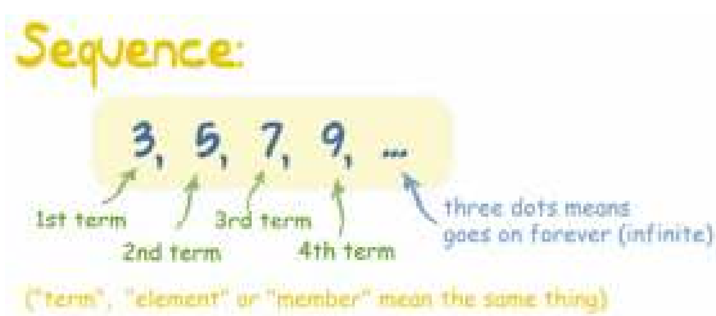
TOPIC 7 – SERIES
(ARITHMETIC PROGRESSION)

- use the expansion of $(a+b)^n$, where n is a positive integer (knowledge of the greatest term and properties of the coefficients are not required, but the notations $\binom{n}{r}$ and $n!$ should be known);
- recognise arithmetic and geometric progressions;
- use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions;
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.

Sequence or Progression

A sequence is a set of things (usually numbers) that are in order.



<u>Arithmetic Progression</u>

In an Arithmetic Sequence **the difference between one term and the next is a constant**.
In other words, you just add the same value each time ... infinitely.

In General you could write an arithmetic sequence like this:

$$\{a, a+d, a+2d, a+3d, \dots\}$$

where:

- **a** is the first term, and
- **d** is the difference between the terms (called the **"common difference"**)

Rule

You can write an Arithmetic Sequence as a rule: $T_n = a_n = a + d(n-1)$

Summing an Arithmetic Series

To sum up the terms of this arithmetic sequence:

$$a + (a+d) + (a+2d) + (a+3d) + ...$$

The formula:

$$S_n = \frac{n}{2}[2a+(n-1)d] \qquad \text{or} \qquad S_n = \frac{n}{2}[a+l]$$

Examples:

Question 1

For each of the following arithmetic progressions, find the common difference, d.

(a) $-4, -1, 2, \ldots$ **(b)** $x + 2y, 2x + 3y, 3x + 4y, \ldots$

Question 2

Write down the first 4 terms of the arithmetic progression whose first term is 2 and common difference is 5.

Question 3

Find the 13^{th} term of the AP, 5, 9, 13, . . .
Find also an expression for the n^{th} term.

Question 4

In an AP, the fifth term is 22 and the tenth term is 47. Find the sixteenth term.

Question 5

Given that x, x^2 and 1 are three successive terms of an AP, find the possible values of x and the common difference.

Question 6

The first 3 terms of an AP are 1, 4 and 7. Find the sum of the first 20 terms.

Question 7

The n^{th} term of an AP is given by $T_n = 7n - 5$. Find the sum of the first 16 terms.

Question 8

An AP has first term 4 and common difference 3. If the sum of all its terms is 375, how many terms are there?

Question 9

In an AP, the sum of the first 10 terms is 155 while the sum of the next 5 terms is 190. Find the sixth term.

Question 10

In an AP consisting of 10 terms, the sum of the last 5 terms is 85. If the fourth term is 9, find the sum of this AP.

Question 11

Find the sum of all the integers between 50 and 150 which are multiples of 4.

Question 12

Senne would like to give a sum of money to a charity each year for 10 years. She decides to give \$100 in the first year, and to increase her contribution by \$20 each year. How much does she give in the last year, and how much does the charity receive from her altogether?

Question 13

A student reading a 426-page book finds that he reads faster as he gets into the subject. He reads 19 pages on the first day, and his rate of reading then goes up by 3 pages each day. How long does he take to finish the book?

Exercise 1 – Arithmetic Progression

20. Find the smallest integer value of n such that $1 + 2 + 3 + 4 + \ldots + n > 600$.

21. The sum of the first n terms of an AP is $6n - 3n^2$. Find
(a) the first term, **(b)** the sixth term, **(c)** the n^{th} term.

9-6n

22. The ninth term of an AP is twice its third term and the sum of the first ten terms is 170. Find the sum of the last ten terms given that the whole progression has 30 terms.

23. In the AP 3, 7, 11, . . . , find the value of n for which the sum of the first $2n$ terms will exceed the sum of the first n terms by 155.

24. Find the sum of all integers between 50 and 400 which end in 3.

25. A piece of string of length 12 m is cut into n pieces in such a way that the lengths of the pieces are in AP. If the lengths of the longest and shortest pieces are 1 m and 20 cm respectively, find n and the total length of the twelve shortest pieces.

26. Find the sum of the first n terms of the AP whose r^{th} term is $8r + 5$. How many terms must be taken for the sum to be just under 1 000?

***27.** In the AP 1, 2, 3, 4, 5, 6, . . . , every third term is multiplied by 3 so that the progression becomes 1, 2, 9, 4, 5, 18, ... Find the sum of the first 60 terms of the new progression.

Answers

20. 35	**21.** **(a)** 3	**(b)** -27	**(c)** $9 - 6n$
22. 570	**23.** 5	**24.** 7 805	**25.** 20, $5\frac{17}{95}$ m
26. $4n^2 + 9n$; 14	**27.** 3 090		

Exercise 2 – Arithmetic Progression (Textbook- Exercise 8C – page 124)

8 A squirrel is collecting nuts. It collects 5 nuts on the first day of the month, 8 nuts on the second, 11 on the third and so on in arithmetic progression.

(a) How many nuts will it collect on the 20th day?

(b) After how many days will it have collected more than 1000 nuts?

9 Kulsum is given an interest-free loan to buy a car. She repays the loan in unequal monthly instalments; these start at \$30 in the first month and increase by \$2 each month after that. She makes 24 payments.

(a) Find the amount of her final payment. (b) Find the amount of her loan.

10 (a) Find the sum of the natural numbers from 1 to 100 inclusive.

(b) Find the sum of the natural numbers from 101 to 200 inclusive.

(c) Find and simplify an expression for the sum of the natural numbers from $n+1$ to $2n$ inclusive.

11 An employee starts work on 1 January 2000 on an annual salary of \$30,000. His pay scale will give him an increase of \$800 per annum on the first of January until 1 January 2015 inclusive. He remains on this salary until he retires on 31 December 2040. How much will he earn during his working life?

Exercise 3 – Arithmetic Progression (Textbook-Mis. Exercise 8C – page 125)

6 The sum of the first two terms of an arithmetic progression is 18 and the sum of the first four terms is 52. Find the sum of the first eight terms. (OCR)

7 The sum of the first twenty terms of an arithmetic progression is 50, and the sum of the next twenty terms is -50. Find the sum of the first hundred terms of the progression. (OCR)

8 An arithmetic progression has first term a and common difference -1. The sum of the first n terms is equal to the sum of the first $3n$ terms. Express a in terms of n. (OCR)

9 Find the sum of the arithmetic progression $1, 4, 7, 10, 13, 16, \ldots, 1000$.

Every third term of the above progression is removed, i.e. 7, 16, etc. Find the sum of the remaining terms. (OCR)

10 The sum of the first hundred terms of an arithmetic progression with first term a and common difference d is T. The sum of the first 50 odd-numbered terms, i.e. the first, third, fifth, … , ninety-ninth, is $\frac{1}{2}T - 1000$. Find the value of d. (OCR)

11 In the sequence $1.0, 1.1, 1.2, \ldots, 99.9, 100.0$, each number after the first is 0.1 greater than the preceding number. Find

(a) how many numbers there are in the sequence,

(b) the sum of all the numbers in the sequence. (OCR)

13 A small company producing children's toys plans an increase in output. The number of toys produced is to be increased by 8 each week until the weekly number produced reaches 1000. In week 1, the number to be produced is 280; in week 2, the number is 288; etc. Show that the weekly number produced will be 1000 in week 91.

From week 91 onwards, the number produced each week is to remain at 1000. Find the total number of toys to be produced over the first 104 weeks of the plan. (OCR)

14 In 1971 a newly-built flat was sold with a 999-year lease. The terms of the sale included a requirement to pay 'ground rent' yearly. The ground rent was set at £28 per year for the first 21 years of the lease, increasing by £14 to £42 per year for the next 21 years, and then increasing again by £14 at the end of each subsequent period of 21 years.

(a) Find how many complete 21-year periods there would be if the lease ran for the full 999 years, and how many years there would be left over.

(b) Find the total amount of ground rent that would be paid in all of the complete 21-year periods of the lease. (OCR)

15 An arithmetic progression has first term a and common difference 10. The sum of the first n terms of the progression is 10 000. Express a in terms of n, and show that the nth term of the progression is

$$\frac{10\,000}{n} + 5(n-1).$$

Given that the nth term is less than 500, show that $n^2 - 101n + 2000 < 0$ and hence find the largest possible value of n. (OCR)