1. Let $$\frac{3}{(3r+2)(3r+5)} = \frac{A}{3r+2} + \frac{B}{3r+5}$$

$$= \frac{A(3r+5) + B(3r+2)}{(3r+2)(3r+5)}$$

$$3 = A(3r+5) + B(3r+2)$$
$$= (3A+3B)r + 5A + 2B$$

$3A + 3B = 0 \qquad 5A + 2B = 3$

$A + B = 0$

$3A = 3$

$A = 1$

$B = -1$

$$\therefore \frac{3}{(3r+2)(3r+5)} = \frac{1}{3r+2} - \frac{1}{3r+5}$$

Since $$\frac{3}{(3r+2)(3r+5)} = \frac{1}{3r+2} - \frac{1}{3r+5}$$

$$\sum_{r=1}^{n} \frac{2}{(3r+2)(3r+5)} = \frac{2}{3}\sum_{r=1}^{n} \frac{3}{(3r+2)(3r+5)}$$

$$= \frac{2}{3}\sum_{r=1}^{n} \frac{1}{3r+2} - \frac{1}{3r+5}$$

$$= \frac{2}{3}\left(\frac{1}{5} - \frac{1}{8}\right.$$

$$+ \frac{1}{8} - \frac{1}{11}$$

$$+ \frac{1}{11} - \frac{1}{14}$$

$$\vdots$$

$$+ \frac{1}{3n+4} - \frac{1}{3n-1}$$

$$+ \frac{1}{3n-1} - \frac{1}{3n+2}$$

$$\left.+ \frac{1}{3n+2} - \frac{1}{3n+5}\right)$$

$$= \frac{2}{3}\left(\frac{1}{5} - \frac{1}{3n+5}\right)$$

$$= \frac{2}{3}\left(\frac{3n+5-5}{5(3n+5)}\right)$$

$$= \frac{2}{3}\left(\frac{3n}{5(3n+5)}\right)$$

$$= \frac{2n}{5(3n+5)}$$

2. $4x^4 - 5x^3 + 6x^2 + x - 9 = 0$

$\alpha, \beta, \gamma, \delta$ are the roots

$2\alpha^2 + 1, 2\beta^2 + 1, 2\gamma^2 + 1, 2\delta^2 + 1$

Let $u = 2\alpha^2 + 1$

$$\alpha = \pm\sqrt{\frac{u-1}{2}}$$

α is a root

$$4\alpha^4 - 5\alpha^3 + 6\alpha^2 + \alpha - 9 = 0$$

$$4\left(\pm\sqrt{\frac{u-1}{2}}\right)^4 - 5\left(\pm\sqrt{\frac{u-1}{2}}\right)^3$$

$$+ 6\left(\pm\sqrt{\frac{u-1}{2}}\right)^2 \pm \sqrt{\frac{u-1}{2}} - 9 = 0$$

$$4\left(\frac{u-1}{2}\right)^2 - 5\left(\pm(u-1)\sqrt{\frac{u-1}{2}}\right)$$

$$+ 6\left(\frac{u-1}{2}\right) \pm \sqrt{\frac{u-1}{2}} - 9 = 0$$

$$4\left(\frac{u^2 - 2u + 1}{4}\right) + 3(u-1) - 9$$

$$= 5\left(\pm(u-1)\sqrt{\frac{u-1}{2}}\right) - \left(\pm\sqrt{\frac{u-1}{2}}\right)$$

$$u^2 - 2u + 1 + 3u - 3 - 9$$

$$= \pm \sqrt{\frac{u-1}{2}} \left(\frac{5(u-1)}{2} - 1 \right)$$

$$u^2 + u - 11 = \pm \sqrt{\frac{u-1}{2}} \frac{(5u-6)}{2}$$

$$(u^2 + u - 11)^2 = \left(\frac{u-1}{2}\right)\left(\frac{5u-7}{2}\right)^2$$

$$u^4 + u^3 - 11u^2 + u^3 + u^2 - 11u$$

$$-11u^2 - 11u + 121$$

$$= \left(\frac{u-1}{2}\right)\frac{(25u^2 - 70u + 49)}{4}$$

$$8(u^4 + 2u^3 - 21u^2 - 22u + 121)$$

$$= (u-1)(25u^2 - 70u + 49)$$

$$8u^4 + 16u^3 - 168u^2 - 176u + 968$$

$$= 25u^3 - 70u^2 + 49u - 25u^2 + 70u - 49$$

$$8u^4 - 9u^3 - 7u^2 - 295u + 1017 = 0$$

The equation having roots

$2\alpha^2 + 1, 2\beta^2 + 1, 2\gamma^2 + 1, 2\delta^2 + 1$ is

$$8u^4 - 9u^3 - 7u^2 - 295u + 1017 = 0$$

3. $a_n = 2 \cdot 5^{2n-1} + 2^{n-1} \cdot 3^{n+1}$

$a_1 = 2 \cdot 5^1 + 2^0 3^2 = 10 + 9 = 19$

$a_2 = 2 \cdot 5^3 + 2^1 3^3 = 250 + 54 = 304 = 19 \times 16$

$a_3 = 2 \cdot 5^5 + 2^2 3^4 = 6574 = 19 \times 346.$

a_n is divisible by 19

When $n = 1$ $a_1 = 19$ $\therefore$ True when $n = 1$

Suppose $a_K = 2 \cdot 5^{2K-1} + 2^{K-1} \cdot 3^{K+1}$ is divisible by 19.

Consider $a_{K+1} = 2 \cdot 5^{2(K+1)-1} + 2^{(K+1)-1} \cdot 3^{(K+1)+1}$

$$= 2 \cdot 5^{2K+1} + 2^K \cdot 3^{K+2}$$

$$= 2 \cdot 5^2 \cdot 5^{2K-1} + 2 \cdot 2^{K-1} \cdot 3 \cdot 3^{K+1}$$

$$= 50 \cdot 5^{2K-1} + 6 \cdot 2^{K-1} \cdot 3^{K+1}$$

$$= \underbrace{6\left(2 \cdot 5^{k-1} + 2^{k-1} \cdot 3^{K+1}\right)}_{\text{divisible by } 19} + 38 \cdot 5^{2K-1}$$

$$= 6(a_K) + 19 \times 2 \times 5^{2K-1}$$

$\therefore$ a_{K+1} is also divisible by 19

$\Rightarrow$ a_n is divisible by $n \geq 1$

4.

$$\left.\begin{aligned} x + y + z &= 0 \\ x + ky + z &= 0 \\ x + y + k^2 z &= 0 \end{aligned}\right\}$$

$$\begin{aligned} -(1) + (2): \quad & x + y + z = 0 \\ -(1) + (3): \quad & (k-1)y = 0 \\ & (k^2 - 1)z = 0 \end{aligned}$$

When $k = 1$: $0z = 0$

Let $z = s, s \in R$

Let $y = t, t \quad R$

$x = -s - t$

When $k = -1$: $0z = 0$

Let $z = s, s \in R$

$y = 0$

$x = s$

when $k \neq -1, 1$: $z = 0$

$y = 0$

$x = 0$

5 a) $$\left.\begin{aligned} x + 2y - 4z &= 3 \\ 2x + 3y - 5z &= 1 \end{aligned}\right\}$$

$-2 \times ① + ②$:

$$\left.\begin{aligned} x + 2y - 4z &= 3 \\ -y + 3z &= -5 \end{aligned}\right\}$$

$$-y + 3z = -5$$

Let $z = s$, $s \in R$

$$y = 3s + 5$$

$$x + 6s + 10 - 4s = 3$$

$$x = -2s - 7$$

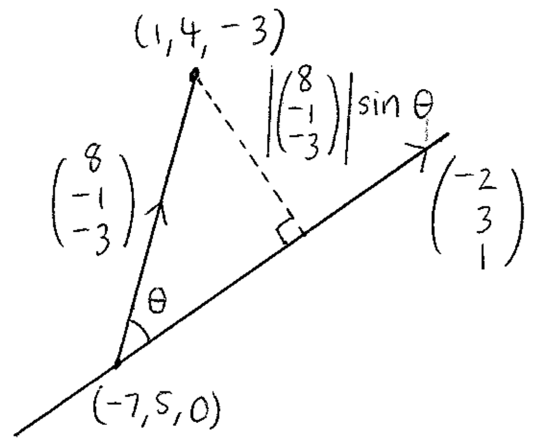
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2s - 7 \\ 3s + 5 \\ s \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ 5 \\ 0 \end{pmatrix} + s\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

$\therefore$ The equation of the line of intersection of the planes $x + 2y - 4z = 3$ and $2x + 3y - 5z = 1$ is

$$\underset{\sim}{r} = \begin{pmatrix} -7 \\ 5 \\ 0 \end{pmatrix} + s\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

b)



The distance of the point $(1,4,-3)$ from the line of intersection of the two planes is $\left\|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\right\|\sin\theta$

$$\begin{pmatrix}8\\-1\\-3\end{pmatrix}\times\begin{pmatrix}-2\\3\\1\end{pmatrix}=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ 8 & -1 & -3\\ -2 & 3 & 1\end{vmatrix}$$

$$=\begin{pmatrix}8\\-2\\22\end{pmatrix}$$

$$=2\begin{pmatrix}4\\-1\\11\end{pmatrix}$$

$$\left|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\times\begin{pmatrix}-2\\3\\1\end{pmatrix}\right| = \left|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\right|\left|\begin{pmatrix}-2\\3\\1\end{pmatrix}\right|\sin\theta$$

$$\left|2\begin{pmatrix}4\\-1\\11\end{pmatrix}\right| = \sqrt{14}\left|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\right|\sin\theta$$

$$2\sqrt{138} = \sqrt{14}\left|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\right|\sin\theta$$

$$\left|\begin{pmatrix}8\\-1\\-3\end{pmatrix}\right|\sin\theta = \frac{2\sqrt{138}}{\sqrt{14}}$$

$$= \frac{2\sqrt{69}}{\sqrt{7}}$$

$$= 2\sqrt{\frac{69}{7}}$$

6 i) $y = \dfrac{2x^2 - 8x + 8}{x^2 - 16} = 2 + \dfrac{40 - 8x}{x^2 - 16}$

Let $\dfrac{40 - 8x}{(x-4)(x+4)} = \dfrac{Q}{x-4} + \dfrac{R}{x+4}$

$\therefore \; 40 - 8x = Q(x+4) + R(x-4)$

$x = 4, \quad 8 = Q(8) \quad \therefore Q = 1$

$x = -4, \quad 72 = R(-8) \quad \therefore R = -9$

$\therefore \; y = 2 + \dfrac{1}{x-4} - \dfrac{9}{x+4}$

ii) Asymptotes : $y = 2$

$x = 4$

$x = -4$

iii) $\dfrac{dy}{dx} = -\dfrac{1}{(x-4)^2} + \dfrac{9}{(x+4)^2}$

$\therefore \; 9(x-4)^2 - (x+4)^2 = 0$

$8x^2 - 80x + 128 = 0$

$x^2 - 10x + 16 = 0$

$(x-2)(x-8) = 0$

iii) (continue) $x = 2,\ y = 0 \quad (2, 0)$

$x = 8,\ y = \frac{3}{2} \quad \left(8, \frac{3}{2}\right)$