Exercise 3f Arrangements, permutations, combinations

1. In how many ways can the letters of the word FACETIOUS be arranged in a line?
What is the probability that an arrangement begins with F and ends with S?

2. (a) In how many ways can seven people sit at a round table?
(b) What is the probability that a husband and wife sit together?

3. On a shelf there are four mathematics books and eight English books.
(a) If the books are to be arranged so that the mathematics books are together, in how many ways can this be done?
(b) What is the probability that all the mathematics books will not be together?

4. The letters of the word PROBABILITY are arranged at random. Find the probability that the two Is are separated.

5. If the letters in the word ABSTEMIOUS are arranged at random, find the probability that the vowels and consonants appear alternately.

6. Nine children play a party game and hold hands in a circle.
(a) In how many different ways can this be done?
(b) What is the probability that Mary will be holding hands with her friends Natalie and Sarah?

7. (a) In how many different ways can the letters in the word ARRANGEMENTS be arranged?
(b) Find the probability that an arrangement chosen at random begins with the letters EE.

8. From a group of ten boys and eight girls, two pupils are chosen at random. Find the probability that they are both girls.

9. From a group of six men and eight women, five people are chosen at random. Find the probability that there are more men chosen than women.

10. From a bag containing six white counters and eight blue counters, four counters are chosen at random. Find the probability that two white counters and two blue counters are chosen.

11. From a group of ten people, four are to be chosen to serve on a committee.
(a) In how many different ways can the committee be chosen?
(b) Among the ten people there is one married couple. Find the probability that both the husband and the wife will be chosen.
(c) Find the probability that the three youngest people will be chosen.

12. In a group of six students, four are female and two are male. Determine how many committees of three members can be formed containing one male and two females. (*L*)

13. Four persons are chosen at random from a group of ten persons consisting of four men and six women. Three of the women are sisters. Calculate the probabilities that the four persons chosen will:
(a) consist of four women,
(b) consist of two women and two men,
(c) include the three sisters. (*NEAB*)

14. A touring party of 20 cricketers consists of nine batsmen, eight bowlers and three wicket keepers. A team of 11 players must have at least five batsmen, four bowlers and one wicket keeper. How many different teams can be selected, (a) if all the players are available for selection, (b) if two batsmen and one bowler are injured and cannot play?

15. Find the number of ways in which ten different books can be shared between a boy and a girl if each is to receive an even number of books.

16. Four letters are picked from the word BREAKDOWN. What is the probability that there is at least one vowel among the letters?

17. Eight people sit in a minibus: four on the sunny side and four on the shady side. If two people want to sit on opposite sides to each other and another two people want to sit on the shady side, in how many ways can this be done?

18. Disco lights are arranged in a vertical line. How many different arrangements can be made from two green, three blue and four red lights (a) if all nine lights are used, (b) if at least eight lights are used?

19. A group consisting of 10 boys and 11 girls attends a course for special games coaching.
(a) When they are introduced, each person hands a card containing his or her photograph and name and address to every other member of the group. State the total number of cards which are exchanged.
(b) 5 boys are selected for basketball and 6 girls for netball. Find the number of different possible selections for each of these.
(c) 5 particular boys and 5 particular girls are selected and placed in mixed pairs for tennis. Find the total number of different mixed pairs which can be made using these 10 children.
(d) If 4 children are chosen at random from the whole group find the probability that there is a majority of girls in the 4 selected.
(*L Additional*)

20. A competition has a first prize, a second prize, a third prize and a fourth prize. Ten competitors enter this competition and the prizes are awarded for the first, second, third and fourth competitors in order of merit.

(a) Find the number of different ways in which these prizes could be won.

Smith and Jones are two of the ten competitors. Find the number of different ways in which the prizes could be won if

(b) neither Smith nor Jones wins a prize,
(c) each of Smith and Jones wins a prize. (C)

21. The number of applicants for a job is 15. Calculate the number of different ways in which six applicants can be selected for interview.

The six selected applicants are interviewed on a particular day. Calculate the number of ways in which the order of the six interviews can be arranged.

Of the six applicants interviewed, three have backgrounds in business, two have backgrounds in education and one has a background in recreation. Calculate the number of ways in which the order of the six interviews can be arranged, when applicants having the same background are interviewed successively. (C)

22. Each of seven children, in turn, throws a ball once at a target. Calculate the number of ways the children can be arranged in order to take the throws.

Given that three of the children are girls and four are boys, calculate the number of ways the children can be arranged in order that

(a) successive throws are made by boys and girls alternately,
(b) a girl takes the first throw and a boy takes the last throw. (C)

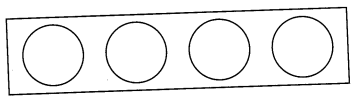
23. To enter a cereal competition, competitors have to choose the eight most important features of a new car, from a possible 12 features, then list the eight in order of preference. Each cereal packet entry form contains space for five entries. A correct entry wins a new car.

(a) What is the probability that a woman wins a new car if she completes the entry form from one packet?
(b) How many entry forms would she need to complete, each entry showing different arrangements, if the probability that she wins a car is to be at least 0.8?

24. Three letters are selected at random from the word SCHOOL. Find the probability that the selection (a) does not contain the letter O, (b) contains both the letters O.

25. How many even numbers can be formed with the digits 3, 4, 5, 6, 7 by using some or all of the numbers (repetitions are not allowed)?

26.

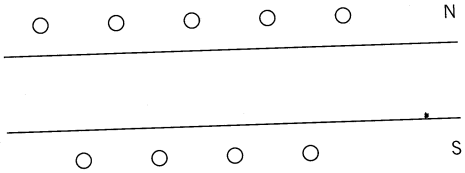


Different coloured pegs, each of which is painted in one and only one of the six colours red, white, black, green, blue and yellow, are to be placed in four holes, as shown in the figure, with one peg in each hole. Pegs of the same colour are indistinguishable. Calculate how many different arrangements of pegs placed in the four holes so that they are all occupied can be made from

(a) six pegs, all of different colours,
(b) two red and two white pegs,
(c) two red, one white and one black peg,
(d) twelve pegs, two of each colour.
(*L Additional*)

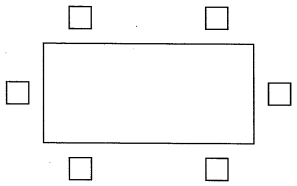
27. (a) Calculate how many different numbers altogether can be formed by taking one, two, three and four digits from the digits 9, 8, 3 and 2, repetitions not being allowed.
(b) Calculate how many of the numbers in part (a) are odd and greater than 800.
(c) If one of the numbers in part (a) is chosen at random, calculate the probability that it will be greater than 300. (*L Additional*)

28. The positions of nine trees which are to be planted along the sides of a road, five on the north side and four on the south side, are shown in the figure.



(a) Find the number of ways in which this can be done if the trees are all of different species.
(b) If the trees in (a) are planted at random, find the probability that two particular trees are next to each other on the same side of the road.
(c) If there are three cupressus, four prunus and two magnolias, find the number of different ways in which these could be planted assuming that trees of the same species are identical.
(d) If the trees in (c) are planted at random, find the probability that the two magnolias are on the opposite sides of the road.
(*L Additional*)

29. A committee consisting of six persons is to be selected from five women and six men.
    (a) Calculate the number of ways in which the chosen committee will contain exactly two men.
    (b) Given that the committee is to contain at least two men, show that it can be selected in 456 ways.
    (c) Given that these 456 ways are equally likely, calculate the probability that there will be more men than women on the committee.
    (d) At a meeting the members of the chosen committee sit at a rectangular table in the fixed seats illustrated in the diagram:



    (i) Given that each may sit in any of the six places, calculate the number of different ways they may be seated at the table.
    (ii) Given that the committee consists of three men and three women and that the men and women must sit alternately round the table, calculate in how many different ways they may be seated. (*L Additional*)

30. A committee of eight members consists of one married couple together with four other men and two other women. From the committee a working party of four persons is to be formed. Find the number of different working parties which can be formed.

    Find also the number if the working party
    (a) may not contain *both* the husband and his wife,
    (b) must contain two men and two women,
    (c) must contain at least one man and at least one woman.

    The eight committee members sit round an octagonal table, their positions being decided by drawing lots. Find the probability of
    (d) the man sitting next to his wife,
    (e) the man sitting opposite to his wife,
    (f) the three women sitting together. (*AEB*)

Summary

- Experimental probability

  $P(A) = \lim_{n \to \infty} \left(\frac{r}{n}\right)$ where $\frac{r}{n}$ is the relative frequency of A.

- Equally likely outcomes

  $P(A) = \frac{n(A)}{n(S)}$ where $n(A)$ is the number of outcomes in A

  $n(S)$ is the number of possible outcomes.

  $0 \leqslant P(A) \leqslant 1$ If A is impossible, $P(A) = 0$

  If A is certain, $P(A) = 1$.

  $P(A') = 1 - P(A)$ where A' is the event 'A does not occur.'

- For events A and B

  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- For mutually exclusive events A and B, $P(A \cap B) = 0$

  so $P(A \text{ or } B) = P(A) + P(B)$ **'or' rule for exclusive events**

  i.e. $P(A \cup B) = P(A) + P(B)$

- For exhaustive events A and B

  $P(A \text{ or } B) = 1$, i.e. $P(A \cup B) = 1$

Exercise 3d Tree diagrams (page 200)

Section A

1. (a) 0.0025 (b) 0.095
2. (a) $\frac{5}{14}$ (b) $\frac{17}{42}$
3. (a) 0.24 (b) 0.42
4. (a) (i) $\frac{8}{27}$ (ii) $\frac{4}{9}$ (iii) $\frac{7}{27}$
   (b) (i) $\frac{5}{21}$ (ii) $\frac{15}{28}$ (iii) $\frac{19}{84}$
5. 0.00599, 0.987
6. (a) $\frac{12}{49}$ (b) $\frac{20}{49}$
7. $\frac{7}{16}$
8. $\frac{25}{72}$
9. $\frac{5}{16}$
10. 0.35
11. 0.825
12. (a) 0.5 (b) 0.5 (c) 0.375
13. 0.788
14. (a) 0.02 (b) 0.64
15. (a) $\frac{3}{11}$ (b) $\frac{12}{55}$ (c) $\frac{3}{44}$
16. (a) .0.34 (b) 0.063 (c) 0.19 (d) 0.97; 3 white
17. 0.624
18. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{4}$ (e) $\frac{3}{4}$

Section B

1. (a) $\frac{5}{12}$ (b) $\frac{3}{5}$
2. (a) $\frac{21}{38}$ (b) $\frac{15}{38}$ (c) $\frac{20}{83}$
3. (a) P(A occurs, given that B occurs)
   (i) mutually exclusive (ii) independent
   (b) 0.88, 0.05
4. (a) 0.33 (b) $\frac{7}{11}$
5. (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{8}{9}$
6. (b) $\frac{77}{95}$
7. (a) $\frac{7}{18}$ (b) (i) $\frac{5}{8}$ (ii) $\frac{8}{25}$
8. (a) $\frac{1}{25}$ (b) $\frac{106}{125}$ (c) $\frac{14}{19}$
9. (a) 0.096 (b) 0.156; $\frac{5}{13}$
10. (a) 0.7, 0.68 (b) 0.28 (c) 0.65625
11. (a) $\frac{6}{323}$ (b) $\frac{135}{323}$ (c) $\frac{1}{5}$ (d) $\frac{1}{5}$
    (i) Yes, no (ii) No, yes
12. (a) 0.000877 (b) 0.421 (c) 0.65 (d) 0.642
13. (a) 0.042875 (b) 0.142 (c) 0.1215
    (d) 0.189 (e) 0.334125; 0.642
14. (a) (i) $\frac{9}{22}$ (ii) $\frac{6}{11}$ (iii) $\frac{2}{11}$ (iv) $\frac{4}{7}$
    (b) (i) 0.0303 (ii) 0.450 (iii) 0.0348
    (c) (i) 0.36 (ii) 0.848
15. $\frac{23}{45}$, $\frac{18}{23}$
16. (b) $\frac{9}{32}$ (c) $\frac{83}{128}$ (d) $\frac{17}{37}$
17. (a) 0.36 (b) 0.6875

Exercise 3e Useful methods (page 206)

1. (a) 0.763 (b) 14
2. (a) 5 (b) 6
3. 0.5, 6
4. 0.999
5. 22
6. $\frac{5}{11}$
7. 1 : 8
8. 0.5 (a) $\frac{1}{6}$ (b) $\frac{25}{216}$ (c) $\frac{625}{7776}$; $\frac{6}{11}$
9. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{12}$ (iii) $\frac{2}{3}$ (b) $\frac{7}{12}$

Exercise 3f Arrangements, permutations, combinations (page 219)

1. 9!, $\frac{1}{72}$
2. (a) 6! (b) $\frac{1}{3}$
3. (a) 4! 9! (b) $\frac{54}{55}$
4. $\frac{9}{11}$
5. $\frac{1}{126}$
6. (a) 8! (b) $\frac{1}{28}$
7. (a) $\frac{12!}{(2!)^4}$ (b) $\frac{1}{66}$
8. $\frac{28}{153}$
9. $\frac{49}{143}$
10. $\frac{60}{143}$
11. (a) 210 (b) $\frac{2}{15}$ (c) $\frac{1}{30}$
12. 12
13. (a) $\frac{1}{14}$ (b) $\frac{3}{7}$ (c) $\frac{1}{30}$
14. (a) 65 268 (b) 4263
15. 510
16. $\frac{37}{42}$
17. 4608
18. (a) 1260 (b) 2520
19. (a) 420 (b) B 252, G 462 (c) 120 (d) $\frac{44}{133}$
20. (a) 5040 (b) 1680 (c) 672
21. 5005, 720, 72
22. 5040 (a) 144 (b) 120
23. (a) 2.5×10^{-7} (b) 3 193 344
24. (a) $\frac{2}{7}$ (b) $\frac{2}{7}$
25. 130
26. (a) 360 (b) 6 (c) 12 (d) 1170
27. (a) 64 (b) 18 (c) $\frac{21}{32}$
28. (a) 9! (b) $\frac{7}{36}$ (c) 1260 (d) $\frac{5}{9}$
29. (a) 75 (c) $\frac{181}{456}$ (d) (i) 6! (ii) 72
30. (a) 55
    (b) 30
    (c) 65
    (d) $\frac{2}{7}$
    (e) $\frac{1}{7}$
    (f) $\frac{1}{7}$

Miscellaneous exercise 3g (page 228)

1. (a) 0.36 (b) 0.48 (c) 0.01024 (d) 0.98976
2. (a) C, C′ (b) C, D (c) C, E
3. (a) 0.0902 (b) unsatisfactory test
4. 0.32, 0.467
5. (a) 0.325 (b) $\frac{51}{260}$ (c) $\frac{5}{13}$
6. (a) 0.28 (b) (i) 0.157 (ii) 0.363 (iii) 0.163
   (c) 0.0728 (d) 0.404
7. 0.166, 0.580
8. 5040 (a) 720 (b) 1440
9. (a) $\frac{1}{343}$ (b) $\frac{1}{49}$ (c) $\frac{30}{49}$ (d) $\frac{8}{343}$ (e) $\frac{1}{4}$ (f) 6
10. (a) $\frac{11}{24}$ (b) $\frac{11}{60}$ (c) $\frac{43}{120}$ (d) $\frac{49}{144}$
11. (a) (i) 0.005 (ii) 0.0955 (b) 0.999 (c) 0.136
12. (a) (i) $\frac{1}{3}$ (ii) $\frac{2}{9}$ (iii) $\frac{1}{3}$ (b) $\frac{2}{15}$ (c) $\frac{3}{10}$
13. 5005, 1960, 315 (a) $\frac{9}{56}$ (b) $\frac{27}{56}$
14. (a) 792 (b) 210 (c) $\frac{35}{132}$ (d) 120 (e) 0.1 (f) 0.1
15. (a) 40 320 (b) (i) 1440 (ii) 5760
    (c) (i) $\frac{1}{7}$ (ii) $\frac{6}{7}$ (d) 576 (e) $\frac{3}{35}$
16. (a) $\frac{1}{4}$ (b) $\frac{5}{14}$ (c) independent
    (d) $\frac{1}{7}$, $P(A|C) \neq P(A)$ (e) $\frac{7}{18}$