Poisson and Normal Distribution

Linear combination of random variables

Linear combination of independent random variables

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Learning Outcome

Students should be able to apply:

- Identify the distribution of $Y = aX + b$ where and a, b are constants.
- Identify the distribution of $Z = aX + bY$ where , and a, b are constants. X and Y are independent.

Expectation and Variance

If $Y = X + b$, where b is a constant, then

$$E(Y) = E(X + b) = E(X) + b \qquad and$$

$$Var(Y) = Var(X + b) = Var(X).$$

If $W = aX$, where a is a constant, then

$$E(W) = E(aX) = aE(X) \qquad and$$

$$Var(W) = Var(aX) = a^2 Var(X).$$

Expectation and Variance

For any random variable X,

$$E(aX+b) = aE(X) + b \qquad \textit{and}$$

$$Var(aX+b) = a^2 Var(X).$$

where a and b are constants.

Example 1:

The temperature in degrees Fahrenheit on a remote island is a random variable with mean 59 and variance 27. Find the mean and variance of the temperature in degrees Centigrade, given that to convert degrees Fahrenheit to degree Centigrade you subtract 32 and then multiply by 5/9.

Expectation and Variance of variables X and Y

For random variables X and Y and constants a and b

$$E(aX+bY)=aE(X)+bE(Y) \qquad and$$

$$E(aX-bY)=aE(X)-bE(Y)$$

If X and Y are independent, then

$$Var(aX+bY)=a^2Var(X)+b^2Var(Y)$$

$$Var(aX-bY)=a^2Var(X)+b^2Var(Y)$$

Example 2:

X and Y are independent random variables such that

E(X) = 10, Var(X) = 2, E(Y) = 8, Var(Y) = 3.

Find

$$(a)\quad E(5X+4Y),$$

$$(b)\quad Var(5X+4Y),$$

$$(c)\quad Var\left(\frac{1}{2}X-Y\right),$$

$$(d)\quad Var\left(\frac{1}{2}X+Y\right),$$

Expectation and Variance of more than one observation

Consider the random variable X where $E(X)=\mu$ and $Var(X)=\sigma^2$.

Take two observations X_1 and X_2 from X.

$E(X_1)=\mu, E(X_2)=\mu, Var(X_1)=\sigma^2, Var(X_2)=\sigma^2$.

$$E(X_1+X_2)=2E(X)$$
$$Var(X_1+X_2)=2Var(X)$$

This result can be extended to n observations.

$$E(X_1+X_2+\ldots+X_n)=nE(X)$$
$$Var(X_1+X_2+\ldots+X_n)=nVar(X)$$

Example 3:

Find the expectation and variance of the number of heads obtained when six fair coins are tossed.

Normal Distribution

If X is normal variable such that $X \sim N(\mu, \sigma^2)$

$$\Rightarrow aX \sim N(a\mu, a^2\sigma^2)$$

$$\Rightarrow aX + b \sim N(a\mu + b, a^2\sigma^2)$$

$$\Rightarrow aX - b \sim N(a\mu - b, a^2\sigma^2)$$

Normal Distribution

For twoindependent variables X and Y, where

If $X \sim N(\mu_1, \sigma_1{}^2)$

$Y \sim N(\mu_2, \sigma_2{}^2)$

$\Rightarrow aX \sim N(a\mu_1, a^2\sigma_1{}^2)$

$\Rightarrow bY \sim N(b\mu_2, b^2\sigma_2{}^2)$

$\Rightarrow aX + bY \sim N(a\mu_1 + b\mu_2, a^2\sigma_1{}^2 + b^2\sigma_2{}^2)$

$\Rightarrow aX - bY \sim N(a\mu_1 - b\mu_2, a^2\sigma^2 + b^2\sigma_2{}^2)$

Example 4:

The maximum load of a lift can carry is 450kg. The weights of men are normally distributed with mean 60kg and standard deviation 10kg. The weights of women are normally distributed with mean 55kg and standard deviation 5kg. Find the probability that the lift will be overloaded by five men and two women, if their weights are independent.

Example 5:

X and Y are independent random variables and X~N(100, 8), Y~N(55, 10). Find the probability that an observation from the population of X is more than twice the value of an observation fro the population of Y.

Example 6:

A machine produces rubber balls whose diameters are normally distributed with mean 5.50 cm and standard deviation 0.08cm.

The balls are packed in cylindrical tubes whose internal diameter are normally distributed with mean 5.70cm and standard deviation 0.12cm.

If a ball, selected at random, is placed in a tube, selected at random, what is the distribution of the clearance? (The clearance is the internal diameter of the tube minus the diameter of the ball.) What is the probability that the clearance is between 0.05cm and 0.25cm?