CURVES USED IN ENGINEERING PRACTICE

5.1 INTRODUCTION

In engineering practice, the profiles of some of the objects contain regular curved features. Some are obtained as intersections, when a plane passes through a cone and some are obtained by tracing the locus of a point moving according to the mathematical relationship, applicable to that particular curve. The following types of curves are considered in this chapter:

Conic sections,
Cycloidal curves,
Involutes, and
Helices

5.2 CONIC SECTIONS

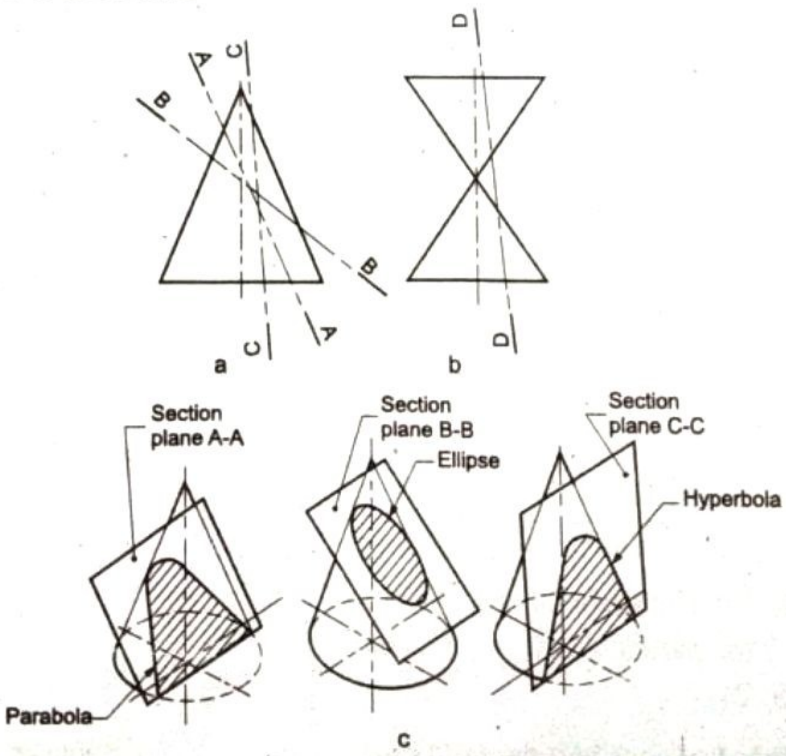

Fig. 5.1 Sections of a cone

The conic sections are the intersections of a right regular cone, by a cutting plane in different positions, relative to the axis of the cone. Considering the apex angle of the cone as 2θ, and the inclination of the cutting plane as α, the following are the possible conic sections (Fig. 5.1a).

5.2.1 Parabola

If the cutting plane angle α is equal to θ, i.e., when the section plane A-A i parallel to a generator of the cone, the curve of intersection is a parabola, which is not a close curve. The size of the parabola depends upon the distance of the section plane from the generator of the cone.

5.2.2 Ellipse

When a cone is cut by a section plane B-B at an angle α, which is more than half of the apex angle θ and less than 90°, the curve of intersection is an ellipse. The size of the ellipse depends upon the angle α and the distance of the section plane from the apex of the cone. Further, it may be noted that only when the section plane cuts all the generators of the cone, the elliptical section formed is a closed curve.

5.2.3 Hyperbola

If the angle α is less than θ (section plane C-C), the curve of intersection is a hyperbola. The curve of intersection is hyperbola, even if $\alpha=0$, i.e., section plane parallel to the axis, provided the section plane is not passing through the apex of the cone. If the section plane passes through the apex, the section produced is an isosceles triangle.

If a double cone is cut by a section plane D-D, on one side of the common axis (Fig. 5.1b), the curves of intersection result in two branches of hyperbola. However, if $\alpha=0$, the two branches of the curve will be symmetric in form.

Figure 5.1c shows the different conic sections, produced by the section planes A-A, B-B and C-C of Fig. 5.1a.

5.3 CONSTRUCTION OF CONICS - ECCENTRICITY METHOD

A conic section may be defined as the locus of a point moving in a plane such that, the ratio of its distance from a fixed point to a fixed straight line is always a constant.

The fixed point is called the focus and the fixed straight line, the directrix.

The ratio, $\dfrac{\text{distance of the point from the focus}}{\text{distance of the point from directrix}}$

is known as eccentricity, e. The value of e is less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola (Fig. 5. 2).

The line passing through the focus and perpendicular to the directrix is called the axis. The point at which the conic section intersects the axis is called its vertex.

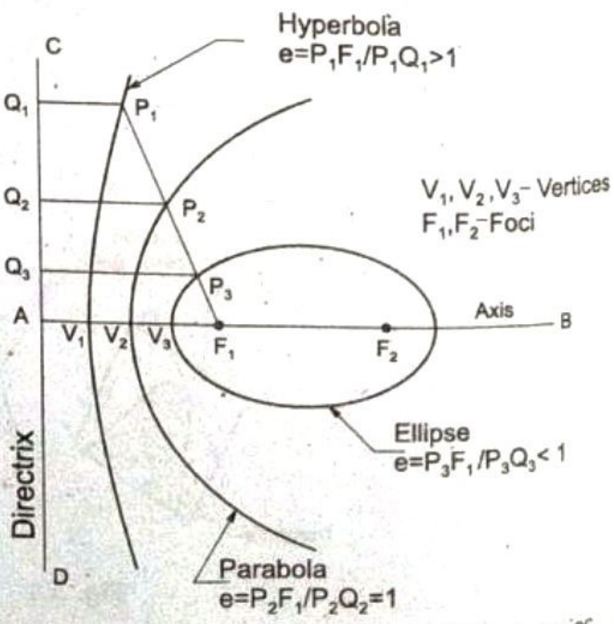

Fig. 5.2 Eccentricity values for different conics

Problem 1 *Construct a parabola, with the distance of the focus from the directrix as 50. Also, draw normal and tangent to the curve, at a point 40 from the directrix. (May/June 2010,2012, JNTU)*

NOTE A parabola is a curve traced by a point, moving such that, at any position, its distance from a fixed point (focus) is always equal to its distance from a fixed straight line (directrix).

Construction (Fig. 5.3)

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark the focus F on the axis such that, AF = 50.
3. Locate the vertex V on AB such that, AV = VF = 25.
4. Draw a line VE, perpendicular to AB such that, VE = VF.
5. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF}{VA} = 1$, the eccentricity.
6. Locate a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equi-distant.

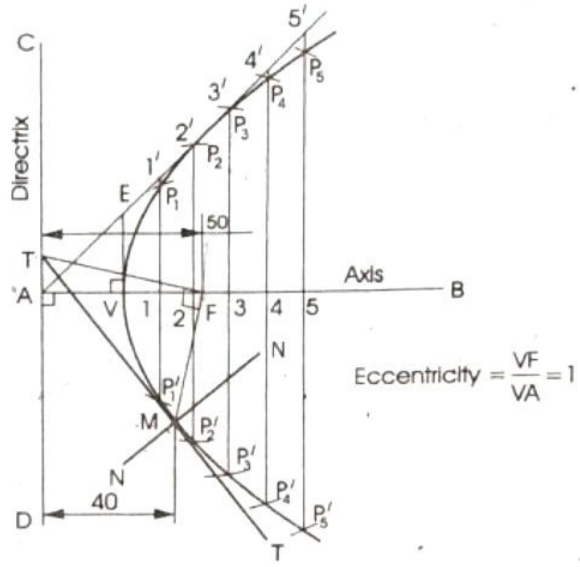

Fig. 5.3 Construction of parabola-Eccentricity method

7. Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at $1'$, $2'$, $3'$, etc.
8. With centre F and radius $1\text{-}1'$, draw arcs intersecting the line through 1 at P_1 and P_1'. P_1 and P_1' are the points on the parabola, because, the distance of P_1 (P_1') from F is $1\text{-}1'$ and from CD, it is A-1 and,

$$\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = 1$$

Similarly, locate the points P_2, P_2' ; P_3', P_3; etc., on either side of the axis.

9. Join the points by a smooth curve, forming the required parabola.

To draw tangent and normal to the parabola, locate the point M, which is at 40 from the directrix. Then, join M to F and draw a line through F, perpendicular to MF, meeting the directrix at T. The line joining T and M and extended (T-T) is the tangent and a line N-N, through M and perpendicular to T-T is the normal to the curve.

Problem 2 *Construct an ellipse, with distance of the focus from the directrix as 50 and eccentricity as 2/3. Also, draw normal and tangent to the curve at a point 40 from the directrix.*

(May/June 2008, 2010,2011, JNTU)

Construction (Fig. 5.4)

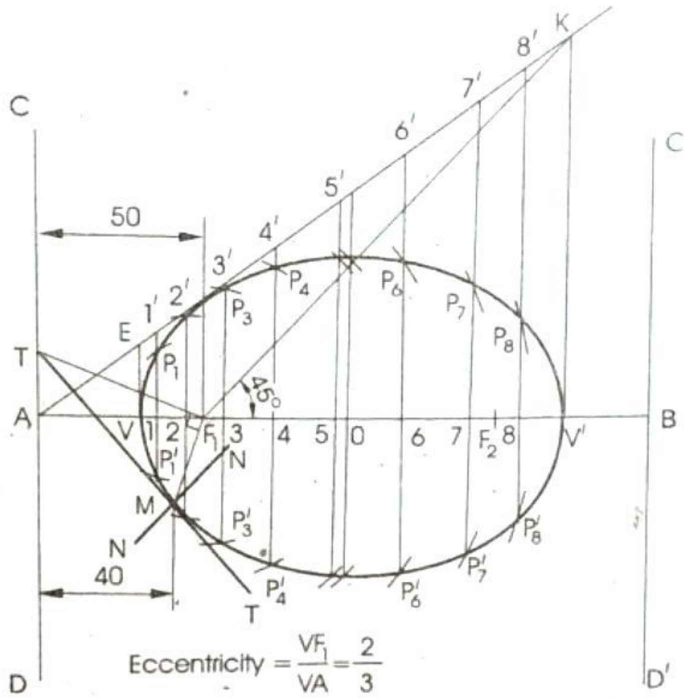

Fig. 5.4 Construction of ellipse-Eccentricity method

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark focus F_1 on the axis such that, $AF_1 = 50$.
3. Divide AF_1 into 5 equal parts.
4. Locate the vertex V on the third division point from A.
5. Draw a line VE, perpendicular to AB such that, $VE = VF_1$.
6. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$, the eccentricity.
7. Mark a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equidistant.
8. Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at 1′, 2′, 3′, etc.
9. With centre F_1 and radius 1-1′, draw arcs intersecting the line through 1 at P_1 and P_1'. P_1 and P_1' are the points on the ellipse, because the distance of P_1 from F_1 is 1-1′ and from CD, it is A-1 and, $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$, the eccentricity.
10. Similarly, locate the points P_2, P_2' ; P_3, P_3' ; etc., on either side of the axis.
11. Join the points by a smooth curve, forming the required ellipse.

NOTE (i) The ellipse is a closed curve and has two foci, two directrices and two vertices. To locate the other vertex V′, draw a line at 45° to the axis passing through F_1 and intersecting AE produced at K. A vertical line drawn from K meets the axis at V′.

(ii) To draw tangent and normal to the ellipse, the construction, similar to the one given for parabola may be followed.

(iii) The second focus F_2 may be located such that, $V' F_2 = VF_1$.

Problem 3 *Construct a hyperbola, with the distance between the focus and the directrix as 50 and eccentricity as 3/2. Also, draw normal and tangent to the curve at a point 30 from the directrix.* (Aug/Sep 2008, JNTU)

Construction (Fig. 5.5)

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark focus F on the axis such that, AF = 50.
3. Divide AF into 5 equal parts.
4. Locate the vertex V on the second division point from A.
5. Draw a line VE, perpendicular to AB such that, VE = VF.
6. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$, the eccentricity.
7. Repeat steps 6 to 8 of Construction: Fig. 5.3. P_1 and P_1' are the points on the hyperbola, because the distance of P_1 from F is 1-1′ and from CD, it is A-1, and $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$, the eccentricity.

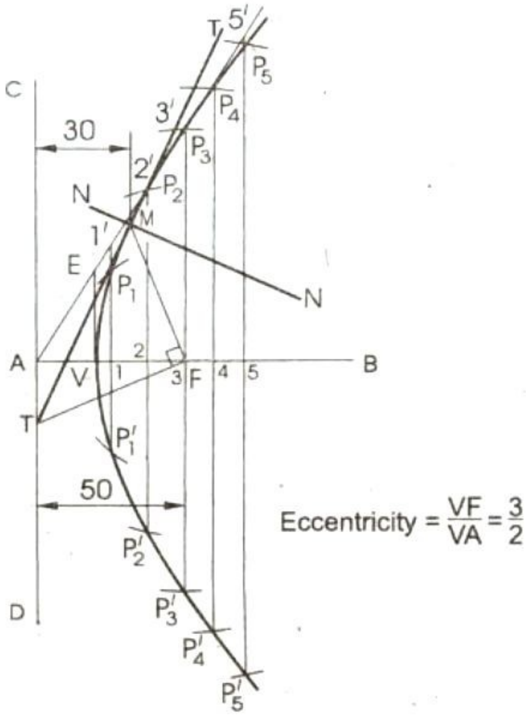

Fig. 5.5 Construction of hyperbola-Eccentricity method

8. Similarly, locate the points P_2, P_2' ; P_3, P_3'; etc., on either side of the axis.
9. Join the points by a smooth curve, forming the required hyperbola.

NOTE (i) A hyperbola is an open curve and has one focus and one directrix.

(ii) To draw tangent and normal to the hyperbola, the construction similar to the one given for parabola may be followed.

Rectangular hyperbola

It is a curve generated by a point which moves in such a way that the product of its distances from two fixed straight lines, the asymptotes at right angles to each other, is a constant.

Problem 20 *Construct a rectangular hyperbola, when a point P on it is at distances of 18 and 34 from two asymptotes. Also, draw a tangent to the curve at a point 20 from an asymptote.*

(May/June 2010, JNTU)

Construction (Fig. 5.28)

(i) Draw the asymptotes OA and OB at right angles to each other and locate the given point P.

(ii) Draw the lines CD and EF; passing through P and parallel to OA and OB respectively.

(iii) Locate a number of points 1,2,3, etc., along the line CD, which need not be equi-distant.

(iv) Join 1, 2, 3, etc., to O and extend if necessary, till these lines meet the line EF at points $1'$, $2'$, $3'$, etc.

(v) Draw lines through 1, 2, 3, etc., parallel to EF and through $1'$, $2'$, $3'$ etc., parallel to CD, to intersect at P_1, P_2, P_3, etc.

A smooth curve passing through these points is the required rectangular hyperbola.

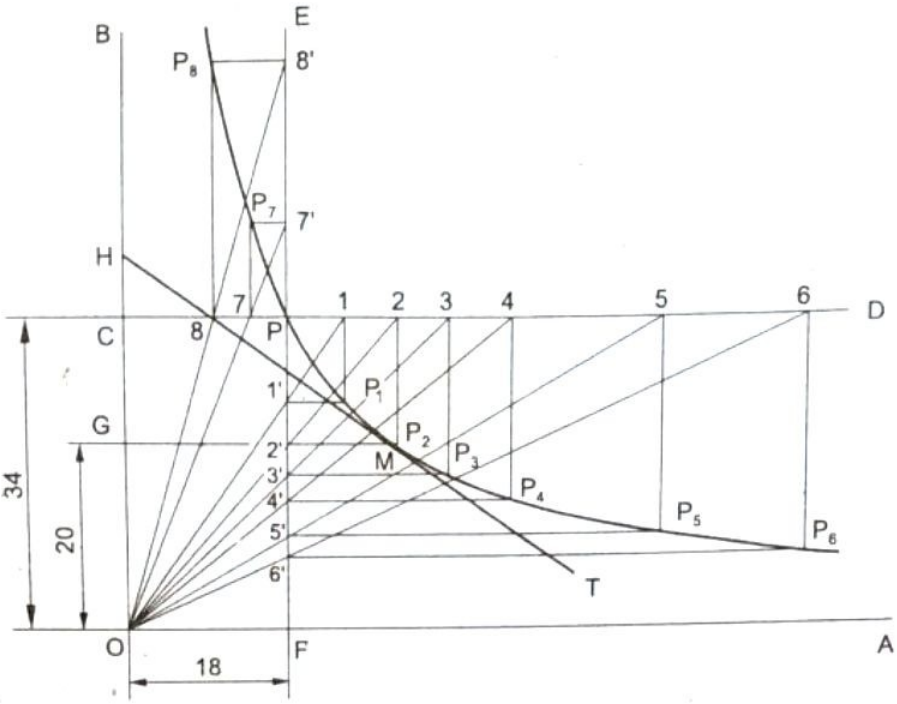

Fig. 5.28 Construction of rectangular hyperbola

To draw tangent to the curve, locate the point M on the curve by drawing a line GM, parallel to OA and at a distance 20 from it. Then, locate the point H on OB such that GH = OG. The line HT passing through M is the required tangent to the curve.

NOTE If the eccentricity for a hyperbola is $\sqrt{2}$, the asymptotes will be at right angles to each other and the hyperbola is known as a rectangular hyperbola.

Problem 21 *The asymptotes of a hyperbola are inclined at 70° to each other. Construct the curve when a point P on it is at distances of 20 and 30 from the two asymptotes.*

(May/June 2010, JNTU)

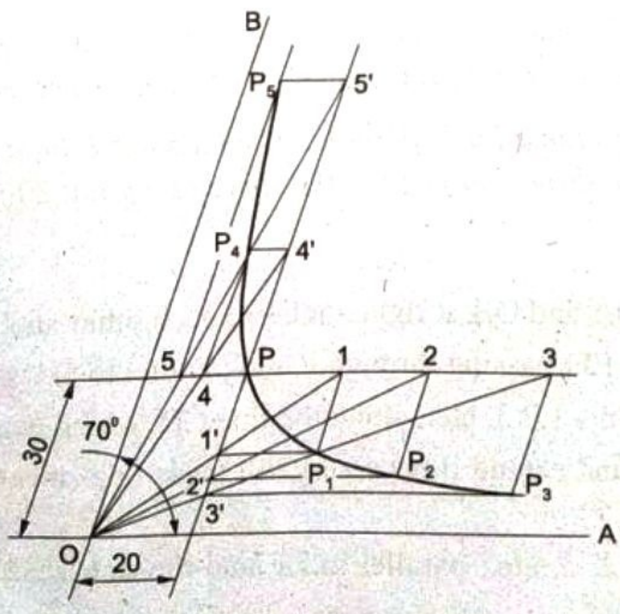

Fig. 5.29 Construction of hyperbola-Given asymptotes and a point on it

1.2 DRAUGHTING TOOLS

The drawing instruments or draughting tools are used to produce drawings quickly and more accurately. To obtain satisfactory results in the form of accurate drawings, the draughting tools used must be of high quality. The students are advised to procure quality draughting tools, which will facilitate to increase efficiency in their draughting work.

The present chapter deals with description of draughting tools used by professional draughtsmen and their methods of use. The following is the list of a majority of draughting tools used by professional draughtsmen:

1. Drawing board
2. Mini-draughter ✓
3. Instrument box, containing the following:
   - (i) Compass ✓
   - (ii) Bow-compass
   - (iii) Spring bow-compass
   - (iv) Divider
   - (v) Bow-divider
   - (vi) Bow-pen
   - (vii) Inking pen
4. 30°-60° and 45°-45° set-squares
5. Protractor ✓
6. Set of scales
7. French curves
8. Flexible curve
9. Templates
10. Drawing sheet ✓
11. Paper fasteners ✓
12. Pencils ✓ — HB, H, 2H pencils.
13. Eraser ✓
14. Erasing shield

5.5 CYCLOIDAL CURVES

Cycloidal curves are generated by a point on the circumference of a circle, when it rolls without slipping along a straight or curved path. The rolling circle is called the generating circle and the fixed straight line / circle is called the directing line/ circle respectively.

5.5.1 Cycloid

A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls along a straight line without slipping (Fig.5.30). Obviously, the size of the curve depends upon the diameter of the generating circle.

Problem 22 *Construct a cycloid, given the diameter of the generating circle as 40. Draw tangent to the curve at a point on it, 35 from the line.* (May/June 2008, 2010, May 2012, JNTU)

Construction (Fig. 5.30)

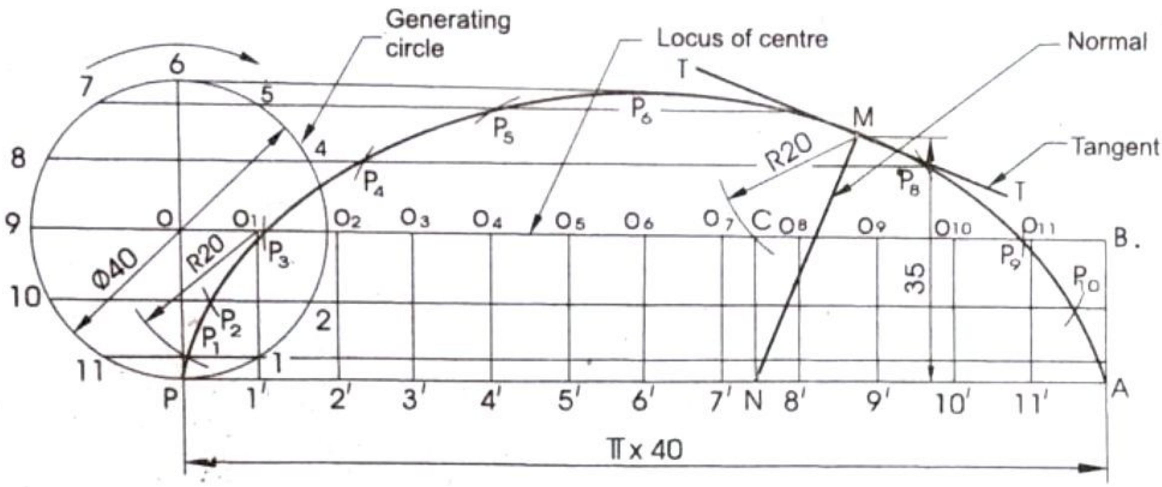

Fig. 5.30 Cycloid

(i) With centre O and radius 20, draw the generating circle.

(ii) Locate the initial position of the generating point P on the circumference of the circle.

(iii) Draw a line PA, tangential and equal to the circumference of the circle.

(iv) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(v) Draw the line OB, parallel and equal to PA, which is the locus of the centre of the generating circle.

(vi) Erect perpendiculars at 1′, 2′, etc., to meet the line OB at O_1, O_2, etc.

(vii) Through the points 1, 2, 3, etc., draw lines parallel to PA.

5.5.2 Epi-cycloid

An epi-cycloid is a curve traced by a point on the circumference of a circle, which rolls without slipping on another circle (directing circle) outside it.

Problem 25 *Draw an epi-cycloid of a circle of 40 diameter, which rolls on another circle of 120 diameter for one revolution clock-wise. Draw a tangent and a normal to it at a point 90 from the centre of the directing circle.* (Aug/Sep 2011, JNTU)

Construction (Fig. 5.33)

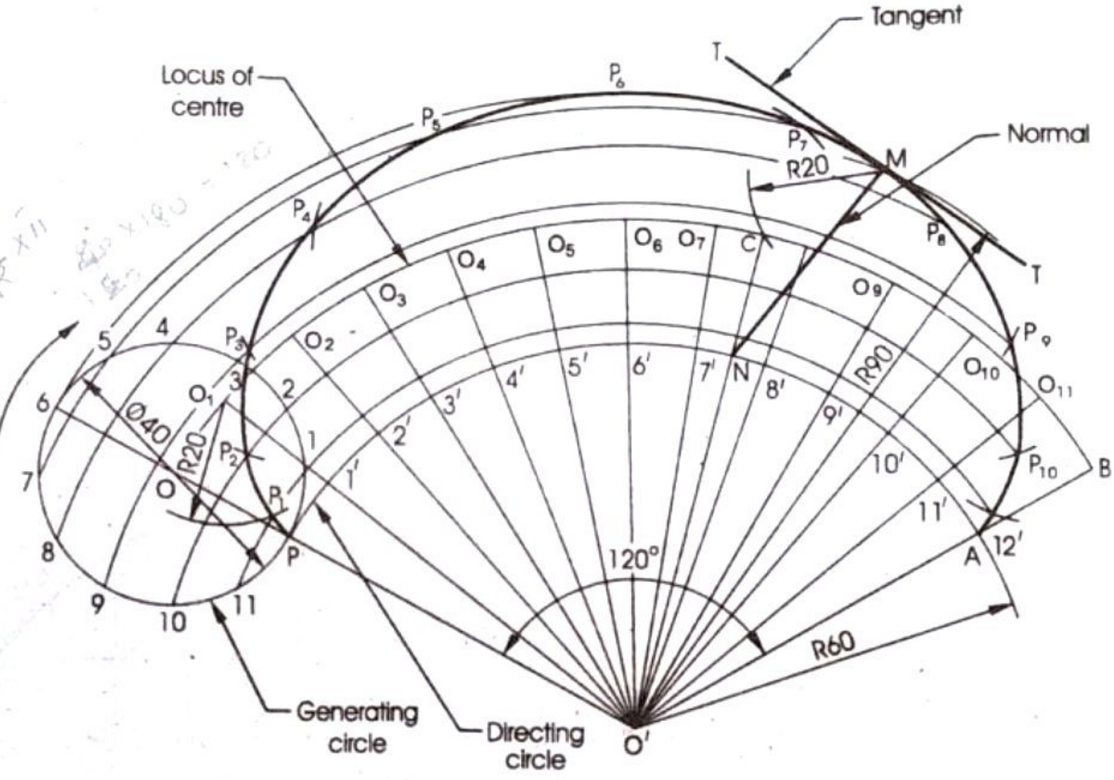

Fig. 5.33 Epi-cycloid

(i) Draw a part of the directing circle with O′ as centre and radius 60.

(ii) Draw any radial line O′ P and extend it.

(iii) Locate the point O on the above line such that, OP = 20, the radius of the generating circle.

(iv) With O as centre and radius 20, draw the generating circle.

(v) Locate the point A on the directing circle such that, the arc length PA is equal to the circumference of the generating circle.

The point A is obtained by setting $\angle PO'A = 360° \times 20/60 = 120°$.

(vi) With centre O′ and radius O′O, draw an arc intersecting the line O′ A produced at B.

The arc OB is the locus of the centre of the generating circle.

(vii) Divide the generating circle and the arc PA into the same number of equal parts and number them as shown.

(viii) Join O', $1'$; O', $2'$; etc., and extend, meeting the arc OB at O_1, O_2, etc.

(ix) Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O' as centre.

(x) With centre O_1 and radius 20, draw an arc intersecting the arc through 1 at P_1.

(xi) In a similar manner, obtain points P_2, P_3, etc.

A smooth curve through these points is the required epi-cycloid.

To draw the tangent and normal

(i) Locate the point M on the curve, which is at 90 from the centre of the directing circle.

(ii) With M as centre and radius 20, draw an arc intersecting the locus of the centre of the generating circle at C.

(iii) Join C to O', intersecting the directing circle at N. The line joining N to M is the required normal and a line T-T, perpendicular to it and passing through M is the required tangent.

5.5.3 Hypo-cycloid

A hypo-cycloid is a curve traced by a point on the circumference of a generating circle, which rolls without slipping on another circle (directing circle), inside it.

Problem 27 *Draw, a hypo-cycloid of a circle of 40 diameter which rolls inside another circle of 160 diameter, for one revolution counter clock-wise. Draw a tangent and a normal to it at a point 65 from the centre of the directing circle.* (Aug/Sep 2008, JNTU)

A procedure similar to the above (Fig. 5.35), may be followed for constructing the hypo-cycloid (Fig. 5.36), keeping in view that the generating circle rolls inside the directing circle.

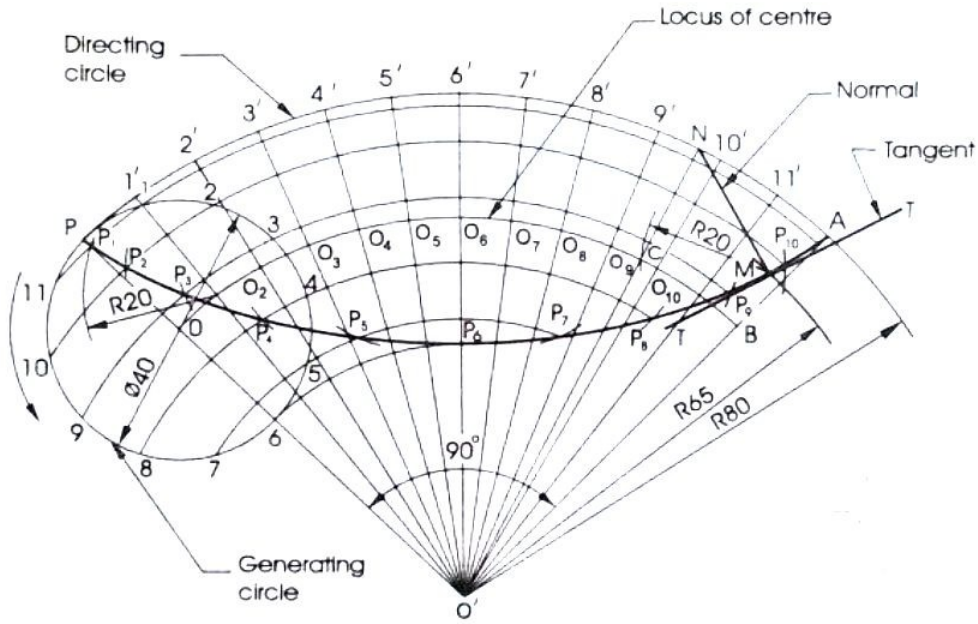

Fig. 5.36 Hypo-cycloid

The method of drawing the tangent and normal to the hypo-cycloid is also similar to the one that is followed for the epi-cycloid.

5.6 INVOLUTE

An involute is a curve traced by a point, on a perfectly flexible thread, while unwinding from a circle or a polygon; the thread being kept tight.

Problem 29 *Draw the involute of an equilateral triangle of side 20 and draw a normal and a tangent at a distance 60 from the centre of the triangle.* (May/June 2010,JNTU)

Construction (Fig. 5.38)

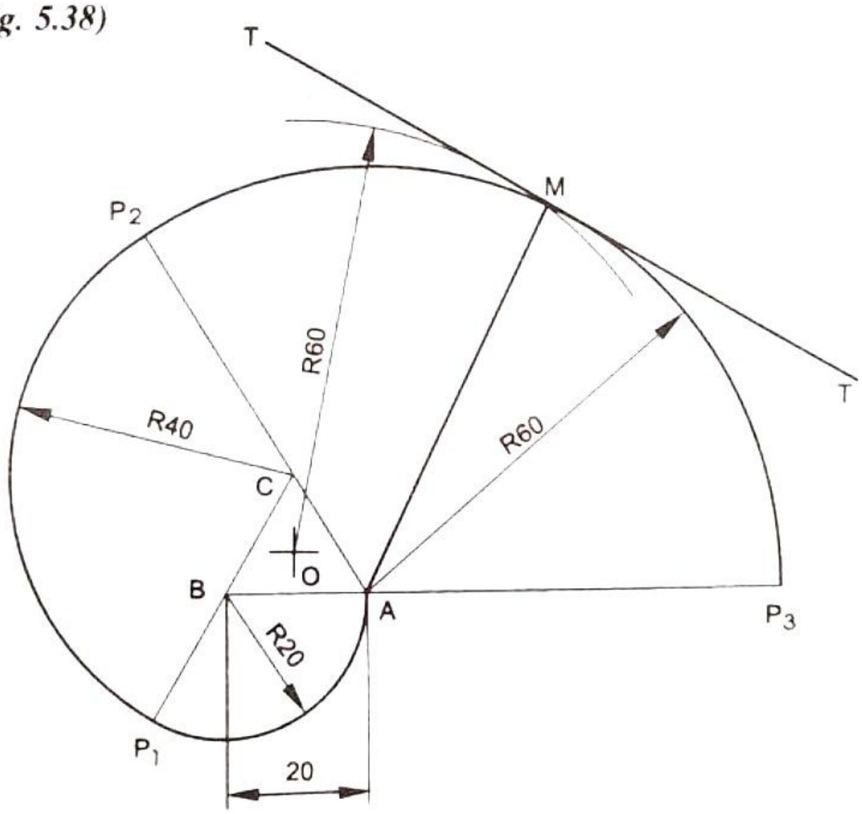

Fig. 5.38 Involute of a triangle

(i) Draw the given triangle ABC of side 20 and locate its centre O.

(ii) Assuming A as the starting point; with B as centre and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 (= $2\times$ 20), draw an arc intersecting the line AC extended at P_2.

(iv) With centre A and radius AP_2 (=$3\times$20), draw an arc intersecting the line BA produced at P_3.

The curve through A, P_1, P_2, P_3 is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre (centre of the triangle) and radius 60, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which A is centre.

(ii) Join A, M; forming the normal to the curve.

(iii) A line T-T, perpendicular to AM at M is the required tangent.

Problem 30 *Draw the involute of a regular hexagon of side 20. Draw a tangent and a normal to the curve at a distance 100 from the centre of the hexagon.*

Construction (Fig. 5.39)

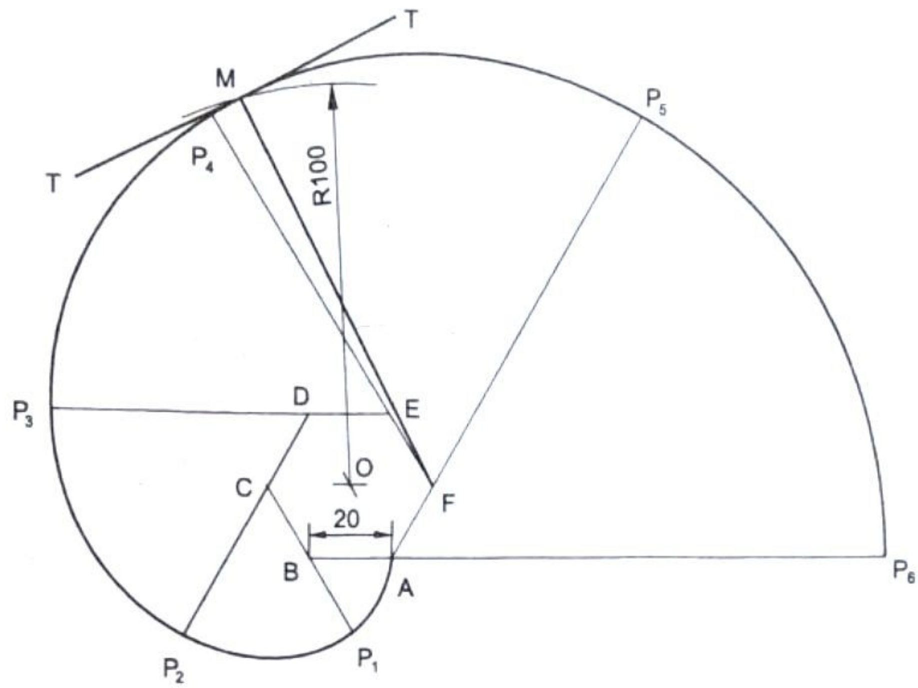

Fig. 5.39 Involute of a hexagon

(i) Draw the hexagon ABCDEF of side 20 and locate its centre O.

Assuming that the thread is unwound from A in the clock-wise direction, the starting point for the involute is A,

(ii) With centre B and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 ($=2 \times 20$), draw an arc intersecting the line DC extended at P_2.

(iv) In a similar way, obtain the other points P_3, P_4, etc.

A smooth curve through the above points is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre and radius 100, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which F is the centre.

(ii) Join F, M; forming normal to the curve.

(iii) A line T-T, perpendicular to FM at M is the required tangent.

Problem 31 *Draw the involute of a circle of 40 diameter. Also, draw a tangent and a normal to the curve at a point 95 from the centre of the circle.* (May 2012, JNTU)

Construction (Fig. 5.40)

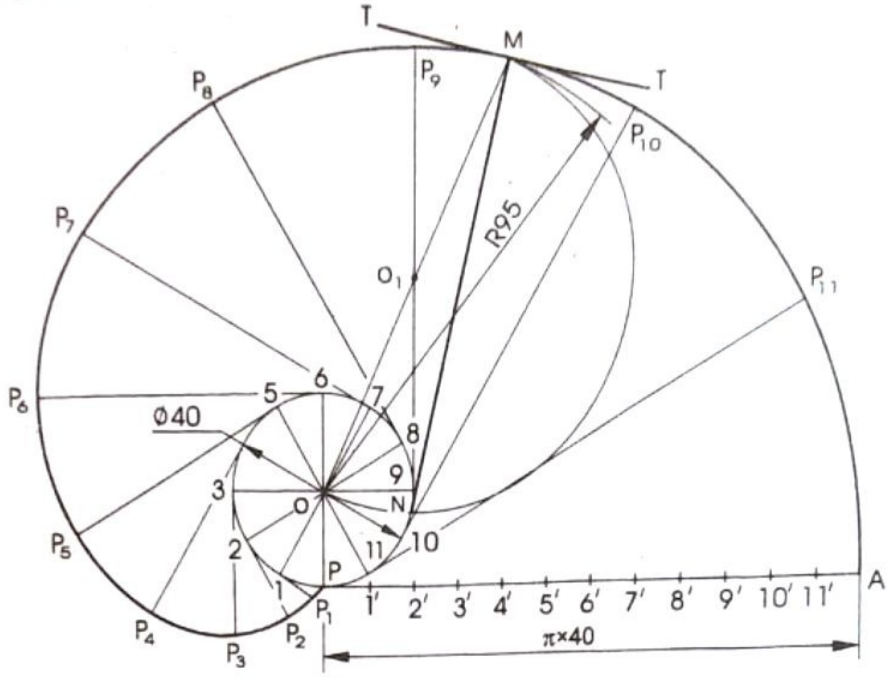

Fig. 5.40 Involute of a circle

(i) With centre O and diameter 40, draw the given circle.

(ii) Assuming P as the starting point, draw a line PA, tangent to the circle and equal to the circumference of the circle.

(iii) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(iv) Draw a tangent to the circle at the point 1 and locate P_1 on it such that, $1P_1 = P1'$.

(v) Draw a tangent to the circle at point 2 and locate P_2 on it such that, $2P_2 = P2'$.

(vi) Locate other points P_3, P_4, etc., in a similar way.

A smooth curve through these points is the required involute.

From the construction, it is obvious that a tangent to the circle is normal to the involute. So, to draw the tangent and normal,

(i) Locate the point M on the curve, which is at 95 from the centre of the circle.

(ii) Join M, O and locate its mid-point O_1.

(iii) With centre O_1 and radius O_1 M, draw a semi-circle intersecting the given circle at N.

(iv) Join N, M; forming the normal to the curve and a line T-T, perpendicular to NM at M is the tangent to the curve

Problem 32 *A thread of length 165 is wound round a circle of 40 diameter. Trace the path of end point of the thread.* (May/June 2011,2012, JNTU)

Construction (Fig. 5.41)

(i) With centre O and radius 20, draw the given circle.

(ii) From point A on the circle, draw a line AP, tangential to the circle and equal to 165, the length of the thread.

(iii) Divide the circle into 12 equal parts and mark the chord lengths along the line AP.

(iv) Draw tangents to the circle at points 1, 2, etc.

(v) Along the tangent through 1, mark P_1 such that, $1P_1 = P1'$.

(vi) Along the tangent through 2, mark P_2 such that, $2P_2 = P2'$.

(vii) In a similar way, locate the points P_3, P_4, etc.

A smooth curve through these points is the required path.

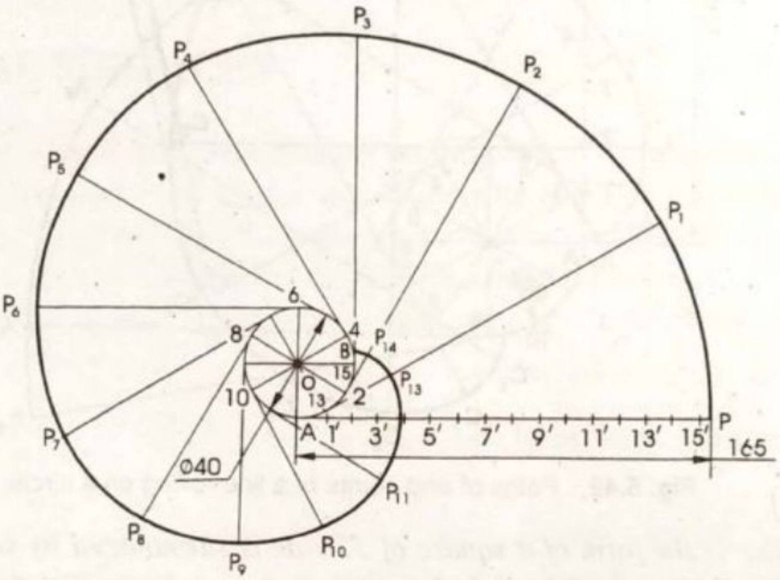

Fig. 5.41 Path of end point of thread, wound round a circle

NOTE (i) The points P_{13}, P_{14} and P_{15} are located along the tangents through 1, 2 and 3.

(ii) The point B on the circle is located such that, the chord length 15-B = 15′ -P.