Curve Sketching in General

Prescription for sketching curve in general:

(a) Obtain all asymptotes i.e. $x \to a^-, x \to a^+, x \to -\infty$ and $x \to +\infty$.

(b) Obtain all stationary points i.e. $\dfrac{dy}{dx} = 0$.

(c) Determine the increasing and decreasing portions, i.e. $\dfrac{dy}{dx} > 0$ and $\dfrac{dy}{dx} < 0$.

(d) Obtain all axial intercepts i.e. $x = 0$ and $y = 0$.

Example:

Sketch the curve $y = \dfrac{1-x}{(x+1)^2}$.

Example:

Sketch the curve $y = \dfrac{x}{(x+2)(x+8)}$.

Example:

Sketch the curve $y = 1 - \dfrac{2x}{(x+2)^2}$.

Example:

Sketch the curve $y = x + \dfrac{1}{x-2}$.

Example:

Sketch the curve $y = \dfrac{x-2}{x-1}$.

Example:

Sketch the curve $y = \dfrac{x^2}{x^2 - 4}$.

Example:

Sketch the curve $y = \dfrac{x^2 - 3x + 16}{x - 3}$.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Let $f(x)=\dfrac{1}{x(x-2)^2}$.

(a) Find the coordinates of the turning points of $y=f(x)$.

(b) State the equation of the asymptote(s) of $y=f(x)$

(c) Sketch $y=f(x)$.

(d) Hence, or otherwise, find the set of values of k where the equation $kx(x-2)^2=1$ has one real root.

Example:

Let $f(x)=\dfrac{2x^2+1}{x^2+1}$.

(a) Find the equation of the asymptotes of $y=f(x)$.

(b) Find the coordinates of the turning point, and determine the nature of each.

(c) Sketch the graph of $y=f(x)$.

(d) Hence, find the set of values of k such that the equation $2x^2+1=k(x^2+1)$ has real solution.

Example:

The curve with equation $y = \dfrac{ax+b}{x(x+2)}$, where a and b are constants, has zero gradient at $(1, -2)$.

(a) Show that $a = -8$, and find the value of b.

(b) Show that there is a second point whose gradient is also zero.

(c) State the equation of all asymptotes of the curve.

(d) Sketch the curve.

(e) Determine the set of value of y for which no part of this curve exists.

Example:

Given that a curve is given by equation $y = \frac{x-2}{x-1}$.

(a) Find the coordinates of the points where the curve crosses x and y – axes.

(b) Determine whether the curve has any stationary point.

(c) Find the equation of the asymptotes of the curve.

(d) Sketch this curve.

Example :

A curve has equation $y = \dfrac{x^2}{x^2 - 5x + 4}$. Find the equations of the asymptotes and the coordinates of the stationary points. Sketch this curve and determine the number of real roots for the equation $a(x-1)^2(x-4) = x^2$ where $a > 0$.

Example:

A curve is given by $f(x)=\dfrac{2-x}{(x-1)^2}, x \neq 1$. Find the asymptotes parallel to the coordinates axes, and determine the coordinates and nature of the turning point, if any. Sketch, in separate diagrams, the curves $y=f(x)$ and $y=|f(x)|$. State the number of roots of the equation $|f(x)|=kx-k$ where $k<0$.

Example:

Given that $f(x)=1-\dfrac{2x}{(x+2)^2}$. State the values of x such that $f(x)$ is defined, and states the equations of all asymptotes. Find the coordinates and the nature of the stationary point. Sketch the graph $y=f(x)$, and use your graph to determine

(a) The number of real roots of the equation $\dfrac{2x}{(x+2)^2}=-1$.

(b) The solution set of x if $f(x)\geq 1$.