TAYLOR'S COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

SEMESTER 1 EXAMINATION DECEMBER 2005

(June 2005 Intake)

Monday 5 December 2005 1.00 pm – 3.00 pm

FURTHER MATHEMATICS 9231/01

PAPER 1 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

1. Prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$.

[5]

2. Prove that $3^{4n-2} + 17^n + 22$ is divisible by 16 for every positive integer n.

[5]

3. Prove that $\sum_{j=1}^{n} \frac{j2^{j-1}}{(j+1)(j+2)} = \frac{2^n}{n+2} - \frac{1}{2}$.

[5]

4. The roots of the equation $x^3 - 4x - 1 = 0$ are α, β and γ. Find the values of S_2, S_4 and $\frac{S_4}{S_2}$ if $S_n = \alpha^n + \beta^n + \gamma^n$. [7]

5. The line l has equation $\mathbf{r} = 4\mathbf{i} - 8\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j})$ and the plane Π_1 has equation $\mathbf{r} = -8\mathbf{i} + 12\mathbf{j} + 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j}) + v(2\mathbf{i} - \mathbf{k})$, where $\lambda, \mu, v \in \mathbb{R}$. The points U and V, which vary independently, are on l and on Π_1, respectively. The point W, which is on the line segment UV, is such that $UW = 3WV$. Find an equation of the locus of W.

[5]

The points L and M have position vectors $-4\mathbf{i} + 13\mathbf{j} + 3\mathbf{k}$ and $15\mathbf{j} + 3\mathbf{k}$, respectively.

i) Determine if the points L and M are in Π_1.

[3]

ii) Find the plane Π_2, which contains the points L and M and also the line l.

[3]

iii) Find an equation of the intersection of Π_2 with the locus of W.

[4]

6. The curve C has equation $y = \frac{2x^2 + 3x + 1}{x - 1}$.

i) Show that $y = 2x + 5 + \frac{6}{x-1}$, and hence find the equations of the asymptotes of C.

[5]

ii) Find the critical points of y.

[10]

iii) Draw a sketch of C.

[8]