Pure Mathematics 1

Tutorial 2 Functions

1	Find the set of values of k for which $k+2x-3x^2$ is negative for all real values of x. [2] $\left[k < -\frac{1}{3}\right]$
2	The curves $y = x^2 - 3x + c$ and $y = k - x - x^2$, where c and k are constants, meet at the point (-2,12). a) Find the values of c and k. [2] b) Hence, find the other point of intersection of the curves. [4] [2, 14 ; (3,2)]
3	(a) If α is a root of the equation $x^2 + 4x + 1 = 0$, show that $\alpha^2 = -4\alpha - 1$ and $\alpha^3 = -4\alpha^2 - \alpha$. Deduce that $\alpha^3 = 15\alpha + 4$. [4] (b) Find the range of values of the constant p for which the function $y = px^2 + 4x + p + 3$ is positive for all real values of x. [4] $[p < -4 \text{ or } p > 1]$
4	(a) Show that if λ is positive but not greater than 3, the roots of the equation $(\lambda - 2)x^2 - (8 - 2\lambda)x - (8 - 3\lambda) = 0$ are real. [4] (b) Prove that for all real values of a, the equation $x^2 + 2ax + 2a^2 + a + 1 = 0$ has no real roots for x. [4]
5	Solve the inequality $2x - 1 \le x^2 - 4 \le 12$. [6] $[3 \le x \le 4]$
6	The function $f : x \mapsto 4 - x^2$ is defined for all real values of x. (i) State the range of f. [2] (ii) Determine the exact values of x for which $ff(x) = 0$. [5] $[f(x) \le 4;\ \pm\sqrt{6}, \pm\sqrt{2}]$
7	Given that $f(x) = ax + b$, ($a > 0$) and $f^2(x) = 4x - 9$, find : (i) an expression for $f^4(x)$, [2] (ii) the value of a and b, [4] (iii) the value of $f^{-1}(7)$. [2]

$\left[16x-45;2,-3;5\right]$

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Given the function $f: x \to x^2 - 4x + 3$, where $x \geq k$,

i) express f in the form $(x+a)^2+b$, [2]

ii) find the least value of k for which f is one-one , [1]

iii) find f^{-1} and sketch the graphs of f and f^{-1} on the same set of axes , making clear the geometrical relationship between the two graphs, [5]

iv) find the point of intersection between f and f^{-1}. [3]

$$\left[(x-2)^2-1; k=2; 2+\sqrt{x+1}; \left(\frac{5+\sqrt{13}}{2}, \frac{5+\sqrt{13}}{2}\right)\right]$$

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The functions f and g are defined by

$$f: x \to x^2 + 3, \quad x \in R,$$
$$g: x \to 2x + 1, \quad x \in R.$$

(a) (i) Express the function fg in the form $a\left(x+\frac{p}{q}\right)^2 + r$ where a, p, q and r are integers. [4]

(ii) Sketch the curve with the equation $y = fg(x)$. [3]

(iii) Determine the range of the function fg. [1]

Another function h is defined by $h: x \to \frac{x}{x-2}$; $x \neq 2$.

(b) Solve the equation $g(x) = h^{-1}(x) + 1$ where h^{-1} is the inverse function of h. [5]

$$\left[4(x+\frac{1}{2})^2+3;\ fg(x) \geq 3; 0,2\right]$$

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a) Show that $x^2 + 4x + 7 = (x+2)^2 + a$, where a is to be determined. [2]

b) Sketch the graph of $y = x^2 + 4x + 7$, indicating its y intercept and vertex. State the axis of symmetry. [3]

c) The function f is defined by $f(x) = x^2 + 4x + 7$ and has as its domain the set of all real numbers. Write down the range of f. [1]

d) Explain, with reference to your sketch, why f has no inverse with its given domain. Suggest a domain for f for which it has an inverse. [2]

$$\left[(x+2)^2+3, a=3; x=-2; f(x) \geq 3; \text{not one-to-one function; if } x \geq -2 \text{ or } x \leq -2\right]$$

11 The function f and g are defined by

$$f : x \to 1+\sqrt{x}, \quad x \geq 0$$
$$g : x \to x^2 \quad , \quad x \in R$$

(i) Find the domain of the inverse function of f. [1]
(ii) Find the expression for f^{-1}. [3]
(iii) Find an expression for fg(x) for the case $x \geq 0$. [2]
(iv) Obtain the value of fg(2). [1]
(v) Sketch the graph of y = fg(x) and its inverse function $y = (fg)^{-1}(x)$. [2]

$[x \geq 1;\ (x-1)^2; 1+x; 3]$

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(a) The function *f* and *g* are defined by:

$$f : x \mapsto 3x-1, \quad x \in \Re$$
$$g : x \mapsto x^2+1, \quad x \in \Re$$

(i) Find range of *g*. **[1]**
(ii) Determine the values of *x* for which $gf(x) = fg(x)$. **[3]**

(b) The function $h : x \mapsto x^2+3x, \quad x \in \Re, \quad x \geq q$, is one-to-one. Find the least value of *q* and the inverse function of *h*, h^{-1}. **[5]**

$$\left[g(x) \geq 1; x = 0,1; q = -\frac{3}{2}, -\frac{3}{2} + \sqrt{x+\frac{9}{4}} \right]$$

13 The functions f and g are defined by

f: $x \to x^2 - 1$, $x \in R$, g: $x \to x - 2$, $x \in R$.

(i) Write down the composite function fg in a similar form. [1]

(ii) Given that h= fg with domain $x \leq k$, write down the greatest possible value of k for which h is a one-one function. [1]

(iii) Find the inverse function h^{-1}. Sketch h and h^{-1} on the same coordinates axes. [5]

(iv) Find the value of x for which $h = h^{-1}$. [4]

$$\left[(x-2)^2 - 1, k = 2; 2 - \sqrt{x+1}; x = 0.7 \right]$$

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Functions f and g are defined by

$$f : x \to 4x^2 - 1,\ x \in R,\ x \geq 0$$
$$g : x \to \sqrt{x+6},\ x \in R,\ x \geq m.$$

a) State the range of f and sketch the graph of f. [2]
b) Explain why the inverse function f^{-1} exists. [1]
c) Find the smallest value of m for which the function g is valid. [2]
d) Find an expression in the terms of x for fg. [2]

$[f(x) \geq -1; f \text{ is one} - \text{to} - \text{one function};\ -6; 4x + 23]$

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(i) The curve with equation $y = 2 + k \sin x$ passes through the point with coordinates $\left(\frac{\pi}{2}, -2\right)$. Find

(a) the value of k, [2]
(b) the values of x in the interval $0 \leq x \leq 2\pi$ for which $y = 2 + 2\sqrt{2}$. [3]

(ii) Write down the maximum and minimum values of the expression $4 - 2\sin 3x$ as x varies.

[2]

$\left[-4;\ \frac{5}{4}\pi, \frac{7}{4}\pi; \max = 6, \min = 2\right]$

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A function is defined by $f(x) = \sqrt{(6+x)}$, $x \geq k$.

(i) State the value of k. [1]

(ii) Write an equation for the inverse function. [2]

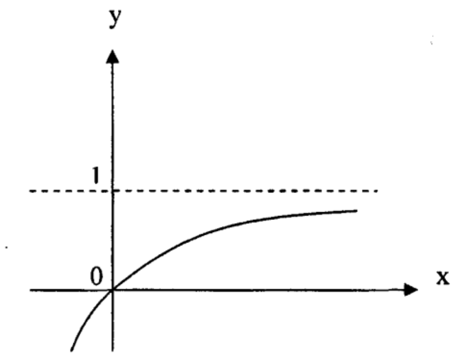
(iii) State the domain and range of f^{-1}. [2]

(iv) Find the value of $[f^{-1}(1)]^{-1}$. [2]

(v) Graph f and f^{-1} on the same axes and find x when $f = f^{-1}$. [4]

$\left[-6;\ x^2 - 6; x \geq 0;\ f^{-1}(x) \geq -6;\ -\frac{1}{5}; x = 3\right]$

17 (a)



The function f is defined for all real values of x, and the diagram shows the graph of $y = f(x)$. The gradient of the graph is always positive, and it decreases as x increases. The line y = 1 is an asymptote to the curve. Sketch the graph of the inverse function $f^{-1}(x)$ and state the domain of $f^{-1}(x)$ [3]

(b) The function $f(x) = x^3$ and $g(x) = 2 - 3x$ are defined for $x \in R$. Find

(i) $fg(x)$ [1]

(ii) $(fg)^{-1}(x)$ [2]

$\left[x < 1;\ (2-3x)^3; \frac{1}{3}(2 - x^{\frac{1}{3}})\right]$

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(i) Express $2x^2 - 8x + 5$ in the form of $a(x + b)^2 + c$, [3]

(ii) For the curve $y = 2x^2 - 8x + 5$, find the set of values of x for which $y \geq 15$. [3]

Given that $f : x \mapsto 2x^2 - 8x + 5$ for the domain $x \geq k$.

(iii) Find the least value of k for which f is one-one. [1]

(iv) State the domain and range of f^{-1}. [2]

(v) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between these two graphs. [3]

$[2(x-2)^2 - 3; x \geq 5, x \leq -1; 2\ ; x \geq -3, f^{-1}(x) \geq 0]$

19 The functions f and g are defined as follows:

$$f : x \rightarrow \sqrt{x+1}, \quad x \geq -1 \ ,$$

$$g : x \rightarrow (x+1)^2, \quad x \geq c \ , x \in R.$$

(i) State the range of f. [1]

(ii) Find the inverse function f^{-1}. [2]

(iii) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between these graphs. [4]

(iv) Find the least value of c for which g has an inverse. [1]

(v) Find the values of x for which $fg(x) = \sqrt{5}$. [3]

$[y \geq 0;\ x^2 - 1;\ -1;\ -3{,}1]$

20 a) Express $3x^2 + 6x - 9$ in the form of $a(x+b)^2 + c$. [3]

b) For the curve $y = 3x^2 + 6x - 9$, state the least value of y and the corresponding value of x. [2]

Given that $f(x) = 3x^2 + 6x - 9$ for the domain $x \leq k$,

c) find the greatest value of k for which $f(x)$ is one-one, [1]

d) express $f^{-1}(x)$ in terms of x in this case. [3]

$\left[3(x+1)^2 - 12\ ;\ -12, -1\ ;\ -1\ ;\ -\frac{\sqrt{x+12}}{3} - 1\right]$