CSL105 : Discrete Mathematics

Major Examination

Indian Institute of Technology Ropar

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April 2017

Total Duration : 3 hours Total Marks : 80 M

Section I [5 Marks each]

1. How many 4 digit decimal numbers are there such that 1 is not in the first place, 2 is not in the second, 3 not in third and 4 not in fourth? Prove.
2. How many positive integer solutions are there for $x_1 + x_2 + x_3 < 10$. Explain.
3. Prove that there exists a k such that 11 divides $2^k - 1$.
4. Is it possible to give an example of a relation which is not a function. Explain.
5. Given two finite sets A, B, we say that $A \times B = B \times A$ iff _________ ?
6. Is complement of a Tree with more than 4 vertices always connected? Give reason for your answer.
7. How many non-isomorphic induced subgraphs does K_6 have?
8. Write a statement that is equivalent to $p \rightarrow q$ and prove it with the help of a truth table.
9. What is the rook polynomial of a 3×3 chess board? Explain.
10. What is the maximum length of a trail in K_{2n}?

Section II [10 Marks each]

1. Show that whenever 25 girls and 25 boys are seated around a circular table, there will always be a person, both of whose neighbors are boys.
2. Show that $\sum_{v \in V} deg(v)^2 = \sum_{v \in V} \sum_{u \in N(v)} deg(u)$, where $N(v)$: { set of vertices adjacent to v}.
3. Let G be an undirected graph with subgraphs G_1, G_2. If $G = G_1 \cup G_2$ and $G_1 \cap G_2 = K_n$ for some $n \in \mathbb{Z}^+$, then show that :

$$P(G, \lambda) = \frac{P(G_1, \lambda)P(G_2, \lambda)}{(\lambda)^n}$$

4. Show that the number of partitions of a positive integer n where no summand appears more than twice equals the number of partitions of n where no summand is divisible by 3.

5. Let $E = (e_{ij})_{mxn}$, $F = (f_{ij})_{mxn}$ be two $m \times n$ (0,1)-matrices. We say that E precedes, or is less than, F, and we write $E \leq F$, if $e_{ij} \leq f_{ij}$, for all $1 \leq i <\leq m$, $1 \leq j \leq n$.
For $m, n \in \mathbb{Z}^+$, let A be the set of all $m \times n$ (0,1)-matrices. Prove that the "precedes" relation makes A into a poset.