TAYLOR'S COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2006

(June 2005 Intake)

Tuesday 29 August 2006 1.30 pm – 4.30 pm

FURTHER MATHEMATICS 9231/01

PAPER 1 **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together

This document consists of **4** printed pages.

1. The curve defined parametrically by the equations

$$x = a\cos t, \quad y = a\sin t, \qquad t = 0, \; t = 2\pi$$

is rotated about the x-axis to form a solid. Find the area of the surface generated. [5]

2. Expressing $\dfrac{r+6}{r(r+2)(r+3)}$ as partial fractions, find S_n if $S_n = \sum_{r=1}^{n} \dfrac{r+6}{r(r+2)(r+3)}$. [6]

3. i) Prove that

$$\cos^5 x = \cos x - 2\sin^2 x\cos x + \sin^4 x\cos x.$$ [2]

ii) Hence, or otherwise find the mean value of $\cos^5 x$ over the interval $0 \le x \le \dfrac{\pi}{2}$. [4]

4. Prove by induction that $\sum_{r=1}^{n} \sin(2r+1)\theta = \sin(n+2)\theta \sin n\theta \operatorname{cosec}\theta$

for every positive integer n. [7]

5. The equation of a curve is

$$x^2 + 9y^2 - 3xy + 11x^2y^2 = 36.$$

Find the mean value of $\dfrac{d^2y}{dx^2}$ over the interval $0 \le x \le 6$. [8]

6. By expressing $\cos^6\theta$ in the form

$$a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$$

or otherwise, find $\int_{-\pi}^{\pi} \cos^6\theta \, d\theta$. [8]

7. i) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} - 3a\frac{dy}{dx} + 2a^2y = 0, \qquad a \neq 0$$

given that $y = 4$ and $\dfrac{dy}{dx} = 5$ when $x = 0$. [4]

ii) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 10\sin x.$$ [6]

8. i) If the roots of the equation

$$x^3 + 3x^2 + 2x + 1 = 0$$

are α, β, γ and $S_n = \alpha^n + \beta^n + \gamma^n$, find S_3. [4]

ii) Draw a diagram to illustrate the region R bounded by the curve whose polar equation is $r = 2 + \cos\theta$ and find the area of R. [7]

9. If I_n denotes $\int_0^{\frac{\pi}{2}} x^n \cos x\,dx$, prove that

i) $I_n = (\frac{\pi}{2})^n - n(n-1)I_{n-2}$, $n \geq 2$ [7]

ii) $I_3 = \frac{\pi^3}{8} - \frac{3\pi}{2} + 6$ p. $I_3 = \frac{\pi^3}{8} - 3\pi + 6$ [5]

10. The curve C has equation

$$y = 2x + 1 + \frac{8}{x+2}.$$

i) Write down the equations of the asymptotes of C. [2]

ii) Find the critical points of C and determine their type. [7]

iii) Sketch C, indicating also the intersection of C with the coordinate axes. [4]

11. Answer only **one** of the following two alternatives.

EITHER

The lines l_1 and l_2 have vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + t(-\mathbf{i} - \mathbf{j} - 3\mathbf{k})$$

and

$$\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

respectively and the point A has coordinates (3, 4, 5). Find

i) the shortest distance between the two lines. [4]

ii) the vector equation of the common perpendicular to the lines. [5]

iii) the distance from the point A to the common perpendicular of the lines. [5]

OR

a) The linear transformation $\mathbf{T}: R^4 \to R^4$ is represented by the matrix $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 6 \\ -1 & -1 & -2 & -4 \\ 2 & 3 & 6 & 11 \\ 3 & 8 & 10 & 21 \end{pmatrix}.$$

Find the rank and nullspace of $\mathbf{A}$.

[5]

b) i) If the square matrices $\mathbf{A}$ and $\mathbf{B}$ have eigenvalues λ and μ respectively, prove that $\lambda + \mu + k$ is an eigenvalue of the matrix $\mathbf{A} + \mathbf{B} + k\mathbf{I}$.

[3]

ii) Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{C}$, where

$$\mathbf{C} = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 8 \\ 0 & 0 & -3 \end{pmatrix}.$$

[6]