Question 1

Sketch on separate diagrams, the following curves for the domain $0° \leq \theta \leq 360°$ and state the corresponding range of y.

a) y = sin x – 2 b) y = cos (x + $\frac{\pi}{4}$) c) y = -3sin x d) y = 2cos x + 1

Question 2

Sketch, on separate diagrams, the following graphs.

(a) $y = 4\cos x - 3$ for $0° \leqslant x \leqslant 180°$

(b) $y = -6\tan x$ for $0° \leqslant x \leqslant 180°$

(c) $y = 3 - 2\sin x$ for $0° \leqslant x \leqslant 270°$

Question 3

On the same diagram, sketch the graphs of $y = 3\cos x$ and $y = 2\sin x - 1$ for the interval $0° \leqslant x \leqslant 360°$. State the number of solutions, in this interval, of the equation $3\cos x + 1 = 2\sin x$.

Question 4

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto x^2 - 2x,$$
$$g : x \mapsto x^2,$$
$$h : x \mapsto \sin x.$$

(i) (a) State whether or not f has an inverse, giving a reason.

(b) Determine the range of the function f.

(ii) (a) Show that gh(x) can be expressed as $\frac{1}{2}(1 - \cos 2x)$.

(b) Sketch the curve C defined by $y = gh(x)$ for $0 \leqslant x \leqslant 2\pi$.

Question 5

Prove the identity

$$\frac{3 - 6\cos^2 x}{\sin x - \cos x} \equiv 3(\sin x + \cos x)$$

Question 6

Prove the identity

$$\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} \equiv \frac{2}{\sin x}.$$

Question 7

The quadratic equation in x, $3x^2 - (4\cos\theta)x + 2\sin\theta = 0$, has equal real roots. Find the value of θ, where $0° < \theta < 360°$.

Question 8

Given that $x = \sin\theta - 2\cos\theta$ and $y = 2\sin\theta + \cos\theta$, find $\sin\theta$ and $\cos\theta$ in terms of x and y. Hence find a relation between x and y independent of θ.

Question 9

If $8\cos^2 x + 2\sin x - 5 = 0$, show that $\sin x = \frac{3}{4}$ and $\sin x = -\frac{1}{2}$. Hence find the possible values of $\cot x$.

Question 10

The diagram shows parts of the graphs of $y = 4\cos 3x$ and $y = 2\sin x + k$. Points P and Q are the respective maximum points on these graphs. Given that the two graphs intersect at the x-axis, find the coordinates of P and of Q. Sketch the two graphs on the same diagram for the interval $0° \leqslant x \leqslant 180°$.