CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST SEPTEMBER 2010

(June 2009 Intake)

Wednesday **25 August 2010** **12.30 pm – 3.30 pm**

FURTHER MATHEMATICS 9231/1

PAPER 1 **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

[Turn over

1 Prove by induction that $169^{2n-1} + 239^n$ is divisible by 34 for all positive integers n. [5]

2 Given $3x^2 + 5xy^2 + 4y^2 = 12$, find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at (1, 1). [5]

3 By considering the sum $z + z^3 + z^5 + \cdots + z^{2N+1}$, where $z = e^{i\theta}$, or otherwise, prove that

$$\sum_{r=0}^{N} \sin(2r+1)\theta = \frac{\sin^2(N+1)\theta}{\sin\theta}.$$ [6]

4 Given $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ bcd & acd & abd & abc \end{pmatrix}$, where a, b, c, d are constants such that $a > b > c > d > 0$,

find the rank of $\mathbf{A}$ and a basis for the range space of $\mathbf{A}$. [6]

5 Find the perpendicular distance between the planes $ax + by + cz = d$ and $ax + by + cz = d^2$, where a, b, c, d are constants. [7]

6 If the equation $5x^4 + 8x^3 + 2 = 0$ has roots α, β, γ, δ, find the equation having roots α^3, β^3, γ^3, δ^3. Hence, or otherwise, find $\alpha^6 + \beta^6 + \gamma^6 + \delta^6$. [8]

7 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 90y = 4\cos 2x + 7\sin 2x.$$ [8]

8 Find the sum

$$(5n+6)(8n+3) + (5n+7)(8n+2) + (5n+8)(8n+1) + \cdots + (9n+8)(4n+1)$$

by expressing it in the form $\sum_{r=5n+6}^{9n+8} r(an + b + cr)$, where a, b, c are constants that need to be determined. [8]

9 If **A** is a square matrix, show that $\mathbf{A}^k\mathbf{x} = \lambda^k\mathbf{x}$ for $k = 2, 3$, where λ is an eigenvalue of **A** with corresponding eigenvector **x.** Given $\mathbf{A} = \begin{pmatrix} 5 & 2 & -4 \\ 7 & -8 & -7 \\ -4 & 12 & 5 \end{pmatrix}$, find a square matrix **P** and a diagonal matrix **D** such that $\mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 = \mathbf{PDP}^{-1}$. [10]

10 The curve C has equation $y = 3 - \dfrac{4}{5(5x+6)} + \dfrac{9}{7(7x+8)}$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Find the coordinates of the stationary points of C and determine their nature. [4]

(iii) Sketch the curve C, marking clearly the stationary points and intercepts. [4]

11 Given $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \; d\theta$, find a reduction formula for I_n. [4]

Hence, or otherwise, find the y-coordinate of the centroid of the region bounded by the curve $y = \tan^3 x$, the x-axis and the line $x = \dfrac{\pi}{4}$. [8]

12 Answer only **one** of the following two alternatives.

EITHER

Given the curve $y = ax^2 + c$, where a and c are constants such that $a > 0$ and $c > 0$. The arc of the curve from $x = 0$ to $x = \dfrac{\sqrt{3}}{2a}$ is denoted C.

(i) Show that the length, S_1, of C is given by

$$S_1 = \int_0^{\frac{\sqrt{3}}{2a}} \sqrt{1 + 4a^2x^2}\, dx.$$

Hence using the substitution $x = \dfrac{1}{2a}\tan\theta$, find S_1 in exact terms. [9]

(ii) Find in exact terms the area, S_2, of the curved surface generated when C is rotated through one revolution about the y-axis. [4]

(iii) Given $S_1 < S_2$, show that $a < \pi$. [1]

OR

The curve C has polar equation $r = \sin\theta(1 - \cos\theta)$, $0 \le \theta \le \pi$.

(i) Sketch C. [3]

(ii) Find the area enclosed by by C from $\theta = 0$ to $\theta = \pi$. [3]

(iii) Find the maximum distance of C from the pole. [8]