The Normal Distribution

S1/5/1: Normal distribution tables
S1/5/2: Using standard normal tables for any normal variable X
S1/5/3: Problems involving the value Miscellaneous questions of μ or σ or both as unknown
S1/5/4: Discussion and evaluation
S1/5/5: The normal approximation to the binomial distribution
S1/5/6: The normal approximation to the binomial distribution

S1/5/1:
Normal distribution tables

Learning Outcome

Students should be able to:

- Associate the use of a normal distribution to model a continuous random variable
- Use the normal distribution tables

Normal Probability Distribution

1. A continuous random variable
2. Normal probability distribution function:
   This is the function for the normal (bell-shaped) curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(x-\mu)}{\sigma}\right]^2} \qquad ,-\infty < x < \infty.$$

3. The probability that x lies in some interval is the area under the curve

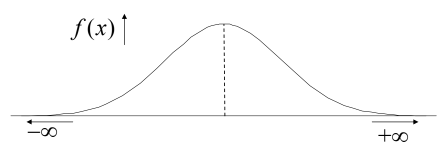
To describe the distribution, write $X \sim N(\mu, \sigma^2)$

mean variance

Normal Probability Distribution

Normal probability curve has the following features:

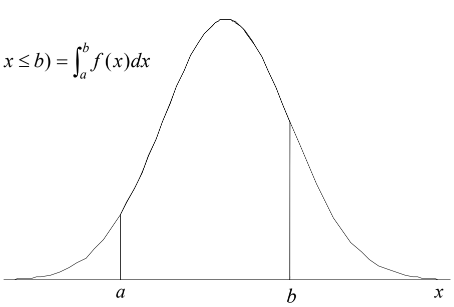
- It is bell-shaped
- It is symmetrical about μ
- It extends from –∞ to +∞
- The total area under the curve is 1



Probabilities for a Normal Distribution

- **Illustration**

$$P(a \le x \le b) = \int_a^b f(x)dx$$

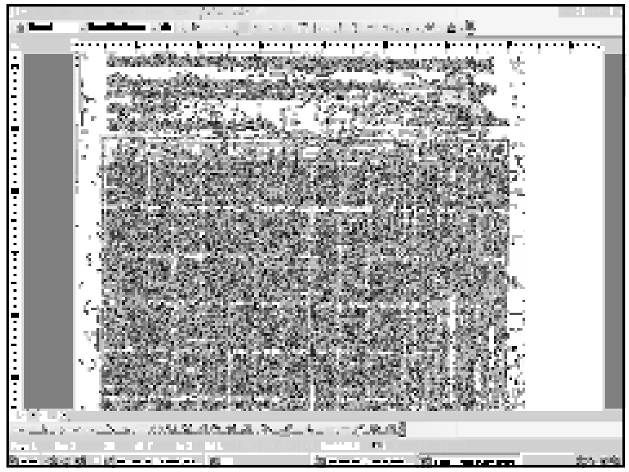


Notes

- We will use a table to find probabilities for normal distributions
- We will learn how to compute probabilities for one special normal distribution: the standard normal distribution
- Transform all other normal probability questions to this special distribution

The Standard Normal Distribution

- The standard normal distribution is the normal distribution of the standard variable *z* (the *z*-score)
- The total area under the normal curve is equal to 1
- The distribution has a mean of 0 and a standard deviation of 1
- The mean divides the area in half, 0.50 on each side
- Nearly all the area is between $z = -3.00$ and $z = 3.00$



Example 1:

Find the area to the left of z = 1.45; P(z < 1.45)

Example 2:

Find the area under the standard normal curve between $z = 0$ and $z = 1.45$

Example 3:

Find the area under the normal curve to the right of $z = 1.45$; $P(z > 1.45)$

Example 4:

Find the area between the mean ($z = 0$) and $z = -1.26$

Example 5:

Find the area to the left of -0.98; $P(z < -0.98)$

Example 6:

Find the area between $z = -2.30$ and $z = 1.80$

Example 7:

Find the area between $z = -1.40$ and $z = -0.50$

Example 8:

Find $P(| Z | < 1.433)$.

Example 9:

Find $P(| Z | > 1.433)$.

Normal Distribution Note

- The normal distribution table may also be used to determine a *z*-score if we are given the area (working backwards)

Example 10:

What z-scores bound the middle 90% of a standard normal distribution?

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<u>Exercise 9A (Page 143)</u>
Q3, Q4

S1/5/2 & S1/5/3:

Using standard normal tables for any normal variable X
&
Problem involving the value of μ or σ or both as unknowns

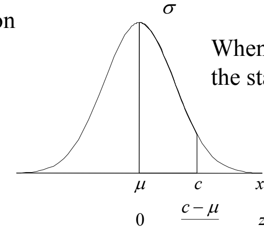
Learning Outcome

Students should be able to:

- Calculate Z by standardizing X
- Interpret and solve problems concerning P(X > x), or a related probability, given the values of x, μ, σ.
- Interpret and solve problems involving the value of μ or σ or both as unknown.

Standardization

- Suppose x is a normal random variable with mean μ and standard deviation σ
- The random variable $z = \frac{x-\mu}{\sigma}$ has a standard normal distribution



When $x = \mu$,
the standard score $z = 0$

Example 11:

Consider the intelligence quotient (I.Q.) score are normally distributed with a mean of 100 and a standard deviation of 16.If a person is picked at random, what is the probability that his or her I.Q. is greater than 90.

Example 12:

A bottling machine is adjusted to fill bottles with a mean of 32.0 oz of soda and standard deviation of 0.02. Assume the amount of fill is normally distributed and a bottle is selected at random:

1) Find the probability the bottle contains between 32.00 oz and 32.025 oz
2) Find the probability the bottle contains more than 31.97 oz

Example 13:

Lengths of metal strips produced by a machine are normally distributed with mean length of 150 cm and a standard deviation of 10 cm.

Find the probability that the length of a randomly selected strip is within 5 cm of the mean.

Notes

- Often we need to find a *cutoff point*: a value of x such that there is a certain probability in a specified interval defined by x

Example 14:

The marks of 500 candidates in an examination are normally distributed with a mean of 45 marks and a standard deviation of 20 marks.

(a) Given that the pass mark is 41, estimate the number of candidates who passed the examination.

(b) If 5% of the candidates obtain a distinction by scoring x marks or more, estimate the value of x.

(c) Estimate the interquartile range of the distribution.

Finding the value of μ or σ or both

Example 15

A radar unit is used to measure the speed of automobiles on an expressway during rush-hour traffic. The speeds of individual automobiles are normally distributed with a mean of 62 mph. Find the standard deviation of all speeds if 3% of the automobiles travel faster than 72 mph.

Example 16:

The speeds of cars passing a certain point on a motorway can be taken to be normally distributed. Observations show that of cars passing the point, 95% are travelling at less than 85 m.p.h. and 10% are travelling at less than 55 m.p.h.

Find the average speed of the cars passing the point.

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<u>Exercise 9B (Page 146)</u>
Q3, Q7, Q12

<u>Exercise 9C (Page 148)</u>
Q2, Q4, Q6, Q8, Q10

S1/5/5 & S1/5/6:

The normal approximation to the binomial distribution

Learning Outcome

Students should be able to:

- Apply the normal approximation to the binomial distribution in problem solving
- Apply the normal approximation to the binomial distribution in solving harder questions

Normal Approximation of the Binomial

The binomial distribution is a probability distribution of the discrete random variable *x*, the number of successes observed in *n* repeated independent trials

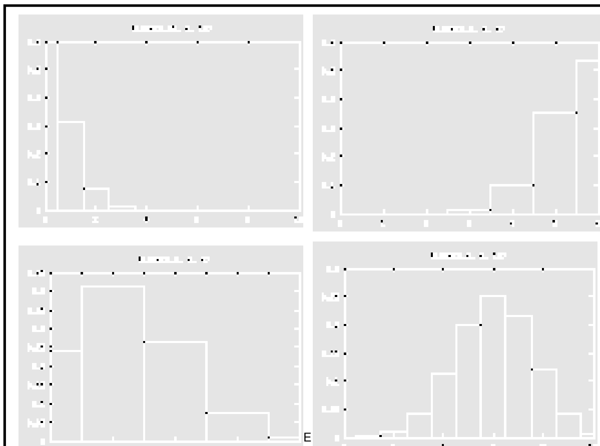
As *n* becomes larger, the distribution appears more and more like the normal distribution.

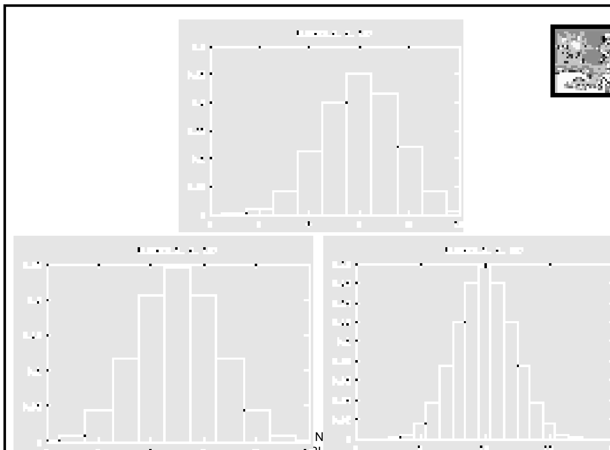
Binomial probabilities can be reasonably estimated by using the normal probability distribution

Normal Approximation of the Binomial

One major difference between the binomial and the normal probability distribution :

The binomial random variable is discrete, whereas the normal random variable is continuous.





Two Problems

1. As p moves away from 0.5, the binomial distribution is less symmetric, less normal-looking

   Solution: The normal distribution provides a reasonable approximation to a binomial probability distribution whenever the values of np and $n(1 - p)$ both exceed 5

2. The binomial distribution is discrete, and the normal distribution is continuous

   Solution: Use the continuity correction factor. Add or subtract 0.5 to account for the width of each rectangle.

If $X \sim B(n, p)$, and if $np > 5$ and $nq > 5$, where $q = 1 - p$, then the distribution of X can reasonably be approximated by $V \sim N(np, npq)$.

Continuity correction factors
Examples:

$P(x = 9) \approx$

$P(x \geq 9) \approx$

$P(x \leq 9) \approx$

$P(x > 9) \approx$

$P(x < 9) \approx$

$P(7 < x < 9) \approx$

$P(7 < x \leq 9) \approx$

$P(7 \leq x < 9) \approx$

$P(7 \leq x \leq 9) \approx$

Example 17:

A random variable X has a binomial distribution with parameters n = 80 and p = 0.4. Use a suitable approximation to calculate the following probabilities.

$(a)\ \ P(X \leq 34)$ $\qquad (b)\ \ P(X \geq 26)$

$(c)\ \ P(X = 33)$ $\qquad (d)\ \ P(30 < X \leq 40)$

Example 18:

Find the normal approximation of the binomial probability p (x = 4,5), where n = 14 and p = 0.5. Compare this to the value of p (x = 4,5) obtained from binomial probability.

Example 19:

Research indicates 40% of all students entering a Certain university withdraw from a course during their first year. What is the probability that fewer than 700 of this year's entering class of 1800 will withdraw from a class?

Example 20:

Find the normal approximation for the binomial probability P($x \geq 9$), where n = 13 and p = 0.7. Compare this to the value of P($x \geq 9$) obtained from binomial probability.

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<u>Exercise 9D (Page 157)</u>
Q2, Q4, Q6, Q8, Q10