Pure Mathematics 1

Tutorial 1 Quadratics

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1	Find the range of values of p for which $(2 - 3p)x^2 + (4 - p)x + 2 = 0$ has no real roots. $[-16 < p < 0]$
2	(i) Express $8 + 6x - x^2$ in the form $a - (x + b)^2$, stating the values of a and b. (ii) Given that $f : x \mapsto 8 + 6x - x^2$ for the domain $x \geq 0$, find the range of f. $[17, -3; y \leq 17]$
3	(i) Express $3 - 6x - x^2$ in the form $a - (x + b)^2$. (ii) Hence or otherwise sketch the graph for the domain $-7 < x < 2$ and state the range of this function. $[-(x+3)^2 + 12; -13 < f(x) \leq 12]$
4	Solve the inequality $(x - 2)(x - 4) \geq 8$. $[x \leq 0 \text{ or } x \geq 6]$
5	a) Express $9x^2 + 12x + 7$ in the form $(ax + b)^2 + c$ where a, b, c are constant whose values are to be found. b) Find the set of values taken by $\dfrac{1}{9x^2 + 12x + 7}$ for real values of x. $[(3x + 2)^2 + 3; 0 < f(x) \leq 1/3]$
6	(a) Solve the inequality $x^2 < 3x + 4$. (b) By means of substitution or otherwise, find two values of x such that $2(4^x) + 4^{-x} = 3$. $[-1 < x < 4; -1/2, 0]$
7	Express $2x^2 + 8x - 10$ in the form $a(x + b)^2 + c$. For the curve $y = 2x^2 + 8x - 10$, write down the co-ordinates of its turning point, and hence, sketch the function. $[2(x + 2)^2 - 18; (-2, -18)]$

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(i) Carry out the process of completing the square for the quadratic polynomial $4x^2 - (5\sqrt{3})x$.
Give your answer in the form $A(x + B)^2 + C$. [3]

(ii) Solve the equation $4x^2 - (5\sqrt{3})x + 3 = 0$, giving your answer in exact form. [3]

$$[\,4(x - \frac{5\sqrt{3}}{8})^2 - \frac{75}{16}; \sqrt{3}, \frac{1}{4}\sqrt{3}\,]$$

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Show that the roots of the equation $\alpha x^2 + (\alpha + \beta)x + \beta = 0$
are real for all values of α and β.

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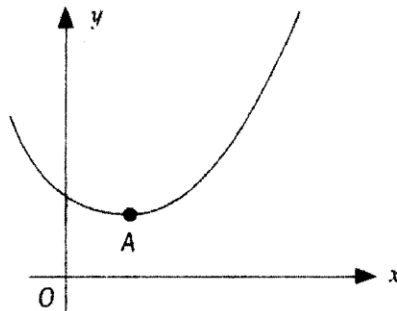
The diagram shows the graph of $y = x^2 - 2px + p$, where p is a positive constant. The point A is the lowest point on the graph which lies above the x-axis.

(i) Using completing the square, express the coordinates of A in terms of p. Hence find the set of possible values for p. [5]

(ii) Given that A lies on the line with equation $y = 2x - 1$, find the exact value of p. [3]

$$\left[(P, -p^2 + p); 0 < p < 1; \frac{-1+\sqrt{5}}{2} \right]$$

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Sketch the curve $y = -x^2 - x$ showing the turning points and the intercepts on the x and y axes.

Determine the greatest and least values of y for $-1 \le x \le 1$. [5]

[1/4 ; -2]

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a) Prove that the expression $(8 - x)(x - 2) - p$ is negative for all real values of x provided that p is greater than 9.

b) Find the range of values of m such that $x^2 - 4mx + m > 0$.

[0 < m < 1/4]