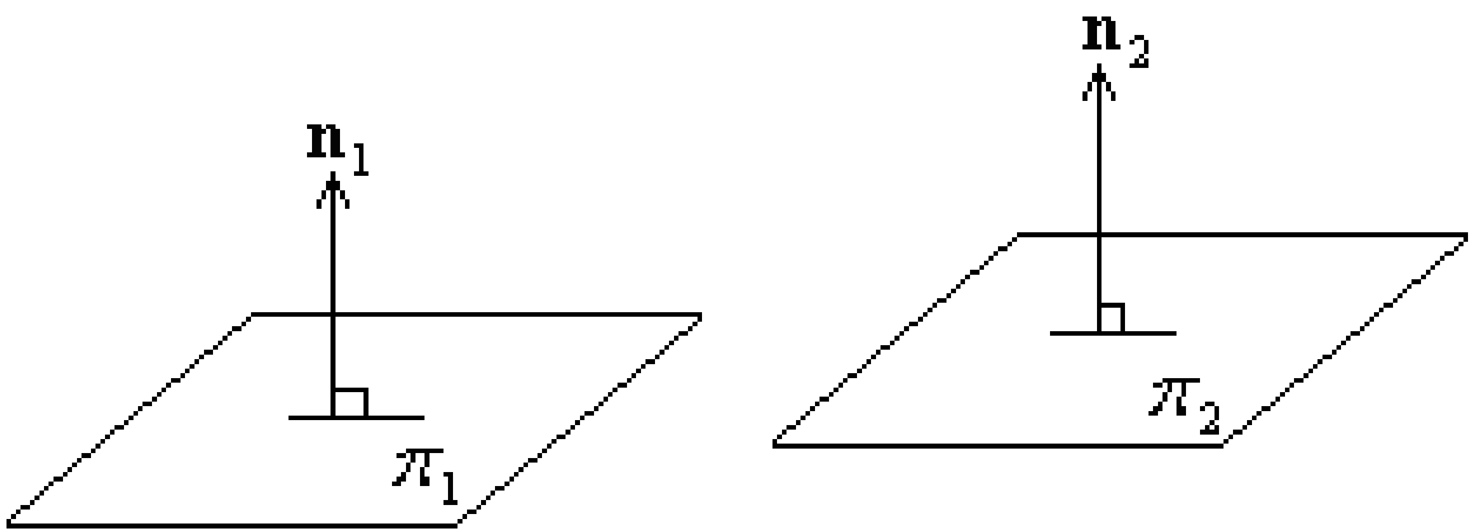
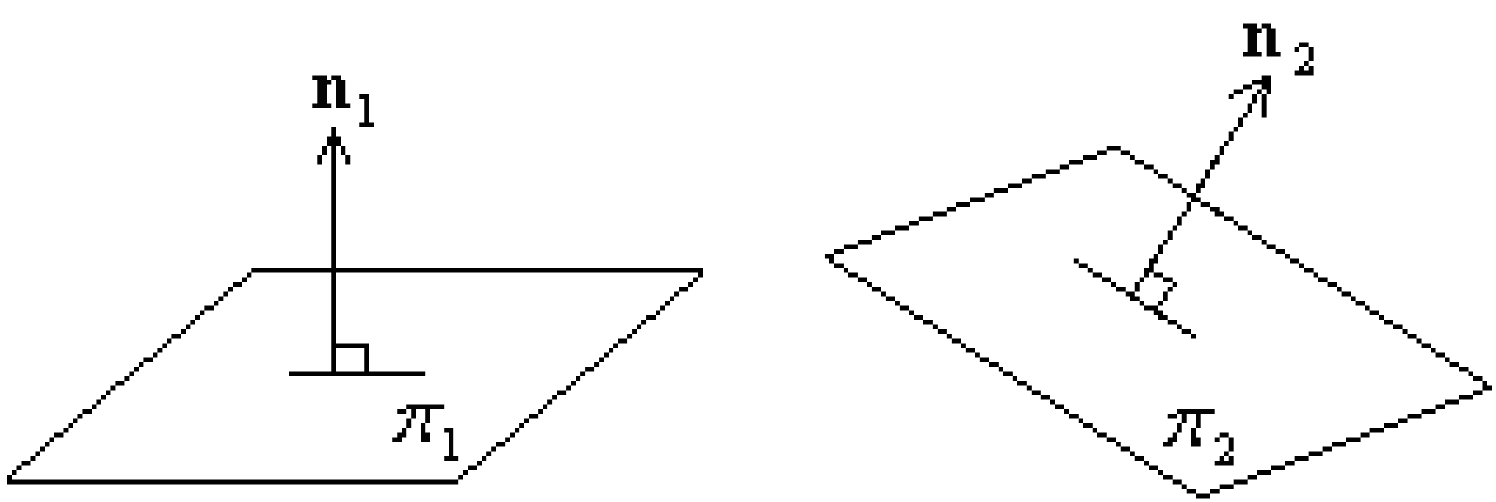
Parallel Planes

Two planes with normal equations $\pi_1 : \mathbf{r} \cdot \mathbf{n}_1 = p_1$ and $\pi_2 : \mathbf{r} \cdot \mathbf{n}_2 = p_2$ are said to be parallel iff $\mathbf{n}_1 = k\mathbf{n}_2$.



$$\mathbf{n}_1 = \mathbf{n}_2 \Leftrightarrow \pi_1 \text{ is parallel to } \pi_2$$



$$\mathbf{n}_1 \neq \mathbf{n}_2 \Leftrightarrow \pi_1 \text{ is not parallel to } \pi_2$$

Example: Determine whether the pair of the following planes are parallel.

$$\text{(i)}\ \pi_1 : \mathbf{r}\cdot(2\mathbf{i}-4\mathbf{j}+8\mathbf{k})=1 \qquad \text{(ii)}\ \pi_1 : \mathbf{r}\cdot(1\quad 3\quad 2)=0$$

$$\pi_2 : \mathbf{r}\cdot(\mathbf{i}-2\mathbf{j}+4\mathbf{k})=1 \qquad \pi_2 : \mathbf{r}\cdot(-1\quad -3\quad 2)=0$$

Example: Determine whether planes π_1 and π_2 are parallel.

(i) $\pi_1 : \mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \lambda_1(2\mathbf{i} + 3\mathbf{k}) + \mu_1(-4\mathbf{j} + 5\mathbf{k})$

$\pi_2 : \mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \lambda_2(3\mathbf{j} + \mathbf{k}) + \mu_2(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(ii) $\pi_1 : \mathbf{r} = s_1\mathbf{i} + (s_1 + t_1)\mathbf{j} + (1 - 2s_1 - t_1)\mathbf{k}$

$\pi_2 : (s_2 - 2t_2)\mathbf{i} + (-3 + 3s_2 + t_2)\mathbf{j} + (-4s_2 + t_2)\mathbf{k}$

Example:

Find the normal equation of the plane passing through the point $A(1,2,3)$ and parallel to the plane $\mathbf{r}\cdot(2\mathbf{i}-\mathbf{j}+\mathbf{k})=1.$

Example:

Find the Cartesian equation of the plane passing through the point $A(1,0,2)$ and parallel to the plane $\mathbf{r} = \lambda\mathbf{i} + (1+\mu)\mathbf{j} + (2\lambda + 3\mu)\mathbf{k}$.

Perpendicular Planes

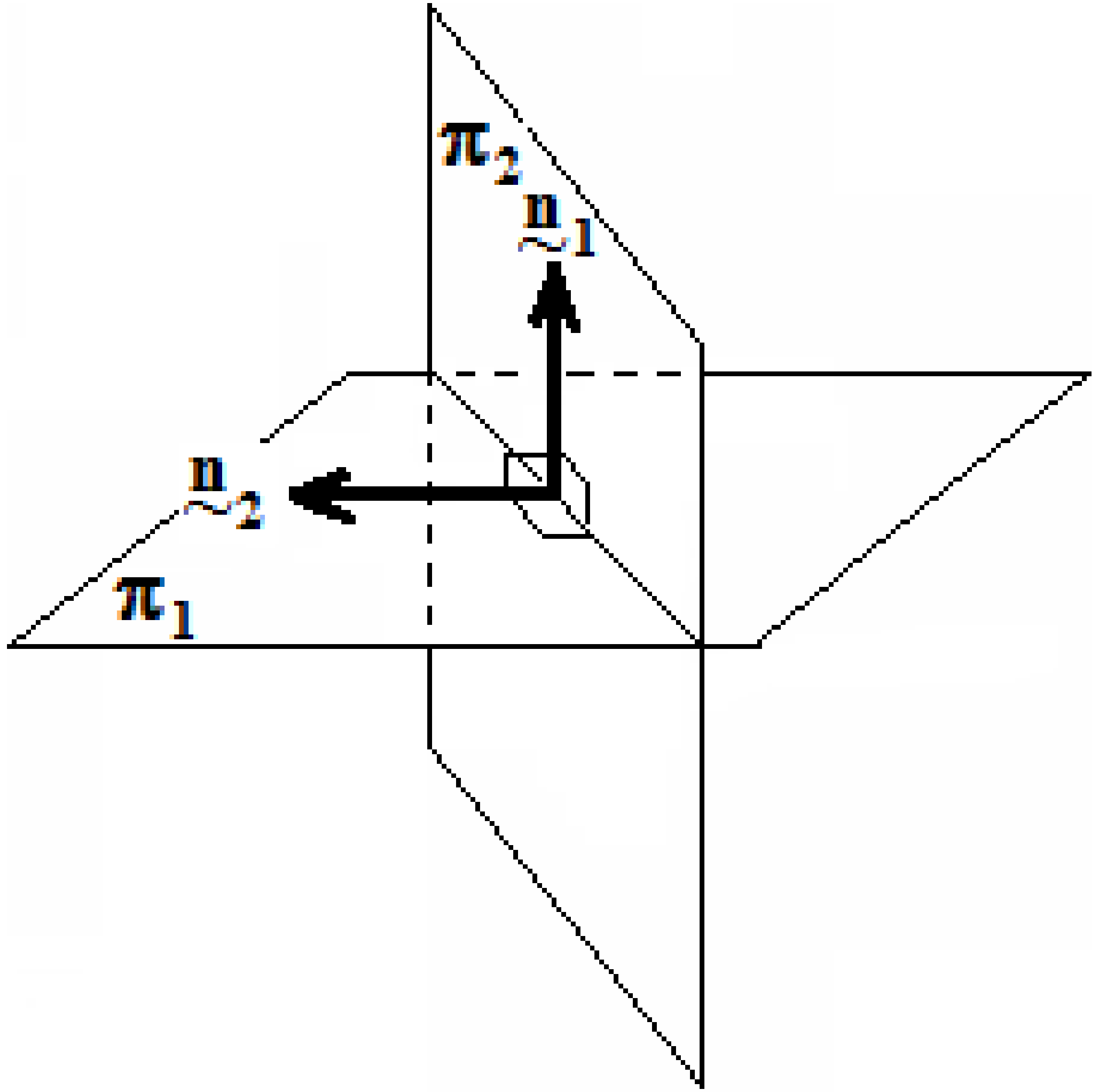
Two planes with normal equations

$\pi_1 : \mathbf{r} \cdot \mathbf{n}_1 = p_1$ and $\pi_2 : \mathbf{r} \cdot \mathbf{n}_2 = p_2$

are said to be perpendicular iff

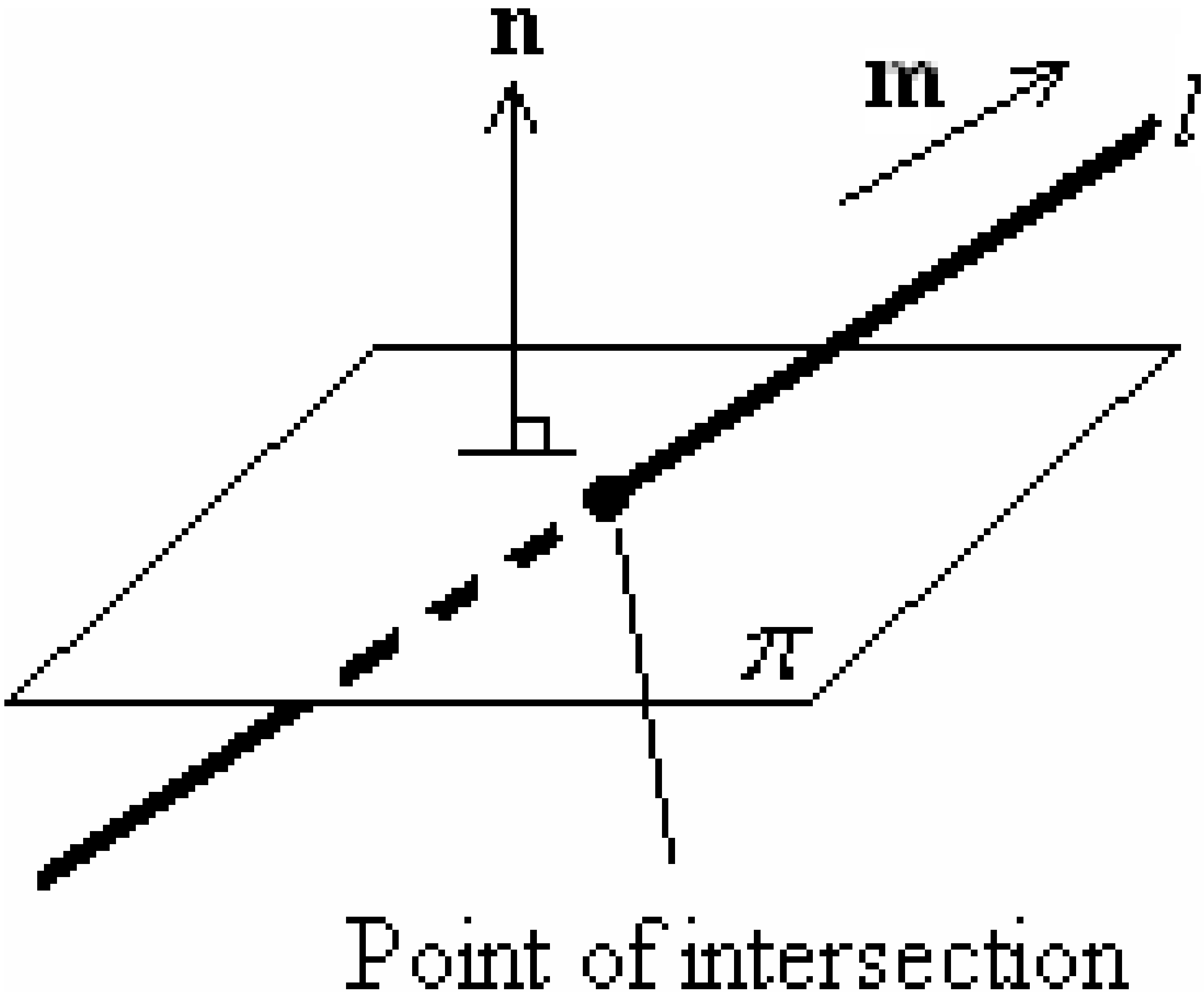
(a) $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$, OR

(b) $\mathbf{n}_1$ is contained in π_2 AND $\mathbf{n}_2$ is contained in π_1


π_2
$\underset{\sim}{n}_1$
$\underset{\sim}{n}_2$
π_1

Intersection of a Line and a Plane

A plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$ and a line l : $\mathbf{r} = \mathbf{a} + \lambda \mathbf{m}$ intersect at a point if there exist ONE UNIQUE value of λ such that $\left(\mathbf{a} + \lambda \mathbf{m}\right) \cdot \mathbf{n} = p$.


n
m
π
Point of intersection

Example:

Find the position vector of the point of intersection between the line $\mathbf{r} = \mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and the plane $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} - 3\mathbf{k}) = 8$.

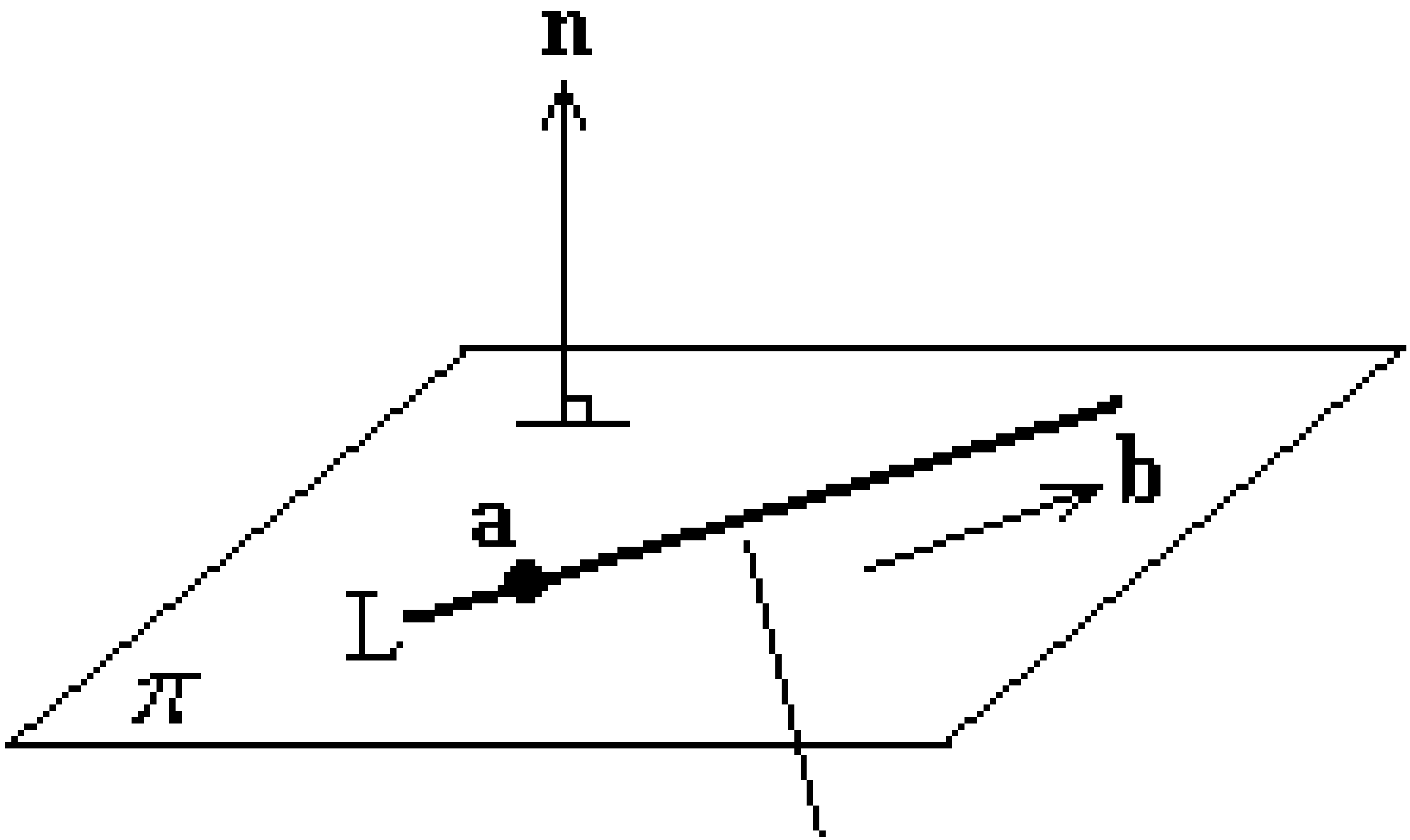
Example:

Find the position vector of the point of intersection between the line $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$ and the plane $\mathbf{r} = \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mu(3\mathbf{j} + \mathbf{k})$.

Example:

Find the coordinates of the point of intersection of the line $\dfrac{x-1}{2} = y = \dfrac{z-1}{-3}$ and the plane $3x + 2y + 4z = 11$.

The line $l : \mathbf{r} = \mathbf{a} + \lambda\mathbf{m}$ is said to LIE IN the plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$ (OR the intersection between l and π is l) if there exists INFINITE values of λ such that $(\mathbf{a} + \lambda\mathbf{m}) \cdot \mathbf{n} = p$.

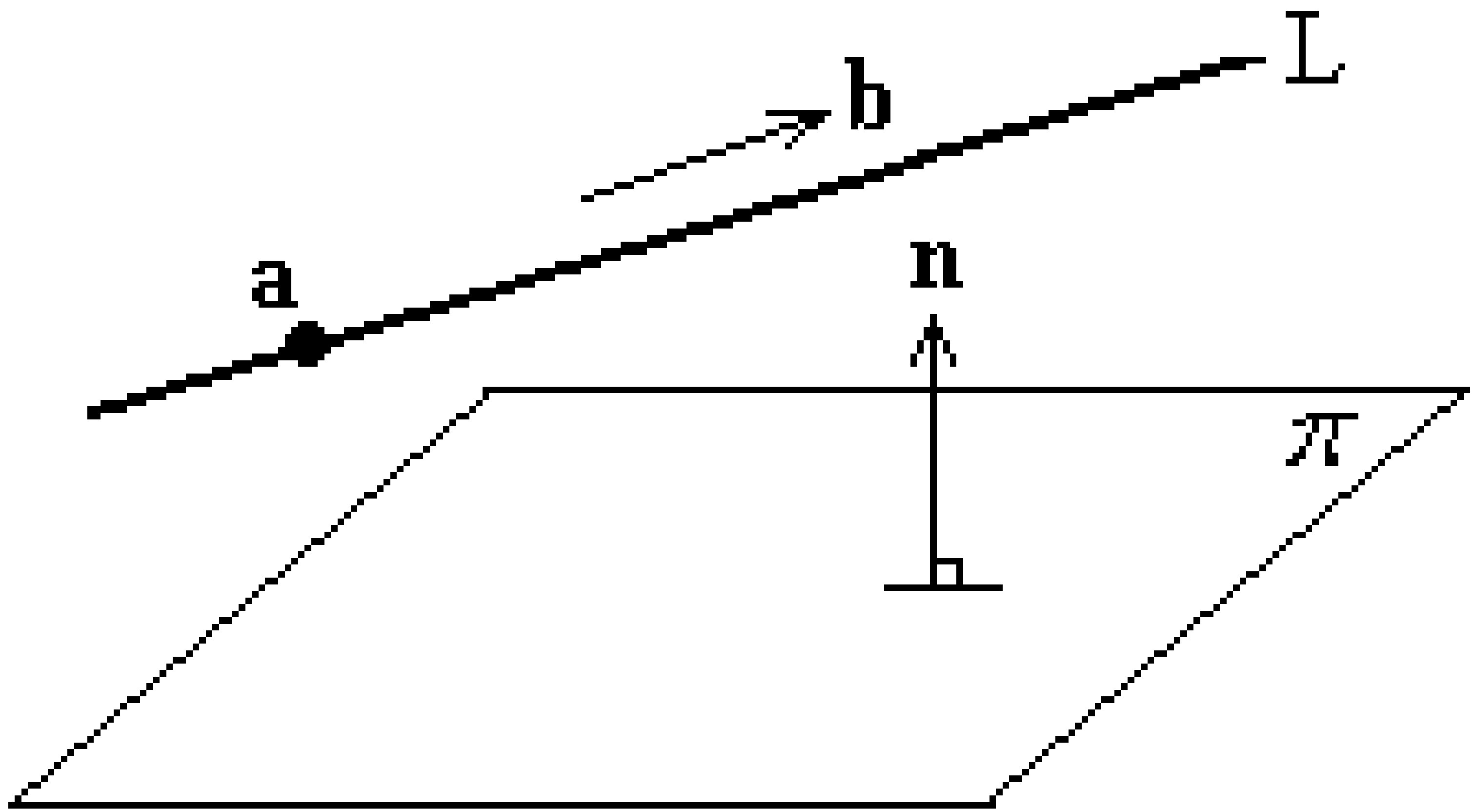


All the points in L are in π

Example:

Find the coordinates of the point of intersection of the line $\dfrac{x-1}{4}=\dfrac{y-2}{3}=\dfrac{z+1}{5}$ and the plane $2x-y-z=1$.

The plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$ and a line l : $\mathbf{r} = \mathbf{a} + \lambda \mathbf{m}$ DO NOT intersect if there exists NO unique value of λ such that $\left(\mathbf{a} + \lambda \mathbf{m}\right) \cdot \mathbf{n} = p$.



L and π do not intersect.

Example:

Determine whether the line $\mathbf{r} = (3+\lambda)\mathbf{i} + (2+2\lambda) - 3\mathbf{k}$ and the plane $\mathbf{r} = (1+t)\mathbf{i} + s\mathbf{j} + (1-s+2t)\mathbf{k}$ intersect.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the Cartesian equation of the plane passing through the point $A(4,1,-2)$ and parallel to the plane $\mathbf{r}\cdot(3\mathbf{i}+4\mathbf{j}-2\mathbf{k})=0$

Example:

Let plane $\pi_1 : \mathbf{r} = 3\mathbf{k} + \lambda_1(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu_1(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and plane $\pi_2 : \mathbf{r} = \mathbf{j} + 3\mathbf{k} + \lambda_2(-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu_2(\mathbf{j} + \mathbf{k})$. Verify that π_1 is parallel π_2. Find the perpendicular distance from the origin to each plane, and hence, find the perpendicular distance between the planes.

Example:

Find the position vector of the point of intersection between the line $\mathbf{r} = \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ and the plane $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - \mathbf{k}) = 7$.

Example:

Find the coordinates of the point of intersection of the line $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{-2}$ and the plane $2x+3y+z=0$.

Example:

Show that the line $\mathbf{r} = \begin{pmatrix} 1+4\lambda & 2+3\lambda & -1+5\lambda \end{pmatrix}$ lies in the plane $\mathbf{r} \cdot \begin{pmatrix} 4 & -3 & 2 \end{pmatrix} = 3$.

Example:

Show that the line $\mathbf{r} = 2\mathbf{i} - 6\mathbf{j} + \lambda(\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$ and the plane $\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$ do not intersect.