Probability and Statistics

Unit-I : Descriptive Mathematics and Methods for data Science

Topics

Data science, Statistics introduction, Population vs Sample, collection of data, primary and Secondary data, dependent and independent categorical and continuous Variables, data visualization, Measures of central tendency, Measures of variability, Skewness, kurtosis correlation, correlation coefficient, Rank correlation, regression coefficients, principle of least squares, Method of least squares, Regression lines

Unit-II : Probability

Topics

probability, probability axioms, Addition law and Multiplicative law of probability, conditional probability, Bayes theorem, Random variables, Probability density functions, properties and Mathematical Expectation

Unit-III : probability Distributions

Binomial Distribution, poison distribution,

poison approximation to the binomial distribution,

Normal distribution and their properties

Unit-IV ; Estimation and testing of Hypothesis,

Large sample test (Part-A)

Topics

Estimation, parameters, statistics, Sampling distribution, Point Estimation, formulation of Null hypothesis, Alternative hypothesis, the critical and acceptance regions, level of significance, two types of errors and power of the test

Large Sample test Part-B

Test for Single proportion, difference of proportions, test for single mean, and difference of Means, confidence interval for parameters in one sample and two sample problems

Unit-V : Small Sample test

Student theory distribution test for single mean, two means and paired t-test, testing of Equality of variances (F-test), χ^2 test for goodness of fit, χ^2 test for independence of attributes

Date
22/01/2020

Unit-II

Probability

Experiment:- the physical process performing the number of times a problem to know the result of that problem is known as an Experiment.

Experiments are classified into two categories. They are

1. Determinastic Experiment
2. Probabilistic or Undeterministic Experiments

Determinastic Experiment:-

When an Experiment is conducted the outcome (or) result of an Experiment is predictable with certainity such Experiment is called determinastic Experiment

Ex: 1 Volume of the sphere having radius r is $V = \frac{4}{3}\pi r^3$.

2. Area of the circle having radius r is $A = \pi r^2$

Probabilistic or Undeterminastic Experiment:-

When an Experiment is conducted the outcome of an experiment is unpredictable with certainity such Experiment is called probabilistic Experiment or undeterminastic Experiment

Ex: Tossing a coin is probabilistic Experiment.

Ex: Throwing a die is also probabilistic Experiment.

Random Experiment:

Random Experiment is an Experiment whose outcome is not unique and therefore cannot be predicted with certainity

Ex: When tossing a coin the possible outcome is head or tail, is an Random experiment

Ex: Throwing a die is also a Random Experiment because the outcome of experiment have more possibilities

Outcome

The result of a Random experiment is called an outcome.

Ex: when tossing a coin there are two possible outcomes that are head (H) and tail (T)

Trail

Single performance of a Experiment is called a trail

Sample space (S)

The set of all possible outcomes of a Random Experiment, is known as Sample space and it is denoted by S.

Ex: When tossing a coin the possibilities of outcome are Head (H) and Tail (T).

So sample space $S = \{H, T\}$

Ex:2 Tossing two coins ~~are~~ (or) a coin is tossing two times the possible outcomes are HH, HT, TH, TT

Therefore Sample space $S = \{HH, HT, TH, TT\}$

Ex:3 Two dice are thrown at a time the possible outcomes

i.e Sample Space $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2),$
$(2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1),$
$(4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5),$
$(5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Note:—

i) Each element in the sample space is known as Sample point

ii) if "n" coins are tossed at a time the Sample Space contains 2^n sample points, where "2" is the possible outcomes of a coin.

iii) If "n" dice are rolled at a time then the sample space contains 6^n sample points where "6" is the possible outcomes of a die

Event:

The set of one or more outcomes of the sample space is called an Event. In other words subset of the sample space is called an Event

Ex: When tossing two coins

sample space $S = \{HH, HT, TH, TT\}$

A is getting two heads, possibilities $= \{HH\}$ is an event

B is getting one head, possibilities $= \{HT, TH\}$ is also an Event

Exhaustive Events: (existed event)

All possible outcomes in any trail of an Experiment is known as an Exhaustive Event. In other words a set is said to be Exhaustive when no other possibilities exist.

Ex:1 In tossing a coin there are two exhaustive events they are Head, Tail

Ex:2 In throwing a die there are six exhaustive events they are 1, 2, 3, 4, 5, 6

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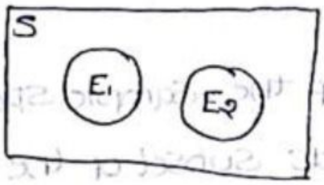
Mutually Exclusive Events (Disjoint Events)

The Events are said to be mutually Exclusive Events (or) disjoint events if the occurance of one event eliminates the occurance of all other events. In other words if no two or more of the events can happen simultaneously in this same trail i.e if E_1 and E_2 are mutually exclusive events then E_1 and E_2 will not happen together therefore

$\boxed{E_1 \cap E_2 = \phi}$

The venn diagram for mutually Exclusive events is



Note:

$E_1, E_2 \cdots E_i, E_j \cdots$ are mutually Exclusive events if and only if $E_i \cap E_j = \phi$ for $i \neq j$

Equally Likely Events:

Two or more events are said to be Equally likely events if the chances of happening that events are Equal.

Ex: In tossing a coin

E_1 is event of getting a Head

E_2 is an event of getting a Tail,

are Equally likely events because when tossing a coin we have Equal chances to get Head and Tail i.e E_1 and E_2 have Equal chances to Exist.

Ex:2 :-

In throwing a die getting 1,2,3,4,5,6 are Equally likely events because when throwing a die the six numbers have Equal chances to come

Complimentary of an Event :-

The complimentary Event of an event E is the Set of basic outcomes in the Sample space S that do not belong to E. The complimentary event of E is denoted by $\bar{E}$ (or) E^c (or) E' i.e

$E \cap \bar{E} = \phi$ and $E \cup \bar{E} = S$

Impossible Event :-

An Event which never happens is called impossible Event

Ex: getting both head and tail in tossing a coin is an impossible event

Null Event :-

An Event which has no Sample point is called a null event and it is denoted by "ϕ"

Simple Event :-

An Event having only one Sample point is called Simple Event.

* 2M imp

Mathematical (or) Classifical definition of probability :-

Probability means chances of happening an Event.

In a Random Experiment, "n" is the total number of Exhaustive outcomes in Sample Space S of the experiment and m is the number of favourable outcomes of happening an event A.

Then the probability of an event A is denoted by P or P(A) and is defined as

$$p = P(A) = \frac{\text{no of favourable cases of } A}{\text{Total no of possible cases}}$$

$$= \frac{m}{n}$$

$$= \frac{n(A)}{n(S)}$$

Where n(A) is the number of elements in the Event A.

n(S) is the number of elements in the Sample space S.

Note :-

* Let $\bar{A}$ denotes the event of not happening an Event A.

So n-m favourable cases to happening an Event $\bar{A}$

Then the probability of happening an event $\bar{A}$ is

$$P(\bar{A}) = \frac{\text{no of favourable cases of an event } \bar{A}}{\text{Total no of possible cases}}$$

$$= \frac{n-m}{n}$$

$$= \frac{n}{n} - \frac{m}{n}$$

$$= 1 - \frac{m}{n}$$

$$\boxed{P(\bar{A}) = 1 - P(A)}$$

$$\boxed{P(A) + P(\bar{A}) = 1}$$

This means probability value of any event is always positive and lies between 0 and 1.

i) Suppose $P(A)$ is the probability of success and $P(\bar{A})$ is the probability of failure in an experiment.

Then we have $P(A)+P(\bar{A})=1$

i.e probability of success + probability of failure = 1

Therefore total probability of any experiment is equal to one

i.e $P(S)=1$

iii) Probability of impossible event ϕ is zero

i.e $P(\phi)=0$

Axiomatic definition of probability :-

Let 'S' be the sample space and A' be any event associated with a Random experiment then the probability is a function defined on a sample space satisfying the following axioms (conditions)

i) $0 \leq P(A) \leq 1$ for any event A in S

This means probability value of every event must be positive and lies between 0 and 1.

ii) $P(S)=1$

This means total probability of every experiment is equal to one

iii) if A_1 and A_2 are two mutually exclusive events then probability of $P(A_1 \cup A_2) = P(A_1)+P(A_2)$

This means probability of sum of two mutually exclusive events is equal to sum of their probabilities.

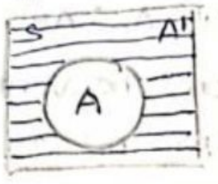
Theorem-1 :-

Statement: if $\bar{A}$ is the complimentary event of A then

$P(\bar{A}) = 1-P(A)$

Proof: Let $\bar{A}$ be the complimentary event of A

So clearly A and $\bar{A}$ are mutually exclusive events (disjoint events)



From the venn diagram we have $A \cap \bar{A} = \phi$ $A \cup \bar{A} = S$

Taking probability on both sides in Above.

$$P(A \cup \bar{A}) = P(S)$$

$$P(A) + P(\bar{A}) = P(S) \quad [\because A \text{ and } \bar{A} \text{ are mutually Exclusive}]$$

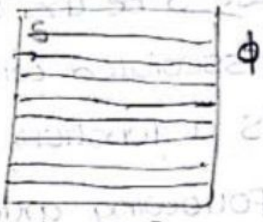
$$P(A) + P(\bar{A}) = 1 \quad [\because \text{By axiom no } 2]$$

$$P(\bar{A}) = 1 - P(A)$$

Theorem-2 :-

Statement:- probability of impossible event is zero i.e $P(\phi) = 0$

Proof: Let S be the sample space and ϕ be the impossible event



So events S and ϕ are mutually Exclusive events. i.e $S \cap \phi = \phi$

From the venn diagram $S \cup \phi = S$ [$\because \phi$ is empty set]

Taking probability on b.s in above.

$$P(S \cup \phi) = P(S) \quad [\because S \text{ and } \phi \text{ are mutually exclusive}]$$

$$P(S) + P(\phi) = P(S)$$

$$1 + P(\phi) = 1$$

$$P(\phi) = 1 - 1$$

$$P(\phi) = 0.$$

Therefore probability of impossible event is zero.

Theorem-3 :-

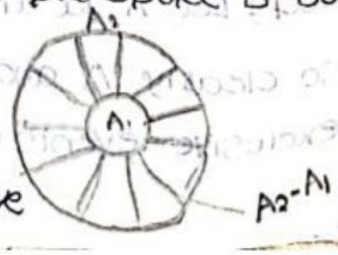
Statement:- If A_1 and A_2 are two events in the Sample Space S Such that $A_1 \subseteq A_2$ then $P(A_2 - A_1) = P(A_2) - P(A_1)$

Proof :-

Let A_1 and A_2 be two events in the Sample Space S, Such that $A_1 \subseteq A_2$

From the venn diagram Clearly

A_1 and $A_2 - A_1$ are mutually Exclusive



and we have $A_1 \cap (A_2 - A_1) = \phi$ and $A_2 = A_1 \cup (A_2 - A_1)$

Taking probability on both sides in above

$$P(A_2) = P(A_1 \cup (A_2 - A_1))$$

$$P(A_2) = P(A_1) + P(A_2 - A_1) \quad [\because A_1 \text{ and } A_2 - A_1 \text{ are mutually exclusive}]$$

$$P(A_2 - A_1) = P(A_2) - P(A_1)$$

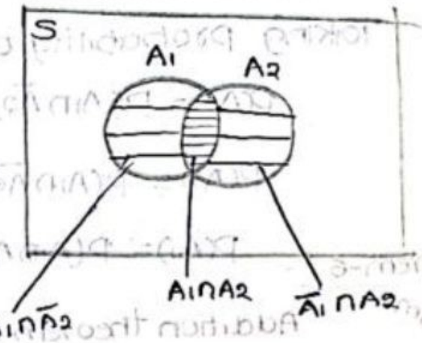
Theorem-4 :-

Statement:- If A_1 and A_2 are two events in the sample space S then

i) $P(A_1 \cap \bar{A}_2) = P(A_1) - P(A_1 \cap A_2)$

ii) $P(\bar{A}_1 \cap A_2) = P(A_2) - P(A_1 \cap A_2)$

Proof: Let A_1 and A_2 be any two events in the sample space S



i) From the venn diagram clearly $(A_1 \cap \bar{A}_2)$ and $(A_1 \cap A_2)$ are mutually exclusive and we have

$(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2) = \phi$

and $A_1 = (A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)$

Taking probability on both sides in above

$$P(A_1) = P((A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2))$$

$$P(A_1) = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2)$$

$$\boxed{P(A_1 \cap \bar{A}_2) = P(A_1) - P(A_1 \cap A_2)}$$

(ii) From the Venn diagram clearly $(A_1 \cap A_2)$ and $(\bar{A}_1 \cap A_2)$ are mutually exclusive and we have

$(\bar{A}_1 \cap A_2) \cap (A_1 \cap A_2) = \phi$

and $A_2 = (\bar{A}_1 \cap A_2) \cup (A_1 \cap A_2)$

Taking probability on both sides

$$P(A_2) = P((\bar{A}_1 \cap A_2) \cup (A_1 \cap A_2))$$

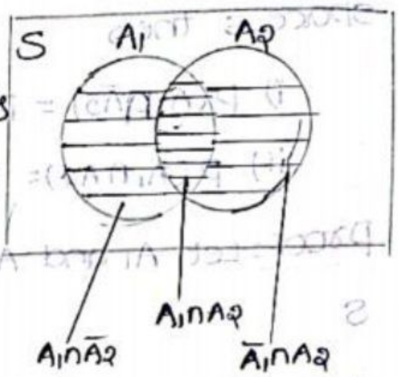
$P(A_2) = P(\bar{A}_1 \cap A_2) + P(A_1 \cap A_2)$

$P(\bar{A}_1 \cap A_2) = P(A_2) - P(A_1 \cap A_2)$

Theorem - 5 :-

Statement: If A_1 and A_2 are two events in the sample space S then $P(A_1) = P(A_1 \cap A_2) + P(A_1 \cap \bar{A}_2)$

Proof: Let A_1 and A_2 be any two events in the sample space S.



From the venn diagram clearly $A_1 \cap \bar{A}_2$ and $(A_1 \cap A_2)$ are mutually exclusive events and we have

$(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2) = \phi$ and

$A_1 = (A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)$

Taking probability on both sides

$P(A_1) = P((A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2))$.

$P(A_1) = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2)$

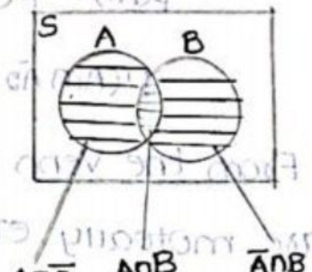
$P(A_1) = P(A_1 \cap A_2) + P(A_1 \cap \bar{A}_2)$

Theorem-6 :-

*** V.V.imp Addition theorem of probability

Statement: If A and B are two events in the sample space S then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof: Let A and B be any two events in the sample space S



From the venn diagram clearly $A \cap \bar{B}$, $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive events (disjoint events).

And we have

$(A \cap \bar{B}) \cap (A \cap B) \cap (\bar{A} \cap B) = \phi$

And $(A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B) = A \cup B$

Taking probability on both sides in above

$P((A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)) = P(A \cup B)$

$P(A\cap\bar{B})+P(A\cap B)+P(\bar{A}\cap B) = P(A\cup B)$ $\quad$ [$\because A\cap B, A\cap\bar{B}, \bar{A}\cap B$ are mutually exclusive events] $\cdots\cdots(1)$

From the venn diagram we have

$A=(A\cap\bar{B})\cup(A\cap B)$

Taking probability on both sides

$P(A)= P((A\cap\bar{B})\cup(A\cap B))$ $\quad$ [$\because (A\cap\bar{B})$ and $(A\cap B)$ are mutually exclusive]

$P(A)= P(A\cap\bar{B}) + P(A\cap B) \rightarrow (2)$

Also from venn diagram we have

$B=(A\cap B)\cup(\bar{A}\cap B)$

Taking probability on b.s

$P(B)= P((A\cap B)\cup(\bar{A}\cap B))$ $\quad$ [$\because A\cap B$ and $\bar{A}\cap B$ are mutually exclusive]

$P(B)= P(A\cap B) + P(\bar{A}\cap B) \rightarrow (3)$

Adding Eqns (2) & (3)

$P(A)+P(B) = P(A\cap\bar{B}) + P(A\cap B) + P(\bar{A}\cap B) + P(A\cap B)$

$= P(A\cap B) + (P(A\cap\bar{B}) + P(A\cap B) + P(\bar{A}\cap B))$

[$\because$ from Eqn (1)]

$P(A)+P(B) = P(A\cap B) + P(A\cup B)$

$$\boxed{P(A\cup B)= P(A)+P(B)-P(A\cap B)}$$

Theorem:7 :-

Statement: If A, B and C are any three events in sample space then $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

Proof: Let A, B, C be any three events in the sample space S

By the addition theorem of probability we know that

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

Now $P(A\cup B\cup C) = P(A\cup(B\cup C))$

$= P(A)+P(B\cup C) - P(A\cap(B\cup C))$

$= P(A)+P(B\cup C) - P((A\cap B)\cup(A\cap C))$

$= P(A)+P(B)+P(C)-P(B\cap C)-[P(A\cap B)+P(A\cap C) - P((A\cap B)\cap(A\cap C))]$

$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(B\cap C)-P(A\cap C)-P(A\cap B)+P(A\cap B\cap C)$

$$\boxed{P(A\cup B\cup C)= P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)}$$

Theorem: 8

Statement: if A_1 and A_2 are any two events in the sample space S Such That $A_1 \subseteq A_2$ then $P(A_1) \leq P(A_2)$

Proof: Let A_1 and A_2 be any two events in the sample space S Such that $A_1 \subseteq A_2$

Since A_1 and A_2 are two events so $A_2 - A_1$ is also an Event in sample space S.

We know that probability of every event must be positive

So $P(A_2 - A_1) \geq 0$

$P(A_2) - P(A_1) \geq 0$ [$\because$ From theorem no 3]

$P(A_2) \geq P(A_1)$

$P(A_1) \leq P(A_2)$

Theorem 9:-

Statement: If A, B are two events in the sample space then

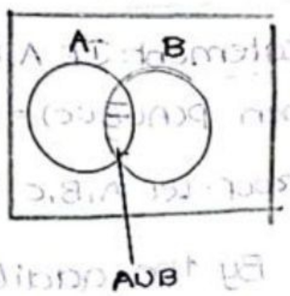
$P(A\cap B) \leq P(A) \leq P(A\cup B) \leq P(A)+P(B)$

Proof: Let A, B be any two events in the sample space S

we know that

$A\cap B \subseteq A \subseteq A\cup B$

Taking Probability on b.s in above



$P(A\cap B) \leq P(A) \leq P(A\cup B)$ [$\because$ From The.⑧] $\to$ ①

Also we know that

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$P(A\cup B) \leq P(A)+P(B) \to$ ②

From Eqns ① & ②

$P(A\cap B) \leq P(A) \leq P(A\cup B) \leq P(A)+P(B)$

Example

$4=3+2-1$

Neglect 1

if it is impossible event $P(A\cap B)=0$.

$4 \leq 5$

Note:
pack of cards

In pack of cards we have 52 cards

Diamond (♦) (13)	Ispade ♠ (13)	Artin ♥ (13)	Kalavour (♣) (club) (13)
Red cards (26)		Black cards (26)	
Ace (A) = 4 (2red + 2black)	Queen (Q) = 4 (2r + 2b)	King (K) = 4 (2r + 2b)	Jackie (J) = 4 (2r + 2b)

Note (ii)

2. $^nc_r = \frac{n!}{r!(n-r)!}$

3. $^np_r = \frac{n!}{(n-r)!}$

Problems

1. Find the probability of getting a head when tossing a coin

Sol: Assume that tossing a coin is an experiment

Possible outcomes i.e. Sample space $S = \{H, T\}$

Total No. of possible cases $n(S) = 2$

Let A be an event of getting a head when tossing a coin

possibilities of event $A = \{H\}$

No. of favourable cases of event $A = n(A) = 1$

The probability of happening an event A is

$$P(A) = \frac{\text{No. of favourable cases of event A}}{\text{Total no of possible cases}}$$

$$= \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{2}$$

2. Find the probability of getting one head in tossing two coins at a time.

Sol: Assume that tossing two coins at a time (or) Tossing a coin two times is an experiment.

Possible outcomes i.e Sample space $S = \{HH, HT, TH, TT\}$

Total no of possible outcomes $= n(S) = 4$ (2^2 - two coins)

Let us Assume that A be an event of getting one head

Possible outcomes of $A = \{HT, TH\}$

No of favourable cases of event $A = n(A) = 2$.

The probability of happening an event A is

$$P(A) = \frac{\text{No. of Favourable cases of A}}{\text{Total no of possible cases}}$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{2}{4}$$

$$P(A) = \frac{1}{2}$$

3. If three coins are tossed find the probability of getting

i) Three heads

ii) Two heads

iii) No heads

iv) Atleast one head

Sol: Assume that tossing three coins is an experiment

Possible outcomes i.e Sample Space

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Total no. of possible cases $n(S) = 2^3 = 8$

i) Let A_1 be an Event of getting three heads when tossing three coins

possible outcomes of event $A_1 = \{HHH\}$

No of favourable cases of event $A_1 = n(A_1) = 1$

The probability of getting an event A_1 is

$$P(A_1) = \frac{\text{no of favourable cases of } A_1}{\text{Total no of possible cases}}$$

$$P(A) = \frac{n(A_1)}{n(S)} = \frac{1}{8}$$

ii) Let A_2 be an event of getting two heads

possible outcomes of event $A_2 = \{HHT, HTH, THH\}$

No. of favourable cases of event $A_2 = n(A_2) = 3$

The probability of happening an event A_2 is

$$P(A_2) = \frac{\text{No. of favourable cases of } A_2}{\text{Total No. of possible cases}}$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{3}{8}$$

iii) Let A_3 be an event of getting No heads i.e all tails

possible outcomes of an event $A_3 = \{TTT\}$

No. of favourable cases of event $A_3 = n(A_3) = 1$

The probability of getting an event A_3 is

$$P(A_3) = \frac{\text{no of favourable cases of } A_3}{\text{Total no of possible cases}}$$

$$P(A_3) = \frac{n(A_3)}{n(S)}$$

$$P(A_3) = \frac{1}{8}$$

iv) Let A_4 be an event of getting atleast one head

possible outcomes of an event $A_4 = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

No of favourable cases of event $A_4 = n(A_4) = 7$

The probability of happening an event A_4 is

$$P(A_4) = \frac{\text{No of favourable cases of } A_4}{\text{Total No of possible cases}}$$

$P(A) = \frac{n(A_4)}{n(S)} = \frac{7}{8}$

4. A Fair coin is tossed 5 times what is the probability of having atleast one head.

Sol: Assume that ω tossing a coin 5 times is an experiment

Total no of possible cases $n(S) = 2^5 = 32$.

Let "A" be the event of getting no heads.

The no of favourable cases of event A $n(A) = 1$

The probability of getting no heads (All Tails) is

$P(A) = \frac{\text{no of favourable cases of } (A)}{\text{Total no of possible cases}}$

$= \frac{n(A)}{n(S)} = \frac{1}{32}$

We know that in any experiment $P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

$= 1 - \frac{1}{32}$.

$P(\bar{A}) = \frac{31}{32}$

Therefore the probability of getting atleast one head $(\bar{A})$ is $P(\bar{A}) = \frac{31}{32}$

5. Find the probability of getting sum of the two numbers is 10 if we throw two Dice.

Sol: Two Dice are thrown at a time (లే) a die is thrown two times is an experiment.

The possible outcomes i.e the Sample Space

S = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

Total no of possible outcomes $n(S) = 6^2 = 36$

Let "A" be an event of getting sum of the two numbers on the upper surface of the dice is 10

The possible outcomes of $A = \{(4,6), (5,5), (6,4)\}$

No. of favourable cases of event $A = n(A) = 3$

The probability of getting an event A is

$$P(A) = \frac{\text{no of favourable cases of A}}{\text{Total no of possible cases}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{36}$$

6. In a single throw of two Dice find the probability of getting sum of the two numbers is perfect square.

Sol: Let us Assume that throwing two Dice or a die is thrown two times is an experiment.

The possible outcomes

The possible outcomes i.e sample space

$$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$$

The total no of possible cases $n(S) = 6^2 = 36$.

Let A be an event of getting sum of the two numbers is perfect square when throwing two dice.

The possibilities of event $A = \{(1,3)\ (2,2)\ (3,1), (3,6), (4,5)\ (5,4), (6,3)\}$

The No. of favourable cases of event $A = 7 \cdot (n(A))$

The probability of happening an event A is

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{36}$$

7. Find the probability of getting same numbers on the upper surface of the dice when three dice are thrown.

Sol:- Let us assume that three dice are thrown at a time is an experiment.

No. of possible cases, $n(S) = 6^3 = 216$.

Let "A" be an event of getting same numbers on the upper surface of dice when three dice are thrown.

The possible outcomes of $A = \{(1,1,1)\ (2,2,2)\ (3,3,3)\ (4,4,4)\ (5,5,5)\ (6,6,6)\}$.

No. of favourable cases of event $A = n(A) = 6$

The probability of happening an event "A" is

$$P(A) = \frac{\text{No. of favourable cases of } A}{\text{Total no of possible cases}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{6}^{1}}{\cancel{216}_{36}} = \frac{1}{36}$$

8. Find the probability of getting one red king if we select a card from pack of cards

Sol:- We know that pack of playing cards contains totally 52 cards.

We can select a card from 52 cards in $52c_1$ ways.

Total No. of possible cases $n(S) = 52c_1$

Let A be an event of selecting a card from pack of 52 cards which is red king. we know that in pack of cards we have only 2 red king cards.

out of 2 red king cards we can select a card in $2c_1$ ways

No. of favourable cases of event $A = n(A) = 2c_1$.

The probability of getting an event A is

$$P(A) = \frac{\text{No of favourable cases of } A}{\text{Total no of possible cases}}$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{2c_1}{52c_1}$$

$$= \frac{\frac{2}{1}}{\frac{52}{1}}$$

$$= \frac{2}{52}$$

$$P(A) = \frac{1}{26}$$

9. Find the probability of getting two ispade cards if we Select two cards from the pack of 52 cards

Sol:- We know that pack of cards contains totally 52 cards

We can select two cards from 52 cards in $52c_2$ ways

Total No. of possible cases $n(S) = 52_{C_2}$

Let "A" be an event of selecting two cards are ispade from pack of 52 cards

We know that in 52 cards we have 13 ispade cards

out of 13 ispade cards we can select 2 cards in 13_{C_2} ways

No. of favourable cases of event $A = n(A) = 13_{C_2}$

The probability of getting an event A is

$$P(A) = \frac{\text{No. of favourable cases}}{\text{Total No. of possible casy}}$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{13_{C_2}}{52_{C_2}} = \frac{\frac{\cancel{13} \times \cancel{12}^{3}}{2 \times 1}}{\frac{\cancel{4}52 \times 51}{2}} = \frac{1}{17}$$

10. What is the probability that a card is drawn from the Pack of plain cards it may be either Queen (or) King

Sol:- We know that pack of cards contains totally 52 cards

We can select a card from 52 cards in $52c_1$ ways

No. of possible cases $n(S) = 52c_1$

Let A be an event of Selected card from pack of cards either king (or) queen

We can select a card from 4 king cards (or) 4 queen cards in

$^4c_1 + {^4c_1}$ ways.

No. of favourable cases of event $A = n(A) = {^4c_1} + {^4c_1}$

The probability of getting event A is

$$P(A) = \frac{\text{No. of favourable cases of event A}}{\text{Total no of possible cases}}$$

$$= \frac{^4c_1 + {^4c_1}}{^{52}c_1}$$

$$= \frac{\frac{4}{1} + \frac{4}{1}}{\frac{52}{1}}$$

$$= \frac{8}{52}$$

$$P(A) = \frac{2}{13}$$

11. Find the probability of a leap year to have 53 sundays or what is the probability of a leap year selected at random will contain 53 sundays?

Sol: We know that a leap year contains 366 days.

out of these 366 days we have 52 full weeks and 2 days extra

i.e 366 days = 52 weeks + 2 days.

In 52 weeks we have 52 sundays and consequent two days may be any one of the following

mon & Tue

Tue & wed

wed & Thu

Thu & Fri

Fri & Sat

Sat & Sun

Sun & mon

Total no of possible cases $n(S) = 7$.

Let A be the event of a leap year contains 53 sundays

for a leap year consists of 53 Sundays we require one Sunday in consequent two days.

The favourable cases of event A = {(Sun, mon), (Sat sun)}

Number of favourable cases of event A = $n(A) = 2$

The probability of happening an event A is

$$P(A) = \frac{\text{No of favourable cases of event A}}{\text{Total no of possible cases}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

12. Find the probability of 53 mondays in a non leap year

Sol: We know that a non leap year (normal year) contains 365 days.

out of these 365 days, we have 52 full weeks and one day extra i.e 365 days = 52 weeks + one day

In 52 weeks we have 52 mondays and one day may be any one of the following

mon
Tues
wed
Thur
Fri
Sat
Sun

The total number of possible cases $n(S) = 7$

Let A be an event of a normal year consists of 53 mondays

For Normal year consists of 53 mondays we require one monday in Extra one day.

The favourable cases of A = {mon}

No. of favourable cases of A = $n(A) = 1$

The probability of getting an event A is

$$P(A) = \frac{\text{No of favourable cases of event A}}{\text{Total no of possible cases}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

13. In a class there are 10 boys and 5 girls. A committee of 4 students is to be selected from the class. Find the probability of the committe contains

i) All are boys

ii) Atleast three girls

iii) exactly 2 girls

Sol: Total no of students in the class room = 10 boys + 5 girls

= 15 students

given that comittee consists of 4 students.

So out of 15 students 4 students can be selected in $^{15}C_4$ ways

The total number of possible cases $n(S) = {}^{15}C_4$

i) Let E_1 be an event that the commitee consists of All boys i.e 4 boys.

out of 10 boys we can select 4 students in $^{10}C_4$ ways.

No. of favourable cases of event $E_1 = n(E_1) = {}^{10}C_4$

The probability of getting four boys (event E_1) is

$$P(E_1) = \frac{\text{No. of favourable cases of event } E_1}{\text{Total no of possible cases}}$$

$$P(E_1) = \frac{n(E_1)}{n(S)}$$

$$= \frac{^{10}C_4}{^{15}C_4} = \frac{\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{10 \times 9 \times 8 \times 7}{15 \times 14 \times 13 \times 12}$$

$$= \frac{2}{13}$$

ii) Let E_2 be an event that the commitee consists atleast 3 girls i.e out of 4 selected students atleast 3 may be girls.

The possibilities of event E_2 are (g, b) (3,1) or (g, b) (4,0)

The favourable cases of event $E_2 = n(E_2) = {}^5C_3 \times {}^{10}C_1 + {}^5C_4 \times {}^{10}C_0$

The probability of comittee consists atleast 3 girls is

$$P(E_2) = \frac{\text{No. of Favourable cases of event } E_2}{\text{Total no of possible cases}}$$

$$= \frac{n(E_2)}{n(S)}$$

$$= \frac{^5C_3 \times {}^{10}C_1 + {}^5C_4 \times {}^{10}C_0}{^{15}C_4}$$

$$= \frac{\frac{5\times4\times3}{3\times2\times1}\times\frac{10}{1}+\frac{5\times4\times3\times2}{4\times3\times2\times1}\times1}{\frac{15\times14\times13\times12}{4\times3\times2\times1}}$$

$$P(E_2) = \frac{100+5}{1365} = \frac{105}{1365} = \frac{1}{13}$$

iii) Let E_3 be an event that the comitee consists exactly 2 girls

i.e. out of 4 selected students exactly 2 members girls and remaining boys.

The possibility of event E_3 is $\overset{g\ \ b}{(2,2)}$

The No. of favourable cases of event E_3 is $n(E_3) = {}^{5}C_2 \times {}^{10}C_2$

The probability of comitee consists 2 girls only is

$$P(E_3) = \frac{\text{No. of favourable cases of event } E_3}{\text{Total no of possible cases}}$$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{{}^{5}C_2 \times {}^{10}C_2}{{}^{15}C_4} = \frac{\frac{5\times4\times10\times9}{2\times1\times2\times1}}{\frac{15\times13\times14\times12}{4\times3\times2\times1}}$$

$$= \frac{5\times4\times10\times9\times3\times2}{15\times14\times13\times12}$$

$$P(E_3) = \frac{30}{91}$$

13. Two cards are selected at random from the ten cards numbered from 1 to 10. Find the probability that the sum is even if

i) The two cards are drawn together

ii) The two cards are drawn one after other with Replacement Process.

Sol: (i) The total number of cards = 10

out of 10 cards 2 cards can be selected in ${}^{10}C_2$ ways

The total number of possible cases $n(S) = {}^{10}C_2$

Let E_1 be the event that the sum is even when two cards are drawn

We know that any 2 odd numbers summation (or) 2 even numbers summation is even

We have in 10 cards 5 are odd number cards and remaining 5 are even number cards

out of 5 even numbers cards 2 cards can be selected at a time in ${}^5C_2$ ways

out of 5 odd number cards two cards can be selected in ${}^5C_2$ ways

The number of favourable cases of event E_1 is $n(E_1) = {}^5C_2 + {}^5C_2$

$$= \frac{5\times4}{2\times1} + \frac{5\times4}{2\times1}$$

$$n(E_1) = 20$$

The probability of getting an event E_1 is

$$P(E_1) = \frac{\text{No. of favourable cases of event } E_1}{\text{Total No. of possible cases}}$$

$$= \frac{n(E_1)}{n(S)} = \frac{20}{{}^{10}C_2} = \frac{20}{\frac{10\times9}{2\times1}} = \frac{4}{9}$$

ii) <u>Replacement process</u>:-

Total number of cards = 10

Here two cards are drawn one by one with Replacement process out of 10 cards 1 card (first card) can be selected in ${}^{10}C_1$ ways

After Replacing the first card there is no change in the total number of cards

The remaining one card (second card) can be selected in ${}^{10}C_1$ ways

The total no of possible cases $n(S) = {}^{10}C_1 \times {}^{10}C_1$

Let E_2 be an event that the sum is even when two cards are drawn with replacement process

we know that any 2 odd numbers sum (or) 2 even numbers sum is even. In 10 cards 5 are odd number cards and remaining 5 are even number cards

out of 5 even number cards 2 cards can be selected one after other with replacement process in ${}^5C_1 \times {}^5C_1$ ways out of 5 odd number cards two cards can be selected one after other with replacement process in $5_{C_1} \times 5_{C_1}$ ways

The number of favourable cases of event E_2 is

$$n(E_2) = 5_{C_1} \times 5_{C_1} + 5_{C_1} \times 5_{C_1}$$

$$= 5 \times 5 + 5 \times 5$$

$$= 25 + 25$$

$$n(E_2) = 50$$

The probability of getting an event "E_2" is

$$P(E_2) = \frac{n(E_2)}{n(S)}$$

$$= \frac{50}{10_{C_1} \times 10_{C_1}} = \frac{\cancel{50}}{\cancel{10} \times \cancel{10}} = \frac{1}{2}$$

14. Three bulbs are chosen at random from the 12 bulbs of which 5 are defective find the probability that

i) All are defective

ii) One is defective

Sol: Total number of bulbs = 12

No. of defective bulbs = 5

No. of non-defective bulbs = 12 - 5 = 7

out of 10 bulbs 3 bulbs can be selected in 10_{C_3} ways

Total number of possible cases $n(S) = 10_{C_3}$

i) Let E_1 be the event that all are defective bulbs when three bulbs are chosen at random from 12 bulbs

out of 5 defective bulbs three bulbs can be selected in 5_{C_3} ways.

The number of favourable cases of event E_1 is $n(E_1) = 5_{C_3}$

$$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

The probability of getting an event E_1 is

$$P(E_1) = \frac{\text{No of favourable cases of event } E_1}{\text{Total no of possible cases}}$$

$$= \frac{n(E_1)}{n(S)} = \frac{10}{220} = \frac{1}{22}$$

ii) Let E_2 be the event that one is defective out of 3 bulbs defective

out of 5 defective bulbs one can be selected in 5_{C_1} ways and remaining 2 bulbs are chosen from non-defective

out of 7 non-defective bulbs 2 can be selected in 7_{C_2} ways

The number of favourable cases of event E_2 is $n(E_2) = 5_{C_1} \times 7_{C_2}$

$$= \frac{5 \times 7 \times 6}{2}$$

$$= 105$$

The probability of getting an event E_2 is $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{105}{220}$

$$P(E_2) = \frac{21}{44}$$

15. Out of 15 items 4 are not in good, 4 are selected at randaum. find the probability that

clues i) $\frac{{}^{4}C_{4}}{{}^{15}C_{4}}$

i) All are not good

ii) $\frac{{}^{4}C_{2} \cdot {}^{11}C_{2}}{{}^{15}C_{4}}$

ii) Two are not good

Sol :-

16. A bag contains 2 white and 4 black balls, two balls are drawn one by one without replacement. find the probability that

i) First ball drawn is white

ii) Both balls drawn are black.

iii) Both balls drawn are white

iv) The first ball drawn is black.

Sol:- Total number of balls in the bag = 2W + 4b = 6 balls

Two balls can be selected one by one from 6 balls,

Total number of possible cases $n(S) = {}^6c_1 \times {}^5c_1 = 6 \times 5 = 30$

i) Let E_1 be the event of first ball drawn is white when two balls are drawn one after other.

The possibilities of event $E_1 = \{(W,W);(W,B)\}$

The no of favourable cases of event E_1 is $n(E_1) = {}^2c_1 \times {}^1c_1 + {}^2c_1 \times {}^4c_1$

$= 2\times1 + 2\times4$

$= 10$

The probability of getting an event E_1 is

$$P(E_1) = \frac{\text{No of favourable cases of event } E_1}{\text{Total no of possible cases}}$$

$$= \frac{n(E_1)}{n(S)} = \frac{10}{30} = \frac{1}{3}$$

ii) Let E_2 be an event of both balls are black when two balls are drawn one after other (one by one)

The possibilities of event $E_2 = \{(b,b)\}$

The no. of favourable cases of event $E_2 = n(E_2) = {}^4c_1 \times {}^3c_1 = 12$

The probability of getting an event E_2 is $P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{12}{30}$

$= \frac{2}{5}$

iii) Let E_3 be an event of both balls are white when two balls are drawn one after other.

The possibilities of event $E_3 = \{(W,W)\}$

The No of favourable cases of event $E_3 = n(E_3) = {}^2c_1 \times {}^1c_1$

The probability of getting an event E_3 is

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{2}{30} = \frac{1}{15}$$

iv) Let E_4 be an event of first ball drawn is black when two balls are drawn one by one.

The possibilities of event $E_4 = \{(b,W),(b,b)\}$

The no. of favourable cases of event E_4 is $n(E_4) =$

$= {}^4c_1 \times {}^2c_1 + {}^4c_1 \times {}^3c_1$

$n(E_4) = 8 + 12 = 20$

The probability of getting an event E_4 is $P(E_4) = \frac{n(E_4)}{n(S)}$

$$P(E_4) = \frac{20}{30} = \frac{2}{3}$$

17. A man pocket contains five 50 ps coins four 25 Ps coins and four 10 ps coins. A boy is asked to draw the two coins at random. what is the probability of drawing

i) maximum possible amount;

ii) minimum possible amount

iii) coins of different values

Sol: Total number of coins in the pocket $= 5+4+4 = 13$

Out of 13 coins 2 coins can be selected in ${}^{13}C_2$ ways

Total number of possible cases $= n(S) = {}^{13}C_2 = \frac{13 \times 12}{2} = 78$

i) Let E_1 is an event of getting maximum possible amount

we know that Available maximum coin is 50 paise

So, out of five 50 ps coins two coins can be selected in ${}^5C_2$ ways

No. of favourable cases of $E_1 = n(E_1) = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$

The probability of getting an event E_1 is

$$P(E_1) = \frac{\text{No. of favourable cases of } E_1}{\text{Total No. of possible cases}} = \frac{n(E_1)}{n(S)} = \frac{10}{78} = \frac{5}{39}$$

ii) Let "E_2" be an event that drawing of minimum possible amount.

W.K.T Available minimum coin is 10 ps

so out of four 10 ps coins two coins can be selected in ${}^4C_2$ ways

No. of Favourable cases of event E_2 is $n(E_2) = {}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$

The probability of getting an event, E_2 is

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{78} = \frac{1}{13}$$

iii) Let "E_3" be the event that selected two coins are different values

W.K.T values of coins in pocket are 50 Ps, 25 Ps, 10 Ps

Possibilities of the event are {(50 Ps, 25 Ps), (25 Ps, 10 Ps) (50 Ps, 10 Ps)}

The No. of favourable cases of event $E_3 = n(E_3) = {}^5C_1 \times {}^4C_1 + {}^4C_1 \times {}^4C_1 + {}^5C_1 \times {}^4C_1$

$= 5\times4 + 4\times4 + 5\times4$

$= 20+16+20$

$= 56$

The probability of getting an event E_3 is

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{56}{78} = \frac{28}{39}$$

19. If $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A\cap B) = \frac{1}{15}$ then find

i) $P(A\cup B)$ ii) $P(A^c\cap B)$ iii) $P(A\cap B^c)$ iv) $P(A^c\cap B^c)$ v) $P(A^c\cup B^c)$

Sol: given that

$P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A\cap B) = \frac{1}{15}$

i) By Addition theorem of probability

we have $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{1}{5} + \frac{2}{3} - \frac{1}{15}$

$= \frac{3+10-1}{15} = \frac{12}{15} = \frac{4}{5}$

ii) $P(A^c\cap B)$

W.K.T $P(A^c\cap B) = P(B) - P(A\cap B)$

$= \frac{2}{3} - \frac{1}{15} = \frac{10-1}{15} = \frac{9}{15} = \frac{3}{5}$

iii) We know that

$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$= \frac{1}{5} - \frac{1}{15} = \frac{3-1}{15} = \frac{2}{15}$

iv) $P(A^c\cap B^c) = P((A\cup B)^c)$ $[\because (A\cup B)^c = A^c\cap B^c]$

$= P(\overline{A\cup B})$

$= 1 - P(A\cup B)$

$= 1 - \frac{4}{5} = \frac{1}{5}$

$P(A^c\cap B^c) = \frac{1}{5}$

v) $P(A^c\cup B^c) = P((A\cap B)^c)$ $\because P(A) + P(\bar{A}) = 1$

$= P(\overline{A\cap B})$ $P(\bar{A}) = 1 - P(A)$

$= 1 - P(A\cap B)$

$= 1 - \frac{1}{15}$

$P(A^c\cup B^c) = \frac{14}{15}$

20. If $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$ then find

i) $P(A\cup B)$ ii) $P(A^c\cap B)$ iii) $P(A\cap B^c)$ iv) $P(A^c\cap B^c)$ v) $P(A^c\cup B^c)$

Sol: given that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$

i) By Addition theorem of probability

We have $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{5}$$

$$=\frac{15+10-6}{30}$$

$$=\frac{19}{30}$$

ii) $P(A^c\cap B)$

W.K.T $P(A^c\cap B)=P(B)-P(A\cap B)$

$$=\frac{1}{3}-\frac{1}{5}$$

$$=\frac{5-3}{15}$$

$$P(A^c\cap B)=\frac{2}{15}$$

iii) $P(A\cap B^c)$

We know that $P(A\cap B^c)=P(A)-P(A\cap B)$

$$=\frac{1}{2}-\frac{1}{5}$$

$$=\frac{5-2}{10}$$

$$P(A\cap B^c)=\frac{3}{10}$$

iv) $P(A^c\cap B^c)=P((A\cup B)^c)$

$$=P(\overline{A\cup B})$$

$$=1-P(A\cup B)$$

$$=1-\frac{19}{30}$$

$$=\frac{30-19}{30}$$

$$P(A^c\cap B^c)=\frac{11}{30}$$

v) $P(A^c\cup B^c)=P((A\cap B)^c)$

$$=P(\overline{A\cap B})$$

$$=1-P(A\cap B)$$

$= 1 - \frac{1}{5}$

$P(A^c \cup B^c) = \frac{4}{5}$

21. If $P(A \cup B) = \frac{4}{5}$, $P(B^c) = \frac{1}{3}$ & $P(A \cap B) = \frac{1}{5}$ then find

i) $P(B)$ ii) $P(A)$ iii) $P(A^c \cap B)$

Sol: Given $P(A \cup B) = \frac{4}{5}$, $P(B^c) = \frac{1}{3}$ & $P(A \cap B) = \frac{1}{5}$

i) we know that

$P(B^c) = 1 - P(B)$ $\left[\because P(A) + P(\bar{A}) = 1 \Rightarrow P(\bar{A}) = 1 - P(A)\right]$

$\Rightarrow P(B) = 1 - P(B^c)$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

ii) By Addition theorem of probability we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{5} = P(A) + \frac{2}{3} - \frac{1}{5}$

$P(A) = \frac{4}{5} + \frac{1}{5} - \frac{2}{3}$

$= \frac{12 - 10 + 3}{15}$

$= \frac{5}{15} = \frac{1}{3}$

iii) $P(A^c \cap B)$

W.K.T

$P(A^c \cap B) = P(B) - P(A \cap B)$ [$\because$ by 4th theorem)

$= \frac{2}{3} - \frac{1}{5} = \frac{10-3}{15} = \frac{7}{15}$

22. If $B \subset A$ & A, B are two events such that

$P(A) = 3P(B)$ and $A \cup B = S$ then find $P(B)$

Sol: given $B \subset A$, so $A \cap B = B$,

Taking probability on b.s in above

$P(A \cap B) = P(B)$

Also given $P(A) = 3P(B)$ and $A \cup B = S$

Taking probability on b.s in above

$P(A\cup B) = P(S)$

$P(A\cup B) = 1 \quad \{\because P(S) = 1\}$

By Addition theorem of probability we have

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(S) = 3P(B) + P(B) - P(B)$

$1 = 3P(B)$

$P(B) = 1/3$

23. Among 150 students 80 are studying maths 40 are studying physics 30 are studying both maths and physics. if a student is chosen at random. find the probability that the student

i) studying maths (or) physics

ii) studying neither maths nor physics

In sets or means union

And means intersection

Sol: Total number of students = 150

Let A be the event of a selected student studying maths.

$P(A) = \frac{80c_1}{150c_1} = \frac{80}{150} = \frac{8}{15}$

Let B be the event of a selected student studying physics

so $P(B) = \frac{40c_1}{150c_1} = \frac{40}{150} = \frac{4}{15}$

Let $A\cap B$ be the event of a selected student studying both maths and physics

$P(A\cap B) = \frac{30c_1}{150c_1} = \frac{30}{150} = \frac{1}{5}$

i) The probability that selected student studying maths or physics is

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{8}{15} + \frac{4}{15} - \frac{1}{5}$

$= \frac{8+4-3}{15}$

$= \frac{9}{15}$

$P(A \cup B) = 3/5$

ii) The probability of selected student studying neither maths nor physics is

$P(A^c \cap B^c) = P((A \cup B)^c)$

$= 1 - P(A \cup B)$ $\quad [\because P(\bar{A}) = 1 - P(A)]$

$= 1 - \frac{3}{5}$

$P(A^c \cap B^c) = 2/5$

24. Two Dice are thrown. Let A be the event that the sum of the numbers on the faces is 9. Let B be any event that atleast one number is 6 then find

i) $P(A \cap B)$ ii) $P(A \cup B)$ iii) $P(A \cap B^c)$ iv) $P(A^c \cap B)$ v) $P(A^c \cap B^c)$

vi) $P(A^c \cup B^c)$

Sol: Assume that throwing of two Dice is an experiment

The possibilities = sample space S =

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

Total number of possible cases $n(S) = 6^2 = 36$

Let A be the event that the sum of the numbers on the faces of Dice is 9

The possibilities of event $A = \{(3,6), (4,5), (5,4), (6,3)\}$

No. of favourable cases of event $A = n(A) = 4$

The probability of getting an event A is $P(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{36}$

$P(A) = 1/9$

Let B be the event that any one number on the two Dice is 6.

The possibilities of event $B = \{(1,6)(2,6),(3,6),(4,6),(5,6),(6,1)$
$(6,2)(6,3)(6,4)(6,5)(6,6)\}$

The number of favourable cases of event $B = n(B) = 11$

The probability of getting an event B is $P(B) = \frac{n(B)}{n(S)}$

$P(B) = \frac{11}{36}$

i) $A \cap B = \{(3,6)(6,3)\}$

No. of favourable cases of event $A \cap B = n(A \cap B) = 2$.

The probability of getting an event $A \cap B$ is $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{9} + \frac{11}{36} - \frac{1}{18}$

$= \frac{4+11-2}{36} = \frac{13}{36}$

iii) $P(A \cap B^c)$

We know that $P(A \cap B^c) = P(A) - P(A \cap B)$

$= \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$

$P(A \cap B^c) = \frac{1}{18}$

iv) $P(A^c \cap B)$

We know that $P(A^c \cap B) = P(B) - P(A \cap B)$

$= \frac{11}{36} - \frac{1}{18}$

$= \frac{11-2}{36}$

$= \frac{9}{36}$

$P(A^c \cap B) = \frac{1}{4}$

v) $P(A^c \cap B^c)$

We know that $P(A^c \cap B^c) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - \frac{13}{36}$

$= \frac{36-13}{36} = \frac{23}{36}$

vi) $P(A^c \cup B^c) = P((A \cap B)^c)$

$= P(\overline{A \cap B})$

$= 1 - P(A \cap B)$

$= 1 - \frac{1}{18}$

$= \frac{18-1}{18}$

$P(A^c \cup B^c) = \frac{17}{18}$

***25. A bag contains 12 balls numbered from 1 to 12. If a ball is drawn at Random what is the probability of having a ball with a number which is multiple of either 2 or 3

Sol: Let S be the sample space

So $S = \{1, 2, 3 \cdots 12\}$

A ball is selected from bag in 12_{C_1} ways

Total no. of possible cases $= n(S) = 12_{C_1} = 12$

Let A be an event that the ball number is multiple of 2

possibilities of $A = \{2, 4, 6, 8, 10, 12\}$

we select a ball from A in 6_{C_1} ways

No. of favourable cases of $A = n(A) = 6_{C_1} = 6$

probability of getting an event $A = P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$.

Let B be an event that the selected ball number is multiple of 3

possibilities of event $B = \{3, 6, 9, 12\}$

we select a ball from B in 4_{C_1} ways

No. of favourable cases of $B = n(B) = 4_{C_1} = 4$

The probability of getting an event $B = \frac{n(B)}{n(S)} = \frac{4}{12} = \frac{1}{3}$

Let $A \cap B$ be an event that selected ball number is multiple of 2 and 3 i.e divisible by L.C.M of 2 and 3 = 6 possibilities of $A \cap B = \{6, 12\}$

we can select a ball from $A \cap B$ in ${}^{2}c_1$ ways

No. of favourable cases of $A \cap B = n(A \cap B) = {}^{2}c_1 = 2$

The probability of getting an event $A \cap B$ is

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

The probability of selected ball number is multiple of 2 or 3

is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}.$$

*** 26. An integer is chosen at Random From 1 to 200 positive integers what is the probability that an integer is chosen at Random which is divisible by both 6 and 8.

Sol: The sample space S contains 1 to 200 positive integers

$S = \{1, 2, 3, \cdots 200\}$

out of 200 sample (positive) integers one can be selected in ${}^{200}c_1$ ways.

The total no of possible cases $n(S) = {}^{200}c_1 = 200$

Let A be an event that an integer is chosen at random which is divisible by 6

$6)200(33$
$\underline{18}$
$\underline{\ 20}$
2

we know that all multiples of 6 are divisible by 6.

The possibilities of event $A = \{6, 12, 18, 24 \cdots 198\}$
$= 33$ numbers

We select an integer from event A in ${}^{33}c_1$ ways

No. of favourable cases of $A = n(A) = {}^{33}c_1 = 33$

The probability of getting an event A is $P(A) = \frac{n(A)}{n(S)}$

$$P(A) = \frac{33}{200}$$

Let B be an event that an integer is chosen at random which is divisible by 8.

We know that all multiples of 8 are divisible by 8

So possibilities of event $B = \{8, 16, 24 \cdots 200\} = 25$ numbers

We can select an integer from B in 25_{C_1} ways

$$8)\,200\,(25 \quad 16 \quad 40 \quad 40 \quad (0)$$

No of favourable cases of event $B = 25_{C_1} = 25$

The probability of getting an event B is

$$P(B) = \frac{n(B)}{n(S)} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$

Let $A \cap B$ be an event that an integer is chosen at random which is divisible by 6 and 8 i.e divisible by L.C.M of 6 and 8 = 24

The possibilities of event $A \cap B = \{24, 48 \cdots 192\}$ = 8 numbers

$$24)\,200\,(8 \quad 192 \quad (8)$$

Out of 8 numbers in $A \cap B$ we can select a number in 8_{C_1} ways

No of favourable cases of $A \cap B = n(A \cap B) = 8_{C_1} = 8$.

The probability of getting $A \cap B = \frac{n(A \cap B)}{n(S)}$

$$P(A \cap B) = \frac{8}{200}$$

$$P(A \cap B) = \frac{1}{25}$$

27. The probabilities that 4 students A, B, C, D to solve the problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}$ and $\frac{1}{4}$ respectively. if all of them try to solve the problem what is the probability that the problem is solved a) Atleast one of them solved

Sol: Let P(A) be the probability that the problem is solved by student A. $P(A) = \frac{1}{3}$

Let P(B) be the probability that the problem is solved by the student B $\Rightarrow P(B) = 2/5$

Let P(C) be the probability that the problem is solved by the student C. $\Rightarrow P(C) = \frac{1}{5}$

Let P(D) be the probability that the problem is solved by the student D $\Rightarrow P(D) = \frac{1}{4}$

Here the events A, B, C, D are independent.
The problem is solved by the any one of the student. $A\cup B \cup C \cup D$
The probability that the problem is solved is $P(A\cup B\cup C\cup D)$

$$P(A\cup B\cup C\cup D) = 1 - P((A\cup B\cup C\cup D)^c) \quad [\because P(A) = 1-P(\bar{A})]$$

$$= 1 - P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}) \quad [\because (A\cup B)^c = A^c\cap B^c]$$

$$= 1 - P(\bar{A})\,P(\bar{B})\cdot P(\bar{C})\cdot P(\bar{D}) \quad [\because A, B, C, D \text{ are independent}]$$

$$= 1 - [(1-P(A))(1-P(B))(1-P(C))(1-P(D))]$$

$$= 1 - [(1-\tfrac{1}{3})(1-\tfrac{2}{5})(1-\tfrac{1}{5})(1-\tfrac{1}{4})]$$

$$= 1 - \frac{2}{3}\cdot\frac{3}{5}\cdot\frac{4}{5}\cdot\frac{3}{4}$$

$$= \frac{300-72}{300} = \frac{228}{300}$$

28. A problem in statistics is given to solve three students A, B, C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solved if all of them try independently. Sol: 29/32

Sol: Let P(A) be the probability that the problem is solved by student A $\Rightarrow P(A) = \frac{1}{2}$

Let P(B) be the probability that the problem is solved by student B $\Rightarrow P(B) = \frac{3}{4}$

Let P(C) be the probability that the problem is solved by student C $\Rightarrow P(C) = \frac{1}{4}$

Here the events A, B, C are independent.

The probability that the problem is solved is $P(A\cup B\cup C)$

$$P(A\cup B\cup C) = 1 - P((A\cup B\cup C)^c) \quad [\because P(A) = 1-P(\bar{A})]$$

$$= 1 - P(\bar{A}\cap\bar{B}\cap\bar{C}) \quad [\because (A\cup B)^c = A^c\cap B^c]$$

$$= 1 - P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C}) \quad [\because A, B, C \text{ are independent so } P(A\cap B\cap C) = P(A)P(B)P(C)]$$

$$= 1 - [(1-P(A))(1-P(B))(1-P(C))]$$

$$= 1 - [(1-\tfrac{1}{2})(1-\tfrac{3}{4})(1-\tfrac{1}{4})]$$

$$= 1 - \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}$$

$P(A\cup B\cup C) = 1 - \frac{3}{32}$

$P(A\cup B\cup C) = \frac{29}{32}$

29. The probability that a student passed in physics test is 2/3 and probability that he passed both physics and English test is 14/45. The probability that he pass atleast one test is 4/5. what is the probability that he passes English test

Sol: Let A be the event that the student passes in physics test.

Let B be the event that the student passes in English test.

Given the probability that the student passes in physics test is $P(A) = \frac{2}{3}$

The probability that the student passes both physics and English test is $P(A\cap B) = \frac{14}{45}$

The probability of student passes atleast one test is

$P(A\cup B) = 4/5$

The probability of student passes in English test is $P(B) = ?$

By the Addition theorem of probability we know that

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(B) = P(A\cup B) + P(A\cap B) - P(A)$$

$$= \frac{4}{5} + \frac{14}{45} - \frac{2}{3}$$

$$= \frac{36+14-30}{45}$$

$$P(B) = \frac{20}{45} = \frac{4}{9}$$

Therefore the probability of the student passes in English test is $P(B) = \frac{4}{9}$

30. A card is drawn from a well shuffled pack of cards what is the probability that it is either spade or an Ace

Sol: we know that in pack of well shuffled cards we have 52 cards

A card is selected from pack of cards in 52_{C_1} ways

Total no of possible cases $= n(S) = 52_{C_1} = 52$

Let A be an event of getting a ispade card

$P(A) = \frac{13_{C_1}}{52_{C_1}} = \frac{13}{52} = \frac{1}{4}$

Let B be an event of getting a Ace card

$P(B) = \frac{n(B)}{n(S)} = \frac{4_{C_1}}{52_{C_1}} = \frac{1}{13}$

Let $A \cap B$ be an event of selected card is ispade and an Ace.

The possibilities of $A \cap B$ — $n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1_{C_1}}{52_{C_1}} = \frac{1}{52}$$

The probability of getting card is ispade or an Ace is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}$

31. From a city 3 newspapers A, B, C are being published. A is read by 20%, B is read by 16%, C is read by 14%, both A and B are read by 8%, both A and C are Read by 5%, both B and C are read by 4% and all three A, B, C are read by 2%. What is the percentage of the population that read atleast one paper.

Sol: Let P(A) be the probability that population read paper A

$P(A) = \frac{20}{100}$

Let P(B) be the probability that population read paper B.

$P(B) = \frac{16}{100}$

Let P(C) be the probability that population read paper C.

$P(C) = \frac{14}{100}$

Let $P(A \cap B)$ be the probability that population read both papers A and B.

$P(A \cap B) = \frac{8}{100}$

Let $P(A \cap C)$ be the probability that population read both the papers A and C

$P(A \cap C) = 5/100$

Let $P(B \cap C)$ be the probability that population read both the papers B and C. $P(B \cap C) = \frac{4}{100}$

Let $P(A \cap B \cap C)$ be the probability that population read all the papers i.e A, B and C $\Rightarrow P(A \cap B \cap C) = \frac{2}{100}$.

The probability of population read atleast one paper i.e is A σ B σ C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{4}{100} - \frac{5}{100} + \frac{2}{100}$

$P(A \cup B \cup C) = \frac{35}{100} = \frac{7}{20} = 35\%$

$\therefore$ The percentage of the population that read atleast one paper is 35%

Conditional probability

If A and B are any two events of an Experiment. The probability of the happening an Event A when another event B has already happened is called conditional probability.

Let A and B are two events in a sample space S and $P(B) \neq 0$ then the conditional probability of event A given B is denoted by $P(A/B)$ and we read it is as probability of A given B and is defined as $\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$

where $P(B) \neq 0$

Similarly the conditional probability of event B given A is

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$ where $P(A) \neq 0$

Independent events

Two Events are said to be independent Events if the occurance of the any one of the events does not dependent on the occurance of any other Events i.e if A and B are independent Events then $P(A/B) = P(A)$ and $P(B/A) = P(B)$
$[\because P(A \cap B) = P(A) \cdot P(B)]$

Mutually independent Events:-

if $A_1, A_2 \cdots A_n$ are mutually independent Events then

$P(A_1 \cap A_2 \cdots A_n) = P(A_1) \cdot P(A_2) \cdot P(A_3) \cdots P(A_n)$

Pair wise independent Events:-

A set of Events $A_1, A_2, A_3 \cdots A_n$ are said to be pair wise independent Events if $P(A_i \cap A_j) = P(A_i) \times P(A_j)\ \forall i \neq j$

***

Multiplication theorem of probability

Statement: if A and B are any two events in the sample spaces then $P(A \cap B) = P(A) \cdot P(B/A)$ & $P(A \cap B) = P(B) \cdot P(A/B)$

Proof Let A and B be any two events in the sample spaces
By definition conditional probability we have

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B) \cdot P(A/B) \rightarrow ①$$

Also by the conditional probability we have

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \cdot P(B/A) \rightarrow ②$$

From Eqns ① & ②

$$P(A \cap B) = P(A) \cdot P(B/A) \text{ \& } P(A \cap B) = P(B) \cdot P(A/B)$$

Hence proved.

Theorem-1:

If A and B are two independent Events then

i) A & $\bar{B}$ ii) $\bar{A}$ & B iii) $\bar{A}$ & $\bar{B}$ are also independent Events

Proof: Let A and B are two independent events in the Sample space S.

So by definition $P(A\cap B) = P(A) \cdot P(B) \longrightarrow ①$

i) we know that $P(A\cap \bar{B}) = P(A) - P(A\cap B)$

$$= P(A) - P(A)\,P(B) \quad [\because \text{From Eq} ①]$$

$$= P(A)\,(1-P(B))$$

$$P(A\cap\bar{B}) = P(A)\,P(\bar{B})$$

$\therefore$ A & $\bar{B}$ are independent Events

ii) Also we know that $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$$= P(B) - P(A)\,P(B) \quad [\because \text{From Eq} ①]$$

$$= P(B)\,[1-P(A)]$$

$$P(\bar{A}\cap B) = P(\bar{A})\,P(B)$$

$\therefore$ $\bar{A}$ & B are independent Events

iii) $P(\bar{A}\cap\bar{B}) = P((A\cup B)^c)$

$$= 1 - P(A\cup B)$$

$$= 1 - P(A) - P(B) + P(A\cap B)$$

$$= 1 - P(A) - P(B) + P(A)\cdot P(B)$$

$$= 1 - P(A) - P(B)\,[1-P(A)]$$

$$= (1-P(A))\,(1-P(B))$$

$$P(\bar{A}\cap\bar{B}) = P(\bar{A})\,P(\bar{B})$$

$\therefore$ $\bar{A}$ & $\bar{B}$ are independent Events

Note: If A and B are independent Events then

$$P(A\cap B) = P(A)\cdot P(B/A)$$

$$P(A\cap B) = P(A)\,P(B) \quad (\because P(A) \text{ \& } P(B) \text{ are independent})$$

Also $P(A\cap B) = P(B)\,P(A/B)$

$$P(A\cap B) = P(B)\,P(A)$$

$$P(A\cap B) = P(A)\,P(B)$$

Th-2 If A, B and C are mutually independent Events then $A\cup B$ & C are also independent.

Proof: Let A, B, C are mutually independent events. So by definition $P(A\cap B\cap C) = P(A)\,P(B)\,P(C)$

Now $P(A\cup B\cap C) = P(A\cap C\cup B\cap C)$

$= P(A\cap C) + P(B\cap C) - P(A\cap C\cap B\cap C)$

$= P(A)\,P(C) + P(B)\,P(C) - P(A\cap B\cap C)$

$= P(A)\,P(C) + P(B)\,P(C) - P(A)\,P(B)\,P(C)$

$= P(C)\,\left(P(A) + P(B) - P(A)(PB)\right)$

$= P(C)\,\left[P(A) + P(B) - P(A\cap B)\right]$

$P(A\cup B\cap C) = P(C)\ P(A\cup B)$

$P(A\cup B\cap C) = P(A\cup B)\,P(C)$

$\therefore$ $A\cup B$ & C are independent Events.

Problems

1. If $P(\bar{A}) = \frac{3}{8}$, $P(\bar{B}) = \frac{1}{2}$, $P(A\cap B) = \frac{1}{4}$ then find

i) $P(A/B)$ ii) $P(B/A)$ iii) $P(\bar{A}/\bar{B})$ iv) $P(\bar{B}/\bar{A})$

Sol: Given $P(\bar{A}) = \frac{3}{8}$

$\Rightarrow P(A) = 1 - P(\bar{A})$

$= 1 - \frac{3}{8} = 5/8$

$P(\bar{B}) = \frac{1}{2} \Rightarrow P(B) = 1 - P(\bar{B})$

$= 1 - \frac{1}{2}$

$P(B) = \frac{1}{2}$

$P(A\cap B) = \frac{1}{4}$

i) $P(A/B)$

We know that $P(A/B) = \frac{P(A\cap B)}{P(B)}$

$P(A/B) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

ii) We know that $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{2}{5}$

iii) $P(\bar{A}/\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{P(\overline{A\cup B})}{P(\bar{B})} = \frac{1 - P(A\cup B)}{P(\bar{B})}$

By Addition theorem of probability we have

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B) = \frac{5}{8} + \frac{1}{2} - \frac{1}{4} = \frac{5+4-2}{8} = \frac{7}{8}$$

Now $P(\bar{A}/\bar{B}) = \dfrac{1-P(A\cup B)}{P(\bar{B})} = \dfrac{1-\frac{7}{8}}{\frac{1}{2}} = \dfrac{\frac{1}{8}}{\frac{1}{2}} = \dfrac{1}{4}$

$$P(\bar{A}/\bar{B}) = \frac{1}{4}$$

iv) $P(\bar{B}/\bar{A}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{A})}$

$$P(\bar{B}/\bar{A}) = \frac{P(\overline{A\cup B})}{P(\bar{A})} = \frac{1-P(A\cup B)}{P(\bar{A})} = \frac{1-\frac{7}{8}}{\frac{3}{8}} = \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$$

2. A married couple (Husband & wife) appearing for an interview for the two vacancies against the same post. The probability of husband is selected is 1/6 and the probability of wife is selected is $\frac{2}{5}$. What is the probability that

i) Both of them will be selected

ii) only one of them will be selected

iii) None of them is selected

iv) Atleast one of them is selected

Sol: Let A be the event that Husband is selected for an interview with probability is $P(A) = \frac{1}{6}$

Let B be the event that wife is selected for an interview with probability is $P(B) = \frac{2}{5}$

Since there are two vacancies so the selection of one doesn't effect on the other.

Hence A and B are independent Events

i) The probability of Both of them are selected is

$$P(A\cap B) = P(A)\cdot P(B) \quad [\because A \text{ and } B \text{ are independent}]$$

$$= \frac{1}{6}\cdot\frac{2}{5} = \frac{2}{30} = \frac{1}{15}$$

ii) only one of them is selected

Favourable cases are $A\cap\bar{B}$ or $\bar{A}\cap B$

Probability of only one of them is selected is

$$P(A\cap\bar{B}) + P(\bar{A}\cap B) = P(A)\,P(\bar{B}) + P(\bar{A})\cdot P(B)$$

$$= P(A)\,(1-P(B)) + (1-P(A))\,P(B)$$

$$= \left(\frac{1}{6}\right)\left(1-\frac{2}{5}\right)+\left(1-\frac{1}{6}\right)\cdot\frac{2}{5}$$

$$= \frac{1}{6}\cdot\frac{3}{5}+\frac{5}{6}\cdot\frac{2}{5}$$

$$= \frac{1}{10}+\frac{1}{3}$$

$$= \frac{13}{30}$$

iii) None of them will be selected that means neither A nor B Selected

The probability of none of them selected is

$$P(\bar{A}\cap\bar{B}) = P(\bar{A})\,P(\bar{B}) \quad [\because A, B \text{ are independent}]$$

$$= (1-P(A))\,(1-P(B))$$

$$= \left(1-\frac{1}{6}\right)\left(1-\frac{2}{5}\right)$$

$$= \frac{5}{6}\cdot\frac{3}{5}$$

$$P(\bar{A}\cap\bar{B}) = \frac{1}{2}$$

iv) Atleast one of them is selected that means either A or B is selected

The probability of Atleast one of them is selected is

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

$$= P(A)+P(B)-P(A)\cdot P(B)$$

$$= \frac{1}{6}+\frac{2}{5}-\frac{1}{6}\cdot\frac{2}{5}$$

H.W

$$= \frac{5+12-2}{30} = \frac{15}{30} = \frac{1}{2}$$

***

3) Box A contains 5 red and 3 white marbles and box B contains 2 red and 6 white marbles. If a marble is drawn from each box. what is the probability that they are both of same colour

***

4) Two Marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with Replacement being made after each drawing. find the probability that

i) Both are white

ii) First is red and second is white

Sol: Total number of marbles = 10 r + 30 w + 20 b + 15 or = 75 marbles

Here the marbles will be drawn one after other with Replacement process

i) Let E_1 be the event of first drawn marble is white

out of 30 white marbles one can be selected in 30_{c_1} ways.

No of favourable cases of $E_1 = n(E_1) = 30_{c_1} = 30$

The probability of first drawn marble is white is

$$P(E_1) = \frac{30_{c_1}}{75_{c_1}} = \frac{30}{75} = \frac{2}{5}$$

Replacing the first marble to draw the second marble in while marbles so there is no changes in the total... no of white marbles to select the second marble

Let E_2 be the event of the second marble drawn is also white.

out of 30 white marbles one can be selected in 30_{c_1} ways

No. of favourable cases of Event $E_2 = n(E_2) = 30_{c_1}$

The probability of second marble drawn is white is

$$P(E_2) = \frac{30}{75} = \frac{2}{5}$$

The probability that both the marbels are white i.e first and second marbels are white is $P(E_1 \cap E_2)$

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$
$$= \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$$

ii) Let E_3 be the event that the first drawn marble is red.

out of 10 red marbles one can be selected in 10_{c_1} ways

No. of favourable cases of event $E_3 = n(E_3) = 10_{c_1}$

The probability that the first drawn marble is red, is

$$P(E_3) = \frac{10_{c_1}}{75_{c_1}} = \frac{10}{75} = \frac{2}{15}$$

Replacing the first marble to draw second marble so there is no changes in the total number of marbles to select second marble.

Let E_4 be the event that second drawn marble is white

out of 30 white marbles one can be selected in 30_{C_1} ways.

No. of favourable cases of event $E_4 = n(E_4) = 30_{C_1}$

The probability that second drawn marble is white is

$$P(E_4) = \frac{30_{C_1}}{75_{C_1}} = \frac{30}{75} = \frac{2}{5}$$

The probability that when two marbels are drawn first one is red and second one is white is

$$P(E_3 \cap E_4) = P(E_3) \cdot P(E_4)$$

$$= \frac{2}{15} \cdot \frac{2}{5} = \frac{4}{75}$$

*** 5) Two Aeroplanes bomb a target in succession. The probability of Each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first missy the target find the probability that.

i) Target is hit

ii) Both fails

Sol: Let A be the event of first plane hitting the target with probability is $P(A) = 0.3 \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - 0.3 = 0.7$

Let B be the event of second plane hitting the target with probability is $P(B) = 0.5 \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.8$

if first Aeroplane hitting the target is fails then we use the second plane to hit the target

i) Now we calculate the probability that the target is hit

$$P(\text{target is hit}) = P(A \text{ hit (or) } (A \text{ fails \& } B \text{ hits}))$$

$$= P(A) + P(\bar{A} \cap B)$$

$$= P(A) + P(\bar{A}) \cdot P(B)$$

$$= 0.3 + 0.7 \cdot 0.2$$

$$= 0.3 + 0.14$$

$$= 0.44$$

ii) Both fails hitting the target

The probability of both fails = P(A fails & B fails)

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A})\, P(\bar{B})$

$= 0.7 \times 0.8$

$= 0.56$

6) Three students A, B, C are in the running Race. A and B have the same probability of winning and Each is twice as lightly to win as C. Find the probability that B or C wins the Race.

Sol: Given A, B, C are three students in the running Race. Also given that A and B have the same probability of winning the Race and Each is twice as to win as "C". So only three students A, B, C are in the running Race so the probability of total space is "1"

So $P(S) = 1 = P(A) + P(B) + P(C) = 1$

$2 \cdot P(C) + 2 \cdot P(C) + P(C) = 1$

$5 \cdot P(C) = 1$

$P(C) = 1/5$

So $P(A) = 2 \cdot P(C) = 2 \cdot \frac{1}{5} = \frac{2}{5}$

$P(B) = 2 \cdot P(C) = 2 \cdot \frac{1}{5} = \frac{2}{5}$

So the probability of winning the Race by B or C is

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{2}{5} + \frac{1}{5} - 0$ (∵ B and C are disjoint Events)

$= \frac{3}{5}$

*** BAYES THEOREM ***

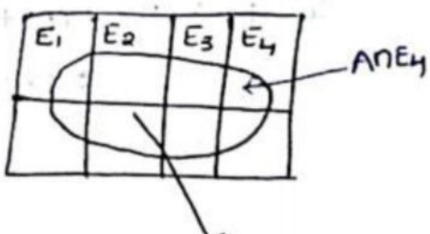
Statement :- Suppose $E_1, E_2 \cdots E_n$ are "n" mutually Exclusive and Exhaustive Events of the sample space 'S'. such that $P(E_i)>0$ $(i=1,2,\cdots n)$ and A is any another Event in S intersecting Every E_i such that $P(A)>0$ and $A \subset \bigcup_{i=1}^{n} E_i$ then

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}$$

Proof :-

Let S be the sample space of the Random Experiment and $E_1, E_2 \cdots E_n$ be the mutually Exclusive events and $S = \bigcup_{i=1}^{n} E_i$

Let A be any other Event in S intersecting Every event and $A \subset S$.



So $A = A \cap S$

$A = A \cap (E_1 \cup E_2 \cup E_3 \cdots \cup E_n)$

$A = (A\cap E_1)\cup(A\cap E_2)\cup(A\cap E_3)\cdots\cup(A\cap E_n)$ —①

From venn diagram $(A\cap E_1), (A\cap E_2), \cdots (A\cap E_n)$ are mutually Exclusive events

And $(A\cap E_1)\cap(A\cap E_2)\cap \cdots \cap(A\cap E_n) = \phi$

Taking probability on both sides in Eqn①

$P(A) = P((A\cap E_1)\cup(A\cap E_2)\cup\cdots(A\cap E_n))$ [$\because$ it is of $P(A\cup B\cup C)$]

$P(A) = P(A\cap E_1) + P(A\cap E_2) + \cdots + P(A\cap E_n)$ [$\because A\cap E_1, (A\cap E_2) \cdots (A\cap E_n)$ are mutually Exclusive

$P(A) = \sum_{i=1}^{n} P(A\cap E_i) \rightarrow$ ②

By the conditional probability we know that

$$P(E_i/A) = \frac{P(A\cap E_i)}{P(A)}$$

$$= \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(A\cap E_i)}$$

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}$$

Hence proved

Problem

1. If 1st box contains 2 white and 3 red balls, 2nd box contains 4 white and 5 red balls. one ball is drawn at random from one of the box and is found to be red. find the probability that red ball is drawn from 1st box

Sol: Let E_1 be the event of choosing Box I

$P(E_1) = 1/2$ (There are two boxes for choosing 1st box)

Let E_2 be the event of choosing box II

$P(E_2) = 1/2$

Let A be the event of drawing a red ball from any one of the box.

The probability of drawing a red ball from box 1 is

$$P(A/E_1) = \frac{3c_1}{5c_1} = \frac{3}{5}$$

	white	Red	
box 1	2	3	5
box 2	4	5	9

The probability of drawing a red ball from box 2 is

$$P(A/E_2) = \frac{5c_1}{9c_1} = \frac{5}{9}$$

The probability (total probability) that Red ball is drawn

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)$$

$$= \frac{1}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{5}{9}$$

$$P(A) = \frac{3}{10} + \frac{5}{18} = \frac{54+50}{180} = \frac{104}{180} = \frac{26}{45}$$

The probability that Red ball is drawn from box 1 is

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(A)}$$

$$= \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)} \quad [\because \text{by bayes theorem}]$$

$$= \frac{\frac{1}{2}\cdot\frac{3}{5}}{\frac{26}{45}} = \frac{\frac{3}{10}}{\frac{26}{45}} = \frac{135}{260} = \frac{27}{52}$$

2. Box 1 contains 1 white, 2 red and 3 green balls, box 2 contains 2 white, 3 red, 1 green balls, Box 3 contains 3 white, 1 red and 2 green balls. Two balls are drawn from a box choosen at random these two are found to be 1 white and 1 red. Determine the probability that balls are drawn from Box 2

Sol: Let E_1 be the event of choosing Box 1

$P(E_1) = 1/3$

Let E_2 be the event of choosing Box 2 $\Rightarrow P(E_2) = 1/3$

Let E_3 be the event of choosing Box 3 $\Rightarrow P(E_3) = 1/3$

Let A be the event of drawing 2 balls ~~for~~ which are one is white and one is red from any box

	white	red	green	
Box 1	1	2	3	6
Box 2	2	3	1	6
Box 3	3	1	2	6

The probability that drawing 1 white and 1 red ball from Box 1

$$P(A/E_1) = \frac{{}^1c_1 \times {}^2c_1}{{}^6c_2} = \frac{1\times 2}{\frac{6\times 5}{2\times 1}} = \frac{4}{30} = \frac{2}{15}$$

The probability that drawing 1 white and 1 red ball from Box 2

$$P(A/E_2) = \frac{{}^2c_1 \times {}^3c_1}{{}^6c_2} = \frac{\frac{2\times 3}{1}}{\frac{6\times 5}{2\times 1}} = \frac{2}{5}$$

The probability that drawing 1 white and 1 red ball from Box 3

$$P(A/E_3) = \frac{{}^3c_1 \times {}^1c_1}{{}^6c_2} = \frac{3\times 2}{6\times 5} = \frac{1}{5}$$

The probability (total probability) that 1 white ball and 1 red ball drawn from Boxes is

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)$$

$$= \frac{1}{3}\cdot\frac{2}{15} + \frac{1}{3}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{1}{5}$$

$$= \frac{2}{45} + \frac{2}{15} + \frac{1}{15}$$

$$P(A) = \frac{11}{45}$$

The probability that 1 white and 1 red balls are drawn from Box 2 is

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(A)}$$

$$= \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$P(E_2/A) = \frac{\frac{1}{3}\cdot\frac{2}{5}}{\frac{11}{45}} = \frac{2}{15}\times\frac{45}{11} = \frac{6}{11}$$

***

3. In a bolt factory missions A, B, C manifacture 20%, 30% and 50% of the total of their output 6%, 3% and 2% bolts are defective. A bolt is drawn at random is found to be defective. find the probability that the defective bolt is manifacture from machine A, machine C

Sol: Let E_1 be the event that bolts produced by the machine A. $\Rightarrow P(E_1) = \frac{20}{100}$

Let E_2 be the event that bolts produced by the machine B $\Rightarrow P(E_2) = \frac{30}{100}$

Let E_3 be the event that bolts produced by the machine C $\Rightarrow P(E_3) = \frac{50}{100}$.

Let D be the event of drawing a defective bolt from any machine

The probability that a defective bolt is choosen which is manufacture by machine A is $P(D/E_1) = \frac{6}{100}$

The probability that a defective bolt is choosen which is manufacture by machine B is $P(D/E_2) = \frac{3}{100}$

The probability that a defective bolt is choosen which is manufacture by machine C is $P(D/E_3) = \frac{2}{100}$

The probability that a defective bolt is drawn which is manufacture by any machine is

$$P(D) = P(E_1)\cdot P(D/E_1) + P(E_2)\cdot P(D/E_2) + P(E_3)\cdot P(D/E_3)$$

$$= \frac{20}{100}\cdot\frac{6}{100} + \frac{30}{100}\cdot\frac{3}{100} + \frac{50}{100}\cdot\frac{2}{100}$$

$$= \frac{120+90+100}{10{,}000} = \frac{310}{10{,}000}$$

The probability that a defective bolt is drawn which is manufacture from machine A is

$$P(E_1/D) = \frac{P(E_1) \cdot P(D/E_1)}{P(D)}$$

$$P(E_1/D) = \frac{\frac{20}{100} \cdot \frac{6}{100}}{\frac{310}{10,000}} = \frac{12}{31}$$

The probability that a defective bolt is drawn which is manufacture from machine C is

$$P(E_3/D) = \frac{P(E_3) \cdot P(D/E_3)}{P(D)}$$

$$= \frac{\frac{50}{100} \cdot \frac{2}{100}}{\frac{310}{10000}} = \frac{10}{31}$$

4. A Business man goes to hotels x, y, z, 20%, 50%, 30% of the times respectively. it is known that 5%, 4%, 8% of the rooms in hotels x, y, z have faulty plumbing. what is the probability that business man room have faulty clumbing is assigned to hotel z

Sol: Let E_1 be the event that bussiness man goes to the hotel 'x'

$P(E_1) = 20/100$

Let 'E_2' be the Event that Business man goes to the hotel 'y'

$P(E_2) = 50/100$

Let 'E_3' be the event that Business man goes to the hotel z

$P(E_3) = 30/100$

Let A be the event that business man room has faulty plumbing

The probability that hotel 'x' has faulty plumbing $P(A/E_1) = 5/100$

The probability that hotel 'y' has faulty plumbing $P(A/E_2) = 4/100$

The probability that hotel 'z' has faulty plumbing

$P(A/E_3) = 8/100$

The probability that (total probability) that hotel room has faulty plumbing is

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)$$

$$P(A) = \frac{20}{100}\times\frac{5}{100} + \frac{50}{100}\times\frac{4}{100} + \frac{30}{100}\times\frac{8}{100}$$

$$= \frac{100}{10,000} + \frac{200}{10,000} + \frac{240}{10,000}$$

$$P(A) = \frac{540}{10,000}$$

The probability that the business man room have faculty plumbing is assigned to hotel z is

$$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(A)}$$

$$= \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{30}{100}\times\frac{8}{100}}{\frac{540}{10,000}}$$

$$= \frac{24\cancel{0}}{\cancel{10,000}}\times\frac{\cancel{10,000}}{54\cancel{0}} = \frac{4}{9}$$

5. In a certain college 25% of boys and 10% of girls are studying mathematics the girls constiute 60% of the Student body. If a student is selected and is found to be studying mathematics.

a) what is the probability that mathematics is being studied

b) Find the probability that the student is a girl.

Sol: Let E_1 be the event that Student is a boy $P(E_1) = \frac{40}{100}$

Let E_2 be the event that the student is a girl $P(E_2) = \frac{60}{100}$

Let A be the event that mathematics is studied by the student.

The probability that the mathematics is being studied is

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)$$

$$= \frac{40}{100}\times\frac{25}{100} + \frac{60}{100}\times\frac{10}{100}$$

$$= \frac{1000}{10,000} + \frac{600}{10,000}$$

$$= \frac{1600}{10,000} = \frac{4}{25}$$

b) The probability that mathematics student is a girl

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(A)}$$

$$= \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{1600}{10,000}} = \frac{600}{10,000} \times \frac{10,000}{1600} = \frac{3}{8}$$

6. suppose 5 men out of 100 and 25 women out of 10000 are colour blind. A colour blind person is choosen at random what is the probability that the person is being a male (Assume female and male are Equal numbers)

Sol: Let E_1 be the Event that selected person is male

$$P(E_1) = 1/2$$

Let E_2 be the Event that selected person is Female

$$P(E_2) = 1/2$$

Let A be the Event of colour-blind is selected at random

The probability that the colour blind person is selected from men $P(A|E_1) = 5/100$

The probability that the colour blind person is selected from ~~P(A)~~ (female) women

$$P(A|E_2) = \frac{25}{10,000}$$

The probability that the colour blind person is selected

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{2} \cdot \frac{5}{100} + \frac{1}{2} \cdot \frac{25}{10000}$$

$$= \frac{5}{200} + \frac{25}{20,000} = \frac{500+25}{20,000} = \frac{525}{20,000}$$

The probability that the colour blind person is male

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(A)}$$

$$\Rightarrow \frac{\frac{1}{\cancel{2}} \cdot \frac{5}{\cancel{100}}}{\frac{525}{\cancel{20000}\ 100}} \Rightarrow \frac{\cancel{500}\ 20}{\cancel{525}\ 21} = \frac{20}{21}$$

7. Of the three men the chances that a politician, a business men and acadamition will be the appointed as a vice chancelar of the university are 0.5, 0.3, 0.2 respectively. The probability that the research is promoted by these persons if they are appointed as vc are 0.3, 0.7, 0.8 respectively.

i) Determine the probability that research is promoted

ii) If A research is promoted what is the probability that VC is an acadimition.

Sol: Let E_1 be the event that politician will be the appointed as VC $P(E_1) = 0.5$

Let E_2 be the event that business men will be appointed as V.C and $P(E_2) = 0.3$

Let E_3 be the event that acadamithian will be appointed as VC $P(E_3) = 0.2$

Let A be the Event that research is promoted by the person

The probability that the research is promoted by the politician if he appointed as VC

$P(A|E_1) = 0.3$

The probability that the research is promoted by the business man if he appointed as VC

$P(A|E_3) = 0.7$

The probability that the research is promoted by the acadamition if he is appointed as vc

$\therefore P(A/E_3) = 0.8$

i) The probability that research is promoted

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)$$

$$= (0.5)(0.3) + (0.3)(0.7) + (0.2)(0.8)$$

$$= 0.52$$

ii) The probability that research is promoted when the vc is acadamition is $P(E_3/A) = \dfrac{P(E_3)\cdot P(A/E_3)}{P(A)}$

$$= \frac{0.2 \times 0.8}{0.52}$$

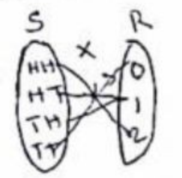
$$= \frac{0.16}{0.52} = 0.3077$$

Random variable : (X) :-

A real number "X" is associated with the outcomes of a Random Experiment. Random variables are usually denoted by capital letters of English Alphabets like X, Y, Z.

For Example in tossing of two coins coming of the no. of heads is a random variable.

i.e sample space (S) = {HH, HT, TH, TT}



Let X denotes the No. of Heads X(HH) = 2

X(HT) = 1

X(s): S→R

[X(HH) means no of heads in HH]

X(TH) = 1

X(TT) = 0

Random Variable X = X(s) = {0, 1, 2}

Random variables are classified into two types according to its characteristics. they are

i) Discrete Random variable

ii) continuous Random variable

i) Discrete Random variable :-

A Random variable is said to be discrete random variable if it takes only finite (or) countable number of values.

Ex :- ① In rolling of two perfect dice, the sum of numbers on the faces of dice is a discrete random variable

ie $X = X(S) = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

② In tossing two coins; getting number of heads is a discrete random variable

sample splace $S = \{HH, HT, TH, TT\}$

Random variable $X = X(S) = \{0, 1, 2\}$

ii) continuous Random variable :-

A Random variable is said to be continuous random variable if it takes all possible values between certain limits.

Ex :- 1. Daily temparature in a city can be mentioned as interval

2. The maximum life time of electric bulbs between 0 to 2000 hrs is a continuous random variable

$$X = X(S) = \{x \mid 0 \le x \le 2000\}$$

Probability Mass Function :-

Let "X" be a discrete random variable taking the values $x_1, x_2, x_3 ----, x_n$ and corresponding probabilities $P(x_1), P(x_2), P(x_3), ----, P(x_n)$ then $P(x_i)$ (or) $P(X = x_i)$ is called the probability mass function of random variable X if $P(x_i)$ satisfies the following two conditions

(i) $P(x_i) \ge 0 \quad \forall i$ (ii) $\sum_{i=1}^{n} P(x_i) = 1$

Probability Distribution Function :-

The probability distribution function of a discrete Random variable "x" is the table that contains the set of all possible values of "x" along with probabilities associated with all possible values of "x"

The probability distribution function of discrete random variable "x" is given by the table

$X=x$	x_1	x_2	x_3	—	—	—	—	—	—	—	x_{n-1}	x_n
$P(X=x)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	—	—	—	—	—	—	—	$P(x_{n-1})$	$P(x_n)$

<u>Ex</u> :- If two coins are tossing suppose consider random variable x is the number of heads getting on two coins

sample space $S = \{HH, HT, TH, TT\}$

$X(HH) = 2$

$X(HT) = 1$ [$\because$ X is no. of heads]

$X(TH) = 1$

$X(TT) = 0$

Random variable $X = \{0, 1, 2\}$

The probability of getting '0' heads $= P(X=0) = \{TT\} = \frac{1}{4}$

The probability of getting '1' head $= P(X=1) = \{HT, TH\} = \frac{2}{4}$

The probability of getting 2 heads $= P(X=2) = \{HH\} = \frac{1}{4}$

The probability distribution function of random discrete variable "X" is

	x_1	x_2	x_3
$X = x$	0	1	2
$P(X=x)$	$1/4$	$2/4$	$1/4$
	$P(x_1)$ (or) $P(X=0)$	$P(x_2)$ (or) $P(X=1)$	$P(x_3)$ (or) $P(X=2)$

probability density function: -

Let "x" be a continuous random variable taking value "x" such that $-\infty \le x \le +\infty$ then the function $f(x)$ is called probability density function of the continuous Random variable "x" and $f(x)$ satisfies the following conditions

i) $f(x_i) \ge 0 \ \forall i$

ii) $\int_{x=-\infty}^{\infty} f(x)\, dx = 1$ (total probability = 1)

Distribution function (or) cumulative distribution function:-

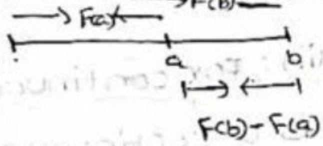
Let "x" be a Random variable then its distribution function is denoted by "F(x)" and is defined as

$$F(x) = P(X \le x) \quad \text{where } -\infty \le x \le \infty$$

The another name of the distribution function is cumulative distribution function.

**Note: -

1) if F(x) is distribution function of Random variable "x" and "a" is less than "b" then



- $\boxed{P(a < x \le b) = F(b) - F(a)}$
- $P(x=2) = P(X \le 1) - P(X \le 2)$
- $P(a \le x \le b) = P(x=a) + P(a < x \le b) = P(x=a) + F(b) - F(a)$
- $P(a \le x < b) = P(X=a) + P(a < x \le b) - P(x=b)$
  $= P(X=a) + F(b) - F(a) - P(X=b)$
- $P(a < x < b) = P(a < x \le b) - P(X=b)$
  $= F(b) - F(a) - P(X=b)$

ii) All distribution functions are monotonically increasing and lies between 0 and 1 i.e $0 \le F(x) \le 1$

iii) $\underset{x \to -\infty}{Lt} F(x) = F(-\infty) = 0$ and $\underset{x \to +\infty}{Lt} F(x) = F(\infty) = 1$

iv) If "x" is a discrete Random variable then the distribution function (or) cumulative distribution function is defined by

$$F(x) = P(X \le x) = \sum_{x_i \le x} P(x_i)$$

v) if x is continuous random variable then the distributive function (or) cumulative distributive function is defined by $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

vi) The derivative of the distribution function is called probability density function

$$\frac{d}{dx} F(x) = f(x)$$

Mean and variance :-

In general Mean (or) Expected value (or) Expectation of Random variable x is denoted by μ (or) $E(X)$ (or) $\bar{x}$ and variance of Random variable "x" is denoted by σ_x^2 (or) $V(x)$

case(i): For discrete Random variable

if x is a discrete Random variable then

$$\text{Mean } \mu = E(x) = \sum_{i=1}^{n} x_i \cdot P(x_i)$$

$$\text{variance } V(X) = E(x^2) - (E(x))^2 \quad \text{where}$$

$$= E(x-\mu)^2 \qquad E(x^2) = \sum_{i=1}^{n} x_i^2 \cdot P(x_i)$$

case(ii): For continuous Random variable

if x is a continuous Random variable then

$$\text{Mean}(\mu) = E(x) = \int_{x=-\infty}^{\infty} x \cdot f(x)\,dx$$

$$\text{Variance } V(X) = \sigma_x^2 = E(x^2) - (E(x))^2 \quad \text{where}$$

$$E(x^2) = \int_{x=-\infty}^{\infty} x^2 f(x)\,dx.$$

Standard Deviation :-

The square root of variance is called standard deviation and it is denoted by σ (or) σ_x i.e $\sigma_x = \sqrt{\text{variance}}$

Problems

1. A Random variable x has the following probability distribution

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

Find i) K ii) $P(X<3)$, $P(X\geq 6)$, $P(1<X\leq 3)$, $P(X=3)$

iii) What is the minimum value of "K" so that $P(X\leq K)>0.3$

iv) Mean and Variance v) cumulative distribution function

Sol: Given probability distribution function

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$X=x$	0	1	2	3	4	5	6
$P(X=x)$	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$
	$p(x_1)$	$p(x_2)$	$p(x_3)$	$p(x_4)$	$p(x_5)$	$p(x_6)$	$p(x_7)$

i) we know that total probability = 1

i.e $\sum_{i=1}^{7} P(x_i)=1$

$$P(x_1)+P(x_2)+P(x_3)+P(x_4)+P(x_5)+P(x_6)+P(x_7)=1$$

$$K+3K+5K+7K+9K+11K+13K=1$$

$$49K=1 \Rightarrow K=1/49$$

The probability distributive function when $K=1/49$ is

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$X=x$	0	1	2	3	4	5	6
$P(X=x)$	$1/49$	$3/49$	$5/49$	$7/49$	$9/49$	$11/49$	$13/49$
	$P(X=0)$	$P(X=1)$	$P(X=2)$	$P(X=3)$	$P(X=4)$	$P(X=5)$	$P(X=6)$

ii) $P(X<3)=P(X=0)+P(X=1)+P(X=2)$

$$=\frac{1}{49}+\frac{3}{49}+\frac{5}{49}=9/49$$

$$P(X\geq 6)=P(X=6)=13/49$$

$$P(1<X\leq 3)=P(X=2)+P(X=3)$$

$$=\frac{5}{49}+\frac{7}{49}=\frac{12}{49}$$

$$P(X=3)=\frac{7}{49}$$

iii) $P(X\leq K)>0.3$

By the trial method put $K=0$

$P(X\leq 0)=P(X=0)=\frac{1}{49}=0.02 \not> 0.3$

put $K=1$

$P(X\leq 1)=P(X=0)+P(X=1)=\frac{1}{49}+\frac{3}{49}=\frac{4}{49}=0.08 \not> 0.3$

put $K=2$

$P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)=\frac{1}{49}+\frac{3}{49}+\frac{5}{49}=\frac{9}{49}=0.18 \not> 0.3$

put $K=3$

$P(X\leq 3)=P(X\leq 2)+P(X=3)=\frac{9}{49}+\frac{7}{49}=\frac{16}{49}=0.32>0.3$

Therefore the minimum value of k when $P(x \le k) > 0.3$ is $k=3$

iv) Mean $\mu = E(x) = \sum_{i=1}^{7} x_i \cdot P(x_i)$ [$\because$ x is discrete Random variable]

$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + x_3 \cdot P(x_3) + x_4 \cdot P(x_4) + x_5 \cdot P(x_5) + x_6 \cdot P(x_6) + x_7 \cdot P(x_7)$

$= 0 \cdot \frac{1}{49} + 1 \cdot \frac{3}{49} + 2 \cdot \frac{5}{49} + 3 \cdot \frac{7}{49} + 4 \cdot \frac{9}{49} + 5 \cdot \frac{11}{49} + 6 \cdot \frac{13}{49}$

$= \frac{3}{49} + \frac{10}{49} + \frac{21}{49} + \frac{36}{49} + \frac{55}{49} + \frac{78}{49}$

$= \frac{203}{49}$

we know that variance $V(x) = E(x^2) - (E(x))^2$

Now $E(x^2) = \sum_{i=1}^{n} x_i^2 \cdot P(x_i)$

$= x_1^2 \cdot P(x_1) + x_2^2 \cdot P(x_2) + x_3^2 \cdot P(x_3) + x_4^2 \cdot P(x_4) + x_5^2 \cdot P(x_5) + x_6^2 \cdot P(x_6) + x_7^2 \cdot P(x_7)$

$= 0^2 \cdot \frac{1}{49} + 1^2 \cdot \frac{3}{49} + 2^2 \cdot \frac{5}{49} + 3^2 \cdot \frac{7}{49} + 4^2 \cdot \frac{9}{49} + 5^2 \cdot \frac{11}{49} + 6^2 \cdot \frac{13}{49}$

$= \frac{3}{49} + \frac{20}{49} + \frac{63}{49} + \frac{144}{49} + \frac{275}{49} + \frac{468}{49}$

$= \frac{973}{49}$

Now variance $V(x) = E(x^2) - (E(x))^2$

$= \frac{973}{49} - \left(\frac{203}{49}\right)^2$

$= 19.857 - 17.163$

$= 2.694$

V) cumulative distribution function is

$X=x$	0	1	2	3	4	5	6
$P(X=x)$	1/49	3/49	5/49	7/49	9/49	11/49	13/49
$F(x)$	1/49	4/49	9/49	16/49	25/49	36/49	49/49

2. A Random variable x has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

then find

i) constant K

ii) Evaluate $P(X<2)$, $P(X\geq 6)$

iii) Determine the minimum value of K so that $P(X\leq k)\geq \frac{1}{2}$

iv) Mean value of X

v) cumulative distribution function (Distribution function)

Sol: given probability distribution function

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$X=x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$
	$P(x_1)$	$P(x_2)$	$P(x_3)$	$P(x_4)$	$P(x_5)$	$P(x_6)$	$P(x_7)$	$P(x_8)$

i) We know that total probability = 1

$$\sum_{i=1}^{8} P(x_i) = 1$$

$$P(x_1)+P(x_2)+P(x_3)+P(x_4)+P(x_5)+P(x_6)+P(x_7)+P(x_8)=1$$

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$10k^2+9k=1$$

$$10k^2+9k-1=0$$

$$10k^2+10k-k-1=0$$

$$10k(k+1)-1(k+1)=0.$$

$$(k+1)(10k-1)=0$$

$$k=-1 \text{ or } k=1/10.$$

Clearly $k\neq -1$ [$\because P(x_i)\geq 0$] and we have $k=1/10$

Therefore the probability distribution function when $k=1/10$ is

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$X=x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$\frac{2}{100}$	$\frac{17}{100}$
	$P(x=0)$	$P(x=1)$	$P(x=2)$	$P(x_3)$	$P(x=4)$	$P(x=5)$	$P(x=6)$	$P(x=7)$

ii) $P(x<2) = P(x=0)+P(x=1)$

$$=0+\frac{1}{10}=\frac{1}{10}$$

$P(x\geq 6) = P(x=6)+P(x=7)$

$$=\frac{2}{100}+\frac{17}{100}=\frac{19}{100}$$

iii) Given $P(x\leq k)\geq 0.5$.

By the trail method put $k=0$

$P(x\leq 0)=P(x=0)=0\ngeq 0.5$

$P(x \leq 1) = P(x=0) + P(x=0) \rightarrow$ put $k=1$

$= 0 + \frac{1}{10} = 0.1 \ngeq 0.5$

put $k=2$

$P(x \leq 2) = P(x \leq 1) + P(x=2)$

$= \frac{1}{10} + \frac{2}{10} = \frac{3}{10} = 0.3 \ngeq 0.5$

put $k=3$

$P(x \leq 3) = P(x \leq 2) + P(x=3)$

$= 0.3 + \frac{2}{10}$

$P(x \leq 3) = 0.5 \geq 0.5$

Therefore the minimum value of k when $P(x \leq k) \geq 1/2$ is $k=3$

iv) By the definition of mean

$$\mu = E(X) = \sum_{i=1}^{8} x_i \, P(x_i)$$

$$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_8 \cdot P(x_8)$$

$$= 0 \cdot 0 + 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{2}{10} + 4 \cdot \frac{3}{10} + 5 \cdot \frac{1}{100} + 6 \cdot \frac{2}{100} + 7 \cdot \frac{17}{100}$$

$$= \frac{10 + 40 + 60 + 120 + 5 + 12 + 119}{100}$$

$$\mu = \frac{366}{100} = 3.66$$

v) Cumulative distribution function is

$x=x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	1/10	2/10	2/10	3/10	1/100	2/100	17/100
$F(x)$	0	1/10	3/10	5/10	8/10	81/100	$\frac{83}{100}$	1

3. Find the mean and variance of uniform probability distribution (or) find the mean and variance of uniform probability distribution $P(x) = \frac{1}{n}$ for $x = 1, 2, 3, \ldots, n$

Sol: The given probability distribution is

$X=x$	1 (x_1)	2 (x_2)	3 (x_3)	-	-	-	n (x_n)
$P(X=x) = P(x)$	1/n ($P(x_1)$)	1/n ($P(x_2)$)	1/n ($P(x_3)$)				1/n ($P(x_n)$)

We know that mean $\mu = E(X) = \bar{X} = \sum_{i=1}^{n} x_i \cdot P(x_i)$

$$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n P(x_n)$$

$$= 1 \cdot (1/n) + 2 \cdot (1/n) + \cdots + n \cdot (1/n)$$

$$= (1+2+\cdots+n)\frac{1}{n}$$

$$= \frac{\cancel{n}(n+1)}{2} \cdot \frac{1}{\cancel{n}}$$

$$E(X) = \frac{(n+1)}{2}$$

We know that

Variance $V(X) = \sigma^2_X = E(X^2) - (E(X))^2$

Now $E(X^2) = \sum_{i=1}^{n} x_i^2 \cdot P(x_i)$

$$= x_1^2 \cdot P(x_1) + x_2^2 \cdot P(x_2) + \cdots x_n^2 \cdot P(x_n)$$

$$= 1^2 \cdot (1/n) + 2^2 \cdot (1/n) + \cdots + n^2 (1/n)$$

$$= (1^2+2^2+\cdots+n^2)\, 1/n$$

$$= \frac{\cancel{n}(n+1)(2n+1)}{6} \cdot \frac{1}{\cancel{n}}$$

$$= \frac{(n+1)(2n+1)}{6}$$

Now $V(X) = E(X^2) - (E(X))^2$

$$= \frac{(n+1)(2n+1)}{6} - \left(\frac{(n+1)}{2}\right)^2$$

$$= \frac{(n+1)}{2}\left(\frac{2(2n+1) - 3 \cdot (n+1)}{6}\right)$$

$$= \frac{(n+1)}{2} \cdot \left(\frac{4n+2-3n-3}{6}\right)$$

$$= \frac{(n+1)}{2} \cdot \frac{(n-1)}{6}$$

Variance $V(X) = \frac{n^2-1}{12}$

4. Find the distribution function which corresponds to probability distribution defined by $f(x) = \frac{x}{15}$ for $x = 1, 2, 3, 4, 5$

Sol: The required probability distribution is

	x_1	x_2	x_3	x_4	x_5
$X = x$	1	2	3	4	5
$P(X=x) = f(x) = \frac{x}{15}$	1/15	2/15	3/15	4/15	5/15
	$P(x_1)$	$P(x_2)$	$P(x_3)$	$P(x_4)$	$P(x_5)$

The cumulative distribution function (distribution function) is

$X=x$	1	2	3	4	5
$P(X=x)=f(x)=\frac{x}{15}$	$1/15$	$\frac{2}{15}$	$3/15$	$4/15$	$5/15$
$F(x)=P(X\le x)$	$1/15$	$3/15$	$6/15$	$10/15$	$15/15$

***5. A random variable X is defined as the sum of the numbers on the phases when two dice are thrown. Find the probability distribution and mean, variance of X.

Sol: When two dice are thrown number of possibilities in sample space

$S = n(S) = 6^2 = 36$

Sample space $S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

Given Random variable x is the sum of numbers on the faces when two dice are thrown.

So $X = X(a,b) = a+b$

Therefore Random variable $X = \{2,3,4,5,6,7,8,9,10,11,12\}$

The probability of getting 2 is $P(X=2) = P\{(1,1)\} = \frac{1}{36}$

The probability of getting 3 is $P(X=3) = P\{(1,2)(2,1)\} = \frac{2}{36}$

The probability of getting 4 is $P(X=4) = P\{(1,3)(2,2)(3,1)\} = \frac{3}{36}$

The probability of getting 5 is $P(X=5) = P\{(1,4),(2,3),(3,2),(4,1)\} = \frac{4}{36}$

The probability of getting 6 is $P(X=6) = P\{(1,5),(2,4),(3,3)(4,2),(5,1)\} = \frac{5}{36}$

The probability of getting 7 is $P(X=7) = P\{(1,6)(2,5)(3,4),(4,3),(5,2),(6,1)\}$
$= \frac{6}{36}$

The probability of getting 8 is $P(X=8) = P\{(2,6)(3,5)(4,4),(5,3),(6,2)\}$
$= \frac{5}{36}$

The probability of getting 9 is $P(X=9) = P\{(3,6),(4,5),(5,4),(6,3)\}$
$= 4/36$

The probability of getting 10 is $P(X=10) = P\{(4,6)(5,5),(6,4)\} = \frac{3}{36}$

The probability of getting 11 is $P(X=11) = \{(5,6)(6,5)\} = \frac{2}{36}$

The probability of getting 12 is $P(X=12) = \{(6,6)\} = 1/36$

The Required probability distribution is

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
$X=x$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	1/36	2/36	3/36	4/36	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
	$P(x_1)$ $P(x=2)$	$P(x_2)$ $P(x=3)$	$P(x_3)$	$P(x_4)$	$P(x_5)$	$P(x_6)$	$P(x_7)$	$P(x_8)$	$P(x_9)$ $P(x=10)$	$P(x_{10})$ $P(x=11)$	$P(x_{11})$ $P(x=12)$

We know that

$\text{Mean}(\mu) = E(X) = \sum_{i=1}^{11} x_i \cdot P(x_i)$

$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + x_3 \cdot P(x_3) + \cdots + x_{11} \cdot P(x_{11})$

$= 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36}$

$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36}$

$= \frac{252}{36} = 7$

We know that

Variance of $X = V(X) = \sigma^2_X = E(X^2) - (E(X))^2$

Now $E(X^2) = \sum_{i=1}^{11} x_i^2 \cdot P(x_i)$

$= x_1^2 \cdot P(x_1) + x_2^2 \cdot P(x_2) + \cdots + x_{11}^2 \cdot P(x_{11})$

$= 4 \cdot \frac{1}{36} + 9 \cdot \frac{2}{36} + 16 \cdot \frac{3}{36} + 25 \cdot \frac{4}{36} + 36 \cdot \frac{5}{36} + 49 \cdot \frac{6}{36} + 64 \cdot \frac{5}{36} + 81 \cdot \frac{4}{36} + 100 \cdot \frac{3}{36} + 121 \cdot \frac{2}{36} + 144 \cdot \frac{1}{36}$

$= \frac{4+18+45+100+180+294+320+324+300+242+144}{36}$

$= \frac{1974}{36} = 54.8333$

Now $V(X) = E(X^2) - (E(X))^2 = 54.8333 - (7)^2$

$= 54.8333 - 49$

$V(X) = 5.8333$

6. Two dice are thrown let x assigned to each point (a,b) in S the maximum of its numbers then find probability distribution and also Expected value of x (Mean)

Sol: when two dice are thrown the number of possible outcomes in the sample space $S = n(S) = 6^2 = 36$.

Sample space $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

Given Random variable x is the maximum of (a,b) in sample space S

i.e $X(a,b) = \max(a,b)$

Ex: $X(1,2)$
$\max(1,2)$
$=2$.
$X(6,5) = \max(6,5)$
$=6$

So Random variable $X = \{1,2,3,4,5,6\}$

The probability of getting $1 = P(X=1) = P\{(1,1)\} = \frac{1}{36}$

The probability of getting $2 = P(X=2) = P\{(1,2)\ (2,1)\ (2,2)\} = \frac{3}{36}$

The probability of getting $3 = P(X=3) = P\{(1,3),(2,3),(3,1),(3,3),(3,2)\} = \frac{5}{36}$

The probability of getting $4 = P(X=4) = P\{(1,4),(2,4),(3,4),(4,1)\ (4,2),(4,3)\ (4,4)\}$

$= \frac{7}{36}$

The probability of getting $5 = P(X=5) = P\{(1,5)\ (2,5)\ (3,5)\ (4,5),(5,1),(5,2),$
$(5,3),(5,4)\ (5,5)\}$

$= \frac{9}{36}$

The probability of getting $6 = P(X=6) = P\{(1,6)\ (2,6)\ (3,6)\ (4,6)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$
$= \frac{11}{36}$

The Required probability distribution is

	x_1	x_2	x_3	x_4	x_5	x_6
$X=x$	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
	$P(x_1)$	$P(x_2)$	$P(x_3)$	$P(x_4)$	$P(x_5)$	$P(x_6)$

we know that Expected value of Random variable X is

$$\mu = E(x) = \sum_{i=1}^{6} x_i \cdot P(x_i)$$

$$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_6 \cdot P(x_6)$$

$$= 1 \cdot \frac{1}{36} + 2 \cdot \frac{3}{36} + 3 \cdot \frac{5}{36} + 4 \cdot \frac{7}{36} + 5 \cdot \frac{9}{36} + 6 \cdot \frac{11}{36}$$

$$= \frac{1+6+15+28+45+66}{36}$$

$$= \frac{161}{36} = 4.472$$

7. Calculate Expectation and variance of x if the probability distribution of the Random variable x is given by

x	-1	0	1	2	3
f	0.3	0.1	0.1	0.3	0.2

Sol: Given probability distribution function

x	x_1: -1	x_2: 0	x_3: 1	x_4: 2	x_5: 3
$P(X=x)$	0.3	0.1	0.1	0.3	0.2
	$P(x_1)$	$P(x_2)$	$P(x_3)$	$P(x_4)$	$P(x_5)$

We know that mean$(\mu) = \sum_{i=1}^{5} x_i\, P(x_i)$

$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_5 \cdot P(x_5)$

$= (-1)(0\cdot3) + (0)(0\cdot1) + 1(0\cdot1) + 2(0\cdot3) + 3(0\cdot2)$

$= -0\cdot3 + 0\cdot1 + 0\cdot6 + 0\cdot6$

$= 1\cdot0$

Variance $(\sigma^2_x) = E(x^2) - E(x)^2$.

Now

$E(x^2) = \sum_{i=1}^{5} x_i^2 \cdot P(x_i)$

$= (-1)^2 \cdot 0\cdot3 + (0)^2\, 0\cdot1 + (1^2)\cdot 0\cdot1 + 2^2(0\cdot3) + 3^2\cdot(0\cdot2)$

$= 0\cdot3 + 0\cdot1 + 1\cdot2 + 1\cdot8 = 2\cdot4$

Now

$V(x) = E(x^2) - E(x)^2$

$= (2\cdot4) - (1)^2$

$= 2\cdot4 - 1$

$V(x) = 1\cdot4$

8. If three cars are selected from a lot of 6 cars containing 2 defective cars. find the probability distribution of the number of defective cars and also find Expected number of defective cars

Sol: Number of cars in lot = 6

Number of defective cars = 2

Number of non defective cars = Total − defective = 6 − 2 = 4

Let 'x' be the Number of defective cars getting when three cars are selected from lot of 6 cars. So Random variable x can take the values 0, 1, 2 [Since our lot contains maximum 2 defective cars]

Therefore $x = x(S) = \{0, 1, 2\}$

The probability of getting 0 defective cars i.e 3 are non-defective.

$$P(X=0) = \frac{4c_3}{6c_3} = \frac{\frac{4\times 3\times 2}{3\times 2\times 1}}{\frac{6\times 5\times 4}{3\times 2\times 1}} = \frac{1}{5}$$

The probability of getting "1" defective car i.e among 3 cars

1 is defective and remaining 2 is non defective

$$P(X=1) = \frac{2c_1 \cdot 4c_2}{6c_3} = \frac{\frac{2\times 4\times 3}{2\times 1}}{\frac{6\times 5\times 4}{3\times 2\times 1}} = \frac{3}{5}$$

The probability of getting "2" defective car, i.e among 3 cars

2 is defective and remaining 1 is non-defective

$$P(X=2) = \frac{2c_2 \cdot 4c_1}{6c_3} = \frac{\frac{1\times 4}{1}}{\frac{6\times 5\times 4}{3\times 2\times 1}} = \frac{1}{5}$$

The probability distribution of number of defective cars is

$X=x$	0	1	2
$P(X=x)$	1/5	3/5	1/5

Expected number of defective cars i.e mean.

$$\mu = \bar{X} = E(x) = \sum_{i=1}^{3} x_i \cdot P(x_i)$$

$$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + x_3 \cdot P(x_3)$$

$$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{5} + 2 \cdot \frac{1}{5}$$

$$E(x) = 1$$

9. Let X denotes the number of heads in a single toss of 4 fair coins. Determine

i) $P(X<2)$, $P(1<X\leq 3)$

ii) Mean of X

Sol:-

When 4 fair coins are tossed the total number of possible outcomes in the sample space $S = n(S) = 16$

Let X be the number of heads when 4 coins are tossed. So Random variable X takes the values $X = X(S) = \{0, 1, 2, 3, 4\}$ [∵ our exp has only 4 coins]

The probability of getting 0 heads $= P(X=0) = \frac{4c_0}{2^4} = \frac{1}{16}$

The probability of getting 1 head = $P(X=1) = \frac{^4C_1}{2^4} = \frac{4}{16}$

The probability of getting 2 heads = $P(X=2) = \frac{^4C_2}{2^4} = \frac{6}{16}$

The probability of getting 3 heads = $P(X=3) = \frac{^4C_3}{2^4} = \frac{\frac{4\times3\times2}{3\times2\times1}}{\frac{16}{1}}$

$= \frac{4}{16}$

The probability of getting 4 heads = $P(X=4) = \frac{^4C_4}{2^4} = \frac{1}{16}$.

The required probability distribution is

	x_1	x_2	x_3	x_4	x_5
$X=x$	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
	$P(X=0)$	$P(X=1)$	$P(X=2)$	$P(X=3)$	$P(X=4)$

i) $P(X<2) = P(X=0) + P(X=1)$

$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

$P(1 < x \leq 3) = P(X=2) + P(X=3)$

$= \frac{6}{16} + \frac{4}{16} = \frac{10}{16}$

ii) Mean of X

we know that mean $\mu = E(X) = \sum_{i=1}^{5} x_i \cdot P(x_i)$

$= x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots x_5 \cdot P(x_5)$

$= 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16}$.

$= \frac{32}{16} = 2$.

10. out of 24 mangoes, 6 are rotten, 2 mangoes are drawn Obtain probability distribution of the number of rotten mangoes

Sol: Total number of mangoes = 24.

Number of rotten mangoes = 6.

Number of Non rotten mangoes = 18

Total no of possibilities $n(S) = {}^{24}C_2$

Let X is a Random variable of getting rotten mangoes.

$\Rightarrow X = \{0, 1, 2\}$

The probability of getting "0" rotten mangoes is $P(X=0) = \frac{^{18}C_2}{^{24}C_2}$

$= 51/92$

" " 1 rotten mango = $\frac{^6C_1 \cdot {}^{18}C_1}{^{24}C_2} = \frac{9}{23}$

The probability of getting 2 rotten mangoes = $\frac{^6C_2}{^{24}C_2} = \frac{5}{92}$

Therefore the probability distribution function of the number of rotten mangoes is

	x_1	x_2	x_3
$X=x$	0	1	2
$P(X=x)$	$\frac{51}{92}$	$\frac{9}{23}$	$\frac{5}{92}$
	$P(X=0)$	$P(X=1)$	$P(X=2)$

11. A discrete Random variable X has the following distribution Function

$$F(x) = \begin{cases} 0 & \text{for } x<1 \\ \frac{1}{3} & \text{for } 1 \le x < 4 \\ \frac{1}{2} & \text{for } 4 \le x < 6 \\ 5/6 & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

Find i) $P(2 < x \le 6)$ ii) $P(X=5)$ iii) $P(X=4)$ iv) $P(x \le 6)$ v) $P(X=6)$

Sol: Given distribution function

$$F(x) = P(X \le x) = \begin{cases} 0 & \text{for } x<1 \\ 1/3 & \text{for } 1 \le x < 4 \\ 1/2 & \text{for } 4 \le x < 6 \\ 5/6 & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

By the definition of distribution function we know that

$$F(x) = P(X \le x)$$

and $P(a < X \le b) = F(b) - F(a)$

i) $P(2 < x \le 6) = F(6) - F(2)$

$= P(X \le 6) - P(X \le 2)$

$= \frac{5}{6} - \frac{1}{3} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$

ii) $P(X=5) = P(X \le 5) - P(X < 5)$

$= P(X \le 5) - P(X \le 4)$

$= F(5) - F(4) = \frac{1}{2} - \frac{1}{2} = 0$

iii) $P(X=4) = P(X \le 4) - P(X<4)$

$= P(X \le 4) - P(X \le 3)$

$= F(4) - F(3)$

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

iv) $P(X \le 6) = F(6)$

$= 5/6$

v) $P(X=6) = P(X \le 6) - P(X<6)$

$= P(X \le 6) - P(X \le 5) = F(6) - F(5)$

$= \frac{5}{6} - \frac{1}{2} = \frac{5-3}{6} = \frac{1}{3}$

Result: –

If X is a discrete Random variable and "a" is constant then

i) $E(a) = a$

ii) $E(aX+b) = a \cdot E(x) + b$

iii) $V(aX+b) = a^2 \cdot V(X)$

Proof: Let "x" be a discrete Random variable and $Y = ax+b$.

By definition of Expectation we know that

$$E(x) = \sum_{i=1}^{n} x_i \cdot P(x_i) \quad [\because x \text{ is discrete Random Variable}]$$

i) Now

$$E(a) = \sum_{i=1}^{n} a \cdot P(x_i)$$

$$= a \cdot \sum_{i=1}^{n} P(x_i)$$

$$= a(1) \quad \left[\because \sum_{i=1}^{n} P(x_i) = \text{total probability} = 1\right]$$

$$= a$$

ii) $$E(Y) = E(ax+b) = \sum_{i=1}^{n} (ax_i+b) \cdot P(x_i)$$

$$= \sum_{i=1}^{n} \left[a x_i \cdot P(x_i) + b\, P(x_i)\right]$$

$$= a \cdot \sum_{i=1}^{n} x_i \cdot P(x_i) + b \cdot \sum_{i=1}^{n} P(x_i)$$

$$= a \cdot E(x) + b \cdot 1$$

$$E(Y) = E(ax+b) = a \cdot E(x) + b$$

iii) By definition of variance we know that

$$V(x) = E(x^2) - (E(x))^2$$

$$V(ax+b) = E((ax+b)^2) - (E(ax+b))^2$$

$$= E(a^2x^2 + b^2 + 2ab\cdot x) - (a\cdot E(x) + b)^2$$

$$= a^2\cdot E(x^2) + E(b^2) + E(2ab\cdot x) - \left(a^2(E(x))^2 + b^2 + 2ab\cdot E(x)\right)$$

$$= a^2\cdot E(x^2) + \cancel{b^2} + \cancel{2ab\,E(x)} - a^2\cdot(E(x))^2 - \cancel{b^2} - \cancel{2ab\,E(x)}$$

$[\because E(ax) = a\cdot E(x)$ and $E(a) = a]$

$$= a^2\cdot[E(x^2) - E(x)^2]$$

$$V(ax+b) = a^2\cdot V(x)$$

[Here $v(b) = 0$

[$\because v(x) = E(x^2) - (E(x))^2$

12) The probability distribution function of the Random variable x is

X	0	1	2
P(x)	1/5	3/5	1/5

then find

i) $E(2x+3)$ ii) $V(2x-3)$

Sol: given probability distribution function

X	0 (x_1)	1 (x_2)	2 (x_3)
P(x)	1/5 ($P(x_1)$)	3/5 ($P(x_2)$)	1/5 ($P(x_3)$)

By definition of mean

$$\mu = E(x) = \sum_{i=1}^{3} x_i\cdot P(x_i)$$

$$= x_1\cdot P(x_1) + x_2\cdot P(x_2) + x_3\cdot P(x_3)$$

$$= 0\cdot\frac{1}{5} + 1\cdot\frac{3}{5} + 2\cdot\frac{1}{5} = 1$$

By def of variance

$$V(x) = E(x^2) - (E(x))^2$$

Now $E(x^2) = \sum_{i=1}^{3} x_i^2\cdot P(x_i)$

$$= x_1^2\cdot P(x_1) + x_2^2\cdot P(x_2) + x_3^2\cdot P(x_3)$$

$$E(x^2) = 0\cdot\frac{1}{5} + 1\cdot\frac{3}{5} + 4\cdot\frac{1}{5} = \frac{7}{5}$$

Now $v(x) = E(x^2) - (E(x))^2$

$= \frac{7}{5} - 1$

$V(x) = \frac{2}{5}$

i) $E(2x+3)$

$= 2 \cdot E(x) + E(3)$ $\quad [\because E(a) = a]$

$= 2 \cdot E(x) + 3$

$= 2 \cdot 1 + 3 = 5$

ii) $V(2x-3)$

$= 2^2 \cdot V(x) - V(3)$ $\quad [\because V(ax+b) = a^2 \cdot V(x) + 0]$

$= 4 \cdot V(x) - 0$ $\quad [\because V(a) = 0]$

$= 4 \cdot \frac{2}{5} = \frac{8}{5}$

(13) The probability distribution function of the discrete random variable x is

X	0	1	2
P(x)	0.55	0.39	0.06

then find

i) Mean, variance

ii) $E(2x+3)$ iii) $V(2x+1)$

Sol: given probability distribution

X	0 (x_1)	1 (x_2)	2 (x_3)
P(X=x)	0.55 ($p(x_1)$)	0.39	0.06 ($p(x_3)$)

By the def of Mean $\mu = \sum_{i=1}^{3} x_i \cdot p(x_i) = 0(0.55) + 1(0.39) + 2(0.06)$

$\mu = 0.51$

Variance $V(x) = E(x^2) - (E(x))^2$

Now $E(x^2) = \sum_{i=1}^{3} x_i^2 \cdot p(x_i) = (0)^2 (0.55) + 1^2 \cdot 0.39 + 2^2 \cdot 0.06$.

$= 0.63$

Now $V(x) = E(x^2) - (E(x))^2$.

$= 0.63 - (0.51)^2 = 0.63 - 0.2601$

$= 0.3699$.

ii) $E(2x+3) = E(2x) + E(3) = 2E(x) + 3 = 2(0.51) + 3 = 4.02$

$V(2x+1) = V(2x) + V(1) = 2^2 V(x) + 0$

$= 4 \cdot V(x)$

$= 4(0.3699)$

$= 1.4796$

Problems on continuous Random variable

1. Find k such that the function "f" is defined by,

$$f(x) = \begin{cases} kx^2 & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

i) $P(\frac{1}{4} < x < \frac{1}{2})$; $P(x<\frac{1}{2})$

ii) $P(x>1)$

iii) distribution function of X

iv) Mean and variance

Sol: Given probability density function

$$f(x) = \begin{cases} kx^2 & \text{if } 0<x<1 \\ 0 & \text{otherwise (or) } -\infty<x<0 \text{ and } 1<x<\infty \end{cases}$$

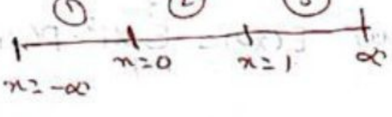
we know that total probability $=1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{1} f(x)\,dx + \int_{x=1}^{\infty} f(x)\,dx = 1$$

$$\int_{x=-\infty}^{0} (0)\,dx + \int_{x=0}^{1} kx^2\,dx + \int_{x=1}^{\infty} (0)\,dx = 1$$

$$0 + k\int_{x=0}^{1} x^2\,dx + 0 = 1$$



$$k\cdot\left(\frac{x^3}{3}\right)_{x=0}^{1} = 1$$

$$\frac{k}{3}\left[(1)^3 - (0)^3\right] = 1$$

$$\frac{k}{3}(1-0) = 1$$

$$\Rightarrow \boxed{k=3}$$

Therefore the probability density function of the continuous Random variable "X" when $k=3$ is

$$f(x) = \begin{cases} 3x^2 & \text{if } 0<x<1 \\ 0 & \text{if } -\infty<x<0 \text{ and } 1<x<\infty \end{cases}$$

i) $P(\frac{1}{4}<x<\frac{1}{2}) = \int_{x=1/4}^{1/2} f(x)\,dx$

$$\boxed{P(a<x<b) = P(a\le x\le b) = \int_a^b f(x)\,dx}$$

$$= \int_{x=1/4}^{1/2} 3x^2\,dx$$

$$= 3\cdot\left(\frac{x^3}{3}\right)_{x=1/4}^{1/2}$$

$$= \left(\tfrac{1}{2}\right)^3 - \left(\tfrac{1}{4}\right)^3$$

$$P\left(\tfrac{1}{4}<x<\tfrac{1}{2}\right) = \frac{1}{8} - \frac{1}{64} = \frac{7}{64}$$

$$P\left(x<\tfrac{1}{2}\right) = P\left(-\infty<x<\tfrac{1}{2}\right)$$

$$= \int_{x=-\infty}^{1/2} f(x)\,dx$$

$$= \int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{1/2} f(x)\,dx$$

$$= \int_{x=-\infty}^{0} 0\cdot dx + \int_{x=0}^{1/2} 3x^2\cdot dx$$

$$= 0 + 3\cdot\left(\frac{x^3}{3}\right)_{x=0}^{1/2}$$

$$P\left(x<\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^3 - 0^3 = \frac{1}{8}$$

ii) $P(X>1) = P(\infty > x > 1)$

$$= P(1<x<\infty)$$

$$= \int_{x=1}^{\infty} f(x)\,dx = \int_{x=1}^{\infty} 0\cdot dx = 0$$

iii) By definition of Distribution function we have

$$F(X) = P(X\le x) = \int_{x=0}^{x} f(x)\,dx$$

$$= 3\cdot\left(\frac{x^3}{3}\right)_{x=0}^{x}$$

$$F(x) = P(x\le x) = x^3 - 0 = x^3$$

$\therefore$ Distribution function of Random variable x is

$$F(x) = P(x\le x) = \begin{cases} x^3 & \text{if } 0<x<1 \\ 0 & \text{if otherwise} \end{cases}$$

IV) By def of mean

$$\mu = E(X) = \int_{x=-\infty}^{\infty} x\, f(x)\,dx$$

$$= \int_{x=-\alpha}^{0} x \cdot f(x)\, dx + \int_{x=0}^{1} x \cdot f(x)\, dx + \int_{x=1}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{x=-\alpha}^{0} 0 \cdot dx + \int_{x=0}^{1} x \cdot (3x^2)\, dx + \int_{x=1}^{\infty} x(0)\, dx$$

$$= 0 + \int_{x=0}^{1} 3x^3\, dx + 0$$

$$E(x) = 3\left(\frac{x^4}{4}\right)_{x=0}^{1} = 3\left(\frac{(1)^4}{4} - \frac{(0)^4}{4}\right) = \frac{3}{4}$$

By definition of variance

$$V(X) = E(x^2) - (E(x))^2$$

Now $$E(x^2) = \int_{x=-\alpha}^{\infty} x^2 \cdot f(x)\, dx$$

$$= \int_{x=-\alpha}^{0} x^2 \cdot f(x)\, dx + \int_{x=0}^{1} x^2 \cdot f(x)\, dx + \int_{x=1}^{\infty} x^2 \cdot f(x)\, dx$$

$$= 0 + \int_{x=0}^{1} x^2 \cdot 3x^2 \cdot dx + 0$$

$$E(x^2) = 3 \cdot \int_{x=0}^{1} x^4 \cdot dx = 3 \cdot \left(\frac{x^5}{5}\right)_0^1 = 3\left(\frac{(1)^5}{5} - \frac{(0)^5}{5}\right) = \frac{3}{5}$$

Now $V(X) = E(x^2) - (E(x))^2 = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{3}{5} - \frac{9}{16} = \frac{48-45}{80} = \frac{3}{80}$

2. For continuous Random variable x whose probability density function is defined by $f(x) = \begin{cases} cx(2-x) & \text{if } 0 < x \leq 2 \\ 0 & \text{if otherwise} \end{cases}$ then find constant "c" and also find mean, variance, distribution function

Sol: Given probability density function of the continuous Random variable x is $f(x) = \begin{cases} c \cdot x(2-x) & \text{if } 0 < x < 2 \\ 0 & \text{if otherwise } -\infty < x < 0 \text{ and } 2 < x < \infty \end{cases}$

We know that total probability = 1

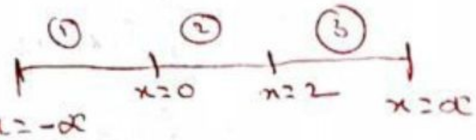
$$\int_{x=-\infty}^{\infty} f(x)\, dx = 1 \quad [\because x \text{ is continuous Random variable}]$$

$$\int_{x=-\alpha}^{0} f(x)\, dx + \int_{x=0}^{2} f(x)\, dx + \int_{x=2}^{\infty} f(x)\, dx = 1$$

$$\int_{x=-\infty}^{0} 0 \cdot dx + \int_{x=0}^{2} cx(2-x)\, dx + \int_{x=2}^{\infty} 0 \cdot dx = 1$$

$$0 + c \cdot \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)_{x=0}^{2} + 0 = 1$$

$$c\left[\left(4 - \frac{8}{3}\right) - \left(0^2 - \frac{0^3}{3}\right)\right] = 1$$



$$c\left(\frac{4}{3}\right) = 1$$

$$\Rightarrow c = 3/4$$

Therefore the probability density function of the continuous Random variable "x" when $c = 3/4$ is

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & \text{if } 0 < x \leq 2 \\ 0 & \text{if } -\infty < x < 0 \text{ and } 2 < x < \infty \end{cases}$$

By the definition of mean we have

$$E(x) = \int_{x=-\infty}^{\infty} x \cdot f(x)\, dx. \qquad [\because x \text{ is continuous R.V}]$$

$$= \int_{x=-\infty}^{0} x \cdot f(x)\, dx + \int_{x=0}^{2} x \cdot f(x)\, dx + \int_{x=2}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x(0)\, dx + \int_{x=0}^{2} x \frac{3}{4}x(2-x)\, dx + \int_{x=2}^{\infty} x(0)\, dx$$

$$= 0 + \frac{3}{4}\int_{x=0}^{2} (2x^2 - x^3)\, dx + 0$$

$$= \frac{3}{4}\left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_{x=0}^{2}$$

$$= \frac{3}{4}\left[\left(\frac{2}{3}(2)^3 - \frac{(2)^4}{4}\right) - \left(\frac{2}{3}(0)^3 - \frac{(0)^4}{4}\right)\right]$$

$$E(x) = \frac{3}{4}\left[\frac{16}{3} - \frac{16}{4}\right] = \frac{3}{4}\left[\frac{64-48}{12}\right] = \frac{3}{\cancel{4}} \times \frac{\cancel{16}^{4}}{\cancel{12}} = 1$$

Now

$$E(x^2) = \int_{x=-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x^2 f(x)\, dx + \int_{x=0}^{2} x^2 f(x)\, dx + \int_{x=2}^{\infty} x^2 f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x^2 (0)\, dx + \int_{x=0}^{2} x^2 \frac{3}{4}x(2-x)\, dx + \int_{x=2}^{\infty} x^2 (0)\, dx$$

$$= 0 + \frac{3}{4}\int_{x=0}^{2} (2x^3 - x^4)\, dx + 0$$

$$= \frac{3}{4}\left(2\frac{x^4}{4} - \frac{x^5}{5}\right)_{x=0}^{2}$$

$$= \frac{3}{4}\left(\left(\frac{2^4}{2} - \frac{2^5}{5}\right) - (0-0)\right)$$

$$= \frac{3}{4}\left(\frac{16}{2} - \frac{32}{5}\right)$$

$$= \frac{3}{4}\left(\frac{80-64}{10}\right)$$

$$E(X^2) = \frac{3}{4}\left(\frac{16}{10}\right) = \frac{6}{5}$$

Now

variance $V(X) = E(X^2) - (E(X))^2$

$$= \frac{6}{5} - (1)^2$$

$$= \frac{6-5}{5}$$

$$V(X) = \sigma_X^2 = \frac{1}{5}$$

We know that $\frac{d}{dx}F(x) = f(x)$

$$\Rightarrow F(x) = \int f(x)\,dx$$

$$F(x) = P(X \le x) = \int_{x=0}^{x} f(x)\,dx$$

$$= \int_{x=0}^{x} \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\cdot\left(2\cdot\frac{x^2}{2} - \frac{x^3}{3}\right)_{x=0}^{x}$$

$$= \frac{3}{4}\cdot\left[\left(x^2 - \frac{x^3}{3}\right) - (0-0)\right]$$

$$= \frac{3}{4}\cdot\left(\frac{3x^2 - x^3}{3}\right)$$

$$F(x) = P(X \le x) = \frac{3x^2 - x^3}{4}$$

The distribution function is $F(x) = \begin{cases} \frac{1}{4}(3x^2 - x^3) & \text{if } 0 < x < 2 \\ 0 & \text{if otherwise} \end{cases}$

...w

H.W

3. Suppose a continuous Random variable x has the probability density function $f(x) = K(1-x^2)$ for $0<x<1$ and $f(x)=0$ otherwise then find constant k, mean and variance.

Sol: Given probability density function

$$f(x) = \begin{cases} K(1-x^2) & \text{for } 0<x<1 \\ 0 & \text{for otherwise} \end{cases}$$

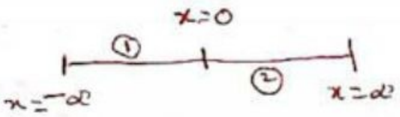
4. A continuous Random variable x has the probability density function $f(x) = Kxe^{-\lambda x}$ when $x \geq 0$, $\lambda > 0$ then find k, mean and variance

Sol: given probability density function of the continuous Random variable x is

$$f(x) = \begin{cases} Kx\,e^{-\lambda x} & \text{when } 0 \leq x \leq \infty \ (x \geq 0) \\ 0 & \text{when } -\infty \leq x \leq 0 \end{cases}$$

we know that total probability $= 1$

$$\int_{x=-\infty}^{\infty} f(x)\, dx = 1$$



$$\int_{x=-\infty}^{0} f(x)\, dx + \int_{x=0}^{\infty} f(x)\, dx = 1$$

$$\int_{x=-\infty}^{0} 0 \cdot dx + \int_{x=0}^{\infty} K\, x \cdot e^{-\lambda x}\, dx = 1$$

$$0 + K \int_{x=0}^{\infty} \underset{f}{x} \cdot \underset{g}{e^{-\lambda x}}\, dx = 1 \quad [\because \text{by ILATE}]$$

$$\boxed{\int f \cdot g\, dx = fg_1 - f'g_2 + f''g_3 - f'''g_4 + \cdots}$$

$$K\left[x\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 1 \cdot \left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right) + 0 \right]_{x=0}^{\infty} = 1$$

$$K \cdot \left[e^{-\lambda x} \left[\frac{-x}{\lambda} - \frac{1}{\lambda^2} \right] \right]_{x=0}^{\infty} = 1$$

$$K \cdot \left[e^{-\infty}\left(\frac{-\infty}{\lambda} - \frac{1}{\lambda^2}\right) - e^{0} \cdot \left(\frac{-0}{\lambda} - \frac{1}{\lambda^2}\right) \right] = 1 \quad \left[\begin{matrix} \because e^{-\infty} = 0 \\ e^{\infty} = \infty \end{matrix} \right]$$

$$K\left(0 + \frac{1}{\lambda^2}\right) = 1$$

$$k = \lambda^2$$

$\therefore$ The probability density function of Random variable x when $K = \lambda^2$ is

$$f(x) = \begin{cases} \lambda^2 x \cdot e^{-\lambda x} & \text{when } 0 \leq x \leq \infty \ (x \geq 0) \\ 0 & \text{when } -\infty \leq x \leq 0 \end{cases}$$

By the definition of mean $\mu = E(x) = \int_{x=-\infty}^{\infty} x \cdot f(x)\, dx$ [$\because x$ is continuous]

$$= \int_{x=-\infty}^{0} x \cdot f(x)\, dx + \int_{x=0}^{\infty} x \cdot f(x)\, dx$$

$$\boxed{\int f \cdot g\, dx = f g_1 - f' g_2 + f'' g_3 - \cdots}$$

$$= \int_{x=-\infty}^{0} x(0)\, dx + \int_{x=0}^{\infty} x \cdot \lambda^2 \cdot x \cdot e^{-\lambda x} \cdot dx$$

$$= 0 + \lambda^2 \int_{x=0}^{\infty} \underset{f}{x^2} \cdot \underset{g}{e^{-\lambda x}} \cdot dx \quad [\because \text{by ILATE}]$$

$$= \lambda^2 \cdot \left[x^2 \left(\frac{e^{-\lambda x}}{-\lambda}\right) - (2x) \cdot \left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right) + 2 \cdot \left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)(-\lambda)}\right)\right]_{x=0}^{\infty}$$

$$= \lambda^2 \left[e^{-\lambda x} \left[\frac{-x^2}{\lambda} - \frac{2x}{\lambda^2} - \frac{2}{\lambda^3} \right]\right]_{x=0}^{\infty}$$

$$= \lambda^2 \left[e^{-\infty} \left(\frac{-\infty^2}{\lambda} - \frac{2\infty}{\lambda^2} - \frac{2}{\lambda^3} \right) - e^0 \left(\frac{-0}{\lambda} - \frac{2(0)}{\lambda^2} - \frac{2}{\lambda^3} \right)\right]$$

$$= \lambda^2 \left[0 \left(-\frac{\infty}{\lambda} - \frac{\infty}{\lambda^2} - \frac{2}{\lambda^3} \right) - 1 \left(0 - 0 - \frac{2}{\lambda^3} \right)\right]$$

$$= \lambda^2 \left(\frac{2}{\lambda^3} \right)$$

$$\mu = E(x) = 2/\lambda$$

By the definition of variance we know that

$$V(x) = \sigma_x^2 = E(x^2) - (E(x))^2$$

Now

$$E(x^2) = \int_{x=-\infty}^{\infty} x^2 \cdot f(x)\, dx \quad [\because x \text{ is continuous}]$$

$$= \int_{x=-\infty}^{0} x^2 \cdot f(x)\, dx + \int_{x=0}^{\infty} x^2 \cdot f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x^2(0)\, dx + \int_{x=0}^{\infty} x^2 (\lambda^2 \cdot x \cdot e^{-\lambda x})\, dx$$

$$= 0 + \lambda^2 \int_{x=0}^{\infty} \underset{f}{x^3} \cdot \underset{g}{e^{-\lambda x}} \cdot dx \quad [\because \text{by ILATE}]$$

$$= \lambda^2 \cdot \left[x^3 \cdot \left(\frac{e^{-\lambda x}}{-\lambda}\right) - (3x^2)\left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right) + 6x \left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)(-\lambda)}\right) - 6 \cdot \frac{e^{-\lambda x}}{(-\lambda)(-\lambda)(-\lambda)(-\lambda)}\right]_{x=0}^{\infty}$$

$$= \lambda^2 \left[e^{-\lambda x} \left(\frac{-x^3}{\lambda} - \frac{3x^2}{\lambda^2} - \frac{6x}{\lambda^3} - \frac{6}{\lambda^4} \right)\right]_{x=0}^{\infty}$$

$$= \lambda^2 \left[e^{-\infty} \left(\frac{-\infty}{\lambda} - \frac{\infty}{\lambda^2} - \frac{\infty}{\lambda^3} - \frac{6}{\lambda^4} \right) - e^{0} \left(0 - 0 - 0 - \frac{6}{\lambda^4} \right)\right]$$

$$= \lambda^2 \left(0 + \frac{6}{\lambda^4} \right)$$

$$E(x^2) = \frac{6}{\lambda^2}$$

Now $V(x) = E(x^2) - \{E(x)\}^2$

$$= \frac{6}{\lambda^2} - \left(\frac{2}{\lambda}\right)^2$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2}$$

$$V(x) = \frac{2}{\lambda^2}$$

5. A continuous Random variable x has the probability density function $f(x) = c \cdot e^{-|x|}$ if $-\infty \le x \le \infty$ show that $c = \frac{1}{2}$ and find the probability that the variant lies between 0 and 4 also find mean and variance

Sol: given probability density function

$$f(x) = c \cdot e^{-|x|} \text{ if } -\infty \le x \le \infty$$

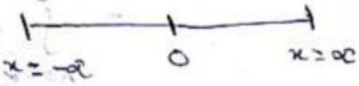
$$= c \cdot e^{-(\pm x)} \text{ if } -\infty \le x \le \infty$$

$$= \begin{cases} c \cdot e^{-(-x)} & \text{if } -\infty \le x < 0 \\ c \cdot e^{-(+x)} & \text{if } 0 \le x \le \infty \end{cases}$$

$$f(x) = c e^{-|x|} = \begin{cases} c \cdot e^{x} & \text{if } -\infty \le x \le 0 \\ c \cdot e^{-x} & \text{if } 0 \le x \le \infty \end{cases}$$

We know that total probability $= 1$

$$\int_{x=-\infty}^{\infty} f(x) \cdot dx = 1$$



$$\Rightarrow \int_{x=-\infty}^{\infty} c \cdot e^{-|x|} \cdot dx = 1$$

$$\Rightarrow 2 \cdot \int_{x=0}^{\infty} c \cdot e^{-|x|} \cdot dx = 1 \quad [\because c\, e^{-|x|} \text{ is even function}$$

$$\Rightarrow 2 \cdot \int_{x=0}^{\infty} c \cdot e^{-x} \, dx = 1$$

$$2c \cdot \left(\frac{e^{-x}}{-1}\right)_{x=0}^{\infty} = 1$$

$-2c \cdot \left[e^{-\alpha} - e^{0}\right] = 1$

$-2c(0-1) = 1$

$2c = 1$

$c = 1/2$

The probability of variant lies between 0 and 4 is

$$P(0 \le x \le 4) = \int_{x=0}^{4} f(x)\, dx$$

$$= \int_{x=0}^{4} c \cdot e^{-|x|} \cdot dx$$

$$= \int_{x=0}^{4} c \cdot e^{-(x)} dx$$

$$= \int_{x=0}^{4} \frac{1}{2} \cdot e^{-x} \cdot dx$$

$$= \frac{1}{2} \cdot \left(\frac{e^{-x}}{-1}\right)_{x=0}^{4}$$

$$= -\frac{1}{2} \cdot (e^{-4} - e^{+0})$$

$$= -\frac{1}{2} \cdot (e^{-4} - 1)$$

$$P(0 \le x \le 4) = \frac{1}{2}(1 - e^{-4})$$

By definition of mean we know that $E(x) = \int_{x=-\infty}^{\infty} x \cdot f(x) \cdot dx$

$$= \int_{x=-\infty}^{\infty} x \cdot c \cdot e^{-|x|} \cdot dx$$

$$= c \int_{x=-\infty}^{\infty} x \cdot e^{-|x|} \cdot dx$$

$$= c(0) \qquad \left[\because x \cdot e^{-|x|} \text{ is Odd function } = 0\right]$$

$$E(x) = 0$$

By the definition of variance we know that

$$V(x) = \sigma_x^2 = E(x^2) - (E(x))^2$$

Now $E(x^2) = \int_{x=-\infty}^{\infty} x^2 \cdot f(x)\, dx \qquad [\because x \text{ is continuous}]$

$$= \int_{x=-\infty}^{\infty} x^2 \cdot c \cdot e^{-|x|} \cdot dx$$

$$= c \cdot 2 \int_{0}^{\infty} x^2 \cdot e^{-|x|} \cdot dx \qquad \left[x^2 e^{-|x|} \text{ is even}\right]$$

$$= 2c \int_0^{\infty} \underset{f}{x^2} \cdot \underset{g}{e^{-x}} \cdot dx \qquad [\because \text{by ILATE}]$$

$$= 2c \left[x^2 \left(\frac{e^{-x}}{-1} \right) - 2x \cdot \frac{e^{-x}}{(-1)(-1)} + 2 \cdot \frac{e^{-x}}{(-1)(-1)(-1)} \right]_{x=0}^{\infty}$$

$$= 2c \left[-x^2 \cdot e^{-x} - 2x \cdot e^{-x} - 2 \cdot e^{-x} \right]_{x=0}^{\infty}$$

$$= 2c \left[e^{-x} (-x^2 - 2x - 2) \right]_{x=0}^{\infty}$$

$$= 2c \left[e^{-\infty} (-(\infty)^2 - 2(\infty) - 2) - e^{-0} (-0^2 - 2(0) - 2) \right]$$

$$= 2c \left[0(-\infty^2 - 2\infty - 2) - 1(-2) \right]$$

$$= 2c(2)$$

$$E(X^2) = 4c = 4 \cdot \frac{1}{2} \quad \left[\because c = \frac{1}{2}\right]$$

$$E(X^2) = 2$$

Now $V(X) = E(X^2) - (E(X))^2 = 2 - (0)^2 = 2$. $\therefore$ variance $V(X) = \sigma_x^2 = 2$

H.W

6. A continuous Random variable x has the probability density function $f(x) = kx^2 \cdot e^{-x}$ where $x \geq 0$ then find

i) constant k ii) Mean and variance

⑦ The probability density function of Random variable x is

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if otherwise.} \end{cases}$$

Find mean and probability of the variant lies between 0 and $\frac{\pi}{2}$.

Sol: Given probability density function

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if otherwise (or) } -\infty \le x \le 0,\ \pi \le x \le \infty \end{cases}$$

By the def of mean we know that

$$E(x) = \mu = \int_{x=-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x\, f(x)\, dx + \int_{x=0}^{\pi} x\, f(x)\, dx + \int_{x=\pi}^{\infty} x\, f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x(0)\, dx + \int_{x=0}^{\pi} x \cdot \frac{1}{2}\sin x \cdot dx + \int_{x=\pi}^{\infty} x(0)\, dx$$

$$= 0 + \frac{1}{2}\int_{x=0}^{\pi} \underset{f}{x} \cdot \underset{g}{\sin x}\, dx + 0 \qquad [\because \text{ by ILATE}$$

$$= \frac{1}{2}\cdot\left[x(-\cos x) - (1)(-\sin x)\right]_{x=0}^{\pi}$$

$$= \frac{1}{2}\left[-x\cdot\cos x + \sin x\right]_{x=0}^{\pi}$$

$$= \frac{1}{2}\left[(-\pi\cos\pi + \sin\pi) - (0 + \sin 0)\right]$$

$$= \frac{1}{2}\left[(-\pi(-1) + 0) - (0+0)\right]$$

$$= \frac{1}{2}\cdot\pi$$

$$= \frac{\pi}{2}$$

The probability of variant lies between 0 and $\frac{\pi}{2}$

$$P(0 \le x \le \tfrac{\pi}{2}) = \int_{x=0}^{\pi/2} f(x)\cdot dx \qquad [\because x \text{ is continuous}]$$

$$= \int_{x=0}^{\pi/2} \frac{1}{2}\cdot\sin x\cdot dx$$

$$= \frac{1}{2}\left[-\cos x\right]_{x=0}^{\pi/2}$$

$$= -\frac{1}{2}\cdot\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$P(0 \le x \le \tfrac{\pi}{2}) = -\frac{1}{2}(0-1) = \frac{1}{2}$$

8) A continuous Random variable x has the probability density function $f(x) = \begin{cases} ax & \text{if } 0 < x < 1 \\ a & \text{if } 1 \le x < 2 \\ -ax + 3a & \text{if } 2 < x < 3 \\ 0 & \text{if otherwise} \end{cases}$

Determine

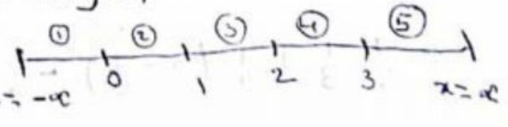
i) Constant "a" ii) Expected value of x

Sol: given probability density function

$$f(x) = \begin{cases} ax & \text{if } 0 < x < 1 \\ a & \text{if } 1 \le x < 2 \\ -ax + 3a & \text{if } 2 < x < 3 \\ 0 & \text{if } -\infty \le x \le 0,\ 3 \le x \le \infty \end{cases}$$

i) we know that total probability $= 1$

$$\int_{x=-\infty}^{\infty} f(x)\, dx = 1$$



$$\Rightarrow \int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{1} f(x)\,dx + \int_{x=1}^{2} f(x)dx + \int_{x=2}^{3} f(x)\,dx + \int_{x=3}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{x=0}^{1} ax\cdot dx + \int_{x=1}^{2} a\cdot dx + \int_{x=2}^{3} (-ax+3a)\,dx + 0 = 1$$

$$\Rightarrow a\cdot\left(\frac{x^2}{2}\right)_0^1 + a(x)_{x=1}^{2} + \left(-a\cdot\frac{x^2}{2} + 3a\cdot x\right)_{x=2}^{3} = 1$$

$$\Rightarrow a\cdot\frac{1}{2} + a(2-1) - a\cdot\frac{9}{2} + 9a + \frac{a}{2}\cdot 4 - 6a) = 1$$

$$\Rightarrow \frac{a}{2} + a - \frac{9a}{2} + 9a + 2a - 6a = 1$$

$$\Rightarrow a + 2a - 9a + 18a + 4a - 12a = 2$$

$$\Rightarrow 4a = 2$$

$$\Rightarrow a = 1/2$$

The probability density function of the Random variable x when $a = 1/2$ is

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } 0 < x < 1 \\ \frac{1}{2} & \text{if } 1 \le x < 2 \\ -\frac{x}{2} + \frac{3}{2} & \text{if } 2 < x < 3 \\ 0 & \text{if otherwise (or) } -\infty \le x \le 0,\ 3 \le x \le \infty \end{cases}$$

By the def of mean $E(x) = \int_{x=-\infty}^{\infty} x\cdot f(x)\cdot dx$

$$= \int_{x=-\infty}^{0} x\cdot f(x)\,dx + \int_{x=0}^{1} x\cdot f(x)\,dx + \int_{x=1}^{2} x\cdot f(x)\,dx + \int_{x=2}^{3} x\cdot f(x)dx + \int_{x=3}^{\infty} x\cdot f(x)dx$$

$$= \int_{x=-\infty}^{0} 0(x)dx + \int_{x=0}^{1} x\cdot\frac{x}{2}\cdot dx + \int_{x=1}^{2} x\cdot\frac{1}{2}dx + \int_{x=2}^{3} x\cdot\left(-\frac{x}{2}+\frac{3}{2}\right)dx + \int_{x=3}^{\infty} x(0)\,dx$$

$$= \frac{1}{2}\cdot\left(\frac{x^3}{3}\right)_{x=0}^{1} + \frac{1}{2}\left(\frac{x^2}{2}\right)_{x=1}^{2} + \left[-\frac{1}{2}\left(\frac{x^3}{3}\right)_2^3 + \frac{3}{2}\cdot\left(\frac{x^2}{2}\right)_2^3\right] + 0$$

$$= \frac{1}{2}\left(\frac{1}{3}-0\right) + \frac{1}{2}\cdot\left(2-\frac{1}{2}\right) - \frac{1}{2}\left(\frac{27}{3} - \frac{8}{3}\right) + \frac{3}{2}\left(\frac{9}{2} - \frac{4}{2}\right)$$

$$= \frac{1}{6} + \frac{1}{2}\left(\frac{4-1}{2}\right) - \frac{1}{2}\left(\frac{19}{3}\right) + \frac{3}{2}\left(\frac{5}{2}\right)$$

$$= \frac{1}{6} + \frac{3}{4} - \frac{19}{6} + \frac{15}{4}$$

$$= \frac{2+9-38+45}{12} = \frac{56-38}{12} = \frac{\cancel{18}^{\,3}}{\cancel{12}_{\,2}}$$

$$E(x) = \frac{3}{2}$$

(9) A continuous Random variable x has the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \le 1 \\ K(x-1)^4 & \text{if } 1 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

then find probability density function and constant k.

Sol: given distribution function

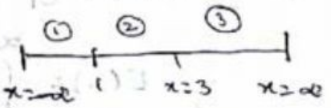
$$F(x) = \begin{cases} 0 & \text{if } x \le 1 \\ K(x-1)^4 & \text{if } 1 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

By the definition of distribution function we know that

$$\frac{d}{dx} F(x) = f(x)$$

The probability density function

$$f(x) = \frac{d}{dx} F(x)$$



$$= \frac{d}{dx} \begin{cases} 0 & \text{if } x \le 1 \\ K(x-1)^4 & \text{if } 1 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

$$= \begin{cases} 0 & \text{if } x \le 1 \\ K\,4(x-1)^3 & \text{if } 1 < x \le 3 \\ 0 & \text{if } x > 3 \end{cases}$$

$$f(x) = \begin{cases} 4K(x-1)^3 & \text{if } 1 \le x \le 3 \\ 0 & \text{if otherwise } -\infty \le x < 1,\ 3 \le x \le \infty \end{cases}$$

we know that $\displaystyle\int_{x=-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{x=-\infty}^{1} f(x)\,dx + \int_{x=1}^{3} f(x)\,dx + \int_{x=3}^{\infty} f(x)\,dx = 1$$

$$\int_{x=-\alpha}^{1} 0 \cdot dx + \int_{x=1}^{3} 4k(x-1)^3 \cdot dx + \int_{x=3}^{\alpha} 0 \cdot dx = 1$$

$$4k \cdot \left(\frac{(x-1)^4}{4}\right)_{x=1}^{3} = 1$$

$$k\left[(3-1)^4 - (1-1)^4\right] = 1$$

$$k \cdot 16 = 1$$

$$\Rightarrow k = 1/16$$

Result:-

If x is a continuous Random variable and a, b are constants then i) $E(a) = a$

ii) $E(aX+b) = a \cdot E(X) + b$ iv) $V(a) = 0$

iii) $V(aX+b) = a^2 \cdot V(X)$

Proof:-

Let X be a continuous Random variable and a, b be any constants so $Y = aX+b$ is also a Random variable

By definition of mean (Expectation) we know that

$$E(X) = \int_{x=-\alpha}^{\alpha} x \cdot f(x)\, dx \quad [\because X \text{ is } c.R.v]$$

i) Now $E(a) = \int_{x=-\alpha}^{\alpha} a \cdot f(x)\, dx$

$$= a \cdot \int_{x=-\alpha}^{\alpha} f(x)\, dx \quad \left[\because \int_{x=-\alpha}^{\alpha} f(x)\, dx = 1\right]$$

$$E(a) = a(1) = a$$

ii) $E(ax+b) = \int_{x=-\alpha}^{\alpha} (ax+b) f(x)\, dx$

$$= \int_{x=-\alpha}^{\alpha} (ax \cdot f(x) + b \cdot f(x))\, dx$$

$$= a \cdot \int_{x=-\alpha}^{\alpha} x \cdot f(x)\, dx + b \cdot \int_{x=-\alpha}^{\alpha} f(x)\, dx$$

$= a \cdot E(X) + b(1)$

$E(aX+b) = a \cdot E(X) + b$

iii) By definition of variance

$$V(X) = E(X^2) - (E(X))^2$$

Now $V(aX+b) = E((aX+b)^2) - (E(aX+b))^2$

$= E(a^2X^2 + b^2 + 2abX) - (a \cdot E(X) + b)^2$ [∵ From ii]

$= E(a^2X^2) + E(b^2) + E(2abX) - (a^2 (E(X))^2 + b^2 + 2ab \cdot E(X))$

$= a^2 \cdot E(X^2) + \cancel{b^2} + \cancel{2ab \cdot E(X)} - a^2 \cdot (E(X))^2 - \cancel{b^2} - \cancel{2ab \cdot E(X)}$

$= a^2 (E(X^2) - (E(X))^2)$

$V(aX+b) = a^2 \cdot V(X)$

iv) $V(a) = E(a^2) - (E(a))^2$

$= a^2 - (a)^2$

$= a^2 - a^2$

$V(a) = 0$

10) The probability density function of the continuous Random variable x is $f(x) = \begin{cases} \frac{1}{2}\sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if otherwise} \end{cases}$ then find

i) Mean ii) variance iii) $E(X^2+2X)$ iv) $V(2X+1)$

Sol: given probability density function

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if otherwise } -\infty < x < 0, \pi \le x \le \infty \end{cases}$$

By the definition of mean we know that

$$E(X) = \mu = \int_{x=-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{x=-\infty}^{0} x \cdot f(x)\, dx + \int_{x=0}^{\pi} x \cdot f(x)\, dx + \int_{x=\pi}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{x=-\infty}^{0} 0 \cdot dx + \int_{x=0}^{\pi} x \cdot \frac{1}{2} \cdot \sin x \cdot dx + \int_{x=\pi}^{\infty} 0 \cdot dx.$$

$$= 0+\frac{1}{2}\cdot\int_{x=0}^{\pi} \underset{f}{x}\cdot \underset{g}{\sin x}\cdot dx + 0 \qquad [\because \text{ by ILATE}$$

$$= \frac{1}{2}\left[x(-\cos x) - (1)(-\sin x)\right]_{x=0}^{\pi}$$

$$= \frac{1}{2}\left[-x\cos x + \sin x\right]_{x=0}^{\pi}$$

$$= \frac{1}{2}\left[(-\pi\cos\pi + \sin\pi) - (-0\cos 0 + \sin 0)\right]$$

$$E(x) = \frac{1}{2}\left[-\pi(-1) + 0 - 0 - 0\right] = \frac{\pi}{2}$$

By the definition of variance we know that

$$V(x) = E(x^2) - (E(x))^2.$$

Now $$E(x^2) = \int_{x=-\infty}^{\infty} x^2\cdot f(x)\,dx$$

$$= \int_{x=-\infty}^{0} x^2\cdot f(x)\,dx + \int_{x=0}^{\pi} x^2\cdot f(x)\,dx + \int_{x=\pi}^{\infty} x^2\cdot f(x)\,dx$$

$$= 0 + \int_{x=0}^{\pi} \underset{f}{x^2}\cdot \frac{1}{2}\cdot \underset{g}{\sin x}\cdot dx + 0 \qquad [\because \text{ ILATE}$$

$$= \frac{1}{2}\cdot\left[x^2\cdot(-\cos x) - 2x\cdot(-\sin x) + 2\cdot(\cos x)\right]_{x=0}^{\pi}$$

$$= \frac{1}{2}\left[(-\pi^2(\cos\pi) + 2\pi\sin\pi + 2\cos\pi) - (0\cos 0 + 2(0)\sin 0 + 2(\cos 0))\right]$$

$$= \frac{1}{2}\left[-\pi^2(-1) + 2\pi(0) + 2(-1) - 0 + 0 - 2\right]$$

$$= \frac{1}{2}\left(\pi^2 - 2 - 2\right)$$

$$E(x^2) = \frac{\pi^2 - 4}{2} \qquad \boxed{\int f\cdot g\,dx = fg_1 - f'g_2 + f''g_3 - f'''g_4 \cdots\cdots}$$

Now $$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{\pi^2 - 4}{2} - \left(\frac{\pi}{2}\right)^2$$

$$= \frac{\pi^2}{2} - 2 - \frac{\pi^2}{4}$$

$$= \frac{2\pi^2 - 8 - \pi^2}{4}$$

$$V(x) = \frac{\pi^2 - 8}{4}$$

iii)

$$E(x^2 + 2X) = E(x^2) + E(2X)$$

$$= E(x^2) + 2\cdot E(x)$$

$$= \frac{\pi^2-4}{2} + 2\cdot\frac{\pi}{2}$$

$$= \frac{\pi^2-4+2\pi}{2}$$

iv) $V(2X+1) = V(2X) + V(1)$

$$= 2^2 \cdot V(X) + 0$$

$$= 4 \cdot \left(\frac{\pi^2-8}{4}\right)$$

$$V(2x+1) = \pi^2 - 8$$

(11) Let $f(x) = 3x^2$ when $0 \le x \le 1$ be the probability density function of a continuous Random variable x. Determine a and b such that i) $P(x \le a) = P(x > a)$ (ii) $P(x > b) = 0.05$

Sol:- The probability Density function

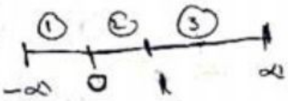
$$f(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{if } -\infty \le x \le 0 \text{ and } 1 \le x \le \infty \end{cases}$$

i) $P(x \le a) = P(x > a)$

$P(-\infty \le x \le a) = P(a < x < \infty)$ $\quad \int_{-\infty}^{0} + \int_{0}^{a} = \int_{a}^{1} + \int_{1}^{\infty}$

(or)

$P(x \le a) = 1 - P(x \le a)$



$$2\,P(x \le a) = 1$$

$$2\,P(-\infty \le x < a) = 1$$

$$2 \cdot \int_{x=-\infty}^{a} f(x)\,dx = 1$$

$$2\left[\int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{a} f(x)\cdot dx\right] = 1$$

$$2\left[\int_{x=-\infty}^{0} 0\cdot dx + \int_{x=0}^{a} 3x^2 \cdot dx\right] = 1$$

$$2\left[0 + 3\cdot\left(\frac{x^3}{3}\right)_{x=0}^{a}\right] = 1$$

$$a^3 - 0 = \frac{1}{2}$$

$$a = \left(\frac{1}{2}\right)^{1/3}$$

$$a = 0.7937$$

Now

$P(x>b)=0.05$

$P(b<x<\infty) = 0.05$

$$\int_{x=b}^{\infty} f(x)\,dx = 0.05$$

$$\int_{x=b}^{1} f(x)\,dx + \int_{x=1}^{\infty} f(x)\,dx = 0.05$$

$$\int_{x=b}^{1} 3x^2\,dx + 0 = 0.05$$

$$\left(\frac{3x^3}{3}\right)_{x=b}^{1} = 0.05$$

$1-b^3 = 0.05.$

$b^3 = 1-0.05$

$b^3 = 0.95$

$b = (0.95)^{1/3}$

$b = 0.98$

$\therefore\ a = 0.7937$ and $b = 0.98$

(P) The students in a class are selected at random one after the other for an examination. Find the probability that the boys and girls are alternate if there are

(i) 5 boys and 4 girls

(ii) 4 boys and 4 girls

Sol :-

(i) Total number of students in the class room

$= 5 \text{ boys} + 4 \text{ girls} = 9$

Here number of boys is morethan number of girls. So we can arrage the boys and girls alternatively (one after other) only in the form

BGBGBGBGB

The probability of arranging boys and girls are alternative is

$$P = \underset{B}{\frac{5c_1}{9c_1}} \cdot \underset{G}{\frac{4c_1}{8c_1}} \cdot \underset{B}{\frac{4c_1}{7c_1}} \cdot \underset{G}{\frac{3c_1}{6c_1}} \cdot \underset{B}{\frac{3c_1}{5c_1}} \cdot \underset{G}{\frac{2c_1}{4c_1}} \cdot \underset{B}{\frac{2c_1}{3c_1}} \cdot \underset{G}{\frac{1c_1}{2c_1}} \cdot \underset{B}{\frac{1c_1}{1c_1}}$$

$$= \frac{\cancel{5}}{\cancel{9}} \cdot \frac{\cancel{4}}{\cancel{8}_2} \cdot \frac{\cancel{4}^1}{7} \cdot \frac{\cancel{3}}{6} \cdot \frac{\cancel{3}}{\cancel{5}} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{1}$$

$$= \frac{1}{126}$$

(ii) Total number of student in the class room $= 4 \text{ boys} + 4 \text{ girls} = 8$

Here number of boys is eaual to number of girls, So, to arrange the boys and girls alternatively we can straits with either boys and girls.

we arrange the boys and girls in the form of BGBGBGBG (or) GBGBGBGB

The probability of arranging boys and girls are alternative is

$$P = BGBGBGBG + GBGBGBGB$$

$$= \frac{4c_1}{8c_1} \cdot \frac{4c_1}{7c_1} \cdot \frac{3c_1}{6c_1} \cdot \frac{3c_1}{5c_1} \cdot \frac{2c_1}{4c_1} \cdot \frac{2c_1}{3c_1} \cdot \frac{1c_1}{2c_1} \cdot \frac{1c_1}{1c_1}$$

$$+ \frac{4c_1}{8c_1} \cdot \frac{4c_1}{7c_1} \cdot \frac{3c_1}{6c_1} \cdot \frac{3c_1}{5c_1} \cdot \frac{2c_1}{4c_1} \cdot \frac{2c_1}{3c_1} \cdot \frac{1c_1}{2c_1} \cdot \frac{1c_1}{1c_1}$$

$$= \frac{\cancel{4}}{\cancel{8}} \cdot \frac{\cancel{4}}{7} \cdot \frac{\cancel{3}}{\cancel{6}} \cdot \frac{\cancel{3}}{5} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{1} + \frac{4}{\cancel{8}} \cdot \frac{4}{7} \cdot \frac{\cancel{3}}{\cancel{6}} \cdot \frac{\cancel{3}}{5} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{1}$$

$$= \frac{1}{70} + \frac{1}{70}$$

$$= \frac{1+1}{70}$$

$$= \frac{\cancel{2}}{\cancel{70}\,35}$$

$$= 1/35$$