AS Final Exam : Revision 2 Functions

P1 June 08

6 The function f is such that $f(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

8 Functions f and g are defined by

$$f : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{2 - x} \quad \text{for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of k for which the equation $fg(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $fg(x) = x$ for the values of k found in part (i). [3]

P1 Nov 08

11 The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

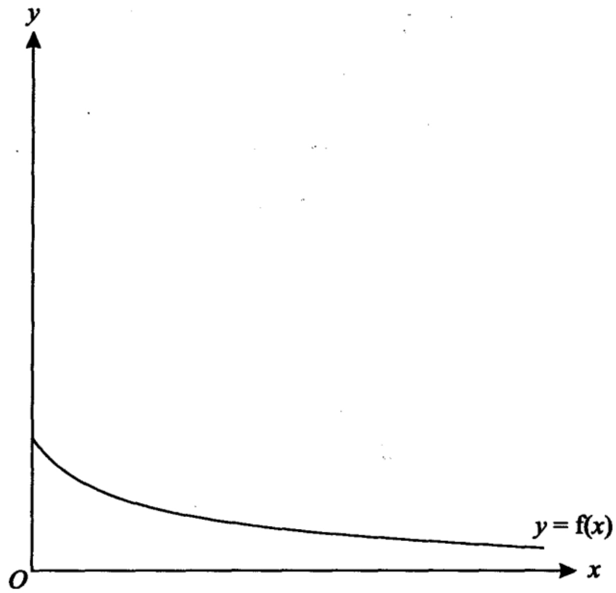
The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

P1 June 07

11



The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \dfrac{6}{2x+3}$ for $x \geqslant 0$.

(i) Find an expression, in terms of x, for $f'(x)$ and explain how your answer shows that f is a decreasing function. [3]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs. [2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$. [3]

P1 Nov 07

11 The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

P1 June 06

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$\mathrm{g} : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R}, \ x \neq -2.$$

(i) Find the values of k for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has two equal roots and solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$ in these cases. [6]

(ii) Solve the equation $\mathrm{fg}(x) = 5$ when $k = 6$. [3]

(iii) Express $\mathrm{g}^{-1}(x)$ in terms of x. [2]

P1 Nov 06

10 The function f is defined by $\mathrm{f} : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) > 4$. [3]

(ii) Express $\mathrm{f}(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

(iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $\mathrm{g} : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $\mathrm{g}(x) = 10$. [3]

P1 June 05

7 A function f is defined by $\mathrm{f} : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

A function g is defined by $\mathrm{g} : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [2]

P1 June 04

10 The functions f and g are defined as follows:

$$f : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $f(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $gf(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$, making clear the relationship between the graphs. [2]

P1 Nov 04

9 The function $f : x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation $ff(x) = 11$. [3]

The function $g : x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation $f(x) = g(x)$ has exactly one real solution. [3]

The function $h : x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $h^{-1}(x)$ and state the domain of h^{-1}. [4]

P1 June 03

5 The function f is defined by $f : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $f(2) = 1$ and $f(5) = 7$.

(i) Find the values of a and b. [2]

(ii) Solve the equation $ff(x) = 0$. [3]

11 The equation of a curve is $y = 8x - x^2$.

(i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]

(iii) Find the set of values of x for which $y \geqslant -20$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 8x - x^2$, for $x \geqslant 4$.

(iv) State the domain and range of g^{-1}. [2]

(v) Find an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [3]

P1 Nov 03

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{4}{2 - x}, \quad x \in \mathbb{R}, \ x \neq 2.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 7$. [3]

(ii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between these two graphs. [3]

P1 June 02

10 The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{6}{2x + 3}, \quad x \in \mathbb{R}, \ x \neq -1.5.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 3$. [3]

(ii) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [3]

(iii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x, and solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

P1 Nov 02

11 **(i)** Express $2x^2 + 8x - 10$ in the form $a(x+b)^2 + c$. [3]

(ii) For the curve $y = 2x^2 + 8x - 10$, state the least value of y and the corresponding value of x. [2]

(iii) Find the set of values of x for which $y \geqslant 14$. [3]

Given that $\mathrm{f} : x \mapsto 2x^2 + 8x - 10$ for the domain $x \geqslant k$,

(iv) find the least value of k for which f is one-one, [1]

(v) express $\mathrm{f}^{-1}(x)$ in terms of x in this case. [3]