TAYLOR'S COLLEGE

Wisdom - Integrity - Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2004

(Jan 2003 & July 2002 [2-Year] Intakes)

Thursday 25 March 2004 8.30 am – 11.30 am

FURTHER MATHEMATICS 9231/01

PAPER 1 3 hours

Additional materials: Answer Booklet/Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Follow the instructions on the front cover of the Answer Booklet.
Write your name, class and student number on all your answer papers.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of a calculator is expected, where appropriate.
Results obtained solely from graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of 5 printed pages.

1. Verify that

$$\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)} = \frac{3}{n(n+1)(n+2)(n+3)}. \qquad [1]$$

Hence find

$$\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)(n+3)}$$

in term of N. [3]

2. Prove by induction that

$$\sum_{n=1}^{N} \frac{(n+1)(n+4)}{(n+2)(n+3)} = \frac{(N+1)(N+2)}{N+3} - \frac{2}{3}$$

for every positive integer N. [7]

3. Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{C}$, where

$$\mathbf{C} = \begin{pmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 2 & 1 & 2 \end{pmatrix}. \qquad [8]$$

4. It is given that $I_n = \int \sec^n x \, dx$.

By writing $\sec^n x$ as $\sec^{n-2} x \sec^2 x$, prove that

$$(n-1)\, I_n = \sec^{n-2} x \tan x + (n-2)\, I_{n-2}. \qquad [6]$$

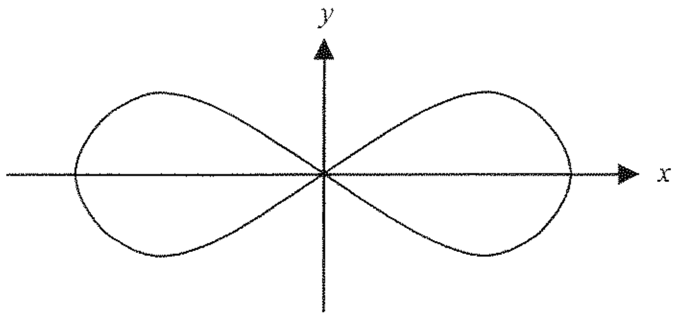
Hence, find $\int \sec^4 x \, dx$. [2]

5. A curve is defined parametrically by $x = \frac{8}{3} t^{\frac{3}{2}}$, $y = t^2 - 2t + 4$ where t is a parameter.

The points A and B on the curve are defined by $t = 0$ and $t = 1$ respectively.

(a) Find the length of the arc AB. [5]

(b) Show that the area of the surface generated by one complete revolution of the arc AB about the y-axis is $\frac{256}{35}\pi$. [4]



6. The diagram shows the curve whose equation, in Cartesian coordinates, is

$$\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right),$$

where a is a positive constant. Show that the equation may be expressed, in polar coordinates, in the form

$$r^2=a^2\cos 2\theta .$$ [3]

Explain how you can deduce, from the polar form of the equation, that the line $y=x$ is tangential to the curve at the pole. [2]

Find the area enclosed by one loop of the curve. [4]

7. By writing the equation $(z+1)^6=z^6$ in the form $\left(\frac{z+1}{z}\right)^6=1$,

show that the value of z satisfying the equation are given by

$$z=\frac{1}{e^{\frac{1}{3}k\pi \mathrm{i}}-1},$$

where k is an integer, and state a set of values of k that gives all the roots of the equation. [5]

Express each root of $(z+1)^6=z^6$ in the form $x+\mathrm{i}\,y$, where x and y are real. [5]

8. (i) Find the value of the constants a and b for which $at + be^{-t}$ is a particular integral for the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 1 - e^{-t}$. [3]

 (ii) Find the solution of the differential equation, given that x and $\frac{dx}{dt}$ are both zero when $t = 0$. [5]

 (iii) Show that, for the solution in part (ii), when t is large and positive, $\frac{dx}{dt} \approx \frac{1}{2}$. [2]

9. The curve C has equation $y = \frac{5(x-1)(x+2)}{(x-2)(x+3)}$.

 (i) Express y is the form $S + \frac{T}{x-2} + \frac{U}{x+3}$. [3]

 (ii) Show that $\frac{dy}{dx} = 0$ for exactly one value of x and find the corresponding value of y. [3]

 (iii) Write down the equations of all the asymptotes of C. [2]

 (iv) Find the set of values of k for which the line $y = k$ does not intersect C. [3]

10. The equation of a plane p and a line l are

$$\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ a+1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

respectively, where a is a constant and λ, μ, t are parameters.

 (i) Write down a set of these simultaneous equations satisfied by λ, μ and t if l and p have a point in common. [1]

 (ii) By considering an appropriate determinant, or otherwise, show that l and p have a unique point of intersection for all value of a except $a = 3$. [3]

 (iii) For the case $a = 0$, verify that l is perpendicular to p, and find the position vector of the point of intersection of l and p. [5]

 (iv) For the case $a = 3$, state the geometrical relationship between l and p, justifying your answer. [3]

11. **EITHER**

The linear transformation $\mathrm{T} : \Re^4 \to \Re^4$ is represented by the matrix $\mathbf{A}$,

where $\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 & 3 \\ 2 & -1 & -1 & 11 \\ 3 & -2 & -3 & 14 \\ 4 & -3 & -5 & 17 \end{pmatrix}$.

Find the rank of $\mathbf{A}$ and a basis for the null space of T. [7]

The vector $\begin{pmatrix} 1 \\ -2 \\ -1 \\ -1 \end{pmatrix}$ is denoted by $\mathbf{e}$. Show that there is a solution of the equation

$\mathbf{Ax} = \mathbf{Ae}$ of the form $\mathbf{x} = \begin{pmatrix} p \\ q \\ 1 \\ 1 \end{pmatrix}$, where p and q are to be found. [5]

OR

(i) The equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has roots α, β, γ and δ such that $\alpha\beta = 1$ and $\gamma\delta = 1$.

(a) Show that $d = 1$ and that $a = c$. [3]

(b) Show that the equation with roots $\alpha + \beta$ and $\gamma + \delta$ is $x^2 + ax + b - 2 = 0$. [3]

(ii) By using the substitution $y = \frac{1}{x}$, or otherwise, show that any equation of the form $x^4 + px^3 + qx^2 + px + 1 = 0$ has roots which occur in reciprocal pairs. [2]

(iii) Solve the equation $x^4 + 2x^3 - x^2 + 2x + 1 = 0$. [4]