1 A sequence of numbers are given by $u_1 = 1$, and $u_n = u_{n-1} + 2$ for all natural numbers n larger than 1.

(i) By stating the first 5 terms in the sequence, conjecture u_n in terms of n for all natural numbers n. [2]

(ii) Prove the conjecture in **(i)** using mathematical induction. [2]

2 In any order, find the set of real values of λ for which the system of linear equations

$$\begin{gathered} x + y + \lambda z = \lambda^2, \\ x + \lambda y + z = \lambda\,, \\ \lambda x + y + z = 1\,, \end{gathered}$$

has a unique solution, has infinite number of solutions and does not have a solution. [5]

Solve the system of linear equation for the case that it has infinite number of solutions. [1]

3 Let $f\;\;r\;\; = \dfrac{2}{r^2-1}$.

(i) Express $f\;\;r$ in sum of partial fractions. [3]

(ii) Hence, or otherwise, evaluate

(a) $S_n = \underset{r=2}{\overset{n}{}} f\;\;r\;\,,$ [3]

(b) $\lim\limits_{n\to\infty} S_n$. [1]

4 The curve C has equation

$$y = 1 - x + \frac{2}{x-2}\,.$$

(i) By using algebraic method, find the range of the function of C. [4]

(ii) Show that C is decreasing. [2]

(iii) Sketch C, stating the coordinates of the intersections with the axes. [3]

5 The planes π_1 and π_2 with equations $x - y + 2z = 1$ and $2x + y - z = 0$ respectively intersect in the line l.

(i) Find a direction of l. Hence, show that the Cartesian equation of l could be written as

$$\frac{x}{-1} = \frac{y - \alpha}{5} = \frac{z - \beta}{3}$$

where α and β are constants to be determined. [5]

Given that the point A has coordinates $(1, 0, 1)$,

(ii) find the Cartesian equation of the plane through A and containing l. [2]

(iii) calculate the perpendicular distance from A to l. [3]

6 **(i)** The cubic equation $x^3 - x^2 - 3x - 10 = 0$ has roots α, β, and γ.

Without evaluating α, β, and γ, find a cubic equation having roots $-\alpha + \beta + \gamma$, $\alpha - \beta + \gamma$, and $\alpha + \beta - \gamma$. [5]

(ii) Given that

$$p + q + r = 0,$$
$$p^2 + q^2 + r^2 = 2,$$
$$p^3 + q^3 + r^3 = 0,$$

find a cubic equation whose roots are p, q and r. [5]

Hence, find possible values for p, q and r. [1]

7 Answer only **one** of the following two alternatives.

EITHER

A system of linear equations is given by

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1,$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2,$$

where a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23} and a_{24} are real numbers, $a_{11} \neq 0$ and $a_{11}a_{22} \neq a_{12}a_{21}$.

Let $\boldsymbol{P}_0$ be the above system of linear equation when $b_1 = b_2 = 0$, and $\boldsymbol{P}_1$ be the one when not all b_1 and b_2 are zero.

(i) **(a)** Show that $\boldsymbol{P}_0$ must be consistent. [1]

(b) If s_1 , s_2 , s_3 , s_4 and t_1 , t_2 , t_3 , t_4 are the non-zero solutions of $\boldsymbol{P}_0$, show that the following sets are also the solutions of $\boldsymbol{P}_0$.

(i) λs_1 , λs_2 , λs_3 , λs_4 where $\lambda \neq 0$. [2]

(ii) $s_1 + t_1$, $s_2 + t_2$, $s_3 + t_3$, $s_4 + t_4$. [2]

(ii) **(a)** If α_1 , α_2 , α_3 , α_4 is a solution of $\boldsymbol{P}_1$ and t_1 , t_2 , t_3 , t_4 is a solution of $\boldsymbol{P}_0$, show that $\alpha_1 + t_1$, $\alpha_2 + t_2$, $\alpha_3 + t_3$, $\alpha_4 + t_4$ is also a solution of $\boldsymbol{P}_1$. [4]

(b) Using the results in **(ii)(a)**, or otherwise, solve the system of linear equations

$$x_1 + x_2 - 2x_3 + x_4 = 1,$$
$$2x_1 + x_2 - x_3 + x_4 = 2.$$ [4]

OR

(i) Prove, by mathematical induction, that

$$\sum_{r=1}^{n} \frac{1}{r^2} < 2 - \frac{1}{n}$$

for every positive integers $n \geq 2$. [4]

(ii) A series is given by $1 \; 1 \; + 4 \; 3 \; + 9 \; 5 \; + 16 \; 7 \; + \cdots + U_n$.

(a) State U_n in terms of n. [1]

(b) Show that

$$\sum_{r=1}^{n} U_r = \frac{1}{6} n \;\; n+1 \;\; 3n^2 + n - 1 \; .$$ [3]

Hence, or otherwise, express, in terms of n, the sum of the first n terms in the series

$$S_n = \ln 2 + 3 \ln 2^4 + 5 \ln 2^9 + 7 \ln 2^{16} + \cdots$$ [3]

Find the value of

$$\lim_{n \to \infty} \; n^{-4} S_n \; .$$ [2]