TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

FIRST SEMESTER EXAMINATION JUNE 2005

(March 2005 Intake)

Wednesday **29 June 2005** **1.00 pm – 3.00 pm**

FURTHER MATHEMATICS 9231

2 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

You are required to hand in the answers for Section A and Section B separately.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be 10 ms^{-2}.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

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Section A: Pure Mathematics 50%

1. Use the relevant standard results in the List of Formulae to prove that

$$S_n = \sum_{k=1}^{n}(8k^3 - 6k^2) = n(n+1)(2n^2 - 1).$$

Hence find an expression for $\sum_{k=n+1}^{3n}(8k^3 - 6k^2)$ [20]

2. The roots of the equation $x^3 - x - 1 = 0$ are α, β, γ, and $S_n = \alpha^n + \beta^n + \gamma^n$.

i) Use the relation $y = x^2$ to show that $\alpha^2, \beta^2, \gamma^2$, are the roots of the equation $y^3 - 2y^2 + y - 1 = 0$. [4]

ii) Hence, or otherwise, find the value of S_4. [1]

iii) Find the values of $S_8 + S_{12}$ and $S_{16} - S_4$. [5]

3. The curve C has equation

$$y = \frac{(x-a)(x-b)}{x-c},$$ where a, b, c are constants, and it is given that $0 < a < b < c$.

i) Express y in the form

$x + P + \dfrac{Q}{x-c}$, giving the constants P and Q in terms of a, b, and c. [3]

ii) Find the equations of the asymptotes of C. [2]

iii) Determine the number of critical points of C. [5]

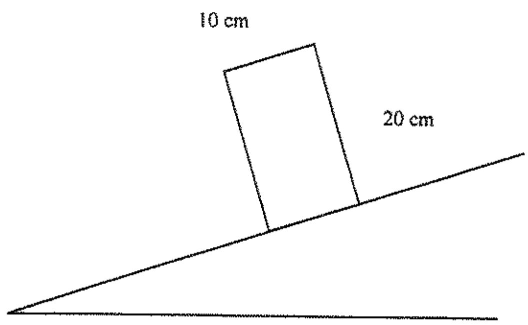
iv) Given also that $a + b > c$, sketch C, showing the asymptotes and the coordinates of the points of intersection of C with the axes. [10]

Students are required to answer questions in Section B using a fresh sheet of answer paper. Answers for Section A and Section B need to be handed in separately.

Section B: Applied Mathematics 50%

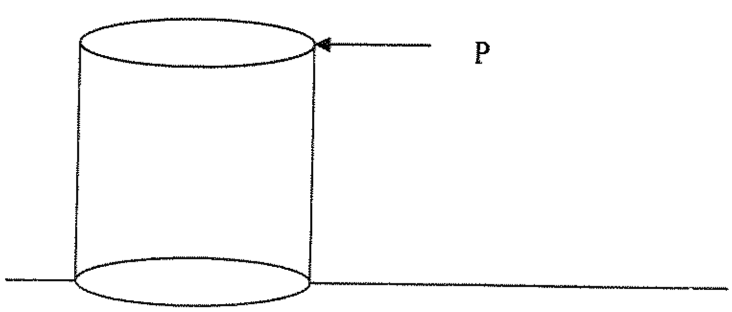
1. The diagram below shows the cross section of a uniform rectangular block. This cross-section has dimensions 20 cm by 10 cm and lies in a vertical plane. The block rests in equilibrium on a rough plane whose inclination α to the horizontal can be varied. The coefficient of friction between the block and the plane is

0.7.Given that α is slowly increased from zero, determine whether equilibrium is broken by toppling or by sliding. [6]



2. A can in the form of a circular cylinder, without a lid, is made of thin metal sheeting of uniform thickness and with a mass per unit area of 1 kg/m^2. The radius of the can is 0.1 m and its height is 0.2 m. The can is placed with its base on a horizontal plane and half-filled with a liquid of density 1.5 kg/m^3. Calculate the height of the centre of gravity of the can together with the liquid, above the base of the can. [8]

A horizontal force P is applied at the top outer edge of the can as shown.



Find the magnitude of the least force P for the can to topple. [4]

3. At time $t = 0$, a particle is projected from a point O with speed u at an angle of elevation α. At time t, the horizontal and vertical distances of the particle from O

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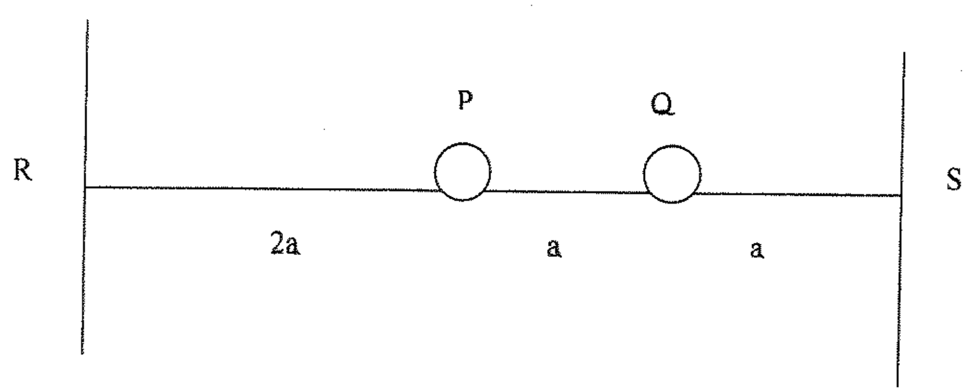
are x and y respectively. Express x and y in terms of u, α, t and g. Hence, show that $y = x\tan\alpha - \dfrac{gx^2(1+\tan^2\alpha)}{2u^2}$. [6]

A golf ball is struck from a point A, leaving A with speed 30 m/s at an angle of elevation θ and lands, without bouncing, in a bunker at point B, which is at the same horizontal level as A. Before landing in the bunker, the ball just clears the top of a tree which is at a horizontal distance of 72 m from A, the top of the tree being 9 m above the level of AB. Show that one of the possible values of θ is $\tan^{-1}\frac{3}{4}$ and find the other values. Given that θ was in fact $\tan^{-1}\frac{3}{4}$, find the distance AB. [10]

4. Two particles P and Q of equal mass m are positioned, as shown in the diagram, on a smooth horizontal surface between two parallel walls, the line joining the particles being perpendicular to the walls.



The coefficient of restitution between the particles is 1/3 and between the particles and the walls is ½.

If P is projected towards R with speed 2u and Q is projected towards S with speed u, show that the speed of P after impact at R is u m/s. Find the speed of Q after impact at S. [4]

Hence,

(a) the time taken until P and Q collide with each other; [8]

(b) the speed and direction of P and Q after they collide. [4]