The Compound Pendulum

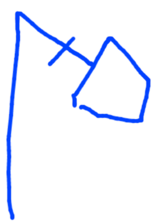
Example 1

A uniform rod of mass 10 kg and length 2.1m can swing freely in a vertical plane round a horizontal pin at one end. Form the equation of rotational motion for the oscillation about the vertical equilibrium position. If the rod swings through a small angle, find the approximate period of oscillation.

$$MI = \frac{1}{3}(10)(1.05)^2 + (10)(1.05)^2$$
$$= 14.7$$
$$\tau = I\alpha$$
$$-10g(1.05\sin\phi) = 14.7\ddot{\phi}$$
$$\ddot{\phi} = -7.143\phi$$
$$\ddot{\phi} = -\omega^2\phi$$
$$T = \frac{2\pi}{\omega} = 2.35s$$

Example 2

A pendulum is made of a strip of wood with a square board screwed to it at one end. It is hung over a nail by a small eye screwed into it at the other end. The wooden strip is 1.2 m long and has a mass of 0.5 kg. The board is 0.4m square and has a mass of 1.5 kg. Find the period when the pendulum makes small oscillations about the vertical.



$$\text{Center of mass} = \frac{0.5g(0.6) + 1.5g(1)}{2g} = 0.9$$
$$MI = \left[\frac{1}{3}(0.5)(0.6)^2 + 0.5(0.6)^2\right] + \left[\frac{1}{3}(1.5)(0.2^2 + 0.2^2) + (1.5)(1^2)\right]$$
$$= 1.78$$
$$-2g(0.9\sin\phi) = 1.78\ddot{\phi}$$
$$\sin\phi \approx \phi$$
$$\ddot{\phi} = -10.112\phi$$
$$\ddot{\phi} = -\omega^2\phi$$
$$T = \frac{2\pi}{\omega} = 1.976s$$

Example 3

A disc of mass m and radius *a* performs small oscillations about a smooth horizontal axis which is tangential to the disc. Find the length of the equivalent simple pendulum.

$$MI \text{ of disk} = \frac{1}{4}ma^2 + ma^2 = \frac{5}{4}ma^2$$
$$-mg(a\sin\phi) = \frac{5}{4}ma^2\ddot{\phi}$$
$$-g\phi = \frac{5}{4}a\ddot{\phi}$$
$$\ddot{\phi} = -\left(\frac{4g}{5a}\right)\phi$$
$$T = 2\pi\sqrt{\frac{5a}{4g}} \quad T = 2\pi\sqrt{\frac{l}{g}}$$
$$l = \frac{5}{4}a$$