TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2010

(January and March 2009 Intakes)

Thursday **1 April 2010** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/13

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

1 Using integration prove that the curved surface area of a cylinder of radius r and height h is $2\pi rh$. [3]

2 Find the perpendicular distance between the planes $3x+8y+5z=4$ and

$3x+8y+5z=6$. [5]

3 If $\left(\dfrac{3iz+5}{2iz+7}\right)^4=1$ find z. [6]

4 Given that $a_r=\dfrac{e-1}{e^r}$ and that $b_r=\dfrac{-1}{r^2+r}$.

(i) Show that $\sum_{r=1}^{n} a_r = 1-e^{-n}$, in terms of n. [2]

(ii) Show that $\sum_{r=1}^{n} b_r = \dfrac{1}{n+1}-1$, in terms of n. [2]

(iii) Find S_∞ given that $S_n=\left(a_1+b_1\right)+\left(a_2+b_2\right)+\left(a_3+b_3\right)+\cdots+\left(a_n+b_n\right)$. [2]

5 Use induction to prove that the sum of the first $2n$ terms of the series

$1^2-3^2+5^2-7^2+\cdots$ is $-8n^2$. [7]

6 The real variables x and y are related by the equation $y=e^{y(x-1)}$. Show that $\dfrac{dy}{dx}=1$

and $\dfrac{d^2y}{dx^2}=3$, when $x=1$. [7]

7 If the equation $x^3+ax+b=0$ has roots α, β and γ, find the equation having roots

$\dfrac{\alpha\beta}{\alpha^2+\beta^2}$, $\dfrac{\alpha\gamma}{\alpha^2+\gamma^2}$ and $\dfrac{\beta\gamma}{\beta^2+\gamma^2}$. [8]

8 If $\mathbf{A}=\begin{pmatrix}3 & 4\\ 6 & 5\end{pmatrix}$, using the eigenvalues and eigenvectors of $\mathbf{A}$, find $\mathbf{A}^{10}$. [9]

9 Given that $y = f(x)$, show that, if the substitution $x = \sqrt[3]{t}$ is made, then

(i) $\frac{dy}{dx} = 3t^{\frac{2}{3}} \frac{dy}{dt}$, [2]

(ii) $\frac{d^2y}{dx^2} = 9t^{\frac{4}{3}} \frac{d^2y}{dt^2} + 6t^{\frac{1}{3}} \frac{dy}{dt}$. [3]

Hence find the general solution of the differential equation

$$\frac{1}{x}\frac{d^2y}{dx^2} - \frac{2}{x^2}\frac{dy}{dx} + 36yx^3 = 0.$$ [5]

10 If $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{pmatrix}$, find the rank and null space of $\mathbf{A}$. [12]

11 Given that

$$I_n = \int \operatorname{cosec}^n x \, dx ,$$

where n is a non-negative integer. Show that for $n \geq 2$,

$$(n-1)I_n = (n-2)I_{n-2} - \cot x \operatorname{cosec}^{n-2} x .$$ [4]

(i) Find I_1, I_2, I_3 and I_4. [4]

(ii) The curve C is defined parametrically by

$$x = 2t^2 , \qquad y = t^4 .$$

Show that the length, S, of the part of C joining the origin to the point where $t = 1$ is given by

$$S = \int_0^1 4t\sqrt{1+t^4}\, dt .$$ [1]

By using the substitution $t^2 = \cot x$ show that

$$S = \sqrt{2} + \ln\left(1+\sqrt{2}\right).$$ [4]

12 Answer only **one** of the following two alternatives.

EITHER

The curve C has equation $y = \dfrac{3(x-2)}{x^2+6x}$.

(i) Write down the equations of all asymptotes of C. [3]

(ii) Show that the curve does not exist for $\frac{1}{6} < y < \frac{3}{2}$. Hence find the coordinates of turning points and determine their nature. [8]

(iii) Sketch the curve C. [3]

OR

The curve C is given by the equation $\left(x^2+y^2\right)^5 = \left(x^4+y^4\right)^2$.

(i) Express C in polar form. [1]

(ii) Sketch C for $0 \le \theta < 2\pi$. [2]

(iii) Find the area enclosed by the curve for $0 \le \theta < 2\pi$. [3]

(iv) Find the coordinates of the points with the minimum distance from the pole. [8]