1 $n = 1$

$$A^1 = 3^{1-1}\begin{pmatrix} 2(1) + 3 & -(1) \\ 4(1) & 3 - 2(1) \end{pmatrix}$$

$$= 3^0 \begin{pmatrix} 2 + 3 & -1 \\ 4 & 3 - 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}$$

Statement is true for $n = 1$

$n = k$

Assume $A^k = 3^{k-1}\begin{pmatrix} 2k + 3 & -k \\ 4k & 3 - 2k \end{pmatrix}$

$n = k + 1$:

$A^{k+1} = A^k A$ $\{$ can also use $A^k \times A \}$

$$= \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix} \times 3^{k-1}\begin{pmatrix} 2k + 3 & -k \\ 4k & 3 - 2k \end{pmatrix}$$

$$= 3^{k-1}\begin{pmatrix} 6k + 15 & -3k - 3 \\ 12k + 12 & 3 - 6k \end{pmatrix}$$

$$= 3^{k-1}\begin{pmatrix} 3(2k+5) & 3(-k-1) \\ 3(4k+4) & 3(1-2k) \end{pmatrix}$$

$$= 3^{k}\begin{pmatrix} 2k+5 & -k-1 \\ 4k+4 & 1-2k \end{pmatrix}$$

$$= 3^{(k+1)-1}\begin{pmatrix} 2(k+1)+3 & -(k+1) \\ 4(k+1) & 3-2(k+1) \end{pmatrix}$$

shown.

2. Let $u = \alpha - 4 \Rightarrow \alpha = u + 4$

$$(u+4)^3 - 17(u+4)^2 + 94(u+4) - 168 = 0$$

$$(u^3 + 12u^2 + 48u + 64)$$

$$-17(u^2 + 8u + 16) + 94u + 376 - 168 = 0$$

$$u^3 - 5u^2 + 6u = 0$$

$$u(u-2)(u-3) = 0$$

$$u = 0, 2, 3$$

$$x = u + 4$$

$$= 4, 6, 7$$

3. $$\frac{2}{(2r-1)(2r+1)} = \frac{1}{2r-1} - \frac{1}{2r+1}$$

$$\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)} = \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$$

$$= \frac{1}{2(1)-1} - \frac{1}{2(n)+1}$$

$$= 1 - \frac{1}{2n+1}$$

$$= \frac{2n}{2n+1}$$

$$\frac{1}{51 \times 53} + \frac{1}{53 \times 55} + \frac{1}{55 \times 57} + \cdots + \frac{1}{199 \times 201}$$

$$= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{199 \times 201}$$

$$- \left(\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{49 \times 51}\right)$$

$$= \frac{1}{2}\left[\sum_{r=1}^{100} \frac{2}{(2r-1)(2r+1)} - \sum_{r=1}^{25} \frac{2}{(2r-1)(2r+1)}\right]$$

$$= \frac{1}{2}\left[\frac{200}{201} - \frac{50}{51}\right]$$

$$= \frac{25}{3417}$$

4 a) $y = \dfrac{3x+2}{x+2} = 3 - \dfrac{4}{x+2}$

Asymptotes $y = 3$, $x = -2$

b) $y = 3 - 4(x+2)^{-1}$

$\dfrac{dy}{dx} = 0 - 4(-1)(x+2)^{-2}(1) = \dfrac{4}{(x+1)^2}$

At the turning points, $\dfrac{dy}{dx} = 0$

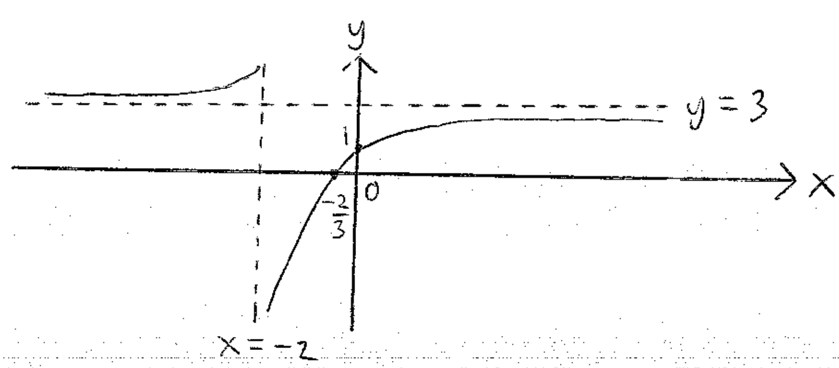
but $\dfrac{4}{(x+2)^2} \neq 0 \quad \forall\, x \in R$

$\therefore$ C does not have any turning points.

c) At the x-axis, $y = 0 \Rightarrow x = \dfrac{-2}{3}$

At the y-axis, $x = 0 \Rightarrow y = 3$

As $x \longrightarrow \pm\infty$, $y \to 3$



5 a) $\left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 1 & k & 2 & 2 \\ -1 & k^2-1 & -k & 2k-5 \end{array}\right)$

$$\xrightarrow[R_3 \; R_3+R_1]{R_2 \; R_2-R_1} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & k+3 & 0 & 1 \\ 0 & k^2-4 & -k+2 & 2k-4 \end{array}\right)$$

No solution provided $k+3=0$

$k=-3$

b) Infinitely many solutions provided

$k^2-4=0 \Rightarrow k=\pm 2$

$-k+2=0 \Rightarrow k=2$

$2k-4=0 \Rightarrow k=2$

Infinitely many solutions when $k=2$

c) when $k=-2$

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -8 \end{array}\right)$$

$4z=-8 \Rightarrow z=-2$

$y=1$

$x-3y+2z=1$

$x = 1+3(1)-2(-2)$

$= 8$