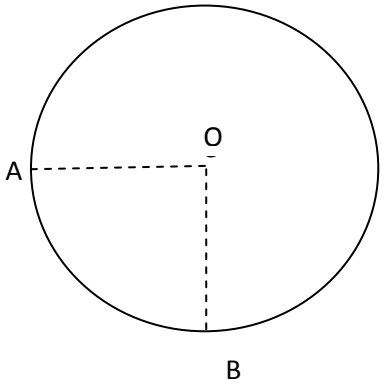
Revision – circular motion (past years Q)

1. Dec 1991



The diagram shows a smooth hollow cylinder, of radius 5 m, fixed with its axis horizontal. The points *A* and *B* are on the inside of the cylinder and are such that *AOB* is a vertical plane, *O* being a point on the axis of the cylinder, *OA* is horizontal and *OB* is vertical. A particle *P*, of mass *m* kg, is being released from rest at *A*. Show that when *P* reaches *B*, its speed is 10 ms^{-1}. [3]

At *B* the particle *P* collides with a second particle *Q*, of mass *4m* kg, which is moving in the opposite direction to *P* with speed 4 ms^{-1}. The coefficient of restitution between *P* and *Q* is $\frac{11}{14}$. Find the velocities of *P* and *Q* immediately after the collision.

[7]

The particle *Q* first comes to instantaneous rest, after the collision, at *C*. Find the angle *BOC*, giving your answer to the nearest 0.1^0. [4]

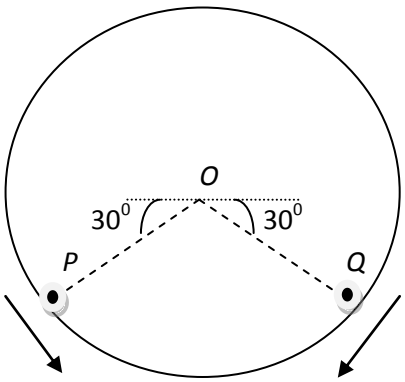
Nov 2010

2. A particle *P* of mass *m* is projected horizontally with speed *u* from the lowest point on the inside of a fixed hollow sphere with centre *O*. The sphere has a smooth internal surface of radius *a*. Assuming that the particle does not lose contact with the sphere, show that when the speed of the particle has been reduced to $\frac{1}{2}u$ the angle θ between *OP* and the downward vertical satisfies the equation

$$8ga\,(1 - \cos\theta) = 3u^2.$$ [2]

Find, in terms of *m*, *u*, *a* and *g*, an expression for the magnitude of the contact force acting on the particle in this position. [4]

3. Dec 1993



A smooth hollow cylinder of internal radius a is fixed with its axis horizontal. Two particles *P* and *Q* having masses *m* and *4m* respectively, are held on the inside surface of the cylinder. The vertical plane containing *P* and Q is perpendicular to the axis of the cylinder and cuts the axis at *O*. Initially the lines *OP* and *OQ* each make an angle 30^0 with the horizontal. At the same instant, *P* and *Q* are each given a speed $2\sqrt{(ga)}$, in directions perpendicular to *OP* and *OQ* respectively and in the plane *OPQ*, as shown in the diagram. Show that, just before the collision, the speed of each particle is $\sqrt{(5ga)}$. [3]

The coefficient of restitution between *P* and *Q* is $\frac{1}{8}$. Find the speed of *P* just after the collision. [5]

Show that *P* loses contact with the inside surface of the cylinder when the line *OP* makes an angle $\cos^{-1}\left(\frac{2}{5}\right)$ with the upward vertical at *O*. [6]

Nov 2009

4.

A particle of mass m is attached to one end A of a light inextensible string of length a. The other end of the string is attached to a fixed point O and the particle hangs in equilibrium under gravity. The particle is projected horizontally so that it starts to move in a vertical circle. The string slackens after turning through an angle of 120°. Show that the speed of the particle is then $\sqrt{\left(\frac{1}{2}ga\right)}$ and find the initial speed of projection. [5]