

# Topic : **P101 Rational Functions**

Subtopic:

- *Proper and Improper Rational Functions*
- *Partial Fractions*

# Rational Functions

Definition :

A rational function is defined by expression in the form  $\frac{P(x)}{Q(x)}$ , where  $P(x)$  and  $Q(x)$  are polynomials.

Examples of rational functions :

$$(a) \frac{1}{2x+1}$$

$$(b) \frac{x^2 + x + 1}{x - 3}$$

$$(c) \frac{2x^2 - x + 1}{x^2 - 2x - 3}$$

Example : Determine whether each of the following is a rational function.

$$(a) \frac{x-1}{4} \quad (b) \frac{2-\sqrt{x}}{4-x} \quad (c) \frac{x^{-3} + x^{-2} + x^{-1} + 1}{1+x}$$

# Proper and Improper Rational Functions

Definition :

A rational function  $\frac{P(x)}{Q(x)}$  is said to be

(a) proper if  $\deg P(x) < \deg Q(x)$ .

(b) improper if  $\deg P(x) \geq \deg Q(x)$ .

Example : Determine whether each of the following rational function is proper.

$$(a) \frac{8x + 1}{x^2 + x - 2}$$

$$(b) \frac{x}{(x - 1)(x - 3)}$$

$$(c) \frac{x^3}{x^2 - 1}$$

$$(d) \frac{x^2 - 3x + 5}{(x + 1)(x - 2)}$$

Long division is carried out to reduce the improper fraction, i.e.

$$\frac{a(x)}{b(x)} = \text{Quotient} + \frac{\text{Remainder}}{b(x)}.$$

Example : Obtain the proper form of the following rational functions.

$$(a) \frac{3x}{x-3}$$

$$(b) \frac{2x^2 + 4x - 3}{x^2 - x - 3}$$

Example:

Express  $\frac{3x^3 - 4x + 5}{x^2 + 1}$  in the proper form.



Example:

Express  $\frac{2x^2 - 5x - 1}{x - 3}$  in the proper form.