Topic : **<u>P106 Systems of Linear Equations</u>**

Subtopic:

- *Solution of a System of Linear Equation(s)*
- *Consistency of a System of Linear Equation(s)*

Linear Equation

Definition :

A linear equation of n unknowns has the form

$a_1x_1 + a_2x_2 + \ldots + a_ix_i + \ldots + a_nx_n = b$ where

$a_i, b \in \mathrm{R}$ and x_i are the unknowns.

Example :

(a) $x = 3$

(b) $2x + 3y = 1$

(c) $\alpha - 2\beta + \gamma = 4$

(d) $x_1 - x_2 + x_3 - x_4 = 2$

Solution of a Linear Equation

Definition :

Let a linear equation of n unknown be

$$a_1x_1 + a_2x_2 + \ldots + a_ix_i + \ldots + a_nx_n = b.$$

$R_1, R_2, \ldots, R_i, \ldots, R_n \in \mathrm{R}$ is called a solution of the above equation if

$$a_1R_1 + a_2R_2 + \ldots + a_iR_i + \ldots + a_nR_n = b.$$

Example:

Determine whether the following statements are true:

(a) $x = 4$ is a solution of $2x = 4$.

(b) $(-1, 4)$ is a solution of $x + y = 3$.

(c) $(1, 0, -1)$ is a solution of $x - y + z = 1$.

Example:

Determine whether the following statements are true:

(a) $\{(x, y) \mid (t, 1-t), t \in \mathrm{R}\}$ is a solution of $x + y = 1$.

(b) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix}, t \in \mathrm{R} \right\}$ is a solution of $x - 2y = 3$.

(c) $(-2\lambda - 3\mu, \lambda, \mu)$ is a solution of $x + 2y + 3z = 4$.

Consistency of a Linear Equation

Definition :

A linear equation is said to be consistent if its solutions set is not ϕ. Otherwise, the system is called inconsistent.

A consistent linear equation has either a unique solution OR infinite number of solutions.

Example:

An equation is given by $ax = 1$ where $a \in \mathrm{R}$.

Solve the equation for $x \in \mathrm{R}$ if

$(\mathrm{a})\ a \neq 0.$

$(\mathrm{b})\ a = 0.$

Example:

An equation is given by $ax = 0$ where $a \in \mathrm{R}$.

Solve the equation completely for x.

Example:

An equation is given by $a(a-1)x = a$ where $a \in \mathrm{R}$. Solve the equation completely for x.

Example:

An equation is given by $ax = bx + c$ where $a, b, c \in \mathrm{R}$. Solve the equation completely for x.

Example:

Solve the following equation completely for $x \in \mathrm{R}$

$(\mathrm{a})\ x+2y=3$ $(\mathrm{b})\ 2x+3y=6$

Example:

Solve the following equation completely for $x \in \mathrm{R}$, where $a, b \in \mathrm{R}$.

(a) $2x + y = a$

(b) $ax + y = b$

Example:

Solve the following equation completely for $x \in \mathrm{R}$.

(a) $x - 2y + 3z = 0$

(b) $-x + y - z = 1$

Example:

An equation is given by $ax + ay + az = b$, where $a, b \in \mathrm{R}$. Solve the equation completely for $x \in \mathrm{R}$.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Solve the following equations, where $a \in \mathrm{R}$, completely for x.

(a) $ax = a^2$ (b) $a(a+1)x = a^2 - 1$

Example:

An equation is given by $a^2 + ax = b^2 + bx$, where $a, b \in \mathrm{R}$. Solve the equation completely for $x \in \mathrm{R}$.

Example:

Solve the following equation completely for $x \in \mathrm{R}$, where $a, b \in \mathrm{R}$.

(a) $ax + by = ab$

(b) $ax + by = c$

Example:

Solve the following equation completely for $x \in \mathrm{R}$, where $a, b \in \mathrm{R}$.

(a) $ax + by + abz = a + b$ (b) $(a-b)x + ay + bz = a + b$

Example:

Solve the following equation completely for $x \in \mathrm{R}$, where $a, b \in \mathrm{R}$.

(a) $(a-1)(b-1)x = a+b-2$

(b) $(a+b-2)x = (a-1)(b-1)$