Conditional Probability

Example 7

Consider a class of 30 students:

	Girls (G)	Boys (B)
Left-Handed (L)	5	6
Right-Handed (R)	12	7

1) If a student is selected at random from the whole class, what is the probability the student is left-handed?

2) If a student is selected at random from the group of girls, what is the probability the girl is left-handed?

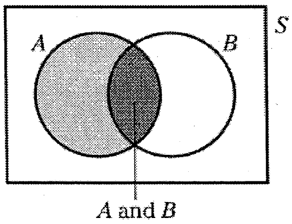
3) If a student is selected at random from the group left-handed, what is the probability the student is a girl?

Conditional probability is used when the probability that an event will occur depends on wheth another event has occurred.

For events A and B, the **conditional probability** that event B occurs, given that event A has alre occurred, i.e. P(B given A) is written $P(B \mid A)$.

Since A has already occurred, the possibility space is reduced to just A.

So, $$P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$$



Now divide the top and bottom of the fraction by $n(S)$:

$$P(B \mid A) = \frac{\frac{n(A \text{ and } B)}{n(S)}}{\frac{n(A)}{n(S)}}$$

$$= \frac{P(A \text{ and } B)}{P(A)}$$

Example 8

There are 5 red counters and 7 blue counters in a bag. Darian takes a counter from the bag, puts it on the table and then takes another counter from the bag. Find the probability that he takes out

(i) a red counter then a blue counter,

(ii) two counters that are the same colour,

(iii) a least one red counter.

Example 9

Two events X and Y are such that $P(X) = 0.2$, $P(Y) = 0.25$, $P(Y \mid X) = 0.4$.

Find (i) $P(X \text{ and } Y)$, (ii) $P(X \mid Y)$, (iii) $P(X \text{ or } Y)$.

Example 10

Of the 120 first year students at a college, 36 study chemistry, 60 study biology and 10 study both chemistry and biology. A first year student is selected at random to represent the college at a conference. Find the probability that the student studies

(i) chemistry, given that the student studies biology,

(ii) biology, given that the student studies chemistry.

Example 11

Last month a consultant saw 60 men and 65 women suspected of having a particular eye condition. Tests were carried out and the following table shows the results. The totals are shown in bold.

	Had eye condition (C)	Did not have eye condition (C')	
Man (M)	25	35	**60**
Woman (W)	20	45	**65**
	45	**80**	**125**

One of these patients was selected at random to take part in a survey. Find the probability that the patient selected

(i) was a woman, given that the patient had the eye condition,

(ii) had the eye condition, given that the patient was a man.

Independent events

You know that, in general, for events A and B

$$P(A \text{ and } B) = P(A) \times P(B \mid A).$$

Now when either of these events can occur without being affected by the outcome of the other, the events are said to be independent.

So, for **independent events** A and B,

$$P(B \mid A) = P(B)$$

Also $\quad P(A \mid B) = P(A)$

This gives the **multiplication rule for independent events**:

$$P(A \textbf{ and } B) = P(A) \times P(B)$$

In set notation $\quad P(A \cap B) = P(A) \times P(B)$

This is sometimes known as the '**and**' rule for independent events.

The rule can be extended to n independent events $A_1, A_2, A_3, \ldots, A_n$ as follows:

$$P(A_1 \text{ and } A_2 \text{ and } A_3 \text{ and } \ldots \text{ and } A_n) = P(A_1) \times P(A_2) \times P(A_3) \times \ldots \times P(A_n).$$

Note:

It is important not to confuse the terms 'mutually exclusive' and 'independent'.

Mutually exclusive events are events that cannot happen together. They are usually the outcomes of one experiment.

Independent events are events that can happen simultaneously or can be seen to happen one after the other.

Example 12

There are 5 red counters and 7 blue counters in a bag. Eliza takes a counter from the bag, notes its colour and then puts it back into the bag. Freddie then takes a counter from the bag. Find the probability that

(i) Eliza takes a red counter and Freddie takes a blue counter,

(ii) Freddie's counter is the same colour as Eliza's counter.

Example 13

Two fair cubical dice are thrown. One is red and the other is blue. Find the probability that

(i) the score is 3 on both dice,

(ii) neither die has a score of 3.

Example 14

The probability that a certain type of machine will break down in the first month of operation is 0.1. Three machines of this type are installed at the same time. The performances of the three machines are independent. Find the probability that at the end of the first month

(i) all three machines have broken down,

(ii) just one machine has broken down,

(iii) at least one machine is working.

Example 15

Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find $P(M)$.

(ii) Find $P(M \text{ and } E)$.

(iii) Are M and E independent events? Justify your answer.

(iv) Given that the person chosen is unemployed, find the probability that the person is female.

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Example 16

Events A and B are such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \text{ or } B) = 0.72$. State, giving a reason in each case, whether events A and B are

(i) mutually exclusive,

(ii) independent.