FURTHER MATHS INTEGRATION SET 2 (Finding Arc Length and Surface Area)

1. The curve defined parametrically by the equations

$$x = a\cos t, \quad y = a\sin t, \quad t = 0, \; t = 2\pi$$

is rotated about the x-axis to form a solid. Find the area of the surface generated. [5]

Ans = $4\pi a^2$

2. The curve C is defined parametrically by

$$x = 3t^2 + 1, \quad y = t^3 - 3t,$$

where $t > 0$.

(i) Show that $\frac{d^2y}{dx^2}$ is positive at every point of C. [5]

(ii) The arc of C joining the point where $t = 1$ to the point where $t = 2$ is rotated through one complete revolution about the y-axis. Find the area of the surface generated. [6]

Ans = (ii) 173 3/5 π

3. The curve C is defined parametrically by the equations

$$\cancel{y = \frac{t^6}{6} + \frac{1}{t^2}}, \; \cancel{x = 6t^2}. \quad y = \frac{t^7}{7} + \frac{7}{t^7}, \; x = 3t^2$$

Find the area of the surface generated by rotating C about the x-axis from $t = 2$ to $t = 1$. [10]

Ans = 357 1095/3136 π

4. . Find the arc length of the curve given by the equations $x = t^2\cos t, y = t^2\sin t$ from $t = 0$ to $t = 2$. [10]

Ans = 16√2 / 3 – 8/3

5. Given the curve $y = ax^2 + c$, where a and c are constants such that $a > 0$ and $c > 0$. The arc of the curve from $x = 0$ to $x = \frac{\sqrt{3}}{2a}$ is denoted C.

(i) Show that the length, S_1, of C is given by

$$S_1 = \int_0^{\frac{\sqrt{3}}{2a}} \sqrt{1 + 4a^2x^2}\, dx.$$

Hence using the substitution $x = \frac{1}{2a}\tan\theta$, find S_1 in exact terms. [9]

(ii) Find in exact terms the area, S_2, of the curved surface generated when C is rotated through one revolution about the y-axis. [4]

(iii) Given $S_1 < S_2$, show that $a < \pi$. [1]

Ans = (i) $\frac{\sqrt{3}}{2a} + \frac{1}{4a}\ln(2+\sqrt{3})$.

(ii) $\frac{7\pi}{6a^2}$.

6. Using integration prove that the curved surface area of a cylinder of radius r and height h is $2\pi rh$. [3]

7.

(i) Find $\int e^{ax}\cos bx\, dx$. $\frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2}$ [3]

(ii) If C: $x = e^t\cos t$ and $y = e^t\sin t$, $t = 0, 1$ find the arc length of C from $t = 0$ to $t = 1$. [4]

(iii) Find the surface area of revolution of C from $t = 0$ to $t = 1$ about the y-axis. [3]

(iv) If $y = e^{2x}\sin 3x$, $x = 4, 5$ and the y coordinate of the centroid of the region bounded by the x-axis, the curve $y = e^{2x}\sin 3x$ and the lines $x = 4$ and $x = 5$ with area A is denoted c, find Ac. [4]