40. $\underset{\sim}{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}1\\-2\\1\end{pmatrix} + s\begin{pmatrix}0\\3\\2\end{pmatrix}$ $\underset{\sim}{r} = \begin{pmatrix}2\\1\\-1\end{pmatrix} + u\begin{pmatrix}0\\0\\5\end{pmatrix} + v\begin{pmatrix}-2\\4\\3\end{pmatrix}$

a) $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1+t\\ -2t+3s\\ 1+t+2s\end{pmatrix}$

$$\left.\begin{aligned} x &= 1+t \\ y &= -2t+3s \\ z &= 1+t+2s \end{aligned}\right\}$$

$$\left.\begin{aligned} t &= x-1 \\ -2t+3s &= y \\ t+2s &= z-1 \end{aligned}\right\}$$

$2\times ① + ②$ 　 $t = x-1$

$-① + ③$: 　 $3s = 2x-2+y$

$2s = z-x$

$$2(2x-2+y) = 3(z-x)$$

$$4x-4+2y = 3z-3x$$

$$7x+2y-3z = 4$$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2 - 2v\\ 1 + 4v\\ -1+5u+3v\end{pmatrix}$

$$\left.\begin{aligned} x &= 2 - 2v \\ y &= 1 + 4v \\ z &= -1+5u+3v \end{aligned}\right\}$$

$$\left.\begin{aligned} -2v &= x-2 \\ 4v &= y-1 \\ 3v+5u &= z+1 \end{aligned}\right\}$$

$$-2(x-2) = y-1$$

$$-2x+4 = y-1$$

$$2x+y=5$$

$$7x+2y-3z=4 \qquad 2x+y=5$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} = 4 \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 5$$

$$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \left|\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}\right| \left|\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\right| \cos\theta$$

$$14+2-0 = \sqrt{62}\sqrt{5}\cos\theta$$

$$\cos\theta = \frac{16}{\sqrt{310}}$$

b) $$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 7 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= 3\underset{\sim}{i} - 6\underset{\sim}{j} + 3\underset{\sim}{k}$$

$$= 3(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$$

$x=0$: $2y - 3z = 4 \qquad y=5$

$$10-3z=4$$

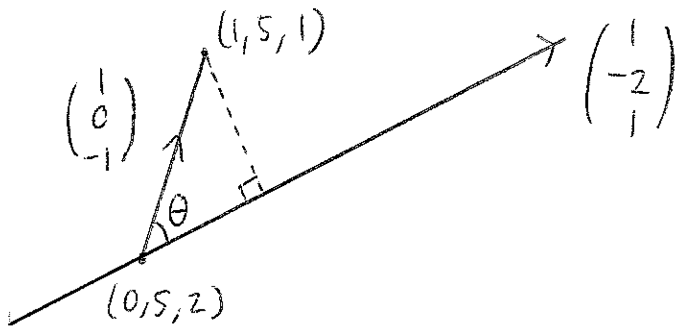
$$3z=6$$

$$z=2$$

$$(0,5,2)$$

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

c)



$$\begin{pmatrix}1\\0\\-1\end{pmatrix} \times \begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= -2\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$$

$$= -2(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$$

$$\left|\begin{pmatrix}1\\0\\-1\end{pmatrix} \times \begin{pmatrix}1\\-2\\1\end{pmatrix}\right| = \left|\begin{pmatrix}1\\0\\-1\end{pmatrix}\right| \left|\begin{pmatrix}1\\-2\\1\end{pmatrix}\right| \sin\theta$$

$$\left|-2(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})\right| = \left|\begin{pmatrix}1\\0\\-1\end{pmatrix}\right| \sqrt{6}\sin\theta$$

$$\left|\begin{pmatrix}1\\0\\-1\end{pmatrix}\right| \sin\theta = \frac{2\sqrt{3}}{\sqrt{6}}$$

$$= \sqrt{2}$$

50. $\underset{\sim}{a} = \underset{\sim}{i} + \sin\theta \underset{\sim}{j} + \cos\theta \underset{\sim}{k}$

$\underset{\sim}{b} = \underset{\sim}{i} + \sin\phi \underset{\sim}{j} + \cos\phi \underset{\sim}{k}, \quad 0 \leq \theta \leq \phi \leq \pi$

a) $$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & \sin\theta & \cos\theta \\ 1 & \sin\phi & \cos\phi \end{vmatrix}$$

$$= \begin{pmatrix} \sin\theta\cos\phi - \cos\theta\sin\phi \\ \cos\theta - \cos\phi \\ \sin\phi - \sin\theta \end{pmatrix}$$

$$|\underset{\sim}{a} \times \underset{\sim}{b}| = \sqrt{(\sin\theta\cos\phi - \cos\theta\sin\phi)^2 + (\cos\theta - \cos\phi)^2 + (\sin\phi - \sin\theta)^2}$$

$$= \sqrt{\sin^2(\theta - \phi) + \cos^2\theta - 2\cos\theta\cos\phi + \cos^2\phi + \sin^2\phi - 2\sin\phi\sin\theta + \sin^2\theta}$$

$$= \sqrt{\sin^2(\theta - \phi) + 2 - 2(\cos\theta\cos\phi + \sin\theta\sin\phi)}$$

$$= \sqrt{\sin^2(\theta - \phi) + 2 - 2\cos(\theta - \phi)}$$

$$= \sqrt{\sin^2(-2\delta) + 2 - 2\cos(-2\delta)}, \quad \delta = \frac{\phi - \theta}{2}$$

$$= \sqrt{\sin^2 2\delta + 2 - 2\cos 2\delta}$$

$$= \sqrt{\sin^2 2\delta + 2 - 2(1 - 2\sin^2\delta)}$$

$$= \sqrt{4\sin^2\delta\cos^2\delta + 2 - 2 + 4\sin^2\delta}$$

$= \sqrt{4\sin^2\delta\cos^2\delta + 4\sin^2\delta}$

$= \sqrt{4\sin^2\delta(1 + \cos^2\delta)}$

$= 2\sin\delta\sqrt{1 + \cos^2\delta}$

b) $\underset{\sim}{a} \times \underset{\sim}{b} = -\frac{\sqrt{3}}{2}\underset{\sim}{i} + \underset{\sim}{j}$

$\sin\theta\cos\phi - \cos\theta\sin\phi = -\frac{\sqrt{3}}{2}$

$\cos\theta - \cos\phi = 1$

$\sin\phi - \sin\theta = 0$

$\sin\phi = \sin\theta$

$\sin\theta\cos\phi - \cos\theta\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta(\cos\phi - \cos\theta) = -\frac{\sqrt{3}}{2}$

$-\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

$\phi = \frac{2\pi}{3}$

55. $P(2,6,3)$ $Q(-1,-15,-6)$

$G(p,q,r)$

i) $\overrightarrow{PQ} = 3\overrightarrow{PG}$

$$\begin{pmatrix}-1\\-15\\-6\end{pmatrix} - \begin{pmatrix}2\\6\\3\end{pmatrix} = 3\left[\begin{pmatrix}p\\q\\r\end{pmatrix} - \begin{pmatrix}2\\6\\3\end{pmatrix}\right]$$

$$\begin{pmatrix}-3\\-21\\-9\end{pmatrix} = 3\begin{pmatrix}p-2\\q-6\\r-3\end{pmatrix}$$

$$\begin{pmatrix}-1\\-7\\-3\end{pmatrix} = \begin{pmatrix}p-2\\q-6\\r-3\end{pmatrix}$$

$p-2=-1$ $\quad q-6=-7$ $\quad r\ 3=-3$

$p=1$ $\quad q=-1$ $\quad r=0$

$\therefore G(1,-1,0)$

$\underset{\sim}{n} = \underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

$\underset{\sim}{r}\cdot\underset{\sim}{n} = \overrightarrow{OG}\cdot\underset{\sim}{n}$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\3\\2\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\3\\2\end{pmatrix}$$

$$x+3y+2z = 1-3+0$$
$$= -2$$

ii) $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 + s \\ 6 + 3s \\ 3 + 2s \end{pmatrix}$$

$x = 2 + s, \; y = 6 + 3s, \; z = 3 + 2s$

$x + 3y + 2z = -2$

$2 + s + 3(6 + 3s) + 2(3 + 2s) = -2$

$2 + s + 18 + 9s + 6 + 4s = -2$

$14s = -28$

$s = -2$

$N(0, 0, -1)$

$\overrightarrow{ON} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$.

iii) $\Pi: \; \underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{u} + \mu \underset{\sim}{v}$

Let $\underset{\sim}{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \; \underset{\sim}{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 0$$

$u_1 + 3u_2 + 2u_3 = 0$

$v_1 + 3v_2 + 2v_3 = 0$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$u_1v_1 + u_2v_2 + u_3v_3 = 0$

$u = -i + j - k \quad v = -5i - j$

56. l_1: $\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$

$\underset{\sim}{a} = -6\underset{\sim}{i} + 3\underset{\sim}{j} + 15\underset{\sim}{k}$ $\quad$ $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$

l_2: $\underset{\sim}{r} = \underset{\sim}{c} + \mu \underset{\sim}{d}$

$\underset{\sim}{c} = 6\underset{\sim}{i} + 15\underset{\sim}{j} + 39\underset{\sim}{k}$ $\quad$ $\underset{\sim}{d} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$.

$\underset{\sim}{r} = -6\underset{\sim}{i} + 3\underset{\sim}{j} + 15\underset{\sim}{k} + \lambda(\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k})$

$\underset{\sim}{r} = 6\underset{\sim}{i} + 15\underset{\sim}{j} + 39\underset{\sim}{k} + \mu(2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k})$

l_1: $\underset{\sim}{r} = (-6 + \lambda)\underset{\sim}{i} + (3 - 2\lambda)\underset{\sim}{j} + (15 + 3\lambda)\underset{\sim}{k}$

l_2: $\underset{\sim}{r} = (6 + \mu)\underset{\sim}{i} + (15 - 3\mu)\underset{\sim}{j} + (39 + 4\mu)\underset{\sim}{k}$

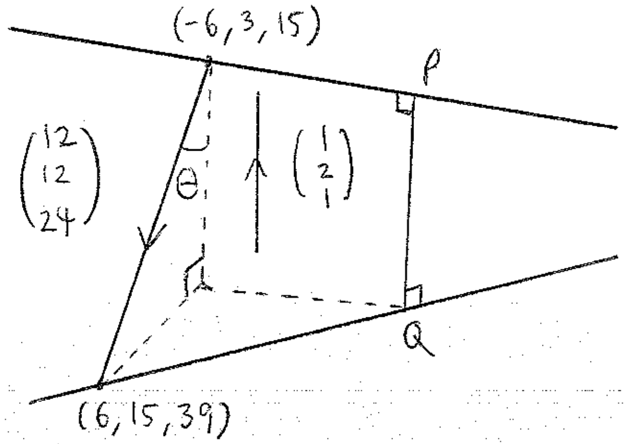
$P(-6 + \lambda, 3 - 2\lambda, 15 + 3\lambda)$

$Q(6 + 2\mu, 15 - 3\mu, 39 + 4\mu)$

i) $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$

ii)



$$\begin{pmatrix}6\\15\\39\end{pmatrix} - \begin{pmatrix}-6\\3\\15\end{pmatrix} = \begin{pmatrix}12\\12\\24\end{pmatrix}$$

$$= 12\begin{pmatrix}1\\1\\2\end{pmatrix}$$

$$12\begin{pmatrix}1\\1\\2\end{pmatrix} \cdot \begin{pmatrix}1\\2\\1\end{pmatrix} = \left|12\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\left|\begin{pmatrix}1\\2\\1\end{pmatrix}\right| \cos\theta$$

$$12(1 + 2 + 2) = \sqrt{6}\left(12\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right| \cos\theta\right)$$

$$12\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right| \cos\theta = \frac{5(12)}{\sqrt{6}}$$

$$= \frac{60\sqrt{6}}{6}$$

$$= 10\sqrt{6}$$

iii) $\overrightarrow{PQ} \,//\, \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$

$\therefore \overrightarrow{PQ} = c\begin{pmatrix}1\\2\\1\end{pmatrix}, \quad c \in R$

$$\begin{pmatrix}6+2\mu\\15-3\mu\\39+4\mu\end{pmatrix} - \begin{pmatrix}-6+\lambda\\3-2\lambda\\15+3\lambda\end{pmatrix} = \begin{pmatrix}c\\2c\\c\end{pmatrix}$$

$$\begin{pmatrix}12+2\mu-\lambda\\12-3\mu+2\lambda\\24+4\mu-3\lambda\end{pmatrix} = \begin{pmatrix}c\\2c\\c\end{pmatrix}$$

$$\left.\begin{aligned} 12 + 2\mu - \lambda &= c \\ 12 - 3\mu + 2\lambda &= 2c \\ 24 + 4\mu - 3\lambda &= c \end{aligned}\right\}$$

$$\left.\begin{aligned} c - 2\mu + \lambda &= 12 \\ 2c + 3\mu - 2\lambda &= 12 \\ c - 4\mu + 3\lambda &= 24 \end{aligned}\right\}$$

$$\begin{array}{ll} -2 \times ① + ② : & \\ - ① + ③ : & \end{array} \quad \left.\begin{aligned} c - \mu + \lambda &= 12 \\ 7\mu - 4\lambda &= -12 \\ -2\mu + 2\lambda &= 12 \end{aligned}\right\}$$

$$\begin{array}{ll} 2 \times ② . & \\ 7 \times ③ . & \end{array} \quad \left.\begin{aligned} c - \mu + \lambda &= 12 \\ 14\mu - 8\lambda &= -24 \\ -14\mu + 14\lambda &= 84 \end{aligned}\right\}$$

$$② + ③ : \quad \begin{aligned} c - \mu + \lambda &= 12 \\ 14\mu - 8\lambda &= -24 \\ 6\lambda &= 60 \\ \lambda &= 10 \\ \mu &= 4 \\ c &= 6 \end{aligned}$$

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -17 \\ 45 \end{pmatrix} \quad \overrightarrow{OQ} = \begin{pmatrix} 14 \\ 3 \\ 55 \end{pmatrix}$$

59. $\Pi_1 : \underset{\sim}{r} = (3 + 6p)\underset{\sim}{i} + (1 + 4p + q)\underset{\sim}{j} + (6 + 2p - 4q)\underset{\sim}{k}$

$= 3\underset{\sim}{i} + \underset{\sim}{j} + 6\underset{\sim}{k} + p(6\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}) + q(\underset{\sim}{j} - 4\underset{\sim}{k})$

$\Pi_2 : \underset{\sim}{r} = (-9 + 3p)\underset{\sim}{i} + (1 + p - 2q)\underset{\sim}{j} + (3 - p + 8q)\underset{\sim}{k}$

$= -9\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k} + p(3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}) + q(-2\underset{\sim}{j} + 8\underset{\sim}{k})$

24W = WV

i) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 + 6p \\ 1 + 4p + q \\ 6 + 2p - 4q \end{pmatrix}$$

$$\left.\begin{aligned} x &= 3 + 6p \\ y &= 1 + 4p + q \\ z &= 6 + 2p - 4q \end{aligned}\right\}$$

$$\left.\begin{aligned} 6p &= x - 3 \\ 4p + q &= y - 1 \\ 2p - 4q &= z - 6 \end{aligned}\right\}$$

$$\left.\begin{aligned} 2p - 4q &= z - 6 \\ 4p + q &= y - 1 \\ 6p \quad\quad &= x - 3 \end{aligned}\right\}$$

$$\left.\begin{aligned} &-2 \times ① + ② : & 2p - 4q &= z - 6 \\ &-3 \times ① + ③ : & 9q &= y - 2z + 11 \\ & & 12q &= x - 3z + 15 \end{aligned}\right\}$$

$12(y - 2z + 11) = 9(x - 3z + 15)$

$12y - 24z + 132 = 9x - 27z + 135$

$9x - 12y - 3z = -3$

$$3x - 4y - z = -1$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -9 + 3p \\ 1 + p - 2q \\ 3 - p + 8q \end{pmatrix}$$

$$\left.\begin{aligned} x &= -9 + 3p \\ y &= 1 + p - 2q \\ z &= 3 - p + 8q \end{aligned}\right\}$$

$$\left.\begin{aligned} 3p &= x + 9 \\ p - 2q &= y - 1 \\ -p + 8q &= z - 3 \end{aligned}\right\}$$

$$\left.\begin{aligned} 3 \times ② :\quad 3p &= x + 9 \\ 3 \times ③ :\quad 3p - 6q &= 3y - 3 \\ -3p + 24q &= 3z - 9 \end{aligned}\right\}$$

$$\left.\begin{aligned} -① + ② :\quad 3p &= x + 9 \\ ① + ③ :\quad -6q &= -x + 3y - 12 \\ 24q &= x + 3z \end{aligned}\right\}$$

$$-4(-x + 3y - 12) = x + 3z$$

$$4x - 12y + 48 = x + 3z$$

$$3x - 12y - 3z = -48$$

$$x - 4y - z = -16$$

$$\underset{\sim}{n}_1 = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} \quad \underset{\sim}{n}_2 = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$$

ii) $$\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}\right| \cos\theta$$

$$3 + 16 + 1 = \sqrt{26}\sqrt{18}\cos\theta$$

$$\cos\theta = \frac{20}{\sqrt{26}\sqrt{18}}$$

$$= \frac{10}{3\sqrt{13}}$$

$$\theta = 22.4^\circ$$

$$90 - \theta = 67.6^\circ$$

iii) $2uw = wv$

Let $w(a, b, c)$

$$2\left[\begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} 3 + 6p \\ 1 + 4p + q \\ 6 + 2p - 4q \end{pmatrix}\right] = \begin{pmatrix} -9 + 3p \\ 1 + p - 2q \\ 3 - p + 8q \end{pmatrix} - \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$2\begin{pmatrix} a - 3 - 6p \\ b - 1 - 4p - q \\ c - 6 - 2p + 4q \end{pmatrix} = \begin{pmatrix} -9 + 3p - a \\ 1 + p - 2q - b \\ 3 - p + 8q - c \end{pmatrix}$$

$$\begin{pmatrix} 2a - 6 - 12p \\ 2b - 2 - 8p - 2q \\ 2c - 12 - 4p + 8q \end{pmatrix} = \begin{pmatrix} -9 + 3p - a \\ 1 + p - 2q - b \\ 3 - p + 8q - c \end{pmatrix}$$

$2a - 6 - 12p = -9 + 3p - a$

$2b - 2 - 8p - 2q = 1 + p - 2q - b$

$2c - 12 - 4p + 8q = 3 - p + 8q - c$

$3a = 15p - 3$

$3b = 9p + 3$

$3c = 3p + 15$

$a = 5p - 1$

$b = 3p + 1$

$c = p + 5$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5p - 1 \\ 3p + 1 \\ p + 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} + p \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

iv) $\underset{\sim}{r} = s\underset{\sim}{k}, \quad s \in R$

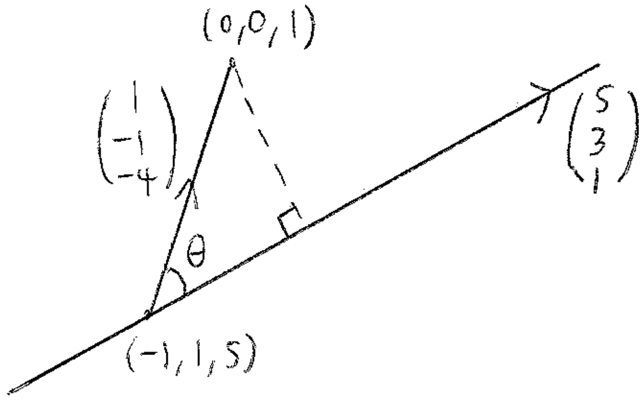
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix}$$

$3x - 4y - z = -1$

$-s = -1$

$s = 1$

$(0, 0, 1)$



$$\begin{pmatrix}1\\-1\\-4\end{pmatrix} \times \begin{pmatrix}5\\3\\1\end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -1 & -4 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= 11\underset{\sim}{i} - 21\underset{\sim}{j} + 8\underset{\sim}{k}$$

$$\left| \begin{pmatrix}1\\-1\\-4\end{pmatrix} \times \begin{pmatrix}5\\3\\1\end{pmatrix} \right| = \left| \begin{pmatrix}1\\-1\\-4\end{pmatrix} \right| \left| \begin{pmatrix}5\\3\\1\end{pmatrix} \right| \sin\theta$$

$$\left| 11\underset{\sim}{i} - 21\underset{\sim}{j} + 8\underset{\sim}{k} \right| = \sqrt{35} \left| \begin{pmatrix}1\\-1\\-4\end{pmatrix} \right| \sin\theta$$

$$\left| \begin{pmatrix}1\\-1\\-4\end{pmatrix} \right| \sin\theta = \frac{\sqrt{626}}{\sqrt{35}}$$

$$\approx 4.23$$

63. $\overrightarrow{OU} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $\overrightarrow{OV} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $\overrightarrow{OW} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$

i) $\overrightarrow{OU} \times \overrightarrow{OV} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \begin{pmatrix} -5 \\ 5 \\ 5 \end{pmatrix} = 5\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \overrightarrow{OU} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$-x + y + z = -1 - 1 + 2$

$x - y - z = 0$

$\therefore$ The equation of Π is $x - y - z = 0$.

Since Y is a point on Π,

let $y = s$ and $z = t$, $s, t \in R$

$\therefore x = y + z$

$= s + t$

$\therefore Y(s+t, s, t)$

If $Z = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, since $2WY = YZ$,

$$2\overrightarrow{WY} = \overrightarrow{YZ}$$

$$2(\overrightarrow{OY} - \overrightarrow{OW}) = \overrightarrow{OZ} - \overrightarrow{OY}$$

$$2\left(\begin{pmatrix} s+t \\ s \\ t \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} s+t \\ s \\ t \end{pmatrix}$$

$$2\begin{pmatrix} s+t-1 \\ s+3 \\ t-2 \end{pmatrix} = \begin{pmatrix} x-s-t \\ y-s \\ z-t \end{pmatrix}$$

$$\begin{pmatrix} 2s+2t-2 \\ 2s+6 \\ 2t-4 \end{pmatrix} = \begin{pmatrix} x-s-t \\ y-s \\ z-t \end{pmatrix}$$

$$\left.\begin{aligned} 2s+2t-2 &= x-s-t \\ 2s+6 &= y-s \\ 2t-4 &= z-t \end{aligned}\right\}$$

$$\left.\begin{aligned} 3s+3t &= x+2 \\ 3s &= y-6 \\ 3t &= z+4 \end{aligned}\right\}$$

$$\therefore y-6+z+4 = x+2$$

$$x-y-z = -4$$

Let $y = s$ and $z = t$, $s, t \in R$

$$x = s+t-4$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s+t-4 \\ s \\ t \end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore$ The equation of the locus of Z can be expressed in the form

$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b} + \mu \underset{\sim}{c}$, $\underset{\sim}{a} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\underset{\sim}{c} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

ii) $$\underset{\sim}{r} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4+s+t \\ s \\ t \end{pmatrix}$$

$x = -4 + s + t$, $y = s$, $z = t$

$x = -4 + y + z$

$x - y - z = -4$

$\therefore$ The locus of Z is a plane Π_1, which is parallel to Π.

iii) $\Pi : x - y - z = 0$

$\Pi_1 : x - y - z = -4$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = -4$$

since $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ is the normal of Π and Π_1,

a vector perpendicular to Π and Π_1 is $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

iv) since $(0,0,0)$ is a point on Π and $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

is perpendicular to Π, a line perpendicular

to Π passing through Π has the equation

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$$= s \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$

when the line meets Π_1,

$x = s$, $y = -s$, $z = -s$

$$s - (-s) - (-s) = -4$$

$$s + s + s = -4$$

$$3s = -4$$

$$s = -\frac{4}{3}$$

$\therefore$ The line meets Π_1 at $\left(\frac{-4}{3}, \frac{4}{3}, \frac{4}{3}\right)$.

$\therefore$ The distance between Π and Π_1 is

$$\sqrt{\left(\frac{-4}{3}-0\right)^2+\left(\frac{4}{3}-0\right)^2+\left(\frac{4}{3}-0\right)^2}$$

$$= \sqrt{\frac{16}{9}+\frac{16}{9}+\frac{16}{9}}$$

$$= \sqrt{\frac{48}{9}}$$

$$= \frac{4\sqrt{3}}{3}.$$

65. $\overrightarrow{OP} = \begin{pmatrix} 4\cos t \\ 4\sin t \\ 3 \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 8\cos t \\ 8\sin t \\ -1 \end{pmatrix}$, $0 \leq t \leq 2\pi$.

i) $\overrightarrow{PQ} = \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix}$

PQ: $\underset{\sim}{r} = \begin{pmatrix} 4\cos t \\ 4\sin t \\ 3 \end{pmatrix} + s \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix}$, $s \in R$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4(s+1)\cos t \\ 4(s+1)\sin t \\ 3-4s \end{pmatrix}$$

$x = 4(s+1)\cos t$, $y = 4(s+1)\sin t$, $z = 3-4s$

$z = 0$

$3 - 4s = 0$

$s = \frac{3}{4}$

$R(7\cos t, 7\sin t, 0)$

ii) $\underset{\sim}{n} = \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix}$

$\underset{\sim}{r} \cdot \underset{\sim}{n} = \overrightarrow{OR} \cdot \underset{\sim}{n}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix} = \begin{pmatrix} 7\cos t \\ 7\sin t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix}$$

$4\cos t\, x + 4\sin t\, y - 4z = 28\cos^2 t + 28\sin^2 t$
$= 28$

$\cos t\, x + \sin t\, y - z = 7$

iii) $A(10,10,40)$

$$\vec{OA} \cdot \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix} = |\vec{OA}| \left| \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix} \right| \cos(180^\circ - \theta)$$

$$\begin{pmatrix} 10 \\ 10 \\ 40 \end{pmatrix} \cdot \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix} = \left| \begin{pmatrix} 10 \\ 10 \\ 40 \end{pmatrix} \right| \left| \begin{pmatrix} 4\cos t \\ 4\sin t \\ -4 \end{pmatrix} \right| (-\cos\theta)$$

$$40\cos t + 40\sin t - 160 = 10\sqrt{18}\sqrt{16\cos^2 t + 16\sin^2 t + 16}$$
$$(-\cos\theta)$$

$$4\cos t + 4\sin t - 16 = 3\sqrt{2}\sqrt{32}\cos\theta$$
$$= -24\cos\theta$$

$$-\cos\theta = \frac{\cos t + \sin t - 4}{6}$$

$$\cos\theta = \frac{4 - \cos t - \sin t}{6}$$

$$= \frac{1}{6}\left(4 - \frac{\sqrt{2}\sin t}{\sqrt{2}} - \frac{\sqrt{2}\cos t}{\sqrt{2}}\right)$$

$$= \frac{1}{6}(4 - \sqrt{2}(\sin t\cos 45^\circ + \cos 45^\circ \sin t)$$

$$= \frac{1}{6}(4 - \sqrt{2}\sin(t + 45^\circ))$$

iv) when $t = 45^\circ$

$$\cos\theta = \frac{4 - \sqrt{2}}{6}$$

$$\simeq 0.431$$

$$\theta \simeq 64.5^\circ$$

66: $\ell_1: \underset{\sim}{r} = \underset{\sim}{j} - 2\underset{\sim}{k} + s(-\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k})$

$\ell_2: \underset{\sim}{r} = \underset{\sim}{i} + 4\underset{\sim}{j} + 6\underset{\sim}{k} + t(3\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$

$\ell_1: \underset{\sim}{r} = -s\underset{\sim}{i} + (1+s)\underset{\sim}{j} + (-2-s)\underset{\sim}{k}$

$\ell_2: \underset{\sim}{r} = (1+3t)\underset{\sim}{i} + (4+t)\underset{\sim}{j} + (6+t)\underset{\sim}{k}$

$P(-s, 1+s, -2-s) \quad Q(1+3t, 4+t, 6+t)$

i) $$\begin{pmatrix}-1\\1\\-1\end{pmatrix} \times \begin{pmatrix}3\\1\\1\end{pmatrix} = \begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ -1 & 1 & -1\\ 3 & 1 & 1\end{vmatrix}$$

$$= 2\underset{\sim}{i} - 2\underset{\sim}{j} - 4\underset{\sim}{k}$$

$$= 2(\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k})$$

ii) $\overrightarrow{PQ} \parallel \underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$

$\overrightarrow{PQ} = c(\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}), \quad c \in R$

$$\begin{pmatrix}1+3t\\4+t\\6+t\end{pmatrix} - \begin{pmatrix}-s\\1+s\\-2-s\end{pmatrix} = c\begin{pmatrix}1\\-1\\-2\end{pmatrix}$$

$$\begin{pmatrix}1+3t+s\\3+t-s\\8+t+s\end{pmatrix} = \begin{pmatrix}c\\-c\\-2c\end{pmatrix}$$

$$\left.\begin{aligned}1+3t+s &= c\\ 3+t-s &= -c\\ 8+t+s &= -2c\end{aligned}\right\}$$

$$\left.\begin{aligned} r - s - 3t &= 1 \\ -r + s - t &= 3 \\ -2r - s - t &= 8 \end{aligned}\right\}$$

$$\begin{aligned} &① + ② \\ &2 \times ① + ③: \end{aligned} \qquad \left.\begin{aligned} r - s - 3t &= 1 \\ -4t &= 4 \\ -3s - 7t &= 10 \end{aligned}\right\}$$

$$t = -1$$
$$s = -1$$
$$r = 0$$

$P(1, 0, -1) \quad Q(-2, 3, 5)$

iii) $\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \overrightarrow{OP} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{aligned} -x + y - z &= -1 + 0 + 1 \\ &= 0 \end{aligned}$$

$$x - y + z = 0$$

$$1 + \lambda + \mu - (-\lambda + \mu) - 1 - 2\lambda = 0$$

The plane through PQ perpendicular to l_1 has equation

$$\underset{\sim}{r} = (1 + \lambda + \mu)\underset{\sim}{i} + (-\lambda + \mu)\underset{\sim}{j} + (-1 - 2\lambda)\underset{\sim}{k}, \quad \lambda, \mu \in \mathbb{R}$$

68. $\underset{\sim}{a} = -2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$

$\underset{\sim}{b} = 7\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$

$\underset{\sim}{c} = -3\underset{\sim}{i} + 10\underset{\sim}{j} + 6\underset{\sim}{k}$

$\underset{\sim}{d} = 12\underset{\sim}{i} - 10\underset{\sim}{j} + 11\underset{\sim}{k}$

$\underset{\sim}{b} - \underset{\sim}{a} = 9\underset{\sim}{i} + 6\underset{\sim}{j} - 6\underset{\sim}{k}$

AB: $\underset{\sim}{r} = -2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k} + s(3i + 2j - 2\underset{\sim}{k})$

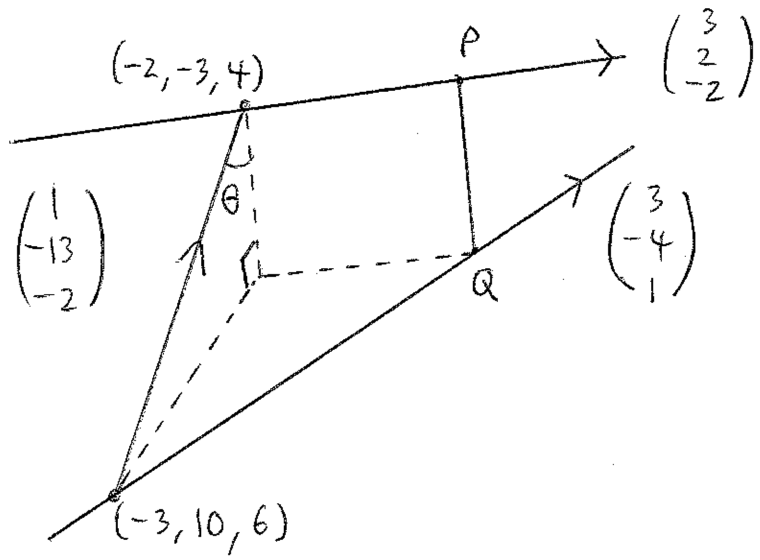
$\underset{\sim}{d} - \underset{\sim}{c} = 15\underset{\sim}{i} - 20\underset{\sim}{j} + 5\underset{\sim}{k}$

CD: $\underset{\sim}{r} = -3\underset{\sim}{i} + 10\underset{\sim}{j} + 6\underset{\sim}{k} + t(3\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k})$

$P(-2 + 3s, -3 + 2s, 4 - 2s)$

$Q(-3 + 3t, 10 - 4t, 6 + t)$

i)



$$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & 2 & -2 \\ 3 & -4 & 1 \end{vmatrix}$$

$$= -6\underset{\sim}{i} - 9\underset{\sim}{j} - 18\underset{\sim}{k}$$

$$\left| \begin{pmatrix} -6 \\ -9 \\ -18 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -13 \\ -2 \end{pmatrix} \right| = \left| \begin{pmatrix} -6 \\ -9 \\ -18 \end{pmatrix} \right| \left| \begin{pmatrix} 1 \\ -13 \\ -2 \end{pmatrix} \right| \cos\theta$$

$$-6 + 117 + 36 = \sqrt{441} \left| \begin{pmatrix} 1 \\ -13 \\ -2 \end{pmatrix} \right| \cos\theta$$

$$\left| \begin{pmatrix} 1 \\ -13 \\ -2 \end{pmatrix} \right| \cos\theta = \frac{147}{21}$$

$$= 7$$

$$\overrightarrow{PQ} \parallel \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$$

$$\overrightarrow{PQ} = c \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} -3 + 3t \\ 10 - 4t \\ 6 + t \end{pmatrix} - \begin{pmatrix} -2 + 3s \\ -3 + 2s \\ 4 - 2s \end{pmatrix} = \begin{pmatrix} 2c \\ 3c \\ 6c \end{pmatrix}$$

$$\begin{pmatrix} 3t - 3s - 1 \\ -4t - 2s + 13 \\ t + 2s + 2 \end{pmatrix} = \begin{pmatrix} 2c \\ 3c \\ 6c \end{pmatrix}$$

$$\left.\begin{aligned} 3t - 3s - 1 &= 2c \\ -4t - 2s + 13 &= 3c \\ t + 2s + 2 &= 6c \end{aligned}\right\}$$

$$\left.\begin{aligned} t + 2s - 6c &= -2 \\ 3t - 3s - 2c &= 1 \\ -4t - 2s - 3c &= -13 \end{aligned}\right\}$$

$-3 \times ① + ②$ $\quad t + 2s - 6c = -2$

$4 \times ① + ③$: $\quad -9s + 16c = 7$

$6s - 27c = -21$

$9 \times ③$: $t + 2s - 6c = -2$

$-9s + 16c = 7$

$54s - 243c = -189$

$6 \times ② + ③$ $\quad t + 2s - 6c = -2$

$-9s + 16c = 7$

$-147c = -147$

$c = 1$

$s = 1$

$t = 2$

$\overrightarrow{OP} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$

$\overrightarrow{OQ} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 8\underset{\sim}{k}$

ii) $\overrightarrow{OB} = 7\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$

$\overrightarrow{BP} = -6\underset{\sim}{i} - 4\underset{\sim}{j} + 4\underset{\sim}{k}$

$\overrightarrow{BQ} = -4\underset{\sim}{i} - \underset{\sim}{j} + 10\underset{\sim}{k}$

$$\overrightarrow{BP} \times \overrightarrow{BQ} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -6 & -4 & 4 \\ -4 & -1 & 10 \end{vmatrix}$$

$$= -36\underset{\sim}{i} + 44\underset{\sim}{j} - 10\underset{\sim}{k}$$

$$\underset{\sim}{r} \cdot \begin{pmatrix} -36 \\ 44 \\ -10 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -36 \\ 44 \\ -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -36 \\ 44 \\ -10 \end{pmatrix} = -252 + 132 + 20$$

$$-36x + 44y - 10z = -100$$

$$18x - 22y + 5z = 50$$

$$\underset{\sim}{r} = \begin{pmatrix} -3 \\ 10 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 18 \\ -22 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 + 18\lambda \\ 10 - 22\lambda \\ 6 + 5\lambda \end{pmatrix}$$

$$18(-3 + 18\lambda) - 22(10 - 22\lambda) + 5(6 + 5\lambda) = 50$$

$$-54 + 324\lambda - 220 + 484\lambda + 30 + 25\lambda = 50$$

$$833\lambda = 294$$

$$\lambda = \frac{6}{17}$$

$$\left(\frac{57}{17}, \frac{38}{17}, \frac{132}{17}\right)$$

$$\sqrt{\left(\frac{57}{17} + 3\right)^2 + \left(\frac{38}{17} - 10\right)^2 + \left(\frac{132}{17} - 6\right)^2} = \sqrt{\frac{29988}{289}} = \frac{42\sqrt{17}}{17}$$

72. $\ell_1 : \underset{\sim}{r} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 7\underset{\sim}{k} + s(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$

$\ell_2 : \underset{\sim}{r} = \underset{\sim}{i} + 6\underset{\sim}{j} - 5\underset{\sim}{k} + t(2\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k})$

$P(4 + s, -3 + s, 7 + s) \quad Q(1 + 2t, 6 + 2t, -5 - 3t)$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 1 & 1 \\ 2 & 2 & -3 \end{vmatrix}$$

$$= -5\underset{\sim}{i} + 5\underset{\sim}{j}$$

$$\overrightarrow{PQ} \parallel \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{PQ} = c\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 + 2t \\ 6 + 2t \\ -5 - 3t \end{pmatrix} - \begin{pmatrix} 4 + s \\ -3 + s \\ 7 + s \end{pmatrix} = c\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 + 2t - s \\ 9 + 2t - s \\ -12 - 3t - s \end{pmatrix} = \begin{pmatrix} -c \\ c \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} -3 + 2t - s &= -c \\ 9 + 2t - s &= c \\ -12 - 3t - s &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} c + 2t - s &= 3 \\ c - 2t + s &= 9 \\ 3t + s &= -12 \end{aligned}\right\}$$

$$-① + ②: \left.\begin{aligned} c + 2t - s &= 3 \\ -4t + 2s &= 6 \\ 3t + s &= -12 \end{aligned}\right\}$$

$$-\frac{1}{2} \times ②: \left.\begin{aligned} c + 2t - s &= 3 \\ 2t - s &= -3 \\ 3t + s &= -12 \end{aligned}\right\}$$

$$② + ③: \begin{aligned} c + 2t - s &= 3 \\ 2t - s &= -3 \\ 5t &= -15 \end{aligned}$$

$$t = -3$$

$$s = -3$$

$$c = 6$$

$$\overrightarrow{OP} = \begin{pmatrix} 1 \\ -6 \\ 4 \end{pmatrix} \quad \overrightarrow{OQ} = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$$

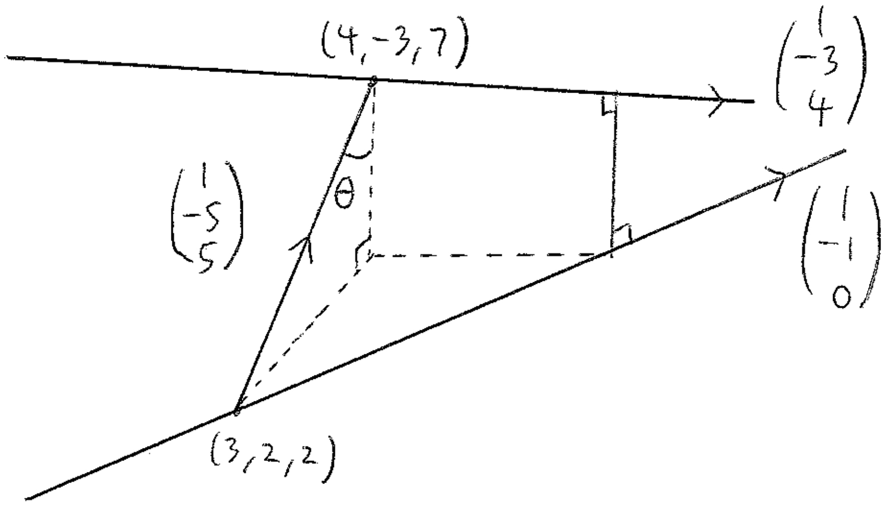
$$\overrightarrow{AB} = -3\underset{\sim}{i} + 9\underset{\sim}{j} - 12\underset{\sim}{k}$$

$$AB: \underset{\sim}{r} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 7\underset{\sim}{k} + \lambda(\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k})$$

$(3,2,2) \quad (5,0,2)$

$$\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$\underset{\sim}{r} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k} + \mu(\underset{\sim}{i} - \underset{\sim}{j})$$



$$\begin{pmatrix}1\\-3\\4\end{pmatrix} \times \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -3 & 4 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \begin{pmatrix}4\\4\\2\end{pmatrix}$$

$$\left|\begin{pmatrix}1\\-5\\5\end{pmatrix} \cdot \begin{pmatrix}4\\4\\2\end{pmatrix}\right| = \left|\begin{pmatrix}1\\-5\\5\end{pmatrix}\right| \left|\begin{pmatrix}4\\4\\2\end{pmatrix}\right| \cos\theta$$

$$|4 - 20 + 10| = 6 \left|\begin{pmatrix}1\\-5\\5\end{pmatrix}\right| \cos\theta$$

$$\therefore \left|\begin{pmatrix}1\\-5\\5\end{pmatrix}\right| \cos\theta = 1$$