Exercise 2.1 – Arrangement in a Line

Section A – Practice with factorials

1 Evaluate the following without using a calculator, then check on your calculator using the $x!$ button.

(i) $7!$ (ii) $\frac{10!}{7!}$

(iii) $\frac{6!}{3!2!}$ (iv) $\frac{8!}{4!3!2!}$

2 Evaluate using the factorial button on your calculator.

(i) $\frac{8!}{5!} \times \frac{4!}{2!}$ (ii) $\frac{10!}{(5!)^2}$

3 Re-write using factorial notation.

(i) $5 \times 4 \times 3 \times 2 \times 1$ (ii) $5 \times 4 \times 3$

(iii) $21 \times 20 \times 19 \times 18$ (iv) $n(n-1)(n-2)$

(v) $\frac{12 \times 11 \times 10 \times 9}{5 \times 4 \times 3}$

Section B – Arrangements in a line

1 Find the number of different arrangements of

(i) 6 different-coloured toy engines on a track,

(ii) 7 people to be seated in a row on a bench,

(iii) 9 paintings to be hung in a row in a gallery.

2 Find the number of ways to arrange the letters of each of the following words.

(i) SPIDER (ii) APRIL

(iii) SANDWICH

3 How many 4-digit numbers can be made with the digits 1, 2, 3, 4

(i) if repetitions are not allowed,

(ii) if repetitions are allowed?

4 The digits 4, 5, 7 and 9 are written on cards, as shown.

[4] [5] [7] [9]

Mari places all the cards in a row to make a 4-digit number.

(i) How many even numbers can be made?

(ii) How many numbers can be made in which 5 and 7 are next to each other?

5 Find the number of ways to arrange the letters of each of the following words:

(i) PENGUIN

(ii) BLACKBERRY

(iii) MATHEMATICAL

6 Find the number of ways in which all eight letters of the word ADVANCED can be arranged if the arrangement must begin and end with an A.

7 Find the number of ways in which all eight letters of the word NEEDLESS can be arranged:

(i) if there are no restrictions,

(ii) if the arrangement must end with N,

(iii) if the three letters E must be placed next to each other,

(iv) if the two letters S must not be placed next to each other,

(v) if the three letters E must be placed together and the two letters S must not be placed together.

8 [1] [3] [3] [8] [8] [8]

The above cards are placed in a line to form a 6-digit number. How many numbers can be made:

(i) if there are no restrictions,

(ii) if the number ends in 3,

(iii) if the number is odd?

9 Eloise is on a diet. Each day she can choose just one item from yoghurt, coffee or cereal for her breakfast. In how many ways can Eloise arrange her breakfast over 5 days?

10 A supermarket uses a five-digit number as a security code on its staff entrance. Digits may be repeated. How many different codes are possible when

(i) any of the ten digits from 0 to 9 may be used,

(ii) the zero cannot be used?

11 Five boys and four girls sit on a bench. In how many ways can they be seated if no two boys sit next to each other?

12 Matilda has eight different textbooks on a bookshelf. Two of them are mathematics books. In how many ways can the books be arranged if the two mathematics books must be placed:

(i) together,

(ii) with one at each end?

13 Three girls and seven boys stand in a line. Calculate the number of different arrangements if:

(i) the two youngest pupils are separated,

(ii) all three girls stand together.

14 There are 10 seats in a row in a theatre. In how many ways can 5 couples be seated in a row if each couple sits together?

15 Find how many different arrangements there are of the eleven letters of the word PROBABILITY if the two letters B are at the beginning and the two letters I are at the end.

16 Four boys and six girls are to be seated on a bench for a photograph. Calculate the number of different arrangements if:

(i) there is to be a boy at each end,

(ii) there is to be a girl at each end.

17 Four identical tins of peaches and six identical tins of pears are arranged in a row on a shelf. Calculate the number of different arrangements if the tin at each end contains the same type of fruit.

18 In how many ways can 5 boys and 3 girls stand in a straight line:

(i) if there are no restrictions,

(ii) if the boys stand next to each other?

Cambridge Paper 6 Q6(b) N03

19 Three identical yellow balloons, two identical red balloons and two identical blue balloons are strung in a row to celebrate Shema's birthday. Calculate the number of arrangements if:

(i) the balloon at each end is the same colour,

(ii) the yellow balloons are next to each other and the blue balloons are not next to each other.

20 Find how many arrangements there are of the nine letters in the words GOLD MEDAL

(i) if there are no restrictions in the order of the letters,

(ii) if the two letters D come first and the two letters L come last.

Cambridge Paper 6 Q7(b) J05

Exercise 2.2 – Permutation of r Items from n Items

1 Evaluate the following without using a calculator. Then check using the ${}_nP_r$ button on the calculator.

(i) ${}_9P_6$ (ii) ${}_6P_2$

(iii) ${}_{10}P_3$ (iv) ${}_9P_9$

2 From a class of 20 pupils, 3 pupils are going to be chosen to be sports officials. The first pupil chosen will be the swimming captain, the second pupil the athletics captain and the third pupil the tennis captain. In how many different ways can the officials be chosen?

3 There are 12 contestants in a singing competition. In how many ways can the first, second, third and fourth prizes be awarded?

4 In a particular minibus there are 16 seats for passengers. How many possible seating arrangements are there for 5 passengers?

5 How many even numbers between 6000 and 7000 can be formed using the digits 1, 2, 3, 6, 7, 9, if no digit is repeated?

6 A security code consists of 3 digits chosen from 4, 5, 6, 7, 8 followed by 2 letters chosen from P, Q, R, S, T, for example, 674TQ. How many different codes are possible

(i) if repetitions are not allowed,

(ii) if repetitions are allowed?

7 Rory is playing a game in which he has to place coloured pegs into holes in a board. He has 6 identical red pegs and the board has 10 holes. How many different arrangements are there for placing the 6 pegs and leaving 4 empty holes?

8 If repetitions are not allowed, how many numbers can be formed with the digits 3, 4, 5, 6, 7

(i) using three of the digits,

(ii) using one or more of the digits?

9 There are 10 seats in the front row at a theatre. Six people are shown to this row. In how many different ways can they be seated if

(i) there are no restrictions,

(ii) two particular people in the group must sit next to each other?

Exercise 2.3 - Permutation of r Items from n Items

1 Evaluate each of the following without using a calculator, then check using the ${}_nC_r$ button on your calculator:

(i) $\binom{11}{4}$ (ii) $\binom{7}{3}$ (iii) $\binom{8}{5}$ (iv) $\binom{10}{8}$

(v) $\binom{9}{6}$ (vi) $\binom{9}{3}$ (vii) $\binom{12}{8}$ (viii) $\binom{12}{4}$

2 How many different teams of 5 people can be chosen, without regard to order, from a squad of 12 people?

3 A committee of four is to be chosen from a group of nine people which includes Mr Green, Mrs Green and Mr Brown. How many different selections are possible if:

(i) there are no restrictions,

(ii) Mr Brown must be on the committee,

(iii) Mr Green and Mrs Green may not both be on the committee,

(iv) Mr Green and Mrs Green must be on the committee but Mr Brown must not be on the committee?

4 Sadie has 6 blouses and 8 skirts. She is going on holiday and decides to pack 4 blouses and 4 skirts. How many different selections are possible?

5 A Spelling-Bee team of 5 students is to be chosen from a class of 12 boys and 9 girls. In how many ways can a team be chosen if the team consists of:

(i) 3 boys and 2 girls,

(ii) 3 girls and 2 boys,

(iii) at least 3 girls?

6 How many different selections of five letters from the eleven letters of the word PROBABILITY contain both letters B and no vowels?

7 In how many ways can a group of 14 people eating at a restaurant be divided between 3 tables seating 6, 5 and 3?

8 A group of 12 guests at a wedding are to travel as passengers from the church to the reception. There are 3 cars: black, silver and blue. Each car holds 4 passengers. Find the number of ways in which the group may travel if Alice and Jack refuse to travel in the same car.

9 A football team consists of 3 players who play in a defence position, 3 players who play in a midfield position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and one forward player,

(ii) if exactly one forward player must be included, together with any two others.

Cambridge Paper 6 Q7(a) part J05

10 Three letters are selected at random from the letters of the word PARABOLA. Find the total number of selections.

11 In a mixed pack of coloured light bulbs there are three red bulbs, one yellow bulb, one blue bulb and one green bulb. Four bulbs are selected at random from the pack. How many different selections are possible?

12 There are 20 teachers at a conference. Of these, 8 are maths teachers, 6 are history teachers, 4 are physics teachers and 2 are geography teachers.

Four of the teachers are to be chosen at random to take part in a quiz. In how many different ways can the teachers be chosen:

(i) if there is to be a teacher from each subject,

(ii) if they all must teach the same subject,

(iii) if there are to be at least two maths teachers?

13 A diagonal of a polygon is defined to be a line joining any two non-adjacent vertices.

(i) Show that the number of diagonals in a 5-sided polygon is $\binom{5}{2} - 5$.

(ii) How many diagonals are there in a 6-sided polygon?

(iii) Show that the number of diagonals in an n-sided polygon is $\frac{n(n-3)}{2}$.

Exercise 2.4 – Permutation and Combination with Probability

1 Each of the 9 letters in the word FACETIOUS is written on a card and the cards are placed in random order in a line.

(i) How many different arrangements are there?

(ii) What is the probability that the arrangement begins with F and ends with S?

2 On a shelf there are 4 different mathematics books and 8 different English books.

(i) The books are to be arranged so that the mathematics books are together. In how many different ways can this be done?

(ii) What is the probability that all the mathematics books are **not** together?

3 The letters of the word ABSTEMIOUS are arranged in a line at random. Find the probability that the vowels and consonants appear alternately.

4 A bag contains six white counters and eight blue counters. Four counters are chosen at random.

Find the probability that:

(i) two white counters and two blue counters are chosen,

(ii) all the counters are the same colour.

5 A committee consists of 4 women and 2 men. A sub-committee is formed consisting of three members of the committee. Find the probability that the sub-committee consists of:

(i) 1 man and 2 women,

(ii) at least 1 man.

6 Two pupils are chosen at random from a class of 10 boys and 8 girls. Find the probability that:

(i) they are both girls,

(ii) they are both boys,

(iii) there is one boy and one girl.

7 The letters of the word PROBABILITY are arranged at random in a line. Find the probability that the two letters I:

(i) are together,

(ii) are separated.

8 Four letters are picked at random from the word BREAKDOWN. Find the probability that there is at least one vowel among the letters.

9 Suan is given a bag of 20 sweets of which 6 are apple flavoured, 6 are lemon flavoured and 8 are orange flavoured. Suan takes out 5 sweets at random and eats them. Find the probability that she eats:

(i) 5 orange flavoured sweets,

(ii) 3 apple flavoured and 2 lemon flavoured sweets,

(iii) exactly 2 apple flavoured sweets,

(iv) no lemon flavoured sweets.

10 Three letters are selected at random from the 9 letters of the word UNIVERSAL. The order in which the letters are selected is unimportant.

(i) Find the number of selections of 3 letters.

(ii) Find the probability that the letter V is included in the selection.

11 When a tetrahedral die, with faces labelled 1, 2, 3, 4 is thrown, the score is the number on which it lands.

Jake throws three fair tetrahedral dice.

(i) List all the possible scores on the three dice which give a total of 6.

(ii) Find the probability that Jake obtains a total score of 6.

12 A plate contains 15 cakes of which 6 have yellow icing, 5 have green icing and 4 have pink icing. Three cakes are taken at random from the plate.

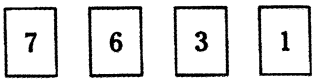
Find the probability that:

(i) exactly two of the cakes have green icing,

(ii) one cake has green icing, one has pink icing and one has yellow icing,

(iii) none of the cakes has yellow icing.

13 Jayne has 4 cards.



She chooses one or more of these cards and places them on a table to form a number. For example, she could form the number 6 or 31 or 731 or 3671.

(i) Find the total number of different numbers that can be formed.

(ii) If one of the numbers in part (i) is chosen at random, what is the probability that it is greater than 300?

14 Peta deals a hand of 10 cards from a well-shuffled pack of ordinary playing cards. Show that the probability that she deals exactly 5 spades is less than 5%.

Exercise 2.5 – Mixed Exercise

1 (i) Find the number of ways in which all 12 letters of the word REFRIGERATOR can be arranged:

(a) if there are no restrictions,

(b) if the Rs must all be together.

(ii) How many different selections of four letters from the 12 letters of the word REFRIGERATOR contain no Rs and two Es?

Cambridge Paper 6 Q5 J07

2 A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopranos, 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen?

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next to each other?

(iii) The 4 tenors and 4 basses stand in a single line with all the tenors next to each other and all the basses next to each other. How many possible arrangements are there if 3 of the tenors refuse to stand next to any of the basses?

Cambridge Paper 6 Q4 J09

3 Nine cards, each of a different colour, are to be arranged in a line.

(i) How many different arrangements of the 9 cards are possible?

The 9 cards include a pink card and a green card.

(ii) How many different arrangements do not have the pink card next to the green card?

Consider all possible choices of 3 cards from the 9 cards with the 3 cards being arranged in a line.

(iii) How many different arrangements in total of 3 cards are possible?

(iv) How many of the arrangements of the 3 cards in part (iii) contain the pink card?

(v) How many of the arrangements of 3 cards in part (iii) do not have the pink card next to the green card?

Cambridge Paper 62 Q7 J10

4 (i) Find the number of ways that a set of 10 different CDs can be shared between Dai and Evan if each receives an odd number of CDs.

(ii) A set of 9 DVDs consists of 3 different horror films and 6 different musicals. In how many ways can these be arranged on a shelf if the horror films are separated from each other?

5 (a) Find how many numbers between 5000 and 6000 can be formed from the digits 1, 2, 3, 4, 5 and 6

(i) if no digits are repeated,

(ii) if repeated digits are allowed.

(b) Find the number of ways of choosing a school team of 5 pupils from 6 boys and 8 girls

(i) if there are more girls than boys in the team,

(ii) if 3 of the boys are cousins and are either all in the team or all not in the team.

Cambridge Paper 61 Q5 N09

6 The digits of the number 1 223 678 can be rearranged to give many different 7-digit numbers.

Find how many different 7-digit numbers can be made if:

(i) there are no restrictions on the order of the digits,

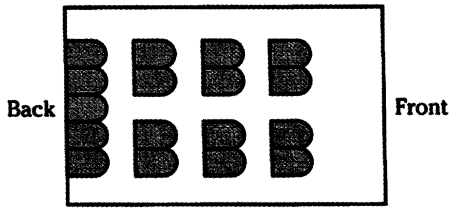
(ii) the digits 1, 3, 7 (in any order) are next to each other,

(iii) these 7-digit numbers are even.

Cambridge Paper 6 Q5 J02

7 Four letters are selected at random from the letters of the word GEOGRAPHY. Find the total number of selections.

8



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.

(i) How many possible seating arrangements are there for the 11 passengers?

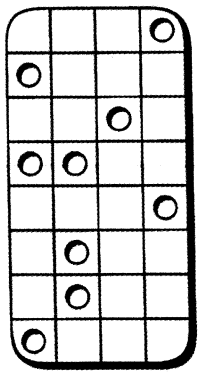
(ii) How many possible seating arrangements are there if 5 particular people sit in the back row?

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

(iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 married couples and 1 other person, who may or may not be married?

Cambridge Paper 6 Q4 J06

9 In a certain hotel, the lock on the door to each room can be opened by inserting a key card. The key card can be inserted only one way round. The card has a pattern of holes punched in it. The card has 4 columns and each column can have either 1 hole, 2 holes, 3 holes or 4 holes punched in it. Each column has 8 different positions for the holes. The diagram illustrates one particular key card with 3 holes punched in the first column, 3 in the second, 1 in the third and 2 in the fourth.



(i) Show that the number of different ways in which a column could have exactly 2 holes is 28.

(ii) Find how many different patterns of holes can be punched in a column.

(iii) How many different possible key cards are there?

Cambridge Paper 6 Q4 N02

Answers

Exercise 2.1

Section A

1 (i) 5040 (ii) 720
(iii) 60 (iv) 140

2 (i) 4032 (ii) 252

3 (i) $5!$ (ii) $\frac{5!}{2!}$ (iii) $\frac{21!}{17!}$
(iv) $\frac{n!}{(n-3)!}$ (v) $\frac{12!}{8!} \times \frac{2!}{5!}$

Section B

1 (i) 720 (ii) 5040 (iii) 362 880
2 (i) 720 (ii) 120 (iii) 40 320
3 (i) $4! = 24$ (ii) $4^4 = 256$
4 (i) $3! = 6$ (ii) 12
5 (i) 2520 (ii) 907 200 (iii) 19 958 400
6 360
7 (i) 3360 (ii) 420 (iii) 360
(iv) 2520 (v) 240
8 (i) 60 (ii) 20 (iii) 30
9 243
10 (i) 10 000 (ii) 59 049
11 2880
12 (i) 10 080 (ii) 1440
13 (i) 2 903 040 (ii) 241 920
14 3840
15 5040
16 (i) 483 840 (ii) 1 209 600
17 98
18 (i) 40 320 (ii) 2880
19 (i) 50 (ii) 18
20 (i) 90 720 (ii) 120

Exercise 2.2

1 (i) 60 480 (ii) 30
(iii) 720 (iv) 362 880
2 6840
3 11 880
4 210
5 12
6 (i) 1200 (ii) 3125
7 151 200
8 (i) 60 (ii) 325
9 (i) 151 200 (ii) 30 240

Exercise 2.3

1 (i) 330 (ii) 35 (iii) 56
(iv) 45 (v) 84 (vi) 84
(vii) 495 (viii) 495
2 792
3 (i) 126 (ii) 56 (iii) 105
(iv) 15
4 1050
5 (i) 7920 (ii) 5544 (iii) 7182
6 10
7 168 168
8 25 200
9 (i) 15 (ii) 75
10 26
11 7
12 (i) 384 (ii) 86 (iii) 2590
13 (ii) 9

Exercise 2.4

1 (i) 362 880 (ii) $\frac{1}{72}$
2 (i) 8 709 120 (ii) $\frac{9}{30}$
3 $\frac{1}{126}$
4 (i) $\frac{60}{143}$ (ii) $\frac{85}{1001}$
5 (i) $\frac{3}{5}$ (ii) $\frac{4}{5}$
6 (i) $\frac{28}{153}$ (ii) $\frac{5}{17}$ (iii) $\frac{80}{153}$
7 (i) $\frac{2}{11}$ (ii) $\frac{9}{11}$
8 $\frac{37}{42}$
9 (i) $\frac{7}{1938}$ (ii) $\frac{25}{1292}$ (iii) $\frac{455}{1292}$ (iv) $\frac{1001}{7752}$
10 (i) 84 (ii) $\frac{1}{3}$
11 (i) (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)
(ii) $\frac{3}{32}$
12 (i) $\frac{20}{91}$ (ii) $\frac{24}{91}$ (iii) $\frac{12}{65}$
13 (i) 64 (ii) $\frac{21}{32}$
14 0.0468... = 4.7% (2 s.f.) < 5%

Exercise 2.5

1 (i) (a) 9 979 200 (b) 181 440
(ii) 15
2 (i) 33 033 000 (ii) 86 400 (iii) 288
3 (i) 362 880 (ii) 282 240 (iii) 504
(iv) 168 (v) 476
4 (i) 512 (ii) 151 200
5 (a) (i) 60 (ii) 216
(b) (i) 1316 (ii) 517
6 (i) 2520 (ii) 360 (iii) 1440
7 91
8 (i) 4.94×10^{11} (ii) 79 833 600 (iii) 21
9 (ii) 162 (iii) $162^4 = 688\,747\,536$