# ADDITIONAL MATHEMATICS

**Paper 0606/11**

**Paper 11**

## Key messages

In order to achieve well in this syllabus, candidates need to be reminded of the necessity of making sure that they have fully answered each part of a question.

Candidates should also be encouraged to work to more than 3 significant figures (where appropriate) whilst working through a question and to give their answer in the form as stated on the front of the question paper, as marks can easily be lost by not working to the required level of accuracy.

Candidates need to ensure that they have their calculator in the correct mode. The importance of distinguishing between radians and degrees must be emphasised e.g. when working with calculus, all angles should be in radians.

## General comments

Candidates have benefitted from the new format of the question paper, answering on the question paper itself. There was less scope for misreading questions and candidates appeared to give more thought as to how to present their answers in the given space on the paper.

Timing did not appear to be an issue, with most candidates having attempted all the questions required with differing levels of success. The standard of the scripts produced showed that most candidates were aware of the syllabus requirements and were able to apply the required mathematical techniques both appropriately and correctly.

## Comments on specific questions

### Question 1

**(a)(i)** Candidates were usually able to obtain the least value correctly, having more difficulty with the greatest value.

**(ii)** Candidates were usually able to obtain the greatest value correctly, having more difficulty with the least value.

**(b)** Most candidates were able to produce a completely correct Venn diagram. However, there were some who omitted to enclose their sets in the rectangle representing the universal set; an omission that was penalised.

*Answers*: **(i)** 7, 0 **(ii)** 22, 15

### Question 2

This was usually done very well, with most candidates attempting a correct method of solution. Most incorrect solutions were the result of arithmetic slips or errors in algebraic manipulation.

*Answer*: – 19

**Question 3**

**(i)** Correct values for the gradient $m$ appeared in the great majority of solutions. Most candidates were able to calculate the value of the intercept on the $y$-axis, but were unsure as to how to deal with the log $c$. Many solutions contained the incorrect statement $c = \log(-0.6)$ and few correct values for $c$ were seen.

**(ii)** Many candidates were able to use their value of $m$ and of $c$ and use a correct approach to obtain $N$ in terms of $t$. Credit was given if this was done correctly.

*Answers*: **(i)** $\lg N = 4\lg t - 0.6$, $c = 0.251$ **(ii)** $N = 0.251t^4$

**Question 4**

Parts **(i)**, **(ii)** and **(iii)** were usually well done. Some candidates split their working into smaller parts, itemising different types of numbers, but most were able to write down their answers straight away, using factorial notation and then going on to evaluate these answers correctly.

**(iv)** This part was rarely done correctly; many candidates did not appreciate the fact that at least two different cases had to be considered i.e. an odd number first, then an even number first or equivalent. A mark allocation of 3 marks should have been an indication that a little more work was needed for this part of the question.

*Answers*: **(i)** 720 **(ii)** 240 **(iii)** 480 **(iv)** 168

**Question 5**

**(i)** Most candidates realised that calculus was involved in this question and attempted differentiation. Some of the notation used was incorrect but the required result was usually obtained correctly

**(ii)** Most candidates realised that they needed to equate their velocity equation from part **(i)** to zero and solve. This was an occasion when candidates needed to realise that when trigonometric functions together with calculus are used then the angles must be in terms of radians unless specifically told otherwise. As a result of this not being realised, there were too many incorrect answers of 45.

**(iii)** Most candidates realised that a substitution of their answer to part **(ii)** was needed. There were instances of calculators being used in the wrong mode, resulting in an incorrect answer. The majority of candidates also did not take into account that when $t = 0$, the particle is already some distance from $O$ and that this needed to be taken into account.

**(iv)** Usually done very well. Most candidates attempted to differentiate with respect to $t$ a second time, make the appropriate substitution and usually obtaining the required result.

*Answers*: **(i)** 2 **(ii)** $\frac{\pi}{4}$ **(iii)** 1 **(iv)** 4

**Question 6**

**(a)** Correct basic equations in terms of $p$ and $q$ were obtained by most candidates. Problems arose when solving these equations with calculators in the wrong mode.

**(b)** Most candidates were able to write down the value of $b$, realising that it was related to the period of the function. Different approaches could be used to obtain the value of $a$ and of $c$. It was intended that candidates recognised $a$ as the amplitude and then go on to use the maximum value of the function to obtain $c$. The constants $a$ and $c$ were often muddled.

*Answers*: **(a)** $p = -2$, $q = \frac{\pi}{12}$ **(b)** $a = 4$, $b = 5$, $c = 7$

**Question 7**

**(i)** An equation involving the term in $x^2$ from the binomial expansion of the given expression and $\frac{3}{5}$ was obtained by most. Most candidates realised that $\frac{x^2}{25}$ was involved, and few made the error of failing to 'square the 5'. This meant that most were able to obtain the equation $\binom{n}{2} = 15$ or equivalent. It had been expected that candidates would form a quadratic equation in $n$ and obtain the result $n = 6$ from this. This method was used by very few, the great majority of candidates were able to recognise, or use trial and improvement, to obtain $n = 6$. This was quite acceptable.

**(ii)** This part proved difficult, with few candidates understanding what was being asked of them. Many candidates worked out the coefficient of $x$. From the minority that did realise that a term not involving $x$ was required there were even fewer who realised that it was necessary to consider three separate terms in order to get the required result. More experience of this type of question would be beneficial to candidates.

*Answers*: **(i)** 6 **(ii)** – 5

**Question 8**

**(a)** Very few candidates attempted to expand out the expression before integration. Most treated it as an inverse form of the chain rule. Most however, were able to achieve a mark for correct use of limits in their result, also correctly using the fact that $e^0 = 1$ and not zero, usually a common error.

**(b)** Usually done extremely well with most recognising that it was an 'inverse chain rule'. There were errors involving the numerical factor but most candidates scored highly.

*Answers*: **(i)** 41.6 **(ii)** $y = \frac{1}{2}\sqrt{4x+1} + 3$

**Question 9**

**(i)** The use of a correct identity was usually made, together with the correct use of cosec*x* or equivalent. Many candidates however were unable either to obtain the equation $\sin^3 x = \frac{1}{8}$ or, if they did, to solve it correctly. There were many examples of poor algebraic manipulation.

**(ii)** This was usually done very well. Most candidates correctly obtained the required form of an equation involving tangent. There were some errors in finding both the required angles because of the negative nature of the trig ratio, but most used a correct order of operations and dealt with radians correctly.

*Answers*: **(i)** $30^{\circ}$, $150^{\circ}$ **(ii)** 1.27, 2.84

**Question 10**

Both of these questions were found difficult by the candidates. They appeared to be unable to formulate a logical plan of action in spite of the structured nature of the questions. More work on longer questions would help candidates to perform more successfully in this type of question.

**EITHER**

**(i)** Subtraction of two small sector areas from the larger sector were not recognised as a viable way of expressing $\theta$ in terms of $r$. Most were able to express $P$ in terms of $\theta$ and $r$, but were then unable to reach the required result.

**(ii)** Differentiation of the given equation, with respect to $r$, was usually correct although the solving of the resulting equation was often done poorly. At this point, most candidates did not complete the task. The question asked for the value of $P$ in this instance. This was rarely done.

**(iii)** Most candidates were successfully able to differentiate a second time and in the majority of cases determined the nature of the turning point correctly. Again, this was another example of the candidates failing to complete the task, as they were asked for the value of $\theta$ for which this stationary value occurred.

*Answers*: **(ii)** $8\sqrt{2}$ **(iii)** minimum, 0.5

**OR**

**(i)** Most candidates were unable to write down the length of $OC$ in terms of $r$. Many started to attempt to use Pythagoras' theorem with little success. Candidates should be aware of the nature of the instructions given in questions. The instruction to 'Write down' means just that. The candidate should be able to look at the problem and be able to write down immediately what is required. The mark allocation to each part is also an indication of the amount of work necessary.

**(ii)** Most started off by obtaining an expression for $\sin\theta$, but were often unable to obtain the required result usually because of poor algebraic manipulation.

**(iii)** Most candidates were able to attempt a correct differentiation of a quotient. They were then expected to find the value of $\theta$ for which $r = \frac{10}{3}$. This was not recognised as the next appropriate step to take by many.

**(iv)** A correct method was used by most who attempted this part of the question.

*Answers*: **(i)** $10 - r$ **(iii)** $\frac{20\sqrt{3}}{9}$ (3.85) **(iv)** $\frac{3\sqrt{3}}{10}$ (0.520)

# ADDITIONAL MATHEMATICS

**Paper 0606/12**

**Paper 12**

## Key messages

In order to achieve well in this syllabus, candidates need to have a good knowledge of the entire syllabus and to appreciate the importance of developing their algebraic skills.

Candidates should also be encouraged to work to more than 3 significant figures (where appropriate) whilst working through a question and to give their answer in the form as stated on the front of the question paper, as marks can easily be lost by not working to the required level of accuracy.

## General comments

Candidates have benefitted from the new format of the question paper, answering on the question paper itself. There was less scope for misreading questions and candidates appeared to give more thought as to how to present their answers in the given space on the paper. Attention must be drawn to the instructions on the front of the paper. It is expected that candidates use either a dark blue or black pen for writing their solutions. Pencil should only be used for diagrams or graphs. Too many candidates were working their solutions in pencil and not in pen. Very often there was work of merit in the pencilled work (this was given full credit where appropriate), but to be sure that all written work is clearly visible, it is important that candidates use the appropriate writing implements when completing their papers.

Candidates found both the options for the final question difficult, with some candidates making little or no attempt at either option. Those with good algebraic skills and a thorough knowledge of the syllabus content usually did well with either of the two options.

## Comments on specific questions

### Question 1

Completely correct solutions were few as many candidates found it difficult to deal with ‘fractions within a fraction’. Most candidates chose to deal with the expression in terms of sine and cosine. A common error was to include the incorrect statement $\frac{1}{\tan\theta + \cot\theta} = \frac{1}{\tan\theta} + \frac{1}{\cot\theta}$. Most candidates however, were able to deal correctly with the resulting incorrect fractions and make use of an appropriate identity. Credit for these correct methods was given.

### Question 2

Candidates appeared to be well prepared for this type of question with many obtaining completely correct solutions. It was easier to obtain a quadratic equation in terms of $y$ rather than $x$ and fewer errors were made when this approach was used. There were of course both algebraic and arithmetic slips in the simplifying of the quadratic equation especially when an equation in $x$ was being attempted. Errors in the expansion of $\left(\frac{x-1}{2}\right)^2$ were the most common, as were mistaking $y$ for $x$ in the solution of the quadratic equation in terms of $y$.

*Answer*: $(5, 2), \left(-\frac{23}{5}, -\frac{14}{5}\right)$

**Question 3**

**(i)** Most candidates were able to obtain the correct result which, as stated in the second part of the question, was meant to help candidates.

**(ii)** In spite of obtaining a correct result for part **(i)**, few candidates were able to use the given substitution correctly and were thus unable to obtain the intended three term quadratic equation. Of those that did complete this first part of the solution, many forgot to carry on and actually obtain a solution for $x$. Many of those who did obtain a solution for $x$ were unable to deal with the base 2 logarithm correctly.

*Answer*: **(i)** $\dfrac{1}{\log_2 x}$ **(ii)** 2, 4

**Question 4**

A correct approach was adopted by most candidates, with the occasional slip in the application of the chain rule when attempting to find the gradient of the given curve. Usually candidates were able to employ a correct method throughout, sometimes being let down by being unable to obtain the correct numerical values of the gradient of the normal and the value of the $y$-coordinate at the given point.

*Answer*: $8y + 3x = 78$

**Question 5**

**(i)** Many completely correct solutions were seen, with most candidates clearly understanding the methods needed to deal with this type of question. Too many candidates though, thought that $y^2 = 5(2^x) + 1$ simplified to $y^2 = 10^x + 1$, and more practice using indices would benefit such candidates.

**(ii)** A correct approach was attempted by most; however, few obtained the required result mainly due to the incorrect simplification mentioned in part **(i)**. Of those that did deal with the correct equation, many did not give their final answer to the required level of accuracy, giving an answer of 2.8 instead of the required 2.81.

*Answers*: **(i)** $y^2 = 5(2^x) + 1$ **(ii)** 2.81

**Question 6**

**(i)** Correct binomial expansions were attempted by most candidates with only the occasional arithmetic or algebraic slip.

**(ii)** Most used their result from part **(i)** to obtain a quadratic equation in either $x^2$ or $y$ and then attempted to solve this equation. Candidates must realise that not all quadratic equations can be solved by factorising and this was an occasion when the quadratic formula or completion of the square needed to be used. Many correct applications of the quadratic formula were seen, but many did not subsequently solve for $x$, having forgotten that the solution was in terms of $x^2$. Of those that did solve for $x$, many did not give both the positive and negative roots.

*Answers*: **(i)** $A = 486$, $B = 540$, $C = 30$ **(ii)** $\pm 1.02$

**Question 7**

**(i)** Poor algebraic skills meant that many candidates were unable to gain full marks for this part of the question.

**(ii)** Many candidates did not recognise that the gradient function of the curve involved was of the form they had been asked to simplify in part **(i)**, choosing erroneously to attempt an 'inverse chain rule' approach. Many candidates misinterpreted the question, choosing to find the equation of the tangent to the curve at the given point.

Answers: **(i)** $16x^{-\frac{1}{2}} - 8 + x^{\frac{1}{2}}$ **(ii)** $y = 32x^{\frac{1}{2}} - 8x + \frac{2}{3}x^{\frac{3}{2}} - 12$

**Question 8**

Although this was quite a basic coordinate geometry question, very few completely correct solutions were seen. Candidates should be encouraged to draw a simple diagram showing all the given information. This can be added to as the question progresses and thus a clearer understanding of the problem would be achieved.

**(i)** As in previous examinations, candidates gave equations for lines that were perpendicular to the given line, but did not bisect this line. Candidates need to ensure that they read the question carefully before starting.

**(ii)** While many were able to find the coordinates of the point *C*, few correct coordinates for the point *D* were seen. A diagram drawn by candidates would have helped with what was a relatively straightforward ratio problem.

**(iii)** A matrix approach was adopted by most and credit was given if this had been applied correctly, as was the less common approach of 'a half the base times the height'.

*Answers*: **(i)** $4y + 3x = 2$ **(ii)** *C* (–2, 2), *D* (10, –7) **(iii)** 37.5

**Question 9**

**(i)** Most candidates recognised that the differentiation of a product was needed and, as a result, this part was usually done well, with only the occasional numerical slip.

**(ii)** This part was found difficult. Most candidates were unable to use their, often correct, result for part **(i)**, by re-writing it and then further manipulating it in order to obtain the required integral. Many 'started again', ignoring the 'Hence' instruction. Most however were able to obtain credit for correct application of limits when integrating.

*Answers*: **(i)** $4x \cos 4x + \sin 4x$ **(ii)** 0.0357

**Question 10**

**(i)** Usually done well, most candidates were able to use a correct trigonometric identity and go on to solve a three term quadratic equation in terms of tangent. Those candidates that chose to deal with the equation in terms of sine and cosine usually fared less well, very often not being able to do enough work to obtain a three term quadratic equation in terms of one variable.

**(ii)** Most candidates were able to use a correct method of approach, dealing with $\sqrt{2}$ correctly and going on to use a correct order of operations which, in most cases, led to the result of –0.524, which is out of the given range. Credit was given for achieving this result. Many candidates were then unable to find a correct solution within the given range, those that did usually found 2.62 with the second solution of 12.0 rarely being seen. As with previous questions, some candidates gave an answer of 2.6 or 2.61, having rounded prematurely and were thus penalised. Answers in terms of $\pi$ were acceptable.

*Answers*: **(i)** 71.6°, 153.4°, 251.6°, 333.4° **(ii)** $2.62^c$, $12.0^c$

 

**Question 11 EITHER**

**(i)** A simple substitution of $x = 0$ and $y = 4$ was required, but many candidates did not recognise this.

**(ii)** Few candidates attempted to differentiate the given equation as a product. Those that did usually went on and achieved the required result, provided a correct value for $A$ had been obtained. Verification of the given value for $B$ was acceptable.

**(iii)** Those candidates who actually attempted this part usually applied a correct method to the problem, obtaining an equation in tan $2x$ which was then solved.

*Answers*: **(i)** $A = 4$ **(ii)** $0.216^c$

**Question 11 OR**

**(i)** Most candidates attempted a differentiation of a quotient with varying levels of success. Most errors occurred when differentiating the logarithmic function. As the answer was given, there were quite a few contrived 'correct' results fortuitously obtained from incorrect work.

**(ii)** Correct applications of small changes were used by most candidates with arithmetic slips involving the use of $x = \sqrt{5}$ being the main source of errors together with inaccurate answers, $-0.11p$ being a common answer.

**(iii)** Most candidates equated the given gradient function to zero. A correct value for $k$ was unnecessary for this part of the question. Few correct solutions were seen as candidates appeared unable to solve either an equation involving a quotient equal to zero, or an equation involving a natural logarithmic function equal to 1. In spite of being asked for the coordinates in terms of e, many still chose to use their calculator to give decimal answers.

*Answers*: **(i)** $k = 2$ **(ii)** $-0.108p$ **(iii)** $\left(\sqrt{1+\mathrm{e}}, \frac{1}{\mathrm{e}}\right)$

# ADDITIONAL MATHEMATICS

**Paper 0606/13**

**Paper 13**

## Key messages

In order to achieve well in this syllabus, candidates need to be reminded of the necessity of making sure that they have fully answered each part of a question.

Candidates should also be encouraged to work to more than 3 significant figures (where appropriate) whilst working through a question and to give their answer in the form as stated on the front of the question paper, as marks can easily be lost by not working to the required level of accuracy.

Candidates need to ensure that they have their calculator in the correct mode. The importance of distinguishing between radians and degrees must be emphasised e.g. when working with calculus, all angles should be in radians.

## General Comments

Questions involving the word "hence" require the candidate to use the preceding result to answer this next part of the question. Candidates will not be credited for alternative methods in such questions.

Candidates should be aware that if they are asked to 'show that...', they will not be able to gain full marks if they appear to miss out necessary steps. They should also remember that method marks are often available if an answer is incorrect, but these cannot be awarded if the method is not clear. The most common example of this occurs if a candidate makes an error forming a quadratic equation. If he/she then factorises and solves, or shows use of the quadratic formula the method mark(s) are earned, but if only the answers appear, no method mark is possible.

## Comments on specific questions

### Question 1

The majority of candidates had some success with this question, most of them evaluating $p$ and/or $q$ correctly. However, a large number of candidates dealt incorrectly with the numbers, with $a$ = 3 being commonly seen.

*Answers*: $a = 648$, $p = \frac{1}{2}$, $q = \frac{21}{5}$

### Question 2

Some candidates turned sec$\theta$ into $\frac{1}{\cos\theta}$ and simplified the resultant expression, others wrote the numerator as $\sin^2\theta$ the denominator as $4\tan^2\theta$ and worked with these.

Common errors included not taking note of the square root sign, misuse of the 4 and the attempt to square root the expression as $\frac{1-\cos\theta}{2\sec\theta - 2}$.

*Answer*: $\frac{1}{2}\cos\theta$

**Question 3**

**(i)** The word inverse did not appear in the question and some candidates did not recognise that this was what was wanted. It is important that candidates understand standard mathematical notation. This part was, in general, well done by those candidates that did recognise what was required. Candidates should be made aware that it was not necessary to take the fraction $\frac{1}{16}$ into the matrix and, indeed, not doing so made the calculations easier in part **(ii)**.

**(ii)** This part of the question started with the word "Hence". Large numbers of candidates chose to ignore this instruction and to create a matrix with four unknown elements and to form, and solve, four simultaneous equations. As remarked upon in the general comments, they could not earn any credit for such a method.

Of the candidates who used the inverse, as required, there was confusion for some who assumed commutativity of matrices and post-multiplied the given matrix by the inverse.

*Answers*: **(i)** $\mathbf{A}^{-1} = \frac{1}{16}\begin{pmatrix} -2 & -3 \\ 8 & 4 \end{pmatrix}$ $\mathbf{M} = \frac{1}{16}\begin{pmatrix} -8 & -17 \\ 16 & 44 \end{pmatrix}$

**Question 4**

**(a)** This part was found difficult. Candidates needed to show their working more clearly in order to gain partial credit for a correct method.

**(b)** In parts **(i)** and **(ii)**, candidates were required to give their answer in terms of π. They should be aware that if a particular form for an answer is required, alternative forms will not be credited.

**(i)** For many candidates, the problem here was not trigonometry in a question on sets but rather the lack of familiarity with the basic trigonometrical functions when linked to angles such as, in this case, $\frac{\pi}{6}$.

**(ii)** Most candidates found this part more difficult as $\cos\theta$ could have two values and therefore there were four elements in set $Y$.

**(iii)** Candidates should be aware that listing the elements of either the intersection or the union of sets $X$ and $Y$ does not "describe the relationship" between them. Only a minority of candidates used the subset idea but even some of these failed to use the correct notation for a proper subset.

*Answers*: **(a)** 6

**(b) (i)** $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **(ii)** $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **(iii)** $X \subset Y$ or $X \cap Y = X$ or $X \cup Y = Y$

**Question 5**

**(i)** Although some candidates were quite comfortable with logarithms, many were less so. Two things were necessary to make progress: splitting $p$ and $q$ and dealing with powers. Most of those who attempted this question managed at least one of these but without both no further progress was possible. Some stayed with powers obtaining $p^7 = q^{21}$ but from this could not answer this part of the question, although they could go straight to part **(ii)** from here.

**(ii)** A simple step for one mark. Candidates either took the latter approach to part **(i)** or divided their lg $q$ by their lg $p$.

*Answer*: **(i)** $\lg p = 3a$, $\lg q = a$ **(ii)** $\frac{1}{3}$

**Question 6**

**(i)** Ostensibly a very easy five marks requiring the knowledge that $\frac{\mathrm{d}}{\mathrm{d}x}\cos kx = -k\sin kx$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sin kx = k\cos kx$, followed by the use of $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

Many candidates made mistakes, some of which (e.g. both signs wrong, or leaving the coefficients as –6 and 4) led to a fortuitously correct value for *x*. Some candidates clearly misunderstood "stationary point" and simply put $y = 0$.

**(ii)** Some candidates did not recognise the term "nature" – a common answer being 'nature is real and positive' or similar. The majority did, however, attempt to find the sign of the second differential. The main problem here was that some had no *x*-value from part **(i)** to substitute, while others omitted to show the substitution merely stating 'positive therefore minimum' or 'negative therefore maximum'. No credit could be given for guesswork; as the question requested candidates to 'determine' rather than 'state', evidence of working was required.

*Answers*: **(i)** $x = 1.18$ **(ii)** maximum

**Question 7**

There were a large number of incorrect approaches to answering this question, together with many candidates making unfounded assumptions. Some assumed the size of angle *A* or *B*, others assumed $BC = AM$, others that the area was half the product of the difference in *x*-values of *A* and *B* with the difference in *y*-values of *B* and *C*.

Two main methods were used for finding point *B* and then point *C*. The first, and more common, required finding the equation of *BC* and finding where this crosses the *x*-axis. The other, having found *B* was to call *C* ($x$, 0) and to apply Pythagoras' Theorem to triangle *ABC*, the solution giving the point *C*.

To find the area, knowing *B* and *C*, was straightforward using half the product of *BC* and *BA*. A large number of candidates used the alternative matrix method.

*Answers*: 20.8.

**Question 8**

**(i)** The great majority of candidates started this question and most were able to form two linear simultaneous equations in two unknowns which they then proceeded to solve. Some made it harder for themselves by substituting from one equation into the other before attempting any simplification. Other candidates filled the space available with some disorganised algebra where the answers were difficult to locate. Candidates need to organise their work logically so that it can be easily followed and credited where appropriate.

**(ii)** The better candidates used their answers for *m* and *n* to work out the vector *b* and found its magnitude. Weaker candidates often used the original form for *b* in terms of *m* and *n* and found the magnitude in general terms, sometimes substituting their *m* and *n* values, sometimes not.

Many candidates obtained something other than $\sqrt{320}$ (which gives $k = 8$) and tried to express their answer as a multiple of $\sqrt{5}$ but this was rarely an integer.

**(iii)** For those candidates who knew what a unit vector is this was an easy mark.

*Answers*: **(i)** $m = 2$, $n = 6$ **(ii)** $k = 8$ **(iii)** $\frac{1}{8\sqrt{5}}\begin{pmatrix} -8 \\ 16 \end{pmatrix}$

**Question 9**

**(i)** Well prepared candidates found this part straightforward. Several candidates, who otherwise used degrees in the question, seemed to think, incorrectly, that the period had to be in radians.

**(ii)** Again, well prepared candidates answered this part well, although finding the angles caused problems for many candidates.

**(iii)** For a sketch, candidates were expected to draw a reasonable curve over the given domain embodying the correct answers to parts **(i)** and **(ii)** with the extra fact that the curve does not begin at the origin but below it at (0,–1).

Attempts ranged from two axes drawn freehand, with no markings, and a curve of irregular amplitude and/or period to carefully drawn and labelled axes with a scale and table of values. Neither of these were appropriate for a sketch.

Some candidates drew three or even four curves on one diagram and compounded this by not labelling the one needed. Drawing over the final curve in ink or rubbing out unwanted lines would have helped.

*Answers*: **(i)** Amplitude = 2, period = 120° **(ii)** 1, x = 30°, 150°, and 270°

**Question 10**

**(a)** While the majority of candidates went for the straightforward approach, some candidates differentiated as a quotient i.e. $\frac{\sin(3x+2)}{\cos(3x+2)}$, with varying degrees of success.

**(b)** As with part **(a)** the main error was the failure to deal with the function within the function, with candidates not multiplying by the differential of $\sqrt{x}$ .

**(c)** The question did not ask for simplification and candidates who tried, unnecessarily, to simplify and made errors doing this were not penalised. Even so, it was often unclear whether candidates were differentiating as a product, as a quotient, or something else. A large number of candidates did not deal correctly with the quotient rule and there were fewer errors made by those using the product rule with the second function being $(2x+3)^{-1}$. The extra complication here was the nature of the first function $\ln(x^3-1)$ itself being a function of a function.

*Answers*: **(a)** $3\sec^2(3x+2)$ **(b)** $\frac{2}{3}\times\frac{1}{2}x^{-\frac{1}{2}}\left[x^{\frac{1}{2}}+1\right]^{-\frac{1}{3}}$ **(c)** $\frac{(2x+3)\frac{3x^2}{x^3-1}-2\ln(x^3-1)}{(2x+3)^2}$.

**Question 11**

**(i)** Some candidates ignored calculus entirely in this question, and assumed either constant velocity or constant acceleration throughout. The majority of candidates used $t = 0$, although some could not deal with $e^0$ , while others assumed the initial velocity to be $v = 0$.

**(ii)** Of those candidates who knew to differentiate not all were able to deal with the first term, with the power being changed in some cases.

**(iii)** Integration was carried out by a majority of candidates but the integration of an exponential term was quite poorly done in many cases.

There was a lack of understanding of "the third second", with many candidates merely substituting $t = 3$ or maybe $t = 3$ and $t = 0$. The difference between the distance covered after three seconds and that after two seconds was required.

Candidates using limits ignored '+ $c$' quite rightly; those not using limits varied in their approach – some never considered a constant of integration while those who did, failed to get it right in many cases.

*Answers*: **(i)** 3 $\text{ms}^{-1}$; **(ii)** 10 $\text{ms}^{-2}$; **(iii)** 533 m.

**Question 12**

A small majority of candidates opted for **EITHER** rather than **OR**. The same errors tended to occur in both with many candidates seemingly lacking confidence working with exponential functions and with natural logarithms. There was also confusion over how to write the answers for range and domain. Some candidates also lost marks by not giving inverse functions in terms of $x$. It is worth noting that a significant number of candidates failed to recognise the notation $f^{-1}(x)$ as being the inverse – some treating it as the differential, others as $\frac{1}{f(x)}$. The majority of candidates would benefit from further familiarity with the functions and the exponential function and natural logarithm sections of the syllabus.

**12 'EITHER'**

**(i)** Few correct answers were seen, candidates failing to realise that the set of real numbers constituted the range.

**(ii)** At some stage candidates had to introduce $e^y$. Having rearranged the equation and interchanged $y$ and $x$ they would have an answer. Some candidates wrote e$y$, seemingly a product not a power of e, an error confirmed by subsequent working.

**(iii)** Apparent lack of familiarity with the topic meant that only a minority of candidates recognised that the range of the inverse function was the domain of the original function. Of those that did recognise this, some wrongly gave $x > 2$ rather than $f^{-1}(x) > 2$.

**(iv)** Many candidates managed $\ln(5x–10) = 0$ but failed to progress. A common incorrect answer was $x = 2$.

**(v)** This was well done by those candidates who progressed this far, the main errors being effectively using fg($x$) rather gf($x$) and treating $f(x^2)$ as $f^2(x)$.

Solution of the quadratic formed by the candidates was often missing with answers simply 'appearing'.

*Answers*: **(i)** $\mathbb{R}$; **(ii)** $\frac{e^x + 10}{5}$ **(iii)** $f^{-1}(x) > 2$ **(iv)** 2.2 **(v)** 5.10

**12 'OR'**

**(i)** An easy mark for candidates but responses were often written using incorrect notation.

**(ii)** Moving from exponentials to natural logarithms caused problems for some candidates, while the negative power caused problems for others. Candidates would benefit from more practice on these areas of the syllabus.

**(iii)** Candidates needed to introduce In, then rearrange and interchange $y$ and $x$. This part was not done well.

 

**(iv)** Many candidates did not appreciate the link between the range of a function and the domain of the inverse.

**(v)** Some candidates used the suggested substitution, others chose not to. Either way they had to deal with a negative power which some found difficult. Candidates, especially weaker ones, would be advised to use hints, such as suggested substitutions, when answering questions as they are designed to help candidates towards efficient methods.

A number of candidates found solutions for $t$ or for $e^x$ then stopped without finding $x$.

*Answers*: **(i)** $f(x) > 2$ **(ii)** $x = -1.79$ **(iii)** $f^{-1}(x) = \ln \frac{4}{x-2}$ **(iv)** $x > 2$ **(v)** 1.27

# ADDITIONAL MATHEMATICS

**Paper 0606/21**

**Paper 21**

## Key messages

In order to succeed in this paper, candidates need to have a good understanding of the whole of the syllabus. They should also ensure that they follow instructions in questions as to the use of calculators and the form in which answers should be given. With the advent of more sophisticated calculators it has become even more important for candidates to show their working. They should be reminded that an incorrect answer with no working will not be awarded any marks, whereas candidates can be awarded marks for working even if they get an incorrect answer.

## General comments

A number of candidates gained good marks in this paper, producing high quality, well presented work and displaying an impressive range of knowledge and skills. However, there were also some candidates who did less well, who would have benefitted from more preparation for the examination.

Facility with the use of a calculator is essential in a mathematics examination. However, there were two questions on this paper where candidates were expressly instructed not to use a calculator. The purpose of these questions was to test knowledge of specific relationships and operations. Candidates should be aware that marks will be lost when the instruction has not been followed.

Candidates also need to be aware of when the use of a calculator is inappropriate; for example, in the option questions, where answers were required in terms of specific quantities (ln2, e). In these situations, the use of decimal equivalents from the calculator does not gain credit.

Centres should be aware that extra paper should not be given to candidates, unless they genuinely need it, for example, to draw a graph again on which a mistake has been made.

## Comments on specific questions

### Question 1

Many candidates realised that there are two solutions to this problem, although some did not. The two equations to be solved were $4x - 5 = 21$ and $4x - 5 = -21$. The solution most commonly missed was $x = -4$.

*Answers*: 6.5, –4

### Question 2

The most common method used was to eliminate $y$, collect terms together on one side, then use the condition for tangency, that is $b^2 = 4ac$. It was necessary to give $k$ explicitly in terms of $c$ to obtain full marks and candidates should ensure that they give the answer in the required form.

*Answer*: $k = c + 9$

**Question 3**

This was one of the two questions referred to above which candidates were instructed to solve without using a calculator. This was done in order to test their ability to expand a bracket containing a surd expression, and to rationalise a fraction containing surd expressions in both the numerator and denominator. All steps in the working had to be shown for full marks to be obtained. The solution begins with the application of the cosine rule, on which some made errors, even though the formula is given on the formulae page. There were also instances of candidates assuming that the triangle was right angled. Candidates would benefit from being familiar with the formulae provided on the question paper. They also need to remember not to make assumptions that cannot be justified.

*Answer*: $\dfrac{-4+3\sqrt{3}}{2}$

**Question 4**

The majority of candidates managed to answer part **(i)** correctly. The result obtained in part **(i)** was then to be used in part **(ii)** as indicated by the word "hence" in the question. Candidates must appreciate the importance of this word in examinations in that it is an instruction to make use of what has just been done in doing the next thing. If they choose to use a different method, they may not be awarded any marks. Those who were able to make the connection between the answer of part **(i)** and the integral to be found in part **(ii)** were able to solve part **(ii)** easily. Those who ignored the instruction, or who did not notice the connection and started again, inevitably went wrong, and presented much incorrect work.

*Answers*: **(i)** $k = -2$ **(ii)** $-\dfrac{3}{x^2+3}+c$, 0.5

**Question 5**

Most answers to part **(a)**, and many answers to part **(b)(i)**, were correct. There were far fewer correct answers to parts **(b)(ii)** and **(iii)**. One approach to questions such as part **(b)** is to try out various composites, and see if they produce the required mapping. It was pleasing to see some candidates checking their answers in this way.

*Answers*: **(a)** 33 **(b)(i)** kh **(ii)** $h^2$ or hh **(iii)** $h^{-1}k^{-1}$ or $(kh)^{-1}$

**Question 6**

Some good work was seen on this question by candidates who were clearly well practised in coordinate geometry. A diagram is extremely helpful in problems such as this, and many candidates did in fact sketch one. From a diagram it then becomes clear that the intersection point of *AD* and *CD* has to be found. Some who did not make a diagram found, and used, the equation of *AB*, which did not help them to solve this particular problem. The equation of *BC*, similarly, was not useful. Candidates would benefit from practice producing sketches for this type of question, as they are invariably helpful.

*Answer*: (7,1)

**Question 7**

The required expression in part **(a)** could be obtained in two short steps, by using the expression for $\text{cosec}^2 x$ from the formulae page, then writing $\cot^2 x$ as $\dfrac{1}{\tan^2 x}$. Some candidates did not use this given identity. Work seen in part **(b)** was of variable quality. The best way to work, as is usually the case with proofs such as this, is from left to right, expanding the brackets, then simplifying the four terms obtained. Weaknesses seen were due both to lack of knowledge of trigonometric relationships, and to deficiencies in skills in basic algebraic manipulation. More practice in this type of question would help candidates achieve higher marks.

*Answer*: **(a)** $\dfrac{p^2+1}{p^2}$

**Question 8**

Few candidates obtained full marks on this question, and some omitted it altogether, indicating a general lack of knowledge of vector methods and a need for more practice. In part **(i)**, an expression for vector *OP* is first found, from the sum of vectors *OA* and *AP*, the given expression for vector *AP* being used to express vector *AP* in terms of **a** and **b**. From the given information in part **(ii)** another expression for vector *OX* is found. The solution is completed by setting the two expression for vector *OX* equal to each other, then equating components of **a** and **b**. Many candidates who had progressed well through the question up to the final stages, setting up a correct relationship involving **a**, **b**, $\mu$ and $\lambda$, seemed unaware of the method of equating components, an important technique in vector work.

*Answers*: **(i)** $\mu\left(\frac{3}{5}\mathbf{a}+\frac{2}{5}\mathbf{b}\right)$ **(ii)** $\mu=\frac{5}{3}$, $\lambda=\frac{2}{3}$

**Question 9**

Graphical work was generally very good, and in part **(iii)** there was good recognition of the relevance of gradient and intercept for finding *a* and *b*. Unfortunately these were sometimes identified the wrong way round. In part **(iv)**, the most straightforward way to estimate the value of *y* was to use the original equation with the values of *a* and *b* which have been found, and substitute $x = 1.5$. The answer was not, as many seemed to assume, the value on the vertical scale of the graph corresponding to the value on the horizontal scale of 1.5. Candidates should remember to check the variable on the graph since, in this case, they were $x\sqrt{x}$ and $y\sqrt{x}$ and not *x* and *y*.

*Answers*: **(i)**

| | | | | | |
|---|---|---|---|---|---|
| $x\sqrt{x}$ | 1.00 | 2.83 | 5.20 | 8.00 | 11.18 |
| $y\sqrt{x}$ | 3.40 | 4.13 | 5.07 | 6.20 | 7.47 |

**(iii)** $a = 3$, $b = 0.4$ **(iv)** 3.05

**Question 10**

This question was very well done, with almost all candidates displaying a sound knowledge of matrices. There were many full mark answers with candidates using the inverse matrix correctly in part **(iv)**.

*Answers*: **(i)** $\begin{pmatrix} 5 & 0 \\ 4 & -13 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 7 & -18 \\ -3 & -19 \end{pmatrix}$ **(iii)** $-\frac{1}{17}\begin{pmatrix} -5 & -2 \\ -1 & 3 \end{pmatrix}$ **(iv)** $x = 9$, $y = -2$

**Question 11**

Some good work was seen on this question, especially in part **(a)(i),** with many candidates displaying good knowledge of the rules of indices. The logarithms created more problems, though many obtained good marks on part **(a)(ii)**, just losing one mark through not rejecting the value of $y = -5$, the other root of the quadratic equation formed. Candidates found part **(b)** more difficult. This was the other question where candidates were instructed not to use a calculator in order to test their knowledge of the rules of logarithms. It was necessary first to write the given expression as $\log_{12}16 - \log_{12}9 + \log_{12}81$ before combining the three logarithms, and it was essential for Examiners to see these steps. Some candidates would benefit from more familiarity with logarithm laws.

*Answers*: **(a)(i)** $x=\frac{1}{3}$ **(ii)** $y = 20$ **(b)** 2

**Question 12**

**EITHER**

This question was answered less often than 12 OR. Some candidates produced very good solutions, but others found the question too difficult. The equations of tangent and normal at *P* had first to be found, so that their axis crossing points would give the required points in part **(i)**. Once these points are known, the areas for the triangles in part **(ii)** are most easily found from the ½ × base × height formula, since the heights of the triangles are already given in the coordinates of *P*. A major weakness amongst candidates knowing how to proceed, and to work correctly, was a tendency to work with decimal values in the equations. Both parts of the question ask for answers in terms of ln2, so full marks could not be awarded for answers given as decimals.

*Answers*: **(i)** $A(1+2\ln 2, 0)$, $B\left(0, \frac{1}{2}+\ln 2\right)$, $C\left(1-\frac{1}{2}\ln 2, 0\right)$, $D(0, -2+\ln 2)$ **(ii)** $k = (\ln 2)^2$

**OR**

This question was more popular than 12 EITHER. Part **(i)** was reasonably well done, many candidates being well practised in finding stationary points. Some however, having successfully found the required point, forgot to determine its nature. Part **(ii)** proved to be more challenging, and there were few fully correct answers. The equation of the normal to the curve at *Q* had to be found, and then its axis crossing points immediately gave the base and height of the triangle whose area was required. There were sometimes similar difficulties in this question to those noted for the other option question above, in that instead of working in terms of e, as the question requires, decimals were used.

*Answers*: **(i)** $\left(1, -\frac{1}{e}\right)$, minimum **(ii)** $\frac{(1+2e^2)^2}{4e}$

# ADDITIONAL MATHEMATICS

**Paper 0606/22**

**Paper 22**

## Key messages

In order to succeed in this paper, candidates need to have a sound knowledge of the entire syllabus. With the advent of more sophisticated calculators it has become even more important for candidates to show their working. They should be reminded that an incorrect answer with no working will not be awarded any marks, whereas candidates can be awarded marks for working even if they get an incorrect answer.

## General comments

There was a good spread of marks across the whole range which showed that the paper provided good differentiation between candidates. Most candidates appeared to have sufficient time to attempt all the questions that they could, and there were some very good scripts seen. **Questions 3** and **4** were generally found to be straightforward. However, a significant number of candidates were unable to make much progress with **Questions 2**, **5**, **8** and **9**.

## Comments on specific questions

### Question 1

**(a)** Few candidates scored full marks on this part. The most common numbers seen were the correct numbers 1, 6, 9, 14 but they were often placed in the wrong regions. Many candidates gained only one mark, for four numbers summing to 30.

**(b)** More candidates were able to answer this part correctly.

### Question 2

**(i)** The sine graph was usually sketched correctly. Although many candidates had some correct points plotted for the cosine graph it was rarely sketched well enough to show that (0, 2) and (180, 2) were maximum turning points on the graph and too often straight lines, or a quadratic curve, were used to join the points.

**(ii)** Many candidates did not seem to appreciate that they should be using their sketches from part **(i)** to answer this part of the question.

**(iii)** The usual, incorrect, response from candidates was to double their answer in part **(ii)** even when parts **(i)** and **(ii)** had been answered correctly, indicating a lack of understanding of what the question was asking.

*Answers*: **(ii)** 2 **(iii)** 2

### Question 3

Many candidates had correctly learned the method for answering this type of question, with completely correct solutions often seen. Marks were sometimes lost when powers of negative numbers were dealt with incorrectly. Some candidates attempted long division rather than using the factor and remainder theorems. Others incorrectly assumed that $(x+1)$ was a factor of $\mathrm{f}(x+1)$ equating $\mathrm{f}(-1)$ to zero.

*Answer*: **18**

**Question 4**

This question was done rather better than many with most candidates having some knowledge of the topic.

**(i)** This was, in general, a well understood question. Many candidates were able to produce the correct order matrix with the correct numbers. In some cases one, two or three entries were incorrect because of problems with signs, but many candidates gained at least one mark.

**(ii)** Again, a majority of candidates were able to produce the 2 by 1 matrix. Of the two entries the –18 was more often incorrect.

**(iii)** A number of candidates calculated –13 but then used +13 in the final fraction. –17 was often seen and more rarely –11. Most candidates knew what to do with the entries of the original matrix to find the inverse. It is useful when calculating an inverse matrix to multiply it by the original matrix to check that no arithmetic errors have been made.

*Answers*: **(i)** $\begin{pmatrix} 9 & -22 & 6 \\ 16 & -29 & 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -18 \\ -2 \end{pmatrix}$ **(iii)** $-\frac{1}{13}\begin{pmatrix} -5 & 2 \\ -1 & 3 \end{pmatrix}$

**Question 5**

In this question many candidates understood the ideas involved and gained most of the marks. A slightly smaller number scored zero. A pictorial representation of each case would have made the question much more straight forward, and candidates should be encouraged to use such representations to aid them in answering this type of question.

In part **(ii)**, many knew to use 4! and 3! although some added rather than multiplied. Most candidates did not appreciate the need to multiply by 2 to indicate the situation where the boys sat together following the girls instead of preceding them.

*Answers*: **(i)** 144 **(ii)** 288 **(iii)** 1440

**Question 6**

The linear and quadratic inequalities in parts **(i)** and **(ii)** were often given correctly. The more able candidates solved these inequalities individually in part **(iii)**, but it was rare to see a correctly combined final solution. The most common error, in the final answer, was to ignore that $x$ should be greater than 40 and quote the final solution as $-50 < x < 60$. Weaker candidates struggled to solve the quadratic inequality at all. It would benefit such candidates to have more practice at solving such inequalities.

*Answers*: **(i)** $4x - 20 > 140$ **(ii)** $x(x - 10) < 3000$ **(iii)** $40 < x < 60$

**Question 7**

There were some good fully correct solutions to this question. Many made good attempts at part **(ii)** scoring the method marks even when they had failed to score in part **(i)**. Candidates seemed more confident in the use of the formula for arc length than in the use of the formula for the area of a sector. In part **(i)** candidates often tried to link the area of just one sector to the given value of 32 rather than the difference in the area of two sectors in an attempt to find the angle.

The attempts to find the length of *FC* in part **(ii)** were split fairly equally between the use of the cosine rule in triangle *OFC*, forming right-angled triangles by using the line of symmetry from O to the midpoint of FC then using standard trigonometry and finding angle *OFC* or *OCF* then using the sine rule.

There were a significant number of candidates who did not use radian formula for sector area and arc length and converted to degrees; this frequently led to inaccuracies.

*Answers*: **(i)** 0.4 **(ii)** 25.5

**Question 8**

There were many good attempts to differentiate in part **(ii)** and integrate in part **(iii)**. However, many candidates worked in degrees rather than radians throughout this question. The most common errors seen were an answer of $t = 241$ in part **(i)** and an acceleration of $-12\sin\left(\frac{t}{3}\right)$ rather than $-4\sin\left(\frac{t}{3}\right)$ in part **(ii)**, together with a final answer using degrees. In part **(i)** it was also common to see candidates incorrectly rearrange $\cos\left(\frac{t}{3}\right) = \frac{1}{6}$ to $\cos t = \frac{1}{2}$ as they tried to solve for *t*. In part **(iii)** some candidates incorrectly assumed that 'at instantaneous rest' meant that *t*, rather than *v*, should equal zero.

*Answers*: **(i)** 4.21 (ii) $-3.37$ **(iii)** 36

**Question 9**

Candidates usually picked up some of the marks available but it was very rare to see a fully correct answer, indicating a need for more practice on this type of question. The process of completing the square was generally attempted with the value of *a* usually correct, *b* correct about half the time and *c* more rarely so. In part **(ii)**, about half the candidates recognized that a sketch of $y = |\mathrm{f}(x)|$ existed solely above the $x$ axis. Too often however, the outer sections were lines or curves that curved the wrong way. In part **(iii)**, many candidates attempted some algebraic manipulation to try to get an answer, rather than using their graph, but only a very few were successful.

*Answers*: **(i)** $a = 2$, $b = -3$, $c = -8$. **(iii)** $0 < k < 8$

**Question 10**

Most candidates seemed to know that they needed to differentiate the given equation to find the gradient function and this they invariably could do correctly. The majority then set it to zero and solved the quadratic though it was clear that not all recognized the significance of having two values for *x* and often did not know which to use for *Q*, going on to use an incorrect *Q* or both of their values for $x$ in part **(ii)**. Many candidates, unnecessarily, found the second differential – some using it to establish which $x$ value gave *Q*, some trying to use it to find the $x$ value of *Q*.

Those candidates, who knew that the way to find the area under a curve was to integrate, usually did the integration itself correctly. Limits, sometimes incorrect ones, were then used to try to find this area. Many candidates possibly thought that the value they had just calculated was the required area and left it as their final answer. Of those who realized that an area needed to be subtracted from it, a few worked out the area of the trapezium by using a simple triangle and rectangle. Many found an equation for *PQ* and either combined it with the curve prior to integrating or integrated it separately to find the required area. Often candidates thought it was just a triangle that needed to be subtracted. Sometimes the gradient of *PQ* was attempted by using a value of $x$ in the gradient function rather than the gradient between two points.

*Answers*: **(i)** $(4, 18)$ **(ii)** 32

**Question 11**

Most candidates picked up a number of marks in this question, even candidates who had scored poorly on the rest of the paper. It was evident that this topic had been covered in some detail and candidates were helped by the structure of the question leading them carefully through what was needed. 11 EITHER was chosen by the majority of candidates.

**EITHER**

A pleasing number of candidates got parts **(i)** and **(ii)** fully correct or scored some marks only making an error with signs. Part **(iii)** proved much more challenging with few getting the correct answers. Some did time consuming rearrangements of the expressions from parts **(i)** and **(ii)** but did not seem to be sure about the direction in which they were heading.

*Answers*: **(i)** $\overrightarrow{OX} = \mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$ **(ii)** $\overrightarrow{OX} = \frac{3}{5}\mathbf{a} + \lambda\left(\frac{7}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}\right)$ **(iii)** $\lambda = \frac{1}{2}$, $\mu = \frac{7}{10}$

**OR**

Many candidates successfully found vector *OP*, but it was common to see vector *OQ* given as 3**d**. Some weaker candidates did not get beyond this part of the question. In the calculation of vector *OR* it was important that candidates showed all their working, so that part-marks could be achieved where a final answer was incorrect. Of those that attempted part **(ii)**, many realised the need to find two of the vectors *PQ*, *QR* or *PR* in order to show that the points were collinear. It was, however, rare to see one vector expressed as a multiple of the other in order to complete the method required.

*Answers*: **(i)** $\overrightarrow{OP} = 3\mathbf{c}$, $\overrightarrow{OQ} = \frac{3}{2}\mathbf{d}$, $\overrightarrow{OR} = 2\mathbf{d} - \mathbf{c}$ **(ii)** $3:1$

# ADDITIONAL MATHEMATICS

**Paper 0606/23**

**Paper 23**

## Key messages

In order to succeed in this paper, candidates need to have a sound knowledge of the entire syllabus. With the advent of more sophisticated calculators it has become even more important for candidates to show their working. They should be reminded that an incorrect answer with no working will not be awarded any marks, whereas candidates can be awarded marks for working even if they get an incorrect answer.

## General comments

Nearly all candidates were able to produce satisfactory or good attempts at several questions. Answers to **Questions 3**, **5**, **7** and **10** normally gained most, if not all, of the marks available, whilst **Questions 4**, **6** and **9** were often less rewarding. However, there were many instances of candidates overlooking part of a question, usually after solving an equation. This applied to **Questions 7(ii)**, **8(ii)**, **9** and **10**.

There was no evidence of any problems with the time allowed to complete the paper.

Overall, the presentation of scripts was quite good. Answering on the question paper helped in this respect. Some candidates provided extra sheets of paper, but Centres are reminded that they should only provide candidates with extra paper if they require more space for an answer, for example, if they have made an error and need to restart their answer.

**Question 12 OR** was slightly more popular than **Question 12 EITHER**. The total mark was usually better for the former, but several candidates produced good responses in part **(i)** of the latter.

## Comments on specific questions

### Question 1

Nearly all candidates rearranged the inequality and factorised the resulting quadratic correctly. The part of the range $x > 3$ caused few problems. The other part proved quite problematic with $x > -2.5$ appearing more often than $x < -2.5$. Nearly all answers involving a small sketch of a quadratic resulted in the correct answer, and candidates should be encouraged to draw such sketches.

*Answer*: $x < -2.5$, $x > 3$

### Question 2

**(i)** This part was usually answered well. Occasionally the negative sign was omitted.

**(ii)** Most candidates understood the idea of an incremental change and obtained the correct answer. Sometimes the power of 4 from part **(i)** was either overlooked or applied to $x$ instead of $12 - 4x$. Weaker attempts either omitted $p$ or substituted $x = p + 0.5$, or gave an answer involving $x$.

*Answers*: **(i)** $-20(12 - x)^4$ **(ii)** $-200\,000p$

**Question 3**

The majority of candidates showed a good understanding of the Binomial Theorem and gained full marks. Weaker candidates often wrote out all of the terms before extracting the one required.

**(i)** Some attempts did not include a minus sign. Several candidates used $-2x^3$ instead of $(-2x)^3$.

**(ii)** Again, considering $(1 + 2x)^7$ rather than $(1 - 2x)^7$ resulted in the loss of marks.

*Answers*: **(i)** −280 **(ii)** −322

**Question 4**

This proved to be a very challenging question for most candidates. Many candidates considered only $b^2 - 4ac = 0$, rather than applying the full given formula. The minus sign in the coefficient of $x$ led to several sign errors. In general, when sign errors were avoided, squaring $(4 + 2\sqrt{2})$ was done correctly. When an expression involving $c + d\sqrt{2}$ in the denominator was obtained, this was usually rationalised correctly. Very few candidates did not reject the negative root.

*Answer*: $2 + \sqrt{2}$

**Question 5**

**(i)** This was usually answered well. The most frequent error was to evaluate ${}^{14}P_6$.

**(ii)** This part proved more difficult than part **(i)**. Fairly common errors were to include the combination of 3 candidates with 3 teachers and to add e.g. 56 and 6, instead of multiplying.

**(iii)** This was usually correct although 15, following ${}^6C_2$, was quite common. A few assumed that the chairperson and the secretary could be the same person, resulting in the answer 36.

*Answers*: **(i)** 3003 **(ii)** 1386 **(iii)** 30

**Question 6**

Answers to this question showed a wide variation. A considerable proportion of the candidates had difficulties in understanding the requirements of the modulus function in this context.

**(i)** There were many correct sketches. Several candidates calculated a table of values, only to either draw a sketch in which $y > 0$ or to abandon the question without drawing a sketch. Others only considered positive values of $x$. Most correct answers followed from a quadratic graph (with a positive coefficient of $x^2$), with that part below the $x$-axis being reflected in the $x$-axis. It was not necessary to include any numbers, only the general shape was required. Curvature had to be in the appropriate direction, with a maximum for a positive value of $x$ and the cusp for positive $x$ further from the origin than the cusp for negative $x$. A few sketches showed the graphs of $y = |2x+3|$ and $y = |2x-7|$.

**(ii)** Nearly all answers did give the number of roots, as requested, rather than their value(s). Several candidates recognised that intersecting graphs was appropriate and added the straight line $y = 2x$ to their sketch in part **(i)**. Very few of those who tried to solve the equation using the quadratic formula were successful. Most only solved one equation without considering which root was appropriate.

*Answers*: **(ii)** 2

**Question 7**

**(i)** Nearly all attempts resulted in the correct gradient. Arithmetic errors led to slightly fewer correct values for the intercept on the $\frac{y}{x}$-axis. Some answers stopped at this point. Most candidates continued to replace $x$ and $y$ with $x$ and $\frac{y}{x}$ respectively, before multiplying through by $x$. A surprising number of candidates divided instead of multiplying.

**(ii)** Often correct. Several candidates solved an equation to obtain a value for $x$ and went on to the next question, forgetting that a value for $y$ was also required.

*Answers*: **(i)** $y = 5x - 2x^2$ **(ii)** $x = 7$, $y = -63$

**Question 8**

Attempts at this question varied widely. Confident candidates quickly gained full marks whilst others, without an overall plan, became confused by several different equations.

**(i)** There were many fully correct answers. An appreciable number of candidates did not understand the equations for arc length and area when using radians. This resulted in solutions involving $2\pi$ and/or 360. Some substitutions involved $r$ in terms of $\theta$, so no further progress could be made.

**(ii)** Only a small proportion of the candidates obtained full marks for this part of the question following difficulties with part **(i)**. Quite often an incorrect expression for the area $A\text{cm}^2$ led to differentiation involving either a product, a quotient, or an expression containing two variables. Some candidates solved their equation to find a value for $r$ and went on to the next question without evaluating a value for $A$.

*Answers*: **(i)** $A = 100r - r^2$ **(ii)** 2500

**Question 9**

There was considerable variation in the answers to this question, with the majority of candidates gaining few, if any, marks, suggesting a lack of familiarity with the topic. Candidates would benefit from more practice in this area of the syllabus. Many attempts were based on a right-angled triangle or one with sides 50, 350 and 400. A few candidates made the error of using the sine or cosine rule with a distance and a speed.

**(i)** Only a small proportion of the candidates produced a correct diagram with the angle of 130° between the speed of 350 $\text{kmh}^{-1}$ and the velocity of 50 $\text{kmh}^{-1}$ from the north. Slightly more used the cosine rule to calculate a resultant speed, nearly all of these continuing to divide 480 by this speed to obtain a time for the flight from $A$ to $B$. Several candidates did not use the cosine rule but were able to obtain some credit for dividing 480 by their incorrect speed following the use of either the sine rule or Pythagoras'.

**(ii)** This part was not as well answered as part **(i)**, nearly always as a result of a poor diagram. Some candidates calculated a correct angle but did not continue to find the bearing of $B$ from $A$.

*Answers*: **(i)** 1.25h **(ii)** 136°

**Question 10**

Candidates usually answered this question well. Most chose to eliminate $y$ rather than $x$. Both usually resulted in a correct quadratic equation. A few errors arose from not collecting like terms carefully. When a quadratic does not give rise to simple roots, candidates would be well advised to perform a quick check on their signs, brackets and collection of terms. This time might well be recouped in later work and more marks gained. Nearly all correct quadratics were followed by the correct coordinates for $A$ and $B$, with only the occasional sign error. Some answers stopped after calculating the coordinates. Most candidates used their coordinates rather than the equation of the line to find the gradient of $AB$. Whilst most candidates used $\frac{-1}{m_{AB}}$ for their gradient they calculated the equation for the line through $A$ or $B$ rather than their mid-point. There were several sign errors made when calculating the mid-point.

*Answer*: $4y + 2x = 15$

**Question 11**

Generally answers to part **(ii)** were better and more complete than those for the other two parts.

**(i)** Strong candidates had little problem answering this part. Weaker candidates made mistakes whilst attempting to rearrange the equation into $\tan\frac{1}{2}x = 4$, often these involved separation of the $\frac{1}{2}$ from the $x$. This mistake was avoided by those candidates who replaced $\frac{1}{2}x$ with, for example, $\theta$. A surprising number of candidates followed $\frac{1}{2}x = 76.0$ with $x = 38.0$.

**(ii)** Most candidates realised the need to obtain a quadratic equation in $\sin y$. Generally, the substitutions made were correct, most mistakes arising either from sign errors or removing the brackets. The solution corresponding to $\sin y = 0$ was often lost when cancelling. Weaker candidates often found the answer 143.1 following 36.9 but had difficulty obtaining the answer 180.

**(iii)** The majority of candidates obtained the correct positive value for either $\cos z$ or $\tan z$ or $\sin z$, often omitting the negative value. Consequently only two answers were given.

*Answers*: **(i)** 151.9 **(ii)** 36.9, 143.1, 180 **(iii)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

**Question 12 EITHER**

**(i)** Nearly all answers to this question included the correct differentiation of $\ln x$. Most continued to use $-\mathrm{e}$ for the gradient of the normal. Those using $-x$ or $-\frac{1}{\mathrm{e}}$ as the gradient made little further progress in this part.

**(ii)** Only a few answers failed to show the use of the product rule. Several of the candidates who had made little progress with part **(i)** noticed that this part of the question could be answered independently of part **(i)**.

**(iii)** Only a small proportion of the candidates produced correct answers to this part. Most candidates attempted to incorporate the given answer to part **(ii)** in their integration of $\ln x$. Quite often the 1 was not integrated. The majority of attempts to calculate the area involved definite integration between $x = 1$ and $x = \mathrm{e} + \frac{1}{\mathrm{e}}$, rather than between $x = 1$ and $x = \mathrm{e}$, with the area of a triangle to be added.

*Answers*: **(iii)** $x\ln x - x\ (+\ c)$, $1 + \frac{1}{2\mathrm{e}}$

**Question 12 OR**

**(i)** Most attempts at this part were correct. A small number of candidates did not use the product rule to differentiate $y = e^x\cos x$. Some used $x = \frac{\pi}{2}$ instead of $x = 0$. A few attempts did not include any differentiation.

**(ii)** Again, only a small number of answers did not include the product rule. Occasionally candidates used the product rule here after not using it in part **(i)**. As before, the marks for this part could be gained without answering part **(i)**.

**(iii)** Quite often the answer to part **(ii)** was ignored. The factor of $\frac{1}{2}$ caused problems in many attempts. The stages required in order to find the shaded area were understood better in this option, with fewer cases of definite integration between the wrong limits.

*Answers*: **(i)** (1, 0) **(iii)** $\frac{1}{2}e^x(\cos x + \sin x)\ (+c)$, $\frac{1}{2}e^{\frac{\pi}{2}} - 1$