

Topic 3
Revision Practice – Trigonometry

Question 1

- (i) Find the exact value of x such that

$$3 \tan^{-1}(x - 2) + \pi = 0. \quad [3]$$

- (ii) Solve, for $-\pi < \theta < \pi$, the equation

$$\cos 2\theta - \sin \theta - 1 = 0,$$

giving your answers in terms of π . [5]

Question 2

- (i) Express $2 \sin x^\circ - 3 \cos x^\circ$ in the form $R \sin(x - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$. [3]

- (ii) Show that the equation

$$\operatorname{cosec} x^\circ + 3 \cot x^\circ = 2$$

can be written in the form

$$2 \sin x^\circ - 3 \cos x^\circ = 1. \quad [1]$$

- (iii) Solve the equation

$$\operatorname{cosec} x^\circ + 3 \cot x^\circ = 2,$$

for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. [4]

Question 3

Find, to 2 decimal places, the solutions of the equation

$$3 \cot^2 x - 4 \operatorname{cosec} x + \operatorname{cosec}^2 x = 0$$

in the interval $0 \leq x \leq 2\pi$. [6]

Question 4

- (i) Prove that, for $\cos x \neq 0$,

$$\sin 2x - \tan x \equiv \tan x \cos 2x. \quad [5]$$

- (ii) Hence, or otherwise, solve the equation

$$\sin 2x - \tan x = 2 \cos 2x,$$

for x in the interval $0 \leq x \leq 180^\circ$. [4]

Question 5

- (i) Given that $\cos x = \sqrt{3} - 1$, find the value of $\cos 2x$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

- (ii) Given that

$$2 \cos (y + 30)^\circ = \sqrt{3} \sin (y - 30)^\circ,$$

find the value of $\tan y$ in the form $k\sqrt{3}$ where k is a rational constant. [5]

Question 6

- (i) Express $4 \sin x + 3 \cos x$ in the form $R \sin (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

- (ii) State the minimum value of $4 \sin x + 3 \cos x$ and the smallest positive value of x for which this minimum value occurs. [3]

- (iii) Solve the equation

$$4 \sin 2\theta + 3 \cos 2\theta = 2,$$

for θ in the interval $0 \leq \theta \leq \pi$, giving your answers to 2 decimal places. [4]

Question 7

- (i) Show that

$$\sin (x + 30)^\circ + \sin (x - 30)^\circ \equiv a \sin x^\circ,$$

where a is a constant to be found. [3]

- (ii) Hence find the exact value of $\sin 75^\circ + \sin 15^\circ$, giving your answer in the form $b\sqrt{6}$. [3]

Question 8

(i) Prove the identity

$$2 \cot 2x + \tan x \equiv \cot x, \quad x \neq \frac{n}{2}\pi, \quad n \in \mathbb{Z}. \quad [5]$$

(ii) Solve, for $0 \leq x < \pi$, the equation

$$2 \cot 2x + \tan x = \operatorname{cosec}^2 x - 7,$$

giving your answers to 2 decimal places. [6]

Question

It is given that $\cos a = \frac{3}{5}$, where $0^\circ < a < 90^\circ$. Showing your working and without using a calculator to evaluate a ,

(i) find the exact value of $\sin(a - 30^\circ)$, [3]

(ii) find the exact value of $\tan 2a$, and hence find the exact value of $\tan 3a$. [4]

Question 10

(i) Express $(\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

(a) $(\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta = -4$, [2]

(b) $(\sqrt{6}) \cos \frac{1}{2}\theta + (\sqrt{10}) \sin \frac{1}{2}\theta = 3$. [4]

Answers – Revision Practice

1.	i) $x = 2 - \sqrt{3}$ ii) $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$
2.	i) $\sqrt{13}$, 56.3 (3sf) iii) $x = 72.4, 220.2$ (1dp)
3.	$x = 0.73, 2.41$ (2dp)
4.	ii) $x = 45^\circ, 63.4^\circ$ (3sf), 135°
5.	i) $7 - 4\sqrt{3}$ ii) $\frac{3}{5}\sqrt{3}$
6.	i) 5, $\alpha = 0.644$ (3sf) ii) -5, $x = 4.07$ (3sf) iii) $\theta = 1.04, 3.03$ (2dp)
7.	i) $a = \sqrt{3}$ ii) $\frac{1}{2}\sqrt{6}$
8.	ii) $x = 0.32, 2.68$ (2dp)
9.	i) $\frac{1}{10}(4\sqrt{3} - 3)$ ii) $-\frac{24}{7}, -\frac{44}{117}$
10.	i) $R = 4, \alpha = 52.24$, ii) a) 232.2 b) 21.7