1. $1^4 - 2^4 + 3^4 - 4^4 + \cdots + (2n-1)^4 - (2n)^4$

$$= \sum_{r=1}^{n} (2r-1)^4 - (2r)^4$$

$$= \sum_{r=1}^{n} 16r^4 - 32r^3 + 24r^2 - 8r + 1 - 16r^4$$

$$= \sum_{r=1}^{n} -32r^3 + 24r^2 - 8r + 1$$

$$= -32\sum_{r=1}^{n} r^3 + 24\sum_{r=1}^{n} r^2 - 8\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$= \frac{-32n^2(n+1)^2}{4} + \frac{24n(n+1)(2n+1)}{6} - \frac{8n(n+1)}{2} + n$$

$$= -8n^2(n+1)^2 + 4n(n+1)(2n+1) - 4n(n+1) + n$$

$$= -8n^2(n^2+2n+1) + 4n(2n^2+3n+1) - 4n^2 - 4n + n$$

$$= -8n^4 - 16n^3 - 8n^2 + 8n^3 + 12n^2 + 4n - 4n^2 - 4n + n$$

$$= -8n^4 - 8n^3 + n$$

2. $$\left.\begin{array}{l} x + y + z = 0 \\ ax + by + cz = 0 \\ (b+c)x + (a+c)y + (a+b)z = 0 \end{array}\right\}$$

$-a \times (1) + (2)$:
$-(b+c) \times (1) + (3)$:

$$\left.\begin{array}{l} x + y + z = 0 \\ (b-a)y + (c-a)z = 0 \\ (a-b)y + (a-c)z = 0 \end{array}\right\}$$

$(2) + (3)$:

$$\left.\begin{array}{l} x + y + z = 0 \\ (b-a)y + (c-a)z = 0 \end{array}\right\}$$

when $a \neq b$: Let $z = s, s \in R$

$$y = \frac{(a-c)s}{b-a}$$

$$x + \frac{(a-c)s}{b-a} + s = 0$$

$$x + \frac{(b-c)s}{b-a} = 0$$

$$x = \frac{(c-b)s}{b-a}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{(c-b)s}{b-a} \\ \frac{(a-c)s}{b-a} \\ s \end{pmatrix}$$

$$= s \begin{pmatrix} \frac{c-b}{b-a} \\ \frac{a-c}{b-a} \\ 1 \end{pmatrix}$$

when $a \neq c$: Let $y = s,\ s \in R$

$$z = \frac{(a-b)s}{c-a}$$

$$x + s + \left(\frac{a-b}{c-a}\right)s = 0$$

$$x + \frac{(c-b)s}{c-a} = 0$$

$$x = \frac{(b-c)s}{c-a}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{(b-c)s}{c-a} \\ s \\ \frac{(a-b)s}{c-a} \end{pmatrix}$$

$$= s\begin{pmatrix} \frac{b-c}{c-a} \\ 1 \\ \frac{a-b}{c-a} \end{pmatrix}$$

3. $x_2 = 2 + \sqrt{x_1 + 7} = 2 + \sqrt{8}$

$x_3 = 2 + \sqrt{2 + \sqrt{8} + 7} = 2 + \sqrt{9 + \sqrt{8}}$

$$2 < \sqrt{8} < 3$$

$$9 < 11 < 9 + \sqrt{8} < 12 < 16$$

$$3 < \sqrt{9 + \sqrt{8}} < 4$$

$$5 < 2 + \sqrt{9 + \sqrt{8}} < 6$$

x_n is true for $n = 3$.

Assume : $5 < x_k < 6$

$$x_{k+1} = 2 + \sqrt{x_k + 7}$$

$$5 < x_k < 6$$

$$12 < x_k + 7 < 13$$

$$3 = \sqrt{9} < \sqrt{12} < \sqrt{x_k + 7} < \sqrt{13} < \sqrt{16} = 4$$

$$3 < \sqrt{x_k + 7} < 4$$

$$5 < 2 + \sqrt{x_k + 7} < 6$$

$$5 < x_{k+1} < 6$$

The statement is true for $k+1$ if it is true for $n = k$.

$\therefore$ $5 < x_{n+1} < 6$ for all $n \geq 3$.

4. $y = \dfrac{2x-1}{(x-2)^2}$

i) Asymptotes : $y = 0$, $x = 2$

ii) $y = \dfrac{2x-1}{(x-2)^2} \Rightarrow yx^2 + (-4y-2)x + 4y + 1 = 0$

$x \in R$: $16y^2 + 16y + 4 - (16y^2 + 4y) \geq 0$

$12y \geq -4$

$y \geq -\frac{1}{3}$

1 turning point

Min point at $y = -\frac{1}{3}$

$-\frac{1}{3}x^2 - \frac{2}{3}x - \frac{1}{3} = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$

$\therefore$ min point $(-1, -\frac{1}{3})$

iii) At the x-axis,

$y = 0$

$x = \frac{1}{2}$

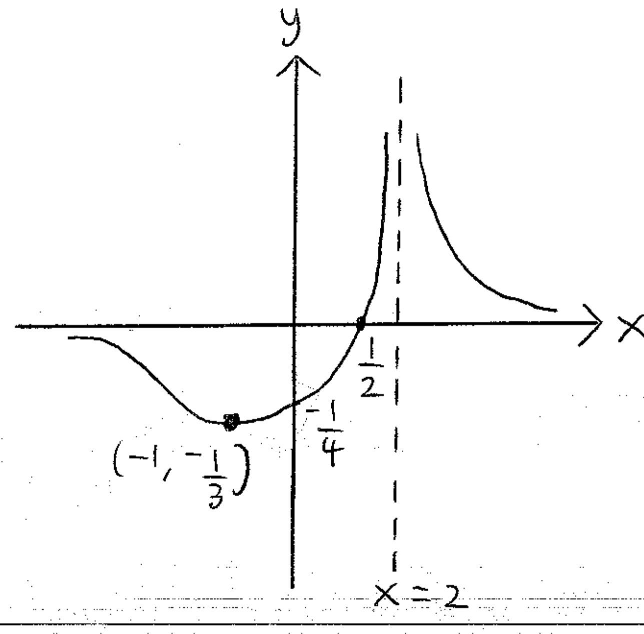
$(\frac{1}{2}, 0)$

At the y-axis

$x = 0$

$y = -\frac{1}{4}$

$(0, -\frac{1}{4})$



5 a) $ax^3 + bx^2 + cx + d = 0$

α, β, γ are the roots.

$\alpha + \beta + \gamma = \frac{-b}{a}$ $\quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ $\quad \alpha\beta\gamma = -\frac{d}{a}$

$$\frac{3\alpha}{\beta + \gamma - \alpha}, \quad \frac{3\beta}{\alpha + \gamma - \beta}, \quad \frac{3\gamma}{\alpha + \beta - \gamma}$$

Let $u = \dfrac{3\alpha}{\beta + \gamma - \alpha}$

$= \dfrac{3\alpha}{\alpha + \beta + \gamma - 2\alpha}$

$= \dfrac{3\alpha}{\frac{-b}{a} - 2\alpha}$, since $\alpha + \beta + \gamma = \frac{-b}{a}$

$= \dfrac{3a\alpha}{-b - 2a\alpha}$

$-bu - 2a\alpha u = 3a\alpha$

$2a\alpha u + 3a\alpha = -bu$

$(2au + 3a)\alpha = -bu$

$\alpha = \dfrac{-bu}{2au + 3a}$

α is a root

$\therefore a\alpha^3 + b\alpha^2 + c\alpha + d = 0$

$$a\left(\frac{-bu}{2au + 3a}\right)^3 + b\left(\frac{-bu}{2au + 3a}\right)^2$$

$$+ c\left(\frac{-bu}{2au + 3a}\right) + d = 0$$

$$\frac{-ab^3u^3}{(2au+3a)^3} + \frac{b^3u^2}{(2au+3a)^2} - \frac{bcu}{2au+3a} + d = 0$$

$$-ab^3u^3 + b^3u^2(2au+3a) - bcu(2au+3a)^2$$
$$+ d(2au+3a)^3 = 0$$

$$-ab^3u^3 + ab^3u^2(2u+3) - a^2bcu(2u+3)^2$$
$$+ a^3d(2u+3)^3 = 0$$

$$-b^3u^3 + b^3u^2(2u+3) - abcu(2u+3)^2$$
$$+ a^2d(2u+3)^3 = 0$$

$$-b^3u^3 + 2b^3u^3 + 3b^3u^2 - abcu(4u^2+12u+9)$$
$$+ a^2d(8u^3+36u^2+54u+27) = 0$$

$$-b^3u^3 + 2b^3u^3 + 3b^3u^2 - 4abcu^3 - 12abcu^2$$
$$-9abcu + 8a^2du^3 + 36a^2du^2 + 54a^2du + 27a^2d = 0$$

$$(8a^2d - 4abc + b^3)u^3 + (36a^2d - 12abc + 3b^3)u^2$$
$$+ (54a^2d - 9abc)u + 27ad^2 = 0$$

$\therefore$ The equation having roots $\dfrac{3\alpha}{\beta+\gamma-\alpha}$,

$\dfrac{3\beta}{\alpha+\gamma-\beta}$, $\dfrac{3\gamma}{\alpha+\beta-\gamma}$ is

$$(8a^2d - 4abc + b^3)u^3 + (36a^2d - 12abc + 3b^3)u^2$$
$$+ (54a^2d - 9abc)u + 27ad^2 = 0$$

b) $2x^4 - 8Ax^3 + 9x^2 - 5x + 3 = 0$

$\alpha, \beta, \gamma, \delta$ are the roots.

$\alpha + \beta + \gamma + \delta = 4A$

$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{9}{2}$

$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{2}$

$\alpha\beta\gamma\delta = \frac{3}{2}$

If $\alpha + \beta + \gamma + \delta = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$,

$$\alpha + \beta + \gamma + \delta = \frac{\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta}{\alpha\beta\gamma\delta}$$

$$4A = \frac{\frac{5}{2}}{\frac{3}{2}}$$

$$= \frac{5}{3}$$

$$\therefore A = \frac{5}{12}$$