Topic 2: Functions

Exercise 3A

Question 7

Find the largest possible domain of each of the following functions.

(a) $\sqrt{x}$ (b) $\sqrt{-x}$ (c) $\sqrt{x-4}$ (d) $\sqrt{4-x}$

(e) $\sqrt{x(x-4)}$ (f) $\sqrt{2x(x-4)}$ (g) $\sqrt{x^2-7x+12}$ (h) $\sqrt{x^3-8}$

(i) $\dfrac{1}{x-2}$ (j) $\dfrac{1}{\sqrt{x-2}}$ (k) $\dfrac{1}{1+\sqrt{x}}$ (l) $\dfrac{1}{(x-1)(x-2)}$

Question 8

The domains of these functions are the set of all positive real numbers. Find their ranges.

(a) $f(x)=2x+7$ (b) $f(x)=-5x$ (c) $f(x)=3x-1$

(d) $f(x)=x^2-1$ (e) $f(x)=(x+2)^2-1$ (f) $f(x)=(x-1)^2+2$

Question 9a), b) and e)

Find the range of each of the following functions. All the functions are defined for all real values of x.

(a) $f(x)=x^2+4$ (b) $f(x)=2(x^2+5)$

(d) $f(x)=-(1-x)^2+7$ (e) $f(x)=3(x+5)^2+2$

Exercise 11A

Question 10

Given that $f: x \mapsto 2x+1$ and $g: x \mapsto 3x-5$, where $x \in \mathbb{R}$, find the value of the following.

(a) gf(1) (b) gf(−2) (c) fg(0) (d) fg(7)

(e) ff(5) (f) ff(−5) (g) gg(4) (h) $gg\left(2\frac{2}{9}\right)$

Question 12

Given that $f: x \mapsto 5-x$ and $g: x \mapsto \dfrac{4}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$ or 5, find the values of the following.

(a) ff(7) (b) ff(−19) (c) gg(1) (d) $gg\left(\frac{1}{2}\right)$

(e) $gggg\left(\frac{1}{2}\right)$ (f) fffff(6) (g) fgfg(2) (h) fggf(2)

Question 15

Given that $f: x \mapsto x+4$, $g: x \mapsto 3x$ and $h: x \mapsto x^2$, where $x \in \mathbb{R}$, express each of the following in terms of f, g, h as appropriate.

(a) $x \mapsto x^2+4$ (b) $x \mapsto 3x+4$ (c) $x \mapsto x^4$

(d) $x \mapsto 9x^2$ (e) $x \mapsto 3x+12$ (f) $x \mapsto 3(x^2+8)$

(g) $x \mapsto 9x+16$ (h) $x \mapsto x^2+8x+16$ (i) $x \mapsto 9x^2+48x+64$

Question 19

For $f: x \mapsto ax+b$, $f(2)=19$ and $ff(0)=55$. Find the possible values of a and b.

Question 20

The functions $f: x \mapsto 4x+1$ and $g: x \mapsto ax+b$ are such that $fg = gf$ for all real values of x. Show that $a = 3b+1$.

Exercise 11B

Question 7

Each of the following functions has domain $x \geq k$. In each case, find the smallest possible value of k such that the function is one–one.

(a) $f: x \mapsto x^2-4$ (b) $f: x \mapsto (x+1)^2$ (c) $f: x \mapsto (3x-2)^2$

(d) $f: x \mapsto x^2-8x+15$ (e) $f: x \mapsto x^2+10x+1$ (f) $f: x \mapsto (x+4)(x-2)$

(g) $f: x \mapsto x^2-3x$ (h) $f: x \mapsto 6+2x-x^2$ (i) $f: x \mapsto (x-4)^4$

Question 12

Find the inverse of each of the following functions.

(a) $f: x \mapsto \dfrac{x}{x-2}$, $x \in \mathbb{R}$ and $x \neq 2$ (b) $f: x \mapsto \dfrac{2x+1}{x-4}$, $x \in \mathbb{R}$ and $x \neq 4$

(c) $f: x \mapsto \dfrac{x+2}{x-5}$, $x \in \mathbb{R}$ and $x \neq 5$ (d) $f: x \mapsto \dfrac{3x-11}{4x-3}$, $x \in \mathbb{R}$ and $x \neq \frac{3}{4}$

Question 13

The function $f: x \mapsto x^2-4x+3$ has domain $x \in \mathbb{R}$ and $x > 2$.

(a) Determine the range of f.

(b) Find the inverse function f^{-1} and state its domain and range.

(c) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

Question 14

The function $f: x \mapsto \sqrt{x-2}+3$ has domain $x \in \mathbb{R}$ and $x > 2$.

(a) Determine the range of f.

(b) Find the inverse function f^{-1} and state its domain and range.

(c) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$.