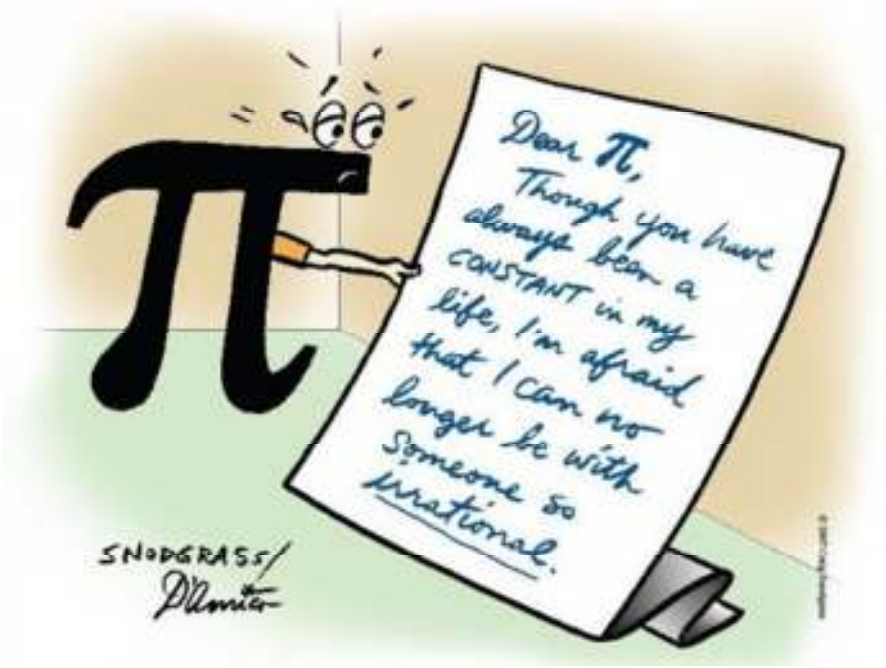
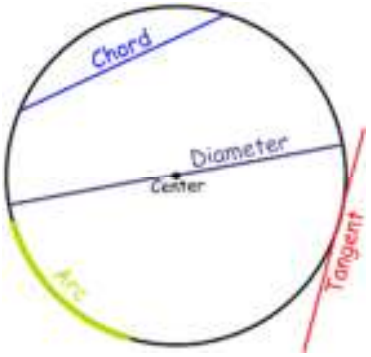
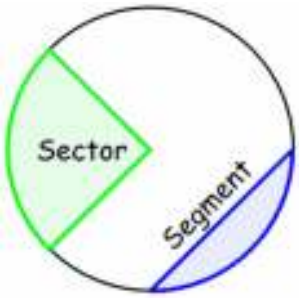
Topic 4 - Circular Measure

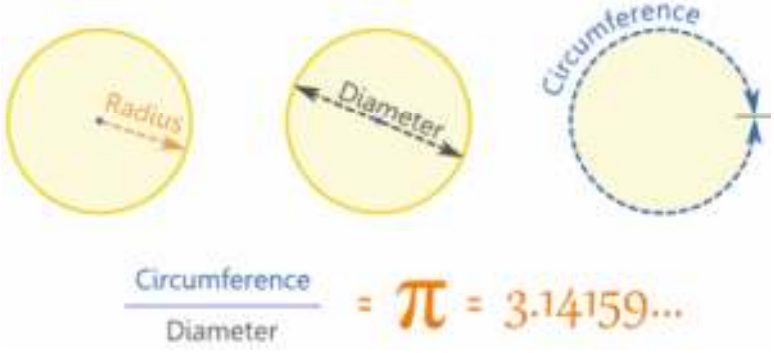


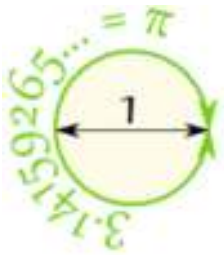
Definition





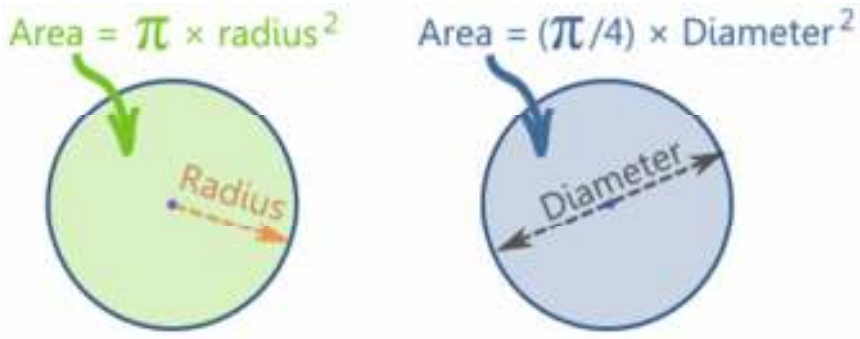
Radius, Diameter and Circumference



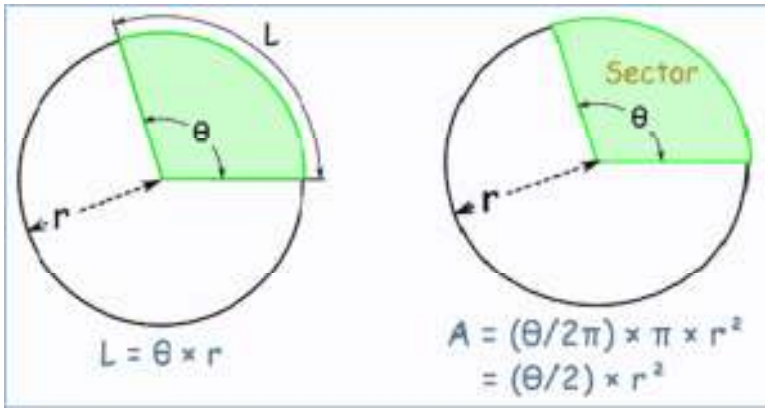


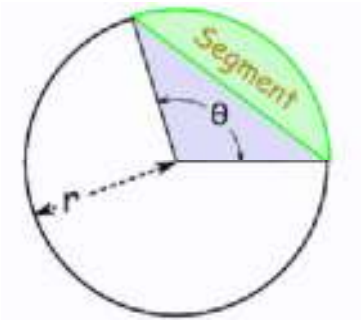
So when the diameter is 1, the circumference is 3.141592654...

Area of a Circle

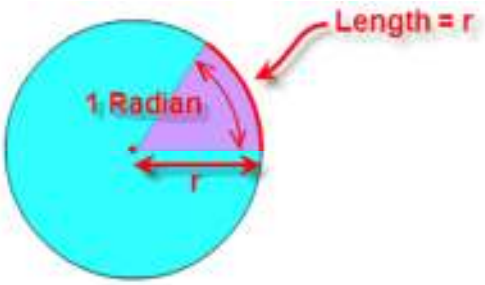


Formulas for Circles





Area of a segment =



A **Radian** "cuts out" a length of a circle's circumference equal to the radius.

Important formula

Converting radians to degrees:

$$\text{degrees} = \text{radians} \times \frac{180}{\pi}$$

Converting degrees to radians:

$$\text{radians} = \text{degrees} \times \frac{\pi}{180}$$

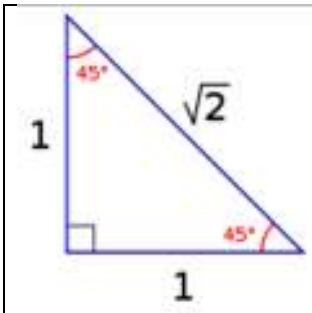
Radians and Degrees

π radians = 180°

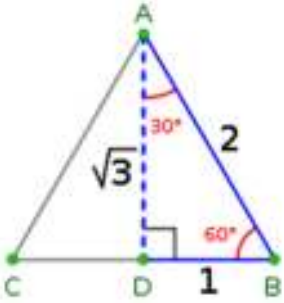
1 radian = 180°/π = 57.2958° (approximately)

30° = π/6 = 0.524	45° = π/4 = 0.785	60° = π/3 = 1.047
90° = π/2 = 1.571	270° 3π/2 = 4.712	360° = 2π = 6.283

Exact values for common angles



Isosceles right-angled triangle



Equilateral triangle

sin 45˚ =

cos 45˚ =

tan 45˚ =

sin 30˚ =

cos 30˚ =

tan 30˚ =

sin 60˚ =

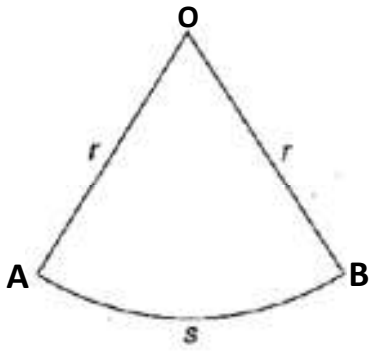
cos 60˚ =

tan 60˚ =

Example 1

The diagram shows a sector OAB of radius 8cm and perimeter of 23cm. Find

a) angle AOB in radian,
b) area of sector AOB.



a) 7/8 rad
b) 28cm^2

Example 2

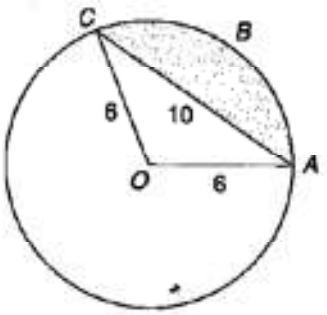
A circle of radius 5.5cm has a sector with area 30.25cm^2. Calculate the perimeter of this sector.

22cm

Example 3

The diagram shows a circle of radius 6 cm with a chord AC of length 10 cm. Calculate, to 3 significant figures, the area of

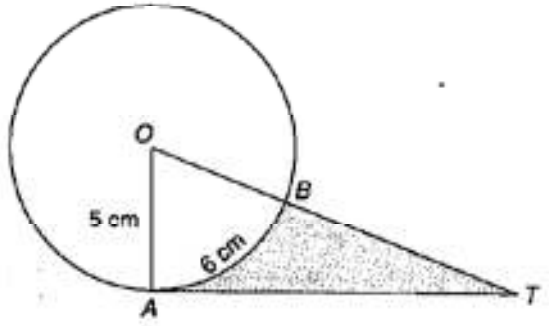
(a) the minor sector AOC,
(b) the shaded segment ABC.



a) 35.5cm^2 b) 18.9cm^2

Example 4

The figure shows a circle, centre O, radius 5 cm. The tangent to the circle at A meets OB produced at T. Given that the length of the arc AB is 6 cm, calculate the area of the shaded region, correct to 2 decimal places.



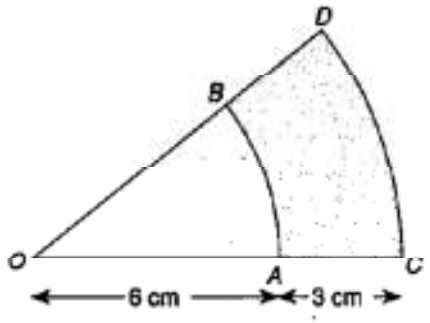
17.15cm²

Example 5

In the figure, arcs AB, CD are arcs of concentric circles, centre O. If $OA = 6$ cm, $AC = 3$ cm and the area of sector AOB is 12 cm², calculate

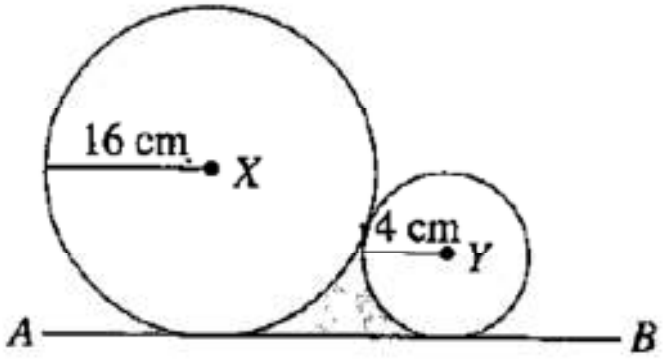
(a) $\angle AOB$ in radians,

(b) the area and perimeter of the shaded region.



a) 2/3 b) 16cm

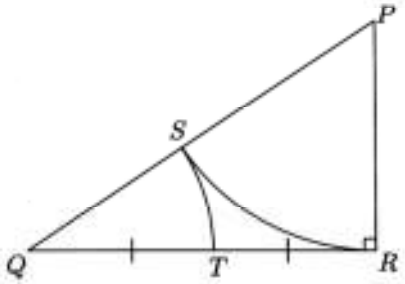
Example 6



Two coins of radii 16cm and 4cm touch each other externally and lie on a straight line AB as shown in the diagram on the right. Find the area enclosed by the two coins and the line AB. *23.6cm²*

Example 7

Triangle PQR is right-angled at R. The circle with centre P and radius PR cuts PQ at S and the circle with centre Q and radius QS cuts QR at T.



If T bisects QR, then the ratio $QS : SP$ equals

(A) 7 : 12 (B) 5 : 12 (C) 5 : 8 (D) 3 : 4 (E) 2 : 3

Exercise 18A – Pure Mathematics 1(page 267) Question 2, 3, 9, 10 and 11

Miscellaneous Exercise 18 – Pure Mathematics 1(page 275) Question 4 and 5