TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

AS TRIAL EXAMINATION MARCH/APRIL 2012

(June 2011 Intake)

Friday **30 March 2012** **8.30 am – 10.15 am**

MATHEMATICS 9709/13

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

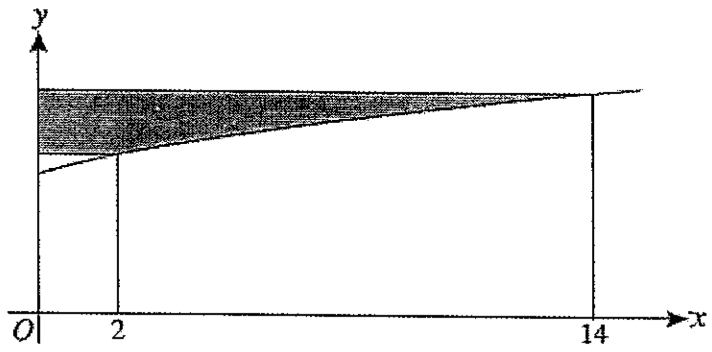
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

1 Given that A and B are points of intersection of the line $y = 4 - x$ and the curve $y = x^2 + 2x$. Find the perpendicular bisector line joining $A$and B. [4]

2



The diagram shows the curve $y = 3 + \sqrt{x+2}$.

The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

Find the exact area of the shaded region. [5]

3 **(i)** Find the terms in x^4 and x^6 in the expansion of $\left(1 - \frac{x}{3}\right)^{10}$. [2]

(ii) Hence find the coefficients of x^6 in the expansion of

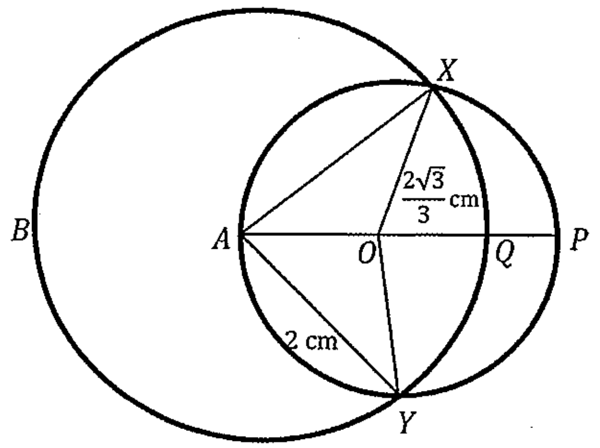
$$(1 + 3x^2)\left(1 - \frac{x}{3}\right)^{10}.$$ [3]

4 **(a)** Prove that

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}.$$ [3]

(b) Solve the equation $3\cos^2\theta - 2\sin\theta - 3 = 0$for $0° \le \theta \le 360°$. [4]

5



The diagram above shows two circles, with centres$A$and O, intersecting at X and Y and $AOQP$ is a straight line. Circle BXQY has centre A with radius 2cm and circle $AXPY$ has centre O with radius $\frac{2\sqrt{3}}{3}$ cm. Angle XAY is $\frac{\pi}{3}$.

Find the

(i) ratio of XPY to XQY [3]

(ii) exact area of $XQYP$ [5]

6 The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k},$$
$$\overrightarrow{OB} = 4\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k},$$
$$\text{and} \quad \overrightarrow{OC} = 5\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}.$$

Find the

(i) unit vector of the mid-point of AB. [3]

(ii) angle of BAC. [4]

(iii) area of triangle ABC. [3]

[Turn over

7 **(a)** The first term of an arithmetic series is 5, and the twenty-fifth term of the series is 6.2. Find the common difference. [2]

Find also the least value of n such that the sum of the first n terms of the series exceeds 1000. [4]

(b) A geometric series has first term a and common ratio r, where $|r| < 1$. The sum to infinity of the series is 8. The sum to infinity of the series obtained by adding all the odd-numbered terms is 6. Find the value of r. [6]

8 An ~~The diagram shows an~~ open-topped cylindrical container is made from cardboard. The cylindrical is of height h cm and base radius r cm.

Given that the area of card used to make the container is $192\pi\ cm^2$,

(i) Show that the capacity of the container, $V\ cm^3$, is given by

$$V = 96\pi r - \frac{1}{2}\pi r^3.$$ [5]

(ii) Find the value of r for which V is stationary. [3]

(iii) Find the corresponding value of V in terms of π. [2]

(iv) Determine whether this is a maximum or a minimum value of V. [2]

9 **(i)** Express $f(x) = 4x^2 - 12x + 4$ in the form $(ax + b)^2 + c$. [3]

(ii) State the least value of $f(x)$ and corresponding value of x. [2]

(iii) Find the set values of x for which $f(x) \geq 20$. [3]

(iv) The function $f(x) = 4x^2 - 12x + 4$, $x \in \mathrm{R}$ and $x \leq k$. Given that f is one to one, determine the least value of k. [1]

(v) When k has this value, Express $f^{-1}(x)$ in terms of x. [3]