AS Final Exam : Revision 6 Vectors

P1 June 08

10 Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

(i) Find the value of p for which OA and OB are perpendicular. [2]

(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree. [3]

(iii) Express the vector $\overrightarrow{AB}$ is terms of p and hence find the values of p for which the length of AB is 3.5 units. [4]

P1 Nov 08

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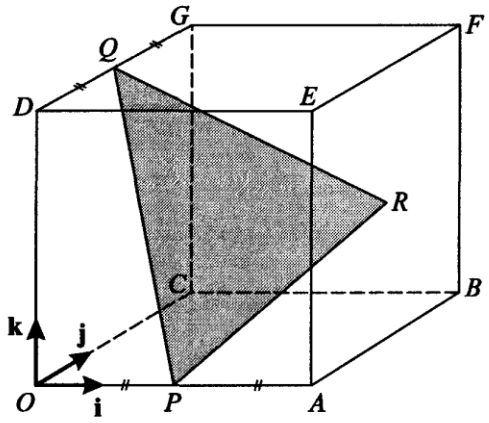

The diagram shows a cube $OABCDEFG$ in which the length of each side is 4 units. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.

(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle QPR. [4]

(iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

P1 June 07

9 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

(i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

(ii) Find the values of m, n and k. [4]

P1 Nov 07

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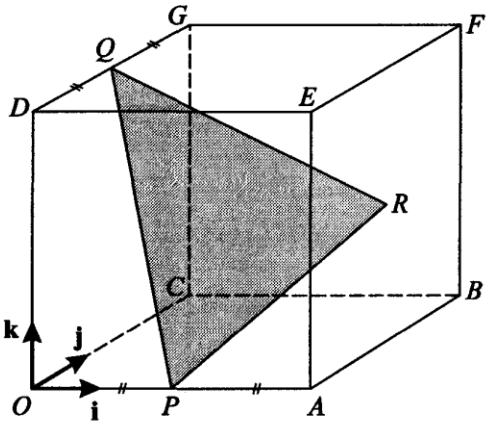

The diagram shows a cube $OABCDEFG$ in which the length of each side is 4 units. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.

(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle QPR. [4]

(iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

P1 June 06

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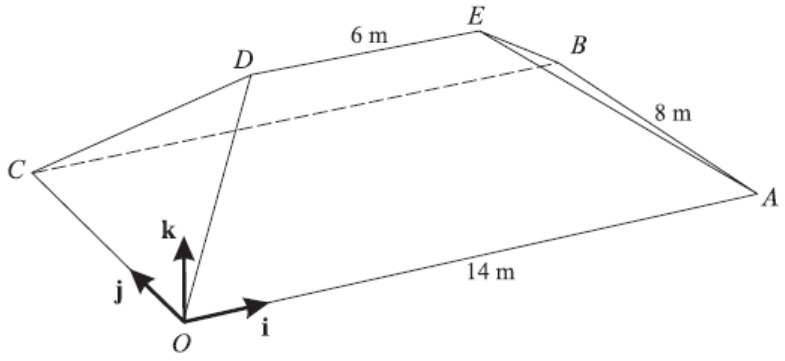

The diagram shows the roof of a house. The base of the roof, $OABC$, is rectangular and horizontal with $OA = CB = 14$ m and $OC = AB = 8$ m. The top of the roof DE is 5 m above the base and $DE = 6$ m. The sloping edges OD, CD, AE and BE are all equal in length.

Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards.

(i) Express the vector $\overrightarrow{OD}$ in terms of **i**, **j** and **k**, and find its magnitude. [4]

(ii) Use a scalar product to find angle DOB. [4]

P1 Nov 06

4 The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB. [3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

P1 June 05

11 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree. [4]

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(iii) The point C is such that $\overrightarrow{OC} = 6\mathbf{j} + p\mathbf{k}$, where p is a constant. Given that the lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are equal, find the possible values of p. [4]

P1 Nov 05

4 Relative to an origin O, the position vectors of points P and Q are given by

$$\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix},$$

where q is a constant.

(i) In the case where $q = 3$, use a scalar product to show that $\cos POQ = \frac{1}{7}$. [3]

(ii) Find the values of q for which the length of $\overrightarrow{PQ}$ is 6 units. [4]

P1 June 04

9 Relative to an origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix},$$

where p and q are constants. Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of p for which angle $AOC = 90°$, [3]

(iii) the values of q for which the length of $\overrightarrow{AD}$ is 7 units. [4]

P1 Nov 04

8 The points A and B have position vectors $\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $-5\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ respectively, relative to an origin O.

(i) Use a scalar product to calculate angle AOB, giving your answer in radians correct to 3 significant figures. [4]

(ii) The point C is such that $\overrightarrow{AB} = 2\overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

P1 June 03

8 The points A, B, C and D have position vectors $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $2\mathbf{j} + 7\mathbf{k}$ and $-2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$ respectively.

(i) Use a scalar product to show that BA and BC are perpendicular. [4]

(ii) Show that BC and AD are parallel and find the ratio of the length of BC to the length of AD. [4]

P1 Nov 03

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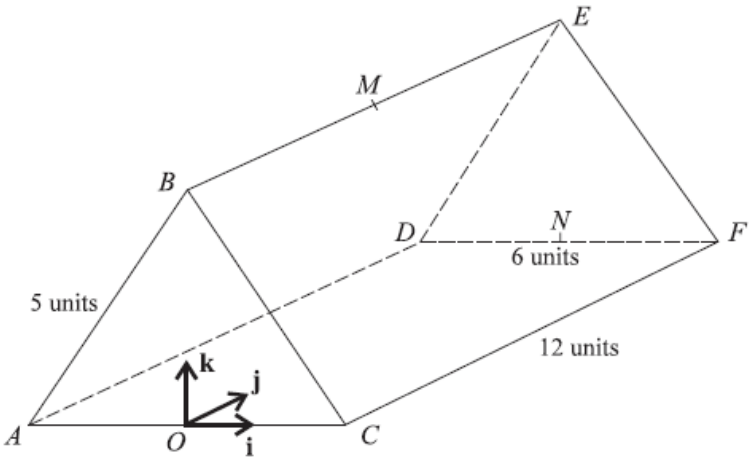


The diagram shows a triangular prism with a horizontal rectangular base $ADFC$, where $CF = 12$ units and $DF = 6$ units. The vertical ends ABC and DEF are isosceles triangles with $AB = BC = 5$ units. The mid-points of BE and DF are M and N respectively. The origin O is at the mid-point of AC.

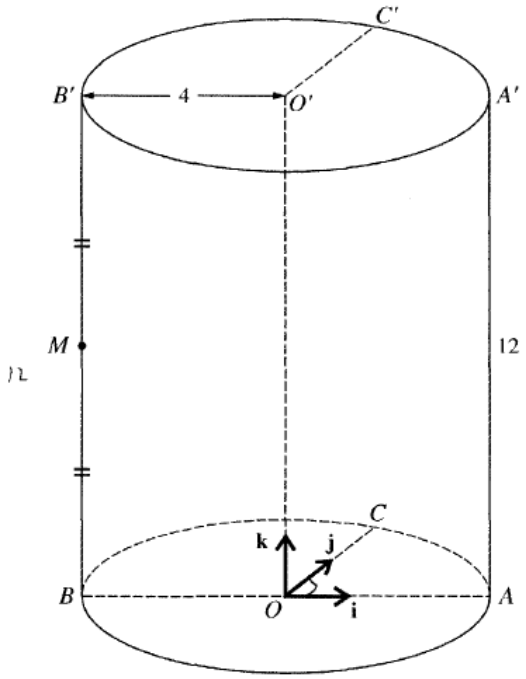
Unit vectors **i**, **j** and **k** are parallel to OC, ON and OB respectively.

(i) Find the length of OB. [1]

(ii) Express each of the vectors $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

(iii) Evaluate $\overrightarrow{MC}.\overrightarrow{MN}$ and hence find angle CMN, giving your answer correct to the nearest degree. [4]

P1 June 02



The diagram shows a solid cylinder standing on a horizontal circular base, centre O and radius 4 units. The line BA is a diameter and the radius OC is at 90° to OA. Points O', A', B' and C' lie on the upper surface of the cylinder such that OO', AA', BB' and CC' are all vertical and of length 12 units. The mid-point of BB' is M.

Unit vectors **i**, **j** and **k** are parallel to OA, OC and OO' respectively.

(i) Express each of the vectors $\overrightarrow{MO}$ and $\overrightarrow{MC'}$ in terms of **i**, **j** and **k**. [3]

(ii) Hence find the angle OMC'. [4]

P1 Nov 02

7 Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} p \\ p \\ p+1 \end{pmatrix}$, find

(i) the angle between the directions of **a** and **b**, [4]

(ii) the value of p for which **b** and **c** are perpendicular. [3]