1 $9^2 + 13^2 + 17^2 + \cdots + (4r+5)^2$

$$= \sum_{r=1}^{n} (4r+5)^2$$

$$= \sum_{r=1}^{n} 16r^2 + 40r + 25$$

$$= 16\sum_{r=1}^{n} r^2 + 40\sum_{r=1}^{n} r + 25\sum_{r=1}^{n} 1$$

$$= \frac{16}{6}n(n+1)(2n+1) + \frac{40}{2}n(n+1) + 25n$$

$$= \frac{8}{3}n(n+1)(2n+1) + 20n(n+1) + 25n$$

$$= \frac{16n^3 + 24n^2 + 8n + 60n^2 + 60n + 75n}{3}$$

$$= \frac{16n^3 + 84n^2 + 143n}{3}$$

$$= \frac{n}{3}(16n^2 + 84n + 143)$$

2 Let $f(n) = 13^n + 8^{2n+1}$

When $n = 1$: $f(1) = 13^1 + 8^{2(1)+1}$

$= 13 + 8^{2+1}$

$= 13 + 8^3$

$= 13 + 512$

$= 525$

$= 3(175)$

$\therefore 3 \mid f(1)$

Assume the statement is true when $n = k$

$n = k$: $f(k) = 13^k + 8^{2k+1}$

$3 \mid f(k)$

$\therefore f(k) = 3s$, s is an integer.

$13^k + 8^{2k+1} = 3s$

When $n = k+1$: $f(k+1) = 13^{k+1} + 8^{2(k+1)+1}$

$= 13^k 13 + 8^{2k+2+1}$

$= 13^k 13 + 8^{2k+1} 8^2$

$= 13^k 13 + 8^{2k+1} 64$

$= 13^k 13 + 8^{2k+1}(13 + 51)$

$= 13^k 13 + 8^{2k+1} 13 + 8^{2k+1} 51$

$$= 13(13^{k} + 8^{2k+1}) + 8^{2k+1}3(17)$$

$$= 13(3s) + 8^{2k+1}3(17)$$

$$= 3(13s + 8^{2k+1}17)$$

Since s is an integer and k is an integer, $13s + 8^{2k+1}17$ is an integer.

$$\therefore 3 \mid f(k+1)$$

$\therefore$ $13^{n} + 8^{2n+1}$ is divisible by 3 for every positive integer n.

3 $2x^3 - 4x^2 + 3x + 9 = 0$

α, β, γ are the roots

$\alpha + \beta, \alpha + \gamma, \beta + \gamma$

$\alpha + \beta + \gamma = 2$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{3}{2}$

$\alpha\beta\gamma = -\frac{9}{2}$

Let $u = \alpha + \beta$

$u + \gamma = \alpha + \beta + \gamma$

$= 2$

$\gamma = 2 - u$

γ is a root

$2\gamma^3 - 4\gamma^2 + 3\gamma + 9 = 0$

$2(2-u)^3 - 4(2-u)^2 + 3(2-u) + 9 = 0$

$2(8 - 12u + 6u^2 - u^3) - 4(4 - 4u + u^2)$

$+ 6 - 3u + 9 = 0$

$16 - 24u + 12u^2 - 2u^3 - 16 + 16u - 4u^2$

$+ 6 - 3u + 9 = 0$

$-2u^3 + 8u^2 - 11u + 15 = 0$

$2u^3 - 8u^2 + 11u - 15 = 0$

$\therefore$ The equation having roots $\alpha + \beta, \alpha + \gamma,$
$\beta + \gamma$ is $2u^3 - 8u^2 + 11u - 15 = 0$.

4 $y = \dfrac{6x - 2}{(2x + 3)(4x - 5)}$

Expressing y in terms of partial fractions,

$$\frac{6x - 2}{(2x + 3)(4x - 5)} = \frac{A}{2x + 3} + \frac{B}{4x - 5}$$

$$= \frac{A(4x - 5) + B(2x + 3)}{(2x + 3)(4x - 5)}$$

$$6x - 2 = A(4x - 5) + B(2x + 3)$$

$$= (4A + 2B)x - 5A + 3B$$

$4A + 2B = 6 \qquad -5A + 3B = -2$

$2A + B = 3$

$11A = 11$

$A = 1$

$B = 1$

$$y = \frac{1}{2x + 3} + \frac{1}{4x - 5}$$

Asymptotes ·

As $x \rightarrow \pm\infty \quad y \rightarrow 0$

As $x \rightarrow \frac{-3}{2} \quad y \rightarrow \pm\infty$

As $x \rightarrow \frac{5}{4} \quad y \rightarrow \pm\infty$

$y = 0$

$x = -\frac{3}{2}$

$x = \frac{5}{4}$

Intersection points:

$x = 0$: $y = \frac{2}{15}$

$y = 0$: $\frac{1}{2x+3} + \frac{1}{4x-5} = 0$

$$\frac{1}{2x+3} = \frac{-1}{4x-5}$$

$$4x - 5 = -(2x+3)$$

$$= -2x - 3$$

$$6x = 2$$

$$x = \frac{1}{3}$$

Critical points:

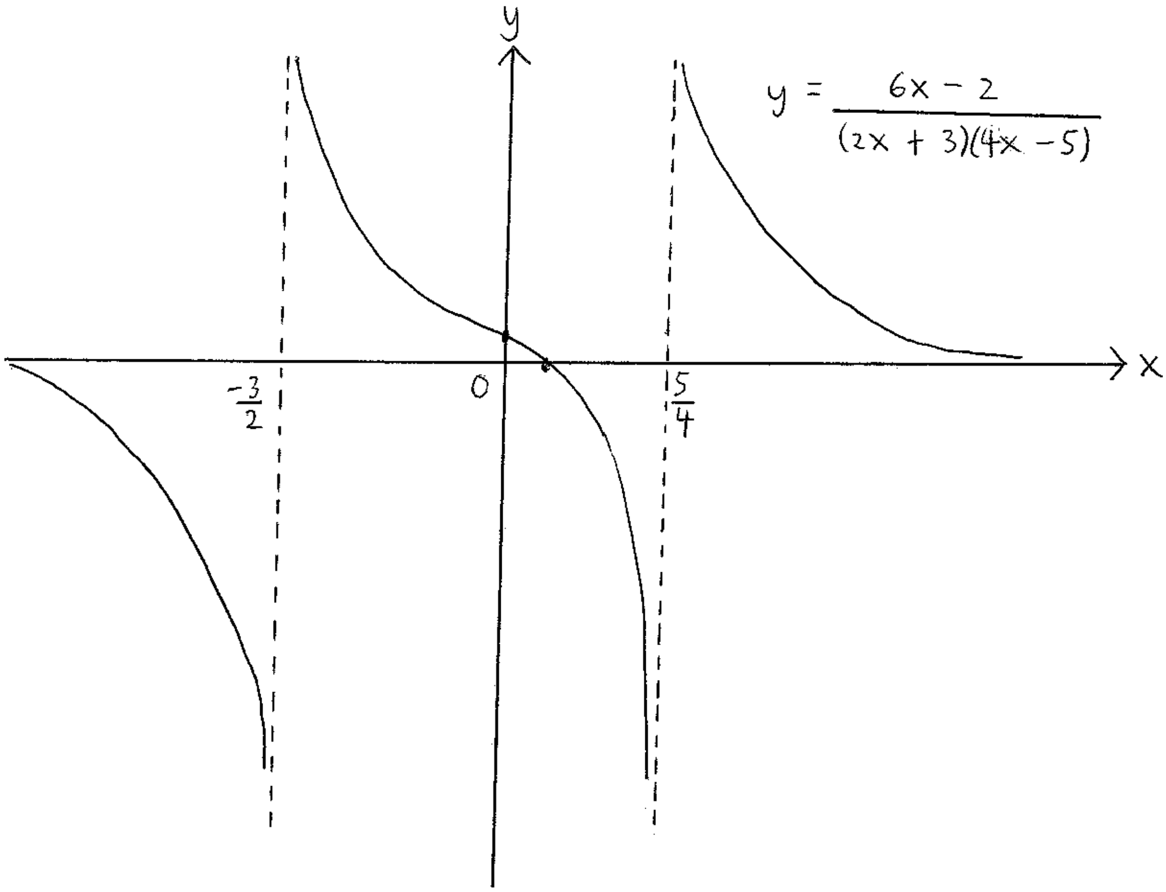
$$\frac{dy}{dx} = \frac{-2}{(2x+3)^2} - \frac{4}{(4x-5)^2}$$

Since $\frac{2}{(2x+3)^2} + \frac{4}{(4x-5)^2} > 0$,

$$\frac{-2}{(2x+3)^2} - \frac{4}{(4x-5)^2} < 0$$

$$\frac{dy}{dx} < 0$$

no critical points



• Intersection points.