Topic 1 – Quadratics (Week 2)

1. Find the range of values of p for which the equation
   (a) $x^2 + 3 = 2x + p$ has real roots,
   (b) $2x^2 + 2x\sqrt{3} + p = p(x^2 + 2)$ has distinct real roots.

2. Find the range of values of x for which
   (a) $(1 + x)(6 - x) \leqslant -8$, (b) $2x(x + 2) < (x + 1)(x + 3)$.

3. Calculate the smallest positive integer k for which the equation $2x^2 + 2kx + 7 = 0$ has two distinct real roots.

4. The line $x + y = a$ and the curve $x^2 + y^2 = b^2$ meet each other. If $b > 0$, show that $-\sqrt{2}b \leqslant a \leqslant \sqrt{2}b$.

5. Given that $3px^2 - 7qx + 3p = 0$ has equal roots and p and q are positive, find the ratio $p : q$ and solve the equation.

6. Sketch the curve $y = (x - 3)(x + 1)$. Hence find the value of p if $(x - 3)(x + 1) = p$ has equal real roots and state the value of these roots.

7. If x is real and $(x + 1)^2 = k(x + 2)$, show that k cannot lie between -4 and 0.

8. Given $y = k + 5x - 2x^2$, find the range of values of k for which y is always negative. When $k = -4$, find the coordinates of the turning point and sketch the curve.

9. Show that the line $y = 5x - 4$ is a tangent to the curve $y = x^2 + x$. Find also the condition for $y = mx - c$ to be a tangent to the curve.

10. A quadratic curve is symmetrical about the line $x = 3$ and passes through the points $(2, 13)$ and $(-1, -2)$. Find its equation and sketch this curve.

11. The equation $\frac{1}{2} + \frac{1}{x + k} = \frac{1}{x}$ has no real roots, find the range of values of k.

12. Show that $x - 3 = k(2x - x^2 + 3)$ has real roots for all non-zero values of k.

***13.** If the x-axis is tangential to the curve $y = a^2x^2 + 6abx + ac + 8b^2$, where a, b and c are constants, show that it is also tangential to the curve $y = ac(x + 1)^2 - 4b^2x$.

14. Find the range of values of h for which $3x^2 + hx + 1 = x - 2hx^2$ has two distinct real roots.

15. Solve for x if $x - 3 < x(x - 3) \leqslant 4$.

16. Find the minimum value of $2x^2 + 3x + 4$ and the corresponding value of x. Sketch the curve $y = 2x^2 + 3x + 4$. Hence, or otherwise, find the range of values of k for which $2x^2 + 3x + 4 \geqslant k$ for all real values of x.

17. Given that the curve whose equation is $y = p - (x - q)^2$ crosses the x-axis at the points $(-1, 0)$ and $(5, 0)$, find the maximum value of y.

18. The line $(k - 2)y = 3x$ meets the curve $xy = 1 - x$ at two distinct points. Find the range of values of k. State also the values of k if the line is a tangent to the curve.

***19.** Find the range of values of k if $kx^2 + 8x > 6 - k$ for all real values of x.

20. Express x in terms of y when $x^2 + 2xy + 5y^2 = 1$. Deduce that x is real only when y lies between certain values (inclusive) and find these values.

21. Sketch the curve $y = 2x^2 - 4x + 1$, indicating the coordinates of the turning point and the exact values of the x-intercepts. Hence, find
(a) the range of values of x for which $2x^2 + 1 \leqslant 4x$,
(b) the range of values of p if $2x^2 - 4x + 1 + p = 0$ has no real roots.

Answers

1. (a) $p \geqslant 2$ (b) $-1 < p < 3, p \neq 2$ **2.** (a) $x \leqslant -2$ or $x \geqslant 7$ (b) $-\sqrt{3} < x < \sqrt{3}$
3. 4
5. 7 : 6; 1
6. $p = -4, x = 1$
8. $k < -\frac{25}{8}$; $\left(\frac{5}{4}, -\frac{7}{8}\right)$
9. $(m - 1)^2 = 4c$
10. $y = 5 + 6x - x^2$
11. $-8 < k < 0$
14. $h < -1$ or $h > 11$, $h \neq -\frac{3}{2}$
15. $-1 \leqslant x < 1$ or $3 < x \leqslant 4$
16. $2\frac{7}{8}$, $x = -\frac{3}{4}$; $k \leqslant 2\frac{7}{8}$
17. 9
18. $k < -10$ or $k > 2$; $k = -10, 2$
19. $k > 8$
20. $x = -y \pm\sqrt{1 - 4y^2}$; $-\frac{1}{2} \leqslant y \leqslant \frac{1}{2}$; $-\frac{1}{2}, \frac{1}{2}$
21. (a) $1 - \frac{1}{\sqrt{2}} \leqslant x \leqslant 1 + \frac{1}{\sqrt{2}}$ (b) $p > 1$