TAYLOR'S COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2005

(July 2004 Intake)

Wednesday 24 August 2005 8.30 am – 11.30 am

FURTHER MATHEMATICS 9231/01

PAPER 1 **3 hours**

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

1 Find the sum of the series

$$\sum_{n=1}^{50} n(n^2+3)$$ [4]

2 Obtain the exact roots of the equation $Z^5 = -32i$, giving your answers in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$ [5]

3 Use De Moivre's theorem to show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$. [3]

Hence, solve the equation

$$32\sqrt{3}x^5 - 40\sqrt{3}x^3 + 10\sqrt{3}x - 3 = 0,$$

giving each root in the form $\sin k\pi$, where k is positive constant. [3]

4 Given that $y = xe^x$.

(i) Find

$$\frac{dy}{dx}, \frac{d^2y}{dx^2} \text{ and } \frac{d^3y}{dx^3}.$$ [2]

(ii) Use induction to establish an expression for

$$\frac{d^ny}{dx^n}.$$ [5]

5 Matrices A and B are given by

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix},$$

where $a \neq 0$.

(i) Find the inverse of A. [4]

(iii) Given that

$$B^{-1} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \end{pmatrix},$$

find the matrix C such that $ACB = I$, where I is the identity matrix. [3]

6 The parametric equations of a curve are given as

$$x = a(\theta - \sin\theta) \quad , \quad y = a(1 - \cos\theta)$$

where a is a constant.
The points A and B on the curve are defined by $\theta = 0$ and $\theta = \pi$ respectively.
Find the length of the arc AB. [7]

7 The equation, in polar coordinates, of a curve is

$$r = a \sin\theta\cos^2\theta$$

where a is a positive constant and $0 \leq \theta \leq \pi/2$.

(i) Sketch the curve. [2]

(ii) Find the exact greatest value of r. [5]

8 The planes π_1 and π_2 have equations, $x + 2y - 3z + 4 = 0$ and $2x + y - 4z - 3 = 0$ respectively. Show that, for all values of λ, every point, which is in both π_1 and π_2 is also in the plane $x + 2y - 3z + 4 + \lambda(2x + y - 4z - 3) = 0$. [2]

The planes π_1 and π_2 meet in the line L.

(i) Find the equation of the plane π_3 which passes though L and the point whose position vector is a**k**. [3]

(ii) Find the value of a if π_2 is perpendicular to π_3. [3]

9 Given the curve

$$y = \frac{3x - 5}{x^2 - 1}$$

(i) State the asymptotes of the curve y. [2]
(ii) Find all the stationary points of the curve and determine their nature. [6]
(iii) Hence, sketch the curve. [3]

[Turn over

4

10 The linear transformation $T: R^4 \rightarrow R^4$ is represented by the matrix

$$\begin{pmatrix} 3 & 0 & -2 & 3 \\ -4 & 2 & 0 & -8 \\ 1 & -2 & 2 & 5 \\ 3 & -3 & 2 & 9 \end{pmatrix}$$

(i) Find the dimension of the range space of T. [3]

(ii) Deduce the dimension of the null space and hence find basis for the null space of T. [5]

(iii) Find the general solution of the equation $AX = \begin{pmatrix} 3 \\ -4 \\ 1 \\ 3 \end{pmatrix}$ [3]

11 Given the matrix A where

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

Find the eigenvalues and the corresponding eigenvectors of A. [9]
Find a matrix P and a diagonal matrix D such that

$$(A-kI)^n = PDP^{-1}$$

where I is the identity matrix, k is a constant and n is a positive integer. [4]

12 Answer only one of the following two alternatives.

EITHER

Given that y is a function of x,where x >0,show that,if the substitution $x = \sqrt{t}$ is made,then

(i) $\dfrac{dy}{dx} = 2\sqrt{t}\,\dfrac{dy}{dt}$,

(ii) $\dfrac{d^2y}{dx^2} = 4t\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt}$ [5]

Hence, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{1}{x}\left[\frac{dy}{dx}\right] + 4x^2(9y+6) = 0$$

[9]

OR

Given that

$$I_n = \int_0^1 x^n e^{-\alpha x}\ dx \quad ,$$

where α is a positive constant and n is non – negative integer, show that for $n \geq 1$,

$$\alpha\, I_n = n\, I_{n-1} - e^{-\alpha}\,. \qquad [3]$$

Hence, find the coordinates of the centroid of the finite region bounded by the x-axis, the line $x = 1$ and the curve $y = xe^{-x}$, giving your answers in terms of e.

[11]