Trigonometry

P1/5/1: Trigonometrical ratios/graphs

P1/5/2: Inverse trigonometric functions

P1/5/3: Identities

P1/5/4: Trigonometrical equations

P1/5/5: Revision

Prepared by

Tan Bee Hong

P1/5/1

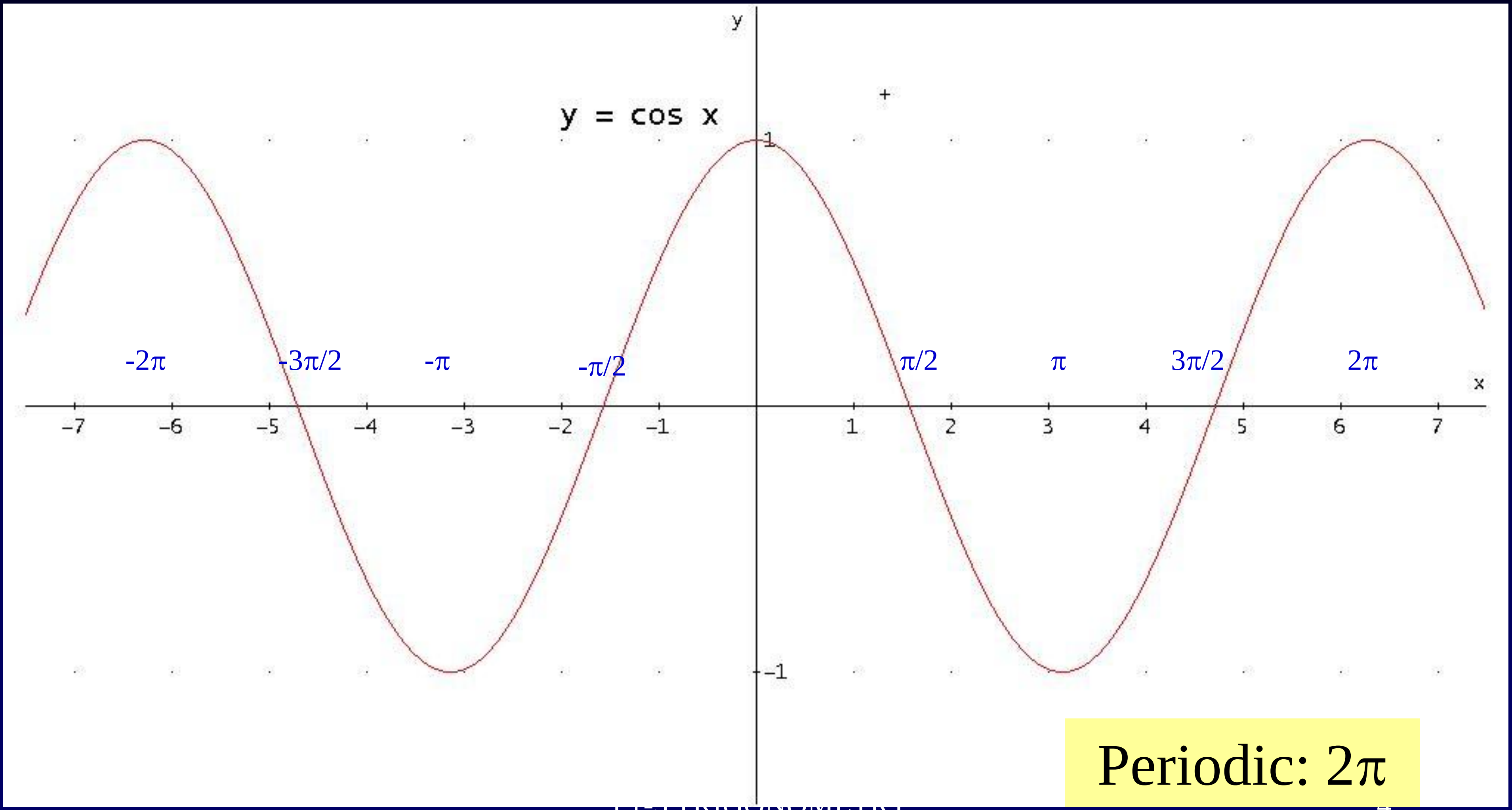
Trigonometrical ratios / graphs

Learning Outcome

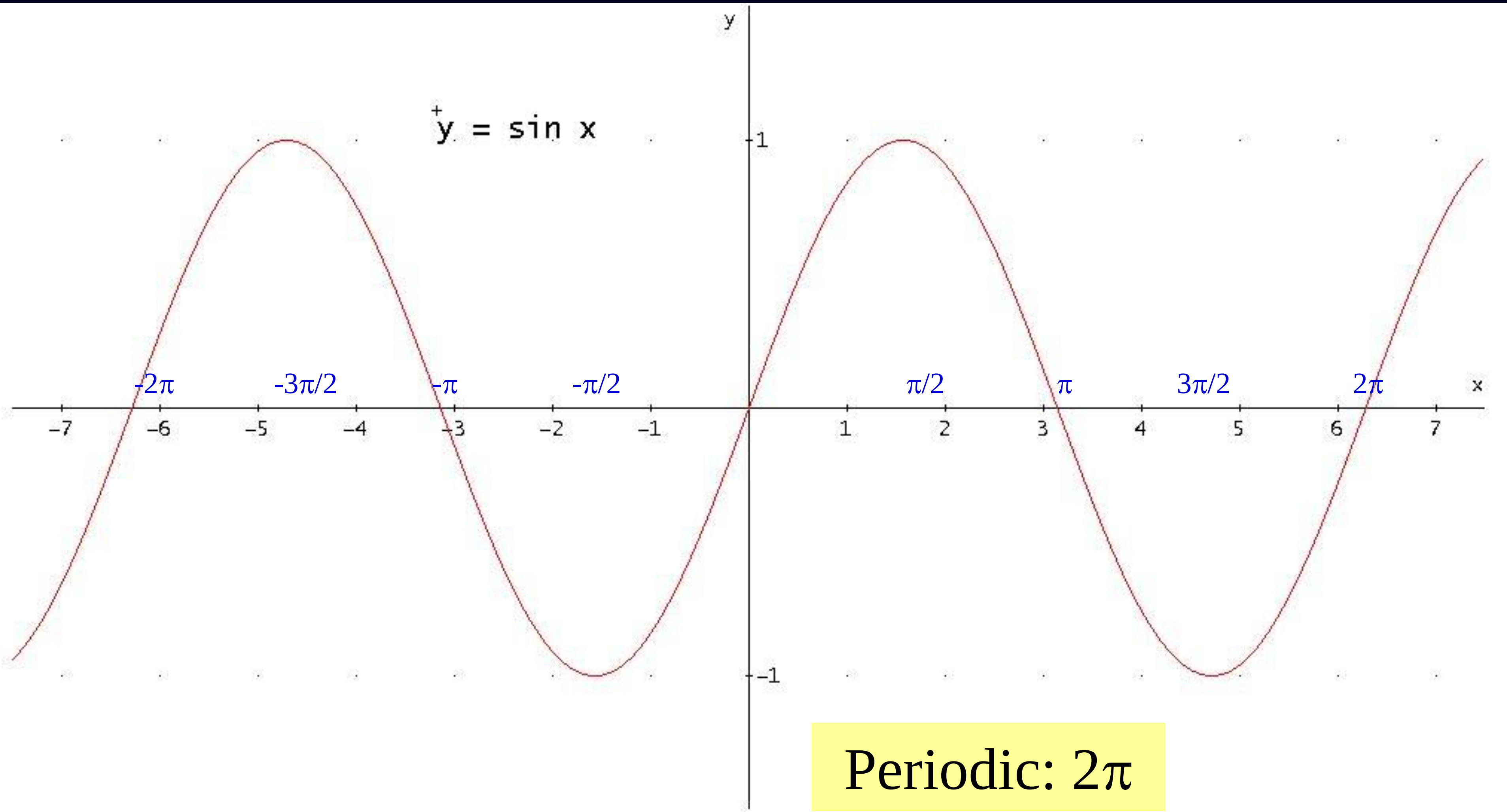
Students should be able to:

- sketch and use graphs of *t*-functions
- use the exact values of sine, cosine and tangent of 30^o, 45^o, 60^o and related angles.

Graphs of the trigonometry functions

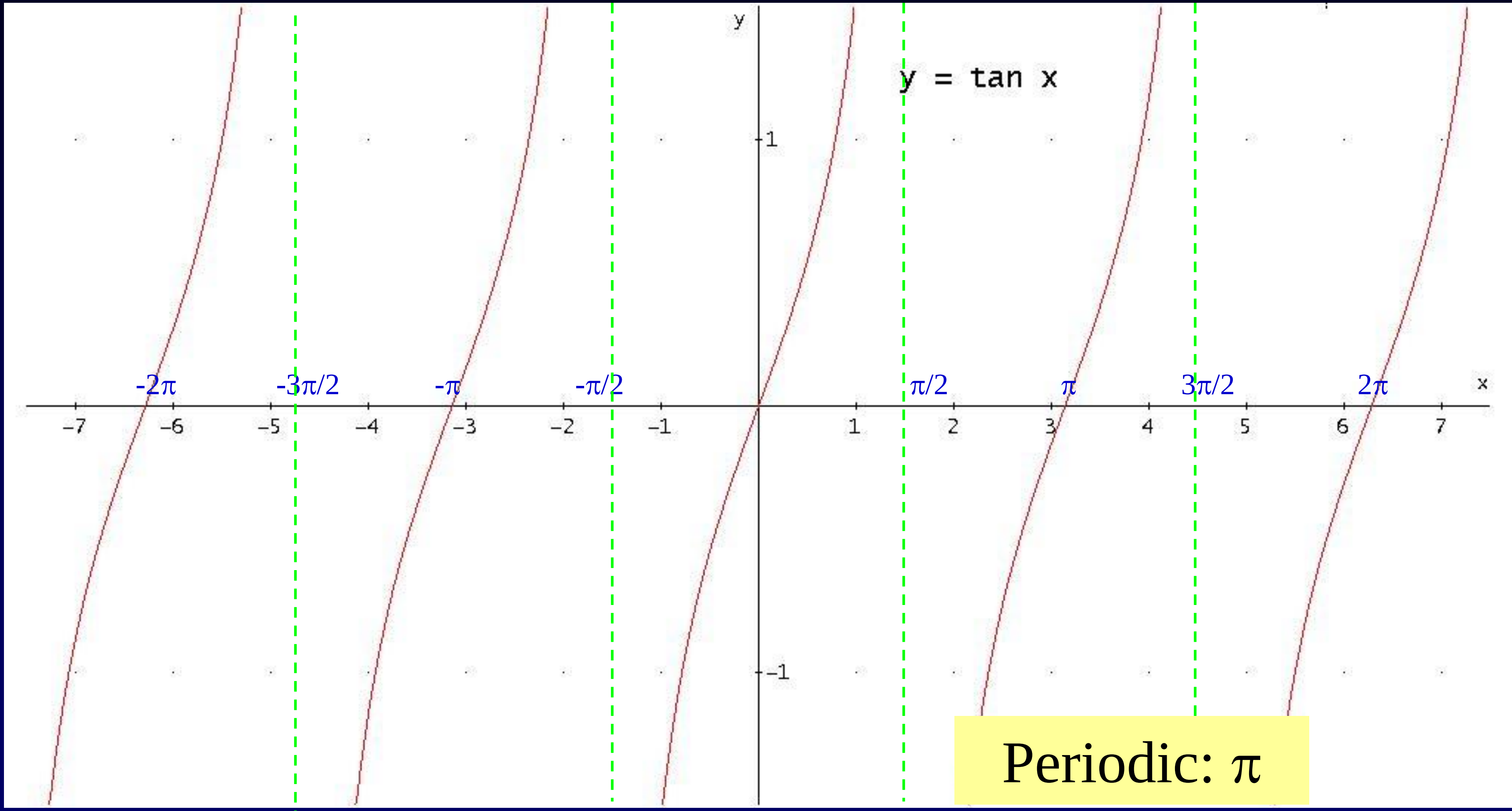


Graphs of the trigonometry functions



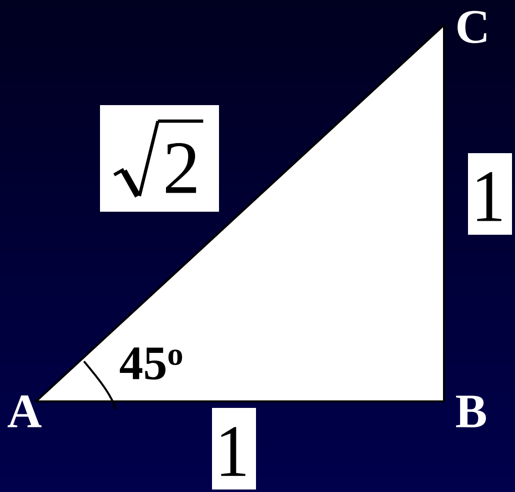
Periodic: 2π

Graphs of the trigonometry functions

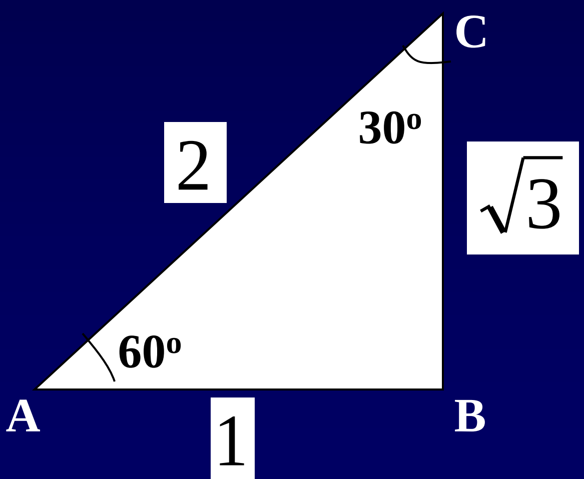


Periodic: π

Exact values of some trigonometric functions



$$\cos 45^{o} = \frac{1}{\sqrt{2}};\ \sin 45^{o} = \frac{1}{\sqrt{2}};\ \tan 45^{o} = 1$$



$$\cos 60^{o} = \frac{1}{2};\ \sin 60^{o} = \frac{\sqrt{3}}{2};\ \tan 60^{o} = \sqrt{3}$$

$$\cos 30^{o} = \frac{\sqrt{3}}{2};\ \sin 30^{o} = \frac{1}{2};\ \tan 30^{o} = \frac{1}{\sqrt{3}}$$

Symmetry properties of the graphs of *cos θ°*, *sin θ°*, and *tan θ°*

Periodic properties:

$$\cos(\theta \pm 360)^o = \cos\theta^o \quad \sin(\theta \pm 360)^o = \sin\theta^o \quad \tan(\theta \pm 180)^o = \tan\theta^o$$

odd/even properties:

$$\cos(-\theta)^o = \cos\theta^o \quad \sin(-\theta)^o = -\sin\theta^o \quad \tan(-\theta)^o = -\tan\theta^o$$

Example 1:

Write down the exact values of the following:

$$(a) \quad \cos\left(150^{o}\right)$$

$$(b) \quad \sin\left(-30\right)^{o}$$

$$(c) \quad \tan\left(-60\right)^{o}$$

$$(a) \quad \cos\left(225^{o}\right)$$

Graphs of the trigonometry functions

$$y = k\cos(m\theta \pm n)$$

amplitude

If + $\Rightarrow$ Shift to the left (n/m) units

If − $\Rightarrow$ Shift to the right (n/m) units

Period:

cosine and sine $\Rightarrow 2\pi/m$

tangent $\Rightarrow \pi/m$

Example 2:

Sketch the graphs:

$$(a)\quad \sin 2\theta$$

$$(b)\quad \tan\left(\frac{\theta}{2}+\frac{\pi}{2}\right)$$

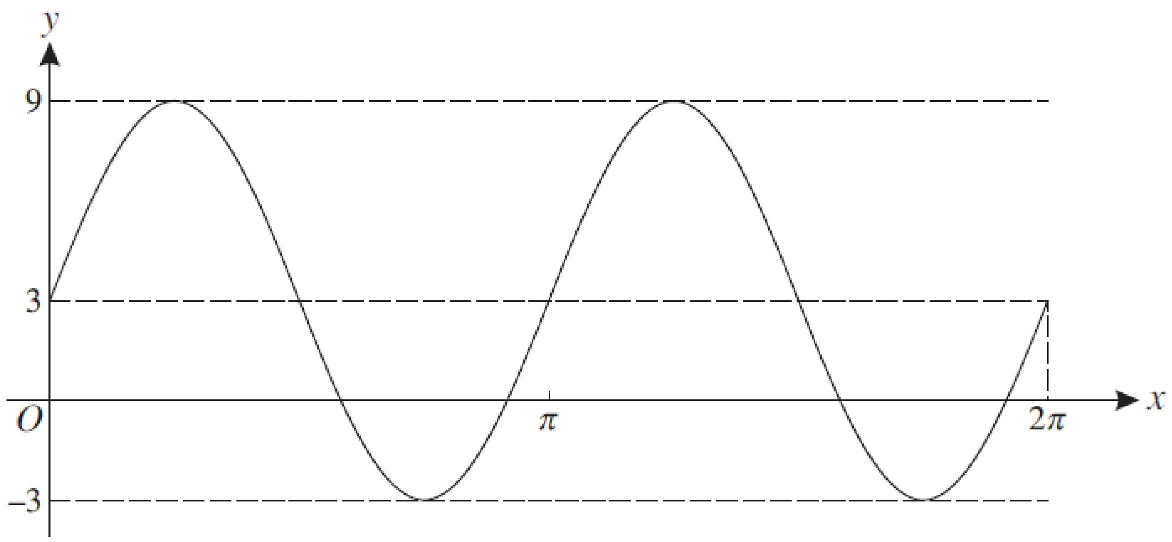
$$(c)\quad 4-2\cos\left(\theta-\frac{\pi}{2}\right)$$

Example 3:

(i) Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

Example 4:



The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$.

(i) Find the values of a, b and c. [3]

(ii) Find the smallest value of x in the interval $0 \leqslant x \leqslant 2\pi$ for which $y = 0$. [3]

Example 5:

The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f.

(ii) Sketch the graph of $y = f(x)$.

(iii) State, with a reason, whether f has an inverse.

Practice Exercise

Pure Mathematics 1 *Hugh Neil & Douglas Quadling (2002)*

Exercise 10A (Page 141)

Q2(e)(f), 5(k)(o), 6(c)(d), 7(f)(g)

Exercise 10C (Page 149)

Q11

P1/5/2

Inverse trigonometric functions

Learning Outcome

Students should be able to:

- use the notation $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to denote the principal values of the inverse trigonometric functions.

Example 6:

Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$,

(ii) $\tan^2 x$.

Example 7:

Given that

$$\boxed{\sin\theta^{o} = \frac{5\sqrt{3}}{14}}$$

and the angle θ^{o} is obtuse, find without using a calculator the values of $\cos\theta^{o}$ and $\tan\theta^{o}$.

Example 8:

Find all values of θ in the required interval which satisfy the following equations, giving your answers correct to 1 decimal place where appropriate.

$(a)\quad \cos\theta^{o} = 0.25$	$-180 \le \theta \le 180$
$(b)\quad \sin\theta^{o} = -0.67$	$-\pi \le \theta \le \pi$
$(c)\quad \cos 2\theta^{o} = \dfrac{1}{3}$	$0 < \theta \le 360$
$(d)\quad \cos(3t + 135)^{o} = \dfrac{1}{2}\sqrt{3}$	$0 < \theta \le 360$
$(e)\quad 4\tan\theta^{o} + 3 = 0$	$-\pi \le \theta \le \pi$
$(f)\quad \tan\left(\dfrac{3}{2}t - 45\right)^{o} = -\sqrt{3}$	$0 < t \le 360$

Example 9:

Find the maximum value and minimum value of *y*:

$$(a)\quad y = 5 + 8\cos 2x^o$$

$$(b)\quad y = \frac{8}{3 - \sin x^o}$$

and give the least positive values of *x* at which they occurs.

Practice Exercise

Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 10C (Page 148)

Q1(k)(n), 3(d)(f), 4(d)(e), 5(e)(f), 7(e)(f), 8(c)

P1/5/3

Identities

P1/5/4:
Trigonometrical equations

Learning Outcome

Students should be able to:

- use identities (basic identities)

Learning Outcome

Students should be able to:

- **Find all solutions of simple trigonometrical equations lying in a specified interval.**

Relations between the trigonometric functions

For all values of θ

$$\tan\theta^o \equiv \frac{\sin\theta^o}{\cos\theta^o}, \qquad \text{provided that } \cos\theta^o \neq 0;$$

$$\cos^2\theta^o + \sin^2\theta^o \equiv 1$$

Example 10:

Prove the identities:

$$(a) \quad \frac{1}{\cos\theta^o} + \tan\theta^o \equiv \frac{\cos\theta^o}{1-\sin\theta^o}$$

$$(b) \quad \frac{1-\tan^2 x}{1+\tan^2 x} \equiv 1-2\sin^2 x$$

$$(c) \quad \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2\tan^2 x.$$

Example 11:

Solve the following equation for θ,

giving solution in the interval $0 \le \theta \le 360$.

$\sin\theta^{o} = \tan\theta^{o}$

Example 12:

Solve the following equation for θ,

giving solution in the interval $0 \le \theta \le 360$.

$$\sin 2\theta^{o} - \sqrt{3}\cos 2\theta^{o} = 0$$

Example 13:

(i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$

(ii) Hence solve the equation

$$3\sin^2(\theta + 70^\circ) - 8\cos(\theta + 70^\circ) - 7 = 0$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$.

Example 14:

(i) Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$.

(ii) Hence solve the equation $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0^\circ \leqslant \theta \leqslant 360^\circ$.

Example 15:

(i) Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0^\circ \leqslant x \leqslant 180^\circ$. [3]

(ii) Verify that $x = 30^\circ$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0^\circ \leqslant x \leqslant 180^\circ$. [2]

(iii) Hence state the set of values of x, for $0^\circ \leqslant x \leqslant 180^\circ$, for which $\sin x < \cos 2x$. [2]

Practice Exercise

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Exercise 10D (Page 152)

Q2, 3(b)(d), 4(c)(d), 5

Practice Exercise

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Miscellaneous exercise 10 (Page 152)

Q5, Q7, Q8, Q10(c)(d), Q11(e)(f), Q12(c)