VECTORS CHALLENGING QUESTIONS

1. The planes π_1 and π_2 have equations, $x + 2y - 3z + 4 = 0$ and $2x + y - 4z - 3 = 0$ respectively. Show that, for all values of λ, every point, which is in both π_1 and π_2 is also in the plane $x + 2y - 3z + 4 + \lambda(2x + y - 4z - 3) = 0$. [2]

The planes π_1 and π_2 meet in the line L.

(i) Find the equation of the plane π_3 which passes though L and the point whose position vector is $a\mathbf{k}$. [3]

(ii) Find the value of a if π_2 is perpendicular to π_3. [3]

2. The plane π with equation $\underset{\sim}{r}.\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}\right) = 7$, meets the line L, which has equation

$\underset{\sim}{r} = \left(3\underset{\sim}{i} + 5\underset{\sim}{j} + \underset{\sim}{k}\right) + \lambda\left(2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}\right)$, at the point P.

(i) Find the coordinates of P. [3]

(ii) Find the shortest distance of π from the origin O. [2]

The line M passes through the point P and the point Q, where Q (4,7,0).

(iii) Determine a vector which is perpendicular to both of the lines L and M. [3]

3. The equations of two planes are

$$\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k}) \qquad \text{and} \qquad 2x + y = 5$$

(i) Find the cosine of the acute angle between the two planes. [3]

(ii) The line of intersection of the two planes is L. Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, the equation of L. [3]

4. The lines l_1 and l_2 have vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + t(-\mathbf{i} - \mathbf{j} - 3\mathbf{k})$$

and

$$\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

respectively and the point A has coordinates (3, 4, 5). Find

i) the shortest distance between the two lines. [4]

ii) the vector equation of the common perpendicular to the lines. [5]

iii) the distance from the point A to the common perpendicular of the lines. [5]

5. The line l_1 passes through the point A, whose position vector is $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, and is parallel to the vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. The line l_2 passes through the point B, whose position vector is $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, and is parallel to the vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. The plane Π_1 contains PQ and l_1, and the plane Π_2 contains PQ and l_2.

(i) Find the length of PQ. [4]

(ii) Find a vector perpendicular to Π_1. [2]

(iii) Find the perpendicular distance from B to Π_1. [3]

(iv) Find the angle between Π_1 and Π_2. [3]

6.

a) Find the equation of the plane containing the line $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + s(\mathbf{i} + 9\mathbf{j} - 3\mathbf{k})$ and the point (4, 8, -3).

[4]

b) Find the equation of the plane containing the lines $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + s(-3\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ and $\mathbf{r} = 7\mathbf{i} + 6\mathbf{j} + 5\mathbf{k} + t(6\mathbf{i} + 4\mathbf{j} - \mathbf{k})$.

[4]

c) Find the equation of the line of intersection of the two planes. [2]

7. Find the intersection point of the planes $3x + 2y - 8z = 1$, $4x + 7y + 9z = 0$ and $5x - 6y + z = 9$. [7]

8. The planes Π_1, Π_2, Π_3 have equations $x + y + kz = 0$, $x + ky + z = 0$ and $kx + y + z = 0$ respectively. Find the common line of intersection of the three planes. [5]

9. Three planes have equations

$$x + 2y + pz = 1$$
$$2x - y + 5z = 11$$
$$3x + y + 8z = q$$

where p and q are constants.

(i) Find the value of p_1 such that when $p \neq p_1$ the planes have exactly one common point. [4]

(ii) Find the values of p_2 and q_2 such that when $p = p_2$ and $q \neq q_2$ the planes have no common points. [2]

(iii) In the case when $p = p_2$ and $q = q_2$ the planes have a common line of intersection, find this line. [3]

10. Find the perpendicular distance between the planes $3x + 8y + 5z = 4$ and $3x + 8y + 5z = 6$. [5]