Hooke's Law and Elastic Potential Energy (EPE)

We have seen that in order for SHM to be performed, the acceleration of a body must be proportional to its displacement from a fixed point and directed towards that point. By Hooke's Law, the tension in an elastic string and the tension or thrust in a spring is $\frac{\lambda x}{l}$ with the usual notation, and acts in such a direction as to attempt restore the string or spring to its natural length. Hence we should expect a particle attached to an elastic, or to a spring, to perform SHM.

Elastic Potential Energy = energy stored in the string or spring

= work done in stretching the string/spring from its natural length

$T = \frac{\lambda x}{l}$, work done = $\int_0^x T\,dx = \int_0^x \frac{\lambda x}{l}\,dx = \frac{\lambda x^2}{2l}$

Example:- An elastic spring has natural length 40 cm and modulus 40 N. Find the work done in compressing it from 35 cm to 30 cm.

Initial compression = 0.4 – 0.35 = 0.05 m

Final compression = 0.4 – 0.3 = 0.1 m

Work done = gain in EPE = $\frac{\lambda}{2l}\left(0.1^2 - 0.05^2\right)$ = 0.375 J

Note:- This is the same as finding the work done in stretching the string from 45cm to 50 cm.

Note:- Example 8

In the case of an elastic string, the string goes slack after its natural length. The attached particle will move under gravity (no tension). The entire motion is still periodic (but not SHM entirely). SHM equations only holds while the string is taut. Max speed remains at the equilibrium position.