## 3.4     Tree Diagram
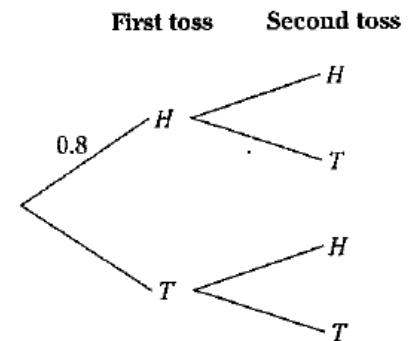
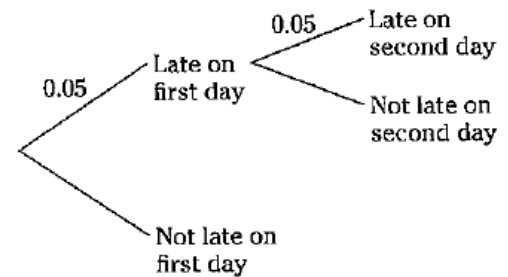**1** A coin is biased so that the probability the coin shows heads when it is tossed is 0.8. The coin is tossed twice.

**First toss      Second toss**

   (i) Copy and complete the tree diagram, showing the probabilities on the branches.

   (ii) Find the probability that the coin shows heads both times.

   (iii) Find the probability that the coin shows heads at least once.

0.8 — H — H, T

T — H, T

**2** The probability that I am late for work on any day is 0.05.

   (i) Copy and compete the tree diagram to show the outcomes on two days.

   (ii) Find the probability that, on two consecutive days, I am late for work

      (a)   on both days,

      (b)   on only one day,

      (c)   on the first day, given that I am late for work on only one day.

0.05 Late on first day — 0.05 Late on second day / Not late on second day

Not late on first day

**3** A mother and her daughter are both entering a cake competition at a show. From past experience they estimate that the probability that the mother will win a prize is $\frac{1}{6}$ and, independently of her mother's result, the probability that the daughter will win a prize is $\frac{2}{7}$.

   (i) Draw a fully labelled tree diagram.

   (ii) Find the probability that

      (a)   either the mother or the daughter, but not both, wins a prize,

      (b)   the mother wins a prize, given that just one of them wins a prize,

      (c)   at least one of them wins a prize.

**4** A manufacturer makes pens. When the process is going well, only 2.5% of the pens are defective. Carmela buys two pens. By drawing a tree diagram, or otherwise, find the probability that

   (i) both pens are defective,

   (ii) exactly one of the pens is defective.

**5** Two golfers, Chris and Toby, are attempting to qualify for a golf tournament. On past performance, the probability that Toby will qualify is 0.8, the probability that Chris will qualify is $p$ and the probability that **both** Toby and Chris will qualify is 0.6. The event 'Chris qualifies' is independent of the event 'Toby qualifies'.
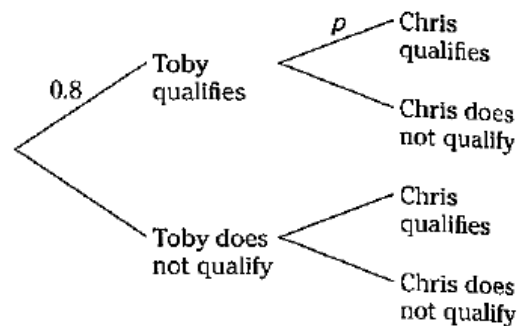
   (i) Find $p$.

  (ii) Find the probability that just one qualifies.

 (iii) Given that just one qualifies, find the probability that it is Chris.

Tree diagram:
- 0.8 — Toby qualifies
  - $p$ — Chris qualifies
  - Chris does not qualify
- Toby does not qualify
  - Chris qualifies
  - Chris does not qualify

**6** Each day I travel to work by route $A$ or route $B$. The probability that I choose route $A$ is $\frac{1}{4}$. The probability that I am late for work if I go via route $A$ is $\frac{2}{3}$ and the probability that I am late if I go via route $B$ is $\frac{1}{3}$.
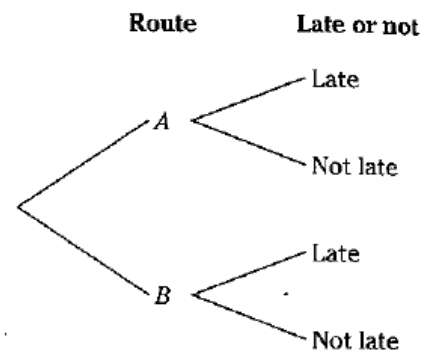
   (i) Find the probability that I am late for work.

  (ii) Given that I am late for work, find the probability that I went via route $B$.

Tree diagram (Route / Late or not):
- $A$
  - Late
  - Not late
- $B$
  - Late
  - Not late

**7** A box contains 20 chocolates, of which 15 have soft centres and 5 have hard centres. Sadie takes two chocolates from the box and eats them.
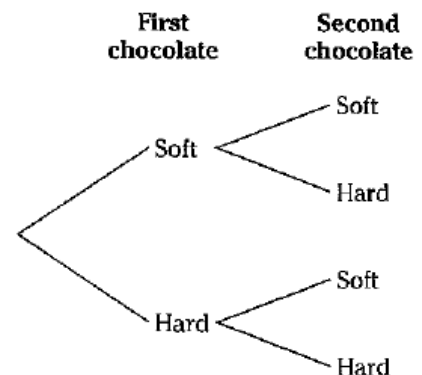
   (i) Find the probability that just one of the chocolates has a soft centre.

  (ii) Find the probability that the first chocolate has a soft centre, given that just one of the chocolates has a soft centre.

Tree diagram (First chocolate / Second chocolate):
- Soft
  - Soft
  - Hard
- Hard
  - Soft
  - Hard

**8** In my bookcase there are four shelves. The number of books on each shelf is shown in the table below.

| | Hardback | Paperback |
|---|---|---|
| Shelf 1 | 11 | 9 |
| Shelf 2 | 8 | 12 |
| Shelf 3 | 16 | 4 |
| Shelf 4 | 9 | 3 |

I choose a shelf at random and then choose a book from the shelf.

   (i) Find the probability that I choose a hardback book.

  (ii) If the book chosen is a hardback, what is the probability that it is from shelf 3?

9 In a large batch of flower seeds 70% have been treated to improve germination. The treated seeds have a probability of 0.8 of germinating, whereas the untreated seeds have a probability of 0.3 of germinating.

A seed is selected at random from the batch.

(i) Draw a fully labelled tree diagram.

(ii) Find the probability that the seed will germinate.

(iii) Find the probability that the seed has been treated, given that it has germinated.

10 Find the probability that a fair cubical die shows 2 on exactly one occasion when it is thrown

(i) twice,

(ii) three times.

11 A box contains 9 pens of which 6 are red and 3 are blue. Patrick is doing an experiment as part of his mathematics homework.

(a) He takes a pen from the box at random, notes its colour and then puts it back in the box. He does this a second time, then a third time.

Find the probability that he takes out

(i) a red pen each time,

(ii) at least one blue pen.

(b) Patrick now repeats the experiment, but this time he does not return the pen to the box each time.

Find the probability that he takes out

(i) a red pen each time,

(ii) at least one blue pen.

12 Some students are answering multiple choice questions. In each question, there are four choices.

(a) Amy does not know the answer to a particular question, so she guesses. What is the probability that Amy guesses the correct answer?

(b) Ben guesses the answers to two of the multiple choice questions. Find the probability that

(i) both answers are correct,

(ii) exactly one answer is correct.

(c) Cathy guesses the answers to three of the multiple choice questions. Find the probability that

(i) all three are incorrect,

(ii) exactly two answers are correct,

(iii) at least two answers are correct,

(iv) fewer than two answers are correct.

(d) Darren guesses the answers to four multiple choice questions. Find the probability that all his answers are correct.

13 Pippa is playing a game at a fete in which she randomly selects a coloured disc from a bag. The bag contains 10 discs of which 9 are blue and 1 is white. If she selects the white disc she will win a teddy bear. She is allowed three attempts and no disc is returned to the bag once it has been chosen.

   (i) Find the probability that Pippa wins the teddy bear.

   (ii) Given that she wins the teddy bear, find the probability that she only needed one attempt.

14 Becky is playing table tennis. Each time that she serves, the probability that she wins the point is 0.6, independently of the result of any preceding serves. At the start of a particular game she serves for the first five points.

   (i) Find the probability that, for the first two points of the game,

      (a) she wins both points,

      (b) she wins exactly one of the points.

   (ii) Calculate the probability that for the first five points of the game she loses all five points.

15 Jodie and Kate are playing a game in which they have two bags containing coloured cubes.
Bag A has 7 red cubes and 3 blue cubes.
Bag B has 4 red cubes and 6 blue cubes.

   (i) Jodie takes a cube at random from bag A and Kate takes a cube at random from bag B. Find the probability that

      (a) both cubes are red,

      (b) just one of the cubes is red.

The cubes are returned to their correct bags.

   (ii) Jodie now takes a cube at random from bag A and, without looking at the colour, puts it in bag B. Kate now takes a cube at random from bag B. Find the probability that it is red.