Series

P1/7/1: **Arithmetic Progression**

P1/7/2: Geometric Progression

P1/7/3: Sum to infinity of a geometric progression

P1/7/4: Binomial Series

P1/7/5: Revision

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P1/7/1:
Arithmetic Progression

Learning Outcome

Students should be able to:

- **Identify an arithmetic progression.**
- **Use the formula for the *n* th term and the sum of the first *n* term in an arithmetic progression.**

Arithmetic sequences

An **arithmetic sequence**, or **arithmetic progression**, is a sequence whose terms go up or down by constant steps.

An arithmetic series of n terms with first term a and common difference d has last term

$$l = a + (n-1)d = T_n$$

and sum

$$S = \frac{1}{2}n(a+l) = \frac{1}{2}n\big(2a + (n-1)d\big).$$

Example 1:

In the following arithmetic progression, the first three terms and the last term are given. Find the number of terms.

$1 - 2x \qquad 1 - x \qquad 1 \qquad \ldots \qquad 1 + 25x$

Example 2:

Find the sum of the given number of terms of the following arithmetic series.

$-3p - 6p - 9p - \ldots$ (100 terms)

Example 3:

The tenth term of an arithmetic progression is 125 and the sum of the first ten terms is 260.

(a) Show that the first term in the progression is -73.

(b) Find the common difference.

Example 4:

Natalie is given an interest-free loan to buy a car. She repays the loan in unequal monthly installments; these start at $30 in the first month and increase by $2 each month after that. She makes 24 payments.

(a) Find the amount of her final payment.

(b) Find the amount of her loan.

<u>Practice Exercise</u>

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<u>Exercise 8C (Page 124)</u>
Q5(h), Q6(h), Q7(f), Q8

P1/7/2:
Geometric Progression

Learning Outcome

Students should be able to:

- **Identify a geometric progression.**
- **Use the formula for the *n* th term and the sum of the first *n* terms in a geometric progression.**

Geometric sequences

A sequence you get from one term to the next by multiplying by a constant is called a **geometric progression**.

Geometric sequences

Formula for the n th term.

$$\boxed{u_n = ar^{n-1}}$$

<u>**Example 5:**</u>

In a geometric progression the fifth term is 100 and the seventh term is 400. Find the first term.

<u>Example 6:</u>

(i) The numbers x, $x + 4$, $2x + 2$ are three successive positive terms of a geometric progression. Calculate the value of x.

(ii) If x is the third term of the progression, calculate the value of the eight term.

Example 7:

An athlete plans a training schedule which involves running 20 km in the first week of training; in each subsequent week the distance is to be increased by 10% over the previous week. Write down an expression for the distance to be covered in the n th week according to this schedule, and find in which week the athlete would first cover more than 100 km.

Summing geometric series

The sum of the geometric series

$$a + ar + ar^2 + ... + ar^{n-1},$$

with *n* terms, is

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}.$$

Summing geometric series

When $r > 1$,

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}.$$

Example 8:

A geometric progression has positive terms. The sum of the first six terms is nine times the sum of the first three terms. The seventh term is 320. Find the common ratio and the first term.

Example 9:

Different numbers x, y, and z are the first three terms of a geometric progression with common ratio r, and also the first, second and fourth terms of an arithmetic progression.

(a) Find the value of r.

(b) Find which of the arithmetic progression will next be equal to a term of the geometric progression.

Practice Exercise

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Exercise 14A (Page 213)

Q3(e), Q6(g), Q7, Q11

P1/7/3:
Sum to infinity of a geometric progression

Learning Outcome

Students should be able to:

- **Apply the condition for the convergence of a geometric progression.**
- **use the formula for the sum to infinity of a convergent geometric progression.**

If $|r| < 1$, the sum of the geometric series with first term a and common ratio r tends to the limit $S_\infty = \dfrac{a}{1-r}$ as the number of terms tends to infinity.

The infinite geometric series is then said to be convergent. S_∞ is called the **sum to infinity** of the series.

Example 10:

Express the following recurring decimals as exact fraction.

$(a)\quad 0.471471471...$

$(b)\quad 0.857142857142857142...$

Example 11:

Find the first term of a geometric series which has a common ratio of $-\frac{3}{5}$ and a sum to infinity of 12.

Example 12:

Find the least value of n for which the sum to n terms of the geometric series $1 + 0.99 + (0.99)^2 + (0.99)^3 + \ldots$ is greater than half the sum to infinity.

Example 13:

A ‘supa-ball’ is thrown upwards from ground level. It hits the ground after 2 seconds and continues to bounce. The time it is in the air for a particular bounce is always 0.8 of the time for the previous bounce. How long does it take for the ball to stop bouncing?

Practice Exercise

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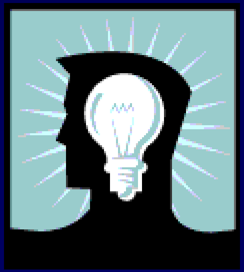
Exercise 14B (Page 217)

Q1(l), Q2(g), Q5, Q10



P1/7/4:

Binomial Series

Learning Outcome

Students should be able to:

- Expand $(a + b)^n$, where n is a positive integer.

The **Binomial theorem** states that, if n is a natural number,

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n.$$

The binomial coefficients

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-(r-1))}{1\times 2\times 3\times\ldots\times r} = \frac{n!}{r!(n-r)!}.$$

Example 14:

(i) Find in ascending powers of x, the first three terms in the expansion of $(1-2x)^5$ and $(1+3x)^9$.

(ii) Hence, find the expansion of $(1-2x)^5(1+3x)^9$ up to the terms in x^2.

Example 15:

Find the coefficient of x^6y^8 in the expansion of $(x+y)^{14}$

Example 16:

(i) Find the value of the coefficient of $\frac{1}{x}$ in the expansion of

$$\left(2x-\frac{3}{x}\right)^5$$

(ii) Find the term independent of x in the expansion of

$$\left(x^2+\frac{2}{x}\right)^6.$$

Example 17:

Find the first three terms in the expansion of $\left(1+2x\right)^8$ in ascending powers of *x*. Find an approximation to 1.02^8.

Practice Exercise

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Exercise 9A (Page 130)

Q1(f), Q4(b), Q10, Q12

Exercise 9B (Page 134)

Q4(c), Q8, Q10