TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

since 1969

CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION DECEMBER 2011

(June 2011 Intake)

Tuesday **6 December 2011** **1.00 pm – 3.00 pm**

FURTHER MATHEMATICS 9231/01

PAPER 1

2 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **3** printed pages.

1 Find the perpendicular distance between the point (3, −4, 5) and the line

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$ [3]

2 Find the value of the integers k and l if the equations $x + ay + a^2 z = 0$, $ax + y + az = 0$ and $a^2 x + ay + z = 0$ have infinitely many solutions when $a = k, l$. [4]

Find also the unique solution of the equations when $a \neq k, l$. [1]

3 Show that $\displaystyle\sum_{n=1}^{N} \frac{1}{n(n+1)} = \frac{N}{N+1}$ and that $\displaystyle\sum_{n=1}^{N} \frac{1}{(n+1)(n+2)} = \frac{N}{2(N+2)}$.

[5]

Express $\dfrac{5n+6}{n(n+1)(n+2)}$ in the form of $\dfrac{a}{n(n+1)}$ and $\dfrac{b}{(n+1)(n+2)}$, where a and b are integers that need to be determined.

Hence evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{5n+6}{n(n+1)(n+2)}$. [4]

4 i) Prove that $\displaystyle\sum_{r=1}^{n} (3^r + 3^{r-1})r = (2n-1)3^n + 1$ for every positive integer n. [5]

ii) Prove that $7^{2n+1} + 9^n$ is divisible by 8 for every positive integer n. [5]

5 The curve C has equation $y = \dfrac{(x-5)(x-1)}{(x+1)(x-3)}$.

i) Find the equations of the asymptotes of C. [3]

ii) Show that C does not have any turning points. [3]

iii) Sketch C, showing clearly the coordinates where C crosses the coordinate axes and the horizontal asymptote. [4]

6 i) If the equation $5x^4 - 6x^3 + 2 = 0$ has roots α, β, γ, δ, find the equation having roots $\alpha^2, \beta^2, \gamma^2, \delta^2$. [4]

If $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n$, find also S_{-4}, S_{-2}, S_2 and S_4. [4]

ii) If the equation $ax^3 + bx^3 + cx^2 + d = 0$ has roots α, β, γ, and

$$\alpha^2 = \frac{\alpha}{\beta\gamma} + \beta\gamma\ ,\quad \beta^2 = \frac{\beta}{\alpha\gamma} + \alpha\gamma \ \text{ and } \ \gamma^2 = \frac{\gamma}{\alpha\beta} + \alpha\beta\ ,$$

show that $b = \pm\sqrt{\dfrac{2a^2c + 3acd}{a+d}}$. [5]