Revision

P105 Roots of Polynomial Equations

CIE CAL FM P1 2008-06 Q5

The equation $x^3 + x - 1 = 0$ has roots α, β, γ.

Show that the equation with roots $\alpha^3, \beta^3, \gamma^3$ is $y^3 - 3y^2 + 4y - 1 = 0$. [4]

Hence find the value of $\alpha^6 + \beta^6 + \gamma^6$. [3]

CIE CAL FM P1 2004-11 Q3

Given that $\alpha + \beta + \gamma = 0$, $\alpha^2 + \beta^2 + \gamma^2 = 14$, $\alpha^3 + \beta^3 + \gamma^3 = -18$, find a cubic equation whose roots are α, β, γ. [4]

Hence find possible values for α, β, γ. [2]

CIE CAL FM P1 2005-11 Q5

In the equation $x^3 + ax^2 + bx + c = 0$, the coefficients a, b and c are real. It is given that all the roots are real and greater than 1.

(i) Prove that $a < -3$. [1]

(ii) By considering the sum of the squares of the roots, prove that $a^2 > 2b + 3$. [2]

(iii) By considering the sum of the cubes of the roots, prove that $a^3 < -9b - 3c - 3$. [4]

CIE CAL FM P1 2002-11 Q2

The equation $x^4 + x^3 + Ax^2 + 4x - 2 = 0$, where A is a constant, has roots $\alpha, \beta, \gamma, \delta$. Find a polynomial equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$. [2]

Given that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$,

find the value of A. [3]

CIE CAL FM P1 2009-11 Q5

The equation $x^3 + 5x + 3 = 0$ has roots α, β, γ.

Use the substitution $x = -\dfrac{3}{y}$ to find a cubic equation in y and show that the roots of this equation are $\beta\gamma, \gamma\alpha, \alpha\beta$. [4]

Find the exact values of $\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2$ and $\beta^3\gamma^3 + \gamma^3\alpha^3 + \alpha^3\beta^3$. [5]

CIE CAL FM P1 2008-11 Q12

The roots of the equation $x^4 - 5x^2 + 2x - 1 = 0$ are $\alpha, \beta, \gamma, \delta$. Let $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n$.

(i) Show that $S_{n+4} - 5S_{n+2} + 2S_{n+1} - S_n = 0$. [2]

(ii) Find the values of S_2 and S_4. [3]

(iii) Find the value of S_3 and hence find the value of S_6. [6]

(iv) Hence find the value of

$$\alpha^2(\beta^4 + \gamma^4 + \delta^4) + \beta^2(\gamma^4 + \delta^4 + \alpha^4) + \gamma^2(\delta^4 + \alpha^4 + \beta^4) + \delta^2(\alpha^4 + \beta^4 + \gamma^4).$$ [3]

ASSIGNMENT

Attempt all the questions in the following slides.

Assignment is to be submitted within **one week** after instruction!!!

Please write your name and your student's id in the script upon submission.

CIE CAL FM P1 2003-06 Q5

The equation $8x^3 + 12x^2 + 4x - 1 = 0$ has roots α, β, γ. Show that the equation with roots $2\alpha + 1, 2\beta + 1, 2\gamma + 1$ is $y^3 - y - 1 = 0$. [3]

The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n$ is denoted by S_n. Find the values of S_3 and S_{-2}. [5]

CIE CAL FM P1 2007-11 Q4

The roots of the equation $x^3 - 8x^2 + 5 = 0$ are α, β, γ. Show that $\alpha^2 = \dfrac{5}{\beta + \gamma}$. [4]

It is given that the roots are all real. Without reference to a graph, show that one of the roots is negative and the other two roots are positive. [3]

CIE CAL FM P1 2006-11 Q6

The roots of the equation $x^3 + x + 1 = 0$ are α, β, γ.

Show that the equation whose roots are

$$\frac{4\alpha + 1}{\alpha + 1}, \quad \frac{4\beta + 1}{\beta + 1}, \quad \frac{4\gamma + 1}{\gamma + 1}$$

is of the form $y^3 + py + q = 0$, where the numbers p and q are to be determined. [5]

Hence find the value of

$$\left(\frac{4\alpha + 1}{\alpha + 1}\right)^n + \left(\frac{4\beta + 1}{\beta + 1}\right)^n + \left(\frac{4\gamma + 1}{\gamma + 1}\right)^n,$$

for $n = 2$ and for $n = 3$. [4]

CIE CAL FM P1 2004-06 Q11

The roots of the equation $x^3 - x - 1 = 0$ are α, β, γ, and $S_n = \alpha^n + \beta^n + \gamma^n$.

(i) Use the relation $y = x^2$ to show that α^2, β^2, γ^2 are the roots of the equation $y^3 - 2y^2 + y - 1 = 0$. [3]

(ii) Hence, or otherwise, find the value of S_4. [2]

(iii) Find the values of S_8, S_{12} and S_{16}. [9]