Constant Series

Theorem :

$$\sum_{r=1}^{n} 1 = n.$$

Example : Evaluate the summation

$$(\mathrm{a}) \sum_{r=1}^{n} e \qquad (\mathrm{b}) \sum_{r=1}^{30} (-\pi)$$

Linear Series

Theorem :

$$\sum_{r=1}^{n} r = \frac{1}{2} n(n+1).$$

Example : Evaluate the summations

$$(\mathrm{a}) \sum_{r=1}^{n} 3r \qquad (\mathrm{b}) \sum_{r=1}^{20} (-r)$$

Example:

Evaluate the following summations:

$$\text{(a)} \sum_{r=0}^{2n} (3r - 5) \qquad \text{(b)} \sum_{r=1}^{n+1} (e - \pi r)$$

Quadratic Series

Theorem :

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1).$$

Example:

Evaluate the following summations:

$$(\text{a}) \sum_{r=1}^{n} r(r+1) \qquad (\text{b}) \sum_{r=1}^{100} \left(3r^2 - 3r + 1\right)$$

Example :

Evaluate $\sum_{r=1}^{50}(2r-1)^2$.

Cubic Series

Theorem :

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2.$$

Example:

Evaluate the following summations:

$$\text{(a)} \sum_{r=1}^{n} r(r+1)(r+2) \qquad \text{(b)} \sum_{r=1}^{50} (2r-1)^3$$

Example:

Given the series $S_n = 1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + 4 \cdot 7 + ... + u_n$.

(a) State u_n.

(b) Express S_n in terms of n.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

If $\sum_{r=n+3}^{2n} r = 312$, find the value of n.

Example :

Find the sum of n terms of the series

$$(p-1)(p+1)+(p-2)(p+2)+(p-3)(p+3)+\ldots$$

Example:

Given the series $S_n = 1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + .. + u_n$.

(a) State u_n.

(b) Express S_n in terms of n.