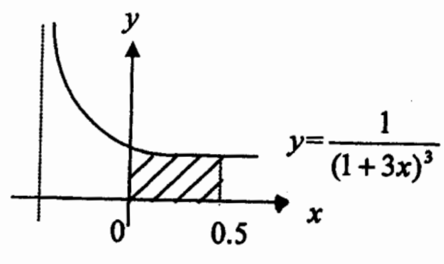
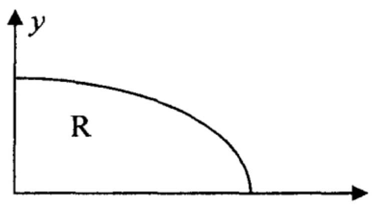
Pure Mathematics 1

Tutorial 9 Integration

1	Evaluate $\int_1^2 3 - 2x^2 + \frac{4}{x^3}\, dx$ $[-\frac{1}{6}]$
2	The finite region bounded by the curve $y = 16 - 4x^2$ and the x – axis is rotated through 2π radians about the x – axis. Find the volume of the solid formed, giving your answer in terms of π. [4] $[V = \frac{8192}{15}\pi]$
3	A curve with equation $y = f(x)$ has $\frac{dy}{dx} = kx(3x+2)$ for each x, where k is a constant. The curve passes through the point $(-1, 1)$ and has gradient 2 at this point. (i) Find the value of k. [2] (ii) Find the equation of the curve. [2] $[2; y = 2x^3 + 2x^2 + 1]$
4	i) Find $\int \frac{1}{(3x+1)^3}\, dx$. [2] ii) Hence find the shaded area under the curve $y = \frac{1}{(1+3x)^3}$ as shown below.  [2] $[-\frac{1}{6(3x+1)^2} + c; \frac{7}{50}]$

5

Diagram below shows the region R in the first quadrant bonded by the curve $y=\frac{1}{2}(4-x^2)$ and the axes.. Calculate the volume of the solid formed when R is rotated through 360^0 about the y axis. [5 marks]



$[4\pi]$

6

A curve is such that $\frac{dy}{dx}=3x^2+6x+k$, where k is a constant. The gradient of the curve at (1,5) is 4. Find the equation of the curve. [5]

$[y=x^3+3x^2-5x+6]$

7

The equation of a curve is $y=(3x+1)^{\frac{1}{3}}+1$.

(i) Find $\int\left((3x+1)^{\frac{1}{3}}+1\right)dx$. [2]

(ii) Hence find the area of the region enclosed by the curve, the x-axis and the lines $x=0$ and $x=1$. [3]

$[y=\frac{1}{4}(3x+1)^{\frac{4}{3}}+x+c\,;2.34]$

8

Find the area bounded by the curves $y = 2x^2-2$ and $y=x^2+x$. [5]

$[\frac{9}{2}]$

9

The gradient at any point (x,y) on a curve is $k\sqrt{x}$. Given that the gradient at point (1,6) is 6, find

(i) the equation of the curve, [5]

(ii) the point at which the curve meets the y-axis. [1]

$\left[y=4x^{\frac{3}{2}}+2\,;(0,2)\right]$

10

a) Find $\int\left(\frac{1}{x^2}-\frac{1}{\sqrt{x}}\right)dx$. [3]

b) A curve passes through the point $(1,0)$ and is such that $\frac{dy}{dx}=\frac{1}{x^2}-\frac{1}{\sqrt{x}}$. Find the equation of the curve. [3]

$[-\frac{1}{x}-2\sqrt{x}+c\,;\,y=-\frac{1}{x}-2\sqrt{x}+3]$

11

The curves $y = \cancel{x^3}\ x^2$ and $y = 4 - 3x^2$ meets at the points A and B.

a) Find the coordinates of A and B and hence sketch the curves. [4]

b) Find the area enclosed by the two curves. [3]

$[(1,1);(-1,1);5\frac{1}{3}]$

12

Show that the tangent to the curve $y = x^2 + 2x + 1$ at point A (1,4) passes through the origin O. [3]

Calculate the area bounded by the curve, the line OA and the x-axis. [4]

$[\frac{2}{3}]$

13

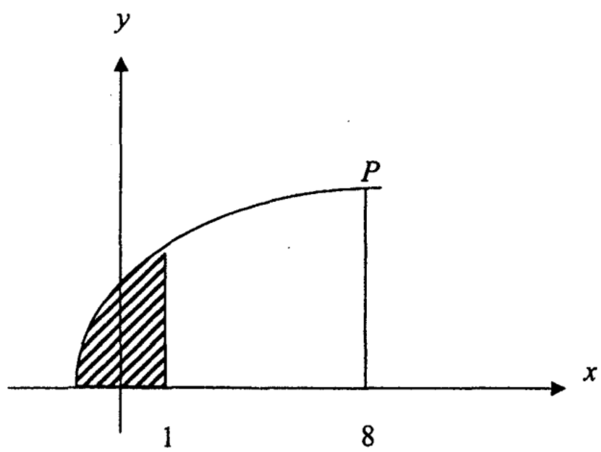
A region is bounded by the curve $y=\frac{1}{x^2}$ and the lines y = x , x = 0 and y = 4.

a) Show this region clearly on a sketch [2]

b) Find the area of this region [5]

$[\frac{5}{2}]$

14



The diagram above shows a curve with equation $y = \sqrt{3x+1}$.

(a) P is a point on the curve. If the x-coordinate of P is 8, find the equation of the normal to the curve at this point. [4]

(b) Using a definite integral, and showing all the steps in your working, calculate the area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [4]

$[10x + 3y = 95; \frac{16}{9}]$

15

(i) Sketch the curve $y = x^2 - 1$. [2]

(ii) Calculate the area bounded by the curve, the x-axis and the line $y = 8$. [3]

(iii) If this area is rotated through 4 right angles about the y –axis,

calculate the volume of the solid that is generated. [3]

$[34\frac{2}{3}; 40\pi]$

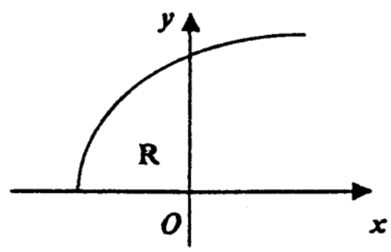
16 The point $P(1,2)$ lies on the curve for which $\frac{dy}{dx} = 2x - 4$. Find

(i) the equation of the normal at P. [4]

(ii) the equation of the curve. [4]

$[2y = x + 3\ ; y = x^2 - 4x + 5]$

17



The diagram shows the region R bounded by the axes and part of the curve

$y = \sqrt{(x+1)}$. Find the exact value of

a) the area of R , [4]

b) the volume of the solid formed when R is rotated completely about the y- axis,

giving your answer as a multiple of π. [4]

$[\frac{2}{3} ; \frac{8}{15}\pi]$

18

Sketch the curve $y = 9 - x^2$ [2]

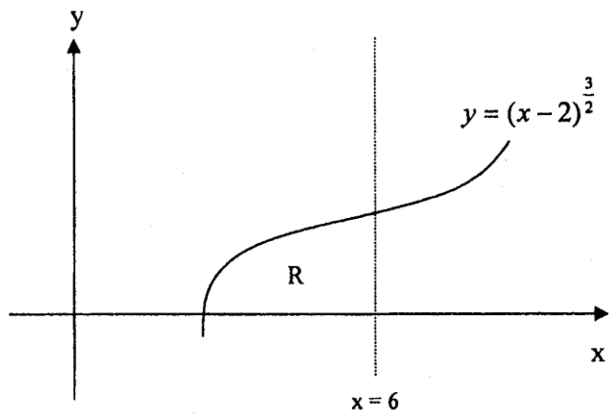
Calculate

(a) The area enclosed between the curve and the x-axis. [3]

(b) The volume enclosed when this area is rotated about the y-axis through 360°

$[36 ; \frac{81}{2}\pi]$

19 The diagram shows the region R in the first quadrant bounded by curve C with equation $y=(x-2)^{\frac{3}{2}}$ and the line x = 6.



a) Calculate the exact area of R. [3]

b) R is rotated completely about the y-axis. Show that the volume of the solid formed is given by

$$V = 288\pi - \pi\int_0^8 (y^{\frac{2}{3}}+2)^2\,dy.$$ [3]

c) Hence, evaluate V exactly. [3]

$[\frac{64}{5}; 124\frac{12}{35}\pi]$

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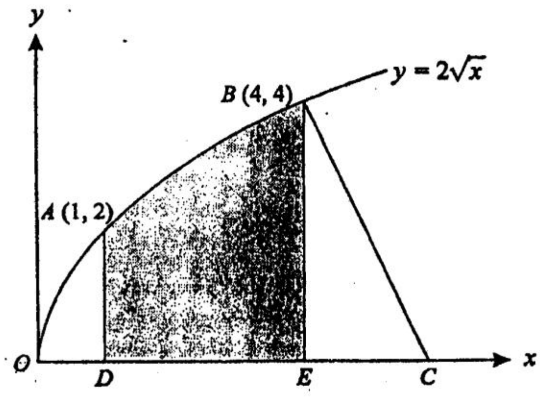


The diagram shows the points A(1,2) and B(4,4) on the curve $y = 2\sqrt{x}$. The line BC is the normal to the curve at B and C lies on the x-axis. Lines AD and BE are perpendicular to the x-axis.

(i) Find the equation of the normal BC. [5]

(ii) Find the area of the region bounded by the curve, lines AD and BE and the x-axis. [4]

$[y + 2x = 12; 9\frac{1}{3}]$

21

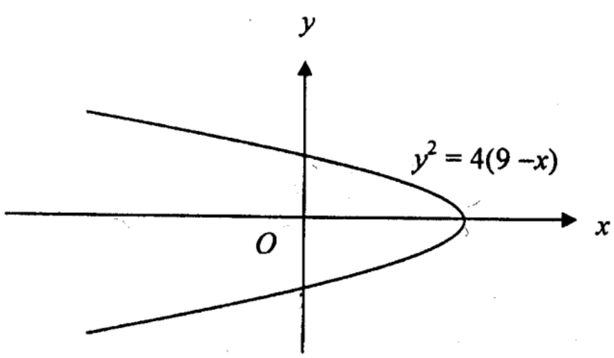


Diagram shows a region bounded by the parabola $y^2 = 4(9 - x)$ and the y-axis.

Find

(i) the points of intersections between the parabola and the two axes; [3]

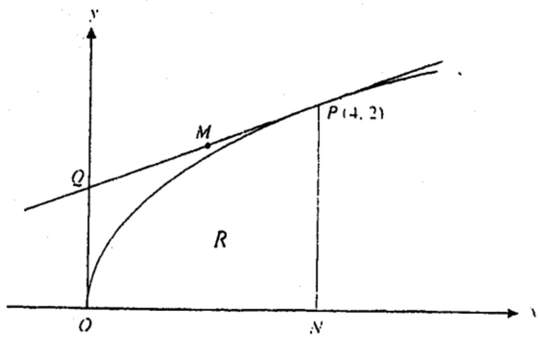
(ii) the enclosed area, and [3]

(iii) the volume generated by rotating the enclosed region about the x-axis. [3]

$[(9,0), (0,6), (0,-6)\ ;\ 72\ ;\ 162\pi\ ]$

22	a) Find $\int\left(6x+\frac{4}{x^3}\right)dx$. [2] b) Show that the area enclosed between the curves $y=9-x^2$ and $y=x^2-7$ is $\frac{128\sqrt{2}}{3}$. [7] $[3x^2-\frac{2}{x^2}+c]$
23	a) i) Evaluate $\int_1^2 x(x^2-2)dx$. [2] ii) Find $\int 6(x+1)^{1/2}dx$. [3] b) Find the volume of revolution, in terms of π, when the region bounded by curve $y=\frac{1}{2}x^3+2$, the x-axis, the lines $x=-1$ and $x=2$ is rotated completely about x-axis. [5] $[\frac{3}{4}; 4(x+1)^{\frac{3}{2}}+c; \frac{675}{28}\pi]$
24	**(a)** Find $\int_{-1}^{0}\frac{1}{(2x+1)^2}dx$. **[3]** **(b)** Find the area bounded by the curves $y=\sqrt{8x}$ and $y=x^2$. Find also the volume of revolution generated when the area is rotated through 360° about the x axis. **[7]** $[-1; \frac{8}{3}; \frac{48}{5}\pi]$

25



The diagram shows the graph of $y = x^{\frac{1}{2}}$. The point P on the graph has coordinates (4, 2).

(i) Find the equation of the tangent at P, giving your answer in the form $ax + by + c = 0$, where a, b, c are integers. [4]

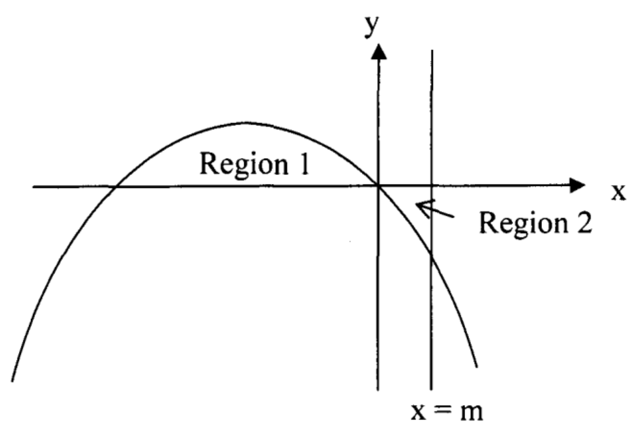
(ii) The tangent at P meets the y – axis at Q. Find the coordinates of the mid-point M of PQ. [3]

(iii) Using integration, find the area of the region R bounded by the curve, the x – axis and PN, where PN is the perpendicular from P to the x – axis. [3]

$[x - 4y + 4 = 0; (2, \frac{3}{2}); \frac{16}{3}]$

26

The graph of $y = -x^2 - 4x$ and $x = m$ where $m > 0$ are shown below:



Region 1 is above the x-axis and is enclosed by the graph of $y = -x^2 - 4x$ and the x-axis. Region 2 is below the x-axis and is enclosed by the graph of $y = -x^2 - 4x$, the x-axis and the line $x = m$.

(i) Find the area of the region 1. [4]

(ii) If $\int_{-4}^{m} (-x^2 - 4x)dx = 0$, state the area of Region 2. [1]

(iii) Find the equation of the tangent to $y = -x^2 - 4x$ at $x = 1$. [5]

$[\frac{32}{3}; \frac{32}{3}; \boldsymbol{y = 6x + 1}]$

27

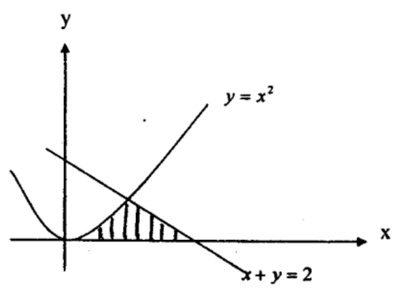
a) Integrate $\dfrac{x^4 - 3x + 1}{2x^3}$ with respect to x. [3]

b) Find the coordinates of point A where the curves $y = (x+1)^2$ and $y = (x-3)^2$ intersect. Sketch both curves on the same pair of axes and hence calculate the area of the region enclosed by the curves and the x-axis.

[8]

$[\frac{1}{4}x^2 + \frac{3}{2x} - \frac{1}{4x^2} + c]$

28



The diagram shows the shaded region enclosed by the curve $y = x^2$, the line $x + y = 2$ and the x-axis. Find

(i) the area of the shaded region. [6]

(ii) the volume of the solid formed when the shaded region is rotated through $360°$ about the x-axis. [5]

$[\frac{5}{6}\,;\frac{8}{15}\pi]$

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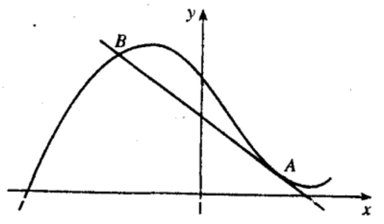

The diagram shows the curve $y = 4x^3 - 4x^2 - 10x + 12$ and the tangent at point A where $x = 1$.

a) Find the equation of this tangent. [5]

b) Show that this tangent meets the curve again at point B where $x = -1$. [3]

c) Calculate the area of the region which lies between the curve and the tangent AB. [3]

$[\boldsymbol{y = -6x + 8\,;5\frac{1}{3}}]$