

CSL105 : Discrete Mathematics  
Minor Examination  
Indian Institute of Technology Ropar  
Instructor: Dr. Sudarshan Iyengar

March 2017

Total Duration : 2 hours

Total Marks : 80 M

Section I

[5 Marks each]

1. Give an example of a relation which is [reflexive and symmetric] but not [transitive and antisymmetric].
2. Write the Hasse diagram of the relation  $R = \{(a, b)/a|b\}$  where  $S = 1, 2, \dots, 10$ .
3. State Pigeon hole principle and its extended/generalized version.
4. What is the condition for a function to be invertible? Explain with an example.
5. Show that in a group of 5 people, we cannot always guarantee of a presence of three mutual friends or three mutual non-friends.
6. A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls. What is the minimum no. of balls we have to choose randomly from the box to ensure that we get 9 balls of same color?
7. Which of the following statements is/are TRUE for undirected graphs? P: Number of odd degree vertices is even. Q: Sum of degrees of all vertices is even.
8. Let  $G = (V, E)$  be a loop free undirected graph. Prove that if  $G$  contains no cycle of odd length then  $G$  is bipartite.

Section II

[10 Marks each]

1. Show that among any  $n + 1$  positive integers not exceeding  $2n$  there must be an integer that divides one of the other integers.
2. Prove that  $n^2 - 1$  is divisible by 8 whenever  $n$  is odd positive integer.
3. Provide a story proof that  $\frac{(n!)}{(3!)^k}$  is an integer, given that  $n = 3k$ . Generalize this result.
4. Enumerate all possible non-isomorphic graphs on 4 vertices.