Exercise 1: Integrating Exponential Functions

Integrate each function with respect to x.

1 e^{4x}

2 $4e^{-x}$

3 $e^{(3x-2)}$

4 $2e^{(1-5x)}$

5 $6e^{-2x}$

6 $5e^{(x-3)}$

7 $e^{\left(2+\frac{x}{2}\right)}$

8 $e^{2x}+\frac{1}{e^{2x}}$

Evaluate the following definite integrals.

9 $\int_0^2 e^{2x}dx$

10 $\int_{-1}^1 2e^{(x+1)}dx$

11 $\int_2^3 e^{(2-x)}dx$

12 $\int_0^2 -e^x dx$

Exercise 2: Integrating $1/x$

Integrate with respect to x

1 $\frac{1}{2x}$

2 $\frac{2}{x}$

3 $\frac{1}{4x}$

4 $\frac{3}{2x}$

5 $\frac{4}{x-1}$

6 $\frac{1}{3x+1}$

7 $\frac{3}{1-2x}$

8 $\frac{6}{2+3x}$

9 $\frac{3}{4-2x}$

10 $\frac{4}{1-x}$

11 $\frac{5}{6-7x}$

Evaluate

12 $\int_1^2 \frac{3}{x+1}dx$

13 $\int_1^2 \frac{1}{2x-1}dx$

14 $\int_4^5 \frac{2}{x-3}dx$

15 $\int_0^1 \frac{1}{2-x}dx$

Exercise 3:Integrating Trigonometric Functions

Integrate each function with respect to x.

1 $\sin 2x$

2 $\cos 7x$

3 $\sec^2 4x$

4 $\sin\left(\frac{1}{4}\pi+x\right)$

5 $3\cos\left(4x-\frac{1}{2}\pi\right)$

6 $\sec^2\left(\frac{1}{3}\pi+2x\right)$

7 $3\sin 5x$

8 $2\sin(3x-\alpha)$

9 $5\cos\left(\alpha-\frac{1}{2}x\right)$

10 $\sin(4x-\pi)$

11 $\cos 3x-\cos x$

12 $\sec^2 2x$

Evaluate

13 $\int_0^{\frac{\pi}{6}} \sin 3x\,dx$

14 $\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \cos\left(2x-\frac{1}{2}\pi\right)dx$

15 $\int_0^{\frac{\pi}{2}} 2\sin\left(2x-\frac{1}{2}\pi\right)dx$

16 $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$

17 Differentiate $\ln\cos x$ with respect to x.

Hence find $\int_0^{\frac{\pi}{3}} \tan x\,dx$

Exercise 4: Integrating of Powers of sin x, cos x and tan x

1 Given that $3\sin x - 4\sin^3 x = \sin 3x$, find $\int(3\sin x - 4\sin^3 x)\,dx$

2 Use the identity $\frac{2\tan x}{1+\tan^2 x} \equiv \sin 2x$ to find the exact value of $\int_0^{\frac{\pi}{4}} \frac{2\tan x}{1+\tan^2 x}\,dx$

3 Use the identity $8\cos^4 x - 8\cos^2 x + 1 \equiv \cos 4x$ to find $\int(8\cos^4 x - 8\cos^2 x + 1)\,dx$

4 (a) Express $2\sin^2 x + 1$ in terms of $\cos 2x$

(b) Use the result of (a) to find $\int \sin^2 x\,dx$

5 Use the double angle formulae for $\cos 2x$ and $\sin 2x$ to show that $2\cos^2 x + 4\sin x\cos x = \cos 2x + 2\sin 2x - 1$

Hence find $\int(2\cos^2 x + 4\sin x\cos x)\,dx$

6 Express $\tan^2 x$ in terms of $\sec^2 x$

Hence find the exact value of $\int_{\frac{\pi}{4}}^{0} 4\tan^2 x\,dx$

7 Differentiate $\sin^3 x$ with respect to x.

Hence find $\int(6\cos x\sin^2 x)\,dx$

Exercise 5: Mixed Questions

Integrate the following functions with respect to x.

1 $3e^{(2x-1)}$

2 $\frac{1}{3x}$

3 $\cos(2x + \pi)$

4 $\frac{2}{1+x}$

5 $\sec^2(2x - 1)$

6 $e^x + e^{-x}$

7 $\sin\left(\frac{\pi}{3} - 3x\right)$

8 $\frac{4}{3x-2}$

Evaluate

9 $\int_0^1 3e^{4x-1}dx$

10 $\int_0^{\frac{\pi}{8}} \cos 4x\,dx$

11 $\int_3^4 \frac{1}{x-2}\,dx$

12 $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\left(2x - \frac{\pi}{4}\right)dx$

13 Use trigonometrical identities to evaluate $\int_0^{\frac{\pi}{4}} (\cos^2 x + 2\sin^2 x)\,dx$

P3 14 (a) Use the expansions of $\sin(4x + x)$ and $\sin(4x - x)$ to show that $2\sin 4x\cos x \equiv \sin 5x + \sin 3x$

(b) Use the result of part (a) or otherwise to find the exact value of $\int_0^{\frac{\pi}{2}} \sin 4x\cos x\,dx$

Exercise 6: Integrating Rational Functions (Partial Fractions)

Express each function in partial fractions and hence find the integral of each function with respect to x.

1 $\frac{2}{x(x+1)}$

2 $\frac{4}{(x-2)(x+2)}$

3 $\frac{x}{(x-1)(x+1)}$

4 $\frac{x-1}{x(x+2)}$

5 $\frac{x-1}{(x-2)(x-3)}$

6 $\frac{1}{x(x-1)(x+1)}$

7 $\frac{x}{x^2-1}$

8 $\frac{2}{x^2-1}$

9 $\frac{2x}{x^2-5x+6}$

10 $\frac{2x-3}{x^2-5x+6}$

Use partial fractions to evaluate

11 $\int_3^4 \frac{x}{(x-2)(x+2)}\,dx$

12 $\int_0^1 \frac{2x}{(x+1)(x-3)}\,dx$

13 $\int_{-1}^1 \frac{5}{x^2+x-6}\,dx$

14 $\int_1^2 \frac{x+1}{x(x+4)}\,dx$

Exercise 7: Integrating functions of the Form $\frac{f(x)}{f'(x)}$

Integrate each function with respect to x

1 $\frac{\cos x}{4+\sin x}$

2 $\frac{e^x}{3e^x-1}$

3 $\frac{x}{(1-x^2)}$

4 $\frac{\cos x}{\sin x}$

5 $\frac{x^3}{1+x^4}$

6 $\frac{2x+3}{x^2+3x-4}$

7 $\frac{1}{x\ln x}$ i.e. $\frac{\frac{1}{x}}{\ln x}$

8 $\frac{2x}{1-x^2}$

9 $\frac{\sin x}{\cos x}$

10 $\frac{\sec x \tan x}{4+\sec x}$

11 $\frac{x-1}{x(x-2)}$

12 $\frac{e^x-1}{(e^x-x)}$

Find the value of

13 $\int_1^2 \frac{2x+1}{x^2+x}\,dx$

14 $\int_0^1 \frac{x}{x^2+1}\,dx$

Exercise 8a): Indefinite Integration by Parts (Integrating Products)

Integrate the following functions with respect to x

1 $x\cos x$

2 xe^{2x}

3 $x^3\ln 3x$

4 xe^{-x}

5 $3x\sin x$

6 $(1-x)e^x$

7 $e^x\sin x$

8 $xe^{(x-1)}$

9 $(1-2x)e^{2x}$

10 $\ln 2x$

11 $e^x(x+1)$

12 $x(1+x)^7$

13 $x\sin\left(x+\frac{1}{6}\pi\right)$

14 $x\cos nx$

15 $x^n\ln x$

16 $3x\cos 2x$

17 $(3x-2)\cos x$

18 $\frac{1}{x}\ln x$

19 $2x\sqrt{e^x}$

20 $x^{\frac{1}{2}}\ln x$

21 x^2e^x

22 $x^2\sin x$

23 x^2e^{4x}

24 $e^{2x}\cos x$

Exercise 8b): Definite Integration by Parts (Integrating Products)

Evaluate

1 $\int_0^{\frac{\pi}{2}} x\sin x\,dx$

2 $\int_1^2 x^5\ln x\,dx$

3 $\int_0^1 (x+1)e^x\,dx$

4 $\int_0^{\pi} 2x\cos x\,dx$

5 $\int_1^2 x\sqrt{x-1}\,dx$

6 $\int_1^2 x\ln x\,dx$

7 $\int_0^1 \ln(1+x)\,dx$

8 $\int_0^1 (1-x)e^{-x}\,dx$

9 $\int_0^{\frac{\pi}{4}} x\sin 2x\,dx$

Exercise 9a): Indefinite Integration using Substitution

Find the following integrals by using the substitution given.

1 $\int x(x^2-3)^4\,dx$; $u = x^2 - 3$

2 $\int x\sqrt{1-x^2}\,dx$; $u = 1 - x^2$

3 $\int \cos 2x\,(\sin 2x + 3)^2\,dx$; $u = \sin 2x + 3$

4 $\int x^2(1 - x^3)\,dx$; $u = 1 - x^3$

5 $\int e^x\sqrt{1+e^x}\,dx$; $u = 1 + e^x$

6 $\int \cos x \sin^4 dx$; $u = \sin x$

7 $\int \sec^2 x \tan^3 x\,dx$; $u = \tan x$

8 $\int x^n(1 + x^{n+1})^2\,dx$; $u = 1 + x^{n+1}$

9 $\int \operatorname{cosec}^2 x \cot^2 x\,dx$; $u = \cot x$

10 $\int \sqrt{x}\sqrt{1+x^{\frac{3}{2}}}\,dx$; $u = 1 + x^{\frac{3}{2}}$

11 $\int (x+1)(x+3)^5\,dx$; $x + 3 = u$

12 $\int \frac{x}{\sqrt{3-x}}\,dx$; $3 - x = u^2$

13 $\int x\sqrt{x+1}\,dx$; $x + 1 = u^2$

14 $\int \frac{2x+1}{(x-3)^6}\,dx$; $x - 3 = u$

15 $\int 2x\sqrt{3x-4}\,dx$; $3x - 4 = u^2$

16 $\int 2x(1-x)^7\,dx$; $u = 1 - x$

17 $\int \frac{x+3}{(4-x)^5}\,dx$ $u = 4 - x$

18 $\int \frac{\sin x}{\sqrt{7+\cos x}}\,dx$; $u^2 = 7 + \cos x$

19 $\int x^3(x^4+4)^2\,dx$; $u = x^4 + 4$

20 $\int e^x(1-e^x)^3\,dx$; $u = 1 - e^x$

21 $\int \sin\theta\sqrt{1-\cos\theta}\,d\theta$; $u^2 = 1 - \cos\theta$

22 $\int (x+1)\sqrt{x^2+2x+3}\,dx$; $u^2 = x^2 + 2x + 3$

23 $\int xe^{x^2+1}\,dx$; $u = x^2 + 1$

24 $\int \sec^2 x(1 + \tan x)\,dx$; $u = 1 + \tan x$

25 $\int \operatorname{cosec} x\,dx$; $u = \cos x$

Exercise 9b): Definite Integration Using Substitution

Evaluate

1 $\int_0^1 x\,e^{x^2}\,dx$; $u = x^2$

2 $\int_0^{\frac{\pi}{2}} \cos x \sin^4 x\,dx$; $u = \sin^4 x$

3 $\int_1^2 \frac{1}{x}\ln x\,dx$; $u = \ln x$

4 $\int_1^2 x^2(x^3-1)^4\,dx$; $u = x^3 - 1$

5 $\int_0^{\frac{\pi}{4}} (\sec^2 x)e^{\tan x}\,dx$; $u = \tan x$

6 $\int_1^2 x(1 + 2x^2)\,dx$; $u = 1 + 2x^2$

7 $\int_2^3 (x-1)\,e^{(x^2-2x)}\,dx$; $u = x^2 - 2x$

8 $\int_0^{\frac{\pi}{6}} \cos x\,(1 + \sin^2 x)\,dx$; $u = \sin x$

9 $\int_1^3 \frac{1}{x}(\ln x)^2\,dx$; $u = \ln x$

10 $\int_0^{\sqrt{3}} x\sqrt{1+x^2}\,dx$; $u^2 = 1 + x^2$

Exercise 10: Mixed Questions

Integrate the following functions with respect to x.

1 $x(1+x^2)^4$; $u = 1 + x^2$

2 xe^{-3x}

3 x^2e^{2x}

4 $\dfrac{x+3}{x+2}$

5 $\dfrac{x^2}{(x^3+1)^2}$; $u = x^3 + 1$

6 $\dfrac{3}{(x-4)(x-1)}$

7 $\dfrac{(x+1)}{x(2x+1)}$

8 $\dfrac{x-1}{(x^2+1)}$; $u = x^2 + 1$

9 $\dfrac{\sin x}{\sqrt{\cos x}}$; $u^2 = \cos x$

Evaluate

10 $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \left(\sin \tfrac{1}{2}x + \cos 2x\right) dx$

11 $\displaystyle\int_2^5 x\sqrt{x-1}\, dx$; $u = x - 1$

12 $\displaystyle\int_0^{\frac{\pi}{4}} \tan x\, dx$

13 $\displaystyle\int_1^2 x\sqrt{5-x^2}\, dx$; $u^2 = 5 - x^2$

14 $\displaystyle\int_4^6 \frac{5}{x^2-x-6}\, dx$

15 $\displaystyle\int_1^2 \frac{2x}{x^2+1}\, dx$

16 $\displaystyle\int_{-2}^{-1} \frac{2-x}{x(1-x)}\, dx$

Exercise 11: Trapezium Rule

Use the trapezium rule with two intervals to find an approximate value for each definite integral.

1 $\displaystyle\int_1^3 \frac{1}{x^2}\, dx$

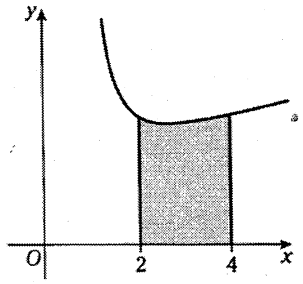
2 $\displaystyle\int_1^3 \ln x\, dx$

3 $\displaystyle\int_0^{\frac{2\pi}{3}} \sqrt{\sin x}\, dx$

4 $\displaystyle\int_1^3 xe^{-x}\, dx$

5 Use sketch graphs to determine whether the results for questions **1** and **2** are underestimates or overestimates.

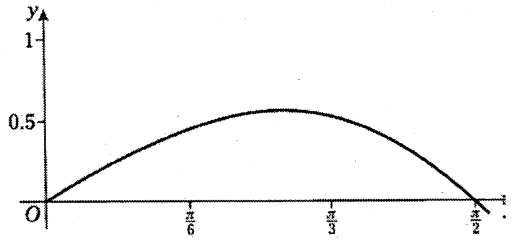
6 The diagram shows a sketch of the curve $y = \dfrac{2x}{\ln x}$



(a) Use the trapezium rule with two intervals to find an approximate value of the shaded area.

(b) Explain whether your approximation is an underestimate or an overestimate of the shaded area.

7 The diagram shows a sketch of part of the curve $y = x\cos x$



(a) Use the trapezium rule with three intervals to find an approximate value of $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\, dx$

(b) Use the sketch to determine whether your result is an underestimate or overestimate.

Answers

Exercise 1

Add a constant to each answer.

1 $\frac{1}{4}e^{4x}$
2 $-4e^{-x}$
3 $\frac{1}{3}e^{(3x-2)}$
4 $-\frac{2}{5}e^{(1-5x)}$
5 $-3e^{-2x}$
6 $5e^{(x-3)}$
7 $2e^{(\frac{x}{2}+2)}$
8 $\frac{1}{2}e^{2x} - \frac{1}{2e^{2x}}$
9 $\frac{e^4 - 1}{2}$
10 $2(e^2 - 1)$
11 $\frac{e-1}{e}$
12 $1 - e^2$

Exercise 2

1 $\frac{1}{2}\ln|x| + K$
2 $2\ln|x| + K$
3 $\frac{1}{4}\ln|x| + K$
4 $\frac{3}{2}\ln|x| + K$
5 $4\ln|x-1| + K$
6 $\frac{1}{3}\ln|3x+1| + K$
7 $-\frac{3}{2}\ln|1-2x| + K$
8 $2\ln|2+3x| + K$
9 $-\frac{3}{2}\ln|4-2x| + K$
10 $-4\ln|1-x| + K$
11 $-\frac{5}{7}\ln|6-7x| + K$
12 $3\ln 1.5$
13 $\frac{1}{2}\ln 3$
14 $2\ln 2 = \ln 4$
15 $\ln 2$

Exercise 3

1 $-\frac{1}{2}\cos 2x + K$
2 $\frac{1}{7}\sin 7x + K$
3 $\frac{1}{4}\tan 4x + K$
4 $-\cos\left(\frac{1}{4}\pi + x\right) + K$
5 $\frac{3}{4}\sin\left(4x - \frac{1}{2}\pi\right) + K$
6 $\frac{1}{2}\tan\left(\frac{\pi}{3} + 2x\right) + K$
7 $-\frac{3}{5}\cos 5x + K$
8 $-\frac{2}{3}\cos(3x - \alpha) + K$
9 $-10\sin\left(\alpha - \frac{1}{2}x\right) + K$
10 $-\left(\frac{1}{4}\right)\cos(4x - \pi) + K$
11 $\frac{1}{3}\sin 3x - \sin x + K$
12 $\frac{1}{2}\tan 2x + K$
13 $\frac{1}{3}$
14 $-\frac{1}{4}$
15 0
16 $\frac{1}{2}$
17 $-\ln\left(\frac{1}{2}\right)$

Exercise 4

1 $-\frac{1}{3}\cos 3x + K$
2 $\frac{1}{2}$
3 $\frac{1}{4}\sin 4x + K$
4 (a) $2 - \cos 2x$ (b) $\frac{x}{2} - \frac{\sin 2x}{4} + K$
5 $-x + \frac{1}{2}\sin 2x - \cos 2x + K$
6 $\pi - 4$
7 $2\sin^3 x + K$

Exercise 5

1 $\frac{3}{2}e^{2x-1} + K$
2 $\frac{1}{3}\ln|x| + K$
3 $\frac{1}{2}\sin(2x + \pi)$
4 $2\ln|1+x| + K$
5 $\frac{1}{2}\tan(2x-1) + K$
6 $e^x - e^{-x}$
7 $\frac{1}{3}\cos\left(\frac{\pi}{3} - 3x\right) + K$
8 $\frac{4}{3}\ln|3x-2| + K$
9 $\frac{3}{4}(e^3 - e^{-1})$
10 $\frac{1}{4}$
11 $\ln 2$
12 $\frac{1}{\sqrt{2}}$
13 $\frac{3\pi}{8} - \frac{1}{4}$
14 $\frac{8}{15}$

Exercise 6

Add a constant to each indefinite integral.

1 $2\ln\left|\frac{x}{x+1}\right|$
2 $\ln\left|\frac{x-2}{x+2}\right|$
3 $\frac{1}{2}\ln|x^2-1|$
4 $\frac{1}{2}\ln\left|\frac{(x+2)^3}{x}\right|$
5 $\ln\frac{(x-3)^2}{|x-2|}$
6 $\frac{1}{2}\ln\frac{|x^2-1|}{x^2}$
7 $\frac{1}{2}\ln|x^2-1|$
8 $\ln\left|\frac{x-1}{x+1}\right|$
9 $\ln\frac{(x-3)^6}{(x-2)^4}$
10 $\ln\left|\frac{(x-3)^3}{x-2}\right|$
11 $\frac{1}{2}\ln\frac{5}{3}$
12 $2\ln 2 - \frac{3}{2}\ln 3$
13 $\ln\frac{1}{6}$
14 $\frac{1}{4}\ln\frac{432}{125}$

Exercise 7

1 $\ln(4 + \sin x)$
2 $\frac{1}{3}\ln|3e^x - 1|$
3 $-\frac{1}{2}\ln|(1-x^2)|$
4 $\ln|\sin x|$
5 $\frac{1}{4}\ln(1+x^4)$
6 $\ln|x^2 + 3x - 4|$
7 $\ln(\ln x)$
8 $-\ln|1-x^2|$
9 $-\ln|\cos x|$
10 $\ln|4 + \sec x|$
11 $\frac{1}{2}\ln|x(x-2)|$
12 $\ln|e^x - x|$
13 $\ln 3$
14 $\ln\sqrt{2}$

Exercise 8a)

1 $x\sin x + \cos x$
2 $\frac{1}{2}(x-2)e^{2x}$
3 $\frac{1}{16}x^4(4\ln|3x| - 1)$
4 $-e^{-x}(x+1)$
5 $3(\sin x - x\cos x)$
6 $e^x(2-x)$
7 $\frac{1}{2}e^x(\sin x - \cos x)$
8 $(x-1)e^{x-1}$
9 $e^{2x}(1-x)$
10 $x(\ln|2x| - 1)$
11 xe^x
12 $\frac{1}{72}(8x-1)(x+1)^8$
13 $\sin\left(x + \frac{1}{6}\pi\right) - x\cos\left(x + \frac{1}{6}\pi\right)$
14 $\frac{1}{n^2}(\cos nx + nx\sin nx)$
15 $\frac{x^{n+1}}{(n+1)^2}[(n+1)\ln|x| - 1]$
16 $\frac{3}{4}(2x\sin 2x + \cos 2x)$
17 $(3x-2)\sin x + 3\cos x$
18 $\frac{1}{2}(\ln|x|)^2$
19 $e^{\frac{x}{2}}(4x-8)$
20 $\frac{2}{3}x^{\frac{3}{2}}(\ln|x| - 2)$
21 $e^x(x^2 - 2x + 2)$
22 $-x^2\cos x + 2x\sin x + 2\cos x$
23 $\frac{e^{4x}}{32}(8x^2 - 4x + 1)$
24 $\frac{e^{2x}}{5}(\sin x + 2\cos x)$

Exercise 8b)

1 1
2 $\frac{32}{3}\ln 2 - \frac{7}{4}$
3 e
4 -4
5 $\frac{16}{15}$
6 $2\ln x - \frac{3}{4}$
7 $2\ln 2 - 1$
8 $e - 2$
9 $\frac{1}{4}$

Exercise 9a)

1 $\frac{1}{10}(x^2-3)^5$
2 $-\frac{1}{3}(1-x^2)^{\frac{3}{2}}$
3 $\frac{1}{6}(\sin 2x+3)^3$
4 $-\frac{1}{6}(1-x^3)^2$
5 $\frac{2}{3}(1+e^x)^{\frac{3}{2}}$
6 $\frac{1}{5}\sin^5 x$
7 $\frac{1}{4}\tan^4 x$
8 $\frac{1}{3(n+1)}(1+x^{n+1})^3$
9 $-\frac{1}{3}\cot^3 x$
10 $\frac{4}{9}\left(1+x^{\frac{3}{2}}\right)^{\frac{3}{2}}$
11 $\frac{1}{21}(x+3)^6(3x+2)$
12 $-\frac{2}{3}(x+6)\sqrt{(3-x)}$
13 $\frac{2}{15}(3x-2)(x+1)^{\frac{3}{2}}$
14 $\frac{1-5x}{10(x-3)^5}$
15 $\frac{4}{135}(9x+8)(3x-4)^{\frac{3}{2}}$
16 $-\frac{1}{36}(8x+1)(1-x)^8$
17 $\frac{5+4x}{12(4-x)^4}$
18 $-2\sqrt{7+\cos x}$
19 $\frac{1}{12}(x^4+4)^3$
20 $-\frac{1}{4}(1-e^x)^4$
21 $\frac{2}{3}(1-\cos\theta)^{\frac{3}{2}}$
22 $\frac{1}{3}(x^2+2x+3)^{\frac{3}{2}}$
23 $\frac{1}{2}e^{(x^2+1)}$
24 $\frac{1}{2}(1+\tan x)^2$
25 $\frac{1}{2}\ln\left|\frac{\cos x-1}{\cos x+1}\right|$

Exercise 9b)

1 $\frac{1}{2}(e-1)$
2 $\frac{1}{5}$
3 $\frac{1}{2}(\ln 2)^2$
4 $\frac{7^5}{15}$
5 $e-1$
6 9
7 $\frac{1}{2}(e^3-1)$
8 $\frac{13}{24}$
9 $\frac{1}{3}(\ln 3)^3$
10 $\frac{7}{3}$

Exercise 10

1 $\frac{1}{10}(1+x^2)^5$
2 $-\frac{1}{9}e^{-3x}(3x+1)$
3 $\frac{1}{4}e^{2x}(2x^2-2x+1)$
4 $x+\ln|x+2|$
5 $\frac{-1}{3(x^3+1)}$
6 $\ln\left|\frac{x-4}{x-1}\right|$
7 $\ln\left|\frac{x}{\sqrt{2x+1}}\right|$
8 $\ln\sqrt{x^2+1}-\tan^{-1}x$
9 $-2\sqrt{\cos x}$
10 $\sqrt{2}$
11 $\frac{256}{15}$
12 $\ln\sqrt{2}$
13 $\frac{7}{3}$
14 $\ln\frac{9}{4}$
15 $\ln\frac{5}{2}$
16 $\ln\frac{3}{8}$

Exercise 11

1 0.806
2 1.24
3 1.462
4 0.529
5 1 is an overestimate, 2 is an underestimate
6 (a) 11.2 (b) overestimate
7 (a) 0.512 (b) underestimate