Partial Fraction Decomposition

A technique used to break a complicated rational function to sum of "smaller" rational functions, called partial fractions.

Example:

$$\frac{7x-8}{(2x-1)(x-2)} \equiv \frac{3}{2x-1} + \frac{2}{x-2}$$

$$\frac{5x-6}{(x+2)(x^2+4)} \equiv -\frac{2}{x+2} + \frac{2x+1}{x^2+4}$$

$$\frac{3x^2+6x+2}{(2x+3)(x+1)^2} \equiv -\frac{1}{2x+3} + \frac{2}{x+1} - \frac{1}{(x+1)^2}$$

Methods to Decompose a Rational Function

If denominator is product of LINEAR factors, then

$$\frac{f(x)}{(a_1x+b_1)(a_2x+b_2)} \equiv \frac{A}{a_1x+b_1} + \frac{B}{a_2x+b_2},$$

where $A, B \in \mathrm{R}$

Example:

Express $\dfrac{x+3}{x(x+1)}$ in partial fractions.

Example:

Express $f(x) = \dfrac{x+3}{(x-2)(x+4)}$ in partial fractions.

Example:

Express $f(x) = \dfrac{x}{(x-4)(x-1)}$ in partial fractions.

Example:

Express $\dfrac{3x}{(x-1)(x-2)(x-3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

If the rational function is IMPROPER, long division is carried out to convert it to a sum of a polynomial and a proper fraction.

Example:

Express $\dfrac{x^2-7}{(x-2)(x+1)}$ in partial fractions.

Example:

$$\text{Express } \frac{x^2+x+1}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

Example:

Express $\dfrac{2x^2+4x-3}{(x+1)(2x-3)}$ in partial fractions.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Express $f(x) = \dfrac{2-x}{(1+x)(1-2x)}$ in partial fractions.

Example:

Split $f(x)=\dfrac{3}{8x^2+6x+1}$ into partial fractions.

Example:

Express $\dfrac{6x^3 + x + 10}{(x-2)(x+2)(2x-1)}$ in partial fractions.