1. $A\underset{\sim}{x} = \lambda \underset{\sim}{x}$

$$A^{-1}A\underset{\sim}{x} = \lambda A^{-1}\underset{\sim}{x}$$

$$\underset{\sim}{x} = \lambda A^{-1}\underset{\sim}{x}$$

Since $\underset{\sim}{x} \neq \underset{\sim}{0}$

$$\lambda \neq 0$$

$$\lambda A^{-1}\underset{\sim}{x} = \underset{\sim}{x}$$

$$A^{-1}\underset{\sim}{x} = \frac{1}{\lambda}\underset{\sim}{x}$$

$\underset{\sim}{x}$ is an eigenvector of A^{-1} with corresponding eigenvalue $\frac{1}{\lambda}$.

2. $\left.\begin{array}{l} kx + y + z = 0 \\ x + ky + z = 0 \\ x + y + kz = 0 \end{array}\right\}$

$\left.\begin{array}{l} x + y + kz = 0 \\ x + ky + z = 0 \\ kx + y + z = 0 \end{array}\right\}$

$-① + ②:$
$-k \times ① + ③:$

$\left.\begin{array}{l} x + y + kz = 0 \\ (k-1)y + (1-k)z = 0 \\ (1-k)y + (1-k^2)z = 0 \end{array}\right\}$

$② + ③:$ $\left.\begin{array}{l} x + y + z = 0 \\ (k-1)y + (1-k)z = 0 \\ (2-k^2-k)z = 0 \end{array}\right\}$

$(2 - k^2 - k)z = 0$

$2 - k - k^2 = 0$

$k^2 + k - 2 = 0$

$(k+2)(k-1) = 0$

$k = 1, -2$

$k = -2$: $0z = 0$

Let $z = s$, $s \in R$

$$-3y + 3z = 0$$

$$y = z$$

$$= s$$

$$x + y - 2z = 0$$

$$x + s - 2s = 0$$

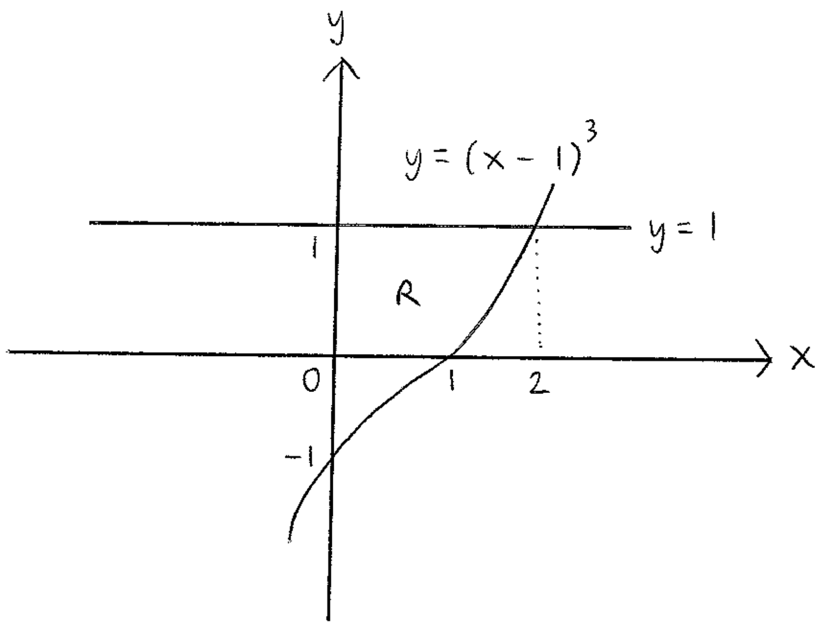
$$x = s$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s \\ s \\ s \end{pmatrix}$$

$$= s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore$ The line of intersection of the planes Π_1, Π_2 and Π_3 is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

3



$$\text{Area}, A = 2 - \int_1^2 (x-1)^3 \, dx$$

$$= 2 - \left[\frac{(x-1)^4}{4}\right]_1^2$$

$$= 2 - \left(\frac{1}{4} - 0\right)$$

$$= \frac{7}{4}$$

$$A\bar{x} = \int_0^2 xy \, dx - \int_1^2 xy \, dx$$

$$= \int_0^2 x \, dx - \int_1^2 x(x-1)^3 \, dx$$

$u = x \qquad dv = (x-1)^3 \, dx$

$du = dx \qquad v = \dfrac{(x-1)^4}{4}$

$$= \left[\frac{x^2}{2}\right]_0^2 - \left(\left[\frac{x(x-1)^4}{4}\right]_1^2 - \int_1^2 \frac{(x-1)^4}{4} dx\right)$$

$$= 2 - 0 - \left(\frac{1}{2} - 0 - \left[\frac{(x-1)^5}{20}\right]_1^2\right)$$

$$= 2 - \left(\frac{1}{2} - \left(\frac{1}{20} - 0\right)\right)$$

$$= 2 - \frac{9}{20}$$

$$= \frac{31}{20}$$

$$\bar{x} = \frac{31}{35}$$

$$A\bar{y} = \int_0^2 \frac{y^2}{2}\,dx - \int_1^2 \frac{y^2}{2}\,dx$$

$$= \int_0^2 \frac{1}{2}\,dx - \int_1^2 \frac{(x-1)^6}{2}\,dx$$

$$= \left[\frac{x}{2}\right]_0^2 - \left[\frac{(x-1)^7}{14}\right]_1^2$$

$$= 1 - \left(\frac{1}{14} - 0\right)$$

$$= \frac{13}{14}$$

$$\bar{y} = \frac{26}{49}$$

$\therefore$ The centroid of R has coordinates $\left(\frac{31}{35}, \frac{26}{49}\right)$

$$\text{Volume, } V = \int_0^1 \pi x^2 \, dy$$

$$= \int_0^1 \pi \left(y^{\frac{2}{3}} + 2y^{\frac{1}{3}} + 1\right) dy$$

$$= \pi \left[\frac{3y^{\frac{5}{3}}}{5} + \frac{3y^{\frac{4}{3}}}{2} + y \right]_0^1$$

$$= \pi \left(\frac{3}{5} + \frac{3}{2} + 1 - 0 \right)$$

$$= \frac{31\pi}{10}$$

$$V\bar{y} = \int_0^1 \pi x^2 y \, dy$$

$$= \int_0^1 \pi y \left(y^{\frac{2}{3}} + 2y^{\frac{1}{3}} + 1 \right) dy$$

$$= \int_0^1 \pi \left(y^{\frac{5}{3}} + 2y^{\frac{4}{3}} + y \right) dy$$

$$= \pi \left[\frac{3y^{\frac{8}{3}}}{8} + \frac{6y^{\frac{7}{3}}}{7} + \frac{y^2}{2} \right]_0^1$$

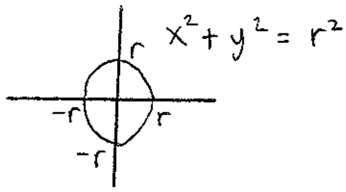
$$= \pi \left(\frac{3}{8} + \frac{6}{7} + \frac{1}{2} - 0 \right)$$

$$= \frac{97\pi}{56}$$

$$\bar{y} = \frac{485}{868}$$

$\therefore$ The coordinates of the centroid of the solid generated when R is revolved around the y-axis is $\left(0, \frac{485}{868}\right)$.

4.



$$x^2+y^2=r^2$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y} = \frac{-x}{\sqrt{r^2-x^2}}$$

i) Circumference $= 2\int_{-r}^{r} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$$= 2\int_{-r}^{r} \sqrt{1+\frac{x^2}{r^2-x^2}}\, dx$$

$$= 2\int_{-r}^{r} \frac{r}{\sqrt{r^2-x^2}}\, dx$$

$$= 2r\left[\sin^{-1}\left(\frac{x}{r}\right)\right]_{-r}^{r}$$

$$= 2r\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2r\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= 2\pi r$$

ii) surface area of sphere

$$= \int_{-r}^{r} 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-r}^{r} 2\pi\sqrt{r^2-x^2} \times \frac{r}{\sqrt{r^2-x^2}}\, dx$$

$$= \int_{-r}^{r} 2\pi r\, dx$$

$$= 2\pi r\left[x\right]_{-r}^{r}$$

$$= 4\pi r^2$$

5. $$\frac{d^2y}{dx^2} + \frac{5dy}{dx} + 4y = 3x^2 - x + 6$$

$$\frac{d^2y}{dx^2} + \frac{5dy}{dx} + 4y = 0$$

$$m^2 + 5m + 4 = 0$$

$$(m+1)(m+4) = 0$$

$$m = -1, -4$$

$\therefore$ The complementary function, y_c, is

$$y_c = Ae^{-x} + Be^{-4x}$$

The particular integral, y_p, is given by

$$y_p = Cx^2 + Dx + E$$

$$\frac{dy_p}{dx} = 2Cx + D$$

$$\frac{d^2y_p}{dx^2} = 2C$$

$$\frac{d^2y_p}{dx^2} + \frac{5dy_p}{dx} + 4y_p = 2C + 5(2Cx + D)$$

$$+ 4(Cx^2 + Dx + E)$$

$$= 4Cx^2 + (10C + 4D)x$$

$$+ 2C + 5D + 4E$$

$$= 3x^2 - x + 6$$

$$4C = 3 \qquad 10C + 4D = -1 \qquad 2C + 5D + 4E = 6$$

$$C = \frac{3}{4} \qquad 5C + 2D = \frac{-1}{2} \qquad \frac{3}{2} - \frac{85}{8} + 4E = 6$$

$$\frac{15}{4} + 2D = \frac{-1}{2} \qquad 4E = \frac{121}{8}$$

$$2D = \frac{-17}{4} \qquad E = \frac{121}{32}$$

$$D = \frac{-17}{8}$$

$$\therefore y_p = \frac{3x^2}{4} - \frac{17x}{8} + \frac{121}{32}$$

$$y = y_c + y_p$$

$$= Ae^{-x} + Be^{-4x} + \frac{3x^2}{4} - \frac{17x}{8} + \frac{121}{32}$$

$\therefore$ The general solution of the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 3x^2 - x + 6$ is $y = Ae^{-x} + Be^{-4x} + \frac{3x^2}{4} - \frac{17x}{8} + \frac{121}{32}$.

6 i) $C: \sin(x+y) = \cos xy$

$$\frac{d}{dx}(\sin(x+y)) = \frac{d}{dx}(\cos xy)$$

$$\cos(x+y)\frac{d}{dx}(x+y) = -\sin xy \frac{d}{dx}(xy)$$

$$\cos(x+y)\left(1+\frac{dy}{dx}\right) = -\sin xy\left(x\frac{dy}{dx}+y\right)$$

At $(\pi, 0)$: $-\left(1+\frac{dy}{dx}\right) = 0$

$$\frac{dy}{dx} = -1$$

$$\frac{d}{dx}\left(\cos(x+y)\left(1+\frac{dy}{dx}\right)\right) = \frac{d}{dx}\left(-\sin xy\left(x\frac{dy}{dx}+y\right)\right)$$

$$-\sin(x+y)\frac{d}{dx}(x+y)\left(1+\frac{dy}{dx}\right) + \cos(x+y)\frac{d^2y}{dx^2}$$

$$= -\cos xy\frac{d}{dx}(xy)\left(x\frac{dy}{dx}+y\right) - \sin xy\frac{d}{dx}\left(x\frac{dy}{dx}+y\right)$$

$$-\sin(x+y)\left(1+\frac{dy}{dx}\right)^2 + \cos(x+y)\frac{d^2y}{dx^2}$$

$$= -\cos xy\left(x\frac{dy}{dx}+y\right)^2 - \sin xy\left(x\frac{d^2y}{dx^2}+2\frac{dy}{dx}\right)$$

At $(\pi, 0)$, $\frac{dy}{dx} = -1$:

$$-\frac{d^2y}{dx^2} = -\pi^2$$

$$\frac{d^2y}{dx^2} = \pi^2$$

ii) $y = x^3e^x$

$$\frac{d^ny}{dx^n} = x^3e^x + 3nx^2e^x + 3n(n-1)xe^x$$
$$+ n(n-1)(n-2)e^x$$

when $n = 1$: $\frac{d^1y}{dx^1} = \frac{dy}{dx}$

$$= x^3e^x + 3x^2e^x$$
$$= x^3e^x + 3\cdot 1x^2e^x + 3\cdot 1\cdot 0xe^x$$
$$+ 1\cdot 0\cdot(-1)e^x$$

Assume the statement is true when $n = k$.

$n = k$: $$\frac{d^ky}{dx^k} = x^3e^x + 3kx^2e^x + 3k(k-1)xe^x$$
$$+ k(k-1)(k-2)e^x$$

when $n = k+1$:

$$\frac{d^{k+1}y}{dx^{k+1}} = x^3e^x + 3(k+1)x^2e^x + 3(k+1)kxe^x$$
$$+ (k+1)k(k-1)e^x$$ (what needs to be proved)

$$\frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)$$

$$= \frac{d}{dx}\left(x^3e^x + 3kx^2e^x + 3k(k-1)xe^x + k(k-1)(k-2)e^x\right)$$

$$= x^3e^x + 3x^2e^x + 3kx^2e^x + 6kxe^x + 3k(k-1)xe^x + 3k(k-1)e^x + k(k-1)(k-2)e^x$$

$$= x^3e^x + 3(k+1)x^2e^x + 3k(k+1)xe^x + (k+1)k(k-1)e^x$$

$$\frac{d^n y}{dx^n} = x^3e^x + 3nx^2e^x + 3n(n-1)xe^x + n(n-1)(n-2)e^x$$

for every positive integer n.

7. i) $\begin{pmatrix} 1 & 2 & -4 & 1 \\ 2 & 3 & -7 & -2 \\ -1 & -3 & 5 & -5 \\ 2 & 5 & -9 & 6 \end{pmatrix} \xrightarrow[R_4 : R_4 - R_2]{R_2 : R_2 - 2R_1,\ R_3 : R_3 + R_1} \begin{pmatrix} 1 & 2 & -4 & 1 \\ 0 & -1 & 1 & -4 \\ 0 & -1 & 1 & -4 \\ 0 & 2 & -2 & 8 \end{pmatrix}$

$\xrightarrow[R_3 : R_3 - R_2,\ R_4 : R_4 + 2R_2]{R_1 : R_1 + 2R_2,\ R_2 : -R_2} \begin{pmatrix} 1 & 0 & -2 & -7 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Dimension of range space of $T = 2$.

ii) Let $x_3 = s$ $x_2 = x_3 - 4x_4$ $x_1 = 2x_3 + 7x_4$

$x_4 = t$ $= s - 4t$ $= 2s + 7t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} 7 \\ -4 \\ 0 \\ 1 \end{pmatrix} t$$

$$\text{Basis} = \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ -4 \\ 0 \\ 1 \end{pmatrix} \right\}$$

iii) Pivots at column 1 and 2

$$\text{Basis} = \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -3 \\ 5 \end{pmatrix} \right\}$$

8. $$\frac{7r-3}{r^3-r} = \frac{2}{r-1} + \frac{3}{r} - \frac{5}{r+1}$$

$$S = 2\sum_{r=2}^{n} \frac{7r-3}{(r-1)r(r+1)}$$

$$= 2\sum_{r=2}^{n} \left(\frac{2}{r-1} + \frac{3}{r} - \frac{5}{r+1}\right)$$

$$= 2\sum_{r=2}^{n} \left[\left(\frac{2}{r-1} - \frac{2}{r}\right) + \left(\frac{5}{r} - \frac{5}{r+1}\right)\right]$$

$$= 2\left[\left(2 - \frac{2}{n}\right) + \left(\frac{5}{2} - \frac{5}{n+1}\right)\right]$$

$$= 9 - \frac{4}{n} - \frac{10}{n+1}$$

As $n \to \infty$, $\frac{4}{n} \to 0$ & $\frac{10}{n+1} \to 0$

$\therefore$ Sum to infinity $= 9$

9. $$\left(\frac{2}{3z-8}+9\right)^4 = 1$$

$$= \cos 0 + i\sin 0$$

$$= \cos 2k\pi + i\sin 2k\pi, \quad k \in \mathbb{Z}$$

$$\frac{2}{3z-8}+9 = (\cos 2k\pi + i\sin 2k\pi)^{\frac{1}{4}}$$

$$= \cos\frac{k\pi}{2} + i\sin\frac{k\pi}{2}, \quad k = 0, 1, 2, 3$$

$$= 1, -1, i, -i$$

$$\frac{2}{3z-8} = -8, -10, -9+i, -9-i$$

$$3z - 8 = \frac{2}{-8}, \frac{2}{-10}, \frac{2}{-9+i}, \frac{2}{-9-i}$$

$$= -\frac{1}{4}, -\frac{1}{5}, \frac{-9-i}{41}, \frac{-9+i}{41}$$

$$3z = \frac{31}{4}, \frac{39}{5}, \frac{319-i}{41}, \frac{319+i}{41}$$

$$z = \frac{31}{12}, \frac{39}{15}, \frac{319-i}{123}, \frac{319+i}{123}$$

$\therefore$ The solutions of the equation

$\left(\frac{2}{3z-8}+9\right)^4$ are

$$\frac{31}{12}, \frac{39}{15}, \frac{319-i}{123}, \frac{319+i}{123}$$

$$\left(\frac{2}{z}-1\right)^n = 1$$

$$= \cos 0 + i\sin 0$$

$$= \cos 2k\pi + i\sin 2k\pi, \quad k \in \mathbb{Z}$$

$$\frac{2}{z}-1 = (\cos 2k\pi + i\sin 2k\pi)^{\frac{1}{n}}$$

$$= \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, \quad k = 0, 1, \cdots, n-1.$$

$$= 2\cos^2\frac{k\pi}{n} - 1 + 2i\sin\frac{k\pi}{n}\cos\frac{k\pi}{n}$$

$$\frac{2}{z} = 2\cos^2\frac{k\pi}{n} + 2i\sin\frac{k\pi}{n}\cos\frac{k\pi}{n}$$

$$\frac{1}{z} = \cos^2\frac{k\pi}{n} + i\sin\frac{k\pi}{n}\cos\frac{k\pi}{n}$$

$$= \cos\frac{k\pi}{n}\left(\cos\frac{k\pi}{n} + i\sin\frac{k\pi}{n}\right)$$

$$z = \sec\frac{k\pi}{n}\left(\cos\frac{k\pi}{n} + i\sin\frac{k\pi}{n}\right)^{-1}$$

$$= \sec\frac{k\pi}{n}\left(\cos\frac{k\pi}{n} - i\sin\frac{k\pi}{n}\right)$$

$$= 1 - i\tan\frac{k\pi}{n}$$

$$|z| = \sqrt{1 + \tan^2\frac{k\pi}{n}}$$

$$= \sqrt{\sec^2\frac{k\pi}{n}}$$

$$= \sec\frac{k\pi}{n}$$

$$\therefore |z|_{min} = 1$$

10 i) $\frac{d}{dx}(\sin^{n-1}x\cos x) = (n-1)\sin^{n-2}x\cos^2 x - \sin^n x$

$$\left[\sin^{n-1}x\cos x\right]_0^{\frac{\pi}{2}} = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x\cos^2 x\,dx - \int_0^{\frac{\pi}{2}} \sin^n x\,dx$$

$$\left(\sin\frac{\pi}{2}\right)^n \cos\frac{\pi}{2} - (\sin 0)^n \cos 0$$

$$= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x(1-\sin^2 x)\,dx - I_n$$

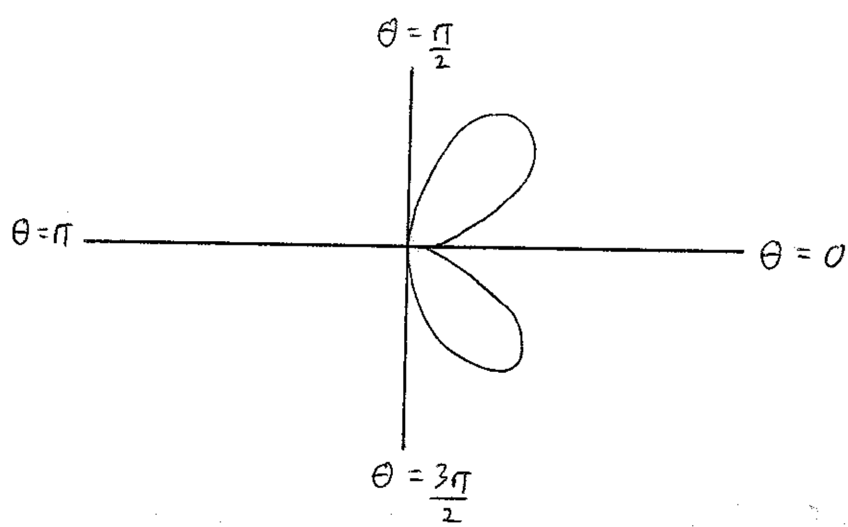
$$0 = (n-1)\int_0^{\frac{\pi}{2}} (\sin^{n-2}x - \sin^n x)\,dx - I_n$$

$$0 = (n-1)I_{n-2} - (n-1)I_n - I_n$$

$$\therefore I_n = \frac{n-1}{n} I_{n-2}$$

ii) a)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	0	$\sqrt{3}$	$2\sqrt{2}$	3	0	-3	$-2\sqrt{2}$	$-\sqrt{3}$	0



b) Area $= \int_0^{\frac{\pi}{2}} \frac{1}{2} \times 16\sin^2 2\theta \sin^2\theta \; d\theta$

$$= \int_0^{\frac{\pi}{2}} 8 \times 4\sin^4\theta\cos^2\theta \; d\theta$$

$$= 32\left[\int_0^{\frac{\pi}{2}} \sin^4\theta \; d\theta - \int_0^{\frac{\pi}{2}} \sin^6\theta \; d\theta\right]$$

$$= 32\left[I_4 - I_6\right]$$

$$= 32\left[I_4 - \frac{5}{6}I_4\right]$$

$$= 32\left(\frac{1}{6}I_4\right)$$

$$= \frac{16}{3}\left(\frac{3}{4}I_2\right)$$

$$= 4\left(\frac{1}{2}I_0\right)$$

$$= 2\int_0^{\frac{\pi}{2}} 1 \; d\theta$$

$$= 2\left[\theta\right]_0^{\frac{\pi}{2}}$$

$$= 2\left(\frac{\pi}{2} - 0\right)$$

$$= \pi$$

11. EITHER

i) $y = \dfrac{2x^2 + 3}{x^2 + 3x - 4} = 2 + \dfrac{1}{x-1} - \dfrac{7}{x+4}$

ii) $y = 2, \quad x = 1, \quad x = -4$

iii) $\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{-1}{(x-2)^2} + \dfrac{7}{(x+4)^2} = 0$

$$-(x^2 + 8x + 16) + 7(x^2 - 2x + 1) = 0$$

$$6x^2 - 22x + 9 = 0$$

$$x = \frac{22 \pm \sqrt{700}}{12}$$

$$= \frac{11 \pm 5\sqrt{7}}{6}$$

$$= 4.03813, \ -0.37146$$

$$\frac{d^2y}{dx^2} = \frac{2}{(x-1)^3} - \frac{14}{(x+4)^3}$$

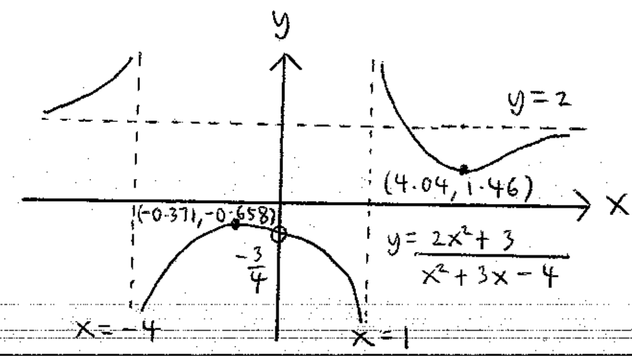
$x = 4.03813$: $y = 1.45830$, $\dfrac{d^2y}{dx^2} = 0.04436 > 0$

$\therefore (4.04, 1.46)$ minimum point.

$x = -0.37146$: $y = -0.65830$, $\dfrac{d^2y}{dx^2} = -1.0684 < 0$

$\therefore (-0.371, -0.658)$ maximum point.

iv)



OR

i) $ax^3 + bx^2 + cx + d = 0$

α, β, γ are the roots.

$\alpha + \beta + \gamma = \frac{-b}{a}$ $\quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ $\quad \alpha\beta\gamma = -\frac{d}{a}$

$$\frac{\gamma}{\beta} = \frac{\beta}{\alpha}$$

$$\alpha\gamma = \beta^2$$

$$\alpha\beta + \beta^2 + \beta\gamma = \frac{c}{a}$$

$$\beta(\alpha + \beta + \gamma) = \frac{c}{a}$$

$$\beta\left(\frac{-b}{a}\right) = \frac{c}{a}$$

$$\beta = -\frac{c}{b}$$

ii)

$$\beta^2\beta = -\frac{d}{a}$$

$$\beta^3 = \frac{-d}{a}$$

$$\beta^3 = \frac{-d}{a}$$

$$\left(\frac{-c}{b}\right)^3 = \frac{-d}{a}$$

$$\frac{-c^3}{b^3} = \frac{-d}{a}$$

$$ac^3 = b^3d$$

A condition for the roots to be in geometric progression is $ac^3 = b^3 d$.

iii) Let $r = \frac{\gamma}{\beta} = \frac{\beta}{\alpha}$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\frac{\alpha + \beta + \gamma}{\beta} = \frac{-b}{a\beta}$$

$$\frac{\alpha}{\beta} + 1 + \frac{\gamma}{\beta} = \frac{-b}{a\beta}$$

$$\frac{1}{\frac{\beta}{\alpha}} + 1 + \frac{\gamma}{\beta} = \frac{-b}{a\left(\frac{-c}{b}\right)}$$

$$\frac{1}{r} + 1 + r = \frac{b^2}{ac}$$

$$1 + r + r^2 = \frac{b^2 r}{ac}$$

$$r^2 + \left(1 - \frac{b^2}{ac}\right) r + 1 = 0$$

$$r^2 + \frac{(ac - b^2) r}{ac} + 1 = 0$$

$$r^2 + \frac{(ac - b^2)r}{ac} + \frac{(ac - b^2)^2}{4a^2c^2} = \frac{(ac - b^2)^2}{4a^2c^2} - 1$$

$$\left(r + \frac{ac - b^2}{2ac}\right)^2 = \frac{b^4 - 2ab^2c + a^2c^2}{4a^2c^2} - 1$$

$$= \frac{b^4 - 2ab^2c - 3a^2c^2}{4a^2c^2}$$

$$r + \frac{ac - b^2}{2ac} = \frac{\pm\sqrt{b^4 - 2ab^2c - 3a^2c^2}}{2ac}$$

$$r = \frac{b^2 - ac}{2ac} \pm \frac{\sqrt{b^4 - 2ab^2c - 3a^2c^2}}{2ac}$$

$\therefore$ The possible common ratios are

$$\frac{b^2 - ac}{2ac} \pm \frac{\sqrt{b^4 - 2ab^2c - 3a^2c^2}}{2ac}.$$