

THE PAIRED t -test

Exercise 4C

- Some psychologists believe that the IQ of the first-born child in a family is significantly greater than the IQ of the last born. In order to investigate this belief, a random sample of 8 families with more than one child agreed to allow their children's IQs to be measured, with the following results.

Family	1	2	3	4	5	6	7	8
IQ of first born	97	121	89	112	138	125	104	114
IQ of last born	101	116	97	108	130	121	101	105

Assuming that the differences have a normal distribution test the psychologists' belief using a 5% significance level.

- The reaction times taken by 10 motorists to apply the brakes of their cars were measured when the motorists had not drunk any alcohol, and after they had drunk a measured amount of alcohol. The reaction times, in hundredths of a second, are given in the table.

Motorist	1	2	3	4	5	6	7	8	9	10
Without alcohol	40	25	19	23	38	37	28	37	41	27
With alcohol	50	37	35	34	52	50	40	46	53	38

Assuming that the population of differences has a normal distribution, test, at the 1% significance level, whether the drinking of alcohol increases the mean reaction time by more than 0.1 s.

- Blood pressure data were obtained from a larger set of 97 people with diabetes. The values of D are summarised by $\sum d = 4092$ and $\sum d^2 = 187\,948$. Carry out a test of the hypothesis $\mu_D > 40$ which does not rely on D having a normal distribution. Use a 5% significance level.
- A person's systolic blood pressure is a measure of the pressure exerted by the heart when it contracts and pushes blood around the body. When the heart has just ceased to contract and is dilating ready for the next contraction, the blood pressure drops and is called the diastolic pressure.

The following table gives the systolic and diastolic blood pressures (measured in mm of mercury) of 6 randomly chosen people with diabetes.

Patient	1	2	3	4	5	6
Systolic pressure	141	129	117	115	93	101
Diastolic pressure	83	76	71	59	51	64

Let D denote the amount by which the systolic pressure exceeds the diastolic pressure of a randomly chosen person with diabetes, and let μ_D denote the mean of D . Assuming that D has a normal distribution, test the hypothesis $\mu_D > 40$ at the 5% significance level.

- An experiment was carried out to compare the difference in the effects of organic and chemical fertilisers on potato yields. Eleven plots of land were selected and two seed potatoes were grown on each plot at a distance of 10 m apart. On one potato an organic fertiliser was used, and on the other, a chemical fertiliser. The choice of which to use was decided by tossing a coin. The differences in yields, d grams, where $d = (\text{mass of organic crop} - \text{mass of chemical crop})$, are summarised by

$$\sum d = -310 \quad \text{and} \quad \sum d^2 = 208\,702.$$

Assuming that the differences have a normal distribution, test, at the 5% significance level, whether there is a difference between the population mean yields.

- 6 A study of the effect of vitamins on attention span was carried out on 40 sets of identical twins of the same age. One twin was randomly chosen to have the vitamin pill and the other was given a placebo (a pill with no vitamin). Each twin was given a puzzle to solve and the time, in minutes, that each twin remained with the puzzle was measured. The summary statistics for the difference in time, d minutes, where $d = (\text{vitamin time} - \text{placebo time})$, are summarised by $\bar{d} = 2.92$ and $s_d^2 = 141.23$.

Without assuming that the differences are distributed normally, test, at the 5% significance level, whether the vitamins increase attention span.

What would be the conclusion of the test if it could be assumed that the differences are distributed normally?

ANSWERS

Exercise 4C (page 91)

- 1 $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 > \mu_2$; $\bar{d} = 2.625$;
 $s^2 = 33.70$; $t = 1.279 < 1.895$, so accept H_0 ,
the belief is not supported.
- 2 $H_0: \mu_D = 10$, $H_1: \mu_D > 10$; $t = 3.162 > 2.821$,
so reject H_0 and accept that $\mu_D > 10$.
- 3 $s^2 = 159.63$, $z = 1.704 > 1.645$ so accept that
 $\mu_D > 40$.
- 4 $t = 2.547 > 2.105$, so accept that $\mu_D > 40$.
- 5 $H_0: \mu_O - \mu_C = 0$, $H_1: \mu_O - \mu_C \neq 0$;
 $t = -0.661$; rejection region is $|T| \geq 2.228$, so
accept H_0 , there is no difference in the effects
of organic and chemical fertilisers.
- 6 $H_0: \mu_D = 0$, $H_1: \mu_D > 0$; sample size large
enough to use normal test; $z = 1.554 < 1.645$,
so accept H_0 , there is no difference in means
of the two populations.
 t -test would be valid; $t = 1.554$; rejection
region is $T \geq a$ where $a > 1.645$, so H_0 still
accepted.