TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST 2012

(June 2011 Intake)

Wednesday **5 September 2012** **12.30 pm – 3.30 pm**

FURTHER MATHEMATICS 9231/12

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

1 If the lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{a} + s\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t\mathbf{d}$, by considering $\vec{PQ} \cdot \vec{AC} = \left|\vec{PQ}\right|\left|\vec{AC}\right| \cos\theta$, where P is on l_1 and Q is on l_2 and PQ is the common perpendicular, show that $\left|\vec{PQ}\right| = \sqrt{k(\mathbf{b} \times \mathbf{d}) \cdot (\mathbf{c} - \mathbf{a})}$, $k \neq 0$. [3]

2 Find the sum $1 \times n^2 + 2 \times (n-1)^2 + 3 \times (n-2)^2 + \cdots + (n-1) \times 2^2 + n \times 1^2$. [8]

3 Prove by induction that $\sum_{r=1}^{n} 2^{n-r} r = 2^{n+1} - 2 - n$ for every positive integer n. [5]

4 Prove by induction that $\int_0^1 y^n (1-y)^r \, \mathrm{d}y = \dfrac{n!}{(r+1)(r+2)\cdots(r+n+1)}$ for every positive integer n. [5]

5 If $x = \dfrac{t^2+1}{t}, y = 2\ln t$, find the surface area of revolution from $t = 1$ to $t = 2$ about the x-axis. [7]

6 Find the sum $1 + \dfrac{\cos 3\theta}{7} + \dfrac{\cos 6\theta}{7^2} + \dfrac{\cos 9\theta}{7^3} + \cdots + \dfrac{\cos 3n\theta}{7^n}$. [8]

7 The curve C has equation $r = a(1 + \cos\theta)$, $a > 0$, $0 \leq \theta \leq 2\pi$.

i) Sketch C. [2]

ii) Find the area enclosed by C. [3]

iii) Find the arc length of C. [3]

8 If $I_n \int \cos^n x \, \mathrm{d}x$, find a reduction formulae for I_n. [6]
Find also I_1, I_2 and I_3. [3]

9 The linear transformations S and T, S: $R^3 \to R^3$, T: $R^3 \to R^3$ are given by the matrices $\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & a+c & a+b \end{pmatrix}$ respectively.

i) Find the rank and a basis for the row space of **M**. [4]

ii) Find the rank of **N**, a basis for the range space and the null space of **N**. [4]

10 If $5y^4 \dfrac{d^2y}{dx^2} + 20y^3 (\dfrac{dy}{dx})^2 + 55y^4 \dfrac{dy}{dx} + 24y^5 = 181\cos x + 83\sin x$ and $v = y^5$,

show that $\dfrac{d^2v}{dx^2} + 11\dfrac{dv}{dx} + 24v = 181\cos x + 83\sin x$. [3]

Find also y in terms of x. [9]

11 The equation $x^3 + ax^2 + bx + c = 0$ has roots p, q, r.

i) Find a, b, c in terms of p, q, r. [3]

ii) Find $p^2q^2 + p^2r^2 + q^2r^2$ in terms of a, b, c. [3]

iii) If [7]

$$p + q + r = \frac{7}{2}$$
$$pq + pr + qr = -\frac{5}{2}$$
$$pqr = -2$$

find p, q, r.

12 EITHER

The curve C has equation $y = \dfrac{a^2x^2 - b^2}{c^2x^2 - d^2}, a, b, c, d > 0$.

i) Find the equations of the asymptotes of C. [2]

ii) Find the stationary points of C and determine their type. [6]

iii) Sketch C, indicating any intersection points with the coordinate axes. [6]

OR

The curve C has equation $y = \dfrac{1}{\sqrt{a^2 - b^2(x-c)^2}}, a, b, c > 0$.

i) Find the mean value of y over the interval $c + \dfrac{a}{2b} \le x \le c + \dfrac{\sqrt{3}a}{2b}$. [5]

ii) Find the coordinates of the centroid of C over $c + \dfrac{a}{2b} \le x \le c + \dfrac{\sqrt{3}a}{2b}$. [9]

