APPROXIMATIONS

1) Small angle approximation – angular SHM

The Simple Pendulum

A simple pendulum consists of a heavy bob of mass *m*, attached to one end of a light string whose other end is fixed. The bob and string make small oscillations in a vertical plane.

The acceleration force towards the mean position (equilibrium position) is $mg\sin\theta$.

The displacement along the path of the bob is *s* where $s = l\theta$

$$\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2}$$

Using *F=ma* in the tangential direction (where *s* is increasing):

$$-mg\sin\vartheta = m\frac{d^2s}{dt^2} \quad \Rightarrow \quad -mg\sin\vartheta = ml\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \quad (\sin\theta \approx \theta\text{, for small }\theta)$$

Which is SHM (angular SHM) of period $2\pi\sqrt{\frac{l}{g}}$

Note: A **seconds pendulum** takes exactly one second to perform one swing. Thus its period is 2 s.

2) Binomial Approximation

a) Two similar light elastic strings each of natural length l and modulus *4mg* are fastened to a particle of mass *m*. Their other ends are attached to two fixed points *P* and *Q*, $4\,l$ apart, on a smooth horizontal table. Suppose that from *O*, being the midpoint of *PQ*, the particle is pulled a small distance on the table at right angles to *PQ* and released. Show that the motion is approximately SHM, and prove that the period is $\pi\sqrt{\frac{l}{g}}$.

b) Consider a bead of mass *m* which is free to move on a smooth wire along the x-axis. It is connected by elastic strings each of natural length *a*, and modulus of elasticity λ to points *A* (a,a) and *B*(-a,-a) on the coordinate axes. It rests in equilibrium at the origin. If the bead is displaced a small distance along the wire and then released and if x^2 and higher powers are neglected, find an approximation to the period.