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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2012

(June 2011 Intake)

Wednesday **29 August 2012** **12.30 pm – 02.15 pm**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

[Turn over

1 Solve the equation $e^{-x} + 1 = 6e^{x}$, giving your answers in exact form. $x = \ln \frac{1}{2}$ or $x = -\ln 2$ [4]

2 Solve the inequality $|3 - 2x| \leq |x + 4|$. [4]

$-\frac{1}{3} \leq x \leq 7$

3 Find the gradient of the tangent to the curve $\ln y = xe^{x^2-1}$ at $x = 1$. $3e$ [5]

4 Express $5\cos\theta + 12\sin\theta$ in the form of $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. $13\cos(\theta - 67.4°)$ [3]

Find the smallest positive angle θ of the equation $5\cos\theta + 12\sin\theta = 10$, giving your answer correct to two decimal places. $27.7°$ [3]

5 The sequence of values is given by the iterative formula $x_{n+1} = \sqrt{\frac{2}{3}x_n{}^2 + \frac{2}{x_n{}^3}}$ and $x_1 = 1$, converges to α.

(i) State an equation satisfied by α, and show that $\alpha = \sqrt[5]{6}$. $\alpha^2 = \frac{2}{3}\alpha^2 + \frac{2}{\alpha^3}$ [3]

(ii) Hence, find the approximate value of $\sqrt[5]{6}$ correct to 3 decimal places. [3]

1.431

6 The polynomial $P(x) = x^4 + ax^3 + bx^2 - 2x - 4$ has factors $(x - 1)$ and $(x + 2)$.

(i) Show that $a = 3$ and $b = 2$. [3]

(ii) Find the third factor of $P(x)$ and show that this factor is positive for all real values of x. $x^2 + 2x + 2$ [5]

7 Plane p_1 has equation $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = 0$ and plane p_2 has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 2$.

(i) Find the angle between p_1 and p_2. 90° [2]

(ii) Planes p_1 and p_2 intersect at line l.

(a) Find a vector in the direction of l. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ [2]

(b) The coordinates of one of the points on line l is $(\alpha, 0, \beta)$. Find the values of α and β. $\alpha = 1, \quad \beta = -1$ [3]

(c) Hence, state an equation of l in the form of $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [1]

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

8 **(i)** z is a complex number such that $(2 + 3i)z = 4 - i$, express z in the form of $a + bi$ where a and b are real numbers. $\frac{5}{13} - \frac{14}{13}i$ [4]

(ii) Find the modulus and argument of the complex number $5 - 3i$. $\sqrt{34}$ [2]
-0.540 rad.

The complex number w is represented in Argand diagram by the point W.

Describe geometrically the locus of W in each of following cases:

(a) $|w| = |5 - 3i|$, [2]

(b) $arg(w - 5 + 3i) = arg(5 - 3i) + \frac{1}{2}\pi$. [3]

9 The Newton's law of cooling states that the rate of change in temperature of an object is proportional to the difference between its temperature and the surrounding temperature. This law could be used in forensic science such as investigation for murder cases:

It was noon on a cold December day in London with a temperature of 16°C.

Detective Chin, a famous detective in the district, arrived at the crime scene to find the sergeant leaning over a body. The sergeant said there were several suspects. If they knew the exact time of death, then they could narrow down the list. Detective Chin measured the temperature of the body, which was 35°C. He then left for lunch. Upon returning at 1:00 pm, he measured the temperature of the body again, which was 34°C.

Let the temperature of the body at time t be θ.

(i) Obtain a differential equation from the Newton's law of cooling. [2]

$\frac{d\theta}{dt} = k(\theta - 16)$

(ii) Express θ in terms of t. [6]

$\theta = 16 + 19\left(\frac{18}{19}\right)^t$

(iii) By assuming that the normal body temperature (37 °C) drops immediately when a person dies, estimate the time of the murder. [3]

10:09 am

10 **(i)** A curve is defined by the parametric equations $x = t - 1$ and $y = \frac{30}{t^2-1}$, where $t > 1$. Show that $\frac{dy}{dt} = \frac{-60t}{(t^2-1)^2}$. [3]

(ii) Let $f(t) = \frac{-60t}{(t-1)(t+1)^2}$.

$\frac{-15}{t-1} + \frac{15}{t+1} + \frac{-30}{(t+1)^3}$

(a) Express $f(t)$ in partial fractions. [4]

(b) Find the exact value of $\int_2^4 f(t)dt$. [5]

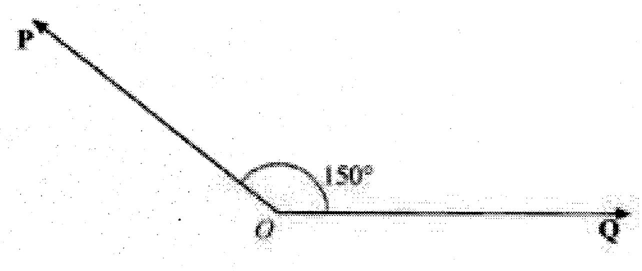
$15\ln\left(\frac{5}{9}\right) - 4$

(June 2011 Intake)

1 A car of mass 1000 kg moves along a horizontal straight road, passing through points A and B. The power of its engine is constant and equal to 15 000 W. The driving force exerted by the engine is 1200 N at A and 750 N at B. Find the increase in the car's kinetic energy as it moves from A to B. [4]

121875 J

2

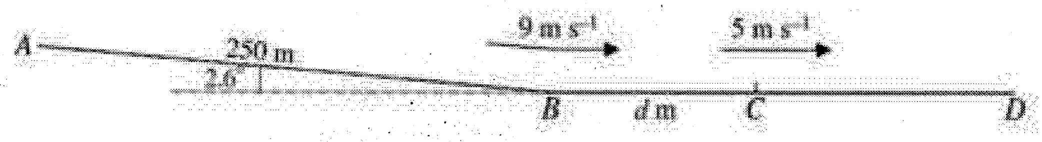


Two forces **P** and **Q** act on a particle at a point O. The force **P** has magnitude 15 N and the force **Q** has magnitude X N. The angle between **P** and **Q** is 150^o, as shown in diagram above. The resultant of **P** and **Q** is **R**. Given that the angle between **R** and **Q** is 50^o, find

(i) the magnitude of **R**, 9.79 N [3]

(ii) the value of X. 19.3N [3]

3



A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest at the top A of a straight path AB, and freewheels down the path to B. The path AB is inclined at 2.6° to the horizontal and is of length 250 m.

(i) Given that the cyclist passes through B with speed 9 ms^{-1}, find the work done against the resistance to motion of the cyclist and his machine. 5830J [3]

The cyclist continues to freewheel along a horizontal straight path BD until he reaches the point C, where the distance BC is d m. His speed at C is 5 ms^{-1}. The resistance to motion is constant, and is the same on BD as on AB.

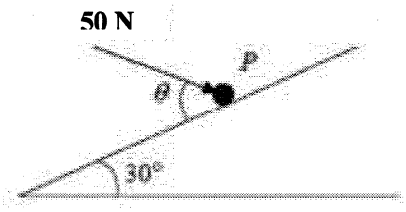
(ii) Find the value of d. 96 [3]

The cyclist starts to pedal at C, generating 450 W of power.

(iii) Find the acceleration of the cyclist immediately after passing through C. [3]

0.833/0.834 ms^{-2}

4



A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle 30^o to the horizontal. The particle is held in equilibrium by a force of magnitude 50 N, acting at an angle θ to the plane, as shown in diagram above. The force acts in a vertical plane through a line of greatest slope of the plane.

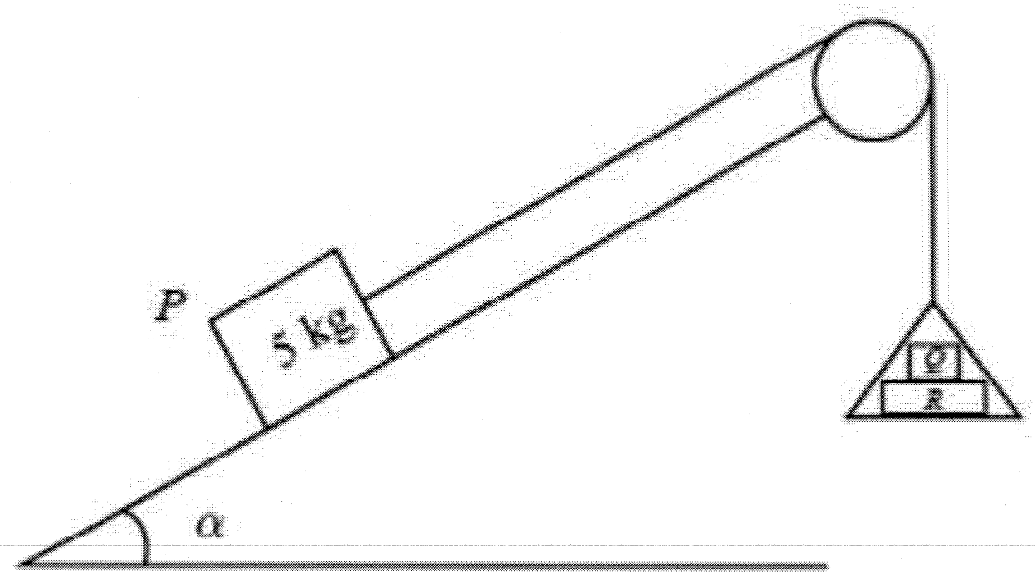
(i) Show that $\cos\theta = \frac{3}{5}$. [3]

(ii) Find the normal reaction between P and the plane. 92 N [3]

The direction of the force of magnitude 50 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(iii) Find the initial acceleration of P. 2.17 ms^{-2} [3]

5



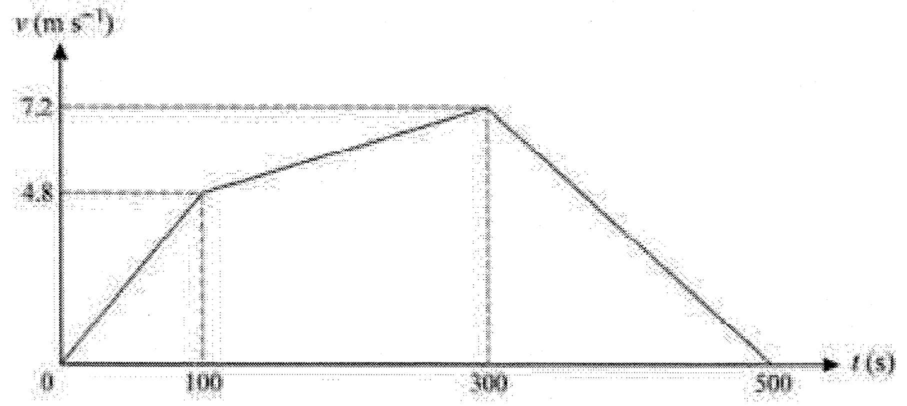
One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 3. The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modeling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(i) the acceleration of the scale pan, and the tension in the string, $6\,ms^{-2}$, $60\,N$ [4]

(ii) the magnitude of the force exerted on block Q by block R, 20 N [3]

(iii) the magnitude of the force exerted on the pulley by the string. 105 N [3]

6



A tractor A starts from rest and travels along a straight road for 500 seconds. The velocity-time graph for the journey is shown above. Another tractor B starts from rest at the same instant as A, and travels along the same road for 500 seconds. Its velocity t seconds after starting is $(0.06t - 0.00012t^2)$ ms^{-1}. Find

(i) how much greater B's initial acceleration is than A's, 0.012ms^{-2} [5]

(ii) how much further B has travelled than A, at the instant when B's velocity reaches its maximum. 155m [7]