Lesson 2

Recall:-

Frequency = no. of cycles in 1 sec

Period = time taken for 1 cycle

⇨ $f = \frac{1}{T}$ -------(1)

ω = angular velocity , rad/s

⇨ Period, $T = \frac{2\pi}{\omega}$ -----------(2)

$\frac{d^2x}{dt^2} = -\omega^2 x$ ----------------------(3)

$v^2 = \omega^2 (a^2 - x^2)$ ---------------(4)

$v_{max} = \omega a$ ------------------------(5)

Example 1

A particle is describing SHM oscillations about x=2 with period 2π seconds, the unit of length being metre. When x=7 the speed of the particle is 12 ms^{-1}. Find the values of x at the extreme points of the oscillation, the speed of the particle as it passes through the origin.

Example 2

A particle is oscillating with SHM. When x=1, v=+7,-7 and when x=5, v=+2,-2, the units being mm and sec. Find the amplitude and the period

(a) If the centre of oscillation is O
(b) If the centre of oscillation is x=1.

Notes (cont'd)---------------------- *CIRCULAR REPRESENTATION OF SHM*

(3) Since $v = \frac{dx}{dt} = \omega \sqrt{a^2 - x^2}$,

$$\frac{1}{\sqrt{a^2-x^2}} dx = \omega \quad dt$$

$$\text{Sin}^{-1} \frac{x}{a} = \omega t + k$$

$$x = a \sin \omega t + k$$

(i) This constant of integration, k, will depend entirely on the point at which $t = 0$ and the value of x at that time. For example, <u>if the particle starts from 0</u>, i.e. when $t = 0$, $x = 0$, therefore $k = 0$ and

$$\boxed{x = a \sin \omega t.}$$

(ii) **<u>If the particle starts from the end</u>**, i.e. when $t = 0$, $x = a$, then $x = a \sin\left(\omega t + \frac{\pi}{2}\right)$

$$\Rightarrow \qquad \boxed{x = a \cos \omega t.}$$

(4) Generally, $x = a \sin(\omega t + k)$ where k depends upon the instant at which we begin measuring x.

By differentiating, this general value for x, i.e., $x = a \sin(\omega t + k)$,
$v = \dfrac{dx}{dt} = a\,\omega \cos(\omega t + k)$ and $a = \dfrac{dv}{dt} = -a\,\omega^2 \sin(\omega t + k) = -\omega^2 x$.

(5) If we substitute $t + \dfrac{2\pi}{\omega}$ for t in the equations for x and v, we will obtain identical results.

This shows that after successive intervals of time $\dfrac{2\pi}{\omega}$, the particle passes through the same position with the same velocity.

Hence, $\boxed{T = \dfrac{2\pi}{\omega}}$ is the period of oscillation.