Real Exponential Functions

Definition :

Let $a \in \mathrm{R}^{+}$. A real exponential function is defined by

$$f(x) = a^{x},\ x \in \mathrm{R}.$$

Properties of $f(x) = a^x, a > 1$:

$(\text{i})\, a^x > 0, \forall x \in R$

$(\text{ii})\, a^x \to \infty$ when $x \to +\infty$ and

$a^x \to 0$ when $x \to -\infty$.

Properties of $f(x) = a^{-x}, a > 1:$

$(\mathrm{i})\ a^{-x} > 0, \forall x \in R$

$(\mathrm{ii})\ a^{-x} \to 0$ when $x \to +\infty$ and

$a^{-x} \to \infty$ when $x \to -\infty$.

Example:

Solve the equation $x(x-1)e^{x} = 0$

Example:

Show that $\dfrac{e^{-x}}{1+x^2} > 0, \forall x \in \mathrm{R}$

Example:

Solve the inequality $x^2(x-1)e^x \leq 0.$

Derivative of Exponential Functions

Theorem :

$$(\text{a})\ \frac{d}{dx}\left(e^{x}\right)=e^{x}\ ,\forall x,$$

$$(\text{b})\ \frac{d}{dx}\left[e^{f(x)}\right]=f'(x)e^{f(x)},\ x\in \mathrm{D}_{f}\ .$$

Example: Find

$$\text{(i)}\ \frac{d}{dx}\left(e^{4x}\right)= \qquad \text{(ii)}\ \frac{d}{dx}\left(e^{x^2}\right)=$$

Example:

Find the gradient of the tangent to the curve $y = e^{2x^3}$ at $x = 1$. Find the equation of the tangent.

Example:

Given that $y = \frac{1}{e^{2x}}$. Show that $\frac{dy}{dx} + 2y = 0$.

Example:

Given that $y = e^{x^2+1}$. Find the stationary point and determine its nature.

$$\text{Example : Find}$$

$$\text{(i)}\ \frac{d}{dx}\left(e^{\sqrt{x}}\right) = \qquad \text{(ii)}\ \frac{d}{dx}\left(\sqrt{e^{x}}\right) =$$

Example:

Given that $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$. Show that $\frac{d^2 y}{dx^2} = y$

Example:

A curve is given by the equation $y = 5x^2 + \dfrac{3}{e^{x^2}}$.

Find the equation of the tangent when $x = 0$.

Example:

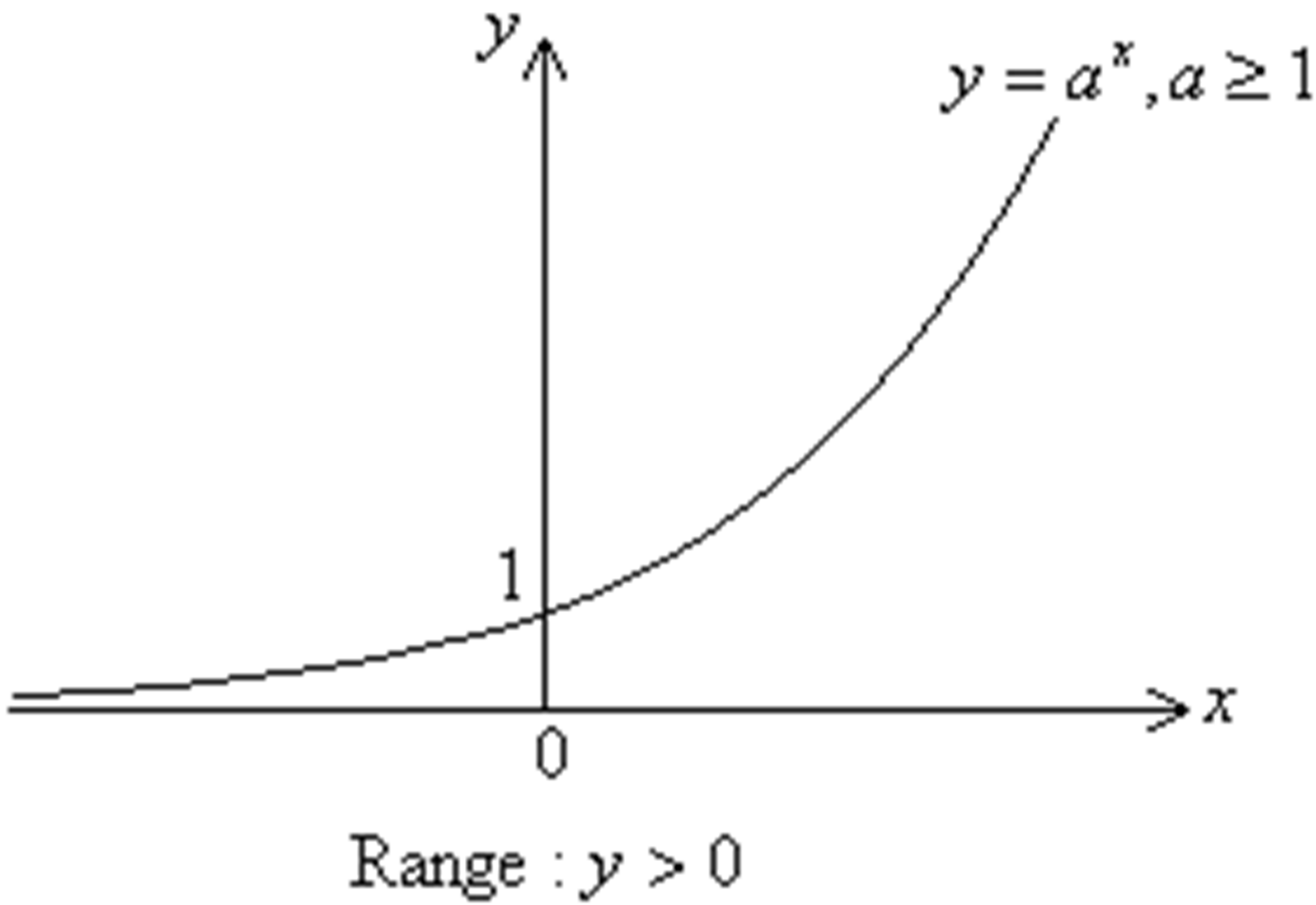
Show that the gradient of the tangent to the curve

$y = \left(e^{-x} + e^{x}\right)^4$ at $x = 1$ is $4\left(e^4 - \frac{1}{e^4}\right) + 8\left(e^2 - \frac{1}{e^2}\right)$.

Example:

Find $\dfrac{d}{dx}\sqrt{\left(e^x - \dfrac{1}{e^x}\right)}$.

Graph of Exponential Functions



Also known as "Growth Model".

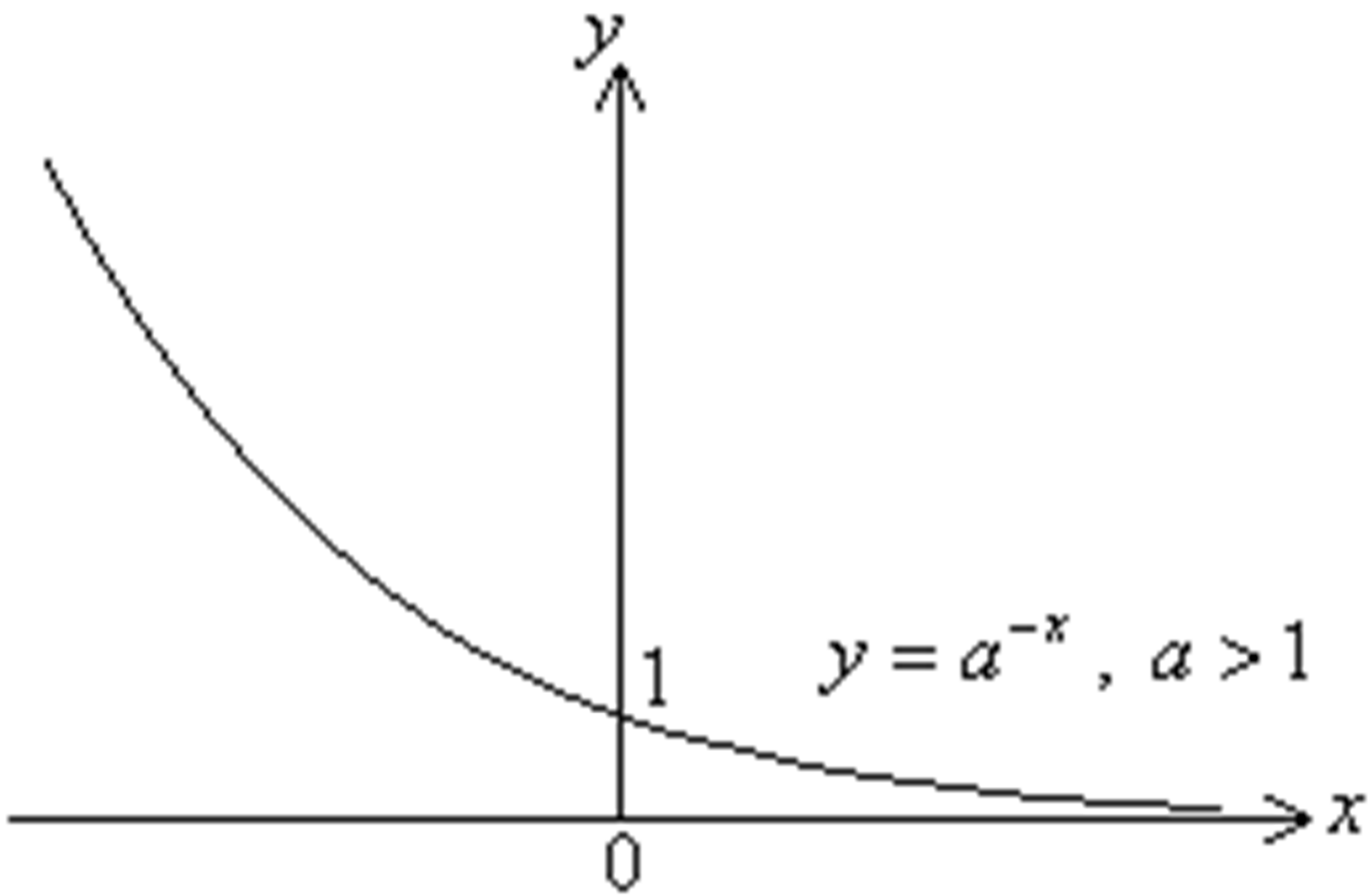
Remark :

When $a > 1$, the curve $y = a^x$

(a) has range $y > 0$, i.e. $a^x > 0, \forall x \in \mathrm{R}$,

(b) is increasing because $\dfrac{d}{dx}\left(a^x\right) > 0, \forall x \in \mathrm{R}$,

(b) is one - to - one.



Range : $y > 0$

Also known as "Decay Model".

Remark :

When $0 < a < 1$, the curve $y = a^x = b^{-x}$ where $b = a^{-1}$

(a) has range $y > 0$, i.e. $a^x > 0, \forall x \in \mathrm{R}$,

(b) is decreasing because $\dfrac{d}{dx}\left(a^x\right) < 0, \forall x \in \mathrm{R}$,

(b) is one - to - one.

Example:

Sketch the curve $y = e^x$.

Example:

Sketch the curve $y = e^{-x}$.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Find the equation of the tangent to the curve $y = x^2 + 2e^{2x}$ at the point where $x = 2$.

Example:

Given that $y = \dfrac{5}{1+e^{3x}}$, find the value of $\dfrac{dy}{dx}$ when $x = 0$.

Example:

Find the coordinates of the three stationary points of the curve $y = e^{x^2\left(x^2-18\right)}$.