Revision

P103 Mathematical Induction

CIE CAL FM P1 2006-06 Q3

Prove by induction, or otherwise, that

$$23^{2n} + 31^{2n} + 46$$

is divisible by 48, for all integers $n \geqslant 0$.

[6]

CIE CAL FM P1 2004-06 Q4

It is given that $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\dfrac{\ln x}{x}\right) = \dfrac{a_n \ln x + b_n}{x^{n+1}}$, where a_n and b_n depend only on n.

(i) Find a_1, a_2 and a_3. [3]

(ii) Use mathematical induction to establish a formula for a_n. [5]

CIE CAL FM P13 2010-06 Q3

The sequence $x_1, x_2, x_3, \ldots$ is such that $x_1 = 3$ and

$$x_{n+1} = \frac{2x_n^2 + 4x_n - 2}{2x_n + 3}$$

for $n = 1, 2, 3, \ldots$. Prove by induction that $x_n > 2$ for all n. [6]

CIE CAL FM P1 2008-11 Q9

Use induction to prove that

$$\sum_{n=1}^{N} \frac{4n+1}{n(n+1)(2n-1)(2n+1)} = 1 - \frac{1}{(N+1)(2N+1)}.$$

[6]

Show that

$$\sum_{n=N+1}^{2N} \frac{4n+1}{n(n+1)(2n-1)(2n+1)} < \frac{3}{8N^2}.$$

[4]

ASSIGNMENT

Attempt all the questions in the following slides.

Assignment is to be submitted within **one week** after instruction!!!

Please write your name and your student's id in the script upon submission.

CIE CAL FM P11 2011-06 Q4

It is given that $f(n) = 3^{3n} + 6^{n-1}$.

(i) Show that $f(n+1) + f(n) = 28(3^{3n}) + 7(6^{n-1})$. [2]

(ii) Hence, or otherwise, prove by mathematical induction that $f(n)$ is divisible by 7 for every positive integer n. [4]

CIE CAL FM P1 2005-11 Q2

The sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 1$ and $u_{n+1} = -1 + \sqrt{(u_n + 7)}$.

(i) Prove by induction that $u_n < 2$ for all $n \geqslant 1$. [4]

(ii) Show that if $u_n = 2 - \varepsilon$, where ε is small, then

$$u_{n+1} \approx 2 - \tfrac{1}{6}\varepsilon.$$ [2]

CIE CAL FM P1 2004-11 Q8

The sequence of real numbers $a_1, a_2, a_3, \ldots$ is such that $a_1 = 1$ and $a_{n+1} = \left(a_n + \dfrac{1}{a_n}\right)^{\lambda}$, where λ is a constant greater than 1. Prove by mathematical induction that, for $n \geqslant 2$, $a_n \geqslant 2^{g(n)}$, where $g(n) = \lambda^{n-1}$. [6]

Prove also that, for $n \geqslant 2$, $\dfrac{a_{n+1}}{a_n} > 2^{(\lambda-1)g(n)}$. [3]

CIE CAL FM P1 2003-11 Q4

Given that $y = x\sin x$, find $\dfrac{d^2y}{dx^2}$ and $\dfrac{d^4y}{dx^4}$, simplifying your results as far as possible, and show that

$$\frac{d^6y}{dx^6} = -x\sin x + 6\cos x. \qquad [3]$$

Use induction to establish an expression for $\dfrac{d^{2n}y}{dx^{2n}}$, where n is a positive integer. [5]