Lesson 1 Rotation of Rigid Body

Derivation of equation of rotational motion:- $\tau = I\alpha$

Compare with equations of linear motion

1. When a torque acts on a body with MI (I) , producing an angular acceleration α, then

$$\tau = I\alpha$$

2. The KE of a body is equal to $\frac{1}{2} I\omega^2$.

3. The angular momentum of the body is $I\omega$.

$\tau = I\alpha$
$150 = 300\alpha$
$\alpha = 0.5 \text{ rads}^{-2}$

Example 1

A flywheel has MI of 300 kgm^2. It is rotating at 2 rads^{-1}. A constant torque of 150 Nm is applied to the flywheel for 10 s, in a sense such as to increase its rate of rotation. Find

a) The angular velocity at the end of this time $w = 2 + (0.5)(10) = 7 \text{ rads}^{-1}$

b) The angular displacement during this time $\phi = 2(10) + \frac{1}{2}(0.5)(10)^2 = 45 \text{ rad}$

c) The increase in KE of the flywheel.

$\frac{1}{2} I w^2 = \frac{1}{2}(300)(7)^2 - \frac{1}{2}(300)(2)^2$
$= 7350 - 600 = 6750$

Example 2

A gate has MI about its hinges of 500 kgm^2. The gate is open and perpendicular to the gateway. It is given an impulse so that it starts to close with an initial angular velocity of 3 rads^{-1}. The hinges provided a resisting torque of 400 Nm. Find

$400 = 500\alpha$

a) The time taken for the gate to shut $\frac{\pi}{2} = 3t + \frac{1}{2}(0.8)t^2$ $\alpha = 0.8$

$t = 0.49 \text{ s}$

b) The angular momentum of the gate at the instant that it shuts.

$w^2 = 3^2 + 2(0.8)(\frac{\pi}{2})$
$w = 3.4$
$Iw = (500)(3.4) = 1700$

Exercise 1

1. A rotary pump achieves its maximum angular speed of 30 rad s^{-1} in 5 seconds. Find the angular acceleration, assuming it is constant, and the number of revolutions turned through in reaching maximum speed.

$\alpha = \frac{w}{t} = \frac{30}{5} = 6 \text{ rad } s^{-2}$

[6 rad s^{-2}, 11.9]

$\phi = \frac{1}{2}(6)(5)^2 = 75$

n of revolutions $= \frac{75}{2\pi} = 11.9$

2. A revolving door rotating at half a revolution per second turns through 3 revolutions in coming to rest. Find the angular deceleration in rad s^{-2}, assuming that it is constant.
[0.262 rad s^{-2}]

$0 = (\pi)^2 + 2\alpha(6\pi)$

$\alpha = -\frac{\pi^2}{12\pi} = -0.262 \text{ rad } s^{-2}$

3. A turntable rotating at 33 revolutions per minute takes 5 seconds to come to rest. How many revolutions does it turn through in doing so?

$\alpha = -0.11$

$S = (0.55)(5) + \frac{1}{2}(-0.11)(5)^2 = 1.375$

If instead the turntable is initially rotating at 78 revolutions per minute, and the angular deceleration is unchanged, how long will it take to come to rest, and how many revolutions will it turn through? [1.375; 11.8s, 7.68]

$0 = 1.3 + (-0.11)t$ $\qquad$ $0 = (1.3)^2 + 2(-0.11)(\phi)$

$t = 11.8s$ $\qquad$ $\phi = 7.68$

4. A shaft is rotating at 1500 rad s^{-1}. After a change of gear, the speed of rotation adjusts to a new angular speed of 1200 rad s^{-1}. If the change takes half a second, calculate the angular deceleration assuming constant.

$1200 = 1500 + \alpha\left(\frac{1}{2}\right)$

[600 rad s^{-2}]

$\alpha = -600 \text{ rad } s^{-2}$

5. A carousel is rotating at 1 rad s^{-1}, and accelerating at a constant rate of 0.4 rad s^{-2}. How fast is it rotating after 5 seconds, and how many revolutions does it turn through in this time? [3 rad s^{-1}, 1.59]

$w = 1 + (0.4)(5) = 3 \text{ rad } s^{-1}$

$\phi = (1)(5) + \frac{1}{2}(0.4)(5)^2 = 10$

$= \frac{10}{2\pi} = 1.59$ revolutions.