Example 1

Find the period of the SHM defined by the equation $\ddot{x} = -25x$.

Example 2

Find the maximum speed and the maximum acceleration of a particle moving with SHM of period $\frac{\pi}{4}$ s and amplitude 25 cm.

Example 3

A particle moves with SHM about a mean position O. When the particle is 50 cm from O its speed is $3{\cdot}6\,\text{m s}^{-1}$ and when it is 120 cm from O its speed is $1{\cdot}5\,\text{m s}^{-1}$. Find the amplitude and the periodic time of the motion.

Example 4

A particle is projected from a point O at time $t = 0$ and performs SHM with O as the centre of oscillation. The motion is of amplitude 20 cm and time period 4 s. Find:
(a) the speed of projection
(b) the speed of the particle when $t = 1{\cdot}5$ s
(c) the value of t when the particle is first at a point 10 cm from O.

Example 5

A particle performs SHM of period 6 s and amplitude 4 cm about a centre O. Find the time it takes the particle to travel to a point P, a distance of 2 cm from O.

Exercise 17A

1. Find the periodic time of the simple harmonic motion governed by each of the following equations. (All the equations use SI units.)
   (a) $\ddot{x} = -x$ (b) $\ddot{x} = -4x$ (c) $\ddot{x} = -9x$.
2. A particle moves with SHM about a mean position O with a periodic time of $\frac{\pi}{2}$ s.
   Find the magnitude of the acceleration of the particle when 1 metre from O.
3. A particle moves with SHM about a mean position O. When the particle is 25 cm from O, it is accelerating at $1\,\text{m s}^{-2}$ towards O.
   Find the periodic time of the motion and the magnitude of the acceleration of the particle when 20 cm from O.
4. A particle moves with SHM of time period $\frac{\pi}{2}$ s and has a maximum speed of $3\,\text{m s}^{-1}$.
   Find the maximum acceleration experienced by the particle.
5. A particle moves with SHM of time period $\frac{\pi}{2}$ s and amplitude 2 m.
   Find the maximum speed of the particle.
6. A particle moves with SHM about a mean position O. The amplitude of the motion is 5 m and the period is 8π s.
   Find the maximum speed of the particle and its speed when 3 m from O.
7. A body of mass 500 g moves horizontally with SHM of amplitude 1 m and time period $\frac{\pi}{2}$ s.
   Find the magnitude of the greatest horizontal force experienced by the body during the motion.
8. A body of mass 100 g moves horizontally with SHM about a mean position O. When the body is 50 cm from O the horizontal force on the body is of magnitude 5 N.
   Find the time period of the motion.
9. A body of mass 5 kg is placed on a rough horizontal surface, coefficient of friction $\frac{11}{49}$.
   State whether or not the body will slide across the surface when the surface is moved horizontally with simple harmonic motion of amplitude 55 cm and:
   (a) a time period of $\frac{\pi}{2}$ s
   (b) a time period of $\frac{3\pi}{2}$ s
   (c) a frequency of 15 oscillations per minute
   (d) a frequency of 20 oscillations per minute.
10. A horizontal platform is made to move vertically up and down with simple harmonic motion according to the relationship $\frac{d^2x}{dt^2} = -49x$, where x m is the vertical displacement of the platform from

its mean position at time t s. Show that any mass placed on the platform will leave the platform if the amplitude of the motion is greater than 20 cm.

11. A particle moves with SHM about a mean position O. The amplitude of the motion is 65 cm and the time period is $\frac{\pi}{4}$ s.
Find how far the particle is from O when its speed is $2\,\text{m s}^{-1}$.

12. A particle moves with SHM about a mean position O. The particle has zero velocity at a point which is 50 cm from O and a speed of $3\,\text{m s}^{-1}$ at O.
Find:
(a) the maximum speed of the particle
(b) the amplitude of the motion
(c) the periodic time of the motion.

13. A particle moves with SHM about a mean position O. When the particle is 60 cm from O, its speed is $1{\cdot}6\,\text{m s}^{-1}$, and when it is 80 cm from O, its speed is $1{\cdot}2\,\text{m s}^{-1}$.
Find the amplitude and period of the motion.

14. The effect of the waves on an empty oil drum floating in the sea is to make it bob up and down with SHM.
If the drum encounters 20 waves every minute and for each wave the vertical distance from peak to trough is 80 cm, find the amplitude and period of the motion and the maximum speed of the drum.

15. A particle moves with SHM about a mean position O. When passing through two points which are 2 m and 2·4 m from O the particle has speeds of $3\,\text{m s}^{-1}$ and $1{\cdot}4\,\text{m s}^{-1}$ respectively.
Find the amplitude of the motion and the greatest speed attained by the particle.

16. A particle moves with SHM about a mean position O. The particle is initially projected from O with speed $\frac{\pi}{6}\,\text{m s}^{-1}$ and just reaches a point A, 2 m from O.
Find how far the particle is from O three seconds after projection.
How many seconds after projection is the particle a distance of 1 m from O:
(a) for the first time
(b) for the second time
(c) for the third time?

17. A particle is released from rest at a point A, 1 m from a second point O. The particle accelerates towards O and moves with simple harmonic motion of time period 12 s and O as the centre of oscillation.
Find how far the particle is from O one second after release.
How many seconds after release is the particle at the mid-point of OA:
(a) for the first time
(b) for the second time?

18. A particle is projected from a point A at time $t = 0$ and performs SHM with A as the centre of oscillation. The amplitude of the motion is 50 cm and the periodic time is 3 s.
Find:
(a) the speed of projection
(b) the speed of the particle when $t = 1$ s
(c) the speed of the particle when $t = 2$ s
(d) the distance of the particle from A when $t = 2$ s.

19. A particle is released from rest at a point A at time $t = 0$ and performs SHM about a mean position B. The particle just returns to A during each oscillation and $\text{AB} = 2\sqrt{2}\,\text{m}$.
If the particle passes through B with speed $\pi\sqrt{2}\,\text{m s}^{-1}$, find the value of t when the particle is first travelling with a speed of $\pi\,\text{m s}^{-1}$. How far from B is the particle then?

20. The head of a piston moves with SHM of amplitude $\frac{\sqrt{3}}{10}$ m about a mean position O.
How far from O is the head of the piston when it is travelling with a speed equal to half of its maximum speed?

21. A particle is fastened to the mid-point of a stretched spring lying on a smooth horizontal surface. The particle is set in motion so that it moves with SHM about a mean position O. If one metre is the greatest distance the particle is from O during the motion, find how far from O the particle is when it is travelling with a speed equal to four-fifths of its greatest speed.

particle performs SHM of period 3 s and amplitude 6 cm about a centre O.
Find the time it takes the particle to travel from O to a point P, a distance of 3 cm from O.

23. A particle performs SHM of period 4 s and amplitude 2 cm about a centre O.
Find the time it takes the particle to travel from O to a point P, a distance of $\sqrt{2}$ cm from O.

24. A particle performs SHM of period 10 s and amplitude 8 cm about a centre O. After passing through O the particle moves through a point A which is 2 cm from O to a point B which is 6 cm from O.
Find the time taken to travel from A to B.

25. A particle performs SHM of period 3 s and amplitude 3 cm about a centre O. After passing through O the particle moves through a point A which is 1 cm from O to a point B which is 2 cm from O.
Find the time taken to travel from A to B.

26. A particle performs SHM of period 4·5 s and amplitude 6 cm about a centre O. The particle passes through a point P which is 3 cm from O, moving away from O.
Find the time which elapses before the particle next passes through P.

27. A particle performs SHM of period 2 s and amplitude 4 cm about a centre O. The particle passes through a point P which is 1 cm from O, moving away from O.
Find the time which elapses before the particle next passes through P.

28. The points A, O, B, C lie in that order on a straight line with AO = OC = 4 cm, and OB = 2 cm. A particle performs SHM of period 6 s and amplitude 4 cm between A and C.
Find the time taken for the particle to travel from A to B.

29. The points A, O, B, C lie in that order on a straight line with AO = OC = 6 cm, and OB = 5 cm. A particle performs SHM of period 3 s and amplitude 6 cm between A and C.
Find the time taken for the particle to travel from A to B.

Exercise 17A *page 439*

1. (a) 2π s (b) π s (c) $\frac{2\pi}{3}$ s **2.** 16 m s^{-2} **3.** π s, 0·8 m s^{-2}

4. 12 m s^{-2} **5.** 8 m s^{-1} **6.** 1·25 m s^{-1}, 1 m s^{-1} **7.** 8 N

8. $\frac{\pi}{5}$ s **9.** (a) Yes (b) No (c) No (d) Yes

11. 60 cm **12.** (a) 3 m s^{-1} (b) 50 cm (c) $\frac{\pi}{3}$ s **13.** 1 m, π s

14. 40 cm, 3 s, 0·84 m s^{-1} **15.** 2·5 m, 5 m s^{-1} **16.** $\sqrt{2}$ m (a) 2 (b) 10 (c) 14

17. $\frac{\sqrt{3}}{2}$ m (a) 2 (b) 10 **18.** (a) $\frac{1}{3}\pi$ m s^{-1} (b) $\frac{1}{6}\pi$ m s^{-1} (c) $\frac{1}{6}\pi$ m s^{-1} (d) $\frac{\sqrt{3}}{4}$ m

19. 0·5 s, 2 m **20.** 15 cm **21.** 60 cm **22.** 0·25 s **23.** 0·5 s

24. 0·948 s **25.** 0·186 s **26.** 1·5 s **27.** 0·839 s **28.** 2 s

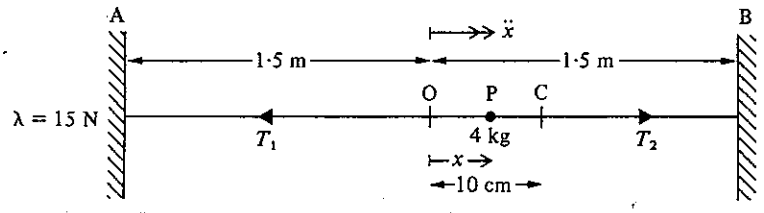
29. 1·22 s

Example 6

One end of a light elastic spring of natural length 2 m and modulus 10 N is fixed to a point A on a smooth horizontal surface. A body of mass 200 g is attached to the other end of the spring and is held at rest at a point B on the surface, causing the spring to be extended by 30 cm. Show that, when released, the body will move with SHM and find the amplitude of the motion.

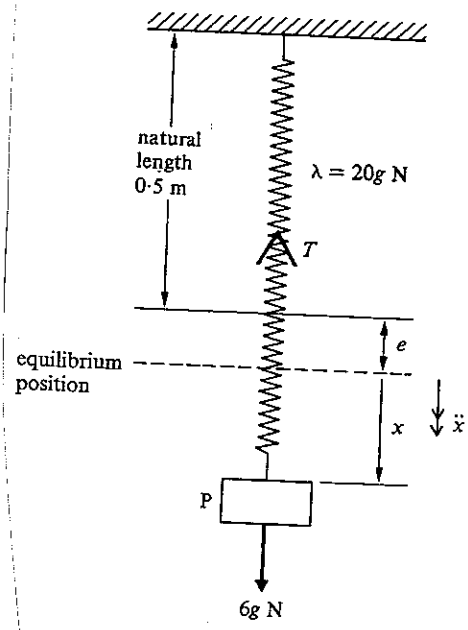
Example 7

A light elastic string of natural length 2·4 m and modulus 15 N is stretched between two points A and B, 3 m apart on a smooth horizontal surface. A body of mass 4 kg attached to the mid-point of the string is pulled 10 cm towards B and then released. Show that the subsequent motion is simple harmonic and find the speed of the body when it is 158 cm from A.



Example 8

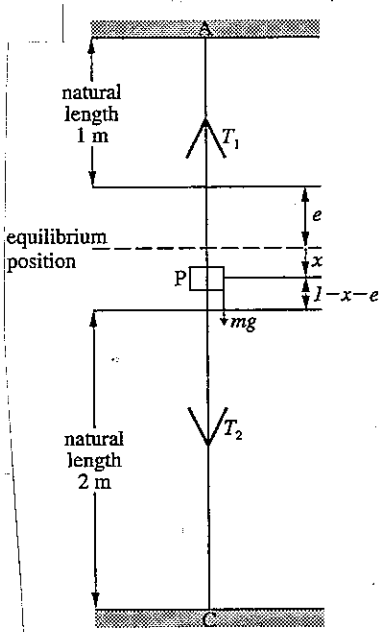
A light elastic spring, of natural length 50 cm and modulus $20g$ N, hangs vertically with its upper end fixed and a body of mass 6 kg attached to its lower end. The body initially rests in equilibrium and is then pulled down a distance of 25 cm and released.
Show that the ensuing motion will be simple harmonic, and find the period of the motion and the maximum speed of the body.
Would the answers have been the same had an elastic string been used in place of the spring?



Example 9

A and C are two fixed points 4 m apart with A vertically above C. A body of mass m kg rests between A and C, held in position by two vertical strings. The upper string has modulus $6mg$ N and natural length 1 m. The lower string has modulus $8mg$ N and natural length 2 m. The body is displaced 20 cm below its equilibrium position and released from rest.

(a) Show that the ensuing motion will be simple harmonic, and find the period of the motion.
(b) Would the answers have been different had the body been displaced by 60 cm?



Exercise 17B

1. A light spring is of natural length 1 m and modulus 2 N. One end of the spring is attached to a fixed point A on a smooth horizontal surface, and to the other end is attached a body of mass 500 g. The body is held at rest on the surface at a distance of 1·25 m from A. Show that on release the body will move with SHM and find the amplitude and time period of the motion.

2. Two points A and B are 1 m apart on a smooth horizontal table. A light spring, of natural length 75 cm and modulus 54 N, has one end fastened to the table at A and the other end fastened to a body of mass 8 kg which is held at rest at B.
Show that when the body is released it moves with SHM and find the maximum speed of the body during the motion.

3. A body of mass 2 kg is fixed to the mid-point of a light elastic string of natural length 1 m and modulus 18 N. The ends of the string are attached to two points A and B 2 m apart on a smooth horizontal surface. The body is pulled a distance y towards A ($y < 50$ cm) and released.
Show that the subsequent motion is simple harmonic and find the time period of the motion.
If the maximum speed of the body is $1{\cdot}5\,\mathrm{m\,s^{-1}}$, find the value of y.

4. A light elastic string of natural length 1·5 m and modulus 12 N is stretched between two points A and B 2 m apart on a smooth horizontal surface. A body of mass 2 kg is attached to the mid-point of the string, pulled 20 cm towards A and released.
Show that the subsequent motion is simple harmonic and find the speed of the body when 88 cm from A.

5. A and B are two fixed points on a smooth horizontal surface with AB = 2 m. A body of mass 4 kg lies on the line AB at a point P and is in equilibrium with a light elastic string of natural length 75 cm and modulus 18 N connecting it to A and a light elastic string of natural length 50 cm and modulus 6 N connecting it to B.
Show that P is midway between A and B.
If the body is then pulled 20 cm towards A and released, show that the subsequent motion is simple harmonic and find the maximum speed of the body during the motion.

6. A light elastic string is of natural length 60 cm and modulus $3mg$ N. The string hangs vertically with its top end fixed and a body of mass m kg fastened to the other end.
Find the extension in the string when the body hangs in equilibrium.
If the body is then pulled vertically downwards a distance of 10 cm and released, show that the ensuing motion will be simple harmonic, and find the time period of the motion and the maximum speed of the body.

7. A body of mass 500 g is attached to end B of a light elastic string AB of natural length 50 cm. The system rests in equilibrium with the string vertical and end A fixed. The body is then pulled vertically downwards a small distance and released.
If the ensuing motion is simple harmonic of time period $\frac{1}{5}\pi$ s, find the modulus of the string.

8. A light spring hangs vertically with its top end fixed and a body of mass m kg attached to the other end. The spring is of natural length 1 metre and modulus $5mg$ N and initially the system rests in equilibrium. The body is then pulled vertically downwards a distance of 30 cm and released.
Show that the subsequent motion is simple harmonic and find the greatest acceleration experienced by the body. Would the motion have been simple harmonic if an elastic string had been used in place of the spring?

9. A light elastic string of natural length 20 cm and modulus 40 N has one end attached to a fixed point A on a smooth horizontal surface and a body of mass 2 kg attached to the other end. The body is held on the surface at a point which is 40 cm from A, and released.
Show that the subsequent motion will be periodic and find the time period of the motion and the speed of the body as it passes through A.

10. A light elastic string of natural length 50 cm hangs vertically with its top end fixed and a body of mass 2 kg attached to the other end. With the body hanging in equilibrium, the string has a total length of 70 cm.
Find the modulus of the string.
If the body is then pulled vertically downwards a distance of 10 cm and released, show that the subsequent motion is simple harmonic and find the speed of the particle when it is 2 cm above the point of release.

A light spring of natural length 40 cm and modulus $2g$ N hangs vertically with its upper end fixed and a particle attached to its other end. When the particle hangs in equilibrium the extension of the string is 5 cm. The spring is now replaced by a different spring of natural length 50 cm. The system is again allowed to settle in a position of equilibrium and then the particle is pulled vertically downwards a short distance and released. If the subsequent motion is simple harmonic with time period $\frac{1}{10}\pi$ s, find the mass of the particle and the modulus of this second spring.

12. A and B are two points 25 cm apart on a smooth horizontal surface. A particle of mass 500 g lies at A and is connected to B by a light spring of natural length 25 cm and modulus 50 N.
If the particle is projected directly towards B with speed $4\,\text{m s}^{-1}$, show that the ensuing motion will be simple harmonic and find the time period and amplitude.

13. A light spring of natural length 50 cm and modulus 147 N hangs vertically with its upper end fixed and a body of mass 1·5 kg attached to the lower end. With the system resting in equilibrium, the body is projected vertically downwards with a speed of $1{\cdot}4\,\text{m s}^{-1}$.
Show that the resulting motion will be simple harmonic and find the amplitude of the motion.

14. A light elastic string of natural length 1 m and modulus $2mg$ N hangs vertically with its upper end fixed and a body of mass m kg attached to its other end.
Find the total length of the string when the mass hangs in equilibrium.
The body is then pulled vertically downwards a distance d metres and released. Show that the body will move with SHM provided $d \leqslant 0{\cdot}5$.

15. A and C are two fixed points 5 m apart with A vertically above C. A body of mass m kg rests between A and C, held in position by two identical vertical strings, each of modulus $4mg$ N and natural length 2 m, one string linking the body to A and the other linking it to C. The body is displaced 10 cm below its equilibrium position and released from rest. Show that the ensuing motion will be simple harmonic, and find the period of the motion.

16. A and C are two fixed points 6 m apart with A vertically above C. A body of mass m kg rests between A and C, held in position by two identical vertical strings, each of modulus $0{\cdot}5mg$ N and natural length 1 m, one string linking the body to A and the other linking it to C. The body is displaced 50 cm below its equilibrium position and released from rest.
Show that the ensuing motion will be simple harmonic, and find the period of the motion.

17. A body of mass 2 kg is attached to the midpoint of a light elastic string of modulus $20g$ N and natural length 2 m. One end of the string is attached to a fixed point P and the other end of the string is attached to a fixed point Q, where P is 3 m vertically above Q. The body rests in equilibrium at a point O. Find the distance PO.
The body is now pulled down to a point R which is a distance of 10 cm below O, and released from rest.
Find the time taken to travel from R to O.

18. A and C are two fixed points 5 m apart with A vertically above C. A body of mass m kg rests in equilibrium between A and C, held in position by two vertical strings. The upper string has modulus $3mg$ N and natural length 1 m. The lower string has modulus $4mg$ N and natural length 2 m. The body is displaced 30 cm below its equilibrium position and released from rest.
Show that the ensuing motion will be simple harmonic, and find the period of the motion.

19. A and C are two fixed points 10 m apart with A vertically above C. A body of mass m kg rests in equilibrium between A and C, held in position by two vertical strings. The upper string has modulus $6mg$ N and natural length 3 m. The lower string has modulus $12mg$ N and natural length 4 m. The body is pulled down a distance of 40 cm and released from rest.
Find the time that elapses until the body is next at rest.

Exercise 17B *page 446*

1. 25 cm, π s
2. $0{\cdot}75\,\text{m s}^{-1}$
3. $\frac{\pi}{3}$ s, 25 cm
4. $0{\cdot}64\,\text{m s}^{-1}$
5. $0{\cdot}6\,\text{m s}^{-1}$
6. 20 cm, $\frac{2\pi}{7}$ s, $0{\cdot}7\,\text{m s}^{-1}$
7. 25 N
8. $14{\cdot}7\,\text{m s}^{-2}$, No
9. $\frac{1}{5}(2+\pi)$ s, $2\,\text{m s}^{-1}$
10. 49 N, $0{\cdot}42\,\text{m s}^{-1}$
11. 250 g, 50 N
12. $\frac{\pi}{10}$ s, 20 cm
13. 10 cm
14. 1·5 m
15. $\frac{\pi}{\sqrt{g}}$ s
16. $\frac{2\pi}{\sqrt{g}}$ s
17. 1·55 m, $\frac{\pi}{28}$ s
18. $\frac{2\pi}{7}$ s
19. $\frac{\pi}{7}$ s