Example:

A sequence of numbers is given by $u_0 = 0, u_1 - 1$, and

$u_{n+2} = u_{n+1} + u_n$ for $n \geq 0$. Prove that $u_{n+1}{}^2 + u_n{}^2 = u_{2n+1}$.

Let $p(n)$: $u_{n+1}^2 + u_n^2 = u_{2n+1}$.

$p(0)$ & $p(1)$ are true because $u_1^2 + u_0^2 = 1 + 0 = 1 = u_1$ &
$u_2^2 + u_1^2 = 1 + 1 = 2 = u_3$.

Hence, we have to prove that $u_{k+2}^2 + u_{k+1}^2 = u_{2k+3}$ if
$\underbrace{u_{k+1}^2 + u_k^2 = u_{2k+1}}_{p(k)}$ & $\underbrace{u_k^2 + u_{k-1}^2 = u_{2k-1}}_{p(k-1)}$

$$
\begin{aligned}
& u_{k+2}^2 + u_{k+1}^2 \\
= \ & (u_{k+1} + u_k)^2 + u_{k+1}^2 \\
= \ & 2u_{k+1}^2 + u_k^2 + \underline{2u_{k+1}u_k} \\
= \ & 2u_{k+1}^2 + u_k^2 + \underline{u_{k+1}u_k + u_{k+1}u_k} \\
= \ & 2u_{k+1}^2 + u_k^2 + \underline{(u_k + u_{k-1})u_k} + \underline{u_{k+1}(u_{k+1} - u_{k-1})} \\
= \ & 3u_{k+1}^2 + 2u_k^2 + u_k u_{k-1} - u_{k+1}u_{k-1} \\
= \ & 3u_{k+1}^2 + 2u_k^2 - u_{k-1}(u_{k+1} - u_k) \\
= \ & 3u_{k+1}^2 + 2u_k^2 - u_{k-1}^2 \\
= \ & 3(u_{k+1}^2 + u_k^2) - (u_k^2 + u_{k-1}^2) \\
= \ & 3u_{2k+1} - u_{2k-1} \\
= \ & \underbrace{u_{2k+1} + \underbrace{u_{2k+1} + \underbrace{u_{2k+1} - u_{2k-1}}_{u_{2k}}}_{u_{2k+2}}}_{} \\
= \ & u_{2k+3}
\end{aligned}
$$

$\therefore$ $p(k+1)$ is true

$\therefore$ $p(0), p(1) \Rightarrow p(2), p(3), \ldots p(n)$ are true ※ .