TAYLOR'S UNIVERSITY COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2007

(Jan2006 Intake) Jan/March 2006 Intake

Wednesday **28 March 2007** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/01

PAPER 1 **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together

This document consists of **5** printed pages.

[Turn over

1 If $S_n = 1\times n + 2(n-1) + 3(n-2) + \cdots + n\times 1$, show that $S_n = \dfrac{n(n+1)(n+2)}{6}$. [5]

2 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 15y = 36\cos x - 12\sin x$$ [5]

3 If the equation $3x^3 + 4x + 2 = 0$ has roots α, β, γ, find the equation having roots

$$\frac{1}{\alpha^2\beta^2} - 2, \frac{1}{\alpha^2\gamma^2} - 2, \frac{1}{\beta^2\gamma^2} - 2.$$ [6]

4 The curve C has equation

$$C:\ y = 3 - \frac{2}{x+9} + \frac{1}{2x-3}.$$

(i) Find the equations of the asymptotes of C. [3]

(ii) Find the coordinates of the critical points of C. [2]

(iii) Find the intersection points of C with the x and y axes. [3]

5 If $I_n = \int \cos^n x\,dx$, find a reduction formula for I_n. [5]

Hence, or otherwise, find an expression for $\int \cos^4 x\,dx$. [3]

6 Prove that $3^n > 2n^2 - 1$ for every positive integer n. [8]

7 Find the roots of the equations

$$z^3 = 4 - 4\sqrt{3}\ i.$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \le \theta \le 2\pi$. [5]

Denoting these roots by z_1 , z_2 , z_3 , show that,

$$z_1^{\ 3} + z_2^{\ 3} + z_3^{\ 3} = \alpha\, e^{i\beta\pi},$$

where α and β are constants.
State the values of α and β . [5]

8 The curve C is defined parametrically by

$$x = 3t^2 + 1, \quad y = t^3 - 3t,$$

where $t > 0$.

(i) Show that $\dfrac{d^2y}{dx^2}$ is positive at every point of C. [5]

(ii) The arc of C joining the point where $t = 1$ to the point where $t = 2$ is rotated through one complete revolution about the y-axis. Find the area of the surface generated. [6]

9 The line l_1 passes through the point A, whose position vector is $\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, and is parallel to the vector $3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. The line l_2 passes through the point B, whose position vector is $2\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$, and is parallel to the vector $\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 . The plane Π_1 contains PQ and l_1 , and the plane Π_2 contains PQ and l_2 .

(i) Find the length of PQ. [4]

(ii) Find a vector perpendicular to Π_1 . [2]

(iii) Find the perpendicular distance from B to Π_1 . [3]

(iv) Find the angle between Π_1 and Π_2 . [3]

[Turn over

10 Prove de Moivre's theorem for a positive integral exponent:
for all positive integers n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. [5]

Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$ [5]

Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x + 1 = 0$$

In the form $\cos k\pi$, where k is a rational number. [3]

11 Answer only **one** of the following two alternatives.

EITHER

(a) The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 3 & -4 & 2 \\ -4 & -1 & 6 \\ 2 & 6 & -2 \end{pmatrix}$$

Find the eigenvalues and corresponding eigenvectors of **A**. [6]
Hence find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^3 = \mathbf{PDP}^{-1}$. [2]

(b) The linear transformation $\mathbf{T} : R^4 \to R^3$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 4 & 9 \\ 2 & 4 & 7 & 15 \\ -1 & 3 & 5 & 13 \end{pmatrix}$$

(i) Show that the dimension of the range space of **T** is 3. [3]

(ii) Write down the basis for R^3. [1]

(iii) Find a basis for the null space of **T**. [2]

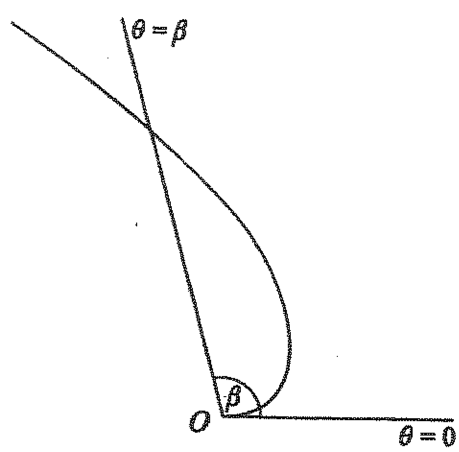
OR

(a) The polar equation of a curve is given as $r = a \sin 2\theta$, where a is a constant.

(i) Sketch the curve from $\theta = 0$ to $\theta = \dfrac{\pi}{2}$ and write down the Cartesian equation of the curve. [3]

(ii) Find the area of the region enclosed by the curve from $\theta = 0$ to $\theta = \dfrac{\pi}{2}$, giving your answer in terms of a and π. [4]

(b)



The curve C has polar equation $\quad r = \theta^{\frac{1}{2}} e^{\theta^2/\pi}$

where $0 \le \theta \le \pi$. The area of the finite region bounded by C and the line $\theta = \beta$ is 3π. Show that

$$\beta = \sqrt{\pi \ln 5}$$ [7]