TAYLOR'S UNIVERSITY COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2008

(Jan/March 2007 Intake)

Thursday **27 March 2008** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/01

PAPER 1 **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together

This document consists of **4** printed pages.

[Turn over

1. Make a conjecture about a number which divides $2^n \,.\, 3^{n+2} + 2 \,.\, 5^{2n+1}$, and prove it by mathematical induction. [5]

2. Express $\dfrac{1}{(r+1)(r+3)}$ in terms of partial fractions. [2]

   Hence, or otherwise find $\displaystyle\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)}$. [4]

3. If the equation $5x^3 - x^2 + 4x - 3 = 0$ has roots α, β, γ find the equation having roots $\alpha^2 + 2, \beta^2 + 2, \gamma^2 + 2$. [7]

4. The integral I_n, where n is a non-negative integer, is defined by

$$I_n = \int_0^1 x^n \sqrt{1-x^2}\, dx.$$

   Show that $(n+2)\, I_n = (n-1)\, I_{n-2}$ [5]
   Find I_5. [2]

5. Find the general solution of the differential equation
$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 10\cos x + 15\sin x.$$ [9]

6. Solve the equation $z^4 + 16 = 0$, giving your answers in the exact form $p + qi$, where p and q are real. Hence, solve the equations $(z + 1)^4 = -16z^4$, giving your answers in the exact form $x + yi$ where x and y are real. [9]

7. a) Find the equation of the plane containing the line $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + s(\mathbf{i} + 9\mathbf{j} - 3\mathbf{k})$ and the point (4, 8, -3).

   [4]

   b) Find the equation of the plane containing the lines $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + s(-3\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ and $\mathbf{r} = 7\mathbf{i} + 6\mathbf{j} + 5\mathbf{k} + t(6\mathbf{i} + 4\mathbf{j} - \mathbf{k})$.

   [4]

   c) Find the equation of the line of intersection of the two planes. [2]

8. A curve has equation $y = \frac{2(x^2 - x - 1)}{x^2 + 2x + 2}$.

(a) Find the coordinates of the points at which the curve cuts the coordinate axes. [2]

(b) Find the equation of the asymptote. [1]

(c) Find the coordinates of the turning points on the curve. [3]

(d) Sketch the graph of the curve. [4]

9. A curve is given by the parametric equations

$$x = a\cos^3 t, \qquad y = a\sin^3 t.$$

Sketch the curve from $t = 0$ to $t = \pi/2$. [1]
Find the surface area generated when the curve is rotated completely about the x-axis. [5]

Find the x-coordinate, $\bar{x}$, of the centroid of this surface area. [5]

[Hint: Given that if $I_n = \int_0^{\pi/2} \sin^n x\,dx$ or $I_n = \int_0^{\pi/2} \cos^n x\,dx$, then $nI_n = (n-1)\,I_{n-2}$.]

10. Sketch the curve with polar equation $r = \sin\theta\ (1 - \cos\theta)$ for $0 \leq \theta \leq \pi$. [2]
Find

(i) the area enclosed by the curve. [6]

(ii) the polar coordinates of the point of the curve furthest from the origin. [4]

11. Answer only **one** of the following two alternatives.

~~EIGHTER~~ EITHER

(i) Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that the series

$$\frac{z}{2} - \frac{z^2}{4} + \frac{z^3}{8} - \frac{z^4}{16} + \ldots = \frac{1 + 2e^{i\theta}}{5 + 4\cos\theta},$$ assuming that $|z| < 1$. [4]

(ii) Use de Moivre's theorem to show

$$\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta.$$

Write the equation $\sin 7\theta = -1$ as an equation in $s = \sin\theta$, and state its roots.
Hence show that

$$1 - 2\sin\frac{\pi}{14} + 2\sin\frac{3\pi}{14} - 2\sin\frac{5\pi}{14} = 0.$$ [10]

OR

a) If $\mathbf{A} = \begin{pmatrix} 1 & 5 & -3 & 8 \\ -1 & -4 & 1 & -12 \\ 2 & 7 & 0 & 28 \\ -3 & -13 & 5 & -32 \end{pmatrix}$, find the rank of **A**.

[5]

b) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are given by $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -7 \\ 6 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -3 \\ 9 \\ -4 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$,

determine if $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbf{R}^3$.

[4]

c) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are given by $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \\ 4 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -3 \\ -5 \\ 7 \\ -9 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 4 \\ 10 \\ -16 \\ 22 \end{pmatrix}$,

$\mathbf{v}_4 = \begin{pmatrix} 7 \\ 9 \\ -11 \\ 13 \end{pmatrix}$, determine if $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly independent. [5]