

**CAMBRIDGE A LEVEL PROGRAMME**  
**SEMESTER ONE EXAMINATION NOV/DEC 2009**  
(June 2010 Intake)

**Thursday**

**9 December 2010**

**1.00 pm – 3.00 pm**

**FURTHER MATHEMATICS**

**9231/01**

**PAPER 1**

**2 hours**

Additional materials: Answer Booklet/Paper  
List of formulae (MF 10)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your name and class on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **50**.

This document consists of **2** printed pages.

- 1 Given that  $f(n) = 2^{2n} - 1$ , prove by induction that  $f(n)$  is divisible by 3 for all positive integers  $n$ . [5]

- 2 (i) Find the sum  $16^2 + 23^2 + 30^2 + \dots + (9n+7)^2$ . [4]

Hence find the minimum value of  $n$  such that

$$16^2 + 23^2 + 30^2 + \dots + (9n+7)^2 > \frac{54n^3 + 201n^2 + 307n}{2}. \quad [1]$$

- (ii) Find the sum  $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots + \frac{n}{(n+1)!}$ . [4]

- 3 The line  $l_1$  passes through the points  $3\mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ . The line  $l_2$  passes through the points  $2\mathbf{i} + 5\mathbf{j}$  and  $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ .

- (i) State the vector equation of line  $l_1$ . [1]

- (ii) State the vector equation of line  $l_2$ . [1]

- (iii) Prove that  $l_1$  and  $l_2$  intersect and find the point of intersection. [5]

- (iv) Determine the equation of the plane that contains  $l_1$  and  $l_2$  in the form  $ax + by + cz = 1$ . [3]

- 4 The curve  $C$  has equation  $y = \frac{2x^2 + 3x - 19}{x - 3}$ .

- (i) Find the equations of the asymptotes of  $C$ . [2]

- (ii) Find the coordinates of the stationary points of  $C$  and determine their nature. [4]

- (iii) Sketch  $C$ , stating the coordinates of the intersection of  $C$  and the coordinate axes. [6]

- 5 If the equation  $5x^4 - 9x^3 + 6 = 0$  has roots  $\alpha, \beta, \gamma, \delta$ , find the equation having roots  $\alpha^4, \beta^4, \gamma^4, \delta^4$ . [6]

- If  $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n$ , find  $S_4$  and  $S_8$  in exact terms. [3]

- Hence, or otherwise, find  $S_{-4}$  and  $S_{-8}$  in exact terms. [5]