Topic 3: Coordinate Geometry

- find the length, gradient and mid-point of a line segment, given the coordinates of the end-points;
- find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient);
- understand and use the relationships between the gradients of parallel and perpendicular lines;
- interpret and use linear equations, particularly the forms $y = mx + c$ and $y - y_1 = m(x - x_1)$;
- understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations (including, in simple cases, the correspondence between a line being tangent to a curve and a repeated root of an equation).

Equation of a Straight Line

The equation of a straight line is usually written as: $y = mx + b$ or $y = mx + c$

What does it stand for?

How do you find "m" and "b"?

- b is easy: just see where the line crosses the y- axis.
- m (the Slope) needs some calculation:

$$m = \frac{change\ in\ y}{change\ in\ x}$$

Knowing this we can work out the equation of a straight line:

Example 1

$$m = \frac{2}{1} = 2$$

b = 1 *(where the line crosses the y-axis)*
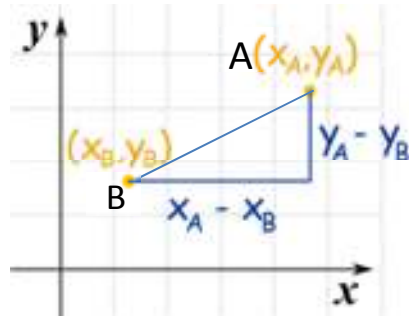
Therefore y = 2x + 1

Linear Equations

1. **Slope- Intercept form**   y = mx + b

2. **Point-slope form** $y - y_1 = m(x - x_1)$

3. **General form**   ax + by + c = 0

Example 2

Find the equation of the line with gradient $\frac{1}{t}$ which passes through the point $(t^2, 2t)$. If this line passes through the point (-2, 1), find the possible values of t.
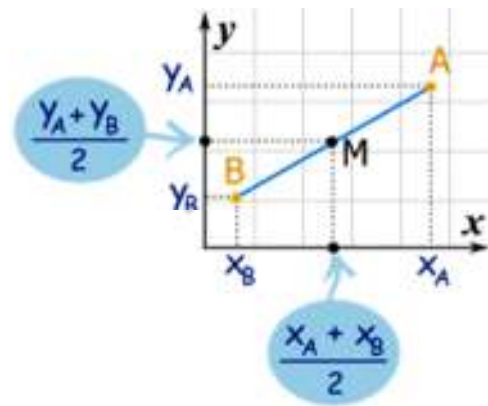
Gradient, Length and Mid-point of a Line Segment

Gradient of a line   Distance between 2 Points

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_A - y_B}{x_A - x_B} \qquad c = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

where c = length AB

Midpoint between Two Points

The midpoint is **halfway** between two points.

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

Example 3

Show that the points A(0, -3), B(4, -2) and C(16, 1) are collinear.

Example 4

The distance between the points (-2, y) and (3, -7) is 13 units. What are the possible values for y?

Example 5

The straight line 3x + 4y = 24 cuts the y axis at the point A and cuts the x axis at the point B. What is the distance AB?

Example 6

The coordinates of the parallelogram ABCD are: A(5,2),   B(2,4),   C(6,7)   and   D(9,5) What is the length of the shorter diagonal of parallelogram ABCD?

Example 7

P(4, -4), Q(9, 6), R(-2, 4) and S are the vertices of a parallelogram. Find

  a) the midpoint of the diagonal PR.
  b) the coordinates of S.

Show that PQRS is a rhombus.

Parallel Lines

How do you know if two lines are **parallel**?

Their slopes are the same!

Perpendicular Lines

Two lines are perpendicular if they meet at a right angle (90°).

How do you know if two lines are perpendicular?

When you multiply their slopes, you get -1

Example 8

What is the equation of the line which is parallel to the line 3x + 2y - 6 = 0 and passes through the point (4, 5)?

Example 9

What is the equation of the line which is perpendicular to the line 4x + 5y - 20 = 0 and passes through the point (8, -3)?

Example 10

The straight line p passes through the point (10, 1) and is perpendicular to the line r with equation $2x + y = 1$.
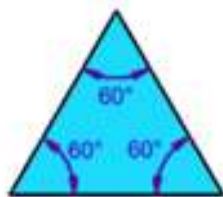
a) Find the equation of p.
b) Find also the coordinates of the point of intersection p and r, and deduce the perpendicular distance from the point (10, 1) to the line r.

Additional Information

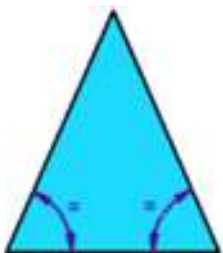1. **Equilateral, Isosceles and Scalene**

   There are three special names given to triangles that tell how many sides (or angles) are equal.

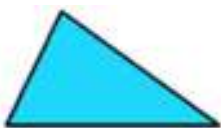   There can be **3**, **2** or **no** equal sides/angles:

Equilateral Triangle

Three equal sides
Three equal angles, always 60°

Isosceles Triangle

Two equal sides
Two equal angles

Scalene Triangle

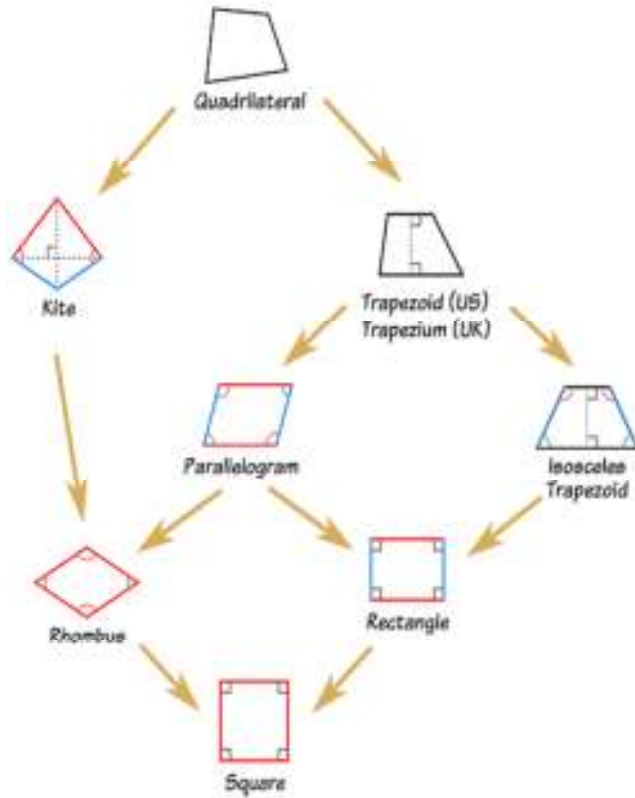No equal sides

Example 11
$P(2, 5)$, $Q(12, 5)$ and $R(8, -7)$ form a triangle.
(a) Find the equations of the altitudes through
(i) R;   (ii) Q.
(b) Find the point of intersection of these altitudes.
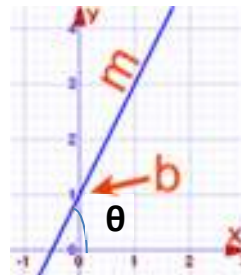(c) Show that the altitude through P also passes through this point.

2. Types of Quadrilaterals and the characteristics of quadrilaterals.

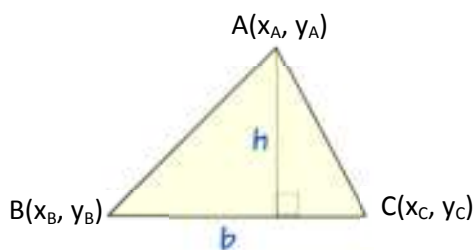**Exercise 1A – Pure Mathematics 1(page 6)** Question 16, 17, 18, 19, 20, 21 and 22

3. Slope

$$m = \frac{change\ in\ y}{change\ in\ x} = \tan \theta$$

Example: Oct/Nov 2005 Examination, Paper 1 – Question **9**

4. Horizontal line – what is the gradient? and   Vertical line - what is the gradient?

5. Area of a triangle

A(x_A, y_A)

B(x_B, y_B)

C(x_C, y_C)

h

b

Area = ½ × b × h or $= \dfrac{1}{2}\begin{vmatrix} x_A & x_B & x_C & x_A \\ y_A & y_B & y_C & y_A \end{vmatrix}$

Exercise 1A – Pure Mathematics 1(page 6) Question 7, 8, 9, 14 and 15

Exercise 1B – Pure Mathematics 1(page 11) Question 5, 7, and 9

Exercise 1C – Pure Mathematics 1(page 14) Question 3 and 5

Miscellaneous Exercise 1 – Pure Mathematics 1(page 15) Question 6, 7, 12, 13, 15 and 16