Topic 1 – Quadratics

Exercise 3C

Question 8

Draw the graphs of

(a) $y = -4x^2 + 3x + 1$,

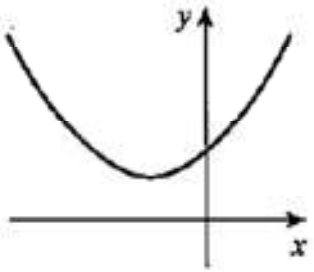
(b) $y = -x^2 + 3x + 1$,

(c) $y = x^2 + 3x + 1$,

(d) $y = 4x^2 + 3x + 1$.

Question 11

Which of the following could be the equation of the curve shown in the diagram?



(a) $y = x^2 - 2x + 5$

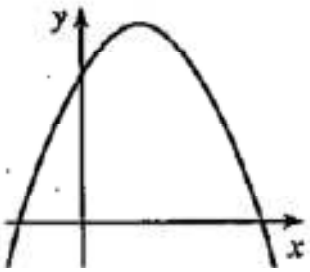
(b) $y = -x^2 - 2x + 5$

(c) $y = x^2 + 2x + 5$

(d) $y = -x^2 + 2x + 5$

Question 12

Which of the following could be the equation of the curve shown in the diagram?



(a) $y = -x^2 + 3x + 4$

(b) $y = x^2 - 3x + 4$

(c) $y = x^2 + 3x + 4$

(d) $y = -x^2 - 3x + 4$

Exercise 4C

Question 1

Solve the following pairs of simultaneous equations.

(a) $y = x + 1$, $x^2 + y^2 = 25$

(b) $x + y = 7$, $x^2 + y^2 = 25$

(c) $y = x - 3$, $y = x^2 - 3x - 8$

(d) $y = 2 - x$, $x^2 - y^2 = 8$

(e) $2x + y = 5$, $x^2 + y^2 = 25$

(f) $y = 1 - x$, $y^2 - xy = 0$

(g) $7y - x = 49$, $x^2 + y^2 - 2x - 49 = 0$

(h) $y = 3x - 11$, $x^2 + 2xy + 3 = 0$

Question 2

Find the coordinates of the points of intersection of the given straight lines with the given curves.

(a) $y = 2x + 1$, $y = x^2 - x + 3$

(b) $y = 3x + 2$, $x^2 + y^2 = 26$

(c) $y = 2x - 2$, $y = x^2 - 5$

(d) $x + 2y = 3$, $x^2 + xy = 2$

(e) $3y + 4x = 25$, $x^2 + y^2 = 25$

(f) $y + 2x = 3$, $2x^2 - 3xy = 14$

(g) $y = 2x - 12$, $x^2 + 4xy - 3y^2 = -27$

(h) $2x - 5y = 6$, $2xy - 4x^2 - 3y = 1$

Question 3

In each case find the number of points of intersection of the straight line with the curve.

(a) $y = 1 - 2x$, $x^2 + y^2 = 1$

(b) $y = \frac{1}{2}x - 1$, $y = 4x^2$

(c) $y = 3x - 1$, $xy = 12$

(d) $4y - x = 16$, $y^2 = 4x$

(e) $3y - x = 15$, $4x^2 + 9y^2 = 36$

(f) $4y = 12 - x$, $xy = 9$

Question 4

Solve the following equations; give irrational answers in terms of surds.

(a) $x^4 - 5x^2 + 4 = 0$

(b) $x^4 - 10x^2 + 9 = 0$

(c) $x^4 - 3x^2 - 4 = 0$

(d) $x^4 - 5x^2 - 6 = 0$

(e) $x^6 - 7x^3 - 8 = 0$

(f) $x^6 + x^3 - 12 = 0$

Question 5

Solve the following equations. (In most cases, multiplication by an appropriate expression will turn the equation into a form you should recognise.)

(a) $x = 3 + \frac{10}{x}$ (b) $x + 5 = \frac{6}{x}$ (c) $2t + 5 = \frac{3}{t}$

(d) $x = \frac{12}{x+1}$ (e) $\sqrt{t} = 4 + \frac{12}{\sqrt{t}}$ (f) $\sqrt{t}\left(\sqrt{t} - 6\right) = -9$

(g) $x - \frac{2}{x+2} = \frac{1}{3}$ (h) $\frac{20}{x+2} - 1 = \frac{20}{x+3}$ (i) $\frac{12}{x+1} - \frac{10}{x-3} = -3$

(j) $\frac{15}{2x+1} + \frac{10}{x} = \frac{55}{2}$ (k) $y^4 - 3y^2 = 4$ (l) $\frac{1}{y^2} - \frac{1}{y^2+1} = \frac{1}{2}$

Question 6

Solve the following equations.

(a) $x - 8 = 2\sqrt{x}$ (b) $x + 15 = 8\sqrt{x}$ (c) $t - 5\sqrt{t} - 14 = 0$

(d) $t = 3\sqrt{t} + 10$ (e) $\sqrt[3]{x^2} - \sqrt[3]{x} - 6 = 0$ (f) $\sqrt[3]{t^2} - 3\sqrt[3]{t} = 4$

Miscellaneous Exercise 4

Question 2

The quadratic polynomial $x^2 - 10x + 17$ is denoted by $f(x)$. Express $f(x)$ in the form $(x-a)^2 + b$ stating the values of a and b.

Hence find the least possible value that $f(x)$ can take and the corresponding value of x. (OCR)

Question 5

By expressing the function $f(x) = (2x+3)(x-4)$ in completed square form, find the range of the function $f(x)$.

Question 6

(a) Solve the equation $x^2 - \left(6\sqrt{3}\right)x + 24 = 0$, giving your answer in terms of surds, simplified as far as possible.

(b) Find all four solutions of the equation $x^4 - \left(6\sqrt{3}\right)x^2 + 24 = 0$ giving your answers correct to 2 decimal places. (OCR)

Question 8

Express $9x^2 - 36x + 52$ in the form $(Ax - B)^2 + C$, where A, B and C are integers.
Hence, or otherwise, find the set of values taken by $9x^2 - 36x + 52$ for real x. (OCR)

Question 10

(a) Express $9x^2+12x+7$ in the form $(ax+b)^2+c$ where a, b, c are constants whose values are to be found.

(b) Find the set of values taken by $\dfrac{1}{9x^2+12x+7}$ for real values of x. (OCR)

Exercise 5A

Question 5

(a) $x-4 \leqslant 5+2x$ (b) $x-3 \geqslant 5-x$ (c) $2x+5<4x-7$

(d) $3x-4>5-x$ (e) $4x \leqslant 3(2-x)$ (f) $3x \geqslant 5-2(3-x)$

(g) $6x<8-2(7+x)$ (h) $5x-3>x-3(2-x)$ (i) $6-2(x+1) \leqslant 3(1-2x)$

Question 6

(a) $\frac{1}{3}(8x+1)-2(x-3)>10$ (b) $\frac{5}{2}(x+1)-2(x-3)<7$

(c) $\dfrac{2x+1}{3}-\dfrac{4x+5}{2} \leqslant 0$ (d) $\dfrac{3x-2}{2}-\dfrac{x-4}{3}<x$

(e) $\dfrac{x+1}{4}+\dfrac{1}{6} \geqslant \dfrac{2x-5}{3}$ (f) $\dfrac{x}{2}-\dfrac{3-2x}{5} \leqslant 1$

(g) $\dfrac{x-1}{3}-\dfrac{x+1}{4}>\dfrac{x}{2}$ (h) $\dfrac{x}{3} \geqslant 5-\dfrac{3x}{4}$

Exercise 5B

Question 4

Use any method you like to solve the following inequalities.

(a) $x^2+5x+6>0$ (b) $x^2-7x+12<0$ (c) $x^2-2x-15 \leqslant 0$

(d) $2x^2-18 \geqslant 0$ (e) $2x^2-5x+3 \geqslant 0$ (f) $6x^2-5x-6<0$

(g) $x^2+5x+2>0$ (h) $7-3x^2<0$ (i) $x^2+x+1<0$

(j) $2x^2-5x+5>0$ (k) $12x^2+5x-3>0$ (l) $3x^2-7x+1 \leqslant 0$

Miscellaneous Exercise 5

2 Solve the inequality $(x+1)^2 < 9$.

3 Solve the inequality $x(x+1) < 12$. (OCR)

4 Solve the inequality $x - x^3 < 0$.

5 Solve the inequality $x^3 \geqslant 6x - x^2$.

Use the discriminant '$b^2 - 4ac$' in answering Questions 6 to 8. You may need to check the value $k = 0$ separately.

6 Find the values of k for which the following equations have two separate roots.

(a) $kx^2 + kx + 2 = 0$ (b) $kx^2 + 3x + k = 0$ (c) $x^2 - 2kx + 4 = 0$

7 Find the values of k for which the following equations have no roots.

(a) $kx^2 - 2kx + 5 = 0$ (b) $k^2x^2 + 2kx + 1 = 0$ (c) $x^2 - 5kx - 2k = 0$

8 Find the range of values of k for which the equation $x^2 + 3kx + k = 0$ has any roots.

9 Find the set of values of x for which $9x^2 + 12x + 7 > 19$. (OCR)

10 Sketch, on the same diagram, the graphs of $y = \frac{1}{x}$ and $y = x - \frac{3}{2}$. Find the solution set of the inequality $x - \frac{3}{2} > \frac{1}{x}$. (OCR)