QUESTION 5i)

arc length XQY $= r\theta = 2\pi/3$ cm

$\angle XAY = \pi/3 \Rightarrow \angle XOY = 2(\pi/3) = 2\pi/3$

arc length XPY $= r\theta = \frac{2\sqrt{3}}{3}(2\pi/3) = \frac{4\sqrt{3}}{9}\pi$

ratio of XPY to XQY

$\frac{4\sqrt{3}}{9}\pi : \frac{2\pi}{3}$

$\frac{2\sqrt{3}}{3} : 1$

QUESTION 5ii)

Area of XQYP

= Area of segment of circle AXPY

− Area of segment of circle BXQY

$= \left\{\frac{1}{2}\left(\frac{2\sqrt{3}}{3}\right)^2\left[2\pi/3 - \sin 2\pi/3\right]\right\} - \left\{\frac{1}{2}(2)^2\left[\pi/3 - \sin \pi/3\right]\right\}$

$= \frac{4}{9}\pi - \frac{2}{3}\left(\frac{\sqrt{3}}{2}\right) - 2\pi/3 + 2\left(\sqrt{3}/2\right)$

$= -\frac{2}{9}\pi + 2/3\sqrt{3}$

$= \frac{2\sqrt{3}}{3} - \frac{2}{9}\pi$

Question 6i)

mid-point of AB $= \frac{1}{2}(OA + OB)$

$= \frac{1}{2}\left[\begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}\right]$

$M_{AB} = \begin{pmatrix} 11/2 \\ 5/2 \\ -7/2 \end{pmatrix}$

$|M_{AB}| = \sqrt{\frac{195}{4}}$

unit vector $= \frac{1}{\sqrt{\frac{195}{4}}} \begin{pmatrix} 11/2 \\ 5/2 \\ -7/2 \end{pmatrix}$

Question 6ii)

$\left.\begin{array}{l} \overrightarrow{AB} = \begin{pmatrix} -3 \\ -1 \\ -1 \end{pmatrix}, \ |\overrightarrow{AB}| = \sqrt{11} \\ \overrightarrow{AC} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}, \ |\overrightarrow{AC}| = 3 \end{array}\right\} \overrightarrow{AB} \cdot \overrightarrow{AC} = 7$

$\angle BAC = \cos^{-1} \frac{7}{\sqrt{11}(3)}$

$= 45.3^\circ$

Question 6iii)

Area of triangle ABC

$= \frac{1}{2}(AB)(AC)\sin(\angle BAC)$

$= \frac{1}{2}(\sqrt{11})(3)\sin 45.3^\circ$

$= 3.54$ units^2

7 (i)

$a = 5$

$T_{25} = 6.2$

$a + 24d = 6.2$

$5 + 24d = 6.2$

$\Rightarrow d = 0.05$

$S_n > 1000$

$\frac{n}{2}[2(5) + (n-1)(0.05)] > 1000$

$\frac{n}{2}(9.95 + 0.05n) > 1000$

$0.05n^2 + 9.95n - 2000 > 0$

$$n = \frac{-9.95 \pm \sqrt{(9.95)^2 - 4(0.05)(-2000)}}{2(0.05)}$$

$n = -322.88$ (rejected) $\qquad n = 123.88$

$\therefore$ least value of $n = 124$

7 (ii)	$S_\infty = \dfrac{a}{1-r}$ $8 = \dfrac{a}{1-r}$ $6 = \dfrac{a}{1-r^2}$ $6 = \left(\dfrac{a}{1-r}\right)\left(\dfrac{1}{1+r}\right)$ $6 = 8\left(\dfrac{1}{1+r}\right)$ $1+r = \dfrac{4}{3}$ $r = \dfrac{1}{3}$

8

(i) volume, $V = \pi r^2 h$

$A = 192\pi$

$\pi r^2 + 2\pi r h = 192\pi$

$h = \dfrac{192 - r^2}{2r}$

$V = \pi r^2 h$

$= \pi r^2 \left(\dfrac{192 - r^2}{2r}\right)$

$= 96\pi r - \frac{1}{2}\pi r^3$ (shown)

(ii) $\dfrac{dV}{dr} = 0$

$96\pi - \frac{3}{2}\pi r^2 = 0$

$r^2 = 64$

$r = \pm 8$

$\therefore r = 8$ cm.

(iii) $V = 96\pi(8) - \frac{1}{2}\pi(8)^3$

$= 512\pi$ cm^3

(iv) $\dfrac{d^2V}{dr^2} = -3\pi r \Big|_{r=8}$

$= -3\pi(8)$

$= -24\pi$ (< 0)

$\therefore$ V is maximum value.

9

(i) $f(x) = 4x^2 - 12x + 4$

$= 4(x^2 - 3x) + 4$

$= 4[(x - \frac{3}{2})^2 - \frac{9}{4}] + 4$

$= 4(x - 3/2)^2 - 5$

$= (2x-3)^2 - 5$

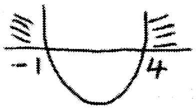
(ii) least value of $f(x) = -5$ when $x = 3/2$

(iii) $f(x) \geq 20$

$(2x-3)^2 - 5 \geq 20$

$4x^2 - 12x - 16 \geq 0$

$(x+1)(x-4) \geq 0$



$x \leq -1$ or $x \geq 4$

(iv) $f(x) = (2x-3)^2 - 5$, $x \leq k$

least value of $k = 3/2$

(v) $y = (2x-3)^2 - 5$

$\pm\sqrt{y+5} = 2x - 3$

$x = \frac{3 \pm \sqrt{y+5}}{2}$

$x \leq 3/2$

$\Rightarrow x = \frac{3 - \sqrt{y+5}}{2}$

$\therefore f^{-1}(x) = \frac{3 - \sqrt{x+5}}{2}$