TAYLOR'S UNIVERSITY COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2008

(July 2007 Intake)

Thursday 28 August 2008 12.30 pm – 3.30 pm

FURTHER MATHEMATICS 9231/01

PAPER 1

3 hours

Additional materials: Answer Booklet/Paper
List of formulae (MF 10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers
The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of 3 printed pages.

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[Turn over

1. Prove that the vectors $\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}5\\4\\8\end{pmatrix}, \begin{pmatrix}7\\-1\\6\end{pmatrix}$ are linearly independent. [5]

2. If $y = \dfrac{1}{ax+b}$, prove that $\dfrac{d^n y}{dx^n} = \dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$ for every positive integer n. [6]

3. Find the intersection point of the planes $3x + 2y - 8z = 1$, $4x + 7y + 9z = 0$ and $5x - 6y + z = 9$. [7]

4. If the equation $3x^4 - x^3 + 9x^2 - 4x + 5 = 0$ has roots $\alpha, \beta, \gamma, \delta$, find the equation having roots $\alpha + \beta + \gamma$, $\alpha + \beta + \delta$, $\alpha + \gamma + \delta$, $\beta + \gamma + \delta$. [7]

5. Find the coordinates of the centroid of the region bounded by the x-axis, the line $x = 1$ and the curve $y = x^3$. [8]

6. The curve C is given by the equation $y = 4 + \dfrac{1}{2x+3} - \dfrac{8}{x-6}$.

   i) Find the asymptotes of C. [3]

   ii) Find the coordinates of the critical points of C. [6]

7. Find the arc length of the curve given by the equations $x = t^2 \cos t, y = t^2 \sin t$ from $t = 0$ to $t = 2$. [10]

8. If $I_n = \int \csc^n x \, dx$, find a reduction formulae for I_n. [5]

   Hence, or otherwise, find I_3 and I_4. [6]

9. Expressing $\sin^6 \theta$ in the form $a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$, or otherwise, find the mean value of $\sin^6 \theta$ over the interval $[-\pi, \pi]$. [11]

10. Simplifying the expression $(r+1)^5-(r-1)^5$, or otherwise, find the sum

$\sum_{r=1}^{n} r^4$. [12]

11. Answer only **one** of the following two alternatives.

EITHER

i) Sketch the curve $r=\sin\theta+\sqrt{3}\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. [4]

ii) Find the maximum value of r and the angle at which it occurs. [5]

iii) Find the area bounded by the curve from $\theta = 0$ to $\theta = \frac{\pi}{2}$. [5]

OR

If $5y^4\frac{d^2y}{dx^2}+20y^3\left(\frac{dy}{dx}\right)^2+25y^4\frac{dy}{dx}+4y^5=3e^{7x}$, using the substitution $z=y^5$, obtain a differential equation in x and z. [5]

Hence, express y in terms of x. [9]