3.5   Miscellaneous exercise

1   On any morning, the probability that I have to wait at the traffic lights on my way to school is $\frac{1}{4}$.

   (i) Find the probability that, on two consecutive mornings, I have to wait at the traffic lights

     (a)   on exactly one morning,

     (b)   on the second morning, given that I have to wait at the traffic lights on exactly one morning,

   (ii) Find the probability that, on three consecutive mornings, I have to wait at least once.

2   A coin is biased so that the probability of obtaining a head is 0.6. The coin is tossed twice.

   (i) Find the probability that at least one head is obtained.

   (ii) Find the probability that exactly one head is obtained, given that at least one head is obtained.

3   A local greengrocer sells fruit, 30% of which is organically grown and 70% is conventionally grown. Sales of apples constitute 20% of the organically grown fruit and 45% of the conventionally grown fruit.

   (i) Show the information on a labelled tree diagram.

A customer who has purchased fruit is chosen at random to take part in a survey.

   (ii) Find the probability that the customer bought apples.

   (iii) Given that the customer bought apples, find the probability that they were organically grown.

4   A choir has 6 sopranos, 5 altos, 4 tenors and 3 basses. The sopranos and altos are women and the tenors and basses are men. Three members of the choir are chosen at random to give out the programmes before the performance.

   (i) Find the probability that

     (a)   all 3 basses are chosen,

     (b)   2 sopranos and 1 tenor are chosen,

     (c)   3 women are chosen.

   (ii) Given that 3 women are chosen, find the probability that they are all sopranos.

5   The ten letters in the word STATISTICS are arranged at random in a line. Find the probability that the three vowels are together.

6   In a certain country 54% of the population is male. It is known that 5% of the males are colour-blind and 2% of the females are colour-blind. A person is chosen at random and found to be colour-blind. By drawing a tree diagram or otherwise, find the probability that the person is male.

<div align="right">Cambridge Paper 6 Q5 N03</div>

7   Alex and Bonnie play each other at table tennis. Each game results in either a win for Alex or a win for Bonnie. If Alex wins a particular game, the probability of her winning the next game is 0.8, but, if she loses, the probability of her winning the next game is 0.3. The probability of Alex winning the first game is 0.7.

   By drawing a tree diagram, or otherwise, find the probability that

   (i)   Alex wins the first game, given that she wins exactly one of the first two games,

   (ii) Bonnie loses two games and wins one game in the first three games.

8   A bag contains 7 yellow discs and 3 red discs. Discs are removed at random, one at a time, **without** replacement.

   Find the probability that

   (i)   the second disc is yellow, given that the first disc was red,

   (ii) the second disc is yellow,

   (iii) the third disc is red, given that the first disc was red.

9   When a particular firm needs to hire a taxi, the receptionist calls one of three firms, X, Y or Z.

   40% of the calls are to X, 50% are to Y and 10% are to Z.

   9% of the taxis hired from X are late, 6% of those hired from Y are late and 20% of those hired from Z are late.

   Find the probability that the next taxi hired

   (i)   will be from X and will not arrive late,

   (ii) will arrive late,

   (iii) is from X, given that it arrives late.

10 At a children's party each of the 12 guests is to receive a toy. James is hoping to get a torch and Emma wants a ball. The 12 toys, consisting of 4 balls, 3 torches and 5 pens, are placed in a bag. James receives the first toy drawn out of the bag and Emma receives the second toy drawn out of the bag. Assume that at each stage each toy has an equal chance of being drawn.

   (i)   Find the probability that Emma will get a ball given that James did not get a torch.

   (ii) When Emma's parents arrive at the end of the party, Emma shows them the ball that she got in the draw. Find the probability that James got a torch.

11 Events A and B are such that $P(A) = 0.5$, $P(A$ and $B) = 0.2$ and $P(A$ or $B) = p$.

Find, in terms of p,

(i) $P(B)$,

(ii) $P(A$ given $B)$

If A and B are independent events

(iii) find the value of p.

12 In a group of 12 international referees there are 4 from Asia, 3 from Africa and 5 from Europe. Three referees are chosen at random from the group to officiate at a tournament. Calculate the probability that

(i) the three referees are chosen from the same continent,

(ii) the three referees are all from Africa, given that they are all from the same continent,

(iii) there is one referee from each continent.

13 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride. If he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he will go on each of the three rides.

The probabilities that Ravi is frightened on each of the rides are as follows:

elephant ride $\frac{6}{10}$,   camel ride $\frac{7}{10}$,   jungle tractor ride $\frac{8}{10}$.

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened.

Ravi goes on a ride.

(iii) Find the probability that he is frightened.

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride.

Cambridge Paper 6 Q5 J09

14 It was found that 68% of passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all the probabilities in decimals.

(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey.

Cambridge Paper 63 Q3 N10

15 The following histogram illustrates the distribution of times, in minutes, that some students spent taking a shower.
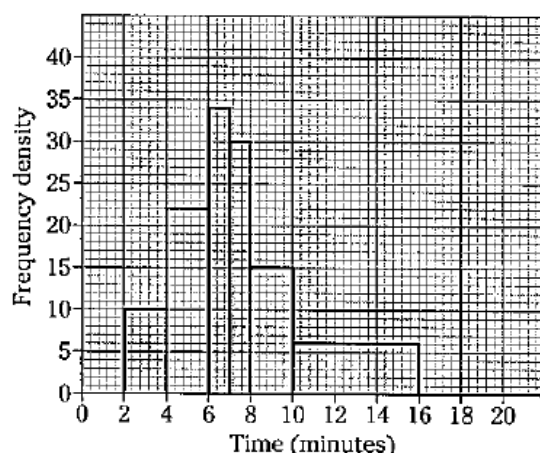
(i) Copy and complete the following frequency table for the data.

Time (t minutes)	$2 < t \leqslant 4$	$4 < t \leqslant 6$	$6 < t \leqslant 7$	$7 < t \leqslant 8$	$8 < t \leqslant 10$	$10 < t \leqslant 16$
Frequency						

(ii) Calculate an estimate of the mean time to take a shower.

(iii) Two of these students are chosen at random. Find the probability that exactly one takes between 7 and 10 minutes to take a shower.

Cambridge Paper 63 Q5 N10

16 The lengths of cars travelling on a ferry are noted. The data are summarised in the following table.

Length of car (x metres)	Frequency	Frequency density
$2.80 \leqslant x < 3.00$	17	85
$3.00 \leqslant x < 3.10$	24	240
$3.10 \leqslant x < 3.20$	19	190
$3.20 \leqslant x < 3.40$	8	a

(i) Find the value of a.

(ii) Draw a histogram on graph paper to represent the data.

(iii) Find the probability that a randomly chosen car on the ferry is less than 3.20 m in length.

Cambridge Paper 6 Q2 N04

17 Two fair 12-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

  Q: the product of the two numbers is 24,

  R: both of the numbers are greater than 8.

(i) Find P(Q).

(ii) Find P(R).

(iii) Are events Q and R exclusive? Justify your answer.

(iv) Are events Q and R independent? Justify your answer.

Cambridge Paper 62 Q5 J10