Exercise 8

1. Given that the moment of inertia of a uniform equilateral triangular lamina, of side $2a$ and mass m, about an axis through a vertex perpendicular to its plane is $\frac{5}{3}ma^2$.

   Deduce that the moment of inertia of a uniform regular hexagonal lamina, of side $2a$ and mass M, about an axis through a vertex perpendicular to the plane of the lamina is $\frac{17}{3}Ma^2$.

   A compound pendulum consists of a uniform regular hexagonal lamina ABCDEF, of side $2a$ and mass M, with a particle of mass $\frac{1}{2}M$ attached to the vertex D. The pendulum oscillates about a smooth horizontal axis which passes through the vertex A and is perpendicular to the plane of the lamina. Show that the period of small oscillations is $\pi\sqrt{\left(\frac{41a}{3g}\right)}$.

2. Three uniform rods AB, BC, CD each of mass m and length l, are rigidly joined so as to form three sides of a square ABCD, and the rods are suspended under gravity from a smooth horizontal axis through D which is perpendicular to the plane of the square. Show that

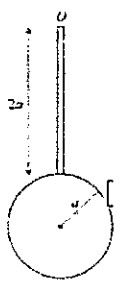
   (a) the moment of inertia of the system about the axis through D is $3ml^2$;

   Given that the distance of the centre of gravity of the system from D is $\frac{5}{6}l$.

   Find the period of small oscillations about the equilibrium position. $\left[2\pi\sqrt{\left(\frac{6l}{5g}\right)}\right]$

3. The figure shows a compound pendulum which consists of a uniform solid sphere of mass m and radius a attached to a uniform rod of mass $\frac{1}{4}m$ and length $2a$. They are attached so that the centre of the sphere lies on the rod produced and so is a distance $3a$ from the pivot O. Find the moment of inertia of the system about a horizontal axis through O and hence the length of the equivalent simple pendulum.

   $\left[\frac{146}{15}ma^2, 2.99a\right]$



4. A pendulum P is modelled as a uniform rod AB, of length $9a$ and mass m, rigidly fixed to a uniform circular disc of radius a and mass $2m$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in the Figure. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through the end A and is perpendicular to the plane of the disc.

(a) Show that the moment of inertia of P about L is $190ma^2$.

The pendulum makes small oscillations about L.

(b) By writing down an equation of motion for P, find the approximate period of these small oscillations.

$\left[\frac{2}{3}\pi\sqrt{\frac{76a}{g}} \right]$