TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME
A2 TRIAL EXAMINATION MARCH/APRIL 2011
(January and March 2010 Intakes)

Tuesday **5 April 2011** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/13

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages.

[Turn over

1 By considering $(r+1)^6-(r-1)^6$, or otherwise, find $\sum_{r=1}^{n} r^5$. [5]

2 Prove by induction that, for all $N \geq 1$,

$$\sum_{n=1}^{N} n \times 3^{n-1} = \frac{1}{4} + \frac{3^N}{4}\ \ 2N-1\ .$$ [6]

3 The arc of the curve $y = 2\sqrt{x-2}$, $x \geq 2$, from the point where $x=3$ to the point $x=5$ is denoted by C. Show that the area of the surface generated when C is rotated through one revolution about the $x-$axis is

$$\frac{16\pi}{3}\ \ 4-\sqrt{2}\ \ .$$ [6]

4 The linear transformation T: $\mathrm{R}^5 \to \mathrm{R}^5$ is given by the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & -5 \\ 2 & 7 & 4 & 15 & -9 \\ 3 & 9 & 7 & 24 & -23 \\ 4 & 17 & 6 & 42 & -26 \\ 5 & 22 & 7 & 58 & -39 \end{pmatrix}$$

(i) Find the rank of **A**. [2]

(ii) Find a basis for the range space of **A**. [3]

(iii) Find the dimension of the null space of **A**. [1]

5 Given that $x = \sec\theta + \tan\theta$ and that $y = \operatorname{cosec}\theta + \cot\theta$.

(i) Show that $x + \frac{1}{x} = 2\sec\theta$ and $y + \frac{1}{y} = 2\operatorname{cosec}\theta$. [2]

(ii) Show that $\frac{dy}{dx} = \frac{-\ \ 1+y^2}{1+x^2}$. [5]

6 By considering $1+\frac{e^{4\theta i}}{5}+\frac{e^{8\theta i}}{5^2}+\frac{e^{12\theta i}}{5^3}+\cdots+\frac{e^{4n\theta i}}{5^n}$, find $\sum_{r=0}^{n}\frac{\sin 4r\theta}{5^r}$. [5]

Find also $\sum_{r=0}^{\infty}\frac{\cos 4r\theta}{5^r}$. [2]

7 The curve C has equation $y=8-\frac{4}{5(5x-3)}+\frac{9}{7(7x+6)}$.

(i) Find the equations of the asymptotes of C. [2]

(ii) Find the coordinates of the stationary points of C. [5]

8 If $I_n=\int \sin^n x\cos^n x\,dx$, find a reduction formula for I_n and hence find I_2, I_3 and I_4.

[9]

9 If $\mathbf{A}=\begin{pmatrix}4 & -4 & 6\\ 2 & -1 & 2\\ -3 & 4 & -5\end{pmatrix}$, find the eigenvalues and eigenvectors of **A**. [6]

If **A** is a non-singular matrix with eigenvalue λ and corresponding eigenvector **x**, find an eigenvalue and corresponding eigenvector of $\mathbf{A}^{-1}$. [1]

Find also a square matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{-1}(\mathbf{A}^{-1})^n\mathbf{P}=\mathbf{D}^n$. [3]

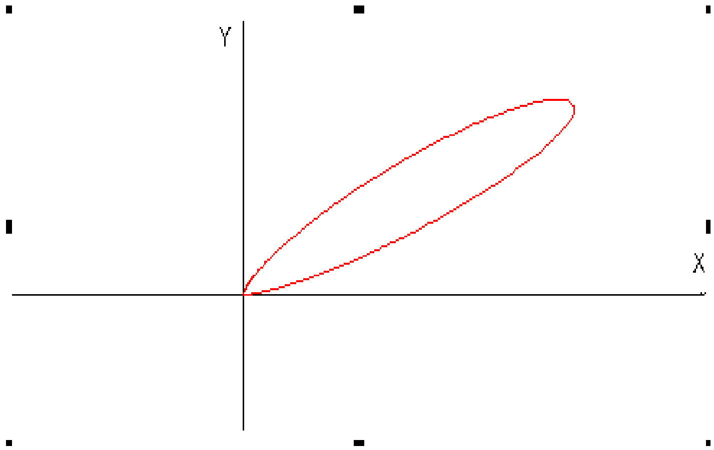
10 Show that the substitution $x=t^2$ transforms the differential equation

$$4x\frac{d^2y}{dx^2}+2\ \ 1-2\sqrt{x}\ \ \frac{dy}{dx}+y=3\sqrt{x}$$

into $$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=3t.$$ [5]

Hence find the general solution for y in terms of x. [6]

11 Find $\frac{dy}{dx}$ given that $x^5 + y^5 = 5x^2y^2$. [2]



The diagram above shows one loop of the curve $x^5 + y^5 = 5x^2y^2$. Show that its polar equation is $r = \frac{5\cos^2\theta\sin^2\theta}{\cos^5\theta + \sin^5\theta}$ and hence the area of the curve enclosed in this loop is

$$A = \frac{25}{2}\int_0^{\frac{\pi}{2}} \frac{\cos^4\theta\sin^4\theta}{\left(\cos^5\theta + \sin^5\theta\right)^2}\,d\theta$$ [5]

By dividing the numerator and denominator by $\cos^{10}\theta$ show that $A = \frac{5}{2}$. [4]

[Question 12 is printed overleaf.]

12 Answer only **one** of the following two alternatives.

EITHER

The position vectors of the points *A*, *B*, *C* and *D* are $-2\mathbf{i}-3\mathbf{j}+4\mathbf{k}$, $7\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, $-3\mathbf{i}+10\mathbf{j}+6\mathbf{k}$ and $12\mathbf{i}-10\mathbf{j}+11\mathbf{k}$ respectively. The points *P* and *Q* are on the lines *AB* and *CD* respectively, and are such that PQ is perpendicular to both *AB* and *CD*.

(i) Show that the lines *AB* and *CD* do not intersect. [5]

(ii) Show that the length of *PQ* is 7 units. [3]

(iii) Find the position vector of the points *P* and *Q*. [6]

OR

The roots of the equation

$$ax^4+bx^3+cx^2+dx+e=0$$

are α, β, γ, δ and

$$S_n=\alpha^n+\beta^n+\gamma^n+\delta^n.$$

(i) Show that $aS_{4+r}+bS_{3+r}+cS_{2+r}+dS_{1+r}+eS_r=0$. [3]

(ii) Show that $S_2=\dfrac{b^2}{a^2}-\dfrac{2c}{a}$ and $S_{-1}=-\dfrac{d}{e}$. [6]

(iii) Find also S_3. [2]

If the equation

$$ax^3+bx^2+cx+d=0, \quad a, b, c, d \neq 0$$

has roots α, β, γ and

$$\alpha^{\alpha\gamma}\beta^{\beta\gamma}\gamma^{\alpha\beta}=k^{\gamma}\ ,\ \ \alpha^{\alpha\beta}\beta^{\alpha\gamma}\gamma^{\beta\gamma}=k^{\beta}\ ,\ \ \alpha^{\beta\gamma}\beta^{\alpha\beta}\gamma^{\alpha\gamma}=k^{\alpha},$$

find *k*.

[3]