TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST 2012

(January & March 2012 Intakes)

Tuesday **26 March 2013** **8.30 am – 11.30 am**

FURTHER MATHEMATICS 9231/12

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

1 If the equation $ax^3 + bx^2 + cx + d = 0$ has roots α, β, γ and $\alpha + \beta = \gamma$, show that $8a^2d + b^3 = 4abc$. [5]

2 Show that the perpendicular distance from the point (l, m, n) to the plane $ax + by + cz = 0$ is $\dfrac{al + bm + cn}{\sqrt{a^2 + b^2 + c^2}}$. [4]

Show that the line r = $\begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix} + s\begin{pmatrix} 10 \\ -9 \\ -2 \end{pmatrix}$ is parallel to the plane $3x + 2y + 6z = 0$. [1]

Find the perpendicular distance from the line to the plane. [1]

3 Show that $(2r+1)^4 - (2r-1)^4 = 64r^3 + 16r^2$. [1]

Hence, or otherwise, find $\sum_{r=1}^{n} r^3$. [5]

4 By considering the identity $a\cos\theta + b\sin\theta = R\cos(\theta - \alpha)$, where $R = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\dfrac{b}{a}$, prove by induction that if $y = e^{ax}\cos bx$, then

$\dfrac{d^n y}{dx^n} = (a^2 + b^2)^{\frac{n}{2}} e^{ax}\cos(bx + n\phi), \phi = \tan^{-1}\dfrac{b}{a}$ for every positive integer n. [7]

5 The curve C has the equation $y = \dfrac{1}{ax+b} + \dfrac{1}{cx+d}, a, b, c, d > 0$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Find the intersection points of C with the axes. [2]

(iii) Show that $\dfrac{dy}{dx} < 0$ at every point of C. [2]

6 If $(1 + \dfrac{2}{z})^n = 1$, find z and $|z|$. [8]

7 If $I_n = \int \tan^n x \ \mathrm{d}x$, find a reduction formula for I_n. [3]

Find also I_3, I_4 and I_5. [5]

8 **(i)** If $y = ax^2$, $a > 0$, find the x-coordinate of the centroid when the curve is revolved about the x-axis from $x = 0$ to $x = b$, $b > 0$. [4]

(ii) If $x = e^t \cos t$ and $y = e^t \sin t$, $t = 0, t = \pi$, find

(a) The arc length from $t = 0$ to $t = \pi$. [6]

(b) The surface area or revolution from $t = 0$ to $t = \pi$ about the y-axis. [3]

9 If the matrix **A** has eigenvalues 3, 5, 7 with corresponding eigenvectors

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, find **A**. [13]

10 The curve C has equation $(x + y)(x^2 + y^2) = xy$.

(i) Express C in polar form. [2]

(ii) Sketch C for $0 \le \theta \le \dfrac{\pi}{2}$. [3]

(iii) Show that $\dfrac{\sin^2 2\theta}{1 + \sin 2\theta} = \sin 2\theta - 1 + \sec^2 2\theta - \tan 2\theta \sec 2\theta$. [3]

Hence find the area enclosed by C from $\theta = 0$ to $\theta = \dfrac{\pi}{2}$. [5]

Answer only **one** of the following two alternatives.

11 EITHER

The linear transformation T: $R^4 \rightarrow R^3$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ b+c+d & a+c+d & a+b+d & a+b+c \end{pmatrix}.$$

(i) Find the rank of **M** if $a > b > c > d > 0$. [4]

(ii) Find a basis for the range space of T if $a > b > c > d > 0$. [2]

(iii) Find the nullspace of T if $a = b = c = d$. [4]

(iv) Find the general solution of $\mathbf{Mx} = \begin{pmatrix} 4 \\ a+b+c+d \\ 3(a+b+c+d) \end{pmatrix}$. [4]

OR

Find the general solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10e^{-2x}$. [10]

Find also the particular solution when $x = 0, y = 5$ and $\frac{dy}{dx} = 1$. [4]