TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2011

(Jan 2012 Intake)

Monday	**6 June 2012**	**1.30 pm – 3.30 pm**

FURTHER MATHEMATICS 9231/01

PAPER 1 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **50**.

This document consists of **3** printed pages.

[Turn over

1 Find the area of the triangle ABC where the position vectors of A, B and C relative to the origin are $\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$. [4]

2 Find, to the nearest degree, the acute angle between the line with equation

$\dfrac{x+1}{3} = \dfrac{y-2}{4} = \dfrac{3-z}{12}$ and the plane with equation $2x + y - 2z = 5$. [4]

3 **(i)** Find the sum $1^3 - 2^3 + 3^3 - 4^3 + \ldots - (n-2)^3 + (n-1)^3 - n^3$. [4]

(ii) Find the sum $1^3 - 2^3 + 3^3 - 4^3 + \ldots + (n-2)^3 - (n-1)^3 + n^3$. [4]

4 **(i)** Prove that

$$\frac{a^{n-1}}{b^{n-1}} + \frac{a^{n-2}}{b^{n-2}} + \cdots + \frac{a^2}{b^2} + \frac{a}{b} + 1 + \frac{b}{a} + \frac{b^2}{a^2} + \cdots + \frac{b^{n-2}}{a^{n-2}} + \frac{b^{n-1}}{a^{n-1}} = \frac{a^{2n-1} - b^{2n-1}}{a^{n-1}b^{n-1}(a-b)}.$$

for every positive integer n. [5]

(ii) If $u_{n+3} = 3u_{n+2} + 4u_{n+1} - 12u_n$, $u_1 = 7, u_2 = 17, u_3 = 43$, show that $u_n = 3^n + 2^{n+1}$ for every positive integer n. [6]

5 The curve C has equation $y = \dfrac{3x}{x^2 - 5x + 4}$.

(i) State all the asymptotes of C. [3]

(ii) Show that C has one minimum and one maximum point and find their coordinates. [5]

(iii) Sketch C, and give the coordinates of any points where C meets the axes. [3]

6 **(i)** If the equation $3x^3 + 5x^2 + 7 = 0$ has roots α, β, γ, find the equation having roots

$\frac{1}{\alpha} + \frac{1}{\beta} + \gamma, \frac{1}{\alpha} + \frac{1}{\gamma} + \beta, \frac{1}{\beta} + \frac{1}{\gamma} + \alpha$. [6]

(ii) If the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, has roots α, β, γ, δ *and* roots

$\beta + \gamma + \delta = \alpha^{k+\beta\gamma\delta}\beta^{\beta\gamma\delta}\gamma^{\beta\gamma\delta}\delta^{\beta\gamma\delta}$, $\alpha + \gamma + \delta = \alpha^{\alpha\gamma\delta}\beta^{k+\alpha\gamma\delta}\gamma^{\alpha\gamma\delta}\delta^{\alpha\gamma\delta}$,

$\alpha + \beta + \delta = \alpha^{\alpha\beta\delta}\beta^{\alpha\beta\delta}\gamma^{k+\alpha\beta\delta}\delta^{\alpha\beta\delta}$, $\alpha + \beta + \gamma = \alpha^{\alpha\beta\gamma}\beta^{\alpha\beta\gamma}\gamma^{\alpha\beta\gamma}\delta^{k+\alpha\beta\gamma}$ show that

$k = \frac{d}{a} + \lg\left(\frac{b^2c - abd + a^2e}{a^3}\right)$. [6]