

**CAMBRIDGE 'A' LEVEL PROGRAMME
SEMESTER ONE EXAMINATION DECEMBER 2006**
(June 2006 Intake)

Tuesday

5 December 2006

1.00 pm – 3.00 pm

FURTHER MATHEMATICS

9231/01

PAPER 1

2 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of 2 printed pages.

1. Prove that $10^{3n} + 38^n + 35$ is divisible by 37 for all positive integers n . [10]

2. Sketch the graph $y = \frac{2x^2 - x - 19}{(x+1)(x-3)}$. [20]

3. The points A, B and C have position vectors $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $-4\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $4\mathbf{i} - \mathbf{k}$ respectively. The point D is on the line segment AB such that $2AD = DB$.

i) Find the position vector of D . [5]

ii) Find the equation of the plane which passes through D and is perpendicular to AB . [5]

iii) Find the position vector of the point E in which the line through C parallel to the vector $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ meets the plane. [5]

iv) Find the perpendicular distance of E from the line AB . [5]

4. If $U_j = \frac{7^j(j-2)}{5(j+3)}$, prove that $U_{j+1} - U_j = \frac{7^j(6j^2 + 12j - 13)}{5(j+3)(j+4)}$. [10]

Hence, or otherwise prove that

$$S_n = \sum_{j=0}^n \frac{7^j(6j^2 + 12j - 13)}{5(j+3)(j+4)} = \frac{7^{n+1}(n-1)}{5(n+4)} + \frac{2}{15}. \quad [10]$$

Find also the limit of $7^{-n}S_n$ as $n \rightarrow \infty$. [5]

5. i) If the equation $2x^3 - x^2 + x + 4 = 0$ has roots α, β and γ , and

$$S_n = \alpha^n + \beta^n + \gamma^n,$$

find the equation having roots α^2, β^2 and γ^2 . Hence, or otherwise, find S_4, S_6 and S_8 . [15]

ii) If the equation $ax^3 + bx^2 + cx + d = 0$ has roots α, β, γ and

$\alpha\beta = \gamma^3, \alpha\gamma = \beta^3$, and $\beta\gamma = \alpha^3$, prove that $d = 0$ or $d = -a$. [10]