Numerical solutions of equations

A2 Trial Examination

August 2010

6 (i) By sketching a suitable pair of graphs, show that the equation $e^x = x + 5$ has one negative root and one positive root. [2]

(ii) Use the iterative formula

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 5}{e^{x_n} - 1}$$

with initial value $x_0 = 1.2$ to determine the positive root correct to 2 decimal places.

Give the result of each iteration to 4 decimal places. [3]

[1.9368]

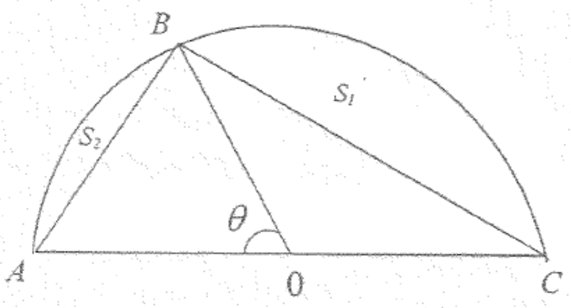
March 2010

3 Show that the equation $\ln x + x - 3 = 0$ has a root in between 1 and e.
Hence, use the iterative formula $x_{n+1} = 3 - \ln x_n$ with initial value $x_1 = 1$ to find the root, correct to two decimal places. [4]

[2.21]

August 2009

5



The diagram shows a semicircle ABC on AC as diameter. The mid-point of AC is 0, and angle $AOB = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$. The area of the segment S_1 bounded by the chord BC is twice the area of the segment S_2 bounded by the chord AB.

Show that $3\theta = \pi + \sin\theta$. [3]

Use an iterative method, based on rearrangement $\theta = \frac{1}{3}(\pi + \sin\theta)$, together with a suitable starting value, to find θ correct to 3 decimal places. [2]

You should show the value of each approximation that you calculate.

[1.374]

March 2009

2 Consider $f(x) = x^3 - 2x - 5$.

(i) Show that there is a root α in the interval $2 \le x \le 3$. [1]

The root α is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{(2 + \frac{5}{x_n})} \quad , \quad x_0 = 2$$

(ii) Calculate the value of α, giving your answer to 4 significant figures. [3]

[2.095]

March 2008

4. Show that the equation $e^{0.8x} = \frac{1}{3-2x}$, $x \neq \frac{3}{2}$ can be written in the form $x = p\ln(3-2x)$, stating the value of p.

Use the iterative formula

$$x_{n+1} = p\ln(3 - 2x_n)$$

with initial value $x_1 = -2.6$ and the value of p found in the earlier part to obtain the root of $e^{0.8x} = \frac{1}{3-2x}$, , to 3 decimal places. [5]

$[-2.643]$

August 2007

10. Given that the curve $y = xe^x - 3e^x - 2x$ has only one turning point, show that the x-coordinate of this turning point lies between $x = 2$ and $x = 3$. [4]

(i) Show that, if a sequence of values given by the iterative formula $x_{n+1} = 2 + 2e^{-x_n}$ converges, then it converges to the x-coordinate of this turning point. [2]

(ii) Use this iterative formula, with initial value $x_0 = 2$, to determine the x-coordinate of this turning point correct to 3 decimal places, showing the result of each iteration. [2]

$[2.218]$

March 2007

6. The equation $x^3 - x - 3 = 0$ has one root, α .

(i) Show that α lies between 1 and 2. (2)

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x_n^3 - 3, \qquad (A)$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \qquad (B)$$

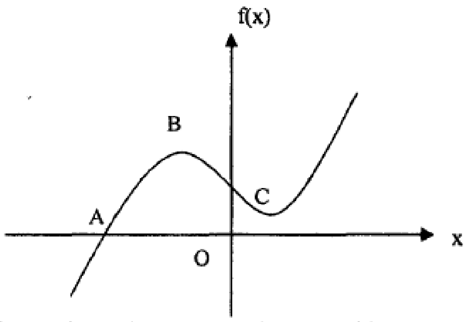
Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. (5)

$[1.67]$

August 2006

8. The diagram below shows the graph of $f(x) = x^3 - 2x + 3\cos x$.



(i) Show that point A is located between $x = -2$ and $x = -1$. [2]

(ii) Find the equation satisfied by the x-coordinates of the two turning points, B and C. [2]

(iii) Show that your equation in (ii) can also be solved numerically using the iterative formula

$$x_{n+1} = \sqrt{\frac{2}{3} + \sin x_n} \qquad [2]$$

(iv) Taking $x_0 = 1$ as a first approximation, find the coordinates of C, correct to 3 decimal places. [3]

$[3x^2 - 2 - 3\sin x = 0; (1.274, 0.397)]$

March 2006

7. (a) By sketching the curves with equations $y = 4 - x^2$ and $y = e^x$, show that the equation $x^2 + e^x - 4 = 0$ has one negative root and one positive root. [3]

(b) Use the iterative formula $x_{n+1} = -\sqrt{(4 - e^{x_n})}$ with $x_0 = -2$ to find an approximation to the negative root of the equation, correct to three decimal places. [3]

$[1.965]$

August 2005

2. (i) Show that the iterative formula $x_{n+1} = \dfrac{2x_n^{\,3}+5}{3x_n^{\,2}+2}$ can be rearranged into the

form $x^3 + ax + b = 0$. [1]

(ii) With these values of a and b, use an iteration based on this rearrangement with initial approximation $x_0 = 1$ to find a solution to the equation. Leave your answer correct to three decimal places [3]

[1.328]

March 2005

4. (i) Show that the equation $2^{1-x} = 4x + 1$ can be arranged in the form

$$x = \frac{1}{2}(2^{-x}) + Q$$ stating the value of the constant Q .

[2]

(ii) Using the iterative formula

$$X_{n+1} = \frac{1}{2}\left(2^{-x_n}\right) + Q, \quad X_0 = 0.2$$

with the value of Q found in part (i), find the root of the equation up to 4 decimal places.

[4]

[0.1887]