

Angle Between Two Lines

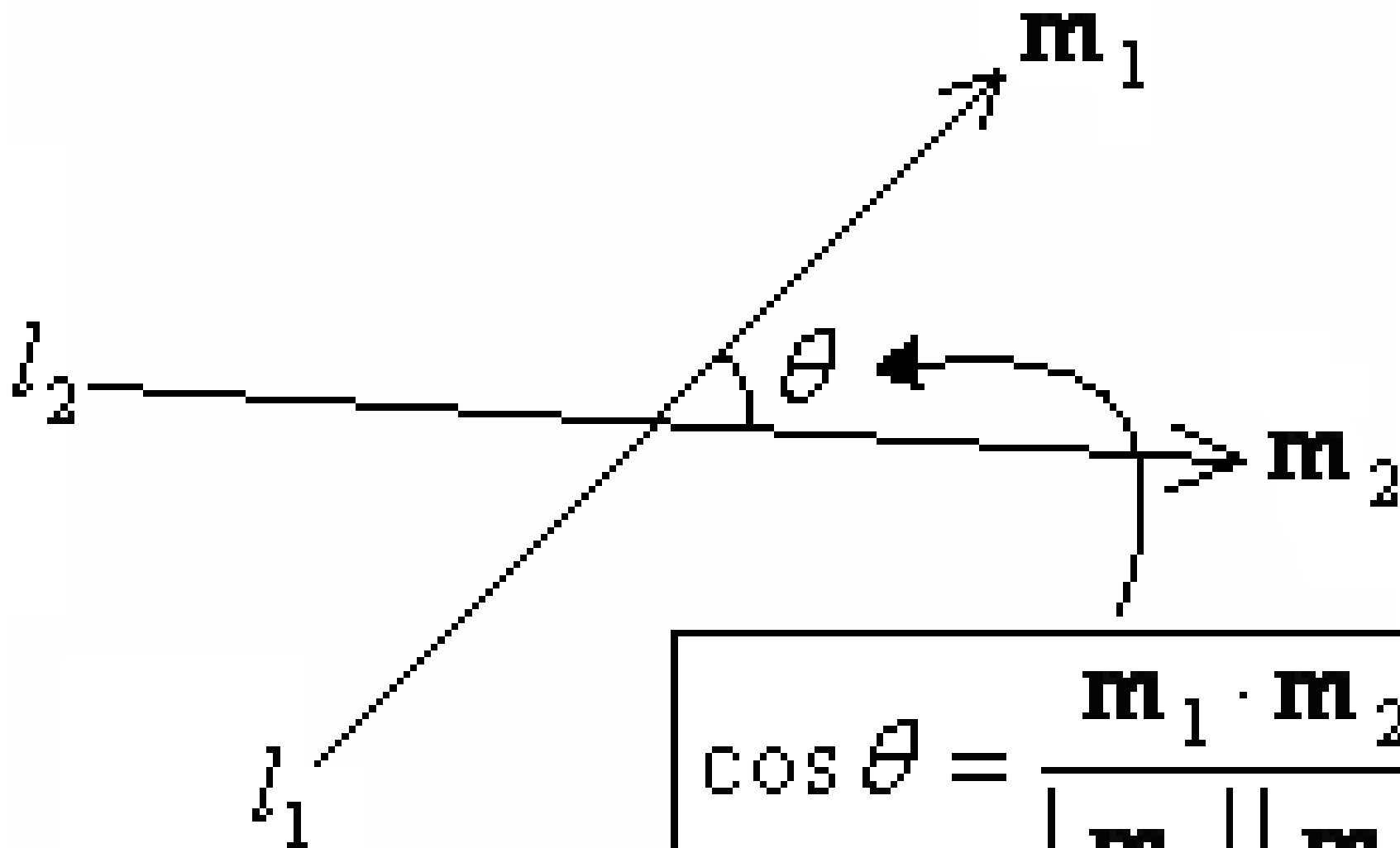
Let two lines have equations

$$l_1 : \mathbf{r} = \mathbf{a}_1 + \lambda_1 \mathbf{m}_1$$

$$l_2 : \mathbf{r} = \mathbf{a}_2 + \lambda_2 \mathbf{m}_2$$

The angle between the lines are

$$\cos \theta = \frac{\mathbf{m}_1 \cdot \mathbf{m}_2}{|\mathbf{m}_1| |\mathbf{m}_2|}, 0^\circ \leq \theta \leq 180^\circ$$



$$\cos \theta = \frac{\mathbf{m}_1 \cdot \mathbf{m}_2}{|\mathbf{m}_1| |\mathbf{m}_2|}$$

Example :

Find the acute angle between the lines

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \text{ and}$$

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}).$$

Example:

Find the acute angle between the lines

$$\mathbf{r} = (2\lambda \quad 1 - 3\lambda \quad -1 - \lambda) \text{ and}$$

$$\mathbf{r} = (1 + \mu \quad -2 \quad -2\mu)$$

Example:

Given that P has coordinates $(3,5,7)$. Find the coordinates of the point Q on the line $l : \frac{x-6}{7} = \frac{y+7}{-6} = z$ such that $PQ \perp l$.

Example:

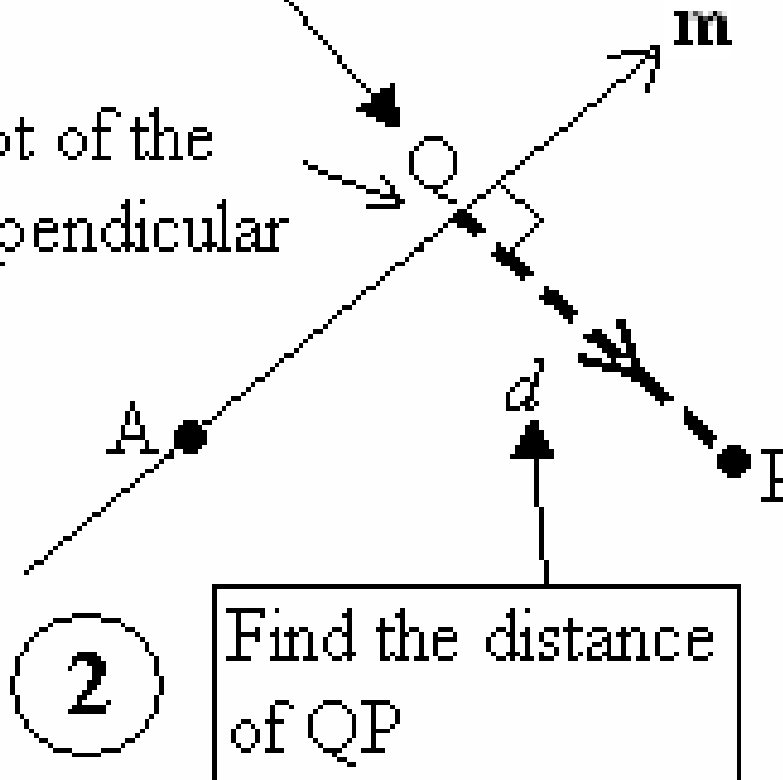
Find the foot of the perpendicular from point $P(1,5,1)$ to the line $\mathbf{r} = (\lambda \quad 5 - 2\lambda \quad 2 + \lambda)$.

Perpendicular Distance from a Point to a Line

1

Find the coordinates of
Q from $\mathbf{m} \cdot \mathbf{QP} = 0$

Foot of the
perpendicular



Example :

Find the perpendicular distance from point $(3,5,7)$ to line $\mathbf{r} = 6\mathbf{i} - 7\mathbf{j} + \lambda(7\mathbf{i} - 6\mathbf{j} + \mathbf{k})$.

Example:

Find the perpendicular distance from point $(1, 5, 1)$
to line $\mathbf{r} = (\lambda \quad 5 - 2\lambda \quad 2 + \lambda)$.

Example:

Find the shortest distance from point $(1,3,2)$ to

line $\frac{x-1}{-1} = \frac{y-2}{-3}, z = 2.$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example :

Find the distance of $(1,0,0)$ from the line
 $\mathbf{r} = t(12\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})$.

Example :

Find an equation of the line l containin the points $(1,3,1)$ and $(1,-3,-1)$. Find the perpendicular distance of the point with coordinates $(2,-1,1)$ from l .

Example:

An aeroplane climbs so that its position relative to the airport control tower t minutes after take-off is given by the vector $\mathbf{r} = -\mathbf{i} - 2\mathbf{j} + t(4\mathbf{i} + 5\mathbf{j} + 0.6\mathbf{k})$, the units being km. the x – and y – axes point towards the east and north respectively. Find the closest distance of the aeroplane from the tower during this flight. How many seconds after leaving the ground is the aeroplane at its closest to the tower?

Example:

$$l_1 : \frac{x-2}{5} = y+3 = \frac{z+1}{2} \text{ and } l_2 : \frac{x-m}{2} = y-2 = \frac{z-5}{-1}.$$

Point $P(7,-2,1)$ and $Q(5,4,3)$.

- (i) Find $\overrightarrow{\mathbf{PQ}}$ and show that $|\overrightarrow{\mathbf{PQ}}| = 2\sqrt{11}$
- (ii) Show that $\overrightarrow{\mathbf{PQ}}$ is perpendicular to both l_1 and l_2 .
- (iii) Show that P lies on l_1 and find m for which Q lies on l_2 .
- (iv) Find the shortest distance between l_1 and l_2 .
- (v) Find the acute angle between l_1 and l_2 .