1 If

$$w_n = (n-1)n(n+1)(n+2)\cdots(n+m),$$

prove that

$$w_n - w_{n+1} = -(m+2)n(n+1)(n+2)\cdots(n+m).$$ [2]

Given

$$v_n = 2n(n+1)(n+2)\cdots(n+m),$$

find $\sum_{n=1}^{N} v_n$ in terms of m and N. [3]

2 Solve the linear system

$$x + b^2y + z = 0\ ,\ \ x + by + z = 0 \ \text{ and } \ x + y - bz = 0,$$

where b is a constant. [5]

3 **Throughout this question the use of a calculator is not permitted.**

If $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{pmatrix}$, find $\mathbf{A}^{-1}$. [6]

4 Prove by mathematical induction that

$$\frac{d}{dx}(u_1u_2\cdots u_n) = u_1u_2\cdots u_n \sum_{r=1}^{n} \frac{1}{u_r}\frac{du_r}{dx}$$

for every positive integer n where $u_1, u_2, \cdots, u_n$ are functions of x. [6]

5 The points A, B, C and D have position vectors $(\mathbf{i} + \mathbf{k})$, $(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, $(-2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$, $(2\mathbf{i} + \mathbf{k})$, respectively, relative to the origin O.

(a) Find the unit vector which is normal to the plane ABC. [3]

(b) Find the cosine of the acute angle between the line AD and the plane ABC. [4]

6 **(a)** The equation $x^4 + px^2 + qx + r = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(i) Express $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ in terms of p. [1]

(ii) Prove that $\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = 2p^2 - 4r$. [2]

(iii) Find the numerical value of $\beta^5 + p\beta^3 + q\beta^2 + r\beta$. [1]

(b) The equation $9x^3 - 9x^2 + mx + 5 = 0$ has roots that are in arithmetic progression. Find the value of m and solve the equation. [5]

7 The curve C has the equation

$$y = \frac{ax - b}{cx - d} \quad, a, b, c, d > 0 \text{ and } bc \neq ad.$$

(a) Find the equations of the asymptotes of C in terms of a, b, c and d. [3]

(b) Show that the curve C has no turning point. [1]

(c) Determine the coordinates of any points where C meets the axes in terms of a, b, c and d. [2]

(d) Sketch C when

(i) $bc > ad$ [3]

(ii) $bc < ad$ [3]