Exercise 1: The derivative of e^x

Differentiate the following.

(i) $y = 3e^x$

(ii) $y = e^{2x}$

(iii) $y = e^{x^2}$

(iv) $y = e^{(x+1)^2}$

(v) $y = xe^{4x}$

(vi) $y = 2x^3e^{-x}$

(vii) $y = \dfrac{x}{e^x}$

(viii) $y = (e^{2x} + 1)^3$

Exercise 2: The Derivative of ln x

1 Write down the derivative of each of the following functions.

(a) $\ln x^3$ (b) $\ln (3x)$

(c) $\ln (x^{-2})$ (d) $\ln \left(\dfrac{3}{\sqrt{x}}\right)$

(e) $\ln \left(\dfrac{1}{x^5}\right)$ (f) $\ln \left(2x^{\frac{1}{2}}\right)$

(g) $\ln \left(x^{-\frac{3}{2}}\right)$ (h) $\ln \left(\dfrac{x^3}{\sqrt{x}}\right)$

2 Locate the stationary points on each curve.

(a) $y = \ln x - x$ (b) $y = x^3 - 2 \ln x^3$

(c) $y = \ln x - \sqrt{x}$

3 Sketch each of the following curves.

(a) $y = -\ln x$ (b) $y = \ln (-x)$

(c) $y = 2 + \ln x$ (d) $y = \ln x^2$

4 Find the derivative of each of the following functions.

(a) $\ln (3x - 1)$ (b) $\ln (4x + 3)$

(c) $\ln (x^2 + 1)$ (d) $\ln (x^2 + 2x)$

(e) $\ln (x^3 + 2)$

Exercise 3: Mixed Questions

1 Express y in terms of x when $\ln y - \ln (y + 1) = \ln x$

2 By sketching two graphs, show that $e^x = x^2$ has only one root.

3 (a) By sketching suitable graphs show that the equation $\ln x = 1 - x$ has only one root.

~~(b) Write down the value of this root~~.

4 Find the equation of the tangent to the curve $y = \ln (x^2)$ at the point where $x = 4$

5 Find the equation of the normal to the curve $y = 3e^{2x}$ where $x = 1$

6 Given that $y = e^{2x} - \ln \sqrt{x}$ find $\dfrac{dy}{dx}$.

7 Write down the derivatives of the following functions.

(a) e^{3x+2} (b) $\ln (x^2 - 3)$

(c) $e^{(x^2-1)}$ (d) $\ln (x^2 - 1)$

(e) $\ln (x^3 + 2x + 1)$

Exercise 4: Differentiating a Product or a Quotient

In each case first identify the type of function and then use the appropriate method to find its derivative. Some of the functions can be differentiated by using a basic rule, so do not assume that a formula is always needed.

1 $x\sqrt{x+1}$

2 $(x^2-8)^3$

3 $\dfrac{x}{x^2+1}$

4 $\sqrt[3]{2-x^4}$

5 $\dfrac{x^2+1}{x^2+2}$

6 $x^2(\sqrt{x}-2)$

7 $(x^2-2)^3$

8 $\sqrt{x-x^2}$

9 $\dfrac{x}{\sqrt{x}+1}$

10 $x^2\sqrt{x-2}$

11 $\dfrac{\sqrt{x+1}}{x^2}$

12 $(x^4+x^2)^3$

13 $\sqrt{x^2-8}$

14 $x^3(x^2-6)$

15 $(x^2-6)^3$

16 $\dfrac{x}{x^2-6}$

17 $(x^4+3)^{-2}$

18 $\sqrt{x}(2-x)^3$

19 $\dfrac{\sqrt{x}}{(2-x)^3}$

20 $(x-1)(x-2)^2$

21 $(2x^3+4)^5$

22 $x\ln x$

23 $(4x-1)^{\frac{2}{3}}$

24 $\dfrac{e^x}{x-1}$

25 $\dfrac{\sqrt{1+x^3}}{x^2}$

26 $\dfrac{\ln x}{\ln(x-1)}$

27 $(\ln x)^2$

28 $\dfrac{(1+2x^2)}{1+x^2}$

29 $e^{-\frac{2}{x}}$

30 $\ln(1-e^x)$

31 $e^{3x}x^3$

32 $\dfrac{2x}{(2x-1)(x-3)}$

33 $\dfrac{e^{\frac{x}{2}}}{x^5}$

34 $\ln\left[\dfrac{x^2}{(x+3)}\right]$

35 $\ln[4x^3(x+3)^2]$

36 $(\ln x)^4$

37 $\dfrac{(x+3)^3}{x^2+2}$

38 $\sqrt{e^x-x}$

39 $4\ln(x^2+1)$

Find and simplify $\dfrac{dy}{dx}$ and hence find $\dfrac{d^2y}{dx^2}$

40 $y=\dfrac{1+2x}{1-2x}$

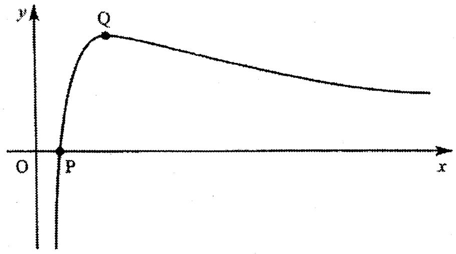
41 $y=\ln\dfrac{x}{x+1}$

42 $y=\dfrac{e^x}{e^x-4}$

Question 43

The diagram shows a sketch of the graph of $y = f(x)$, where

$$f(x) = \frac{\ln x}{x} \quad (x > 0).$$



The graph crosses the x axis at the point P and has a turning point at Q.

(i) Write down the x co-ordinate of P.

(ii) Find the first and second derivatives, $f'(x)$ and $f''(x)$, simplifying your answers as far as possible.

(iii) Hence show that the x co-ordinate of Q is e.
Find the y co-ordinate of Q in terms of e.
Find $f''(e)$, and use this result to verify that Q is a maximum point.

Exercise 5: Differentiating Trigonometric Functions

1 Write down the derivative of each of the following expressions.

(a) $\sin x - \cos x$

(b) $\sin\theta + 4$

(c) $3\cos\theta$

(d) $5\sin\theta - 6$

(e) $2\cos\theta + 3\sin\theta$

(f) $4\sin x - 5 - 6\cos x$

2 Find the gradient of each curve at the point whose x-coordinate is given.

(a) $y = \cos x;\ \frac{1}{2}\pi$

(b) $y = \sin x;\ 0$

(c) $y = \cos x + \sin x;\ \pi$

(d) $y = x - \sin x;\ \frac{1}{2}\pi$

(e) $y = 2\sin x - x^2;\ -\pi$

(f) $y = -4\cos x;\ \frac{1}{2}\pi$

3 For each of the following curves find the smallest positive value of θ at which the gradient of the curve has the given value.

(a) $y = 2\cos\theta;\ -1$

(b) $y = \theta + \cos\theta;\ \frac{1}{2}$

(c) $y = \sin\theta + \cos\theta;\ 0$

(d) $y = \sin\theta + 2\theta;\ 1$

4 Considering only positive values of x, locate the first two turning points on each of the following curves and determine whether they are maximum or minimum points.

(a) $2\sin x - x$ (b) $x + 2\cos x$

In each case illustrate your solution by a sketch.

5 Find the equation of the tangent to the curve $y = \cos\theta + 3\sin\theta$ at the point where $\theta = \frac{1}{2}\pi$

6 Find the equation of the normal to the curve $y = x^2 + \cos x$ at the point where $x = \pi$

7 Find the coordinates of a point on the curve $y = \sin x + \cos x$ at which the tangent is parallel to the line $y = x$

Exercise 6: Mixed Questions

This exercise contains a variety of functions. Consider carefully what method to use in each case and do not forget to check first whether a given function has a standard derivative.

Find the derivative of each function in questions **1** to **12**.

1 (a) $-\sin 4\theta$

(b) $\theta - \cos\theta$

(c) $\sin^3\theta + \sin 3\theta$

2 (a) $x^3 + e^x$

(b) $e^{(2x+3)}$

(c) $e^x \sin x$

3 (a) $3\sin x - e^{-x}$

(b) $x^4 + 4e^x - \ln 4x$

4 $(x+1)\ln x$

5 $\sin^2 3x$

6 $(4x-1)^{\frac{2}{3}}$

7 $\dfrac{(x^4-1)}{(x+1)^3}$

8 $x^2 \sin x$

9 $\dfrac{e^x}{x-1}$

10 $\dfrac{1+\sin x}{1-\sin x}$

11 $x^2\sqrt{x-1}$

12 $\sin x \cos^3 x$

In each question from **13** to **16**, find

(a) the gradient of the curve at the given point,

(b) the equation of the tangent to the curve at that point,

(c) the equation of the normal to the curve at that point.

13 $y = \sin x - \cos x;\ x = \frac{1}{2}\pi$

14 $y = x + e^x;\ x = 1$

15 $y = 1 + x + \sin x;\ x = 0$

16 $y = 3 - x^2 + \ln x;\ x = 1$

17 Considering only positive values of x, locate the first two turning points, if there are two, on each of the following curves and determine whether they are maximum or minimum points.

(a) $y = 1 - \sin x$ (b) $y = \frac{1}{2}x + \cos x$

18 Find the coordinates of a point on the curve where the tangent is parallel to the given line.

(a) $y = 3x - 2\cos x;\ y = 4x$

(b) $y = 2\ln x - x;\ y = x$

Exercise 7: Implicit Functions

Differentiate the following equations with respect to x.

1 $x^2 + y^2 = 4$

2 $x^2 + xy + y^2 = 0$

3 $x(x+y) = y^2$

4 $\dfrac{1}{x} + \dfrac{1}{y} = e^y$

5 $\sin x + \sin y = 1$

6 $xe^y = x + 1$

7 Find $\dfrac{dy}{dx}$ as a function of x if $y^2 = 2x + 1$

8 Find the gradient of $x^2 + y^2 = 9$ at the points where $x = 1$

9 Find the equation of the tangent at $\left(1, \frac{1}{3}\right)$ to the curve whose equation is

$$2x^2 + 3y^2 - 3x + 2y = 0$$

10 The equation of a curve is $\cos x + \sin y = 1$

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) At the point $\left(\frac{\pi}{4}, \alpha\right)$ on the curve, the gradient of the tangent is 1. Find the value of α for $0 \leqslant \alpha \leqslant \pi$

P3 11 The equation of a curve is $x^3 + y^3 = 28$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(b) Find the value of y when $x = 1$ and hence find the equation of the tangent to the curve at this point.

P3 12 The equation of a curve is $x^2 + y^2 = \alpha^2$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(b) Find the equation of the tangent to the curve at the point $(\alpha\sqrt{2}, \alpha\sqrt{2})$.

P3 13 The equation of a curve is $x^2 + 2y^2 = 3$

Find the equation of the normal to the curve at the point on the curve at which $x = y$

Exercise 8: Parametric Equations

1 Find the gradient function of each of the following curves in terms of the parameter.

(a) $x = 2t^2,\ y = t$

(b) $x = \cos\theta, y = \sin\theta$

(c) $x = t,\ y = \frac{4}{t}$

2 If $x = \frac{t}{1-t}$ and $y = \frac{t^2}{1-t}$, find $\frac{dy}{dx}$ in terms of t.

What is the value of $\frac{dy}{dx}$ at the point where $x = 1$?

3 If $x = t^2$ and $y = t^3$, find $\frac{dy}{dx}$ in terms of t.

4 The parametric equations of a curve are $y = e^t - 1$ and $x = 1 - e^{-t}$

Find $\frac{dy}{dx}$ in terms of t.

5 Find the turning points of the curve whose parametric equations are $x = t, y = t^3 - t$, and distinguish between them.

6 A curve has parametric equations

$$x = \theta - \cos\theta, y = \sin\theta.$$

Find the smallest positive value of θ at which the gradient of this curve is zero.

7 Find the equation of the tangent to the curve $x = t^2, y = 4t$ at the point where $t = -1$

8 Find the equation of the normal to the curve

$$x = 2\cos\theta, y = 3\sin\theta$$

at the point where $\theta = \frac{1}{4}\pi$.
Find the coordinates of the point where this normal cuts the curve again.

Exercise 9: Mixed Questions

1 Differentiate with respect to x

(a) y^4 (b) xy^2

(c) $\frac{1}{y}$ (d) $x\ln y$

(e) $\sin y$ (f) e^y

(g) $y\cos x$ (h) $y\cos y$

In questions **2** to **4** find $\frac{dy}{dx}$ in terms of x and y.

2 $x^2 - 2y^2 = 4$

3 $\frac{1}{x} + \frac{1}{y} = 2$

4 $x^2y^3 = 9$

In questions **5** to **10** find $\frac{dy}{dx}$ in terms of the parameter.

5 $x = t^2, y = t^3$

6 $x = (t+1)^2, y = t^2 - 1$

7 $x = \sin^2\theta, y = \cos^3\theta$

8 $x = 4t, y = \frac{4}{t}$

9 $x = e^t, y = 1 - t$

10 $x = \frac{t}{1-t}, y = \frac{t^2}{1-t}$

11 If $x = \sin t$ and $y = \cos 2t$, find $\frac{dy}{dx}$ in terms of x.

12 If $x = e^t - t$ and $y = e^{2t} - 2t$, show that $\frac{dy}{dx} = 2(e^t + 1)$

13 Differentiate $y^2 - 2xy + 3y = 7x$ with respect to x. Hence find the equation of the tangent to the curve at the point where $y = 1$

14 Find the equation of the normal to the curve

$$x = \cos\theta, y = 2\sin\theta$$

at the point where $\theta = \frac{3\pi}{4}$

Answers

Exercise 1

(i) $3e^x$

(ii) $2e^{2x}$

(iii) $2xe^{x^2}$

(iv) $2(x+1)e^{(x+1)^2}$

(v) $e^{4x}(1+4x)$

(vi) $2x^2e^{-x}(3-x)$

(vii) $\frac{1-x}{e^x}$

Exercise 2

1 (a) $\frac{3}{x}$ (b) $\frac{1}{x}$ (c) $-\frac{2}{x}$ (d) $-\frac{1}{2x}$

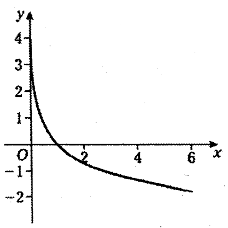
(e) $-\frac{5}{x}$ (f) $\frac{1}{2x}$ (g) $-\frac{3}{2x}$ (h) $\frac{5}{2x}$

2 (a) $(1, -1)$

(b) $\left(2^{\frac{1}{3}}, \{2 - 2\ln 2\}\right)$

(c) $(4, \{\ln 4 - 2\})$

3 (a)



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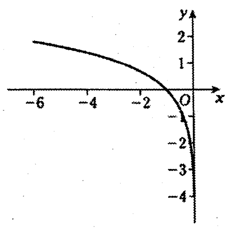

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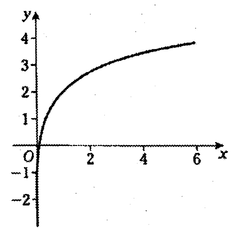
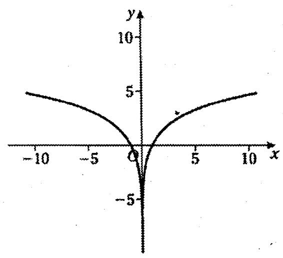


(d)



(a) $\frac{3}{3x-1}$ (b) $\frac{4}{4x+3}$

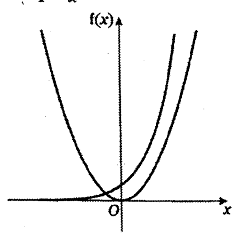
(c) $\frac{2x}{x^2+1}$ (d) $\frac{2x+2}{x^2+2x}$

(e) $\frac{3x^2}{x^3+2}$

Exercise 3

1 $y = \frac{x}{1-x}$

2



there is only one point of intersection

3 1

4 $y - 2\ln 4 = \frac{1}{2}(x-4)$

5 $y - 3e^2 = -\frac{1}{6e^2}(x-1)$

6 $\frac{dy}{dx} = 2e^{2x} - \frac{1}{2x}$

7 (a) $3e^{3x+2}$ (b) $\frac{2x}{x^2-3}$ (c) $2xe^{x^2-1}$

(d) $\frac{2x}{x^2-1}$ (e) $\frac{3x^2+2}{x^3+2x+1}$

Exercise 4

1 $\frac{3x+2}{2\sqrt{x+1}}$

2 $6x(x^2-8)^2$

3 $\frac{1-x^2}{(x^2+1)^2}$

4 $\frac{-4x^3}{3(2-x^4)^{\frac{2}{3}}}$

5 $\frac{2x}{(x^2+2)^2}$

6 $\frac{1}{2}x(5\sqrt{x}-8)$

7 $6x(x^2-2)^2$

8 $\frac{1-2x}{2\sqrt{x-x^2}}$

9 $\frac{\sqrt{x}+2}{2(\sqrt{x}+1)^2}$

10 $\frac{x(5x-8)}{2\sqrt{x-2}}$

11 $\frac{-(3x+4)}{2x^3\sqrt{x+1}}$

12 $6x^5(x^2+1)^2(2x^2+1)$

13 $\frac{x}{\sqrt{x^2-8}}$

14 $x^2(5x^2-18)$

15 $6x(x^2-6)^2$

16 $\frac{-(x^2+6)}{(x^2-6)^2}$

17 $-8x^3(x^4+3)^{-3}$

18 $\frac{(2-x)^2(2-7x)}{2\sqrt{x}}$

19 $\frac{2+5x}{2\sqrt{x}(2-x)^4}$

20 $(x-2)(3x-4)$

21 $30x^2(2x^3+4)^4$

22 $1 + \ln x$

23 $\frac{8}{3}(4x-1)^{-\frac{1}{3}}$

24 $\frac{e^x(x-2)}{(x-1)^2}$

25 $\frac{-(x^3+4)}{2x^3\sqrt{1+x^3}}$

Exercise 4

26 $\frac{(x-1)\ln(x-1) - x\ln x}{x(x-1)\{\ln(x-1)\}^2}$

27 $\frac{2\ln x}{x}$

28 $\frac{2x}{(1+x^2)^2}$

29 $\frac{2}{x^2e^{\frac{2}{x}}}$

30 $\frac{-e^x}{1-e^x}$ or $\frac{e^x}{e^x-1}$

31 $3x^2e^{3x}(x+1)$

32 $\frac{4}{5(2x-1)^2} - \frac{6}{5(x-3)^2} = \frac{2(3-2x^2)}{(2x-1)^2(x-3)^2}$

33 $\frac{e^{\frac{x}{2}}(x-10)}{2x^6}$

34 $\frac{2}{x} - \frac{1}{x+3}$

35 $\frac{5x+9}{x(x+3)}$

36 $\frac{4}{x}(\ln x)^3$

37 $\frac{(x+3)^2(x^2-6x+6)}{(x^2+2)^2}$

38 $\frac{e^x-1}{2\sqrt{e^x-x}}$

39 $\frac{8x}{x^2+1}$

40 $\frac{dy}{dx} = \frac{4}{(1-2x)^2}; \frac{d^2y}{dx^2} = \frac{16}{(1-2x)^3}$

41 $\frac{dy}{dx} = \frac{1}{x(x+1)}; \frac{d^2y}{dx^2} = -\frac{(2x+1)}{x^2(x+1)^2}$

42 $\frac{dy}{dx} = \frac{-4e^x}{(e^x-4)^2}; \frac{d^2y}{dx^2} = \frac{4e^x(e^x+4)}{(e^x-4)^3}$

Question 43

Exercise 5

1 (a) $\cos x + \sin x$ (b) $\cos\theta$

(c) $-3\sin\theta$ (d) $5\cos\theta$

(e) $3\cos\theta - 2\sin\theta$ (f) $4\cos x + 6\sin x$

2 (a) -1 (b) 1 (c) -1

(d) 1 (e) $2(\pi - 1)$ (f) 4

3 (a) $\frac{1}{6}\pi$ (b) $\frac{1}{6}\pi$

(c) $\frac{1}{4}\pi$ (d) π

4 (a) $\left(\frac{1}{3}\pi, \sqrt{3} - \frac{1}{3}\pi\right)$, max; $\left(\frac{5}{3}\pi, -\sqrt{3} - \frac{5}{3}\pi\right)$, min

(b) $\left(\frac{1}{6}\pi, \frac{1}{6}\pi + \sqrt{3}\right)$, max; $\left(\frac{5}{6}\pi, \frac{5}{6}\pi - \sqrt{3}\right)$, min

5 $y + \theta = 3 + \frac{1}{2}\pi$

6 $2\pi y + x = 2\pi^3 - \pi$

7 $(0, 1)$

Exercise 6

1 (a) $-4\cos 4\theta$

(b) $1 + \sin\theta$

(c) $3\sin^2\theta\cos\theta + 3\cos 3\theta$

2 (a) $3x^2 + e^x$

(b) $2e^{(2x+3)}$

(c) $e^x(\sin x + \cos x)$

3 (a) $3\cos x + e^{-x}$

(b) $4x^3 + 4e^x - \frac{1}{x}$

4 $1 + \frac{1}{x} + \ln x$

5 $3\sin 6x$

Exercise 6

6 $\frac{8}{3}(4x-1)^{-\frac{1}{3}}$

7 $\dfrac{(x^4+4x^3+3)}{(x+1)^4}$

8 $2x\sin x + x^2\cos x$

9 $\dfrac{e^x(x-2)}{(x-1)^2}$

10 $\dfrac{2\cos x}{(1-\sin x)^2}$

11 $\dfrac{x(5x-4)}{2\sqrt{x-1}}$

12 $\cos^2 x(4\cos^2 x - 3)$

13 (a) 1 (b) $y - x = 1 - \frac{1}{2}\pi$
(c) $y + x = 1 + \frac{1}{2}\pi$

14 (a) $1 + e$ (b) $y = x(1+e)$
(c) $y(1+e) + x = (1+e)^2 + 1$

15 (a) 2 (b) $y = 2x + 1$
(c) $2y + x = 2$

16 (a) -1 (b) $x + y = 3$
(c) $x - y + 1 = 0$

17 (a) $\left(\frac{1}{2}\pi, 0\right)$, min; $\left(\frac{3}{2}\pi, 2\right)$, max
(b) $\left(\frac{1}{6}\pi, \left\{\frac{1}{12}\pi + \frac{1}{2}\sqrt{3}\right\}\right)$, max;
$\left(\frac{5}{6}\pi, \left\{\frac{5}{12}\pi - \frac{1}{2}\sqrt{3}\right\}\right)$, min

18 (a) $\left(\frac{1}{6}\pi, \left\{\frac{1}{2}\pi - \sqrt{3}\right\}\right)$ (b) $(1, -1)$

Exercise 7

1 $2x + 2y\dfrac{dy}{dx} = 0$

2 $2x + y + (x + 2y)\dfrac{dy}{dx} = 0$

3 $2x + x\dfrac{dy}{dx} + y = 2y\dfrac{dy}{dx}$

4 $-\dfrac{1}{x^2} - \dfrac{1}{y^2}\dfrac{dy}{dx} = e^y\dfrac{dy}{dx}$

5 $\cos x + \cos y\dfrac{dy}{dx} = 0$

6 $e^y + xe^y\dfrac{dy}{dx} = 1$

7 $\dfrac{dy}{dx} = \pm\dfrac{1}{\sqrt{2x+1}}$

8 $\pm\frac{1}{4}\sqrt{2}$

9 $3x + 12y - 7 = 0$

10 (a) $\dfrac{dy}{dx} = \dfrac{\sin x}{\cos y}$
(b) $\dfrac{\pi}{4}$

11 (a) $\dfrac{dy}{dx} = -\dfrac{x^2}{y^2}$ (b) $y = 3, 9y = -x + 28$

12 (a) $\dfrac{dy}{dx} = -\dfrac{x}{y}$ (b) $y = 2a\sqrt{2} - x$

13 $y = 2x + 1$ or $y = 2x - 1$

Exercise 8

1 (a) $\dfrac{1}{4t}$ (b) $-\cot\theta$ (c) $-\dfrac{4}{t^2}$

2 $\dfrac{dy}{dx} = 2t - t^2; \frac{3}{4}$

3 $\frac{3}{2}t$

4 $\dfrac{dy}{dx} = e^{2t}$

5 $\left(-\frac{1}{3}\sqrt{3}, \frac{2}{9}\sqrt{3}\right)$, max; $\left(\frac{1}{3}\sqrt{3}, -\frac{2}{9}\sqrt{3}\right)$, min

6 $\frac{1}{2}\pi$

7 $2x + y + 2 = 0$

8 $6y = 4x + 5\sqrt{2}, \left(-\frac{137}{97}\sqrt{2}, -\frac{21}{194}\sqrt{2}\right)$

Exercise 9

1 (a) $4y^3\dfrac{dy}{dx}$ (b) $y^2 + 2xy\dfrac{dy}{dx}$
(c) $-\dfrac{1}{y^2}\dfrac{dy}{dx}$ (d) $\ln y + \dfrac{x}{y}\dfrac{dy}{dx}$
(e) $\cos y\dfrac{dy}{dx}$ (f) $e^y\dfrac{dy}{dx}$
(g) $\dfrac{dy}{dx}\cos x - y\sin x$
(h) $(\cos y - y\sin y)\dfrac{dy}{dx}$

2 $\dfrac{x}{2y}$

3 $-\dfrac{y^2}{x^2}$

4 $-\dfrac{2y}{3x}$

5 $\dfrac{3t}{2}$

6 $\dfrac{t}{t+1}$

7 $-\frac{3}{2}\cos\theta$

8 $-\dfrac{1}{t^2}$

9 $-\dfrac{1}{e^t}$

10 $2t - t^2$

11 $\dfrac{dy}{dx} = -4x$

13 $2y\dfrac{dy}{dx} - 2x\dfrac{dy}{dx} - 2y + 3\dfrac{dy}{dx} = 7$
$37y = 81x + 1$

14 $4y + 2x = 3\sqrt{2}$