Trigonometry

P1/5/1: Trigonometrical ratios/graphs
P1/5/2: Inverse trigonometric functions
P1/5/3: Identities
P1/5/4: Trigonometrical equations
P1/5/5: Revision

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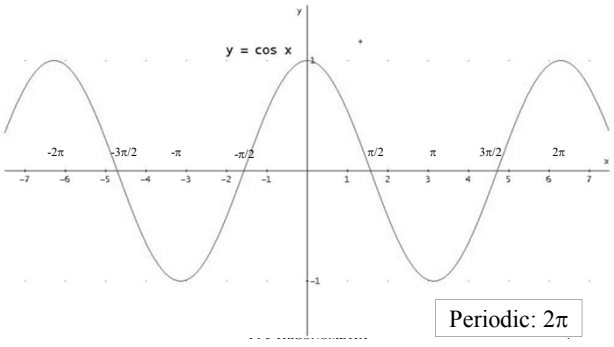
P1/5/1
Trigonometrical ratios / graphs

Learning Outcome

Students should be able to:

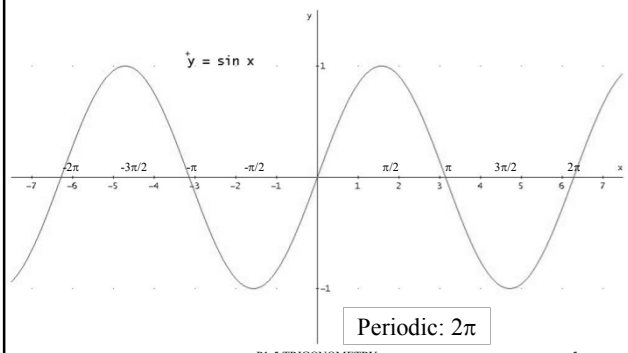
- sketch and use graphs of *t*-functions
- use the exact values of sine, cosine and tangent of 30^o, 45^o, 60^o and related angles.

Graphs of the trigonometry functions



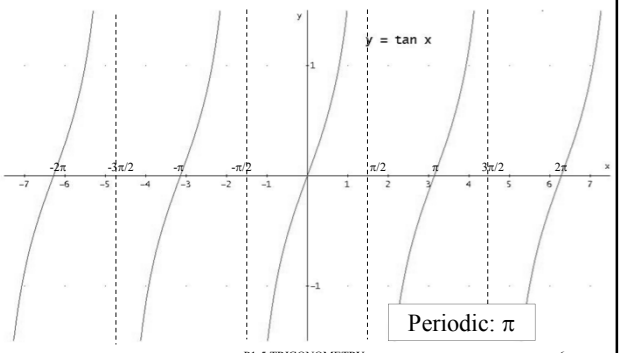
Periodic: 2π

Graphs of the trigonometry functions



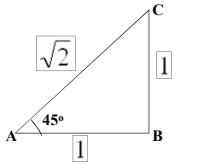
Periodic: 2π

Graphs of the trigonometry functions

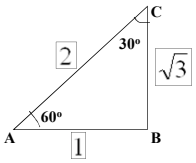


Periodic: π

Exact values of some trigonometric functions



$$\cos 45° = \frac{1}{\sqrt{2}};\ \sin 45° = \frac{1}{\sqrt{2}};\ \tan 45° = 1$$



$$\cos 60° = \frac{1}{2};\ \sin 60° = \frac{\sqrt{3}}{2};\ \tan 60° = \sqrt{3}$$

$$\cos 30° = \frac{\sqrt{3}}{2};\ \sin 30° = \frac{1}{2};\ \tan 30° = \frac{1}{\sqrt{3}}$$

Symmetry properties of the graphs of *cos θ°*, *sin θ°*, and *tan θ°*

Periodic properties :

$\cos(\theta \pm 360)^\circ = \cos\theta^\circ \quad \sin(\theta \pm 360)^\circ = \sin\theta^\circ \quad \tan(\theta \pm 180)^\circ = \tan\theta^\circ$

odd/even properties :

$\cos(-\theta)^\circ = \cos\theta^\circ \quad \sin(-\theta)^\circ = -\sin\theta^\circ \quad \tan(-\theta)^\circ = -\tan\theta^\circ$

Example 1:

Write down the exact values of the following:

$(a)\ \cos(150^\circ)$

$(b)\ \sin(-30)^\circ$

$(c)\ \tan(-60)^\circ$

$(a)\ \cos(225^\circ)$

Graphs of the trigonometry functions

$$y = k\cos(m\theta \pm n)$$

amplitude

If + ⇒Shift to the left (n/m) units

If − ⇒Shift to the right (n/m) units

Period:

cosine and sine ⇒2π/m

tangent ⇒π/m

Example 2:

Sketch the graphs:

$(a)\ \sin 2\theta$

$(b)\ \tan\left(\frac{\theta}{2}+\frac{\pi}{2}\right)$

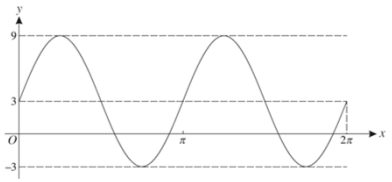
$(c)\ 4-2\cos\left(\theta-\frac{\pi}{2}\right)$

Example 3:

(i) Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

Example 4:



The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$.

(i) Find the values of a, b and c. [3]

(ii) Find the smallest value of x in the interval $0 \leqslant x \leqslant 2\pi$ for which $y = 0$. [3]

Example 5:

The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f.

(ii) Sketch the graph of $y = f(x)$.

(iii) State, with a reason, whether f has an inverse.

Practice Exercise

Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 10A (Page 141)
Q2(e)(f), 5(k)(o), 6(c)(d), 7(f)(g)

Exercise 10C (Page 149)
Q11

P1/5/2
Inverse trigonometric functions

Learning Outcome

Students should be able to:

- use the notation $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to denote the principal values of the inverse trigonometric functions.

Example 6:

Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$,

(ii) $\tan^2 x$.

Example 7:

Given that

$$\sin\theta^\circ = \frac{5\sqrt{3}}{14}$$

and the angle θ^o is obtuse, find without using a calculator the values of $\cos\theta^o$ and $\tan\theta^o$.

Example 8:

Find all values of θ in the required interval which satisfy the following equations, giving your answers correct to 1 decimal place where appropriate.

$(a)\ \cos\theta^\circ = 0.25$	$-180 \le \theta \le 180$
$(b)\ \sin\theta^\circ = -0.67$	$-\pi \le \theta \le \pi$
$(c)\ \cos 2\theta^\circ = \frac{1}{3}$	$0 < \theta \le 360$
$(d)\ \cos(3t+135)^\circ = \frac{1}{2}\sqrt{3}$	$0 < \theta \le 360$
$(e)\ 4\tan\theta^\circ + 3 = 0$	$-\pi \le \theta \le \pi$
$(f)\ \tan\left(\frac{3}{2}t - 45\right)^\circ = -\sqrt{3}$	$0 < t \le 360$

Example 9:

Find the maximum value and minimum value of y:

$$(a)\quad y = 5 + 8\cos 2x^{o}$$
$$(b)\quad y = \frac{8}{3 - \sin x^{o}}$$

and give the least positive values of x at which they occurs.

Practice Exercise

Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 10C (Page 148)
Q1(k)(n), 3(d)(f), 4(d)(e), 5(e)(f), 7(e)(f), 8(c)

P1/5/3
Identities

P1/5/4:
Trigonometrical equations

Learning Outcome

Students should be able to:

- use identities (basic identities)

Learning Outcome

Students should be able to:

- **Find all solutions of simple trigonometrical equations lying in a specified interval.**

Relations between the trigonometric functions

For all values of θ

$$\tan\theta^\circ \equiv \frac{\sin\theta^\circ}{\cos\theta^\circ}, \qquad \text{provided that } \cos\theta^\circ \neq 0,$$

$$\cos^2\theta^\circ + \sin^2\theta^\circ \equiv 1$$

Example 10:

Prove the identities:

$$(a) \quad \frac{1}{\cos\theta^\circ} + \tan\theta^\circ \equiv \frac{\cos\theta^\circ}{1-\sin\theta^\circ}$$

$$(b) \quad \frac{1-\tan^2 x}{1+\tan^2 x} \equiv 1 - 2\sin^2 x$$

$$(c) \quad \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2\tan^2 x.$$

Example 11:

Solve the following equation for θ, giving solution in the interval $0 \le \theta \le 360$.

$\sin \theta^o = \tan \theta^o$

Example 12:

Solve the following equation for θ, giving solution in the interval $0 \le \theta \le 360$.

$\sin 2\theta^o - \sqrt{3} \cos 2\theta^o = 0$

Example 13:

(i) Given that

$$3 \sin^2 x - 8 \cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$

(ii) Hence solve the equation

$$3 \sin^2(\theta + 70^\circ) - 8 \cos(\theta + 70^\circ) - 7 = 0$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$.

Example 14:

(i) Prove the identity $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta}\right)^2 \equiv \frac{1 - \cos \theta}{1 + \cos \theta}$.

(ii) Hence solve the equation $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta}\right)^2 = \frac{2}{5}$, for $0^\circ \leqslant \theta \leqslant 360^\circ$.

Example 15:

(i) Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0° \leqslant x \leqslant 180°$. [3]

(ii) Verify that $x = 30°$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0° \leqslant x \leqslant 180°$. [2]

(iii) Hence state the set of values of x, for $0° \leqslant x \leqslant 180°$, for which $\sin x < \cos 2x$. [2]

Practice Exercise

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Exercise 10D (Page 152)
Q2, 3(b)(d), 4(c)(d), 5

Practice Exercise

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Miscellaneous exercise 10 (Page 152)
Q5, Q7, Q8, Q10(c)(d), Q11(e)(f), Q12(c)