TAYLOR'S
COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2012

(March 2012 Intake)

Tuesday **12 June 2012** **8.30 – 10.30 am**

FURTHER MATHEMATICS 9231

PAPER 1 and 2 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **50**.

This document consists of **4** printed pages.

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Section A: Pure Mathematics (50%)

1 Prove that

$$\frac{a^{n-1}}{b^{n-1}}+\frac{a^{n-2}}{b^{n-2}}+\cdots+\frac{a^2}{b^2}+\frac{a}{b}+1+\frac{b}{a}+\frac{b^2}{a^2}+\cdots+\frac{b^{n-2}}{a^{n-2}}+\frac{b^{n-1}}{a^{n-1}}=\frac{a^{2n-1}-b^{2n-1}}{a^{n-1}b^{n-1}(a-b)}.$$

for every positive integer n. [5]

2 If the equation $3x^3+5x^2+7=0$ has roots α, β, γ, find the equation having roots

$\frac{1}{\alpha}+\frac{1}{\beta}+\gamma, \frac{1}{\alpha}+\frac{1}{\gamma}+\beta, \frac{1}{\beta}+\frac{1}{\gamma}+\alpha$. [6]

3 The curve C has equation $y=\frac{3x}{x^2-5x+4}$.

(i) State all the asymptotes of C. [3]

(ii) Show that C has two turning points. [3]

4 **(i)** Find the sum $1^3-2^3+3^3-4^3+\ldots-(n-2)^3+(n-1)^3-n^3$. [4]

(ii) Find the sum $1^3-2^3+3^3-4^3+\ldots+(n-2)^3-(n-1)^3+n^3$. [4]

Students are required to answer questions in Section B using a fresh sheet of answer paper. Answer for Section A and Section B need to be handed in separately.

Section B: Mechanics (50%)

1 A ball drops vertically onto a smooth plane inclined to the horizontal at an angle of α. It hits the plane with speed 8 ms^{-1} and rebounds horizontally. The coefficient of restitution between the ball and the plane is $\frac{1}{3}$. Find the value of α and the speed with which the ball rebounds. [5]

2 A smooth sphere P of mass m is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass $2m$ is at rest on the table. The sphere P collides directly with Q. After the collision the direction of motion of P is unchanged. The spheres have the same radii and the coefficient of restitution between P and Q is e.

Show that the speed of Q immediately after the collision is $\frac{1}{3}(1+e)u$. [5]

Find the range of possible values of e. [3]

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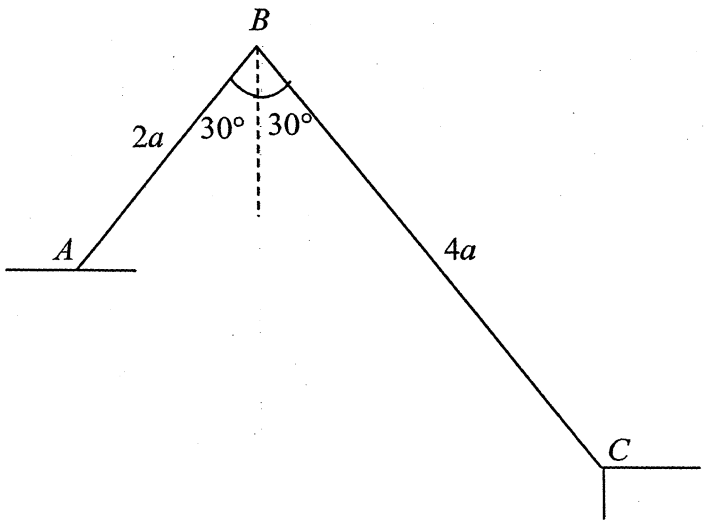
3



Two uniform bars AB and BC have lengths $2a$ and $4a$ and weights W and $2W$ respectively. They are freely hinged together at B and C is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with A resting on a rough horizontal surface.The normal contact force and the frictional force acting at A are denoted by R and F respectively. The rods make an angle of 30° with the vertical (see diagram). By taking moment about C for the system show that an equation relating F and R is given by

$$\sqrt{3}F + 3R = \frac{9}{2}W.$$ [3]

Find another equation relating F and R and hence find show that $R = \frac{5}{4}W$. [3]

The coefficient of friction between the rod AB and the horizontal surface is μ. Find the set of possible values of μ. [3]

Find the horizontal and vertical component of the reaction at B. [3]