

Tutorial (Discussion 1) – Topic 2

Question 1

For each of the following, state the largest domain for which the function is defined.

- (a) $f(x) = x^2 - 1$ (b) $f(t) = \frac{t+3}{t+4}$ (c) $g(x) = \sqrt{x+3}$ (d) $h(z) = \sqrt[3]{z+3}$
- (a) \mathbb{R}
 (b) $t \in \mathbb{R}, t \neq -4$
 (c) $g(x) = \sqrt{x+3}$
 (d) \mathbb{R}

Question 2

- (a) If $f(x) = 3x + 2$ and $g(x) = 2x - 3$, obtain and simplify the formulae for: (i) $fg(x)$ (ii) $gf(x)$ (iii) $gg(x)$
 (b) If $h(x) = 2 - x$
 (i) obtain the formula for $hh(x)$,
 (ii) explain why the function h is called self-inverse.
- (a) (i) $6x - 7$ (ii) $6x + 1$
 (iii) $4x - 9$
 (b) (i) x
 (ii) $h(x) = h^{-1}(x)$

Question 3

The functions f and g are defined by

$$f(x) = x^2 - 3, \quad x \in \mathbb{R}; \quad g(x) = x + 3, \quad x \in \mathbb{R}$$

- (a) Obtain the formula for the function $gf(x)$.
 (b) State the range for each of the following functions.
 (i) $f(x)$ (ii) $g(x)$ (iii) $gf(x)$
- (a) x^2
 (b) (i) $f(x) \in \mathbb{R}, f(x) \geq -3$
 (ii) \mathbb{R}
 (iii) $gf(x) \geq 0$

Question 4

The formulae for functions f and g are defined by

$$f(x) = x^2 + 2, \quad g(x) = \sqrt{3 - x}$$

- (a) State the largest possible domain for:
 (i) $f(x)$ (ii) $g(x)$
 (b) State the largest possible domain and corresponding range for $gf(x)$.
- (a) (i) \mathbb{R} (ii) $x \leq 3$
 (b) $-1 \leq x \leq 1$ $0 \leq gf(x) \leq 1$

Question 5

The formulae for functions f and g are defined by

$$f(x) = \frac{x+3}{x-4}, \quad g(x) = \frac{x-1}{x+2}$$

- (a) Show that $gf(x) = \frac{7}{3x-5}$.

Question 6

11 The functions f , g and h are defined by

$$f(x) = 2 - 3x, \quad x \in \mathbb{R}; \quad g(x) = 1 - x, \quad x \in \mathbb{R};$$

$$h(x) = \frac{2-x}{3}, \quad x \in \mathbb{R}$$

- (a) Show that (i) $fg(x) = gf(x)$, (ii) $fh(x) = hf(x)$.
 (b) Does $gh(x) = hg(x)$?

Question 7

12 If $f(x) = \frac{x+1}{x-2}$,

(a) show that $f(x) = 1 + \frac{3}{x-2}$

(b) write down a valid domain and range for $f(x)$, $x \in \mathbb{R}, x \neq 2, f(x) \in \mathbb{R}, f(x) \neq 1$

(c) show that $f^{-1}(x) = \frac{1+2x}{x-1}$

(d) write down a valid domain and range for $f^{-1}(x)$.

$$x \in \mathbb{R}, x \neq 1, f^{-1}(x) \in \mathbb{R}, f^{-1}(x) \neq 2.$$

Question 8

The functions g and h are defined by

$$g(x) = \sqrt{x-4}, \quad x \in \mathbb{R}, x > 4; \quad h(x) = \sqrt{1-x^2}, \quad x \in \mathbb{R}, -1 \leq x \leq 1$$

State the domain and range for the function $hg(x)$.

$$x \in \mathbb{R}, 4 < x \leq 5 \\ hg(x) \in \mathbb{R}, 0 \leq hg(x) < 1$$

Question 9

Functions f and g are defined, for $x \geq 0$, by

$$f: x \mapsto \frac{12}{x+1},$$

$$g: x \mapsto \sqrt{x+1}.$$

(i) Write down the range of f and of g .

$$0 < f(x) \leq 12 \quad [2]$$

(ii) Find $fg(3)$.

$$g(x) \geq 1 \quad [2]$$

(iii) Solve $gf(x) = 3$.

$$\frac{1}{2} \quad [3]$$

Question 10

The function f is defined by

$$f: x \mapsto 9 - x^2, \quad x \in \mathbb{R}.$$

Find the exact solutions of the equation $ff(x) = 0$.

$$\pm 6, \quad \pm 2\sqrt{3}$$