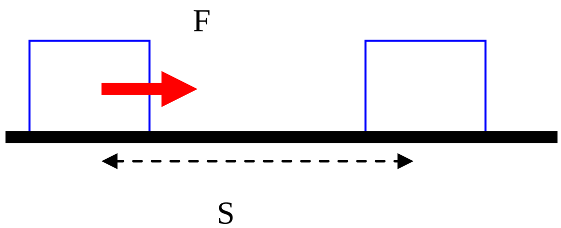
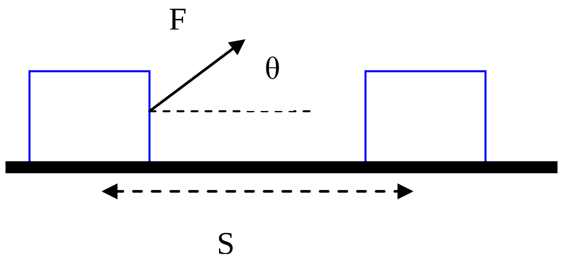
TOPIC 4 - Work, Energy & Power

Work Done

Suppose a force **F** acts on a body, causing it to move in a particular direction. Then the **work done** by the force is the component of **F** in the direction of motion × the distance the body moves as a result. Work done is measured in joules (which has symbol J).



So if we have a constant force of magnitude F newtons, which moves a body a distance s (meters) along a flat surface, the **work done is (F × s) Joules**.



Now suppose that this force is at an angle of **θ** to the horizontal. If the body moves a distance of **s** metres along the ground, then the **work done is (F cos θ × s**)

(since F cos θ is the component of the force in the direction of motion).

Mechanical Energy

The energy of a body is a measure its ability to do work (position and motion).

Kinetic Energy

The kinetic energy (K.E.) of a body is the energy a body has as a result of its motion. A body which isn't moving will have zero kinetic energy, therefore.

- K.E. = $\frac{1}{2} mv^2$ where m is the mass and v is the velocity of the body.

Gravitational Potential Energy

Gravitational potential energy (G.P.E.) is the energy a body has because of its height above the ground.

- G.P.E. = mgh where h is the height of the body above the ground.

There are also other types of potential energy (such as elastic potential energy). Basically, the total potential energy measures the energy of the body due to its position.

Example

Find the change in the potential energy of a child of mass 48 kg when

(a) ascending a vertical distance of 2m
(b) descending a vertical distance of 2 m.

Conservation of Energy

The Law of Conservation of Energy

In a closed system the amount of energy is constant. Or in other words 'energy can never be created nor destroyed', it merely changes from one form into another.

If gravity is the only external force which does work on a body, then the total energy of the body will remain the same, a property known as the conservation of energy.

Therefore, providing no work is done:

- Initial (PE + KE) = final (PE + KE)

Example 1

A pump forces up water at a speed of $8ms^{-1}$ from a well into a reservoir at a rate of 50 kg s^{-1}. If the water is raised a vertical height of 40 m, what is the work done per second?(assume g=10 ms^{-2})

[21600J]

Connection Between Energy and Work Done

If energy is not conserved, then it is used to do work.

In other words, the work done is equal to the change in energy. For example, the work done against gravity is equal to the change in the potential energy of the body and the work done against all resistive forces is equal to the change in the total energy.

Note:

- Work-Energy Principle – if a constant force acts on an object over a certain distance, the work done by the force is equal to the **gain in the kinetic energy** of the object.

Example 2

A car and driver have a total mass of 1000kg. The car gains speed from $7ms^{-1}$ to 13 ms^{-1} with constant acceleration over a distance of 200 m. Calculate the driving force.

[300N]

Example 3

In a science experiment, a 50g mass slides down a 60^o incline of length 0.5m. If the mass is given an initial speed of 2 ms^{-1} down the plane and its final speed is measured as 3 ms^{-1}, what is the magnitude of the frictional force opposing the mass? (assume $g=10\ ms^{-2}$, answer to 2d.p.)

[0.183N]

- ➔ Extended Work-Energy Principle – the work done by the force acting on an object, minus the work done against resistance, is equal to the gain in the kinetic energy of the object.

Example 4

A cyclist and her machine together have a mass of 100kg. She free-wheels down a hill inclined ($\sin \theta = 1/20$) for a distance of 500 metres. If her speed at the top was 5 ms^{-1} and there is air resistance of 40 N, how fast will she be going at the bottom of the hill?

[11.2 ms^{-1}]

Work Done Against Gravity

Now suppose that the force we are considering is one which causes a body to be lifted off of the ground. We call the work done by the force the "work done against gravity". This is equal to mgs joules, where s is the vertical distance moved by the body, m is the mass of the body and g is the acceleration due to gravity. [Compare with "gravitational potential energy"].

Power

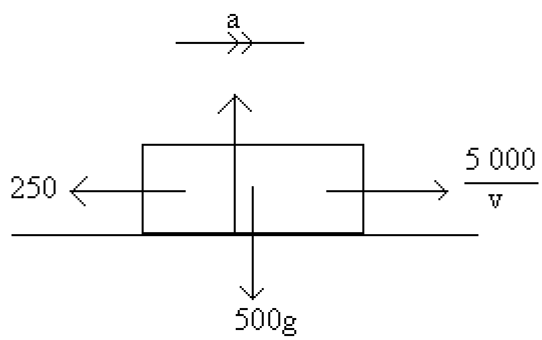
Power is the rate at which work is done (measured in watts (W)), in other words the work done per second.

It turns out that: Power = Force × Velocity

For example, if the engine of a car is working at a constant rate of 10kW, the forward force generated is power/velocity = 10 000 / v, where v is the velocity of the car (the 10 was changed to 10 000 so that we are using the standard unit of W rather than kW).

Example 5

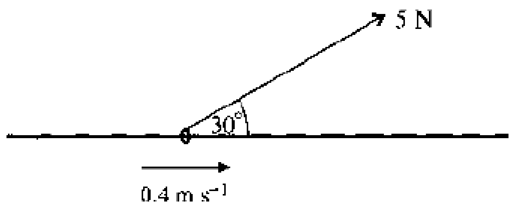
A car of mass 500kg is travelling along a horizontal road. The engine of the car is working at a constant rate of 5kW. The total resistance to motion is constant and is 250N. What is the acceleration of the car when its speed is 5m/s?



[1.5ms^{-2}]

Exercise 1: Exam Based Questions

Question 1



One end of a light inextensible string is attached to a ring which is threaded on a fixed horizontal bar. The string is used to pull the ring along the bar at a constant speed of $0.4\ \mathrm{m\,s^{-1}}$. The string makes a constant angle of $30°$ with the bar and the tension in the string is 5 N (see diagram). Find the work done by the tension in 10 s. [3]

Question 2

A car of mass 1500 kg arrives at the foot of a straight hill travelling at $30\ \mathrm{ms^{-1}}$. It reaches the top of the hill 40 seconds later travelling at $10\ \mathrm{ms^{-1}}$. The length of the hill is 1 km and the gain in height is 120 metres. The average resistance to the motion is 500 N.

a) Find the loss in kinetic energy. [3]
b) Find the work done by the car to get to the top of the hill. [3]
c) Find the average power developed by the engine. [3]

Question 3

A car of mass 1200 kg travels along a horizontal straight road. The power of the car's engine is 20 kW. The resistance to the car's motion is 400 N.

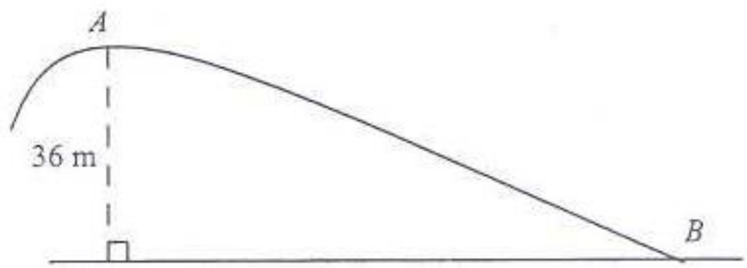
(i) Find the speed of the car at an instant when its acceleration is $0.5\ \mathrm{m\,s^{-2}}$. [4]

(ii) Show that the maximum possible speed of the car is $50\ \mathrm{m\,s^{-1}}$. [2]

The work done by the car's engine as the car travels from a point A to a point B is 1500 kJ.

(iii) Given that the car is travelling at its maximum possible speed between A and B, find the time taken to travel from A to B. [2]

Question 4



The diagram shows the vertical plane through the portion AB of a road along which a car of mass 750 kg travels against a resistance. The point A is 36 m higher than the lowest point B.

The car travels from A to B, a distance of 150 m, with no power being transmitted to its wheel. Given that its speed at A is 12 ms^{-1} and its speed at B is 24 ms^{-1}.Find

(i) the loss in potential energy, [1]

(ii) the gain in kinetic energy, [2]

(iii) the work done against the resistance. [2]

Question 5

(i) A car C of mass 1200 kg climbs a hill of length 500 m at a constant speed. The hill is inclined at an angle of 6° to the horizontal. The driving force exerted by C's engine has magnitude 1800 N. Find the work done against the resistance to the motion of C, as it climbs from the bottom of the hill to the top. **[4]**

(ii) Another car D, also of mass 1200 kg, climbs the same hill with increasing speed. The speed at the bottom is $8\,\mathrm{m\,s^{-1}}$ and the speed at the top is $20\,\mathrm{m\,s^{-1}}$. Assuming the resistance to the motion of D is constant and has magnitude 700 N, find the work done by D's engine as D climbs from the bottom of the hill to the top. **[4]**

(iii) The driving force exerted by D's engine is 4 times as great when D is at the top of the hill as it is when D is at the bottom. Find the ratio of the power developed by D's engine at the top of the hill to the power developed at the bottom. **[3]**

Question 6

. A car has mass 800 kg and its engine has constant power 40 kW. The car moves in a straight horizontal line, starting from rest. Ignoring all resistance, find the time taken to reach a speed of 20 ms^{-1}. [3]

The car reaches a slope with an angle of $\sin^{-1}\left(\frac{1}{10}\right)$ which it climbs at a constant speed of 25 ms^{-1} against a resistance of 1600N.

a) What extra power must the car engine produce? [3]

While the car is climbing the slope at a speed of 25 ms^{-1} the power is suddenly removed and the car slows down and comes to rest.

b) How far along the slope does the car travel before coming to a stop? [4]

Question 7

A car of mass 900 kg is moving on a horizontal road at a constant speed of 30 ms^{-1}, with its engine working at its maximum rate of 78 kW.

(i) Calculate the total resistance acting on the car. [2]

The car moves against this same resistance when moving up a hill inclined at 10° to the horizontal. Given that the car is travelling up the hill at a constant speed and working at its maximum rate.

(ii) calculate the speed of the car. [3]

The engine is switched off.

(iii) Calculate the further distance that the car travels up the hill before coming to rest, assuming that the resistance remains unaltered. [4]

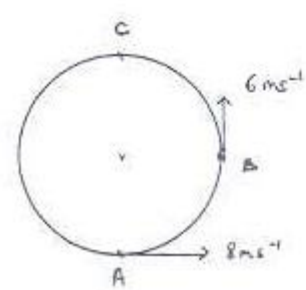
Question 8

The diagram shows a rough circular wire, centre O, radius 1.2 m, fixed in a vertical plane with the top point C vertically above the bottom point A. A particle of mass 0.6 kg is threaded on the wire and projected from A with speed 8 ms^{-1}. The point B is at the same level as O and the particle passes through B with a speed of 6 ms^{-1}. Calculate

i) the loss of kinetic energy as the particle travels from A to B, [2]

ii) the gain in potential energy as the particle travels from A to B, [2]

iii) the work done in overcoming the frictional resistance of the wire from A to B. [3]



Question 9

A straight road is inclined at an angle α to the horizontal where $\sin \alpha = 1/20$.
A lorry of mass 4800 kg moves up the road at a constant speed of 12 ms^{-1}.
The resistance to the motion of the lorry is constant and has a magnitude of 2000 N.

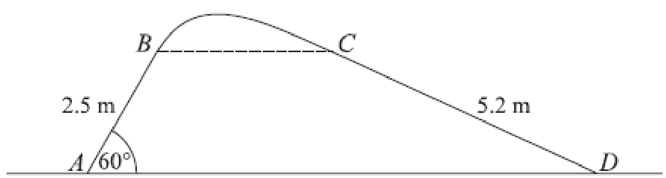
(i) Find, in kW to 3 significance figures, the rate of working of the lorry's engine. [3]

The road becomes horizontal. The lorry's engine continues to work at the same rate and the resistance to the motion remains the same. Find

(ii) the acceleration of the lorry immediately after the road becomes horizontal, [3]

(iii) the maximum speed, in ms^{-1} to 3 significant figures, at which the lorry can travel along the horizontal road. [3]

Question 10



The diagram shows a vertical cross-section $ABCD$ of a surface. The parts AB and CD are straight and have lengths 2.5 m and 5.2 m respectively. AD is horizontal, and AB is inclined at 60° to the horizontal. The points B and C are at the same height above AD. The parts of the surface containing AB and BC are smooth. A particle P is given a velocity of $8\,\mathrm{m\,s^{-1}}$ at A, in the direction AB, and it subsequently reaches D. The particle does not lose contact with the surface during this motion.

(i) Find the speed of P at B. [4]

(ii) Show that the maximum height of the cross-section, above AD, is less than 3.2 m. [2]

(iii) State briefly why P's speed at C is the same as its speed at B. [1]

(iv) The frictional force acting on the particle as it travels from C to D is 1.4 N. Given that the mass of P is 0.4 kg, find the speed with which P reaches D. [4]