TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST 2013

(July ~~2013~~ 2012 Intakes)

Thursday **22 August 2013** **12.30 pm – 2.15 pm**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

[Turn over

1 Solve the inequality $|x + 2a| \geq 2x + a$, where $a > 0$. $x \leq a$ [4]

2 The polynomial $6x^4 - 7x^3 + ax^2 + bx - 12$, where a and b are constants is denoted by $\mathrm{p}(x)$. It is given that $(x - 1)$ is a factor of $\mathrm{p}(x)$ and when $\mathrm{p}(x)$ is divided by $(x + 1)$, it gives a remainder of -50. Find the values of a and b. $a = -19$, $b = 32$ [5]

3 **(i)** Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constant A and B. 2, -2 [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,d\theta,$$

giving your answer in the form $a\pi - \ln b$. $\frac{\pi}{2} - \ln 2$ [5]

4 It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants now is 42. The number is treated as a continuous variable and denoted as N. The number of years from now is denoted by t.

(i) The expression giving N in term of t is

$$N = Ae^{mt}.$$

Determine the values of A and m. 42, $\frac{\ln 2}{9}$ [3]

(ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures. 11.3 [2]

(iii) Find the rate at which the number of plants will be increasing at a time 35 years from now. 479 [3]

5 A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

(i) Write down a differential equation which models this situation. $\frac{dA}{dt} = kA^2$ [1]

(ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. $\frac{7}{3}$ [7]

6 **(i)** Use the substitution $x = \sin^2\theta$ to show that

$$\int \sqrt{\frac{x}{1-x}}\,dx = \int 2\sin^2\theta\,d\theta.$$ [3]

(ii) Hence find the exact value of

$$\int_0^1 \sqrt{\frac{x}{1-x}}\,dx.$$ $\frac{\pi}{2}$ [5]

7 The gradient of the curve $y = (2x^2+9)^{\frac{5}{2}}$ at the point $\boldsymbol{P}$ is 100.

(i) Show that the x- coordinate of $\boldsymbol{P}$ satisfies the equation $x = 10(2x^2+9)^{-\frac{3}{2}}$. [3]

(ii) Show by calculation that the x- coordinate of $\boldsymbol{P}$ lies between 0.3 and 0.4. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the x- coordinate of $\boldsymbol{P}$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$x \approx 0.36$ (2 d.p.)

8 The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. $(1,-4)$ [8]

$(-1, 4)$

– **[Turn over**

9 The complex number $4 - 4i$ is denoted by u.

(i) Find the modulus and argument of u. $\sqrt{32}$, $\frac{-\pi}{4}$ [2]

(ii) Sketch an Argand diagram showing the region $\boldsymbol{R}$ consisting of points representing the complex number z where

$$|z - u| \leq 2.$$ [2]

(iii) For the complex numbers represented by points in the region $\boldsymbol{R}$, it is given that

$$p \leq |z + i| \leq q \quad \text{and} \quad \alpha \leq \arg z \leq \beta,$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [5]

$p = 3$, $q = 7$ $\alpha = -0.424$, $\beta = -1.15$

10 The line l has equation $\boldsymbol{r} = \begin{pmatrix} a \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$, where a is a constant. The plane p has the equation $2x - 3y + 6z = 10$.

(i) Given that l does not lie in p, show that l is parallel to p. [2]

(ii) Find the value of a for which l lies in p. -4 [2]

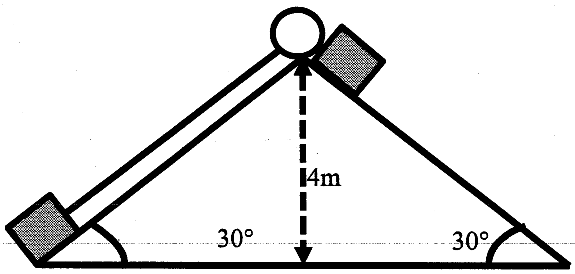
(iii) It is now given that the distance between l and p is 2. Find the possible values of a. [5]

$3, -11$

July 2012 Intake
(1.5 years)
Paper 4

1 A ball is dropped from a building of height 40m and a stone is thrown vertically upwards from the ground 2s later than the ball. The stone is thrown with initial speed of $15ms^{-1}$. Find the distance below the top of the building where the ball and stone collide. 33.06m [5]

2



Particle P and Q, of mass m kg and n kg respectively, are connected by a light string of length 8m passing over a smooth pulley at the top of a double 30° slope, as shown in the above diagram. The coefficient of friction for both particles is 0.1. The system starts from rest with the string taut and P at the top of the slope. The velocity of the particles when Q reaches the top of the slope is $5.12\ ms^{-1}$. Using the work-energy principle express m in terms of n. $m = 3.01n$ [6]

3 A car of mass 1200kg travels up a hill, inclined at 10° to the horizontal, against a constant resistance force of 250N. Its maximum speed is $50kmh^{-1}$. Find

(i) the power output of the engine, 32413 N [3]

(ii) the initial acceleration when the car reaches level road at top of the hill, assuming that the power stays constant. $1.74\ ms^{-2}$ [2]

The car reaches a point A at a velocity $15ms^{-1}$. The engine is switched off at point A and the car continues along a straight line with constant acceleration until it comes to rest at point B.

(iii) Calculate the distance AB. 540m [2]

4 A particle moves in a straight line so that at time t s its acceleration is $(7 - 4t)$ ms $^{-2}$. When t = 0 the particle is at a point O and has velocity 4ms $^{-1}$.

(i) Find an expression for the velocity v at time t. $V = -7t - 2t^2 + 4$ [2]

(ii) Show that the maximum velocity of the particle is $10\frac{1}{8}$ ms $^{-1}$. [2]

(iii) Find the greatest positive displacement of the particle from O during the motion. 29.3m [3]

5 A lift travels up and down between the ground floor and the 5th floor of a building. It starts from rest at the ground floor and moving up, takes 3s to increase its speed uniformly to 5ms^{-1}, maintains this speed for 5s and then slows down uniformly to rest in another 5s reaching the 3rd floor. The lift travels down at uniform acceleration of 2ms^{-2} for 5s and decelerates for 5s to reach the first floor. A woman of mass 60kg enters the lift at the 1st floor and travels to the 5th floor with an acceleration of 5ms^{-2} for 5s.

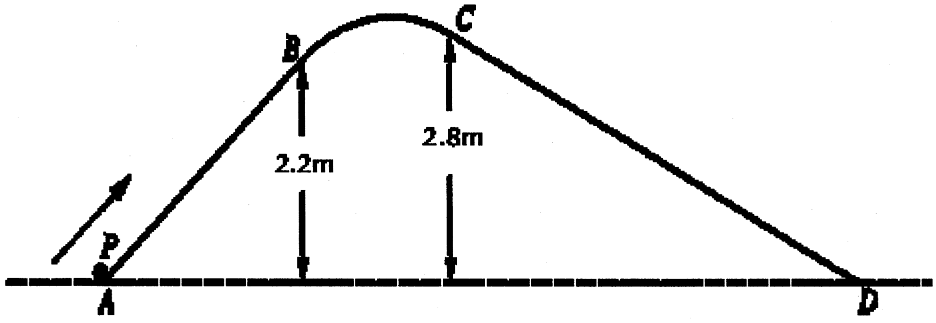
(i) Draw the velocity-time graph describing the motion of the lift for first 23s. [4]

(ii) Find the normal reaction of the floor of the lift on the woman in the final 5s. 900N [2]

The lift has mass 300kg, all the resistance to motion may be neglected, the mass of each occupant is 60kg and the tension in the supporting cable should not exceed 15000N.

(iii) What is the greatest number of occupants that can be carried safely if the magnitude of the acceleration does not exceed 3ms^{-2}. 14 [2]

6



The diagram shows the vertical cross-section ABCD of a surface. BC is a circular arc, and AB and CD are tangents to BC at B and C respectively. A and D are at the same horizontal level, and B and C are at heights 2.2m and 2.8m respectively above the level of A and D. A particle P of mass 0.1kg is given a velocity of $10ms^{-1}$ at A, in the direction of AB. The parts of the surface containing AB and BC are smooth.

(i) Find the decrease in the speed of particle P as P moves along the surface from B to C. $0.850ms^{-1}$ [4]

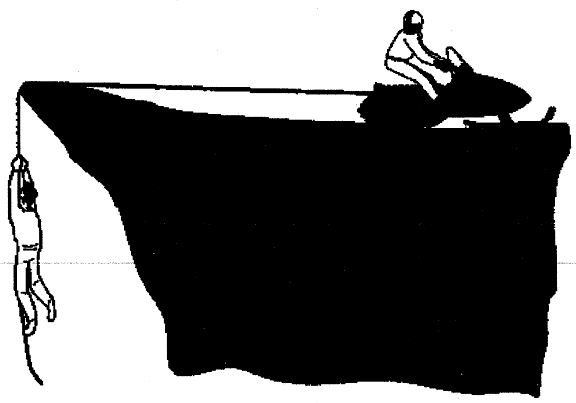
The part of the surface containing CD exerts a constant frictional force on P, as it moves from C to D and P comes to rest as it reaches D. Find the

(ii) work done against the frictional force when the particle move from C to D. 5 J [2]

(iii) coefficient of friction between particle P and plane CD if CD makes an angle 25° to the horizontal line AD. [2]

0.833

7 Edward is using a snowmobile to pull Kelvin out of a crevasse. His rope passes over a smooth block of ice at the top of the crevasse as shown in the figure below and Kelvin hangs freely away from the side. At rest, Kelvin's height from the ground is 10km. Edward's mass is 80kg, his snowmobile has a mass of 300kg and Kelvin's mass is 75kg. Ignoring any resistance to motion,



(i) calculate the driving force required for the snowmobile to give Kelvin an upward acceleration of 0.4ms^{-2} and tension in the rope for this acceleration, 780N, 932N [4]

(ii) how long will it take for Kelvin to reach a distance of 1km starting from rest? 70.7s [2]

After travelling 1km the rope snaps and Kelvin fall freely.

(iii) Find the time taken to hit the ground. 49.8 s [3]