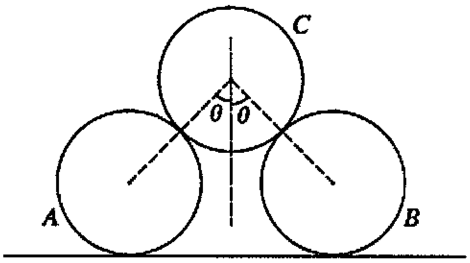
<u>PAST YEARS' QUESTIONS</u>

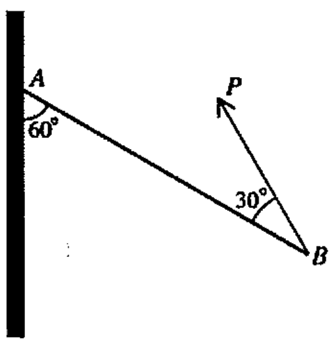
Nov2012 P22



Two identical uniform rough spheres A and B, each of weight W and radius a, are at rest on a rough horizontal plane, and are not in contact with each other. A third identical sphere C rests on A and B with its centre in the same vertical plane as the centres of A and B. The line joining the centres of A and C and the line joining the centres of B and C are each inclined at an angle θ to the vertical (see diagram). The coefficient of friction between each sphere and the plane is μ. The coefficient of friction between C and A, and between C and B, is μ'. The system remains in equilibrium. Show that

$$\mu \geqslant \frac{\sin\theta}{3(1+\cos\theta)} \quad \text{and} \quad \mu' \geqslant \frac{\sin\theta}{1+\cos\theta}. \qquad [14]$$

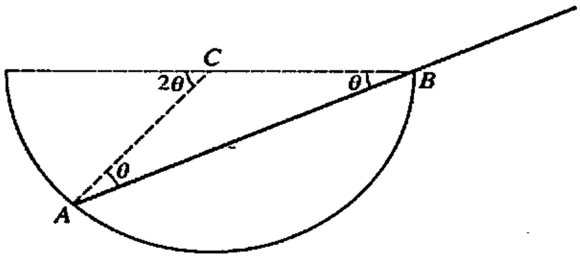
June2010 P21



A uniform rod AB of weight W rests in equilibrium with A in contact with a rough vertical wall. The rod is in a vertical plane perpendicular to the wall, and is supported by a force of magnitude P acting at B in this vertical plane. The rod makes an angle of 60° with the wall, and the force makes an angle of 30° with the rod (see diagram). Find the value of P. [3]

Find also the set of possible values of the coefficient of friction between the rod and the wall. [4]

NOV2010



A hemispherical bowl of radius r is fixed with its rim horizontal. A thin uniform rod rests in equilibrium on the rim of the bowl with one end resting on the inner surface of the bowl at A, as shown in the diagram. The rod has length $2a$ and weight W. The point of contact between the rod and the rim is B, and the rim has centre C. The rod is in a vertical plane containing C. The rod is inclined at θ to the horizontal and the line AC is inclined at 2θ to the horizontal. The contacts at A and B are smooth.

In any order, show that

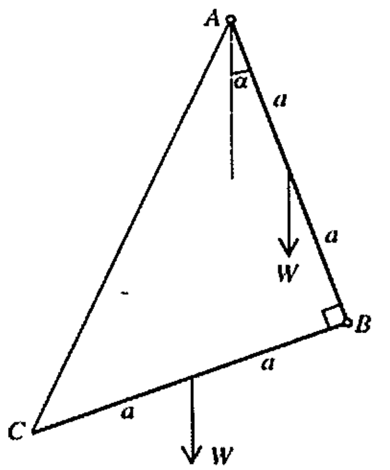
(i) the contact force acting on the rod at A has magnitude $W \tan \theta$,

(ii) the contact force acting on the rod at B has magnitude $\dfrac{W \cos 2\theta}{\cos \theta}$,

(iii) $2r \cos 2\theta = a \cos \theta$.

[9]

NOV09

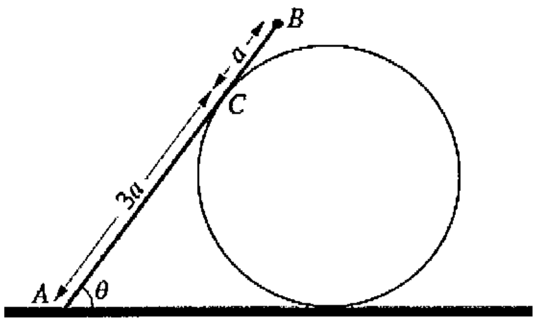


Two uniform rods, AB and BC, each have length $2a$ and weight W. They are smoothly jointed at B, and A is attached to a smooth fixed pivot. A light inextensible string of length $(2\sqrt{2})a$ joins A to C so that angle $ABC = 90°$. The system hangs in equilibrium, with AB making an angle α with the vertical (see diagram). By taking moments about A for the system, or otherwise, show that $\alpha = 18.4°$, correct to the nearest $0.1°$. [3]

Find the tension in the string in the form kW, giving the value of k correct to 3 significant figures. [3]

Find, in terms of W, the magnitude of the force acting on the rod BC at B. [6]

June 2012



The diagram shows a uniform rod AB, of length $4a$ and weight W, resting in equilibrium with its end A on rough horizontal ground. The rod rests at C on the surface of a smooth cylinder whose axis is horizontal. The cylinder rests on the ground and is fixed to it. The rod is in a vertical plane perpendicular to the axis of the cylinder and is inclined at an angle θ to the horizontal, where $\cos\theta = \frac{3}{5}$. A particle of weight kW is attached to the rod at B. Given that $AC = 3a$, show that the least possible value of the coefficient of friction μ between the rod and the ground is $\dfrac{8(2k+1)}{13k+19}$. [9]

Given that $\mu = \frac{9}{10}$, find the set of values of k for which equilibrium is possible. [3]