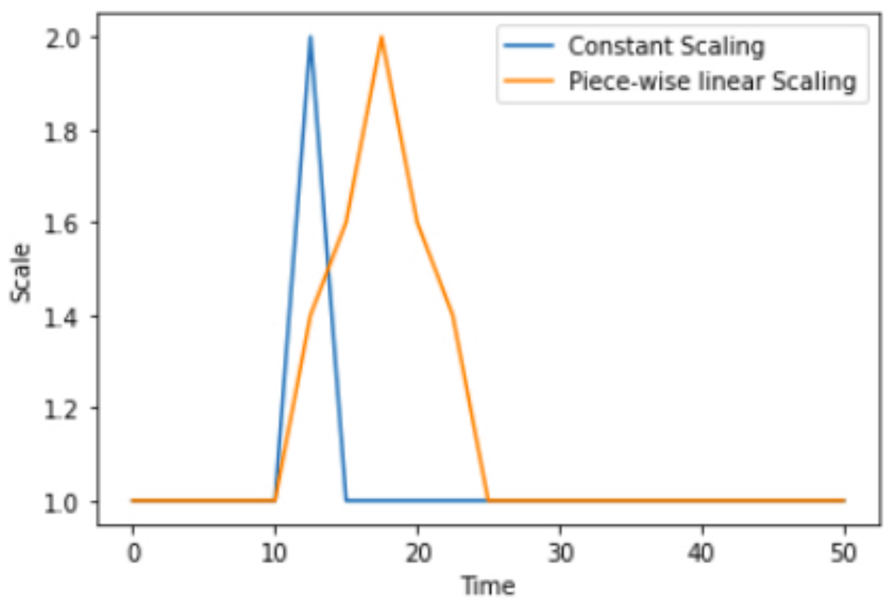
Time scaling:

2a) To ensure smooth robot trajectories we need to create a scaling transformation function that leads to smooth acceleration/deceleration or higher order derivatives of the robot.

One possible solution is presented below.



When we can replace a constant scaling with a piece-wise linear scale, this would ensure smooth transition

A better solution would be to include an exponential function such as $S = e^{kt}$ this scaling function can be continuously differentiable. Derivation for a two robot case:

(1b) $$d^2 = |\vec{r}|^2 - \frac{(\vec{r} \cdot \vec{V}_{1/2})^2}{|\vec{V}_{1/2}|^2} \geq R^2, \quad \vec{r} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$

relative velocity vector

$$\vec{V}_{1/2} = (S\dot{x}_1 - \dot{x}_2)\hat{i} + (S\dot{y}_1 - \dot{y}_2)\hat{j}$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 - R^2$$

$$- \frac{\left((S\dot{x}_1 - \dot{x}_2)(x_1 - x_2) + (S\dot{y}_1 - \dot{y}_2)(y_1 - y_2)\right)^2}{S(\dot{x}_1 - \dot{x}_2)^2 + (S\dot{y}_1 - \dot{y}_2)^2} \geq 0$$

$$\boxed{(x_1 - x_2)^2 + (y_1 - y_2)^2 - R^2 - \frac{\left((S\dot{x}_1 - S\dot{x}_2)(x_1 - x_2) + (S\dot{y}_1 - \dot{y}_2)(y_1 - y_2)\right)^2}{(\dot{x}_1 - \dot{x}_2)^2 + (S\dot{y}_1 - \dot{y}_2)^2}} \geq 0 \qquad (1b1)$$

$$K_1 = (x_1 - x_2)^2 + (y_1 - y_2) - R^2$$

$$(s\dot{x}_1 - \dot{x}_2)(x_1 - x_2) + (s\dot{y}_1 - \dot{y}_2)(y_1 - y_2)$$

$$= s(\dot{x}_1 x_1 - \dot{x}_1 x_2 + \dot{y}_1 y_1 - \dot{y}_1 y_2)$$

$$+ (\dot{x}_2 x_2 - \dot{x}_2 x_1 + \dot{y}_2 y_2 - \dot{y}_2 y_1)$$

$$K_2 = \dot{x}_2 x_1 - \dot{x}_1 x_2 + \dot{y}_1 y_1 - \dot{y}_1 y_2$$

$$K_3 = \dot{x}_2 x_2 - \dot{x}_2 x_1 + \dot{y}_2 y_2 - \dot{y}_2 y_1$$

$$(s\dot{x}_1 - \dot{x}_2)^2 + (s\dot{y}_1 - \dot{y}_2)^2$$

$$(s\dot{x}_1)^2 + (\dot{x}_2)^2 - 2s\dot{x}_1\dot{x}_2$$

$$+ (s\dot{y}_1)^2 + (\dot{y}_2)^2 - 2s\dot{y}_1\dot{y}_2$$

$$s^2(\dot{x}_1^2 + \dot{y}_1^2) + (\dot{x}_2^2 + \dot{y}_2^2)$$

$$-2s(\dot{x}_1\dot{x}_2 + \dot{y}_1\dot{y}_2)$$

$$\dot{x}_1^2 + \dot{y}_1^2 = K_1 D$$

$$\dot{x}_2 + \dot{y}_2^2 = K_3 D$$

$$\dot{x}_1 + \dot{y}_1 + \dot{y}_1\dot{y}_2 = K_2 D$$

$$K_1 - \frac{(sK_2 + K_3)^2}{(s^2 K_1 D - 2sK_2 D + K_3 D)}$$

$$s^2 K_1 K_{1D} - 2s K_{2D} K_1 + K_{2D} K_1$$

$$- (s^2 K_2^2 + K_3^2 + 2sK_2 K_3)$$

$$s^2(K_1 K_{1D} - K_2^2) - 2s(K_{2D} K_1 + K_2 K_3)$$

$$+ (K_{3D} K_1 - K_3^2) \geq 0$$

representing the above in the form

$$as^2+bs+c \geq 0$$

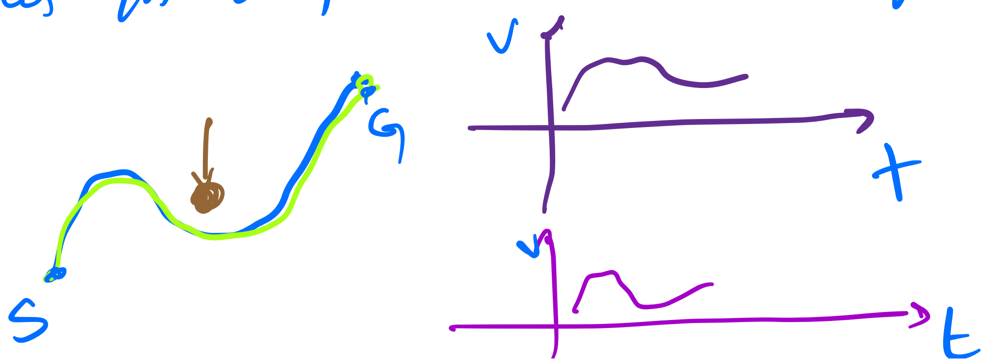
gives us the solution space set S

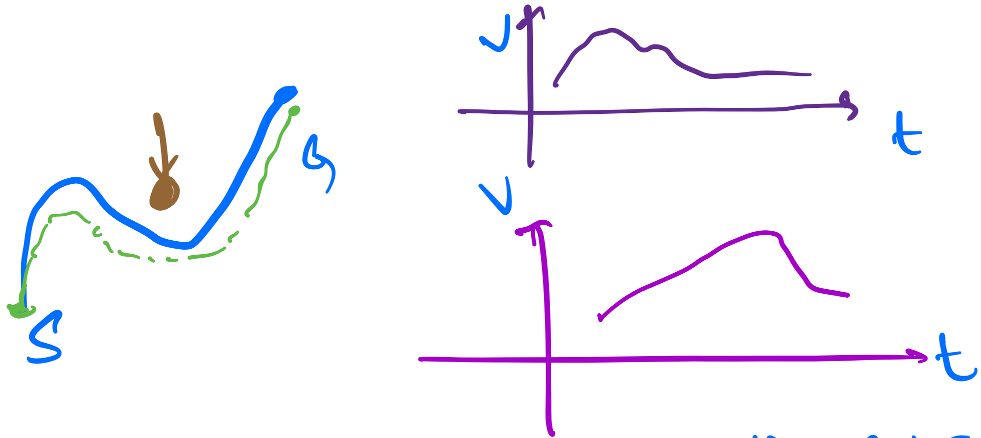
$$S_{sol} = \begin{cases} [S_{min},\infty) \cap \left(\left(-\infty, \frac{-b-\sqrt{b^2-4ac}}{2a}\right) \cup \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\right), & a>0,\ dis>0 \\ [S_{min},\infty), & a>0,\ d<0 \\ [S_{min},\infty) \cap \left[\frac{-b-\sqrt{b^2-4ac}}{2a}, \frac{-b+\sqrt{b^2-4ac}}{2a}\right], & a<0,\ d>0 \\ & a<0,\ d>0 \end{cases}$$

if d is the discriminant

2) MPC based collision avoidance:

b) As we know the initial trajectories of the robot & the dynamic obstacle, a time scaled trajectory will be closer to the original trajectory in terms of path. but the velocity & acceleration profiles will change due to time scaling





Example trajectory path (left) velocity (right)

An MPC based trajectory generation for collision avoidance solves optimization problem to fit the given trajectory to the actual trajectory, this may or may not represent the actual trajectories but they can be modified to add additional constraints to achieve goal reaching & obstacle avoidance.

A time scaled trajectory is the closest trajectory to the given trajectory
An MPC trajectory is more capable but may not be very close to the original trajectory

(3) Multi-Robot obstacle avoidance:-

in the case of multirobot obstacle avoidance, the velocities that belong to the intersection velocity space of all the obstacles are the ideal pick. If there exists no such velocity, then the velocity closest to the intersection velocity space is picked.

these velocity space refers to the collision free velocity space.

generally the solution space is of the form

$$\left[\max\left\{ \frac{-b_i - \sqrt{b_i^2 - 4a_ic_i}}{2a_i} \right\}, \min\left\{ \frac{-b_i + \sqrt{b_i^2 - 4a_ic_i}}{2a_i} \right\} \right]$$

if max is < 0

$$\left[0, \min\left\{ \frac{-b_i + \sqrt{b_i^2 - 4a_ic_i}}{2a_i} \right\} \right]$$

if $\max\left\{ \frac{-b_i + \sqrt{b_i^2 - 4a_ic_i}}{2a_i} \right\} < 0$

no solutions exist

here

If s is the scale factor solution which makes $\vec{V}_{1/2}$ come out of $C_{1/2}$, then $1/s$ is the scale factor solution which makes $\vec{V}_{2/1}$ come out of $C_{2/1}$

for getting collision avoidance condition for robot 2 $\vec{V}_{2/1}$ is expressed in the following manner

$$(x_2-x_1)^2+(y_2-y_1)^2-R^2-\frac{\left((s'\dot{x}_2-\dot{x}_1)(x_2-x_1)+(s'\dot{y}_2-\dot{y}_1)(y_2-y_1)\right)^2}{(s'\dot{x}_2-\dot{x}_1)^2+(s'\dot{y}_2-\dot{y}_1)^2}\geq 0$$

taking $1/s'$ common

$$(x_1-x_2)^2+(y_1-y_2)^2-R^2-\frac{\left((\frac{1}{s'}\dot{x}_1-\dot{x}_2)(x_1-x_2)+(\frac{1}{s'}\dot{y}_1-\dot{y}_2)(y_1-y_2)\right)^2}{(\frac{1}{s'}\dot{x}_1-\dot{x}_2)^2+(\frac{1}{s'}\dot{y}_1-\dot{y}_2)^2}\geq 0$$

Comparing equation (iii) with the above equation, we can infer that scale is $1/s$,