Revision Exercise : Probability

1 (a) Events A and B are such that $P(A) = 0.3$ and $P(B) = 0.4$.
Find $P(A \cup B)$ when A and B are

(i) mutually exclusive,

(ii) independent.

[4]

(b) Events C and D are such that $P(C) = 0.6$, $P(D) = 0.3$ and $P(C \cup D) = 0.7$. Find

(i) $P(C \cap D)$,

(ii) $P(C \cap D')$,

(iii) $P(C|D')$.

[6]

2 Peter and Andrew play tennis. Peter serves throughout the first game, Andrew serves throughout the second game and so on, alternately. When Peter serves, the probability that he wins the game is 0.8. When Andrew serves, the probability that Peter wins the game is 0.4. A game cannot be drawn.

(i) Find the probability that, after 3 games, Peter has won 2 games and Andrew has won one game. [5]

After 6 games Peter and Andrew have each won 3 games. They will continue playing until one of them has won 6 games.

(ii) Find the probability that Peter will win by either 6 games to 3 or by 6 games to 4. [5]

3 A bag contains 4 red balls and 4 black balls. Whenever a red ball is drawn it is replaced in the bag and an extra red ball is added to the bag. Whenever a black ball is drawn it is not replaced and no extra balls are added. 3 balls are drawn, one after another. Find the probability that

(i) all three balls are red, [2]

(ii) at least one of the balls is red, [2]

(iii) exactly two of the balls are red. [4]

Given that exactly 2 of the 3 balls drawn are red, find

(iv) the probability that the first ball drawn was red. [3]

4 The weather on any day is classified as wet or dry. If it is wet on any particular day the probability that it will be wet the next day is 0.4. If it is dry on any particular day the probability that it will be dry the next day is 0.7.

Given that in a particular week Monday is dry, find the probability that

(i) both Tuesday and Wednesday of that week will be dry, [2]

(ii) Wednesday of that week will be dry. [3]

Given also that Wednesday of that week is dry,

(iii) find the probability that Tuesday of that week was also dry. [2]

(a) The independent events A and B are such that $P(A) = 0.6$ and $P(B) = 0.3$. Find

(i) $P(A \cap B')$, (ii) $P((A \cup B)')$. [5]

(b) The events C and D are such that $P(C) = 0.5$, $P(C \cup D) = 0.8$ and $P(C|D) = 0.25$. Find

(i) $P(C \cap D)$, (ii) $P(D|C)$. [5]

6 (a) The events A and B are such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.64$. Find $P(A \mid B)$. [4]

(b) The events C and D are independent and are such that $P(C) = 0.6$, $P(D) = x$ and $P(C \cup D) = 0.7$. Find x. [5]

7 Three bags each contain balls, identical in shape and size.
Bag A contains 5 red, 3 blue and 2 green balls.
From this bag 2 balls are chosen without replacement.

(i) Find the probability that the 2 balls are the same colour. [4]

The 2 balls which were chosen are replaced in bag A.
Bag B contains 4 red, 2 blue and 2 green balls.
Bag C contains 3 red, 1 blue and 1 green ball.
One of the 3 bags, A, B or C, is chosen at random and a ball is chosen at random from this bag.

(ii) Find the probability that the ball is red. [4]

(iii) Given that the ball is red, find the probability that bag C was chosen. [2]

12 (a) Events A and B are such that $P(A) = 1/3$, $P(B \mid A) = 1/4$ and $P(A' \cap B') = 1/6$. Find

(i) $P(A \cup B)$, (ii) $P(B)$.

(b) A man writes 5 letters, one each to A, B, C, D and E. Each letter is placed in a separate envelope and sealed. He then addresses the envelopes, at random, one each to A, B, C, D and E.

(i) Find the probability that the letter to A is in the correct envelope and the letter to B is in an incorrect envelope. [1]

(ii) Find the probability that the letter to A is in the correct envelope, given that the letter to B is in an incorrect envelope. [3]

(iii) Find the probability that both of the letters to A and B are in incorrect envelopes. [5]

16 In the UK, the failure rate for treatment by IVF is 80%. Find the probability that there are exactly 6 failures in 10 randomly chosen patients receiving the treatment.

It is given that there are fewer than 8 failures in the 10 treatments. Find the conditional probability that there are exactly 6 failures.