AS Final Exam : Revision 8 Differentiation

P1 June 08

4 The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

P1 Nov 08

8 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. **[3]**

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. **[5]**

P1 June 07

10 The equation of a curve is $y = 2x + \frac{8}{x^2}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. **[3]**

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. **[3]**

(iv) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. **[3]**

P1 Nov 07

8 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. **[3]**

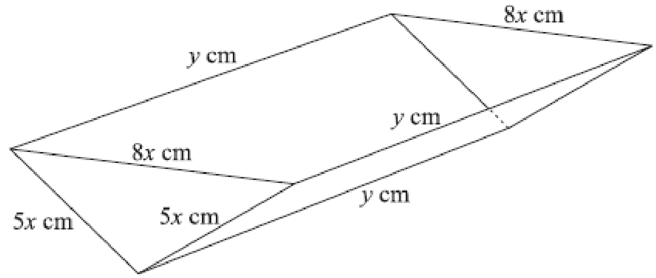
(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. **[5]**

P1 June 06

1 A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k. [3]

P1 Nov 06

9



The diagram shows an open container constructed out of $200\,\text{cm}^2$ of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

(ii) Show that the volume, $V\,\text{cm}^3$, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

P1 June 05

2 Find the gradient of the curve $y = \dfrac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

10 The equation of a curve is $y = x^2 - 3x + 4$.

(i) Show that the whole of the curve lies above the x-axis. [3]

(ii) Find the set of values of x for which $x^2 - 3x + 4$ is a decreasing function of x. [1]

The equation of a line is $y + 2x = k$, where k is a constant.

(iii) In the case where $k = 6$, find the coordinates of the points of intersection of the line and the curve. [3]

(iv) Find the value of k for which the line is a tangent to the curve. [3]

P1 Nov 05

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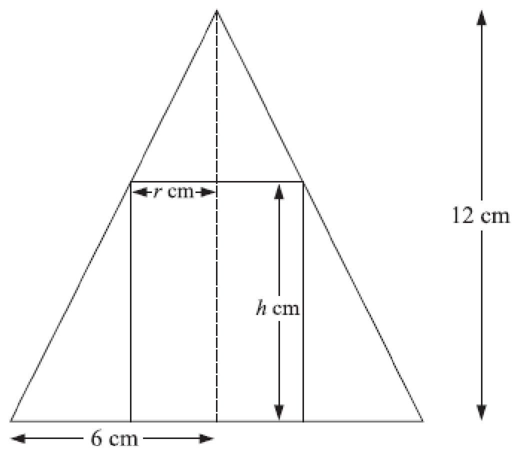


The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, $V\text{ cm}^3$, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]

8 A function f is defined by $\mathrm{f} : x \mapsto (2x-3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

(i) Find an expression, in terms of x, for $\mathrm{f}'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $\mathrm{f}^{-1}(x)$ and find the domain of f^{-1}. [4]

9 The equation of a curve is $xy = 12$ and the equation of a line l is $2x + y = k$, where k is a constant.

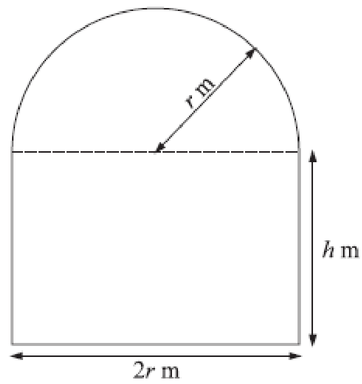
(i) In the case where $k = 11$, find the coordinates of the points of intersection of l and the curve. [3]

(ii) Find the set of values of k for which l does not intersect the curve. [4]

(iii) In the case where $k = 10$, one of the points of intersection is $P\ (2, 6)$. Find the angle, in degrees correct to 1 decimal place, between l and the tangent to the curve at P. [4]

P1 June 04

8



The diagram shows a glass window consisting of a rectangle of height h m and width $2r$ m and a semicircle of radius r m. The perimeter of the window is 8 m.

(i) Express h in terms of r. [2]

(ii) Show that the area of the window, A m^2, is given by

$$A = 8r - 2r^2 - \tfrac{1}{2}\pi r^2.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value is a maximum or a minimum. [2]

P1 Nov 04

5 The equation of a curve is $y = x^2 - 4x + 7$ and the equation of a line is $y + 3x = 9$. The curve and the line intersect at the points A and B.

(i) The mid-point of AB is M. Show that the coordinates of M are $\left(\frac{1}{2}, 7\frac{1}{2}\right)$. [4]

(ii) Find the coordinates of the point Q on the curve at which the tangent is parallel to the line $y + 3x = 9$. [3]

(iii) Find the distance MQ. [1]

P1 Nov 03

8 A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is 72 cm^3.

(i) Express y in terms of x and show that the total surface area, $A\text{ cm}^2$, of the block is given by

$$A = 4x^2 + \frac{216}{x}.$$ [3]

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

P1 June 02

8 A hollow circular cylinder, open at one end, is constructed of thin sheet metal. The total external surface area of the cylinder is $192\pi\text{ cm}^2$. The cylinder has a radius of r cm and a height of h cm.

(i) Express h in terms of r and show that the volume, $V\text{ cm}^3$, of the cylinder is given by

$$V = \tfrac{1}{2}\pi(192r - r^3).$$ [4]

Given that r can vary,

(ii) find the value of r for which V has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

P1 Nov 02

8 A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]