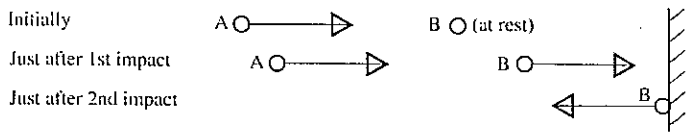
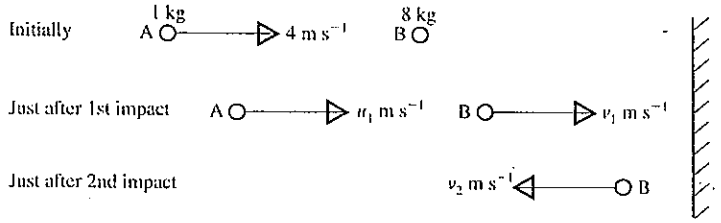
Momentum Ex 4.5

In questions 1 to 3, two particles, A and B, lie on a smooth horizontal plane in a line that is perpendicular to a vertical wall. Initially B is at rest when A strikes it directly. B then goes on to strike the wall and rebounds.

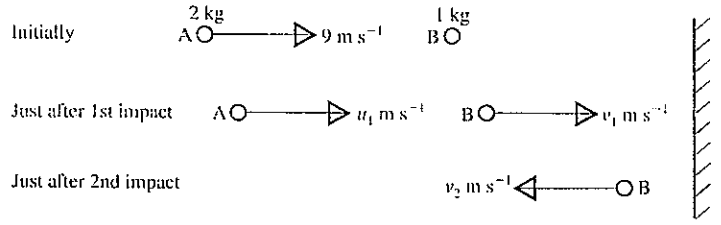


1. The mass of A is 1 kg and it strikes B with speed $4\,\text{m s}^{-1}$. B's mass is 8 kg.



Given that the coefficient of restitution at each impact is $\frac{1}{2}$, find u_1, v_1 and v_2. Explain why there is no further collision.

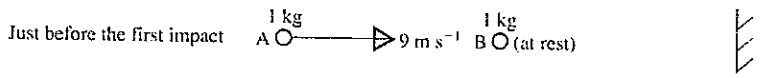
2. The masses of A and B are 2 kg and 1 kg respectively. A strikes B with speed $9\,\text{m s}^{-1}$. The coefficient of restitution between A and B is 1, and that between B and the wall is $\frac{1}{2}$.



(a) Find the values of u_1 and v_1.

(b) Show that there is a second collision between A and B and find their velocities after it. Is there a further collision?

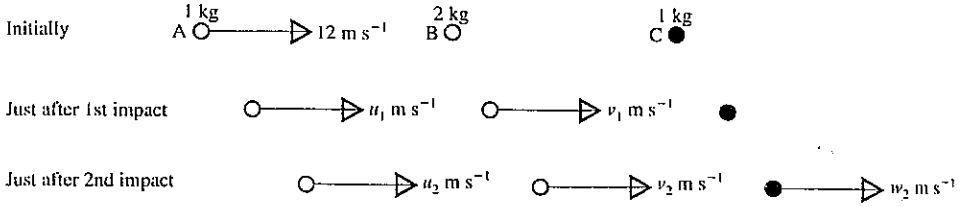
3. Particle A, of mass 1 kg, strikes B, also of mass 1 kg, at $9\,\text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{3}$, and that between B and the wall is $\frac{1}{2}$.



Find the velocities of A and B after each impact until no further collisions can occur.

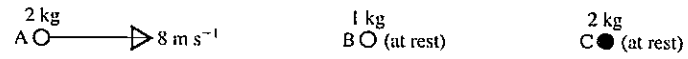
In questions 4 to 7 three small smooth spheres, all with the same radius, lie in a straight line on a smooth horizontal plane. A is projected directly towards B which is at rest and after that collision B goes on to collide directly with C, also at rest.

4. The masses of A, B and C are 1 kg, 2 kg and 1 kg respectively. The coefficient of restitution between A and B is 1 and that between B and C is $\frac{1}{2}$.



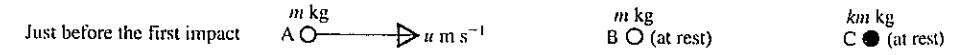
Find the velocities of each sphere after each possible impact.

5. The situation just before the first collision is shown in the diagram.



At each impact the coefficient of restitution is $\frac{1}{2}$. Find the velocities of A, B and C after each of the collisions that can occur.

6. The masses of A, B and C are m kg, m kg and km kg respectively and A's initial speed is u m s^{-1}. At each impact the coefficient of restitution is 1.



(a) Find, in terms of u, the velocities of A and B after the first collision.

(b) Find, in terms of u and k, the velocities of B and C after the second collision.

(c) State, with reasons, whether there will be further collisions if
(i) $k > 1$ (ii) $k < 1$.

ANSWERS

1. $u_1 = -\frac{4}{3}$, $v_1 = \frac{2}{3}$, $v_2 = \frac{1}{3}$; A travelling away from the wall faster than B so B will not catch A to collide again.
2. (a) $u_1 = 3$, $v_1 = 12$
(b) A: -3 m s^{-1}, B: 6 m s^{-1}
(c) B will collide with the wall again.
3. After A strikes B: A → 3 m s^{-1}; B → 6 m s^{-1}
After B strikes wall: 3 m s^{-1} ← B
After B strikes A: 1 m s^{-1} ← A; B → 1 m s^{-1}
After B strikes wall again: $\frac{1}{2}$ m s^{-1} ← B
There will be no more collisions.
4. After 1st collision
4 m s^{-1} ← A; B → 8 m s^{-1}
After 2nd collision
4 m s^{-1} ← A; B → 4 m s^{-1}; C → 8 m s^{-1}
5. 1st: A → 4 m s^{-1}; B → 8 m s^{-1}; C → 0
2nd: A → 4 m s^{-1}; B → 0; C → 4 m s^{-1}
3rd: A → 2 m s^{-1}; B → 4 m s^{-1}; C → 4 m s^{-1}
6. (a) A → 0; B → u
(b) B → $\frac{u(1-k)}{1+k}$; C → $\frac{2u}{k+1}$
(c) (i) Velocity of B is negative so B hits A.
(ii) Velocity of B is positive and < velocity of C; no further collisions