Revision Exercise 1

(Quadratics, Inequalities, Coordinate Geometry, Functions and Circular Measure)

1 The line l_1 passes through the points $A(4,8)$ and $B(10,26)$. Show that an equation for l_1 is $y = 3x - 4$.

The line l_1 intersects the line l_2, which has equation $y = 5x + 4$, at C. Find the coordinates of C.

2 Show that any root of the equation $5 + x - \sqrt{3 + 4x} = 0$ is also a root of the equation $x^2 + 6x + 22 = 0$. Hence show that the equation $5 + x - \sqrt{3 + 4x} = 0$ has no solutions.

3 Write $x^2 + 10x + 38$ in the form $(x + b)^2 + c$ where the values of b and c are to be found.

(a) State the minimum value of $x^2 + 10x + 38$ and the value of x for which this occurs.

(b) Determine the values of x for which $x^2 + 10x + 38 \geqslant 22$.

4 Simplify $\left(4x^{\frac{1}{2}}y\right)^2 + \left(2x^{-1}y^2\right)$.

5 Solve the inequalities (a) $2x^2 - 5x + 2 \leqslant 0$, (b) $(2x - 3)^2 < 16$, (c) $\frac{1}{3}x - \frac{1}{4}(2x - 5) < \frac{1}{5}$.

6 Show that the equation $2^{x+1} + 2^{x-1} = 160$ can be written in the form $2.5 \times 2^x = 160$. Hence find the value of x which satisfies the equation.

7 Find the values of k such that the straight line $y = 2x + k$ meets the curve with equation $x^2 + 2xy + 2y^2 = 5$ exactly once.

8 Display on the same axes the curves with equations $y = x^3$ and $y = \sqrt[3]{x}$, and give the coordinates of their points of intersection.

9 A mail-order photographic developing company offers a picture-framing service to its customers. It will enlarge and mount any photograph, under glass and in a rectangular frame. Its charge is based on the size of the enlargement. It charges \$6 per metre of perimeter for the frame and \$15 per square metre for the glass. Write down an expression for the cost of enlarging and mounting a photograph in a frame which is x metres wide and y metres high.

A photograph was enlarged and mounted in a square frame of side z metres at a cost of \$12. Formulate and solve a quadratic equation for z.

10 Find the equation of the straight line through $A(1,4)$ which is perpendicular to the line passing through the points $B(2,-2)$ and $C(4,0)$. Hence find the area of the triangle ABC, giving your answer in the simplest possible form.

11 Solve the inequalities

(a) $2(3 - x) < 4 - (2 - x)$, (b) $(x - 3)^2 < x^2$, (c) $(x - 2)(x - 3) \geqslant 6$.

12 The quadratic equation $(p - 1)x^2 + 4x + (p - 4) = 0$ has a repeated root. Find the possible values of p.

13 Solve the simultaneous equations

$$2x + 3y = 5,$$
$$x^2 + 3xy = 4.$$

14 Prove that the triangle with vertices at the points $(1,2)$, $(9,8)$ and $(12,4)$ is right-angled, and calculate its area.

15 Find where the line $y = 5 - 2x$ meets the curve $y = (3-x)^2$. What can you deduce from your answer?

16 A rhombus has opposite vertices at $(-1,3)$ and $(5,-1)$. Find the equations of its diagonals. One of the other vertices is $(0,-2)$. Find the fourth vertex.

17 Points A and B have coordinates $(-1,2)$ and $(7,-4)$ respectively.

(a) Write down the coordinates of M, the mid-point of AB.

(b) Calculate the distance MB.

(c) The point P lies on the circle with AB as diameter and has coordinates $(2,y)$ where y is positive. Calculate the value of y, giving your answer in surd form.

18 Solve the inequalities (a) $x^2 - x - 2 > 0$, (b) $(x+1)(x-2)(x-3) > 0$.

19 Two of the sides of a triangle have lengths 4 cm and 6 cm, and the angle between them is $120°$. Calculate the length of the third side, giving your answer in the form $m\sqrt{p}$, where m and p are integers, and p is prime.

20 A triangle has vertices $O(0,0)$, $A(2,6)$ and $B(12,6)$. Write down the equation of the perpendicular bisector of AB, and find the perpendicular bisector of OA. Find the coordinates of the point C where these lines meet, and calculate the distances of C from O, A and B.

Write down the area of triangle OAB. Hence find the length of the perpendicular from A to OB, and deduce that angle AOB is $45°$. (MEI, adapted)

21 A quadrilateral has vertices $A(-1,1)$, $B(1,2)$, $C(4,1)$ and $D(3,4)$. Find the lengths and the equations of the two diagonals AC and BD. (OCR)

22 The quadratic function $f(x) = px^2 + qx + r$ has $f(0) = 35$, $f(1) = 20$ and $f(2) = 11$. Find the values of the constants p, q and r.

Express $f(x)$ in the form $a(x+b)^2 + c$. Use your answer to find the smallest value of $f(x)$. (OCR, adapted)

23 Use the substitution $y = 3^x$ to find the values of x which satisfy the equation $3^{2x+2} - 10 \times 3^x + 1 = 0$.

24 Show that $\sqrt{N+1} - \sqrt{N} = \dfrac{1}{\sqrt{N+1} + \sqrt{N}}$. Use this to explain why $\sqrt{101}$ is close to, but slightly less than, 10.05.

Without using a calculator, find the roots of $x^2 + 7x - 13 = 0$, giving your answers correct to 2 decimal places.

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Answers (Revision Exercise 1)

1 $(-4,-16)$

3 $(x+5)^2+13$;
(a) 13, −5 (b) $x \leq -8$ or $x \geq -2$

4 $8x^2$

5 (a) $\frac{1}{2} \leq x \leq 2$ (b) $-\frac{1}{2} < x < \frac{7}{2}$ (c) $x > 6.3$

6 6

7 $\pm\sqrt{65}$

8 $(0,0),(-1,-1),(1,1)$

9 $\$\left(12(x+y)+15xy\right)$; $5z^2+8z-4=0$, 0.4

10 $x+y=5$; 7

11 (a) $x > \frac{4}{3}$ (b) $x > \frac{3}{2}$ (c) $x \leq 0$ or $x \geq 5$

12 0, 5

13 $x=1$, $y=1$ or $x=4$, $y=-1$

14 25

15 (2,1); the line is a tangent to the curve.

16 $2x+3y=7, 3x-2y=4$; (4,4)

17 (a) (3,−1) (b) 5 (c) $2\sqrt{6}-1$

18 (a) $x < -1$ or $x > 2$ (b) $-1 < x < 2$ or $x > 3$

19 $2\sqrt{19}$ cm

20 $x=7$, $3y+x=10$; (7,1); all $5\sqrt{2}$; 30; $2\sqrt{5}$

21 5, $y=1$; $2\sqrt{2}$, $y=x+1$

22 3, −18, 35; $3(x-3)^2+8$; 8

23 −2, 0

24 1.52, −8.52