Unit-5
Test of Significance (Small Samples)

Test of Significance for small samples:

Definition: If the sample size less than 30, it is called a small sample.

The following are some important tests for small samples.

(i) Student's 't' Test
(ii) F - Test
(iii) χ^2 - Test.

Test of Hypothesis for Single Mean (μ):

Test Statistic $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ when $\bar{x}$ = Sample mean, μ = population mean, s = Sample S.D, n = Sample size

if 's' is not given directly

$t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$ if 's' is given directly.

Here Degrees of freedom $=(n-1)$

Note:-

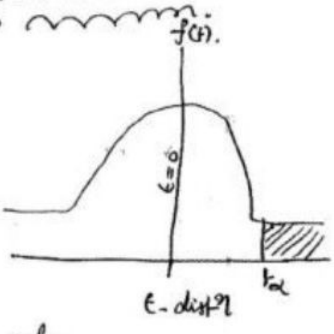
① For one tailed Test $|t_{tab}|$ = See t-Table $(n-1)$ at α

② For two tailed Test $|t_{Tab}|$ = See t-Table $(n-1)$ at $\frac{\alpha}{2}$

Confidence interval for μ:

Confidence interval for μ is $\left(\bar{x}-t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}},\ \bar{x}+t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right)$

Properties of t- distribution: (or) characteristics of t-distribution



1. The shape of t- distribution is bell-shaped, which is similar to that of a normal distribution and is symmetrical about the mean.

2. The t- distribution curve is also asymptotic to t-axis. i.e The two tails of the curve on both sides of $t=0$ Extends to infinity

3. It is Symmetrical about the line $t=0$.

4. The form of probability Curve Varies with degrees of freedom.

5. It is unimodel with Mean = Median = Mode.

6. The shape of t- distribution depends on Sample Size n.

Applications of t- distribution (or) Uses of t- distribution

(1) To test the Significance of the Sample mean, when population Variance is not given

(2) To test. the Significance of the mean of the Sample i.e To test if the sample mean differs significantly from the population mean

(3) To test the Significance of the difference between two Sample means.

(4) To test the significance of an observed Sample correlation Co-efficient and Sample regression Co-efficient.

① A Random Sample of six steel beams has mean compressive strength of 58,392 P.S.i (pounds per square inch) with standard deviation of 648 P.S.i. Use this information and the level of significance $\alpha = 0.05$ to test whether the true average compressive strength of steel from which this sample came is 58,000 P.S.i. Assume normality.

Sol Given sample size $n = 6$ (Small Sample), Degrees of freedom $= n-1 = 5$

Sample mean $\bar{x} = 58,392$

sample sandard deviation $s = 648$.

population mean $\mu = 5800$

(i) Null Hypothesis $H_0 : \mu = 58000$

(ii) Alternative Hypothesis $H_1 : \mu \neq 58000$ <two tailed Test>

(iii) Level of Significance : $\alpha = 0.05$

(iv) Test statistic : $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$ <S.D of sample given directly>

$$= \frac{58,392 - 58000}{\frac{648}{\sqrt{6-1}}} = \frac{392}{648} \times \sqrt{5} = 1.353$$

$\therefore |t_{cal}| = 1.353$

(v) Conclusion: $|t_{tab}| = 2.571$. < See t-table $(n-1)$ at $\frac{\alpha}{2}$. See t-Table 5 at $\frac{0.05}{2} = 0.025$

$\therefore |t_{cal}| < |t_{tab}|$

$\therefore H_0$ is accepted

Hence the average compressive strength of steel beem is 58000 P.S.i

② A sample of 26 bulbs gives a mean life of 990 hours S.D of 20hours. The manufacturer claims that the mean life of bulbs is 1000 hours. Is the sample not up to the standard.

Sol Given $n = 26$ (Small sample), $\bar{x} = 990$, $s = 20$, $\mu = 1000$

d.o.f $= 26-1 = 25$.

(i) $H_0 : \mu = 1000$, (ii) $H_1 : \mu < 1000$ (iii) $\alpha = 5\% = 0.05$ (iv) $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = -2.5$ <S.D of Sample given directly>

$\therefore |t_{cal}| = 2.5$

(v) Conclusion: $|t_{Tab}| = 1.708$ < See t-Table $(n-1)$ at α. See t-Table 25 at 0.05

$\therefore H_0$ is rejected (i.e sample is not up to standard).

(3)[K] A new process of producing synthetic diamonds can be operated at a profitable level only if the average weight of the diamonds is greater than 0.5 carat. To test the profitability of the process, 6 diamonds are produced with weights 0.45, 0.60, 0.52, 0.49, 0.58 and 0.54 carat respectively. Do the 6 measurements present sufficient evidence to indicate that the average weight of the diamonds produced by the process is in excess of 0.5 carat.

⟨Dec. 2014⟩

Sol Given $n = 6$ (small sample), $\mu = 0.5$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1}{6}(0.45 + 0.60 + 0.52 + 0.49 + 0.58 + 0.54) = 0.53.$$

Sample Variance $s^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{(n-1)}$

$$= \left[(0.45 - 0.53)^2 + (0.60 - 0.53)^2 + (0.52 - 0.53)^2 + (0.49 - 0.53)^2 + (0.58 - 0.53)^2 + (0.54 - 0.53)^2\right] / 5$$

$$s^2 = 0.0156/5$$

$$s = \frac{\sqrt{0.0156}}{\sqrt{5}}$$

$\boxed{s = 0.0558}$, Degrees of freedom $= n - 1 = 6 - 1 = 5$

(i) Null Hypothesis H_0: $\mu = 0.5$ (ii) Alternative Hypothesis H_1: $\mu > 0.5$ ⟨one tailed⟩

(iii) Level of Significance: $\alpha = 5\% = 0.05$

(iv) Test Statistic : $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ ⟨∵ S.D of sample not given directly⟩

$$t\ cal = \frac{0.53 - 0.5}{\frac{0.0558}{\sqrt{6}}} = 1.3155$$

$\therefore |t\ cal| = 1.3155.$

(v) Conclusion: $|t\_Tab| = 2.015$ ⟨ See t-Table (n-1) at α ; See t-Table 5 at 0.05 ⟩

$\therefore |t\ cal| < |t\_Tab|$

$\therefore$ H_0 is accepted.

$\therefore$ The mean of the population of the diamonds produced by the new process is $\mu = 0.5$ //

④ Labels on one-gallon cans of paint indicate the drying time and the area that can be covered in one coat.

A manufacture of paints claims that the brand of paint they manufacture will cover 420 sq.ft per gallon. To test this, a random sample of 10 one-gallon cans of paints was tested. The actual areas painted in sq ft are 362, 356, 413, 422, 372, 416, 376, 434, 388 and 421. Do the data present sufficient evidence to indicate that the average differs from 420 sq.ft?

Sol

Given $\mu = 420$, $n = 10$ < small sample >

Mean and S.D are not given, first we have to find them.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{1}{10}(362+356+413+422+372+416+376+434+388+421)$$

$$\boxed{\bar{x} = 396}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{9}\Big[(362-396)^2 + (356-396)^2 + (413-396)^2 + (422-396)^2 + (372-396)^2 + (416-396)^2 + (376-396)^2 + (434-396)^2 + (388-396)^2 + (421-396)^2\Big]$$

$$s^2 = \frac{7230}{9} = 803.3333$$

$$s = \sqrt{803.3333} = 28.343$$

Degrees of freedom $= n-1 = 10-1 = 9$.

(i) Null Hypothesis $H_0: \mu = 420$ (ii) Alternative Hypothesis $H_1: \mu \neq 420$

(iii) Level of Significance: $\alpha = 5\% = 0.05$ (iv) Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \langle \because \text{S.D of sample not given directly} \rangle$$

$$= \frac{396 - 420}{\frac{28.3333}{\sqrt{10}}} = -2.678$$

$\therefore |t\text{-cal}| = 2.678$

(v) Conclusion: $|t\text{-tab}| = 2.262$ < See t-Table 9 at $\frac{0.05}{2} = 0.025$

$\therefore |t\text{-cal}| > |t\text{-Tab}|$

$\therefore$ H_0 is rejected.

(5) Find 95% Confidence limits for the mean of a normality distributed population from which the following Sample was taken 15, 17, 10, 18, 16, 9, 7, 11, 13, 14.

Sol Given $n = 10$, $\bar{x} = \frac{1}{10}(15+17+10+18+16+9+7+11+13+14) = 13$.

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{(n-1)} = \frac{40}{3}$$

$s = \frac{\sqrt{40}}{\sqrt{3}}$, degrees of freedom $= n-1 = 10-1 = 9$

Given Confidence limit = 95%.

i.e Level of Significance $\alpha = 5\% = 0.05$

$t_{\frac{\alpha}{2}} = 2.262$ ⟨ See t-Table $(n-1)$ at $\frac{\alpha}{2}$; See t-Table 9 at $\frac{0.05}{2} = 0.025$

$\therefore$ 95% Confidence limits for μ is $\left(\bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right)$

$$\left(13 - (2.262)\frac{\sqrt{40}}{\sqrt{3}},\ 13 + (2.262)\frac{\sqrt{40}}{\sqrt{3}}\right)$$

$$(13 - 2.6,\ 13 + 2.6)$$

$$(10.4,\ 15.6) \,//$$

2M

(6) A Sample of Size 10 and standard deviation 0.03 is taken from a population. Find Maximum error with 99% Confidence. ⟨May 2015⟩

Sol Given $n = 10$, $s = 0.03$. Confidence limit = 99%. $\therefore \alpha = 1\% = 0.01$

degrees of freedom $= 10 - 1 = 9$.

$t_{\frac{\alpha}{2}} = 3.25$. ⟨ See t-Table $(n-1)$ at $\frac{\alpha}{2}$; See t-Table 9 at $\frac{0.01}{2} = 0.005$

$\therefore$ Maximum Error $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = \frac{3.25\,(0.03)}{\sqrt{9}} = 0.0325 \,//$

(7) A Random Sample of Size 16 Values from a normal population showed a mean of 53 and a Sum of Squares of deviations from the mean equals to 150. Can this Sample be regarded as taken from the population having 56 as mean? Obtain 95% confidence limits of the mean of the population.

Sol Given $n = 16$, $\bar{x} = 53$ $\mu = 56$ and $\Sigma(x_i - \bar{x})^2 = 150$ $\therefore s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = 10$

d.o.f $= 16 - 1 = 15$

(I) $H_0: \mu = 56$ (ii) $\mu \neq 56$ (iii) $\alpha = 5\% = 0.05$ (IV) $t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -3.79$ $|t_{tab}| = 2.131$

(V) $|t_{cal}| > |t\text{-Tab}|$ and 95% Confidence limits $(51.31,\ 54.68)$

Testing of Two means:

Test Statistic $t = \dfrac{\bar{x} - \bar{y}}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

where $\bar{x} = \dfrac{\Sigma x_i}{n_1}$, $\bar{y} = \dfrac{\Sigma y_i}{n_2}$

$s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ (or)

Degrees of freedom $= n_1 + n_2 - 2$

$s^2 = \dfrac{1}{n_1 + n_2 - 2}\left[\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2\right]$

Note: Confidence limits for the difference of two population means are $\left((\bar{x}-\bar{y}) - t_{\frac{\alpha}{2}} \cdot \sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},\ (\bar{x}-\bar{y}) + t_{\frac{\alpha}{2}} \sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right)$

5.

① The IQs of 16 students from one area of a city showed a mean of 107 with standard deviation of 10, while the IQs of 14 students from another area of city showed a mean of 112 with standard deviation of 8. Is there a significant difference between the IQs of the two groups at a 0.05 level of significance.

⟨June 2015⟩

Sol Given $n_1 = 16$, $\bar{x} = 107$, $s_1 = 10$ and $n_2 = 14$, $\bar{y} = 112$, $s_2 = 8$

(i) Null Hypothesis: $H_0: \mu_1 = \mu_2$ (ii) Alternative Hypothesis $H_1: \mu_1 \neq \mu_2$

(iii) Level of significance: $\alpha = 0.05$

(iv) Test statistic: $t = \dfrac{\bar{x} - \bar{y}}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$= \dfrac{16 \times 100 + 14 \times 16}{16 + 14 - 2}$

$s^2 = 89.14$

$\boxed{s = 9.44}$

$\therefore t\ cal = \dfrac{107 - 112}{\sqrt{89.14\left(\frac{1}{16} + \frac{1}{14}\right)}} = -1.447$

$\therefore |t\ cal| = 1.447$

(v) Conclusion: $|t_{tab}| = 2.048$

See t-Table (n_1+n_2-2) at $\frac{\alpha}{2}$

See t-Table 28 at $\frac{0.05}{2} = 0.025$

$\therefore |t_{cal}| < |t_{tab}|$

$\therefore$ Ho is accepted.

Hence The difference between the IQs of two groups is not significant.

(2) In a manufacturing Company, a one-month training programme is Conducted to each employee. A new training programme has been developed. To test the new programme, 2 groups of 9 employees each were assigned a job, the first group trained under the old System and second trained under the new programme. Their performance timings in minutes are recorded as follows.

old programme :	31	38	36	30	42	34	32	33	39
New programme :	34	33	26	36	31	28	27	29	35

Do the job execution times present sufficient evidence to indicate that the mean time is less for the new programme?

Sol Given $n_1 = 9$, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{31+38+36+30+42+34+32+33+39}{9}$

$\therefore \boxed{\bar{x} = 35}$

$n_2 = 9$, $\bar{y} = \frac{\Sigma y_i}{n} = \frac{34+33+26+36+31+28+27+29+35}{9}$

$\therefore \boxed{\bar{y} = 31}$

$\Sigma(x_i - \bar{x})^2 = (-4)^2 + 3^2 + 1^2 + (-5)^2 + 7^2 + (-1)^2 + (-3)^2 + (-2)^2 + 5^2 = 130$

$\Sigma(y_i - \bar{y})^2 = 3^2 + 2^2 + (-5)^2 + 5^2 + 0^2 + (-3)^2 + (-4)^2 + (-2)^2 + 4^2 = 108$

now $s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right]$

$= \frac{1}{16}[130+108] = \frac{238}{16} = 14.875$

$\boxed{s^2 = 14.875}$

(i) Null Hypothesis $H_0: \mu_1 = \mu_2$

(ii) Alternative Hypothesis $H_1: \mu_1 > \mu_2$ $\langle \because$ one tailed Test$\rangle$

(iii) Level of Significance : $\alpha = 0.05$.

(iv) Test Statistic : $t = \dfrac{\bar{x} - \bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \dfrac{35-31}{\sqrt{14.875\left(\frac{1}{9}+\frac{1}{9}\right)}}$

$$= \frac{4 \times 3}{\sqrt{14.875 \times 2}} = 2.2$$

$\therefore |t\,cal| = 2.2$

(v) Conclusion : $|t\,tab| = 1.746$ $\langle$ See t-Table n_1+n_2-2 at α; See t-Table 16 at 0.05

$\therefore |t\,cal| > |t\,Tab|$

$\therefore$ Ho is rejected.

5 (3) Two independent samples of 8 and 7 items respectively had the following values.

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	–

Is the difference between the means of samples significant? $\langle$Dec 2017$\rangle$

Sd Given $n_1 = 8$, $n_2 = 7$ and $\bar{x} = \frac{1}{8}(11+11+13+11+15+9+12+14) = 12$

$\bar{y} = \frac{1}{7}(9+11+10+13+9+8+10) = 10$

$\Sigma(x_i-\bar{x})^2 = (-1)^2+(-1)^2+1^2+(-1)^2+3^2+(-3)^2+0^2+2^2 = 26$

$\Sigma(y_i-\bar{y})^2 = (-1)^2+1^2+0^2+3^2+(-1)^2+(-2)^2+0^2 = 16$.

$$s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right] = \frac{1}{13}(42) = 3.23.$$

(i) Null Hypothesis $H_0: \mu_1 = \mu_2$ (ii) Alternative Hypothesis $H_1: \mu_1 \neq \mu_2$

(iii) L.O.S : $\alpha = 0.05$ (iv) Test statistic : $t = \dfrac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = 2.15$

$\therefore |t\,cal| = 2.15$

(v) Conclusion: $|t\,Tab| = 2.16$ $\langle$ See t-Table 13 at $\frac{0.05}{2} = 0.025$.

$\therefore |t\,cal| < |t\,Tab|$ $\therefore$ Ho is accepted.

paired Sample t-Test:

paired observations arise in many practical situations where each homogeneous experimental unit receives both population Conditions. As result, Each experimental unit has a pair of observations, one for each population.

For instance, to test the effectiveness of "drug" some 11 persons blood presure is measured "before" and "after" the intake of certain drug. Here the individual person is the experimental unit and two populations are blood pressure "before" and "after" the drug is given.

Test Statistic $t = \dfrac{\bar{d} - \mu}{\frac{s}{\sqrt{n}}}$, Here $\mu = 0$.

$$\boxed{t\ cal = \dfrac{\bar{d}}{\frac{s}{\sqrt{n}}}}$$

where $\bar{d} = \frac{1}{n}\Sigma d_i$ and $s^2 = \frac{1}{n-1} \Sigma (d_i - \bar{d})^2$ ✓

(a) $s^2 = \frac{1}{n-1}\left[\Sigma d^2 - n(\bar{d})^2\right]$

Degrees of freedom $= (n-1)$.

Note: ① By using paired Sample t-Test, we test Two means.

② confidence interval for difference of two population means $\left(\bar{d} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}},\ \bar{d} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right)$

① The following are the average weekly losses of work-hours due to accidents in 10 industrial units before and after introduction of certain safety measures.
use 0.05 LOS to test whether the safety measures are effective.

Before	43	66	109	35	56	85	36	38	49	42
After	36	50	96	37	53	78	28	48	41	42

Sol Given $n = 10$

differences d_i = (43-36), (66-50), (109-96), (35-37), (56-53)
(85-78), (36-28), (38-48), (49-41), (42-42)

$$d_i = 7, 16, 13, -2, 3, 7, 8, -10, 8, 0$$

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{1}{10}(7+16+13-2+3+7+8-10+8+0)$$

$$\therefore \bar{d} = 5.$$

now $s^2 = \frac{1}{(n-1)} \Sigma (d_i - \bar{d})^2$

$$= \frac{1}{9}\left[2^2 + 11^2 + (8)^2 + (-7)^2 + (-2)^2 + 2^2 + 3^2 + (-15)^2 + 3^2 + (-5)^2\right]$$

$$s^2 = 57.1111$$

$$\boxed{s = 7.5571}$$

Degrees of freedom = $n-1 = 10-1 = 9$.

(i) Null Hypothesis $H_0 : \mu_1 = \mu_2$

(ii) Alternative Hypotheses $H_1 : \mu_1 > \mu_2$ < one tailed >

(iii) Level of significance : $\alpha = 0.05$

(iv) Test statistic : $t = \frac{\bar{d} - \mu}{\frac{s}{\sqrt{n}}}$ $< \mu = 0$

$$= \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{5}{7.5771} \times \sqrt{10} = 2.0922$$

$$\therefore |t\ cal| = 2.0922$$

(v) Conclusion : $|t_{Tab}| = 1.833$ ⟨ See t-Table $n-1$ at α; See t-Table 9 at 0.05.

$\therefore |t_{cal}| > |t_{Tab}|$

$\therefore$ H_0 is rejected.

Hence safty measures are effective.

(K) ② Find 90% confidence interval for the mean improvement in lost worker-hours for above problem.

Sol Given $n = 10$, From ① problem $\bar{d} = 5$, $s = 7.56$

Given Confidence limit = 90%.

i.e α = level of Significance = 10% = 0.1

$\therefore t_{\frac{\alpha}{2}} = 1.833.$ ⟨ See t-Table $(n-1)$ at $\frac{\alpha}{2}$; See t-Table 9 at $\frac{0.1}{2} = 0.05$

90% Confidence interval is

$$\left(\bar{d} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{d} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right)$$

$$\left(5 - 1.833 \cdot \frac{7.56}{\sqrt{10}},\ 5 + 1.833 \cdot \frac{7.56}{\sqrt{10}}\right)$$

$$(0.62,\ 9.38)$$

(S) (3) Memory capacity of 10 students were tested before and after training. State whether the training was effective or not from the following scores.

Before Training	12	14	11	8	7	10	3	0	5	6
After Training	15	16	10	7	5	12	10	2	3	8

⟨June 2017⟩

Sol

Same as ①, Given $n = 10$, $\bar{d} = -1.2$, $s = 2.78$

(i) $H_0 : \mu_1 = \mu_2$ (ii) $H_1 : \mu_1 < \mu_2$ (iii) LOS : $\alpha = 0.05$ (iv) $t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = -1.365$

$\therefore |t_{cal}| = 1.365$

(v) conclusion : $|t_{Tab}| = 1.833$ $\therefore |t_{cal}| < |t_{Tab}|$

$\therefore$ H_0 is accepted.

F – Test :

This is very useful in testing the equality of population means by comparing sample variances.

To test whether there is any significant difference between two estimates of population variance (or) to test if the two samples have come from same population, we use F-Test.

In this case, we set up Null Hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ (i.e populations variances are same).

The Test Statistic is $F = \frac{s_1^2}{s_2^2}$ where $s_1^2 = \frac{\Sigma (x_i - \bar{x})^2}{n_1 - 1}$

and $s_1^2 > s_2^2$. $\quad s_2^2 = \frac{\Sigma (y_i - \bar{y})^2}{n_2 - 1}$

Degrees of freedom are $\nu_1 = n_1 - 1$, $\quad n_1$ = first sample size

$\nu_2 = n_2 - 1$. $\quad n_2$ = second sample size

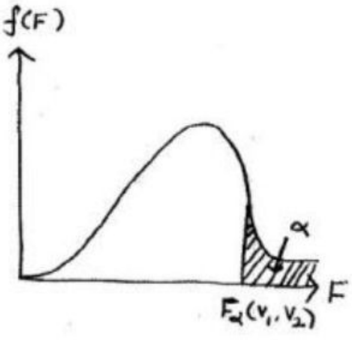
Note:- ① we will take greater of the variances s_1^2 or s_2^2 in the numerator and adjust for the degrees of freedom.

i.e $F = \frac{\text{Greater Variance}}{\text{Smaller Variance}}$.

② If sample variance s^2 is given, we can obtain population variance σ^2 by using the relation $n\sigma^2 = (n-1)s^2$ and vice versa.

③ $F_\alpha(\nu_1, \nu_2)$ is the value of F with ν_1 and ν_2 degrees of freedom such that the area under the F-distribution to the right of F_α is α. In Tables F_α tabulated for $\alpha = 0.05$ and $\alpha = 0.01$ for various combinations of d.o.f ν_1 and ν_2. Clearly, the value of F at 5% significance is lower than 1%..

properties of F- distribution:



① F- distribution curve lies entirely in first quadrant.

② The F - curve depends not only on the two parameters but also on the order in which They are stated.

③ $F_{1-\alpha}(v_1, v_2) = \dfrac{1}{F_\alpha(v_2, v_1)}$ where $F_\alpha(v_1, v_2)$ is the Value of F with v_1 and v_2 d.o.f such that the area under the F- distribution curve to the right of F_α is α.

(4) The mode of F distribution is less than unity.

Examples

① For an F- distribution, find (a) $F_{0.05}$ with $v_1 = 7$ and $v_2 = 15$

(b) $F_{0.99}$ with $v_1 = 28$ and $v_2 = 12$.

Sol ⓐ From The Table, $F_{0.05}$ with $v_1 = 7$ and $v_2 = 15$ is 2.71

ⓑ $F_{0.99}(28, 12) = \dfrac{1}{F_{0.01}(12, 28)} = \dfrac{1}{2.90} = 0.34482$ //

5 ① The measurements of the output of two units have given the following results. Assuming the both samples have been obtained from the populations at 10% significant level, Test the whether the two populations have the Same Variance.

Unit - A	14.1	10.1	14.7	13.7	14.0
Unit - B	14.0	14.5	13.7	12.7	14.1

⟨June 2017⟩

Sol Given $n_1 = 5$, $n_2 = 5$, $\bar{x} = \frac{\Sigma x_i}{n_1} = \frac{1}{5}(14.1+10.1+14.7+13.7+14) = 13.32$

$$\bar{y} = \frac{\Sigma y_i}{n_2} = \frac{1}{5}(14+14.5+13.7+12.7+14.1) = 13.8$$

calculation for mean's and S.D's of Samples:-

x_i	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	y_i	$(y_i-\bar{y})$	$(y_i-\bar{y})^2$
14.1	0.78	0.6084	14.0	0.2	0.04
10.1	−3.22	10.3684	14.5	0.7	0.49
14.7	1.38	1.9044	13.7	−0.1	0.01
13.7	0.38	0.1444	12.7	−1.1	1.21
14.0	0.68	0.4624	14.1	0.3	0.09
		$\Sigma(x_i-\bar{x})^2 = 13.488$			$\Sigma(y_i-\bar{y})^2 = 1.84$

$$s_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1-1} = \frac{13.488}{4} = 3.372$$

$$s_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2-1} = \frac{1.84}{4} = 0.46.$$

(i) Null Hypothesis $H_0: \sigma_1^2 = \sigma_2^2$

(ii) Alternative Hypothesis $H_1: \sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance: $\alpha = 10\% = 0.01$.

(iv) Test Statistic : $F = \frac{S_1^2}{S_2^2}$ $\langle \because S_1^2 > S_2^2$

$$F_{cal} = \frac{3.372}{0.46} = 7.33.$$

$\therefore |F_{cal}| = 7.33.$

(v) Conclusion: Degrees of freedom are (n_1-1, n_2-1).

i.e $(4,4)$.

$\therefore$ Tabulated F at 10% level for $\overset{v_1 v_2}{(4,4)}$ degrees of freedom is 15.98

i.e $F_{0.01}(4,4) = 15.98$

i.e $|F_{tab}| = 15.98$

$\therefore |F_{cal}| < F_{tab}|$

$\therefore$ H_0 is Accepted

i.e There is no significant difference between the Variances.

② In one sample of 10 observations, The sum of the squares of the deviations of the sample values from sample mean was 120 and in the other sample of 12 observations, it was 314. Test whether the difference is significant at 5% level.

Sol Given $n_1 = 10$, $n_2 = 12$, $\Sigma(x_i - \bar{x})^2 = 120$, $\Sigma(y_i - \bar{y})^2 = 314$.

$$\therefore S_1^2 = \frac{\Sigma(x_i - \bar{x})^2}{n_1 - 1} = \frac{120}{9} = 13.33$$

$$S_2^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_2 - 1} = \frac{314}{11} = 28.54.$$

(i) Null Hypothesis H_0: $\sigma_1^2 = \sigma_2^2$ (ii) Alternative Hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance : $\alpha = 5\% = 0.05$ (iv) Test Statistic $F = \frac{S_2^2}{S_1^2}$ $\langle \because S_2^2 > S_1^2 \rangle$

$\therefore F_{cal} = \frac{28.54}{13.33} = 2.14$. $\therefore |F_{cal}| = 2.14$

(v) Conclusion: Degrees of freedom are $(n_2-1, n_1-1) = (11, 9)$

$\therefore$ Tabulated F at 5% level for $\underset{v_1\ v_2}{(11,9)}$ is 2.90

i.e $|F_{tab}| = 2.9$

$\therefore |F_{cal}| < |F_{tab}|$

$\therefore$ H_0 is accepted.

5

(3) Two random samples gave the following results

Sample	Size	Sample mean	Sum of squares of deviation from mean
1	10	15	90
2	12	14	108

check whether the samples came from the same normal population. ⟨June 2016⟩

Sol

To test whether the two samples came from same normal population i.e ($\mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$), we have to test (I) The equality of variances by F-test (II) The equality of means by using t-Test.

(I) F-test (To test Equality of Variances)

Given $n_1 = 10$, $\bar{x} = 15$, $\Sigma(x_i - \bar{x})^2 = 90$

$n_2 = 12$, $\bar{y} = 14$, $\Sigma(y_i - \bar{y})^2 = 108$

$$S_1^2 = \frac{\Sigma(x_i - \bar{x})^2}{n_1 - 1} = \frac{90}{9} = 10$$

$$S_2^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_2 - 1} = \frac{108}{11} = 9.82$$

Null Hypothesis H_0: $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

Level of Significance: $\alpha = 5\% = 0.05$

Test Statistic : $F = \frac{S_1^2}{S_2^2}$ $\langle \because S_1^2 > S_2^2$

$$F_{cal} = \frac{10}{9.82} = 1.018$$

$$\therefore |F_{cal}| = 1.018$$

Conclusion: Degree of freedom $(n_1 - 1, n_2 - 1) = (9, 11)$

$\therefore$ Tabulated F at 5% level for (9,11) d.o.f is 2.90

i.e $|F_{tab}| = 2.90$

$\therefore |F_{cal}| < |F_{Tab}|$

$\therefore$ H_0 is accepted.

i.e $\boxed{\sigma_1^2 = \sigma_2^2}$

(II) t – Test (To test Equality of means)

Given $\bar{x} = 15$, $\bar{y} = 14$. $n_1 = 10$, $n_2 = 12$

$\Sigma (x_i - \bar{x})^2 = 90$, $\Sigma (y_i - \bar{y})^2 = 108$

$$\therefore \quad S^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma (x_i - \bar{x})^2 + \Sigma (y_i - \bar{y})^2\right]$$

$$= \frac{1}{20}\left[90 + 108\right] = 9.9$$

$\boxed{S^2 = 9.9}$

Null Hypothesis H_0: $\mu_1 = \mu_2$

Alternative Hypothesis H_1: $\mu_1 \neq \mu_2$

Level of Significance : $\alpha = 5\% = 0.05$.

Test Statistic : $t = \dfrac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{15 - 14}{\sqrt{9.9\left(\frac{1}{10} + \frac{1}{12}\right)}} = 0.74$

$\therefore |t_{cal}| = 0.74$

Conclusion: Degrees of freedom $= n_1 + n_2 - 2 = 20$

$|t\ Tab| = 2.086$ $\langle$ See t-Table 20 at $\frac{0.05}{2} = 0.025$

$\therefore |t_{cal}| < |t\text{-}Tab|$

$\therefore$ H_0 is accepted.

i e $\boxed{\mu_1 = \mu_2}$

Hence From I and II, The given Samples have been drawn from the Same normal population

Hence We accept the Null Hypothesis $\sigma_1^2 = \sigma_2^2$ and $\mu_1 = \mu_2$

(4) The nicotine contents in milligrams in two samples of tobacco were found to be as follows.

Sample A	24	27	26	21	25	—
Sample B	27	30	28	31	22	36

Can it be said that the two samples have come the same normal population? < DEC 2016 >

Sol To test whether the two samples came from same population (i.e $\sigma_1^2 = \sigma_2^2$ and $\mu_1 = \mu_2$), we have to test (I) the equality of variances by using F-Test (II) The equality of means by using t-Test.

Given $n_1 = 5$, $n_2 = 6$

Calculation for mean & S.D of samples.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	y_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
24	0.6	0.36	27	−2	4
27	2.4	5.76	30	1	1
26	1.4	1.96	28	−1	1
21	3.6	12.96	31	2	4
25	0.4	0.16	22	−7	49
			36	7	49
123		21.2	174		108

$$\therefore \bar{x} = \frac{\Sigma x_i}{n_1} = \frac{123}{5} = 24.6, \quad \bar{y} = \frac{\Sigma y_i}{n_2} = \frac{174}{6} = 29$$

$$\Sigma (x_i - \bar{x})^2 = 21.2, \quad \Sigma (y_i - \bar{y})^2 = 108.$$

$$\therefore s_1^2 = \frac{\Sigma (x_i - \bar{x})^2}{n_1 - 1} = \frac{21.2}{4} = 5.3$$

$$s_2^2 = \frac{\Sigma (y_i - \bar{y})^2}{n_2 - 1} = \frac{108}{5} = 21.6.$$

(I) F-Test (To test equality of Variances)

Null Hypothesis H_0: $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

Level of Significance : $\alpha = 5\% = 0.05$

Test statistic: $F = \frac{s_2^2}{s_1^2}$ $\langle \because s_2^2 > s_1^2 \rangle$

$$F_{cal} = \frac{21.6}{5.3} = 4.075$$

$$\therefore |F_{cal}| = 4.075$$

Conclusion: Degrees of freedom $(n_2-1, n_1-1) = (\overset{v_1}{5}, \overset{v_2}{4})$.

Tabulated value of F at 0.05 with (5,4) d.o.f is 6.26

i.e $F_{0.05}(5,4) = 6.26$

i.e $|F_{tab}| = 6.26$

$\therefore |F_{cal}| < |F_{tab}|$

$\therefore$ H_0 is accepted.

i.e $\boxed{\sigma_1^2 = \sigma_2^2}$

(II) t - Test (To test equality of means)

From the given data $\bar{x} = 24.6$, $\bar{y} = 29$

$\Sigma(x_i - \bar{x})^2 = 21.2$, $\Sigma(y_i - \bar{y})^2 = 108$

$$s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right]$$

$$s^2 = \frac{1}{5+6-2}\left[21.2+108\right] = 14.35$$

Null Hypothesis H_0: $\mu_1 = \mu_2$

Alternative Hypothesis H_1: $\mu_1 \neq \mu_2$

Level of Significance : $\alpha = 5\% = 0.05$

Test statistic: $t_{cal} = \frac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{24.6-29}{\sqrt{14.35\left(\frac{1}{5}+\frac{1}{6}\right)}} = -1.92$

$$|t_{cal}| = 1.92$$

Conclusion: Degree of freedom $= n_1 + n_2 - 2 = 9$

$|t_{Tab}| = 2.26$ $\langle$ See t-Table 9 at $\frac{0.05}{2} = 0.025$.

$\therefore |t_{cal}| < |t_{Tab}|$.

$\therefore$ H_0 is accepted

i.e $\boxed{\mu_1 = \mu_2}$

$\therefore$ From (I) and (II) we conclude that two samples came from the same normal population.

χ^2 – Test for goodness of fit:

Suppose we are given a set of "observed frequencies" obtained under some experiment and we want to test if the experimental results support a particular hypothesis or theory. Karl pearson developed a test for testing the significance of discrepancy between experimental values and theoretical values obtained under some theory or hypothesis.

This Test known as χ^2 – test of goodness of fit. Karl pearson proved the test statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}, \text{ where } O = \text{observed frequency (Experimental)}$$
$$E = \text{Expected frequency (Theoretical)}.$$

χ^2 – Test is used to "test whether the difference between observed and Expected frequencies are significant"

χ^2

Note:- ① If the data is given in series of "n" numbers then $d.o.f = n-1$.

② In case of Binomial distribution $d.o.f = n-1$

In case of poisson distribution $d.o.f = n-2$

In case of Normal distribution $d.o.f = n-3$.

properties of χ^2 – distribution:

① χ^2 – distribution curve is not symmetrical, lies entirely in the first quadrant and hence not a normal curve, since χ^2 varies from 0 to ∞.

② It depends on the degrees of freedom V.

③ If V_1, V_2 are the degree of freedom of χ_1^2 and χ_2^2 then (V_1+V_2) is the degree of freedom of $\chi_1^2 + \chi_2^2$

(4) Mean $= v$ and Variance $= 2v$.

Applications of χ^2 - distribution:

(i) To test the goodness of fit

(ii) To test the independence of attributes.

(iii) To test if the population has a specified value of the variance σ^2.

① The number of automobile accidents per week in a certain community are as follows: 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period.

Sol: Expected frequency of accidents each week $= \frac{100}{10} = 10$

Null Hypothesis H_0: The accidents conditions were the same during the 10 week.

Alternative Hypothesis H_1: The accidents conditions are different during the 10 week.

Level of significance: $\alpha = 5\%$

Observed Frequency (O)	Expected Frequency (E)	$(O-E)$	$\frac{(O-E)^2}{E}$
12	10	2	0.4
8	10	−2	0.4
20	10	10	10.0
2	10	−8	6.4
14	10	4	1.6
10	10	0	0.0
15	10	5	2.5
6	10	−4	1.6
9	10	−1	0.1
4	10	−6	3.6
100	100		26.6

Test statistic:

Now $\psi^2 = \sum \frac{(O-E)^2}{E}$

$\psi^2 = 26.6$

$\therefore |\psi^2 cal| = 26.6$

Conclusion: Degrees of freedom $= n-1 = 10-1 = 9$

Since no. of observations $= n = 10$.

The Tabulated value of ψ^2 for 9 d.o.f at 5% level of significance is 16.9

$\therefore |\psi^2 Tab| = 16.9$ < See ψ^2 Table 9 at 0.05 $= 16.919$

$\therefore |\psi^2 cal| > |\psi^2 Tab|$.

$\therefore$ H_0 is rejected.

i.e The accidents conditions are different during the 10 week. period

ψ^2

(3) A sample analysis of Examination results of 500 Students was made. It was found that 220 students had failed, 170 has secured a third class, 90 were placed in second class and 20 got a first class. Do these figures commensurate with the general Examination result which is in the ratio of 4 : 3 : 2 : 1 for the various categories respectively.

Sol Total frequency $= 220 + 170 + 90 + 20 = 500$

Expected frequencies are in the ratio of $4:3:2:1$

Sum of Ratio $= 4+3+2+1 = 10$

If we devide the total frequency 500 in the ratio $4:3:2:1$, we get Expected freequencies as $\frac{4}{10} \times 500 = 200$, $\frac{3}{10} \times 500 = 150$, $\frac{2}{10} \times 500 = 100$, and $\frac{1}{10} \times 500 = 50$.

i.e Expected freequencies are 200, 150, 100, 50.

Class	Observed Frequency (O)	Expected Frequency (E)	(O - E)	$\frac{(O-E)^2}{E}$
Failed	220	200	20	2.00
Third	170	150	20	2.667
second	90	100	-10	1.000
First	20	50	-30	18.00
	500	500		23.667

Null Hypothesis H_0: The observed results commensurate with the general Examination results.

Alternative Hypothesis H_1: The observed results don't commensurate with the general Examination results.

Level of Significance: $\alpha = 5\%$.

Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E} = 23.667$

$\therefore |\chi^2 cal| = 23.667$

Conclusion: Degrees of freedom $= (n-1) = 4-1 = 3$.

Tabulated value of χ^2 at 5% level for 3 d.o.f $= 7.81$

ie $|\chi^2 Tab| = 7.81$. See χ^2 Table 3 at 0.05 $= 7.815$

$\therefore |\chi^2 cal| > |\chi^2 Tab|$

$\therefore$ H_0 is rejected.

i.e The observed results are not commensurate with the general Examination results.

② A die is thrown 264 times with the following results. Show that the die is biased (Given $\psi^2_{0.05} = 11.07$ for 5 d.o.f).

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

Sol: Expected frequency of each of the numbers 1, 2, 3, 4, 5, 6 is $\frac{264}{6} = 44$ $\left\langle \because \frac{\text{Sum of frequency}}{\text{no. of units}} \right.$

Null Hypothesis H_0: The die is unbiased.

Alternative Hypothesis H_1: The die is biased

Level of Significance : $\alpha = 5\% = 0.05$

Test statistic: $\psi^2 = \sum \frac{(O-E)^2}{E}$

ψ^2-3

No. appeared on the die	observed frequency (O)	Expected frequency (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	40	44	16	0.3636
2	32	44	144	3.2727
3	28	44	256	5.8181
4	58	44	196	4.4545
5	54	44	100	2.2727
6	52	44	64	1.4545
	264	264		17.6362

$\therefore \psi^2 = \sum \frac{(O-E)^2}{E} = 17.6362$

ie $|\psi^2 \text{cal}| = 17.6362$

Conclusion: Degrees of freedom $= n-1 = 5$

Tabulated Value of ψ^2 for 5 d.o.f at 5% level = 11.07

i.e $|\psi^2 \text{ Tab}| = 11.07$ $\left\langle \text{See } \psi^2 \text{ Table 5 at } 0.05 = 11.07 \right.$

$|\psi^2 \text{ Tab}| > |\psi^2 \text{ Tab}|$

$\therefore$ H_0 is rejected

i.e The die is biased.

(4) A pair of dice are Thrown 360 times and frequency of each Sum is indicated below.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

would you say that the dice are fair on the basis of the χ^2 test at 0.05 level of significance.

Sol

Null Hypothesis H_0: The dice are fair

Sum of frequency = 360

Alternative Hypothesis H_1: The dice are not fair

Level of Significance : $\alpha = 5\% = 0.05$.

The probabilities of a getting Sum 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Sum	observed Frequency (O)	Expected Frequency $E = 360 \times P(x)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
2	8	10	4	0.4
3	24	20	16	0.8
4	35	30	25	0.833
5	37	40	9	0.225
6	44	50	36	0.72
7	65	60	25	0.417
8	51	50	1	0.02
9	42	40	4	0.1
10	26	30	16	0.53
11	14	20	36	1.8
12	14	10	16	1.6
	360	360		7.445

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 7.445 \quad \text{ie } |\chi^2 cal| = 7.445$$

Conclusion:

Degrees of freedom $= n-1 = 11-1 = 10$

$|\chi^2 Tab| = 18.3 \quad \Rightarrow \quad |\chi^2 cal| < |\chi^2 Tab|$

$\therefore$ Ho is accepted

i.e The dice are fair.

See χ^2 Table 10 at 0.05 $= 18.307$

⑤ Fit a poisson distribution to the following data and test the goodness of fit at 0.05 LOS:

x	0	1	2	3	4	5	6	7
$f(x)$	305	366	210	80	28	9	2	1

Sol Null Hypothesis H_0: poisson distribution is a good fit
Alternative Hypothesis H_1: poisson distribution is not a good fit.
Level of Significance : $\alpha = 0.05$

$$\text{Mean} = \lambda = \frac{\sum x f(x)}{\sum f(x)} = \frac{0\times305+1\times366+2\times210+3\times80+4\times28+5\times9+6\times2+7\times1}{1001}$$

$$= \frac{1202}{1001}$$

$$\therefore \boxed{\lambda = 1.2}$$

By poisson dist^n $P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $x = 0,1,2 \ldots 7$.

$$P(x) = \frac{e^{-1.2}(1.2)^x}{x!}$$

$\therefore$ Expected frequence $E = N \times P(x)$

$$E = \frac{1001 \times e^{-1.2}(1.2)^x}{x!} = \frac{1001\times0.3\,(1.2)^x}{x!}$$

$$\therefore \boxed{E = (300.3)\left(\frac{(1.2)^x}{x!}\right)}$$

χ^2-4.

x	Observed Frequency (O).	Expected Frequency (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	305	$300.3 \times \frac{1.2^0}{0!} = 300.3$	22.09	0.0736
1	366	$300.3\times \frac{1.2^1}{1} = 360.36$	31.810	0.0883
2	210	$300.3\times \frac{1.2^2}{2!} = 216.216$	38.639	0.1787
3	80	$300.3\times \frac{1.2^3}{3!} = 86.48$	41.990	0.4355
4	28	$300.3\times \frac{1.2^4}{4!} = 25.946$	4.219	0.1626
5	9	$300.3\times \frac{1.2^5}{5!} = 6.227$	7.689	1.2348
6	2	$300.3\times \frac{1.2^6}{6!} = 1.245$	0.570	0.4578
7	1	$300.3\times \frac{1.2^7}{7!} = 0.213$	0.619	2.9061
	1001			5.5874

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 5.5874$$

$\therefore \ |\chi^2 cal| = 5.5874$

Conclusion: Degrees of freedom $= n-2 = 8-2 = 6$ See χ^2 Table 6 at 0.05

Tabulated Value of χ^2 at 5% level for 6 d.o.f $= 12.592$

$\therefore \ |\chi^2 Tab| = 12.592$

$\therefore \ |\chi^2 cal| < |\chi^2 Tab|$

$\therefore$ H_0 is accepted

i.e poisson distribution is a good fit.

χ^2 Test for Independent of Attributes:

Attribute: An Attribute means a quality or characteristic.

Examples of attributes are drinking, Smoking, honesty, beauty etc.

An Attribute may be marked by its presence (position) or absence in a number of a given population

Let us consider two attributes A and B.
A is divided into two classes and B is divided into two classes.
The Various cell frequencies can be Expressed in the following table known as 2x2 Contingency table.

A	a	b
B	c	d

a	b	a+b
c	d	c+d
a+c	b+d	N=a+b+c+d

I A-1

The Expected frequencies are given by

$E(a) = \frac{(a+c)(a+b)}{N}$	$E(b) = \frac{(b+d)(a+b)}{N}$	a+b
$E(c) = \frac{(a+c)(c+d)}{N}$	$E(d) = \frac{(b+d)(c+d)}{N}$	c+d
a+c	b+d	N= a+b+c+d

Note:- In this χ^2 Test, The Test of two Attributes A and B under consideration are independent or not.

Null Hypothesis H_0: Attributes are independent.

Degrees of freedom = $(r-1)(s-1)$ where r = no. of rows
s = no. of columns.

1. On the basis of information given below about the treatment of 200 patients suffering from a disease, state whether the new treatment is comparatively superior to the conventional treatment.

	Favourable	Not Favourable	Total
New	60	30	90
Conventional	40	70	110

Sol (i) Null Hypothesis H_0: No difference between new and conventional treatment (or) New and conventional treatments are independent.

(ii) Alternative Hypothesis H_1: New and conventional treatments are not independent.

(iii) Level of Significance : $\alpha = 5\% = 0.05$.

(iv) Computations

	Favourable	Not favourable	Total
New	60	30	90
Conventional	40	70	110
Total	100	100	200

Table of Expected frequencies

$\frac{100 \times 90}{200} = 45$	$\frac{100 \times 90}{200} = 45$	90
$\frac{100 \times 110}{200} = 55$	$\frac{100 \times 110}{200} = 55$	110
100	100	200

Observed Frequency (O)	Expected Frequency (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
60	45	225	5
30	45	225	5
40	55	225	4.09
70	55	225	4.09
			$\Sigma\left[\frac{(O-E)^2}{E}\right] =$ 18.18

$$\therefore \chi^2 = \Sigma\left[\frac{(O-E)^2}{E}\right]$$

$$= 18.18$$

$$\therefore |\chi^2_{cal}| = 18.18$$

Conclusion: Degrees of freedom $= (r-1)(s-1) = (2-1)(2-1) = 1$

$|\chi^2_{Tab}| = 3.841.$ < See χ^2 Table | at 0.05 = 3.841

$\therefore |\chi^2_{cal}| > |\chi^2_{Tab}|$

$\therefore$ H_0 is rejected.

i.e New and Conventional treatment are not independent.

(2) From the following data, find whether there is any significant liking in the habit of taking soft drinks among the categories of employees.

soft drinks	employees Clerks	Teachers	officers
pepsi	10	25	65
Thumsup	15	30	65
Fanta	50	60	30

<Dec 2017, 15>

Sol

(i) Null Hypothesis H_0: There is no significant liking in the habit of taking soft drinks

(ii) Alternative Hypothesis H_1: There is significant liking.

(iii) Level of Significance: $\alpha = 0.05$.

(iv) Computations

I A-2

Soft drinks	employees Clerks	Teachers	officers	Total
pepsi	10	25	65	100
Thumsup	15	30	65	110
Fanta	50	60	30	140
Total	75	115	160	350

Table of Expected frequencies.

$\frac{75\times100}{350} = 21.4$	$\frac{115\times100}{350} = 32.9$	$\frac{160\times100}{350} = 45.7$	100
$\frac{75\times110}{350} = 23.6$	$\frac{115\times110}{350} = 36.1$	$\frac{160\times110}{350} = 50.3$	110
$\frac{75\times140}{350} = 30$	$\frac{115\times140}{350} = 46$	$\frac{160\times140}{350} = 64$	140
75	115	160	350

Observed Frequency (O)	Expected Frequency (E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
10	21.4	129.96	6.073
25	32.9	62.41	1.897
65	45.7	372.49	8.151
15	23.6	73.96	3.134
30	36.1	37.21	1.031
65	50.3	216.09	4.3
50	30	400	13.333
60	46	196	4.261
30	64	1156	18.062
			60.2425

Test Statistic $\psi^2 = \sum \frac{(O-E)^2}{E} = 60.2425$

$\therefore |\psi^2 \text{cal}| = 60.2425.$

Conclusion: Degrees of freedom $= (r-1)(s-1) = (3-1)(3-1) = 4.$

$|\psi^2 \text{Tab}| = 9.488$ See ψ^2 Table 4 at 0.05 $= 9.488$

$\therefore |\psi^2 \text{cal}| > |\psi^2 \text{Tab}|.$

$\therefore$ Ho is rejected.

i.e The Habit of taking soft drink depends on categories of employees.

H.W

(3) Given the following contingency table for hair colour & eye colour. Find the value of ψ^2. Is there good Association between the two?

		Hair Colour			
		Fair	Brown	Black	Total
Eye colour	Blue	15	5	20	40
	Grey	20	10	20	50
	Brown	25	15	20	60
	Total	60	30	60	150

Sol
Hints

H_0: Hair and eye colour are independent

H_1: Hair and eye colour are dependent

L: $\alpha = 0.05$

$|\chi^2 cal| = 3.6458$

$\langle$ D.o.f $= (3-1)(3-1) = 4$.

$|\chi^2 Tab| = 9.488$

Ho is accepted

i.e Hair and Eye colour are independent.

i.e Hair and eye colour are not associated.

IA-4

χ^2 - Test for population Variance:

Suppose that a random sample is drawn from a normal population with mean μ and Variance σ^2

under the Null Hypothesis H_0 that the population Variance σ^2 has a specified value σ_0^2. Test statistic is

$$\chi^2 = \sum \frac{(x_i - \bar{x})^2}{\sigma_0^2} = \frac{ns^2}{\sigma_0^2}$$

It follows χ^2 - distribution with $(n-1)$ d.o.f.

① A firm manufacturing rivets wants to limit variations in their length as much as possible. The lengths (in cms) of rivets manufactured by new process are.

2.15	1.99	2.05	2.12	2.17
2.01	1.98	2.03	2.25	1.93.

Examine whether the new process can be considered superior to the old if the old population has standard deviation 0.145 cm?

Sol Given $n = 10$, $\bar{x} = \frac{\sum x_i}{n} = \frac{20.68}{10} = 2.068$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{0.09096}{9} = 0.0101$$

and $\sigma_0 = 0.145$.

(i) Null Hypothesis H_0: $\sigma^2 = \sigma_0^2$

(ii) Alternative Hypothesis H_1: $\sigma^2 \neq \sigma_0^2$

(iii) Level of Significance : $\alpha = 0.05$

(iv) Test Statistic : $\chi^2 = \frac{ns^2}{\sigma_0^2} = \frac{10 \times 0.0101}{(0.145)^2} = 4.8$

$\therefore |\chi^2 cal| = 4.8$

(v) Conclusion: d.o.f $= n-1 = 9$ — See χ^2 Table 9 at 0.05 $= 16.919$

$|\chi^2 Tab| = 16.919$

$\therefore |\chi^2 cal| < |\chi^2 Tab|$

$\therefore$ Ho is accepted.

② A random sample of size 20 from a normal population gives a mean of 42 and variance 25. Test the hypothesis that the population standard deviation is 8 at 5% level.

Sol Given $n = 20$, $\bar{x} = 42$, $s^2 = 25$, $\sigma_0 = 8$

(i) Null Hypothesis $H_0: \sigma^2 = \sigma_0^2$

(ii) Alternative Hypothesis $H_1: \sigma^2 \neq \sigma_0^2$

(iii) Level of significance : $\alpha = 0.05$

(iv) Test Statistic : $\psi^2 = \frac{ns^2}{\sigma_0^2} = \frac{20 \times 25}{8^2} = 7.8125$

$\therefore \; |\psi^2 \text{ cal}| = 7.8125$

(v) Conclusion: $d.o.f = n-1 = 19.$

$\therefore \; |\psi^2 \text{ Tab}| = 30.14$ ⟨ see ψ^2 Table 19 at 0.05 $= 30.14$

$\therefore \; |\psi^2 \text{ cal}| < |\psi^2 \text{ Tab}|$

$\therefore$ Ho is accepted.

*

2. t_α- Critical values of t-distribution

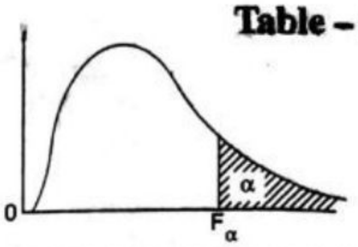
v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	20920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

3. t_α-Critical values of t-distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.051
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.125	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

Critical Values of the F-Distribution

Table – 5



Values of $F_{0.05}(\nu_1, \nu_2)$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Values of $F_{0.05}(v_1, v_2)$

v_2	v_1 = 10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.75	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Values of $F_{0.01}(v_1, v_2)$

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	405.2	4999.5	5403	5625	5764	5859	5928	5981	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.67	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.80	2.51	2.41

Values of $F_{0.01}(v_1, v_2)$

v_2	v_1: 10	12	15	20	24	30	40	60	120	∞
1	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	99.40	99.42	99.43	99.46	99.46	99.47	99.47	99.48	99.49	99.50
3	27.23	27.05	26.87	26.60	26.60	26.50	26.41	26.32	26.22	26.13
4	14.55	14.37	14.20	13.93	13.93	13.84	13.75	13.65	13.56	13.46
5	10.05	9.89	9.72	9.47	9.47	9.38	9.29	9.20	9.11	9.02
6	7.87	7.72	7.56	7.31	7.31	7.23	7.14	7.06	6.97	6.88
7	6.62	6.47	6.31	6.07	6.07	5.99	5.91	5.82	5.74	5.65
8	5.81	5.67	5.52	5.28	5.28	5.20	5.12	5.03	4.95	4.86
9	5.26	5.11	4.96	4.73	4.73	4.65	4.57	4.48	4.40	4.31
10	4.85	4.71	4.56	4.33	4.33	4.25	4.17	4.08	4.00	3.91
11	4.54	4.40	4.25	4.02	4.02	3.94	3.86	3.78	3.69	3.60
12	4.30	4.16	4.01	3.78	3.78	3.70	3.62	3.54	3.45	3.36
13	4.10	3.96	3.82	3.59	3.59	3.51	3.43	3.34	3.25	3.17
14	3.94	3.80	3.66	3.43	3.43	3.35	3.27	3.18	3.09	3.00
15	3.80	3.67	3.52	3.29	3.29	3.21	3.13	3.05	2.96	2.87
16	3.69	3.55	3.41	3.18	3.18	3.10	3.02	2.93	2.84	2.75
17	3.59	3.46	3.31	3.08	3.08	3.00	2.92	2.83	2.75	2.65
18	3.51	3.37	3.23	3.00	3.00	2.92	2.84	2.75	2.66	2.57
19	3.43	3.30	3.15	2.92	2.92	2.84	2.76	2.67	2.58	2.49
20	3.37	3.23	3.09	2.86	2.86	2.78	2.69	2.61	2.52	2.42
21	3.31	3.17	3.03	2.80	2.80	2.72	2.64	2.55	2.46	2.36
22	3.26	3.12	2.98	2.75	2.75	2.67	2.58	2.50	2.40	2.31
23	3.21	3.07	2.93	2.70	2.70	2.62	2.54	2.45	2.35	2.26
24	3.17	3.03	2.89	2.66	2.66	2.58	2.49	2.40	2.31	2.21
25	3.13	2.99	2.85	2.62	2.62	2.54	2.45	2.36	2.27	2.17
26	3.09	2.96	2.81	2.58	2.58	2.50	2.42	2.33	2.23	2.13
27	3.06	2.93	2.78	2.55	2.55	2.47	2.38	2.29	2.20	2.10
28	3.03	2.90	2.75	2.52	2.52	2.44	2.35	2.26	2.17	2.06
29	3.00	2.87	2.73	2.49	2.49	2.41	2.33	2.23	2.14	2.03
30	2.98	2.84	2.70	2.47	2.47	2.39	2.30	2.21	2.11	2.01
40	2.80	2.66	2.52	2.29	2.29	2.20	2.11	2.02	1.92	1.80
60	2.63	2.50	2.35	2.12	2.12	2.03	1.94	1.84	1.73	1.60
120	2.47	2.34	2.19	1.95	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.32	2.18	2.04	1.79	1.79	1.70	1.59	1.47	1.32	1.00

4. χ_α^2- Critical values of the chi-squared distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.75	0.50
1	0.04393	0.03157	0.03628	0.03982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.103	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.352	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.711	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	1.145	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.635	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	2.167	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.733	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	3.325	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.940	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	4.575	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	5.226	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.892	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	6.571	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	7.261	7.261	8.547	10.307	11.036	11.721	14.339
16	5.142	5.812	6.614	7.962	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	8.672	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	9.390	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	10.117	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	10.851	10.851	12.443	14.578	14.452	16.266	19.337
21	8.034	8.897	9.915	11.591	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	12.338	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	13.091	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	13.848	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	14.611	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	15.379	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	16.151	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	16.928	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	17.708	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	18.493	18.493	20.599	23.364	24.478	25.508	29.336

χ_α^2- Critical values of the chi-squared distribution

v	α									
	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.545	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302
30	33.530	34.800	36.250	10.256	43.773	46.979	47.962	50.892	53.672	59.703