Example:

Prove by mathematical induction, for $n \in \mathrm{N}$, that $f(n) = 3^{2n} + 11$ is divisible by 4.

Example:

Prove by mathematical induction, for $n \in \mathrm{N}$, that $f(n) = 7^n + 12n + 5$ is a multiple of 6.

Example:

Given that $f(n) = 13^n - 6^n, n \in \mathrm{N}$.

(a) Express $f(k+1) - 6f(k)$ in terms of k.

(b) Hence, prove by mathematical induction, for $n \in \mathrm{N}$, that $f(n)$ is divisible by 7.

Example:

Given that $f(n) = 3^{4n-2} + 17^n + 22,\ n \in \mathrm{N}$.

(a) Find $f(1)$, $f(2)$ and $f(3)$.

(b) Make a conjecture about the largest factor of $f(n)$.

(c) Prove (b) by mathematical induction.

Example:

Prove by mathematical induction that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

is true for $n \in \mathrm{N}$.

Example:

Prove by mathematical induction that if

$$y = \frac{1}{1+x}, \text{ then } \frac{d^n y}{dx^n} = \frac{(-1)^n (n!)}{(1+x)^{n+1}} \text{ for } n \in \mathrm{N}.$$

Example:

Prove by mathematical induction, for $n \in \mathrm{N}$, that

$$\frac{d^n}{dx^n}(\sin x) = \sin\left(x + \frac{1}{2}\pi x \right).$$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Given that $f(n) = 5^{2n} - 6n + 8, n \in \mathrm{N}$.

(a) Express $f(k+1) - 25f(k)$ in terms of k.

(b) Hence, prove by mathematical induction, for $n \in \mathrm{N}$, that $f(n)$ is a multiple of 9.

Example:

Given that $f(n) = 1957^n + 8(31)^n$, $n \in \mathrm{N}$.

(a) Find $f(1)$, $f(2)$ and $f(3)$.

(b) Make a conjecture about the largest factor of $f(n)$.

(c) Hence, prove (b) by mathematical induction.

Example:

Prove by mathematical induction, for $n \in \mathrm{N}$, that $f(n) = 2^{6n} + 3^{2n-2}$ is divisible by 5.

Example:

Show that $\sqrt{2}\sin\left[x+\frac{1}{4}\pi(k+1)\right]\equiv\sin\left(x+\frac{1}{4}\pi k\right)+\cos\left(x+\frac{1}{4}\pi k\right)$.

Hence, or otherwise, prove by mathematical induction that

if $y=e^x\sin x$, then $\frac{d^n y}{dx^n}=\left(\sqrt{2}\right)^n e^x\sin\left(x+\frac{1}{4}\pi n\right)$ for $n\in\mathrm{N}$.

Example:

By evaluating $\frac{d}{dx}\left(xe^{x}\right)$, $\frac{d^{2}}{dx^{2}}\left(xe^{x}\right)$, $\frac{d^{3}}{dx^{3}}\left(xe^{x}\right)$, state a conjecture about $\frac{d^{n}}{dx^{n}}\left(xe^{x}\right)$ for $n \in \mathrm{N}$. Then, prove it by mathematical induction.