1. a) $x^2 - 2xy + 2y^2 = 5$

$$\frac{d}{dx}(x^2 - 2xy + 2y^2) = \frac{d}{dx}(5)$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(2xy) + \frac{d}{dx}(2y^2) = 0$$

$$2x - 2x\frac{dy}{dx} - y\frac{d}{dx}(2x) + \frac{dy}{dx}\frac{d}{dy}(2y^2) = 0$$

$$2x - 2x\frac{dy}{dx} - 2y + 4y\frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(2x - 2x\frac{dy}{dx} - 2y + 4y\frac{dy}{dx}\right) = 0$$

$$\frac{d}{dx}(2x) - \frac{d}{dx}\left(2x\frac{dy}{dx}\right) - \frac{d}{dx}(2y) + \frac{d}{dx}\left(4y\frac{dy}{dx}\right) = 0$$

$$2 - 2x\frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}\frac{d}{dx}(2x) - \frac{dy}{dx}\frac{d}{dy}(2y)$$

$$+ 4y\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(4y) = 0$$

$$2 - 2x\frac{d^2y}{dx^2} - \frac{2dy}{dx} - \frac{2dy}{dx} + 4y\frac{d^2y}{dx^2}$$

$$+ \frac{dy}{dx}\left(\frac{dy}{dx}\right)\frac{d}{dy}(4y) = 0$$

$$2 - 2x\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right)^2 = 0$$

b) $2xy^2 - x^2y^3 = 1$

$$\frac{d}{dx}(2xy^2 - x^2y^3) = \frac{d}{dx}(1)$$

$$2x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(2x) - x^2\frac{d}{dx}(y^3) - y^3\frac{d}{dx}(x^2) = 0$$

$$2x\frac{dy}{dx}\frac{d}{dy}(y^2) + 2y^2 - x^2\frac{dy}{dx}\frac{d}{dy}(y^3) - 2xy^3 = 0$$

$$4xy\frac{dy}{dx} + 2y^2 - 3x^2y^2\frac{dy}{dx} - 2xy^3 = 0$$

$$\frac{d}{dx}\left(4xy\frac{dy}{dx} + 2y^2 - 3x^2y^2\frac{dy}{dx} - 2xy^3\right) = 0$$

$$\frac{d}{dx}\left(4xy\frac{dy}{dx}\right) + \frac{d}{dx}(2y^2) - \frac{d}{dx}\left(3x^2y^2\frac{dy}{dx}\right)$$

$$-\frac{d}{dx}(2xy^3) = 0$$

$$4xy\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(4xy) + \frac{dy}{dx}\frac{d}{dy}(2y^2)$$

$$-3x^2y^2\frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}\frac{d}{dx}(3x^2y^2)$$

$$-2x\frac{d}{dx}(y^3) - y^3\frac{d}{dx}(2x) = 0$$

$$4xy\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(4x\frac{dy}{dx} + y\frac{d}{dx}(4x)\right) + 4y\frac{dy}{dx}$$

$$-3x^2y^2\frac{d^2y}{dx^2} - \frac{dy}{dx}\left(3x^2\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(3x^2)\right)$$

$$-2x\frac{dy}{dx}\frac{d}{dy}(y^3) - 2y^3 = 0$$

$$4xy\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(4x\frac{dy}{dx} + 4y\right) + 4y\frac{dy}{dx}$$

$$-3x^2y^2\frac{d^2y}{dx^2} - \frac{dy}{dx}\left(3x^2\frac{dy}{dx}\frac{d}{dy}(y^2) + 6xy^2\right)$$

$$-6xy^2\frac{dy}{dx} - 2y^3 = 0$$

$$4xy\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 + 4y\frac{dy}{dx} + 4y\frac{dy}{dx}$$

$$-3x^2y^2\frac{d^2y}{dx^2} - \frac{dy}{dx}\left(6x^2y\frac{dy}{dx} + 6xy^2\right) - 6xy^2\frac{dy}{dx} - 2y^3 = 0$$

$$4xy\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 + 8y\frac{dy}{dx} - 3x^2y^2\frac{d^2y}{dx^2}$$

$$-6x^2y\left(\frac{dy}{dx}\right)^2 - 6xy^2\frac{dy}{dx} - 6xy^2\frac{dy}{dx} - 2y^3 = 0$$

$$4xy\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 + 8y\frac{dy}{dx} - 3x^2y^2\frac{d^2y}{dx^2}$$

$$-6x^2y\left(\frac{dy}{dx}\right)^2 - 12xy^2\frac{dy}{dx} - 2y^3 = 0$$

c) $x^3 + y^3 = 3xy$

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3xy)$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 3x\frac{dy}{dx} + y\frac{d}{dx}(3x)$$

$$3x^2 + \frac{dy}{dx}\frac{d}{dy}(y^3) = 3x\frac{dy}{dx} + 3y$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$$

$$x^2 + y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{d}{dx}(x^2 + y^2\frac{dy}{dx}) = \frac{d}{dx}(x\frac{dy}{dx} + y)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2\frac{dy}{dx}) = \frac{d}{dx}(x\frac{dy}{dx}) + \frac{dy}{dx}$$

$$2x + y^2\frac{d}{dx}(\frac{dy}{dx}) + \frac{dy}{dx}\frac{d}{dx}(y^2)$$

$$= x\frac{d}{dx}(\frac{dy}{dx}) + \frac{dy}{dx}\frac{d}{dx}(x) + \frac{dy}{dx}$$

$$2x + y^2\frac{d^2y}{dx^2} + (\frac{dy}{dx})\frac{dy}{dx}\frac{d}{dy}(y^2)$$

$$= x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx}$$

$$2x + y^2\frac{d^2y}{dx^2} + 2y\left(\frac{dy}{dx}\right)^2 = x\frac{d^2y}{dx^2} + \frac{2dy}{dx}$$

d) $(x^2 + y^2)^2 = x^2 - y^2$

$$\frac{d}{dx}\left((x^2+y^2)^2\right) = \frac{d}{dx}(x^2 - y^2)$$

$$2(x^2+y^2)\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(x^2) - \frac{d}{dx}(y^2)$$

$$2(x^2+y^2)\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)\right) = 2x - \frac{dy}{dx}\frac{d}{dy}(y^2)$$

$$2(x^2+y^2)\left(2x + \frac{dy}{dx}\frac{d}{dy}(2y)\right) = 2x - 2y\frac{dy}{dx}$$

$$2(x^2+y^2)\left(2x + \frac{2dy}{dx}\right) = 2x - 2y\frac{dy}{dx}$$

$$2(x^2+y^2)\left(x + y\frac{dy}{dx}\right) = x - y\frac{dy}{dx}$$

$$\frac{d}{dx}\left(2(x^2+y^2)\left(x + y\frac{dy}{dx}\right)\right) = \frac{d}{dx}\left(x - y\frac{dy}{dx}\right)$$

$$2(x^2+y^2)\frac{d}{dx}\left(x + y\frac{dy}{dx}\right) + \left(x + y\frac{dy}{dx}\right)\frac{d}{dx}\left(2(x^2+y^2)\right)$$

$$= \frac{d}{dx}(x) - \frac{d}{dx}\left(y\frac{dy}{dx}\right)$$

$$2(x^2+y^2)\left(\frac{d}{dx}(x) + \frac{d}{dx}\left(y\frac{dy}{dx}\right)\right)$$

$$+ \left(x + y\frac{dy}{dx}\right)\left(\frac{d}{dx}(2x^2) + \frac{d}{dx}(2y^2)\right)$$

$$= 1 - y\frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}\left(\frac{dy}{dx}\right)$$

$$2(x^2+y^2)\left(1+y\frac{d}{dx}\left(\frac{dy}{dx}\right)+\frac{dy}{dx}\left(\frac{dy}{dx}\right)\right)$$

$$+\left(x+y\frac{dy}{dx}\right)\left(4x+\frac{dy}{dx}\frac{d}{dy}(2y^2)\right)=1-y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2$$

$$2(x^2+y^2)\left(1+y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$$

$$+\left(x+y\frac{dy}{dx}\right)\left(4x+4y\frac{dy}{dx}\right)=1-y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2$$

e) $4y-x^2+2x^2y=4x$

$$\frac{d}{dx}(4y-x^2+2x^2y)=\frac{d}{dx}(4x)$$

$$\frac{d}{dx}(4y)-\frac{d}{dx}(x^2)+\frac{d}{dx}(2x^2y)=4$$

$$4\frac{dy}{dx}-2x+2x^2\frac{dy}{dx}+y\frac{d}{dx}(2x^2)=4$$

$$4\frac{dy}{dx}-2x+2x^2\frac{dy}{dx}+4xy=4$$

$$2\frac{dy}{dx}-x+x^2\frac{dy}{dx}+2xy=2$$

$$\frac{d}{dx}\left(2\frac{dy}{dx}-x+x^2\frac{dy}{dx}+2xy\right)=\frac{d}{dx}(2)$$

$$\frac{d}{dx}\left(2\frac{dy}{dx}\right)-\frac{d}{dx}(x)+\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+\frac{d}{dx}(2xy)=0$$

$$2\frac{d^2y}{dx^2} - 1 + x^2\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(x^2)$$

$$+ 2x\frac{dy}{dx} + y\frac{d}{dx}(2x) = 0$$

$$2\frac{d^2y}{dx^2} - 1 + x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = 0$$

$$2\frac{d^2y}{dx^2} - 1 + x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$$

f) $x^2 + 4xy + 5y^2 = 9$

$$\frac{d}{dx}(x^2 + 4xy + 5y^2) = \frac{d}{dx}(9)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(4xy) + \frac{d}{dx}(5y^2) = 0$$

$$2x + 4x\frac{dy}{dx} + y\frac{d}{dx}(4x) + \frac{dy}{dx}\frac{d}{dy}(5y^2) = 0$$

$$2x + 4x\frac{dy}{dx} + 4y + 10y\frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(2x + 4x\frac{dy}{dx} + 4y + 10y\frac{dy}{dx}\right) = 0$$

$$\frac{d}{dx}(2x) + \frac{d}{dx}\left(4x\frac{dy}{dx}\right) + \frac{d}{dx}(4y) + \frac{d}{dx}\left(10y\frac{dy}{dx}\right) = 0$$

$$2 + 4x\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(4x) + \frac{4dy}{dx}$$

$$+ 10y\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\left(10\frac{dy}{dx}\right) = 0$$

$$2 + 4x\frac{d^2y}{dx^2} + \frac{4dy}{dx} + \frac{4dy}{dx} + 10y\frac{d^2y}{dx^2} + 10\left(\frac{dy}{dx}\right)^2 = 0$$

$$1 + 2x\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5y\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 = 0$$

9) $x^2 + y^2 = e^{x+y} - 1$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(e^{x+y} - 1)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(e^{x+y}) - \frac{d}{dx}(1)$$

$$2x + \frac{dy}{dx}\frac{d}{dy}(y^2) = e^{x+y}\frac{d}{dx}(x+y)$$

$$2x + 2y\frac{dy}{dx} = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left(2x + 2y\frac{dy}{dx}\right) = \frac{d}{dx}\left(e^{x+y}\left(1 + \frac{dy}{dx}\right)\right)$$

$$\frac{d}{dx}(2x) + \frac{d}{dx}\left(2y\frac{dy}{dx}\right) = e^{x+y}\frac{d}{dx}\left(1 + \frac{dy}{dx}\right)$$

$$+ \left(1 + \frac{dy}{dx}\right)\frac{d}{dx}(e^{x+y})$$

$$2 + 2y\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(2y) = e^{x+y}\frac{d^2y}{dx^2}$$

$$+ \left(1 + \frac{dy}{dx}\right)e^{x+y}\frac{d}{dx}(x+y)$$

$$2 + 2y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\frac{dy}{dx}\frac{d}{dy}(2y)$$

$$= e^{x+y}\frac{d^2y}{dx^2} + \left(1+\frac{dy}{dx}\right)e^{x+y}\left(1+\frac{dy}{dx}\right)$$

$$2 + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = e^{x+y}\frac{d^2y}{dx^2} + e^{x+y}\left(1+\frac{dy}{dx}\right)^2$$

h) $y = x + 2y^4$

$$\frac{dy}{dx} = \frac{d}{dx}(x + 2y^4)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(2y^4)$$

$$= 1 + \frac{dy}{dx}\frac{d}{dy}(2y^4)$$

$$= 1 + 8y^3\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(1 + 8y^3\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(1) + \frac{d}{dx}\left(8y^3\frac{dy}{dx}\right)$$

$$= 0 + 8y^3\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(8y^3)$$

$$= 8y^3\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\frac{dy}{dx}\frac{d}{dy}(8y^3)$$

$$= 8y^3\frac{d^2y}{dx^2} + 24y^2\left(\frac{dy}{dx}\right)^2$$

i) $\sin x \cos y = \cos x + \sin y$

$$\frac{d}{dx}(\sin x \cos y) = \frac{d}{dx}(\cos x + \sin y)$$

$$\sin x \frac{d}{dx}(\cos y) + \cos y \frac{d}{dx}(\sin x)$$

$$= \frac{d}{dx}(\cos x) + \frac{d}{dx}(\sin y)$$

$$\sin x \frac{dy}{dx}\frac{d}{dy}(\cos y) + \cos y \cos x$$

$$= -\sin x + \frac{dy}{dx}\frac{d}{dy}(\sin y)$$

$$-\sin x \sin y \frac{dy}{dx} + \cos x \cos y = -\sin x + \cos y \frac{dy}{dx}$$

$$\frac{d}{dx}\left(-\sin x \sin y \frac{dy}{dx} + \cos x \cos y\right)$$

$$= \frac{d}{dx}\left(-\sin x + \cos y \frac{dy}{dx}\right)$$

$$-\sin x \sin y \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(-\sin x \sin y)$$

$$+ \frac{d}{dx}(\cos x \cos y) = \frac{d}{dx}(-\sin x) + \frac{d}{dx}\left(\cos y \frac{dy}{dx}\right)$$

$$-\sin x \sin y \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(-\sin x \frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}(-\sin x)\right)$$

$$+ \cos x \frac{d}{dx}(\cos y) + \cos y \frac{d}{dx}(\cos x)$$

$$= -\cos x + \cos y \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(\cos y)$$

$$-\sin x \sin y \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(-\sin x \frac{dy}{dx}\frac{d}{dy}(\sin y) - \sin y \cos x\right)$$

$$+ \cos x \frac{dy}{dx}\frac{d}{dy}(\cos y) + \cos y\,(-\sin x)$$

$$= -\cos x + \cos y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\frac{dy}{dx}\frac{d}{dy}(\cos y)$$

$$-\sin x \sin y \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(-\sin x \cos y \frac{dy}{dx} - \sin y \cos x\right)$$

$$-\sin y \cos x \frac{dy}{dx} - \sin x \cos y$$

$$= -\cos x + \cos y \frac{d^2y}{dx^2} - \sin y \left(\frac{dy}{dx}\right)^2$$

$$-\sin x \sin y \frac{d^2y}{dx^2} - \sin x \cos y \left(\frac{dy}{dx}\right)^2 - \sin y \cos x \frac{dy}{dx}$$

$$-\sin y \cos x \frac{dy}{dx} - \sin x \cos y$$

$$= -\cos x + \cos y \frac{d^2y}{dx^2} - \sin y \left(\frac{dy}{dx}\right)^2$$

$$-\sin x \sin y \frac{d^2y}{dx^2} - \sin x \cos y \left(\frac{dy}{dx}\right)^2 - 2\sin y \cos x \frac{dy}{dx}$$

$$-\sin x \cos y = -\cos x + \cos y \frac{d^2y}{dx^2} - \sin y \left(\frac{dy}{dx}\right)^2$$

j) $\sin(\ln x) = \ln(\cos y)$

$$\frac{d}{dx}(\sin(\ln x)) = \frac{d}{dx}(\ln \cos y)$$

$$\cos(\ln x)\frac{d}{dx}(\ln x) = \frac{\frac{d}{dx}(\cos y)}{\cos y}$$

$$\frac{\cos(\ln x)}{x} = \frac{\frac{dy}{dx}\frac{d}{dy}(\cos y)}{\cos y}$$

$$= -\frac{\sin y}{\cos y}\frac{dy}{dx}$$

$$= -\tan y \frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{\cos(\ln x)}{x}\right) = \frac{d}{dx}\left(-\tan y \frac{dy}{dx}\right)$$

$$\frac{x\frac{d}{dx}\cos(\ln x) - \cos(\ln x)\frac{d}{dx}(x)}{x^2}$$

$$= -\tan y \frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}\frac{d}{dx}(\tan y)$$

$$\frac{-x\sin(\ln x)\frac{d}{dx}(\ln x) - \cos(\ln x)}{x^2}$$

$$= -\tan y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)\frac{dy}{dx}\frac{d}{dy}(\tan y)$$

$$\frac{-\sin(\ln x) - \cos(\ln x)}{x^2} = -\tan y \frac{d^2y}{dx^2} - \sec^2 y \left(\frac{dy}{dx}\right)^2$$

$$\frac{\sin(\ln x) + \cos(\ln x)}{x^2} = \tan y \frac{d^2y}{dx^2} + \sec^2 y \left(\frac{dy}{dx}\right)^2$$

2. a) $x = t^3 \qquad y = 2t$

$$\frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 2$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2}{3t^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{2}{3t^2}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{2}{3t^2}\right)$$

$$= \frac{1}{3t^2}\left(\frac{-4}{3t^3}\right)$$

$$= \frac{-4}{9t^5}$$

b) $x = 2\cos t \qquad y = 3\sin t$

$$\frac{dx}{dt} = -2\sin t \qquad \frac{dy}{dt} = 3\cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{3\cos t}{-2\sin t}$$

$$= -\frac{3}{2}\cot t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{-3\cot t}{2}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{-3\cot t}{2}\right)$$

$$= \frac{-1}{2\sin t}\left(\frac{3\csc^2 t}{2}\right)$$

$$= \frac{-3}{4\sin^3 t}$$

c) $x = t^3 + t$, $y = t^2 - t$

$$\frac{dx}{dt} = 3t^2 + 1 \qquad \frac{dy}{dt} = 2t - 1$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2t-1}{3t^2+1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{2t-1}{3t^2+1}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{2t-1}{3t^2+1}\right)$$

$$= \frac{1}{3t^2+1}\left(\frac{(3t^2+1)2 - (2t-1)6t}{(3t^2+1)^2}\right)$$

$$= \frac{6t^2+2 - 12t^2 + 6t}{(3t^2+1)^3}$$

$$= \frac{-6t^2+6t+2}{(3t^2+1)^3}$$

d) $x = 1 - \frac{1}{\sin t}$ $y = 1 + \frac{1}{\cos t}$

$\frac{dx}{dt} = \csc t \cot t$ $\frac{dy}{dt} = \sec t \tan t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\sec t \tan t}{\csc t \cot t}$$

$$= \frac{\sin^3 t}{\cos^3 t}$$

$$= \tan^3 t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(\tan^3 t)$$

$$= \frac{dt}{dx}\frac{d}{dt}(\tan^3 t)$$

$$= \sin t \tan t\,(3\tan^2 t \sec^2 t)$$

$$= 3\tan^4 t \sec t$$

e) $x = \cos^2 t \quad y = \sin^3 t$

$$\frac{dx}{dt} = -2\cos t \sin t \quad \frac{dy}{dt} = 3\sin^2 t \cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{3\sin^2 t \cos t}{-2\cos t \sin t}$$

$$= -\frac{3\sin t}{2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{-3}{2}\sin t\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{-3\sin t}{2}\right)$$

$$= \frac{1}{-2\sin t \cos t}\left(\frac{-3\sin t}{2}\right)$$

$$= \frac{3}{4\cos t}$$

f) $x = \cos^3 t \qquad y = \sin^3 t$

$$\frac{dx}{dt} = -3\cos^2 t \sin t \qquad \frac{dy}{dt} = 3\sin^2 t \cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{3\sin^2 t \cos t}{-3\cos^2 t \sin t}$$

$$= -\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(-\tan t)$$

$$= \frac{dt}{dx}\frac{d}{dt}(-\tan t)$$

$$= \frac{1}{-3\cos^2 t \sin t}(-\sec^2 t)$$

$$= \frac{1}{3\cos^4 t \sin t}$$

9) $x = t - \cos t \qquad y = \sin t$

$$\frac{dx}{dt} = 1 + \sin t \qquad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\cos t}{1 + \sin t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{\cos t}{1 + \sin t}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{\cos t}{1 + \sin t}\right)$$

$$= \frac{1}{1 + \sin t}\left(\frac{(1 + \sin t)(-\sin t) - \cos t(\cos t)}{(1 + \sin t)^2}\right)$$

$$= \frac{-\sin t - \sin^2 t - \cos^2 t}{(1 + \sin t)^3}$$

$$= \frac{-\sin t - 1}{(1 + \sin t)^3}$$

$$= \frac{-(1 + \sin t)}{(1 + \sin t)^3}$$

$$= \frac{-1}{(1 + \sin t)^2}$$

h) $x = t + e^{-t}$ $\qquad$ $y = 1 - e^{-t}$

$\frac{dx}{dt} = 1 - e^{-t}$ $\qquad$ $\frac{dy}{dt} = e^{-t}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{e^{-t}}{1 - e^{-t}}$$

$$= \frac{1}{e^{t} - 1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{e^{t} - 1}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{1}{e^{t} - 1}\right)$$

$$= \frac{e^{t}}{e^{t} - 1}\left(\frac{e^{t}}{(e^{t} - 1)^{2}}\right)$$

$$= \frac{e^{2t}}{(e^{t} - 1)^{3}}$$

i) $x = e^t \tan t$ $\qquad$ $y = e^t \cot t$

$\frac{dx}{dt} = e^t \tan t + e^t \sec^2 t$ $\qquad$ $\frac{dy}{dt} = e^t \cot t - e^t \csc^2 t$

$$\frac{dy}{dt} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{e^t \cot t - e^t \csc^2 t}{e^t \tan t + e^t \sec^2 t}$$

$$= \frac{\cot t - \csc^2 t}{\tan t + \sec^2 t}$$

$$= \frac{\frac{\cos t \sin t - 1}{\sin^2 t}}{\frac{\sin t \cos t + 1}{\cos^2 t}}$$

$$= \frac{\cos^2 t(\cos t \sin t - 1)}{\sin^2 t \cdot (\sin t \cos t + 1)}$$

$$= \frac{\cos^3 t \sin t - \cos^2 t}{\sin^3 t \cos t + \sin^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{\cot t - \csc^2 t}{\tan t + \sec^2 t}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{\cot t - \csc^2 t}{\tan t + \sec^2 t}\right)$$

$$= \frac{1}{e^t \tan t + e^t \sec^2 t} \frac{\left(\begin{array}{l}(\tan t + \sec^2 t)(-\csc^2 t + 2\csc^2 t \cot t \\ -(\cot t - \csc^2 t)(\sec^2 t + 2\sec^2 t \tan t)\end{array}\right)}{(\tan t + \sec^2 t)^2}$$

$$= \frac{\begin{array}{l}-\tan t \csc^2 t - \sec^2 t \csc^2 t \\ + 2\csc^2 t + 2\sec^2 t \csc^2 t \cot t \\ -(\cot t \sec^2 t - \sec^2 t \csc^2 t \\ \quad + 2\sec^2 t - 2\sec^2 t \csc^2 t \tan t)\end{array}}{e^t(\tan t + \sec^2 t)^3}$$

$$= \frac{\begin{array}{l}-\sec t \csc t + 2\csc^2 t - \sec t \csc t - 2\sec^2 t \\ + 2\sec^2 t \csc^2 t(\cot t + \tan t)\end{array}}{e^t(\tan t + \sec^2 t)^3}$$

$$= \frac{2\sec^3 t \csc^3 t - 2\sec t \csc t + 2\csc^2 t - 2\sec^2 t}{e^t(\tan t + \sec^2 t)^3}$$

j) $x = \ln\left(t + \frac{1}{t}\right) \qquad y = \ln\left(t - \frac{1}{t}\right)$

$$\frac{dx}{dt} = \frac{1 - \frac{1}{t^2}}{t + \frac{1}{t}} \qquad \frac{dy}{dt} = \frac{1 + \frac{1}{t^2}}{t - \frac{1}{t}}$$

$$= \frac{\frac{t^2-1}{t^2}}{\frac{t^2+1}{t}} \qquad = \frac{\frac{t^2+1}{t^2}}{\frac{t^2-1}{t}}$$

$$= \frac{t^2-1}{t(t^2+1)} \qquad = \frac{t^2+1}{t(t^2-1)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\frac{t^2+1}{t(t^2-1)}}{\frac{t^2-1}{t(t^2+1)}}$$

$$= \frac{(t^2+1)^2}{(t^2-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{(t^2+1)^2}{(t^2-1)^2}\right)$$

$$= \frac{dt}{dx}\frac{d}{dt}\left(\frac{(t^2+1)^2}{(t^2-1)^2}\right)$$

$$= \frac{t(t^2+1)}{t^2-1}\left(\frac{(t^2-1)^2 2(t^2+1)2t - (t^2+1)^2 2(t^2-1)2t}{(t^2-1)^4}\right)$$

$$= \frac{t(t^2+1)}{(t^2-1)^4}\left((t^2-1)2(t^2+1)2t - (t^2+1)^2 2(2t)\right)$$

$$= \frac{4t^2(t^2+1)^2\left(t^2-1-(t^2+1)\right)}{(t^2-1)^4}$$

$$= \frac{-8t^2(t^2+1)^2}{(t^2-1)^4}$$