OCTOBER / NOVEMBER 2010

1. $C : y = \dfrac{e^{2x} + e^{-2x}}{4}$, $x = 0$ $x = \dfrac{1}{2}$

$$\frac{dy}{dx} = \frac{e^{2x} - e^{-2x}}{2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{e^{2x} - e^{-2x}}{2}\right)^2$$

$$= 1 + \frac{e^{4x} - 2 + e^{-4x}}{4}$$

$$= \frac{e^{4x} + 2 + e^{-4x}}{4}$$

$$= \left(\frac{e^{2x} + e^{-2x}}{2}\right)^2$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{e^{2x} + e^{-2x}}{2}$$

The arc length of C from $x = 0$ to $x = \dfrac{1}{2}$ is $\displaystyle\int_0^{\frac{1}{2}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$$= \int_0^{\frac{1}{2}} \frac{e^{2x} + e^{-2x}}{2}\, dx$$

$$= \left[\frac{e^{2x} - e^{-2x}}{4}\right]_0^{\frac{1}{2}}$$

$$= \frac{e - e^{-1}}{4} - \left(\frac{1 - 1}{4}\right)$$

$$= \frac{e^2 - 1}{4e}$$

2. $$S_N = \sum_{n=1}^{N} \frac{1}{n(n+2)}$$

$$= \sum_{n=1}^{N} \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right.$$

$$+ \frac{1}{2} - \frac{1}{4}$$

$$+ \frac{1}{3} - \frac{1}{5}$$

$$+ \frac{1}{4} - \frac{1}{6}$$

$$\vdots$$

$$+ \frac{1}{N-3} - \frac{1}{N-1}$$

$$+ \frac{1}{N-2} - \frac{1}{N}$$

$$+ \frac{1}{N-1} - \frac{1}{N+1}$$

$$\left. + \frac{1}{N} - \frac{1}{N+2}\right)$$

$$= \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{N+1} - \frac{1}{N+2}\right)$$

$$= \frac{1}{2}\left(\frac{3}{2} - \frac{1}{N+1} - \frac{1}{N+2}\right)$$

$$= \frac{3}{4} - \frac{1}{2(N+1)} - \frac{1}{2(N+2)}$$

$$\lim_{N\to\infty} S_N = \lim_{N\to\infty}\left(\frac{3}{4} - \frac{1}{2(N+1)} - \frac{1}{2(N+2)}\right)$$

$$= \frac{3}{4}$$

3. $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, $x = 1$ $x = 4$

The area of the region bounded by $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, the x-axis from $x=1$ to $x=4$ and the line $x=4$, A, is $\int_1^4 y \, dx$

$$= \int_1^4 \sqrt{x} - \frac{1}{\sqrt{x}} \, dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3} - 2x^{\frac{1}{2}} \right]_1^4$$

$$= \frac{2(4^{\frac{3}{2}})}{3} - 2(4^{\frac{1}{2}}) - \left(\frac{2}{3} - 2\right)$$

$$= \frac{2(8)}{3} - 2(2) - \frac{2}{3} + 2$$

$$= \frac{16}{3} - 4 - \frac{2}{3} + 2$$

$$= \frac{8}{3}$$

The y-coordinate of the region bounded by $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, the x-axis from $x=1$ to $x=4$ and the line $x=4$, R, is

$$\frac{\int_1^4 \frac{y^2}{2} \, dx}{A}$$

$$= \frac{\int_1^4 \frac{1}{2}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx}{\frac{8}{3}}$$

$$= \frac{3}{8} \int_1^4 \frac{1}{2}\left(x - 2 + \frac{1}{x}\right) dx$$

$$= \frac{3}{16} \int_1^4 x - 2 + \frac{1}{x} \, dx$$

$$= \frac{3}{16} \left[\frac{x^2}{2} - 2x + \ln x \right]_1^4$$

$$= \frac{3}{16} \left(\frac{4^2}{2} - 2(4) + \ln 4 - \frac{1}{2} + 2 - \ln 1 \right)$$

$$= \frac{3}{16} \left(8 - 8 + 2\ln 2 - \frac{1}{2} + 2 - 0 \right)$$

$$= \frac{3}{16} \left(\frac{3}{2} + 2\ln 2 \right)$$

$$= \frac{9}{32} + \frac{3}{8} \ln 2$$

CONFIDENTIAL

4. Let $f(n) = 7^{2n+1} + 5^{n+3}$

when $n=0$. $f(0) = 7^{2(0)+1} + 5^{0+3}$

$= 7^{0+1} + 5^3$

$= 7^1 + 5^3$

$= 7 + 125$

$= 132$

$= 44(3)$

$\therefore 44 \mid f(0)$

Assume the statement is true when $n=k$.

$n=k$: $f(k) = 7^{2k+1} + 5^{k+3}$

$44 \mid f(k)$

$\therefore f(k) = 44s$, s is an integer

$7^{2k+1} + 5^{k+3} = 44s$

When $n=k+1$:

$f(k+1) = 7^{2(k+1)+1} + 5^{k+1+3}$

$= 7^{2k+2+1} + 5^{k+1+3}$

$= 7^{2k+1}7^2 + 5^{k+3}5^1$

$= 7^{2k+1}49 + 5^{k+3}5$

$= 7^{2k+1}(44+5) + 5^{k+3}5$

$= 7^{2k+1}44 + 7^{2k+1}5 + 5^{k+3}5$

$= 7^{2k+1}44 + 5(7^{2k+1} + 5^{k+3})$

$= 7^{2k+1}44 + 5(44s)$

$= 44(7^{2k+1} + 5s)$

Since s is an integer and k is an integer, $7^{2k+1} + 5s$ is an integer.

$\therefore 44 \mid f(k+1)$

$\therefore 7^{2n+1} + 5^{n+3}$ is divisible by 44 for every integer $n \geq 0$.

CONFIDENTIAL

5. $I_n = \int_0^1 (1-x)^n \sin x \, dx, \quad n \geq 0$

$I_{n+2} = \int_0^1 (1-x)^{n+2} \sin x \, dx$

$u = (1-x)^{n+2} \qquad dv = \sin x \, dx$

$du = -(n+2)(1-x)^{n+1} dx \qquad v = -\cos x$

$= \left[-(1-x)^{n+2}\cos x\right]_0^1 - \int_0^1 (n+2)(1-x)^{n+1} \cos x \, dx$

$= 0 - (-1) - (n+2)\int_0^1 (1-x)^{n+1} \cos x \, dx$

$= 1 - (n+2)\int_0^1 (1-x)^{n+1} \cos x \, dx$

$u = (1-x)^{n+1} \qquad dv = \cos x \, dx$

$du = -(n+1)(1-x)^n dx \qquad v = \sin x$

$= 1 - (n+2)\Big(\left[(1-x)^{n+1}\sin x\right]_0^1$

$+ \int_0^1 (n+1)(1-x)^n \sin x \, dx\Big)$

$= 1 - (n+2)\left(0 + (n+1)\int_0^1 (1-x)^n \sin x \, dx\right)$

$= 1 - (n+2)(n+1)\int_0^1 (1-x)^n \sin x \, dx)$

$= 1 - (n+2)(n+1) I_n$

when $n = 4$: $I_6 = 1 - 6(5) I_4$

$= 1 - 30 I_4$

$I_4 = 1 - 4(3) I_2$

$= 1 - 12 I_2$

$I_2 = 1 - 2(1) I_0$

$= 1 - 2 I_0$

$I_0 = \int_0^1 (1-x)^0 \sin x \, dx$

$= \int_0^1 \sin x \, dx$

$= \left[-\cos x\right]_0^1$

$= -\cos 1 - (-1)$

$= 1 - \cos 1$

$I_2 = 1 - 2(1 - \cos 1)$

$= 1 - 2 + 2\cos 1$

$= 2\cos 1 - 1$

$I_4 = 1 - 12(2\cos 1 - 1)$

$= 1 - 24\cos 1 + 12$

$= 13 - 24\cos 1$

$I_6 = 1 - 30(13 - 24\cos 1)$

$= 1 - 390 + 720\cos 1$

$= 720\cos 1 - 389$

$\approx 0.017.$

6. $T: R^4 \longrightarrow R^4$

$$A = \begin{pmatrix} 1 & 2 & -1 & \alpha \\ 2 & 3 & -1 & 0 \\ 2 & 1 & 2 & -2 \\ 0 & 1 & -3 & -2 \end{pmatrix}$$

$$\xrightarrow[-2r_1 + r_3]{-2r_1 + r_2} \begin{pmatrix} 1 & 2 & -1 & \alpha \\ 0 & -1 & 1 & -2\alpha \\ 0 & -3 & 4 & -2\alpha - 2 \\ 0 & 1 & -3 & -2 \end{pmatrix}$$

$$\xrightarrow[r_2 + r_4]{-3r_2 + r_3} \begin{pmatrix} 1 & 2 & -1 & \alpha \\ 0 & -1 & 1 & -2\alpha \\ 0 & 0 & 1 & 4\alpha - 2 \\ 0 & 0 & -2 & -2\alpha - 2 \end{pmatrix}$$

$$\xrightarrow{2r_3 + r_4} \begin{pmatrix} 1 & 2 & -1 & \alpha \\ 0 & -1 & 1 & -2\alpha \\ 0 & 0 & 1 & 4\alpha - 2 \\ 0 & 0 & 0 & 6\alpha - 6 \end{pmatrix}$$

If the dimension of the range space of T is 4, $6\alpha - 6 \neq 0$

$\therefore \alpha \neq 1$.

$$\alpha = 1: \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & -1 & 0 \\ 2 & 1 & 2 & -2 \\ 0 & 1 & -3 & -2 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2 + r_3} \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

since if

$$a\begin{pmatrix} 1 \\ 2 \\ 2 \\ 0 \end{pmatrix} + b\begin{pmatrix} 2 \\ 3 \\ 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} -1 \\ -1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\left.\begin{aligned} a + 2b - c &= 0 \\ 2a + 3b - c &= 0 \\ 2a + b + 2c &= 0 \\ b - 3c &= 0 \end{aligned}\right\}$$

$$\begin{aligned} & & a + 2b - c &= 0 \\ -2 \times ① + ②&: & -b + c &= 0 \\ -2 \times ① + ③&: & -3b + 4c &= 0 \\ & & b - 3c &= 0 \end{aligned}$$

$$\begin{aligned} -3 \times ② + ③&: & a + 2b - c &= 0 \\ ② + ④&: & -b + c &= 0 \\ & & c &= 0 \\ & & -2c &= 0 \end{aligned}$$

$$c = 0$$
$$b = 0$$
$$a = 0$$

the vectors $\begin{pmatrix}1\\2\\2\\0\end{pmatrix}$, $\begin{pmatrix}2\\3\\1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\-1\\2\\-3\end{pmatrix}$ are

linearly independent.

$\therefore \left\{ \begin{pmatrix}1\\2\\2\\0\end{pmatrix}, \begin{pmatrix}2\\3\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\\2\\-3\end{pmatrix} \right\}$

forms a basis for the range space of T.

If $\begin{pmatrix}p\\1\\1\\q\end{pmatrix}$ is in the range space of T,

$$\begin{pmatrix}p\\1\\1\\q\end{pmatrix} = a\begin{pmatrix}1\\2\\2\\0\end{pmatrix} + b\begin{pmatrix}2\\3\\1\\1\end{pmatrix} + c\begin{pmatrix}-1\\-1\\2\\-3\end{pmatrix}$$

$$= \begin{pmatrix}a+2b-c\\2a+3b-c\\2a+b+2c\\b-3c\end{pmatrix}.$$

$$\left.\begin{aligned} a+2b-c &= p \\ 2a+3b-c &= 1 \\ 2a+b+2c &= 1 \\ b-3c &= q \end{aligned}\right\}$$

$$\left.\begin{aligned} -2\times 1 + 2:\quad a+2b-c &= p \\ -2\times 1 + 3:\quad -b+c &= 1-2p \\ -3b+4c &= 1-2p \\ b-3c &= q \end{aligned}\right\}$$

$$\left.\begin{aligned} -3\times 2 + 3:\quad a+2b-c &= p \\ 2 + 4:\quad -b+c &= 1-2p \\ c &= -2+4p \\ -2c &= 1-2p+q \end{aligned}\right\}$$

$$-2(-2+4p) = 1-2p+q$$
$$4-8p = 1-2p+q$$
$$6p+q = 3$$

7. $x^3 + 4x - 1 = 0$

α, β, γ are the roots.

$y = \frac{1}{1+x}$

$x + 1 = \frac{1}{y}$

$x = \frac{1}{y} - 1$

$\left(\frac{1}{y} - 1\right)^3 + 4\left(\frac{1}{y} - 1\right) - 1 = 0$

$\frac{1}{y^3} - \frac{3}{y^2} + \frac{3}{y} - 1 + \frac{4}{y} - 4 - 1 = 0$

$\frac{1}{y^3} - \frac{3}{y^2} + \frac{7}{y} - 6 = 0$

$1 - 3y + 7y^2 - 6y^3 = 0$

$6y^3 - 7y^2 + 3y - 1 = 0$

$\therefore$ The equation $6y^3 - 7y^2 + 3y - 1$ has roots $\frac{1}{\alpha+1}, \frac{1}{\beta+1}, \frac{1}{\gamma+1}$.

$\frac{1}{\alpha+1} + \frac{1}{\beta+1} + \frac{1}{\gamma+1} = \frac{7}{6}$

$\frac{1}{(\alpha+1)(\beta+1)} + \frac{1}{(\alpha+1)(\gamma+1)} + \frac{1}{(\beta+1)(\gamma+1)} = \frac{1}{2}$

$\frac{1}{(\alpha+1)(\beta+1)(\gamma+1)} = \frac{1}{6}$

$\frac{1}{(\alpha+1)^n} + \frac{1}{(\beta+1)^n} + \frac{1}{(\gamma+1)^n}$

$n=1: \frac{1}{\alpha+1} + \frac{1}{\beta+1} + \frac{1}{\gamma+1} = \frac{7}{6}$

$n=2: \frac{1}{(\alpha+1)^2} + \frac{1}{(\beta+1)^2} + \frac{1}{(\gamma+1)^2}$

$= \left(\frac{1}{\alpha+1} + \frac{1}{\beta+1} + \frac{1}{\gamma+1}\right)^2$

$- 2\left(\frac{1}{(\alpha+1)(\beta+1)} + \frac{1}{(\alpha+1)(\gamma+1)} + \frac{1}{(\beta+1)(\gamma+1)}\right)$

$= \left(\frac{7}{6}\right)^2 - 2\left(\frac{1}{2}\right)$

$= \frac{49}{36} - 1$

$= \frac{13}{36}$

Since $6y^3 - 7y^2 + 3y - 1 = 0$ has roots

$\frac{1}{\alpha+1}, \frac{1}{\beta+1}, \frac{1}{\gamma+1}$, if $S_n = \frac{1}{(\alpha+1)^n} + \frac{1}{(\beta+1)^n} + \frac{1}{(\gamma+1)^n}$,

$$6S_3 - 7S_2 + 3S_1 - S_0 = 0$$

$$S_0 = \frac{1}{(\alpha+1)^0} + \frac{1}{(\beta+1)^0} + \frac{1}{(\gamma+1)^0} = 1+1+1 = 3$$

$$S_1 = \frac{7}{6}$$

$$S_2 = \frac{13}{36}$$

$$6S_3 - 7\left(\frac{13}{36}\right) + 3\left(\frac{7}{6}\right) - 3 = 0$$

$$6S_3 - \frac{91}{36} + \frac{7}{2} - 3 = 0$$

$$6S_3 = \frac{73}{36}$$

$$S_3 = \frac{73}{216}$$

$$\therefore \frac{1}{(\alpha+1)^3} + \frac{1}{(\beta+1)^3} + \frac{1}{(\gamma+1)^3} = \frac{73}{216}$$

$$\frac{(\beta+1)(\gamma+1)}{(\alpha+1)^2} + \frac{(\gamma+1)(\alpha+1)}{(\beta+1)^2} + \frac{(\alpha+1)(\beta+1)}{(\gamma+1)^2}$$

$$= \frac{(\alpha+1)(\beta+1)(\gamma+1)}{(\alpha+1)^3} + \frac{(\alpha+1)(\beta+1)(\gamma+1)}{(\beta+1)^3}$$

$$+ \frac{(\alpha+1)(\beta+1)(\gamma+1)}{(\gamma+1)^3}$$

$$= (\alpha+1)(\beta+1)(\gamma+1)\left(\frac{1}{(\alpha+1)^3} + \frac{1}{(\beta+1)^3} + \frac{1}{(\gamma+1)^3}\right)$$

$$= 6S_3$$

$$= \frac{73}{36}$$

CONFIDENTIAL

8. $C_1 : r = 3\sin\theta, \quad 0 \le \theta < \pi$

$C_2 : r = 1 + \sin\theta, \quad -\pi < \theta \le \pi$

i) When C_1 meets C_2,

$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$r = \frac{3}{2}$

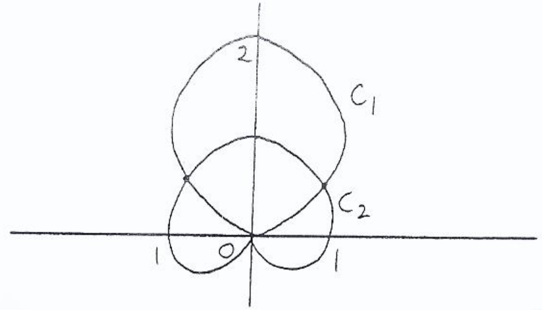
$\therefore C_1$ and C_2 meet at $\left(\frac{3}{2}, \frac{\pi}{6}\right)$ and $\left(\frac{3}{2}, \frac{5\pi}{6}\right)$.

ii)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
$3\sin\theta$	0	$\frac{3}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3\sqrt{3}}{2}$	3	$\frac{3\sqrt{3}}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3}{2}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$1+\sin\theta$	1	$\frac{3}{2}$	$1+\frac{1}{\sqrt{2}}$	$1+\frac{\sqrt{3}}{2}$	2

θ	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$\frac{\pi}{3}$	$-\frac{\pi}{2}$
$1+\sin\theta$	$\frac{1}{2}$	$1-\frac{1}{\sqrt{2}}$	$1-\frac{\sqrt{3}}{2}$	0



iii) The area of the region inside C_1 and outside C_2 is

$$2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(3\sin\theta)^2}{2} - \frac{(1+\sin\theta)^2}{2}\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\sin^2\theta - (1 + 2\sin\theta + \sin^2\theta)\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 8\sin^2\theta - 1 - 2\sin\theta\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4 - 4\cos 2\theta - 1 - 2\sin\theta\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 3 - 4\cos 2\theta - 2\sin\theta\, d\theta$$

$$= \Big[3\theta - 2\sin 2\theta + 2\cos\theta\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{3\pi}{2} - 2\sin\pi + 2\cos\frac{\pi}{2} - \left(\frac{\pi}{2} - 2\sin\frac{\pi}{3} + 2\cos\frac{\pi}{6}\right)$$

$$= \frac{3\pi}{2} - \frac{\pi}{2} + 2\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \pi$$

9. $$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{pmatrix}$$

$$|A - \lambda I| = (3-\lambda) \begin{vmatrix} 2-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 0 & 3-\lambda \end{vmatrix}$$

$$+ 0 \begin{vmatrix} -1 & 2-\lambda \\ 0 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[(2-\lambda)(3-\lambda) - 1\right] + 1(-3+\lambda) - 0$$

$$+ 0$$

$$= (3-\lambda)(6 - 5\lambda + \lambda^2 - 1) - 3 + \lambda$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 5) - (3-\lambda)$$

$$= (3-\lambda)(\lambda^2 - 5x + 5 - 1)$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 4)$$

$$= (3-\lambda)(\lambda - 1)(\lambda - 4)$$

when $|A - \lambda I| = 0$:

$$(3-\lambda)(\lambda - 1)(\lambda - 4) = 0$$

$$\lambda = 1, 3, 4.$$

$\lambda = 1$: $$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right)$$

$$\xrightarrow{r_1 \longleftrightarrow r_2} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right)$$

$$\xrightarrow{2r_1 + r_2} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right)$$

$$\xrightarrow{r_2 + r_3} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Let $z = s$, $s \in R$

$y = 2s$

$-x + y - z = 0$

$-x + 2s - s = 0$

$x = s$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s \\ 2s \\ s \end{pmatrix}$$

$$= s \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

CONFIDENTIAL

$\lambda = 3$: $\begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{-r_2 + r_3} \left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$y = 0$

Let $z = s$, $s \in R$

$x = -s$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -s \\ 0 \\ s \end{pmatrix} = s\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 4$: $\begin{pmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ -1 & -2 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right)$$

$$\xrightarrow{-r_1 + r_2} \left(\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right)$$

$$\xrightarrow{-r_2 + r_3} \left(\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Let $z = s$, $s \in R$

$y = -s$

$x + y = 0$

$x = s$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s \\ -s \\ s \end{pmatrix} = s\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$\therefore$ The eigenvalues of A are $1, 3, 4$ with corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

If $A^n = PD^nP^{-1}$, where P is a square matrix and D is a diagonal matrix, $P = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$

and $D = \begin{pmatrix} 1^n & 0 & 0 \\ 0 & 3^n & 0 \\ 0 & 0 & 4^n \end{pmatrix}$.

CONFIDENTIAL

$$A\underset{\sim}{x} = \lambda \underset{\sim}{x}$$

$$(A - 2I)\underset{\sim}{x} = A\underset{\sim}{x} - 2I\underset{\sim}{x}$$

$$= \lambda \underset{\sim}{x} - 2\underset{\sim}{x}$$

$$= (\lambda - 2)\underset{\sim}{x}$$

$\therefore$ If A has an eigenvalue λ with corresponding eigenvector $\underset{\sim}{x}$, $A - 2I$ has an eigenvalue $\lambda - 2$ with corresponding eigenvector $\underset{\sim}{x}$.

If $(A - 2I)^3 = MDM^{-1}$, where M is a non-singular matrix and D is a diagonal matrix,

$$M = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } D = \begin{pmatrix} (1-2)^3 & 0 & 0 \\ 0 & (3-2)^3 & 0 \\ 0 & 0 & (4-2)^3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix}.$$

10. $(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta$
$$- 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta$$
$$+ 5\cos\theta\sin^4\theta + i\sin^5\theta$$

$$\cos 5\theta + i\sin 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$
$$+ i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$$

$$\therefore \cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$$

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta}$$

$$= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$$

$$= \frac{\dfrac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta}}{\dfrac{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}{\cos^5\theta}}$$

$$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

$$= \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}, \quad t = \tan\theta.$$

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when $\tan 5\theta = 0$: $\dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4} = 0$

$$5t - 10t^3 + t^5 = 0$$

$$t^5 - 10t^3 + 5t = 0$$

$$t(t^4 - 10t^2 + 5) = 0$$

$$t = 0,\ t^4 - 10t^2 + 5 = 0$$

$$\tan 5\theta = 0$$

$$5\theta = n\pi$$

$$\theta = \frac{n\pi}{5},\ n = 0, 1, 2, 3, 4$$

$$t = \tan\theta$$

$$= \tan\frac{n\pi}{5}$$

$\therefore$ The roots of the equation $x^4 - 10x^2 + 5 = 0$ are $\tan\frac{n\pi}{5}$, $n = 1, 2, 3, 4$.

$$\tan\frac{\pi}{5} + \tan\frac{2\pi}{5} + \tan\frac{3\pi}{5} + \tan\frac{4\pi}{5} = 0$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} + \tan\frac{\pi}{5}\tan\frac{3\pi}{5} + \tan\frac{\pi}{5}\tan\frac{4\pi}{5}$$

$$+ \tan\frac{2\pi}{5}\tan\frac{3\pi}{5} + \tan\frac{2\pi}{5}\tan\frac{4\pi}{5} + \tan\frac{3\pi}{5}\tan\frac{4\pi}{5} = 10$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5} + \tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{4\pi}{5}$$

$$+ \tan\frac{\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5} + \tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5} = 0$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5} = 5$$

Since $\tan\frac{3\pi}{5} = \tan\left(\pi - \frac{2\pi}{5}\right) = -\tan\frac{2\pi}{5}$

and $\tan\frac{4\pi}{5} = \tan\left(\pi - \frac{\pi}{5}\right) = -\tan\frac{\pi}{5}$,

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\left(-\tan\frac{2\pi}{5}\right)\left(-\tan\frac{\pi}{5}\right) = 5$$

$$\tan^2\frac{\pi}{5}\tan^2\frac{2\pi}{5} = 5$$

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = \sqrt{5}$$

11.

$$x\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 4xy = 8x^2 + 16, \ x \neq 0$$

If $z = xy$

$$\frac{dz}{dx} = \frac{d}{dx}(xy)$$

$$= x\frac{dy}{dx} + y$$

$$\frac{d^2z}{dx^2} = \frac{d}{dx}\left(x\frac{dy}{dx} + y\right)$$

$$= x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx}$$

$$= x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}$$

$$\frac{d^2z}{dx^2} + 4z = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy$$

$$= 8x^2 + 16$$

$$\frac{d^2z}{dx^2} + 4z = 0$$

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$\therefore$ The complementary function, z_c, is

$z_c = A\cos 2x + B\sin 2x$.

The particular integral, z_p, is given by

$$z_p = Cx^2 + Dx + E$$

$$\frac{dz_p}{dx} = 2Cx + D$$

$$\frac{d^2z_p}{dx^2} = 2C$$

$$\frac{d^2z_p}{dx^2} + 4z_p = 2C + 4(Cx^2 + Dx + E)$$

$$= 4Cx^2 + 4Dx + 2C + 4E$$

$$= 8x^2 + 16$$

$4C = 8 \quad 4D = 0 \quad 2C + 4E = 16$

$C = 2 \quad D = 0 \quad E = 3$

$\therefore z_p = 2x^2 + 3$

$$z = z_c + z_p$$

$$= A\cos 2x + B\sin 2x + 2x^2 + 3$$

$$xy = A\cos 2x + B\sin 2x + 2x^2 + 3$$

$\therefore$ The general solution is

$$y = \frac{A\cos 2x}{x} + \frac{B\sin 2x}{x} + 2x + \frac{3}{x}$$

$$\frac{dy}{dx} = \frac{-2A\sin 2x}{x} - \frac{A\cos 2x}{x^2} + \frac{2B\cos 2x}{x} - \frac{B\sin 2x}{x^2} + 2 - \frac{3}{x^2}$$

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$x = \frac{\pi}{2}\ \ y = 0: \quad 0 = \frac{2}{\pi}(-A) + \pi + \frac{6}{\pi}$

$$\frac{2A}{\pi} = \frac{\pi^2 + 6}{\pi}$$

$$A = \frac{\pi^2 + 6}{2}$$

$x = \frac{\pi}{2}\ \ \frac{dy}{dx} = -2: \quad -2 = A\left(\frac{4}{\pi^2}\right) + 2B\left(\frac{-2}{\pi}\right) + 2 - \frac{12}{\pi^2}$

$$= \left(\frac{\pi^2+6}{2}\right)\frac{4}{\pi^2} - \frac{4B}{\pi} + 2 - \frac{12}{\pi^2}$$

$$= \frac{2\pi^2+12}{\pi^2} - \frac{4B}{\pi} + 2 - \frac{12}{\pi^2}$$

$$\frac{4B}{\pi} = \frac{2\pi^2+12}{\pi^2} + 4 - \frac{12}{\pi^2}$$

$$= 6$$

$$B = \frac{3\pi}{2}$$

$$\therefore\ y = \frac{(\pi^2+6)\cos 2x}{2x} + \frac{3\pi \sin 2x}{2x} + 2x + \frac{3}{x}$$

12. EITHER

$C: \ y = \frac{x^2 + 2\lambda x}{x^2 - 2x + \lambda}, \quad \lambda \neq -1.$

i)
$$\begin{array}{r|l} & \ \ 1 \\ \hline x^2 - 2x + \lambda & x^2 + 2\lambda x \\ & x^2 - 2x + \lambda \\ \hline & 2(\lambda+1)x - \lambda \end{array}$$

$$= 1 + \frac{2(\lambda+1)x - \lambda}{x^2 - 2x + \lambda}$$

$$\frac{dy}{dx} = \frac{2(\lambda+1)}{x^2 - 2x + \lambda} - \frac{(2(\lambda+1)x - \lambda)(2x-2)}{(x^2-2x+\lambda)^2}$$

when $\frac{dy}{dx} = 0$

$$\frac{2(\lambda+1)}{x^2-2x+\lambda} - \frac{(2(\lambda+1)x - \lambda)(2x-2)}{(x^2-2x+\lambda)^2} = 0$$

$$2(\lambda+1)(x^2 - 2x + \lambda) - (2(\lambda+1)x - \lambda)(2x-2) = 0$$

$$\lambda x^2 - 2\lambda x + \lambda^2 + x^2 - 2x + \lambda$$

$$-2(\lambda+1)x^2 + \lambda x + 2(\lambda+1)x - \lambda = 0$$

$$-(1+\lambda)x^2 + \lambda x + \lambda^2 = 0$$

$$(1+\lambda)x^2 - \lambda x - \lambda^2 = 0$$

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$a = 1 + \lambda \quad b = -\lambda \quad c = -\lambda^2$

$b^2 - 4ac = \lambda^2 - 4(1+\lambda)(-\lambda^2)$

$= \lambda^2 + 4\lambda^2 + 4\lambda^3$

$= 4\lambda^3 + 5\lambda^2$

$$x = \frac{\lambda \pm \lambda\sqrt{4\lambda + 5}}{2(1+\lambda)}$$

$\therefore$ C has at most two stationary points.

ii) If C has exactly two stationary points,

$b^2 - 4ac > 0$

$4\lambda^3 + 5\lambda^2 > 0$

$\lambda^2(4\lambda + 5) > 0$

$4\lambda + 5 > 0$

$\lambda > -\frac{5}{4}$

iii) If C has two vertical asymptotes,

the equation $x^2 - 2x + \lambda = 0$ has two roots

$a = 1 \quad b = -2 \quad c = \lambda$

$b^2 - 4ac = 4 - 4\lambda$

$b^2 - 4ac > 0$

$4 - 4\lambda > 0$

$\lambda < 1.$

iv) a) when $y = 0$: $\dfrac{x^2 + 2\lambda x}{x^2 - 2x + \lambda} = 0$

$x^2 + 2\lambda x = 0$

$x(x + 2\lambda) = 0$

$x = 0, -2\lambda$

b) As $x \to \pm\infty \quad y \to 1$

$\therefore$ $y = 1$ is the horizontal asymptote.

when $y = 1$:

$$\frac{x^2 + 2\lambda x}{x^2 - 2x + \lambda} = 1$$

$x^2 + 2\lambda x = x^2 - 2x + \lambda$

$2(\lambda + 1)x = \lambda$

$x = \dfrac{\lambda}{2(\lambda + 1)}$

v) a) $\lambda < -2$

If C has no stationary points,

$b^2 - 4ac < 0$

$4\lambda^3 + 5\lambda^2 < 0$

$\lambda^2(4\lambda + 5) < 0$

$4\lambda + 5 < 0$

$\lambda < -\frac{5}{4}$

$\therefore$ no stationary points.

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As $x \to \pm\infty$ $y \to 1$

As $x \to 1 \pm \sqrt{1-\lambda}$ $y \to \pm\infty$

$y = 1$

$x = 1 \pm \sqrt{1-\lambda}$

when $x = 0$: $y = 0$

Since $\lambda < -2$

$-\lambda > 2$

$1-\lambda > 3$

$\sqrt{1-\lambda} > \sqrt{3}$ $\quad -\sqrt{1-\lambda} < -\sqrt{3}$

$1+\sqrt{1-\lambda} > 1+\sqrt{3}$ $\quad 1-\sqrt{1-\lambda} < 1-\sqrt{3}$

Also, $\dfrac{\lambda}{2\lambda+2} > 0$

Also since $\lambda < -2$,

$\lambda(4\lambda+5) > 0$

$4\lambda^2 + 5\lambda > 0$

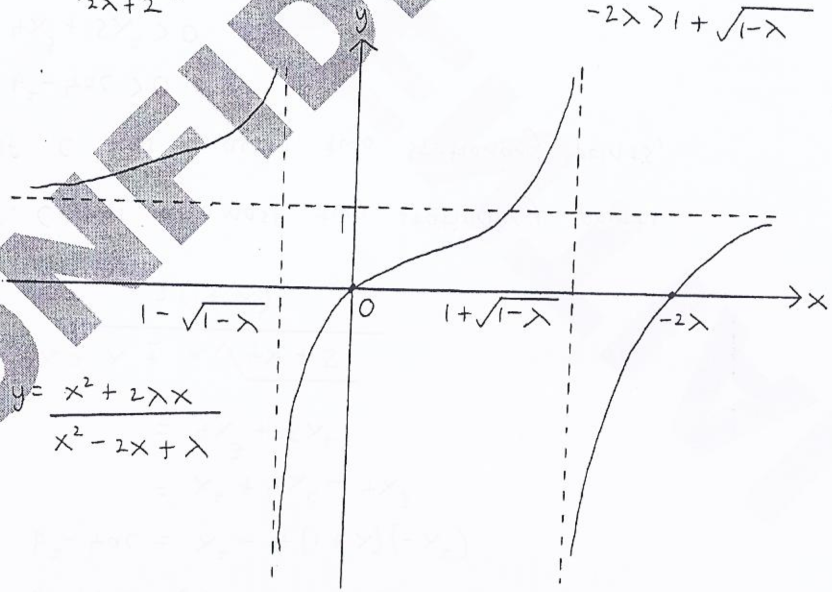
$4\lambda^2 + 4\lambda + 1 > 1-\lambda$

$(2\lambda+1)^2 > 1-\lambda$

$2\lambda+1 < -\sqrt{1-\lambda}$

$-2\lambda-1 > \sqrt{1-\lambda}$

$-2\lambda > 1+\sqrt{1-\lambda}$



b) $\lambda > 2$

when $\dfrac{dy}{dx} = 0$:

$(1+\lambda)x^2 - \lambda x - \lambda^2 = 0$

$a = 1+\lambda \quad b = -\lambda \quad c = -\lambda^2$

$$x = \frac{\lambda \pm \sqrt{\lambda^2 - 4(1+\lambda)(-\lambda^2)}}{2(1+\lambda)}$$

$$= \frac{\lambda \pm \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^3}}{2(1+\lambda)}$$

$$= \frac{\lambda \pm \sqrt{4\lambda^3 + 5\lambda^2}}{2(1+\lambda)}$$

$$= \frac{\lambda \pm \lambda\sqrt{4\lambda+5}}{2(1+\lambda)}$$

As $x \to \pm\infty$ $y \to 1$

$y = 1$

Since $\lambda > 2$

$-\lambda < -2$

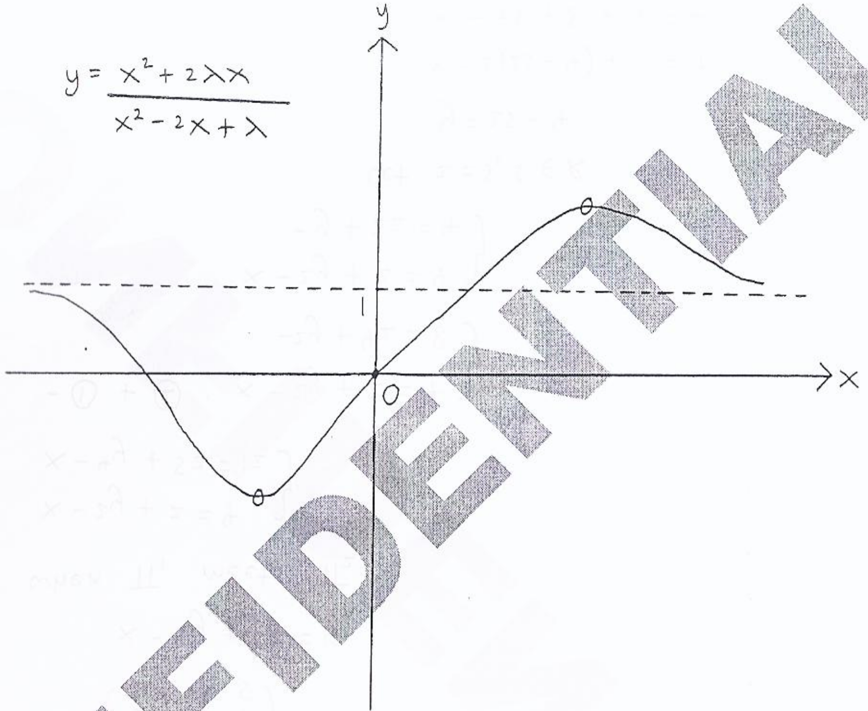
$1-\lambda < -1$

Also, $\dfrac{\lambda}{2\lambda+2} > 0$

Also since $\lambda > 2$,

$-2\lambda < -4$

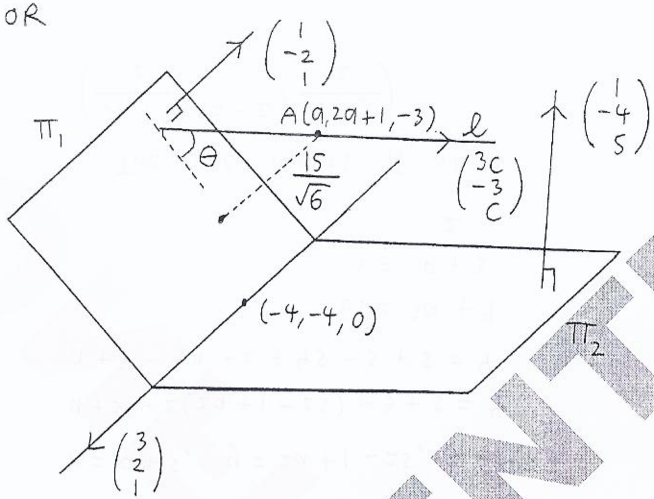
$$y = \frac{x^2 + 2\lambda x}{x^2 - 2x + \lambda}$$



o : Stationary point

• : Intersection point.

OR



$$\Pi_1 : \underset{\sim}{r} = \begin{pmatrix}2\\1\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\4\end{pmatrix} + \mu\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2 + 2\lambda - \mu\\1 + 3\lambda\\4 + 4\lambda + \mu\end{pmatrix}$$

$$\left.\begin{aligned} x &= 2 + 2\lambda - \mu \\ y &= 1 + 3\lambda \\ z &= 4 + 4\lambda + \mu \end{aligned}\right\}$$

$$\left.\begin{aligned} -\mu + 2\lambda &= x - 2 \\ 3\lambda &= y - 1 \\ \mu + 4\lambda &= z - 4 \end{aligned}\right\}$$

$$① + ③ : \left.\begin{aligned} -\mu + 2\lambda &= x - 2 \\ 3\lambda &= y - 1 \\ 6\lambda &= x + z - 6 \end{aligned}\right\}$$

$$x + z - 6 = 2(y-1)$$
$$= 2y - 2$$
$$x - 2y + z = 4$$

$\therefore$ The cartesian form of Π_1 is $x - 2y + z = 4$

Π_2: $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 12$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 12$$

$$x - 4y + 5z = 12$$

when Π_1 meets Π_2,

$$\left.\begin{aligned} x - 2y + z &= 4 \\ x - 4y + 5z &= 12 \end{aligned}\right\}$$

$-① + ②$: $\left.\begin{aligned} x - 2y + z &= 4 \\ -2y + 4z &= 8 \end{aligned}\right\}$

$$\left.\begin{aligned} x - 2y + z &= 4 \\ -y + 2z &= 4 \end{aligned}\right\}$$

Let $z = s,\ s \in R$

$$y = 2s - 4$$
$$x - 2(2s-4) + s = 4$$
$$x - 4s + 8 + s = 4$$
$$x = 3s - 4.$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3s - 4 \\ 2s - 4 \\ s \end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$\therefore$ The line of intersection of Π_1 and Π_2 is

$$\underset{\sim}{r} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.$$

The line perpendicular to Π_1 and passing through A has equation $\underset{\sim}{r} = \begin{pmatrix} a \\ 2a+1 \\ -3 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

since $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is normal to Π_1.

When the line meets Π_1,

$x = a + s$, $y = 2a + 1 - 2s$, $z = -3 + s$

$$a + s - 2(2a + 1 - 2s) - 3 + s = 4$$
$$a + s - 4a - 2 + 4s - 3 + s = 4$$
$$6s = 3a + 9$$
$$s = \frac{a+3}{2}$$

$\therefore$ The line meets Π_1 at

$$\left(\frac{3a+3}{2},\ a-2,\ \frac{a-3}{2}\right).$$

If the perpendicular distance from A to Π_1 is $\frac{15}{\sqrt{6}}$,

$$\sqrt{\left(\frac{3a+3}{2}-a\right)^2+\left(a-2-(2a+1)\right)^2+\left(\frac{a-3}{2}-(-3)\right)^2}=\frac{15}{\sqrt{6}}$$

$$\left(\frac{a+3}{2}\right)^2+(-a-3)^2+\left(\frac{a+3}{2}\right)^2=\frac{75}{2}$$

$$\frac{(a+3)^2}{4}+(a+3)^2+\frac{(a+3)^2}{4}=\frac{75}{2}$$

$$\frac{3(a+3)^2}{2}=\frac{75}{2}$$

$$(a+3)^2=25$$

$$a+3=5$$

$a=2$, since $a>0$

$\therefore A(2,5,-3)$

$$l: \underset{\sim}{r}=\begin{pmatrix}2\\5\\-3\end{pmatrix}+t\begin{pmatrix}3c\\-3\\c\end{pmatrix}$$

If the acute angle between l and Π_1 is $\sin^{-1}\left(\frac{2}{\sqrt{6}}\right)$, since $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ is perpendicular to Π_1

$$\begin{pmatrix}3c\\-3\\c\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}=\left|\begin{pmatrix}3c\\-3\\c\end{pmatrix}\right|\left|\begin{pmatrix}1\\-2\\1\end{pmatrix}\right|\cos\left(90^\circ-\sin^{-1}\frac{2}{\sqrt{6}}\right)$$

$$3c+6+c=\sqrt{9c^2+9+c^2}\sqrt{1+4+1}\cos\left(90^\circ-\sin^{-1}\frac{2}{\sqrt{6}}\right)$$

$$3c+6+c=\sqrt{9c^2+9+c^2}\sqrt{1+4+1}\left(\frac{2}{\sqrt{6}}\right)$$

$$4c+6=\sqrt{10c^2+9}\sqrt{6}\left(\frac{2}{\sqrt{6}}\right)$$

$$=2\sqrt{10c^2+9}$$

$$2c+3=\sqrt{10c^2+9}$$

$$(2c+3)^2=10c^2+9$$

$$4c^2+12c+9=10c^2+9$$

$$6c^2-12c=0$$

$$c^2-2c=0$$

$$c(c-2)=0$$

$c=2$, since $c>0$

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