Revision

P102 Summation

CIE CAL FM P1 2004-06 Q1

Use the relevant standard results in the List of Formulae to prove that $S_N = \sum_{n=1}^{N}(8n^3 - 6n^2) = N(N+1)(2N^2 - 1)$. [2]

Hence show that $\sum_{n=N+1}^{2N}(8n^3 - 6n^2)$ can be expressed in the form $N(aN^3 + bN^2 + cN + d)$, where the constants a, b, c, d are to be determined. [2]

CIE CAL FM P1 2005-06 Q1

Use the method of differences to find S_N, where

$$S_N = \sum_{n=N}^{N^2} \frac{1}{n(n+1)}.$$ [3]

Deduce the value of $\lim_{N\to\infty} S_N$. [1]

CIE CAL FM P13 2011-11 Q1

Verify that $\dfrac{1}{n^2} - \dfrac{1}{(n+1)^2} = \dfrac{2n+1}{n^2(n+1)^2}$. [1]

Let $S_N = \displaystyle\sum_{r=1}^{N} \frac{2r+1}{r^2(r+1)^2}$. Express S_N in terms of N. [2]

Let $S = \displaystyle\lim_{N\to\infty} S_N$. Find the least value of N such that $S - S_N < 10^{-16}$. [3]

ASSIGNMENT

Attempt all the questions in the following slides.

Assignment is to be submitted within **one week** after instruction!!!

Please write your name and your student's id in the script upon submission.

CIE CAL FM P13 2011-06 Q1

Find $2^2 + 4^2 + \ldots + (2n)^2$. [2]

Hence find $1^2 - 2^2 + 3^2 - 4^2 + \ldots - (2n)^2$,
simplifying your answer. [3]

CIE CAL FM P1 2003-06 Q3

Let $v_1, v_2, v_3, \ldots$ be a sequence and let

$u_n = nv_n - (n+1)v_{n+1}$, for $n = 1, 2, 3, \ldots$.

Find $\sum_{n=1}^{N} u_n$. [2]

In each of the following cases determine whether the series $u_1 + u_2 + u_3 + \ldots$ is convergent, and justify your conclusion. Give the sum to infinity where this exists.

(i) $v_n = n^{-\frac{1}{2}}$. [2]

(ii) $v_n = n^{-\frac{3}{2}}$. [2]

CIE CAL FM P1 2003-11 Q2

Given that $u_n = \dfrac{1}{n^2 - n + 1} - \dfrac{1}{n^2 + n + 1}$,

find $S_N = \sum_{n=N+1}^{2N} u_n$ in terms of N. [3]

Find a number M such that $S_N < 10^{-20}$ for all $N > M$. [3]

Example:

Prove that $\displaystyle\sum_{r=1}^{n} \frac{1}{\sqrt{r}+\sqrt{r-1}} = \sqrt{n}.$

Deduce that $\displaystyle\sum_{r=1}^{n} \frac{1}{\sqrt{r}} < 2\sqrt{n}.$