Example:

A sequence of numbers is given by $u_1 = 2, u_2 = 6$ and $u_n = 3u_{n-1} - 2u_{n-2}$ for $n \geq 3$. Prove by mathematical induction that $u_n = 2(2^n - 1)$.

Example:

A sequence of numbers is given by $u_1 = 1, u_2 = 5$ and $u_{n+2} = 5u_{n+1} - 6u_n$ for $n \geq 0$. Prove by mathematical induction that $u_n = 3^n - 2^n$.

Example:

A sequence of numbers is given by $u_1 = -1, u_2 = 0$ and $u_{n+2} = 6u_{n+1} - 9u_n$ for $n \geq 0$. Prove by mathematical induction that $u_n = (n-2)3^{n-1}$.

Example:

Prove by induction that $\sum_{r=1}^{n} 2r > n^2$.

Example:

Prove by induction that $\sum_{r=1}^{n} \frac{1}{\sqrt{r}} < 2\sqrt{n}$.

Example:

A sequence of numbers is given by $u_1 = 3$ and $u_{n+1} = \sqrt{2 + u_n}$ for $n \geq 0$. Prove by mathematical induction that $u_n > 2$ for $n \in \mathrm{N}$.

Example : (Bernoulli's Inequality)

Prove by induction that $(1+x)^n > 1+nx$ for all $x>0$ and for integers $n>1$.

<u>Homework</u>

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

A sequence of numbers is given by $u_0 = u_1 = -1$ and $u_{n+2} = 5u_{n+1} - 6u_n$ for $n \geq 0$. Prove by mathematical induction that $u_n = 3^n - 2^{n+1}$.

Example:

A sequence of numbers is given by $u_1 = 1$, and $u_{n+1} = u_n + (2n+1)$ for $n \geq 1$.

(a) Write down u_2, u_3 and u_4.

(b) Conjecture u_n in terms of n.

(c) Prove (b) by mathematical induction.

Example:

A sequence of numbers is given by $u_0 = 1, u_1 = 3$ and $u_n = 2u_{n-1} - 6u_{n-2}$ for $n \geq 2$. Prove by mathematical induction that $u_n = \sqrt{2}^k \left(\cos \frac{\pi}{4} k + 2 \sin \frac{\pi}{4} k \right)$.

Example:

Prove by induction that $\sum_{r=1}^{n-1} \frac{1}{r} > \ln n$ for $n \geq 2$.

$\left[\text{Hints}: e^{\frac{1}{k}} > 1 + \frac{1}{k}\right].$

Example:

A sequence of numbers is given by $u_0 = 0, u_1 - 1$, and $u_{n+2} = u_{n+1} + u_n$ for $n \geq 0$. Prove that $u_{n+1}{}^2 + u_n{}^2 = u_{2n+1}$.