# Derivative of Rational Functions

# (Quotient Rule)

Let $u$ and $v$ be functions of $x$.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\text{Example: Find}$$

$$\text{(i)}\ \frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \qquad\qquad \text{(ii)}\ \frac{d}{dx}\left(\frac{\sqrt{x}-1}{x}\right) =$$

Example:

$$\text{Find } f(x) \text{ if } \int f(x)\,dx = \frac{x}{\ln x} + C,\ C \in \mathrm{R}.$$

Example:

Determine the number of stationary points of the curve $y = \dfrac{e^x}{x-1}$. Determine the nature of each.

Example:

Find the equation of tangent to the curve

$$y = \frac{e^x}{\sin x} \text{ when } x = \frac{\pi}{2}.$$

Example:

Show that the $x$-coordinates of the stationary points of the curve $y=\dfrac{\cos x}{\sqrt{x}}$ satisfy the equation $\cot x=-2x, x>0$.

# Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example:

Differentiate $y$ with respect to $x$.

$$\text{(a) } y = \frac{\sin x}{x} \qquad\qquad \text{(b) } y = \frac{e^x + 5x}{e^x - 2}$$

Example:

Find the turning point of the curve $y = \dfrac{e^{x}}{2x+1}$.

Example:

Find the equation of the tangent and normals to the curve $y = \dfrac{\sin x}{1 - 2\cos x}$ when $x = 0$.

Example:

If $f(x)=\dfrac{x^2-3x}{x+1}$, find $f'(x)$. Hence, find the values of $x$ for which $f(x)$ is decreasing.

Example:

Given that $y = xe^{-3x}$, find $\dfrac{dy}{dx}$. Hence, find the coordinates of the stationary point on the curve $y = xe^{-3x}$.