CHALLENGING COMPLEX NUMBER QUESTIONS

1. The complex number z satisfies the equation $|z - (7 - 3\mathrm{i})| = 4$.

   (i) Sketch an Argand diagram to illustrate this equation. [2]

   (ii) Given that $|z|$ is as small as possible,

   (a) find the exact value of $|z|$, [2]

   (b) hence find an exact expression for z, in the form $x + \mathrm{i}y$. [2]

   (iii) It is given instead that $-\pi < \arg z \leqslant \pi$ and that $|\arg z|$ is as large as possible. Find the value of $\arg z$ in radians, correct to 4 significant figures. [3]

2. **The complex number z satisfies $|z - 2 - 5\mathrm{i}| \leqslant 3$.**

   (i) On an Argand diagram, sketch the region in which the point representing z can lie. [3]

   (ii) Find exactly the maximum and minimum possible values of $|z|$. [2]

   (iii) It is given that $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$. With this extra information, find the maximum value of $|z - 6 - \mathrm{i}|$. Label the point(s) that correspond to this maximum value on your diagram with the letter P. [3]

3. (a) The complex number z satisfies $|z - 3 - 4i| < 5$.

   (i) On an Argand diagram, sketch and shade the region in which the point representing z can lie. [3]

   (ii) Find the range of possible values of $|z + 1|$. [2]

   (b) Using a single Argand diagram, sketch the loci given by

   (i) $|z - 2i| = 1$,

   (ii) $\arg(z - i) = \frac{1}{4}\pi$. [3]

   Hence, find the value of z, in cartesian coordinate form $x + iy$, that satisfy both (i) and (ii). [1]

4. In a single Argand diagram, sketch the following loci, labelling each locus clearly.

(i) $|z| = 5$,

(ii) $|z+8| = |z-8\mathrm{i}|$.

The two complex numbers that satisfy the above equations are represented by p and q, where $\arg\left(\dfrac{p}{q}\right) > 0$. Find p and q exactly. [5]

State the exact value of $\arg\dfrac{(5-p)}{(5-q)}$. [1]

5. The complex number z is given by $z = x + \mathrm{i}y$, where $x > 0$ and $y > 0$. Sketch an Argand diagram, with the origin O, showing the points P, R and Q representing z, $3\mathrm{i}z$ and $(z + 3\mathrm{i}z)$ respectively. [2]

Describe the geometrical relationship between O, P, Q and R. [1]

(a) Given that $y = 3x$, show that the point representing z^2 is collinear with the origin and the point Q. [2]

(b) Given that $|z| \le 3$ and $\tan^{-1}\left(\dfrac{1}{3}\right) \le \arg z \le \tan^{-1}(3)$, illustrate both of these relations on a single Argand diagram. Find the area of the region in which the point P can lie. [3]