

Revision 4 & 5 Discrete random variables and Normal distributions

P6 Mac 08

3. Potatoes are packed in sacks which it is claimed contain 25kg. In fact the mean mass of the sacks is 25.14kg with a standard deviation of 0.15kg. You may assume that the masses are normally distributed. A sack of potatoes is considered underweight if its mass is less than 25kg.
- a) Calculate the proportion of sacks which are underweight. [3]
- b) Assuming that standard deviation remains unchanged calculate, correct to 2 decimal places, to what value must the mean be adjusted if only 5% of the potato sacks are allowed to be underweight? [3]

P6 Mac 08

4. A discrete random variable X takes the values 0, 1 and 2 with probabilities a , b , and c respectively. If expectation and variance of X are $4/3$ and $5/9$ respectively, find the values of a , b , and c . [7]

P6 Mac 08

7. A farmer uses a machine to dig up potatoes and the probability that a potato will be damaged by the machine is 0.1.
- a) Find the probability that a random selection of 12 potatoes will include 3 damaged potatoes. [3]
- b) A random sample of n potatoes is selected. Let X be the random variable representing the number of damaged potatoes from this selection.
- i) Write down, in terms of n , the mean and standard deviation of X . [2]
- ii) Use the normal approximation to show that for $P(X \geq 3)$ to exceed 0.95, the sample size, n , must satisfy the (approximate) inequality $n - 25 > 4.935\sqrt{n}$. Hence show that the smallest value of n is 65. [6]

P6 Aug 07

- 4.(a) The random variable X is normally distributed with mean 10 and standard deviation σ . It is given that $P(X > 11) = 0.35$. Find the standard deviation. [3]
- (b) A normal distribution has mean μ and standard deviation σ . If 1000 observations are taken from this distribution, how many would you expect to be between $\mu - 2\sigma$ and $\mu + 2\sigma$? [2]

P6 Aug 07

6. Two red balls and two white balls are placed in a bag. Balls are drawn one by one at random and without replacement. The random variable X is the number of white balls drawn before the first red ball is drawn.
- (a) Show that $P(X = 1) = \frac{1}{3}$ [2]
- (b) Draw up the probability distribution table for X . [3]
- (c) Find $E(X)$ and show that $Var(X) = \frac{5}{9}$ [4]

P6 Aug 07

8. A reader of a magazine enters for a competition in the magazine, in which the competitors have to choose the correct answers to a number of questions. There are five suggested answers for each question, but the reader is completely unskillful and selects an answer at random to each question, so that, for each question, the probability of choosing the correct answer is $\frac{1}{5}$.
- (a) For a competition with 12 questions, find the probability of the reader getting more than 2 correct answers. [4]
- (b) For a competition with 100 questions, use a suitable approximation to show that the probability of the reader getting more than 26 correct answers is 0.052. [6]

P6 Mac 07

3. When a certain type of cell is subjected to radiation, the cell may die, survive as a single cell or divide into two cells with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively.
- Two cells are independently subjected to radiation. The random variable X represents the total numbers of cells in existence after this experiment.
- (a) Find the probability distribution of X . [3]
- (b) Evaluate $E(X)$. [2]
- (c) Find $Var(X)$. [2]

P6 Mac 07

6. In a large population the proportion having blood group A is 35%. Specimens of blood from the first five people attending a clinic are to be tested. It can be assumed that these five people are a random sample from the population. The random variable X denotes the number of people in the sample who are found to have blood group A .
- (a) Show that $P(X \leq 2) = 0.765$, correct to 3 decimal places. [2]
 - (b) Three such samples of five people are taken. Find the probability that each of these three samples has more than two people with blood group A . [3]
 - (c) One hundred such samples of five people are taken. Using a suitable approximation, find the probability that the number of these samples that contain two or fewer people with blood group A will be at least 70. [4]

P6 Mac 07

7. A college student investigated how long he actually had to spend on assignments, which were nominally for half-hour periods. He found that the times were approximately normally distributed, with mean 35 minutes and standard deviation 8 minutes. Using this model, and assuming independence between assignments, find
- (a) the probability that one particular assignment will take less than 25 minutes, [2]
 - (b) the time in which 90% of all assignments can be completed, [3]
 - (c) the probability that three assignments each take more than 40 minutes. [4]

P6 Aug 06

5. Ashley and Kim take part in a fishing competition. Each catches a different type of fish. Ashley catches an adult mullet measuring 38 centimetres in length. Kim catches an adult bream measuring 45 centimetres in length.
- i. The length of adult mullet can be modelled by a normal distribution with a mean of 30 centimetres and a standard deviation of 3.5 centimetres. Find the proportion of adult mullet measuring 38 centimetres or more in length. [3]
 - ii. The length of adult bream can be modelled by a normal distribution with mean of 40 centimetres and standard deviation 5 centimetres. Find the proportion of adult bream measuring 45 centimetres or less in length [3]
 - iii. Comparing Ashley's catch to Kim's catch, who do you think has performed better? Justify your answer. [2]

P6 Aug 06

6. Analysis of the purchase at a snack bar over a long period shows that 55% of customers buy a hot drink, and that this figure is made up of 32% who buy coffee, 19% who buy tea and 4% who buy chocolate. No one buys two drinks.
- Find the probability that at least three out of a random sample of ten customers buy coffee. [4]
 - Using a suitable approximation, find the probability that in a random sample of 120 customers, 77 or more will buy a hot drink. [5]

P6 Aug 06

7. In each round of a particular game, Albert draws 3 marbles without replacement from a box containing 2 red marbles and 8 blue marbles. The number of red marbles drawn in each round is denoted R . Find the probability distribution for R and show that the mean of R is $\frac{3}{5}$. [5]

If Albert scores 4 points for each red marble drawn but loses 1 point for every blue marble drawn, calculate the number of points Albert expects to have at the end of 5 rounds. [5]

P6 Aug 05

1. The mass of a certain grade of potato is normally distributed with mean 50 grams and standard deviation 6 grams. If one potato is chosen at random, find the probability that its mass will exceed 35 grams. [3]

P6 Aug 05

2. X is a binomial variable with mean 4 and variance $\frac{4}{3}$. Show that the largest value that X can take is 6, and find $P(X = 5)$. [5]

P6 Aug 05

3. The discrete random variable X is such that it only takes values 0, 1, 2, 3, 4, 5.

The probability distribution of X is given by the following:

| | | | | | | |
|------------|-----|-----|-----|-----|-----|-----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| $P(X = x)$ | a | a | a | b | b | b |

also $P(X \geq 2) = 3P(X < 2)$, where a and b are constants.

- (i) Determine the values of a and b , [3]
- (ii) Show that the expectation of X is $\frac{23}{8}$. [2]

P6 Aug 05

5. If there is a probability of 0.2 of failure to get through in any attempt to make a telephone call, calculate the ~~most probable~~^{expected} number of failures in 10 attempts.

Calculate the probability of 3 or more failures in 10 attempts. [5]

Use a suitable approximation to estimate the probability of less than 75 successes in 100 attempts. [4]

P6 Mac 05

3. In a company, the number of hours the part-time workers work in a week is normally distributed with mean 20 and standard deviation 6.9.
- (i) Calculate the proportion of workers who work more than 25 hours. [3]
- (ii) Given that only 3% of the workers worked for less than T hours, calculate T . [3]

P6 Mac 05

4. In a large city one person in five is left-handed.
- (i) 10 people are randomly chosen in this city. Find the probability that fewer than three people are left-handed. [3]
- (ii) How large must a random sample be if the probability that it contains at least one left-handed person is to be greater than 0.95. [4]

P6 Mac 05

5. A statistical-minded parent is discussing pocket money with a young child. The parent has a £1 coin and two 50p coins. All the three coins are spun and if a coin lands with heads uppermost the child can have it as pocket money, otherwise the parent keeps it. The child's parent is generous and, rather than disappointing the child, in the event of all the three coins landing tails the child will receive 50p.

The random variable X denotes the amount of pocket money in pounds the child receives.

- (i) Show that $P(X = 1) = \frac{1}{4}$. [2]
- (ii) Draw up a table to show the probability distribution of X . [3]
- (iii) Find $E(X)$. [2]

P6 Mac 05

7. A large bag of seeds contains three varieties, A, B and C in the ratios 4 : 2 : 1 and their germination rates are 50%, 60% and 80% respectively.
- (i) A seed is chosen at random from the bag. Using a tree diagram, or otherwise, calculate the probability that it will germinate. [3]
 - (ii) Given that a seed does not germinate, find the probability that it is of variety C. [3]
 - (iii) Given that 150 seeds are chosen at random from the bag, use a suitable approximation to find the probability that less than 90 of them will germinate. [4]

P6 Aug 04

1. It is given that 40% of the population support the Gamboge Party. One hundred and fifty members of the population are selected at random. Use a suitable approximation to find the probability that more than 55 out of the 150 support the Gamboge Party. [5]

P6 Aug 04

3. The quality of articles manufactured in a certain process, classified as 'poor', 'fair' or 'good' on the basis of a measured quantity X may be assumed to have a normal distribution with mean 50 units and standard deviation 5 units. Articles are 'poor' if $X < 44$, and the proportion of 'fair' and 'good' are equal. Calculate the boundary value of X separating 'fair' and 'good', to the nearest integer value. [6]

P6 Aug 04

6. A random variable X has the probability distribution given in the following table.

| | | | | |
|------------|-----|----------------|----------------|-----|
| x | 2 | 3 | 4 | 5 |
| $P(X = x)$ | p | $\frac{2}{10}$ | $\frac{3}{10}$ | q |

- a) Given that $E(X) = 4$, find p and q . [4]
 b) Show that $\text{Var}(X) = 1$. [2]
 c) Ten independent observations of X are taken. Find the probability that the value 3 is obtained at most three times. [3]

P6 Mac 04

5. Two discs are drawn, without replacement, from a box containing 3 red discs and 4 white discs. The discs are drawn at random. If X is the random variable "the number of red discs drawn",
- (i) show that $P(X = 1) = \frac{4}{7}$ [2]
- (ii) tabulate the probability distribution of X . [4]
- (iii) calculate the expected number of red discs. [2]

P6 Mac 04

7. (i) Tests on 2 types of electric light bulb show the following :
- Type A has lifetime distributed normally with an average lifetime of 1150 hours and a standard deviation of 30 hours.
- Type B has lifetime distributed normally with an average lifetime of 1900 hours and a standard deviation of 50 hours.
- (a) What percentage of bulbs of type A could be expected to have a lifetime of more than 1200 hours ? [2]
- (b) What percentage of bulbs of type B would you expect to last longer than 1800 hours ? [3]
- (ii) Find the probability of obtaining between 4 and 7 heads inclusive with 12 tosses of a fair coin,
- (a) using the binomial distribution, [3]
 (b) using the normal approximation to the binomial distribution. [3]

P6 Aug 03

2. A fair coin is tossed 100 times. The event that the number of heads obtained is less than 45 is denoted by A . By using a suitable approximation to the binomial distribution, calculate $P(A)$. [4]

P6 Aug 03

5. Ten playing cards, two of which are Aces, are lying face down on a table. A player turns the cards over, one by one in random order, and the N th card turned over is the first Ace found ($1 \leq N \leq 9$). Calculate the probabilities $P(N=2)$ and $P(N=9)$. [4]

A player pays a stake of 10p to try his luck at finding an Ace; if the first card is an Ace he gets his stake of 10p back and in addition he gets a prize of 30p. If he fails to find an Ace with his first card, he turns a second card, and if this is an Ace his 10p stake is returned, but otherwise he gets nothing at all and his stake is lost. Calculate a player's expectation, stating whether it is a gain or a loss. [4]

P6 Aug 03

7. A school student investigated how long he actually had to spend on homework assignments, which were nominally for half-hour periods. He found that the times were approximately normally distributed, with mean 35 minutes and standard deviation 8 minutes. Using this model, and assuming independence between assignments, find
- (i) the probability that one particular assignment takes less than 25 minutes, [3]
 - (ii) the time in which 90% of all assignments can be completed, [3]
 - (iii) the probability that three assignments each take more than 40 minutes, [3]
 - (iv) the probability that three out of ten assignments given will each take more than 40 minutes. [3]

P6 Mac 03

- 3 (i) In a sample of 500 girls, the heights are found to be normally distributed with mean 161.2 cm and standard deviation 4.7 cm. Estimate how many will have heights more than 165 cm. [2]
- (ii) X is a random variable with mean and standard deviation as μ and σ respectively. Given $P(X < 30) = 0.14$ and $P(X < 60) = 0.79$, calculate the values of μ and σ . [4]

P6 Mac 03

6. The six faces of a fair cubical dice are numbered 1, 2, 2, 3, 3, 3. When the dice is thrown once, the score is the number appearing on the top face. This is denoted by X .
- (i) Find the mean and the standard deviation of X . [2]
- (ii) The dice is thrown twice and Y denotes the sum of the scores obtained. Draw up a table to show the probability distribution of Y . [3]
- (iii) Find the expectation and variance of Y . [4]

P6 Mac 03

7. Compact discs are packed in a box which contains 20 discs. 5 % of the discs are faulty. The boxes are packed in crates which contain 50 boxes. Find the probabilities that
- (i) (a) a box contains four faulty discs, [3]
- (b) a box contains at least one faulty disc. [3]
- (ii) Using a suitable approximation, find the probability that a crate contains between 35 and 39 (inclusive) boxes with at least one faulty disc. [4]