



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 1

Systems of linear equations

Singular and non-singular matrices

Determinants

Rank of a matrix

Row reduction

Null space

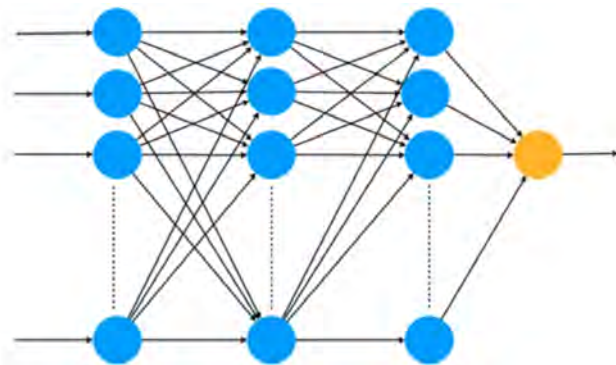


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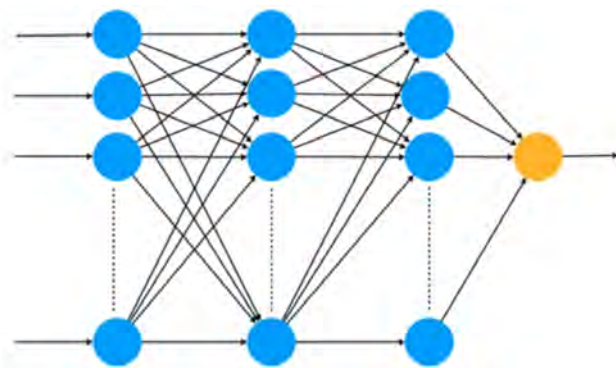
System of Linear Equations

Machine learning motivation

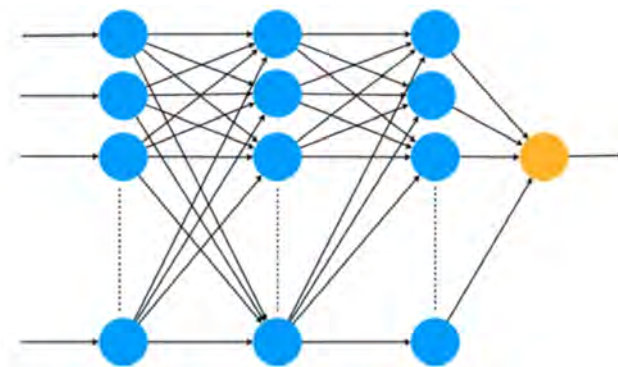
Neural networks - Matrix operations



Neural networks - Matrix operations

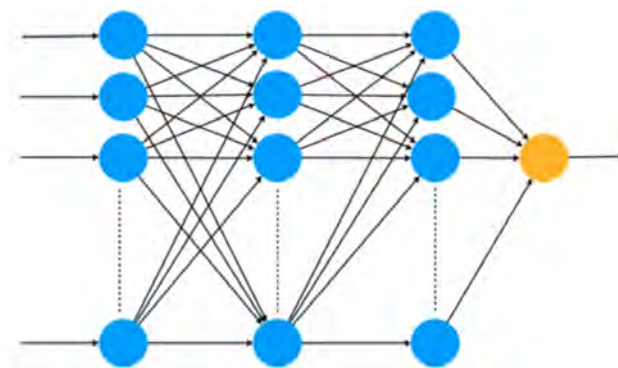


Neural networks - Matrix operations



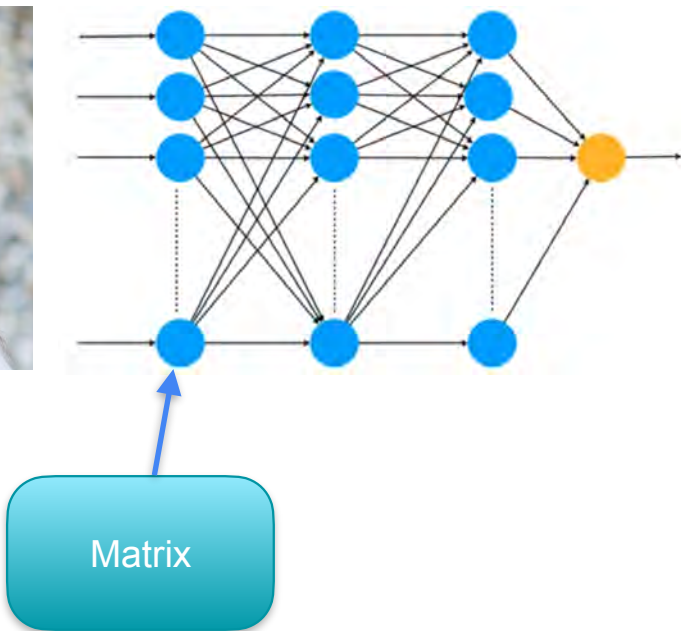
Yes, it's a face

Neural networks - Matrix operations



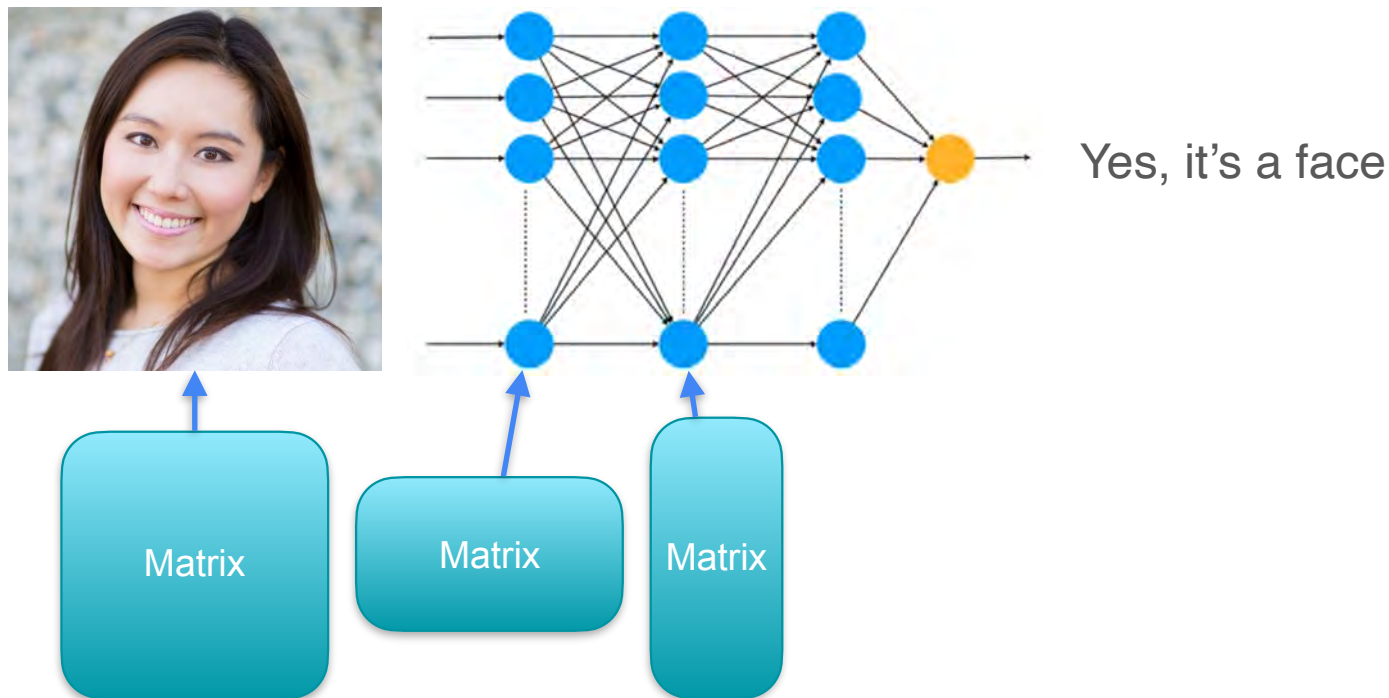
Yes, it's a face

Neural networks - Matrix operations

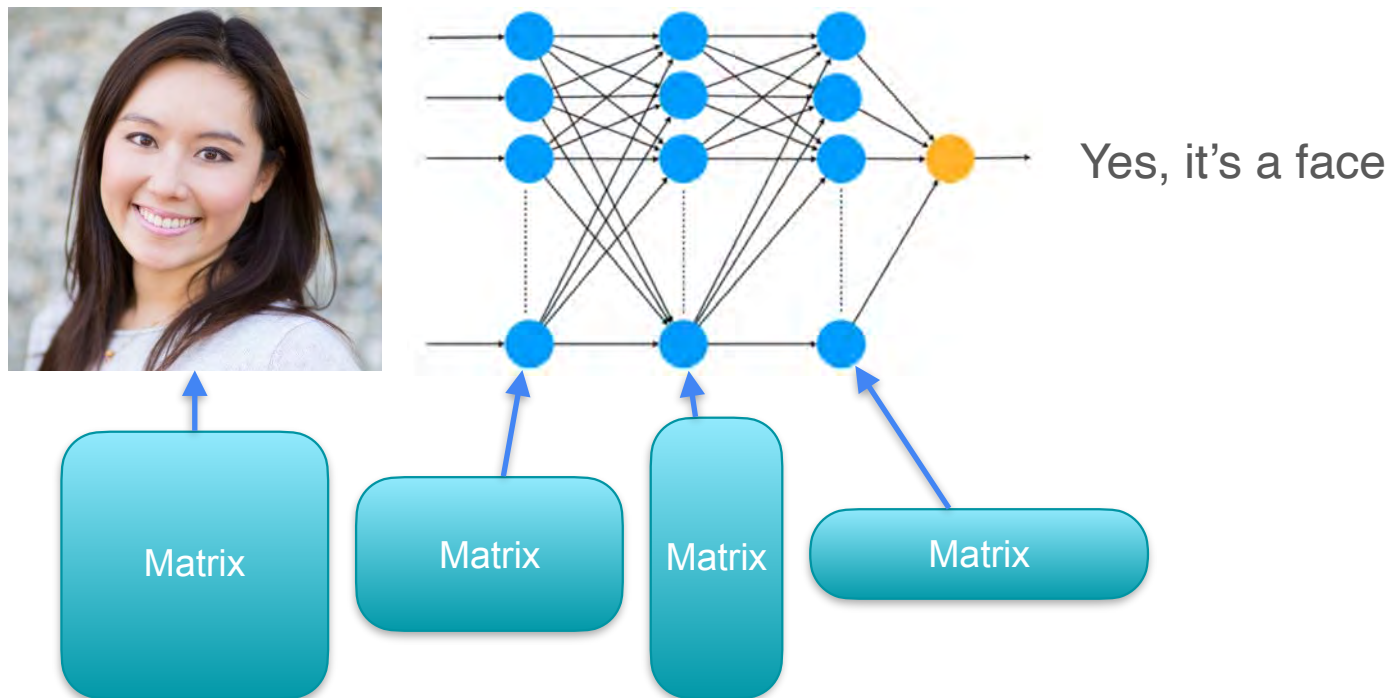


Yes, it's a face

Neural networks - Matrix operations



Neural networks - Matrix operations



Neural networks - image recognition

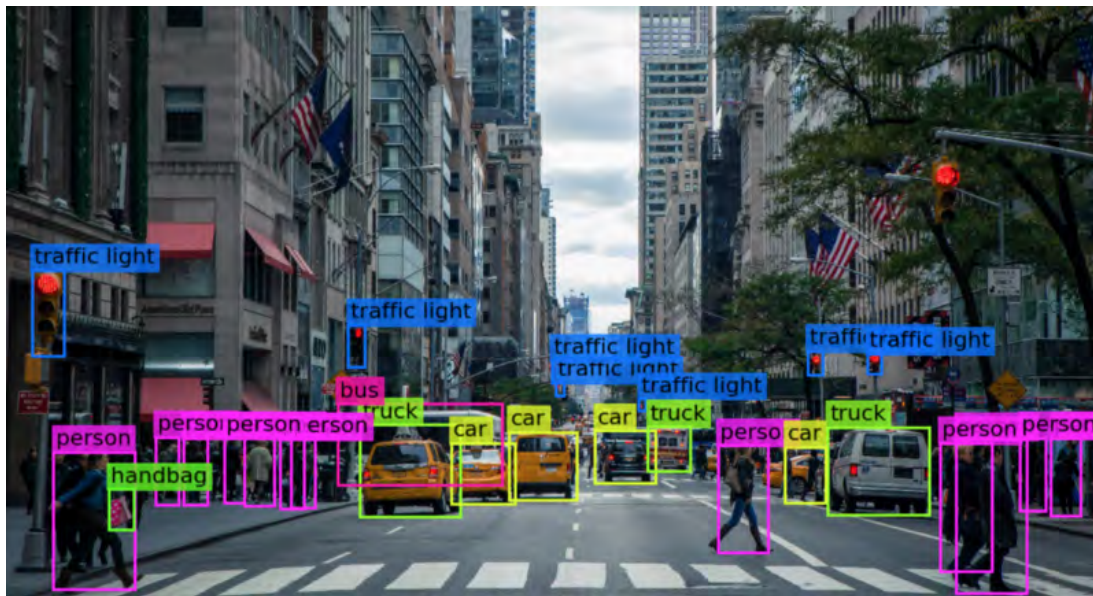


Image recognition in a busy street in New York.

- Image recognition: Getting the computer to see images and recognize what is on them.



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System of Linear Equations

System of sentences

Systems of sentences



Systems of sentences

System 1



 The dog is **black**
 The cat is **orange**

Systems of sentences

System 1



 The dog is **black**
 The cat is **orange**

System 2



 The dog is **black**
 The dog is **black**

Systems of sentences



System 1

 The dog is **black**
 The cat is **orange**

System 2



 The dog is **black**
 The dog is **black**

System 3



 The dog is **black**
 The dog is **white**

Systems of sentences



System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**



System 3

 The dog is **black**
 The dog is **white**

Complete



Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**



Complete

System 2

 The dog is **black**
 The dog is **black**



Redundant

System 3

 The dog is **black**
 The dog is **white**



Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**



Complete

System 2

 The dog is **black**
 The dog is **black**

Redundant



System 3

 The dog is **black**
 The dog is **white**

Contradictory



Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

Complete



System 2

 The dog is **black**
 The dog is **black**

Redundant

Singular

System 3



 The dog is **black**
 The dog is **white**

Contradictory

Singular

Systems of sentences



System 1

 The dog is **black**
 The cat is **orange**

Complete

Non-singular



System 2

 The dog is **black**
 The dog is **black**

Redundant

Singular

System 3




 The dog is **black**
 The dog is **white**

Contradictory




Singular

Systems of sentences




System 1

 The dog is **black**
 The cat is **orange**
 The bird is **red**




System 2

 The dog is **black**
 The dog is **black**
 The bird is **red**

System 3

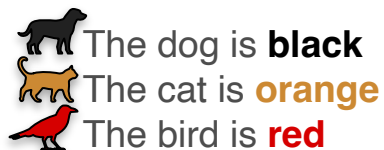
 The dog is **black**
 The dog is **black**
 The dog is **black**

System 4

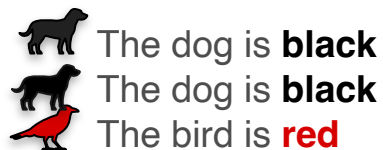
 The dog is **black**
 The dog is **white**
 The bird is **red**

Systems of sentences

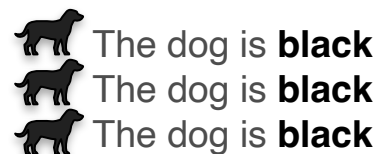
System 1



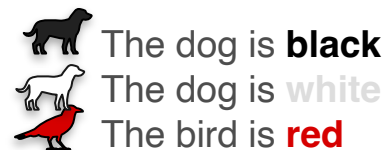
System 2



System 3



System 4

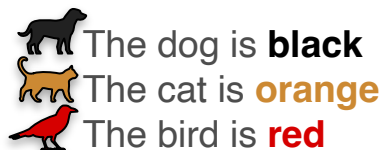


Complete

Non-singular

Systems of sentences

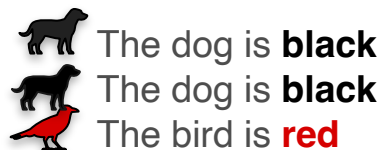
System 1



Complete

Non-singular

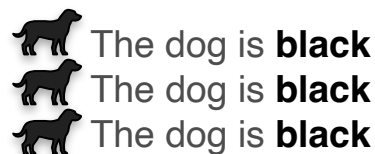
System 2



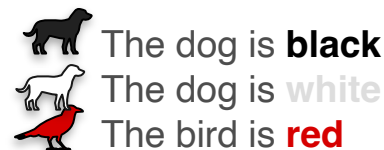
Redundant

Singular

System 3

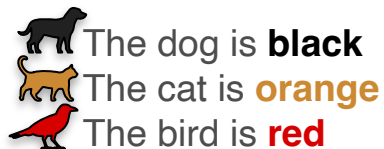


System 4



Systems of sentences

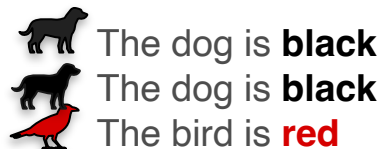
System 1



Complete

Non-singular

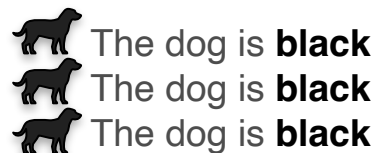
System 2



Redundant

Singular

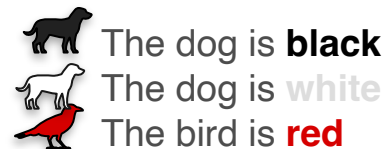
System 3



Redundant

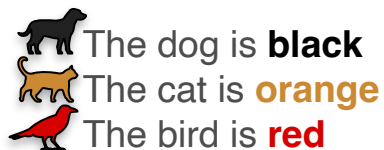
Singular

System 4



Systems of sentences

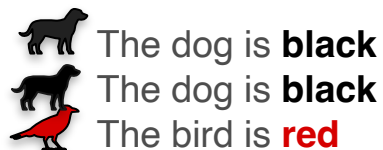
System 1



Complete

Non-singular

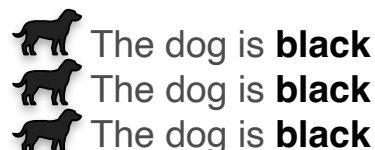
System 2



Redundant

Singular

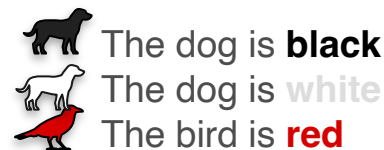
System 3



Redundant

Singular

System 4



Contradictory

Singular

Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

Problem 1:

What color is the bird?

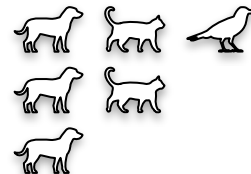
Problem 2:

Is this system singular or non-singular?

Solution: Systems of information

Given this system:

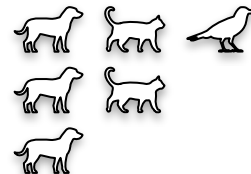
- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

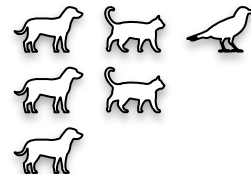


Solution 1:

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



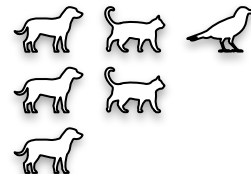
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



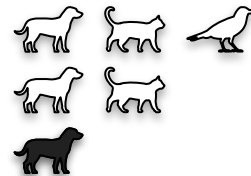
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



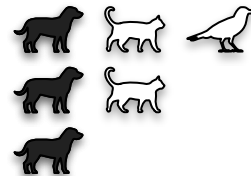
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



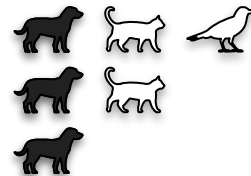
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



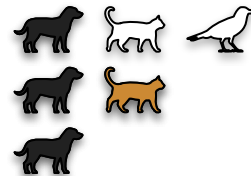
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



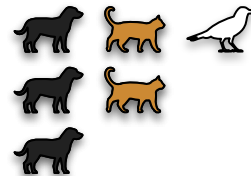
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

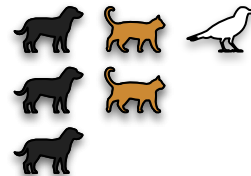
The bird is red. 

Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

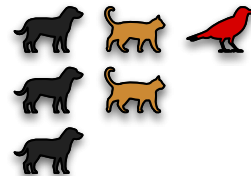
The bird is red. 

Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

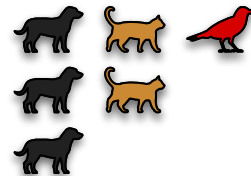
The bird is red. 

Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

The bird is red. 

Solution 2:

It is non-singular.   



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System of Linear Equations

System of equations

Sentences \rightarrow Equations

Sentences

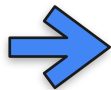
Between the dog and
the cat, one is black.



Sentences \rightarrow Equations

Sentences

Between the dog and
the cat, one is black.



Sentences with numbers

The price of an apple
and a banana is \$10.



Sentences → Equations

Sentences

Between the dog and
the cat, one is black.



Sentences with numbers

The price of an apple
and a banana is \$10.



Equations

$$a + b = 10$$



Quiz: Systems of equations 1

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.

Question: How much does each fruit cost?

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \text{🍌} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$\boxed{12}$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$\boxed{12}$$

↙
\$2

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{Apple} + \text{Banana} = \$10$$

A blue curved arrow points from the banana icon to the text "\$2" below it.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{Apple} + \text{Banana} + \text{Banana} = \$12$$

A blue square box is drawn around the second banana icon. A blue curved arrow points from the bottom of this box to the text "\$2" below it.

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

Diagram illustrating the addition: An apple is shown above the number 8, and a banana is shown above the number 2. Arrows point from the numbers 8 and 2 to the apple and banana respectively. The equation is shown as: Apple + Banana = \$10.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

Diagram illustrating the addition of fruit prices:

- 1 apple + 1 banana + 1 banana = \$2
- The second banana and the result '\$2' are highlighted with blue boxes and an arrow, indicating the total cost for the bananas.

- **Solution:** An apple costs \$8, a banana costs \$2.

Quiz: Systems of equations 2

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

Question: How much does each fruit cost?

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍏🍏} + \text{🍌🍌} = \$20$$

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍏🍏} + \text{🍌🍌} = \$20$$

Same thing!!!

Solution: Systems of equations 2

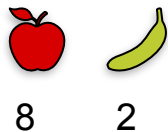
- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \text{ apples} + 2 \text{ bananas} = \$20$$

Same thing!!!



Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



8 2

5 5

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



| | |
|-----|-----|
| 8 | 2 |
| 5 | 5 |
| 8.3 | 1.7 |
| 0 | 10 |

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \text{ apples} + 2 \text{ bananas} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

0 10

Infinitely many solutions!

Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

Question: How much does each fruit cost?

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$

Contradiction!

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$



Contradiction!



No solutions!

Systems of equations

Systems of equations





System 1

- $a + b = 10$
 







- $a + 2b = 12$
 

Systems of equations

System 1





- $a + b = 10$
 
- $a + 2b = 12$
 

System 2







- $a + b = 10$
 
- $2a + 2b = 20$
   

Systems of equations







System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

System 2





- $a + b = 10$
 
- $2a + 2b = 20$
   

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
   






Systems of equations

System 1






- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:

System 2





- $a + b = 10$
 
- $2a + 2b = 20$
  

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1






- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






$$\text{🍏 } a = 8$$

$$\text{🍌 } b = 2$$

System 2





- $a + b = 10$
 
- $2a + 2b = 20$
  

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:

$$\text{🍏 } a = 8$$






$$\text{🍌 } b = 2$$

Complete

System 2





- $a + b = 10$
 
- $2a + 2b = 20$
  

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1






- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






$$\text{🍏 } a = 8$$

$$\text{🍌 } b = 2$$

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Complete

Non-singular

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






$$\text{apple } a = 8$$

$$\text{banana } b = 2$$

Complete






Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  

Infinite solutions

System 3


- $a + b = 10$
 
- $2a + 2b = 24$
  


Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






 $a = 8$

 $b = 2$


Complete


Non-singular

System 2






- $a + b = 10$
 
- $2a + 2b = 20$
  

Infinite solutions

 $a = 8$





 $b = 2$

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






$$\text{🍏 } a = 8$$

$$\text{🍌 } b = 2$$

Complete

Non-singular

System 2






- $a + b = 10$
 
- $2a + 2b = 20$
  

Infinite solutions

$$\text{🍏 } a = 8 \quad , \quad 7$$





$$\text{🍌 } b = 2 \quad 3$$

System 3


- $a + b = 10$
 
- $2a + 2b = 24$
  


Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






 $a = 8$

 $b = 2$


Complete

Non-singular

System 2






- $a + b = 10$
 
- $2a + 2b = 20$
  

Infinite solutions

 $a = 8, 7, 6$





 $b = 2, 3, 4$

System 3


- $a + b = 10$
 
- $2a + 2b = 24$
  


Systems of equations

System 1






- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:


 $a = 8$

 $b = 2$

System 2






- $a + b = 10$
 
- $2a + 2b = 20$
  

Infinite solutions

 $a = 8, 7, 6, \dots$

 $b = 2, 3, 4, \dots$

System 3





- $a + b = 10$
 
- $2a + 2b = 24$
  

Complete


Non-singular


Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:






 $a = 8$

 $b = 2$


Complete

Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  






Infinite solutions

 $a = 8, 7, 6, \dots$

 $b = 2, 3, 4, \dots$





Redundant

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique solution:

$$\begin{aligned} \text{🍏 } a &= 8 \\ \text{🍌 } b &= 2 \end{aligned}$$

Complete

Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  






Infinite solutions

$$\begin{aligned} \text{🍏 } a &= 8, 7, 6, \dots \\ \text{🍌 } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant





Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 






Unique solution:

$$\begin{aligned} \text{🍏 } a &= 8 \\ \text{🍌 } b &= 2 \end{aligned}$$

Complete

Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  






Infinite solutions

$$\begin{aligned} \text{🍏 } a &= 8, 7, 6, \dots \\ \text{🍌 } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

No solution

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 







Unique solution:

$$\begin{aligned} \text{apple } a &= 8 \\ \text{banana } b &= 2 \end{aligned}$$

Complete

Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
   







Infinite solutions

$$\begin{aligned} \text{apple } a &= 8, 7, 6, \dots \\ \text{banana } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
   

No solution

Contradictory

Systems of equations

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 






Unique solution:

$$\begin{aligned} \text{apple } a &= 8 \\ \text{banana } b &= 2 \end{aligned}$$

Complete

Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
  






Infinite solutions

$$\begin{aligned} \text{apple } a &= 8, 7, 6, \dots \\ \text{banana } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
  

No solution

Contradictory

Singular

What is a linear equation?

Linear

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



Non-linear

$$a^2 + b^2 = 10$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

$$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$$



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System of Linear Equations

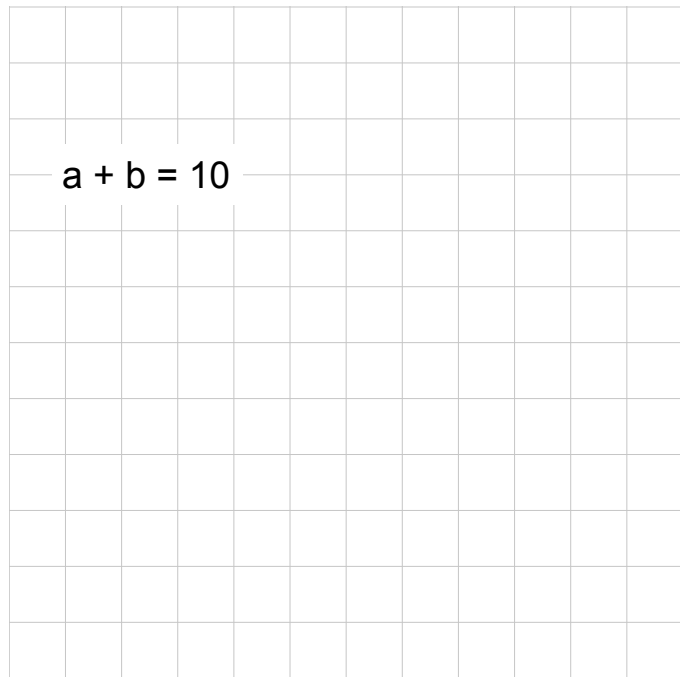
System of equations as lines

Linear equation \rightarrow line

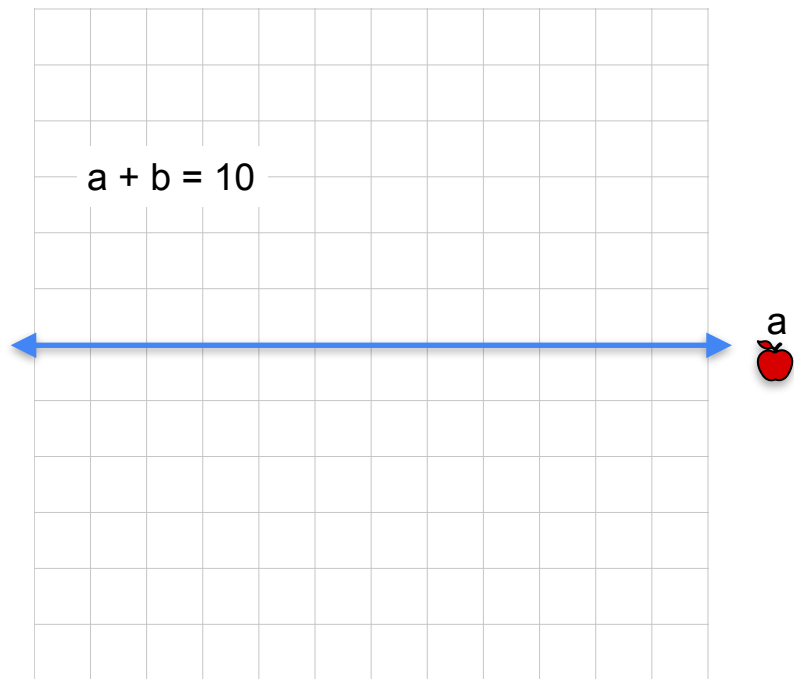
Linear equation \rightarrow line

$$a + b = 10$$

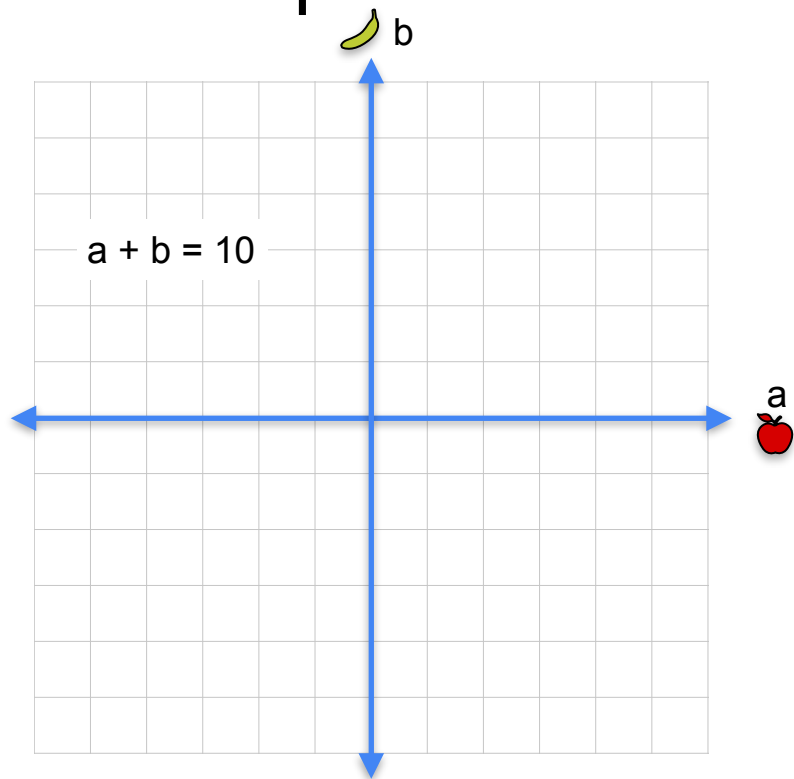
Linear equation \rightarrow line



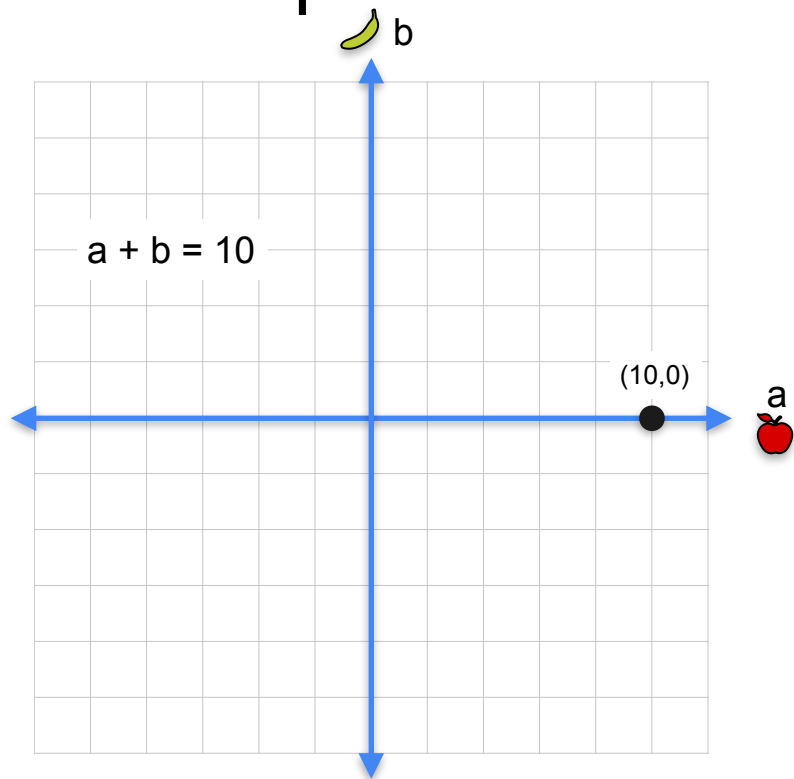
Linear equation \rightarrow line



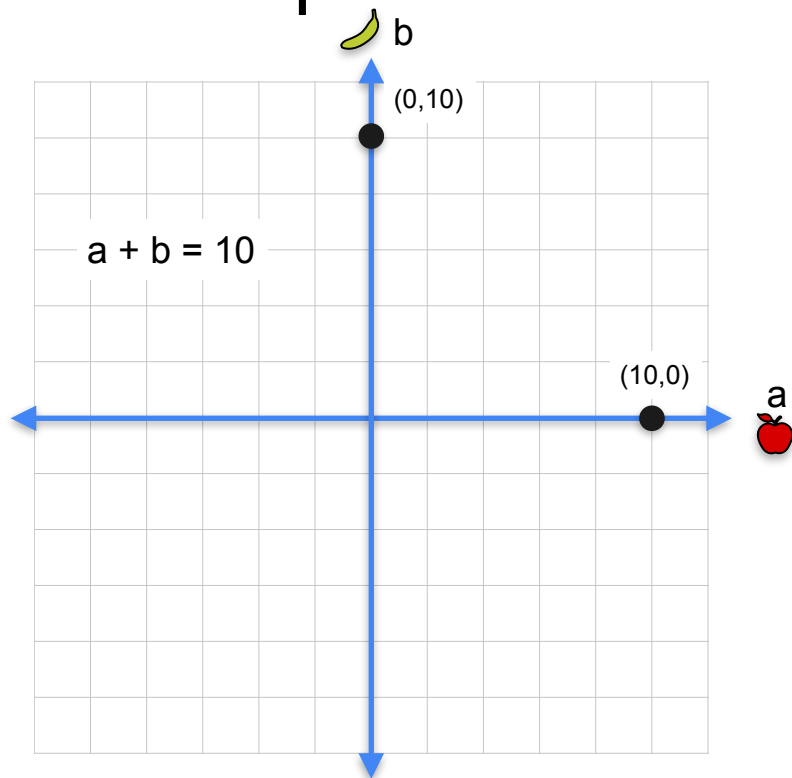
Linear equation \rightarrow line



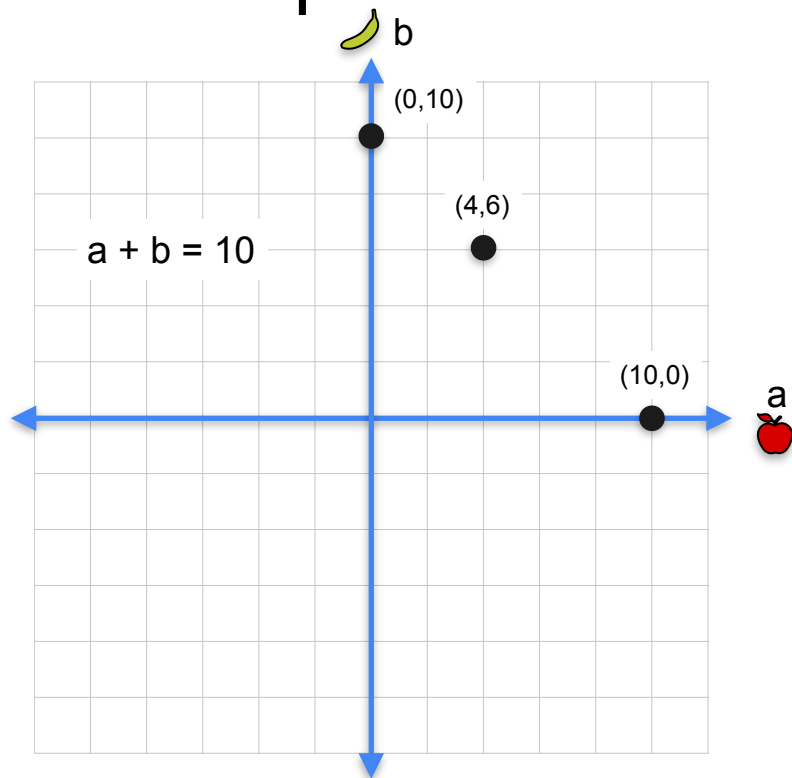
Linear equation \rightarrow line



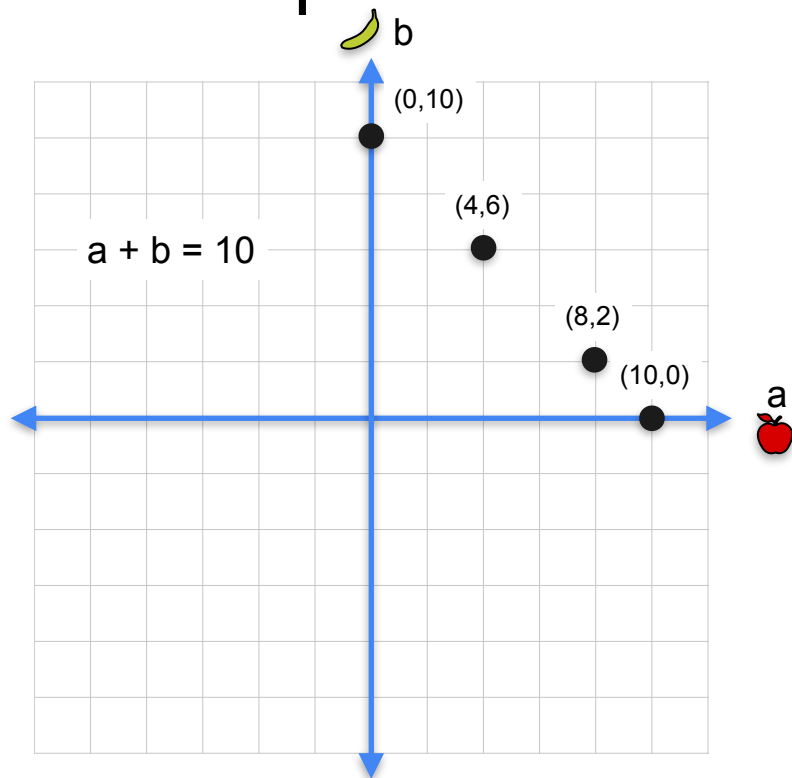
Linear equation \rightarrow line



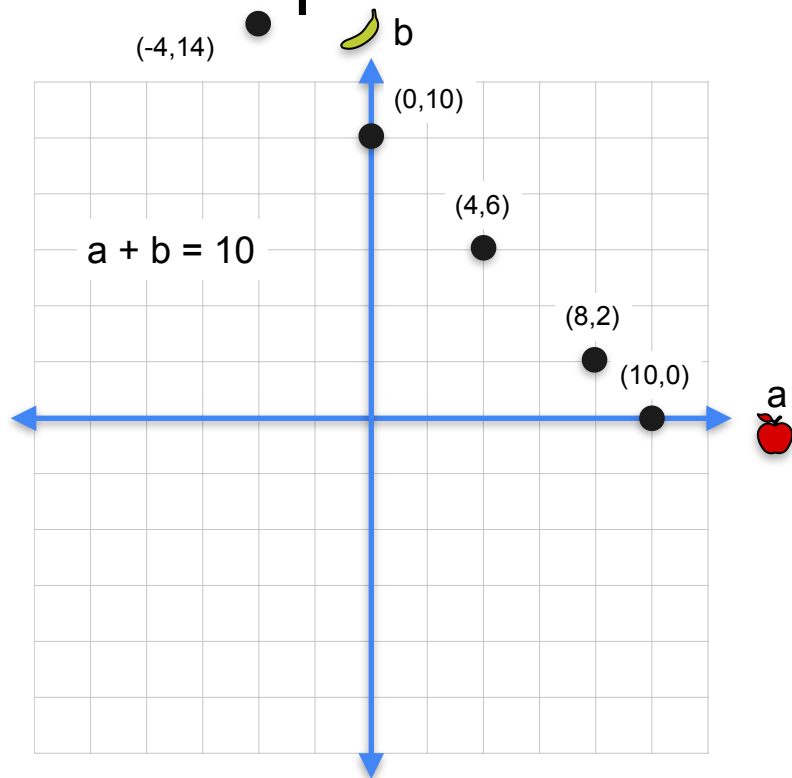
Linear equation \rightarrow line



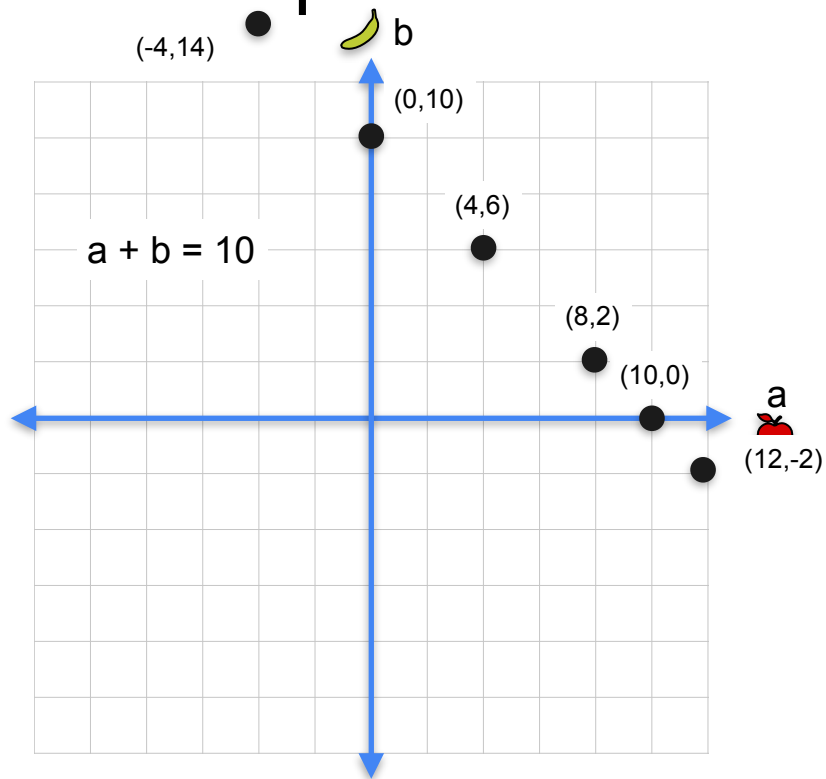
Linear equation \rightarrow line



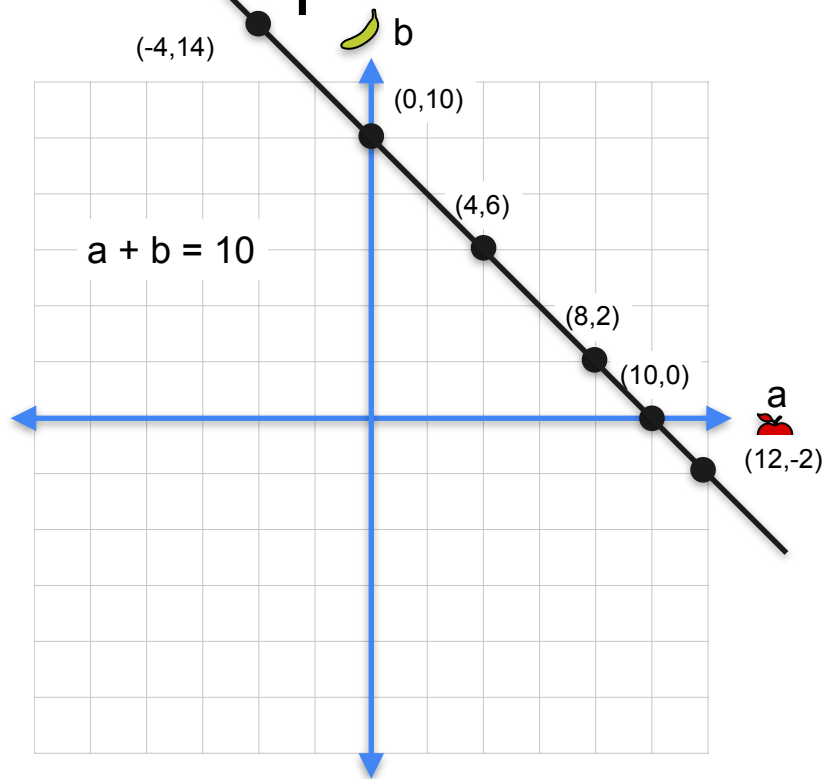
Linear equation \rightarrow line



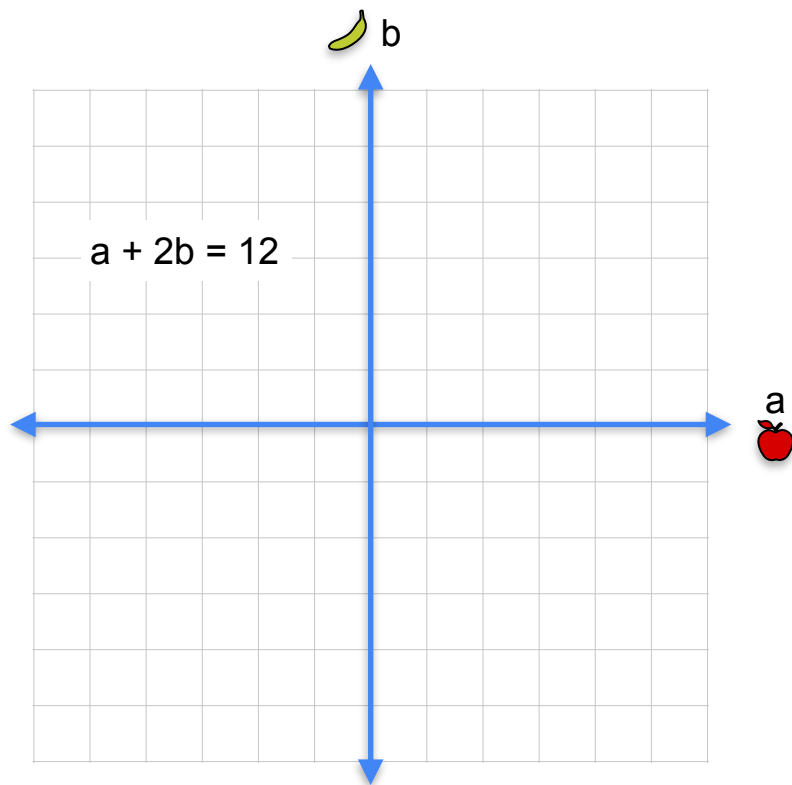
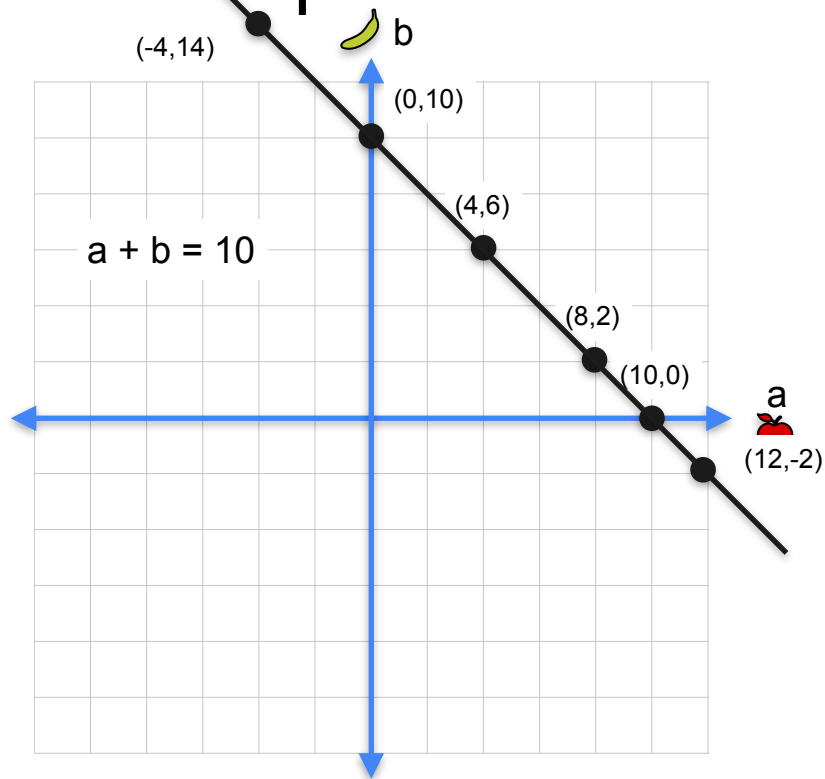
Linear equation \rightarrow line



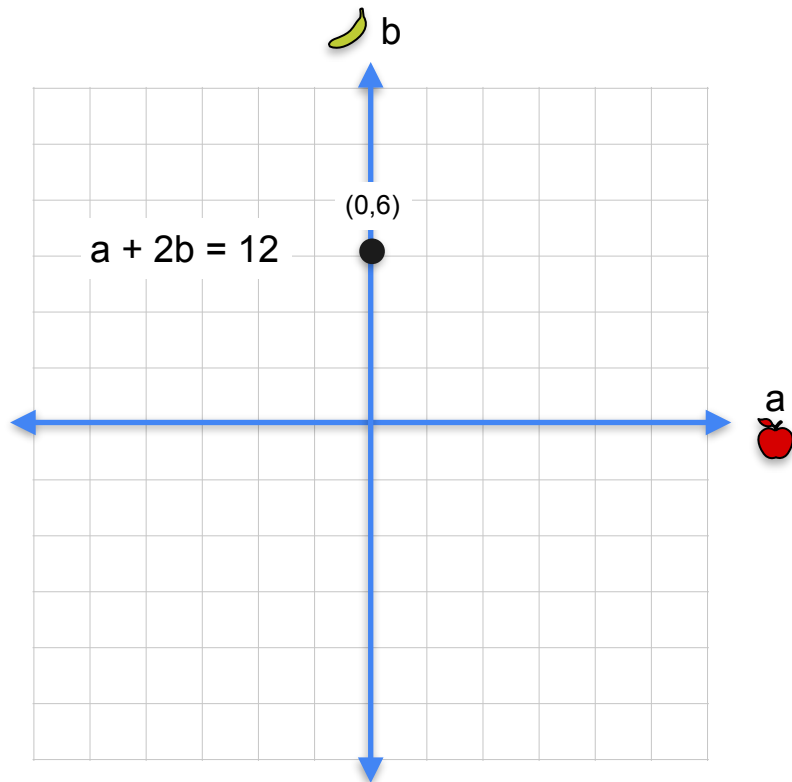
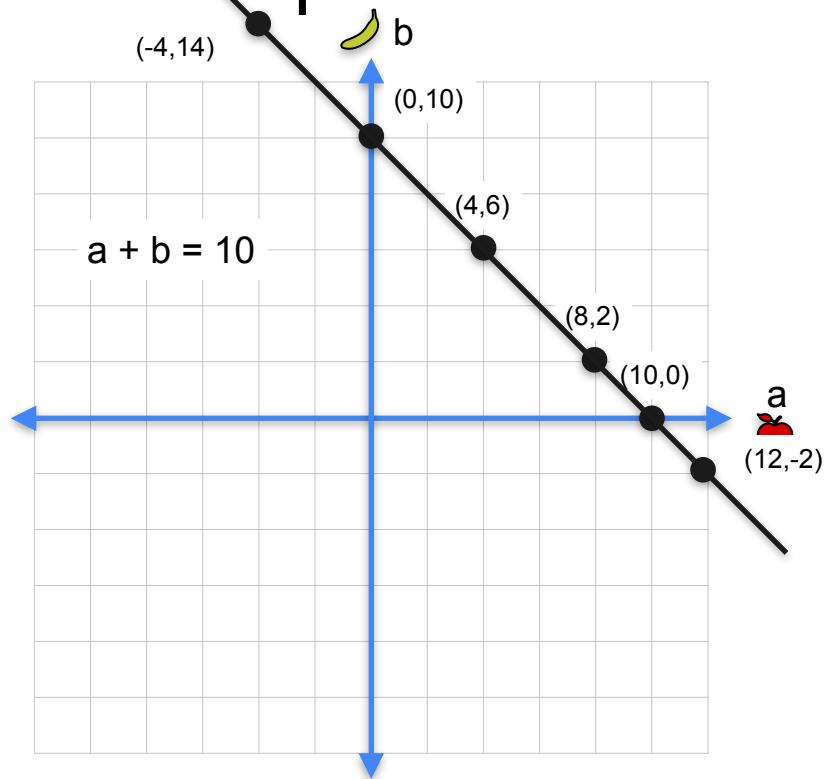
Linear equation \rightarrow line



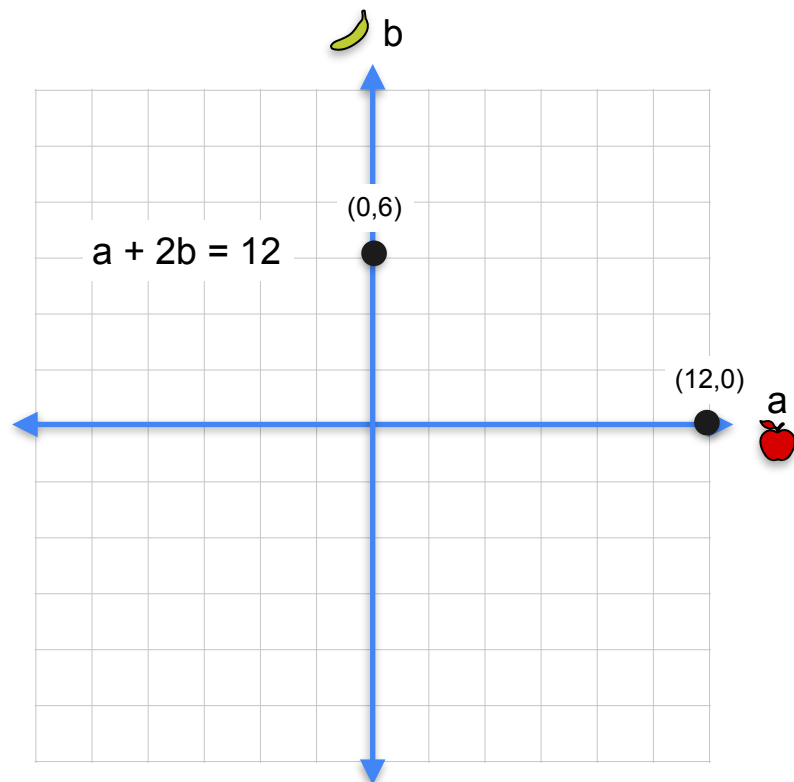
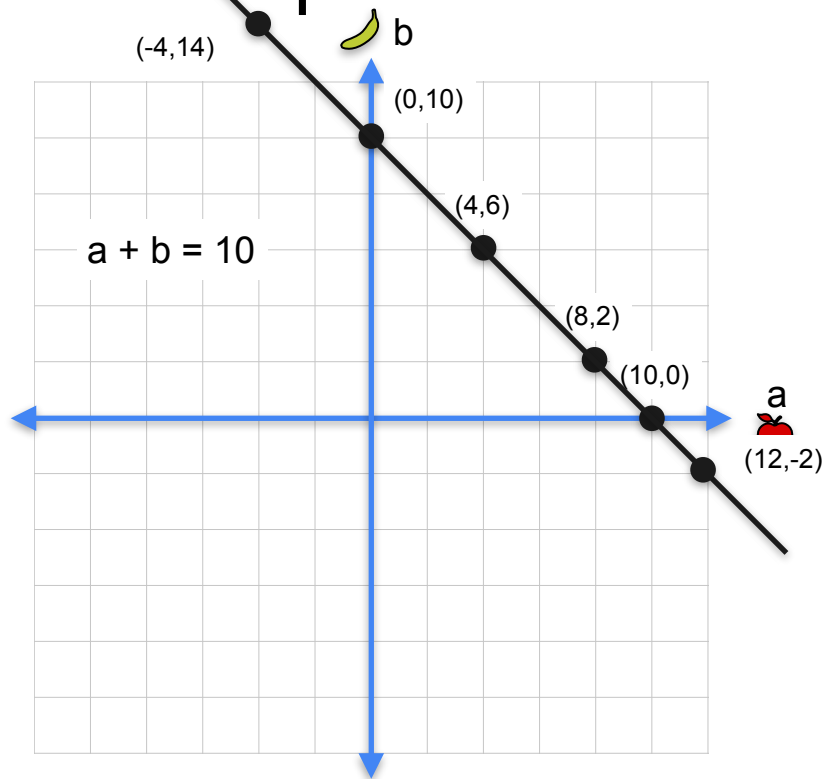
Linear equation \rightarrow line



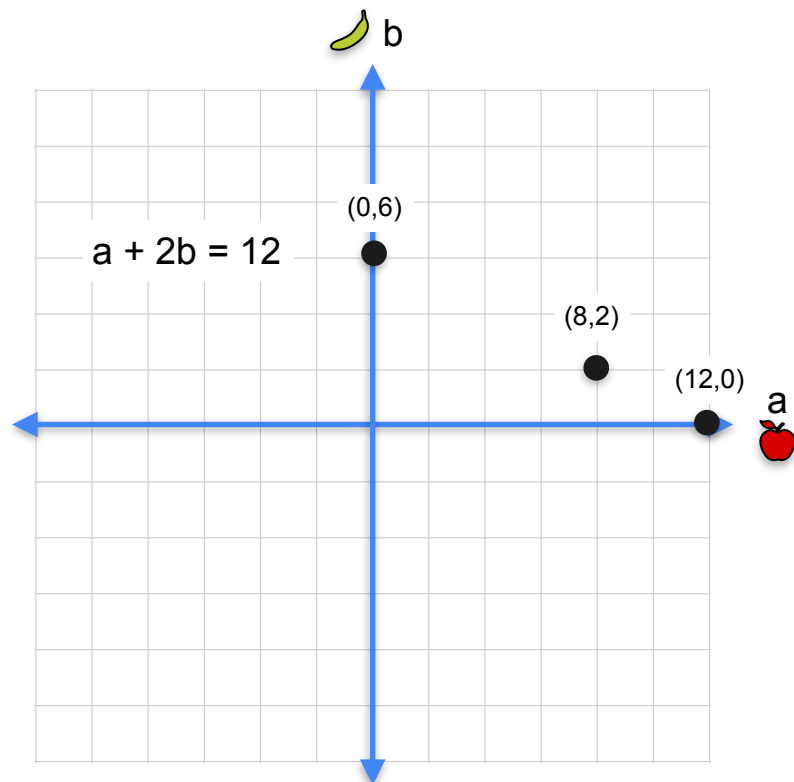
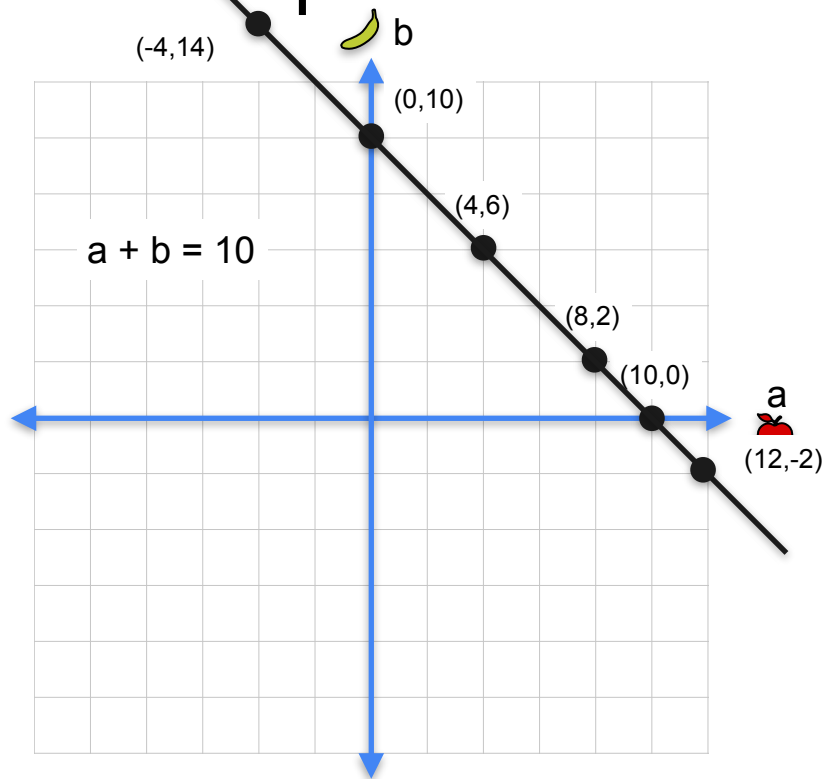
Linear equation \rightarrow line



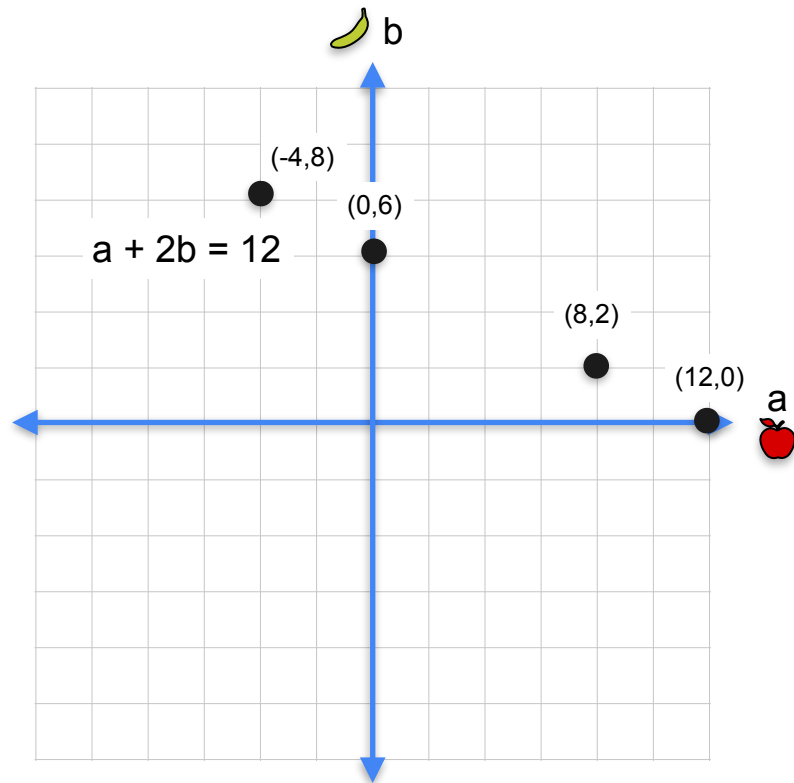
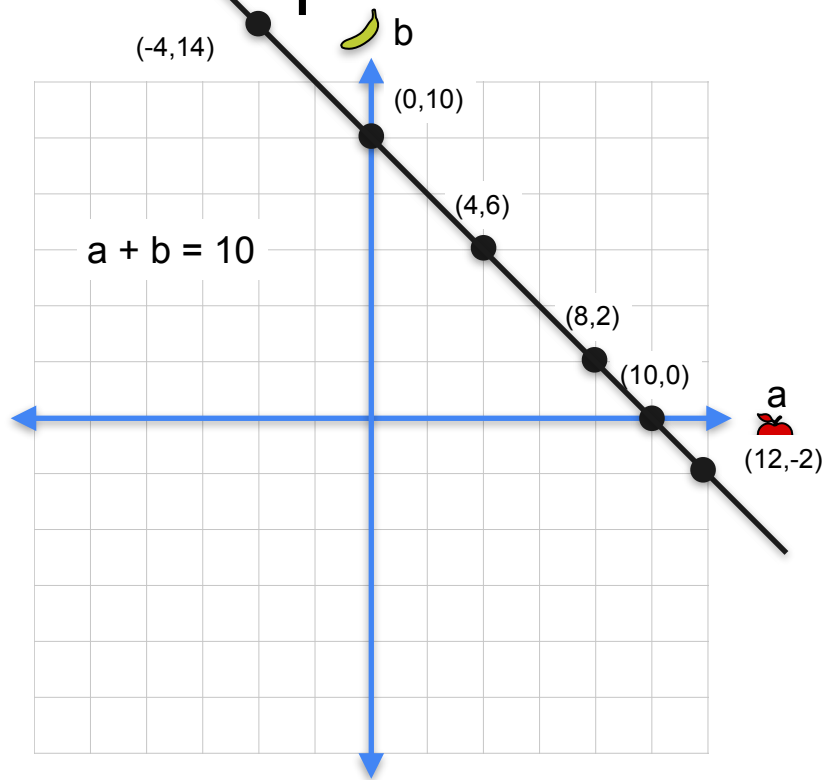
Linear equation \rightarrow line



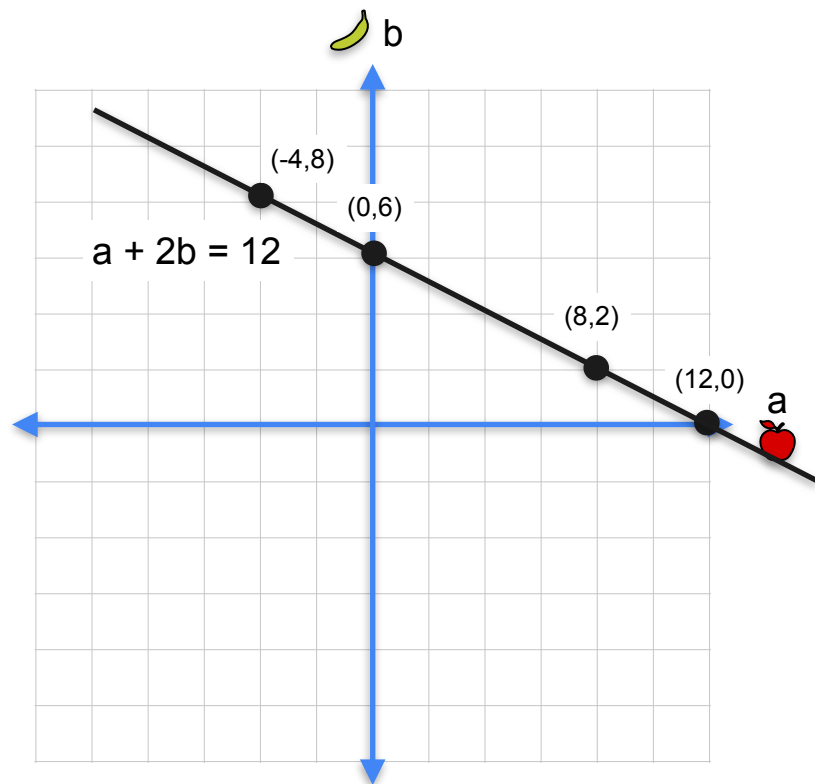
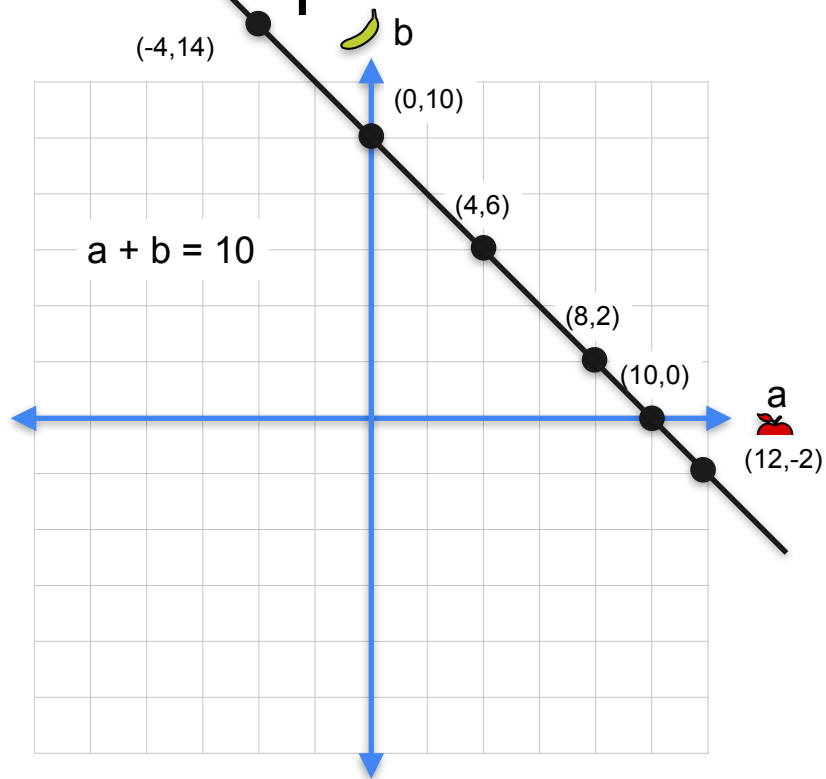
Linear equation \rightarrow line



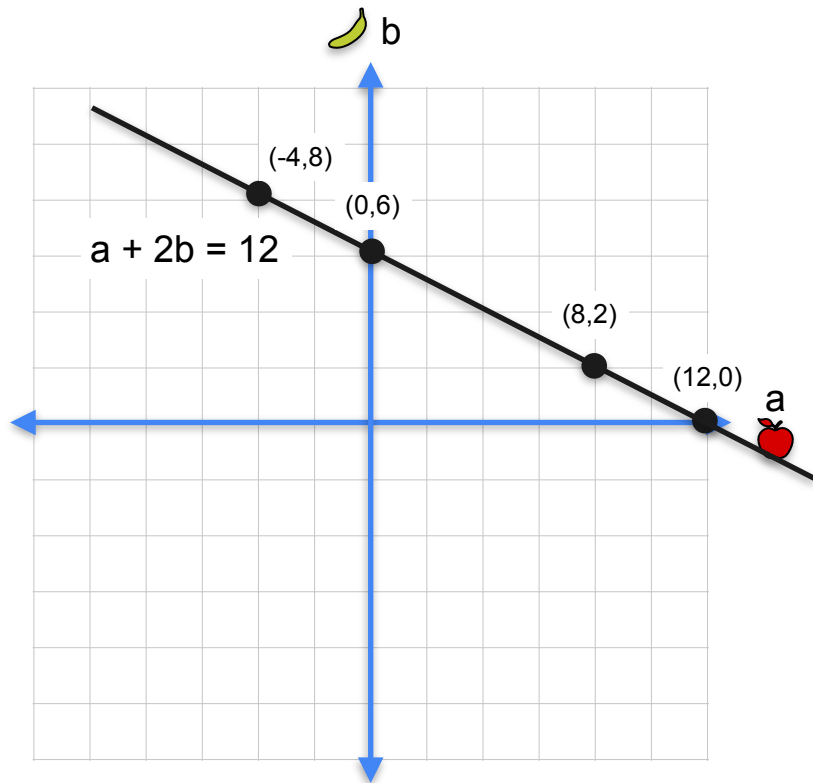
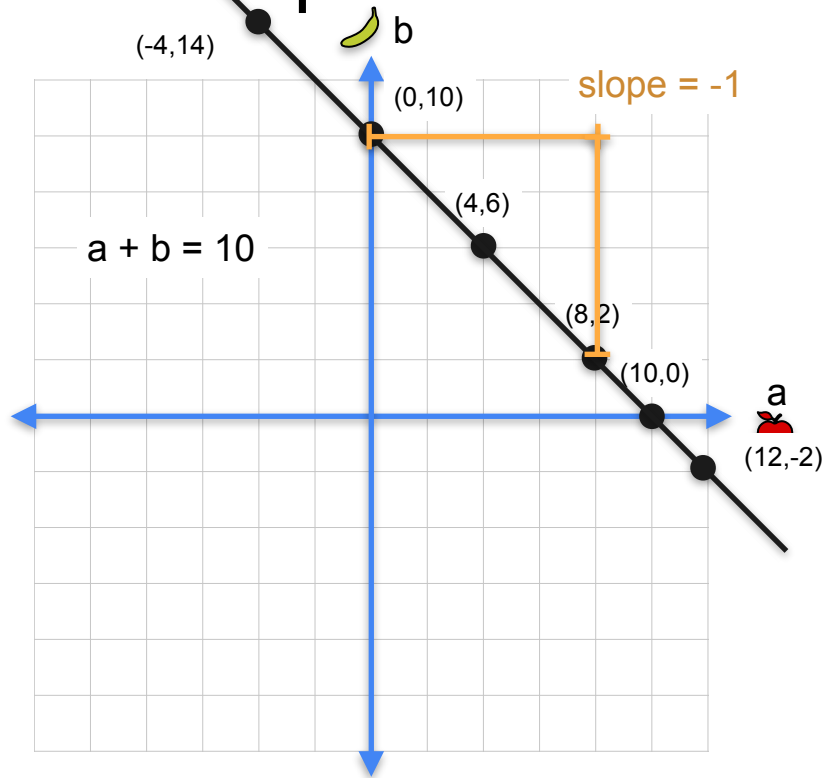
Linear equation \rightarrow line



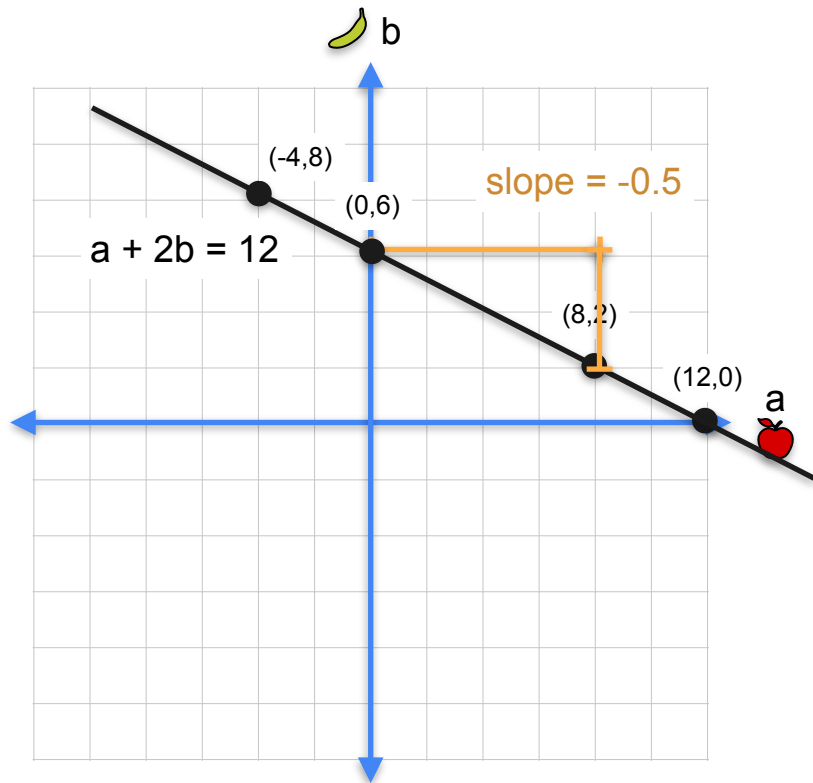
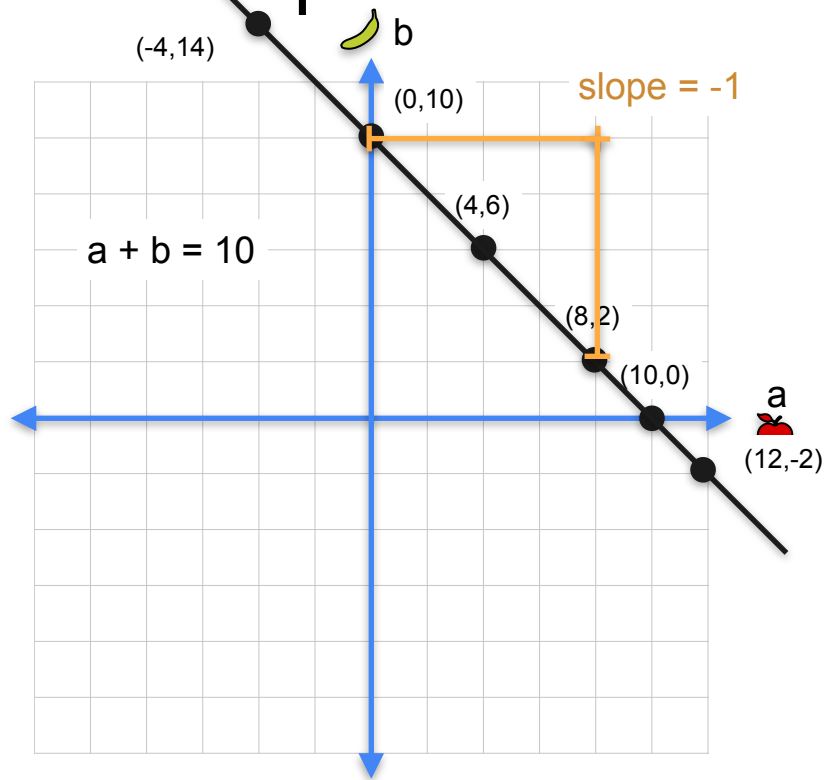
Linear equation \rightarrow line



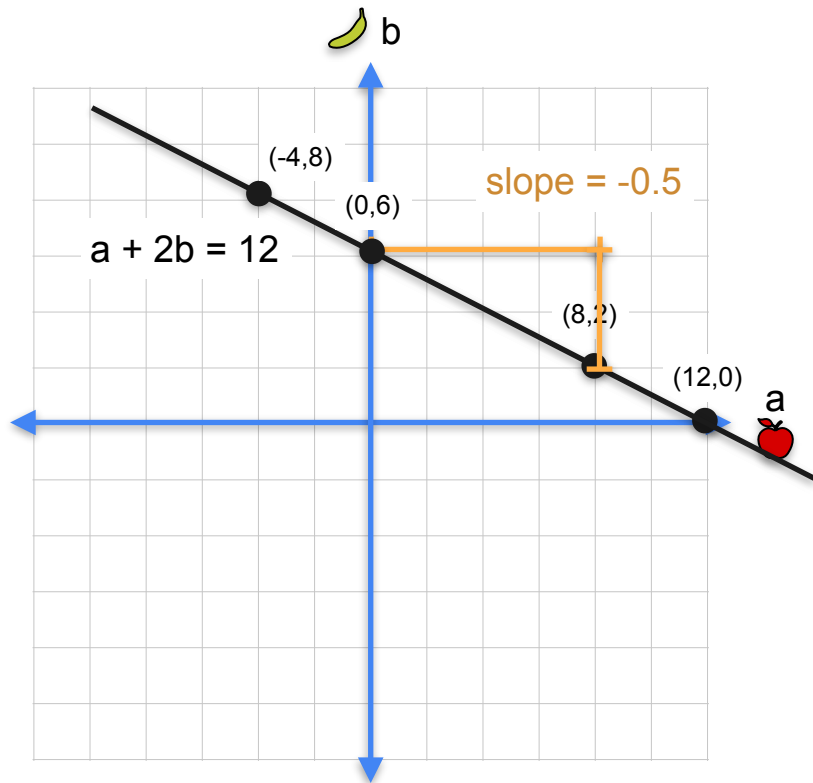
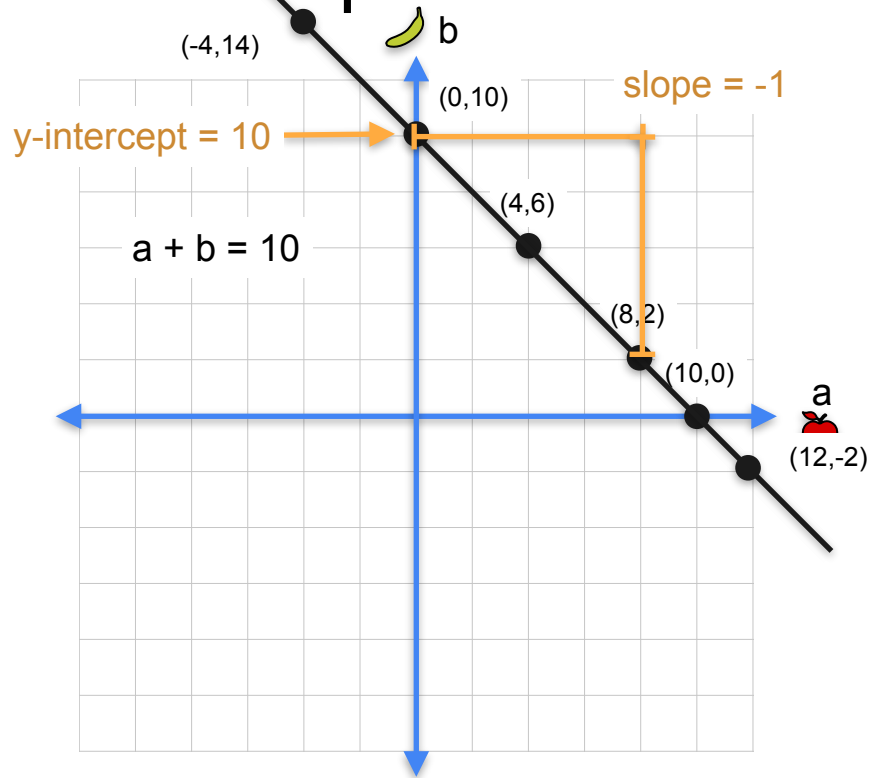
Linear equation \rightarrow line



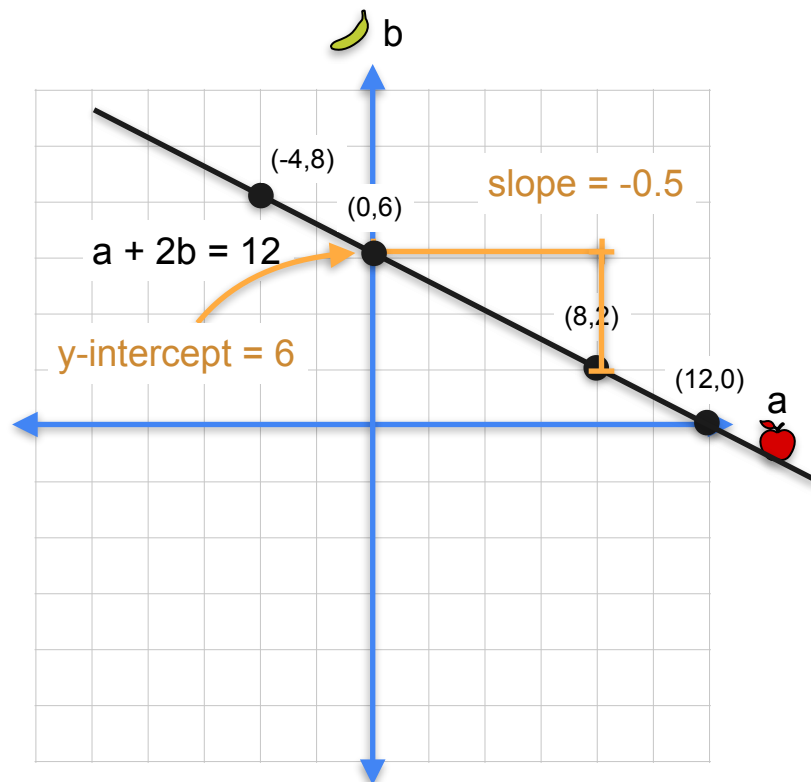
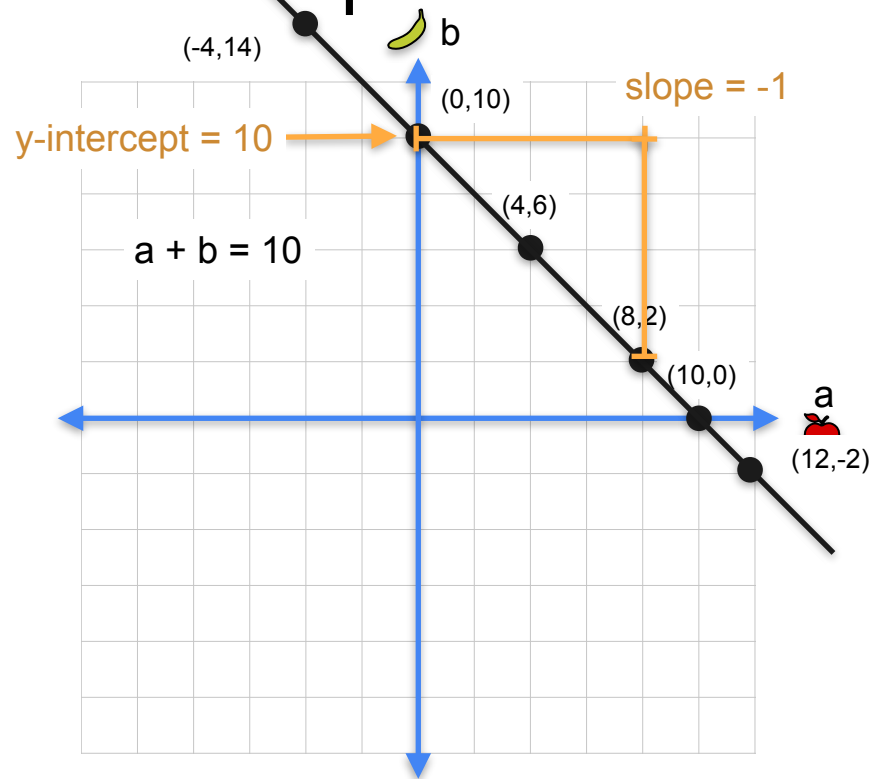
Linear equation \rightarrow line



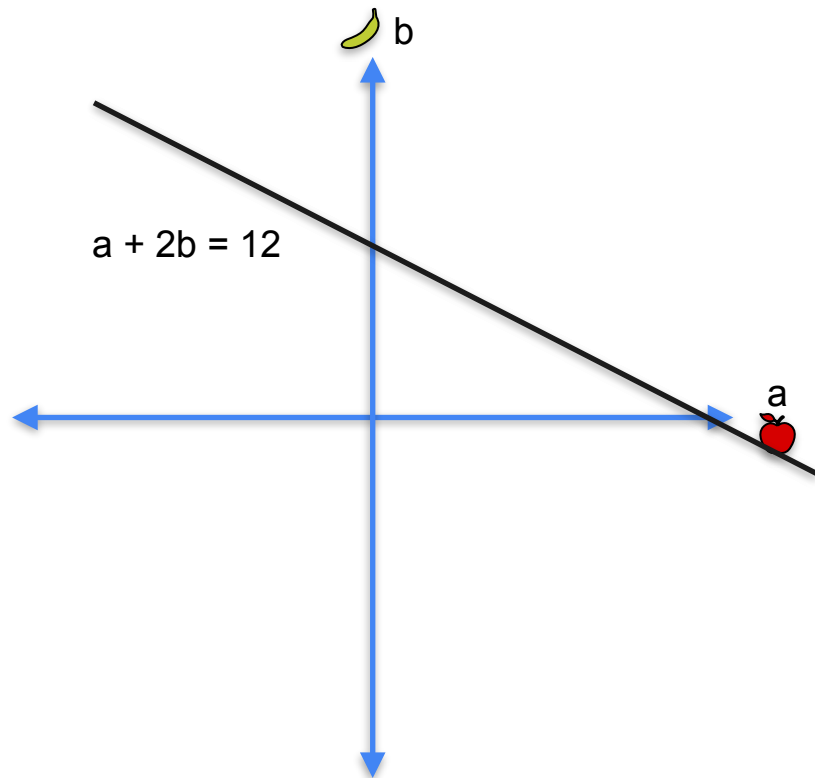
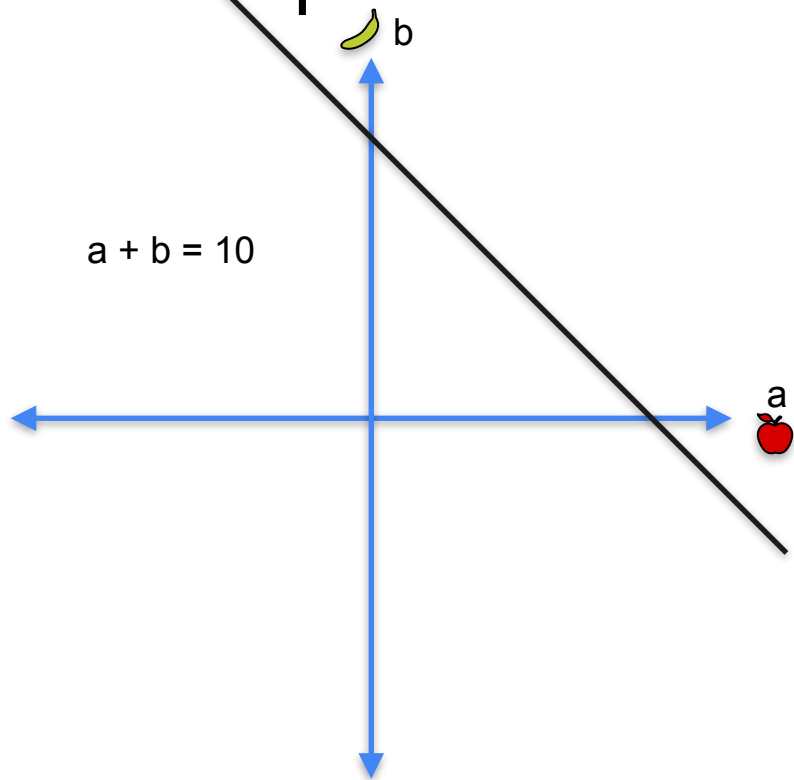
Linear equation \rightarrow line



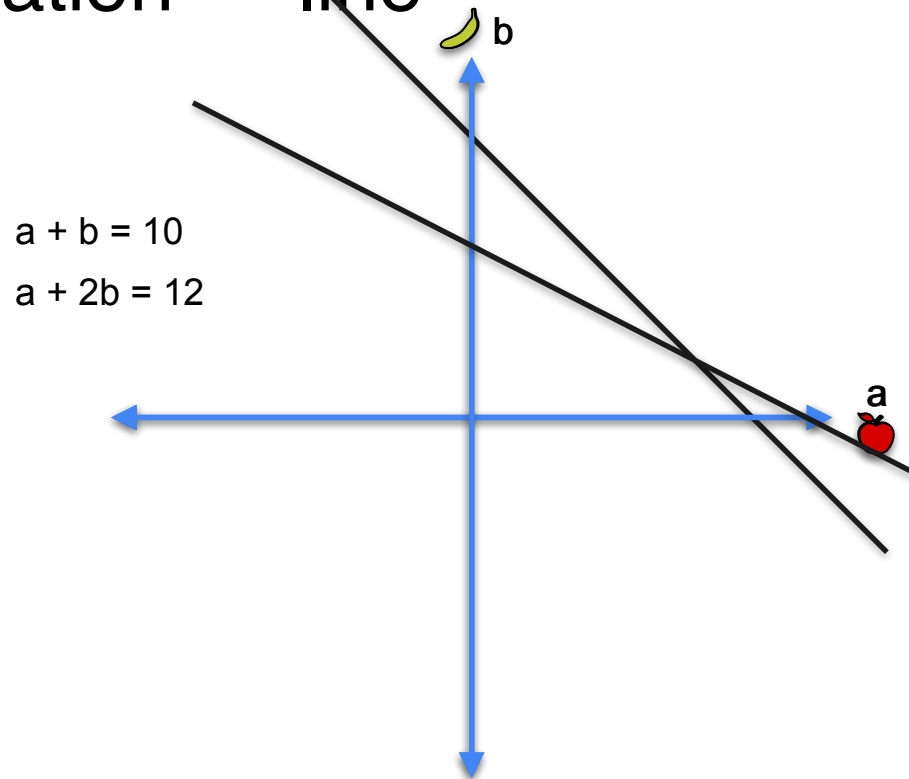
Linear equation \rightarrow line



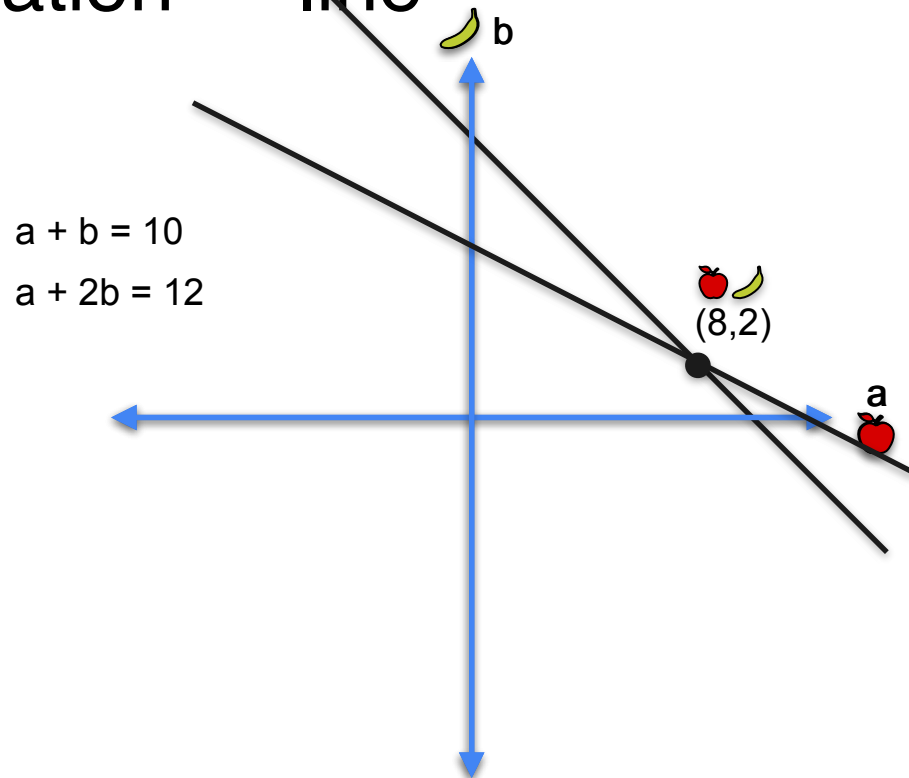
Linear equation \rightarrow line



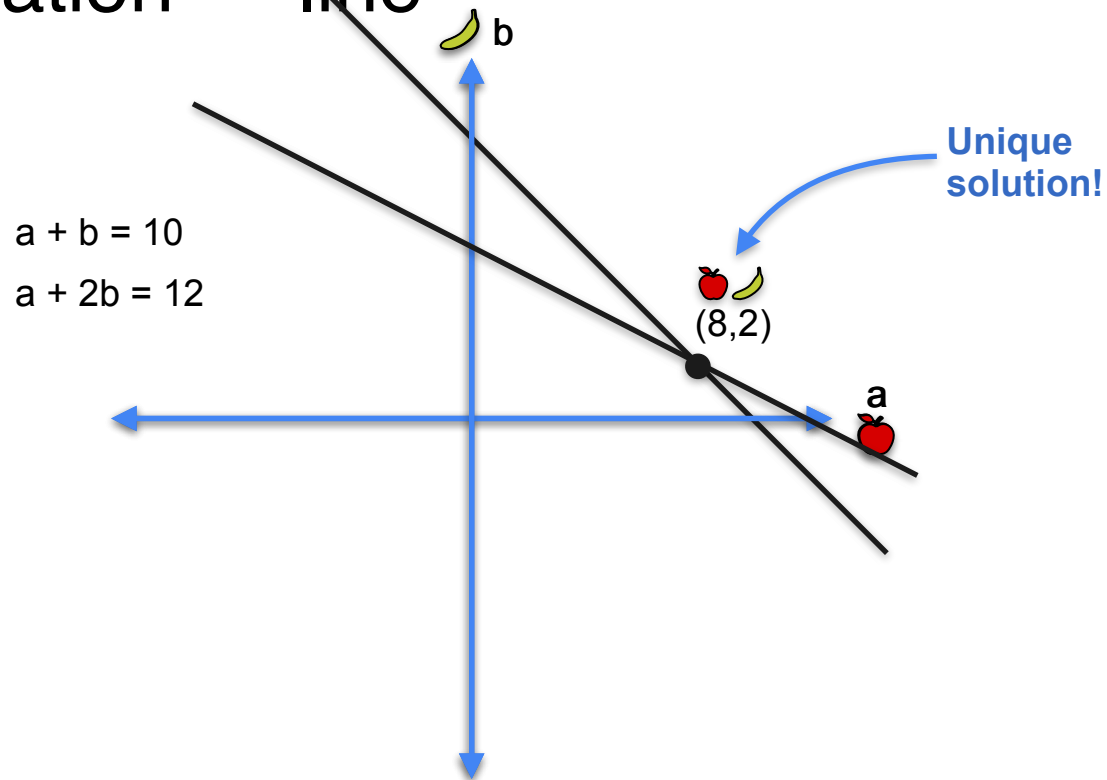
Linear equation \rightarrow line



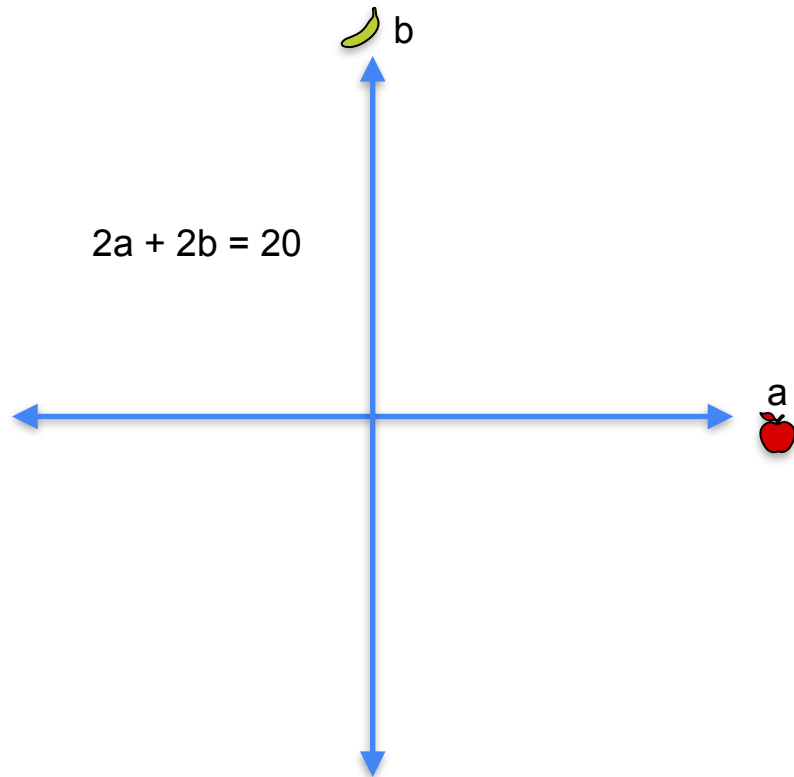
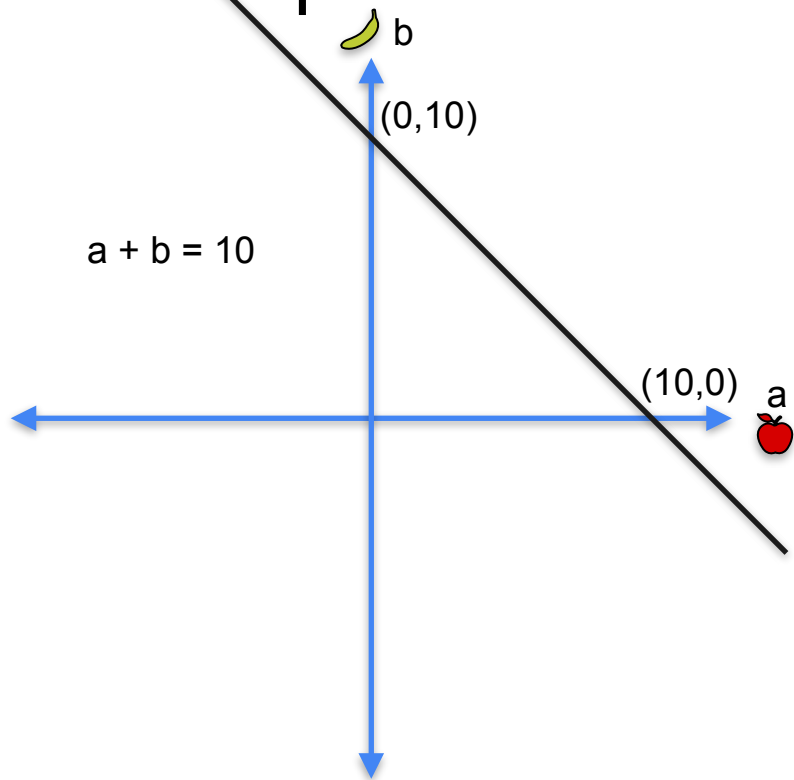
Linear equation \rightarrow line



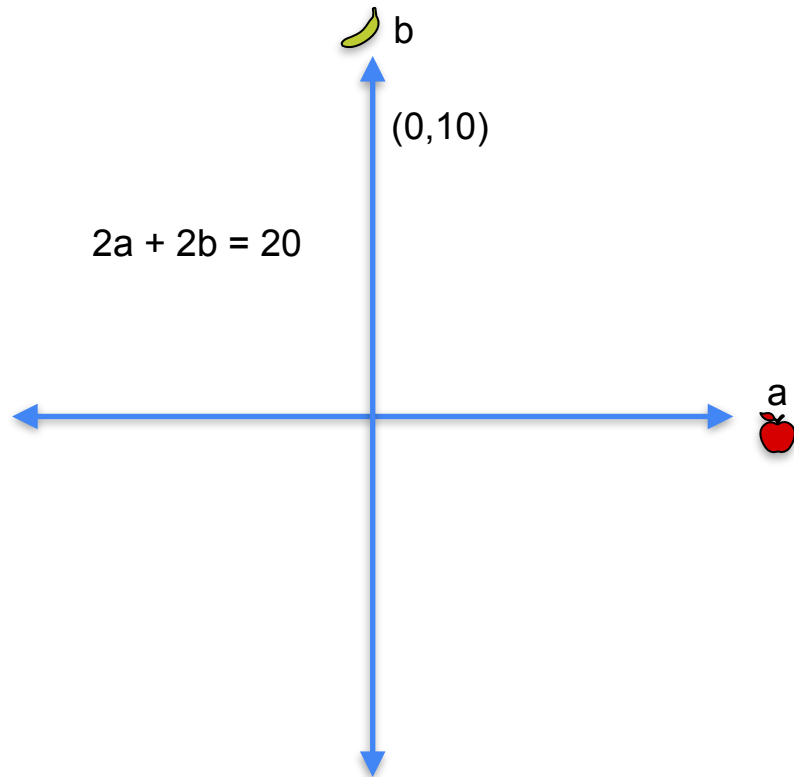
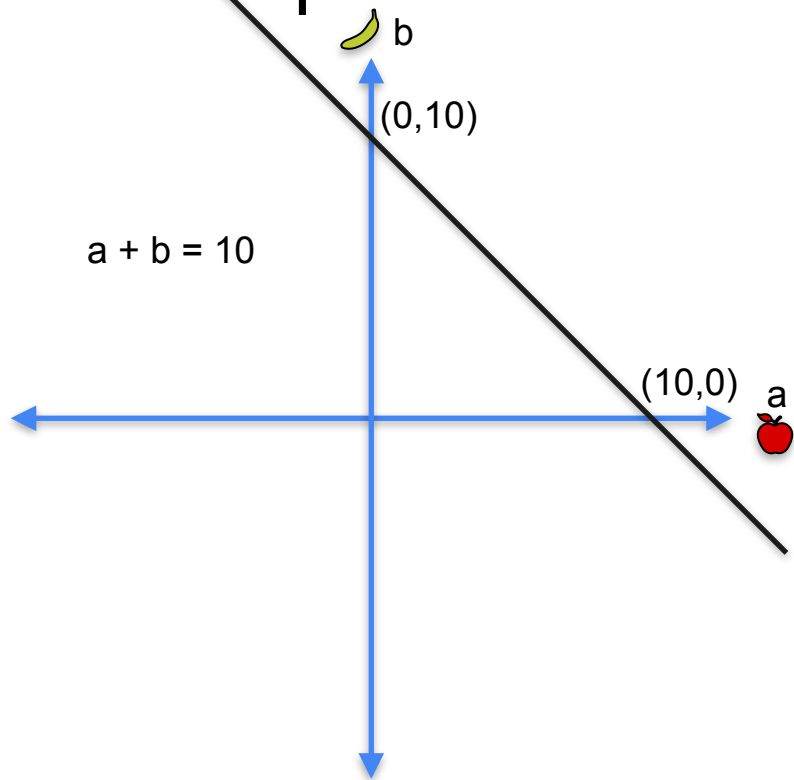
Linear equation \rightarrow line



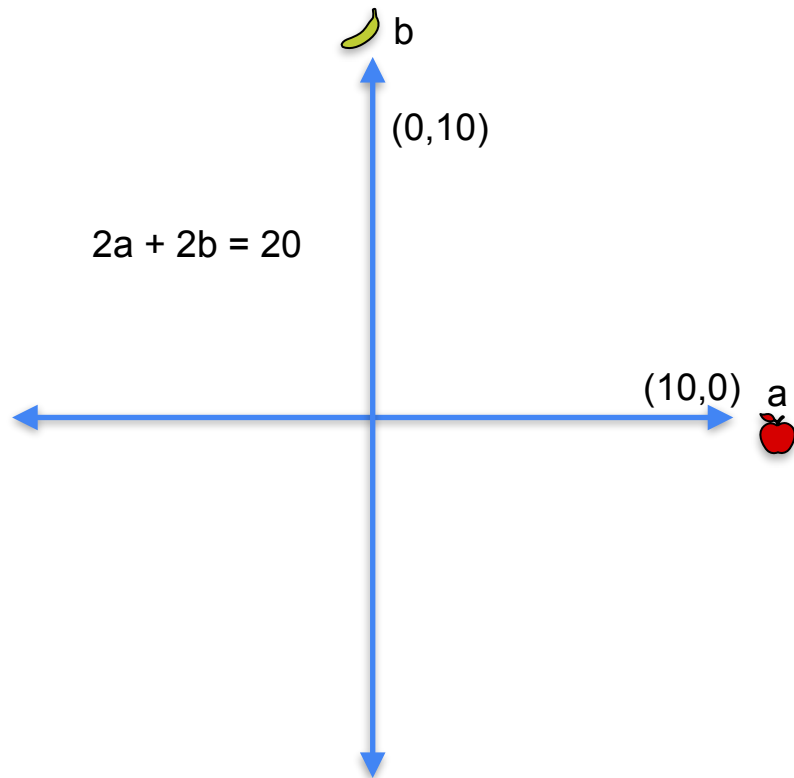
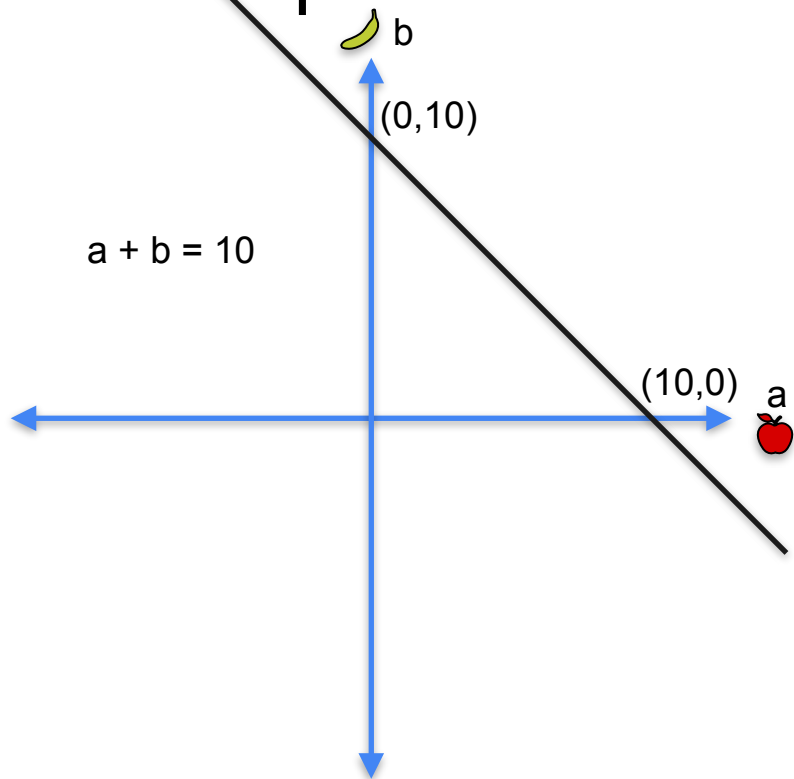
Linear equation \rightarrow line



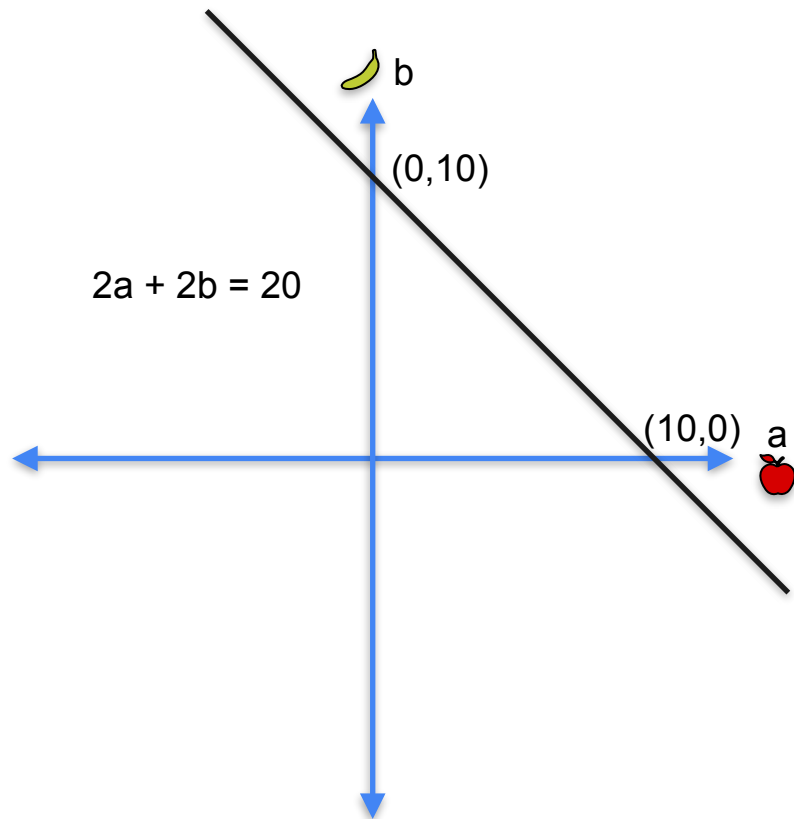
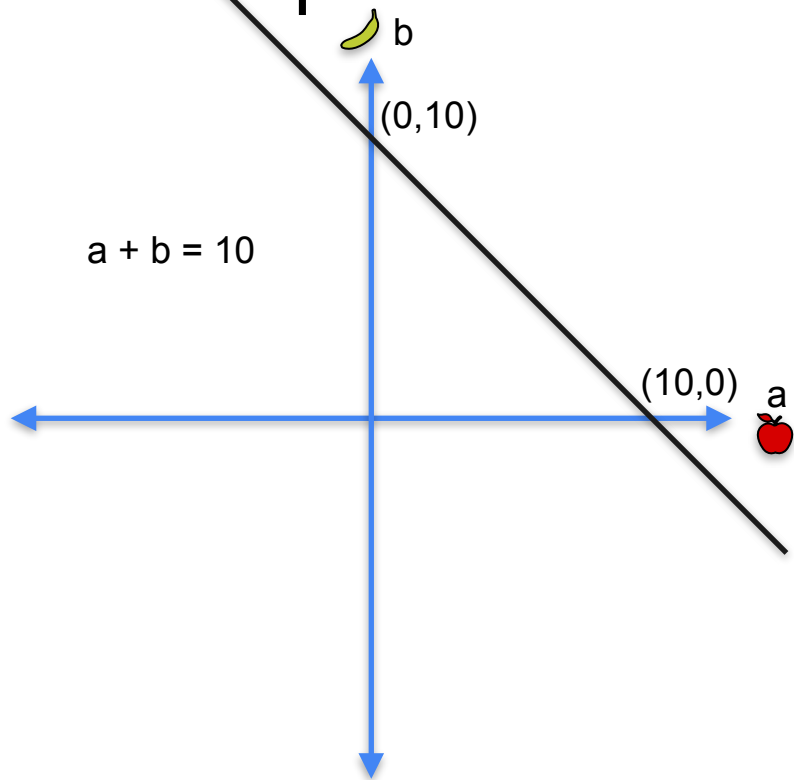
Linear equation \rightarrow line



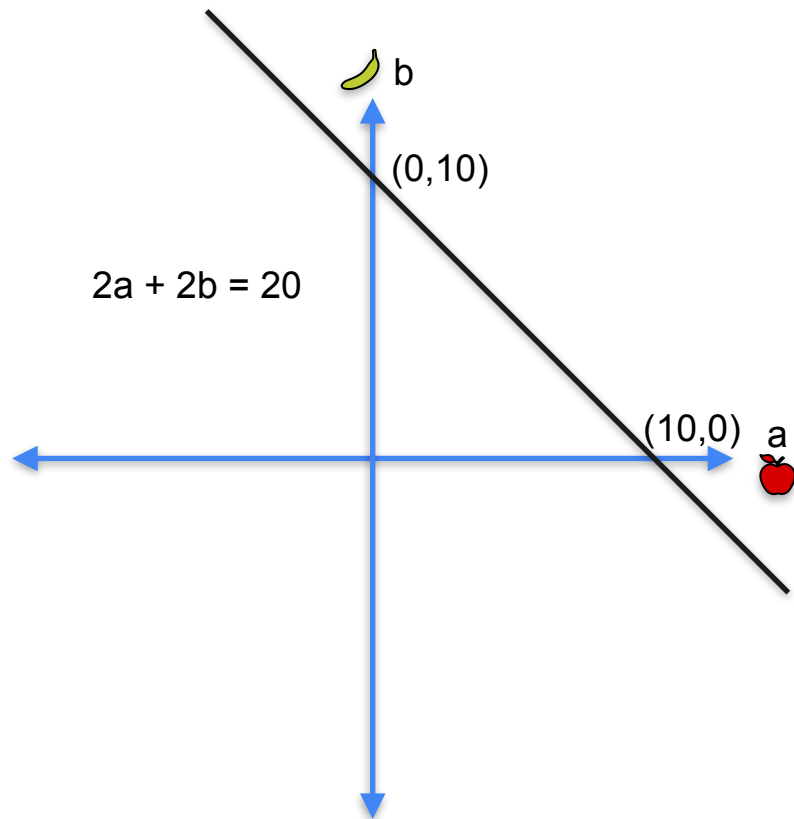
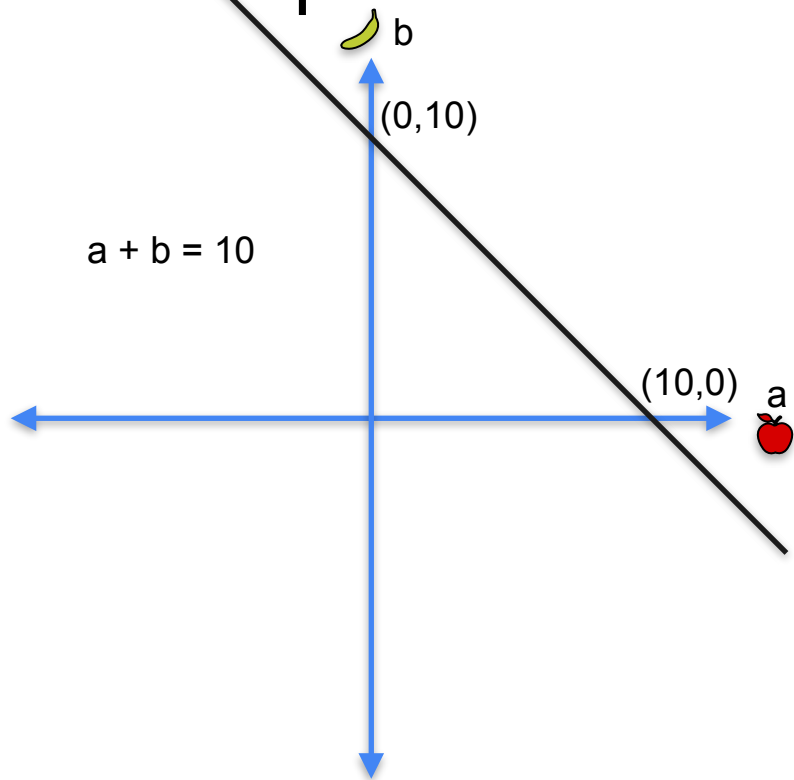
Linear equation \rightarrow line



Linear equation \rightarrow line



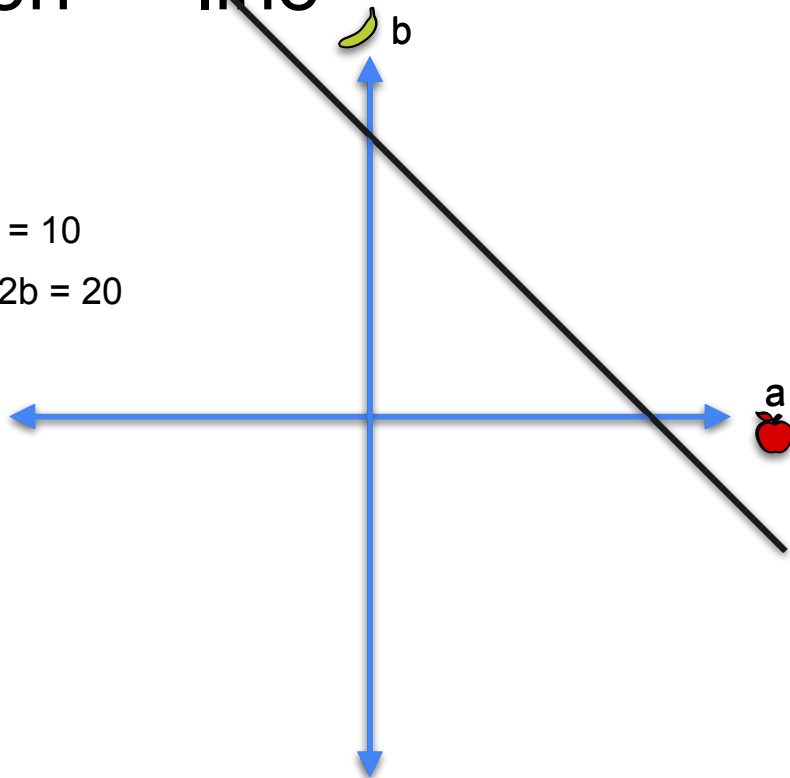
Linear equation \rightarrow line



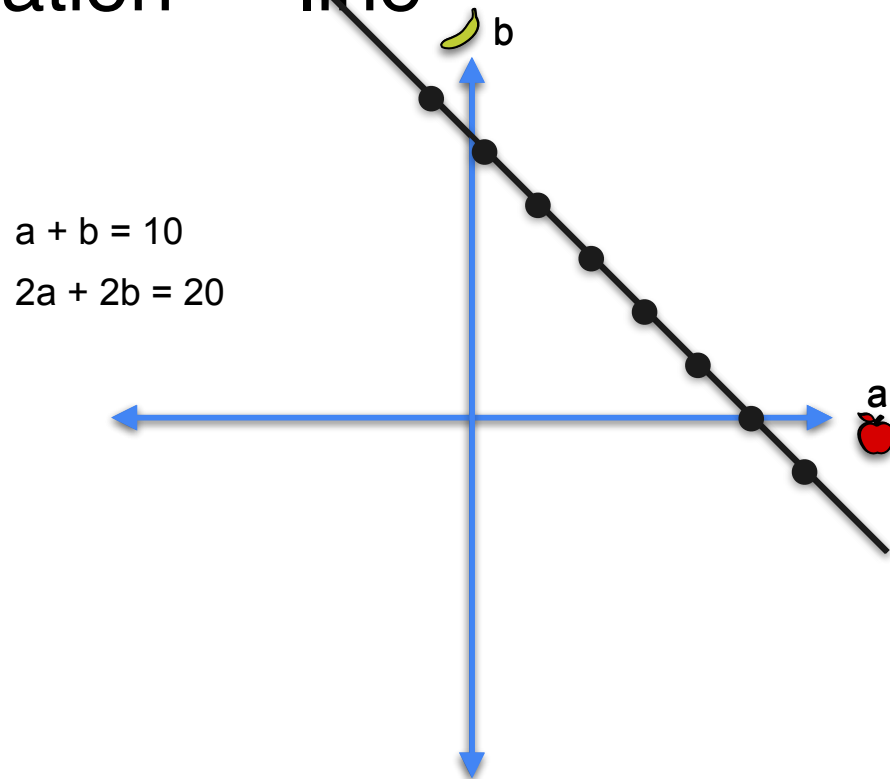
Linear equation \rightarrow line

$$a + b = 10$$

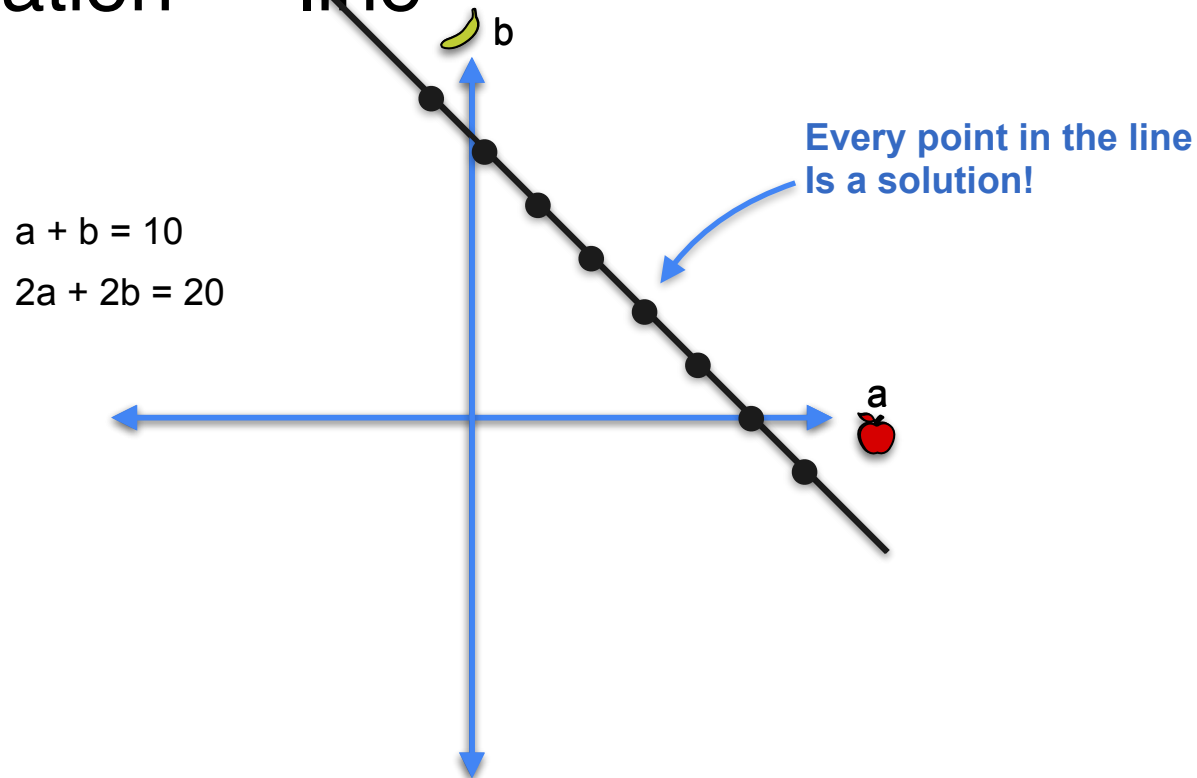
$$2a + 2b = 20$$



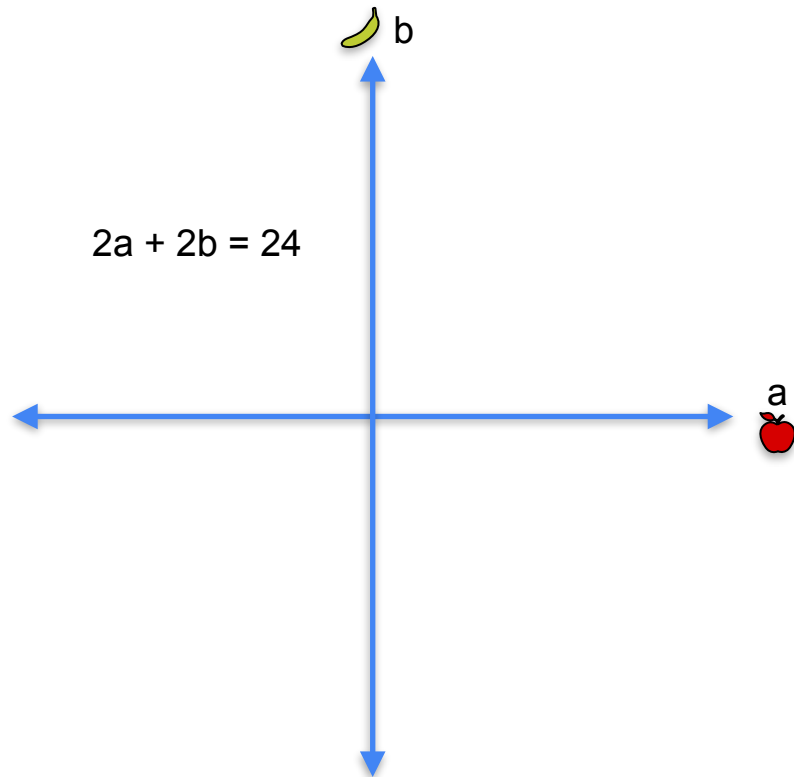
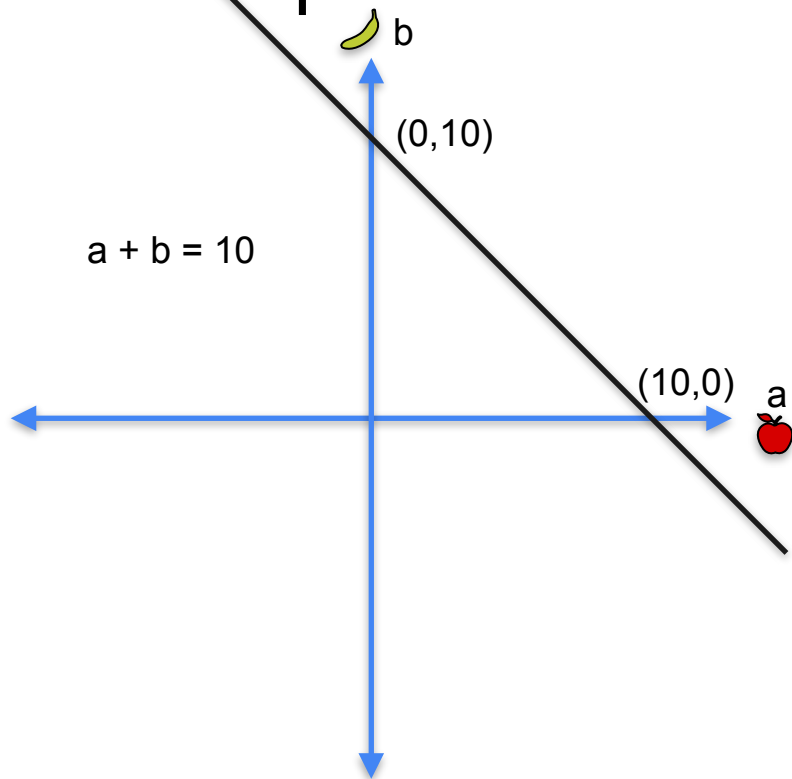
Linear equation \rightarrow line



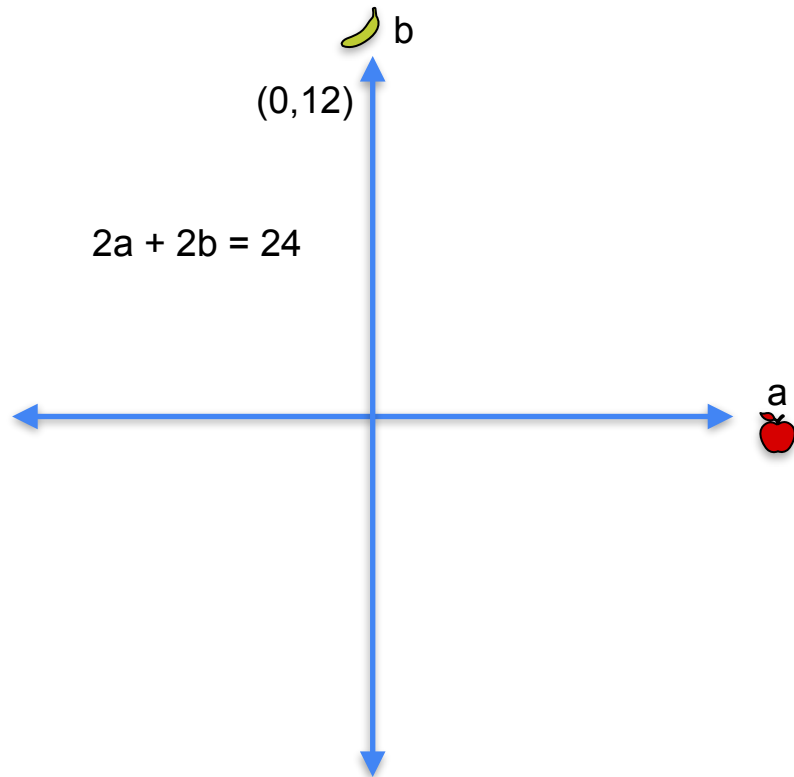
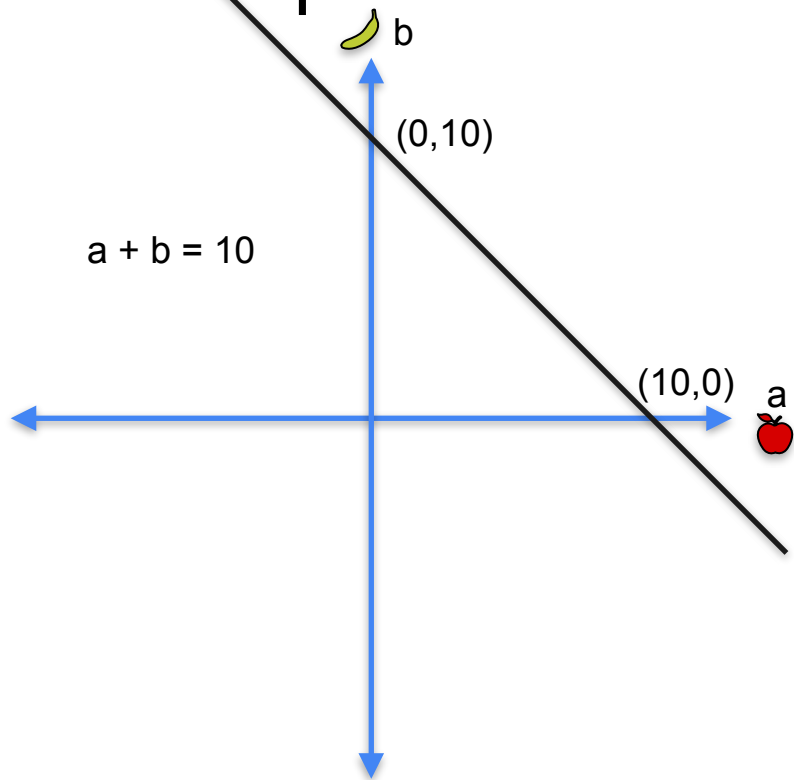
Linear equation \rightarrow line



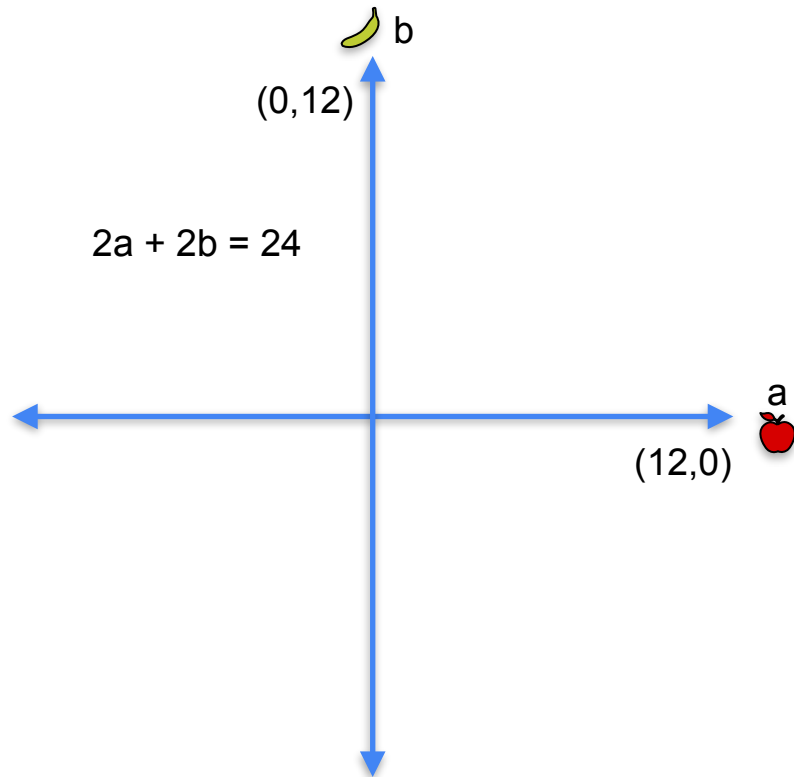
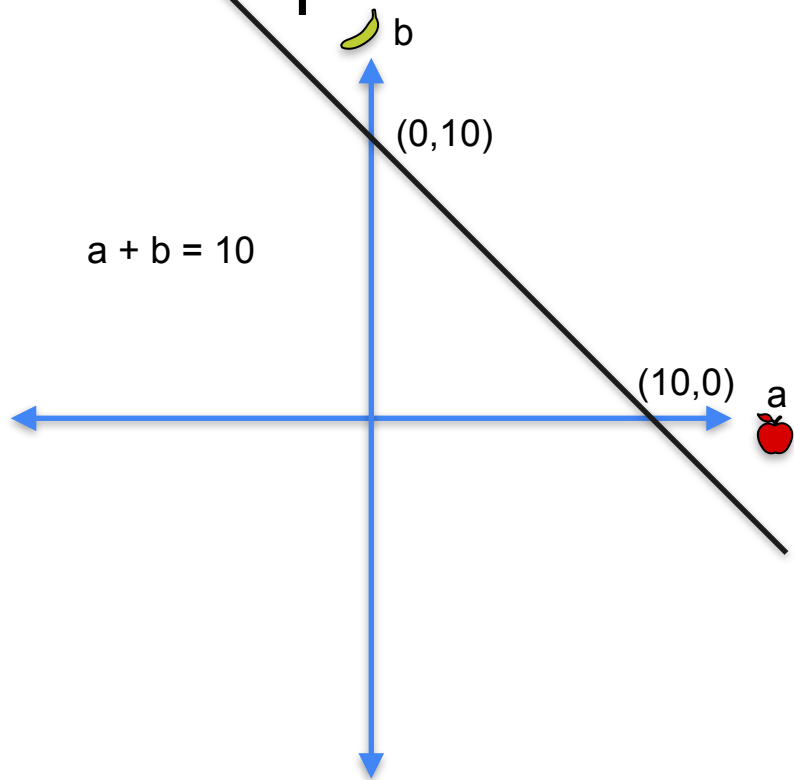
Linear equation \rightarrow line



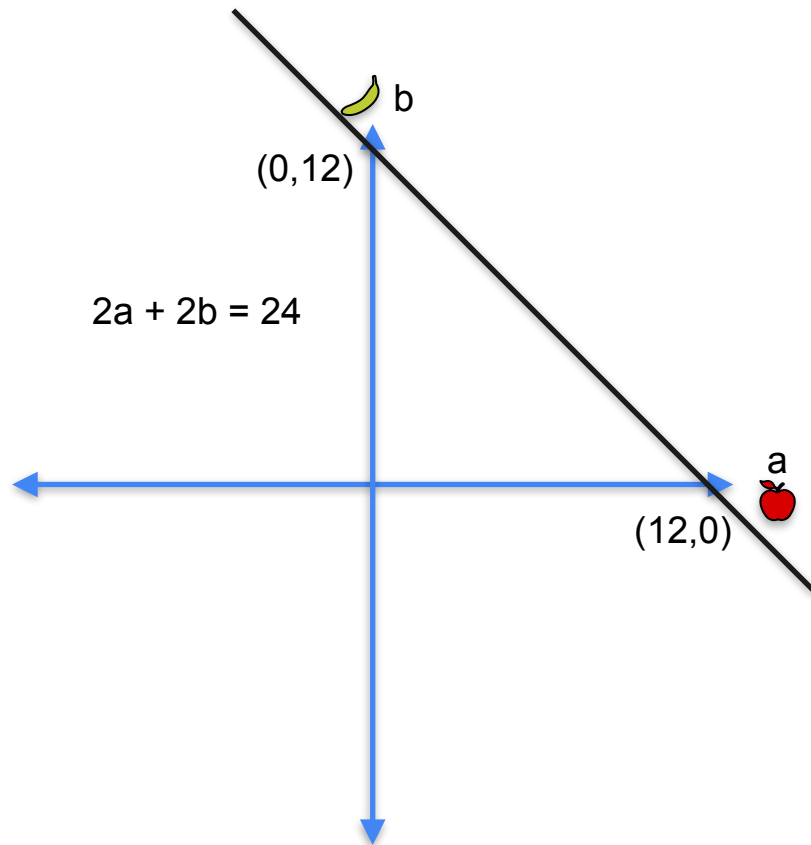
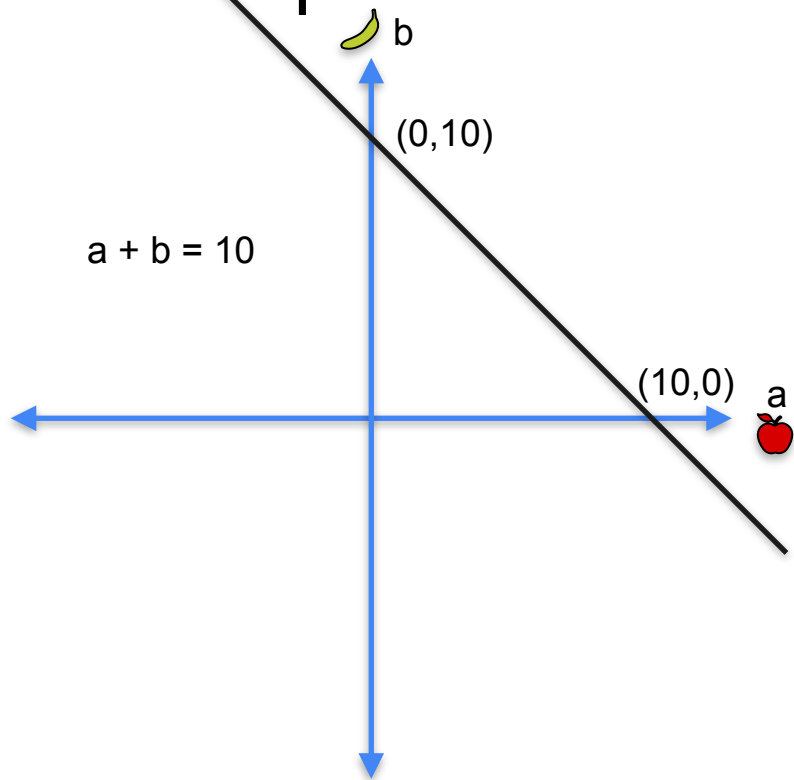
Linear equation \rightarrow line



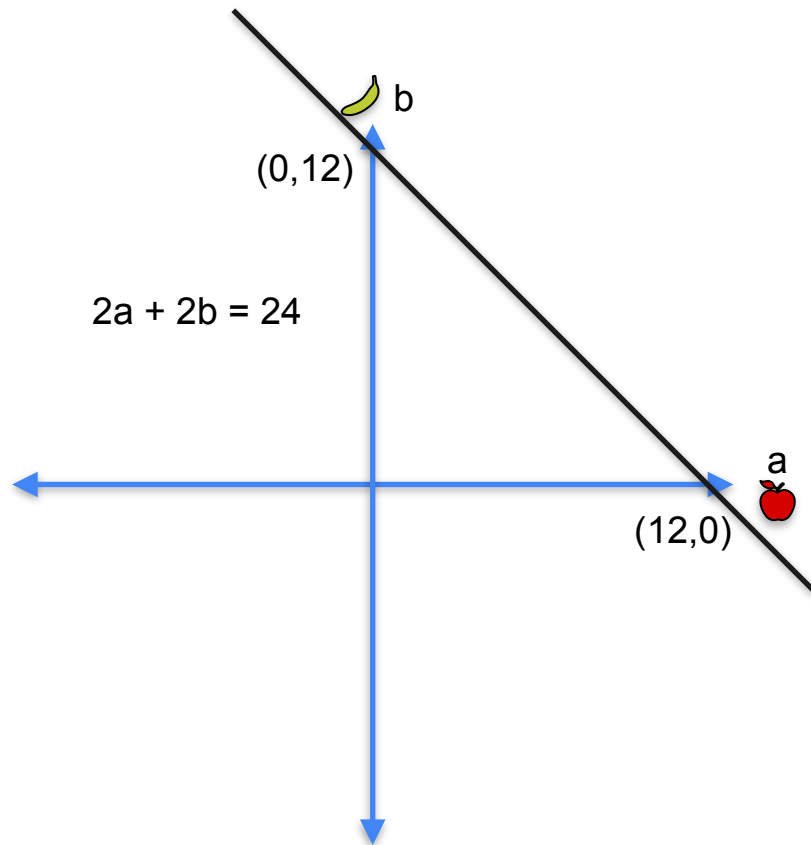
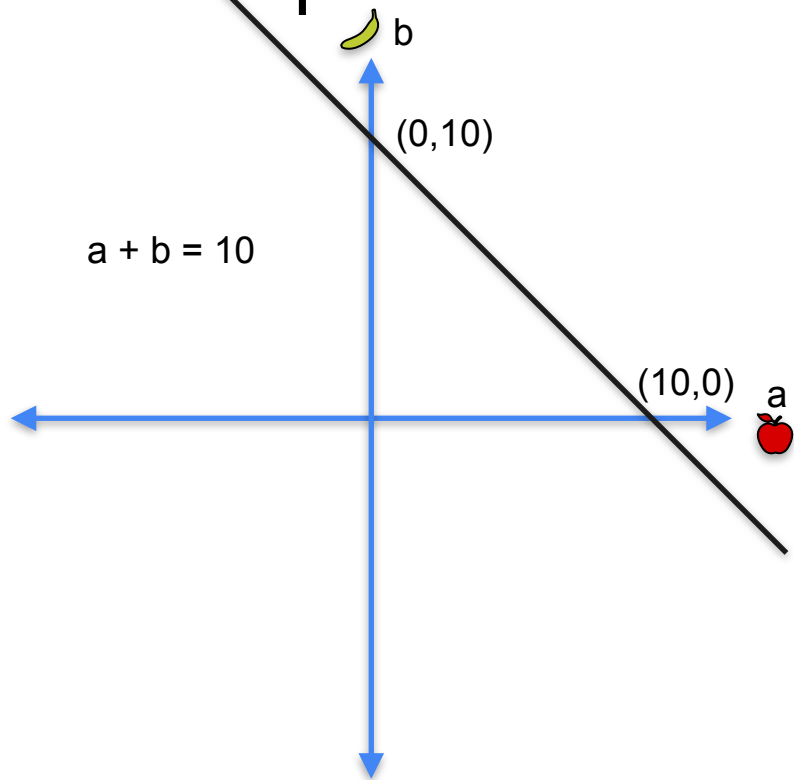
Linear equation \rightarrow line



Linear equation \rightarrow line



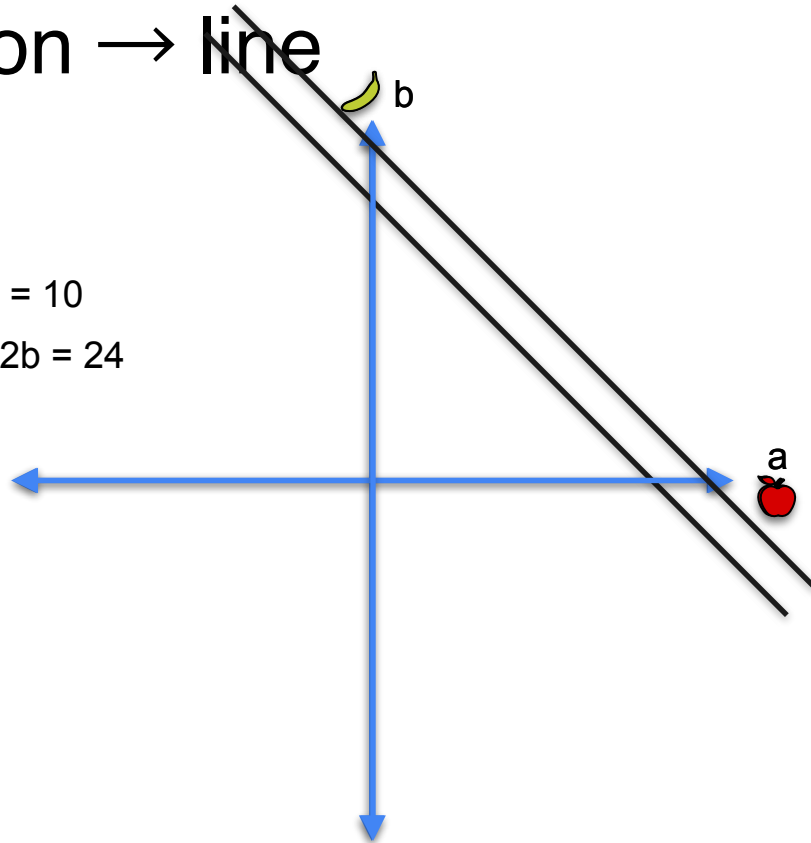
Linear equation \rightarrow line



Linear equation \rightarrow line

$$a + b = 10$$

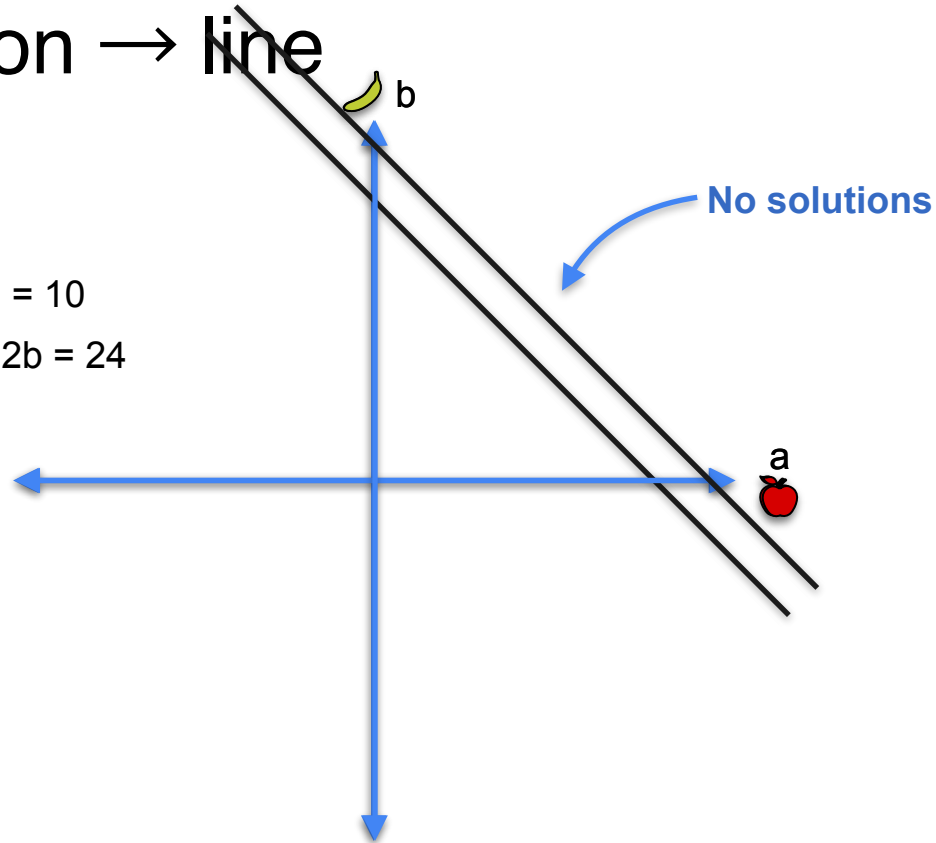
$$2a + 2b = 24$$



Linear equation → line

$$a + b = 10$$





$$2a + 2b = 24$$



Systems of equations as lines





Systems of equations as lines

System 1






- $a + b = 10$
 
- $a + 2b = 12$
 

Systems of equations as lines

System 1





- $a + b = 10$
 
- $a + 2b = 12$
 

System 2





- $a + b = 10$
 
- $2a + 2b = 20$
  

Systems of equations as lines





System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 

Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$



 b

a 

System 2

- $a + b = 10$



- $2a + 2b = 20$



 b

a 

System 3

- $a + b = 10$



- $2a + 2b = 24$



 b

a 

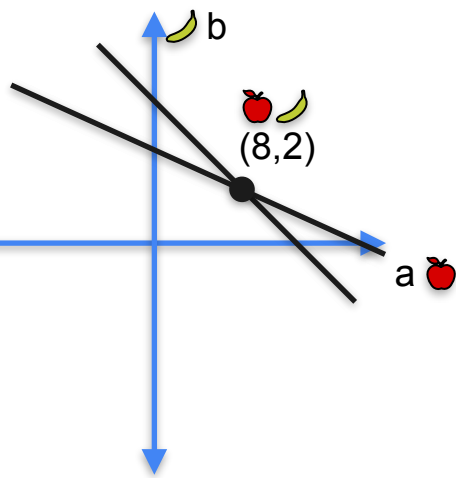
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

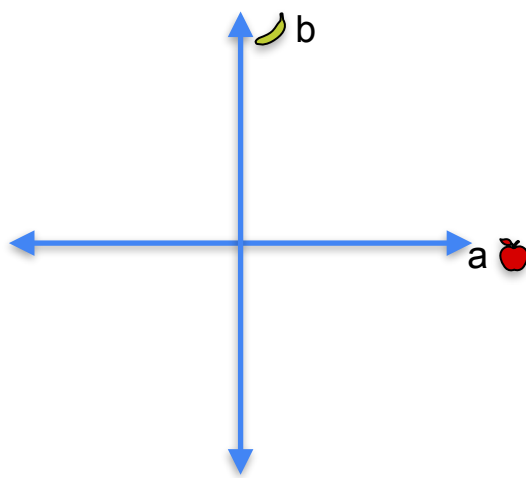


System 2

- $a + b = 10$



- $2a + 2b = 20$

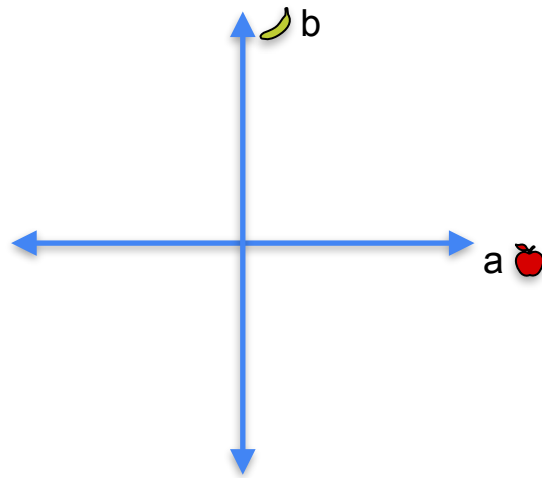


System 3

- $a + b = 10$



- $2a + 2b = 24$



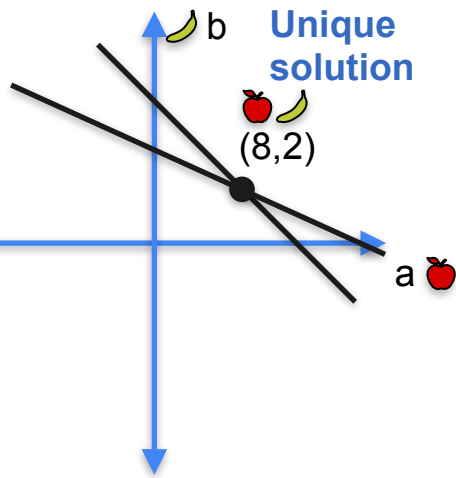
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

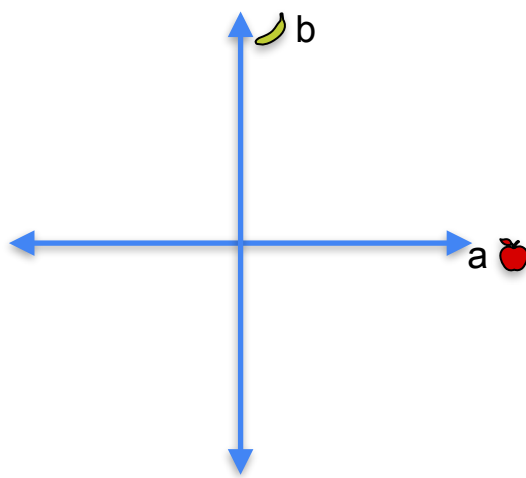


System 2

- $a + b = 10$



- $2a + 2b = 20$

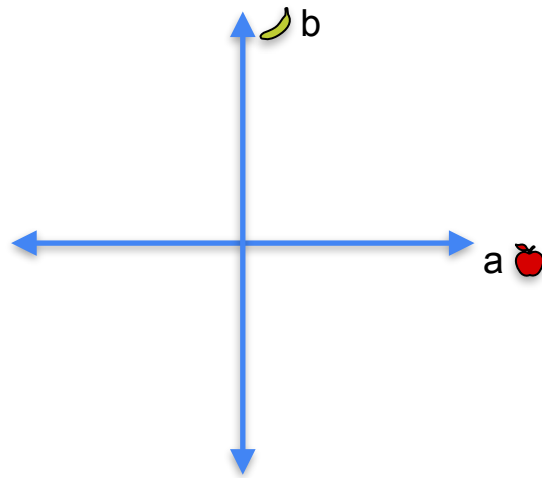


System 3

- $a + b = 10$



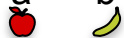
- $2a + 2b = 24$



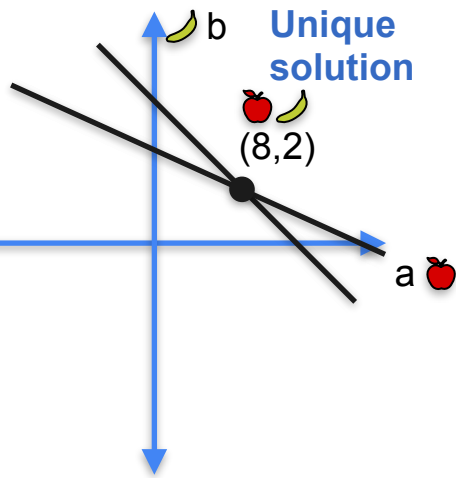
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

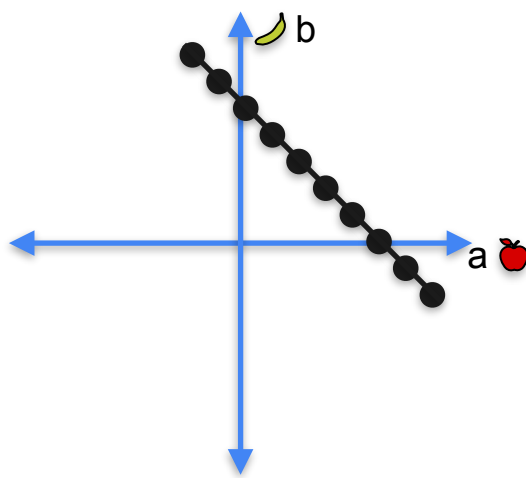


System 2

- $a + b = 10$



- $2a + 2b = 20$

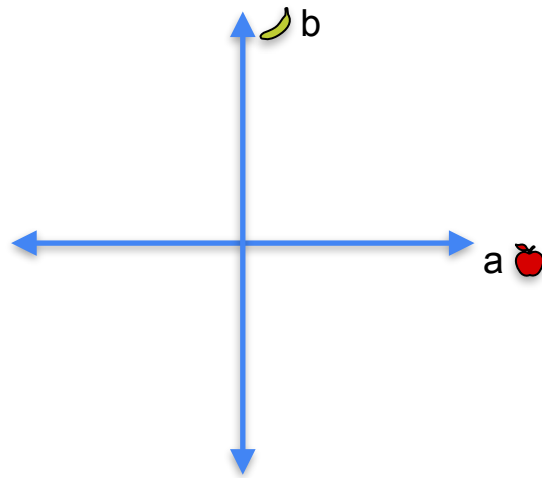


System 3

- $a + b = 10$



- $2a + 2b = 24$



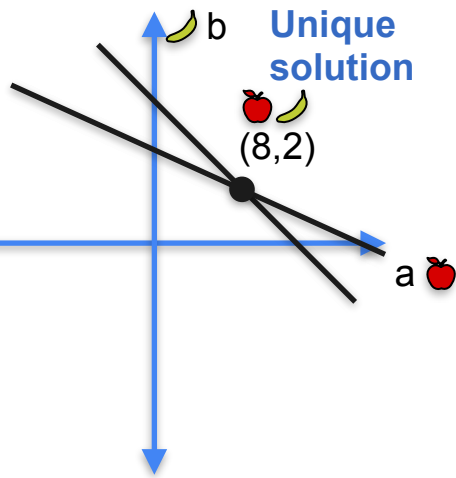
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

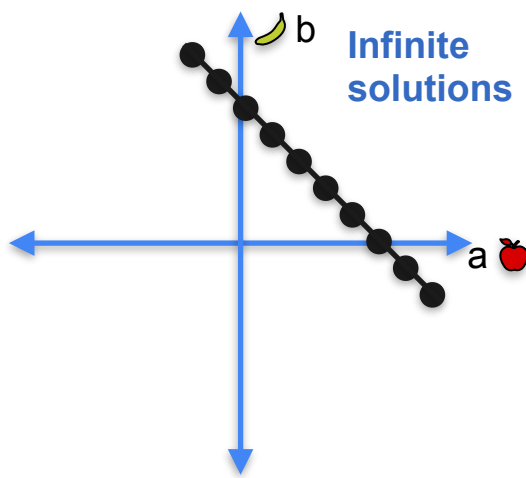


System 2

- $a + b = 10$



- $2a + 2b = 20$

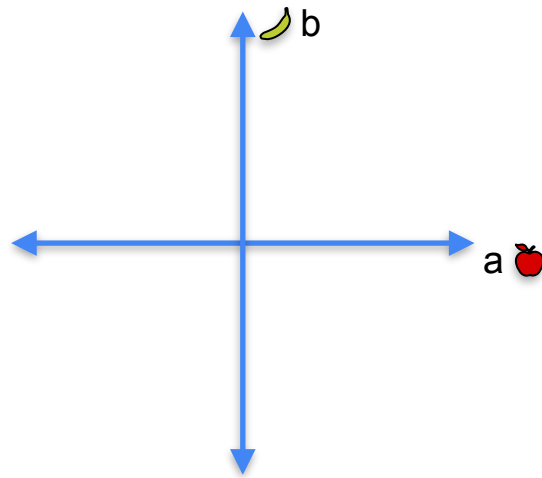


System 3

- $a + b = 10$



- $2a + 2b = 24$



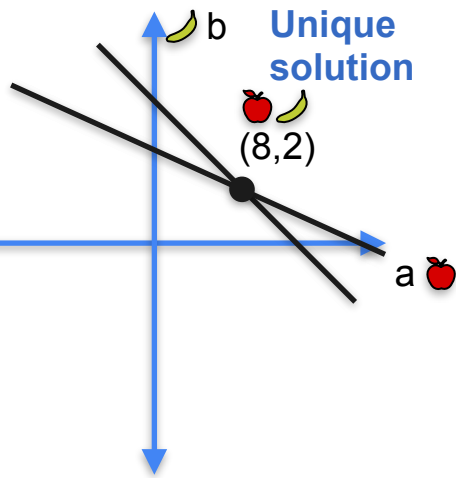
Systems of equations as lines

System 1

- $a + b = 10$

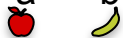


- $a + 2b = 12$

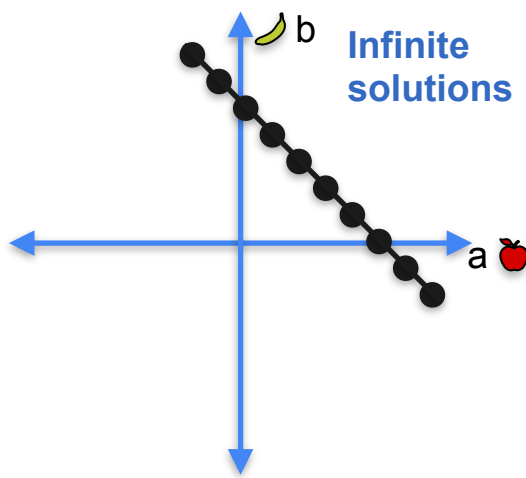


System 2

- $a + b = 10$



- $2a + 2b = 20$

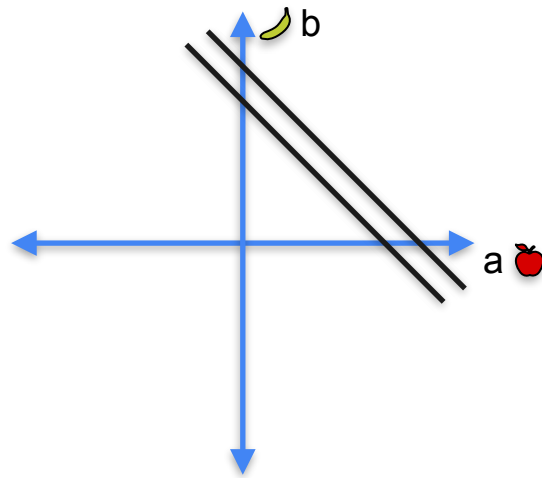


System 3

- $a + b = 10$



- $2a + 2b = 24$



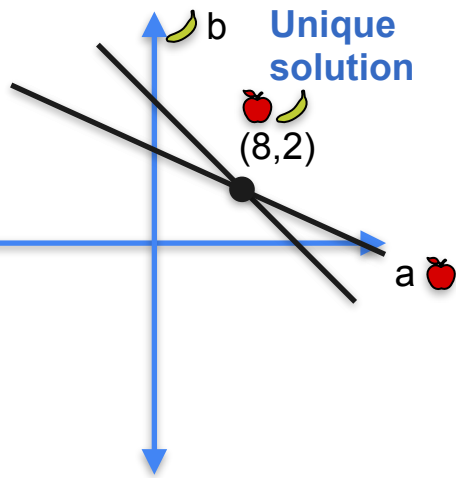
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

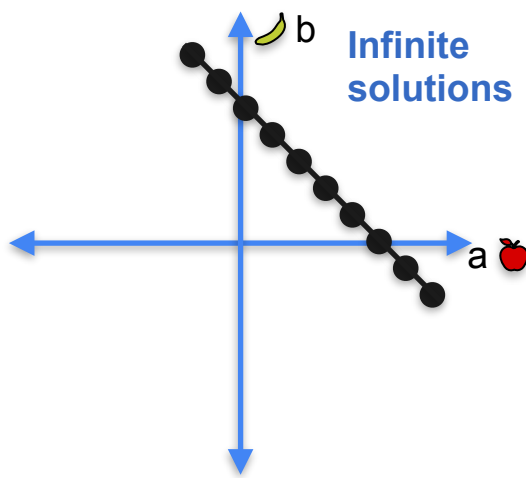


System 2

- $a + b = 10$



- $2a + 2b = 20$

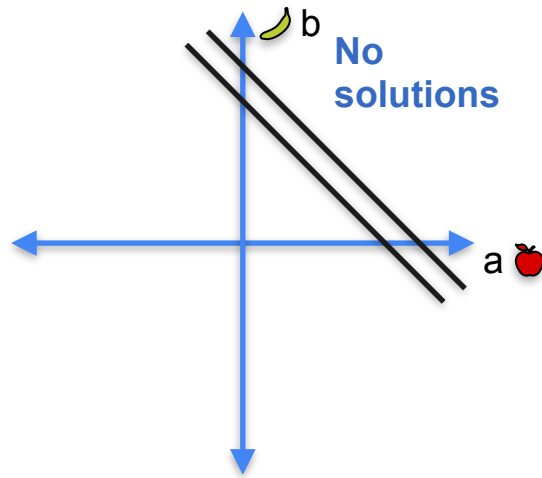


System 3

- $a + b = 10$



- $2a + 2b = 24$



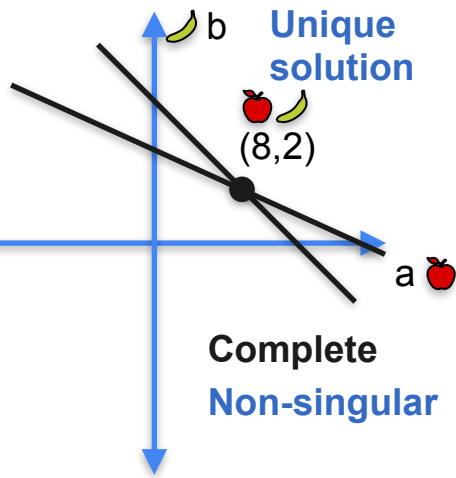
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

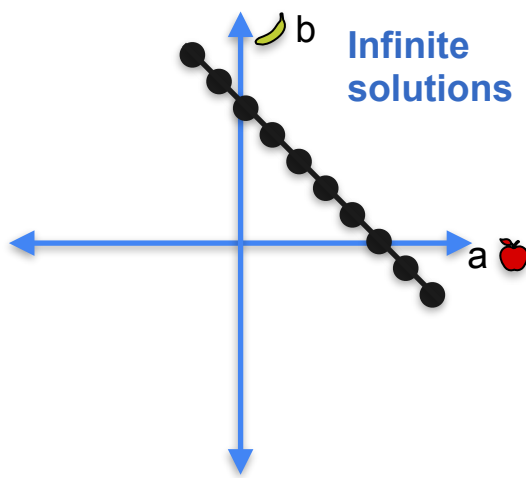


System 2

- $a + b = 10$



- $2a + 2b = 20$

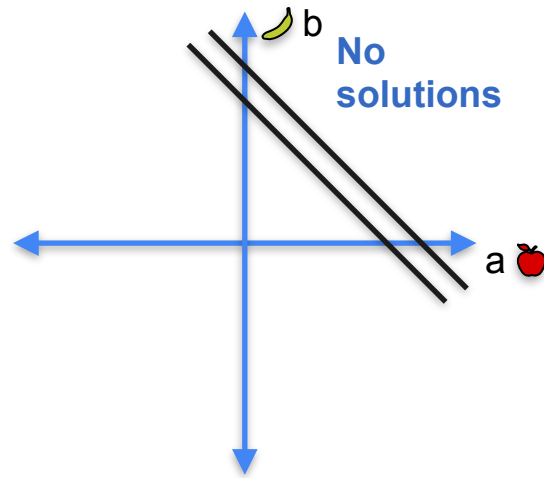


System 3

- $a + b = 10$



- $2a + 2b = 24$



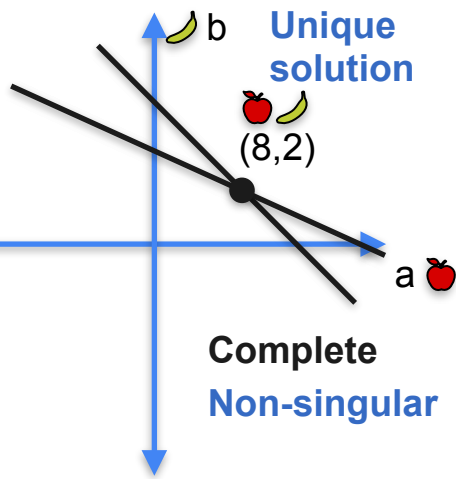
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

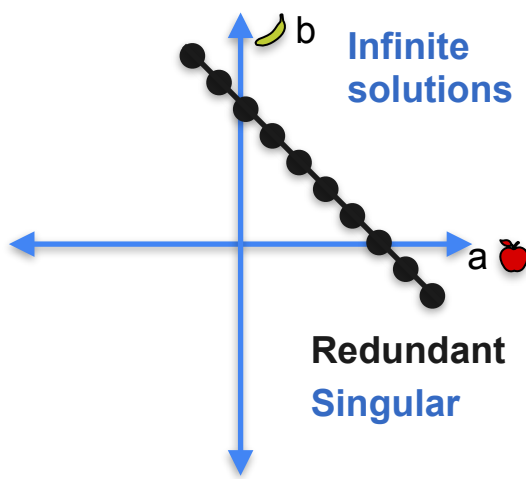


System 2

- $a + b = 10$



- $2a + 2b = 20$

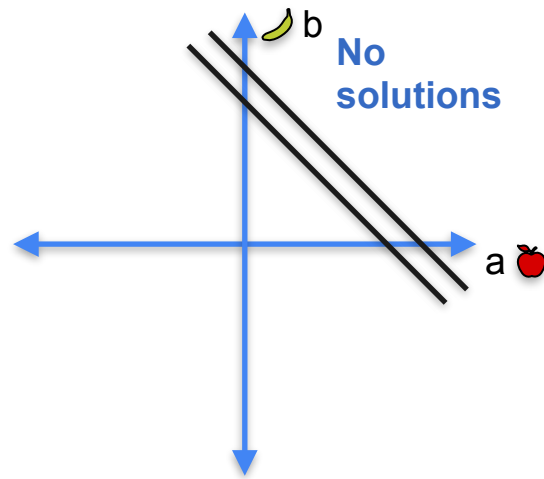


System 3

- $a + b = 10$



- $2a + 2b = 24$



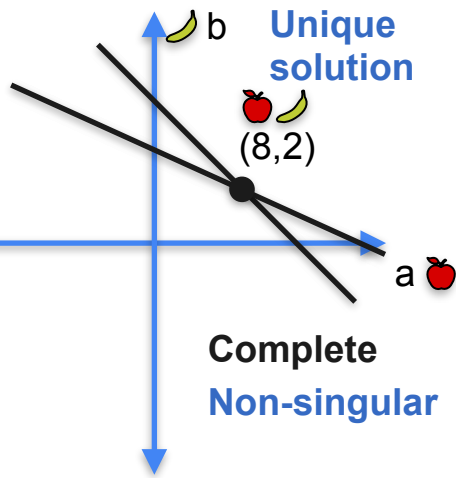
Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

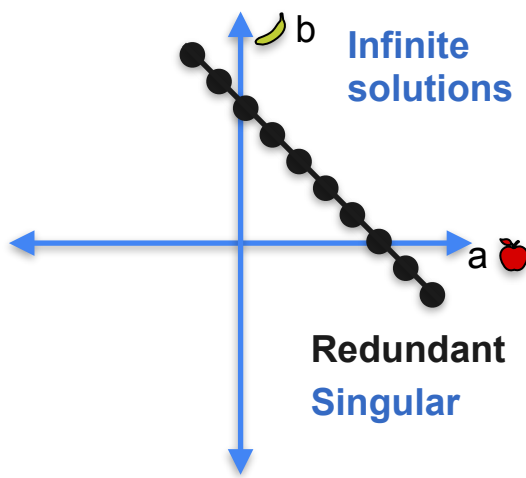


System 2

- $a + b = 10$



- $2a + 2b = 20$

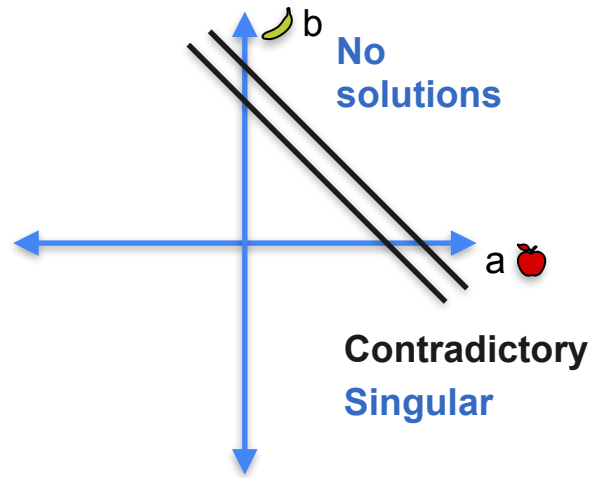


System 3

- $a + b = 10$



- $2a + 2b = 24$

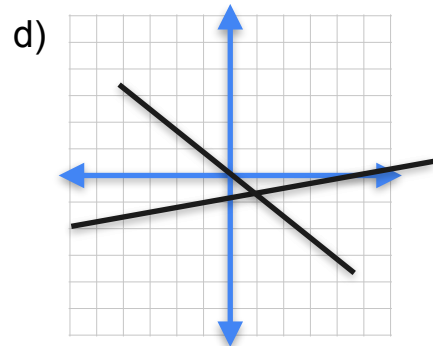
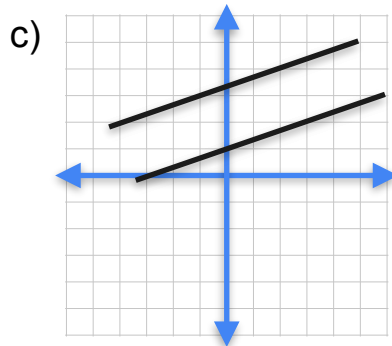
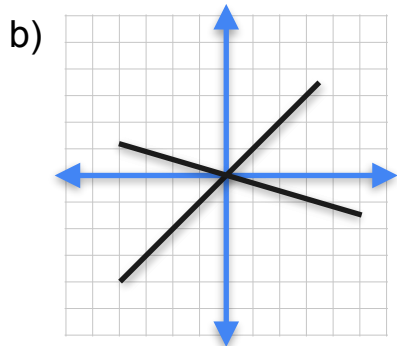
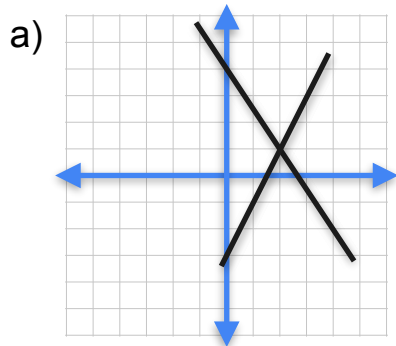


Quiz

Problem 1

Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$

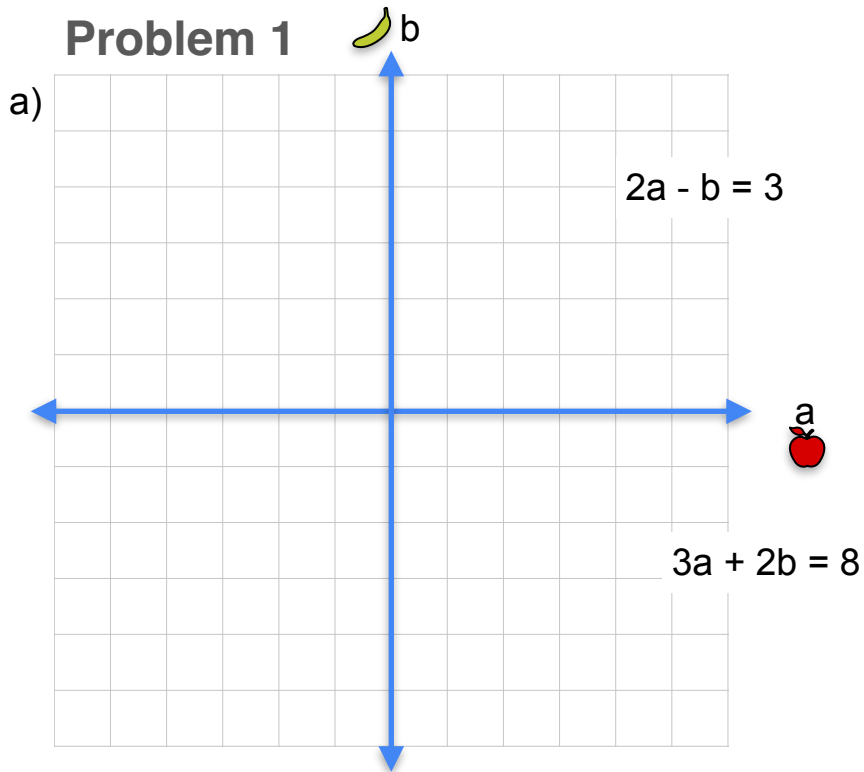


Problem 2

Is this system singular or non-singular?

Solution

Problem 1

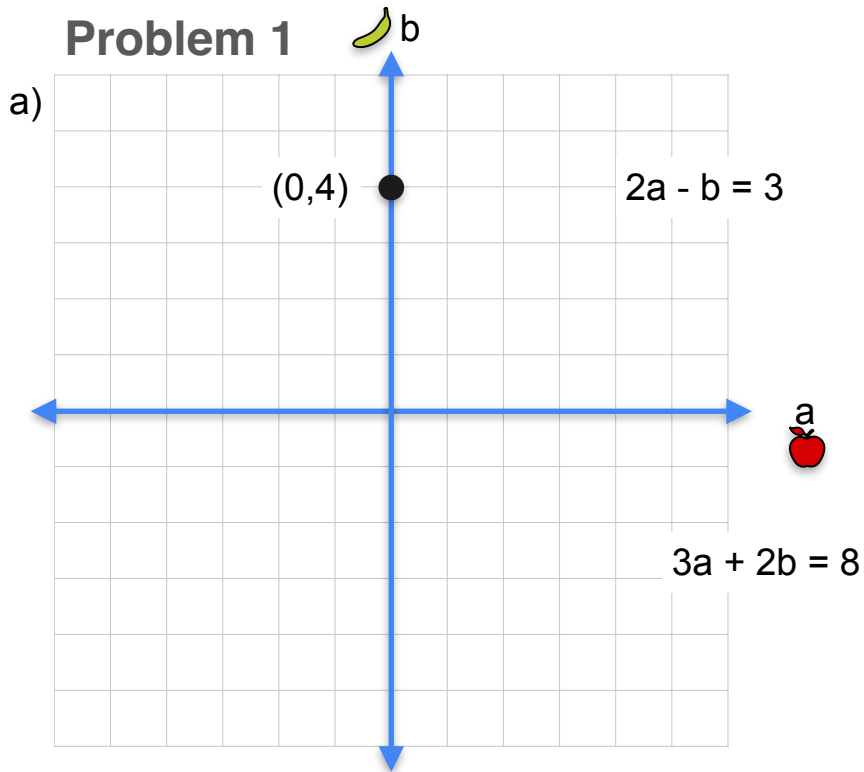


Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1



Problem 2

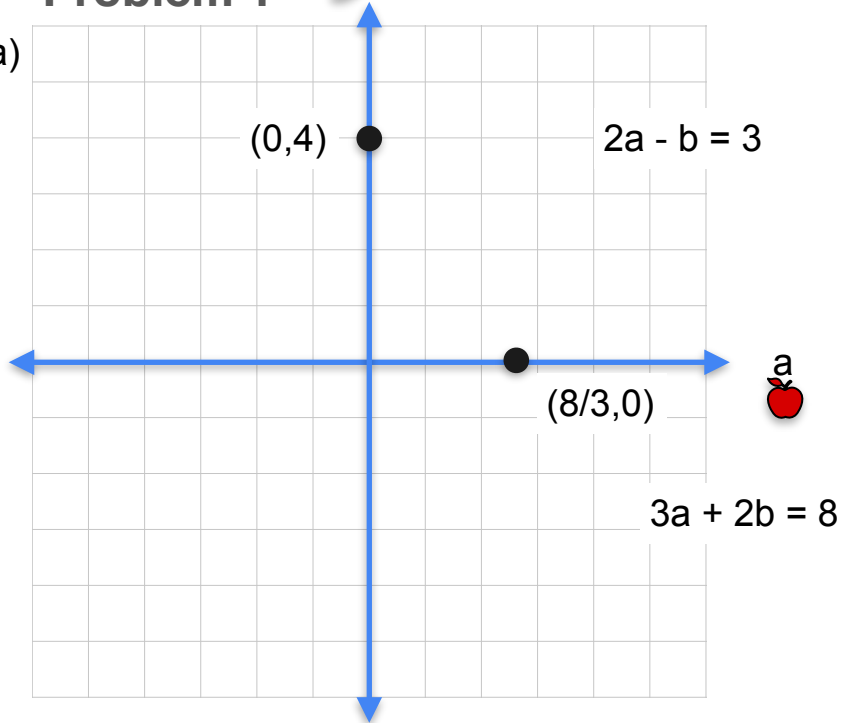
Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1



a)

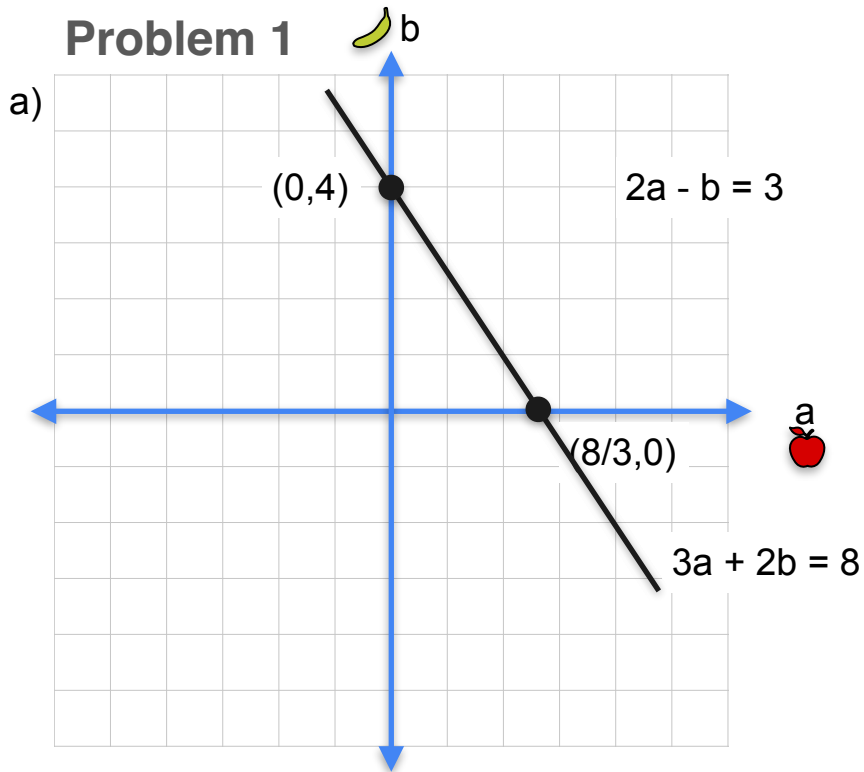


Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

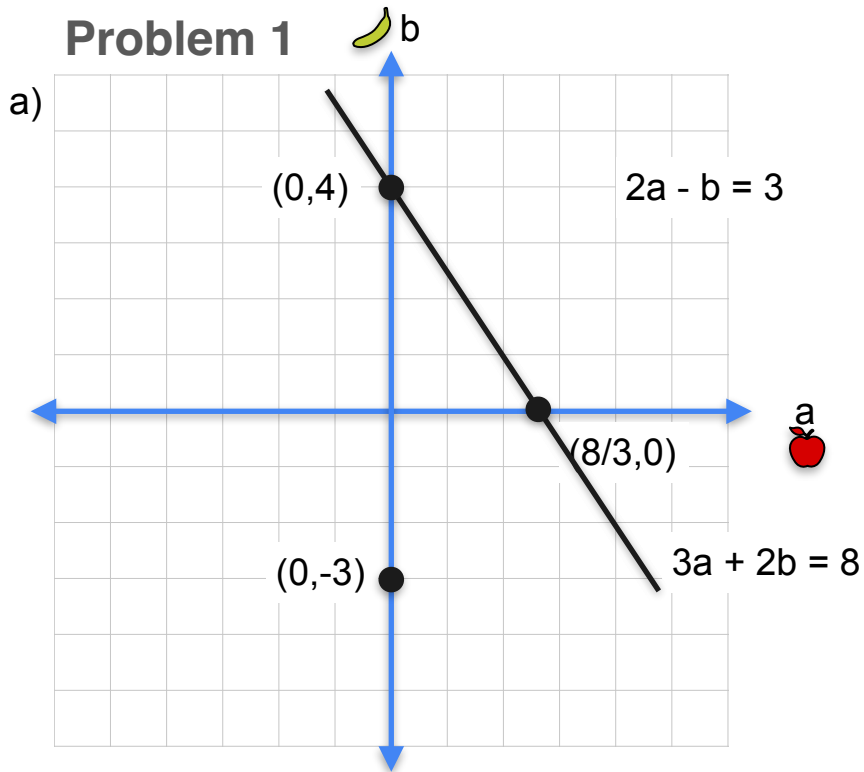


Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1



Problem 2

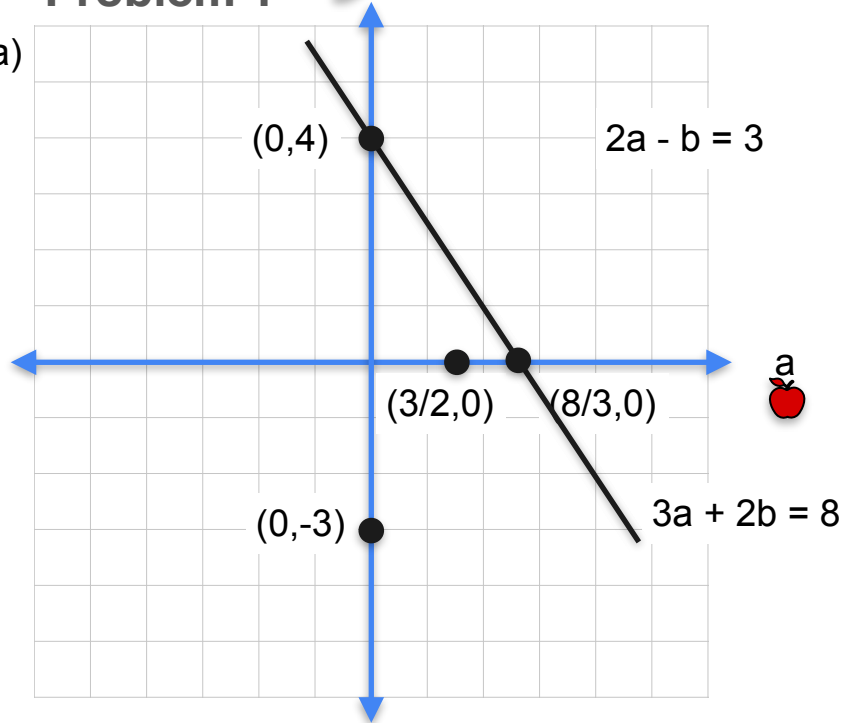
Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

 b

a)



Problem 2

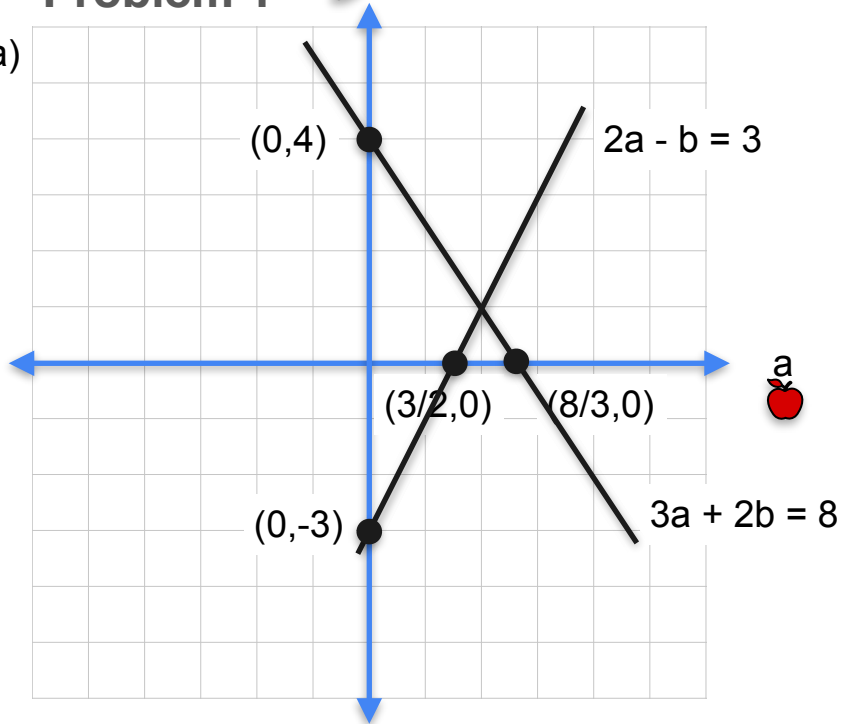
Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

 b

a)



Problem 2

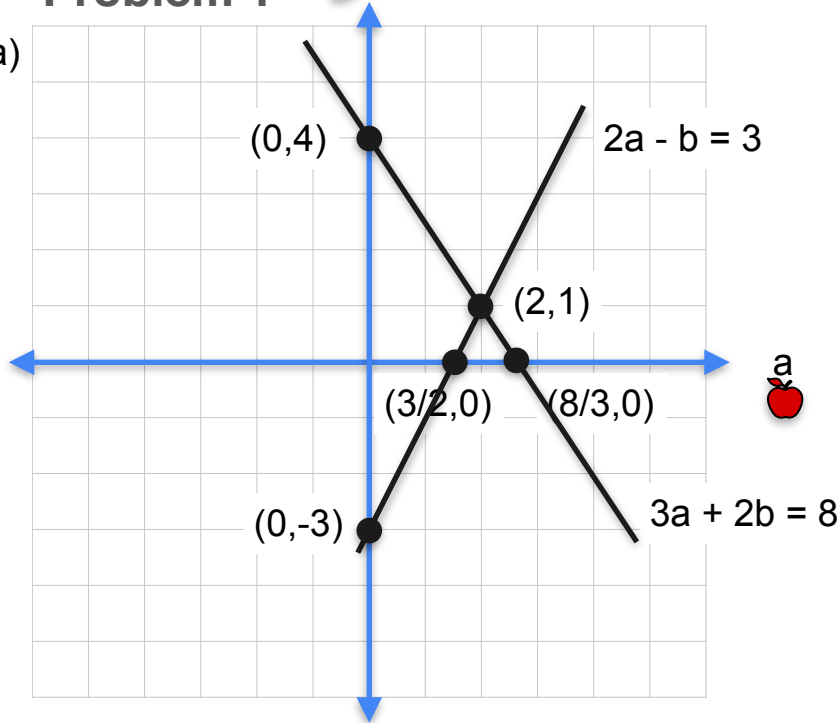
Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

 b

a)



Problem 2

Since the lines cross at a unique point, the system is non-singular.



DeepLearning.AI

System of Linear Equations

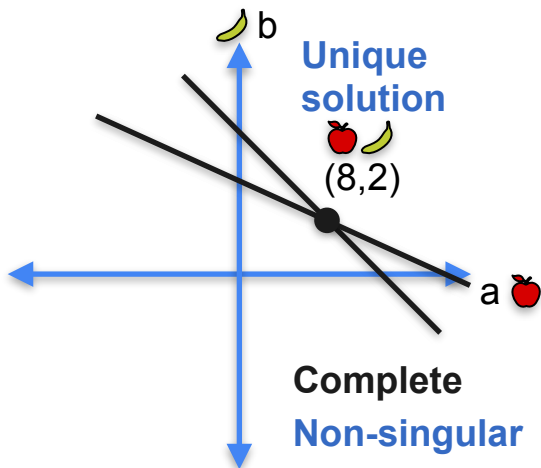
**A geometric notion of
singularity**

Systems of equations as lines

System 1

- $a + b = 10$

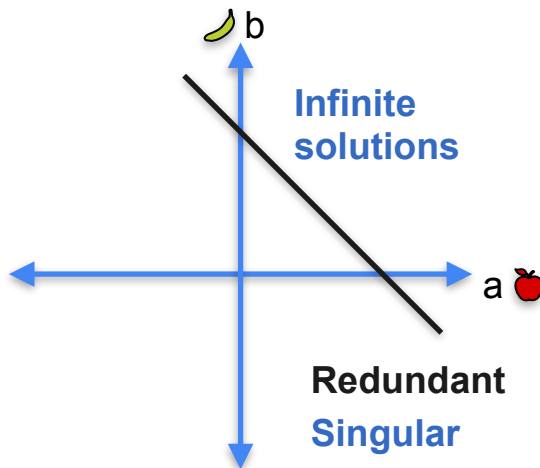
- $a + 2b = 12$



System 2

- $a + b = 10$

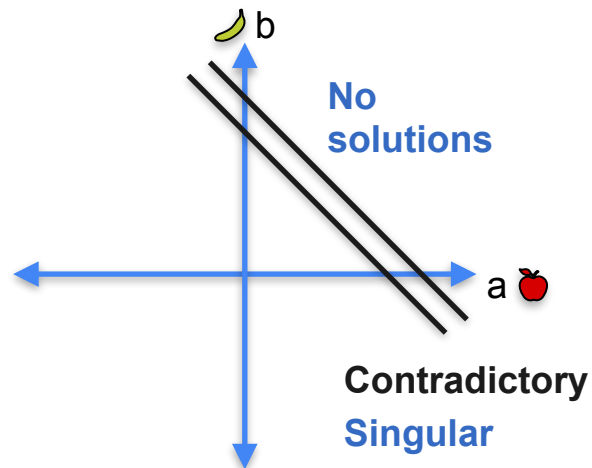
- $2a + 2b = 20$



System 3

- $a + b = 10$

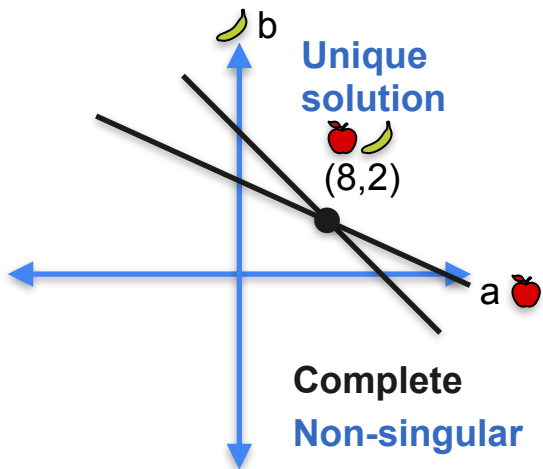
- $2a + 2b = 24$



Systems of equations as lines

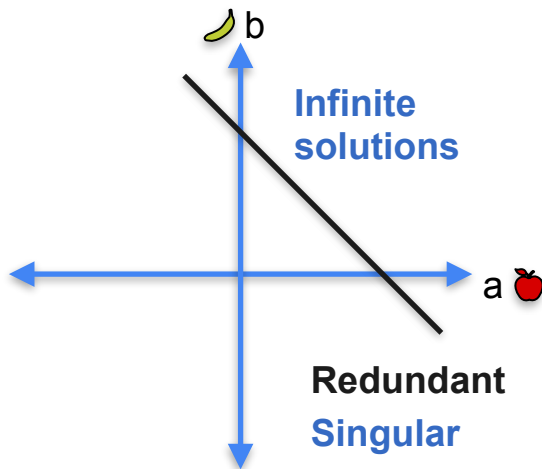
System 1

- $a + b = 10$
- $a + 2b = 12$



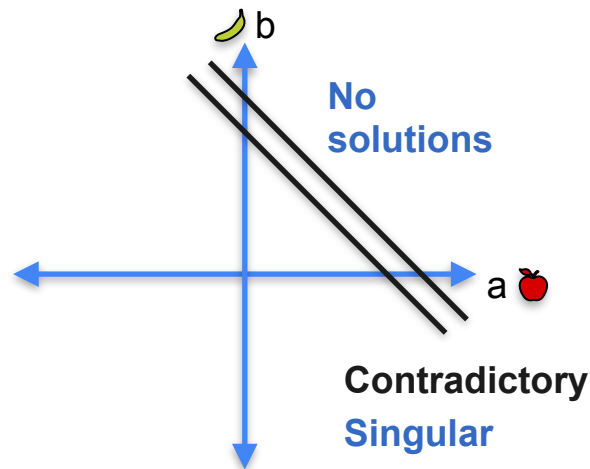
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

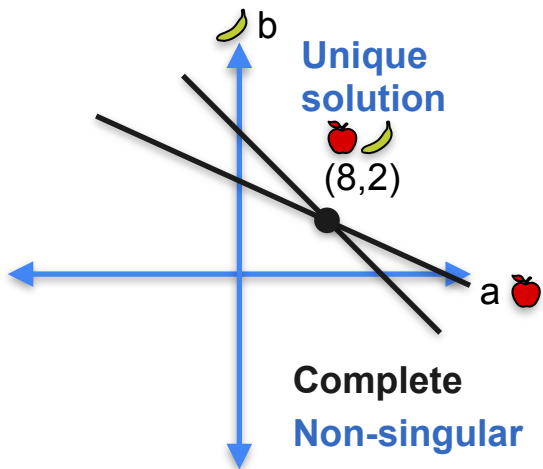
- $a + b = 10$
- $2a + 2b = 24$



Systems of equations as lines

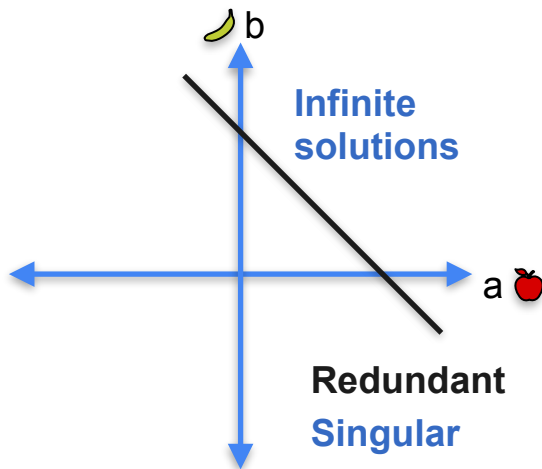
System 1

- $a + b = 0$
- $a + 2b = 0$



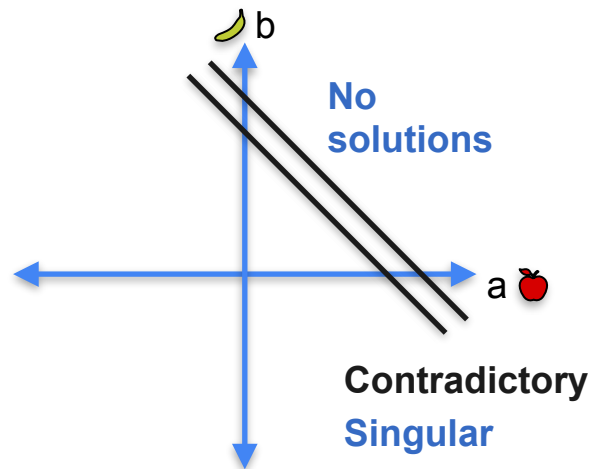
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

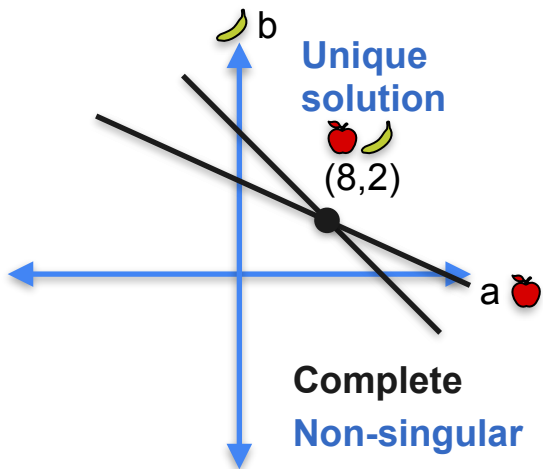
- $a + b = 10$
- $2a + 2b = 24$



Systems of equations as lines

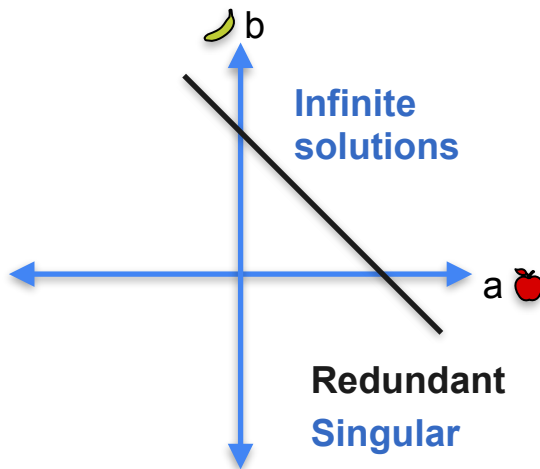
System 1

- $a + b = 0$
- $a + 2b = 0$



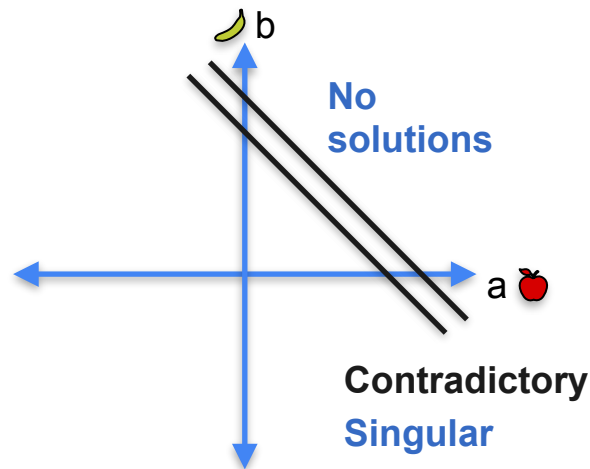
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

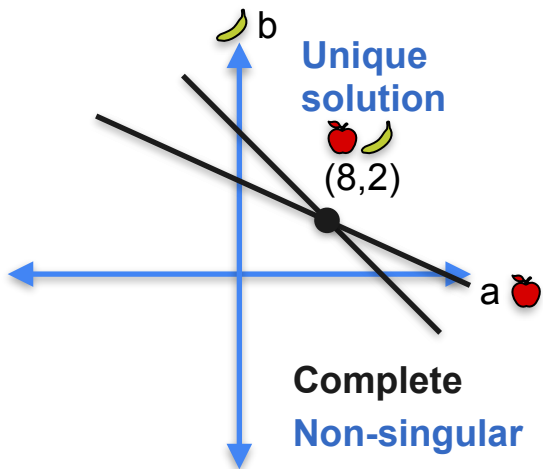
- $a + b = 10$
- $2a + 2b = 24$



Systems of equations as lines

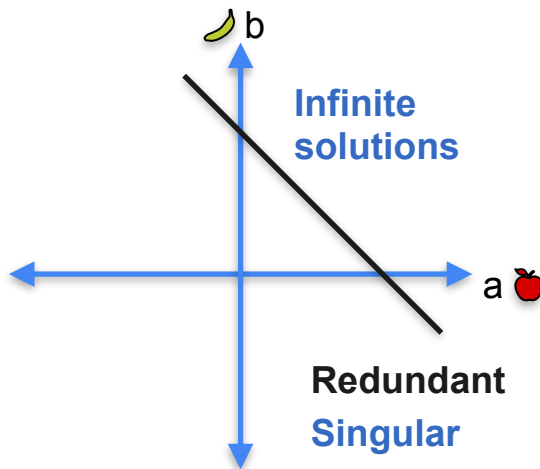
System 1

- $a + b = 0$
- $a + 2b = 0$



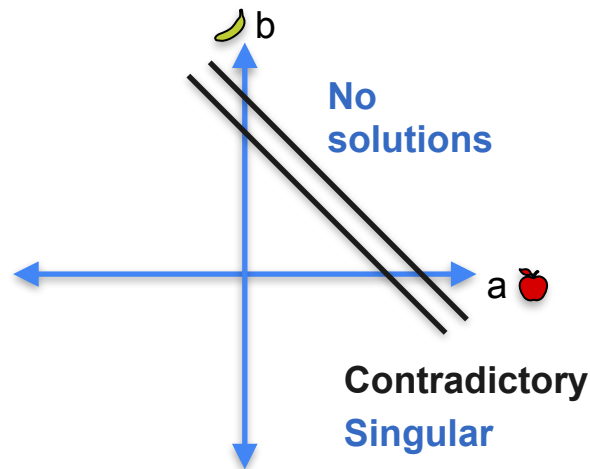
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $a + b = 0$
- $2a + 2b = 0$



Systems of equations as lines

System 1

- $a + b = 0$
- $a + 2b = 0$

b

Unique
solution

a

Complete
Non-singular

System 2

- $a + b = 0$
- $2a + 2b = 0$

b

Infinite
solutions

a

Redundant
Singular

System 3

- $a + b = 0$
- $2a + 2b = 0$

b

Infinite
solutions

a

Redundant
Singular







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System of Linear Equations






**Singular vs nonsingular
matrices**

Systems of equations as matrices

System 1

- $a + b = 0$
 
- $a + 2b = 0$
 



System 2

- $a + b = 0$
 
- $2a + 2b = 0$
  

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |



System 2

- $a + b = 0$
- $2a + 2b = 0$

Systems of equations as matrices



System 1

- $a + b = 0$
- $a + 2b = 0$

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |





System 2

- $a + b = 0$
- $2a + 2b = 0$

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |



Systems of equations as matrices

System 1





-  $a +$  $b = 0$
-  $a + 2$  $b = 0$



**Non-singular
system**

(Unique solution)

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

System 2

-  $a +$  $b = 0$
-  $2a + 2$  $b = 0$



|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

Non-singular
system



|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

Non-singular
matrix

(Unique solution)





System 2

- $a + b = 0$
- $2a + 2b = 0$



|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Systems of equations as matrices

System 1

-  $a +$  $b = 0$
-  $a + 2$  $b = 0$





**Non-singular
system**

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |



**Non-singular
matrix**

(Unique solution)

System 2

-  $a +$  $b = 0$
-  $2a + 2$  $b = 0$





**Singular
system**

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |



(Infinitely many solutions)

Systems of equations as matrices

System 1

-  $a +$  $b = 0$
-  $a + 2$  $b = 0$





**Non-singular
system**

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |



**Non-singular
matrix**

(Unique solution)

System 2

-  $a +$  $b = 0$
-  $2a + 2$  $b = 0$

**Singular
system**

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

**Singular
matrix**

(Infinitely many solutions)



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System of Linear Equations

**Linear dependence and
independence**

Linear dependence between rows



Non-singular

- $a + b = 0$
- $a + 2b = 0$

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular system



- $a + b = 0$
- $2a + 2b = 0$

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

Linear dependence between rows



Non-singular

- $a + b = 0$
- $a + 2b = 0$

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular system

- $a + b = 0$
- $2a + 2b = 0$



| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

Second equation is
a multiple of the
first one

Linear dependence between rows

Non-singular



- $a + b = 0$
- $a + 2b = 0$

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular system

- $a + b = 0$
- $2a + 2b = 0$





Second equation is
a multiple of the
first one



| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

Second row is a
multiple of the first
row







Linear dependence between rows

Non-singular



- $a + b = 0$
 
- $a + 2b = 0$
 

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
   

Second equation is
a multiple of the
first one

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

Second row is a
multiple of the first
row



Rows are
linearly dependent

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$



No equation is a multiple of the other one

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |

Second row is a multiple of the first row



Rows are
linearly dependent

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

No equation is a multiple of the other one





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

No row is a multiple of the other one

Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Second row is a multiple of the first row


Rows are
linearly dependent

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

No equation is a multiple of the other one



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

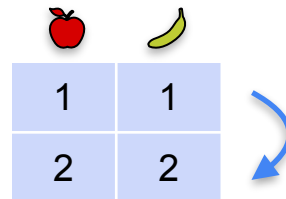
No row is a multiple of the other one

Rows are
linearly independent

Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Second row is a multiple of the first row

Rows are
linearly dependent



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System of Linear Equations

The determinant

Linear dependence between rows

Non-singular matrix



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |



Singular matrix





| | |
|---|---|
| 1 | 1 |
| 2 | 2 |


Linear dependence between rows

Non-singular matrix

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |



Singular matrix

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |






Linear dependence between rows

Non-singular matrix

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular matrix



| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |





| | |
|---|---|
| 1 | 1 |
|---|---|


Linear dependence between rows

Non-singular matrix

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |





| | |
|---|---|
| 1 | 1 |
|---|---|



 $\times 2 =$


Linear dependence between rows

Non-singular matrix

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular matrix



| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |





| | | | | | |
|---|---|-----|---|---|---|
| 1 | 1 | x 2 | = | 2 | 2 |
|---|---|-----|---|---|---|


Linear dependence between rows

Non-singular matrix

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |






$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent



Linear dependence between rows


Non-singular matrix

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |



Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |






| | | | | | |
|---|---|-----|---|---|---|
| 1 | 1 | x 2 | = | 2 | 2 |
|---|---|-----|---|---|---|

Rows linearly dependent

Linear dependence between rows



Non-singular matrix


| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |



| | |
|---|---|
| 1 | 1 |
|---|---|

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |



| | |
|---|---|
| 1 | 1 |
|---|---|



 $\times 2 =$


| | |
|---|---|
| 2 | 2 |
|---|---|

Rows linearly dependent

Linear dependence between rows



Non-singular matrix


| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |



$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? =$$

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |






$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Linear dependence between rows



Non-singular matrix


| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |



$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |






$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix



| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |




$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Rows linearly independent

Singular matrix

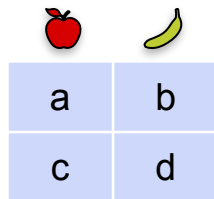
| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |





$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Determinant




| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| a | b |
| c | d |

Matrix is singular if

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} * k = \begin{vmatrix} c & d \end{vmatrix}$$

Determinant



| | |
|---|---|
| a | b |
| c | d |

$$ak = c$$

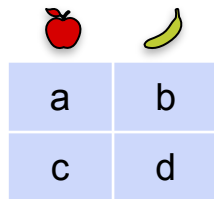
Matrix is singular if

| | |
|---|---|
| a | b |
|---|---|

 * k =

| | |
|---|---|
| c | d |
|---|---|

Determinant



| | |
|---|---|
| a | b |
| c | d |


$$ak = c$$

$$bk = d$$

Matrix is singular if

| | | | | | |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

Determinant



| | |
|---|---|
| a | b |
| c | d |

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$



Matrix is singular if

| | |
|---|---|
| a | b |
|---|---|

 * k =

| | |
|---|---|
| c | d |
|---|---|

Determinant

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| a | b |
| c | d |

Matrix is singular if

| | | | | | |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|


$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

Determinant



| | |
|---|---|
| a | b |
| c | d |

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

$$ak = c$$

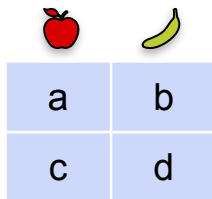
$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

$$ad - bc = 0$$

Determinant



| | |
|---|---|
| a | b |
| c | d |

Matrix is singular if

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$


$$ak = c$$

$$bk = d$$



$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

Determinant


$$ad - bc = 0$$

Determinant

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| a | b |
| c | d |

$$\text{Determinant} = ad - bc$$

$$ak = c$$

$$bk = d$$

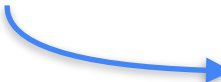
$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

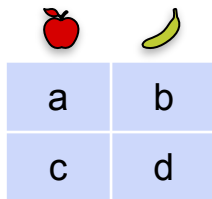
| | | | | | |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

Determinant

$$ad = bc$$

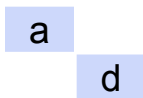

$$ad - bc = 0$$

Determinant



| | |
|---|---|
| a | b |
| c | d |

$$\text{Determinant} = ad - bc$$



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot k = \begin{bmatrix} c & d \end{bmatrix}$$

Determinant

$$ad = bc$$


$$ad - bc = 0$$

Determinant

| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| a | b |
| c | d |

$$\text{Determinant} = ad - bc$$

$$\begin{array}{c} a \\ - \\ d \end{array}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

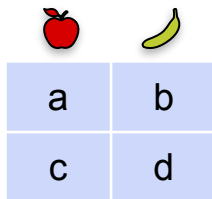
$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant



| | |
|---|---|
| a | b |
| c | d |

$$\text{Determinant} = ad - bc$$

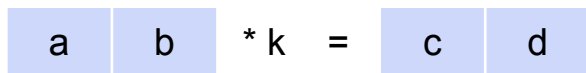

$$\begin{array}{cc} a & b \\ & d \end{array} - \begin{array}{cc} & c \\ b & \end{array}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$



Determinant

$$ad = bc$$




$$ad - bc = 0$$

Determinant

Non-singular matrix



|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

Singular matrix

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Determinant



Non-singular matrix

|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

Determinant



$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 1 & & \end{array}$$

Singular matrix

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Determinant

Non-singular matrix



|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 1 | |



$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Determinant

Non-singular matrix



|  |  |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | 1 |
| 1 | 2 |

Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 1 | |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix



|  |  |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| 1 | 1 |
| 2 | 2 |

Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 2 | |

Determinant

Non-singular matrix



| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 1 | |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |



Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 2 | |

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix



| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 1 | |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |



Determinant

| | | | | |
|---|---|---|---|---|
| 1 | | - | | 1 |
| | 2 | | 2 | |

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix



| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 1 | 2 |

Determinant

$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 1 & & \end{array}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

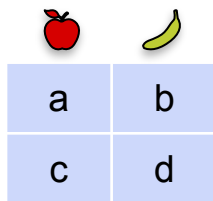
| | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
| 1 | 1 |
| 2 | 2 |



Determinant

$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 2 & & \end{array}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

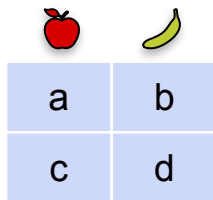
Determinant and singularity



| | |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| a | b |
| c | d |

$$ad - bc$$

Determinant and singularity



| | |
|---|---|
| a | b |
| c | d |

Matrix is singular



$$ad - bc$$

Determinant is zero

Quiz: Determinant

Problem 1: Find the determinant of the following matrices

Matrix 1

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Matrix 2

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |

Problem 2: Are these matrices singular or non-singular?

Solutions: Determinant

Matrix 1: $\det = 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Non-singular

Matrix 2: $\det = 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |

Singular



DeepLearning.AI

System of Linear Equations

System of equations (3x3)

Quiz: Systems of equations

Problem 1: You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid \$10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid \$15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid \$12.

How much does each fruit cost?

Solution: Systems of equations

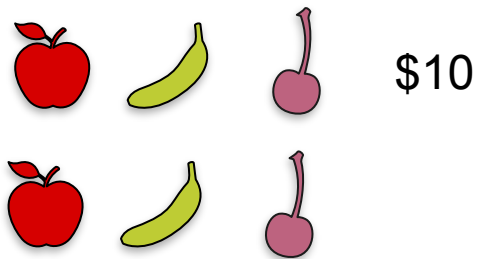
Solution: Systems of equations



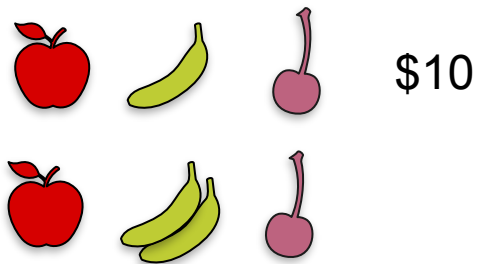
Solution: Systems of equations



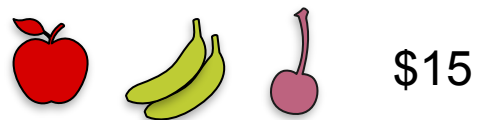
Solution: Systems of equations



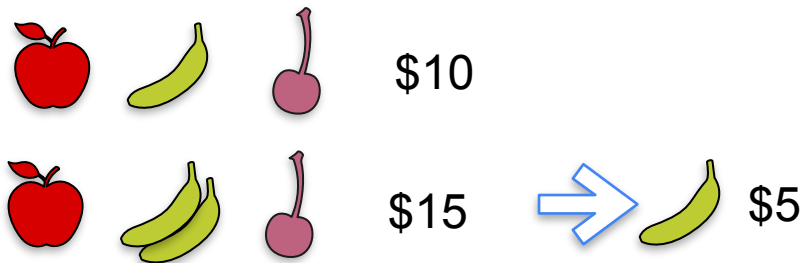
Solution: Systems of equations



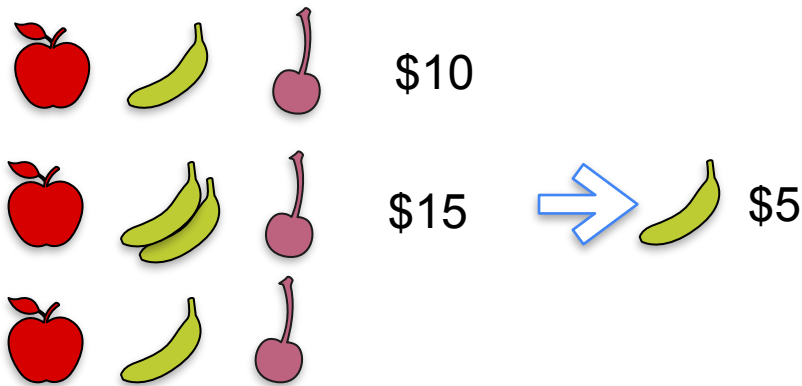
Solution: Systems of equations



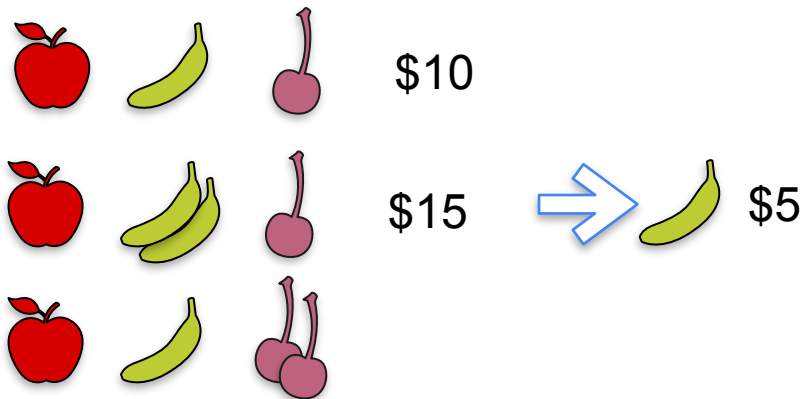
Solution: Systems of equations



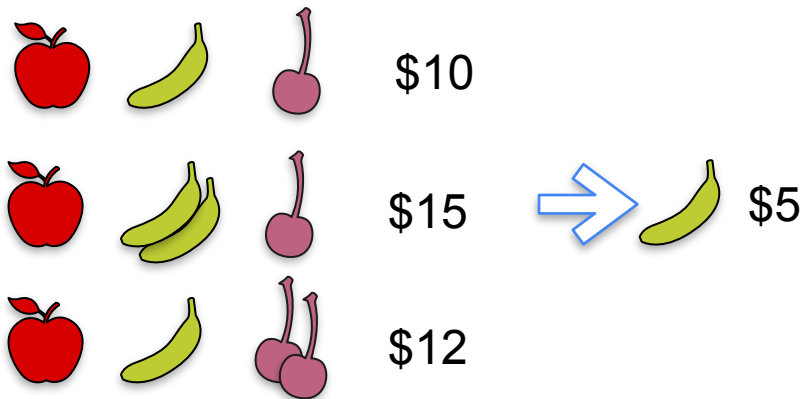
Solution: Systems of equations



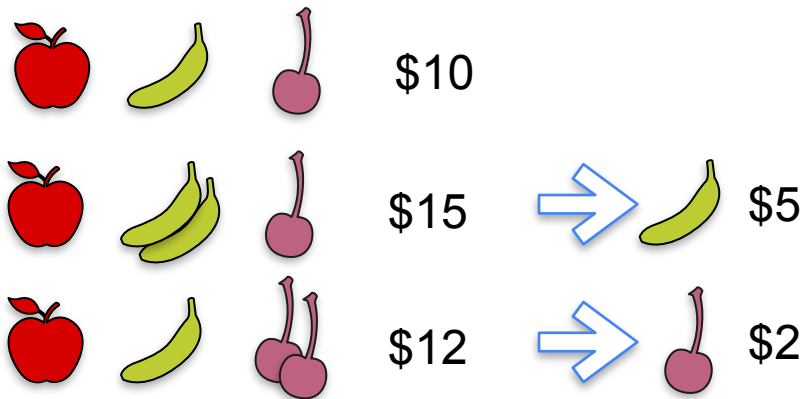
Solution: Systems of equations



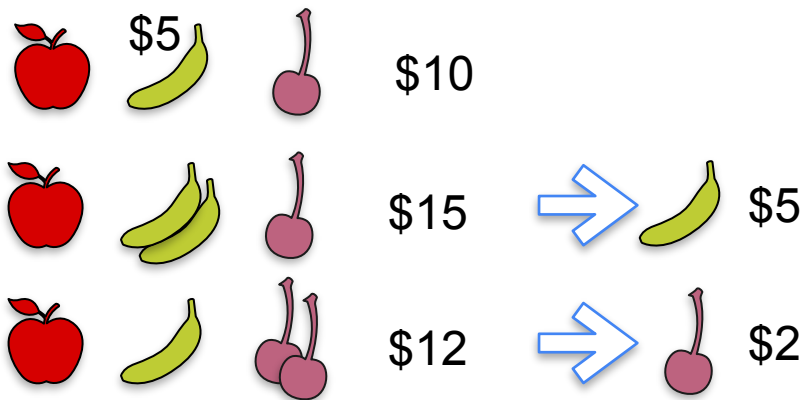
Solution: Systems of equations



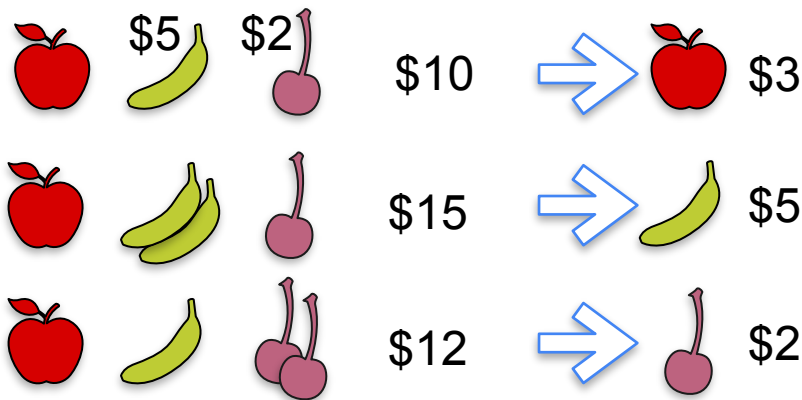
Solution: Systems of equations



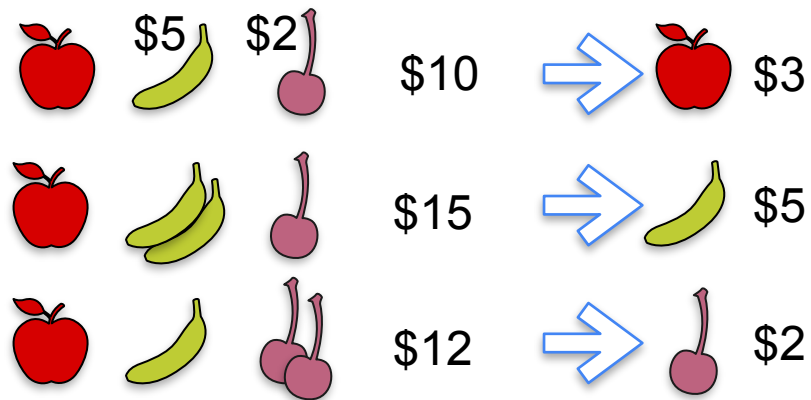
Solution: Systems of equations



Solution: Systems of equations



Solution: Systems of equations



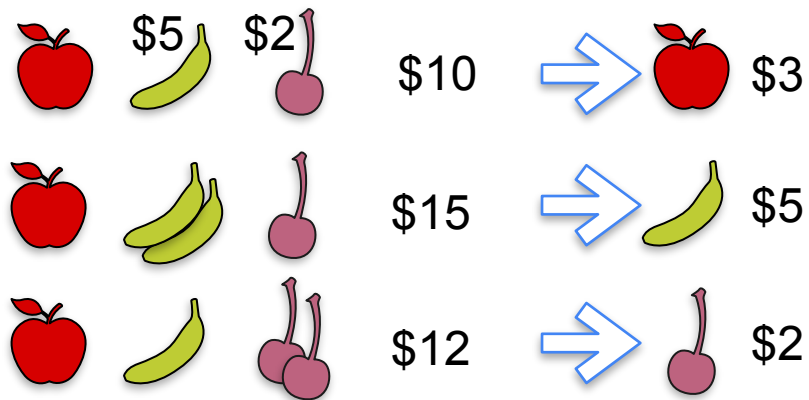
System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Solution: Systems of equations



System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Solution

$$a = 3$$

$$b = 5$$

$$c = 2$$

Quiz: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Infinitely many sols.

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinitely many sols.

$$c = 5$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Infinitely many solutions

Solutions: More systems of equations

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

Infinitely many solutions

Any 3 numbers that add to 10 work.

$(0,0,10), (2,7,1), \dots$



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System of Linear Equations

**Singular vs non-singular
matrices**

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

Singular

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

Singular

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

Singular

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

Singular

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

Singular

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Singular

Constants don't matter for singularity

System 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

Redundant

Singular

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

Redundant

Singular

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Infinite solutions:

$$a + b + c = 0$$

(i.e., $c = -a - b$)

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

Redundant

Singular

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Infinite solutions:

$$a + b + c = 0$$

(i.e., $c = -a - b$)

Redundant

Singular

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Non-singular

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Singular

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Constants don't matter for singularity

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Non-singular

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Singular

System 3

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Singular

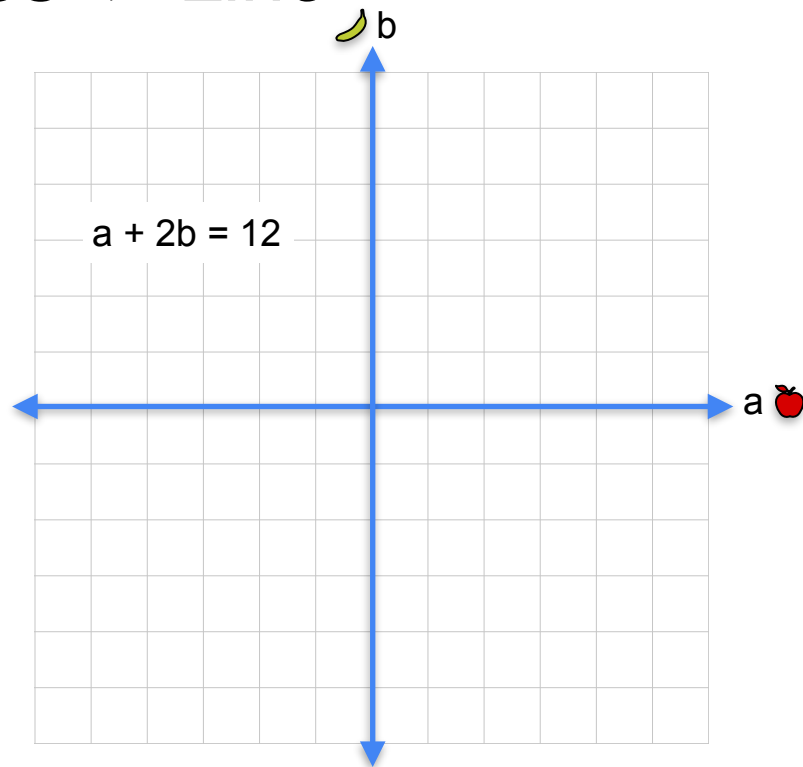
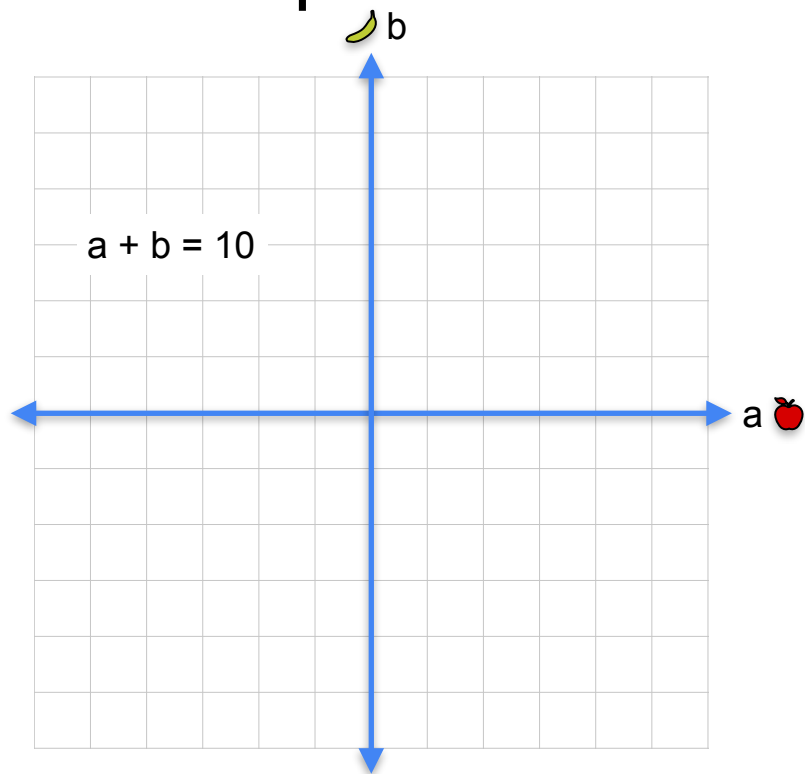


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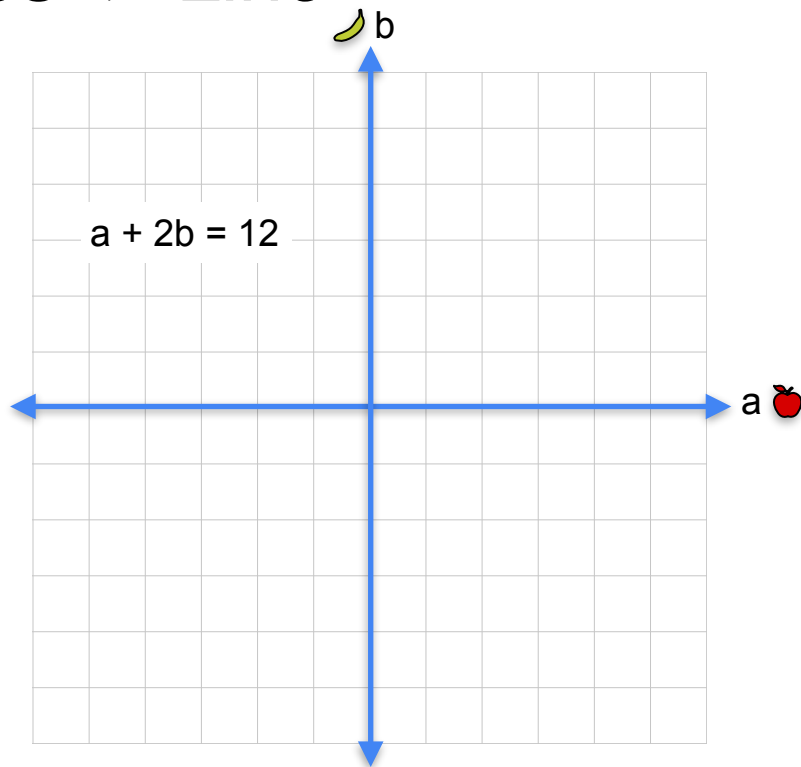
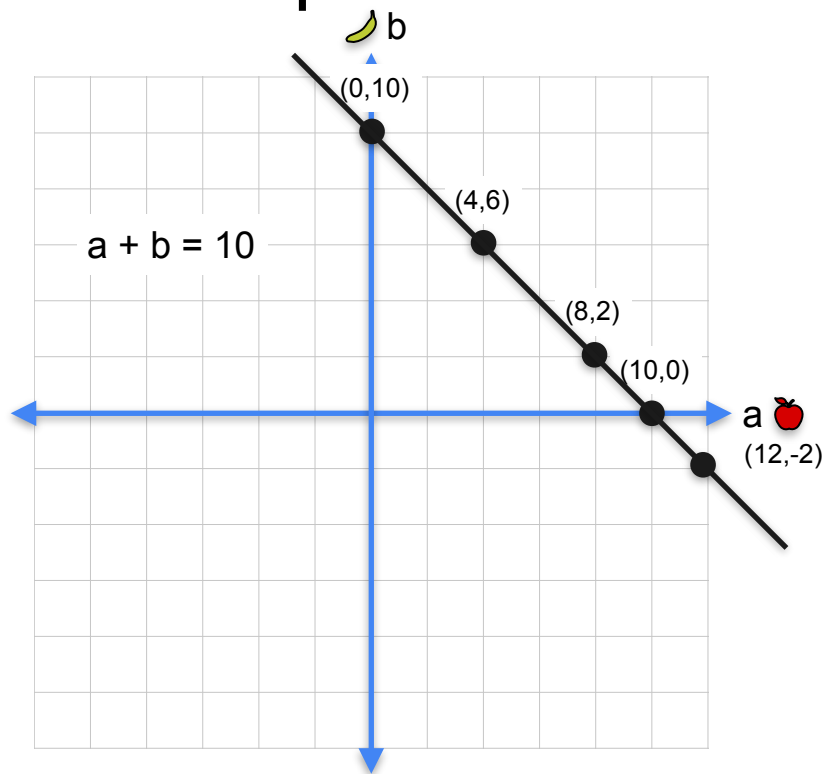
System of Linear Equations

**System of equations as
planes (3x3)**

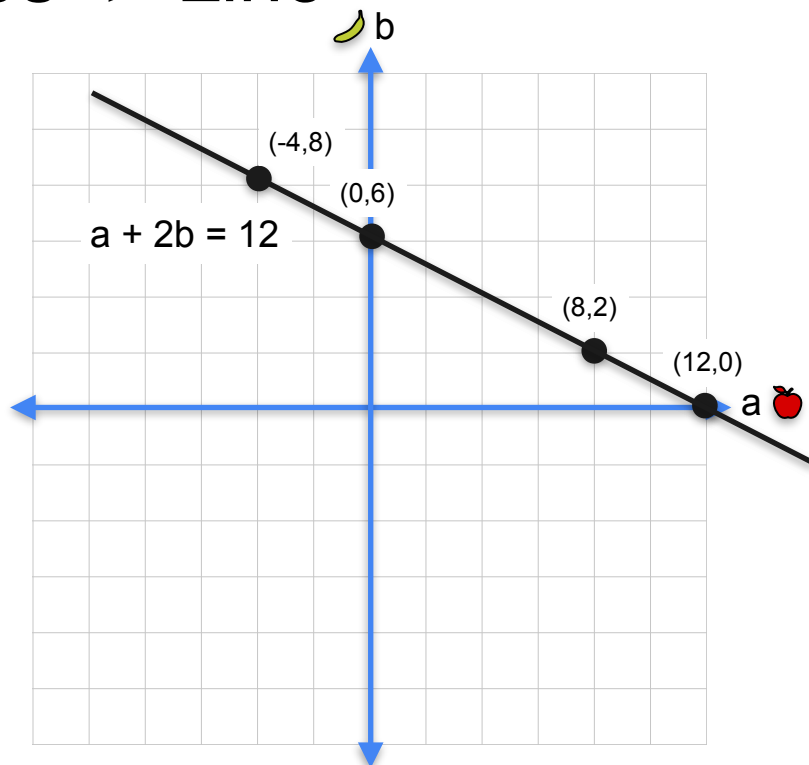
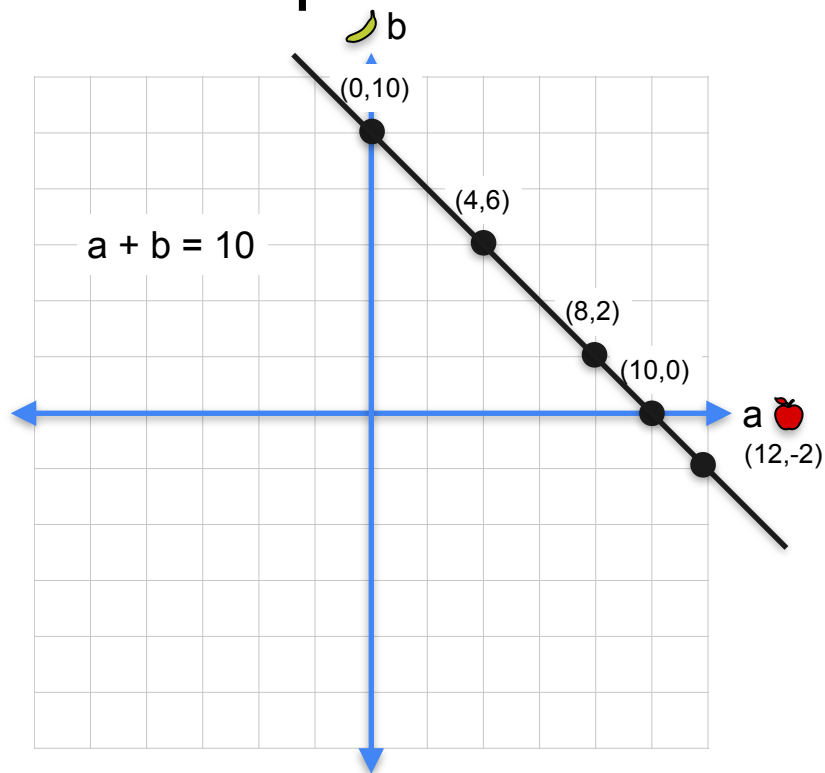
Linear equation in 2 variables -> Line



Linear equation in 2 variables -> Line

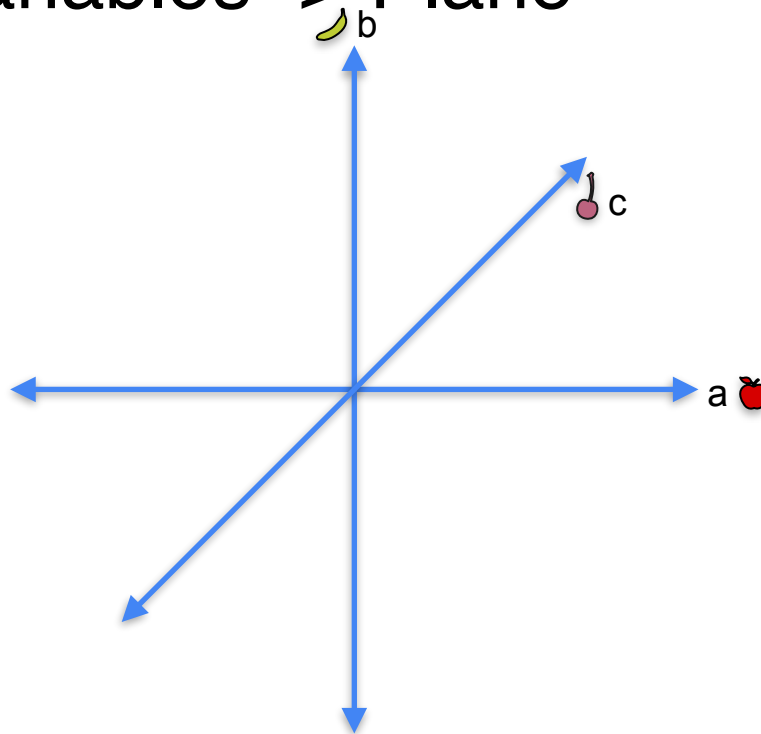


Linear equation in 2 variables -> Line



Linear equation in 3 variables -> Plane

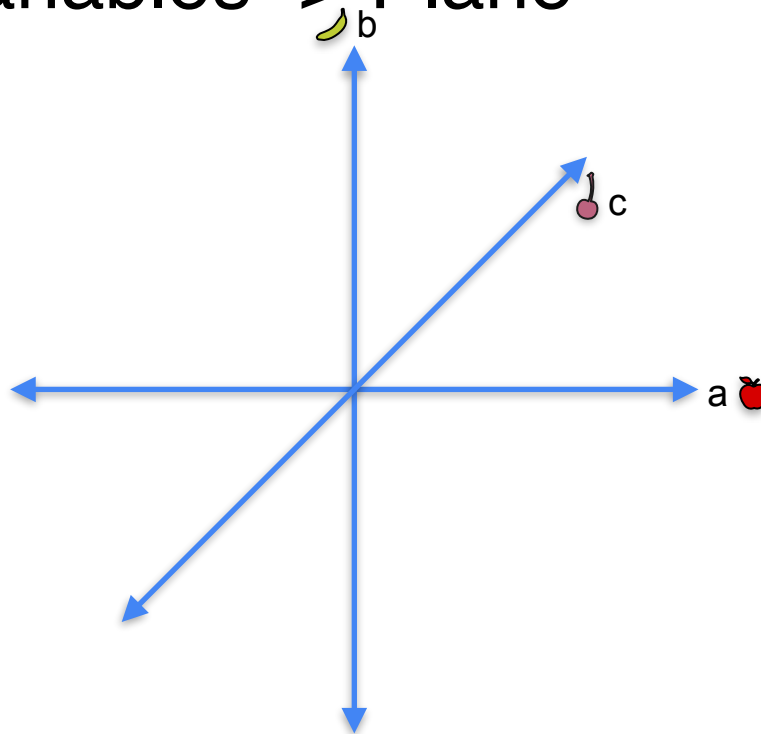
$$a + b + c = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

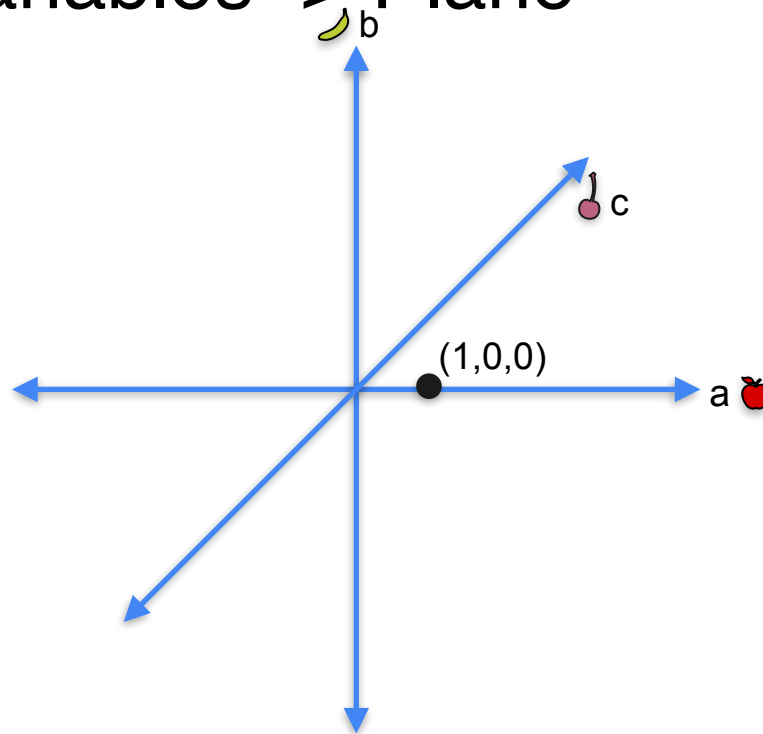
$$1 + 0 + 0 = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

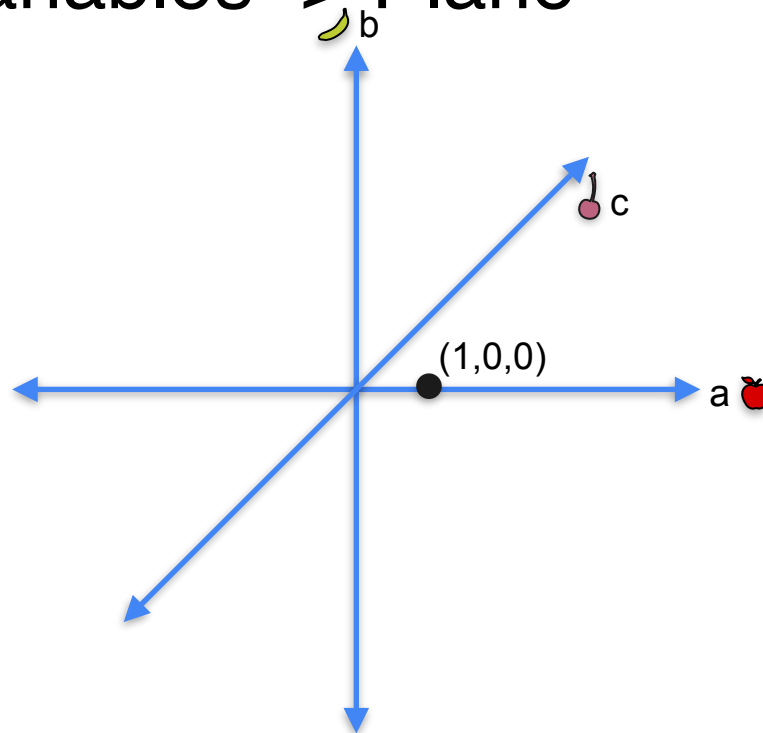


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

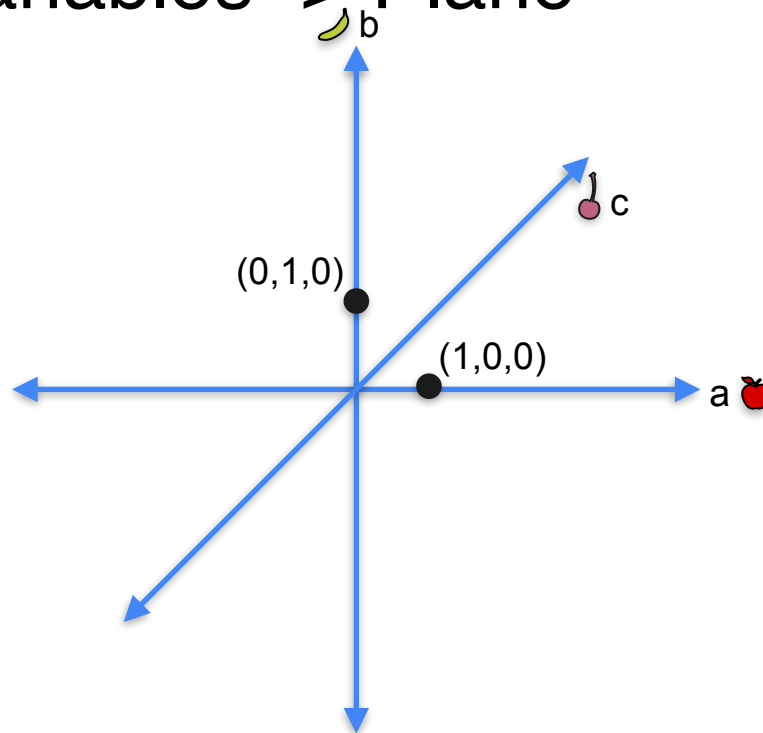


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



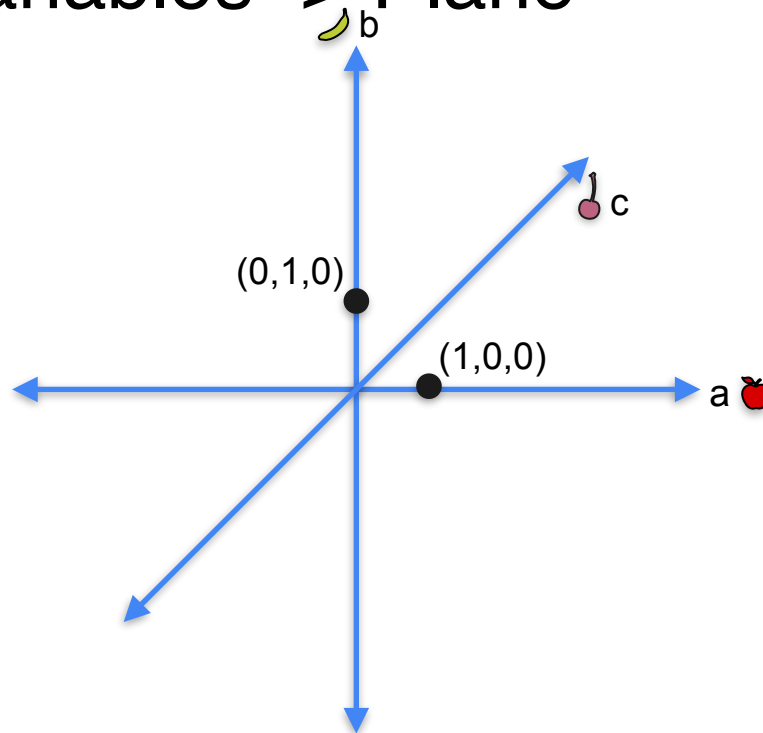
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



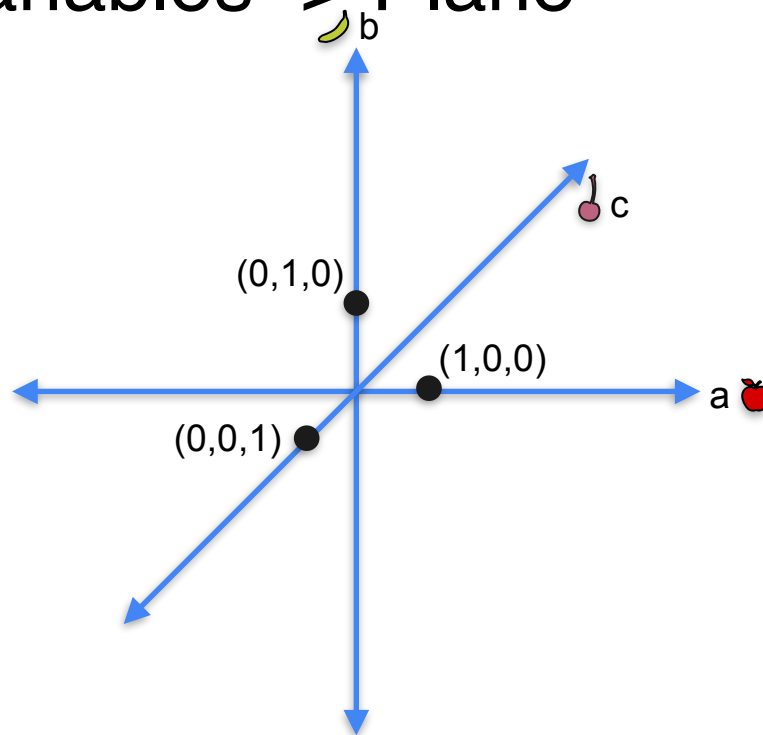
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



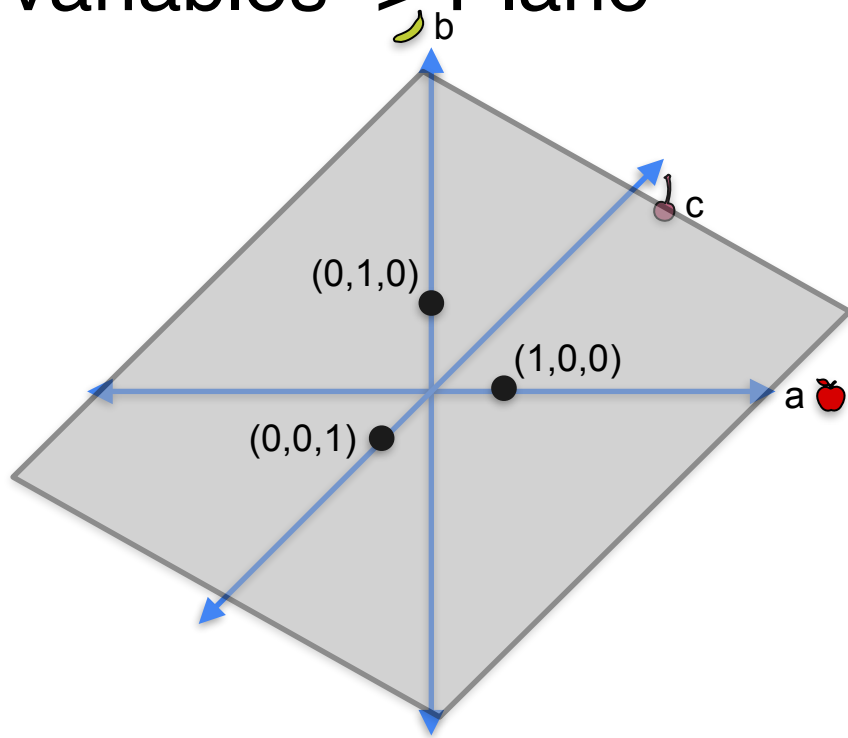
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

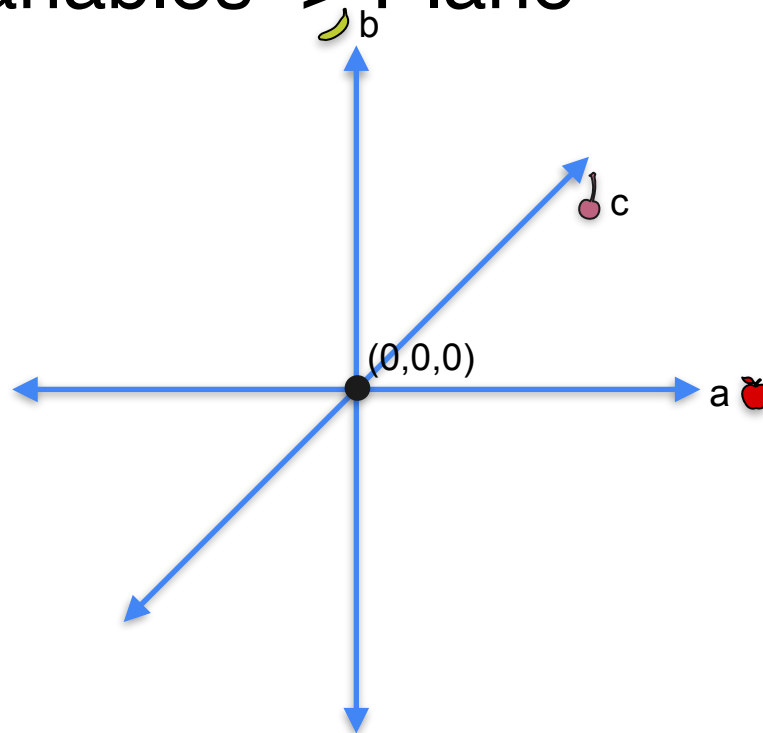
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



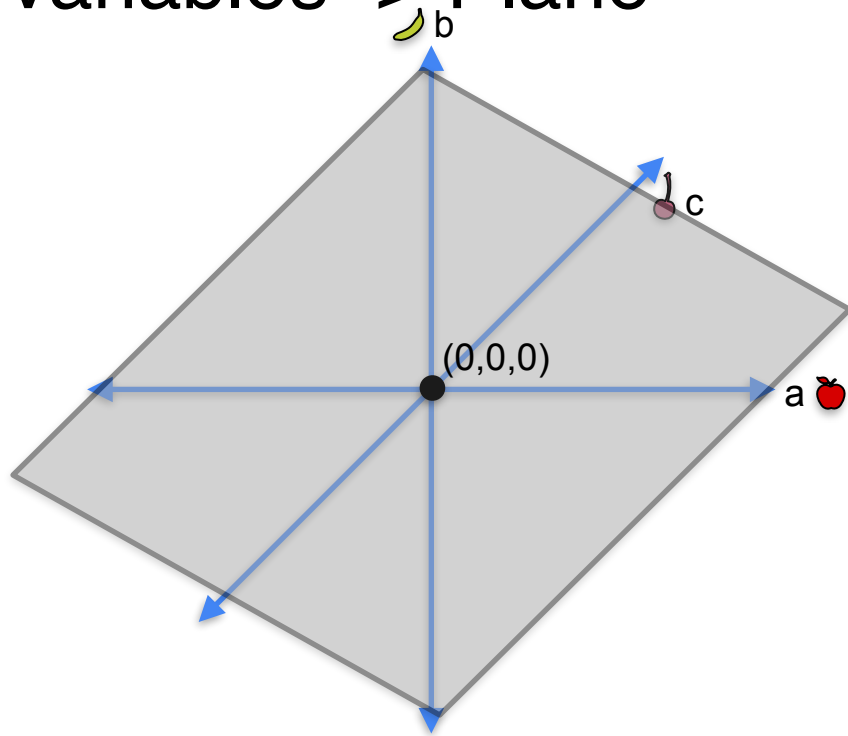
Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$



Linear equation in 3 variables -> Plane

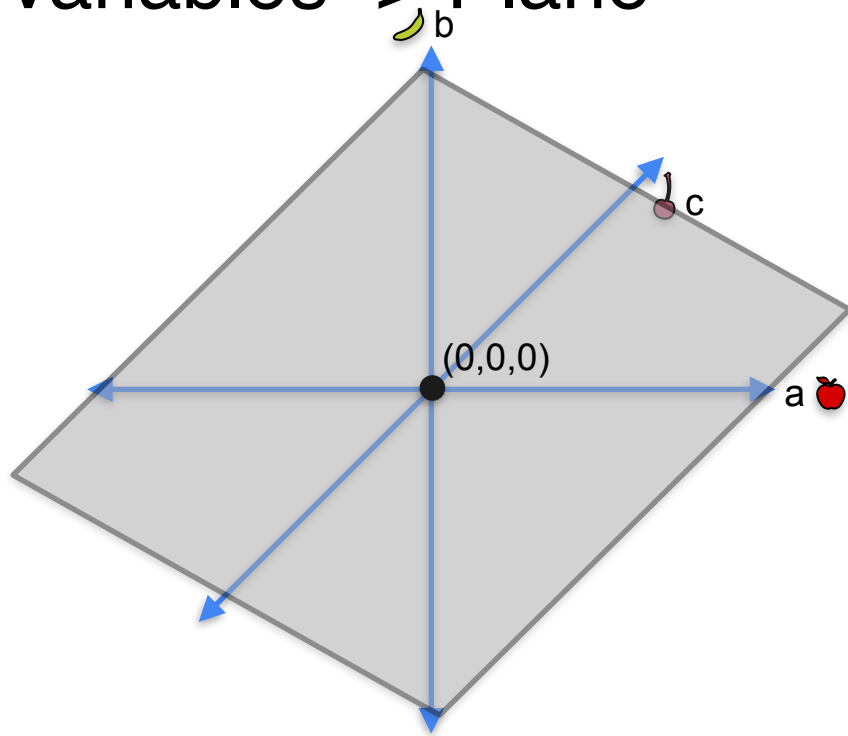
$$3a - 5b + 2c = 0$$



Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$

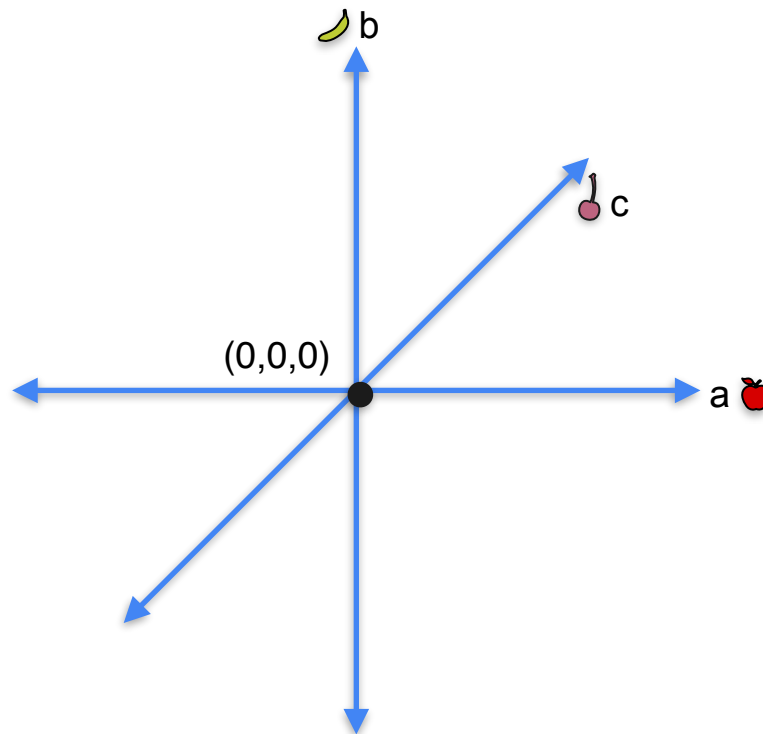
$$3(0) + 5(0) + 2(0) = 0$$



System 1

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



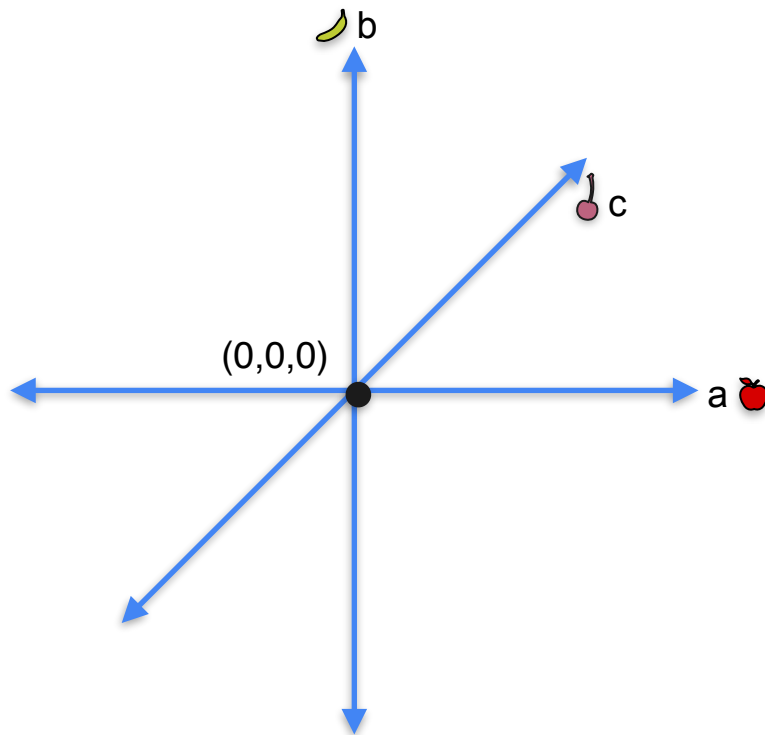
System 1

System 1

- $a + b + c = 0$

- $a + 2b + c = 0$

- $a + b + 2c = 0$



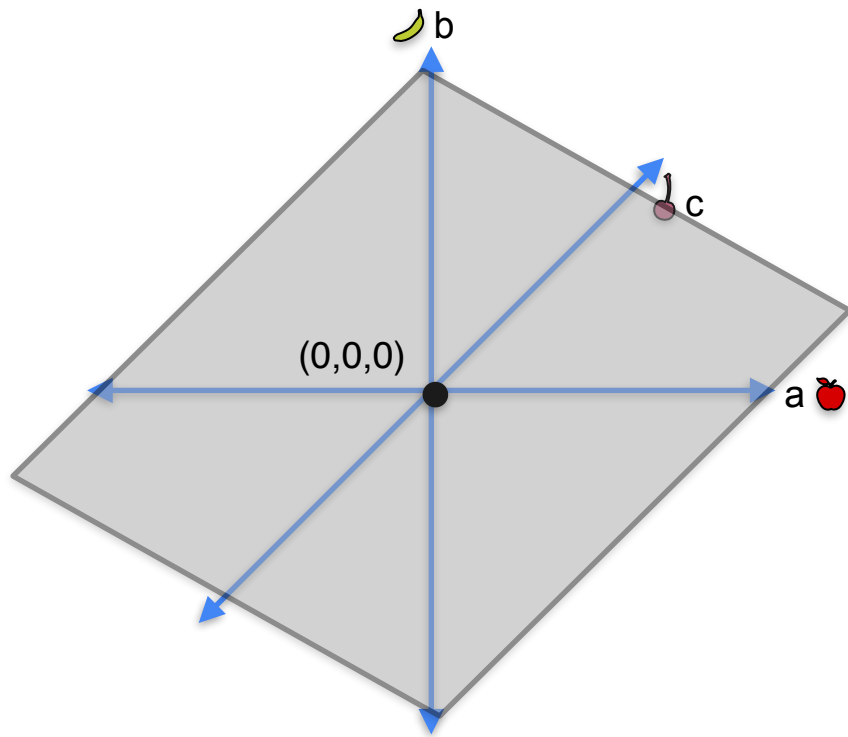
System 1

System 1

- $a + b + c = 0$

- $a + 2b + c = 0$

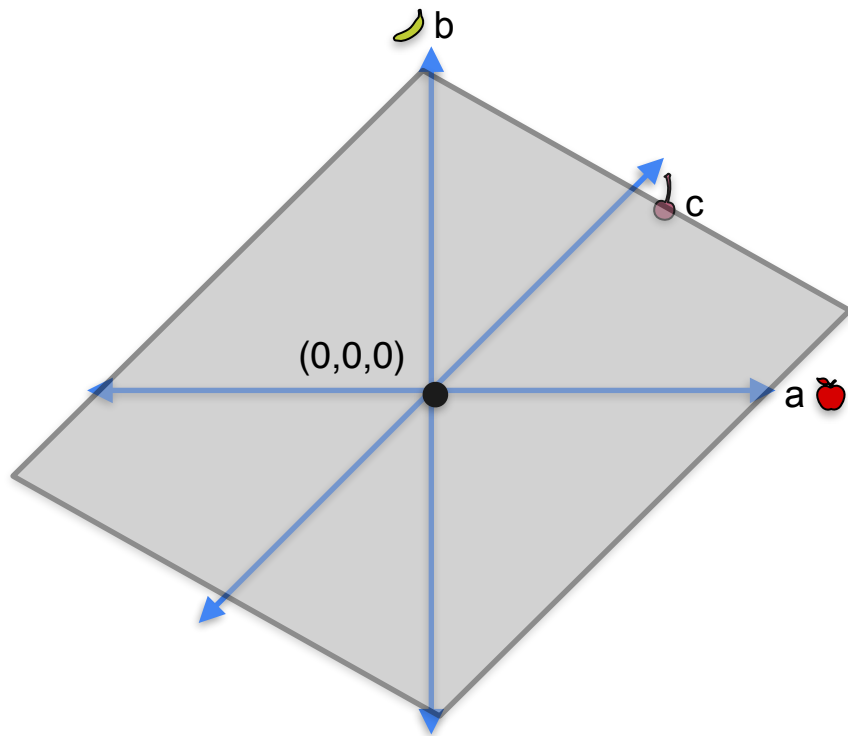
- $a + b + 2c = 0$



System 1

System 1

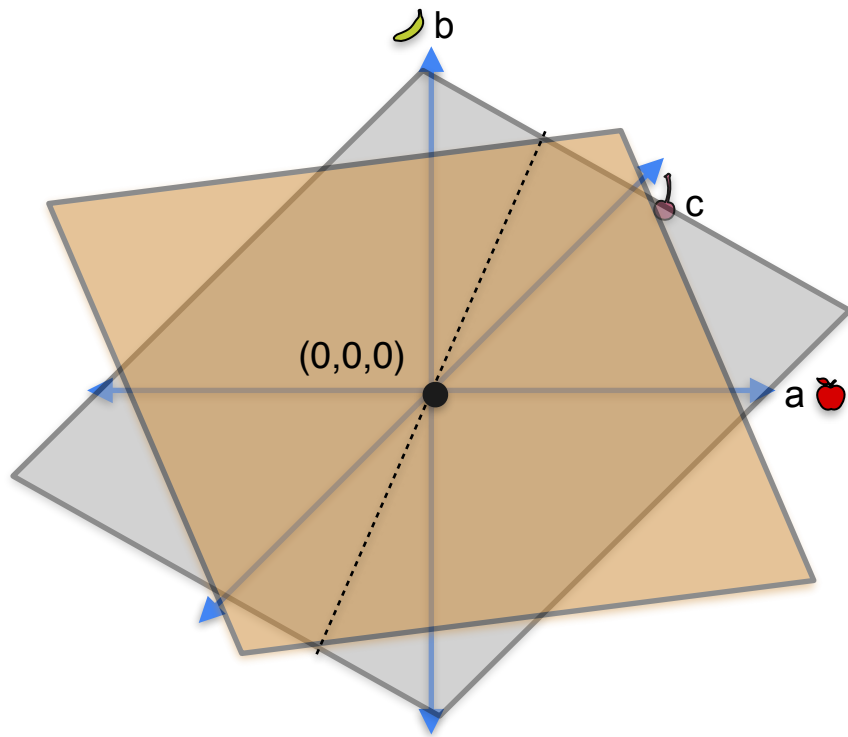
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



System 1

System 1

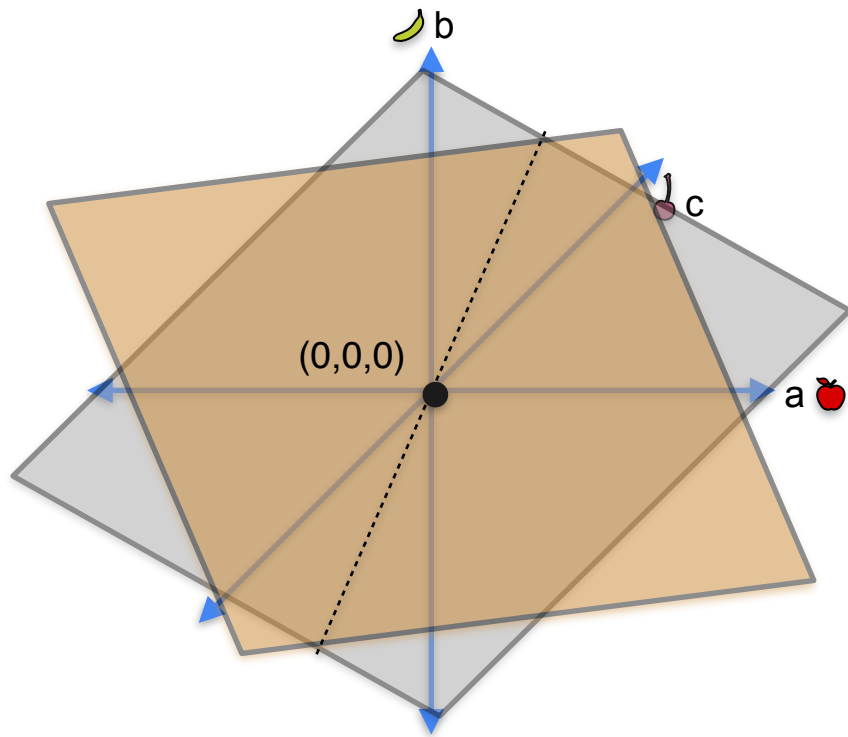
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



System 1

System 1

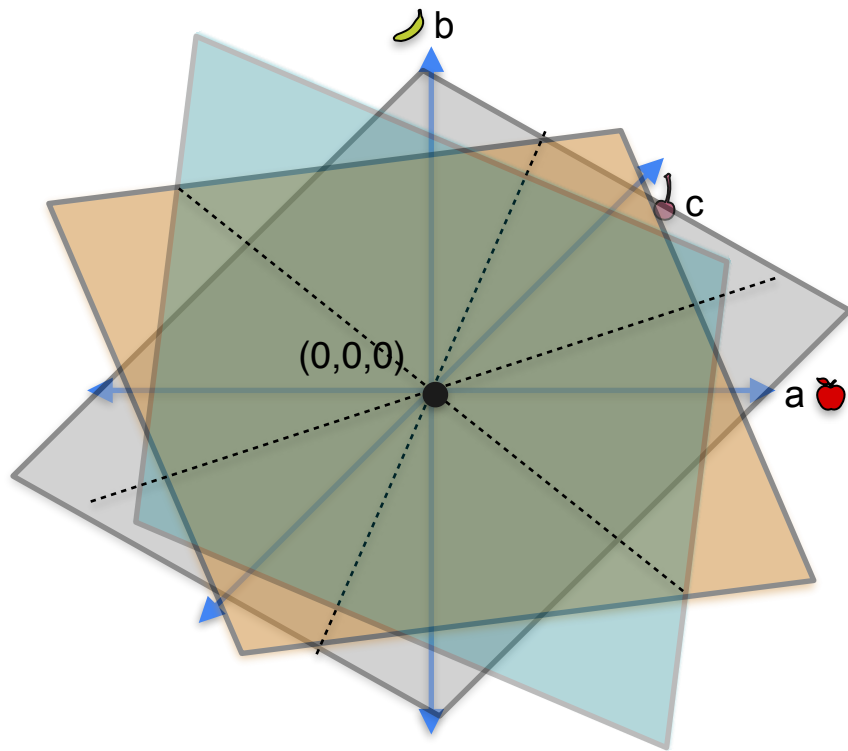
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



System 1

System 1

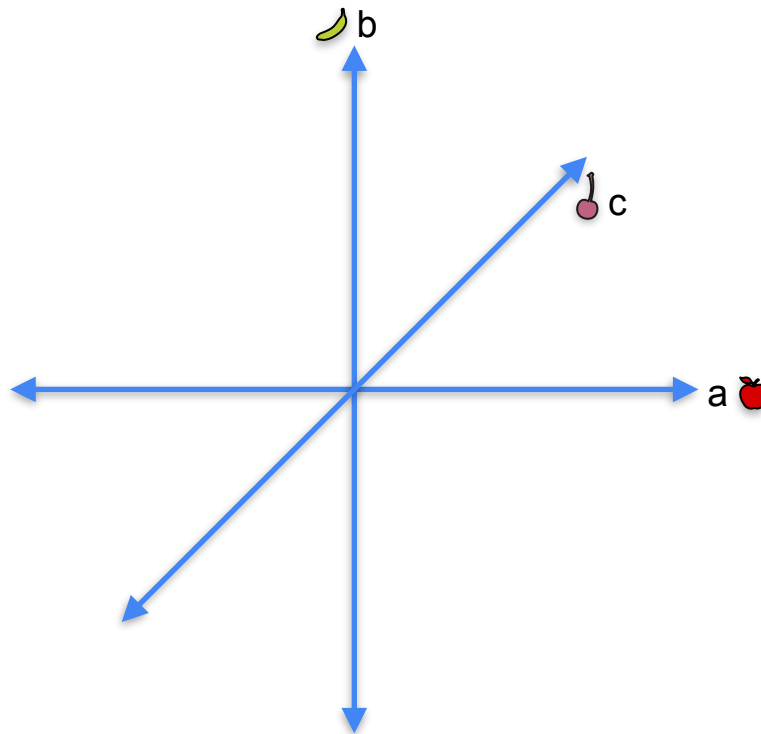
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



System 2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



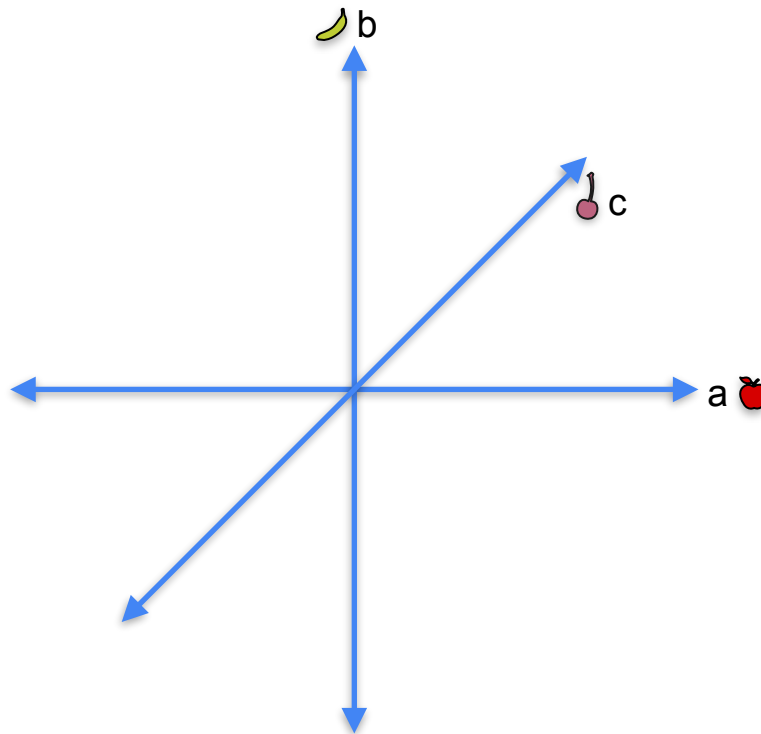
System 2

System 2

- $a + b + c = 0$

- $a + b + 2c = 0$

- $a + b + 3c = 0$



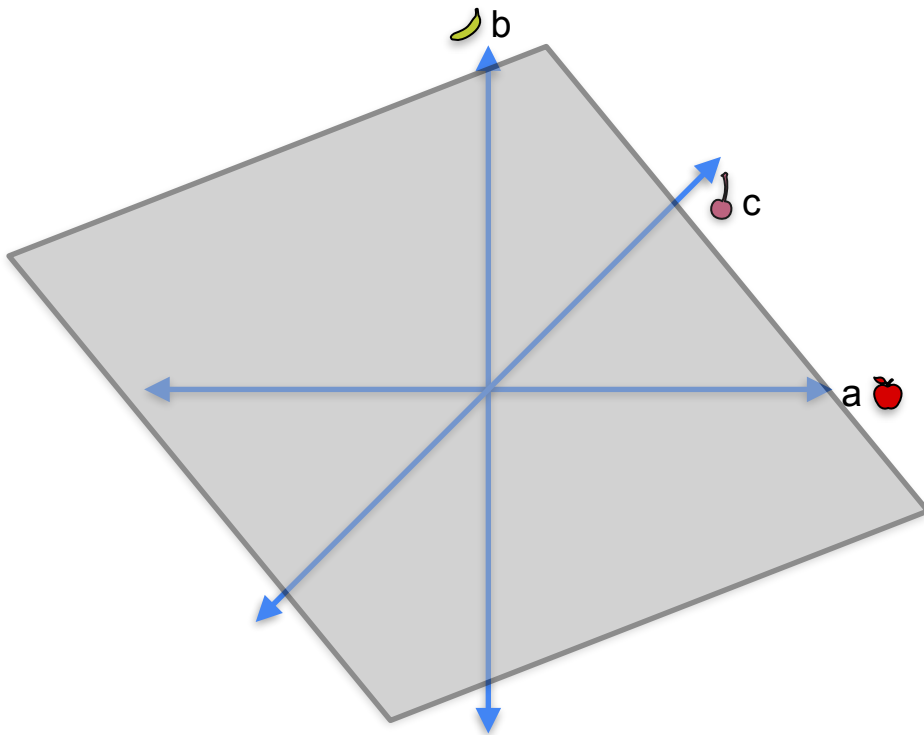
System 2

System 2

- $a + b + c = 0$

- $a + b + 2c = 0$

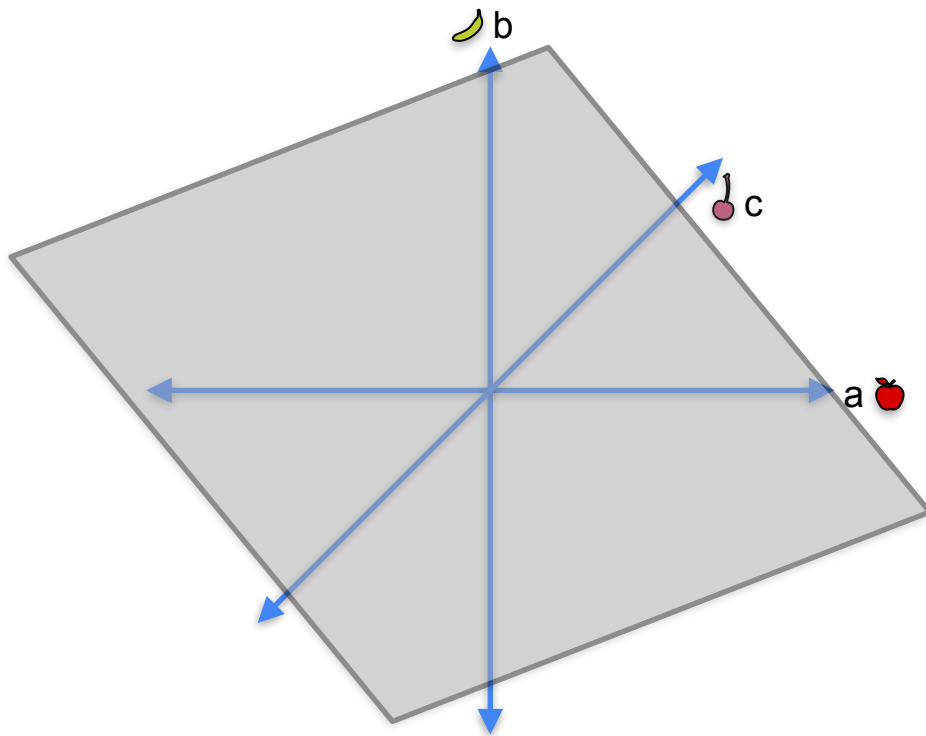
- $a + b + 3c = 0$



System 2

System 2

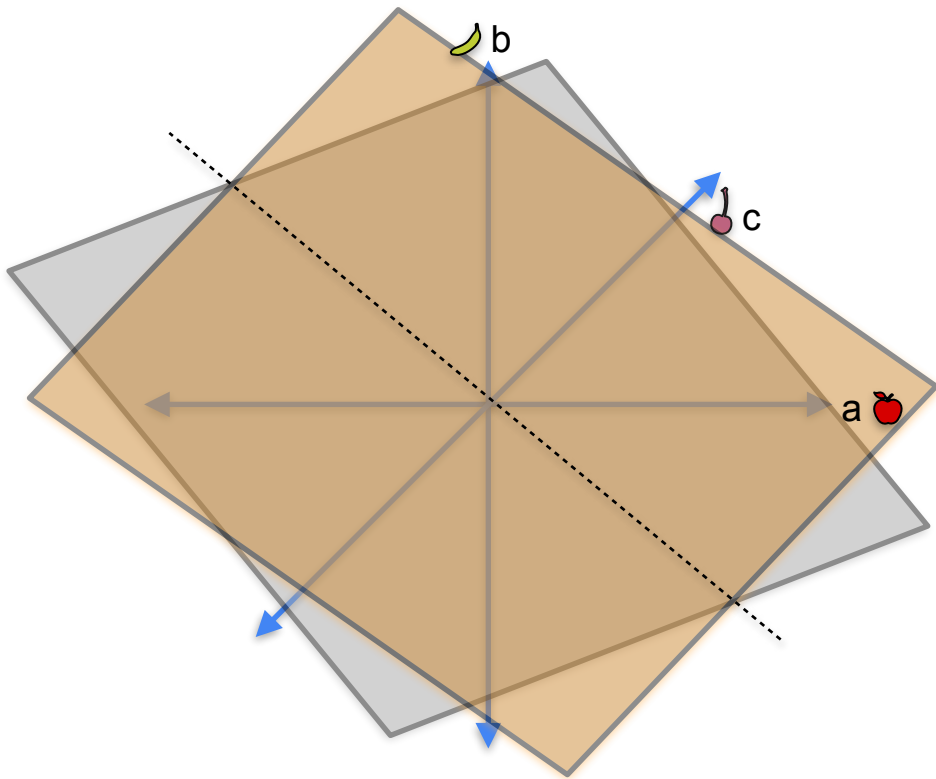
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

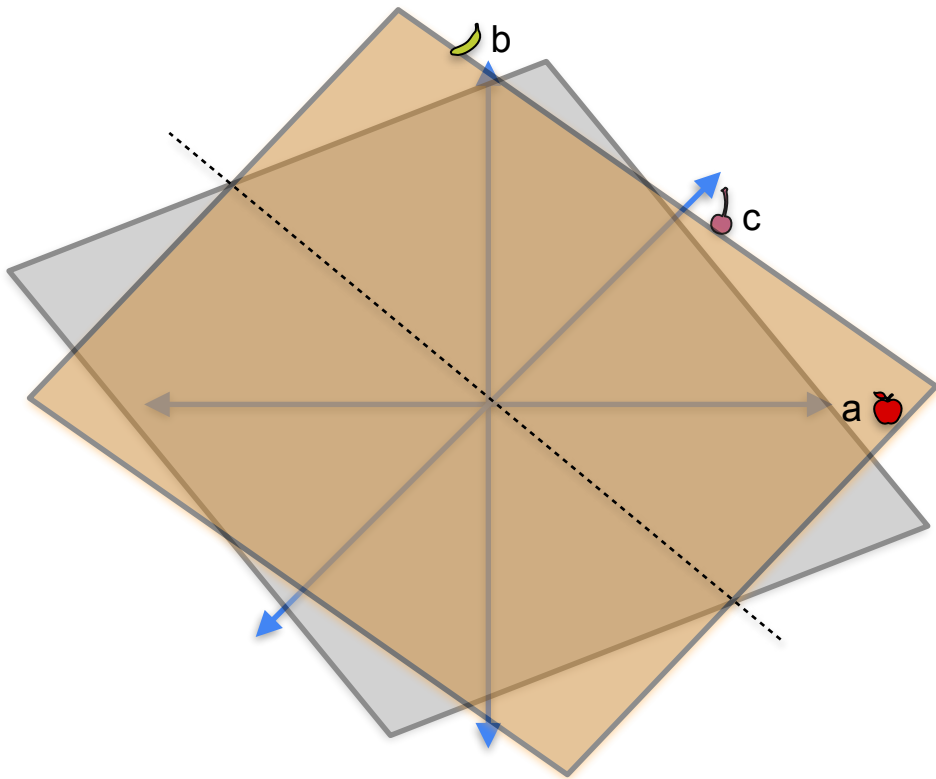
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

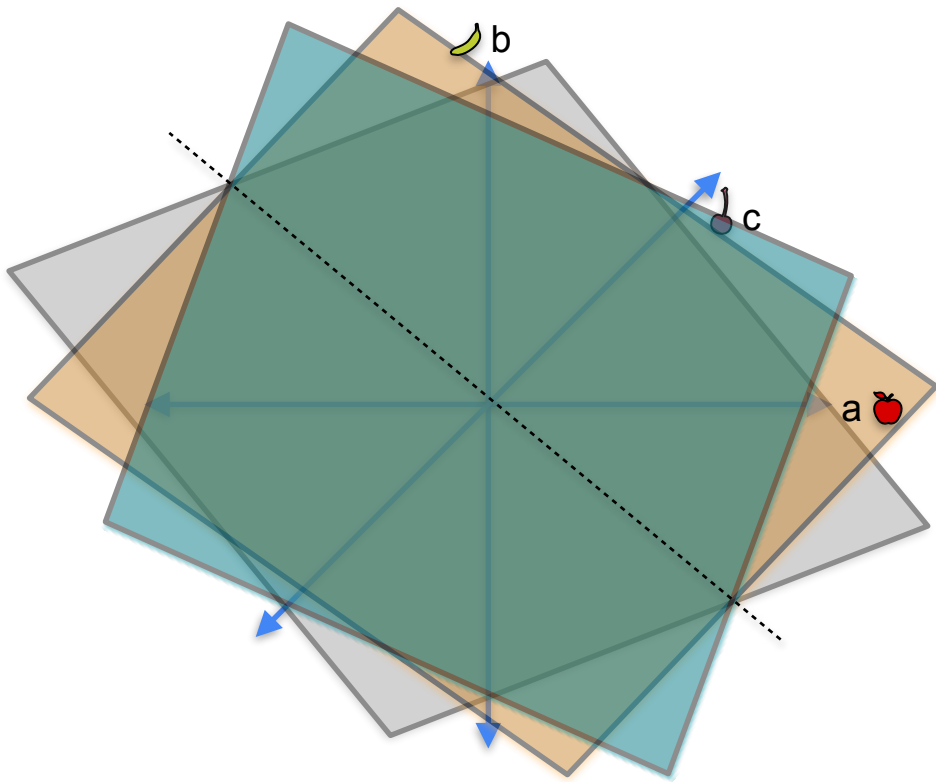
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

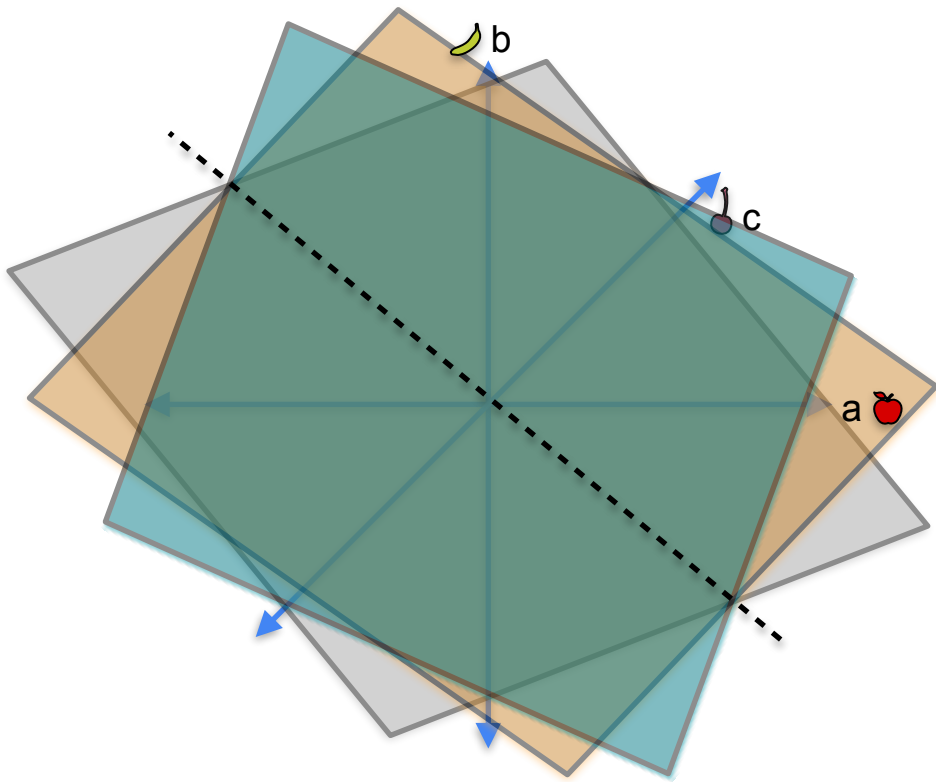
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

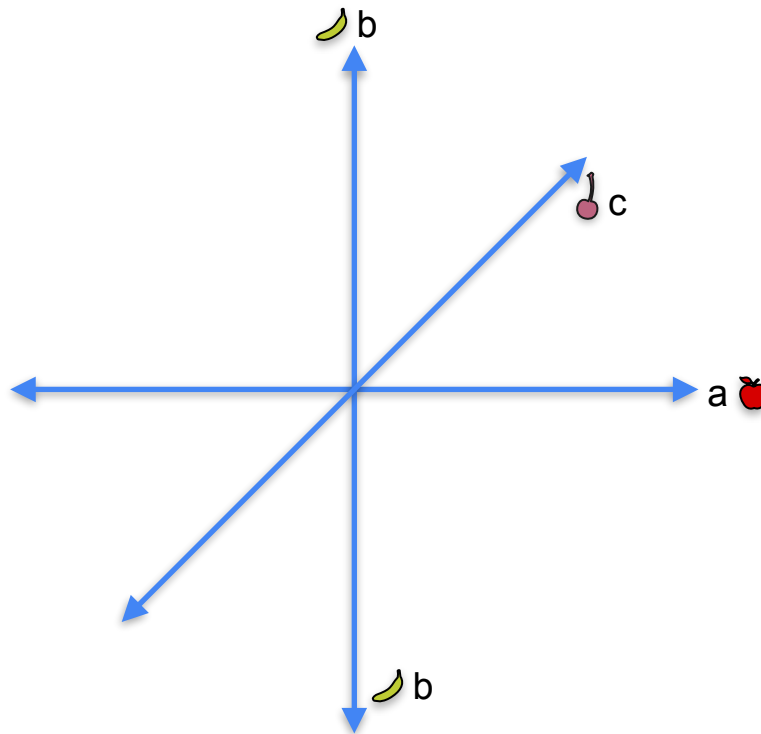
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 3

System 3

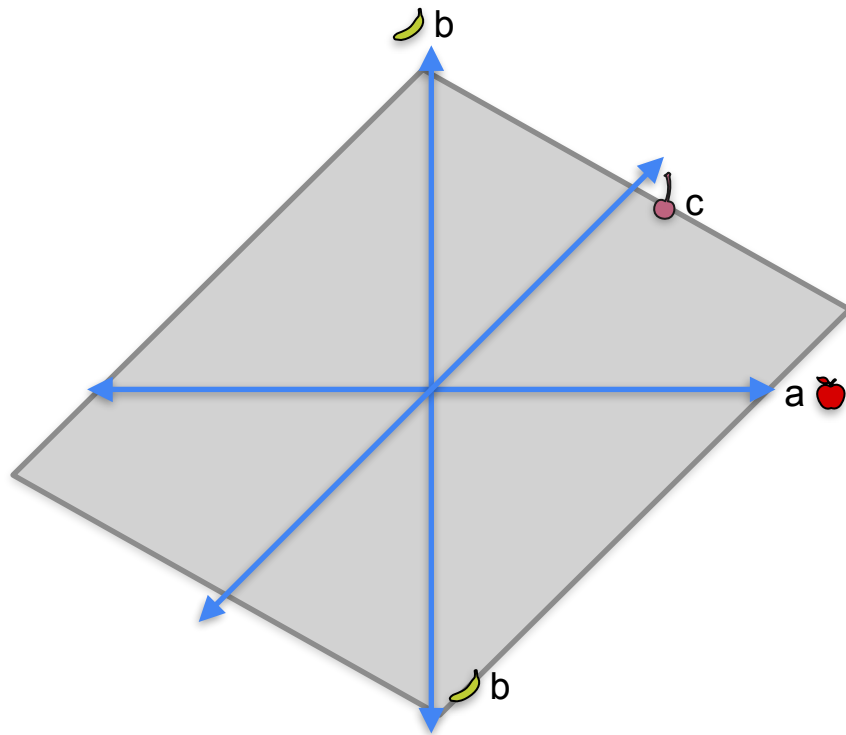
- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$



System 3

System 3

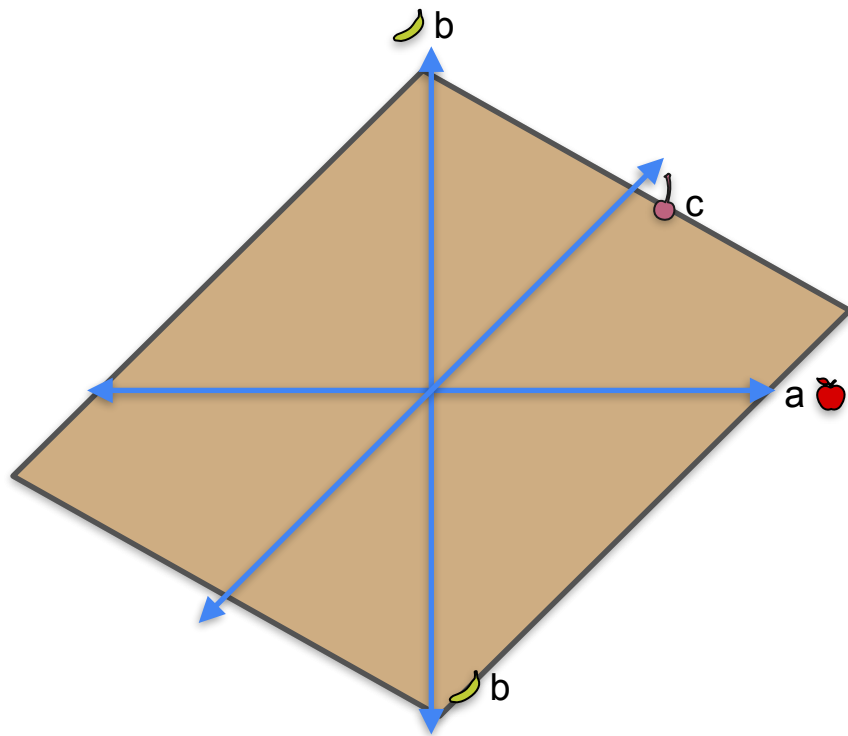
- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$



System 3

System 3

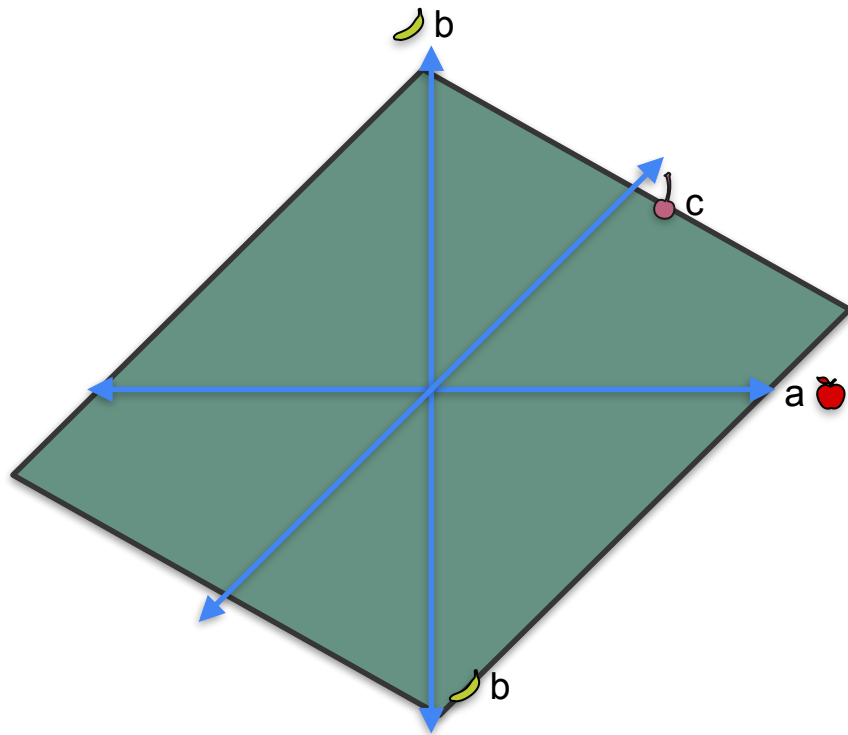
- $a + b + c = 0$
- $2a + 2b + 2c = 0$ ←
- $3a + 3b + 3c = 0$



System 3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$





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System of Linear Equations

**Linear dependence and
independence (3x3)**

Linear dependence and independence

$$a = 1$$

$$b = 2$$

$$a + b = 3$$

Linear dependence and independence


$$\begin{aligned} a &= 1 \\ b &= 2 \\ a + b &= 3 \end{aligned} \qquad a + 0b + 0c = 1$$

Linear dependence and independence

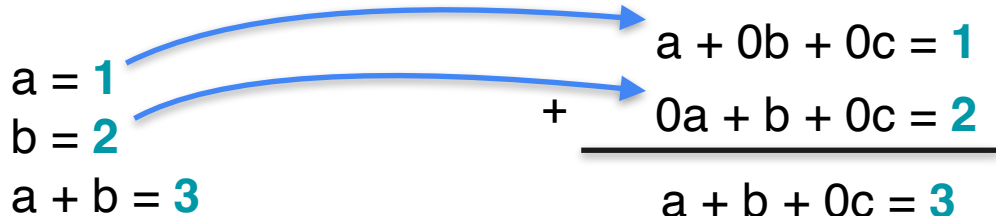
The diagram illustrates the substitution of values for variables a and b into two linear equations. On the left, the values are given as $a = 1$, $b = 2$, and $a + b = 3$. On the right, the equations are $a + 0b + 0c = 1$ and $0a + b + 0c = 2$. Blue curved arrows point from the value 1 in $a = 1$ to the constant term 1 in the first equation, and from the value 2 in $b = 2$ to the constant term 2 in the second equation.

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \end{array}$$

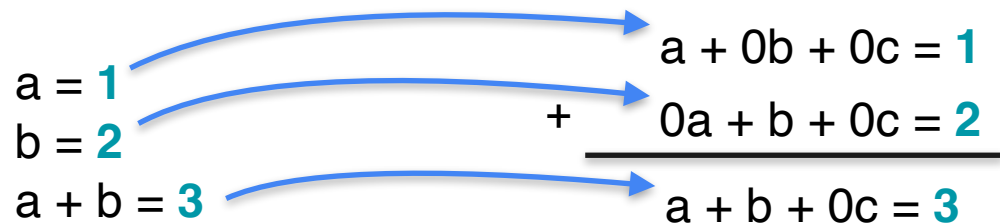
Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{1.5cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1.5cm}} 0a + b + 0c = 2 \\ + \\ \hline \end{array}$$

Linear dependence and independence


$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \qquad \begin{array}{r} a + 0b + 0c = 1 \\ + \quad 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

Linear dependence and independence



The diagram illustrates the process of substituting values for variables a and b into a linear equation. On the left, three equations are listed: $a = 1$, $b = 2$, and $a + b = 3$. On the right, two equations are shown: $a + 0b + 0c = 1$ and $0a + b + 0c = 2$. These two equations are added together, as indicated by a plus sign and a horizontal line. The result of the addition is $a + b + 0c = 3$. Blue curved arrows show the mapping from the left equations to the right equations: from $a = 1$ to the first equation, from $b = 2$ to the second equation, and from $a + b = 3$ to the final result equation.

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ a + b = 3 & \rightarrow & \hline a + b + 0c = 3 \end{array}$$

Linear dependence and independence

$$\begin{array}{lcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & \hline a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & \hline a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & \hline a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence


$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence

$$\begin{aligned}a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0\end{aligned}$$

$$a + b + c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence

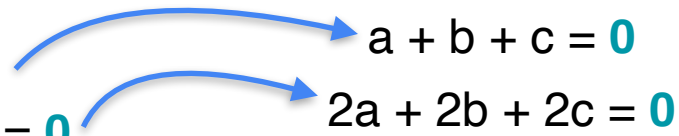
$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array}$$


Diagram illustrating linear dependence. The first two equations are shown on the left, and the third equation is shown on the right. Blue arrows indicate that the first two equations are linearly dependent on the third equation.

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence

$$\begin{array}{rcl} a + b + c & = & 0 \\ 2a + 2b + 2c & = & 0 \\ 3a + 3b + 3c & = & 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ \xrightarrow{\quad} \end{array} \begin{array}{rcl} a + b + c & = & 0 \\ + & & \\ \hline 2a + 2b + 2c & = & 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad} & + \quad 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & & \hline & & 3a + 3b + 3c = 0 \end{array}$$

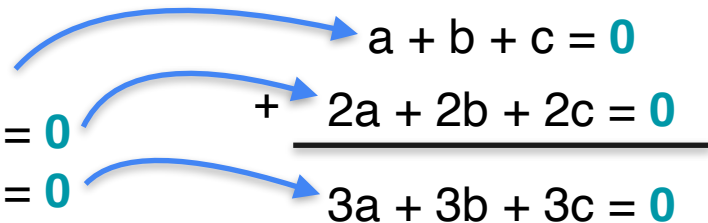
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad} & + \quad 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Linear dependence and independence



The diagram illustrates the linear dependence of three equations. On the left, three equations are listed vertically: $a + b + c = 0$, $2a + 2b + 2c = 0$, and $3a + 3b + 3c = 0$. On the right, the same three equations are shown, but the second equation is preceded by a plus sign and the third equation is underlined. Blue curved arrows point from each of the three equations on the left to the corresponding equation on the right, indicating that they are being summed together. The result of the summation is $3a + 3b + 3c = 0$.

$$\begin{array}{rcl} a + b + c & = & 0 \\ 2a + 2b + 2c & = & 0 \\ 3a + 3b + 3c & = & 0 \end{array} \quad \begin{array}{rcl} & + & \\ & + & \\ & + & \end{array} \quad \begin{array}{rcl} a + b + c & = & 0 \\ 2a + 2b + 2c & = & 0 \\ \hline 3a + 3b + 3c & = & 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3

Linear dependence and independence

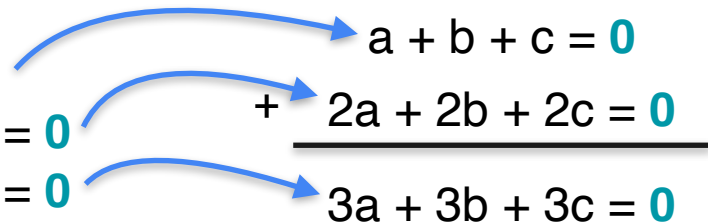
$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad + \quad} & 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad + \quad} & 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$


| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

$$a + b + c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{\quad} a + b + 3c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \xrightarrow{\quad} \end{array} \quad \begin{array}{l} a + b + c = 0 \\ \hline a + b + 3c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{rcl} a + b + c & = & 0 \\ a + b + 2c & = & 0 \\ a + b + 3c & = & 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \xrightarrow{\quad} \\ \hline \end{array} \quad \begin{array}{rcl} a + b + c & = & 0 \\ a + b + 3c & = & 0 \\ \hline 2a + 2b + 4c & = & 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{rcl} a + b + c & = & 0 \\ a + b + 2c & = & 0 \\ a + b + 3c & = & 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \xrightarrow{\quad} \\ \hline \end{array} \quad \begin{array}{rcl} a + b + c & = & 0 \\ a + b + 3c & = & 0 \\ \hline 2a + 2b + 4c & = & 0 \\ \downarrow \div 2 & & \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$
$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 2c = 0 \\ \hline 2a + 2b + 3c = 0 \\ \downarrow \div 2 \\ a + b + 1.5c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

Diagram illustrating the elimination of a variable from a system of linear equations:

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$

Operations shown:

$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \\ \downarrow \div 2 \\ a + b + 2c = 0 \end{array}$$

Matrix representation of the coefficients:

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$
$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \\ \downarrow \div 2 \\ a + b + 2c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$
$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \\ \downarrow \div 2 \\ a + b + 2c = 0 \end{array}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3
Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

No relations between rows

Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

No relations between rows

Rows are **linearly independent**

Quiz: Linear dependence and independence

Problem: Determine if the following matrices have linearly dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

Dependent (singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

No relations

**Independent
(Non-singular)**

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

Dependent (singular)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

No relations

**Independent
(Non-singular)**

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

$$2\text{Row1} = \text{Row3}$$

Dependent (singular)

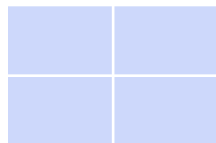


DeepLearning.AI

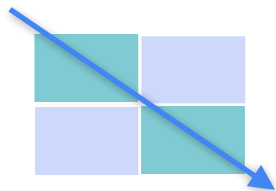
System of Linear Equations

The determinant (3x3)

Diagonals in a 3x3 matrix



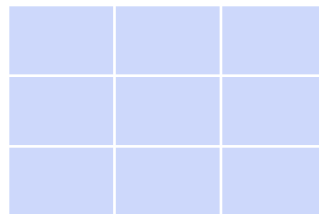
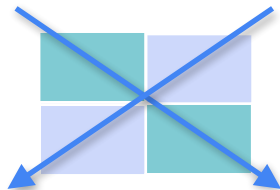
Diagonals in a 3x3 matrix



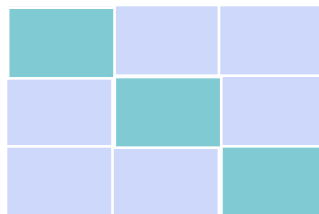
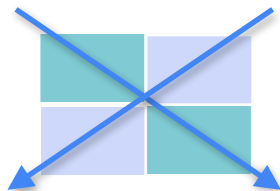
Diagonals in a 3x3 matrix



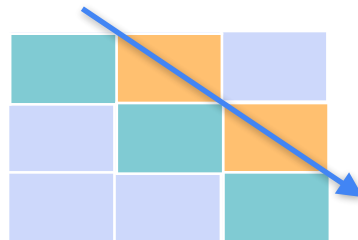
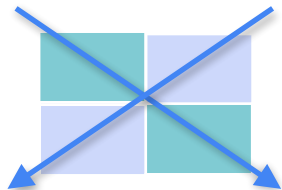
Diagonals in a 3x3 matrix



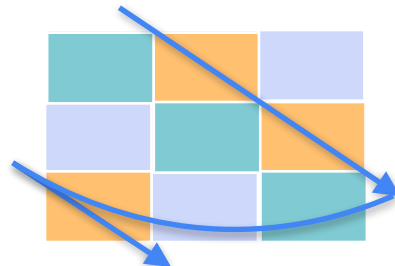
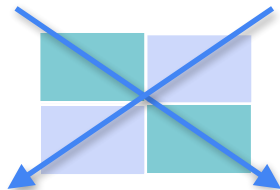
Diagonals in a 3x3 matrix



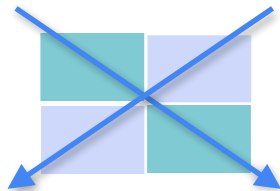
Diagonals in a 3x3 matrix



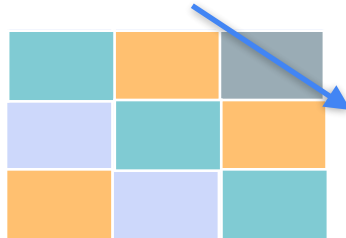
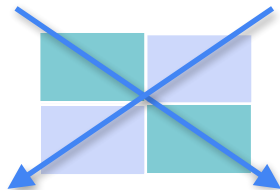
Diagonals in a 3x3 matrix



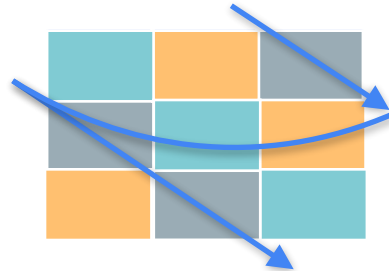
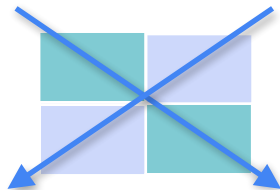
Diagonals in a 3x3 matrix



Diagonals in a 3x3 matrix



Diagonals in a 3x3 matrix



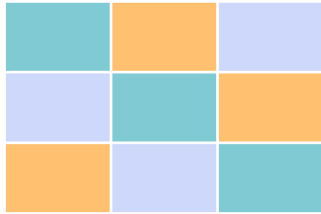
Diagonals in a 3x3 matrix



Determinant

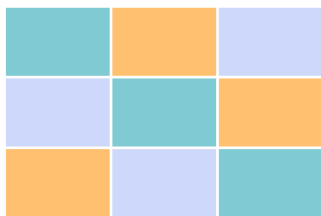
Determinant

Add

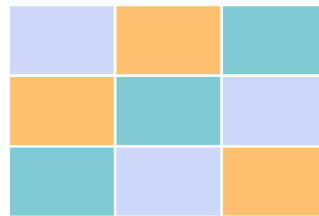


Determinant

Add



Subtract



The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

| | | |
|---|---|---|
| 1 | | |
| | 2 | |
| | | 2 |

$$+ 1 \cdot 2 \cdot 2$$

The determinant

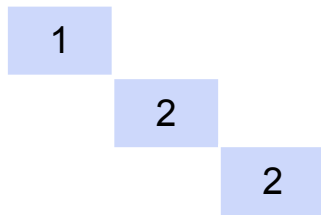
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

| | | | |
|---|---|---|--|
| 1 | | | |
| | 2 | | |
| | | 2 | |

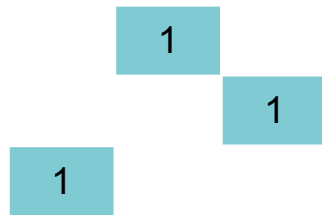
$$+ 1 \cdot 2 \cdot 2$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



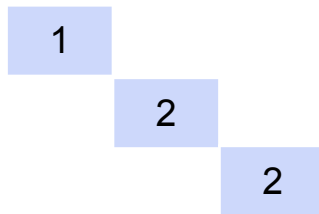
$$+ 1 \cdot 2 \cdot 2$$



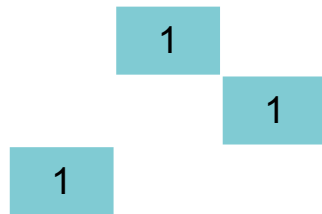
$$+ 1 \cdot 1 \cdot 1$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



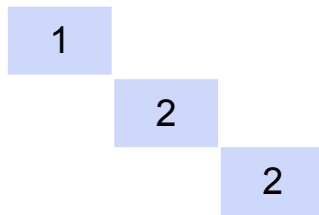
$$+ 1 \cdot 2 \cdot 2$$



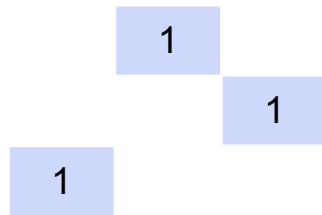
$$+ 1 \cdot 1 \cdot 1$$

The determinant

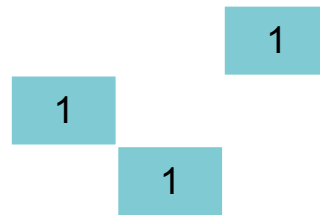
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$+ 1 \cdot 2 \cdot 2$$



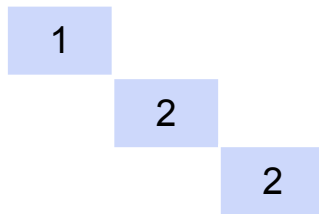
$$+ 1 \cdot 1 \cdot 1$$



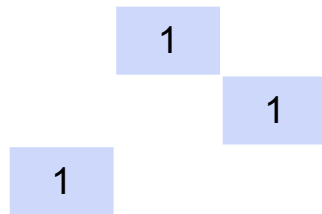
$$+ 1 \cdot 1 \cdot 1$$

The determinant

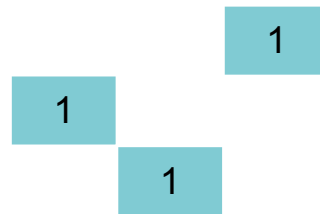
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$+ 1 \cdot 2 \cdot 2$$



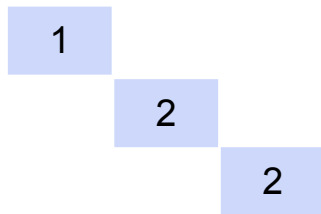
$$+ 1 \cdot 1 \cdot 1$$



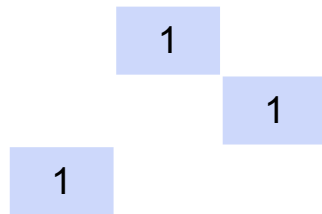
$$+ 1 \cdot 1 \cdot 1$$

The determinant

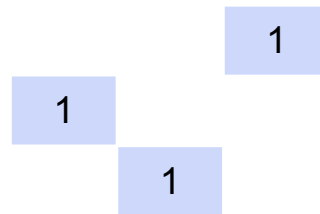
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



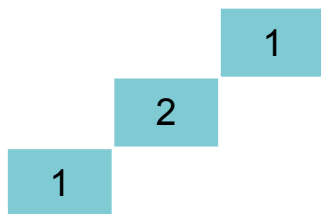
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



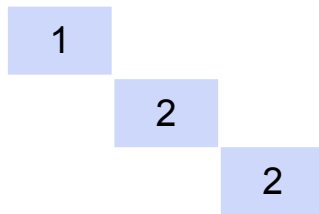
$$+ 1 \cdot 1 \cdot 1$$



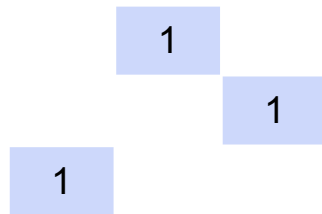
$$- 1 \cdot 2 \cdot 1$$

The determinant

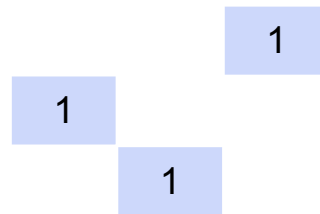
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



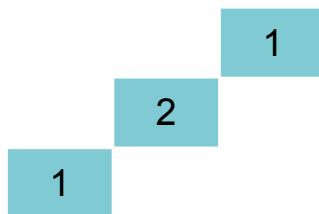
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



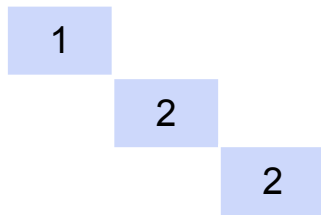
$$+ 1 \cdot 1 \cdot 1$$



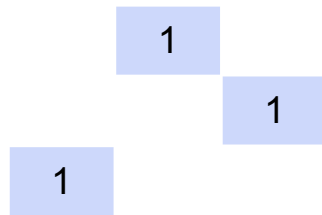
$$- 1 \cdot 2 \cdot 1$$

The determinant

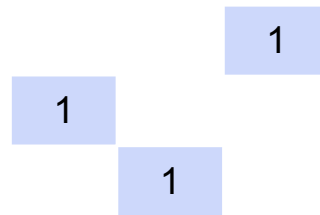
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



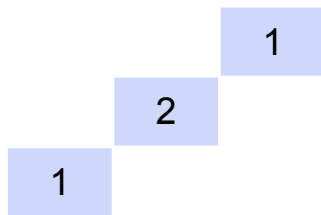
$$+ 1 \cdot 2 \cdot 2$$



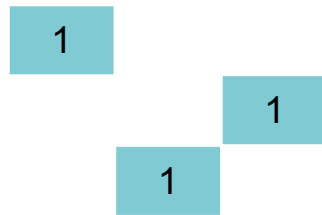
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



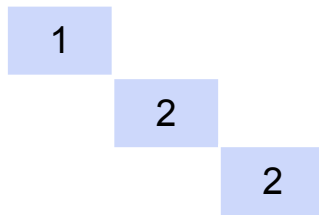
$$- 1 \cdot 2 \cdot 1$$



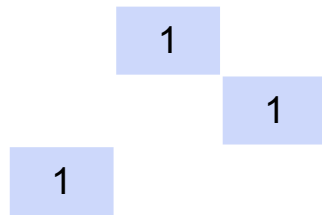
$$- 1 \cdot 1 \cdot 1$$

The determinant

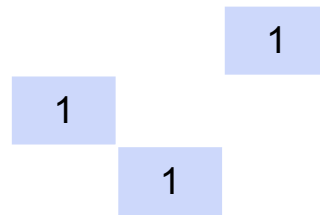
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



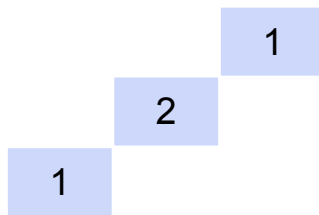
$$+ 1 \cdot 2 \cdot 2$$



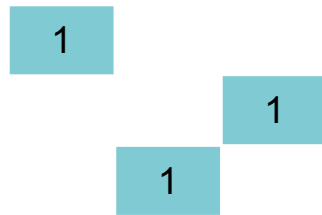
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



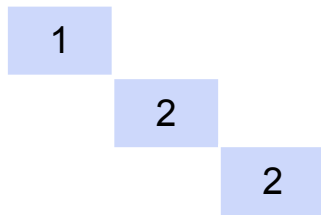
$$- 1 \cdot 2 \cdot 1$$



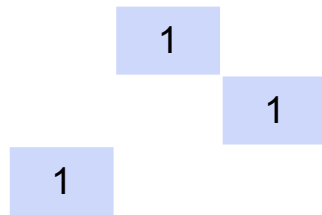
$$- 1 \cdot 1 \cdot 1$$

The determinant

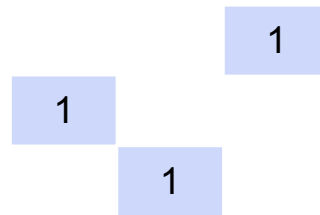
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



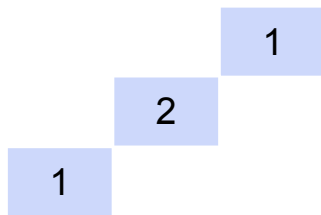
$$+ 1 \cdot 2 \cdot 2$$



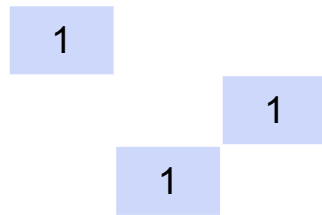
$$+ 1 \cdot 1 \cdot 1$$



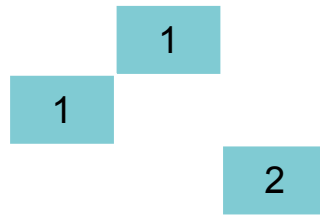
$$+ 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 2 \cdot 1$$



$$- 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 1 \cdot 2$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

$$+ 1 \cdot 2 \cdot 2$$

$$+ 1 \cdot 1 \cdot 1$$

$$+ 1 \cdot 1 \cdot 1$$

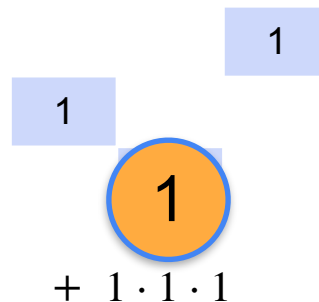
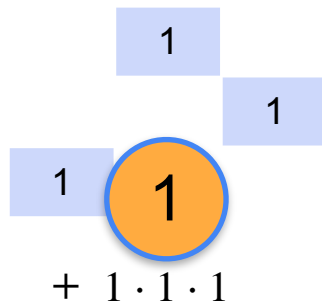
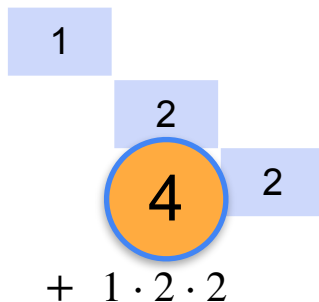
$$- 1 \cdot 2 \cdot 1$$

$$- 1 \cdot 1 \cdot 1$$

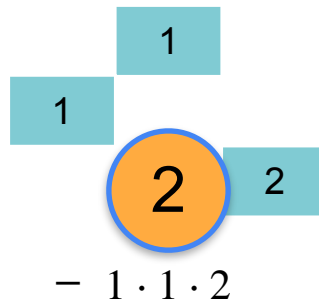
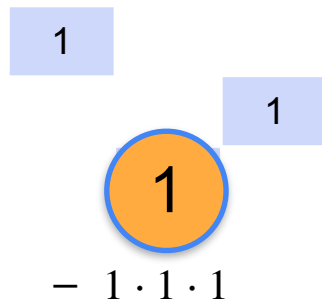
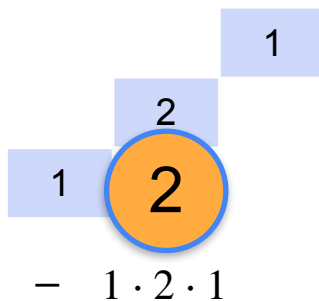
$$- 1 \cdot 1 \cdot 2$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

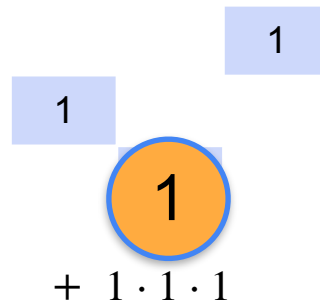
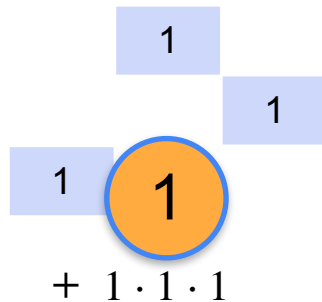
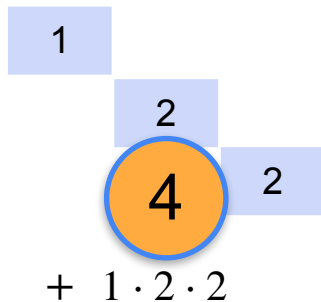


$$\text{Det} = 4 + 1 + 1 - 2 - 1 - 2$$

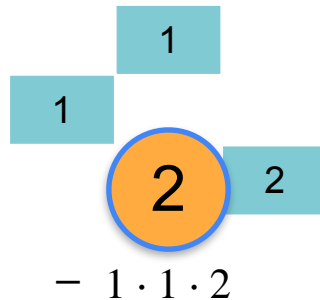
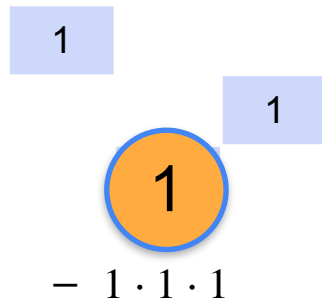
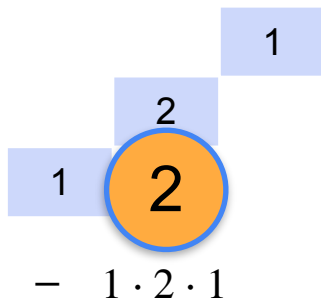


The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$\begin{aligned} \text{Det} &= 4 + 1 + 1 \\ &\quad - 2 - 1 - 2 \\ &= 1 \end{aligned}$$



Quiz: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz).
Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz).
Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz).
Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Determinant = 0

Singular

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz).
Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

Singular

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

Determinant = 0

Singular

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz).
Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

Singular

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

Determinant = 0

Singular

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

Determinant = 6

Non-singular

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

Singular

| | | |
|---|---|----|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | 0 | -1 |

Determinant = 0

Singular

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

Determinant = 6

Non-singular

| | | |
|---|---|----|
| 1 | 2 | 5 |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Determinant = 0

Singular

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

$$\begin{aligned}\text{Det} &= 6+0+0-0-0-0 \\ &= 6\end{aligned}$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|---|
| 1 | | |
| | 2 | |
| | | 3 |

$$+ 1 \cdot 2 \cdot 3$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|---|
| 1 | | |
| | 2 | |
| | | 3 |

$$+ 1 \cdot 2 \cdot 3$$

| | | |
|---|---|---|
| | 1 | |
| | | 2 |
| 0 | | |

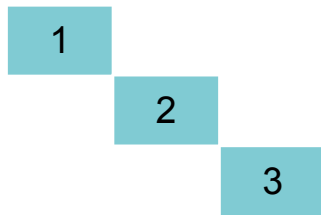
$$+ 1 \cdot 2 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

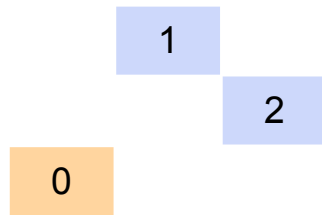
$$= 6$$

The determinant

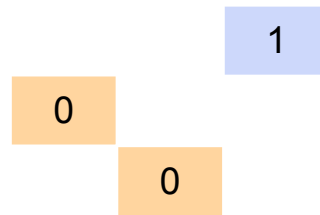
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



$$+ 1 \cdot 2 \cdot 0$$



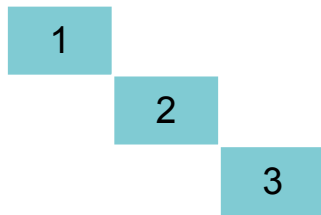
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



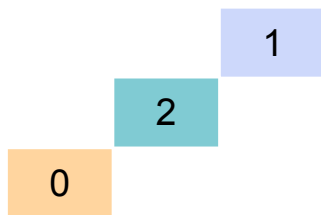
$$+ 1 \cdot 2 \cdot 0$$



$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

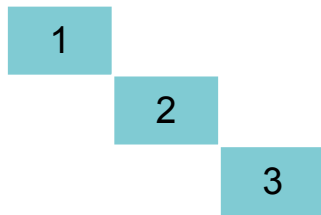
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



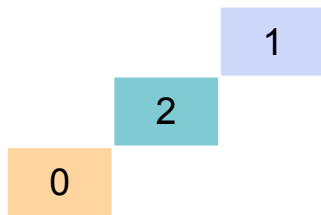
$$+ 1 \cdot 2 \cdot 0$$



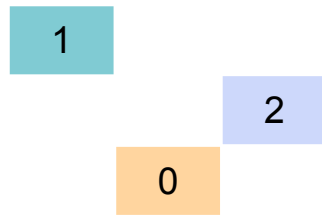
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$



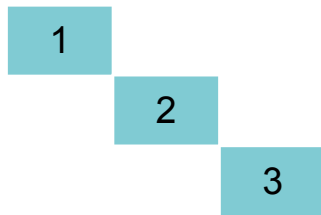
$$- 1 \cdot 2 \cdot 0$$



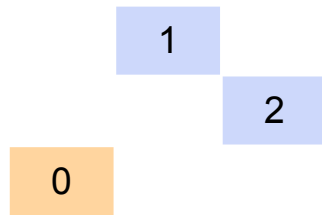
$$- 1 \cdot 2 \cdot 0$$

The determinant

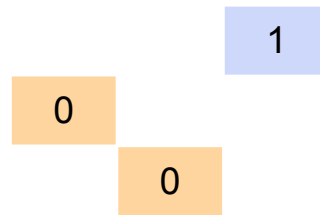
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



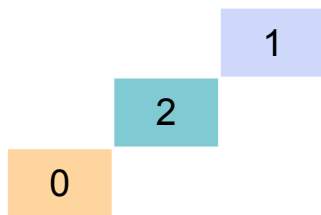
$$+ 1 \cdot 2 \cdot 0$$



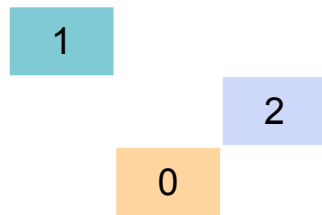
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

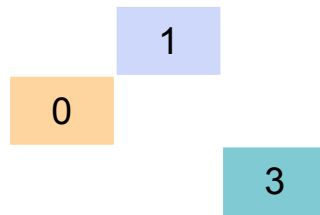
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 3$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

| | | |
|---|---|---|
| 1 | | |
| | 2 | |
| | | 3 |

$$+ 1 \cdot 2 \cdot 3$$

| | | |
|---|---|---|
| | 1 | |
| | | 2 |
| 0 | | |

$$+ 1 \cdot 2 \cdot 0$$

| | | |
|---|---|---|
| | | 1 |
| 0 | | |
| | 0 | |

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

| | | |
|---|---|---|
| | | 1 |
| | 2 | |
| 0 | | |

$$- 1 \cdot 2 \cdot 0$$

| | | |
|---|---|---|
| 1 | | |
| | | 2 |
| | 0 | |

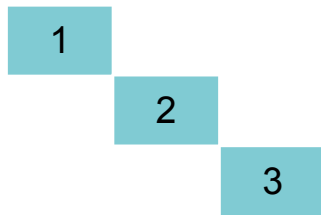
$$- 1 \cdot 2 \cdot 0$$

| | | |
|---|---|---|
| | 1 | |
| 0 | | |
| | | 3 |

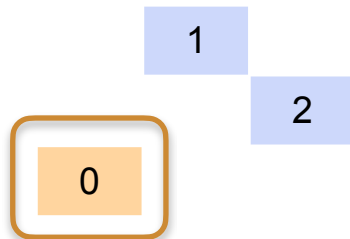
$$- 1 \cdot 0 \cdot 3$$

The determinant

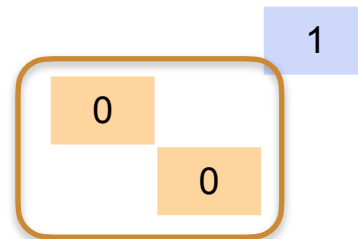
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



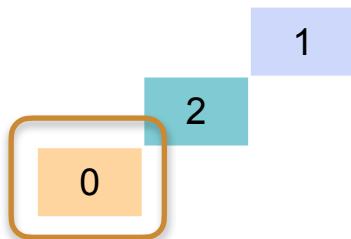
$$+ 1 \cdot 2 \cdot 0$$



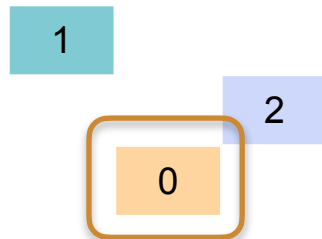
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

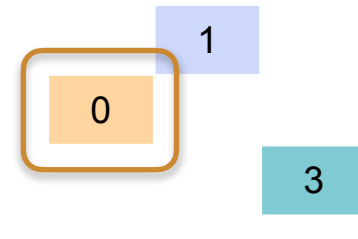
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



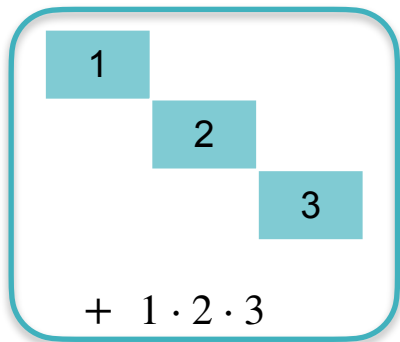
$$- 1 \cdot 2 \cdot 0$$

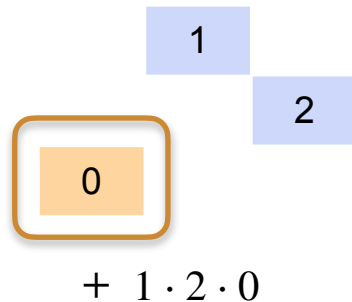


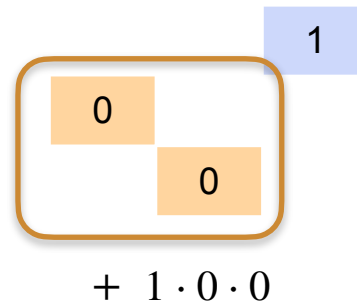
$$- 1 \cdot 0 \cdot 3$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

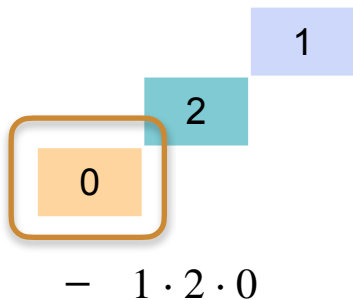

$$+ 1 \cdot 2 \cdot 3$$

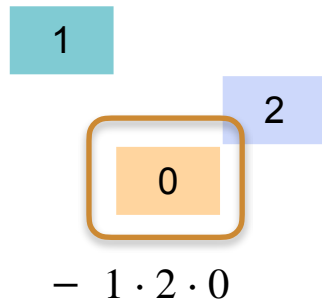

$$+ 1 \cdot 2 \cdot 0$$

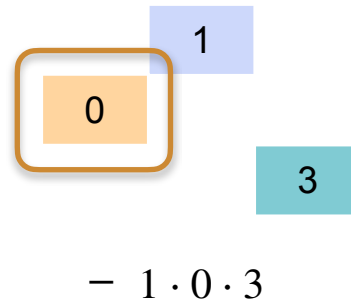

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

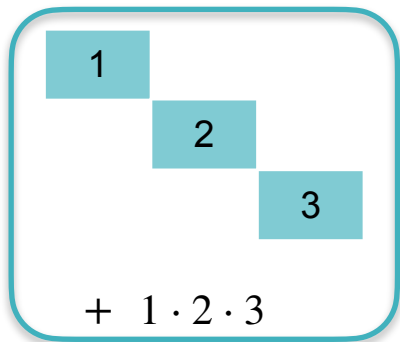

$$- 1 \cdot 2 \cdot 0$$

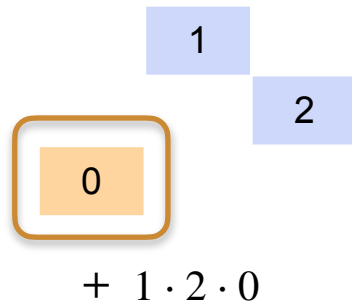

$$- 1 \cdot 2 \cdot 0$$

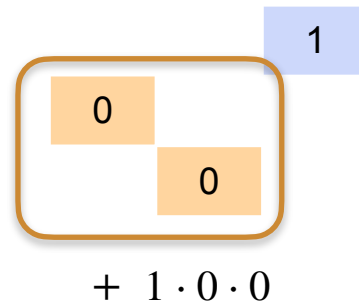

$$- 1 \cdot 0 \cdot 3$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

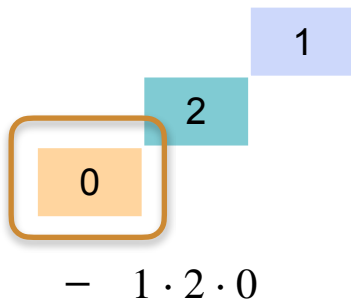

$$+ 1 \cdot 2 \cdot 3$$

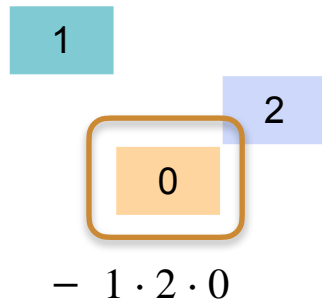

$$+ 1 \cdot 2 \cdot 0$$

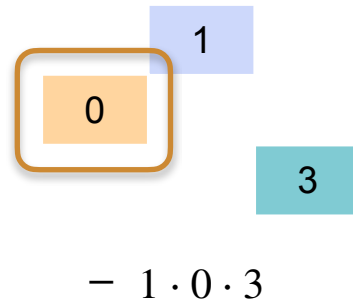

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$

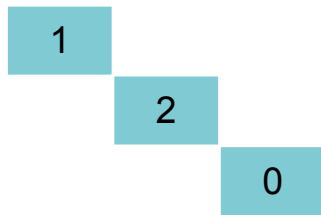

$$- 1 \cdot 0 \cdot 3$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |



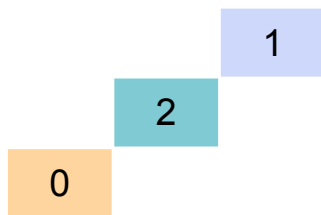
$$+ 1 \cdot 2 \cdot 0$$



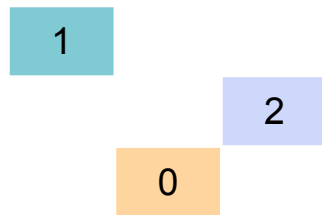
$$+ 1 \cdot 2 \cdot 0$$



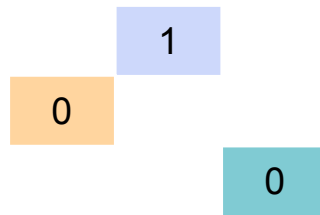
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



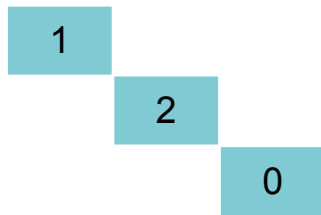
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 0$$

The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |



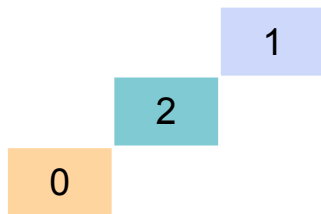
$$+ 1 \cdot 2 \cdot 0$$



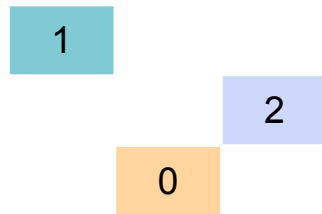
$$+ 1 \cdot 2 \cdot 0$$



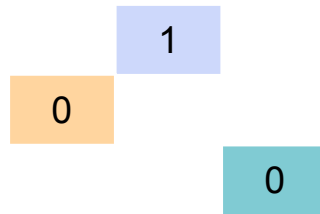
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



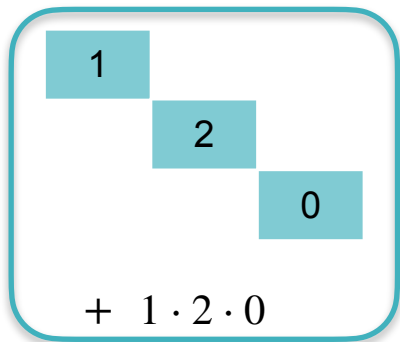
$$- 1 \cdot 2 \cdot 0$$

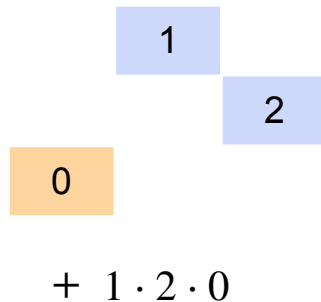


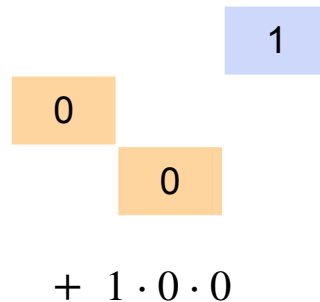
$$- 1 \cdot 0 \cdot 0$$

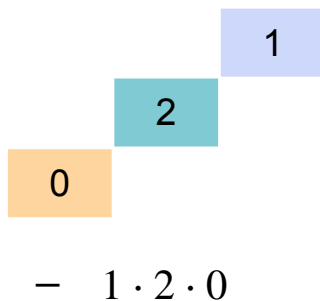
The determinant

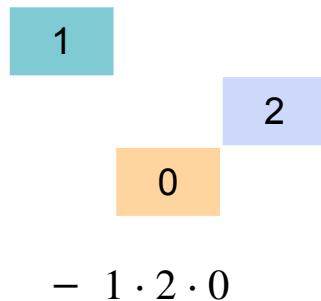
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

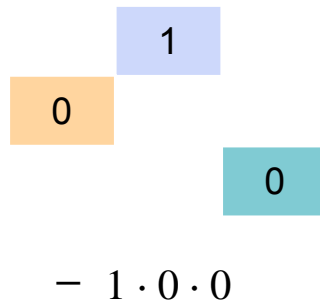

$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 0 \cdot 0$$

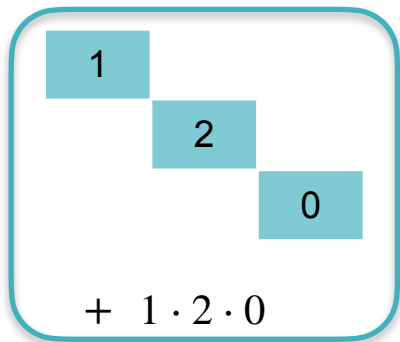

$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 0 \cdot 0$$

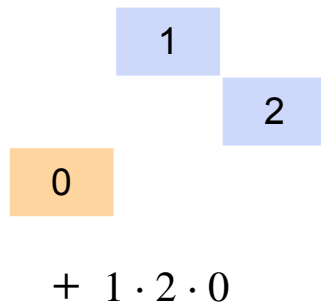
The determinant

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

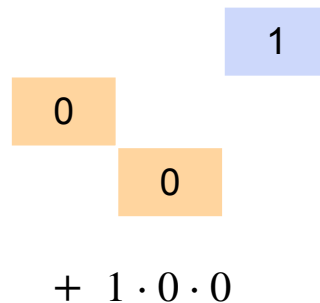


$$+ 1 \cdot 2 \cdot 0$$

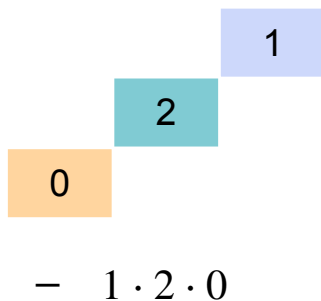
$$\text{Det} = 0+0+0-0-0-0$$



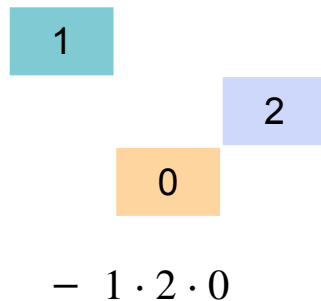
$$+ 1 \cdot 2 \cdot 0$$



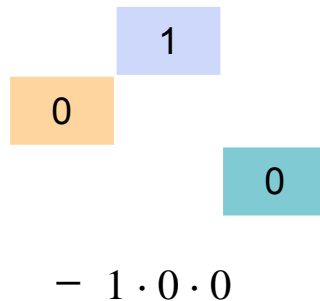
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



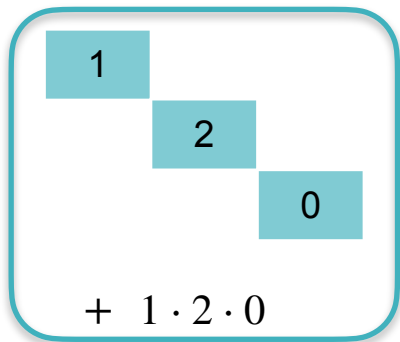
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 0$$

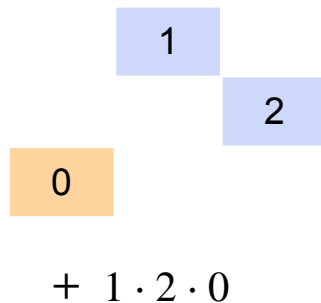
The determinant

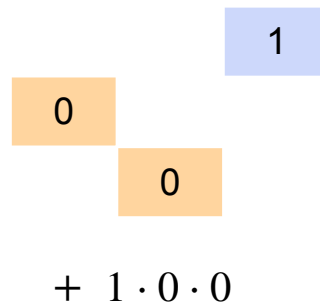
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

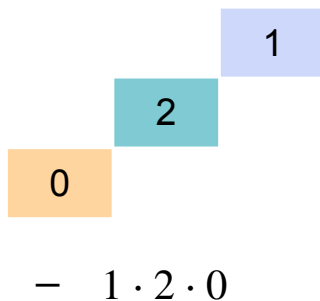

$$+ 1 \cdot 2 \cdot 0$$

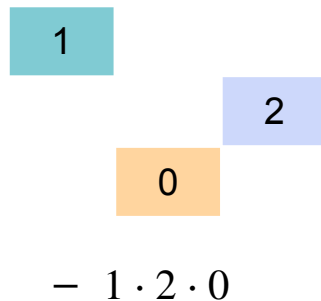
$$\text{Det} = 0+0+0-0-0-0$$

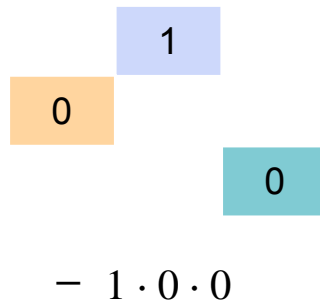
$$= 0$$


$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 0 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 0 \cdot 0$$



DeepLearning.AI

System of Linear Equations

Conclusion