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Question 1

A) The SVD Decomposition of a matrix is given by $M = UDV^T$ and the eigen Decomposition is given by $M = PDP^{-1}$.

* The SVD is more generalizable to matrices for the following reasons.

→ SVD is valid for any rectangular matrix, whereas the eigen decomposition is possible only for square matrix.

and there exsist ~~Situations~~ Situations ~~where~~ such as Singular matrix where the eigen decomposition is not possible even for square matrix.

→ In SVD U and V^T need not be inverse of each other, but in case of eigen it is a ~~a requir~~ requirement that they have to be inverse of each other. Hence eigen decomposition can be a specific case of ~~SVD~~ SVD.

3) Find SVD of

$$M = \begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix}$$

We need to decompose M as

$$M = U D V^T \rightarrow (1)$$

Where U ~~is~~ and V are orthogonal matrix i.e. their ~~cols~~ colouns form an orthonormal set and

$UU^T = U^T U = I$

$VV^T = V^T V = I$

and D is a diagonal matrix with eigen values as diognal entries.

[D also has zero's appended at the botom as rows, if the matrix is rectangular].

From (1) we get

$$M^T M = V D^T D V^T$$

The matrix V^T is given by the ~~set~~ eigenvectors of $(M^T M)$.

$$M = \begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix} ; \quad M^T = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$$

$$M^TM = \begin{bmatrix} 333 & 81 \\ 81 & 117 \end{bmatrix} \longrightarrow (2)$$

Determine eigen values of Equation (2)

$\det(M^TM - \lambda I) = 0$

$$\begin{bmatrix} 333-\lambda & 81 \\ 81 & 117-\lambda \end{bmatrix} = 0$$

$$(333-\lambda)(117-\lambda) - (81)^2 = 0$$

$$38961 - 450\lambda + \lambda^2 - 6561 = 0$$

$$\lambda^2 - 450\lambda + 32400 = 0$$

$$\lambda^2 - 360\lambda - 90\lambda + 32400 = 0$$

$$\lambda(\lambda - 360) - 90(\lambda - 360) = 0$$

$$(\lambda - 360)(\lambda - 90) = 0$$

$$\underline{\underline{\lambda = 360, 90}} \longrightarrow (3)$$

→ Determine eigenvector for $\lambda = 360$

$$V_1 = \begin{bmatrix} 333-360 & 81 \\ 81 & 117-360 \end{bmatrix} \begin{bmatrix} 'v_1 \\ 'v_2 \end{bmatrix} = 0$$

where $'v_1$ and $'v_2$ are the components of the basis of first eigen vector.

$$\begin{bmatrix} -27 & 81 \\ 81 & -243 \end{bmatrix} \begin{bmatrix} {}^1v_1 \\ {}^1v_2 \end{bmatrix} = 0$$

We need to determine $[{}^1v_1 \ {}^1v_2]^T$ which exist in the null space of the matrix.

We can observe that the second column (c_2) is 3 times in magnitude of first column c_1.

$\Rightarrow$ $c_2 = -3c_1$. The vector in null space can be given as

$$\begin{bmatrix} {}^1v_1 \\ {}^1v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$ we obtain this matrix as unit vector

$$\begin{bmatrix} {}^1v_1 \\ {}^1v_2 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ -1 \end{bmatrix} \longrightarrow (4)$$

$\rightarrow$ Determine Eigen values for $\lambda = 90$

$$V_2 = \begin{bmatrix} 333-90 & 81 \\ 81 & 117-90 \end{bmatrix} \begin{bmatrix} {}^1v_2 \\ {}^2v_2 \end{bmatrix} = 0$$

Where ${}^1V_2$ and ${}^2V_2$ are components of the basis vectors.

Date
Page

$$= \begin{bmatrix} 243 & 81 \\ 81 & 27 \end{bmatrix} \begin{bmatrix} {}^2v_1 \\ {}^2v_2 \end{bmatrix} = 0$$

The vetor $[{}^2v_1 \ {}^2v_2]^T$ is in the null space of matrix and it can be observed that
$c_1 = 3c_2$.

Therefore the null space vector is given by

$$\begin{bmatrix} {}^2v_1 \\ {}^2v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

~~V~~ V can be given by stacking them as ~~row~~ coloumns

$$V = \begin{bmatrix} {}^1v_1 & {}^2v_1 \\ {}^1v_2 & {}^2v_2 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} -3 & 1 \\ -1 & -3 \end{bmatrix}$$

$$V^T = \frac{1}{\sqrt{10}} \begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix} \quad \longrightarrow \quad 5$$

→ Determine "D" matrix:-

* Matrix M is of shape (3x2), therefore D has to be of shape (3 x 2)
~~The~~ there will exist one row of zeros in D.

* As $M^T M = V D^T D V^T$
the eigen values of $(M^T M)$ are squared

Hence we must take the square root of eigen values for the matrix D.

$$D = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix}$$

$$\sigma_1 = \sqrt{360} = 6\sqrt{10}$$
$$\sigma_2 = \sqrt{90} = 3\sqrt{10}$$

$$D = \begin{bmatrix} 6\sqrt{10} & 0 \\ 0 & 3\sqrt{10} \\ 0 & 0 \end{bmatrix}$$ //

→ Determine matrix U.

$$M = U D V^T$$

$$M V = U D \qquad [V^T V = V V^T = I]$$

$$M V D^{-1} = U$$ [Inverting only the square diagonal matrix D]. the 0's row is not included.

$$U = \frac{1}{\sqrt{10}}\begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 1/6 & 0 \\ 0 & 1/3 \end{bmatrix} \frac{1}{\sqrt{10}}$$

$\cancel{U = \begin{bmatrix} -1/3 \\ \\ \end{bmatrix}}$

$$U = \begin{bmatrix} -1/3 & -2/3 \\ -2/3 & -1/3 \\ -2/3 & 2/3 \end{bmatrix} \rightarrow (6)$$

$$U = \begin{bmatrix} -0.333 & -0.666 \\ -0.666 & -0.333 \\ -0.666 & 0.666 \end{bmatrix} \rightarrow (6)$$

We need to determine another vector in the null space of M^T

3^{rd} column is given by $= \frac{NS(A^T)}{|NS(A^T)|}$

Where NS is null space of A^T

$$\rightarrow M^T = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$$

reduce M^T to row echelon form we get

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \rightarrow (7)$$

Extract the last column and append 1 to it to obtain the null space vector

i.e $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \xrightarrow[\text{vector}]{\text{unit}} \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$

append to equation (6).

$$U = \begin{bmatrix} -0.333 & -0.666 & -0.666 \\ -0.666 & -0.333 & 0.666 \\ -0.666 & 0.666 & 0.333 \end{bmatrix}$$

$$D = \begin{bmatrix} 6\sqrt{10} & 0 \\ 0 & 3\sqrt{10} \\ 0 & 0 \end{bmatrix}$$

$$V^T = \frac{1}{\sqrt{10}} \begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix}$$

Question 2 :-

A) Correct options are :-

B and C

B) False :- LDA projects the data points on a line such that it would be helpful for classification.
. PCA does not use Labels.

Question 3

A) ~~Prior~~ :-

$$\underbrace{\underset{\text{Posterior}}{P(A|B)} = \frac{P(B|A) \cdot P(A)}{P(B)}}_{\text{Bayes formula}} \leftarrow \text{Prior}$$

Prior :- It represents the probability (or uncertainity) over "A" before any observations are made

Posterior :- It ~~represents~~ represents the Probability after ~~after~~ observation is made and ~~[illegible]~~ includes the prior. it is a conditional distribution.

B) ~~B~~. Using Bayes theorem

$$P(F \mid H_+ S_+) = \frac{P(H_+ S_+ \mid F) \cdot P(F)}{P(H_+ S_+)}$$

$P(F)$ = Prior Probablity of having Flu in General

= 0.05

$P(H_+ S_+ \mid F)$ = Probablity of having headace and Sour throat Given you have Flu = 0.9

$P(H_+ S_+)$ = Probality of having headace & Sour throat independent of ~~the~~ flu = 0.2

$$= \frac{0.9 \times 0.05}{0.2} = \underline{\underline{0.225}} = \underline{\underline{22.5\%}}$$