o1 d1 b1 c1
f e o d a c b

Fig.11.41

**Problem 36** *A pentagonal prism of side of base 25 and axis 40 long, is resting on H.P on a corner of its base. Draw the projections of the prism, when the base is inclined at 60° to H.P and the axis appears to be inclined at 30° to V.P. Follow the change of position method.*

***Construction (Fig. 11.43)***

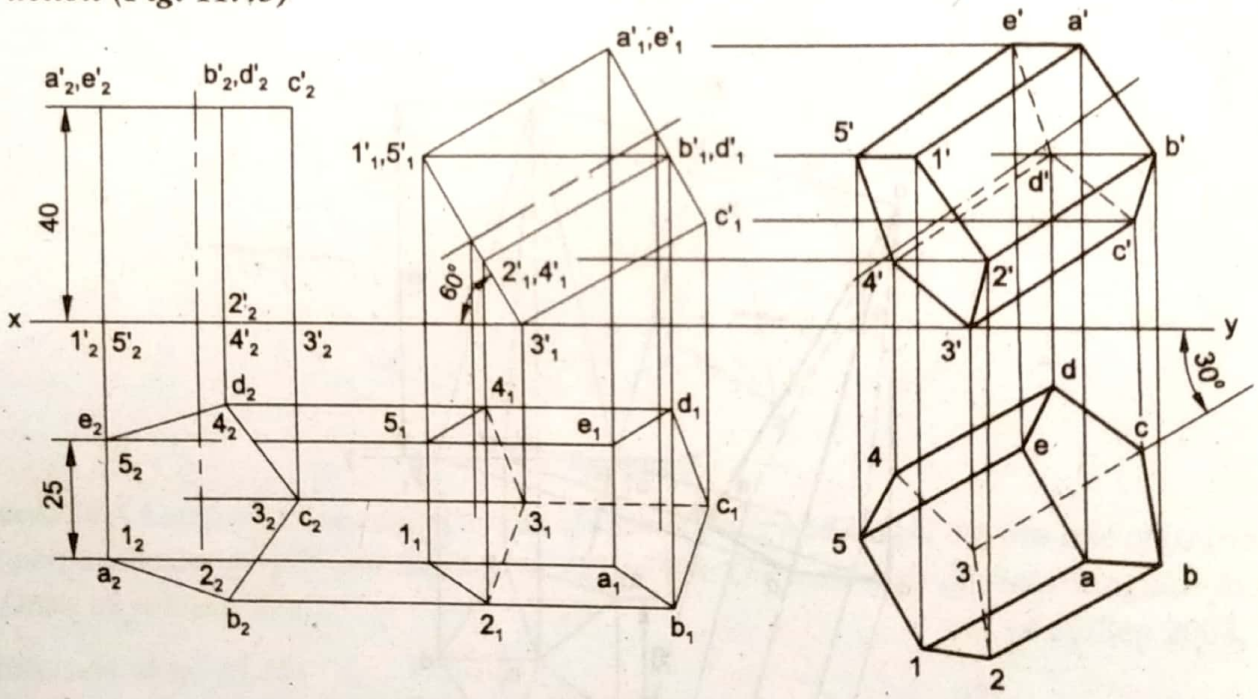
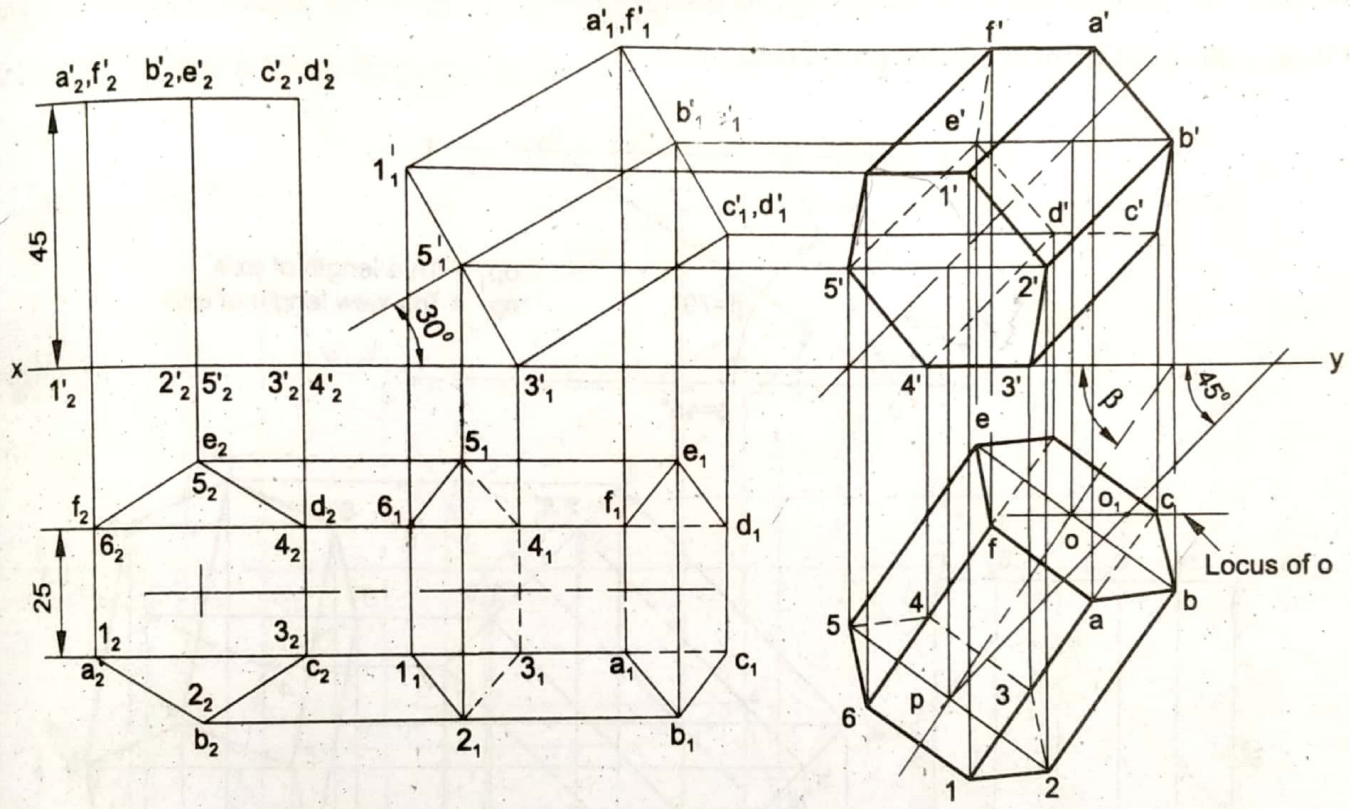



**Fig.11.43**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with two adjacent edges of the base equally inclined to V.P.
2. Redraw the front view such that, the corner $3_1'$ lies on xy and the front view of the base 1-2-3-4-5, makes an angle of 60° with xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, its axis makes an angle of 30° with xy.
5. Obtain the final front view, by projection.

**Problem 37** *A hexagonal prism of base 25 side and axis 45 long, is positioned with one of its base edges on H.P such that, the axis is inclined at 30° to H.P and 45° to V.P. Draw its projections. Follow the change of position method.*

***Construction* (Fig. 11.44)**




$po_1$ = True length of axis
po = Top view length of axis

**Fig.11.44**

1. Draw the projections of the prism, assuming that it is resting on its base on H.P and with an edge of the base perpendicular to V.P.
2. Redraw the front view such that, the front view of the base edge 3-4, lies on xy and the axis makes an angle of 30° with xy.
3. Obtain the second top view, by projection.
4. Determine the apparent angle $\beta$, the inclination, the axis makes with xy in the final top view.
5. Redraw the top view such that, its axis makes the angle $\beta$ with xy.
6. Obtain the final front view, by projection.

**Problem 41** *A regular square prism lies with its axis inclined at 60° to H.P and 30° to V.P. The prism is 60 long and has a face width of 25. The nearest corner is 10 away from V.P and the farthest one is 100 from H.P. Draw the projections of the solid.* (May/June 2008, JNTU)

***Construction (Fig. 11.48)***

1. Draw the projections of the solid, assuming it to be resting on its base on H.P, with the vertical faces equally inclined to V.P.
2. Redraw the front view such that, the axis makes an angle 60° with xy and the farthest corner $a_1'$ is 100 above xy.
3. Obtain the second top view, by projection.
4. Redraw the top view such that, the axis is perpendicular to xy and the nearest corner c is 10 away from xy. This is the final top view.
5. Obtain the final front view, by projection.

**NOTE** The axis of the prism is making an angle of 60° with the H.P. and 30° with V.P. As $\theta+\phi=90°$, the front and top views of the axis should lie on a single projector.

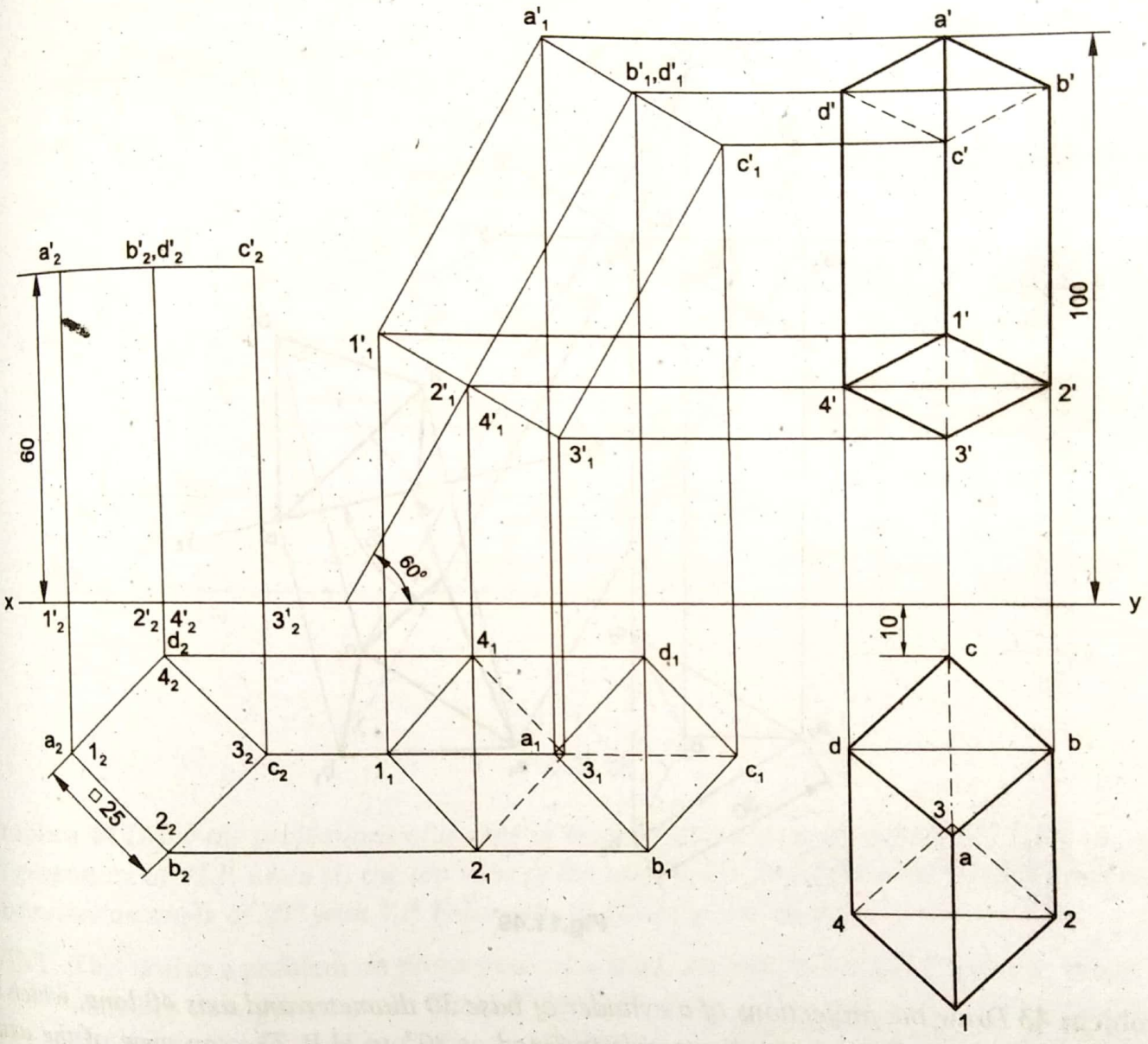

a'1
a'
b'1,d'1
b'
d'
c'1
c'
a'2
b'2,d'2
c'2
1'1
1'
100
2'1
4'1
4'
2'
3'1
3'
60
60°
x
y
1'2
2'2
4'2
3'2
d2
4
1
d1
10
c
a2
1
2
3
2
a1
3
1
c1
d
b
25
c2
2
3
a
b2
2
1
b1
4
2
1

**Fig.11.48**