Functions

P1/2/1: Definitions

P1/2/2: Inverse functions

P1/2/3: Composite functions

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P1/2/1
Definitions

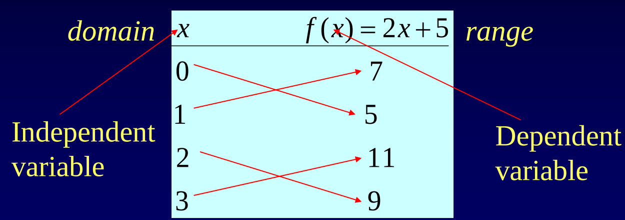
Learning Outcome

Students should be able to:

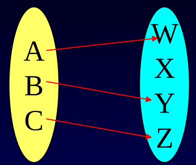
- define the terms functions, domain and range
- state the range of various functions with given domains
- identify one-one and many to one functions

Definitions

Function: A function from a set D (the domain) to a set R (the range) is a rule that assigns a unique element in R to each element in D.

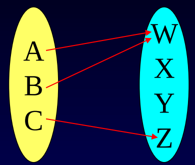


One-to-one function



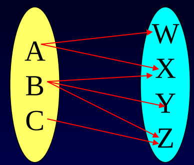
$Example: f(x) = 2x$

Many-to-one function



$Example: f(x) = x^2$

One-to-many relationship



$Example: y^2 = x$

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Example 1:

Find the domain of the following functions.

$(a)\quad g(x) = \sqrt{x-4}$

$(b)\quad g(x) = \sqrt{x(x-4)}$

$(c)\quad g(x) = \dfrac{1}{x-2}$

$(d)\quad g(x) = \dfrac{1}{\sqrt{x-2}}$

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Example 2:

All functions are defined for the largest possible domain of values of x. Find their ranges.

$(a)\quad f(x) = 2x+7$

$(b)\quad f(x) = (x-1)^2 + 2$

$(c)\quad f(x) = \dfrac{1}{x^4}$

$(d)\quad f(x) = \sqrt{4-x^2}$

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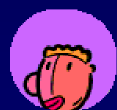
Practice Exercise

Pure Mathematics 1 *Hugh Neil & Douglas Quadling (2002)*

Exercise 3A (Page 35)

Q7(d)(f)(j)(k)(l), Q8(b)(e), Q10(d)



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P1/2/2
Inverse functions

P1/2/3
Composite functions

Learning Outcome

Students should be able to:

- define an inverse of a function
- specify the domain and range of an inverse function
- determine the existence of an inverse function from the given domain of a function
- determine the domain for which the inverse exist
- illustrate in graphical terms the relation between a one one function and its inverse

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Learning Outcome

Students should be able to:

- find the compositions of 2 functions
- to illustrate by using a mapping diagram the relation between two given functions and their compositions
- to determine the domain ad range of composite functions given the domain and range of their original functions.

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Inverse functions

A function *f* defined for some domain D is **one-one** if, for each number *y* in the range R of f there is only one number $x \in$ D such that $y = f(x)$.
The function with domain R define by $f^{-1} : y \mapsto x$ where $y = f(x)$, is the inverse function of *f*.

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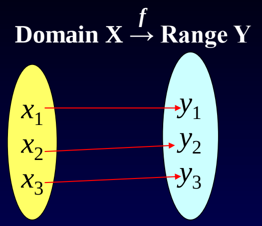

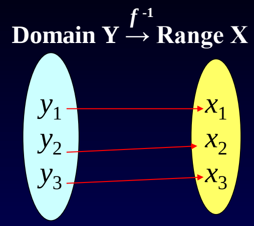

f is one-to-one function

f^{-1} is one-to-one function

	Domain	Range
$f(x)$	D	R
$f^{-1}(x)$	R	D

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Graphing Inverse functions

If f is **one-one** function, the graph of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of each other in the line $y = x$.

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Example 3:

Find the inverse of these functions and sketch the graph of the functions.

$(a) \quad f(x) = 2x + 1 \text{ for } x \in \Re$

$(b) \quad f(x) = x^2 - 4, \text{ for } x \le 0$

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Example 4:

The function $f(x) = x^2 + 2x + 6, \quad x \in \Re \text{ and } x \le k$

Given that f is one-one, determine the greatest possible value of k. When k has this value,

(a) determine the range of f,

(b) find the inverse function f^{-1} and state its domain and range

(c) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

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Example 5:

The function f is defined by $f(x) = 2x^2 - 8x + 11,\ x \in \Re$

(i) Express $f(x)$ in the form $a(x+b)^2 + c,$ where a, b and c are constants.
(ii) State the range of f.
(iii) Explain why f does not have an inverse.

The function g is defined by $g(x) = 2x^2 - 8x + 11,$ for $x \le A,$
Where A is a constant.

(iv) State the largest value of A for which g has an inverse.
(v) When A has this value, obtain an expression, in terms of $g^{-1}(x)$ and state the range of $g^{-1}(x)$.

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Practice Exercise

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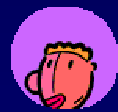
Exercise 11B (Page 169)
Q7(f), Q10(c)(e), Q14, Q15

Miscellaneous exercise 11 (Page 172)
Q10



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Composite functions

To form the **composite function** $g\,f$, the domain D must be chosen so that the whole of the range of f is included in the domain of g.
The function $g\,f$ is then defined as

$$gf : x \mapsto g\big(f(x)\big), \qquad x \in D$$

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Example 6:

$f(x) = 2x + 1$ and $g(x) = 3x - 5,$ where $x \in \Re$

Find the value of the following:

(a) $gf(1)$
(b) $fg(7)$
(c) $ff(-5)$
(d) $gff(1)$
(e) $ff^{-1}(5)$
(f) $f^{-1}f(-4)$

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Practice Exercise

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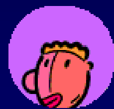
Exercise 11B (Page 170)
Q4

Miscellaneous Exercise 11 (Page 172)
Q3, Q11, Q15



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