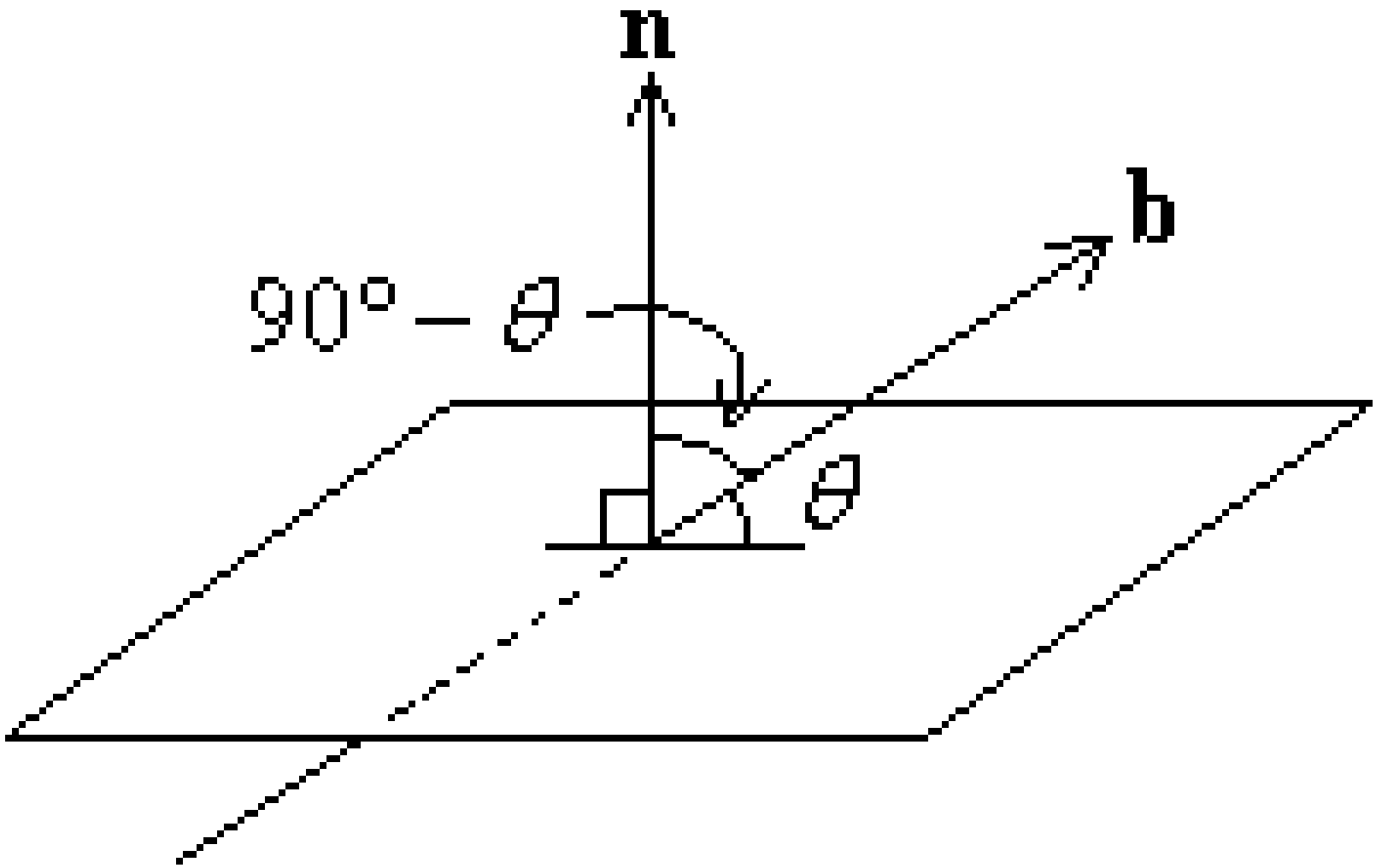
Angle between a Line and a Plane

The angle between a line $\mathrm{L} : \mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and a plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$ is given by

$$\sin\theta = \frac{\mathbf{b} \cdot \mathbf{n}}{|\mathbf{b}||\mathbf{n}|}$$


n
b
90° − θ
θ

Example:

Find the angle between the line $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ and the plane $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$.

Example:

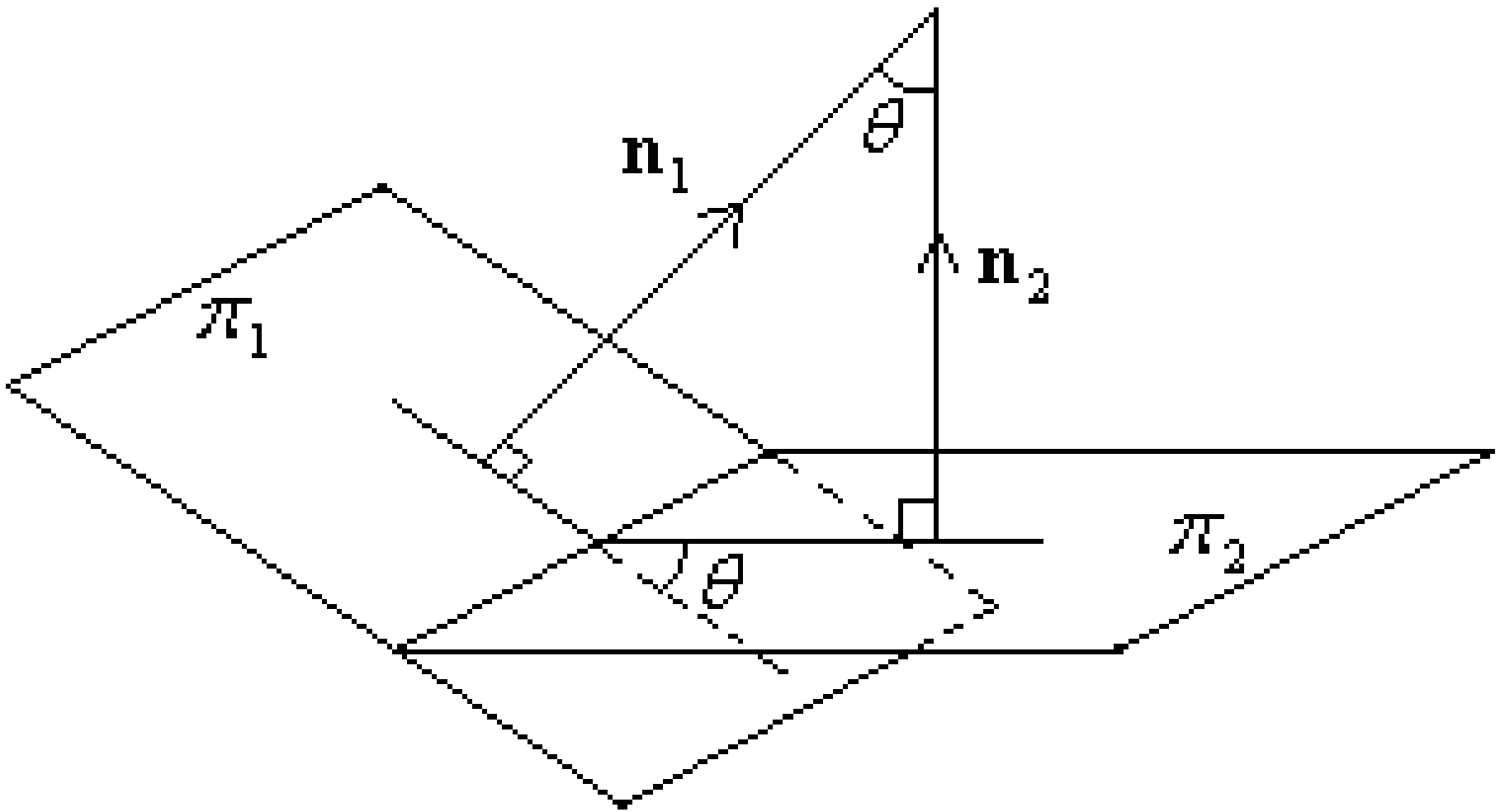
Find the angle between the line $x-1=2-z,\ y=-1$ and the plane $y-z=1$.

Angle between Two Planes

The angle between two planes

$\pi_1 : \mathbf{r} \cdot \mathbf{n}_1 = p_1$ and $\pi_2 : \mathbf{r} \cdot \mathbf{n}_2 = p_2$

is given by $\cos\theta = \dfrac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$


$\mathbf{n}_1$
θ
$\mathbf{n}_2$
π_1
π_2
θ

Example :

Find the acute angle between the planes

$\mathbf{r} \cdot \left(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}\right) = 3$ and $\mathbf{r} \cdot \left(\mathbf{i} + \mathbf{j} - 2\mathbf{k}\right) = 0.$

Example:

Find the acute angle between the planes

$2x - y = 1$ and $x + y + z = 0.$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the angle between the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-3}{3}$

and the plane $3x - 2y + z = 5$.

Example :

Find the acute angle between the planes

$\mathbf{r} \cdot (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) = 4$ and $\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 5$.

Example:

Find the acute angle between the planes

$\mathbf{r} = (2\lambda_1 + 2\mu_1)\mathbf{i} - \lambda_1\mathbf{j} + (-1 + 3\mu_1)\mathbf{k}$ and

$\mathbf{r} = \mu_2\mathbf{i} + (\lambda_2 + 2\mu_2)\mathbf{j} + (3 + 4\lambda_2)\mathbf{k}$

Example:

Find the Cartesian equation of the plane that contains points $(1,-1,1),(2,1,2)$ and $(3,1,-1)$. Find the cosine acute angle of this plane with the plane $2x - y + 3z = 0$.