Pure Mathematics 1

Tutorial 7 Series

1

Find the term in the expansion of $(3+\sqrt{x})^{11}$ that contains x^4. [3]

[$4455x^4$]

2

Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x-\frac{3}{x}\right)^5$. **[4]**

[-1080]

3

i) Find the first four terms in the expansion of $(1+3x)^6$, in ascending powers of x. [3]

ii) Hence find an estimate for $(1.12)^6$ leaving your answer correct to 4 decimal places. [3]

[$1+18x+135x^2+540x^4+\ldots$;1.9706]

4

A man arranged to pay off a debt of £3600 by 40 annual installments which form an arithmetic series. However, when 30 of the installments had been paid, he died, leaving behind a third of his debt unpaid. Find the value of the last installment he paid just before he died. [5]

[109]

5

The sum of the first six terms of an arithmetic progression is 21, and the seventh term is three times the sum of the third and the fourth terms. Find the first term and the common difference. [5]

[-9; 5]

6

(a) John wishes to build up a triangular pile of bricks so that there is 1 brick on the top row, 2 on the next, 3 on the next, and so on. If he has 100 bricks,

(i) how many rows can he complete, [2]

(ii) how many bricks has he left? [1]

(b) Find the fraction that is equivalent to the recurring decimal 2.343434….. [2]

[13;9; 232/99]

7	a) Write and simplify the first 3 terms in the expansions of $(3+4x)^5$. [4] b) Hence or otherwise obtain the coefficient of x^2 in the expansion of $(2-x)(3+4x)^5$. [2] [$243+1620x+4320x^2$;7020]
8	If the first , fifth and tenth terms of an arithmetic progression are three consecutive terms in a geometric progression and the sum of the second and eighth terms is 20, find the first term and the (non-zero) common difference. [8] [8;1/2]
9	a) Given that the first and second terms of an arithmetic progression are 12 and 6 respectively, find the sum of the first hundred terms. [3] b) A geometric progression has positive terms. The sum of the first six terms is nine times the sum of the first three terms. The seventh term is 320. Find the common ratio and the first term. [4] [-28500; a = 5; r = 2]
10	A rope is cut into 10 pieces whose lengths are in geometrical progression measured 20metres, 18 metres, 16.2 metres in decreasing order. (i) find the length of the rope to the nearest metres [3 marks] (ii) Find the length of the last piece that is more than 10 metres long [4 marks] [130; 7]
11	The first three terms of a converging geometric progression are $x+4$, x and $x-3$ respectively. Calculate a) the value of x, [3] b) the common ratio, [2] c) the difference between the sum to infinity and the sum of the first 25 terms. [3] [12; 0.75; 0.0482]

12

a) Find the term independent of x in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$. [3]

b) The first term of a geometric progression exceeds the second term by 4 and the sum of the second and third terms is $2\frac{2}{3}$. Find the first three terms of this these geometric progression. [5]

[-8064; 6, 2, 2/3 or 4/3, -8/3, 16/3]

13

(a) Given that the first and second terms of an arithmetic progression are 12 and 6 respectively, find the sum of the first hundred terms. [3]

(b) I save $\$c$ every month, at 0.5% per month interest on the total amount saved. It is given that the total at the end of n months is

$$\$\,(1.005c + 1.005^2c + 1.005^3c + \ldots\ldots + 1.005^n c).$$

Find the number of months at the end of which the total first exceeds $\$100c$. [5]

[-28500; 81]

14

a) Expand $(1+px)^4$ and $(1+qx)^3$ in ascending powers of x up to and including the term in x^2. [4]

b) Given that the coefficient of x^2 in the expansion of $(1+px)^4(1+qx)^3$ is -6 and that $p+q=1$, find the values of p and q. [5]

[$1+3qx+3q^2x^2+\ldots; -1,3; 2,-2$]

15

i) In an arithmetic progression, the sum of the first and fifth term is 18 and the fifth term is 6 more than the third term. Show that the sum of the first ten terms of the progression is 165. [6]

ii) Find the sum to infinity of the geometric progression

$$1+\frac{x}{x+1}+\frac{x^2}{(x+1)^2}+\ldots$$ [3]

[165; $x+1$]

16	**(a)** Find the sum of the positive integers less than 200 which are multiples of 4 or 7. Hence, or otherwise, find the sum of the positive integers less than 200 which are not multiples of 4 or 7. **[5]** **(b)** In a geometric progression, the sum of the first five terms is 44 and the sum of the next five terms is $-\frac{11}{8}$. Find the common ratio and first term of this series. **[5]** [12942; -1/2 ; 64]
17	(a) A geometric progression has first term $\frac{1}{4}$ and common ratio $\frac{1}{\sqrt{2}}$. Find the sum to infinity of the progression, correct to 3 decimal places. [4] (b) The sum S_n of the first n terms of an arithmetic progression is given by $S_n = an + bn^2$. Given that $S_8 = 20$ and $S_{10} = 27$, find the values of a and b. Hence, or otherwise, find the nth term of this arithmetic progression. [6] [0.854; 1.7;0.1; 1.6+0.2*n*]
18	(i) A building society offers 6% interest per annum on investments. Someone deposits £ 4000 in an account and leaves the interest to accumulate. (a) Find the total value of the investment (to the nearest pound) after 6 years. [2] (b) Find how many years have elapsed before the investment has at least doubled its value. [3] (ii) The first term of an arithmetic progression is 3 and the sum of the first 6 terms is 4 times the sum of the first 3 terms. Find the common difference. [5] [5674; 12; 6]
19	The sum of the first 100 terms of an arithmetic progression is 10000; the first, second and fifth terms of this progression are 3 consecutive terms of a geometric progression, a) Find the first term, a and the non-zero common difference, d of the arithmetic progression. [8] b) Find the least value of *n* for which the sum to *n* terms of the geometric series $1 + 0.99 + (0.99)^2 + (0.99)^3 + \ldots$ is greater than half the sum to infinity. [5] [1 , 2 ; 69]