QUADRATICS

P1/1/1: Quadratic expressions of the form $ax^2 + bx + c$ and their graphs

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P1/1/1

Quadratic expressions of the form $ax^2 + bx + c$ and their graphs

Learning Outcome

Students should be able to:

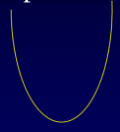
- carry out the process of completing the square for a quadratic polynomial.
- locate the vertex of the quadratics graphs from the completed form.
- sketch the quadratic graph

Quadratics expression

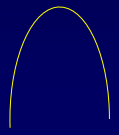
$ax^2 + bx + c$
where $a (a \neq 0)$, b and c are constants (coefficients).

The graph is a parabola.

If $a > 0$ or



If $a < 0$



Completed square form

$f(x) = x^2 - 10x + 21$ can be written as

$f(x) = (x-7)(x-3)$ factor form ← *x*-intercept?

$f(x) = (x-5)^2 - 4$ completed square form ← *Vertex?* ← *Range of f(x)?*

Graph

Example 1:

Express the following functions in completed square form.

(*i*) $y = x^2 + 14x + 50$

(*ii*) $y = 2x^2 + 12x - 5$

(*iii*) $y = 3 - 7x - 3x^2$

Example 2:

Express the following function in the form of

$y = (ax + b)^2 + c$

$y = 9x^2 - 6x + 5$

Example 3:

Sketch the graph of the following quadratic functions.

(a) Locate the vertex and the axis of symmetry.

(b) Find the least or greatest value of the expression, and the value of *x* for which this occurs.

(c) Find the range of $f(x)$.

(*i*) $f(x) = x^2 + 14x + 50$

(*ii*) $f(x) = 2x^2 + 12x - 5$

(*iii*) $f(x) = 3 - 7x - 3x^2$

(*iv*) $f(x) = 9x^2 - 6x + 5$

(*v*) $f(x) = (x-1)(x-2)$

Practice Exercise

Pure Mathematics 1 *Hugh Neil & Douglas Quadling (2002)*

Exercise 4A (Page 54)
Q6(e)(f), Q8(c)(f), Q9(c)

P1/1/2

Solving quadratic equation in one unknown

Learning Outcome

Students should be able to:

- use an appropriate method to solve a given quadratic equation.
- solve equations which can be reduced to quadratic equations.

Solving Quadratic equation in one unknown

Example 4:

Solving the following quadratic equations, Leave your answers in surd form if any.

$(a)\quad 2x^2+7x+3=0$ $(b)\quad 2x^2-3x+4=0$

(i) Factorization OR

(ii) Completing the square method OR

(iii) Quadratic formula

The solution of $ax^2+bx+c=0$, where $a\neq 0$, is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Equation which reduce to quadratic equations

Example 5:

Solve the following equations:

$(i)\quad x^4 - 5x^2 + 4 = 0$

$(ii)\quad \sqrt{x} = 6 - x$

Practice Exercise

Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 4B (Page 58)
Q1(e)(g)(i)

Exercise 4C (Page 61)
Q4(d), Q5(f), Q6(d)

P1/1/3
Nature of roots of quadratic expression

Learning Outcome

Students should be able to:

- evaluate the discriminant of a quadratic polynomial.
- use the discriminant to determine the nature of the roots.
- relate the nature of roots to the quadratic graph.

Nature of roots of quadratic expression

The discriminant $b^2 - 4ac$

$ax^2 + bx + c = 0$

$\Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(ii) If $b^2 - 4ac > 0$, the equation $ax^2 + bx + c = 0$ will have two roots.

(iii) If $b^2 - 4ac < 0$, there will be no roots.

(iv) If $b^2 - 4ac = 0$, there is one root only or a repeated root.

Example 6:

What can you deduce from the values of discriminants of the quadratics in the following equations?

$(a)\quad 2x^2 - 7x + 3 = 0$

$(b)\quad x^2 - 3x + 4 = 0$

$(c)\quad x^2 + 2x + 1 = 0$

Example 7:

What can you deduce about the value of the constant k? If the equation

(i) $3x^2 + 5x - k = 0$ has two real roots.

(ii) $3x^2 + 5x - k = 0$ has two distinct real roots.

Example 8:

(i) Find the set of values of k for which $k + 2x - 3x^2$ is negative for all real values of x.

(ii) Find the range of values of the constant p for which the following function is positive for all real values of x.

$y = px^2 + 4x + p + 3$

Practice Exercise

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Exercise 4B (Page 58)
Q4(e)(f)(g), Q5(e)(h)(i)

P1/1/4

Simultaneous equations of which one is linear and one is quadratic

Learning Outcome

Students should be able to:

- solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic.

Simultaneous equations of which one is linear and one is quadratic

Example 14:

At how many points does the line $3y - x = 15$ meet the curve $4x^2 + 9y^2 = 36$? State the coordinates of

Practice Exercise

Pure Mathematics 1 *Hugh Neil & Douglas Quadling (2002)*

Exercise 4C (Page 61)
Q2(h), Q3(d)

P1/1/5

Linear inequalities and quadratic inequalities

Learning Outcome

Students should be able to:

- solve linear inequalities
- solve quadratic inequalities

Algebraic method

If $a > 0$, then these statements are equivalent

$$x^2 \le a^2 \Leftrightarrow -a \le x \le a$$

$$x^2 \ge a^2 \Leftrightarrow x \le -a \text{ or } x \ge a$$

Example 15:

Solve the linear inequalities.

$(i)\quad 3x+7>-5$

$(ii)\quad -3x\geq -12$

$(iii)\quad -1<2x+3<6$

$(iv)\quad x-1<2x+2<3x+1$

Example 16:

Solve the quadratic inequalities

$(i)\quad x^2-3x-4\geq 0$

$(ii)\quad -12x^2-7x+12>0$

$(iii)\quad 8-3x-x^2>0$

$(iv)\quad x^2+3x-5>0$

Practice Exercise

Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 5A (Page 68)
Q3(g), Q4(h), 5(i), 6(g)

Exercise 5B (Page 71)
Q1(i)(k), Q4(e)(f)(g)(l)