Vectors

A2 Trial Examination

August 2010

12 The lines l_1 and l_2 have vector equations

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + s(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} - \mathbf{k})$$

where s and t are variable parameters.

(i) Show that the lines intersect and are perpendicular to each other. [4]

(ii) Find a vector equation of the straight line l_3 in terms of λ, which passes through the point of intersection of l_1 and l_2 and the point with position vector $4\mathbf{i} + \lambda\mathbf{j} - \mathbf{k}$, where λ is a real number. [2]

(iii) The line l_3 makes an angle θ with the plane containing l_1 and l_2. Find $\sin\theta$ in terms of λ. [4]

$$\left[r = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 9 \\ \lambda - 4 \\ -3 \end{pmatrix} ; \frac{-5\lambda - 13}{\sqrt{42}\sqrt{\lambda^2 - 8\lambda + 106}} \right]$$

March 2010

9 The position vectors of three points A, B, C on a plane ski-slope are $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}, \mathbf{b} = -2\mathbf{i} + 26\mathbf{j} + 11\mathbf{k}, \mathbf{c} = 16\mathbf{i} + 17\mathbf{j} + 2\mathbf{k}$, where the units are in metres.

(i) Show that the vector $2\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ is perpendicular to $\overrightarrow{AB}$ and also perpendicular to $\overrightarrow{AC}$. Hence find the equation of the plane of the ski-slope. [4]

The track for an overhead railway lies along the straight edge DEF, where D and E have position vectors

$\mathbf{d} = 130\mathbf{i} - 40\mathbf{j} + 20\mathbf{k}$ and $\mathbf{e} = 90\mathbf{i} - 20\mathbf{j} + 15\mathbf{k}$,

and F is a point on the ski-slope.

(ii) Find the equation of the straight line DE. [3]

(iii) Find the position vector of the point F. [3]

(iv) Show that $\overrightarrow{DF} = 15(-8\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and hence find the length of the track. [3]

$$\left[2x - 3y + 7z = -5\,; r = \begin{pmatrix}130\\-40\\20\end{pmatrix} + t\begin{pmatrix}-40\\20\\-5\end{pmatrix}\,;\ \begin{pmatrix}10\\20\\5\end{pmatrix}\,; 135\right]$$

August 2009

6 Two planes have vector equations

$$r.(2i - 3j - k) = 14 \text{ and } r.(11i + j - 2k) = 42$$

(i) Find the acute angle between these two planes. [2]

(ii) Determine a vector equation for the line of intersection of these two planes. [5]

$$\left[60\,; r = \begin{pmatrix}4\\-2\\0\end{pmatrix} + t\begin{pmatrix}1\\-1\\5\end{pmatrix}\right]$$

March 2009

10 Lines L_1 , L_2 and L_3 has vector equations

$$L_1 : \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k})$$
$$L_2 : \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$
$$L_3 : \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} + c\mathbf{j} + \mathbf{k})$$

(i) Calculate the angle between L_1 and L_2. [3]

(ii) Given that L_2 and L_3 intersect, find the value of c. [3]

(iii) Find the equation of the plane Π containing the point $A(5,-1,-2)$ and is perpendicular to L_2. [3]

(iv) Determine the perpendicular distance of $B(6,0,3)$ to the plane Π. [4]

$[68.5\,;\ -3;\ x + 3y + 2z = -2;\ \sqrt{14}]$

August 2008

9 The points A, B and C have position vectors, with respect to the origin, given by

$\vec{OA} = (5i - j - 3k)$, $\vec{OB} = (-4i + 4j - k)$ and $\vec{OC} = (5i - 2j + 11k)$,

Find (i) a vector equation for the line BC, [2]

(ii) a vector equation for the plane OAB, [3]

(iii) the cosine of the acute angle between the lines OA and OB [2]

Obtain, the Cartesian equation for Π, the plane which passes through A and is perpendicular to BC. [3]

$$\left[\mathbf{r} = (-4\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + t(3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}); \mathbf{r} \bullet (13\mathbf{i} + 17\mathbf{j} + 16\mathbf{k}); 3x - 2y + 4z = 5; \frac{21}{\sqrt{35}\sqrt{33}}\right]$$

March 2008

8. The line l has equation $r = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. The plane p has equation $x + 2y + 3z = 5$.

i) Show that the line l lies in the plane p. [2]

ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

$[4x + y - 2z = 1]$

August 2007

12. The line l has equation $\mathbf{r} = 7\boldsymbol{i} + 3\boldsymbol{k} + t\,(5\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k})$, and the plane π has equation $\mathbf{r} \cdot (\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}) = 1$. Find the coordinates of the point A where l meets π. The point B has coordinates (7, 0, 3), and C is the foot of the perpendicular from B to π. Find the coordinates of C. Find a vector equation for the line AC, and calculate the angle BAC, giving your answer to the nearest degree. [10]

$$\left[A(2,-3,1); C(5,-2,-1); \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}; 53^{\circ} \right]$$

March 2007

9. The line has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. (3)

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. (6)

$$[4x + y - 2z = 1]$$

August 2006

10. The position vectors **a**, **b**, **c**, **d**, relative to the origin O, of the points A, B, C and D are given by $\mathbf{a} = 2\mathbf{i}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j}$, $\mathbf{c} = 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

(i) Obtain a unit vector normal to the plane Π_1, the plane containing B, C and D. [4]

(ii) Calculate the angle between the planes Π_1 and Π_2, the plane containing A, B and D. [4]

(iii) Find the position vector of K, the foot of the perpendicular from C to the plane Π_2, and hence find the distance of C from this plane. [4]

$$\left[\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; 71.6^{0}; \frac{\mathbf{8}}{\mathbf{5}}\mathbf{i} + 4\mathbf{j} + \frac{4}{5}\mathbf{k}; \frac{4\sqrt{5}}{5} \right]$$

March 2006

11. (a) The points A and B have position vectors $\mathbf{a} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ respectively relative to a fixed point O. The line L_1 has vector equation $\mathbf{r} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Write down a vector equation for the line L_2 which passes through the points A and B. [1]

(ii) Show that the lines L_1 and L_2 intersect and state the position vector of the point of intersection. [3]

(iii) Calculate the acute angle between the lines L_1 and L_2. [2]

(b) Two planes have vector equations

$$\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} - \mathbf{k}) = 14 \quad \text{and } \mathbf{r}.(11\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 42 .$$

(i) Find the acute angle between these two planes. [2]

(ii) Determine a vector equation for the line of intersection of these two planes. [3]

$$\left[\begin{array}{l} \mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} \text{or } \mathbf{r} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix}; \begin{pmatrix} 4 \\ 8 \\ -6 \end{pmatrix}; 45.6^0; 60^0; \\ \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ -20 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \text{or } \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -10 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \text{or } \mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \end{array}\right]$$

August 2005

10. Two lines l_1 and l_2 have equations given by

$$\vec{r} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{r} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \text{ respectively.}$$

(i) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection. [5]

(ii) Show that the vector equation of the plane π_1 which contains l_2 and is perpendicular to the plane $-2x + 4y + 7z = 36$ is $\vec{r} . \begin{pmatrix} 7 \\ -14 \\ 10 \end{pmatrix} = 12$. [4]

(iii) Find the angle between π_1 and l_1 . [3]

$$\left[(2,3,4); 83.1^0\right]$$

March 2005

10 The equation of the plane Π_1 is $y + z = 0$ and the equation of the line L is

$$\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + t(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}).$$ Find :

(i) the position vector of the point of intersection of L and the plane Π_1, [3]

(ii) the length of the perpendicular distance from the origin to the line L, [3]

(iii) an equation of the plane Π_2 which contains the line L and the origin, [3]

(iv) the acute angle between the planes Π_1 and Π_2, giving your answer correct to the nearest tenth of a degree. [3]

$$\left[(1,4,-4);\ \sqrt{19};\ 8x - 11y - 9z = 0;\ 29.9^0\right]$$