1. $$\frac{2}{n^2-2n} = \frac{2}{n(n-2)} = \frac{1}{n-2} - \frac{1}{n}$$

$$\sum_{n=3}^{N} \frac{1}{n^2-2n} = \frac{1}{2}\sum_{n=3}^{N} \frac{2}{n(n-2)}$$

$$= \frac{1}{2}\sum_{n=3}^{N}\left(\frac{1}{n-2} - \frac{1}{n-1} + \frac{1}{n-1} - \frac{1}{n}\right)$$

$$= \frac{1}{2}\left[1 - \frac{1}{N-1} + \frac{1}{2} - \frac{1}{N}\right]$$

$$= \frac{1}{2}\left(\frac{3}{2} - \frac{1}{N-1} - \frac{1}{N}\right)$$

$$= \frac{3}{4} - \frac{1}{2(N-1)} - \frac{1}{2N}$$

2. $n=2$: $LHS = 3^2$ $\quad$ $RHS = 1+2(2)$

$= 9$ $\quad$ $= 5$

$9 > 5$

$\therefore$ statement is true for $n=2$

$n=k$: Assume $3^k > 1 + 2k$

$n=k+1$: Need to show that $3^{k+1} > 1+2(k+1) = 3+2k$

$3^{k+1} = 3(3^k) > 3(1+2k) = 3+6k$

since $k>1 \Rightarrow 3+6k > 3+2k$

$\therefore 3^{k+1} > 1+2(k+1)$

If statement is true for $n=k$ then it is true for $n=k+1$.

By induction $3^n > 1+2n$ for integers $n>1$.

3. $2x^3 - 5x^2 + 7x - 4 = 0$

roots $\Rightarrow \alpha, \beta, \gamma$

New roots : $\alpha^2, \beta^2, \gamma^2$

let, $u = \alpha^2 \Rightarrow \alpha = \pm\sqrt{u}$

①: $2u(\pm\sqrt{u}) - 5u + 7(\pm\sqrt{u}) - 4 = 0$

$(5u+4)^2 = (2u+7)(\pm\sqrt{u})^2$

$25u^2 + 40u + 16 = (4u^2 + 28u + 49)u$

$4u^3 + 3u^2 + 9u - 16 = 0$

$\alpha^4 + \beta^4 + \gamma^4 = (\alpha^2+\beta^2+\gamma^2)^2 - 2(\alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2)$

$= \left(\frac{-3}{4}\right)^2 - 2\left(\frac{9}{4}\right)$

$= -\frac{63}{16}$

4. i) $y = \dfrac{x^2 - 2x - 1}{x - 2} = x - 1 - \dfrac{2}{x - 1}$

Asymptote : $y = x - 1$, $x = 1$

ii) $y = x - 1 - \dfrac{2}{x - 1}$

$\dfrac{dy}{dx} = 1 + \dfrac{2}{(x-1)^2}$

For turning points to exist, $\dfrac{dy}{dx} = 0$

but $1 + \dfrac{2}{(x-1)^2} \neq 0$

since both are positive, hence no turning points.

At the x - axis,

$y = 0$

$x^2 - 2x - 1 = 0$

$x = -0.414$

@ 2.414

At the y - axis

$x = 0$

$y = \dfrac{1}{2}$