PAST YEAR QUESTIONS

NOV 2009

With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

The mid-point of AB is M. The point N lies on AC between A and C and is such that $AN = 2NC$.

(i) Find a vector equation of the line MN. [4]

(ii) It is given that MN intersects BC at the point P. Find the position vector of P. [4]

The plane p has equation $2x - 3y + 6z = 16$. The plane q is parallel to p and contains the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

(i) Find the equation of q, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Calculate the perpendicular distance between p and q. [3]

(iii) The line l is parallel to the plane p and also parallel to the plane with equation $x - 2y + 2z = 5$. Given that l passes through the origin, find a vector equation for l. [5]

JUNE 2009

The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. It is given that l lies in the plane with equation $2x + by + cz = 1$, where b and c are constants.

(i) Find the values of b and c. [6]

(ii) The point P has position vector $2\mathbf{j} + 4\mathbf{k}$. Show that the perpendicular distance from P to l is $\sqrt{5}$. [5]

NOV 2008

Two planes have equations $2x - y - 3z = 7$ and $x + 2y + 2z = 0$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for their line of intersection. [6]

JUNE 2008

The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. [4]

(ii) The point P lies on l and is such that angle PAB is equal to $60°$. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. [6]

NOV 2007

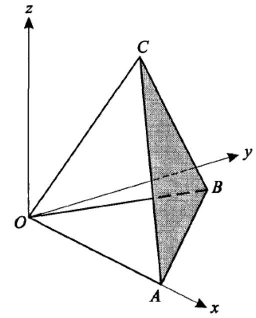
The straight line l has equation $\mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $(\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find a vector equation for the line which lies in p, passes through A and is perpendicular to l. [5]

JUNE 2007



The diagram shows a set of rectangular axes Ox, Oy and Oz, and three points A, B and C with position vectors $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Calculate the acute angle between the planes ABC and OAB. [4]

NOV 2006

The line l has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

JUNE 2006

The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

(i) State a vector equation for the line l. [1]

(ii) Find the position vector of N and show that $BN = 3$. [6]

(iii) Find the equation of the plane containing A, B and N, giving your answer in the form $ax + by + cz = d$. [5]