



**CAMBRIDGE A LEVEL PROGRAMME  
SEMESTER ONE EXAMINATION JUNE 2011  
(March 2011 Intake)**

**Wednesday**

**8 June 2011**

**12.30 pm – 2.30 pm**

**FURTHER MATHEMATICS**

**9231**

**PAPER 1 & 2**

**2 hours**

Additional materials: Answer Booklet/Paper  
List of formulae (MF 10)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your name and class on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **50**.

This document consists of 4 printed pages.

**Section A: Pure Mathematics (50%)**

1 Prove that  $89^{2n+1} + 316^n$  is divisible by 45 for every positive integer  $n$ . [4]

2 Find the sum  $11^2 + 14^2 + 17^2 + \cdots + (3n+8)^2$ . [2]

Hence find  $\sum_{r=4n+1}^{6n} (3r+8)^2$  and the minimum value of  $n$  such that

$$\sum_{r=4n+1}^{6n} (3r+8)^2 > 456n^3 + 570n^2 + 1035. \quad [3]$$

3 The curve  $C$  has equation  $y = 7 + \frac{9}{4x+3} + \frac{5}{x+6}$ .

(i) Find the equations of the asymptotes of  $C$ . [3]

(ii) Sketch  $C$ , indicating any intersection points with the coordinate axes.

[5]

4 If the equation  $9x^4 - 2x^3 + 5x^2 - 8x + 1 = 0$  has roots  $\alpha, \beta, \gamma, \delta$ , find the equation having roots  $\alpha^2, \beta^2, \gamma^2, \delta^2$ . [5]

Find also  $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$  and  $\alpha^{-2} + \beta^{-2} + \gamma^{-2} + \delta^{-2}$ . [3]

Students are required to answer questions in Section B using a fresh sheet of answer paper. Answers for Section A and Section B need to be handed in separately.

### **Section B: Mechanics (50%)**

- 1 A snooker ball of mass  $m$  is moving with constant speed  $U$  in a straight line on a smooth horizontal table. It moves towards a straight smooth cushion, in a direction making an angle  $\theta$  with the cushion. The coefficient of restitution between the ball and the cushion is  $e$ . Find the components, parallel and perpendicular to the cushion, of the velocity of the ball as it leaves the cushion. After colliding with the cushion it rebounds in a direction making an angle  $\phi$  with the cushion. Show that

$$\tan \phi = e \tan \theta. \quad [3]$$

Show that the kinetic energy lost in the impact is

$$\frac{1}{2} m U^2 \sin^2 \theta (1 - e^2). \quad [3]$$

- 2 A particle  $A$  of mass  $2m$  is moving with speed  $2u$  on a smooth horizontal table. The particle collides directly with a particle  $B$  of mass  $4m$  moving with speed  $u$  in the same direction as  $A$ . The coefficient of restitution between  $A$  and  $B$  is  $\frac{1}{2}$

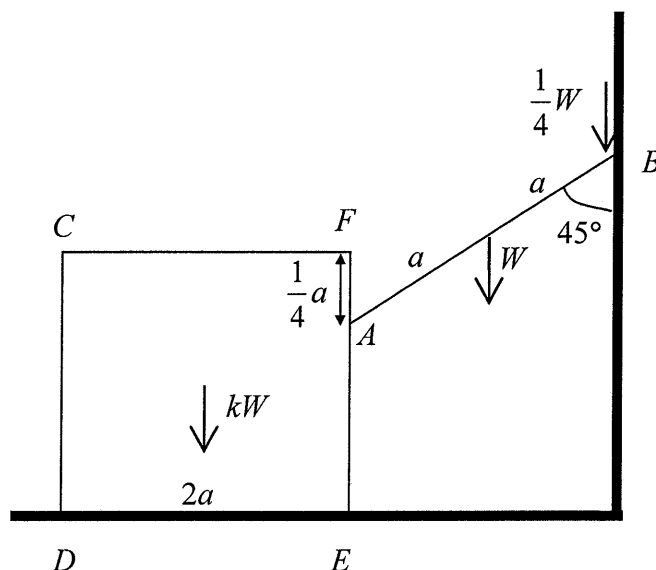
(i) Show that the speed of  $B$  after the collision is  $\frac{3}{2}u$ . [3]

(ii) Find the speed of  $A$  after the collision. [1]

Subsequently  $B$  collides directly with a particle  $C$  of mass  $m$  which is at rest on the table. The coefficient of restitution between  $B$  and  $C$  is  $e$ . Given that there are no further collisions,

(iii) find the range of possible values for  $e$ . [4]

3



The diagram shows a central cross-section  $CDEF$  of a uniform solid cube of weight  $kW$  with edges of length  $2a$ . The cube rests on a rough horizontal floor. One of the vertical faces of the cube is parallel to a smooth vertical wall. A uniform ladder, of length  $2a$  and weight  $W$ , is represented by  $AB$ . The ladder rests in equilibrium with  $A$  in contact with a rough vertical surface of the cube and  $B$  in contact with the wall and makes an angle of  $45^\circ$  with the wall. The distance  $AF$  is  $\frac{1}{4}a$  and the vertical plane containing  $AB$  is perpendicular to the wall. The coefficients of friction between the ladder and the cube is equal to  $\mu$ . A vertical force of  $\frac{1}{4}W$  is applied at the top of the ladder. Show that  $\mu \geq \frac{5}{3}$ .

[5]

Given that the coefficient of friction between the cube and the floor is equal  $\frac{2}{5}$ , find the largest value of  $k$  for which the cube does not slide.

[3]

Show that the cube will not turn about the edge  $D$  for any value of  $k$ .

[3]