Tutorial (Discussion 1) – Topic 3

1 The points $A(-1, 1)$, $B(p, 7)$ and $C(5, 10)$ are collinear. Find the value of p and the ratio $AB : BC$.

2 The perpendicular bisector of $(-3, 3)$ and $(7, 11)$ passes through the point $(6, p)$. Find the value of p.

3 Find the perpendicular distance from $(7, 4)$ to the line $y = -\frac{x}{2} + 5$.

4 Find the area of the triangle formed by the intersection of the lines $2y - 3x + 4 = 0$, $x = 0$ and $y = 0$.

***5** Show that the triangle whose vertices are at $A(3, 2)$, $B(1, 5)$, $C(4, 3)$ is isosceles and find the coordinates of point D such that $ABCD$ is a rhombus.

6 A, B and C have coordinates $(5, 7)$, $(3, 3)$ and $(7, 4)$ respectively. A point D lies on BC produced such that $BD = 2BC$. Find the equation of the line AD.

7 Show that $A(3, 4)$, $B(2, 7)$ and $C(8, 9)$ are three vertices of a rectangle. Find the coordinates of the fourth vertex and the area of the rectangle.

8 The straight lines $y = k^2x + 12$ and $2ky = 4x + 5$ are perpendicular ($k \neq 0$).

(a) Find the value of k.
(b) Find the point of intersection of the two lines.

9 A triangle is formed by the intersection of the straight lines $y = 2x + 3$, $2y + x = 26$ and $11y = 7x + 18$.

(a) Find the coordinates of the vertices of the triangle.
(b) Show that the triangle is right-angled.
(c) Find the area of the triangle.

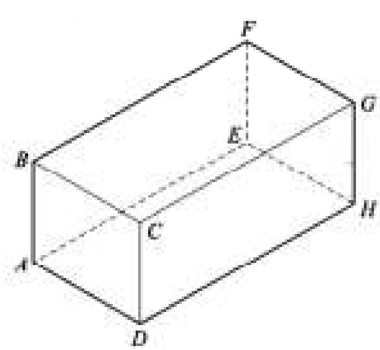
10 A straight line has the equation $\frac{2 - x}{3} = \frac{y - 4}{5}$.

(a) Write the equation in the form $y = mx + c$.

(b) Find the equation of the straight line which is perpendicular to $\frac{2 - x}{3} = \frac{y - 4}{5}$ and which passes through the point $(2, 7)$.

11 Three points have coordinates $A(-5, 2)$, $B(1, 14)$ and $C(3, 6)$. Find the coordinates of the point of intersection of the line AB and the perpendicular bisector of AC.

12 The cuboid $ABCDEFGH$ is shown. The coordinates of A, B and H are $(0, 3, 5)$, $(0, 3, 9)$ and $(4, 9, 5)$ respectively. Write down the coordinates of G and hence find the length of AG.



***13** Find the perpendicular distance between the lines $y = 2x + 7$ and $y = 2x + 2$.

Answers

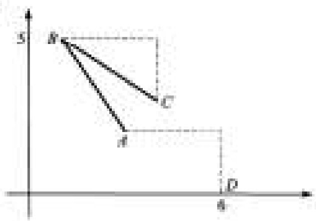
1 3; 2 : 1

2 2

3 $\sqrt{5}$ units

4 $\frac{4}{5}$ units^2

5



$\overrightarrow{BC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \therefore \overrightarrow{AD} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \therefore D(6, 0)$

6 $x + 3y = 26$

7 (9, 6), 20 units^2

8 (a) $-\frac{1}{2}$ (b) $(-4, 11)$

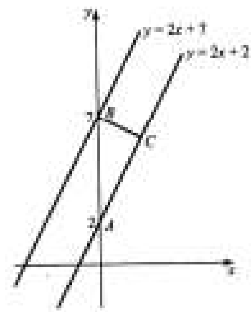
9 (a) (4, 11), (10, 8), (−1, 1)
(c) 37.5 units^2

10 (a) $y = -\frac{3}{5}x + \frac{22}{5}$
(b) $5y = 3x + 29$

11 (−2.5, 7)

12 (4, 9, 9), $\sqrt{68}$ units

13



Gradient of $BC = -\frac{1}{2}$ and it intersects the y-axis at (0, 7).
Equation of BC is $y = -\frac{x}{2} + 7$.

C is the intersection of $y = -\frac{x}{2} + 7$ and $y = 2x + 2$ (1)

$$-\frac{x}{2} + 7 = 2x + 2$$

$$5 = \frac{5x}{2}$$

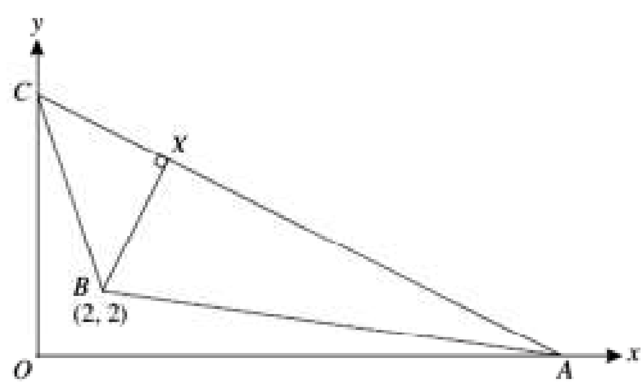
$$x = 2$$

Substituting in (1) gives $y = 6$, hence C is the point (2, 6).

Length $BC = \sqrt{(7-6)^2 + (0-2)^2}$
$= \sqrt{5}$ units

14 (a) (3, 0)
(b) $\sqrt{28}$ units

Additional Question



In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates (2, 2). The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]