Exercise 1.5 – Scaling/Combining Sets of Data

To answer these questions you need to be familiar with the formulae for the mean and the standard deviation.

1 Cartons of orange juice are advertised as containing 1 litre. A random sample of 100 cartons gave the following results for the volume, x litres.

$$\sum x = 101.4 \text{ and } \sum x^2 = 102.83$$

Calculate the mean and standard deviation of the volume of orange juice in these 100 cartons.

2 For a set of 10 numbers, $\sum x = 290$ and $\sum x^2 = 8469$.

Find (i) the mean, (ii) the standard deviation, (iii) the variance.

3 For a particular set of data,

$n = 100$, $\sum x = 584$ and $\sum x^2 = 23\,781$.

Find

(i) the mean, $\bar{x}$

(ii) the variance.

4 For a set of 9 numbers, $\sum(x - \bar{x})^2 = 234$. Find the standard deviation of the numbers.

5 For a set of 12 numbers, it is given that

$\sum(x - \bar{x})^2 = 60$, where $\bar{x}$ is the mean.

(i) Find the standard deviation.

It is also given that $\sum x^2 = 285$.

(ii) Find $\bar{x}$.

6 From the information given about each of the following sets of data, work out the missing values in the table:

	n	$\sum x$	$\sum x^2$	$\bar{x}$	s.d.
(i)	63	7623	924 800		
(ii)		152.6		10.9	1.7
(iii)	52		57 300	33	
(iv)	18			57	4

7 A machine cuts lengths of wood. A sample of 20 rods gave the following results for the length, x cm.

$$\sum xf = 997 \qquad \sum x^2 f = 49\,711$$

(i) Find the mean length of the 20 rods.

(ii) Find the variance of the lengths of the 20 rods.

8 For a particular set of observations,

$\sum f = 20$, $\sum x^2 f = 16\,143$, $\sum xf = 563$.

Calculate the standard deviation.

9 The mean of the numbers 3, 6, 7, a, 14, is 8. Find the standard deviation of the set of numbers.

10 (i) Calculate the mean and the standard deviation of the four numbers 2, 3, 6, 9.

(ii) When two numbers, a and b, are added to this set of four numbers, the mean is increased by 1 and the **variance** is increased by 2.5. Find a and b.

11 The numbers a, b, 8, 5, 7 have mean 6 and variance 2. Find the values of a and b, if $a < b$.

12 A test is taken by 30 students. Their scores, x, have a mean of 60 and a standard deviation of 20.

(i) Find $\sum x$ and show that $\sum x^2 = 120\,000$.

Another 20 students take the test. Their scores, y, are such that $\sum y = 1400$ and $\sum y^2 = 100\,000$.

(ii) Show that the mean score of the combined group of 50 students is 64.

(iii) Calculate the standard deviation of the scores of the 50 pupils.

13 The number of errors, x, on each of 200 pages of typescript was monitored. The results were summarised as follows:

$\sum x = 920 \qquad \sum x^2 = 5032$

(i) Calculate the mean and standard deviation of the number of errors on a page.

A further 50 pages were monitored and it was found that, for these pages, the mean was 4.4 errors and the standard deviation was 2.2 errors.

(ii) Find the mean and standard deviation of the number of errors per page for the 250 pages.

14 The manager of a car showroom monitored the numbers of cars sold during two successive five-day periods.

During the first five days the numbers of cars sold per day had mean 1.8 and standard deviation 0.6. During the next five days the numbers of cars sold per day had mean 2.8 and standard deviation 0.81.

Find the mean and standard deviation of the numbers of cars sold per day during the full ten days.

15 For a particular set of data,

$n = 100$, $\sum(x - 50) = 123.5$ and $\sum(x - 50)^2 = 238.4$.

Find the mean and standard deviation of x.

16 Salt is packed in bags which the manufacturer claims contain 25 kg. Eighty bags are examined and the weight, x kg, of each bag is found. The results are summarised as follows.

$\sum(x - 25) = 27.2$ and $\sum(x - 25)^2 = 85.1$

(i) Without doing any calculations, state whether the mean weight of the 80 bags is less than 25 kg, equal to 25 kg or more than 25 kg.

(ii) Calculate the mean weight.

(iii) Find the standard deviation of the weights of the 80 bags.

17 A summary of 15 observations of x gave the following information:

$$\sum(x - 2.5) = 200$$

Find (i) the mean $\bar{x}$, (ii) $\sum(x - \bar{x})$.

18 It is known that, for 100 observations of x, the mean $\bar{x} = 25$.

Find

(i) $\sum x$ (ii) $\sum(x - 20)$ (iii) $\sum(x - 27)$.

Given also that the standard deviation of x is 3, find

(iv) $\sum x^2$ (v) $\sum(x - 20)^2$ (vi) $\sum(x - 27)^2$