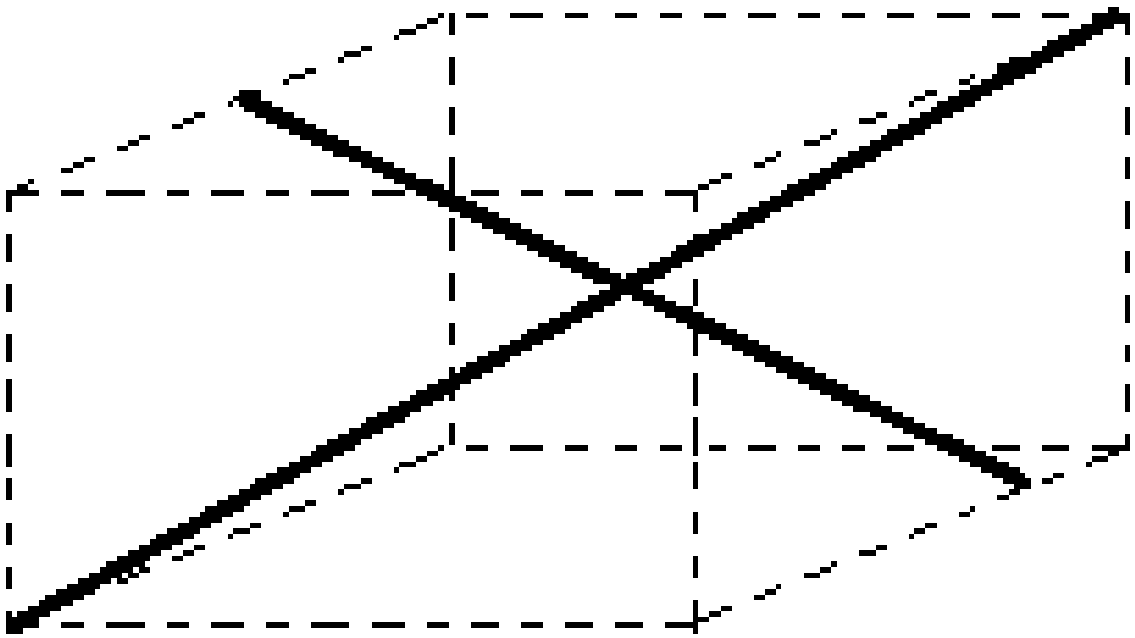
Intersection of Two Straight Lines

Two straight lines $l_1 : \mathbf{r} = \mathbf{a}_1 + \lambda_1 \mathbf{m}_1$ and $l_2 : \mathbf{r} = \mathbf{a}_2 + \lambda_2 \mathbf{m}_2$ intersect at a point if there exist unique values of λ_1 and λ such that $\mathbf{a}_1 + \lambda_1 \mathbf{m}_1 = \mathbf{a}_2 + \lambda_2 \mathbf{m}_2$.



Intersecting lines

Example:

Find the position vector of the intersection point of lines $\mathbf{r} = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 & -2 & 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 & 2 & 2 \end{pmatrix} + \mu \begin{pmatrix} -3 & -4 & 1 \end{pmatrix}$.

Example:

Find the position vector of the intersection point of lines $\frac{x-1}{-4} = y-1 = z-1$ and $\frac{x}{2} = \frac{y-1}{-1} = z-2.$

Example:

Find the value of α if following lines intersect:

$$l_1 : \mathbf{r} = s\mathbf{i} + (1+2s)\mathbf{j} + (-1+s)\mathbf{k}$$

$$l_2 : \mathbf{r} = (1+t)\mathbf{i} + 7\mathbf{j} + (-4+\alpha t)\mathbf{k}$$

Find the coordinates of the intersection point.

Situation :

Find the point of intersection between the two lines :

$l_1 : (2-t)\mathbf{i} + (1+2t)\mathbf{j} + (5+2t)\mathbf{k}$

$l_2 : (1+s)\mathbf{i} + (2-s)\mathbf{j} + (1-3s)\mathbf{k}.$

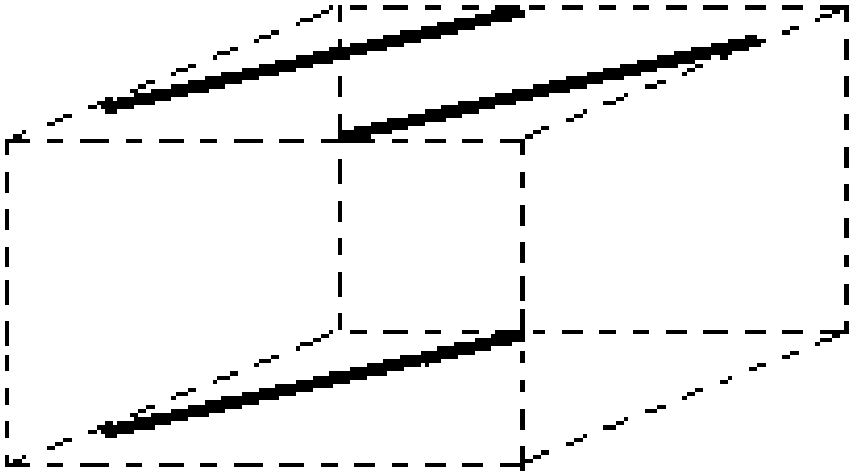
Explain what happen.

Parallel Lines and Skew Lines

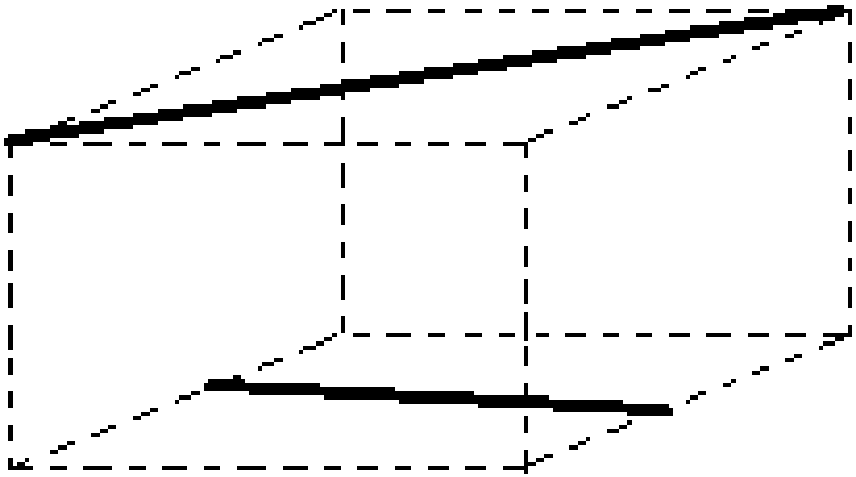
Two straight lines $l_1 : \mathbf{r} = \mathbf{a}_1 + \lambda_1 \mathbf{m}_1$ and $l_2 : \mathbf{r} = \mathbf{a}_2 + \lambda_2 \mathbf{m}_2$ do not intersect if there exist no unique values of λ_1 and λ_2 such that $\mathbf{a}_1 + \lambda_1 \mathbf{m}_1 = \mathbf{a}_2 + \lambda_2 \mathbf{m}_2$.

If $\mathbf{m}_1$ is the multiple of $\mathbf{m}_2$, the two lines are said to be **parallel**.

If the two lines are not parallel and do not intersect, they are said to be **skew**.



Parallel lines



Skew lines

Example:

Show that the following lines intersect at a point.

$$l_1 : \mathbf{r} = \begin{pmatrix} 1+2\lambda & 2-\lambda & -3+4\lambda \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} 2+\mu & 4-\mu & 4+\mu \end{pmatrix}$$

Example:

Determine whether the lines $\frac{x-1}{-1}=\frac{y}{2}=\frac{z-1}{-1}$ and $\frac{x-2}{2}=\frac{y-1}{-4}=\frac{z-3}{2}$ intersect.

Example:

Determine whether the lines

$\mathbf{r} = (2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + s(-9\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and

$\mathbf{r} = (7\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) + t(-15\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ intersect.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the position vector of the intersection point of lines $\mathbf{r} = (3 + 2\lambda)\mathbf{i} + (2 + \lambda)\mathbf{j} + (4 + \lambda)\mathbf{k}$ and $\mathbf{r} = (-1 + 2\mu)\mathbf{i} + \mathbf{j} + 3\mu\mathbf{k}$.

Example :

Find the value of k such that the straight line joining the points $(-2, k, -9)$ and $(2,1,7)$ intersects that joining the points $(-2,-4,4)$ and $(7,2,1)$.

Find the coordinates of the intersection point.

Example: Determine whether the lines AB and CD are parallel, intersect each other, or are skew.

(a) $A(3,2,4), B(-3,-7,-8), C(0,1,3), D(-2,5,9)$.

(b) $A(3,1,0), B(-3,1,3), C(5,0,-1), D(1,0,1)$

(c) $A(-5,-4,-3), B(5,1,2), C(-1,-3,0), D(8,0,6)$.

Example:

Two straight lines have equations $\frac{x-1}{4} = \frac{y-3}{-2} = z-2$ and $\frac{x-3}{2} = \frac{y-8}{-3} = \frac{z-7}{-1}$ respectively. Show that both lines intersect, and find the position vector of the point of intersection.