

ANSWERS

1. POLYNOMIAL EQUATION

1. $\pm \frac{1}{\sqrt{3}}, \frac{(-1 \pm \sqrt{7})}{3}$
2. (a) 1, 2, $\frac{1}{2}(-3 + i\sqrt{19})$
 $\frac{1}{2}(-3 - i\sqrt{19})$
3. $\{t \in \mathbb{R} : t \leq \frac{1}{4}\}; \alpha = -1 - t + \dots,$
 $\beta = -t^{-1} + 1 + t + \dots;$
 $\alpha \rightarrow -1, \beta \rightarrow -\infty$
4. (a) $a^2x^3 + 4c^2x^2 + ac(2 - a)x + ac = 0$
5. (a) $a(\cos \theta \pm i \sin \theta)$
 (b) $x^2 + x - 1 = 0$
6. $(2x - 3)^2(x - 6)$ or $\frac{1}{3}(2x - 9)^2(3x - 2)$
7. $-2(x - y)(y - z)(z - x)(x + y + z)$
8. (a) $\frac{1}{5(x - 2)} - \frac{x + 2}{5(x^2 + 1)};$
 (b) $\frac{2x}{x^2 - 4}; P(x) = x^3 - 2x^2 + x - 2$
9. $\frac{1}{2}\sqrt{2}, \sqrt{2}, 2\sqrt{2}$
10. (a) -5, $\frac{1}{2}$, 6; (b) (i) $5\frac{1}{4}$, (ii) $-8\frac{3}{8}$
 (iii) $2x^3 - 5x^2 - x + 7 = 0$
11. -a, $a^2 - 2b$; $-a^3 + 3ab - 3c$,
 $x^3 - 4x^2 + 2x - 8 = 0$; 4, $\pm i\sqrt{2}$
12. (a) (i) $-4 < k < -3, -3 < k < 1$;
 (ii) $-3 < k < \frac{1}{2}$;
 (iii) $-4 < k < -3, \frac{1}{2} < k < 1$,
 (b) (i) $\frac{21}{25}$; (ii) $-\frac{2}{5}$
13. (a) $z^4 - 6z^3 + 18z^2 - 30z + 25 = 0$;
 (b) $3 \pm i, \pm \frac{1}{2}$
14. (a) $\frac{1}{2}, \frac{4}{3}, -6$;
 (b) $y^3 + 9y^2 + 2y - 48 = 0$
15. $a = b = -4, x = 2 \pm \sqrt{2}$ or $-1 \pm i$
16. (a) $a = 6, b = 4, \beta = 1 + i\sqrt{3}, \gamma = -\frac{1}{2}$
17. (a) $y = 2 - i, w = 3 - 4i$;
 (b) $\pm (3 - 2i)$; (c) $1 - i, -5 + 3i$;
 (d) $z^4 + 8z^3 + 16z^2 - 48z + 68 = 0$
18. $p = -3A, q = \frac{3G^3}{H}, r = -G^3$;
 $2, 5 \pm \sqrt{21}$
19. (a) -5, -5, $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, -2\frac{1}{2}$
20. $p = -4, q = 7, r = -1$
21. (b) $q^3 = rp^3$
22. -1; $(x^2 + 2x + 2)(3x^2 - 2x - 3)$;
 $-1 \pm i, \frac{1}{3}(1 \pm \sqrt{10})$
23. (a) $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$;
 (b) $0, \frac{2}{5}\pi, \frac{4}{5}\pi, \frac{6}{5}\pi, \frac{8}{5}\pi$;
 (c) $\frac{1}{4}(\sqrt{5} - 1)$
24. (a) 54; (b) 0, 1, 3; (c) 1; 1, 2
25. (a) -2p; (c) 0
26. (a) -5, $2 \pm \sqrt{3}$; (b) 754
27. (i) 5; (ii) 8
28. 8, 32
29. 3
30. 3
31. (i) -2
32. -1
33. -
34. $x^3 - 15x^2 + 67x - 125 = 0$;
 67, 91
35. 17, 66

2. FUNCTIONS

	Injective	Surjective	Bijective	Inverse	Domain and Range
(a)	No	No	No	No	-
(b)	Yes	No	No	Yes	$[-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$
(c)	No	No	No	No	-
(d)	Yes	Yes	Yes	Yes	$\mathbb{R} \rightarrow \mathbb{R}$

2. $f^{-1}: x \rightarrow 1 - x, g^{-1}: x \rightarrow \frac{1}{1-x}, h: x \rightarrow \frac{1}{x}$

3. (a) $A = \{x: 0 \leq x \leq \frac{\pi}{2}\}, \mathbb{R}^+ \cap \mathbb{O};$
 $B = \{x: x \geq \frac{1}{2}\}, y \geq 1$
 (b) (i) $\theta: x \rightarrow -e^{-x}$, (ii) $\phi: x \rightarrow \sin 2x$

4. -

	Injective	Surjective	Range	Inverse
(a)	No	No	$(1, \infty)$	No
(b)	No	Yes	\mathbb{C}	No
(c)	No	Yes	\mathbb{C}	No
(d)	Yes	Yes	\mathbb{C}	Yes

$\rho \circ h: x \rightarrow (5 \sin x + 12 \cos x)^3 + 5 \sin x + 12 \cos x + 1, 2211$

6. -
 7. (a) $f^{-1}(x) = \begin{cases} x, & 0 \leq x < 1 \\ \frac{1}{2}(5-x), & 1 \leq x \leq 3 \end{cases}$
 8. (a) bijective; (b) bijective;
 (c) injective; (d) none of these;
 $x \rightarrow \frac{3}{x^2 + 4} + 2$; Range $(2, \frac{3}{4})$
 9. (a) $n = 3$;
 (b) (i) $A = [-1, 0]$, $B = \mathbb{R}$;
 (ii) $A = [-1, 1]$, $B = \{x \in \mathbb{R} / x \leq 1\}$
 10. $0, \pm \frac{1}{2}, \pm \frac{1}{2}\sqrt{3}, \pm 1$

3. CURVE SKETCHING

1. Zeros at $0, \pi, 2\pi, 3\pi, 4\pi$;
 maxima at $\frac{\pi}{3}, \frac{7\pi}{3}$; minima at
 $\frac{5\pi}{3}, \frac{11\pi}{3}$
 2. $-\frac{5}{3} - \frac{19h}{12} - \frac{7h^2}{576}$
 3. $\frac{8}{9(x-3)} - \frac{8}{9x} + \frac{1}{3x^2}$; inflexion point
 4. -
 5. $y = 2, 18\frac{2}{3}$
 6. $(\frac{1}{3}, -20\frac{20}{27})$
 7. (b) $\frac{2n\pi}{3} + \frac{\pi}{18}$ or $\frac{2n\pi}{3} - \frac{\pi}{6}$
 8. $4 + 3\sqrt{3}$
 9. $(-3, -5), (\frac{3}{2}, 4)$
 10. -
 11. $1 - \frac{1}{x+1} + \frac{1}{x-2}$
 12. $(-1, 0), (1, 8)$; $0 \leq a \leq 8$
 13. -
 14. $\frac{dy}{dx} = \frac{t^2 - 1}{t^2 + 1}$; $\frac{d^2y}{dx^2} = \frac{4t^3}{(t^2 + 1)^3}$
 There is a maximum at $(0, -2)$,
 and a minimum at $(0, 2)$
 15. (a) $\frac{4}{5} < x < 2$; (b) $-3 \leq x \leq \frac{1}{3}$ or
 $x \geq 2$
 16. -
 17. 5, 1
 18. $x = 0, y = x$
 19. $x = -1, y = x - 1$
 20. (i) $x = 1, y = 2x + 5$;
 (ii) $x = 1 \pm \sqrt{3}$
 21. $\bar{x} = 1, \bar{y} = 2$
 22. (i) $x = -\frac{1}{2}$ and $y = 2x + 1$

23. (i) $x = -1$ and $y = x - 3$;
 (ii) $(1, 0)$ min.
 $(-3, 8)$ max.
 24. $a = 1, b = 1$
 25. (i) $x = -1, y = x - 3$;
 (ii) $x = 1$ or -3
 26. (i) $x = 1, y = x + 4$;
 (ii) $(3, 9)$;
 (iv) 1 root
 27. (i) $x = -1, y = x - 1$;
 (ii) 1, -3

4. TRIGONOMETRY

1. (a) $+5, -5$
 2. $R = 13, \alpha = 1.18 (\approx 67.4^\circ)$
 (b) $-3.142, -0.790, 3.142, 5.493,$
 $9.425, 11.78$
 (c) no solution set
 3. (a) $0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, 2\pi$
 (b) $\theta = 90^\circ n + 16.85^\circ, n \in \mathbb{Z}$
 4. $-287.6^\circ, -139.8^\circ, 72.4^\circ,$
 220.2°
 5. $\theta = 360^\circ n \pm 41.4^\circ$ or $180^\circ n \pm (-1)^n 30^\circ$
 6. $\theta = 360^\circ n + 60.5^\circ$ or $360^\circ n - 166.7^\circ$
 7. (b) $60^\circ n + 5^\circ$ or $60^\circ n - 25^\circ$
 8. (a) $34^\circ 36', 163^\circ 50', 214^\circ 36',$
 $343^\circ 50'$; (b) $210^\circ, 270^\circ, 330^\circ$
 9. (a) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$
 (b) $R = 6, \alpha = \frac{\pi}{6}; 2n\pi + \frac{\pi}{6}$
 10. -
 11. (a) $\frac{1}{3}\pi$; (b) $A + B = 2n\pi$ or
 $A - B = 4n\pi, n \in \mathbb{Z}^+$
 12. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}, y = \frac{3\sqrt{3}}{2}$ (max.),
 $y = -\frac{3\sqrt{3}}{2}$ (min.), $y = 0$ (neither);
 (a) $k > \frac{1}{\pi}$, (b) $0 < k \leq \frac{1}{\pi}$
 13. $p = 4$; Roots are $4 \cos 50^\circ,$
 $4 \cos 70^\circ, 4 \cos 170^\circ$
 $\sec 50^\circ + \sec 70^\circ + \sec 170^\circ = 2\sqrt{3}$
 14. $a = 5, b = 10, \theta = \tan^{-1}(\frac{3}{4})$;
 $0 \leq x < 0.725, 1.77 < x \leq \frac{3}{4}\pi$
 15. (c) $\frac{56}{65}$; (d) $0.785, 1.99, 3.93, 5.86$
 16. (b) $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$;
 $12 - 5x^2 + \frac{1}{40}x^6$

$$17. \frac{1}{4}(\sqrt{5} - 1); \left(\frac{23 - 5\sqrt{5}}{40}\right)$$

$$18. (b) \alpha = 0.785, \beta = -0.464$$

$$19. (a) \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4};$$

$$(b) 8 \cos x \cos^3 \left(\frac{1}{2}a\right); 1$$

$$20. (a) 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3; a=1, b=-1;$$

$$(b) \frac{5}{12}, 1 - e^{-\frac{1}{2}}$$

$$21. (a) x + \frac{1}{3}x^3 + \frac{2}{15}x^5; -\frac{1}{20};$$

$$(b) -3x - \frac{5}{2}x^2 - 3x^3 - \frac{17}{4}x^4; -0.328$$

$$22. (a) y \leq -\sqrt{7}, y \geq \sqrt{7}; -\frac{3}{\sqrt{7}}$$

$$23. (a) (i) \frac{1}{6}\pi, \frac{1}{2}\pi, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{3}{2}\pi, \frac{11}{6}\pi;$$

$$(ii) \frac{1}{6}\pi, \frac{1}{2}\pi, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{3}{2}\pi, \frac{11}{6}\pi;$$

$$(iii) \frac{1}{6}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi, \frac{2}{3}\pi, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{4}{3}\pi, \frac{3}{2}\pi, \frac{5}{3}\pi, \frac{11}{6}\pi,$$

$$(b) -2 \pm \sqrt{5}$$

$$24. \frac{1}{4}[3 \sin \theta - \frac{1}{3n} \sin(3^{n+1}\theta)]$$

5. MATHEMATICAL INDUCTION

$$1. e^a(a-1) + 1$$

$$2. -$$

$$3. a = 1, b = 4, S_n = \frac{n(3n+4)}{(n+1)(n+2)}$$

$$4. -$$

$$5. (a) -1; (b) 2^{n-1}n; (c) n(n+1)2^{n-2}$$

$$6. 12$$

$$7. (a) y = \frac{1}{n+1}[(1+x)^{n+1} - 1];$$

$$(b) \frac{(n-r+1)x}{r}, r = 9$$

$$8. \frac{1}{4}N(N+1)(N+2)(N+3),$$

$$\frac{1}{5}N(N+1)(N+2)(N+3)(N+4);$$

$$a = 1, b = -6, c = 7, d = -1;$$

$$\frac{1}{5}N(N+1)(N+2)(N+3)(N+4)$$

$$- \frac{3}{2}N(N+1)(N+2)(N+3) +$$

$$\frac{7}{3}N(N+1)(N+2) - \frac{1}{2}N(N+1)$$

$$9. -$$

$$10. (a) \frac{1}{x} = \frac{2}{x+1} + \frac{1}{x+2}$$

$$11. -$$

$$12. (b) (ii) (n+\frac{1}{2})^2 + \frac{3}{4}, (n-\frac{1}{2})^2 + \frac{3}{4}$$

$$(iii) S_N = \frac{1}{2}(1 - \frac{1}{N^2+N+1}) \checkmark$$

$$13. (a) (i) S_N = \frac{1}{N+1} - e^{-N}$$

$$14. -$$

$$15. -$$

$$16. -$$

6. HYPERBOLIC FUNCTIONS

$$1. -$$

$$2. \frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}; 0.131$$

$$3. \frac{1}{2} \ln \frac{27}{16}$$

$$4. (b) \ln [x + 2 + \sqrt{(x^2 + 4x - 5)}] + c$$

$$5. (a) x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + c;$$

$$(b) 1 + \frac{1}{3}x^2 + \frac{1}{5}x^4;$$

$$1 + \alpha \beta x^2 + \frac{\alpha^2 \beta (\beta - 1)}{2} x^4;$$

$$\alpha = -\frac{13}{15}, \beta = -\frac{5}{13}$$

$$6. (a) (i) -\operatorname{csch}^2 x;$$

$$(iii) A - 2 \tanh x - \frac{1}{\sinh x \cosh x}$$

$$7. \sinh(\ln 2) = \frac{3}{4}, \cosh(\ln 2) = \frac{5}{4}$$

$$8. 16 \cosh^5 x - 20 \cosh^3 x + 5 \cosh x; 0.332$$

$$9. (a) -\frac{1}{2}, \frac{2}{11};$$

$$(b) (i) x \sinh^{-1} x - \sqrt{(x^2 + 1)}; (ii) 0.66$$

$$10. \ln(\cosh x) - \frac{1}{2} \tanh^2 x - \frac{1}{4} \tanh^4 x + c; 0.125$$

$$11. (a) x = \ln 3, y = \ln 2$$

$$12. (a) \frac{1}{\sqrt{(x^2-1)}}; (b) \frac{3}{4}$$

$$13. -$$

$$14. \frac{\sqrt{(1+t^2)}}{1-t^2}$$

$$15. -$$

$$16. \frac{1024}{5625}$$

$$17. -$$

$$18. -$$

7. FURTHER DIFFERENTIATION AND INTEGRATION

$$1. I_0 = \frac{2}{3}, I_6 = \frac{32}{315}$$

2. (0.61919, 0.7956)
3. $\frac{bncos nt - u}{ansin nt}$; $\frac{abn^3 - aun^2 \cos nt}{sin nt}$
4. -
5. $I_1 = \frac{1}{2}(1 - \frac{1}{e})$, $I_9 = 12 - \frac{65}{2e}$
6. $\frac{\sqrt{3}}{2\sqrt{3t+4}}$, $t < -2$ or $0 < t < 2$
7. $x + \frac{1}{6}x^3 + \frac{3}{40}x^5$; -1
8. -
9. $I_4 = \frac{3}{16}\pi$, $I_5 = \frac{8}{15}$; $\int_0^{\frac{1}{2}\pi} \sin^n \theta d\theta$
10. (a) (i) $x + \frac{1}{6}x^3 + \frac{3}{40}x^5$
11. Area = $\frac{1}{r}$, $4.100 < \sum_{r=2}^{99} \frac{1}{r} < 4.195$
12. (a) $2 \ln 2$, (b) $2[\cot \frac{\pi}{12} - \cot(\frac{2n+1}{12}\pi)]$;
 $n = 6k$ ($k \in \mathbb{Z}^+$)
13. $\frac{1}{32}(5e^4 - 1)$; 0.467
14. $1 + \frac{1}{3}x^2 - \frac{4}{45}x^4$; 0.513
15. (a) $\frac{5\pi}{32}$; (b) $\frac{16}{15}$; (c) 0; (d) $\frac{35\pi}{256}$
16. 2611
17. (a) $\frac{1}{13}(-3 \cos 3x - 2 \sin 3x)e^{-2x} + K$;
 (b) $1 - 2x - \frac{5}{2}x^2 + \frac{23}{3}x^3$
18. (a) 0.127; (b) 0.154
19. $x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{24}x^5$
20. (a) $\frac{1}{2}$; (b) $-\frac{1}{2}$; (c) $t \tan^{-1}(2t) - \frac{1}{4} \ln(1 + 4t^2) + c$
21. $y \approx mx + \frac{1}{6}m(1 - m^2)x^3$; $\frac{21}{25}$
22. (a) $I_1 = k \sin^{-1} k + \sqrt{(1 - k^2)} - 1$;
 $I_2 = \frac{1}{2} \sin^{-1} k + \frac{1}{2}k \sqrt{(1 - k^2)}$;
 (b) $\frac{1}{4}\pi + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{1}{12}x^3$
23. (a) $\frac{1}{\sqrt{2}} \cosh^{-1}(\frac{2x+1}{\sqrt{3}}) + c$; (b) $\frac{5}{16}\pi$
24. (a) $-\frac{108}{125}$; (b) $\frac{\pi^3}{2} - 12\pi + 24$
25. -
26. -
27. -
28. -
29. $I_4 = 9e - 24$
30. -
31. $I_4 = \frac{25}{162} + \frac{3}{8} \ln 3$; 0.504

32. (i) $1 - \frac{5}{2e}$

33. (a) -60; (b) $\frac{1}{4} + \frac{3}{32}\pi$

8. APPLICATIONS OF INTEGRATION

1. -
2. (a) a ; (b) $\frac{1}{2}b\pi$, $\frac{1}{2}c\pi$;
 (c) $\frac{1}{2}\pi(2a^2 + b^2 + c^2)$
3. -
4. $8a$,
5. -
6. -
7. (a) $\frac{9}{2}a^2\pi$; (b) 49
8. $\frac{\pi}{2} - 1$
9. -
10. 3.6, 49
11. $1 - \frac{8}{3\pi} \approx 0.151$;
 $\pi(\frac{1}{4}\pi - \frac{76}{105}) \approx 0.193$
12. -
13. $2\pi + \frac{3}{2}\sqrt{3}$
14. -
15. $(\ln \sqrt{2} - \frac{\pi}{12})$
16. $\frac{dy}{dx} = \frac{\sin \theta}{1 - \cos \theta}$; $\frac{d^2y}{dx^2} = -\frac{1}{y^2}$
17. $r^2 = a^2 \cos 2\theta$
18. (a) $\frac{1}{2} + \frac{1}{8} \sinh 4$; (b) $8a$
19. -
20. $(\frac{205}{162} - \frac{1}{8} \ln 3)\pi$
21. $3\sqrt{3}$
22. (b) 1.15; (c) $\frac{28\pi}{15}$
23. (b) $6a$; (c) $\frac{3\pi a^2}{8}$
24. (a) (i) $\frac{1}{3}(x^2 + a^2)^{\frac{3}{2}}$;
 (ii) $\frac{1}{2}x \sqrt{(x^2 + a^2)} + \frac{1}{2}a^2 \sinh^{-1}(\frac{x}{a})$;
 (b) $70a^2$
25. 4.84
26. (a) $\frac{dy}{dx} = \frac{t(2 - t^3)}{1 - 2t^3}$; $(2^{\frac{1}{3}}, 2^{\frac{2}{3}})$,
 $(2^{\frac{2}{3}}, 2^{\frac{1}{3}})$;
 (b) $1\frac{1}{2}$
27. -
28. $\frac{2\sqrt{2}\pi}{5}(e^{\pi} - 2)$

29. (c) $x = 1 + \frac{\pi}{2}$, $-1 + \frac{3\pi}{2}$, (d) $2 - \frac{\pi}{2}$
 30. $\frac{1}{8}\pi (\sinh 4a + 4a - 4 \sinh 2a)$
 31. (a) 0.084
 33. $\frac{-x(x^3 - 2by^2)}{y(y^3 - 2bx^2)}$; $\frac{5b^2}{2}$
 34. (a) $-\frac{1}{4a} \csc^4(\frac{t}{2})$; (c) $8a$
 35. (a) $\sin t$, $\frac{\cos^5 t}{3 \sin t}$
 36. (a) $\tanh^{-1} x = x + \frac{1}{3}x^3 + \frac{1}{5}x^5$;
 (b) $\sqrt{2} a (e^{\frac{1}{2}\pi} - 1)$
 37. (a) $\frac{1}{2}x \sqrt{4 + x^2} + 2 \sinh^{-1}(\frac{1}{2}x) + c$
 38. -
 39. $2 - \frac{1}{2}\pi$
 40. (i) $-(t^3 + t)(t^2 - 1)^{-3}$
 41. (0.418, 0.342)
 42. $\frac{35}{96}$
 43. (i) $\frac{8}{3}$, (ii) (2.4, 1.5)
 44. 2
 45. (i) 5.7; (ii) 10; (iii) 93.6π
 46. (i) $\frac{1}{8}a^2(e^{\pi}-1)$; (ii) $\frac{\sqrt{5}}{4}a(e^{\frac{1}{2}\pi}-1)$
 47. (i) 4; (ii) $(\frac{20}{7}, 5)$; (iii) 16.6

9. DIFFERENTIAL EQUATION

1. $x = \sinh^2 t$; $v = \sinh 2t$
 2. $\frac{dz}{dt} = \frac{n}{A} \{1 - (\frac{z}{h})^{\frac{1}{2}}\}$;
 $z = \frac{16h}{9}$ when $t = \frac{2h}{n} A (\frac{2}{3} + \ln 3)$
 3. (a) $x^2 y = \sec^2 x + c$;
 (b) $x - 1 = \ln |\frac{x+2y}{x-2y}|$
 13. $x = e^{-t} \sin 2t + 2$; $x \rightarrow 2$ as $t \rightarrow \infty$
 16. $\frac{dz}{dx} - 2z = -2x$
 17. (a) $y = 2 \sin x$; (b) $x = (1-t)e^{-t}$
 18. $y = 2e^t + 2e^{-2t}$
 19. (a) $y \cos x = x^2 + c$; (b) $\sqrt{5} \text{ ms}^{-1}$
 20. $y = \cos x - \cos 2x$; $x = 2n\pi$ or
 $2n\pi \pm \frac{2\pi}{3}$; $\cos x = \frac{1}{4}$ or ± 1
 21. (a) $y = \frac{1}{2}x^3 \sin 2x + \frac{1}{4}x^2 \cos 2x + cx^2$
 (b) $x^2 = 2y^2 \ln Ay$
 22. $x = 2 \sin 3t - 6 \cos 3t(1 - e^{-2t})$
 23. (a) $y = (1 - 2x)e^{-2x}$; (b) $x^2 e^{-2x}$
 24. $x = 100 - 99 e^{-\frac{1}{50}t^2}$; 5.89

25. $8\sqrt{2}$; 7
 26. $y = x^2 + Ce^{x^2}$
 27. (b) $y = (\frac{x-1}{x})e^{-x}$
 28. (b) $y = x^3(\ln x - 1) + 3x^2$
 29. $y = A \cos 3x^2 + b \sin 3x^2 - \frac{2}{3}$
 30. (a) $x = t$;
 (b) $y = \tan^{-1} x - 1 + ke^{-\tan^{-1} x}$
 31. $x = t - 2 + 2(t+1)e^{-t}$
 32. $y = \cos x + k \cos^2 x$; $\frac{1}{4}$
 33. $y = \frac{3}{13} \cos 2x + \frac{2}{13} \sin 2x + Ae^{-3x}$,
 $y = \frac{1}{39}(9 \cos 2x + 6 \sin 2x + 26 + 4e^{-3x})$
 34. (a) 0; (b) $y = \{x - 2 \tan^{-1}(\frac{1}{2}x) + c\} e^{\tan^{-1}(\frac{1}{2}x)}$; $c = \frac{1}{2}\pi - 1$
 35. $y = e^{-x}(1 + A \cos 2x + B \sin 2x)$;
 $y = e^{-x}(2 \sin 2x + 1)$;
 $\frac{11}{12}\pi$, $\frac{19}{12}\pi$, $\frac{23}{12}\pi$
 36. (a) $\frac{1}{5}e^{2x}(2 \cos X + \sin X) + k$;
 (b) $y = \frac{1}{5}(2 \cos X + \sin X + 3t^{-2x})$;
 (c) $x = e^{-t}(A \cos 3t + B \sin 3t) + \frac{1}{85}(9 \sin t - 2 \cos t)$
 37. $y = (Ax + B)e^{3x} + x + \frac{2}{3}$;
 (b) $y \sqrt{a^2 - x^2} = c - a^2 \sin^{-1}(\frac{x}{a})$
 38. (a) $y = (x^2 + x + c)(1 - 2x)(1 + 2x)$;
 (b) $x = \frac{1}{4}t \cos 2t$
 39. (b) $y = \frac{1}{x}[A \sin(2 \ln x) + B \cos(2 \ln x) + \frac{x^3}{20}]$
 40. (a) $y = (4 + 4x + e^x)/(4 + e^x)$;
 (b) $y = \frac{2}{5} \sin 2x - \frac{3}{10} \cos 2x + Ae^{-x} + Be^{-2x}$; $R = 0.5$, $\phi = 0.644$
 41. (a) $y = \frac{1}{2 \ln x} + \frac{e^2}{2x^2 \ln x}$;
 (b) $y = \frac{1}{\sqrt{x}}[e^{-x}(A \cos x + B \sin x) + \frac{1}{2}x - \frac{1}{2}]$
 42. (a) $y^2(1+x^2)^2 = (6x+2x^3)+1, \frac{3}{2}$;
 (b) $b = e^{-2x}(\cos 4x + \sin 4x) + 3x + 7$
 43. (i) $\bar{y} = \tanh x + \operatorname{sech} x$;
 (ii) $a = 3$, $b = 2$, $y = (Ae^{-x} + Be^{-2x})^{\frac{1}{3}}$

$$44. (a) y = \frac{1}{2}x^3 + \frac{1}{2}x,$$

$$y = e^{-\frac{x}{2}} \left(A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right) + 3 \sin x - \cos x$$

$$45. (a) y = \frac{1 + \sinh^{-1}x}{\sqrt{1+x^2}};$$

$$(b) y = Ae^{-x} + Be^{-ax} + e^{-2x};$$

$$y = \frac{1}{a-1}e^{-x} + \frac{1}{1-a}e^{-ax} + e^{-2x}$$

$$46. (a) y = \left(\frac{1}{3} + \frac{A}{x^3} \right) \sin x;$$

$$(b) y = e^{-3x} (A \cos 4x + B \sin 4x) + \sin 2x$$

$$47. (a) y = \frac{3(1+x)^2}{(1+x)^3+2};$$

$$(b) y = ae^{-x} + be^{-3x} - 8 \cos 2x - \sin 2x$$

$$48. (a) y = (x + \frac{\pi}{2}) \sin x;$$

$$(b) y = \frac{1}{x} [A \cos(3 \ln x) + B \sin(3 \ln x)] + 4 \ln x$$

10. COMPLEX NUMBER (I) -

DE MOIVRE'S THEOREM

1. -

$$2. (a) 2^{19}(-1 - i\sqrt{3}); (b) 2i, \sqrt{3} - i, -\sqrt{3} - i$$

$$14. (a) (i) (\sqrt{2}, -\frac{1}{4}\pi); (ii) (2, \frac{1}{6}\pi);$$

$$(iii) (\sqrt{2}, \frac{5}{12}\pi); (iv) (64, \pi)$$

$$15. (a) A = 64, B = -112, C = 56,$$

$$D = -7; x = 4 \cos^2 \frac{1}{14}\pi,$$

$$4 \cos^2 \frac{3}{14}\pi, 4 \cos^2 \frac{5}{14}\pi$$

$$18. (a) x^2 + 2x \cos \theta + 1;$$

$$(b) (x^2 + 1)(x^2 + \sqrt{3}x + 1)$$

$$(x^2 - \sqrt{3}x + 1)$$

19. -

$$20. (a) \cos 2\theta + i \sin 2\theta; \cos \frac{2\pi}{9}$$

$$+ i \sin \frac{2\pi}{9}, \cos \frac{8\pi}{9} + i \sin \frac{8\pi}{9},$$

$$\cos \frac{14\pi}{9} + i \sin \frac{14\pi}{9};$$

$$(b) z = 0, 4, \frac{2}{5} \pm \frac{6}{5}i$$

21. -

$$22. \tan 7\theta = \frac{(7t - 35t^3 + 21t^5 - t^7)}{(1 - 21t^2 + 35t^4 - 7t^6)};$$

$$\sec^2(\frac{1}{7}\pi) + \sec^2(\frac{2}{7}\pi) + \sec^2(\frac{3}{7}\pi) = 24$$

$$23. (a) 5 - i, -5 + i; (b) 1 - i, -1 + i;$$

$$(c) -192 - 128i; (d) -30 + 45i$$

$$24. (a) \frac{1}{32} \cos 6\theta - \frac{1}{16} \cos 4\theta +$$

$$\frac{1}{32} \cos 2\theta + \frac{1}{16}; \frac{1202}{729};$$

$$(b) -3, -\frac{1}{3}, -\frac{3}{5} + \frac{4}{5}i; -\frac{3}{5} - \frac{4}{5}i$$

$$25. (a) 8\sqrt{2} - 9 \leq |z - w| \leq 8\sqrt{2} + 9;$$

$$(b) -1, 6r + 3 (r \in \mathbb{Z})$$

$$26. (a) 7 - 14t + 7t^2 - t^3, x = \pm 2 \sin \frac{\pi}{7},$$

$$\pm 2 \sin \frac{2\pi}{7}, \pm 2 \sin \frac{3\pi}{7};$$

$$(b) \cot \theta$$

$$27. e^{\frac{2}{5}ir\pi}, (r = 0, \dots, 4);$$

$$\text{together with } e^{\frac{1}{4}(2r+1)i\pi}, (r = 0, \dots, 3)$$

$$28. (a) 1, \pi; (b) \frac{21}{32}\sqrt{3}, \frac{\pi}{6}$$

$$29. (a) e^{\frac{2ik\pi}{5}} (k = 0, \dots, 4);$$

$$(c) \tan(\frac{k\pi}{5}) (k = 0, \dots, 4)$$

$$30. (a) (i) \sqrt{2}, -\frac{1}{4}\pi; (ii) 4, \pi;$$

$$(iii) 1, \frac{1}{3}\pi;$$

$$(b) 2^{\frac{1}{6}}e^{\frac{-i\pi}{12}}, 2^{\frac{1}{6}}e^{\frac{7\pi}{12}}, 2^{\frac{1}{6}}e^{\frac{5\pi}{4}}$$

$$31. (a) e^{-\frac{13}{9}i\pi}, e^{-\frac{7}{9}i\pi}, e^{-\frac{1}{9}i\pi}, e^{\frac{1}{9}i\pi},$$

$$e^{\frac{7}{9}i\pi}, e^{\frac{13}{9}i\pi};$$

$$(b) a = 1, b = -2, c = -2, d = 6$$

$$32. n = 9, 13, 17$$

$$33. e^{\frac{2}{7}\pi i}$$

$$34. (b) -1024, (c) 1, 4\theta$$

$$35. (c) \sin(n+1)\theta$$

$$36. e^{\frac{1}{9}ni\pi} \text{ for } n = 1, 5, 7, 11, 13, 17$$

$$37. 2 \cos \frac{\theta}{2} (\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}),$$

$$2 \cos \frac{\theta}{2} [\cos(-\frac{\theta}{2}) + i \sin(-\frac{\theta}{2})]$$

38. -

$$39. (a) \cos(\frac{2\pi k}{9}) + i \sin(\frac{2\pi k}{9}), \text{ where}$$

$$k = 0, 1, 2, \dots, 8; \text{ Sum} = 0$$

$$40. -$$

$$41. -$$

$$42. -$$

43. -
 44. -
 45. (a) $z = e^{i(\frac{2\pi k}{6})}$, $k = 0, 1, 2, 3, 4, 5$;
 3.23, 1.24;
 (b) $16s^5 - 20s^3 + 5s$;
 $\sin \frac{\pi}{30}$, $\sin \frac{\pi}{6}$, $\sin \frac{13\pi}{30}$

46. -

11. COMPLEX NUMBER (II) -

LOCI AND TRANSFORMATION

1. $z_1 = 1 + i\sqrt{3}$, $z_2 = 1 - i\sqrt{3}$
 If $n \equiv 0 \pmod{6}$, $z_1^n = z_2^n = 2^n$;
 If $n \equiv 1 \pmod{6}$,
 $z_1^n = 2^{n-1}(1 + i\sqrt{3})$;
 $z_2^n = 2^{n-1}(1 - i\sqrt{3})$
2. -
 3. (b) $W \in \{w : \frac{1}{4} < \operatorname{Re} w < \frac{1}{2}\}$
 4. -
 5. -
 6. -
 15. $x + 3y = -4$; $A : x + y = 0$,
 $B : x = \frac{1}{2}$, $C : x = y$
16. -
17. $w\bar{w} + \frac{(1+i)}{2}w + \frac{(1-i)}{2}\bar{w} = 0$,
 a circle centre $-\frac{1}{2} + \frac{1}{2}i$,
 radius $\frac{1}{\sqrt{2}}$
18. (a) 2, 8; $\frac{24}{7}$; (b) $-\frac{13}{4}$, $-\frac{13}{4}$, 0;
 $z = \frac{13}{2}$
19. $\frac{4}{3} - \frac{4}{9}i$
20. (a) $z = 2 - i$, $w = 1 + 3i$
21. circle centre (5, 0), radius 4
22. A line which is perpendicular to the line joining the origin and the point $(\frac{24}{25}, \frac{18}{25}i)$
23. $a = 4$, $b = 9$, $c = -4$;
 (a) Centre 4, radius 5
24. $w - \bar{w} = 4ib^2$
25. $(2+i)\bar{z} + (2-i)z - 7 = 0$ or $4x + 2y - 7 = 0$
26. Centre $-2 + i$, radius $\sqrt{5}$
27. $4.8 \leq |z| \leq 10$
28. (a) $-2 + i$; (b) (i) $\frac{2}{3}i, \frac{4}{3}$;
 (ii) $w(1+2i) + \bar{w}(1-2i) = 3$

29. $\sqrt{\alpha^2 + \beta^2 - b}$; (a) 1; (b) -1;
 (c) $-1 \pm 2\sqrt{2}$
30. (a) $(1-i)w + iz$, $2w - z$,
 $(1+i)w - iz$
31. $\operatorname{Re} w = 1$; $(1+i)a$
32. (a) $1 - i$, 2; (b) $e^{-i\pi/6}$, $e^{i\pi/2}$,
 $e^{-i5\pi/6}$; (c) $n = 4k + 2$ where $k \in \mathbb{Z}$
33. (a) $(3+i)z^* + (3-i)z - 10 = 0$;
 (b) $zz^* - \frac{1}{2}(3+i)z^* - \frac{1}{2}(3-i)z = 0$
34. (c) \Rightarrow (b); $20 \leq |z| \leq 40$;
 $-\frac{6}{17} \leq \arg z \leq \frac{3}{4}$
35. (a) $(a_1 - a_2)z^* + (a_1 - a_2)^*$
 $z + (b_2 - b_1) = 0$
36. (a) $-4 + 3i$, 5; (b) $(3-4i)z^* + (3+4i)z + 48 = 0$
37. (a) Centre $(\frac{5}{3}, 0)$, radius $\frac{4}{3}$;
 (b) Centre $(-1, -\frac{1}{2})$, radius $\frac{\sqrt{5}}{2}$
38. (i) The point Q moves from C, which represents $0 + 2i$, along the imaginary axis to 0 and the back along the imaginary axis to C;
 (ii) The point Q starts at C and moves once round the circle $|w| = 2$ in an anticlockwise sense to return to C.
39. (a) (i) $z = \cos(\frac{2}{3}k\pi) + i \sin(\frac{2}{3}k\pi)$
 where $k = 0, 1, 2$
 (ii) $z = \cos(\frac{1+3k}{3}\pi) \pm i \sin(\frac{1+3k}{3}\pi)$
 where $k = 0, 1$
- (b) (ii) $\frac{1}{25}(28 + 4i)$
40. -
41. $k = \frac{1}{7}$
42. $\frac{2\sqrt{2}}{3}$
43. (i) Q moves along the imaginary axis from O to the point representing $-i$,
 (ii) Q moves anti-clockwise around the circle $|z| = 1$ from the point representing the point $-i$;
44. Q starts at a point representing $\frac{3}{4}$ and moves vertically up to a point representing $(\frac{3}{4} + \frac{1}{4}i)$
45. (i) move along the straight line

from $(1 - i)$ to $(1 - \frac{1}{3}i)$;

(ii) moves through an arc of the circle from O to $(\frac{1}{2} - \frac{1}{2}i)$

12. VECTOR

1. -
2. -
3. (a) $\underline{r} = 2\underline{i} + 3\underline{j} + \underline{k} + \lambda(-\underline{i} - 2\underline{j} - 2\underline{k})$;
(b) $\frac{11}{3\sqrt{10}}$;
(c) $\underline{r} = 2\underline{i} + 3\underline{j} + \underline{k} + \lambda(-\underline{i} - 2\underline{j} - 2\underline{k}) + \mu(-\underline{j} - 3\underline{j})$;
(d) $\cos^{-1}(\frac{10}{\sqrt{41}\sqrt{29}})$
20. $P = (5, 4, -1)$, $Q = (6, 2, 0)$
21. $\underline{r} = (1, -1, 1) + \lambda(1, 2, 1) + \mu(1, 3, 3)$
or $3x - 2y + z = 6$;
 $\underline{r} = (1, -1, 1) + \lambda(3, -1, -4)$
or $\frac{x-1}{3} = \frac{y+1}{-1} = \frac{z-1}{-4}$
24. (a) $\pm \frac{1}{3}(2\underline{i} - \underline{j} + 2\underline{k})$; (b) $\frac{2}{3}$;
(c) $\frac{1}{3\sqrt{5}}$
25. Magnitude 5 at $\sin^{-1}(-\frac{3}{5})$ to direction of \underline{i} ; $t = 2$; $4\sqrt{5}$
26. (a) False; (b) False; (c) True;
(d) False; (e) True
27. (b) $\underline{r} \cdot (2\underline{i} - 3\underline{j} + 6\underline{k}) = 24$; 12
28. (a) $\pm (-\underline{i} + 2\underline{j} + \underline{k})/\sqrt{6}$; (b) $2\sqrt{6}$;
(c) $\underline{i} + 7\underline{j} - 2\underline{k}$
29. (a) $3x - 2y + z = 6$ or
 $\underline{r} = (1, -1, 1) + \lambda(1, 2, 1) + \mu(2, 2, -2)$;
(b) $RB : BC = 1 : 3$
30. (b) (i) True; (ii) False; (iii) True
31. $\underline{r} \cdot (2\underline{i} - 6\underline{j} + 3\underline{k}) = 4$; $\frac{5\sqrt{21}}{63}$
32. (a) $\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$; (b) $\underline{r} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 3$
33. (a) $\underline{r} \cdot (3, -4, -1) = -7$;
(b) $(3, 3, 4)$; (c) $(1, 3, -2)$
34. $2 : -1$
35. $(1, 1, -1)$; $\underline{r} \cdot (2, -2, 1) = -1$;
(a) $\frac{5}{\sqrt{26}}$; (b) $\frac{5}{\sqrt{1066}}$
36. $6\underline{a} - 5\underline{c}, \frac{8}{3}\underline{b} - \frac{5}{3}\underline{c}$
37. $3\underline{i} - 4\underline{j} + \underline{k}$
38. (a) True; (b) False; (c) False;
(d) False
39. (b) $\frac{bc}{\sqrt{(b^2 + c^2)}}, \frac{ca}{\sqrt{(a^2 + c^2)}}$
40. (a) $\frac{16}{\sqrt{310}}$; (b) $\begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$
41. (a) (i) $\underline{r} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$;
(ii) No solutions;
(b) $\underline{x} = \frac{1}{2}\underline{c} - (\frac{\underline{b} \cdot \underline{c}}{2 + \underline{a} \cdot \underline{b}})\underline{a}$
42. $\underline{r} \cdot (2, 3, -6) = -4$; $(5, 3, 12)$;
 $\sqrt{21}$
43. $\lambda^2 \left| \frac{\lambda - 1}{\lambda + 1} \right|$
44. $\mu = \frac{2}{3}\underline{b} \cdot (\underline{u} - 2\underline{v})$;
(a) $\underline{r} = \underline{a} + \lambda\underline{u} + \mu\underline{v}$;
(b) $\frac{2}{3}\underline{u} \times \underline{v}$;
(c) $\underline{r} = t \{ \underline{a} + \frac{2}{3}\underline{b} \cdot (2\underline{u} - \underline{v}) - \underline{u} \}$
45. (a) $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$; (b) $\frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$; (c) $\begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix}$,
 $\begin{pmatrix} -7 \\ -7 \\ 0 \end{pmatrix}$
46. -
47. $\frac{1}{7}\sqrt{629} \approx 3.58$
48. (a) $\sqrt{3}/\sqrt{2}$
49. (a) $A(6, 0, -1), B(2, 2, 1)$;
(c) $(3, 6, -1)$
50. (a) $2 \sin \delta \sqrt{1 + \cos^2 \delta}$; $\theta = \frac{1}{3}\pi$,
 $\phi = \frac{2}{3}\pi$
51. (a) $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$; (b) $\cos^{-1}(\frac{1}{\sqrt{10}})$;
(c) $\frac{4}{5}\sqrt{5}$; (d) $2\sqrt{2}$
52. $\frac{1}{3}(\underline{a} + \underline{b} + \underline{c})$
53. (a) $k = 2, \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix}$;
(b) $\underline{r} = \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$
54. $9\underline{i} + 6\underline{j} - 18\underline{k}$; $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$;
 $\underline{r} = (\frac{7}{2} + 2\lambda)\underline{i} + 9\underline{j} + (\lambda - 15)\underline{k}$; 9

55. (i) $x + 3y + 2z = -2$; (ii) $-k$;
 (iii) $-i + j - k$, $-5i - j + 4k$
 56. (i) $i + 2j + k$; (ii) $4i - 17j + 45k$,
 $14i + 3j + 55k$
 57. $-i + 4j$, $-4i + 16j$; $-7i - 8j - 18k$
 58. -
 59. (a) $(-9, 12, 3)$, $(-3, 12, 3)$;
 (b) 67.6° ;
 (d) 4.23
 60. -
 61. $(2, -4, 1)$
 62. $3\sqrt{6}$; (i) $\begin{pmatrix} 9 \\ -6 \\ -3 \end{pmatrix}$; (ii) $\begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$
 63. (iii) $-5i + 5j + 5k$
 64. (ii) $p = 2i + 3j - 4k$, $q = -i + j + k$;
 (iii) $7x - 8y + z + 14 = 0$
 65. (i) $(7 \cos t)i + (7 \sin t)j$;
 (iv) 64.5°
 66. (i) $2i - 2j - 4k$
 67. -
 68. (i) $PQ = 7$, $p = i - j + 2k$,
 $q = 3i + 2j + 8k$
 69. (i) $\frac{1}{2}a$, $\frac{1}{3}(2a + b)$, $\frac{3}{4}c$;
 (iv) $\frac{3}{8}$
 70. (i) 4;
 (ii) 20
 71. $\vec{OL} = \frac{1}{4}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{4}\vec{c}$;
 (i) $\frac{1}{5}\vec{a} + \frac{2}{5}\vec{b}$;
 (ii) $\frac{1}{6}\vec{a} + \frac{1}{3}\vec{b} + \frac{1}{6}\vec{c}$
 72. $P(1, -6.4)$, $Q(-5, 0, 4)$; 1
 73. Locus of W is a plane containing the point $(-5, 9, 1)$ and parallel to π_1 ;
 (iii) $r = \frac{1}{2}\begin{pmatrix} 8 \\ 13 \\ -21 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$
 74. (ii) 1.20; (iii) 54.7

13. SET AND RELATION

1. (a) Each equivalence class contains only one element; (b) Nothing

2.

ρ	x_1	x_2	x_3	x_4	x_5
x_1	1	0	1	0	0
x_2	1	1	1	1	1
x_3	0	0	1	0	0
x_4	1	0	1	1	1
x_5	1	0	1	0	1

ρ is a total ordering: $x_2 \rho x_4$
 $\rho x_5 \rho x_1 \rho x_3$

3. -
 4. (a) Symmetric; (b) equivalence relation, only class is S;
 (c) order relation: $2 < 4 < 8 < 16$;
 $3 < 9 < 18$; $2, 3 < 6 < 12 < 18$
 etc; (d) equivalence relation,
 classes: $\{2, 4, 8\}$, $\{3, 9\}$, $\{5\}$,
 $\{7\}$
 11. -
 12. $\{a\}$, $\{b, e\}$, $\{c, d, f, g\}$,
 $\{f, g, c, d\}$
 13. -
 14. $\{1, 7\}$, $\{2, 8\}$, $\{3, 9\}$, $\{4, 10\}$,
 $\{5\}$, $\{6\}$
 15. (a) ρ_1 : Transitive, ρ_2 = reflexive
 and transitive, ρ_3 = symmetric;
 (b) (i) $\{a, 2 - a\}$; (ii) $\{a, a + 2n\}$,
 $(2n + 1)\pi - a$
 16. -
 17. (ii) $\{\text{Even numbers}\}$, $\{\text{odd numbers}\}$;
 (iv) $\{\text{Reduced fractions with the same denominator}\}$; (v) $\{f\}$,
 $\{a, c\}$, $\{b, d, e\}$
 18. -
 19. -
 20. (a) $\{a, c, e, g\}$, $\{b, d, f\}$
 21. (a) -2, 4; (b) $1\frac{1}{4}$
 22. (b) 1, 3, 7, 9; (d) 10, 20, 40, 80
 23. (ii) The operation $*$ is not distributive over the operation \circ ;
 (iii) Identity in C with respect to $*$ is \circ . Inverse of z with respect to $*$ is $-\frac{z}{z+1}(z \neq -1)$;
 (iv) -1, $\pm i$.
 24. (i) $(0, 0)$, $(1, 1)$; (iii) $\{(1, 1)$,
 $(1, -1)$, $(-1, 1)$, $(-1, -1)\}$; (iv) 12
 25. (i) 15; (ii) 112; (iv) 126 and 104
 26. (ii) $P = \begin{pmatrix} \pm 1 & x \\ 0 & \pm 1 \end{pmatrix}$
 27. (i) $I = \{ax+b \in S: a \equiv 1(\text{mod } 5) \text{ and } b \equiv 1(\text{mod } 7)\}$
 (ii) $Q = \{ax+b \in S: a \equiv 2(\text{mod } 5) \text{ and } b \equiv 3(\text{mod } 7)\}$
 (iii) $R = \{ax+b \in S: a \equiv 0(\text{mod } 5)\} \cup \{ax+b \in S: b \equiv 0(\text{mod } 7)\}$, 11
 28. (ii) 0;
 (iii) $-\frac{1}{k}$,
 (iv) $k = \pm 1$
 29. -
 30. (ii) $\{x^2+2x+3, x^2+3x+2, 2x^2+x+3,$
 $2x^2+2x+2, 2x^2+3x+1, 3x^2+x+2,$
 $3x^2+2x+1\}$

(iii)	an element of the equivalence class	no. of elements
	$x^2 + x + 1$	1
	$x^2 + x + 2$	3
	$x^2 + x + 3$	6
	$x^2 + 3x + 3$	6
	$2x^2 + 3x + 3$	3
	$3x^2 + 3x + 3$	1

31. (iii) $\{(x, y)/y=0\}$;
 (iv) $\{(-1, \frac{1}{2}), (-1, -\frac{1}{2})\}$
 32. R_1 and R_3 are not equivalence relation,
 R_2 is an equivalence relation

14. MATRICES AND GROUP

1. $ad \neq 0$
 2. -
 3. -
 4. $H = \{0, 1, 2, 3, 4\}$ under addition modulo 5
 5. $\{1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i\}$
 6. $f: x \rightarrow 0$ for all $x \in \mathbb{Z}$ and $g: 0, 3 \rightarrow 0; 1, 4 \rightarrow 1; 2, 5 \rightarrow 2$
 7. -
 8. -
 9. -
 10. -
 34. 81; (a) 20, (b) $\left\{ \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}, \begin{pmatrix} \pm 1 & 0 \\ 0 & \mp 1 \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} \mp 1 & 0 \\ 0 & \pm 1 \end{pmatrix}, \begin{pmatrix} \pm 1 & 0 \\ 0 & \mp 1 \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} 0 & \pm 1 \\ \mp 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix} \right\}$
 35. (a) $H = K$; (b) $H = K$
 36. -
 37. -
 39. -
 41. $I = (1, 0), A^{-1} = \left(\frac{a_1}{a_1^2 + a_2^2}, -\frac{a_2}{a_1^2 + a_2^2} \right); X = (0, \pm 1), (\pm 1, 0)$

42. $C = \{aI\}, (a \in \mathbb{R}, a \neq 0)$

43. *	u	v	w	x	y	z
u	v	w	u	z	x	y
v	w	u	v	y	z	x
w	u	v	w	x	y	z
x	y	z	x	w	v	u
y	z	x	y	v	u	w
z	x	y	z	u	w	v

44. (a) $\{1, -1\}, \{1, i, -1, -i\}$;

(c) $\theta = \frac{k\pi}{4}$ where $k = 0, 1, 2, \dots, 7$

45. -

46. $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, D = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

	C	D	A	B
C	C	D	A	B
D	D	C	B	A
A	A	B	C	D
B	B	A	D	C

47. 5; (a) $\{e, a, a^2, a^3\}$;
 (b) $\{e, a^2b, a^2b\}$ or $\{e, a^2, ab, a^3b\}$

48. -

49. $t(x) = 1 - x, u(x) = \frac{x-1}{x}$;

$(p, r), (p, s), (p, t), (p, q, u)$

50. (b) Multiplication; \mathbb{Z} ; addition

51. -

52. 6

53. (iii) $w = x \neq 0, y = z = 0$

54. (c)	0	1	5	7	11
1	1	5	-7	11	11
5	5	1	11	7	7
7	7	11	1	5	5
11	11	7	5	1	1

(d) The groups in (a) and (c) are isomorphic

55. $\frac{1}{3}$

56. $s = 1, t = -1; \alpha = -\frac{1}{\mu}(\lambda^2 + \frac{3}{4}),$
 $\beta = \lambda - \frac{1}{2}; \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 7 \\ -1 & -3 \end{pmatrix}, \begin{pmatrix} -3 & -7 \\ 1 & 2 \end{pmatrix} \right\}$

57. *	e	a	a ²	b	ab	a ² b
e	e	a	a ²	b	ab	a ² b
a	a	a ²	e	ab	a ² b	b
a ²	a ²	e	a	a ² b	b	ab
b	b	a ² b	ab	e	a ²	a
ab	ab	b	a ² b	a	e	a ²
a ² b	a ² b	ab	b	a ²	a	e

Required equivalence class is $\{a^2, ab\}$.

58. (i)	I	P	Q	R	S	T
I	I	P	Q	R	S	T
P	P	Q	I	T	R	S
Q	Q	I	P	S	T	R
R	R	S	T	I	P	Q
S	S	T	R	Q	I	P
T	T	R	S	P	Q	I

(ii) $\{I, R\}, \{I, S\}, \{I, T\}, \{I, P, Q\}$;

(iii)

Element	P	Q	R	S	T
Order	3	3	2	2	2

59. (ii) $\{f_1, f_2\}, \{f_1, f_3\}, \{f_1, f_5\}, \{f_1, f_4, f_5\}$

60. (iii) $\{1, 7, 9, 15\}$

61. (i) 1, 6, 3, 2, 3, 6

(ii) 1, 3, 3, 2, 2, 2, $\{e, x_1, x_2\}, \{e, x_3\}, \{e, x_4\}, \{e, x_5\}$

(iii) (a) L is isomorphic to G;
Define $f: G \rightarrow L$ such that
 $f(e) = 0, f(a) = 1, f(a^2) = 2, f(a^3) = 3, f(a^4) = 4, f(a^5) = 5$
(b) M is isomorphic to G;
Define $f: G \rightarrow M$ such that
 $g(e) = 1, g(a) = 3, g(a^2) = 2, g(a^3) = 6, g(a^4) = 4, g(a^5) = 5$

62. (ii)

o	e	a	a ²	a ³	b	ab	a ² b	a ³ b
e	e	a	a ²	a ³	b	ab	a ² b	a ³ b
a	a	a ²	a ³	e	ab	a ² b	a ³ b	b
a ²	a ²	a ³	e	a	a ² b	a ³ b	b	ab
a ³	a ³	e	a	a ²	a ³ b	b	ab	a ² b
b	b	a ³ b	a ² b	ab	a ²	a	e	a ³
ab	ab	b	a ³ b	a ² b	a ³	a ²	a	e
a ² b	a ² b	ab	b	a ³ b	e	a ³	a ²	a
a ³ b	a ³ b	a ² b	ab	b	a	e	a ³	a ²

(iii) $\{e, a\}, \{e, a, a^2, a^3\}$

(iv) not isomorphic

63. (i)

*	e	a	b	c	ab	ac	bc	abc
e	e	a	b	c	ab	ac	bc	abc
a	a	e	ab	ac	b	c	abc	bc
b	b	ab	e	bc	a	abc	c	ac
c	c	ac	bc	e	abc	a	b	ab
ab	ab	b	a	abc	e	bc	ac	c
ac	ac	c	abc	a	bc	e	ab	b
bc	bc	abc	c	b	ac	ab	e	a
abc	abc	bc	ac	ab	c	b	a	e

(ii) $\{e, a\}, \{e, b\}, \{e, c\}, \{e, ab\}, \{e, ac\}, \{e, bc\}, \{e, abc\}, \{e, a, b, ab\}$

(iii) It is not isomorphic to G

64. (i)

*	I	P	Q	R	S	T
I	I	P	Q	R	S	T
P	P	I	T	S	R	Q
Q	Q	S	I	T	P	R
R	R	T	S	I	Q	P
S	S	Q	R	P	T	I
T	T	R	P	Q	I	S

(ii) $\{I, P\}, \{I, Q\}, \{I, R\}, \{I, S, T\}$

(iii) 2, 2, 2, 3, 3

(iv) not isomorphic

65. (iii) $\{e, a, a^2, a^3\}, \{e, b\}, \{e, ab\}, \{e, a^2b\}, \{e, a^3b\}$;

(iv) H and G are not isomorphic

66. (i) 1, 4, 2, 4, 4, 2, 4, 2;

(ii) $\{1, 2, 4, 8\}, \{1, 4, 7, 13\}, \{1, 4, 11, 14\}$;

(iii) not isomorphic

15. LINEAR SPACE

1. $A^{-1} = \frac{1}{6} \begin{pmatrix} -13 & 2 & 5 \\ 21 & -3 & -6 \\ -11 & 1 & 4 \end{pmatrix}$,

$$x = \frac{1}{6} \begin{pmatrix} 13 \\ -12 \\ 8 \end{pmatrix}$$

2. $\begin{pmatrix} 1 & -a & ac & -b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{pmatrix}$;

$$\begin{pmatrix} 3 & -3a & -1 & 3ac & -3b & c & +2 \\ 1 & & -a & & ac & -b & +5 \end{pmatrix}$$

18. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$,

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix},$$

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

21. (a) The point (bk, dk)

22. $\left\{ \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \right\}$ (or any two

linearly independent vectors of the form $\begin{pmatrix} 2\lambda - \mu \\ \lambda + 3\mu \\ \mu \end{pmatrix}$; $\left\{ \begin{pmatrix} 10 + 3\lambda \\ \lambda \\ -2 - \lambda \end{pmatrix} \right\}$; $\lambda \in \mathbb{R}$); This is not a subspace.

23. (b) 2, $\left\{ \begin{pmatrix} 3 \\ -2 \\ 9 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix} \right\}$ (or any two

linearly independent vectors of the form $\begin{pmatrix} 3\lambda \\ -2\lambda \\ 9\lambda + 2\mu \\ \mu \end{pmatrix}$)

24. (a) $x = 0, y = 3t, z = t; t \in \mathbb{R};$

(b) $x = -\frac{1}{2}, y = 3t - \frac{3}{2}, z = t;$
 $t \in \mathbb{R}$

25. (b)(i) No; (ii) Yes $\{(1, 1, 1)\}$; (iii) Yes $\{(1, 7, 0), (0, 0, 1)\}$; (iv) Yes $\{(0, 0, 1)\}$

26. (b) $2; \{(-3, 4, 1)\}$

27. $\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix}, \begin{pmatrix} \frac{4}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{5}{3} \end{pmatrix},$

$\begin{pmatrix} -\frac{4}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{5}{3} \end{pmatrix}$

29. (b) $1; x = 3 - 5\lambda, y = \lambda,$
 $z = 4\lambda - 2 (\lambda \in \mathbb{R})$

30. $\{(a, 2a, -a) : a \in \mathbb{R}\}, 1; 1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
 $\frac{1}{2} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}; -1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \frac{7}{2} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$
 $-\frac{3}{2} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

31. Basic for V is $\begin{pmatrix} -1 \\ 4 \\ -1 \\ 1 \end{pmatrix}, X = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} +$
 $\begin{pmatrix} -1 \\ 4 \\ -1 \\ 1 \end{pmatrix}$

32. (b) (i), (iii) not possible; (ii) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix};$

(c) (i) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\};$

(ii), (iii) not possible

33. (a) $\{(0, 1, 0, 2)\};$ (b) $\{(-14, 7, 0, 3), (0, 0, 1, 0)\};$ (c) $\{(5, -3, 1)\}$

34. (a) $P = \begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$

$Q = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix};$

(b) $P = \begin{pmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ -2 & \frac{3}{2} & -\frac{1}{2} \end{pmatrix},$

$Q = \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix},$

$A^{-1} = \begin{pmatrix} 6 & -4 & \frac{3}{2} \\ 3 & -\frac{5}{2} & 1 \\ -2 & \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

(c) $P = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix},$

$Q = \begin{pmatrix} 1 & \frac{1}{2} & -\frac{3}{2} \\ 0 & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

35. (b) $S : \{(1, 0, 1, 1), (1, 0, 0, 0)\},$
 $T : \{(1, -1, 0, 1), (0, 1, 1, 0)\},$

$S \cap T : \{(1, 0, 1, 1)\},$
 $S + T : \{(1, 0, 1, 1), (1, 0, 0, 0),$
 $(0, 1, 1, 0)\}$
 $U : \{(0, 1, 0, 0), (0, 0, 1, 0)\}$

36. Magnification

37. $a = -3; (a) x = -\frac{(9t+2)}{5}, y = t,$

$z = -\frac{(11t+8)}{5};$ (b) no solutions

38. $\begin{pmatrix} 11 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix},$

$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} -11 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right\}$

39. (b) $\begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix};$ (c) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

40. (a) $\{x^2, x\};$ (b) Not a subspace,
(c) $\{x^2 - 1, x - 1\},$ (d) $\{x^2 - 1\}$

41. (a) False; (b) True; (c) False;
(d) False; (e) True

42. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -4 \\ 1 \\ 1 \end{pmatrix} \right\},$

(a) $X = \begin{pmatrix} \lambda \\ 2\lambda - 4\mu \\ -2\lambda + \mu \end{pmatrix};$

(b) $X = \begin{pmatrix} 1 + \lambda \\ 1 + 2\lambda - 4\mu \\ -2\lambda + \mu \end{pmatrix};$

(c) No solution

43. $\begin{pmatrix} 11 & -12 & -7 \\ -8 & 9 & 5 \\ 7 & -8 & -4 \end{pmatrix};$

(a) $\begin{pmatrix} 11 & -8 & 7 \\ -12 & 9 & -8 \\ -7 & 5 & -4 \end{pmatrix};$

- (b) $\begin{pmatrix} 5\frac{1}{2} & -12 & -7 \\ -4 & 9 & 5 \\ 3\frac{1}{2} & -8 & -4 \end{pmatrix}$
- (c) $\begin{pmatrix} -2 & -4 & -1\frac{1}{2} \\ -1\frac{1}{2} & -2\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -2 & -1\frac{1}{2} \end{pmatrix}$
44. $x = \lambda c, y = \mu + \lambda d, A = \lambda M + \mu I;$
 (d) $\begin{pmatrix} 5 & 5 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} -2 & 5 \\ 2 & -5 \end{pmatrix}$
45. (a) $\lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, (b) No solution ;
 (c) $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$
46. Null space has basis $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$; range
 has basis $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$;
 Line $\underline{r} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$; (b) Point
 $\begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$; (c) Line $\underline{r} = \lambda \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$
47. (a) $\frac{1}{2}(e_1 + e_2)$; (b) $-e_1 + e_2 + e_3$;
 (c) $\frac{1}{2}e_3 + t(-e_1 + e_2 + e_3) (t \in \mathbb{R})$
48. $P = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 \\ -2 & -3 & 1 \end{pmatrix}, Q = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{5} & \frac{1}{5} & 0 \\ 11 & -2 & 5 \end{pmatrix}$
 $B^{-1} = \begin{pmatrix} -20 & 4 & -9 \\ -5 & 1 & -2 \\ 11 & -2 & 5 \end{pmatrix}$;
 $\alpha = 2, \beta = 3, \gamma = -1$
49. (a) 3; (b) $5x - 6y - 7z = 0$;
 dimension = 2; (c) $\frac{7}{9}$
50. (a) $a = -3$; (b) $a = 7$; $\begin{pmatrix} 19\lambda - 1 \\ 1 & 9\lambda \end{pmatrix}$;
 (c) $a \neq 7, -3$
51. (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, (b) $\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$;
 (c) $\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}, A^{-1} = \begin{pmatrix} -2 & -9 & -15 \\ -1 & -5 & -9 \\ 1 & 4 & 7 \end{pmatrix}$
52. $\left\{ \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ -6 \\ -4 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} -6t \\ t+1\frac{1}{2} \\ 4t \end{pmatrix} \right\}$
53. (b) P, R, S
54. $\begin{pmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}; -4\underline{p} - 4\underline{q} + 5\underline{r},$
 $-7\underline{p} - 6\underline{q} + 10\underline{r}; \underline{u} = \underline{r} - \underline{p}$
55. (a) $p = a + d, q = ad - bc$;
 (b) $\begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{pmatrix}$
56. (a) True; (b) False; (c) True;
 (d) True; (e) False
57. $\begin{pmatrix} 1 & 1 & -4 \\ -2 & -1 & 7 \\ 1 & -1 & -1 \end{pmatrix}; \begin{pmatrix} -8 \\ 13 \\ 0 \end{pmatrix}$; (b) (5, 0, -13)
58. $\left\{ \begin{pmatrix} -5 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -9 \\ 4 \\ 4 \\ 2 \end{pmatrix} \right\}; \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\};$
 $\begin{pmatrix} -2 \\ 0 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 1 \\ 3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -9 \\ 4 \\ 4 \\ 2 \end{pmatrix}$
59. Dimension = 4; $x^3 = (x+1)^3 - 3(x+1)^2 + 3(x+1) - 1$; $k = 8$; $\{x^2, 1\}$
60. (b) $3C, -2C$
61. $\alpha = 7x - 3y, \beta = y - 2x$; $\begin{pmatrix} -1 \\ 1 \end{pmatrix},$
 $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \begin{pmatrix} -10 & 7 & 8 \\ 3 & -2 & -2 \end{pmatrix}$
62. Dimension = 4; $x^3 = (x+1)^3 - 3(x+1)^2 + 3(x+1) - 1$; $k = 8$; $\{x^2, 1\}$
63. (a) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ k & 0 & 1 \end{pmatrix}$; (b) $p =$
 $\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{7}{2} & -\frac{1}{2} & 0 \\ 2 & -3 & 1 \end{pmatrix}$
 $x = 1, y = 2, s = 0, t = 1, X =$
 $\begin{pmatrix} 2 + \lambda \\ 1 - \lambda \\ \lambda \\ 0 \end{pmatrix}$
64. (a) $\{e^{2x}, e^x\}$; (d) $\{(1, 0, -1), (0, 1, -1)\}$; (e) $\{1, i\}$
65. $r(2, 3, -6) = -4$;
 $(5, 3, 12); \sqrt{21}$
66. (a) (1), 1; (b) $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, 2$; (c)
 $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -3 & 1 \end{pmatrix}, 3, \begin{pmatrix} 8 & -1 & -5 \\ 2 & 0 & -1 \\ 11 & -1 & -7 \end{pmatrix}$;
 (d) $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}, 2$

$$67. (a) 2; (b) \begin{pmatrix} 3 \\ -4 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \\ -3 \end{pmatrix}; (c) \begin{pmatrix} 2 \\ 4 \\ 3 \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 4 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$$68. \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 \\ -3 & 1 & 1 \end{pmatrix}; x_1 = 2\lambda - 5\mu + v,$$

$$x_2 = 2\mu + v, x_3 = \lambda, x_4 = \mu,$$

$$x_5 = v;$$

$$3; \{ (2, 0, 1, 0, 0), (-5, 2, 0, 1, 0), (1, 1, 0, 0, 1) \};$$

$$a = 2; x_1 = 2\lambda - 5\mu + v,$$

$$x_2 = 2 + 2\mu + v, x_3 = \lambda,$$

$$x_4 = \mu, x_5 = v$$

$$69. (a) 2, 2; (b) 5x - 3y - 2z = 0;$$

$$(c) (3, -3, -2); (d) \{(0, 2, -3)\}$$

$$70. \frac{1}{9} \begin{pmatrix} -2 & 4 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & -2 \end{pmatrix}; (a) \begin{pmatrix} 2\lambda \\ \lambda \\ -4\lambda \end{pmatrix},$$

$$\text{dimension } 1; (b) \frac{1}{3} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \frac{10}{3} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$+ \frac{1}{3} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}; 4 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$71. \{(1, 1, 0), (0, 1, 1)\}; \{(2, -2, 1)\};$$

$$(i) r = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};$$

$$(ii) r = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$72. A^{-1} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -4 & 0 & 3 \end{pmatrix}, B^{-1} = \begin{pmatrix} 12 & -6 & -24 \\ -6 & 12 & 0 \\ -24 & 0 & 72 \end{pmatrix}$$

$$73. (a) a = 7; (b) a = 5; x = 24t - 4,$$

$$y = 13t - 3, z = t$$

$$(c) a \neq -5$$

$$74. (a) 4a - 5b - 2c = 0$$

$$75. (b) \alpha = 3y - 4x, \beta = 3x - 2y;$$

$$\begin{pmatrix} 11 \\ 1 \end{pmatrix}, \begin{pmatrix} 15 \\ 2 \end{pmatrix}; \begin{pmatrix} -41 & -54 \\ 31 & 41 \end{pmatrix}$$

$$76. 2; (a) x = 3t, y = 11t, z = 10t,$$

$$t \in \mathbb{R}; (b) x = 3t, y = 11t - \frac{5}{3},$$

$$z = 10t - \frac{4}{3}, t \in \mathbb{R};$$

$$\{ \begin{pmatrix} 4 \\ 3 \\ 6 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -8 \end{pmatrix} \}; (c) \text{No Solution}$$

$$77. (a) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} \lambda + \mu - \frac{1}{2} & \frac{1}{2} \\ \lambda & \mu \end{pmatrix};$$

$$(b) \begin{pmatrix} x + y \\ 2x - 3y \end{pmatrix}$$

$$78. (a) 2; \begin{pmatrix} 1 \\ 2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -3 \\ 5 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 4 \\ 0 \\ 1 \end{pmatrix};$$

$$(b) (i) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ -4 & 2 & 5 \end{pmatrix};$$

$$79. (a) (-7 \ 6 \ 12), (b) x = \lambda, y = 7\lambda - 10,$$

$$z = 4 - 6\lambda, t = 11 - 12\lambda$$

$$80. (a) (2, -1, 3);$$

$$(b) 2, x^3 + 3x^2 - 3, 2x^2 + x - 2$$

$$81. \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix},$$

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

$$82. 3, 10; \begin{pmatrix} 1+3\lambda \\ 1-2\lambda \\ \lambda \end{pmatrix}$$

$$83. \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \text{dimension}; \text{coefficients}$$

$$1, -2, 4, -1 \text{ and } 1, -1, 2, 0$$

$$84. 2, \{ \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \}; \{ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \}; \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

$$85. \begin{pmatrix} 6 & -1 & -4 \\ -7 & 3 & 5 \\ 15 & -3 & -10 \end{pmatrix}; (a) (26 \ -4 \ -17);$$

$$(b) x = 7 - 26t, y = 2 + 4t, z = -4 + 17t,$$

$$86. (a) (1); (b) \begin{pmatrix} 1 & 1 \\ 0 & -\frac{1}{2} \end{pmatrix};$$

$$(c) \begin{pmatrix} 1 & 1 & -2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix},$$

$$A^{-1} = \begin{pmatrix} -3 & -2 & 1 \\ -2 & 0 & 1 \\ 3 & 1 & -1 \end{pmatrix};$$

$$(d) \begin{pmatrix} 1 & 1 & -2\lambda \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$87. (a) 1; (b) 10, -1, -7; (c) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, 2x - y + 8z = 1$$

$$88. P = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix},$$

$$Q = \begin{pmatrix} 1 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \end{pmatrix},$$

$$A^{-1} = \begin{pmatrix} -4 & 3 & 4 \\ 12 & -9 & -11 \\ -1 & 1 & 1 \end{pmatrix}; B \text{ has no}$$

$$\text{inverse}; (a) \begin{pmatrix} 7 \\ -16 \\ 2 \end{pmatrix}; (b) \begin{pmatrix} 3t + 1 \\ -9t + 2 \\ t \end{pmatrix}$$

$$89. \alpha = x - 2y + z, \beta = y - 2z, \gamma = z$$

$$L \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix}, L \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ 5 \end{pmatrix};$$

$$\alpha = 16, \beta = -10, \gamma = 3;$$

$$L \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 22 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 15 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix};$$

$$L \begin{pmatrix} 4 \\ 6 \end{pmatrix} = 32 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 20 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix};$$

$$L \begin{pmatrix} 5 \\ 7 \end{pmatrix} = 38 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 25 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 8 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix};$$

$$L \begin{pmatrix} 9 \\ 13 \end{pmatrix} = 70 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 45 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 14 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix};$$

$$90. (i) \text{Rank of } A = 2, \text{ Dimension of}$$

$$K = 2; (ii) e_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ -1 \end{pmatrix};$$

$$(iii) e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, e_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$91. (ii) 2; (iii) \text{not linearly independent}; (iv) \text{given vector does belong to the range of } T$$

$$92. (i) 3; (ii) \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 7 \\ 4 \end{pmatrix} \right\};$$

$$(iii) W \text{ not a vector space since it does not contain the zero vector}$$

$$93. (i) 2;$$

$$(ii) \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}, r = -150.5, s = 151.5$$

$$94. (ii) \text{Any two columns of } M \text{ may be selected as a basis for } R_T;$$

$$\begin{pmatrix} -4 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

$$95. (ii) e_1 = \begin{pmatrix} 7 \\ -6 \\ -5 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 7 \\ -11 \\ 0 \\ 0 \end{pmatrix}$$

$$96. (ii) e_1 = \begin{pmatrix} 1 \\ 0 \\ a \\ -a \end{pmatrix}, (iii) e_2 = \begin{pmatrix} 1 \\ -a \\ 0 \\ 0 \end{pmatrix}$$

$$97. (i) \text{Rank of } M \text{ is } 2;$$

$$(ii) p = -3, q = -1, r = 4, s = 0$$

$$(iv) X = \begin{pmatrix} 6 \\ -5 \\ 10 \\ 0 \end{pmatrix}$$

$$98. (ii) \left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} \right\}; (iv) \text{No value}$$

$$99. (i) \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} \right\}; (ii) \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\};$$

$$(iii) \left\{ \begin{pmatrix} 7 \\ 3 \\ -24 \\ 15 \end{pmatrix} \right\}$$

16. EIGENVALUES AND EIGENVECTORS

$$1. P = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}, Q = \begin{pmatrix} 5 & 0 \\ 4 & 3 \end{pmatrix},$$

$$R = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

2. (a) λ^2 ; (b) $\lambda + 3$; (c) λ^{-1}
 3. -
 9. $\lambda_1 = -1$, $\lambda_2 = 2$; $e_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$,
 $e_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $\begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}$;
 $\begin{pmatrix} -3068 & 4092 \\ -3069 & 4093 \end{pmatrix}$
 10. $P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix}$,
 $B^n = \begin{pmatrix} 2^{n+1} & 1 & -2^{n+1} + 2 & 0 \\ 2^n & -1 & -2^n + 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
 11. -
 12. $P = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$,
 $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -27 \end{pmatrix}$
 13. Eigenvalues are 1, -1, -2; corresponding eigenvectors are
 $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ respectively.
 $P = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & -1 & \frac{1}{3} \\ -\frac{11}{3} & 4 & -\frac{14}{3} \end{pmatrix}$,
 $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 16 \end{pmatrix}$
 14. 1, -1, 2 and $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ respectively;
 (i) 11, 9, 12 and $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
 respectively;
 (ii) $P = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & -1 & -1 \end{pmatrix}$
 15. C: -1, 3, 5 and $\begin{pmatrix} -17 \\ 6 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
 respectively
 D: -3, -1 and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ respectively;
 $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, -9

16. -1, 1, 2 and $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix}$ respectively;
 $Q = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -4 & -4 \\ -1 & -1 & -4 \end{pmatrix}$; $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
 -40
 17. $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
 18. 1, 3, -4 and $\begin{pmatrix} 11 \\ -5 \\ -6 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -7 \\ -5 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ respectively;
 $Q = \begin{pmatrix} 11 & 5 & 1 \\ -5 & -7 & 0 \\ -6 & -5 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$
 19. 4, 5, 7 and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ respectively;
 $Q = \begin{pmatrix} 2 & -3 & -1 \\ -3 & -2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$
 20. 1, 2, -3; $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 8 \\ -5 \end{pmatrix}$;
 $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{2}$; $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 8 \\ -5 \end{pmatrix}$
 21. $Q = \begin{pmatrix} 13 & 24 & 2 \\ 1 & -5 & 0 \\ 6 & 12 & 1 \end{pmatrix}$, $D = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

17. STATICS

1. -
 2. -
 5. $\frac{5}{6\sqrt{3}} \approx 0.481$, 1.58a
 9. -
 10. (a) $\cos \theta = \frac{4}{5}$; (b) no equilibrium position; B will move to C
 11. $W(1 - 2 \sin \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta)^{\frac{1}{2}}$
 12. -
 13. 48°
 14. (a) $P < 2\mu W$; (b) $P < \mu W$; (c) $P < Wa/(a+h)$; (d) $P < Wa/2h$
 15. $\frac{3}{2} \rho l$; $\frac{5\rho g l^2}{6a}$; $\frac{\rho g l |18a - 5l|}{12a}$; $\frac{5}{18}$
 16. $12a$; $\frac{1}{8}$