Revision

P107 Lines and Planes

CIE CAL FM P13 2011-11 Q9

Find a cartesian equation of the plane Π containing the lines $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + s(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - 7\mathbf{j} + 10\mathbf{k} + t(\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$. [4]

The line l passes through the point P with position vector $6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$. Find

(i) the position vector of the point where l meets Π, [3]

(ii) the perpendicular distance from P to Π, [3]

(iii) the acute angle between l and Π. [3]

CIE CAL FM P12 2011-11 Q2

The position vectors of points A, B, C, relative to the origin O, are $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{c} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$.

Find $\mathbf{a} \times \mathbf{b}$ and deduce the area of the triangle OAB. [3]

Hence find the volume of the tetrahedron $OABC$, given that the volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ perpendicular height. [2]

CIE CAL FM P13 2011-06 Q10

The lines l_1 and l_2 have equations

l_1: $\mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$ and

l_2: $\mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \mu(4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$.

Find a cartesian equation of the plane Π containing l_1 and l_2. [4]

Find the position vector of the foot of the perpendicular from the point with position vector $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$ to Π. [4]

The line l_3 has equation $\mathbf{r} = \mathbf{i} + 10\mathbf{j} + 3\mathbf{k} + \nu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$.

Find the shortest distance between l_1 and l_3. [5]

CIE CAL FM P12 2011-06 Q6

The line l_1 passes through the point with position vector $8\mathbf{i} + 8\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$. The line l_2 passes through the point with position vector $7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and is parallel to the vector $4\mathbf{i} - \mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. In either order,

(i) show that $PQ = 13$,

(ii) find the position vectors of P and Q.

CIE CAL FM P12 2010-06 Q7

The lines l_1 and l_2 have vector equations

$\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ and

$\mathbf{r} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} - \mathbf{k})$ respectively.

(i) Show that l_1 and l_2 intersect. [3]

(ii) Find the perpendicular distance from the point P whose position vector is $3\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$ to the plane containing l_1 and l_2. [3]

(iii) Find the perpendicular distance from P to l_1. [4]

CIE CAL FM P1 2008-06 Q12

The position vectors of the points A, B, C, D are $7\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$, $2\mathbf{i} + 7\mathbf{j} + \lambda\mathbf{k}$ respectively. It is given that the shortest distance between the line AB and the line CD is 3.

(i) Show that $\lambda^2 - 5\lambda + 4 = 0$. [7]

(ii) Find the acute angle between the planes through A, B, D corresponding to the values of λ satisfying the equation in part **(i)**. [7]

ASSIGNMENT

Attempt all the questions in the following slides.

Assignment is to be submitted within **one week** after instruction!!!

Please write your name and your student's id in the script upon submission.

CIE CAL FM P11 2010-11 Q12

The plane Π_1 has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + \mu(-\mathbf{i} + \mathbf{k})$. Obtain a cartesian equation of Π_1 in the form $px + qy + rz = d$. [4]

The plane Π_2 has equation $\mathbf{r}.(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = 12$. Find a vector equation of the line of intersection of Π_1 and Π_2. [3]

The line l passes through the point A with position vector $a\mathbf{i} + (2a + 1)\mathbf{j} - 3\mathbf{k}$ and is parallel to $3c\mathbf{i} - 3\mathbf{j} + c\mathbf{k}$, where a and c are positive constants. Given that the perpendicular distance from A to Π_1 is $\dfrac{15}{\sqrt{6}}$ and that the acute angle between l and Π_1 is $\sin^{-1}\left(\dfrac{2}{\sqrt{6}}\right)$, find the values of a and c. [7]

CIE CAL FM P1 2008-11 Q11

The plane Π_1 has equation

$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \theta(2\mathbf{j} - \mathbf{k}) + \phi(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

Find a vector normal to Π_1 and hence show that the equation of Π_1 can be written as $2x + 3y + 6z = 14$. [4]

The line l has equation $\mathbf{r} = 3\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} + t(4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k})$.

The point on l where $t = \lambda$ is denoted by P. Find the set of values of λ for which the perpendicular distance of P from Π_1 is not greater than 4. [4]

The plane Π_2 contains l and the point with position vector $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find the acute angle between Π_1 and Π_2. [4]

CIE CAL FM P1 2003-11 Q9

The line l_1 passes through the point A with position vector $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and is parallel to the vector $3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$. The variable line l_2 passes through the point $(1 + 5\cos t)\mathbf{i} - (1 + 5\sin t)\mathbf{j} - 14\mathbf{k}$, where $0 \leqslant t < 2\pi$, and is parallel to the vector $15\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$. The points P and Q are on l_1 and l_2 respectively, and PQ is perpendicular to both l_1 and l_2.

(i) Find the length of PQ in terms of t. [4]

(ii) Hence show that the lines l_1 and l_2 do not intersect, and find the maximum length of PQ as t varies. [3]

(iii) The plane Π_1 contains l_1 and PQ; the plane Π_2 contains l_2 and PQ. Find the angle between the planes Π_1 and Π_2, correct to the nearest tenth of a degree. [4]

CIE CAL FM P1 2002-11 Q9

The planes Π_1 and Π_2, which meet in the line l, have vector equations $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \theta_1(2\mathbf{i} + 3\mathbf{k}) + \phi_1(-4\mathbf{j} + 5\mathbf{k})$,
$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \theta_2(3\mathbf{j} + \mathbf{k}) + \phi_2(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, respectively.
Find a vector equation of the line l in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [5]

Find a vector equation of the plane Π_3 which contains l and which passes through the point with position vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find also the equation of Π_3 in the form $ax + by + cz = d$. [4]

Deduce, or prove otherwise, that the system of equations
$6x - 5y - 4z = -32$, $5x - y + 3z = 24$, $9x - 2y + 5z = 40$,
has an infinite number of solutions. [3]