Pure Mathematics 1

Tutorial 3 Coordinate geometry

1	A triangle is formed by the points A(-1,3), B(5,7) and C(0,8). Find the coordinates of the point where the line through B parallel to AC cuts the x-axis. **[4]** $\left[\left(\frac{18}{5},0\right)\right]$
2	The points A, B have the coordinates (2,2) and (- 4,3) respectively. Find the equation of the perpendicular bisector of the line AB [5 marks] $[2y = 12x + 17]$
3	Find the equation of the tangent and the equation of the normal to the curve $y = x^2 + \frac{1}{x}$ at the point P where $x = 2$. [5] $[4y = 15x - 12 ; 30y + 8x = 151]$
4	The points P, Q and R have coordinates $(3,-1)$, $(2,5)$ and $(6,3)$ respectively. Find the equation of the straight line joining P to the mid-point of QR. [4] $[y = 5x - 16]$
5	The line whose equation is $y = x + 5$ is the perpendicular bisector of the line joining the points P(3,10) and Q(a,b) (i) Write down in terms of a and b the coordinates of the mid-point of PQ. **[2]** (ii) Show that $a + b = 13$ **[3]** $\left[\left(\frac{3+a}{2},\frac{10+b}{2}\right)\right]$
6	The straight line $3x - 4y + 7 = 0$ meets the curve $8y = 3x^2 + 5$ at points A and B . Calculate : [4] (i) the gradient of AB . (ii) the mid-point M of AB. [1] (iii) Hence write down the equation of the normal to AB at the point M. [3] $\left[\frac{3}{4};(1,\frac{5}{2});6y = -8x + 23\right]$
7	**The point P (x, y) lies on the line $7y = x + 23$ and is 5 units from the point (2,0).** **Calculate the coordinates of the two possible positions of P.** [5] $[(-2,3); (5,4)]$

8	A triangle ABC has A at the point (7, 9), B at (3, 5), C at (5, 1). Find the equation of the line joining the midpoints of AB and AC. Find also the area of the triangle enclosed by this line and the axes. [5] $[y=-2x+17; 72.25]$
9	P and Q are the points of intersection of the line $\frac{x}{a}+\frac{y}{b}=1,\ (a>0,\ b>0),$ with the x-axis and y-axis respectively. The distance PQ is 8 and the gradient of PQ is -3. Find the value of a and b. [6] $\left[\sqrt{\frac{32}{5}}; 3\sqrt{\frac{32}{5}}\right]$
10	Three points have coordinates $A(1,7)$, $B(7,5)$ and $C(0,-2)$. Find (i) the mid-point of AB; [1] (ii) the equation of the perpendicular bisector of AB, and [3] (iii) the area of the triangle ABC. [2] $[(4,6); y=3x-6; 28]$
11	Three points A, B and C have coordinates (2,9) (4,3) and (2, −5) respectively. The line through C with gradient $\frac{1}{2}$ meets the line AB produced at D. Find i) the coordinates of D, [5] ii) the equation of line through D perpendicular to the line $5y-4x=17$. [2] $[(6,-3); 4y=-5x+18]$
12	The line l has equation $2x-y-1=0$. The line m passes through the point A(0,4) and is perpendicular to the line l. a) Find the equation of m and show that the lines l and m intersect at the point P(2,3). [6] b) Find the length of AP. [3] $[2y=-x+8; \sqrt{5}]$
13	A rectangle ABCD has points A(3,2) and B(1,6) a) Find the equation of BC [4] Given that the equation of AC is $y=x-1$, find b) the coordinates of C [3] c) the perimeter of the rectangle ABCD [3] $[2y=x+11; (13,12); 35.78]$

14 The straight line L_1 has equation $2y - x + 7 = 0$. The straight line L_2 passes through the point P(-1,6) and is perpendicular to L_1.

(i) Find the equation of L_2, giving your answer in the form of $ax + by + c = 0$. [3]

(ii) Find the coordinates of the point of intersection of L_1 and L_2. [3]

(iii) Show that the perpendicular distance from P to L_1 is $4\sqrt{5}$. [2]

$[y + 2x - 4 = 0 \,;\, (3, -2)]$

15 The straight line l has equation $2y - x + 7 = 0$. The straight line l' passes through the point P(-1, 6) and is perpendicular to l.

(a) Find the equation of l', giving your answer in the form $ax + by + c = 0$.[3]

(b) Find the coordinate of the point of intersection of l and l'. [2]

(c) Show that the perpendicular distance from P to l is $4\sqrt{5}$. [2]

(d) It is given that the points Q(-7, -7) and R(9, 1) lie on l. Find the exact area of the triangle PQR. [3]

$[y + 2x - 4 = 0; (3, -2); 80]$

16 . The vertex A of a triangle ABC has coordinates (3, 7) and the equation of the side BC is $4x - 3y = 11$. Show that the length of the perpendicular from A to BC is 4 units. [4]

(i) Given that the area of the triangle is 40 unit^2, find the length of BC. [1]

ii) If the coordinates of the mid-point of BC are (5, 3), find the coordinates of B and C. [4]

$[20 \,;\, (11, 11), (-1, -5)]$

17 PRQS is a rhombus where PQ and RS are perpendicular to each other. The coordinates of P, Q and R are (t, 2t) , (3t, 0) and (x,y) respectively.

(i) Find the equation of RS. [3]

Given that PQ = ½ RS,

(ii) show that $x^2 - 4tx = 0$, and [3]

(iii) find the coordinates of R and S in terms of t. [3]

$[y = x - t \,;\, (0, -t); (4t, 3t)]$

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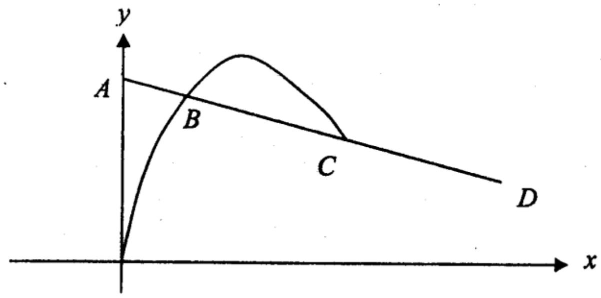

The diagram above shows part of the curve with equation $y = 16x - kx^2$, where k is a constant. The points A and D have coordinates (0 , 18) and (6 , 15) respectively.

(a) Calculate the length of AD, giving your answer correct to 3 significant figures. [2]

The line l passes through the points A and D and intersects the curve at points B and C. Given that C has coordinates (4 , 16),

(b) show that $k = 3$, [1]

(c) calculate the x-coordinate of B. [5]

$\left[6.71; \frac{3}{2}\right]$

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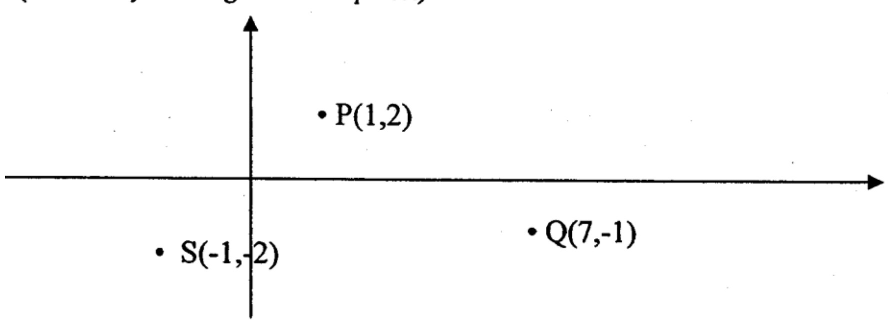
(*Solution by drawing is not acceptable)*



PQRS is a parallelogram in which the coordinates of P, Q and S are (1,2), (7,-1) and (-1,-2) respectively.

(i) Prove that the angle QPS is a right angle. [2]

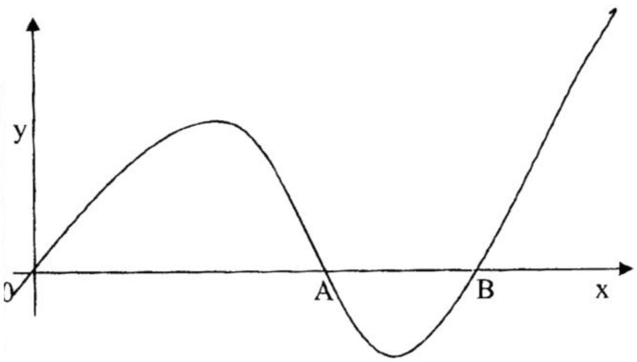
(ii) Find the equation of the straight line parallel to PS and passes through Q. [2]

(iii) Find the equation of the straight line parallel to PQ and passes through S. Hence find the coordinates of R. [4]

(iv) Find the distance of RS. Leave your answer in surd form. [2]

$\left[y = 2x - 15\ ; (5, -5); 3\sqrt{5}\right]$

20 . The figure below shows a sketch of part of the curve C with equation $2y = 3x^3 - 7x^2 + 4x$ which meets the x-axis at the origin O, point A(1,0) and point B.



(a) Find the coordinates of B. [4]

The normals to the curve C at the points O and A meet at the point N.

(b) Find the coordinates of N. [6]

(c) Calculate the area of triangle OAN. [2]

$\left[\left(\frac{4}{3}, 0\right); \left(\frac{4}{5}, -\frac{2}{5}\right); \frac{4}{5}\right]$