8. $$n(n+1)(n+2)\cdots(n+k-1)$$
$$-(n-1)n(n+1)\cdots(n+k-2)$$
$$= n(n+1)(n+2)\cdots(n+k-2)(n+k-1-(n-1))$$
$$= n(n+1)(n+2)\cdots(n+k-2)(k)$$
$$= kn(n+1)(n+2)\cdots(n+k-2)$$

$k=3$: $$n(n+1)(n+2) - (n-1)n(n+1)$$
$$= n(n+1)(n+2-(n-1))$$
$$= 3n(n+1)$$

$$\sum_{m=1}^{N} 3n(n+1) = \sum_{n=1}^{N} n(n+1)(n+2) - (n-1)n(n+1)$$

$$3\sum_{n=1}^{N} n(n+1) = N(N+1)(N+2)$$
$$-(N-1)N(N+1)$$
$$+(N-1)N(N+1)$$
$$-(N-2)(N-1)N$$
$$+(N-2)(N-1)N$$
$$-(N-3)(N-2)(N-1)$$
$$\vdots$$

$$+\ 4\cdot 3\cdot 2 - 3\cdot 2\cdot 1$$

$$+\ 3\cdot 2\cdot 1 - 2\cdot 1\cdot 0$$

$$= N(N+1)(N+2)$$

$$\therefore \sum_{n=1}^{N} n(n+1) = \frac{N(N+1)(N+2)}{3}$$

$$k=4.\quad n(n+1)(n+2)(n+3) - (n-1)n(n+1)(n+2)$$

$$= n(n+1)(n+2)(n+3-(n-1))$$

$$= 4n(n+1)(n+2)$$

$$\sum_{n=1}^{N} 4n(n+1)(n+2)$$

$$= \sum_{n=1}^{N} n(n+1)(n+2)(n+3)$$

$$-(n-1)n(n+1)(n+2)$$

$$= N(N+1)(N+2)(N+3)$$

$$-(N-1)N(N+1)(N+2)$$

$$+(N-1)N(N+1)(N+2)$$

$$-(N-2)(N-1)N(N+1)$$

$$+ (N-2)(N-1)N(N+1)$$
$$- (N-3)(N-2)(N-1)N$$

$$+ 5 \cdot 4 \cdot 3 \cdot 2 - 4 \cdot 3 \cdot 2 \cdot 1$$
$$+ 4 \cdot 3 \cdot 2 \cdot 1 - 3 \cdot 2 \cdot 1 \cdot 0$$
$$= N(N+1)(N+2)(N+3)$$

$$\sum_{n=1}^{N} n(n+1)(n+2) = \frac{N(N+1)(N+2)(N+3)}{4}$$

$$K = 5: \; n(n+1)(n+2)(n+3)(n+4)$$
$$- (n-1)n(n+1)(n+2)(n+3)$$
$$= n(n+1)(n+2)(n+3)(n+4-(n-1))$$
$$= 5n(n+1)(n+2)(n+3)$$

$$\sum_{n=1}^{N} 5n(n+1)(n+2)(n+3)$$

$$= \sum_{n=1}^{N} \begin{matrix} - (n-1)n(n+1)(n+ \\ n(n+1)(n+2)(n+3)(n+4) \end{matrix}$$

$$= \sum_{n=1}^{N} \begin{matrix} 5n(n+1)(n+2) \\ - (n-1)n(n+1)(n+2)(n+3) \end{matrix}$$

$$= N(N+1)(N+2)(N+3)(N+4)$$

$$-(N-1)N(N+1)(N+2)(N+3)$$

$$+(N-1)N(N+1)(N+2)(N+3)$$

$$-(N-2)(N-1)N(N+1)(N+2)$$

$$+(N-2)(N-1)N(N+1)(N+2)$$

$$-(N-3)(N-2)(N-1)N(N+1)$$

$$\vdots$$

$$+ 6\cdot 5\cdot 4\cdot 3\cdot 2 - 5\cdot 4\cdot 3\cdot 2\cdot 1$$

$$+ 5\cdot 4\cdot 3\cdot 2\cdot 1 - 4\cdot 3\cdot 2\cdot 1\cdot 0$$

$$= N(N+1)(N+2)(N+3)(N+4)$$

$$\sum_{n=1}^{N} (n(n+1)(n+2)(n+3)$$

$$= \frac{N(N+1)(N+2)(N+3)(N+4)}{5}$$

$$an(n+1)(n+2)(n+3) + bn(n+1)(n+2)$$
$$+ cn(n+1) + dn = n^4$$

$$a(n^3 + 3n^2 + 2n)(n+3) + b(n^3 + 3n^2 + 2n)$$
$$+ c(n^2 + n) + dn = n^4$$

$$a(n^4 + 6n^3 + 11n^2 + 6n) + b(n^3 + 3n^2 + 2n)$$
$$+ c(n^2 + n) + dn = n^4$$

$$an^4 + (6a + b)n^3 + (11a + 3b + c)n^2$$
$$+ (6a + 2b + c + d)n = n^4$$

$a = 1$

$6a + b = 0$

$11a + 3b + c = 0$

$6a + 2b + c = 0$

$b = -6$

$c = 7$

$d = -1$

$$n^4 = n(n+1)(n+2)(n+3) - 6n(n+1)(n+2)$$
$$+ 7n(n+1) - n$$

$$\sum_{n=1}^{N} n^4 = \sum_{n=1}^{N} n(n+1)(n+2)(n+3) - 6n(n+1)(n+2) + 7n(n+1) - n$$

$$= \sum_{n=1}^{N} n(n+1)(n+2)(n+3) - 6\sum_{n=1}^{N} n(n+1)(n+2) + 7\sum_{n=1}^{N} n(n+1) - \sum_{n=1}^{N} n$$

$$= \frac{N(N+1)(N+2)(N+3)(N+4)}{5} - \frac{3N(N+1)(N+2)(N+3)}{2} + \frac{7N(N+1)(N+2)}{3} - \frac{N(N+1)}{2}, \quad \text{since } \sum_{n=1}^{N} n = \frac{N(N+1)}{2}$$

10. a) $$\frac{2}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$= \frac{A(x+1)(x+2) + Bx(x+2) + Cx(x+1)}{x(x+1)(x+2)}$$

$$2 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$$

$$= A(x^2+3x+2) + B(x^2+2x) + C(x^2+x)$$

$$= (A+B+C)x^2 + (3A+2B+C)x + 2A$$

$A + B + C = 0$ $\quad$ $3A + 2B + C = 0$ $\quad$ $2A = 2$

$A = 1$

$$B + C = -1$$
$$2B + C = -3$$
$$B = -2$$
$$C = 1$$

$$\frac{2}{x(x+1)(x+2)} = \frac{1}{x} - \frac{2}{x+1} + \frac{1}{x+2}$$

$$\sum_{n=1}^{N} \frac{2}{n(n+1)(n+2)} = \sum_{n=1}^{N} \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}$$

$$= \frac{1}{1} - \frac{2}{2} + \frac{1}{3}$$

$$+ \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$$

$$+ \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$$

$$+ \frac{1}{4} - \frac{2}{5} + \frac{1}{6}$$

$$+ \frac{1}{5} - \frac{2}{6} + \frac{1}{7}$$

$$\vdots$$

$$+ \frac{1}{n-4} - \frac{2}{n-3} + \frac{1}{n-2}$$

$$+ \frac{1}{n-3} - \frac{2}{n-2} + \frac{1}{n-1}$$

$$+ \frac{1}{n-2} - \frac{2}{n-1} + \frac{1}{n}$$

$$+ \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}$$

$$+ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}$$

$$= \frac{1}{1} - \frac{2}{2} + \frac{1}{2}$$

$$+ \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$$

$$= \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$$

$$= \frac{1}{2} + \frac{n+1-(n+2)}{(n+1)(n+2)}$$

$$= \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

$$\therefore \sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

b) Let $f(n) = 3^{4n-2} + 17^n + 22$

$n=1: \; f(1) = 3^{4(1)-2} + 17^1 + 22$

$$= 3^{4-2} + 17 + 22$$

$$= 3^2 + 17 + 22$$

$$= 9 + 17 + 22$$

$$= 48$$

$$= 16(3)$$

$$\therefore 16 \mid f(1)$$

Assume the statement is true when $n = k$.

$n = k$: $f(k) = 3^{4k-2} + 17^k + 22$

$16 \mid f(k)$

$\therefore f(k) = 16s$, s is an integer.

when $n = k + 1$:

$$f(k+1) = 3^{4(k+1)-2} + 17^{k+1} + 22$$
$$= 3^{4k+4-2} + 17^k 17 + 22$$
$$= 3^{4k-2} 3^4 + 17^k 17 + 22$$
$$= 3^{4k-2} 81 + 17^k 17 + 22$$
$$= 3^{4k-2}(80 + 1) + 17^k(16 + 1) + 22$$
$$= 3^{4k-2} 80 + 3^{4k-2} + 17^k 16 + 17^k + 22$$
$$= 3^{4k-2} 80 + 17^k 16 + 3^{4k-2} + 17^k + 22$$
$$= 16(3^{4k-2} 5 + 17^k) + 16s$$
$$= 16(3^{4k-2} 5 + 17^k + s)$$

Since s is an integer and k is an integer, $3^{4k-2} 5 + 17^k + s$ is an integer.

$\therefore 16 \mid f(k+1)$

$\therefore 3^{4n-2} + 17^n + 22$ is divisible by 16 for every positive integer n.

11. $u_1, u_2, u_3, \dots, u_1 = 1, u_{n+1} = \dfrac{5u_n + 4}{u_n + 2}, n \geq 1$

$u_n < 4$

$u_n > 0$

when $n = 1$: $u_1 = 1 > 0$

Assume the statement is true when $n = k$.

$n = k$: $u_k > 0$

when $n = k+1$: $u_k > 0$

$u_k + 2 > 0$

$5u_k > 0$

$5u_k + 4 > 0$

$\dfrac{5u_k + 4}{u_k + 2} > 0$

$u_{k+1} > 0$

$\therefore u_n > 0$ for every positive integer n.

$u_n < 4$

when $n = 1$: $u_1 = 1 < 4$

Assume the statement is true when $n = k$.

$n = k$: $u_k < 4$

when $n = k+1$: $u_k < 4$

$5u_k + 4 - 4u_k - 8 < 0$

$5u_k + 4 < 4u_k + 8$

$\dfrac{5u_k + 4}{u_k + 2} < 4$

$u_{k+1} < 4$

$\therefore u_n < 4$ for every positive integer n.

12. a) $y = \frac{1}{1 + x}$, $\frac{d^n y}{dx^n} = \frac{(-1)^n n!}{(1 + x)^{n+1}}$

When $n = 1$:

$$\frac{d^1 y}{dx^1} = \frac{dy}{dx} = \frac{-1}{(1 + x)^2} = \frac{(-1)1}{(1 + x)^2} = \frac{(-1)^1 1!}{(1 + x)^{1+1}}$$

Assume the statement is true when $n = k$.

$n = k$: $\frac{d^k y}{dx^k} = \frac{(-1)^k k!}{(1 + x)^{k+1}}$

$n = k + 1$. $\frac{d^{k+1} y}{dx^{k+1}} = \frac{(-1)^{k+1} (k + 1)!}{(1 + x)^{k+2}}$

(what needs to be proved)

$$\frac{d^{k+1} y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)$$

$$= \frac{d}{dx}\left(\frac{(-1)^k k!}{(1 + x)^{k+1}}\right)$$

$$= (-1)^k k! \frac{d}{dx}\left(\frac{1}{(1 + x)^{k+1}}\right)$$

$$= (-1)^k k!(-k-1)(1+x)^{-k-1-1}$$

$$= (-1)^k k!(-1)(k+1)(1+x)^{-k-2}$$

$$= \frac{(-1)^{k+1}(k+1)!}{(1+x)^{k+2}}$$

$$\therefore \frac{d^n y}{dx^n} = \frac{(-1)^n n!}{(1+x)^{n+1}}$$

for every positive integer n

b) i) $$\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}$$

$$= \frac{n^2 + n + 1 - (n^2 - n + 1)}{(n^2 + n + 1)(n^2 - n + 1)}$$

$$= \frac{n^2 + n + 1 - n^2 + n - 1}{n^4 - n^3 + n^2 + n^3 - n^2 + n + n^2 - n + 1}$$

$$= \frac{2n}{n^4 + n^2 + 1}$$

ii) $$n^2 + n + 1 = \left(n + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$n^2 - n + 1 = \left(n - \frac{1}{2}\right)^2 + \frac{3}{4}$$

iii)

$$S_N = \sum_{n=1}^{N} \frac{n}{n^4 + n^2 + 1}$$

$$= \sum_{n=1}^{N} \frac{1}{2}\left(\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}\right)$$

$$= \sum_{n=1}^{N} \frac{1}{2}\left(\frac{1}{(n - \frac{1}{2})^2 + \frac{3}{4}} - \frac{1}{(n + \frac{1}{2})^2 + \frac{3}{4}}\right)$$

$$= \frac{1}{2}\left(\frac{1}{(\frac{1}{2})^2 + \frac{3}{4}} - \frac{1}{(\frac{3}{2})^2 + \frac{3}{4}}\right.$$

$$+ \frac{1}{(\frac{3}{2})^2 + \frac{3}{4}} - \frac{1}{(\frac{5}{2})^2 + \frac{3}{4}}$$

$$+ \frac{1}{(\frac{5}{2})^2 + \frac{3}{4}} - \frac{1}{(\frac{7}{2})^2 + \frac{3}{4}}$$

$$\vdots$$

$$+ \frac{1}{(N - \frac{5}{2})^2 + \frac{3}{4}} - \frac{1}{(N - \frac{3}{2})^2 + \frac{3}{4}}$$

$$+ \frac{1}{(N - \frac{3}{2})^2 + \frac{3}{4}} - \frac{1}{(N - \frac{1}{2})^2 + \frac{3}{4}}$$

$$+ \frac{1}{\left(N - \frac{1}{2}\right)^2 + \frac{3}{4}} - \frac{1}{\left(N + \frac{1}{2}\right)^2 + \frac{3}{4}}\Bigg)$$

$$= \frac{1}{2}\left(\frac{1}{\left(\frac{1}{2}\right)^2 + \frac{3}{4}} - \frac{1}{\left(N + \frac{1}{2}\right)^2 + \frac{3}{4}}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\frac{1}{4} + \frac{3}{4}} - \frac{1}{N^2 + N + \frac{1}{4} + \frac{3}{4}}\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{N^2 + N + 1}\right)$$

$$= \frac{1}{2} - \frac{1}{2(N^2 + N + 1)}$$

Since $\frac{1}{2(N^2 + N + 1)} > 0$,

$$\frac{1}{2} - \frac{1}{2(N^2 + N + 1)} < \frac{1}{2}$$

$$\therefore \; S_N < \frac{1}{2}.$$

14. a) $\displaystyle\sum_{n=1}^{N} \frac{1}{\sqrt{n}+\sqrt{n-1}}$

$$= \sum_{n=1}^{N} \frac{\sqrt{n}-\sqrt{n-1}}{(\sqrt{n}+\sqrt{n-1})(\sqrt{n}-\sqrt{n-1})}$$

$$= \sum_{n=1}^{N} \sqrt{n}-\sqrt{n-1}$$

$$\begin{aligned}
= & \sqrt{N} - \sqrt{N-1} \\
& + \sqrt{N-1} - \sqrt{N-2} \\
& + \sqrt{N-2} - \sqrt{N-3} \\
& \vdots \\
& + \sqrt{3} - \sqrt{2} \\
& + \sqrt{2} - \sqrt{1} \\
& + \sqrt{1} - \sqrt{0}
\end{aligned}$$

$$= \sqrt{N}$$

Since $\displaystyle\frac{1}{\sqrt{n}+\sqrt{n}} < \frac{1}{\sqrt{n}+\sqrt{n-1}}$,

$$\frac{1}{2\sqrt{n}} < \frac{1}{\sqrt{n}+\sqrt{n-1}}$$

$$\sum_{n=1}^{N} \frac{1}{2\sqrt{n}} < \sum_{n=1}^{N} \frac{1}{\sqrt{n}+\sqrt{n-1}}$$

$$\sum_{n=1}^{N} \frac{1}{2\sqrt{n}} < \sqrt{N}$$

$$\sum_{n=1}^{N} \frac{1}{\sqrt{n}} < 2\sqrt{N}$$

OR

$$\sum_{n=1}^{N} \frac{1}{\sqrt{n}} < 2\sqrt{N}$$

when $N = 1$: $\displaystyle\sum_{n=1}^{1} \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1 < 2 = 2 \cdot 1 = 2\sqrt{1}$

Assume the statement is true when $N = k$.

$N = k$: $\displaystyle\sum_{n=1}^{k} \frac{1}{\sqrt{n}} < 2\sqrt{k}$

when $N = k+1$: $\displaystyle\sum_{n=1}^{k+1} \frac{1}{\sqrt{n}} = \sum_{n=1}^{k} \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{k+1}}$

$$\sum_{n=1}^{k} \frac{1}{\sqrt{n}} < 2\sqrt{k}$$

$$\sum_{n=1}^{k} \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{k+1}} < 2\sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\sum_{n=1}^{k+1} \frac{1}{\sqrt{n}} < 2\sqrt{k} + \frac{1}{\sqrt{k+1}}$$

Since $k > 0$,

$$4k^2 + 4k + 1 > 4k^2 + 4k$$

$$(2k+1)^2 > 4k(k+1)$$

$$2k+1 > 2\sqrt{k}\sqrt{k+1}$$

$$2k+2 > 2\sqrt{k}\sqrt{k+1} + 1$$

$$2(k+1) > 2\sqrt{k}\sqrt{k+1} + 1$$

$$\frac{2(k+1)}{\sqrt{k+1}} > \frac{2\sqrt{k}\sqrt{k+1} + 1}{\sqrt{k+1}}$$

$$2\sqrt{k+1} > 2\sqrt{k} + \frac{1}{\sqrt{k+1}}$$

Since $\displaystyle\sum_{n=1}^{k+1} \frac{1}{\sqrt{n}} < 2\sqrt{k} + \frac{1}{\sqrt{k+1}}$ and

$2\sqrt{k} + \dfrac{1}{\sqrt{k+1}} < 2\sqrt{k+1}$, $\displaystyle\sum_{n=1}^{k+1} \frac{1}{\sqrt{n}} < 2\sqrt{k+1}$.

$\displaystyle\sum_{n=1}^{N} \frac{1}{\sqrt{n}} < 2\sqrt{N}$ for every positive integer n.

b) $u_1, u_2, u_3, \dots, \quad u_1 = 5, \quad u_{n+1} = \left(u_n + \frac{1}{u_n}\right)^2, \ n \geq 1$

$u_n > 2^m, \quad m = 2^n$

when $n=1$: $u_1 = 5 > 4 = 2^2 = 2^{2^1} = 2^m, m = 2$

Assume the statement is true when $n=k$.

$n=k$: $u_k > 2^m, \ m = 2^k$

$u_k > 2^{2^k}$

when $n=k+1$: $u_k > 2^{2^k} > 0$

$$\frac{1}{u_k} > 0$$

$$u_k + \frac{1}{u_k} > 2^{2^k}$$

$$\left(u_k + \frac{1}{u_k}\right)^2 > \left(2^{2^k}\right)^2$$

$$u_{k+1} > 2^{2^k 2}$$

$$u_{k+1} > 2^{2^{k+1}}$$

$$u_{k+1} > 2^m, \ m = 2^{k+1}$$

$\therefore u_n > 2^m, m = 2^n$ for every positive integer n.

15. a) $$\frac{1}{1+a^{n-1}} - \frac{1}{1+a^n} = \frac{1+a^n-(1+a^{n-1})}{(1+a^{n-1})(1+a^n)}$$

$$= \frac{1+a^n-1-a^{n-1}}{(1+a^{n-1})(1+a^n)}$$

$$= \frac{a^n-a^{n-1}}{(1+a^{n-1})(1+a^n)}$$

$$\therefore \sum_{n=1}^{N} \frac{a^n-a^{n-1}}{(1+a^{n-1})(1+a^n)} = a^{n-1}(a$$

$$= \sum_{n=1}^{N} \frac{1}{1+a^{n-1}} - \frac{1}{1+a^n}$$

$$\sum_{n=1}^{N} \frac{a^{n-1}(a-1)}{(1+a^{n-1})(1+a^n)}$$

$$= \frac{1}{1+a^{0-1}} - \frac{1}{1+a^1}$$

$$+ \frac{1}{1+a^{2-1}} - \frac{1}{1+a^2}$$

$$+ \frac{1}{1+a^2} - \frac{1}{1+a^3}$$

$\vdots$

$$+\frac{1}{1+a^{N-3}} - \frac{1}{1+a^{N-2}}$$

$$+\frac{1}{1+a^{N-2}} - \frac{1}{1+a^{N-1}}$$

$$+\frac{1}{1+a^{N-1}} - \frac{1}{1+a^{N}}$$

$$= \frac{1}{1+a^{0}} - \frac{1}{1+a^{N}}$$

$$= \frac{1}{1+1} - \frac{1}{1+a^{N}}$$

$$= \frac{1}{2} - \frac{1}{1+a^{N}}$$

$$= \frac{1+a^{N}-2}{2(a^{N}+1)}$$

$$= \frac{a^{N}-1}{2(a^{N}+1)}$$

$$\sum_{n=1}^{N} \frac{a^{n-1}}{(1+a^{n-1})(1+a^{n})} = \frac{a^{N}-1}{2(a-1)(a^{N}+1)}$$

when $a = 2$:

$$\sum_{n=1}^{N} \frac{2^{n-1}}{(1 + 2^{n-1})(1 + 2^{n})} = \frac{2^{N} - 1}{2(2^{N} + 1)}$$

$$\sum_{n=1}^{N} \frac{2^{n}}{(1 + 2^{n-1})(1 + 2^{n})} = \frac{2^{N} - 1}{2^{N} + 1}$$

Since $\dfrac{2^{N} - 1}{2^{N} + 1} < 1$,

$$\therefore \sum_{n=1}^{N} \frac{2^{n}}{(1 + 2^{n-1})(1 + 2^{n})} < 1.$$

b) Let $f(n) = 10^{3n} + 38^{n} + 35$

when $n = 1$: $f(1) = 10^{3(1)} + 38^{1} + 35$

$= 10^{3} + 38^{1} + 35$

$= 1000 + 38 + 35$

$= 1073$

$= 37(29)$

Assume the statement is true when $n = k$.

$n = k$: $f(k) = 10^{3k} + 38^{k} + 35$

$37 \mid f(k)$

$\therefore f(k) = 37s$, s is an integer.

when $n = k + 1$:

$$f(k+1) = 10^{3(k+1)} + 38^{k+1} + 35$$

$$= 10^{3k+3} + 38^{k+1} + 35$$

$$= 10^{3k}10^{3} + 38^{k}38 + 35$$

$$= 10^{3k}1000 + 38^{k}38 + 35$$

$$= 10^{3k}(999 + 1) + 38^{k}(37 + 1) + 35$$

$$= 10^{3k}999 + 10^{3k} + 38^{k}37 + 38^{k} + 35$$

$$= 10^{3k}999 + 38^{k}37 + 10^{3k} + 38^{k} + 35$$

$$= 10^{3k}(37)27 + 38^{k}37 + 37s$$

$$= 37(10^{3k}27 + 38^{k} + s)$$

Since s is an integer and k is an integer,

$10^{3k}27 + 38^{k} + s$ is an integer.

$$\therefore 37 \mid f(k+1)$$

$\therefore 10^{3n} + 38^{n} + 35$ is divisible by 37

for every non-negative integer n.