Paper 6 -Topic 3 - Probability

All probabilities must lie between 0 and 1 inclusive. The sum of the probabilities of all possible outcomes is 1.

The sample space, S is the name given to all possible outcomes.

The probability P(A) of an event A is the sum of the probabilities of the outcomes which make up A.

$$P(A) = n(A)/n(S)$$

Example 1

Two fair coins are tossed. Show the possible outcomes on a possibility space diagram and find the probability that two heads are obtained.

If A is an event, the A′ is the complement of A ('not A') and

$$P(A) + P(A') = 1$$

Example 2

A group of 20 university students contains eight who are in their first year of study. A student is picked at random to represent the group at a meeting. Find the probability that the student is not in the first year of study.

ILLUSTRATING TWO OR MORE EVENTS USING VENN DIAGRAMS

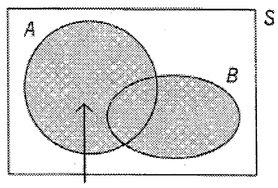
Suppose A and B are two events associated with the same experiment. Consider the outcomes described below

(a) $A \cup B$

In set language, the set that contains the outcomes that are in **A or B or both** is called the **union** of A and B and is written **$A \cup B$**.

To represent $A \cup B$ on the Venn diagram, shade the whole of the coloured 'figure-of-eight' shape.

Remember that although this outcome is written A or B it includes the events that are in both A and B as well.

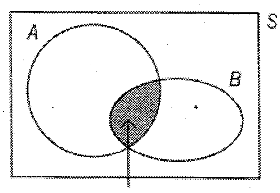


$A \cup B$ means A or B or both.

(b) $A \cap B$

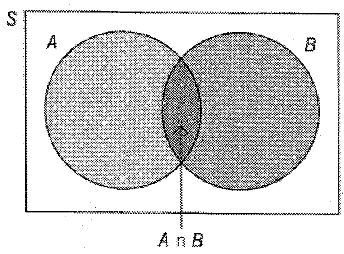
In set language, the set that contains the outcomes that are in **both A and B** is called the **intersection** of A and B and is written **$A \cap B$**.

To represent $A \cap B$ on the Venn diagram, shade the overlap of A and B. This outcome is often written A and B.



$A \cap B$ means A and B.

PROBABILITY RULE FOR COMBINED EVENTS



If the number of outcomes in A is $n(A)$ and the number of outcomes in B is $n(B)$, then for two overlapping sets A and B, if you add $n(A)$ and $n(B)$ together you will count the overlap twice.

So to find the number of outcomes in $A \cup B$ you have to take one overlap away like this:

$n(A \cap B) = n(A) + n(B) - n(A \cap B)$

Dividing by $n(S)$, this becomes

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)\ .$$

Alternatively

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Remember that the word
or means **A or B or both.**

Example 3

In a class of 20 children, 4 of the 9 boys and 3 of the 11 girls are in the athletics team. A person from the class is chosen to be in the 'egg and spoon' race on Sports Day. Find the probability that the person chosen is

(a) in the athletics team,
(b) female,
(c) a female member of the athletics team,
(d) a female or in the athletics team.

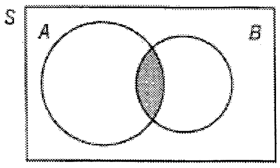
Example 4

Events C and D are such that $P(C) = \dfrac{19}{30}$, $P(D) = \dfrac{2}{5}$ and $P(C \cup D) = \dfrac{4}{5}$.

Find $P(C \cap D)$.

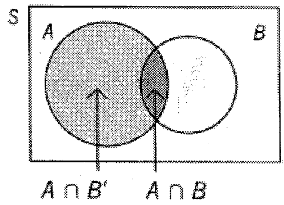
Other useful results relating two events A and B

(a)



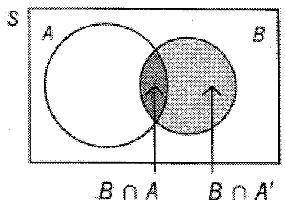
$$\underbrace{P(A \cap B)}_{P(A \text{ and } B)} = \underbrace{P(B \cap A)}_{P(B \text{ and } A)}$$

(b)



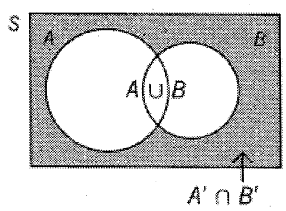
$$P(A) = \underbrace{P(A \cap B)}_{P(A \text{ and } B)} + \underbrace{P(A \cap B')}_{P(A \text{ but not } B)}$$

(c)



$$P(B) = \underbrace{P(B \cap A)}_{P(B \text{ and } A)} + \underbrace{P(B \cap A')}_{P(B \text{ but not } A)}$$

(d)



$P(\text{neither } A \text{ nor } B) = 1 - P(A \text{ or } B)$

i.e. $\quad P(A' \cap B') = 1 - P(A \cup B)$

Example 5

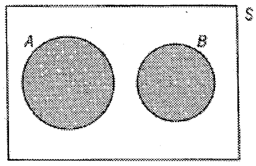
In a survey, 15% of the participants said that they had never bought lottery tickets or a premium bonds, 73% had bought lottery tickets and 49% had bought premium bonds.

Find the probability that a person chosen at random from those taking part in the survey

(a) had bought lottery tickets or premium bonds,
(b) had bought lottery tickets and premium bonds,
(c) had bought lottery tickets only.

EXCLUSIVE (OR MUTUALLY EXCLUSIVE) EVENTS

Consider events, A and B, of the same experiment.
A and B are said to be **exclusive** (or **mutually exclusive**) if they cannot occur at the same time. For example, with one throw of a die you cannot score a three and a five at the same time, so the events 'scoring a 3' and 'scoring a 5' are exclusive events.



If A and B are exclusive, then $P(A \cap B) = 0$ since $A \cap B$ is an impossible event. There is no overlap of A and B.

For exclusive events, the rule for combined events becomes

$$P(A \cup B) = P(A) + P(B)$$

This is known as the **addition rule** for exclusive events.

It is also known as the '**or**' rule for exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)$$

Extending this result to n exclusive events,

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \ \dots \text{ or } A_n) = P(A_1) + P(A_2) + P(A_3) + \dots + P(A_n)$$

Example 6

In a race in which there are no dead heats, the probability that John wins is 0.3, the probability that Paul wins is 0.2 and the probability that Mark wins is 0.4.

Find the probability that

(a) John or Mark wins,
(b) John or Paul or Mark wins,
(c) someone else wins.