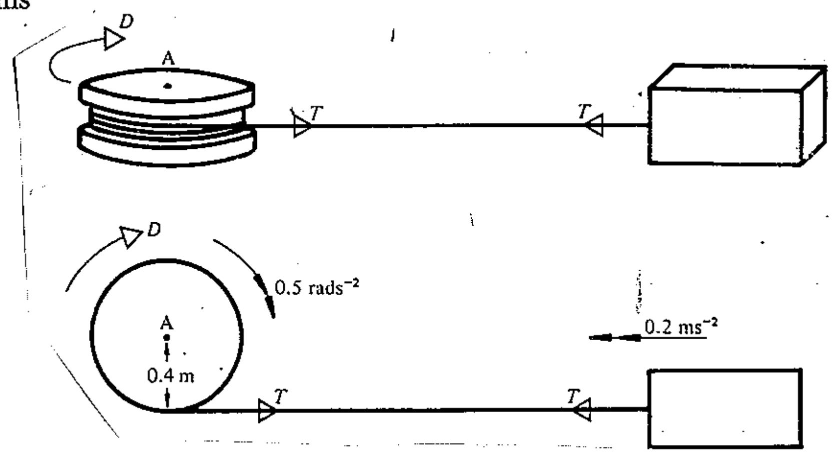
Rotation of Rigid Bodies : Exercise 4

1. A uniform cylinder of mass 2 kg and radius 5 cm can rotate about its axis. A thread is wound round the cylinder and pulled with a force of 2 N tangentially to the cylinder at right angles to the axis. The motion is opposed by a frictional couple of 4 N cm. Find the angular acceleration of the cylinder. [24 rad s^{-2}]

2. A crate of mass 100 kg is dragged across smooth horizontal ground by a light rope whose other end is being wrapped round a pulley of radius 0.4 m. The pulley has a fixed smooth vertical axle and its moment of inertia about this axis is 24 kg m^2.
If the crate moves with an acceleration of 0.2 ms^{-2}, find the magnitude of the torque driving the pulley. Find also the total kinetic energy of the moving system when the speed of the crate is 2ms^{-1} [20 Nm, 500 J]



3. A heavy pulley, which may be regarded as a uniform circular disc of mass $6m$, centre O and radius a, can turn freely in a vertical plane about a fixed horizontal axis through O. A light inextensible string passes over the pulley and particles of mass $2m$ and $3m$ are attached at its free ends. The system is released from rest and the string does not slip on the pulley. Show that the heavier particle falls with acceleration $g/8$.
After time t a constant frictional couple is applied to the pulley and in consequence motion the system comes to rest again in a *further* time t.
Assuming that the lighter particle does not reach the pulley throughout the motion, calculate the total distance covered by the heavier particle, [$gt^2/8$]

4. Find the moment of inertia of a uniform square lamina, of mass M and side a about a diagonal. A uniform square lamina $ABCD$ of mass M and side a is free to rotate about a fixed vertical axis which coincides with the diagonal AC. The lamina is given an initial angular velocity ω_0 and, under the action of a constant driving torque G against a constant frictional torque L, completes 10 revolutions in the first second and 20 revolutions in the next second. Show that $\omega_0 = 10\pi$ rad s^{-1}.
The constant driving torque G is then removed and the lamina is brought to rest by the frictional torque L which has been constant throughout the motion. The lamina is thus brought to rest in a further 15 revolutions. Find L and G.
[125/36 $Ma^2\pi$, 185/36 $Ma^2\pi$]