TAYLOR'S COLLEGE

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CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2005

(Jan 2004 Intake)

Monday 4 April 2005 1.00 pm – 4.00 pm

FURTHER MATHEMATICS 9231/01

PAPER 1 3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

1. If $\sin y = x$, show that

$$\left(1-x^2\right)\frac{d^2y}{dx^2} = x\frac{dy}{dx}$$ [4]

2. Express $\dfrac{2}{x(x+1)(x+2)}$ in partial fractions. [2]

Hence, find

$$\sum_{n=1}^{N}\frac{1}{n(n+1)(n+2)}$$ [2]

Show that the sum to infinity exists and find it. [2]

3. Given that n is a positive integer greater than 2, use induction to prove that $2^n > 2n$. [6]

4. Given that $I_n = \int_0^{\pi/3} \sec^n x \, dx$, show that for $n \geq 2$, $I_n = \dfrac{2^{n-2}}{n-1}\sqrt{3} + \dfrac{n-2}{n-1} I_{n-2}$. [4]

Hence, evaluate I_7, giving your answer in exact form. [4]

5. The plane π with equation $\underset{\sim}{r}.\left(2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}\right) = 7$, meets the line L, which has equation $\underset{\sim}{r} = \left(3\underset{\sim}{i} + 5\underset{\sim}{j} + \underset{\sim}{k}\right) + \lambda\left(2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}\right)$, at the point P.

(i) Find the coordinates of P. [3]

(ii) Find the shortest distance of π from the origin O. [2]

The line M passes through the point P and the point Q, where Q (4,7,0).

(iii) Determine a vector which is perpendicular to both of the lines L and M. [3]

6. Given that the matrix A, where

$$A = \begin{bmatrix} 3 & 0 & -2 & 3 \\ -4 & 2 & 0 & -8 \\ 1 & -2 & 2 & 5 \\ 3 & -3 & 2 & 9 \end{bmatrix}$$

Find:

(i) the rank of A. [3]

(ii) A basis for the subspace V defined by $V = \{x \in \Re^4 : A\mathbf{X} = 0\}$ [3]

(iii) the general solution of the equation

$$AX = \begin{bmatrix} 3 \\ -4 \\ 1 \\ 3 \end{bmatrix}$$ [2]

7. If α, β, γ are the roots of the equation $x^3 + px + q = 0$ $(q \neq 0)$, find in terms of p and q:

(i) $\alpha^4 + \beta^4 + \gamma^4$ [4]

(ii) $\alpha^5 + \beta^5 + \gamma^5$ [2]

Prove also that $2(\alpha^7 + \beta^7 + \gamma^7) = 7\alpha\beta\gamma(\alpha^4 + \beta^4 + \gamma^4)$ [4]

8. The curve with parametric equations

$x = t - \sin t,\ y = 1 - \cos t$

is defined from $t = 0$ to $t = \pi$.

(i) Find the length of the curve. [5]

(ii) Find the area of the surface generated when the curve is rotated completely about the x-axis. [6]

[Turn over

9. Sketch the curve with polar equation $r = \theta \sin\theta$ for $0 \le \theta \le 2\pi$. [4]
Hence, find the area bounded between the 2 loops. [7]

10. Use De Moivre's theorem to show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. [4]

Deduce an expression for $\sin 5\theta$ in terms of $\sin\theta$. [2]

Hence, find an expression for $\tan 5\theta$ in terms of $\tan\theta$. [4]

Given now that, $T_n = \cos^n\theta + \sin^n\theta$ deduce that, when $\theta = \pi/15$,
$16T_5 - 20\,T_3 + 5T_1 = (1+\sqrt{3})/2$. [2]

11. ANSWER **EITHER**

Given that

$$x^2\frac{d^2y}{dx^2} + (2x^2 + x)\frac{dy}{dx} + (2x^2 + x - 0.25)y = x^{5/2} \qquad (x > 0)$$

and that $y = x^{-1/2}z$, show that

$$\frac{d^2z}{dx^2} + a\frac{dz}{dx} + bz = cx$$

where a, b, c are constants to be found. Hence, find the particular solution for y in terms of x, if given that when $x = \pi$, $y = 0$ and $\frac{dy}{dx} = 0$.

[16]

[The second alternative for Question 11 is printed on the next page.]

OR

Given

$$M = \begin{pmatrix} -\frac{3}{2} & 4 & \frac{11}{3} \\ 1 & -\frac{1}{2} & -2 \\ -1 & 2 & \frac{7}{3} \end{pmatrix}$$

(i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{M}$. [7]

(ii) Deduce that as the positive integer n tends to infinity, all the elements of $\mathbf{M}^n$ tend to 0. [3]

(iii) Show that the eigenvectors of $\mathbf{M}$ are the same as $\mathbf{N}$ where $\mathbf{N} = k\mathbf{I} + q\mathbf{M}$.
eigenvalues
Hence, find the corresponding ~~eigenvectors~~, leaving your answer in terms of k and q where necessary. [6]