The Trapezium Rule

The area, 'under' a curve y = f(x) can be found **exactly** by integration.

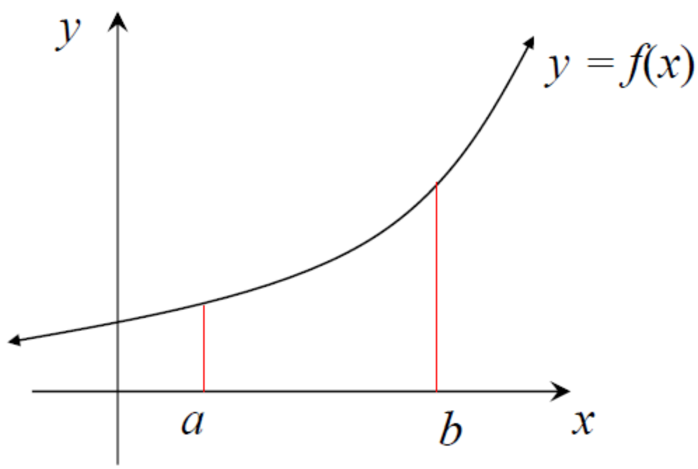
Sometimes y = f(x) cannot be integrated.

Examples: $y = e^{x^2}$, $y = \cos x^3$, $y = \dfrac{1}{1+x^2}$

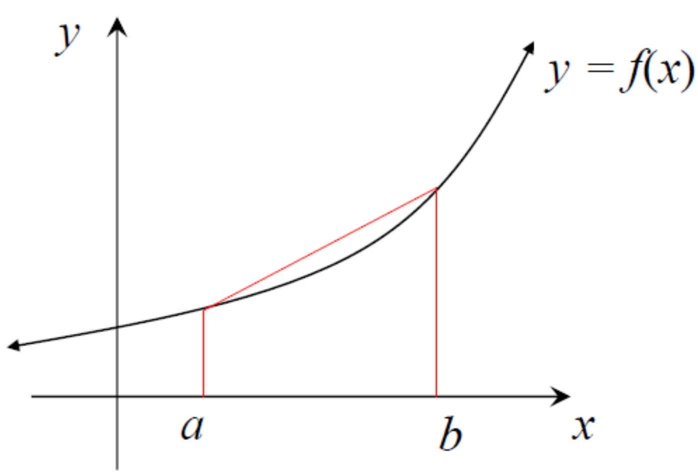
The area under these graphs can only be **approximated**, using numerical methods ~ Trapezium Rule.

Approximation by using Trapezium Rule

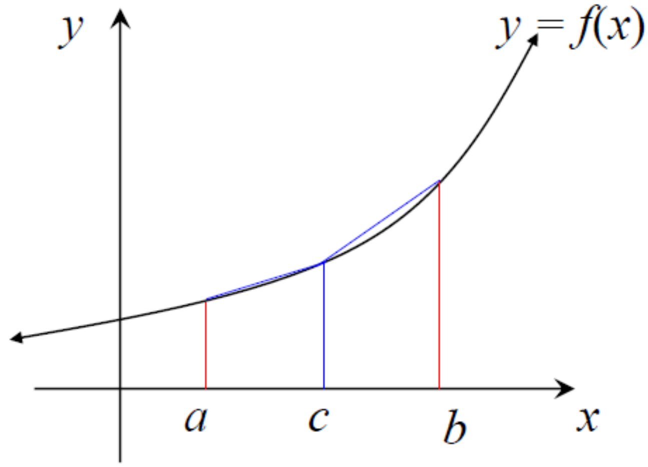
To find an approximation to the area, A, between y = f(x), the x-axis, x = a and x = b.



$$\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

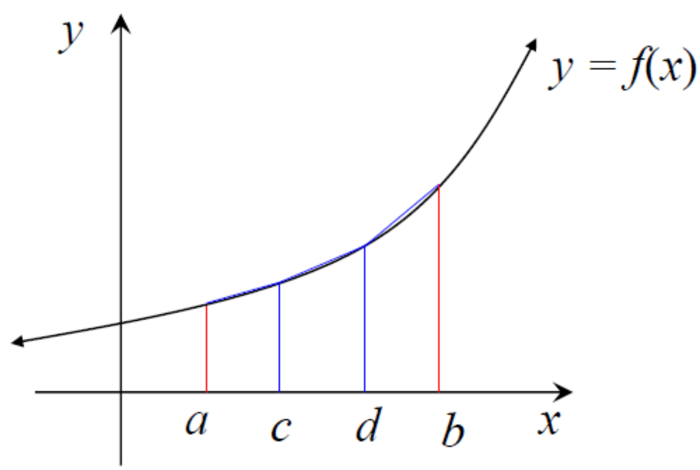


$$A \approx \frac{b-a}{2}\{f(a)+f(b)\}$$



$$A \approx \frac{c-a}{2}\{f(a)+f(c)\}+\frac{b-c}{2}\{f(c)+f(b)\}$$

$$=\frac{c-a}{2}\{f(a)+2f(c)+f(b)\}$$



Area =

In general;

$$\text{Area} = \int_a^b f(x)dx$$

$$\approx \frac{h}{2}\{y_0 + 2y_{others} + y_n\}$$

where $h = \dfrac{b-a}{n}$

$n =$ number of trapeziums

NOTE: there is always one more function value than interval

Area ≈ $\frac{h}{2}(y_0 + y_n + 2(y_1 + y_2 + .. + y_{n-1}))$

Example 1

Use the Trapezoidal Rule with 4 intervals to estimate the area under the curve $y = (4 - x^2)^{\frac{1}{2}}$, between $x = 0$ and $x = 2$ (correct to 3 decimal points)

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

	$\boxed{1}$	$\boxed{2}$	$\boxed{2}$	$\boxed{2}$	$\boxed{1}$
x	0	0.5	1	1.5	2
y	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{h}{2}\{y_0 + 2y_{others} + y_n\}$$

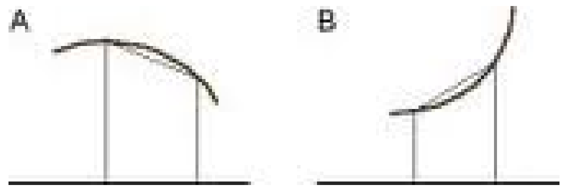
$$= \frac{0.5}{2}\{2 + 2(1.9365 + 1.7321 + 1.3229) + 0\}$$

$$= 2.996 \text{ units}^2 \qquad (\textit{exact value} = \pi)$$

Example 2

Use the trapezium rule with 6 ordinates, to estimate $\int_0^1 \sqrt{\tan x}\, dx$.
Give the answer correct to 3 decimal places.

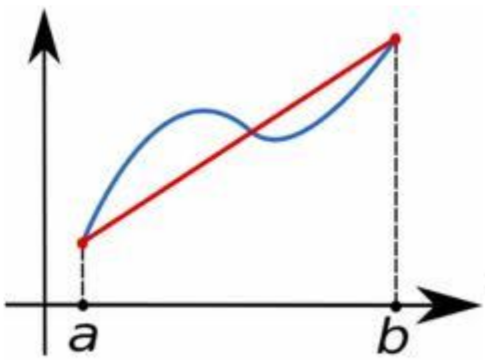
NOTE:



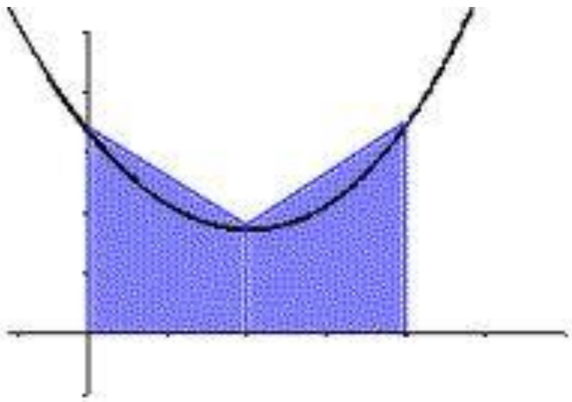
Some trapeziums give an Underestimate or Overestimate of the area under the curve.

Example

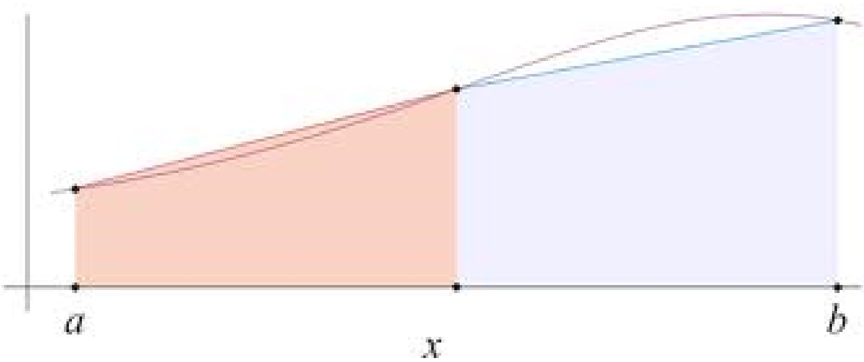
1.



2.



3.

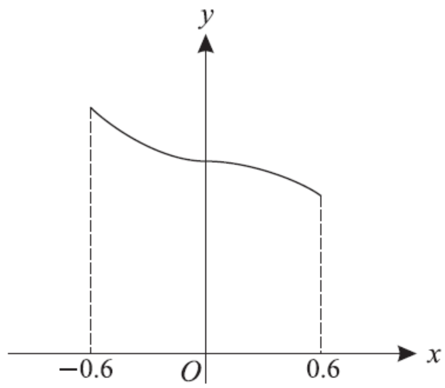


Exercise

1. Find an approximation for $\int_{-1}^{1} e^{x^2}\,dx$ using the trapezium rule with 6 ordinates.

(6 ordinates means 5 strips).

Question 2(June 2005)



The diagram shows a sketch of the curve $y = \dfrac{1}{1 + x^3}$ for values of x from -0.6 to 0.6.

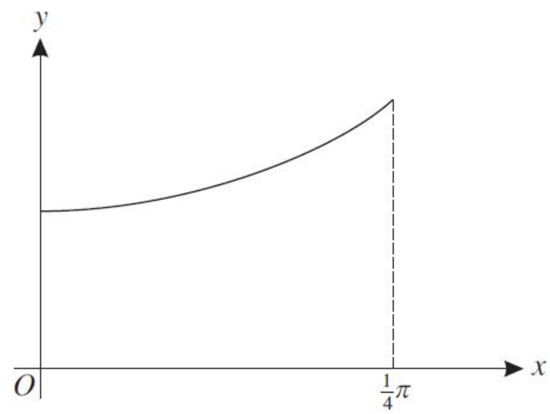
(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-0.6}^{0.6} \frac{1}{1 + x^3}\, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

Question 3(June 2009)



The diagram shows the curve $y = \sqrt{(1 + 2\tan^2 x)}$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$.

(i) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 + 2\tan^2 x)}\, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) The estimate found in part **(i)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with six intervals would be greater than E or less than E. [1]