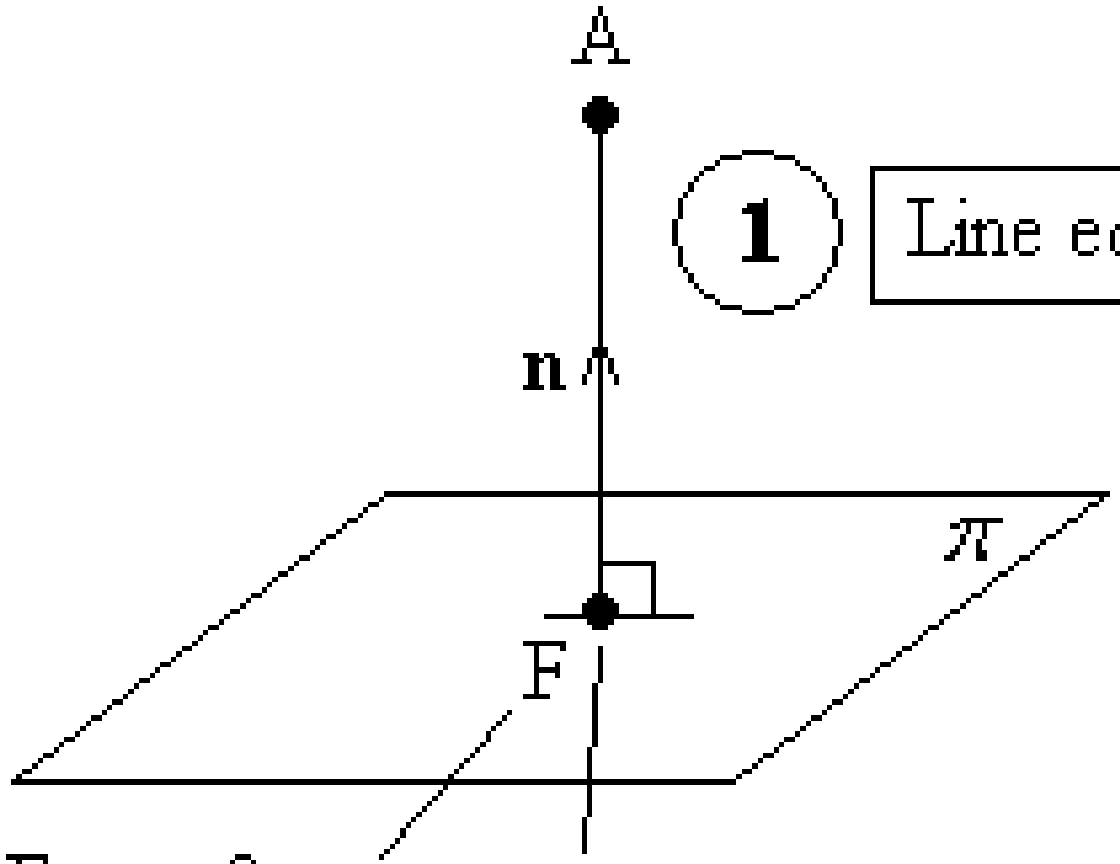
Foot of the Perpendicular from a Point to a Plane

The foot of the perpendicular from a point A to a plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$ can be found by solving the equation $(\mathbf{a} + \lambda\mathbf{n}) \cdot \mathbf{n} = p$ for λ.



1 Line equation : $\mathbf{r} = \mathbf{a} + \lambda\mathbf{n}$

Foot of perpendicular

2 Solve equation : $(\mathbf{a} + \lambda\mathbf{n}) \cdot \mathbf{n} = p$

Example :

Find the foot of the perpendicular from point $A(2,3,1)$ to plane $\mathbf{r} \cdot (\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = 4.$

Example:

Find the foot of the perpendicular from point $A(9,-5,2)$ to plane $3x-4y-z=-7$.

Example:

Find the foot of the perpendicular from point $\mathrm{A}(0,0,0)$ to plane $\mathbf{r} = s\mathbf{i} + (-2 + s + t)\mathbf{j} + (-2 - s - 2t)\mathbf{k}$.

Example:

Find the perpendicular distance from point $A(2,3,1)$ to plane $\mathbf{r} \cdot (\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = 4.$

Example:

Find the shortest distance from point $\mathrm{A}(9,-5,2)$ to plane $\mathbf{r} = -\mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{k})$.

Example:

A plane is given by $x + 4y - 2z = 1$. P is a point with coordinates $(1, -2, 3)$ and M is the foot of the perpendicular from P to the plane. Find the

(a) coordinates of M.

(b) coordinates of reflective image of P by the plane.

Example : (Point - plane distance formula)

Let a plane $\pi : \mathbf{r} \cdot \mathbf{n} = p$, and a point $Q : h\mathbf{i} + k\mathbf{j} + l\mathbf{h}$.

(a) Show that the perpendicular distance from the origin to π is $\frac{p}{|\mathbf{n}|}$.

(b) Find the normal equation of the plane which is parallel to π and contains Q. Deduce the perpendicular distance from the origin to this plane.

(c) Hence, find the perpendicular distance from Q to π.

$\therefore$ Show that the perpendicular distance from point (h, k, l) to a plane $ax + by + cz + d = 0$ is $\left| \frac{ah + bk + cl + d}{\sqrt{a^2 + b^2 + c^2}} \right|$ units.

Example:

Use the point - plane distance formula to find the distance from point A to plane π.

$(a)\ \pi : 2x - y - 3z = 4$

$A : (0, -1, 1)$

$(\text{b})\ \pi : 3x + 2y - 6z = 1$

$A : (5, 6, 2)$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the foot of the perpendicular from point with position vector $(1 \quad 3 \quad 2)$ to plane $2x + 2y - z = -3$.

Example:

Find the perpendicular distance from point $(0,-1,1)$ to plane $\mathbf{r}\cdot(2\mathbf{i}-\mathbf{j}-3\mathbf{k})=4.$

Example :

Find the shortest distance of the plane containing points with position vectors $2\mathbf{i}, 3\mathbf{i} + \mathbf{j}$ and $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, from O.

Example:

Three points on a plane are given by $A(1,2,1)$, $B(0,1,-2)$ and $(3,0,-1)$.

(a) Find the normal equation of the plane.

(b) Find the coordinates of the reflective image of point $(3,-2,4)$ by the plane.