# Complex Numbers

# A2 Trial Examination

**August 2010**

11 (a) A complex number $z$ satisfies $|z| = |z - u|$, where $u$ is denoted by $-2+2i$.

(i) Sketch the locus of the point which represents $z$ in an Argand diagram. [2]

(ii) Find the least value of $|z|$. [2]

(iii) Express $u$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

(b) The complex number $w$ is such that $ww^* + 2w = 3 + 4i$, where $w^*$ is the complex conjugate of $w$. Find $w$ in the form $a + bi$, where $a$ and $b$ are real. [4]

$$\left[\sqrt{2}\,,\ \sqrt{8}\,(\cos\frac{3\pi}{4} + i\,sin\frac{3\pi}{4})\,;\ -1+2i\right]$$

**March 2010**

10 (a) Solve the equation $\left(\frac{3-i}{2+i}\right)z = \frac{5+5i}{-1+2i}$, giving your answer in the form $a + ib$. [4]

(b) The roots of the equation $z^2 + 6z + 13 = 0$ are denoted by $z_1$ and $z_2$, where $\arg(z_1) > 0$. Find $z_1$ and $z_2$, and show these roots on a sketch of an Argand diagram. [3]

Find the modulus and argument of $(z_1 + 1)$. [2]

On the same diagram, sketch the loci given by $|z+1| = 2\sqrt{2}$ and $\arg(z+1) = \frac{\pi}{4}$. Hence, find in the form of $a + ib$, the complex number satisfying the above two loci. [4]

$$\left[2 - i\,;\ -3+2i, -3-2i\,;\, 2\sqrt{2}\,, \tfrac{3\pi}{4}\,,\ \ 1+2i\right]$$

**August 2009**

11 (a) A complex number z satisfies $|z - 3 - 4i| = 2$. Describe in geometrical terms, with the aid of a sketch, the locus of the point which represents z in an Argand diagram. Find [2]

(i) the greatest value of $|z|$, [2]

(ii) the difference between the greatest and least values of arg z. [2]

(b) Given that $z = 1 + i\sqrt{3}$,

(i) find $|z|$ and arg z. [2]

(ii) Hence, or otherwise, show that $z^5 - 16z^* = 0$. [5]

$$\left[circle\ centre\ at\ (3,4)\ with\ radius\ 2, \quad 7, 0.823, \quad 2, \frac{\pi}{3}\right]$$

**March 2009**

9 (i) Suppose $z = \frac{1}{2}(\cos\theta + i\sin\theta)$.

(a) State $|z|$ and find $(1-z)^*$ [3]

(b) Find the imaginary part of $\frac{1}{1-z}$ [4]

(ii) Given the set of the complex numbers $z$ is such that $|z-2i| = 1$.

Sketch $|z-2i| = 1$ on an Argand diagram. Find the largest possible value of arg $z$. [4]

$$\left[\frac{1}{2}, \left(1 - \frac{1}{2}cos\theta\right) + \frac{1}{2}sin\theta i; \frac{2sin\theta}{5 - 4cos\theta}; \frac{2\pi}{3}\right]$$

**August 2008**

6 (i) Find the modulus and argument of each of the two complex numbers $z$ satisfying the equation $\frac{1+z^2}{1-z^2} = i$ [4]

(ii) Sketch in an Argand diagram the set of points satisfying both

$|z| < |z-1|$ and $-\frac{\pi}{4} < \arg z < \frac{\pi}{4}$. [3]

$$\left[|z_1| = 1, \arg z_1 = \frac{\pi}{4}; |z_2| = 1, \arg z_2 = -\frac{3\pi}{4}\right]$$

**March 2008**

9. i) The complex number $w$ has modulus $\sqrt{2}$ and argument $-\frac{3}{4}\pi$, and the complex number $z$ has modulus 2 and argument $-\frac{1}{3}\pi$. Find the modulus and principal argument of $wz$, in exact form. [3]

ii) On a single Argand diagram, sketch the following loci.

a) $|z-2i| = 4$ [1]

b) $\arg(z+2) = \frac{1}{4}\pi$ [1]

Hence or otherwise, find the exact value of $z$ satisfying both equations in parts a) and b). [5]

$$\left[2\sqrt{2}; \frac{11}{12}\pi; \text{circle centre at } (0,2) \text{ and radius } 4; \text{half line from } (-2,0) \text{ making angle of } \frac{\pi}{4}; 2\sqrt{2}, 2+2\sqrt{2}\right]$$

**August 2007**

11. The complex number $\dfrac{2}{-1+i}$ is denoted by $u$.

(i) Find the modulus and argument of $u$ and $u^2$. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers $u$ and $u^2$. Shade the region whose points represent the complex number $z$ which satisfy both the inequalities $|z| < 2$ and $-\dfrac{\pi}{4} \le \arg(z - u^2) \le 0$.

[4]

$$\left[\sqrt{2}, -\frac{3\pi}{4}; 2, \frac{\pi}{2}\right]$$

**March 2007**

8. Find the modulus of the complex number $\dfrac{7+3i}{5-2i}$, and show that its argument is $\frac{1}{4}\pi$. (4)

Describe with the aid of a sketch the locus of the point in an Argand diagram representing the complex number $z$, where $\arg\left(\dfrac{z}{5-2i}\right) = \frac{1}{4}\pi$. (4)

$$\left[\sqrt{2}; \arg z = 0.4049\right]$$

**August 2006**

9. (a) Find the argument of the complex number $\dfrac{5+i}{2+3i}$. [3]

(b) Given that w and z are two complex numbers such that
$|w - 6 + 4i| = 5$ and $|z + 2 - 4i| \le 4$.

(i) Sketch and label the loci of w and z. [2]

(ii) Obtain the exact least possible value of $|w - z|$. [1]

(c) Find, in the form a + bi, where a and b are real and exact, the product of
$(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6})^2$ and $2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$. [4]

$$\left[-\frac{\pi}{4}; \text{circle centre (6,-4) and radius 5; shaded circle centre (-2,4) and radius 4}; 8\sqrt{2} - 9; \sqrt{3} - i\right]$$

**March 2006**

10. It is given that $\alpha = 1 + \sqrt{3}\,i$ is a root of the cubic equation $3z^3 - 4z^2 + 8z + 8 = 0$.

(a) Write down another complex root $\beta$, and hence find the real root of the equation. [3]

(b) Find the modulus and argument of each of the complex numbers $\alpha$, $\beta$, $\alpha\beta$ and $\frac{\alpha}{\beta}$. [4]

(c) Sketch the locus of points in the Argand diagram representing the complex numbers z for which $|z - \alpha| = \sqrt{3}$. [2]

$$\left[1-\sqrt{3}i;\ -\frac{2}{3};\ 2 \text{ and } \frac{\pi}{3};\ 2 \text{ and } -\frac{\pi}{3};\ 4 \text{ and } 0;\ 1 \text{ and } \frac{2\pi}{3}\right]$$

**August 2005**

6. ( i ) Express $10\,i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0^0 < \theta < 180^0$.

[ 1 ]

( ii ) The complex number $5 - 2i$ is denoted by $a$. Express $\frac{a}{a^*}$ in the form $p + iq$.

[ 2 ]

( iii ) In each of the following cases, show by a clear drawing in an Argand diagram, the set of points representing z, given that

( a ) $|z-1| = |z+i|$ ( b ) $Re(z+2) = 3$. [ 4 ]

$$\left[10(\cos 90^0 + i\sin 90^0);\ \frac{21}{29} - \frac{20}{29}i\right]$$

**March 2005**

9 The complex number $z_1$ has modulus 2 and argument $\frac{\pi}{3}$.

(i) Write $z_1$ in the form $a + bi$, where $a, b \in \Re$. [1]

(ii) Write down the modulus and argument of :

(a) $z_1^3$ [2]

(b) $\frac{1}{z_1^2}$ [2]

(iii) Given that $z_2 = 3 - i$ and $z_3 = 2 + 3i$,

(a) find $\frac{z_3}{z_2}$ in the form $c + di$ where $c, d \in \Re$ [2]

(b) Use an Argand diagram to find, in the form $a + bi$, the complex number which satisfy the following pairs of equation :

$|z - z_2| = 3$ and argument $(z - z_2) = \frac{\pi}{3}$.

[5]

$$\left[1+\sqrt{3}i; 8 \text{ and } \pi; \frac{1}{4} \text{ and } -\frac{2\pi}{3}; \frac{3}{10}+\frac{11}{10}i; z = 4.5+\left(\frac{3\sqrt{3}}{2}-1\right)i\right]$$