Example:

Show algebraically that the graph $y = \dfrac{x-1}{x^2+3}$ can only take values in the interval $-\dfrac{1}{2} \le y \le \dfrac{1}{6}$.

Example:

A curve is given by equation $y=\dfrac{2x^2-5x+4}{x-2}$.

(a) Find all the equations of the asymptotes.

(b) Without using any diagram, find the values of $y$ for which there are no points on the curve.

Example:

Find, algebraically, the range of the function

$$f(x) = \frac{x^2 - x - 2}{(x-1)(x-3)}.$$

Example:

Without using any diagram, show that $\dfrac{1-2x-x^2}{x^2} \geq -2$

for all real values of $x$.

Example:

The equation of a curve is $y = \dfrac{2x-5}{x^2-4}, x \in \mathrm{R}.$

(a) Find the asymptotes and the stationary point.

(b) Prove algebraically that $y \leq \dfrac{1}{4}$ or $y \geq 1.$

(c) Sketch the curve.

(d) Determine the number of real roots of the equation

$$k\left(x^2-4\right) = 2x-5 \text{ where } k > \frac{1}{2}.$$

# Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

A curveis given by equation $y = \frac{4}{x^2} + x.$

(a) Show algebraically that the entire curve lies above the line $y = x$.

(b) Find the coordinates and the nature of the turning point.

(c) Sketch the curve.

Example:

Find the set of values of $k$ such that $y = \dfrac{x-k}{x^2-4x+k}$ takes all values as $x$ varies.

Example:

A curve is given by equation $y = \dfrac{x^2 + px}{x^2 + p}$. Given that $y \in \mathrm{R}$ for $x \in \mathrm{R}$, find the set of values of $p$.

Example:

A curve $C$ has equation $y=\frac{(x+1)(x-1)}{(x-2)^2}$. Find the coordinates of the turning points of $C$. Sketch the graph of $C$, indicatiing clearly the asymptotes, axial intercepts and stationary points. Hence, on separate diagrams, sketch the following curves.

(a) $y=\frac{|x+1||x-1|}{(x-2)^2}$ (b) $y=\frac{\sqrt{(x+1)(x-1)}}{x-2}$ (c) $y=\frac{|x+1||x-1|}{(x+2)^2}$