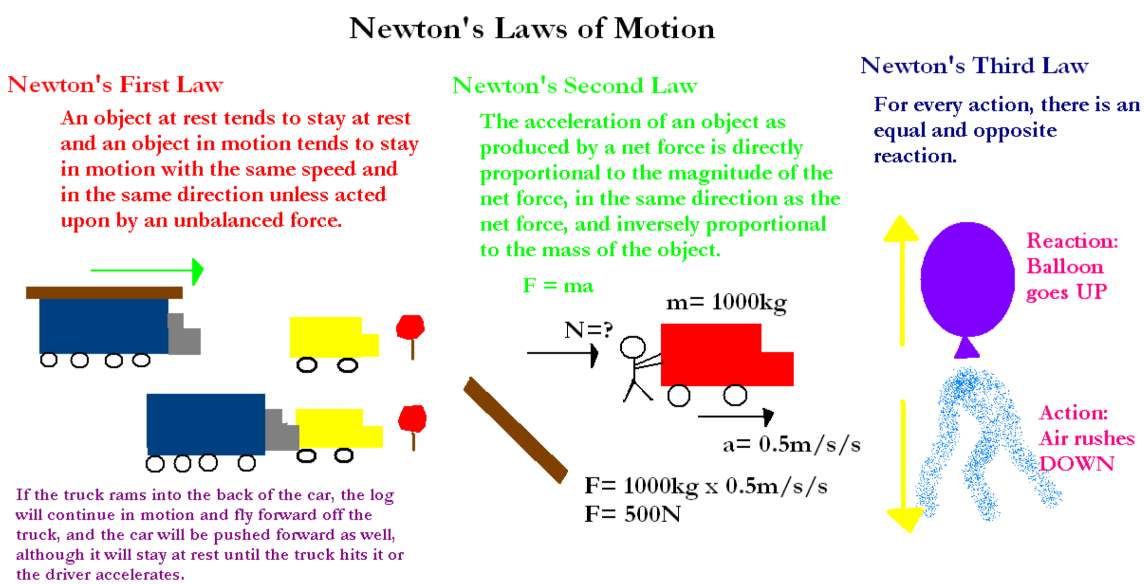
Newton's Laws of Motion

statement	consequence(s)
1st law Every body will remain at rest or continue to move in a straight line at constant speed unless an external force acts on it.	(a) If a body has an acceleration, then there must be a force acting on it. (b) If a body has no acceleration, then the forces acting on it must be in equilibrium.
2nd law The rate of change of momentum of a moving body is proportional to the external force acting on it and takes place in the direction of that force. So when an external force acts on a body of constant mass, the force produces an acceleration which is directly proportional to the force.	(a) The basic equation of motion for constant mass is: **Force = mass × acceleration** (in N) (in kg) (in m s^{-2}) (b) The force and acceleration of the body are both in the same direction. (c) A constant force on a constant mass gives a constant acceleration.
3rd law If a body A exerts a force on a body B, then B exerts an equal and opposite force on A.	These forces between bodies are often called reactions. In a rigid body the internal forces occur as equal and opposite pairs and the net effect is zero. So only external forces need to be considered.

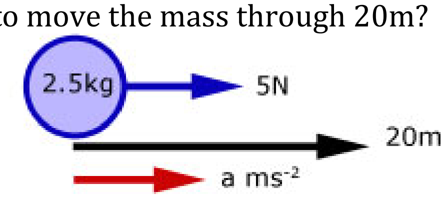


Linear acceleration

Here the mass is either stationary and is accelerated by a force in a straight line or is initially moving at constant velocity before the force is applied.

Example 1

A 5N force acts on a 2.5kg mass, making it accelerate in a straight line.
 i) What is the acceleration of the mass?
 ii) How long will it take to move the mass through 20m?



Linear retardation

Here the mass is already moving at constant velocity in a straight line before the force is applied, opposing the motion.

Example 2

A 4 kg mass travelling at constant velocity 15 ms^{-1} has a 10 N force applied to it against the direction of motion.

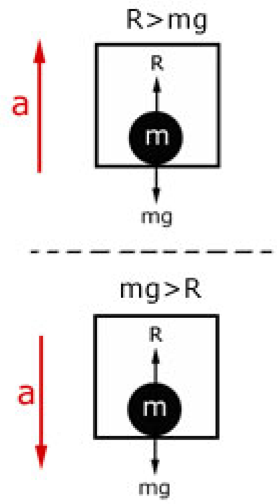
i) What is the deceleration produced?

ii) How long will it take before the mass is brought to rest?

Example 3

A sky diver with mass 80kg is falling at a constant velocity of 70 ms^{-1} . When he opens his parachute he experiences a constant deceleration of 3g for 2 seconds.
i) What is the magnitude of the decelerating force?
ii) What is his rate of descent at the end of the 2 seconds deceleration?

Mass Ascending or Descending in a Lift

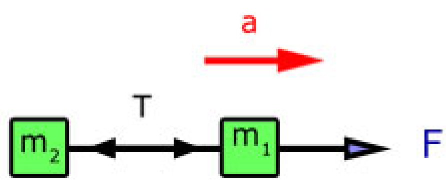


Example 4
A person of mass 100 kg stands in a lift. What is the force exerted by the lift floor on the person when the lift is:

i) moving upwards at 3 ms^{-2}

ii) moving downwards at 4 ms^{-2} [1300, 600]

Tow-bar/Tow-rope/Chains



Usually one body pulled horizontally by another with each linked by a tow-bar or similar. This is similar to the pulley but drawn out in a line.

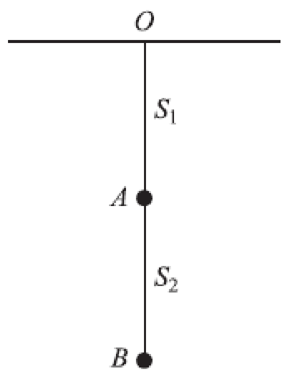
Example 5

A car of mass 600 kg tows a trailer of mass 250 kg in a straight line using a rigid tow-bar. The resistive force on the car is 200N. The resistive force on the trailer is 80N. If the forward thrust produced by the engine of the car is 800 N, what is (to 3 d.p.)

i) the acceleration of the car

ii) the tension in the tow-bar [0.612, 233]

Additional Question



S_1 and S_2 are light inextensible strings, and A and B are particles each of mass 0.2 kg. Particle A is suspended from a fixed point O by the string S_1, and particle B is suspended from A by the string S_2. The particles hang in equilibrium as shown in the diagram.

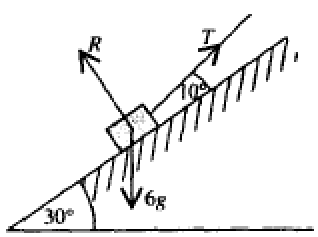
(i) Find the tensions in S_1 and S_2. [3]

The string S_1 is cut and the particles fall. The air resistance acting on A is 0.4 N and the air resistance acting on B is 0.2 N.

(ii) Find the acceleration of the particles and the tension in S_2. [5]

Inclined Plane

Example 6



A body of mass 6 kg is sliding down a smooth plane inclined at 30° to the horizontal. Its speed is controlled by a rope inclined at 10° to the plane as shown; the tension in the rope is 10 N. Given that the body starts from rest, find how far down the plane it travels in 5 seconds. Taking g as 9.8, give the answer corrected to 2 significant figures. [41m]

Additional question

A cyclist is riding up a hill inclined at 20° to the horizontal. His speed at the foot of the hill is 10 m/s but after 30 seconds it has dropped to 4 m/s. The total mass of the cyclist and his machine is 100 kg and there is a wind of strength 15 N down the slope. Find, corrected to 3 significant figures, the constant driving force exerted by the cyclist up the slope. [330N]

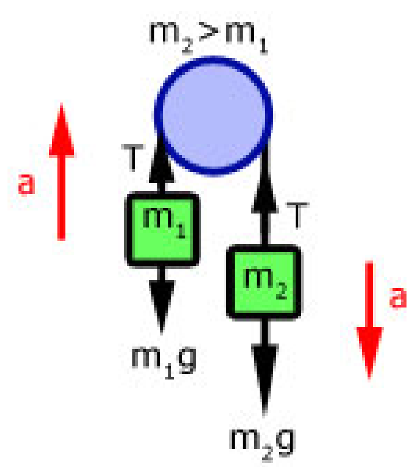
CONNECTED PARTICLES (Pulley System)

Two particles connected by a light inextensible string which passes over a fixed light smooth frictionless pulley are called **connected particles.**
The tension in the string is the same throughout its length, so each particle is acted upon by the same tension.

Problems concerned with connected particles usually involve finding the acceleration of the system and the tension in the string.

To solve problems of this type:

1. Draw a clear diagram showing the forces on each particle and the common acceleration.
2. Write down the equation of motion, i.e. force = mass × acceleration for each particle separately.
3. Solve the two equations to find the common acceleration, a, and/or the tension, T, in the string.



Example 7

A 3 kg mass and a 5 kg mass are connected over a pulley by a light inextensible string. When the masses are released from rest, what is:
i) the acceleration of each mass?
ii) the tension in the string (Take g = 10 ms^{-2}. Answer to 3 d.p.)

Example 8

Two particles of masses 1 kg and 3 kg are attached to the ends of a long light inelastic string which passes over a fixed smooth pulley. The system is held with both particles hanging at a height of 2m above the ground and is released from rest. In the ensuing motion the heavier particle hits the ground and does not rebound. Find the greatest height by the mass of 1 kg. [5m]

Example 9

A light inextensible string passing over a smooth pulley connects a particle A of mass 1.5 kg with a lighter particle B of mass M kg. The particles are held at rest at the same level and then released. When B has ascended a distance of 0.8 m its speed is 2 ms^{-1}; at this instant A strikes the floor and becomes disconnected from the string. Calculate

i) the initial acceleration of the particles, $2 \cdot 5\ ms^{-1}$ [2]

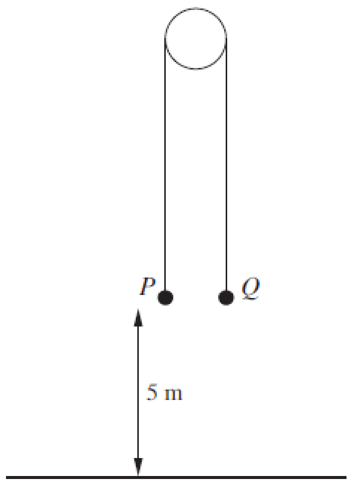
ii) the tension in the string whilst both A and B are moving, $11 \cdot 25$ N [2]

iii) the value of M, $0 \cdot 9$ kg [2]

iv) the further time that elapses before B strikes the floor. [3]

$t = 0 \cdot 8$ s

Example 10



Particles P and Q, of masses 0.55 kg and 0.45 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The particles are held at rest with the string taut and its straight parts vertical. Both particles are at a height of 5 m above the ground (see diagram). The system is released.

(i) Find the acceleration with which P starts to move. [3]

The string breaks after 2 s and in the subsequent motion P and Q move vertically under gravity.

(ii) At the instant that the string breaks, find

(a) the height above the ground of P and of Q, [2]

(b) the speed of the particles. [1]

(iii) Show that Q reaches the ground 0.8 s later than P. [4]

Example 11

Two particles A and B, of mass 0.3 kg and 0.4 kg respectively, are connected by a light inextensible string which passes over a smooth, light, fixed pulley. The particles are released from rest with the string taut and the hanging parts vertical. Calculate

(i) the acceleration of A, [4]

The particles continue to move in this system until the instant when they each acquire a speed of 3.5 ms^{-1}. At this instant both A and B are 1.5 m above horizontal ground and the string is cut.

(ii) Find the time, measured from the instant when the string is cut, for A to reach the ground. [4]

[1.43, 1]

CONNECTED PARTICLES

Case 1

Common situations The simplest situation in which connected particles occur is illustrated above. There are several other situations in which the motion of connected particles is considered. The most common are shown below.

One particle on a **smooth horizontal table** as shown.

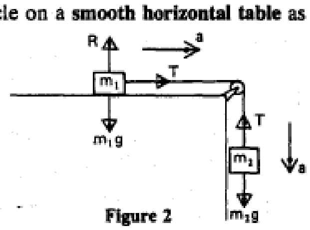


Figure 2

For m_1:
Resolving ⊥ to table: $R = m_1 g$
Equation of motion: $T = m_1 a$
For m_2:
Equation of motion: $m_2 g - T = m_2 a$

One particle on a **rough horizontal table** as shown.

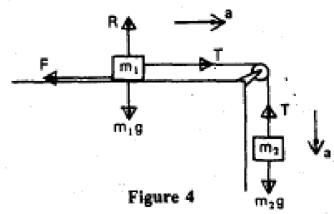


Figure 4

For m_1:
Resolving ⊥ to table: $R = m_1 g$
Equation of motion: $T - F = m_1 a$
For m_2:
Equation of motion: $m_2 g - T = m_2 a$

Case 2

One particle on a **smooth inclined plane** as shown.

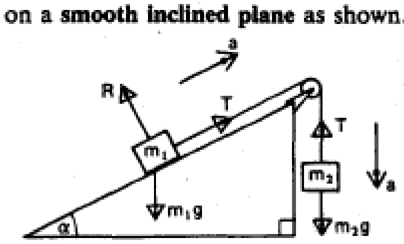


Figure 3

For m_1:
Resolving ⊥ to plane: $R = m_1 g \cos \alpha$
Equation of motion: $T - m_1 g \sin \alpha = m_1 a$
For m_2:
Equation of motion: $m_2 g - T = m_2 a$

One particle on a **rough inclined plane** as shown.

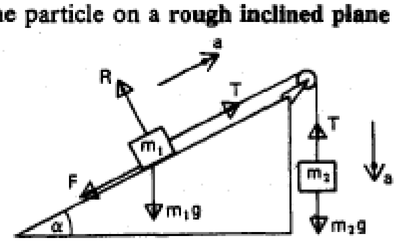
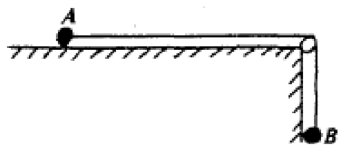


Figure 5

For m_1:
Resolving ⊥ to plane: $R = m_1 g \cos \alpha$
Equation of motion: $T - F - m_1 g \sin \alpha = m_1 a$
For m_2:
Equation of motion: $m_2 g - T = m_2 a$

Example 12

[Take the acceleration due to gravity to be 10 ms^{-2}.]
The diagram represents a particle A, of mass 3 kg, and a particle B, of mass 2 kg, which are connected by a light inextensible string. Particle A rests on a rough horizontal table, the string passes over a smooth fixed pulley at the edge of the table, and particle B hangs freely. The system is released from rest, and each particle moves a distance of 1 metre in the first second of the motion. Find the tension in the string and the coefficient of friction between A and the table.



$T = 16\text{ N}$
$\mu = \frac{1}{3}$

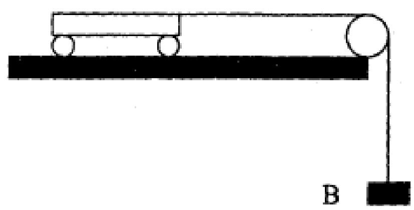
Example 13

A 2 kg mass on a smooth 30^{0} plane is connected to a 5 kg mass by a light inextensible string passing over a pulley at the top of the plane. When the particles are released from rest the 2 kg mass moves up the plane.

i) what is the acceleration of the 2 kg & 5 kg masses?

ii) What is the tension in the string?

Example 14

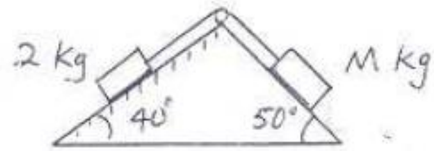


A trolley of mass 2 kg can move on a horizontal table. One end of a light inextensible string is fixed to the trolley. The string passes over a smooth pulley at the edge of the table, and a wooden block B of mass 0.5 kg hangs freely at the other end of the string. The part of the string between the trolley and the pulley is horizontal. Resistances to the motion of the system, from all causes, are modeled as a constant horizontal force of magnitude F newtons acting on the trolley.

a) The system is released is released from rest with B at a height of 1 m above the floor, and B hits the floor 2.5 s later. Use this information to calculate the acceleration of B while it is falling, and the speed with which it hits the floor. [3]
[0·32, 0·8] [4·84, 4·2]

b) Hence find the value of F and the tension in the string while B is falling. [5]

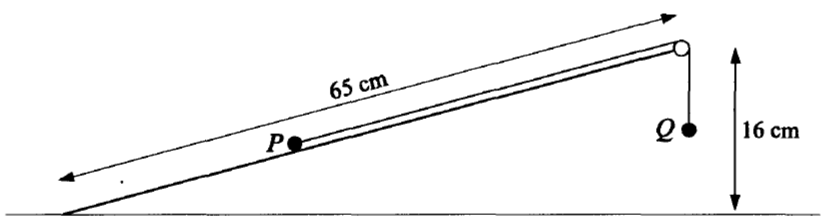
Example 15



Two planes, one rough and one smooth, inclined at angles 40° and 50° respectively, are placed back to back. Particles of masses 2 kg and M kg are placed on the slopes and are joined by a light inextensible string that passes over a smooth pulley as shown. The 40° slope has a coefficient of friction of 0.5 and air resistance can be ignored. The system is released from rest and the masses accelerate at 0.5 ms^{-2}. Find

(i) the two possible values of M, [4]
(ii) the tension of the string for each case. [4]

Additional Question
Question 1



A rough inclined plane of length 65 cm is fixed with one end at a height of 16 cm above the other end. Particles *P* and *Q*, of masses 0.13 kg and 0.11 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley at the top of the plane. Particle *P* is held at rest on the plane and particle *Q* hangs vertically below the pulley (see diagram). The system is released from rest and *P* starts to move up the plane.

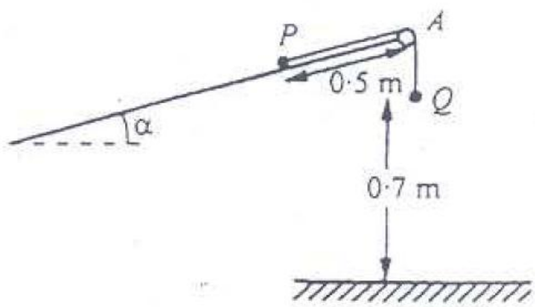
(i) Draw a diagram showing the forces acting on *P* during its motion up the plane. [1]

(ii) Show that $T - F > 0.32$, where T N is the tension in the string and F N is the magnitude of the frictional force on *P*. [4]

The coefficient of friction between *P* and the plane is 0.6.

(iii) Find the acceleration of *P*. [6]

Question 2

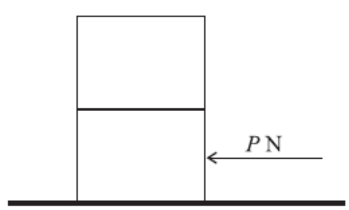


Two particles P and Q, of masses 0.2 kg and m kg respectively, are attached to the ends of a light inextensible string. The particle P is placed on a plane inclined to the horizontal at an angle α, where sin α = 0.2. The coefficient of friction between P and the plane is $\frac{1}{\sqrt{6}}$. The string passes over a smooth pulley at A, and AQ hangs vertically (see diagram). The system is at rest with the string taut and the part AP parallel to the line of greatest slope of the inclined plane. The particle P is at a distance of 0.5 m from A and the particle Q is a height of 0.7 m above the ground. Show that, for the system to remain at rest, $0 < m \leq 0.12$. [5]

It is given that m = 0.3 and the system is released from rest in the position shown in the diagram. Show that the speed of P as it reaches A is 1.90 ms^{-1}, correct to 3 significant figures. [5]

Just before P reaches A, the string breaks. Find the speed with which Q strikes the ground. [2]

Question 3



Two identical boxes, each of mass 400 kg, are at rest, with one on top of the other, on horizontal ground. A horizontal force of magnitude P newtons is applied to the lower box (see diagram). The coefficient of friction between the lower box and the ground is 0.75 and the coefficient of friction between the two boxes is 0.4.

(i) Show that the boxes will remain at rest if $P \leqslant 6000$. [2]

The boxes start to move with acceleration a m s^{-2}.

(ii) Given that no sliding takes place between the boxes, show that $a \leqslant 4$ and deduce the maximum possible value of P. [7]