TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

FIRST SEMESTER EXAMINATION JUNE 2006

(Jan 2006 Intake)

Tuesday 6 June 2006 1.00 pm – 3.00 pm

FURTHER MATHEMATICS 9231/01

PAPER 1 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be 10 ms^{-2}.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages.

Semester 1 (June 2006)

1. Find the value of the determinant $\begin{vmatrix} 5 & 3 & 4 \\ \sin\theta & \cos\theta & \sin\theta \\ \cos\theta & -\sin\theta & \cos\theta \end{vmatrix}$ [7]

2.The three points on a plane are A(1,2,3), B(2,-1,2) and C(3,1,-1). Write down $\boldsymbol{p}$, the direction vector of AB and $\boldsymbol{q}$, the direction vector of AC. [2]

(i) Express the vector equation of the plane in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. [2]

(ii) Hence, write down the Cartesian equation of the plane. [4]

3. **(i)** The nth member a_n of a sequence is defined by $a_n = 5^{2n+1} - 12n + 7$. By considering $a_{n+1} - 25a_n$, prove that a_n is divisible by 12. [6]

(ii) Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{1}{2r-1} - \frac{1}{2r+1} = \frac{2n}{2n+1}.$$ [6]

4. **(i)** Given that the rational function $f(x) = \dfrac{x^2-2x+1}{x+2}$.

(a) Find the equations of the two asymptotes. [2]

(b) Find the coordinates of the turning points on the graph. [5]

(c) Sketch the graph of $y = f(x)$ showing the asymptotes and the turning points clearly. [4]

(ii) Given that $f(x) = k$, show that $x^2 - (2+k)x + (1-2k) = 0$. Write down the condition for the roots of this quadratic equation to be real. Hence find the range of values of k. [4]

5. Given that the straight lines l_1 and l_2 have equations $\boldsymbol{r} = \mathbf{i} + 3\mathbf{j} + \mathbf{k} + s(-2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\boldsymbol{r} = -2\mathbf{j} + 8\mathbf{k} + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively.

(i) Find the acute angle between l_1 and l_2. [2]

(ii) Find the vector perpendicular to both l_1 and l_2. [2]

(iii) Hence, write down the Cartesian equation of the plane Π containing l_1 and l_2.

[3]

The point (2,1,5) lies on a line l_3 which is normal to the plane Π.

(iv) Write down the vector equation of the line l_3 . [2]

(v) Find the coordinates of the foot of the perpendicular from the point (2,1,5) to the plane Π. [3]

(vi) Hence, find the distance from the point (2,1,5) to the plane Π. [3]

6. If $u_n = \dfrac{1}{n(n+1)...(n+k)}$, where n and k are integers such that $n \geq 1$ and $k \geq 2$, and that $v_n = (n+k)u_n$. Show that $v_n - v_{n+1} = ku_n$.

[4]

Deduce that

$$\sum_{n=1}^{N} u_n = \frac{1}{k}[\frac{1}{k!} - \frac{N!}{(N+k)!}].$$

[8]

Find $\lim\limits_{N \to \infty} (\sum_{n=1}^{N} u_n)$ in terms of k.

[1]

7. The equation

$$3x^4 + 4x^3 - x^2 - 10x - 6 = 0$$

has roots $\alpha, \beta, \gamma, \delta$. Given that $\alpha\beta = 2$, express $3x^4 + 4x^3 - x^2 - 10x - 6$ as a product of two quadratic factors $(3x^2 + ax + b)(x^2 + cx + d)$ where a, b, c and d are constants.

[12]

8. The roots of the equation

$$x^3 + px + q = 0$$

are α, β, γ. Find the equation whose roots are $\alpha - \lambda, \beta - \lambda, \gamma - \lambda$, expressing your answer in the form $y^3 + Ay^2 + By + C = 0$.

[8]

Denoting $(\alpha - \lambda)^k + (\beta - \lambda)^k + (\gamma - \lambda)^k$ by S_k, where k is a nonnegative integer, show that $S_2 = 3\lambda^2 - 2p$ and find S_3 in terms of p, q and λ. [10]