Derivative of Trigo Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}\sin(ax+b)=a\cos(ax+b)$$

$$\frac{d}{dx}\cos(ax+b)=-a\sin(ax+b)$$

$$\frac{d}{dx}\tan(ax+b)=a\sec^2(ax+b)$$

Example: Find

$$(\mathrm{i})\ \frac{d}{dx}\sin 4x = \qquad (\mathrm{ii})\ \frac{d}{dx}\cos 6x =$$

Example:

Find the gradient function of the curve

$y = \tan 3x$.

Example:

Find the gradient of the curve $y = \cos(3x+1)$ when $x = \pi$.

Example:

Show that $y = \sin(x-4) \Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = 1$

Example:

If $y = \tan(7x - 2)$, show that $\frac{dy}{dx} - 7y^2 = 7$.

$$\frac{d}{dx}\left[\sin f(x)\right] = \cos f(x) \times \frac{d}{dx} f(x)$$

$$\frac{d}{dx}\left[\cos f(x)\right] = -\sin f(x) \times \frac{d}{dx} f(x)$$

$$\frac{d}{dx}\left[\tan f(x)\right] = \sec^2 f(x) \times \frac{d}{dx} f(x)$$

Example:

Find $\dfrac{d}{dx}\sin\left(x^2+1\right).$

Example:

Show that the tangent of the curve $y = \cos\left(x^3 + x^2 + 1\right)$ at the origin is parallel to the $x-$axis.

Example:

Show that the curve $y = \tan(4x^2 + 4)$ has only one turning point for all values of x.

In general,

$$\frac{d}{dx}\left[\sin^n f(x)\right] = n\sin^{n-1} f(x) \times \frac{d}{dx}\sin f(x)$$

$$\frac{d}{dx}\left[\cos^n f(x)\right] = n\cos^{n-1} f(x) \times \frac{d}{dx}\cos f(x)$$

$$\frac{d}{dx}\left[\tan^n f(x)\right] = n\tan^{n-1} f(x) \times \frac{d}{dx}\tan f(x)$$

Examples:

Find $\dfrac{d}{dx}\sin^2 4x.$

Example:

Given that $f(x)=\cos^5(x+1)$. Find the values of x such that $f'(x)=0$ for $0 \le x \le \pi$.

Example:

Find the $x-$coordinates of the turning points of the curve $y=\tan^4\left(x^2+x\right)$ in the domain $-\dfrac{\pi}{2}\leq x\leq\dfrac{\pi}{2}$. Determine the nature of each turning point.

Example:

Find the equation of the tangent to the curve

$y = \sec^3 x$ when $x = \dfrac{\pi}{4}$.

Example:

Find the equation of the normal to the curve

$y = \operatorname{cosec}^4 3x$ when $x = \dfrac{\pi}{4}$.

Example:

Show that

$$\frac{d}{dx}\left[\cot^5\left(x^3+1\right)\right] = -15x^2\cot^6\left(x^3+1\right)\sec^2\left(x^3+1\right)$$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the equation of the tangent and normal where $x = \frac{1}{3}\pi$ on the curve $y = \cos 3x$.

Example:

Find the equation of the tangent and normal where $x = \frac{1}{2}\pi$ on the curve $y = 3\sin^2 2x$.

Example:

The gross national product (GNP) of a country, \$ P billion, at t years after the year 2000 is given by $P = 1 + 0.02t + 0.05\sin 0.6t$. At what rate is the GNP changing in the year (a) 2000 (b) 2005?

Example:

P, Q and R are the points on the graph of $y = \cos x$ for which

$x = 0,\ x = \dfrac{1}{4}\pi$ and $x = \dfrac{1}{2}\pi$ respectively.

(a) Find the point S where the normal at Q meets the $y-$axis.

(b) Compare the distances SP, SQ and SR. Hence, use a sketch to show how the curve $y = \cos x$, over $-\dfrac{1}{2}\pi < x < \dfrac{1}{2}\pi$, is related to the circle with centre S and radius SQ.

Example:

Given that $f(x)=\sin x+\sin 2x$.

(a) Find, in terms of π, the $x-$intercepts of the curve $y=f(x)$ for $0\le x\le 2\pi$.

(b) Show that $f(\pi-\theta)=\sin\theta-\sin 2\theta$, and that $f(\pi-\theta)+f(\pi+\theta)=0, \forall\theta$.

(d) Show that the greatest value of f, for $0\le x\le 2\pi$, occurs when $\cos x=\dfrac{-1+\sqrt{33}}{8}$.

Example:

A length of channel of given depth d is to be made from a rectangular sheet of metal of width $2a$. The metal is to be bent in such a way that the cross-section is shown in the figure.

(a) Show that $BC = 2(a - d \operatorname{cosec} \theta)$

(b) Show that area $ABCD = 2ad + d^2(\cot\theta - 2\operatorname{cosec}\theta)$.

(c) Show that MAX area of $ABCD$ is $d(2a - d\sqrt{3})$.

(d) Show that $2d \le a\sqrt{3}$, considering BC.