Revision

P104 Curve Sketching

CIE CAL FM P1 2009-06 Q10

The curve C has equation $y = \dfrac{x^2}{x+\lambda}$, where λ is a non-zero constant. Obtain the equation of each of the asymptotes of C. [3]

In separate diagrams, sketch C for the cases $\lambda > 0$ and $\lambda < 0$. In both cases the coordinates of the turning points must be indicated. [8]

CIE CAL FM P11 2011-11 Q7

The curve C has equation $y = \dfrac{x^2 + px + 1}{x - 2}$, where p is a constant. Given that C has two asymptotes, find the equation of each asymptote. [3]

Find the set of values of p for which C has two distinct turning points. [5]

Sketch C in the case $p = -1$. Your sketch should indicate the coordinates of any intersections with the axes, but need not show the coordinates of any turning points. [3]

CIE CAL FM P13 2011-11 Q10

A curve C has equation $y = \dfrac{5(x^2 - x - 2)}{x^2 + 5x + 10}$. Find the coordinates of the points of intersection of C with the axes. [2]

Show that, for all real values of x, $-1 \leqslant y \leqslant 15$. [4]

Sketch C, stating the coordinates of any turning points and the equation of the horizontal asymptote. [7]

CIE CAL FM P1 2003-11 Q11

The curve C has equation $y = \dfrac{5(x-1)(x+2)}{(x-2)(x+3)}$.

(i) Express y in the form $P + \dfrac{Q}{x-2} + \dfrac{R}{x+3}$. [3]

(ii) Show that $\dfrac{dy}{dx} = 0$ for exactly one value of x and find the corresponding value of y. [4]

(iii) Write down the equations of all the asymptotes of C. [3]

(iv) Find the set of values of k for which the line $y = k$ does not intersect C. [4]

ASSIGNMENT

Attempt all the questions in the following slides.

Assignment is to be submitted within **one week** after instruction!!!

Please write your name and your student's id in the script upon submission.

CIE CAL FM P11 2010-06 Q11

The curve C has equation $y = \dfrac{x(x+1)}{(x-1)^2}$.

(i) Obtain the equations of the asymptotes of C. [3]

(ii) Show that there is exactly one point of intersection of C with the asymptotes and find its coordinates. [2]

(iii) Find $\dfrac{dy}{dx}$ and hence

(a) find the coordinates of any stationary points of C,

(b) state the set of values of x for which the gradient of C is negative. [6]

(iv) Draw a sketch of C. [3]

CIE CAL FM P1 2005-06 Q12

The curve C has equation $y = \dfrac{ax^2 + bx + c}{x + 4}$, where a, b and c are constants. It is given that $y = 2x - 5$ is an asymptote of C.

(i) Find the values of a and b. [3]

(ii) Given also that C has a turning point at $x = -1$, find the value of c. [3]

(iii) Find the set of values of y for which there are no points on C. [4]

(iv) Draw a sketch of the curve with equation

$$y = \frac{2(x-7)^2 + 3(x-7) - 2}{x - 3}.$$ [3]

[You should state the equations of the asymptotes and the coordinates of the turning points.]

CIE CAL FM P1 2005-06 Q12

The curve Γ, which has equation $y = \dfrac{ax^2 + bx + c}{x^2 + px + q}$, has asymptotes $x = 1$, $x = 4$ and $y = 2$. Find the values of a, p and q. [4]

It is given that Γ has a stationary point at $x = 2$.

(i) Find the value of c. [3]

(ii) Show that if $b \neq -10$ then Γ has exactly 2 stationary points. [2]

(iii) Draw a sketch of Γ for the case where $b = -6$. [4]

CIE CAL FM P11 2011-06 Q11

The curve C has equation $y = \dfrac{x^2 + \lambda x - 6\lambda^2}{x + 3}$, where λ is a constant such that $\lambda \neq 1$ and $\lambda \neq -\frac{3}{2}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and deduce that if C has two stationary points then $-\frac{3}{2} < \lambda < 1$. [5]

(ii) Find the equations of the asymptotes of C. [3]

(iii) Draw a sketch of C for the case $0 < \lambda < 1$. [3]

(iv) Draw a sketch of C for the case $\lambda > 3$. [3]

CIE CAL FM P1 2002-11 Q11

The curve C has equation $y = \dfrac{(x-a)(x-b)}{x-c}$, where a, b, c are constants, and it is given that $0 < a < b < c$.

(i) Express y in the form $x + P + \dfrac{Q}{x-c}$, giving the constants P and Q in terms of a, b and c. [3]

(ii) Find the equations of the asymptotes of C. [2]

(iii) Show that C has two stationary points. [5]

(iv) Given also that $a + b > c$, sketch C, showing the asymptotes and the coordinates of the points of intersection of C with the axes. [4]