CONTENTS

ADDITIONAL MATHEMATICS

Paper 0606/01
Paper 1

General comments

There were many very good scripts, but equally many from candidates who should not have been entered for the examination. The standard of presentation was variable and some scripts were difficult to mark with attempts at particular questions in several different places on the script. Candidates should also be warned of the difficulties, both to candidates and to Examiners, of giving working in two columns. The topics of matrices and relative velocity continue to present most candidates with difficulties.

Comments on specific questions

Question 1

The vast majority of candidates found this to be a successful starting question. Most differentiated the given equation by a correct quotient formula, though occasionally the numerator was seen as (*uv' −vu')* instead of (*vu' − uv'*). Some candidates preferred to express *y* as $(3x-2)(x^2+5)^{-1}$ but were equally successful. The most common error was to expand $-(3x-2)2x$ as $-3x^2-4x$. Virtually all candidates recognised the need to set $\frac{dy}{dx}$ to 0, though only about a half of all attempts were successful. Many errors were caused by failure to realise that the numerator itself is 0.

Answers: **(i)** $\frac{dy}{dx}=\frac{15+4x-3x^2}{(x^2+5)^2}$; **(ii)** $x = 3$ or $-1\frac{2}{3}$.

Question 2

There was a poor response to a question that normally causes few problems. Many candidates assumed that the formula for solving a quadratic equation applied equally to a cubic. Similarly there were a depressing number of solutions in which $x^3-5x=-2$ was followed by $x = -2$ or $(x^2-5)=-2$. About a half of all solutions realised that $x = 2$ was a solution and proceeded to divide by $(x-2)$ to obtain a quadratic. The fact that the cubic had no term in x^2 presented problems to some candidates. Of those arriving at correct solutions by applying the formula to $x^2+2x-1=0$, most failed to answer the question fully by failing to realise that $\sqrt{8}=2\sqrt{2}$.

Answers: $x = 2$, $x=-1+\sqrt{2}$ and $x=-1-\sqrt{2}$.

Question 3

Many candidates offered perfectly correct solutions – others failed to recognise the basic properties of the modulus function. Most candidates sketched $y=x-1$ correctly, but a large minority of solutions failed to realise that the graph of $y=|2x+3|$ was 'V-shaped' with the base of the 'V' at (−1.5, 0). Part **(ii)** was often correct even from candidates failing to realise the 'V-shape' of $y=|2x+3|$. Most successful attempts to part **(ii)** came from realisation that two equations could be obtained by expressing $|2x+3|$ as either $2x + 3$ or as $-(2x+3)$. Candidates attempting to square both sides usually failed through poor algebra. A large number of attempts were seen in which $x + 2x + 3$ was equated with either +1 or to −1. A majority of attempts just offered one solution ($x=-\frac{2}{3}$).

Answers: **(ii)** $x=-\frac{2}{3}$ or $x=-4$.

Question 4

This was poorly answered with relatively few candidates achieving full marks. Most candidates realised that "*a*" was equal to the amplitude. Only a small number realised that because the period of f was120°, then the value of *b* was 3 and not $\frac{1}{3}$. Only a few were able to link the 'minimum value of −1' with the correct value of *c*. The sketch graphs were again variable with only a small number of candidates realising that there were 3 full cycles in the given domain. Many good candidates obtained a correct sketch for the interval 0° to 120° but failed to include the other 2 cycles.

Answers: **(i)** $a = 2$, $b = 3$; **(ii)** $c = 1$.

Question 5

This was well answered and a source of high marks for most candidates. The solution of the simultaneous equations (in either *x* or *y*) was impressive and most candidates proceeded to obtain a correct value for the length of *AB*. A surprising number of candidates eliminated *x* to obtain a quadratic in *y* and solved this correctly only to take the answers as values of *x* rather than *y*.

Answer: 16.7.

Question 6

Many candidates had little idea of how to relate this question to matrices and performed several numerical calculations, with varying degrees of success. The question did request matrix multiplication and purely numerical attempts gained no credit. Of those who could produce correct matrices, many were unable to match relevant columns to rows to obtain a correct product. Of greater concern was the large number of candidates who 'match' matrices that are not compatible for multiplication. Correct notation is essential in a question of this type and candidates who finished the calculation by multiplying a 3 × 1 matrix with a 1 × 3 matrix lost marks, even if the answer was correctly given as a 1 × 1 matrix. The order in which the candidates write their two matrices must be correct. Many candidates ignored the 'percentages' and gave an answer that was 100 times too large.

Answer: $2748.

Question 7

This proved to be a difficult question and there were very few correct answers. Many candidates made no attempt at it. Of the attempts seen, only a few realised that the required triangle was the one with sides 12 and 7 with the angle of 135° opposite the 12. Many others were seen in which the 135° (or 45°) was given as the included angle between the 7 and the 12.

Answer: 020.6°.

Question 8

This proved to be a difficult question with full marks rarely seen. The question required 3 separate calculations – **(i)** solving $(ax+3)\ln x = 0$, **(ii)** differentiating $y = (ax+3)\ln x$, and **(iii)** coping with the gradient of the normal. Most candidates scored some of the available marks through correctly attempting one or two of these processes. The solution of $(ax+3)\ln x = 0$ caused most problems, with most candidates failing to realise that $\ln x = 0$ yielded the solution $x = 1$. The differentiation was generally correct, though failure to use the 'product' formula was common in many scripts. Most candidates realised that the gradient of the line $5y + x = 2$ was $-\frac{1}{5}$ but at least a large minority of them equated this with $\frac{dy}{dx}$, rather than proceeding to the gradient of the normal as 5.

Answer: $a = 2$.

Question 9

(a) This question presented more problems than usual. Most candidates attempted to write down a few terms of the progression – only a minority realised that the 4th term was the term independent of *x*. Errors over the binomial coefficient (which was also occasionally omitted) and in taking $(-2)^3$ as ±2 or as +8 rather than −8 was also common.

(b) Surprisingly this part often led to more marks than part **(a)**. A majority of attempts realised that the value of *k* was $\frac{{}_nC_2}{{}_nC_3}$. The expansion and simplification of this to the required answer however proved to be too difficult for all but a few candidates.

Answers: **(a)** −102; **(b)** $\frac{3}{n-2}$.

Question 10

This was very well answered with most candidates obtaining full marks or very high marks. The use of radian measure was much improved over previous years and the trigonometry needed to evaluate *AB* and subsequently *DB* was generally accurate. In part **(i)** candidates often lost marks through premature approximation of angle *ACB* to either 100° or 1.7 radians. A few weaker candidates also in part **(ii)** took the perimeter of the shaded region as being equivalent to the perimeter of triangle *ABC* − perimeter of the sector *ACD*.

Answers: **(i)** 57.1 cm²; **(ii)** 44.6 cm.

Question 11

Most candidates realised the need to integrate $-\frac{a}{x^3}$ but having obtained $y=\frac{a}{2x^2}$, many omitted the constant of integration or failed to realise that the two given points led to simultaneous equations for *a* and for *c*. Many at this stage started again by putting *a* = 20, integrating and using one of the points to evaluate *c*. In part **(ii)**, many candidates realised the need to integrate but instead of using the expression for *y*, returned to the expression for $\frac{dy}{dx}$ and integrated this again. The use of the limits 2 to *p* and *p* to 5 was very good. There were however only a small minority of correct solutions.

Answers: **(i)** $y=\frac{10}{x^2}+1$; **(ii)** *p* = 3.16 or √10.

Question 12 EITHER

This was the less popular of the two alternatives. Solutions generally fitted into two categories – the question was either completely correct or earned the first 2 marks in both parts **(a)** and **(b)**.

(a) Solutions to part **(i)** were generally correct but it was rare for candidates to realise in part **(ii)** that there were only two possible cases – 'trigonometry and calculus' or 'algebra and calculus' and that these were ${}_8C_8$ and ${}_9C_8$ respectively.

(b) Again part **(i)** was generally correct and there were more candidates obtaining correct solutions to parts **(ii)** and **(iii)**. In part **(ii)** many realised that the required answer was ${}_7P_4$; less however realised that the answer to part **(iii)** came directly from ${}_5P_1 \times {}_7P_4$ or from ${}_8P_5 - {}_7P_5$. Many weaker candidates used combinations rather than permutations for this part of the question.

Answers: **(a)(i)** 495, **(ii)** 10; **(b)(i)** 6720, **(ii)** 840, **(iii)** 4200.

Question 12 OR

This was the more popular alternative and solutions were generally very good. The graphs of lg y against x were accurate and most candidates realised that the equation $y = Ab^x$ reduced to $\lg y = \lg A + x\lg b$. These candidates usually realised that the gradient was lg b and that the intercept on the lg y axis was lg A and subsequent values of a and of B were usually within the acceptable range. A considerable number of candidates reduced $y = Ab^x$ to $y = x\lg Ab$ and eventually decided that the gradient was lg A and that the intercept was lg b.

Part **(iii)** was rarely correct. Many candidates realised that $y = 2^x$ reduced to $\lg y = x\lg 2$, but then plotted this on the original graph with a gradient of 2 rather than a gradient of lg 2.

Answers: **(ii)** A = 5.0 (± 0.2), b = 1.4 (± 0.05); **(iii)** 4.5 (± 0.2).

Paper 0606/02

Paper 2

General comments

The general standard of performance appeared to be not quite as good as in the previous year. Many candidates had difficulties with the first half of the paper, but compensated for this by faring better on the later, and more heavily marked, questions.

Comments on specific questions

Question 1

This proved to be a very demanding question with only a small percentage of candidates able to obtain the correct answer. Although the vector $\overrightarrow{AB}$ was frequently found correctly there were also many errors, usually $3\mathbf{i} + (k - 7)\mathbf{j}$ or $5\mathbf{i} + (k - 7)\mathbf{j}$. Many candidates found k to be –6.2 either from $3\mathbf{i} + (k + 7)\mathbf{j} = 0.6\mathbf{i} + 0.8\mathbf{j}$ or from $(\mathbf{i} - 7\mathbf{j}) + (0.6\mathbf{i} + 0.8\mathbf{j}) = 4\mathbf{i} + k\mathbf{j}$. Very few used the idea of ratio embodied in $3\mathbf{i} + (k + 7)\mathbf{j} = \lambda(0.6\mathbf{i} + 0.8\mathbf{j})$.

The length of $\overrightarrow{AB}$, $\sqrt{3^2 + (k+7)^2}$, was often involved, although this was sometimes taken to be $3 + (k + 7)$.

Few could successfully evaluate k from $\dfrac{3\mathbf{i} + (k+7)\mathbf{j}}{\sqrt{k^2 + 14k + 58}}$ and those who reached k = –11 or –3 almost invariably failed to reject k = –11.

Answer: –3.

Question 2

Here again there were very few correct answers. Weak candidates lost their way very quickly by saying, for example, $10\cos(x + 1) = 6$ or $5\cos x + 5\cos 1 = 6$. Many were able to find the angle corresponding to $\cos^{-1}0.3$, although this sometimes led to $\dfrac{x+1}{2}$ = 72.5° and x = 144°. The value 1.53 was frequently given as the answer - perhaps candidates merely read 'the smallest value of x' and overlooked 'x > 10'. Those who attempted to find larger values of x usually did not know how to proceed correctly.

Answer: 14.1.

Question 3

Perhaps candidates did not quite understand what was required in parts **(i)** and **(ii)**, in that many used words which simply described the set notation e.g. in **(i)** 'intersection', 'null set' and in **(ii)** 'the complement of C'. In part **(i)** some merely interpreted $A \cap B$, stating 'Students who are over 180 cm tall and who are vegetarian', while in part **(ii)** a considerable number of statements contained a reference to vegetarians. Most candidates included $B \cap C$ in their answers to part **(iii)** but usually failed to complete it correctly, often simply leaving a non-statement e.g. $(B \cap C) \cap A$ or $(B \cap C) \cup A$. Among the acceptable alternative answers $(B \cap C) \cap A = \varnothing$ was often seen.

Answers: **(i)** Some vegetarians are over 180 cm tall; **(ii)** No cyclists are over 180 cm tall; **(iii)** $(B \cap C) \subset A'$.

Question 4

The more able candidates had little difficulty, initially converting the left hand side entirely into terms of $\sin\theta$ and $\cos\theta$. Some candidates merely changed $\cot\theta$ to $\dfrac{1}{\tan\theta}$ and then persisted in retaining $\tan\theta$ throughout subsequent manipulation, which almost invariably petered out. Some of the weaker candidates used incorrect substitution e.g. $\sec\theta = \dfrac{1}{\sin\theta}$ or, more commonly, $\sec\theta = 1 + \tan\theta$. Frequently candidates made matters more complicated by failing to find the lowest common denominator, e.g. $\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin\theta - \sin\theta\cos\theta}{\sin^2\theta}$; this process produced even lengthier denominators when early multiplication led to four fractions to be combined.

Question 5

This proved to be a difficult question with many candidates making no attempt or trying to make use of the discriminant '$b^2 - 4ac$', while others substituted c and d but failed to make any headway. Candidates successfully solving the quadratic equation usually continued to complete the question correctly, the exceptions being those who replaced the surds with decimals or 'simplified' them e.g. $\sqrt{20} + \sqrt{12} = \sqrt{32}$ and $\sqrt{20} - \sqrt{12} = \sqrt{8}$. Some candidates understood the relationship between the roots and coefficients of a quadratic equation and neatly obtained the required result via $c + d = \sqrt{20}$, $cd = 2$, although others who obtained these equations could not successfully complete the question.

Question 6

(a) The vast majority of candidates obtained the critical values of -2 and 3.5. Some went no further while many others could not find the correct values, proceeding from $(2x - 7)(x + 2) > 0$ to $x > 3.5$, $x > -2$.

(b) Many candidates answered this correctly by eliminating y and then using '$b^2 - 4ac = 0$'. A few candidates eliminated x but then were usually defeated by the subsequent algebra. Others misused the discriminant taking it to be > 0 or $\geqslant 0$ while, despite the instruction to find ' … the values of k … ', the single answer $\sqrt{3}$ was frequently obtained from $k^2 = 3$. Candidates trying to compare gradients were mostly unsuccessful in that the gradient of $y + kx = 8$ was usually taken as k, but a more serious error, having obtained $\dfrac{dy}{dx} = -\dfrac{1}{2}x$, was to assume $(\pm)k = -\dfrac{1}{2}$.

Answers: **(a)** $\{x : x < -2\} \cup \{x : x > 3.5\}$; **(b)** $\pm\sqrt{3}$.

Question 7

Candidates usually scored quite well on this question with few applying the functions in the wrong order or taking, for instance, fg to be $e^x(2x - 3)$.

(i) Quite a number of candidates appeared to be unaware of the instruction to give ' … non-exact answers correct to 3 significant figures' and so offered 2.5 as the answer. A few candidates evaluated fg (7).

(ii) Although h was usually correct, many candidates did not offer any comment as to the range and correct answers were rarely seen.

(iii) Most candidates understood that natural logarithms were involved but these were sometimes misapplied e.g. $x = 2e^y - 3 \Rightarrow \ln x = \ln 2e^y - \ln 3$ or $x + 3 = 2e^y \Rightarrow \ln(x + 3) = 2\ln e^y$.

Answers: **(i)** 2.47; **(ii)** $2e^x - 3$; $h(x) > -3$; **(iii)** $\ln\left(\frac{x+3}{2}\right)$.

Question 8

(i) There were many correct solutions. Weak candidates simply removed logarithms so that $\log_3(2x + 1) - \log_3(3x - 11)$ became $(2x + 1) - (3x - 11)$; others took $\log_3(2x + 1) - \log_3(3x - 11)$ to be $\frac{\log_3(2x+1)}{\log_3(3x-11)}$ and then 'cancelled' $\log_3$. A fairly common error resulted in $(3x - 11)$ becoming $(3x - 1)$ at some stage and occasionally $\log_3[\ ] = 2$ led to $[\ ] = 2^3$ rather than 3^2.

(ii) Candidates were generally much less successful with this part of the question. The change of base formula was frequently quoted and applied correctly but some candidates simply replaced $\log_4 y$ by the incorrect $2\log_2 y$ or, similarly, $\log_2 y$ by $\frac{1}{2}\log_4 y$. Candidates using lg often replaced lg4 and lg2 by approximate decimal forms and arrived at an incorrect answer. Some applied the change of base formula correctly i.e. $\log_4 y = \frac{\log_2 y}{\log_2 4}$ but then took this to be $\log_2(y - 4)$.

Answers: **(i)** 4; **(ii)** 64.

Question 9

The initial part of this question was rarely done correctly. Most appreciated that $(x \pm 2)^2$ was involved but the constant a was often given as 2, 4 or 6. Some candidates dealt with $x^2 - 4x - 6$ but when finally changing the sign $-\{(x - 2)^2 - 10\}$ sometimes became $10 - (x + 2)^2$; others simply took $x^2 - 4x - 6$ to be the same as $6 + 4x - x^2$.

(i) Most candidates answered this correctly by using the calculus; the y-coordinate or the nature of the turning point was occasionally omitted. Very rarely did candidates take the value of a and of b obtained previously to be the required y- and x-coordinates. In fact very many seemed completely unaware of this feature in that they accepted without comment the fact that their answers to part **(i)** did not tally with their values of a and b.

(ii) Some candidates merely calculated a set of values of f corresponding to the values 0 to 5 of x, offering no specific answer. The values calculated were usually correct although some treated $-x^2$ as if it was $(-x)^2$. The most frequently offered answer was $1 \leqslant f(x) \leqslant 6$.

(iii) Most candidates decided that f did not have an inverse, but reasons were not always given or clear.

Answers: $10 - (x - 2)^2$; **(i)** (2, 10), maximum; **(ii)** $1 \leqslant f(x) \leqslant 10$; **(iii)** No inverse; f is not one-one.

Question 10

This question proved to be a good source of marks with many candidates scoring full marks.

Some candidates had to find the equations of AC and BC in order to obtain the gradients of these lines. Most understood that '$m_1m_2 = -1$' was involved but many of the weaker candidates, having found the gradient of BC to be $\frac{3}{5}$, took the gradient of CD to be $-\frac{5}{3}$ which usually led to (3, 7) being obtained as the coordinates of D; very rarely did a candidate appear to consider this somewhat odd in view of the given diagram. Some of the weakest candidates first obtained spurious coordinates of D and then used these to find the equations of AD and CD. The idea of length was understood by most candidates. Some candidates took each of angles ADC and DAC to be x and then, via $x + x + 90° = 180° \Rightarrow x = 45°$, assumed they had proved triangle ACD was isosceles.

Answers: **(i)** $5y = 3x + 26$, $y = 4x - 22$; **(ii)** (8, 10).

Question 11

Differentiation was very good, posing little difficulty for all but the weakest candidates. However, relatively few obtained the value 5 for k, the difficulty being in understanding that $(2x - 3)^{3/2}$ can be written as $(2x - 3)\sqrt{(2x - 3)}$. A commonly offered value for k was 3. The rest of the question was poorly answered. In part **(ii)**, which was frequently not attempted, common errors were to substitute $x = 6 + p$ in the expression for y or for $\frac{dy}{dx}$, or to assume δx was $6 + p$. Only a minority of candidates obtained $90p$ and few of these realised it was necessary to add this to 189. Those candidates who appreciated the significance of the word 'Hence' understood how to deal with part **(iii)** but there were some answers of 930, i.e., 186×5, rather than 37.2 i.e., $186 \div 5$. Far too many candidates attempted to integrate $x(2x - 3)^{½}$ directly, e.g. $\frac{x^2}{2} \times \frac{(2x-3)^{3/2}}{3}$.

Answers: **(i)** 5; **(ii)** $189 + 90p$; **(iii)** 37.2.

Question 12 EITHER

This was by far the more popular of the two alternatives. A few very weak candidates attempted to apply the constant acceleration formulae but most understood that $a = \frac{dv}{dt}$ and $s = \int v\,dt$. Differentiation and integration of the exponential function proved beyond the capabilities of most of the candidates, so that correct answers were infrequent. Candidates who obtained $a = 5e^{-½ t}$ almost always found it necessary to evaluate t when $v = 8$, rather than make use of the step $e^{-½ t} = 0.2$. Many candidates lost marks after integrating by either saying '$s = 0$ when $t = 0$ therefore $c = 0$' or by failing to use their lower limit, $t = 0$, with the definite integral. The idea that $e^{-½ t}$ tends to zero as t becomes large was clearly not familiar to many candidates and answers such as 0, 9.5, infinity and 'v increases' were fairly common. Graphs rarely gained full marks. Many plotted values of v corresponding to the first few integer values of t; these points were often joined by straight lines. Some candidates produced two straight lines whilst a few offered three straight lines forming a trapezium with the x-axis.

Answers: **(i)** 1 ms^{-2}; **(ii)** 41 m; **(iii)** 10.

Question 12 OR

Most candidates completed part **(i)** successfully using either the quotient rule or the chain rule; weaker candidates trying to reach the given answer failed to include minus signs. Again, most candidates dealt with part **(ii)** correctly. In part **(iii)** quite a number of candidates failed to see the connection with part **(i)** and virtually repeated the work in dealing with $\frac{d}{d\theta}\left(\frac{2\sec\theta}{3}\right)$; the standard result $\frac{d}{d\theta}(\tan\theta) = \sec^2\theta$ was clearly not familiar to all candidates. Some did not realise that $\frac{dT}{d\theta}$ was necessary. Some used $PX = 1.5$ km to find the corresponding value of θ and attempted to verify that $\frac{dT}{d\theta}$ was 0 for this value of θ.