TAYLOR'S UNIVERSITY COLLEGE
Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2007

(Jan 2007 Intake)

Friday **15June 2007** **8.30 am – 10.30 am**

FURTHER MATHEMATICS 9231/01

PAPER 1 2 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages.

[Turn over

1. By considering the sum $(r+1)^5-(r-1)^5$, or otherwise, find a formula for $\sum_{r=1}^{n} r^4$. [10]

2. Prove that $4^n > 3n^2 + 5$ for every positive integer $n > 2$. [10]

3. If the equation $7x^3-6x^2+5x+9=0$ has roots α, β, γ find the value of

$$(\frac{3}{\alpha\beta}-\frac{\gamma}{4})^2+(\frac{3}{\alpha\gamma}-\frac{\beta}{4})^2+(\frac{3}{\beta\gamma}-\frac{\alpha}{4})^2.$$ [10]

4. The line l_1 passes through the points with position vectors $4\mathbf{i}-\mathbf{j}-9\mathbf{k}$ and $9\mathbf{i}-7\mathbf{k}$. The line l_2 has equation $\mathbf{r} = \mathbf{i}+2\mathbf{j}-3\mathbf{k}+t(\mathbf{i}+3\mathbf{j}-\mathbf{k})$. The line l_3 is perpendicular to l_1 and l_2, and passes through the point with position vector $2\mathbf{i}+\mathbf{j}-5\mathbf{k}$.

   (i) Find the equation of the plane which contains l_2 and l_3, giving your answer in the form $ax+by+cz=d$. [6]

   (ii) Show that l_1 and l_3 intersect. [5]

   (iii) Find the shortest distance between l_1 and l_2. [4]

5. Find the acute angle between the planes with equations

$$x + y - z + 2 = 0 \quad \text{and} \quad x - 2y + z - 9 = 0. \qquad [4]$$

The planes meet in the line l, and P is the point on l whose position vector is $\alpha\mathbf{i} + \beta\mathbf{j} + 10\mathbf{k}$.

(i) Find α and β. [4]

(ii) Find the vector equation for l. [3]

The planes Π_1 and Π_2 are both perpendicular to l. The perpendicular distance from P to Π_1 is $\sqrt{14}$ and the perpendicular distance from P to Π_2 is also $\sqrt{14}$. Find the equations for Π_1 and Π_2 in the form $ax + by + cz = d$. [6]

6. The curve **C** has equation

$$y = \frac{2x^2 - 6x + 8}{x^2 - 5x + 4}.$$

(i) Write down the equations of the asymptotes of **C**. [4]

(ii) Find the coordinates of the stationary points of **C**. [5]

(iii) Draw a sketch of **C**, stating the coordinates of all the points of intersection with the coordinate axes. [4]