INTEGRATION FURTHER MATHS PURE SET 1

1. It is given that $I_n = \int \sec^n x \, dx$.

By writing $\sec^n x$ as $\sec^{n-2} x \sec^2 x$, prove that

$$(n-1) I_n = \sec^{n-2} x \tan x + (n-2) I_{n-2}. \qquad [6]$$

Hence, find $\int \sec^4 x \, dx$. [2]

2. The integral I_n is defined by $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, \mathrm{d}x$.

By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan x \sec^n x\right)$, or otherwise, show that

$$(n+1) I_{n+2} = 2^{\frac{1}{2}n} + n I_n. \qquad [4]$$

Find the value of I_6. [4]

3. Given that $I_n = \int_0^{\pi/3} \sec^n x \, \mathrm{d}x$, show that for $n \geq 2$, $I_n = \frac{2^{n-2}}{n-1}\sqrt{3} + \frac{n-2}{n-1} I_{n-2}$.

[4]

Hence, evaluate I_7, giving your answer in exact form. [4]

4. (i) Show that

$$\frac{d}{dx}\left[x(1-x^2)^n\right] = (2n+1)(1-x^2)^n - 2n(1-x^2)^{n-1} \qquad [3]$$

(ii) It is given that

$$I_n = \int_0^1 (1-x^2)^n \, dx \qquad (n \geq 0)$$

Use the result in part (i) to show that

$$I_n = \frac{2n}{2n+1} I_{n-1} \qquad (n \geq 1) \qquad [2]$$

Hence evaluate I_5, leaving your answer as a fraction in its lowest terms. [2]

5. If I_n denotes $\int_0^{\frac{\pi}{2}} x^n \cos x\,dx$, prove that

i) $I_n = (\frac{\pi}{2})^n - n(n-1)I_{n-2} \quad , n \geq 2$ [7]

ii) $I_3 = \frac{\pi^3}{8} - \frac{3\pi}{2} + 6$. $I_3 = \frac{\pi^3}{8} - 3\pi + 6$ [5]

6. . The integral I_n, where n is a non-negative integer, is defined by

$$I_n = \int_0^1 x^n \sqrt{1-x^2}\,dx.$$

Show that $(n+2) I_n = (n-1) I_{n-2}$ [5]
Find I_5. [2]

7. **(i)** Given that

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,dx,$$

where n is a non-negative integer. By considering $\frac{d}{dx}\left(\sin^{n-1} x \cos x\right)$ show that for $n \geq 2$,

$$I_n = \frac{n-1}{n} I_{n-2}.$$ [5]

8. Given that

$$I_n = \int \operatorname{cosec}^n x\,dx,$$

where n is a non-negative integer. Show that for $n \geq 2$,

$$(n-1)I_n = (n-2)I_{n-2} - \cot x \operatorname{cosec}^{n-2} x.$$ [4]

(i) Find I_1, I_2, I_3 and I_4. [4]

9. If $I_n = \int \sec^2 y \sec^n(\tan y) \csc^n(\tan y)\, dy$, by using the substitution $x = \tan y$, show that $I_n = \int \sec^n x \csc^n x\, dx$ and find a reduction formula for I_n, in terms of x. [7]

Hence find I_2, I_3, and I_4. [5]

10. If $I_n = \int_0^{\frac{\pi}{2}} e^x \cos^n x\, dx$, show that $(n^2+1)I_n = n(n-1)I_{n-2} - 1$. [6]
Find also I_1, I_2 and I_3. [7]

11. Let $I_n = \int (\ln x)^n\, dx$. Show that $I_n = x(\ln x)^n - nI_{n-1}$. [3]

Hence, find I_2. [4]