TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2011

(Jan 2011 Intake)

Monday **6 June 2011** **1.30 pm – 3.30 pm**

FURTHER MATHEMATICS 9231/01

PAPER 1 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **50**.

This document consists of **3** printed pages.

[Turn over

1 Prove by induction that

$$7^{2n-1}+3^{2n}$$

is divisible by 8 for all integers $n \geq 1$. [6]

2 Use the method of difference to find S_N, where

$$S_N = \sum_{n=1}^{N} \frac{2}{(n+1)(n+3)}.$$ [5]

Hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$. [2]

3 Prove by induction that, for all $N \geq 1$,

$$\sum_{n=1}^{N} n(n+1)\left(\frac{1}{2}\right)^{n-1} = 16-\left(N^2+5N+8\right)\left(\frac{1}{2}\right)^{N-1}.$$ [7]

4 Given that the equations of 3 planes are

$$\begin{aligned} x+3y+2z &= 1 \\ x-y-z &= 0 \\ 2x+2y+\theta z &= 3\theta+\phi-2 \end{aligned}$$

where θ and ϕ are constant.

(i) Show that if $\theta \neq 1$ then the planes have a unique point of intersection. Find this point in the case $\phi = 0$. [3]

(ii) Show that if $\theta = 1$ and $\phi = 0$ then the planes have an infinite number of intersection points. [3]

(iii) Show that if $\theta = 1$ and $\phi \neq 0$ the planes do not have any common points. [3]

5 If the equation $4x^3 + 8x + 5 = 0$ has roots α, β, γ find the equation having roots $\alpha\beta + \gamma$, $\alpha\gamma + \beta$, $\beta\gamma + \alpha$. [10]

6 The curve C has equation $y = 5 - \frac{2}{9x+4} - \frac{3}{7x+6}$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that $\frac{dy}{dx} > 0$ at every point of C. [2]

(iii) Sketch C, stating the coordinates of the intersection of C and the coordinate axes. [6]