Integration

A2 Trial Examination

August 2010

5 Given that $\frac{dy}{dx} = e^{x-y}$ and $y = 1$ when $x = 0$, find the exact value of y when $x = 1$. [5]

$[ln(2e-1)]$

8 Show that $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx = \frac{1}{8}\pi^2 - \frac{1}{2}$. [6]

10 (i) Express $\frac{x+4}{(x+1)^2(x+2)}$ in partial fractions. [5]

(ii) The finite region R is bounded by the curve $y = \frac{x+4}{(x+1)^2(x+2)}$, the x-axis, the y-axis and the line $x = 3$. Find the area R, giving your answer in the form $p + \ln q$, where p and q are rational numbers to be found. [4]

$$\left[\frac{2}{x+2} - \frac{2}{x+1} + \frac{3}{(x+1)^2}\ ; \ln\frac{25}{64} + \frac{9}{4}\right]$$

March 2010

4 The gradient of the curve $y = f(x)$ is inversely proportional to the square root of x and the curve passes through $(0, 3)$ and $(4, 23)$. Find the equation of the curve. [5]

$[y = 10\sqrt{x} + 3]$

5 Show that $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx = \frac{1}{5}(2e^{\pi} + 1)$ [7]

8 Given that $\frac{x^2 - 2x - 9}{(2x-1)(x^2+3)} \equiv \frac{A}{2x-1} + \frac{Bx+C}{x^2+3}$

(i) Determine the values of A, B and C. [4]

(ii) Hence, evaluate $\int_1^2 \frac{x^2 - 2x - 9}{(2x-1)(x^2+3)} \, dx$, giving your answer correct to two decimal places. [5]

$[-3\,, 2\,, 0\ ;\ -1.09]$

August 2009

1 An engineer estimated the area of the vertical cross-section of water flowing under a bridge. For her model she measured the depth of water at 4 m intervals from one end of the bridge to the other end. Her results are given in the table.

Distance from one end (m)	0	4	8	12	16	20	24
Depth (m)	1.2	2.3	3.8	4.9	3.2	1.9	0.6

She used the trapezium rule to estimate the area of the cross-section. Calculate the estimate she obtained. [2]

[68]

3 Use integration by parts to find $\int 4xe^{-2x}\,dx$. [4]

$[-2xe^{-2x} - e^{-2x} + c]$

4 Solve the differential equation $\frac{dy}{dx} = \sqrt{y}\sec^2 3x$ given that $y = 1$ when $x = 0$, expressing your answer in the form $y = f(x)$. [5]

$\left[y = \left(\frac{\tan 3x}{6} + 1\right)^2\right]$

March 2009

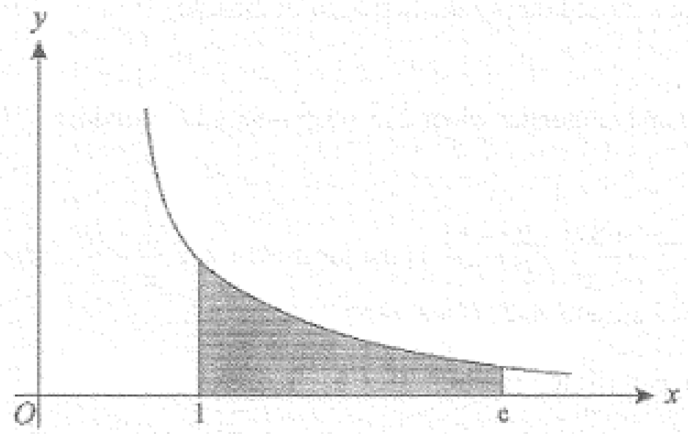
3 Find the exact value of $\int_0^1 3x^2 e^x\,dx$. [5]

$[3e - 6]$

6 (i) Given that $y = \dfrac{4\ln x - 3}{4\ln x + 3}$

Show that $\dfrac{dy}{dx} = \dfrac{24}{x(4\ln x + 3)^2}$. [3]

(ii)



The diagram shows part of the curve with equation $y = \dfrac{2}{\sqrt{x}(4\ln x + 3)}$.

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = e$ and $y = 0$.

Find the exact volume of the solid produced when this shaded region is rotated 360^0 about the x – axis. [4]

$\left[\dfrac{4\pi}{21}\right]$

8 (i) Find the quotient and remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence, express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii)Use the result in part (ii) to show that [5]

$$\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}\,dx = 14 + \frac{1}{2}\ln\left(\frac{13}{5}\right).$$

$[2x + 3, x\ ; 2,3,1,0\ ]$

August 2008

5 Show that $\int_0^2 xe^{2x}\,dx = \frac{1}{4}(3e^4+1)$ [4]

Hence, find the exact value of

$$\int_0^2 x^2 e^{2x}\,dx$$ [3]

[$\frac{1}{4}(5e^4-1)$ **]**

7 (i) Given that $I = \int_{-1}^{1} \frac{1}{1+e^{-x}}\,dx$, show that the estimate of I obtained by using the trapezium rule with three ordinates is 1(to the nearest integer). [3]

(ii) By means of substitution $u = e^x$, show that the estimate obtained in (i) is a good estimate. [4]

10 (i) Find the constants A, B and C in the identity $\frac{5x^2+4x-20}{(x+2)(x^2+4)} \equiv \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+4)}$ [3]

(ii) Find $\int_0^2 \frac{x}{x^2+4}\,dx$ [2]

(iii) Use the substitute $x = 2\tan\theta$ to find $\int_0^2 \frac{1}{x^2+4}\,dx$, [4]

(iv) Use the answers to parts (i), (ii) and (iii) above to show that

$\int_0^2 \frac{5x^2+4x-20}{(x+2)(x^2+4)}\,dx = a\ln 2 - b\pi$, where a and b are positive integers.

Find a and b. [5]

[-1, 6, -8; ½ ln2 ; $\pi/8$; $2\ln 2-\pi$ **]**

March 2008

6. i) Show that $\int_1^e x\ln x\,dx = \frac{1}{4}\left(e^2+1\right)$. [4]

ii) Hence evaluate $\int_1^e x(\ln x)^2\,dx$ correct to 3 decimal places. [3]

[1.597]

August 2007

3. Use the trapezium rule, with 4 intervals, to estimate the value of $\int_1^3 \ln x \, dx$, giving your answer correct to 4 significant figures. [4]

[1.282]

8. (i) Express $\dfrac{1}{y^2 - y}$ in partial fractions. [3]

(ii) Given the differential equation $\dfrac{1}{2}\left(\dfrac{dy}{dx}\right) + xy = xy^2$ and that when $x = 0$, $y = 2$, find y in terms of x. [5]

$$[\frac{1}{y-1} - \frac{1}{y}; y = \frac{2}{2 - e^{x^2}}]$$

9. Using integration by parts show that $\int_0^{\frac{\pi}{3}} x \sin 6x \, dx = -\dfrac{\pi}{18}$. [5]

Hence, show that $\int_0^{\frac{\pi}{3}} x^2 \cos 6x \, dx = \dfrac{\pi}{54}$. [3]

$$[2.218]$$

March 2007

1. Evaluate $\int_0^1 \dfrac{x^2}{\sqrt{4 - x^2}} dx$ using the trapezium rule with 5 intervals. (4)

[0.186]

5. Find the exact value of $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx$. (7)

$$\left[\frac{1}{5}(2e^{\pi} + 1)\right]$$

August 2006

2. Use integration by parts to show that the exact value of $\int_1^e (\ln x)^2 dx$ is $e - 2$. [4]

6. Given that $\dfrac{x^2 + 4}{x^2 + 2x}$ can be written as $A + \dfrac{B}{x} + \dfrac{C}{x + 2}$.

Find the values of A, B and C. [5]

Hence evaluate $\int_1^4 \dfrac{x^2 + 4}{x^2 + 2x} \, dx$. [4]

[1,2,-4; 3]

7. The curve $y = x\sqrt{(1 + x)}$ cuts the x-axis at two points.

(i) Find the coordinates of these two points of intersection. [3]

(ii) Use the substitution $u = 1 + x$ to find the area enclosed by the curve and the x-axis. [6]

$$\left[(0,0),(-1,0);\frac{4}{15}\right]$$

March 2006

9. The equation of a curve is given as $y = x^2e^{2x}$.

(i) Find the exact value of $\frac{dy}{dx}$ at the point $x = 2$. [4]

(ii) Use integration by parts to find the exact area between the graph of y and the x-axis from $x = 0$ to $x = 2$. [5]

$$\left[12e^4;\frac{5}{4}e^4-\frac{1}{4}\right]$$

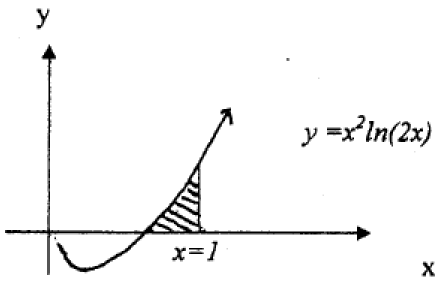
August 2005

5. By means of the substitution $x = \tan\theta$ or otherwise, find the exact value of

$$\int_0^\infty \frac{x^2}{(1+x^2)^2}\,dx.$$ [7]

$$\left[\frac{\pi}{4}\right]$$

8. The sketch below shows the curve with equation $y = x^2 \ln(2x)$



(i) Find the exact coordinates of the stationary point. [5]

(ii) Find the x-coordinate of the point where the curve cuts the x- axis .

Hence calculate the area of the shaded region. [5]

$$\left[\left(\frac{1}{2\sqrt{e}}, -\frac{1}{8e}\right); \frac{1}{2}; \frac{1}{3}\ln 2 - \frac{7}{72}\right]$$

<u>March 2005</u>

2 . Find the exact value of $\int_1^e (\ln x)^2 \, dx$.

[4]

$$[e-2]$$

6 Evaluate $\int_0^1 \frac{1}{1+x^2} \, dx$ using

(i) the trapezium rule with 5 intervals , [4]

(ii) integration with the substitution $x = \tan \theta$. [3]

[0.784; 0.785]

8 Given that $f(x) = 2x^3 + 9x^2 + 10x + 3$ and one of the roots of the equation $f(x) = 0$ is -3 ,

(i) factorise $f(x)$ completely. [2]

(ii) Express $\frac{10}{f(x)}$ in partial fractions. [3]

(iii) Evaluate $\int_0^1 \frac{10}{f(x)}\,dx$, giving your answer in the form of $\ln k$,

where k is a rational number. [4]

$$\left[(x+3)(2x+1)(x+1);\frac{1}{x+3}-\frac{5}{x+1}+\frac{8}{2x+3};\ln\frac{27}{8}\right]$$