1. $$\sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(n+1)2^n}$$

$n=1$: $$\sum_{r=1}^{1} \frac{r+2}{r(r+1)2^r} = \frac{1+2}{1(1+1)2^1} = \frac{3}{2\ 2} = \frac{3}{4}$$

$$1 - \frac{1}{(1+1)2^1} = 1 - \frac{1}{2\cdot 2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$n=k$. Assume $$\sum_{r=1}^{k} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(k+1)2^k}$$

$n=k+1$ $$\sum_{r=1}^{k+1} \frac{r+2}{r(r+1)2^r} = \frac{k+1+2}{(k+1)(k+1+1)2^{k+1}} + \sum_{r=1}^{k} \frac{r+2}{r(r+1)2^r}$$

$$= \frac{k+3}{(k+1)(k+2)2^{k+1}} + 1 - \frac{1}{(k+1)2^k}$$

$$= 1 + \frac{k+3}{(k+1)(k+2)2^{k+1}} - \frac{2(k+2)}{(k+1)(k+2)2^{k+1}}$$

$$= 1 + \frac{k+3-2k-4}{(k+1)(k+2)2^{k+1}}$$

$$= 1 + \frac{-k-1}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{\cancel{(k+1)}}{\cancel{(k+1)}(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+2)2^{k+1}} = 1 - \frac{1}{(k+1+1)2^{k+1}}$$

$$\sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(n+1)2^n}$$ for every positive integer n

$$\frac{r+2}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$$

$$= \frac{A(r+1) + Br}{r(r+1)}$$

$$\therefore\ r+2 = A(r+1) + Br$$

$$= (A+B)r + A$$

$$A + B = 1 \qquad A = 2$$

$$B = -1$$

$$\frac{r+2}{r(r+1)} = \frac{2}{r} - \frac{1}{r+1}$$

$$\frac{r+2}{r(r+1)2^r} = \left(\frac{2}{r} - \frac{1}{r+1}\right)\frac{1}{2^r}$$

$$\sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = \sum_{r=1}^{n}\left(\frac{2}{r} - \frac{1}{r+1}\right)\frac{1}{2^r}$$

$$= \left(\frac{2}{1} - \frac{1}{1+1}\right)\frac{1}{2^1}$$

$$+ \left(\frac{2}{2} - \frac{1}{2+1}\right)\frac{1}{2^2}$$

$$+ \left(\frac{2}{3} - \frac{1}{3+1}\right)\frac{1}{2^3}$$

$$\vdots$$

$$+ \left(\frac{2}{n-1} - \frac{1}{n-1+1}\right)\frac{1}{2^{n-1}}$$

$$+ \left(\frac{2}{n} - \frac{1}{n+1}\right)\frac{1}{2^n}$$

$$= \frac{2}{2} - \frac{1}{2 \cdot 2}$$

$$+ \frac{2}{2 \cdot 2^2} - \frac{1}{3 \cdot 4}$$

$$+ \frac{2}{3 \cdot 8} - \frac{1}{4 \cdot 8}$$

$$\vdots$$

$$+ \frac{2}{(n-1)2^{n-1}} - \frac{1}{n\,2^{n-1}}$$

$$+ \frac{2}{n\,2^n} - \frac{1}{(n+1)2^n}$$

$$= 1 - \frac{1}{4}$$

$$+ \frac{1}{4} - \frac{1}{12}$$

$$+ \frac{1}{12} - \frac{1}{32}$$

$$\vdots$$

$$+ \frac{1}{(n-1)2^{n-2}} - \frac{1}{n2^{n-1}}$$

$$+ \frac{1}{n\,2^{n-1}} - \frac{1}{(n+1)2^n}$$

$$= 1 - \frac{1}{(n+1)2^n}$$

$$\therefore \sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(n+1)2^n}$$

2 $$S_n = \sum_{r=1}^{n} (-1)^{r-1} r^3$$

$$= 1^3 - 2^3 + 3^3 - 4^3 + \dots + (-1)^{n-1} n^3$$

$$S_{2n} = \sum_{r=1}^{2n} (-1)^{r-1} r^3$$

$$= 1^3 - 2^3 + 3^3 - 4^3 + \dots + (-1)^{2n-2}(2n-1)^3 + (-1)^{2n-1}(2n)^3$$

$$= 1^3 - 2^3 + 3^3 - 4^3 + \dots + (2n-1)^3 - (2n)^3$$

$$= \sum_{r=1}^{n} (2r-1)^3 - (2r)^3$$

$$= \sum_{r=1}^{n} (4r^2 - 4r + 1)(2r-1) - 8r^3$$

$$= \sum_{r=1}^{n} (8r^3 - 8r^2 + 2r - 4r^2 + 4r - 1 - 8r^3)$$

$$= \sum_{r=1}^{n} (-12r^2 + 6r - 1)$$

$$= -12 \sum_{r=1}^{n} r^2 + 6 \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$= -12 \ \frac{n}{6}(n+1)(2n+1) + \frac{6n}{2}(n+1) - n$$

$$= -2n(n+1)(2n+1) + 3n(n+1) - n$$

$$= -2n(n+1)(-4n - 2 + 3) - n$$

$$= n(n+1)(-4n+1) - n$$

$$= n(-4n^2 + n - 4n + 1 - 1)$$

$$= n(-4n^2 - 3n)$$

$$= -n^2(4n+3)$$

$$S_{2n+1} = \sum_{r=1}^{2n+1} (-1)^{r-1} r^3$$

$$= (-1)^{2n}(2n+1)^3 + \sum_{r=1}^{2n} (-1)^{r-1} r^3$$

$$= (2n+1)^3 + S_{2n}$$

$$= -n^2(4n+3) + (2n+1)^3$$

$$= -n^2(4n+3) + 8n^3 + 12n^2 + 6n + 1$$

$$= 8n^3 + 12n^2 + 6n + 1 - 4n^3 - 3n^2$$

$$= 4n^3 + 9n^2 + 6n + 1$$

$$\frac{2S_{2n+1}}{n^3} = \frac{2}{n^3}(4n^3 + 9n^2 + 6n + 1)$$

$$= 8 + \frac{18}{n} + \frac{12}{n^2} + \frac{2}{n^3}$$

$$\lim_{n\to\infty} \frac{2S_{2n+1}}{n^3} = \lim_{n\to\infty}\left(8 + \frac{18}{n} + \frac{12}{n^2} + \frac{2}{n^3}\right)$$

$$= 8$$

3. $8x^3 + 12x^2 + 4x - 1 = 0$

$\alpha + \beta + \gamma = -\frac{12}{8} = -\frac{3}{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{4}{8} = \frac{1}{2}$

$\alpha\beta\gamma = \frac{1}{8}$

$2\alpha - 1, \ 2\beta - 1, \ 2\gamma - 1$

Let $u = 2\alpha - 1$

$\therefore \alpha = \frac{u+1}{2}$

Since α is a root $\therefore 8\alpha^3 + 12\alpha^2 + 4\alpha - 1 = 0$

$8\left(\frac{u+1}{2}\right)^3 + 12\left(\frac{u+1}{2}\right)^2 + 4\left(\frac{u+1}{2}\right) - 1 = 0$

$\frac{8(u^3 + 3u^2 + 3u + 1)}{8} + 12\frac{(u^2 + 2u + 1)}{4} + 2(u+1) - 1 = 0$

$u^3 + 3u^2 + 3u + 1 + 3(u^2 + 2u + 1) + 2u + 2 - 1 = 0$

$u^3 + 3u^2 + 3u + 1 + 3u^2 + 6u + 3 + 2u + 1 = 0$

$u^3 + 6u^2 + 11u + 5 = 0$

$S_n = (2\alpha - 1)^n + (2\beta - 1)^n + (2\gamma - 1)^n$

$u^3 + 6u^2 + 11u + 5 = 0$

$a = 1 \quad b = 6 \quad c = 11 \quad d = 5$

$aS_{3+r} + bS_{2+r} + cS_{1+r} + dS_r = 0$

$S_{3+r} + 6S_{2+r} + 11S_{1+r} + 5S_r = 0$

$r=0$: $S_3 + 6S_2 + 11S_1 + 5S_0 = 0$

$S_0 = 3$

$S_1 = 2\alpha - 1 + 2\beta - 1 + 2r - 1 = 2(\alpha+\beta+r) - 3 = -6$

$S_2 = (2\alpha-1)^2 + (2\beta-1)^2 + (2r-1)^2$

$= 4\alpha^2 - 4\alpha + 1 + 4\beta^2 - 4\beta + 1 + 4r^2 - 4r + 1$

$= 4(\alpha^2+\beta^2+r^2) - 4(\alpha+\beta+r) + 3$

$= 4\left[(\alpha+\beta+r)^2 - 2(\alpha\beta + \alpha r + \beta r)\right] - 4(\alpha+\beta+r) + 3$

$= 4\left[\frac{9}{4} - 2\left(\frac{1}{2}\right)\right] - 4\left(\frac{-3}{2}\right) + 3$

$= 9 - 4 + 6 + 3$

$= 14$

$S_3 + 6(14) + 11(-6) + 5(3) = 0$

$S_3 = -33$

$r=-2$: $S_1 + 6S_0 + 11S_{-1} + 5S_{-2} = 0$

$-6 + 6(3) + 11S_{-1} + 5S_{-2} = 0$

$r=-1$: $S_2 + 6S_1 + 11S_0 + 5S_{-1} = 0$

$14 + 6(-6) + 11(3) + 5S_{-1} = 0$

$S_{-1} = \frac{-11}{5}$

$\therefore -6 + 6(3) + 11\cancel{S_{-1}}\left(\frac{-11}{5}\right) + 5S_{-2} = 0$

$5S_{-2} = \frac{61}{5}$

$S_{-2} = \frac{61}{25}$

4. $C: \quad y = \dfrac{x^2+2x-3}{(\lambda x+1)(x-7)}$

i) $\lambda = 0: \quad y = \dfrac{x^2+2x-3}{x-7}$

$$\frac{x^2+2x-3}{x-7} = Ax+B + \frac{C}{x-7}$$
$$= \frac{(Ax+B)(x-7)+C}{x-7}$$

$$x^2+2x-3 = (Ax+B)(x-7)+C$$
$$= Ax^2+Bx-7Ax-7B+C$$
$$= Ax^2+(B-7A)x+C-7B$$

$A=1 \quad B-7A=2 \quad C-7B=-3$

$B=9 \quad C=60$

$$y = x+9+\frac{60}{x-7}$$

$x \to \pm\infty \quad y \to x+9$

$x \to 7 \quad y \to \pm\infty$

Asymptotes: $y = x+9$

$x = 7$

ii) $y = \dfrac{x^2+2x+3}{(\lambda x+1)(x-7)}$

$$= \frac{x^2+2x+3}{\lambda x^2+x-7\lambda x-7}$$

$$= \frac{x^2 + 2x - 3}{\lambda x^2 + (1 - 7\lambda) x - 7}$$

$$= \frac{\dfrac{x^2 + 2x - 3}{x^2}}{\dfrac{\lambda x^2 + (1-7\lambda) x - 7}{x^2}}$$

$$= \frac{1 + \dfrac{2}{x} - \dfrac{3}{x^2}}{\lambda + \dfrac{(1-7\lambda)}{x} - \dfrac{7}{x^2}}$$

$x \to \pm\infty \qquad y \to \dfrac{1}{\lambda}$

$x \to 7 \qquad y \to \pm\infty$

$x \to -\dfrac{1}{\lambda} \qquad y \to \pm\infty$

Asymptotes : $y = \dfrac{1}{\lambda}$

$x = 7$

$x = -\dfrac{1}{\lambda}$

iii) $\lambda = -1$

$$y = \frac{x^2 + 2x - 3}{(-x+1)(x-7)}$$

$$= \frac{(x+3)(x-1)}{(-x+1)(x-7)}$$

$$= \frac{(x+3)(x-1)}{-(x-1)(x-7)}$$

$$= \frac{x+3}{-(x-7)} \quad \text{if} \quad x \neq 1$$

$x \neq 1$: $\quad y = \dfrac{x+3}{7-x}$

$$= \frac{\frac{x+3}{x}}{\frac{7-x}{x}}$$

$$= \frac{1+\frac{3}{x}}{\frac{7}{x}-1}$$

$x \to \pm\infty \quad y \to \frac{1}{-1} = -1$

$$y = \frac{x+3}{7-x}$$

$$(7-x)y = x+3$$

$$7y - xy = x+3$$

$$x + xy = 7y - 3$$

$$x(1+y) = 7y-3$$

$$x = \frac{7y-3}{1+y}$$

$$= \frac{\frac{7y - 3}{y}}{\frac{1+y}{y}}$$

$$= \frac{7 - \frac{3}{y}}{\frac{1}{y} + 1}$$

$y \to \pm\infty \quad x \to 7$

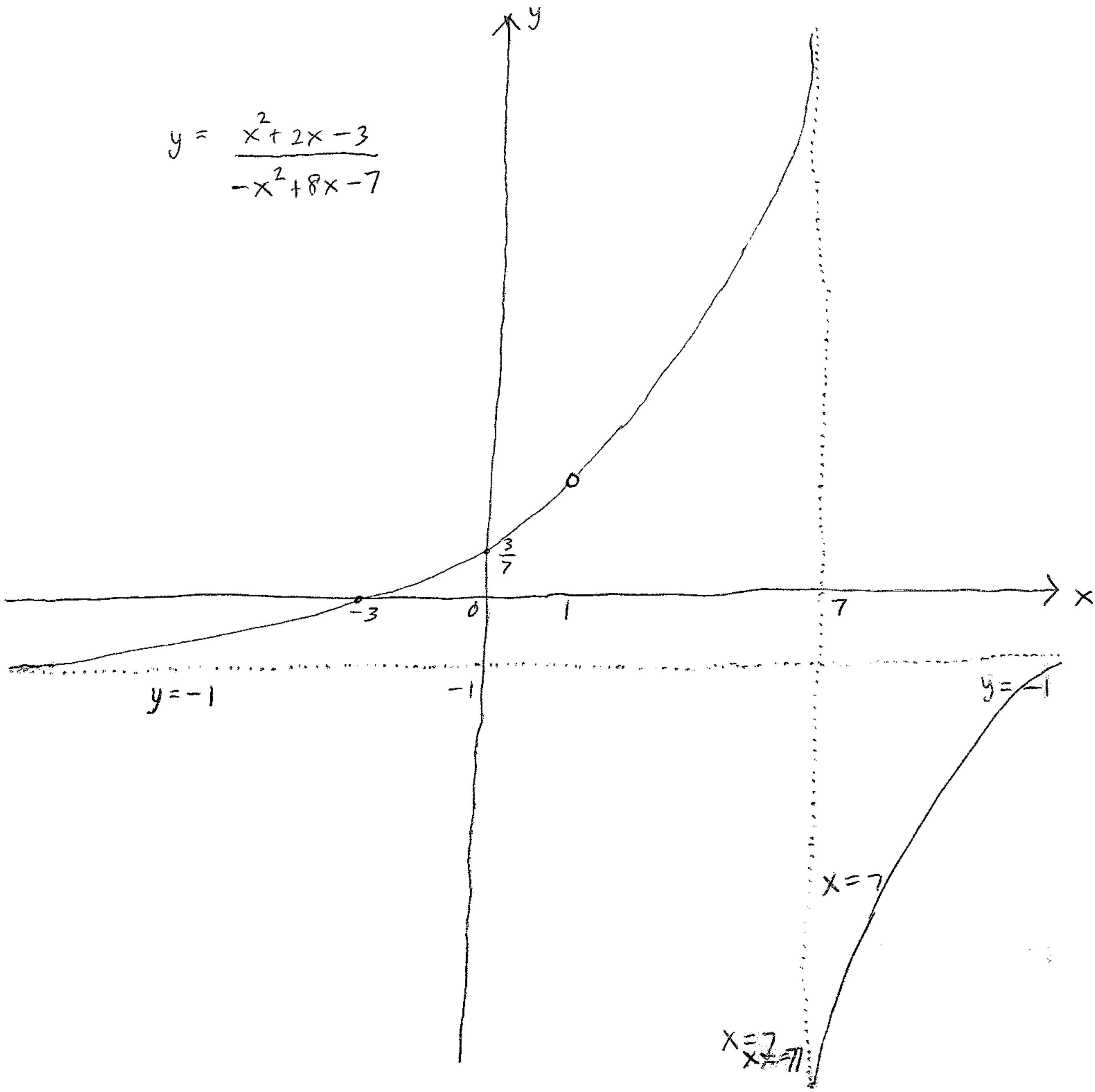
Asymptotes . $x = 7$
$y = -1$
$x = 1$

$$y = \frac{x+3}{7-x}$$

$x = 0 \quad y = \frac{3}{7}$

$y = 0 \quad x = -3$

Intersection points $\left(0, \frac{3}{7}\right)$ $(-3, 0)$



- Intersection points
- Undefined point

5. i) $\ell_1: \ \underset{\sim}{r} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} + s\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix}$

$\ell_2: \ \underset{\sim}{r} = \begin{pmatrix} 1+5\cos t \\ -1-5\sin t \\ -14 \end{pmatrix} + u\begin{pmatrix} 15 \\ 8 \\ -3 \end{pmatrix}$

Let $A(1,-2,-2)$ and $B(1+5\cos t, -1-5\sin t, -14)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 1+5\cos t \\ -1-5\sin t \\ -14 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5\cos t \\ 1-5\sin t \\ -12 \end{pmatrix}$

$\because$ $PQ \perp \ell_1$ and $PQ \perp \ell_2$

$\therefore$ $\overrightarrow{PQ}$ // direction of $\ell_1 \times \ell_2$

$\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 15 \\ 8 \\ -3 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & -4 & -2 \\ 15 & 8 & -3 \end{vmatrix} = \begin{pmatrix} 28 \\ -21 \\ 84 \end{pmatrix} = 7\begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$

$\overrightarrow{PQ} = \lambda\begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$

$|\overrightarrow{PQ}| = \lambda\sqrt{4^2 + (-3)^2 + 12^2} = \lambda\sqrt{169} = 13\lambda$

$\overrightarrow{AB} \cdot \overrightarrow{PQ} = |\overrightarrow{AB}|\,|\overrightarrow{PQ}| \cos\theta$

$\begin{pmatrix} 5\cos t \\ 1-5\sin t \\ -12 \end{pmatrix} \cdot \lambda\begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} = |\overrightarrow{AB}|\ 13\lambda \cos\theta$

$(20\cos t + 15\sin t - 147)\lambda = 13\lambda|\overrightarrow{AB}|\cos\theta$

$\therefore |\overrightarrow{AB}|\cos\theta = \dfrac{20\cos t + 15\sin t - 147}{13}$

$$PQ = |\vec{PQ}| = \frac{|\vec{AB} \cdot \vec{PQ}|}{|\vec{PQ}|}$$

$$= \left| |\vec{AB}| \cos\theta \right|$$

$$= \left| \frac{20\cos t + 15\sin t - 147}{13} \right|$$

$$= \frac{|20\cos t + 15\sin t - 147|}{13}$$

ii) $20\cos t + 15\sin t = R\sin(t+\alpha)$

$$= R\sin t\cos\alpha + R\cos t\sin\alpha$$

$R\sin\alpha = 20 \qquad R\cos\alpha = 15$

$R^2\sin^2\alpha + R^2\cos^2\alpha = 20^2 + 15^2$

$R^2(\sin^2\alpha + \cos^2\alpha) = 400 + 225$

$R^2(1) = 625$

$R = \sqrt{625}$

$= 25$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{20}{15}$

$= \frac{4}{3}$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$$\therefore \quad PQ = \frac{|20\cos t + 15\sin t - 147|}{13}$$

$$= \frac{\left|25\sin\left(t + \tan^{-1}\frac{4}{3}\right) - 147\right|}{13}$$

$$\because \quad -1 \leq \sin\left(t + \tan^{-1}\frac{4}{3}\right) \leq 1$$

$$-25 \leq 25\sin\left(t + \tan^{-1}\frac{4}{3}\right) \leq 25$$

$$-25 - 147 \leq 25\sin\left(t + \tan^{-1}\frac{4}{3}\right) - 147 \leq 25 - 147$$

$$-172 \leq 25\sin\left(t + \tan^{-1}\frac{4}{3}\right) - 147 \leq -122$$

$$-(-172) \geq -\left(25\sin\left(t + \tan^{-1}\frac{4}{3}\right) - 147\right) \geq -(-122)$$

$$172 \geq 147 - 25\sin\left(t + \tan^{-1}\frac{4}{3}\right) \geq 122$$

$$\frac{122}{13} \leq \frac{147 - 25\sin\left(t + \tan^{-1}\frac{4}{3}\right)}{13} \leq \frac{172}{13}$$

$$PQ = \frac{\left|25\sin\left(t + \tan^{-1}\frac{4}{3}\right)\right| - 147|}{13}$$

$$= \frac{\left|-\left(147 - 25\sin\left(t + \tan^{-1}\frac{4}{3}\right)\right)\right|}{13}$$

$$= \frac{\left|147 - 25\sin\left(t + \tan^{-1}\frac{4}{3}\right)\right|}{13}$$

$$\frac{122}{13} \leqslant \frac{|147 - 25\sin(t + \tan^{-1}\frac{4}{3})|}{13} \leqslant \frac{172}{13}$$

$$\frac{122}{13} \leqslant PQ \leqslant \frac{172}{13}$$

$$\therefore PQ_{min} = \frac{122}{13} \quad \& \quad PQ_{max} = \frac{172}{13}$$

$PQ \neq 0$

$\therefore$ l_1 & l_2 do not intersect.

iii) Since Π_1 contains l_1 and PQ

$\therefore \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$ is normal to Π_1

$$\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \hat{k} \\ 3 & -4 & -2 \\ 4 & -3 & 12 \end{vmatrix} = \begin{pmatrix} -54 \\ -44 \\ 7 \end{pmatrix}$$

Since Π_2 contains l_2 and PQ

$\therefore \begin{pmatrix} 15 \\ 8 \\ -3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$ is normal to Π_2

$$\begin{pmatrix} 15 \\ 8 \\ -3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \hat{k} \\ 15 & 8 & -3 \\ 4 & -3 & 12 \end{vmatrix} = \begin{pmatrix} 87 \\ -192 \\ -77 \end{pmatrix}$$

$$\begin{pmatrix} -54 \\ -44 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 87 \\ -192 \\ -77 \end{pmatrix} = \left|\begin{pmatrix} -54 \\ -44 \\ 7 \end{pmatrix}\right| \left|\begin{pmatrix} 87 \\ -192 \\ -77 \end{pmatrix}\right| \cos\theta$$

$$-3211 = \sqrt{4901}\sqrt{50362}\cos\theta$$

$$\cos\theta = \frac{-3211}{\sqrt{4901}\sqrt{50362}}$$

$$90^\circ < \theta < 180^\circ$$

$$\therefore \quad 0^\circ < 180^\circ - \theta < 90^\circ$$

$$\cos(180^\circ - \theta) = -\cos\theta$$

$$= \frac{3211}{\sqrt{4901}\sqrt{50362}}$$

$$\sin(180^\circ - \theta) = \frac{\sqrt{236513641}}{\sqrt{4901}\sqrt{50362}}$$

$$\tan(180^\circ - \theta) = \frac{\sqrt{236513641}}{3211}$$

6. i) $\ell_1: \underset{\sim}{r} = \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} + s\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

$\ell_2: \underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$

$\overrightarrow{OP} = \begin{pmatrix} 3+3s \\ -5+4s \\ -4+2s \end{pmatrix} \qquad \overrightarrow{OQ} = \begin{pmatrix} 2+t \\ 3-t \\ 5-4t \end{pmatrix}$

$\therefore \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 2+t \\ 3-t \\ 5-4t \end{pmatrix} - \begin{pmatrix} 3+3s \\ -5+4s \\ -4+2s \end{pmatrix}$

$= \begin{pmatrix} t-3s-1 \\ -t-4s+8 \\ -4t-2s+9 \end{pmatrix}$

Since $\overrightarrow{PQ}$ is $\perp$ to ℓ_1 and $\overrightarrow{PQ}$ is $\perp$ to ℓ_2

$\therefore \overrightarrow{PQ}$ // direction of $\ell_1 \times \ell_2$

$\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & 4 & 2 \\ 1 & -1 & -4 \end{vmatrix} = \begin{pmatrix} -14 \\ 14 \\ -7 \end{pmatrix} = -7\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

$\overrightarrow{PQ} = c\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} t-3s-1 \\ -t-4s+8 \\ -4t-2s+9 \end{pmatrix} = \begin{pmatrix} 2c \\ -2c \\ c \end{pmatrix}$

$$\left.\begin{aligned} t-3s-1 &= 2c \\ -t-4s+8 &= -2c \\ -4t-2s+9 &= c \end{aligned}\right\}$$

$$\left.\begin{aligned} t - 3s - 2c &= 1 \\ -t - 4s + 2c &= -8 \\ -4t - 2s - c &= -9 \end{aligned}\right\}$$

$$\left.\begin{aligned} t - 3s - 2c &= 1 \\ -7s &= -7 \\ -14s - 9c &= -5 \end{aligned}\right\}$$

$s = 1 \quad c = -1 \quad t = 2$

$P(6,-1,-2) \quad Q(4,1,-3)$

$$\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

$$|\overrightarrow{PQ}| = \sqrt{(-2)^2 + 2^2 + (-1)^2}$$
$$= 3$$

(iii) $A(3,-5,-4)$

$$\overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP}$$
$$= \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$$

Since Π_1 contains $\overrightarrow{PA}$ and $\overrightarrow{PQ}$

$\overrightarrow{PQ} \times \overrightarrow{PA}$ is normal to Π_1

$$\overrightarrow{PQ} \times \overrightarrow{PA} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -2 & 2 & -1 \\ -3 & -4 & -2 \end{vmatrix} = \begin{pmatrix} -8 \\ -1 \\ 14 \end{pmatrix}$$

iii) Since Π_1 contains P and $\begin{pmatrix}-8\\-1\\14\end{pmatrix}$ is normal to Π_1

$$\underset{\sim}{r} \cdot \begin{pmatrix}-8\\-1\\14\end{pmatrix} = \overrightarrow{OP} \cdot \begin{pmatrix}-8\\-1\\14\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}8\\1\\-14\end{pmatrix} = \begin{pmatrix}6\\-1\\-2\end{pmatrix} \cdot \begin{pmatrix}8\\1\\-14\end{pmatrix}$$

$$8x + y - 14z = 75$$

$B(2, 3, 5)$

The normal of Π_1 passing through B is

$$\underset{\sim}{r} = \begin{pmatrix}2\\3\\5\end{pmatrix} + r\begin{pmatrix}8\\1\\-14\end{pmatrix}$$

This perpendicular intersects Π_1 at

$$8(2 + 8r) + 3 + r - 14(5 - 14r) = 75$$

$$16 + 64r + 3 + r - 70 + 196r = 75$$

$$r = \frac{14}{29}$$

The perpendicular distance from B to Π_1 is

$$\frac{14}{29}\left|\begin{pmatrix}8\\1\\-14\end{pmatrix}\right| = \frac{14}{29}\sqrt{8^2 + 1^2 + (-14)^2}$$

$$= \frac{14}{29}\sqrt{261}$$

iv) $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$

$$= \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ 4 \\ 7 \end{pmatrix}$$

Since Π_2 contains PB and PQ

$\overrightarrow{PQ} \times \overrightarrow{PB}$ is normal to Π_2

$$\overrightarrow{PQ} \times \overrightarrow{PB} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -2 & 2 & -1 \\ -4 & 4 & 7 \end{vmatrix} = \begin{pmatrix} 18 \\ 18 \\ 0 \end{pmatrix} = 18\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -1 \\ 14 \end{pmatrix} = \left|\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right| \left|\begin{pmatrix} -8 \\ -1 \\ 14 \end{pmatrix}\right| \cos\theta$$

$$-8 - 1 = \sqrt{2}\sqrt{261} \cos\theta$$

$$\cos\theta = \frac{-9}{\sqrt{2}\sqrt{261}}$$

$$= \frac{-9}{\sqrt{2}(3)\sqrt{29}}$$

$$= \frac{-3}{\sqrt{2}\sqrt{29}}$$

$$90^\circ < \theta < 180^\circ$$

$$\therefore 0^\circ < 180^\circ - \theta < 90^\circ$$

$$\cos(180^\circ - \theta) = -\cos\theta = \frac{3}{\sqrt{2}\sqrt{29}}$$

$$\cancel{\sin(180^\circ - \theta) = \frac{3}{\sqrt{2}\sqrt{29}}}$$

$$\sin(180^\circ - \theta) = \sqrt{2}\sqrt{29}$$

$$\tan(180^\circ - \theta) = \frac{\sqrt{2}\sqrt{2}}{2}$$