Paper 3
TOPIC 1: Algebra

THE MODULUS FUNCTION

Objective:

- understand the meaning of $|x|$, and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x-a| < b \Leftrightarrow a-b < x < a+b$ in the course of solving equations and inequalities;

Definition:

The notation $|x|$ means the magnitude of x, ignoring the sign. Eg: $|-2| = 2$ and $|5| = 5$.

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$ graph

Exercise 1:

Sketch the following graphs:

1. $y = |x+3|$
2. $y = |2x|$
3. $y = |x| - 2$
4. $y = 2 - |x|$
5. $y = 4|x|$
6. $y = |x^2 - 4|$

Solving Modulus Equations

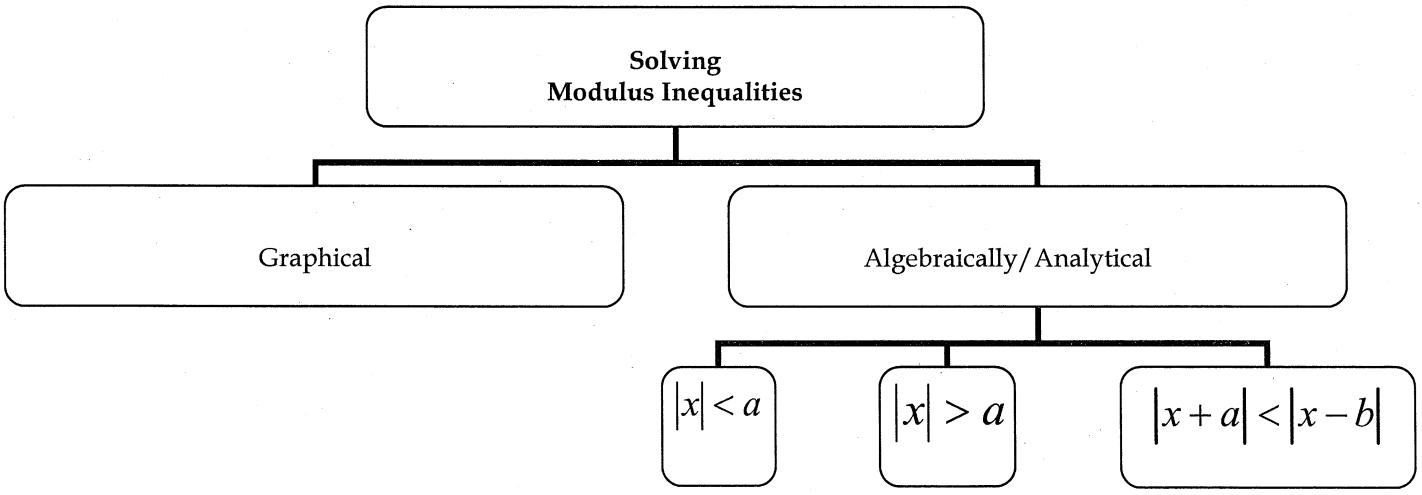
Solve the equations:

a) $|x+1| = 4$

b) $|3-x| = 0$

c) $|2x-3| = x+3$

d) $|x-2| = |2x-1|$



Examples

1. Solve the following inequalities using both methods, analytical and graphical.
   a) $|x+3|<1$ b) $|2x-3|\le x+3$ c) $|2x-4|<x$

2. Solve the following inequalities using analytical method.
   a) $|2x-1|\ge 5$ b) $|3x+2|\ge 2-x$ c) $|2x-3|\ge x+1$

3. Solve the following inequalities using analytical method.
   a) $|x+3|<|3x-1|$ b) $|x+3|\ge 2|x+1|$ c) $|1-x|-|2x+1|\le 0$

Exercise 2 – The Modulus Function

1. Using the same axes and scales, sketch the graphs of $y = |x+2|$ and $y = |x^2+2x-3|$, indicating clearly the points where they meet the two axes. Hence state the numbers of solutions to the equation $|x^2+2x-3|=|x+2|$. *[4 solutions]*
2. Find the ranges of values of x for which $3|x-1|\ge|x+2|$. *[$x \ge 5/2$ or $x \le 1/4$]*
3. Sketch the graph of $y=|x+2|$ and hence, or otherwise, solve the inequality
   $|x+2|>2x+6, \quad x\in R$.
4. Solve the inequality $|y-1|>6$. Hence solve the inequality $|2^x-1|>6$.

Exercise 3 – The Modulus Function

Question 1

Solve the inequality $|x+1|<|x-2|$.

[$x < ½$]

Question 2

Sketch, on a single diagram, the graphs of $x+2y=6$ and $y=|x+2|$. Hence, or otherwise, solve the inequality $|x+2|<\frac{1}{2}(6-x)$.

[-10 < x < 2/3]

Question 3

Solve the equation $|x|=|2x+1|$.

[-1/3 , -1]

Question 4

Solve the equation $4|x|=|x-1|$.

On the same diagram sketch the graphs of $y=4|x|$ and $y=|x-1|$, and hence, or otherwise, solve the inequality $4|x|>|x-1|$.

[-1/3 , 1/5; x < -1/3 or x > 1/5]

Question 5

Solve the inequality $-3<|1+x|<2$.

[-3 < x < 1]