Topic：**P102 Summation**

Subtopic:

- *Sigma Notation*
- *Summation Involving Polynomials*
- *Telescoping Series*
- *Method of Differences*

Sigma Notation

Definition :

$$\sum_{r=a}^{b} f(r)$$

$$= \underline{\underline{f(a)+ f(a+1)+ f(a+2)+ \ldots + f(b-1)+ f(b)}}$$

$$\Uparrow \qquad \Uparrow \qquad \Uparrow \qquad \Uparrow \qquad \Uparrow$$

$$\underline{r=a} \quad \underline{r=a+1} \quad \underline{r=a+2} \qquad \underline{r=b-1} \quad \underline{r=b}$$

$$(\text{Start}) \qquad\qquad\qquad\qquad (\text{End})$$

Example:

Write down all the terms in each summation.

(a) $\sum_{r=1}^{5} r$

(b) $\sum_{r=3}^{r=8} \frac{1}{r}$

(c) $\sum_{r=5}^{r=10} 2$

(d) $\sum_{r=1}^{n} 2^r$

Example :

Write down the first three terms and the last term of each summation.

$$(\text{a}) \sum_{r=1}^{n} \frac{1}{r(r+1)}$$

$$(\text{b}) \sum_{r=2}^{n} \left(r^2 - r\right)$$

Example :

Write down first three terms and the last three terms of $\sum_{r=3}^{92}(2r+1)$. Hence, evaluate the sum.

Example:

Show that $\sum_{r=10}^{n} 4^r = \frac{4^{10}}{3}\left(4^{n-9} - 1\right)$

Example :

Express $\sum_{r=1}^{n} \frac{1}{3^r}$ in terms of n. Hence, find the value of the limit if $n \to \infty$.

Example:

Rewrite the following series using sigma notation.

$$(a)\left(1+\frac{1}{1}\right)+\left(2+\frac{1}{2}\right)+\left(3+\frac{1}{3}\right)+\ldots+\left(100+\frac{1}{100}\right).$$

$$(b)1+2x+3x^2+4x^3+\ldots$$

$$(c)1+{}^nC_1x+{}^nC_2x^2+\ldots+x^n$$

Theorem :

$$\sum_{r=1}^{n}\left[f(r)\pm g(r)\right]=\sum_{r=1}^{n}f(r)\pm\sum_{r=1}^{n}g(r)$$

$$\sum_{r=1}^{n}\lambda f(r)=\lambda\sum_{r=1}^{n}f(r)$$

Example:

If $\sum_{r=1}^{n} f(r) = 4$ and $\sum_{r=1}^{n} g(r) = -3$, find

(a) $\sum_{r=1}^{n} [f(r) + g(r)]$

(b) $\sum_{r=1}^{n} [3f(r) - g(r)]$

Example:

Express $\sum_{r=1}^{n}\left(3r-3^{r}\right)$ in terms of n.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Evaluate the following summation.

$$\text{(a)} \sum_{r=3}^{90} (-3) \qquad \text{(b)} \sum_{r=10}^{n} 4$$

Example:

Rewrite the following series using sigma notation.

$(a) 1-2+3-4+...+101.$

$(b) 3^2+5^2+7^2+...+21^2.$

$(c) -\frac{1}{2}+\frac{1}{5}-\frac{1}{8}+\frac{1}{11}.....$

Example:

If $\sum_{r=1}^{n} f(r) = \alpha$ and $\sum_{r=1}^{n} g(r) = \beta$, justify the validity of

(a) $\sum_{r=1}^{n} [f(r) \times g(r)] = \alpha\beta.$

(b) $\sum_{r=1}^{n} [f(r)]^2 = \alpha^2.$

(c) $\sum_{r=1}^{n} \frac{1}{f(r)} = \frac{1}{\alpha}.$

Example:

Justify the validity of each of the following statements.

$$(\text{a})\sum_{r=1}^{2n} r^3 = \sum_{r=0}^{2n-1} (r+1)^3. \qquad (\text{b})\sum_{r=1}^{n} 1 = \frac{1}{2}n(n+1).$$