Minor Exam CSL105

March 2017

1. Prove by Induction that $1 + \frac{1}{2} + \frac{1}{3} + \ldots = \infty$.
2. Show that among any n + 1 positive integers not exceeding 2n there must be an integer that divides one of the other integers.
3. Show that
$$1.2.3 + 2.3.4 + 3.4.5 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$
.
4. Prove that $n^2 - 1$ is divisible by 8 whenever n is odd positive integer.
5. You need to choose a password which is at least 6 characters and at most 8 characters in length with an added condition that each character is an uppercase letter or a digit. Also, your password must contain at least one digit. In how many ways can you choose your password?
6. Provide a story proof that $\frac{(n!)}{(3!)^k}$ is an integer, given that $n = 3k$. Generalize this result.
7. Let $S = \{1, 2, 3, \ldots\}$. Consider a relation $R = \{(a, b)/a + b \leq 10\}$. Prove or disprove that R satisfies reflexive, symmetric, antisymmetric and transitive properties.
8. Give an example of a relation which is [reflexive and symmetric] but not [transitive and antisymmetric].
9. In the matrix representation of a relation, how does one find if the relation is transitive or not? Prove that your method works.
10. Write the Hasse diagram of the relation $R = \{(a, b)/a|b\}$ where $S = 1, 2, \ldots, 10$.
11. Every sequence of n2 + 1 distinct real numbers contains a subsequence of length n + 1 that is either strictly increasing or strictly decreasing.
12. State Pigeon hole principle and its extended/generalized version.

13. Define a bijection with an example.
14. State well ordering principle. State and Prove Mathematical Induction.
15. What is the condition for a function to be invertible? Explain with an example.
16. Show that in a group of 5 people, we cannot always guarantee of a presence of three mutual friends or three mutual non-friends.
17. Enumerate all possible non-isomorphic graphs on 4 vertices.
18. In how many ways can 12 different books be distributed among 4 children (all 4 of different age) so that (a) each child gets three books ? (b) The two oldest children get four books each and the two youngest get two books each.
19. When n is a positive integer greater than 1, show that ${}^nC_2 +{}^{(n-1)} C_2$ is a perfect square.
20. Assignment 1.2
21. Assignment 1.5
22. Assignment 1.16
23. Assignment 2.1
24. Assignment 2.15
25. Assignment 2.18
26. Assignment 3.1
27. Assignment 3.14
28. Assignment 3.15