

Topic 3 – Trigonometric Functions

Exercise 1

- 1 (i) $x = 90^\circ$
 (ii) $x = 60^\circ, 300^\circ$
 (iii) $x = 14.0^\circ, 194.0^\circ$
 (iv) $x = 109.5^\circ, 250.5^\circ$
 (v) $x = 135^\circ, 315^\circ$
 (vi) $x = 210^\circ, 330^\circ$
- 2 (i) -1
 (ii) $-\frac{2}{\sqrt{3}}$
 (iii) $-\frac{2}{\sqrt{3}}$
 (iv) $-\frac{2}{\sqrt{3}}$
 (v) 0
 (vi) $-\sqrt{2}$
- 3 (i) $B = 60^\circ, C = 30^\circ$
 (ii) $\sqrt{3}$
- 4 (i) $L = 45^\circ, N = 45^\circ$
 (ii) $\sqrt{2}, \sqrt{2}, 1$
- 5 (ii) 14.0°
- 6 (i) $0 \leq \alpha \leq 90^\circ$
 (ii) No, for each of the second, third and fourth quadrants a different function is positive.
 (iii) No, the graphs of the three functions do not intersect at a single point.
- 7 (i) $x = 0^\circ, 180^\circ, 360^\circ$
 (ii) $x = 45^\circ, 225^\circ$

Exercise 2

- 1 $\tan^4 A$
- 2 $\sec^2 \theta$
- 3 $\tan \theta$
- 4 $\sin^3 \theta$
- 5 $x^2 - y^2 = 16$
- 6 $x^2(b^2 - y^2) = a^2b^2$
- 7 $x^2(b^2 - y^2) = a^2b^2$
- 11 $38.2^\circ, 141.8^\circ$
- 12 $57.7^\circ, 122.3^\circ, 237.7^\circ, 302.3^\circ$
- 13 $30^\circ, 150^\circ$
- 14 $30^\circ, 150^\circ$
- 15 $45^\circ, 166.2^\circ, 225^\circ, 346^\circ$
- 16 $199^\circ, 341^\circ$

Exercise 3

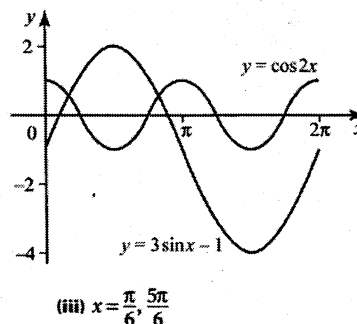
- 1 (i) $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$
 (ii) $-\frac{1}{\sqrt{2}}$
 (iii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
 (iv) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
- 2 (i) $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta)$
 (ii) $\frac{1}{2}(\sqrt{3} \cos \theta + \sin \theta)$
 (iii) $\frac{1}{2}(\sqrt{3} \cos \theta - \sin \theta)$
 (iv) $\frac{1}{\sqrt{2}}(\cos 2\theta - \sin 2\theta)$
- (v) $\frac{\tan \theta + 1}{1 - \tan \theta}$
 (vi) $\frac{\tan \theta - 1}{1 + \tan \theta}$
- 3 (i) $\sin \theta$
 (ii) $\cos 8\phi$
 (iii) 0
 (iv) $\cos 2\theta$
- 4 (i) $\theta = 15^\circ$
 (ii) $\theta = 157.5^\circ$
 (iii) $\theta = 0^\circ$ or 180°
 (iv) $\theta = 111.7^\circ$
 (v) $\theta = 165^\circ$
- 5 (i) $\theta = \frac{\pi}{8}$
 (ii) $\theta = 2.79$ radians
- 6 (i) $\frac{1}{\sqrt{5}}$
 (ii) $\sin \beta = \frac{3}{5}, \cos \beta = \frac{4}{5}$

Exercise 4

- 1 (i) $\theta = 14.5^\circ, 90^\circ, 165.5^\circ, 270^\circ$
 (ii) $\theta = 0^\circ, 35.3^\circ, 144.7^\circ, 180^\circ, 215.3^\circ, 324.7^\circ, 360^\circ$
 (iii) $\theta = 90^\circ, 210^\circ, 330^\circ$
 (iv) $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
 (v) $\theta = 0^\circ, 138.6^\circ, 221.4^\circ, 360^\circ$

Exercise 4

- 2 (i) $\theta = -\pi, 0, \pi$
 (ii) $\theta = -\pi, 0, \pi$
 (iii) $\theta = \frac{-2\pi}{3}, 0, \frac{2\pi}{3}$
 (iv) $\theta = \frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$
 (v) $\theta = \frac{-11\pi}{12}, \frac{-3\pi}{4}, \frac{-7\pi}{12}, \frac{-\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$
- 3 $3 \sin \theta - 4 \sin^3 \theta$
 $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$
- 4 $\theta = 51^\circ, 309^\circ$
- 5 $\cot \theta$
- 6 $\frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3 \tan^2 \theta}$
- 8 (ii) $\theta = 63.4^\circ$
- 9 (i)



Exercise 5

- 1 (i) $\sqrt{2} \cos(\theta - 45^\circ)$
 (ii) $29 \cos(\theta - 46.4^\circ)$
 (iii) $2 \cos(\theta - 60^\circ)$
 (iv) $3 \cos(\theta - 41.8^\circ)$
- 2 (i) $\sqrt{2} \cos(\theta + \frac{\pi}{4})$
 (ii) $2 \cos(\theta + \frac{\pi}{6})$
- 3 (i) $\sqrt{5} \sin(\theta + 63.4^\circ)$
 (ii) $3 \sin(\theta + 48.2^\circ)$
- 4 (i) $\sqrt{2} \sin(\theta - \frac{\pi}{4})$
 (ii) $3 \sin(\theta - 0.49 \text{ rad})$

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Exercise 5

5 (i) $2 \cos(\theta - (-60^\circ))$

(ii) $4 \cos(\theta - (-45^\circ))$

(iii) $2 \cos(\theta - 30^\circ)$

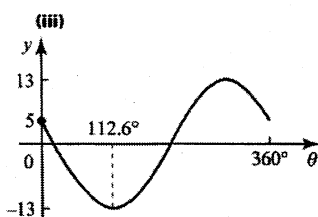
(iv) $13 \cos(\theta - 22.6^\circ)$

(v) $2 \cos(\theta - 150^\circ)$

(vi) $2 \cos(\theta - 135^\circ)$

6 (i) $13 \cos(\theta + 67.4^\circ)$

(ii) Max 13, min -13

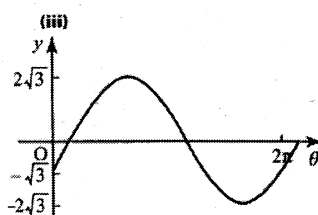


(iv) $\theta = 4.7^\circ, 220.5^\circ$

7 (i) $2\sqrt{3} \sin(\theta - \frac{\pi}{6})$

(ii) Max $2\sqrt{3}$, $\theta = \frac{2\pi}{3}$;

min $-2\sqrt{3}$, $\theta = \frac{5\pi}{3}$

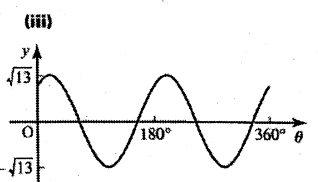


(iv) $\theta = \frac{\pi}{3}, \pi$

8 (i) $\sqrt{13} \sin(2\theta + 56.3^\circ)$

(ii) Max $\sqrt{13}$, $\theta = 16.8^\circ$;

min $-\sqrt{13}$, $\theta = 106.8^\circ$

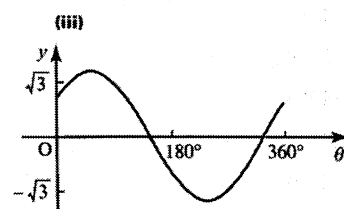


(iv) $\theta = 53.8^\circ, 159.9^\circ, 233.8^\circ, 339.9^\circ$

Exercise 5

9 (i) $\sqrt{3} \cos(\theta - 54.7^\circ)$

(ii) Max $\sqrt{3}$, $\theta = 54.7^\circ$;
min $-\sqrt{3}$, $\theta = 234.7^\circ$



(iv) Max $\frac{1}{3 - \sqrt{3}}$, $\theta = 234.7^\circ$;

min $\frac{1}{3 + \sqrt{3}}$, $\theta = 54.7^\circ$

10 (ii) $\theta = 30.6^\circ, 82.0^\circ$

11 (i) $\cos x \cos a - \sin x \sin a$

(ii) $r = \sqrt{29}$, $a = 68.2^\circ$

(iii) Max $\sqrt{29}$ when $x = 291.8^\circ$;

min $-\sqrt{29}$ when $x = 111.8^\circ$

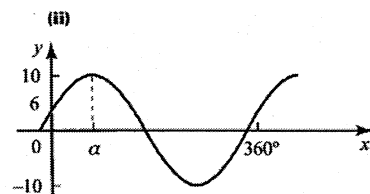
(iv) $x = 235.7^\circ, 347.9^\circ$

12 (i) 30.96°

(ii) $x = 15.7^\circ, 282.4^\circ$

(iii) $x = 7.9^\circ, 141.2^\circ, 187.9^\circ, 321.2^\circ$

13 (i) $R = 10$, $a = 53.13^\circ$



(iii) $x = 119.55^\circ, 346.71^\circ$

(iv) $\theta = 103.29^\circ, 330.45^\circ$

14 (i) $c = \sqrt{a^2 + b^2}$

(ii) $\tan a = \frac{b}{a}$

(iii) $a = 36.87^\circ$

(iv) $\theta = 103.29^\circ, 330.45^\circ$

Exercise 6

1 $y = 1 - 2x^2$

4 $\frac{56}{65}, -\frac{16}{65}$

5 $x = 2y - 1$

7 $5 \sin(\theta - \alpha)$ where $\tan \alpha = \frac{3}{4}$, 7, -3

8 $\sqrt{2} \sin(2\theta - 45^\circ)$, 67.5°

9 $-\pi, 0, \pi$

11 $\cot^2 x$

12 $90^\circ, 270^\circ$

13 (a) $2 - \cos 2\theta$ (b) $2 + 2 \cos 4A$

14 0

15 40.2°