TAYLOR'S UNIVERSITY COLLEGE
Wisdom • Integrity • Excellence

**CAMBRIDGE A LEVEL PROGRAMME**

# A2 TRIAL EXAMINATION MARCH/APRIL 2009

**(Jan/March 2008 Intake)**

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| **Monday** | **30 March 2009** | **8.30 am – 11.30 am** |

## FURTHER MATHEMATICS 9231/01

**PAPER 1** **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.



**[Turn over**

**1** The vector $\mathbf{x}$ is an eigenvector of the square matrix $\mathbf{A}$ with corresponding eigenvalue $\lambda$. Show that if $\mathbf{A}^{-1}$ exists, then $\lambda \neq 0$. Show also that the vector $\mathbf{x}$ is an eigenvector of $\mathbf{A}^{-1}$ and find the corresponding eigenvalue. [4]

**2** The planes $\Pi_1$, $\Pi_2$, $\Pi_3$ have equations $x + y + kz = 0$, $x + ky + z = 0$ and $kx + y + z = 0$ respectively. Find the common line of intersection of the three planes. [5]

**3** Find the centroid of the region $R$ in the first quadrant bounded by the $x$ and $y$ axes, the line $y = 1$ and the curve $y = (x-1)^3$. [5]

Find also the centroid of the solid generated when $R$ is rotated completely about the $y$-axis. [2]

**4** The equation of a circle is given by $y^2 + x^2 = r^2$, where $r$ is a positive real number.

**(i)** Find, using integration, the circumference of the circle, in terms of $r$. [5]

**(ii)** Find, in terms of $r$, the surface area of a sphere of radius $r$. [3]

**5** Find the general solution of the differential equation $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 4y = 3x^2 - x + 6$.

[9]

**6** **(i)** The curve $C$ is defined implicitly by the equation $\sin(x+y) = \cos xy$. Find the value of $\dfrac{d^2y}{dx^2}$ at the point $(\pi, 0)$. [5]

**(ii)** If $y = x^3e^x$, prove that

$$\frac{d^n y}{dx^n} = x^3e^x + 3nx^2e^x + 3n(n-1)xe^x + n(n-1)(n-2)e^x$$

for every positive integer $n$. [4]

 

**7** The linear transformation $\mathrm{T}:\mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{L}$, where

$$\mathbf{L}=\begin{pmatrix} 1 & 2 & -4 & 1 \\ 2 & 3 & -7 & -2 \\ -1 & -3 & 5 & -5 \\ 2 & 5 & -9 & 6 \end{pmatrix}$$

**(i)** Find the dimension of the range space of $\mathrm{T}$. [4]

**(ii)** Find a basis of the null space of $\mathrm{T}$. [3]

**(iii)** Find a basis of the column space of $\mathrm{T}$. [3]

**8** Express $\dfrac{7r-3}{r^3-r}$ in partial fractions. [3]

Hence, find the sum of the following series

$$S=\frac{22}{1\times2\times3}+\frac{36}{2\times3\times4}+\frac{50}{3\times4\times5}+\cdots\frac{14n-6}{n^3-n}\ .$$ [6]

Deduce the sum to infinity of the above series. [1]

**9** Find all solutions of the equation $\left(\dfrac{2}{3z-8}+9\right)^4=1$. [6]

Given $\left(\dfrac{2}{z}-1\right)^n=1$, where $n$ is a positive integer. By expressing $z$ in the form $r\left(\cos\dfrac{k\pi}{n}-i\sin\dfrac{k\pi}{n}\right)$, show that $|z|=\sec\dfrac{k\pi}{n}$, $k=0,\ 1, ..., n-1$ and state the least value of $|z|$. [6]

**10** **(i)** Given that

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx \,,$$

where $n$ is a non-negative integer. By considering $\frac{d}{dx}\left(\sin^{n-1} x \cos x\right)$ show that for $n \geq 2$,

$$I_n = \frac{n-1}{n} I_{n-2} \,.$$ [5]

**(ii)** The curve $C$ has polar equation

$$r = 4\sin 2\theta \sin\theta \,,$$

where $0 \leq \theta < 2\pi$.

**(a)** Draw a sketch of $C$. [3]

**(b)** Using the results from parts **(i)** and **(ii)(a)** above, find the area of the region enclosed by one loop of the curve $C$. [4]

**11** Answer only **one** of the following two alternatives.

**EITHER**

The curve $C$ has equation $y = \frac{2x^2+3}{x^2+3x-4}$.

**(i)** Express $y$ in the form $P + \frac{Q}{x-1} + \frac{R}{x+4}$. [3]

**(ii)** Write down the equations of all asymptotes of $C$. [3]

**(iii)** Find the coordinates of turning points and determine their nature. [5]

**(iv)** Sketch the curve $C$. [3]

**OR**

The roots of the equation $ax^3 + bx^2 + cx + d = 0$, $\alpha$, $\beta$, $\gamma$, are in geometric progression.

**(i)** Show that one of the roots is $-\frac{c}{b}$. [3]

**(ii)** Show that for the roots to be in geometric progression, $ac^3 = b^3 d$. [4]

**(iii)** Find the possible common ratios in terms of $a$, $b$, $c$, and $d$. [7]