Circular Measure

P1/4/1: The measure of degree and radian

P1/4/2: The measure of arc length

P1/4/3: Area of sector and segment

P1/4/4: Revision

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P1/4/1

The measure of degree and radian

P1/4/2

The measure of arc length

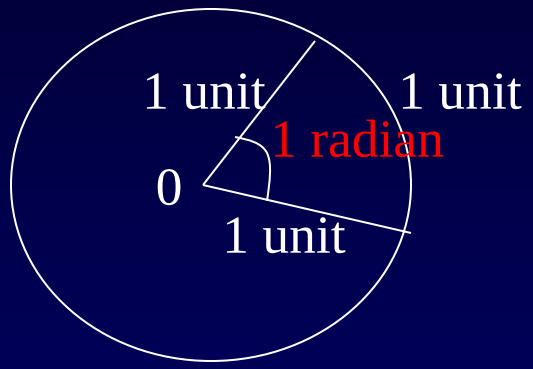
Learning Outcome

Students should be able to:

- define the term radian
- use the relationship between radians and degrees

The measure of degree and radian

In a circle of radius 1 unit, radii joining the centre O to the ends of an arc of length 1 unit form an angle called 1 radian.



radian = rad

$$\pi \text{ rad} = 180^{\circ}$$

There are 2π radians in an unit circle.

Example 1:

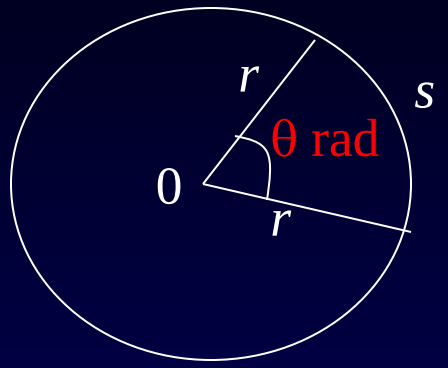
Write each of the following angles in radians, leaving your answers as a multiple of π

$(a)\ \ 135^{o}$ $\qquad$ $(b)\ \ 600^{o}$

Each of the following is an angle in radians. Without using a calculator change these to degrees.

$(a)\ \ \frac{3}{4}\pi$ $\qquad$ $(b)\ \ 6\pi$

The measure of arc length



S = arc length

A = area of sector

The length of circular arc, $S = r\theta$

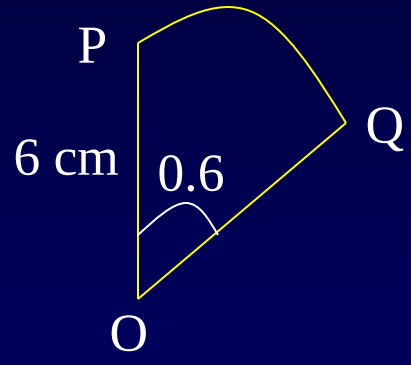
$$A = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$$

Example 2:

The diagram shows a sector of a circle with centre O and radius 6 cm.

Angle POQ = 0.6 radians

Calculate the length of arc PQ and the area of sector POQ.



Practice Exercise

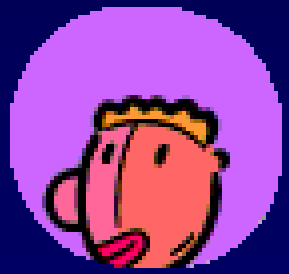
Pure Mathematics 1 ***Hugh Neil & Douglas Quadling (2002)***

Exercise 18A (Page 268)

Q4(d)(g)

Miscellaneous Exercise 18 (Page 275)

Q2, 3, 5



P1/4/3

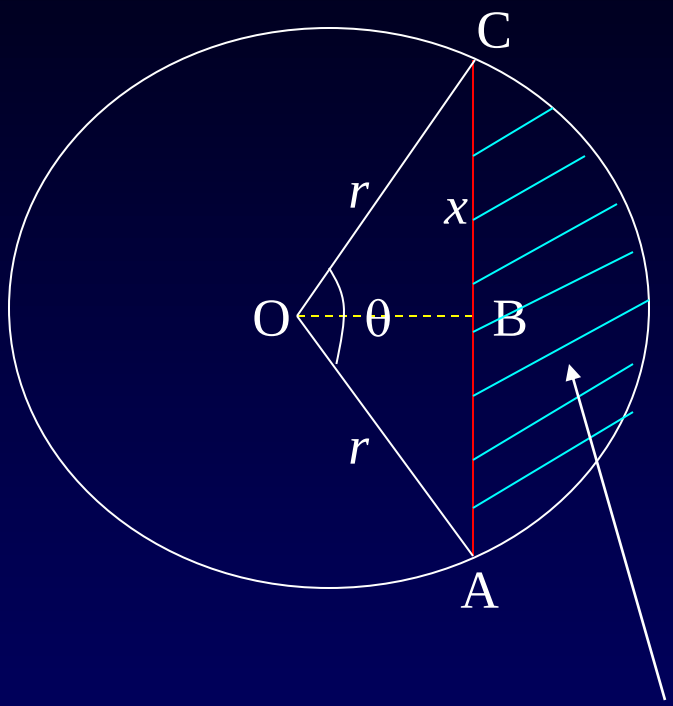
Area of sector and segment

Learning Outcome

Students should be able to:

- define a sector and a segment
- calculate the area of a sector and a segment.

Area of sector and segment



$$\sin\frac{\theta}{2}=\frac{x}{r}\Rightarrow x=r\sin\frac{\theta}{2}$$

$$\text{Chord AC}=2x=2r\sin\frac{\theta}{2}$$

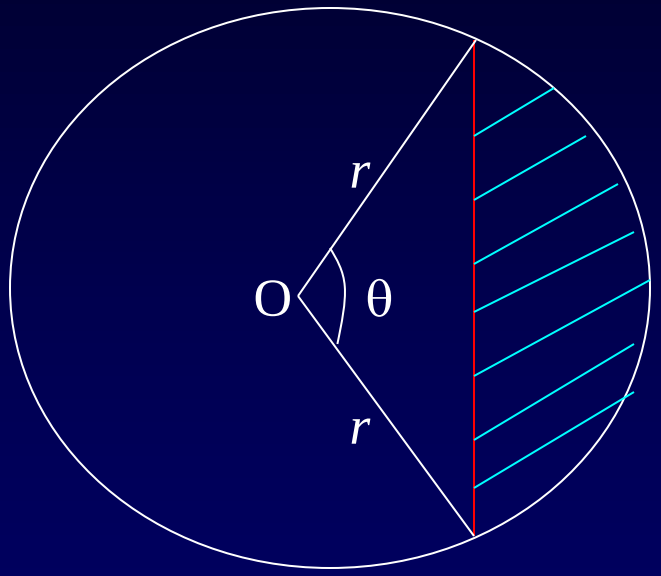
$$OB=r\cos\frac{\theta}{2}$$

$$\text{Area of the segment}=\frac{1}{2}r^2(\theta-\sin\theta)$$

Example 3:

Find the area of the shaded segment for the following case:

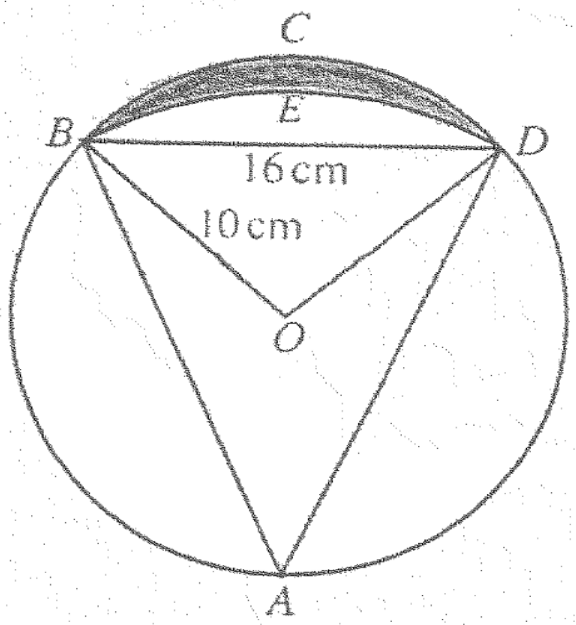
$$r = 28\,cm, \theta = \frac{5}{6}\pi$$



Example 4:

Find the area of the segment cut off by a chord of length 10 cm from a circle of radius 13 cm.

Example 5:

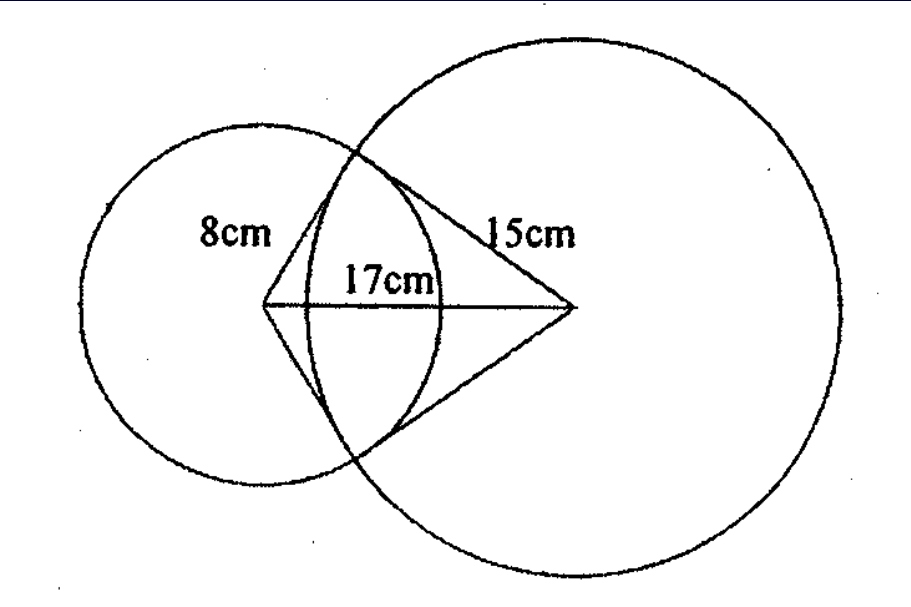


The diagram, which is not drawn to scale, shows a circle *ABCDA*, centre *O* and radius 10 cm. The chord *BD* is 16 cm long. *BED* is an arc of a circle, centre *A*.

Given that the length of *AB* is approximately 17.9 cm, find the perimeter of the shaded region enclosed by the arcs *BCD* and *BED*. [4]

Example 6:

Two circles of radii 8cm and 15cm are drawn, partly overlapping. Their centers are 17cm apart. Find the area common to the two circles.



Example 7:

A solar eclipse can be modeled by two overlapping circles each with radius r.

Show that when the distance between the centers of the circles is $\frac{3}{2}r$, the area of overlapping is approximately 14.4% of the area of one of the circles. [5]

Practice Exercise

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Exercise 18A (Page 268)
Q5(e)

Miscellaneous Exercise 18 (Page 276)
Q4