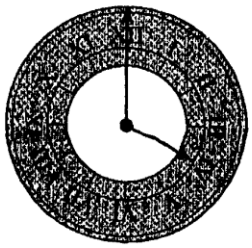
Pure Mathematics 1

Tutorial 4 Circular Measure

1 A chord of a circle subtends an angle of θ radians at the centre of the circle. If the area of the minor segment cut off by the chord is one sixth of the area of the circle, prove that $\sin\theta = \theta - \frac{\pi}{3}$. [3]

2 The face of a town hall clock carries a design of concentric circles. The inner circle has radius 0.5 m and the outer circle has radius 1 m, which is also the length of the minute hand. The region between the circles is shaded. Find the area of the shaded region swept out by the minute hand in ten minutes. (Leave π in the answer.) [3]

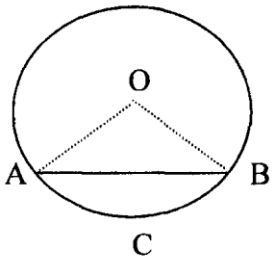


$\left[\frac{\pi}{8}\right]$

3 **The distance between the centers of two coplanar circles, each with a radius of 5cm, is 6cm. Calculate the common area shared by the two circles, giving your answer in cm^2 correct to two significant figures. [4]**

[22]

4 Diagram below is a circle with centre O radius 2cm.Given that chord AB = 2 cm, find the area of the segment ACB of the circle. [3 marks]



$\left[2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)\right]$

5

a) A chord PQ of a circle, radius 6 cm, subtends an angle of 2.5 radians at the centre of the circle. Taking π to be 3.14, calculate, correct to one decimal place,

i) the length of the **major** arc PQ, [2]
ii) the area enclosed by the chord PQ and the **minor** arc PQ. [3]

$[22.7;34.2]$

6

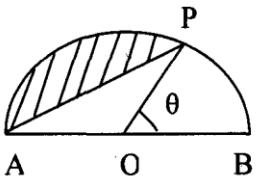
A and B are two points on the circumference of a circle, centre O and radius **r**, and **AOB** = θ radians.

(i) State the perimeter of the sector OAB. [1]

(ii) Given that the square of the perimeter is 16 times the area of the sector OAB, find the value of θ. [3]

$[2r+r\theta\,;2\,\text{rad}]$

7



The diagram shows a semicircle APB on AB as diameter. The midpoint of AB is O. The point P on the semicircle is such that the area of the sector POB is equal to twice the area of the shaded segment. Given that angle POB is θ radians, show that $3\theta = 2(\pi - \sin\theta)$. [5]

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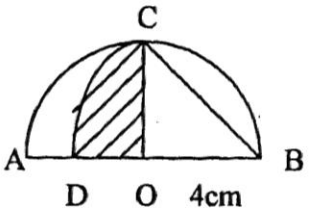


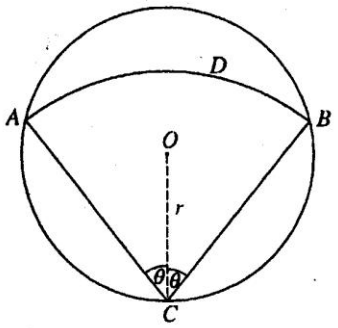
The diagram above shows a semicircle ABC with centre O and radius 4cm such that angle BOC = 90^0.Given that CD is an arc of a circle with centre B, calculate

i) the exact length of the arc CD, in terms of π, [2]
ii) the area of the shaded region, in terms of π. [3]

$\left[\sqrt{2}\pi; 4\pi - 8\right]$

9

(a)



The points A, B and C lie on a circle with centre O and radius r. The lines AC and BC each make an angle θ radians with OC. The circular arc ADB has centre C and radius CA.

(i) Express the area of the sector $CADB$ in terms of r and θ. [3]

(ii) Hence show that, if the area of sector $CADB$ is half of the area of the circle ABC, then $\theta \cos^2\theta = \frac{1}{8}\pi$ [2]

$\left[4r^2\theta\cos^2\theta\right]$

10

The area of the sector AOB of a circle, whose centre is O and has a radius of 5 cm, is 5π cm^2.

(a) Calculate the value of the acute angle AOB.

(b) The point A is joined to the point C on OB, 3 cm from O. Calculate the area of the region bounded by BC, CA and the arc AB. [3]

$\left[\frac{2\pi}{5} rad; 8.58\right]$

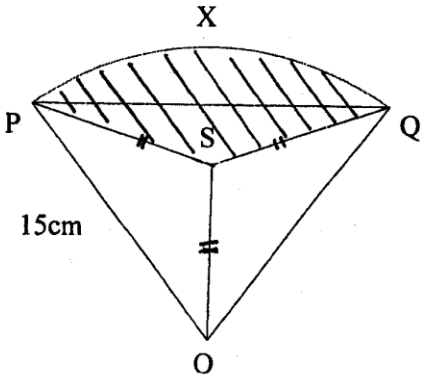
11

a) A sector OAB of a circle of radius r and centre O has $\prec AOB = \theta$ *radians*. Given that the area of the sector OAB is twice the square of the length of the arc AB, find θ. [3]

b) Find the area of the segment cut of by the chord AB if the radius of the circle is 10 cm. [3]

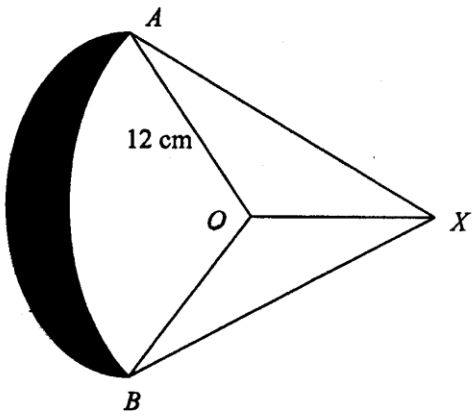
$\left[\frac{1}{4}; 0.1298\right]$

12 The diagram shows an equilateral triangle OPQ of side 15cm, and the point S such that OS=PS=QS. The arc PXQ has centre O and radius 15cm. Find the perimeter of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]



$\left[10\sqrt{3}+5\pi\right]$

13



The left edge of the shaded region, shown in the figure above consists of an arc of a circle of radius 12 cm with centre O. The angle $AOB = \frac{2}{3}\pi$ radians. The right edge of the shaded region is a circular arc with centre X, where $OX = 12$ cm.

(i) Write down the value of the angle AXB. [1]

(ii) Calculate the area of the shaded region. [6]

$\left[\frac{\pi}{3};49.3\right]$

14 A conical paper cup is constructed by removing the major sector from a circle of radius 10 cm and attaching radii. Find the angle subtended at the centre of the circle by the major arc length so that the cup has a depth of 8 cm.(Leave your answer in π.) [6]

$\left[\frac{6}{5}\pi\right]$

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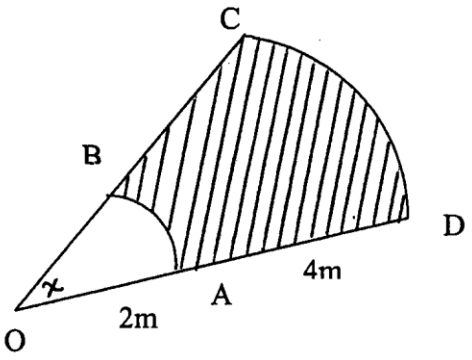


AB and DC are arcs of concentric circles centre O, where OA = 2 m and AD = 4 m. Given that angle AOB = x radians and the perimeter of the figure ABCD, shown in the diagram, is 12 m, calculate :

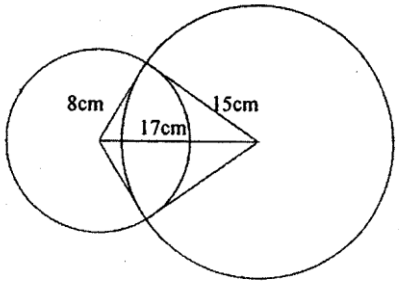
(i) the value of x. [4]

(ii) the shaded area ABCD. [4]

[0.5; 8]

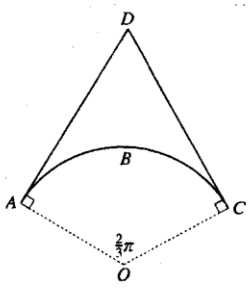
16

Two circles of radii 8cm and 15cm are drawn, partly overlapping. Their centers are 17cm apart. Find the area common to the two circles. [7]



[59.4]

17



In the diagram, ABC is an arc of a circle with centre O and radius 5cm. The lines AD and CD are tangents to the circle at A and C respectively. Angle AOC = $\frac{2}{3}\pi$ radians.

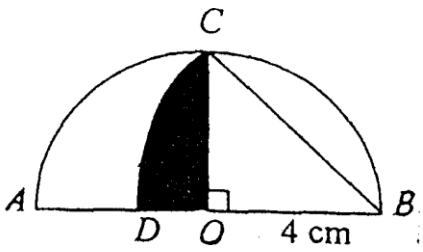
a) Show that the exact length of AD is $5\sqrt{3}$ cm. [2]

b) Find the area of the sector AOC, giving your answer in terms of π. [2]

c) Calculate the area of the region enclosed by AD, DC and the arc ABC, giving your answer correct to 2 significant figures. [3]

$\left[\frac{25}{3}\pi ; 17\right]$

18



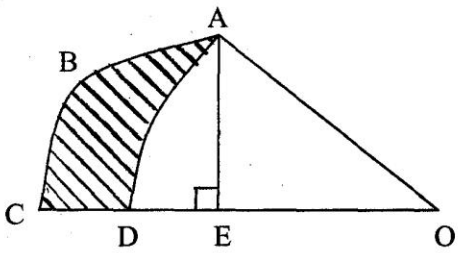
The diagram above shows a semi-circle ABC with centre O and radius 4cm such that angle BOC=90°. Given that CD is an arc of a circle with centre B, calculate

a) the length of the arc CD, in terms of π and $\sqrt{2}$, [4]

b) the area of the shaded region, in terms of π. [4]

$\left[\pi\sqrt{2} ; 4\pi - 8\right]$

19



AOD is a circular sector with centre O and ABCE is a quadrant with centre E as shown in the diagram above. Given that OD = 10cm, EC = 8 cm and the arc length of AD = 9 cm.

a) Show that $\angle AOE$ is 0.9 rad. [2]

b) Find the area of the shaded region. [7]

[29.27]