TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2012

(January & March 2011 Intakes)

Thursday **29 March 2012** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/13

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

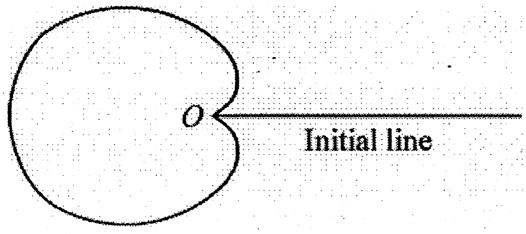
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages.

[Turn over

1



The diagram above shows the curve C with polar equation

$$r = a\sin\frac{1}{2}\theta, \quad 0 \le \theta < 2\pi .$$

Find the area bounded by C. [4]

2 Find the perpendicular distance from the point (9, 1, 4) to the line
$\mathbf{r} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k} + s(3\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$. [3]

Find also the equation of the plane containing the line and the point. [3]

3 Prove that $\sum_{r=2}^{n} r(r-1)x^r = \dfrac{n(n-1)x^{n+3} - 2(n^2-1)x^{n+2} + (n+1)nx^{n+1} - 2x^2}{(x-1)^3}$ for every positive integer $n \ge 2$. [6]

4 Given that

$$\tan y = \sqrt{x} \ , \quad x > 0, \ 0 < y < \frac{\pi}{2} .$$

Show that

$$2x(x+1)\frac{d^2y}{dx^2} + (3x+1)\frac{dy}{dx} = 0 .$$ [7]

5 If $z = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$, $k = 0, 1, 2, \ldots, n-1$, show that $z^n - 1$ is a factor of the

equation $z^{3n} - (\sqrt{3}+1)z^{2n} + (\sqrt{3}+1)z^n - 1 = 0$. [2]

Show that the other factor is $z^{2n} + az^n + b$, where a and b are constants that need to be found. [2]

Hence, or otherwise, find all solutions of the equation

$$z^{3n} - (\sqrt{3}+1)z^{2n} + (\sqrt{3}+1)z^n - 1 = 0.$$ [4]

6 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 16y = 144\cos 8x.$$ [8]

7 Express $\frac{7r-3}{r^3-r}$ in partial fractions. [3]

Hence find the sum of the first n terms of the following series

$$\frac{22}{6} + \frac{36}{24} + \frac{50}{60} + \cdots$$ [7]

8 The linear transformation T: $R^3 \to R^3$ is given by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$,

Where a, b, c are constants and $a > b > c > 0$.

(i) Find $\mathbf{M}^{-1}$. [4]

(ii) If

$$\begin{aligned} x + y + z &= |M| \\ ax + by + cz &= |M| \\ a^2x + b^2y + c^2z &= |M| \end{aligned},$$

where $|M|$ is the determinant of **M**, find x, y, and z. [5]

(iii) Find a basis of the range space of T. [3]

9 The curve C has equation $y = \dfrac{x^2 + 2x - 3}{x + 2}$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that C has no turning points. [2]

(iii) Draw a sketch of C and its asymptotes. Give the coordinates of the points of intersection of C with the coordinate axes, [4]

(iv) In the same diagram draw a sketch of $y = \dfrac{1}{x}$ and hence find the number of real roots of the equation $x^3 + 2x^2 - 4x - 2 = 0$. [3]

10 If $I_n = \int_0^{\frac{\pi}{2}} e^x \cos^n x \, dx$, show that $(n^2 + 1)I_n = n(n-1)I_{n-2} - 1$. [6]

Find also I_1, I_2 and I_3. [7]

11 Answer only one of the following two alternatives.

EITHER

If $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $bc - ad > 0$, find the eigenvalues of **A**. [4]

The eigenvalues of **A** are λ_1, λ_2 with corresponding eigenvectors $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

Find u_1, u_2, v_1 and v_2 in terms of λ_1, λ_2, a and b. [6]

If $\mathbf{P} = \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$, show that **AP** = **PD**. [4]

OR

Given that α, β, γ and δ are the roots of the quartic equation $2x^4 + 4x^3 - 6x^2 + x - 1 = 0$, form the quartic equation whose roots are $2\alpha - 1$, $2\beta - 1$, $2\gamma - 1$ and $2\delta - 1$. [5]

The sum $(2\alpha-1)^n + (2\beta-1)^n + (2\gamma-1)^n + (2\delta-1)^n$ is denoted S_n.

Find the value of S_3. [9]