1 $8^2 + 13^2 + 18^2 + \cdots + (5n+3)^2$

$$= \sum_{r=1}^{n} (5r+3)^2$$

$$= \sum_{r=1} 25r^2 + 30r + 9$$

$$= 25\sum_{r=1}^{n} r^2 + 30\sum_{r=1}^{n} r + 9\sum_{r=1}^{n} 1$$

$$= \frac{25n(n+1)(2n+1)}{6} + \frac{30n(n+1)}{2} + 9n$$

$$= \frac{25n(n+1)(2n+1)}{6} + 15n(n+1) + 9n$$

$$= \frac{5n(n+1)(5(2n+1)+18)}{6} + 9n$$

$$= \frac{5n(n+1)(10n+5+18)}{6} + 9n$$

$$= \frac{5n(n+1)(10n+23)}{6} + 9n$$

$$= \frac{5n}{6}(10n^2 + 23n + 10n + 23) + 9n$$

$$= \frac{5n}{6}(10n^2 + 33n + 23) + \frac{54n}{6}$$

$$= \frac{n}{6}(50n^2 + 165n + 115) + \frac{54n}{6}$$

$$= \frac{n}{6}(50n^2 + 165n + 115 + 54)$$

$$= \frac{n}{6}(50n^2 + 165n + 169)$$

2. Let $f(n) = 17^n + 3^{2n-1}$

When $n = 1$: $f(1) = 17^1 + 3^{2(1)-1}$

$= 17 + 3^{2-1}$

$= 17 + 3^1$

$= 17 + 3$

$= 20$

$= 5(4)$

$\therefore 4 \mid f(1)$

Assume that $f(n)$ is divisible by 4 when $n = k$.

$n = k$: $4 \mid f(k)$

$f(k) = 4s, \ s \in N$

$17^k + 3^{2k-1} = 4s$

When $n = k + 1$:

$f(k+1) = 17^{k+1} + 3^{2(k+1)-1}$

$= 17^k 17 + 3^{2k+2-1}$

$= 17^k 17 + 3^{2k-1} 3^2$

$= 17^k 17 + 3^{2k-1} 9$

$= 17^k (16 + 1) + 3^{2k-1}(8 + 1)$

$= 17^k 16 + 17^k + 3^{2k-1} 8 + 3^{2k-1}$

$= 17^k 16 + 3^{2k-1} 8 + 17^k + 3^{2k-1}$

$= 8(17^k 2 + 3^{2k-1}) + 4s$

$= 4(17^k 4 + 3^{2k-1} 2 + s)$

Since s is an integer and k is an integer, $17^k 4 + 3^{2k-1} 2 + s$ is an integer.

$\therefore 4 \mid f(k+1)$

Since $17^n + 3^{2n-1}$ is divisible by 4 when $n = 1$ and $17^n + 3^{2n-1}$ is divisible by 4 when $n = k+1$ if it is divisible by 4 when $n = k$. $17^n + 3^{2n-1}$ is divisible by n for every positive integer n.

3. $2x^4 - 3x^3 + 5x^2 - 9x + 7 = 0$

$\alpha, \beta, \gamma, \delta$ are the roots.

$\alpha^2 + 2, \beta^2 + 2, \gamma^2 + 2, \delta^2 + 2$

Let $u = \alpha^2 + 2$

$\therefore \alpha^2 = u - 2$

$\alpha = \pm\sqrt{u-2}$

α is a root

$\therefore 2\alpha^4 - 3\alpha^3 + 5\alpha^2 - 9\alpha + 7 = 0$

$2(\pm\sqrt{u-2})^4 - 3(\pm\sqrt{u-2})^3$

$+ 5(\pm\sqrt{u-2})^2 - 9(\pm\sqrt{u-2}) + 7 = 0$

$2(u-2)^2 - 3(\pm(u-2)\sqrt{u-2})$

$+ 5(u-2) - 9(\pm\sqrt{u-2}) + 7 = 0$

$2(u-2)^2 + 5(u-2) + 7$

$-3(\pm(u-2)\sqrt{u-2}) - 9(\pm\sqrt{u-2}) = 0$

$2(u^2 - 4u + 4) + 5u - 10 + 7$

$\pm 3\sqrt{u-2}\,(-(u-2) - 3) = 0$

$$2u^2 - 8u + 8 + 5u - 10 + 7$$

$$\pm 3\sqrt{u-2}(-u-1) = 0$$

$$2u^2 - 3u + 5 \pm 3\sqrt{u-2}(-u-1) = 0$$

$$2u^2 - 3u + 5 = \pm 3\sqrt{u-2}(u+1)$$

$$(2u^2 - 3u + 5)^2 = \left[\pm 3\sqrt{u-2}(u+1)\right]^2$$

$$= 9(u-2)(u+1)^2$$

$$(2u^2 - 3u)^2 + 10(2u^2 - 3u) + 25$$

$$= 9(u-2)(u^2 + 2u + 1)$$

$$14u^4 - 12u^3 + 9u^2 + 20u^2 - 30u + 25$$

$$= 9(u^3 + 2u^2 + u - 2u^2 - 4u - 2)$$

$$4u^4 - 12u^3 + 29u^2 - 30u + 25$$

$$= 9(u^3 - 3u - 2)$$

$$4u^4 - 12u^3 + 29u^2 - 30u + 25$$

$$= 9u^3 - 27u - 18$$

$$4u^4 - 21u^3 + 29u^2 - 3u + 43 = 0$$

The equation having roots

$\alpha^2 + 2, \beta^2 + 2, \gamma^2 + 2$ and $\delta^2 + 2$ is

$$4u^4 - 21u^3 + 29u^2 - 3u + 43 = 0$$

4. $y = \dfrac{5x + 3}{2x - 4}$

$$\begin{array}{r|l} & \frac{5}{2} \\ \hline 2x - 4 & 5x + 3 \\ & 5x - 10 \\ \hline & \quad 13 \end{array}$$

$= \dfrac{5}{2} + \dfrac{13}{2x - 4}$

As $x \to \pm\infty$ $y \to \frac{5}{2}$

As $x \to 2$ $y \to \pm\infty$

$\therefore$ The asymptotes of y are $y = \frac{5}{2}$ and $x = 2$

When $x = 0$: $y = \frac{-3}{4}$

When $y = 0$: $\dfrac{5x + 3}{2x - 4} = 0$

$5x + 3 = 0$

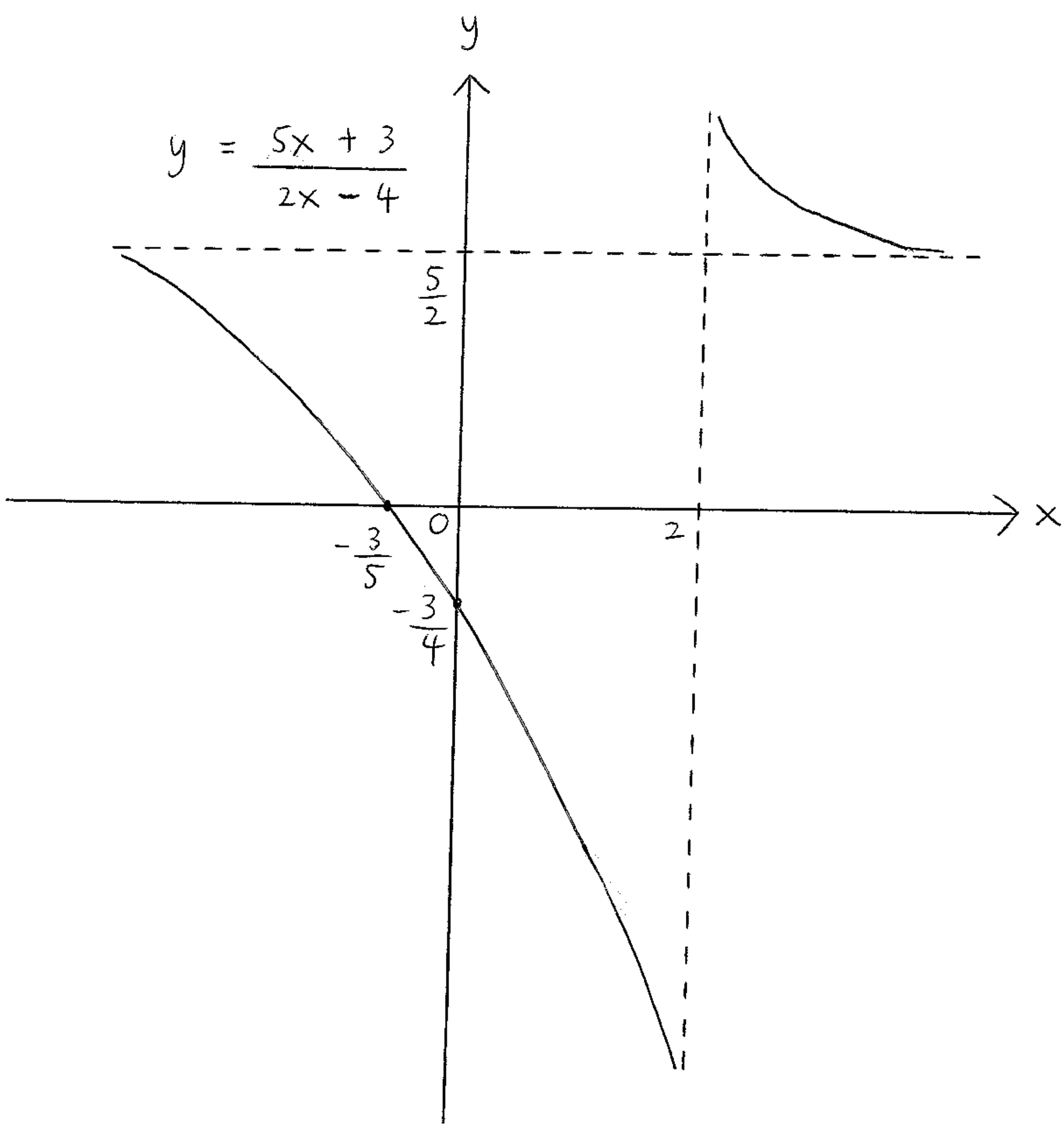
$x = \frac{-3}{5}$

$\therefore$ The intersection points of y are $(0, \frac{-3}{4})$ and $(\frac{-3}{5}, 0)$.

$$\frac{dy}{dx} = \frac{-26}{(2x-4)^2}$$

$$\neq 0$$

$\therefore$ There are no critical points.



• Intersection point.