

# Topic : P107 Lines and Planes

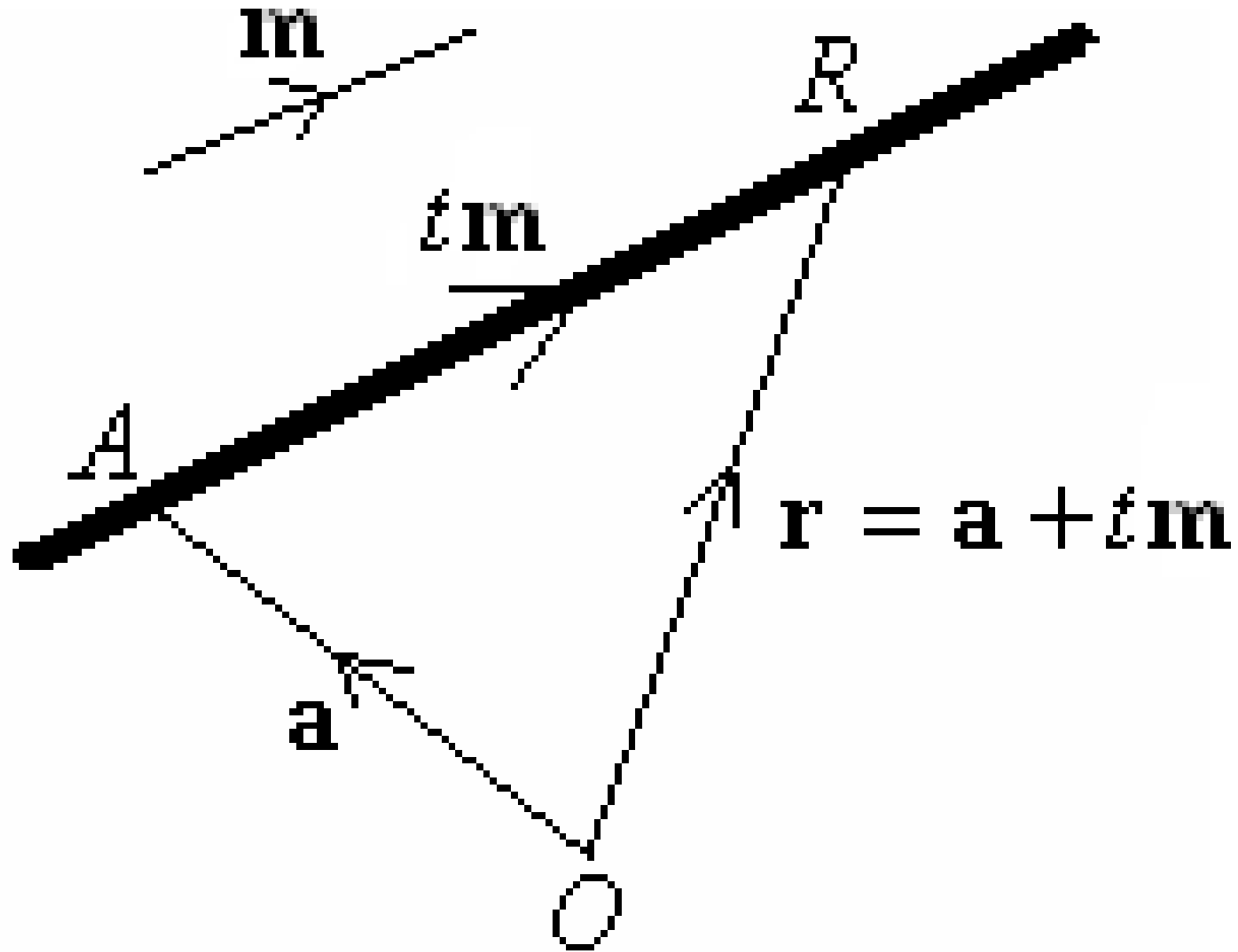
Subtopic:

- *Equations of a Straight Line and a Plane*
- *Parallel, Intersection, Skew*
- *Perpendicular Distances*
- *Angles*

# **Vector Equation of a Straight Line**

If a straight line in the direction of  $\mathbf{m}$  and passing through a point  $A$ , then its equation is given by

$$\mathbf{r} = \mathbf{a} + t\mathbf{m}.$$



Example:

Find, the form of  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ , the vector equation of the straight line which is parallel to the vector  $2\mathbf{i} - \mathbf{j}$  and passes through  $(3, -2)$ .

Example :

Find, in parametric form, the vector equation of the straight line which is parallel to the vector  $(\mathbf{i} - \mathbf{j})$  and passes through point  $A(4, -1)$ .

$B$  and  $C$  are points  $(3, \alpha)$  and  $(\beta, 3)$  which are located on the above line. Find  $\alpha$  and  $\beta$ .

Example:

Find the vector equation of the straight line which is parallel to the vector  $\begin{pmatrix} 1 & 3 & -4 \end{pmatrix}$  and passes through the point with position vector  $\begin{pmatrix} 2 & -5 & 6 \end{pmatrix}$ .

Example:

State the direction of the following straight lines.

$$(a) \mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \end{pmatrix} \qquad (b) \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$(c) \mathbf{r} = \mathbf{i} + 3t\mathbf{j} \qquad (d) \mathbf{r} = \mathbf{i} + 3\mathbf{k} + s(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Example:

State the direction of the following straight lines.

$$(a) \mathbf{r} = \begin{pmatrix} 1 + 2t \\ t \end{pmatrix} \quad (b) \mathbf{r} = \begin{pmatrix} 3 - t \\ t \\ 2 \end{pmatrix}$$

$$(c) \mathbf{r} = -t\mathbf{i} - t\mathbf{j} \quad (d) \mathbf{r} = (2t - 3)\mathbf{j} + t\mathbf{k}$$



# Cartesian Equations of a Straight Line

The Cartesian equation of a straight line with vector equation  $\mathbf{r} = \mathbf{a} + t\mathbf{m}$  can be obtained by replacing  $\mathbf{r}$

with  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ , and then express  $t$  in terms of  $x$ ,  $y$  and  $z$

independently.

Example :

Obtain the Cartesian form of the following equations.

$$(a) \mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \end{pmatrix} \quad (b) \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$(c) \mathbf{r} = t\mathbf{i} - 3t\mathbf{j}$$

$$(d) \mathbf{r} = 2\mathbf{j} + s(-2\mathbf{i} + \mathbf{k})$$

Example :

Find the Cartesian equation of the line which passes through the point  $A(1, -1, 0)$  and is parallel to  $5\mathbf{i} - \mathbf{j} - \mathbf{k}$ . Show that the point  $B(-14, 2, 3)$  lies on the line.

Example :

$l$  is a straight line which is parallel to the vector  $(1 \ 0 \ -3)$  and passes through the point  $A(0, 2, 1)$ . Points  $B$  and  $C$  are located on the line which has  $x$  - coordinates of 1 and  $z$  - coordinates of 4 respectively

(a) Find the Cartesian equation of  $l$ .

(b) Find the coordinates of  $B$  and  $C$ .

Example :

Find the Cartesian equation of the straight line passing through points  $(1, 2, 0)$  &  $(-5, 4, 2)$ .

Determine whether  $(19, -4, 3)$  is located on the line.

Example :

Obtain the parametric form or vector form of the equation of the following straight line. State also their direction.

(a)            (b)

(c)            (d)

# **Homework**

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example :

Find the equation of the straight line that passes the points  $(0, 2, -1)$  and  $(2, 3, 1)$ . Two points A and B on the straight line have parameters 0 and  $t$  respectively. Find the distance AB.



Example :

A tunnel is to be excavated through a hill. In order to define position, coordinates  $(x, y, z)$  are taken relative to an origin  $O$  such that  $x$  is the distance east from  $O$ ,  $y$  the distance north and  $z$  the vertical distance upwards, with one unit equal to 100 m. The tunnel starts at point  $A(2,3,5)$  and runs in a straight line in the direction  $\mathbf{i} + \mathbf{j} - 0.5\mathbf{k}$ .

(a) Write down the vector equation of the tunnel.

(b) An old tunnel through the hill is

$\mathbf{r} = (4 + 7\mu)\mathbf{i} + (1 + 15\mu)\mathbf{j} + 2\mathbf{k}$ . Show that the point  $P$  on the new tunnel where  $x = 7.5$  is directly above a point  $Q$  in the old tunnel. Find the vertical separation  $PQ$  of the tunnels at this point.

Example :

ABCD is a parallelogram with vertices  $A(1,1,2)$ ,  $B(2,0,-1)$ ,  $C(3,3,0)$ . Find the vector equations and Cartesian equations of AB and AC, and the coordinates of D.