`A2 TRIAL MAR/APR 2011

1. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \alpha(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \beta(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where α and β are scalar parameters.

(i) Show that l_1 and l_2 meet and find the position vector of their point of intersection. [5]

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(ii) Show that point A lies on l_1. [1]

(iii) Find an equation for the plane containing l_1 and is perpendicular to l_2, giving your answer in the form $ax + by + cz = d$. [2]

A2 TRIAL MAR/APR 2010

2. The position vectors of three points A, B, C on a plane ski-slope are $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}, \mathbf{b} = -2\mathbf{i} + 26\mathbf{j} + 11\mathbf{k}, \mathbf{c} = 16\mathbf{i} + 17\mathbf{j} + 2\mathbf{k}$, where the units are in metres.

(i) Show that the vector $2\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ is perpendicular to $\overrightarrow{AB}$ and also perpendicular to $\overrightarrow{AC}$. Hence find the equation of the plane of the ski-slope. [4]

The track for an overhead railway lies along the straight edge DEF, where D and E have position vectors

$\mathbf{d} = 130\mathbf{i} - 40\mathbf{j} + 20\mathbf{k}$ and $\mathbf{e} = 90\mathbf{i} - 20\mathbf{j} + 15\mathbf{k}$,

and F is a point on the ski-slope.

(ii) Find the equation of the straight line DE. [3]

(iii) Find the position vector of the point F. [3]

(iv) Show that $\overrightarrow{DF} = 15(-8\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and hence find the length of the track. [3]

A2 TRIAL MAR/APR 2008

3\. The line l has equation $r = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. The plane p has equation $x + 2y + 3z = 5$.

i) Show that the line l lies in the plane p. [2]

ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

A2 TRIAL AUG/SEPT 2008

4\. The points A, B and C have position vectors, with respect to the origin, given by

$\overrightarrow{OA} = (5\boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k})$, $\overrightarrow{OB} = (-4\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$ and $\overrightarrow{OC} = (5\boldsymbol{i} - 2\boldsymbol{j} + 11\boldsymbol{k})$,

Find (i) a vector equation for the line BC, [2]

(ii) a vector equation for the plane OAB, [3]

(iii) the cosine of the acute angle between the lines OA and OB [2]

Obtain, the Cartesian equation for Π, the plane which passes through A and is perpendicular to BC. [3]

ANSWER

1\. (i) (-3,3,7) (iii) $3x - y + 5z = 23$

2\. (i) $2x - 3y + 7z = -5$ (ii) $\mathbf{r} = \begin{matrix} 130 \\ -40 \\ 20 \end{matrix} + t \begin{matrix} -40 \\ 20 \\ -5 \end{matrix}$ (iii) $\begin{matrix} 10 \\ 20 \\ 5 \end{matrix}$ (iv) DF = 135m

3\. (ii) $4x + y - 2z = 1$

4\. (i) **r** = (=4i + 4j – k) + t(3i – 2j + 4k) (ii) **r**. (13i + 17j + 16k) = 0 (iii)

A2 TRIAL AUG/SEPT 2007

5. The line l has equation $\mathbf{r} = 7\mathbf{i} + 3\mathbf{k} + t\,(5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$, and the plane π has equation $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 1$. Find the coordinates of the point A where l meets π. The point B has coordinates (7, 0, 3), and C is the foot of the perpendicular from B to π. Find the coordinates of C. Find a vector equation for the line AC, and calculate the angle BAC, giving your answer to the nearest degree. [10]

Ans: A(2,-3,1) C(5,-2,-1) $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ $\theta = 53°$

A2 TRIAL MARCH/APR 2012

6. Relative to the origin, the position vectors of points A and B are $4\mathbf{i} - 11\mathbf{j} + 4\mathbf{k}$ and $7\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ respectively.

(i) Find a vector equation for the line l passing through points A and B. [2]

(ii) Find the position vector of the point P on l such that OP is perpendicular to l. Hence find the perpendicular distance from O to l. [5]

(iii) The planes π_1 and π_2 have equations $3x - y - z = 2$ and $x + 5y + z = 14$ respectively. Find the line of intersection of π_1 and π_2. [4]

Ans: (i) $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 7 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$ (ii) OP = $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ 9 units (iii) $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -6 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

A2 TRIAL MAR/APR 2007

7. The line has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. (3)

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. (6)

Ans: (ii) 4x + y -2z = 1

A2 TRIAL MAR/APR 2009

8. Lines L_1 , L_2 and L_3 has vector equations

$$L_1 : \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k})$$
$$L_2 : \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$
$$L_3 : \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} + c\mathbf{j} + \mathbf{k})$$

(i) Calculate the angle between L_1 and L_2. [3]

(ii) Given that L_2 and L_3 intersect, find the value of c. [3]

(iii) Find the equation of the plane Π containing the point $A(5,-1,-2)$ and is perpendicular to L_2. [3]

(iv) Determine the perpendicular distance of $B(6,0,3)$ to the plane Π. [4]

Ans : (i) 68.5° (ii) -3 (iii) x + 3y + 2z = -2 (iv) √14