TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

FIRST SEMESTER EXAMINATION JUNE 2005

(Jan 2005 Intake)

Monday **6 June 2005** **1.00 pm – 3.00 pm**

FURTHER MATHEMATICS 9231/01

PAPER 1 **2 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be 10 ms^{-2}.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages.

[Turn over

1. Prove that for every positive integer n,

$$\sum_{r=1}^{n} \frac{r+2}{r(r+1)2^r} = 1 - \frac{1}{(n+1)2^n}$$

[5]

2. If $S_n = \sum_{r=1}^{n}(-1)^{r-1}r^3$ find expressions for S_{2n} and S_{2n+1} in terms of n and find the limit of $\frac{2S_{2n+1}}{n^3}$ as $n \to \infty$. [12]

3. The equation $8x^3 + 12x^2 + 4x - 4 = 0$ has roots α, β and γ. Show that the equation with roots $2\alpha - 1$, $2\beta - 1$, $2\gamma - 1$ is $u^3 + 6u^2 + 11u + 2 = 0$. The sum $(2\alpha - 1)^n + (2\beta - 1)^n + (2\gamma - 1)^n$ is denoted by S_n. Find the values of S_3 and S_{-2}.

[12]

4. The curve C has equation

$$y = \frac{x^2 + 2x - 3}{(\lambda x + 1)(x - 7)}$$

where λ is a constant.

i) Find the equations of the asymptotes of C for the case $\lambda = 0$.

[3]

ii) Find the equations of the asymptotes of C for the case where λ is not equal to any of -1, $-\frac{1}{7}$, 0, $\frac{1}{3}$. [6]

iii) Sketch C for the case where $\lambda = -1$. Show, on your diagram, the equations of the asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

[6]

5. The line l_1 passes through the point A with position vector $\mathbf{i}$ – [redacted]$\mathbf{j} - 2\mathbf{k}$ and is parallel to the vector $3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$. The variable line l_2 passes through the point $(1 + 5\cos t)\mathbf{i} - (1 + 5\sin t)\mathbf{j} - 14\mathbf{k}$, where $0 \leq t < 2\pi$, and is parallel to the vector $15\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$. The points P and Q are on l_1 and l_2 respectively, and PQ is perpendicular to both l_1 and l_2.

i) Find the length of PQ in terms of t. [4]

ii) Hence show that the lines l_1 and l_2 do not intersect [6]

iii) The plane Π_1 contains l_1 and PQ; the plane Π_2 contains l_2 and PQ. Find the [redacted] angle between the planes Π_1 and Π_2. [6]

6. The line l_1 passes through the point A, whose position vector is $3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$ and is parallel to the vector $3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. The line l_2 passes through the point B, whose position vector is $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, and is parallel to the vector $\mathbf{i} - \mathbf{j} - 4\mathbf{k}$. The point P on l_1 and Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. The plane Π_1 contains l_1 and PQ; the plane Π_2 contains l_2 and PQ.

i) Find the length of PQ. [5]

ii) Find a vector perpendicular to Π_1. [4]

iii) Find the perpendicular distance from B to Π_1. [5]

iv) Find the [redacted] cosine [redacted] of the angle between Π_1 and Π_2. [6]