Oct/Nov 2011 (9231/11)

5 The point $P(2, 1)$ lies on the curve with equation

$$x^3 - 2y^3 = 3xy.$$

Find

(i) the value of $\frac{dy}{dx}$ at P, [3]

(ii) the value of $\frac{d^2y}{dx^2}$ at P. [4]

11.

Given that

$$x^2\frac{d^2y}{dx^2} + 4x(1+x)\frac{dy}{dx} + 2(1+4x+2x^2)y = 8x^2$$

and that $x^2y = z$, show that

$$\frac{d^2z}{dx^2} + 4\frac{dz}{dx} + 4z = 8x^2. \quad [4]$$

Find the general solution for y in terms of x. [8]

Describe the behaviour of y as $x \to \infty$. [2]

Oct/Nov 2011 (9231/13)

4 A curve has parametric equations

$$x = 2\sin 2t, \quad y = 3\cos 2t,$$

for $0 < t < \frac{1}{2}\pi$. For the point on the curve where $t = \frac{1}{3}\pi$, find the value of

(i) $\frac{dy}{dx}$, [3]

(ii) $\frac{d^2y}{dx^2}$. [4]

6 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = \sin 2t. \quad [6]$$

Describe the behaviour of x as $t \to \infty$, justifying your answer. [2]

May/June 2011 (9231/11)

7 The variables x and y are related by the differential equation

$$y^2\frac{d^2y}{dx^2} + 2y^2\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right)^2 - 5y^3 = 8e^{-x}.$$

Given that $v = y^3$, show that

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} - 15v = 24e^{-x}.$$ [4]

Hence find the general solution for y in terms of x. [7]

May/June 2011 (9231/13)

4 The curve C has equation

$$2xy^2 + 3x^2y = 1.$$

Show that, at the point $A\ (-1,\ 1)$ on C, $\frac{dy}{dx} = -4$. [3]

Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

8 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10\sin t.$$ [6]

Find the particular solution, given that $x = 5$ and $\frac{dx}{dt} = 2$ when $t = 0$. [4]

State an approximate solution for large positive values of t. [1]

Oct/Nov 2010 (9231/01)

11 It is given that $x \neq 0$ and

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 8x^2 + 16.$$

Show that if $z = xy$ then

$$\frac{d^2z}{dx^2} + 4z = 8x^2 + 16.$$ [3]

Find y in terms of x, given that $y = 0$ and $\frac{dy}{dx} = -2$ when $x = \frac{1}{2}\pi$. [9]

May/June 2010 (9231/11)

1 The variables x and y are such that $y = -1$ when $x = 1$ and

$$x^2 + y^2 + \left(\frac{dy}{dx}\right)^3 = 29.$$

Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 1$. [5]

11 Answer only **one** of the following two alternatives.

EITHER

The variables z and x are related by the differential equation

$$3z^2\frac{d^2z}{dx^2} + 6z^2\frac{dz}{dx} + 6z\left(\frac{dz}{dx}\right)^2 + 5z^3 = 5x + 2.$$

Use the substitution $y = z^3$ to show that y and x are related by the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 5x + 2.$$ [3]

Given that $z = 1$ and $\frac{dz}{dx} = -\frac{2}{3}$ when $x = 0$, find z in terms of x. [9]

Deduce that, for large positive values of x, $z \approx x^{\frac{1}{3}}$. [2]

May/June 2010 (9231/13)

7 It is given that

$$x = t^2e^{-t^2} \quad \text{and} \quad y = te^{-t^2}.$$

(i) Show that

$$\frac{dy}{dx} = \frac{1 - 2t^2}{2t - 2t^3}.$$ [3]

(ii) Find $\frac{d^2y}{dx^2}$ in terms of t. [5]

8 Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 10\sin 3x - 20\cos 3x. \qquad [5]$$

Show that, for large positive x and independently of the initial conditions,

$$y \approx R\sin(3x + \phi),$$

where the constants R and ϕ, such that $R > 0$ and $0 < \phi < 2\pi$, are to be determined correct to 2 decimal places. [4]

Oct/Nov 2009 (9231/01)

4 It is given that

$$x = t + \sin t, \quad y = t^2 + 2\cos t,$$

where $-\pi < t < \pi$. Find $\frac{dy}{dx}$ in terms of t. [2]

Show that

$$\frac{d^2y}{dx^2} = \frac{2t\sin t}{(1+\cos t)^3}. \qquad [4]$$

Show that $\frac{dy}{dx}$ increases with x over the given interval of t. [2]

9 Show that if y depends on x and $x = e^u$ then

$$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}. \qquad [4]$$

Given that y satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 3y = 30x^2,$$

use the substitution $x = e^u$ to show that

$$\frac{d^2y}{du^2} + 4\frac{dy}{du} + 3y = 30e^{2u}. \qquad [2]$$

Hence find the general solution for y in terms of x. [5]

May/June 2009 (9231/01)

6 A curve has equation

$$(x + y)(x^2 + y^2) = 1.$$

Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(0, 1)$. [7]

8 Find the general solution of the differential equation

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 65y = 65x^2 + 8x + 73. \qquad [6]$$

Show that, whatever the initial conditions, $\frac{y}{x^2} \to 1$ as $x \to \infty$. [2]

Oct/Nov 2008 (9231/01)

5 The curve C has equation

$$x^2 - xy - 2y^2 = 4.$$

Show that, at the point A $(2, 0)$ on C, $\frac{dy}{dx} = 2$. [2]

Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

8 Find y in terms of t, given that

$$5\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 15 + 12t + 5t^2,$$

and that $y = \frac{dy}{dt} = 0$ when $t = 0$. [9]

May/June 2008 (9231/01)

11 Show that, with a suitable value of the constant α, the substitution $y = x^{\alpha}w$ reduces the differential equation

$$2x^2\frac{d^2y}{dx^2} + (3x^2 + 8x)\frac{dy}{dx} + (x^2 + 6x + 4)y = f(x)$$

to

$$2\frac{d^2w}{dx^2} + 3\frac{dw}{dx} + w = f(x). \qquad [5]$$

Find the general solution for y in the case where $f(x) = 6\sin 2x + 7\cos 2x$. [6]

Oct/Nov 2007 (9231/01)

5 The positive variables x and y are related by

$$y = x^2 + 2\ln(xy).$$

Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when both x and y are equal to 1. [7]

12.

Show that the substitution $y = \frac{1}{w}$ reduces the differential equation

$$y\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^2 - 5y^2 = (5x^2 + 4x + 2)y^3$$

to

$$\frac{d^2w}{dx^2} + 2\frac{dw}{dx} + 5w = -5x^2 - 4x - 2.$$ [4]

Find the general solution for w in terms of x. [6]

Find a function f such that $\lim_{x\to\infty}\left(\frac{y}{f(x)}\right) = 1$. [3]

May/June 2007 (9231/01)

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 58x + 37.$$ [6]

4 Given that the variables x and y are related by

$$y = x + e^{-xy},$$

find the value of $\frac{d^2y}{dx^2}$ when $x = 0$. [6]

Oct/Nov 2006 (9231/01)

8 Given that

$$2y^3\frac{d^2y}{dx^2} + 12y^3\frac{dy}{dx} + 6y^2\left(\frac{dy}{dx}\right)^2 + 17y^4 = 13e^{-4x}$$

and that $v = y^4$, show that

$$\frac{d^2v}{dx^2} + 6\frac{dv}{dx} + 34v = 26e^{-4x}. \qquad [4]$$

Hence find the general solution for y in terms of x. [5]

10 The curve C has equation

$$y = x^2 + \lambda\sin(x+y),$$

where λ is a constant, and passes through the point $A\left(\frac{1}{4}\pi, \frac{1}{4}\pi\right)$. Show that C has no tangent which is parallel to the y-axis. [5]

Show that, at A,

$$\frac{d^2y}{dx^2} = 2 - \frac{1}{64}\pi(4-\pi)(\pi+2)^2. \qquad [5]$$

May/June 2006 (9231/01)

8 Obtain the general solution of the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 80e^{-3t}. \qquad [5]$$

Given that $y = 8$ and $\frac{dy}{dt} = -8$ when $t = 0$, show that $0 \leqslant ye^{3t} \leqslant 10$ for all t. [5]

Oct/Nov 2005 (9231/01)

4 Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 24e^{2x},$$

given that $y = 1$ and $\frac{dy}{dx} = 9$ when $x = 0$. [7]

May/June 2005 (9231/01)

2 Given that

$$x = t - \sin t \qquad \text{and} \qquad y = 1 - \cos t,$$

where $0 < t < 2\pi$, show that

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\operatorname{cosec}^4\left(\frac{1}{2}t\right). \qquad [5]$$

12. It is given that

$$\frac{d^2y}{dx^2} + (2a-1)\frac{dy}{dx} + a(a-1)y = 2a - 1 + a(a-1)x,$$

where a is a constant. Find y in terms of a and x, given that y and $\frac{dy}{dx}$ are both zero when $x = 0$. [8]

Hence show that if $a > 1$ then $y \approx x$ as $x \to \infty$. [2]

It is given that

$$\frac{d^2z}{dx^2} + (2a-1)\frac{dz}{dx} + a(a-1)z = e^x,$$

where the constant a is positive. Find $\lim_{x\to\infty} e^{-x}z$. [3]

Oct/Nov 2004 (9231/01)

7 The curve C has equation

$$xy + (x+y)^5 = 1.$$

(i) Show that $\frac{dy}{dx} = -\frac{5}{6}$ at the point A $(1, 0)$ on C. [3]

(ii) Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

12 Answer only **one** of the following two alternatives.

EITHER

The variable y depends on x, and the variables x and t are related by $x = e^t$. Show that

$$x\frac{dy}{dx} = \frac{dy}{dt} \quad \text{and} \quad x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}.$$ [5]

(i) Given that y satisfies the differential equation

$$4x^2\frac{d^2y}{dx^2} + 16x\frac{dy}{dx} + 25y = 50(\ln x) - 1,$$

find a differential equation involving only t and y. [2]

(ii) Show that the complementary function of the differential equation in t and y may be written in the form

$$Re^{-\frac{3}{2}t}\sin(2t + \phi),$$

where R and ϕ are arbitrary constants. [3]

(iii) Find a particular integral of the differential equation in t and y. [3]

(iv) Hence find the general solution of the differential equation in x and y. [1]

May/June 2004 (9231/01)

9 The variable y depends on x and the variables x and t are related by $x = \frac{1}{t}$. Show that

$$\frac{dy}{dx} = -t^2\frac{dy}{dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = t^4\frac{d^2y}{dt^2} + 2t^3\frac{dy}{dt}. \qquad [4]$$

The variables x and y are related by the differential equation

$$x^5\frac{d^2y}{dx^2} + (2x^4 - 5x^3)\frac{dy}{dx} + 4xy = 14x + 8.$$

Show that

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 8t + 14. \qquad [2]$$

Hence find the general solution for y in terms of x. [5]

Oct/Nov 2003 (9231/01)

7 Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-\alpha t},$$

where α is a constant and $\alpha \neq 2$. [7]

Show that if $\alpha < 2$ then, whatever the initial conditions, $ye^{\alpha t} \to \frac{1}{(2-\alpha)^2}$ as $t \to \infty$. [2]

May/June 2003 (9231/01)

9 The variables x and t, where $x > 0$ and $0 \leqslant t \leqslant \frac{1}{2}\pi$, are related by

$$x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 5x\frac{dx}{dt} + 3x^2 = 3\sin 2t + 15\cos 2t,$$

and the variables x and y are related by $y = x^2$. Show that

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 6\sin 2t + 30\cos 2t. \qquad [3]$$

Hence find x in terms of t, given that $x = 2$ and $\frac{dx}{dt} = -\frac{3}{2}$ when $t = 0$. [10]

Oct/Nov 2002 (9231/01)

6 A curve has equation $x^3 + xy^2 - y^3 = 3$.

(i) Show that there is no point of the curve at which $\frac{dy}{dx} = 0$. [4]

(ii) Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. [5]

8 The value of the assets of a large commercial organisation at time t, measured in years, is $\$(10^8 y + 10^9)$. The variables y and t are related by the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 15\cos 3t - 3\sin 3t.$$

Find y in terms of t, given that $y = 3$ and $\frac{dy}{dt} = -2$ when $t = 0$. [9]

Show that, for large values of t, the value of the assets is less than $\$9.5 \times 10^8$ for about a third of the time. [3]

May/June 2002 (9231/01)

4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 15x + 16.$$ [6]

Show that, whatever the initial conditions, $y \approx 3x + 2$ when x is large and positive. [1]

7 Given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = t^3 e^{-t},$$

where $t \neq 0$, find $\frac{dy}{dx}$ in terms of t. [3]

Find $\frac{d^2y}{dx^2}$ in terms of t, and hence find the values of t for which $\frac{d^2y}{dx^2} = 0$. [5]