



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Ordinary Level

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**ADDITIONAL MATHEMATICS**

4037/01, 0606/01

Paper 1

October/November 2009

MARK SCHEME

Maximum Mark : 80

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**IMPORTANT NOTICE**

Mark Schemes have been issued on the basis of **one** copy per Assistant examiner and **two** copies per Team Leader.



UNIVERSITY of CAMBRIDGE  
International Examinations

**[Turn over**

<p><b>1(i)</b> <math>2a^3 - 7a^2 + 7a^2 + 16 = 0</math> leading to <math>a^3 = -8</math>, <math>a = -2</math></p> <p><b>(ii)</b> <math>2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16</math> <math>= 21</math></p>	<p>M1 A1 [2]</p> <p>M1 A1 [2]</p>	<p>M1 for use of <math>x = a</math></p> <p>M1 for substitution of <math>x = -\frac{1}{2}</math></p>
<p><b>2 (i)</b> <b>(ii)</b></p> $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$	<p>B1,B1 [2]</p> <p>B2,1,0 [2]</p>	<p>B1 for each matrix, must be in correct order</p> <p>-1 for each error</p>
<p><b>3</b> <math>4(2k+1)^2 = 4(k+2)</math> <math>4k^2 + 3k - 1 = 0</math> leading to <math>k = \frac{1}{4}, -1</math></p>	<p>M1 A1</p> <p>M1 A1 [4]</p>	<p>M1 for use of '<math>b^2 - 4ac</math>' Correct quadratic equation</p> <p>M1 for correct attempt at solution A1 for both 1 values</p>
<p><b>4</b> <math>(13-3y)^2 + 3y^2 = 43</math> (or <math>x^2 + \frac{(13-x)^2}{3} = 43</math>) <math>6(2y^2 - 13y + 21) = 0</math> (or <math>2(2x^2 - 13x + 20) = 0</math>) <math>(2y-7)(y-3) = 0</math> (or <math>(2x-5)(x-4) = 0</math>) <math>y = 3</math> or <math>\frac{7}{2}</math> <math>\left(x = \frac{5}{2} \text{ or } 4\right)</math> ( or <math>x = 4</math> or <math>\frac{5}{2}</math> <math>\left(y = \frac{7}{2} \text{ or } 3\right)</math>)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1,A1 [5]</p>	<p>M1 for eliminating one variable</p> <p>A1 for correct quadratic</p> <p>M1 for correct attempt at solving quadratic</p> <p>A1 for each correct pair</p>
<p><b>5 (i)</b> <math>(3+\sqrt{2})^2 + (3-\sqrt{2})^2 = 22</math> <math>AC = \sqrt{22}</math></p> <p><b>(ii)</b> <math>\tan A = \frac{3-\sqrt{2}}{3+\sqrt{2}}</math></p> $\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$	<p>M1</p> <p>A1 [2]</p> <p>M1</p> <p>M1, A1 [3]</p>	<p>M1 for use of Pythagoras</p> <p>M1 for correct ratio</p> <p>M1 for rationalising</p>

<p><b>6 (i)</b> <math>3x^2 - 10x - 8 = 0</math>  <math>(3x + 2)(x - 4) = 0</math>  critical values <math>-\frac{2}{3}, 4</math>  <math>A = \{x : -\frac{2}{3} \leq x \leq 4\}</math></p> <p><b>(ii)</b>  <math>B = \{x : x \geq 3\}</math>  <math>A \cap B = \{x : 3 \leq x \leq 4\}</math></p>	<p>M1</p> <p>A1</p> <p>A1 [3]</p> <p>B1 B1 [2]</p>	<p>M1 for attempt to solve quadratic</p> <p>A1 for critical values</p> <p>B1 for values of <math>x</math> that define <math>B</math>.  B1 for attempt to combine the sets correctly and correct use of notation</p>
<p><b>7 (i)</b> <math>{}^{13}C_8 = 1287</math></p> <p><b>(ii)</b> 7 teachers, 1 student : 6  6 teachers, 2 students <math>{}^7C_6 \times {}^6C_2 : 105</math>  5 teachers, 3 students <math>{}^7C_5 \times {}^6C_3 : 420</math>  531</p>	<p>M1, A1 [2]</p> <p>B1 B1 B1 B1 [4]</p>	<p>M1 for correct C notation</p>
<p><b>8 (i)</b> When <math>t = 0, N = 1000</math></p> <p><b>(ii)</b> <math>\frac{dN}{dt} = -1000ke^{-kt}</math>  when <math>t = 0, \frac{dN}{dt} = -20</math> leading to  <math>k = \frac{1}{50}</math></p> <p><b>(iii)</b> <math>500 = 1000e^{-kt}</math>  <math>t = -50 \ln \frac{1}{2}</math> leading to 34.7 mins</p>	<p>B1 [1]</p> <p>M1</p> <p>B1</p> <p>A1 [3]</p> <p>M1</p> <p>M1 A1 [3]</p>	<p>M1 for differentiation</p> <p>B1 for <math>\frac{dN}{dt} = -20</math> stated</p> <p>M1 for attempt to formulate equation using half life  M1 for a correct attempt at solution</p>
<p><b>9 (i)</b> <math>20 \times -2(1 - 2x)^{19}</math></p> <p><b>(ii)</b> <math>x^2 \frac{1}{x} + 2x \ln x</math></p> <p><b>(iii)</b>  <math>\frac{x(2 \sec^2(2x + 1)) - \tan(2x + 1)}{x^2}</math></p>	<p>B1, B1 [2]</p> <p>M1 B1 A1 [3]</p> <p>M1 B1 A1 [3]</p>	<p>B1 for 20 and <math>(1 - 2x)^{19}</math>  B1 for -2</p> <p>M1 for attempt to differentiate a product.  B1 for <math>\frac{1}{x}</math></p> <p>M1 for attempt to differentiate a quotient.  B1 for differentiation of <math>\tan(2x + 1)</math></p>

<p><b>10 (i)</b> <math>\frac{dy}{dx} = 9x^2 - 4x + 2</math>  at <math>P</math> grad = 7  tangent <math>y - 3 = 7(x - 1)</math></p> <p><b>(ii)</b> at <math>Q</math>, <math>7x - 4 = 3x^3 - 2x^2 + 2x</math>  leading to <math>3x^3 - 2x^2 - 5x + 4 = 0</math>  <math>(x - 1)(3x^2 + x - 4) = 0</math>  <math>(x - 1)(3x + 4)(x - 1) = 0</math>  leading to <math>x = -\frac{4}{3}, y = -\frac{40}{3}</math></p>	<p>M1  A1  M1  A1  [4]</p> <p>M1  B1, M1  M1  A1  [5]</p>	<p>M1 for differentiation and attempt to find gradient  M1 for attempt to find tangent equation, allow unsimplified</p> <p>M1 for equating tangent and curve equations  B1 for realising <math>(x - 1)</math> is a factor and attempt to factorise  M1 for factorisation and attempt to solve quadratic  A1 for both</p>
<p><b>11 (a)</b> <math>\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}</math>  <math>= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}</math>  <math>= \frac{1}{\cos \theta \sin \theta}</math>  <math>= \operatorname{cosec} \theta \sec \theta</math></p> <p><b>(b)(i)</b> <math>\tan x = 3 \sin x</math>  <math>\frac{\sin x}{\cos x} = 3 \sin x</math>  <math>\sin x - 3 \sin x \cos x = 0</math>  leading to <math>\cos x = \frac{1}{3}, \sin x = 0</math>  <math>x = 70.5^\circ, 289.5^\circ</math> and <math>x = 180^\circ</math></p> <p><b>(ii)</b> <math>2 \cot^2 y + 3 \operatorname{cosec} y = 0</math>  <math>2(\operatorname{cosec}^2 y - 1) + 3 \operatorname{cosec} y = 0</math>  <math>2 \operatorname{cosec}^2 y + 3 \operatorname{cosec} y - 2 = 0</math>  <math>(2 \operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0</math>  leading to <math>\sin y = -\frac{1}{2}, y = \frac{7\pi}{6}, \frac{11\pi}{6}</math></p>	<p>M1  M1  A1  [3]</p> <p>M1  A1  A1  B1  [4]</p> <p>M1  M1  A1, A1  [5]</p>	<p>M1 for attempt to get in terms of sin and cos and attempt to get one fraction  M1 for use of identity</p> <p>M1 for use of <math>\tan x = \frac{\sin x}{\cos x}</math> and correct attempt to solve</p> <p><math>\sqrt{A1}</math> on their <math>x = 70.5^\circ</math>  B1 for <math>x = 180^\circ</math></p> <p>M1 for use of correct identity  M1 for attempt to solve quadratic  M1 for dealing with cosec</p>

<p><b>12 EITHER</b></p> <p>(i) <math>\pi r^2 h = 1000</math>, leading to</p> $h = \frac{1000}{\pi r^2}$ <p>(ii) <math>A = 2\pi r h + 2\pi r^2</math> leading to given answer</p> $A = 2\pi r^2 + \frac{2000}{r}$ <p>(iii) <math>\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2}</math> when <math>\frac{dA}{dr} = 0</math>, <math>4\pi r = \frac{2000}{r^2}</math> leading to <math>r = 5.42</math></p> <p>(iv) <math>\frac{d^2 A}{dr^2} = 4\pi + \frac{4000}{r^3}</math>  + ve when <math>r = 5.42</math> so min value <math>A_{\min} = 554</math></p>	<p>M1 A1 [2]</p> <p>M1 A1 [2]</p> <p>M1 A1 DM1 A1 [4]</p> <p>M1 A1 A1 [3]</p>	<p>M1 for attempt to use volume</p> <p>M1 for attempt to use surface area GIVEN ANSWER</p> <p>M1 for attempt to differentiate and set to 0 DM1 for solution</p> <p>M1 for second derivative method or gradient method'</p> <p>A1 for minimum, can be given if <math>r</math> incorrect but + ve</p>
<p><b>12 OR</b> (i) <math>y = x + \cos 2x</math></p> $\frac{dy}{dx} = 1 - 2 \sin 2x$ <p>when <math>\frac{dy}{dx} = 0</math>, <math>\sin 2x = \frac{1}{2}</math> leading to <math>x = \frac{\pi}{12}, \frac{5\pi}{12}</math></p> <p>(ii) Area = <math>\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} x + \cos 2x dx</math></p> $= \left[ \frac{x^2}{2} + \frac{1}{2} \sin 2x \right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$ $= \frac{\pi^2}{12}$	<p>M1 A1</p> <p>M1 M1 A1, A1 [6]</p> <p>M1</p> <p>A1, A1 DM1 A1 [5]</p>	<p>M1 for attempt to differentiate</p> <p>M1 for setting to 0 and attempt to solve M1 for correct order of operations</p> <p>M1 for attempt to integrate</p> <p>A1 for each term correct DM1 for correct use of limits (Trig terms cancel out)</p>