TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH 2013

(January & March 2012 Intakes)

Friday **29 March 2012** **8.30 am – 10.15 am**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

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[Turn over

1 Solve the equation

$$|x+3|<2x. \qquad x \geqslant 3$$ [2]

2 The variables x and y are related by the equation $x^3y+3y^3=2$. Find, in terms of x and y, an expression for $\frac{dy}{dx}$. $\frac{-3x^2y}{x^3+9y^2}$ [2]

Hence, find the value of $\frac{dx}{dy}$ at the point (-1, 1). $-\frac{8}{3}$ [1]

3 The parametric equations of a curve are

$$x=3+2\sin\theta, \quad y=4-2\cos\theta,$$

where $0 \le \theta < 2\pi$.

(i) Express $\frac{dy}{dx}$ in terms of θ. $\tan\theta$ [3]

(ii) Find the cartesian equation of this circle. $x^2+y^2-6x-8y+5=0$ [2]

4 Let $f(x)=\frac{3x-1}{(1-2x)^2}$.

(i) Express $f(x)$ in partial fractions. $-\frac{3}{2(1-2x)}+\frac{1}{2(1-2x)^2}$ [2]

(ii) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3, simplifying each term. [3]

$-1-x+4x^3$

5 Let $f(x)=\ln(x+2)-x+1, \quad x>-2, x\in\Re$.

(i) Show that there is a root for $f(x)=0$ in the interval $2<x<3$. [2]

(ii) Use the iterative formula

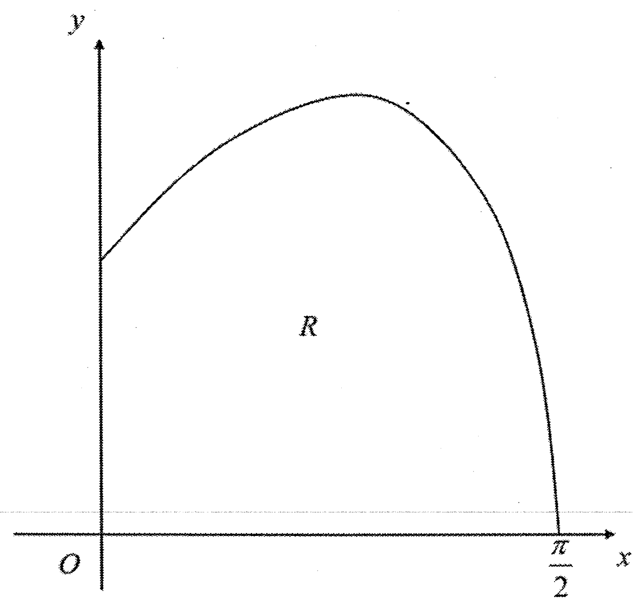
$$x_{n+1}=\ln(x_n+2)+1, \; x_0=2.5$$

with initial value $x_1=1$, find the root for $f(x)=0$ correct to 3 decimal places. Give the result of each iteration to 5 decimal places [3]

$\alpha \approx 2.505$ (3d.p.)

6 One root of the equation $x^3 + kx + 11 = 0$, where k is a constant, is 1. Find the other two roots of the equation, giving your answers in an exact form. $-\frac{1}{2} \pm \frac{3\sqrt{5}}{2}$ [6]

7



The diagram shows part of the curve with equation

$$y = e^x \cos x, \quad 0 \le x \le \frac{\pi}{2}.$$

The finite region R is bounded by the curve and the coordinate axes.

(i) Use the trapezium rule with three equal intervals to find an approximation for the area of R. 1.77 units [4]

(ii) State, with a reason, whether your approximation underestimates or overestimates the area of R. (underestimates) [2]

8 Given that

$$3\cos x - 4\sin x \equiv R\cos(x + \alpha),$$

where $R > 0$ and $0^\circ < \alpha < 90^\circ$, find the values of R and α, giving the value of α correct to 2 decimal places. $R = 5$, $\alpha = 53.13^\circ$ [2]

Hence solve the equation

$$3\cos 2\theta - 4\sin 2\theta = 2,$$

for $0^\circ < \theta < 360^\circ$, giving your answers correct to 1 decimal place. [6]

~~6.6~~ 6.7°, 120.2°, 186.6°, 300.2°

9 **(a)** Given that $z_1 = 2+i$ and that $z_2 = -2+4i$, find in the form $a+bi$, the complex number z which is such that

$z = \frac{6}{5} + \frac{8}{5}i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}.$$

Find the modulus and argument of z, giving your answer for arg z in the interval $-\pi < \arg z < \pi$, correct to three decimal places. $|z| = 2$, $\arg z = 0.927$ rad [5]

(b) A complex number z satisfies the inequality $|z+2-2\sqrt{3}i| \le 2$. Describe, in geometrical terms, with the aid of a sketch, the corresponding region in an Argand diagram. [1]

Find

(i) the least possible value of $|z|$, 2 units [1]

(ii) the greatest possible value of $\arg z$. $\frac{5}{6}\pi$ [2]

10 Relative to a fixed origin O, the vector equations of the two lines l_1 and l_2 are

$$l_1 : \mathbf{r} = 9\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} + t(-8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}),$$

and

$$l_2 : \mathbf{r} = -16\mathbf{i} + \alpha\,\mathbf{j} + 10\mathbf{k} + s(\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}),$$

where α is a constant.
The two lines intersect at the point A.

(i) Find the position vector of the point A. $\begin{pmatrix} -15 \\ -7 \\ 19 \end{pmatrix}$ [5]

(ii) Prove that the acute angle between l_1 and l_2 is 60°. [2]

Point B lies on l_1 and point C lies on l_2. The triangle ABC is equilateral with sides of length $14\sqrt{2}$.

(iii) Find one of the possible position vectors for the point B and the corresponding position vector for the point C. [6]

$\vec{OB} = \begin{pmatrix} 1 \\ -1 \\ 9 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} -17 \\ 1 \\ 1 \end{pmatrix}$

11 A drop of oil is modeled as a circle of radius r cm. At time t

$$r = 4(1 - e^{-\lambda t}),\ t > 0,$$

where λ is a positive constant.

(i) Show that the area A of the circle satisfies

$$\frac{dA}{dt} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t}).$$ [5]

In an alternative model of the drop of oil its area A at time t satisfies

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{t^2},\ t > 0.$$

Given that the area of the drop is 1 cm^2 at $t = 1$,

(ii) find an expression for A in terms of t for this alternative model. $A = \dfrac{4t^2}{(1+t)^2}$ [6]

(iii) Show that, in the alternative model, the value of A cannot exceed 4. [2]

$t \to \infty$, $A \to 4$

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A2 TRIAL EXAMINATION MARCH 2013

(January and March 2012 Intakes)

Wednesday **27 March 2013** **8.30 am – 9.45 am**

MATHEMATICS 9709/43

PAPER 4 Mechanics 1 (M1) 1 hour 15 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use 10 ms^{-2}.
At the end of the examination, fasten all your work securely together.

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The total marks for this paper is 50.
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[Turn over

1 A particle starts at a point O and moves along a straight line. Its velocity t s after leaving O is $(2.4t - 0.24t^2)$ ms^{-1}. Find the displacement of the particle from O when its deceleration is 0.6ms^{-2}. 27.3 m [5]

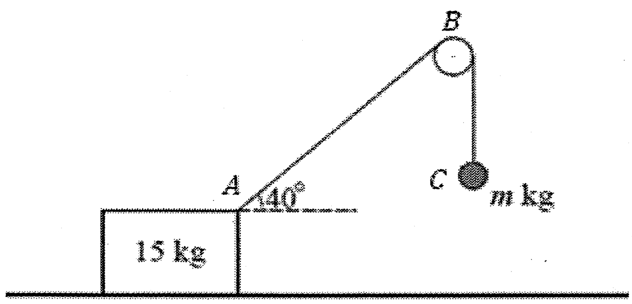
2 A car of mass 1000kg is moving along a straight horizontal ~~road~~ with a constant acceleration of a ms^{-2}. The resistance to motion is modeled as a constant force of magnitude 1200N. When the car is travelling at 12ms^{-1}, the power generated by the engine of the car is 24kW.

(i) Calculate the value of a. $0.8ms^{-2}$ [3]

When the car is travelling at 14 ms^{-1}, the engine is switched off and the car comes to rest, without braking, in a distance of d metres. Assuming the resistance is the same,

(ii) Use the work-energy principle to calculate the value of d. 81.7m [3]

3



The figure shows a block of mass 15kg on a rough horizontal plane. A light string is fixed to the block at A, passes over a smooth fixed pulley B and is attached at C to a sphere. The section of the string between the block and the pulley is inclined at 40° to the horizontal and the section between the pulley and the sphere is vertical. The system is in equilibrium and the tension in the string is 58.8N.

(i) The sphere has a mass of m kg. Calculate the value of m. 5.88m [2]

(ii) Find the frictional and normal components of the force exerted on the block by the plane. $F_r = 45N$, $R = 112.7 N$ [4]

4 A man drives a car on a horizontal straight road. At $t = 0$, where the time t is in seconds, the car runs out of petrol. At this instant the car is moving at 12ms^{-1}. The car decelerates uniformly, coming to rest when $t = 8$. The man then walks back along the road at 0.7ms^{-1} until he reaches a petrol station a distance of 420m from his car. After his arrival at the petrol station it takes him 250s to obtain a can of petrol. He is then given a lift back to his car on a motorcycle. The motorcycle starts from rest and accelerates uniformly until its speed is 20ms^{-1}; it then decelerates uniformly, coming to rest at the stationary car at time $t = T$.

(i) Sketch the shape of the (t, v) graph for the man for $0 \le t \le T$.
[Your sketch need not be drawn to scale; numerical values need not be shown.] [4]

(ii) Find the value of T. 900s [3]

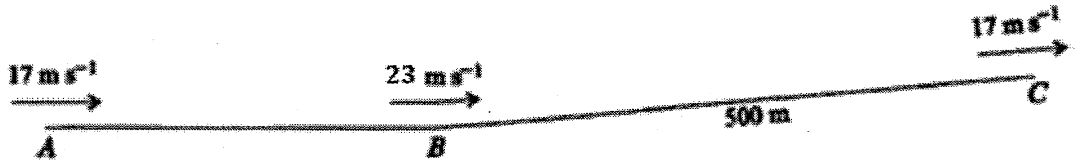
5 A particle of mass 0.04 kg is acted on by a force of magnitude P N in a direction at an angle α to the upward vertical. The resultant of the weight of the particle and the force applied to the particle acts horizontally. Given that $\alpha = 20°$ find

(i) the value of P, 0.426 [3]

(ii) the magnitude of the resultant, 0.146N [2]

(iii) the magnitude of the acceleration of the particle. 3.64ms^{-2} [2]

6



A lorry of mass 12500 kg travels along a road that has a straight horizontal section AB and a straight inclined section BC. The length of BC is 500 m. The speeds of the lorry at A, B and C are 17 ms^{-1}, 23 ms^{-1} and 17ms^{-1} respectively (see diagram).

(i) The work done by the driving force as the lorry travels from A to B, is 7000kJ. Find the work done against the resistance to motion of the lorry, as it travels from A to B. 5500 kJ [4]

(ii) As the lorry travels from B to C, the resistance to motion is 4800 N and the height of C above the level of AB is 24 m. Find the work done by the driving force. 3900 kJ [4]

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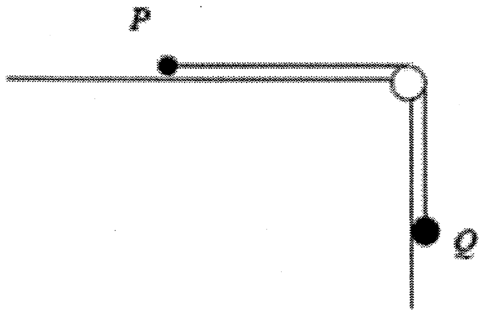

Two particles P and Q, of masses 3kg and 5kg respectively, are connected by a light inextensible string which passes over a small smooth light pulley fixed at the edge of a rough horizontal table. Particle P lies on the table and particle Q hangs freely below the pulley (see diagram). The coefficient of friction between P and the table is 0.6. The system is released from rest with the string taut.

(i) Find the tension in the string and the acceleration of the particles while both are moving. $4\,\mathrm{ms}^{-2}$, $30\,\mathrm{N}$ [6]

When Q has moved a distance h, it hits the floor and the string becomes slack. Given that P remains on the table during the subsequent motion and does not reach the pulley.

(ii) Find, in terms of h, the distance moved by P after the string becomes slack until P comes to rest. $\frac{2}{3}h$ [5]