TAYLOR'S UNIVERSITY COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2007

(June 2006 Intake)

Thursday 30 August 2007 12.30 pm – 3.30 pm

FURTHER MATHEMATICS 9231/01

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers
The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages.

1 Express the polar equation $r = \sec^n\theta + \operatorname{cosec}^n\theta$ in Cartesian form. [5]

2 Find the sum $13^3 + 21^3 + 29^3 + \ldots + (8n+5)^3$. [6]

3 Find the y coordinate of the centroid of the area bounded by the curve $y = e^x$, the line $x = 1$ and the x and y axes. [7]

4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 25y = 32e^x.$$

[8]

5 If the square matrix $\mathbf{A}$ has an eigenvalue λ and eigenvector $\mathbf{x}$, show that

a) i) $\mathbf{A}^n$ has an eigenvalue λ^n and eigenvector $\mathbf{x}$ for every positive integer n. [4]

ii) $k\mathbf{A}$ has an eigenvalue $k\lambda$ and eigenvector $\mathbf{x}$, where k is a scalar. [2]

b) the matrix $a_n\mathbf{A}^n + a_{n-1}\mathbf{A}^{n-1} + \ldots + a_2\mathbf{A}^2 + a_1\mathbf{A} + a_0\mathbf{I}$ has eigenvalue $a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_2\lambda^2 + a_1\lambda + a_0$ and eigenvector $\mathbf{x}$. [3]

6 The planes Π_1, Π_2, Π_3 are given by the equations

Π_1: $x + y + z = 0$

Π_2: $ax + by + cz = 0$

Π_3: $a^2x + b^2y + c^2z = 0$.

Find the possible lines of intersection between Π_1, Π_2, Π_3. [9]

7 If the equation $3x^4 - 2x^3 + x^2 - 9x + 7 = 0$ has roots $\alpha, \beta, \gamma, \delta$ find the equation having roots $\frac{1}{\alpha^2+4}, \frac{1}{\beta^2+4}, \frac{1}{\gamma^2+4}, \frac{1}{\delta^2+4}$. [10]

8 The curve C is defined parametrically by the equations

$$\cancel{y = \frac{t^6}{6} + \frac{1}{t^3}, \quad x = 6t^2.} \quad y = \frac{t^7}{7} + \frac{3}{t^3}, \quad x = 3t^2$$

Find the area of the surface generated by rotating C about the x-axis from $t = 2$ to $t = 1$.

[10]

9 If $I_n = \int \sec^n x \, dx$, show that $(n-1)I_n = (n-2)I_{n-2} + \tan x \sec^{n-2} x$.

[6]

Hence, or otherwise, find I_3 and I_4.

[4]

10 Sketch the graph $y = 5 - \dfrac{3}{x-1} + \dfrac{48}{x+9}$.

[12]

11 Answer only **one** of the following two alternatives.

EITHER

By considering the sum $z + z^2 + z^3 + \ldots + z^n$, where $z = e^{i\theta}$, or otherwise, find the sum $\sum_{r=1}^{n} \cos r\theta$.

[14]

OR

a) Find the equation of the tangent to the curve

$$\tan x + \operatorname{cosec} y = \cot x + \sec y$$

at $(\frac{\pi}{4}, \frac{\pi}{4})$.

[7]

b) If $e^x \ln y = x^e$, find the value of $\dfrac{d^2y}{dx^2}$ at the point (e, e).

[7]