Pure Mathematics 1

Tutorial 5 Trigonometry

1	Prove the following identity : $\frac{1}{\tan x} + \tan x \equiv \frac{1}{\sin x \cos x}$.
2	Prove the identity $\frac{1-2\sin^2\theta}{\cos\theta+\sin\theta} = \cos\theta - \sin\theta$
3	Prove the identity $(\frac{1}{\cos x} - \tan x)(\frac{1}{\sin x} + 1) = \frac{1}{\tan x}$
4	(a) Find the set of values of x for which $4x(x-1) > 3$. [2] (b) Using your answer in part (a), find the range of values of θ, where $0° < \theta < 360°$, for which $4\sin\theta(\sin\theta - 1) > 3$. [3] $\left[x < -\frac{1}{2}, x > \frac{3}{2}; 210 < \theta < 330\right]$
5	.a) Given that $f(x) = \cos x$ for the domain $0 \le x \le k$, state the largest value of k, in radians, for which f has an inverse. [1] b) Sketch and label, on the same diagram, the graphs of $y = \cos x$ and $y = \sin 2x$ in the interval $0 \le x \le 2\pi$. [4] $[\pi]$
6	(a) Solve $\sin 2\alpha = \sin 30°$ for $0° \le \alpha \le 360°$. [2] (b) State the least value of $2\cos(3\theta + \frac{\pi}{4})$. For $0 \leqslant \theta \leqslant \pi$, state the corresponding values of θ, that give this least value. [3] $\left[15, 75, 195, 255; \frac{\pi}{4}, \frac{11}{12}\pi\right]$

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Solve $2\cos^2 2x + \sin 2x = 1$ for $-90° \le x \le 90°$.

$[-15, -75, 45]$

8

a) By completing the square, find the greatest value of $\cos^2\theta - \cos\theta + 6$ and the value of θ when this occurs. [4]

b) Solve the equation $\sin 4x = \frac{1}{2}$, giving all solutions such that $0 \le x \le 180$ [2]

$\left[\frac{23}{4}, 60; 7.5, 37.5, 97.5, 127.5\right]$

9

Solve the equation $4\sin^2 2\theta \cos 2\theta = \tan^2 2\theta$, giving all the solutions in the interval $0^0 \le \theta \le 360^0$. State your answers in **1 decimal place.** [7]

$[0, 90, 180, 270, 360, 25.5, 154.5, 205.5, 334.5]$

10

Find all values of x for which $0° < x < 360°$ that satisfy the equation

(a) $\sin\left(\frac{1}{2}x\right) = \frac{1}{4}$. [3]

(b) $1 - \cos^2 x = 3\sin x - 4\sin^3 x$. [4]

$[29, 331; 48.6, 131.4, 180, 270]$

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(a) Sketch the graph of $2\sin\theta$ for $0° \le \theta \le 360°$. [2]

(b) Solve the following equation for all values of θ from -180° to + 180°:

$4 - \sin\theta = 4\cos^2\theta$ [5]

$[0, 14.5, 180, 165.5]$

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(a) Find all the values of y which satisfy $\tan 2y = 2$ in the interval $0° < y < 360°$ [3]

(b) Show that the equation $\cos\theta + \sin\theta = 2(\sin\theta - \cos\theta)$ can be express as $\tan\theta = 3$ and hence solve the equation for $0° \le \theta \le 360°$ [4]

$[31.7, 121.7, 211.7, 301.7; 71.6, 251.6]$

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a) Prove the identity $(1+\sin\theta)(\frac{1}{\cos\theta}-\tan\theta)\equiv\cos\theta$. [4]

b) Hence, solve the equation $(1+\sin\theta)(\frac{1}{\cos\theta}-\tan\theta)=2\cos^2\theta-1$ for $0\le\theta\le 360°$. [4]

[120,240,0,360]

14

The level of water, Y metres above sea level, at time t hours after midnight is given by $Y = 4.3 + 2.1\cos(15t - 50)°$, $0 < t \le 24$.

(a) Find the maximum height of the water level, and the time when this happens. [3]

(b) Find the values of t, correct to one decimal place, when $Y = 5.5$. [5]

[6.4,3.20*am*;7.0,23.7]

15

(i) On the same axes, sketch $y = \sin 2x$ and $y = \cos 3x$ for $0 \le x \le \pi$. Hence find the number of solutions to the equation $\sin 2x = \cos 3x$ in the same interval. [3]

(ii) Find all the values of x, $0° \le x \le 360°$, which satisfy the equation

$$3\sin x = 2 + \frac{1}{\sin x}.$$ [4]

[3 ; 199.5 , 340.5 , 90]

16

a) Find all the solutions in the interval $0° < \theta < 180°$ of the equation $2\sin(3\theta - 48°) - 1 = 0$. [5]

b) Show that $\sqrt{2\sin^2\theta + 6\cos^2\theta - 2} \equiv 2\cos\theta$. [3]

[26 ,66 , 146]

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a) Prove that $(1 + \sin\theta + \cos\theta)^2 \equiv 2(1 + \sin\theta)(1 + \cos)$. [4]

b) Find all values of θ, such that $0 \le \theta \le \pi$, which satisfy the equation

$4\sin\theta\cos\theta = \tan\theta$. [5]

$\left[0 , \pi , \frac{1}{3}\pi , \frac{2}{3}\pi\right]$

18

(a) If $\sin^{-1}x = \frac{\pi}{5}$, find the value of $\cos^{-1}x$. [2]

(b) A line goes through (a,b) and makes an angle of θ with the x-axis. Show that the equation of the line can be written as
$y\cos\theta - x\sin\theta = b\cos\theta - a\sin\theta$. [3]

(c) Solve the equation $2\tan\theta - \frac{4}{\tan\theta} = \frac{1}{\sin\theta}$, giving your answers in radians in the range $-\pi < \theta < \pi$, correct to 2 significant figures. [4]

$\left[\frac{3}{10}\pi\ ;\ \pm 2.3\ ,\ 1.0\right]$

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(i) Sketch the graph of the equation $y = -2\sin\frac{1}{2}x$ for $-\pi \le x \le \pi$. State the period. [3]

(ii) Prove the identity $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha\sin^2\alpha$. [3]

(iii) Find the solutions of the equation $\tan^2 x\sin x = \sin x$ in the interval $0° \le x < 360°$. [3]

[4π ; 0 , 180 ,45 , 135 , 225 , 315]

20

a) Solve the equation $\cos(3x) + \sqrt{3}\sin(3x) = 0$ for which $0 < x < \pi$. [4]

b) Sketch the graph $y = 3\sin x$ for $0 \le x \le 2\pi$. [2]

c) Find the values of θ such that $4 + \sin\theta = 6\cos^2\theta$ for $-180^0 \le \theta \le 180^0$. [5]

$\left[\frac{5}{18}\pi, \frac{11}{18}\pi, \frac{17}{18}\pi; 30, 150, -138.2, -41.8\right]$

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(i) The curve with equation $y = 2 + k\sin x$ passes through the point with coordinates $\left(\frac{\pi}{2}, -2\right)$. Find

(a) the value of k, [2]

(b) the values of x in the interval $0 \le x \le 2\pi$ for which $y = 2 + 2\sqrt{2}$. [3]

(ii) Write down the maximum and minimum values of the expression $4 - 2\sin 3x$ as x varies. [2]

$\left[-4;\ \frac{5}{4}\pi, \frac{7}{4}\pi;\ \max = 6, \min = 2\right]$