REVISION EXERCISE 1

1. Solve the simultaneous equations $10y - 6x = 3$, $(x - 2)^2 + (2y - 3)^2 = 61$.

2. Given that $\lg 2 = a$ and $\lg 3 = b$, express $\lg \sqrt[3]{972}$ in terms of a and b. Find x if $\lg x = 3a - 4b + 1$.

3. (a) Find the exact value of x if $8^x = \sqrt{2\sqrt{8\sqrt{2}}}$.
   (b) Solve the following equations.
      (i) $2 \log_2 x = 4 + \log_2 (x + 5)$
      (ii) $4e^{2y} - 21 = 0$
      (iii) $\lg (4^z + 2) - z \lg 2 = \lg 3$

4. (a) When the expression $2x^2 - (8 - p)x + (p + 1)(p - 3)$ is divided by $x + p$, the remainder is p. Calculate the possible values of p.
   (b) The expressions $x^3 + ax^2 - x + b$ and $x^3 + bx^2 - 5x + 3a$ have a common factor $x + 2$. Find the value of a and of b.

5. (a) Given that $f(x) = 8x^3 + 4x - 3$, find the remainder, if it exists, when
      (i) $f(x)$ is divided by $2x + 1$,
      (ii) $f(x)$ is divided by $2 - x$,
      (iii) $\dfrac{1}{f(x)}$ is divided by $x + 1$,
      (iv) $f(x + 1)$ is divided by $x + 2$.
   (b) Solve the equation $4x^3 + 3x^2 - 16x = 12$. Hence, find to three significant figures, the value of x such that $4e^{3x} + 3e^{2x} - 16e^x = 12$.

6. (a) Find the range of values of x for which $x^2 - 3x + 8$ has values between 6 and 12.
   (b) Factorise $2x^2 - 3x - 5$ and hence sketch the curve $y = 2x^2 - 3x - 5$.
   (c) Express $-2x^2 + 8x + 9$ in the form $a(x - h)^2 + k$. Hence state the maximum value of $-2x^2 + 8x + 9$ and sketch the curve $y = -2x^2 + 8x + 9$.

7. If the equation $x^2 - 2kx + k^2 - 2k - 6 = 0$ has real roots, show that the roots of the equation $x^2 + 6x = 3 + k$ has two distinct real roots.

1. (a) Given that $f(x) = \frac{1}{2}\left[(x + 6)^2 + (x - 4)^2\right]$, express $f(x)$ in the form $(x + h)^2 + k$. Hence, sketch the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 1$.
   (b) Solve the simultaneous inequalities $x(x - 1) < 6$, $13x - 6 \leqslant 2x^2$.

2. (a) Find the range of values of k for which the equation $2kx^2 + (8 - 4k)x + k + 1 = 0$ has real roots. State the largest integer value of k for which this equation has no real roots.
   (b) If each of the equations $px^2 + qx + 2r = 0$, $rx^2 + px - q + 1 = 0$ have equal real roots, find a relation between p and q.

3. Solve the simultaneous equations $x + 2y = 7$, $x^2 + 4y^2 = 37$. Hence, find the possible values of a and b, correct to three significant figures where necessary, which satisfy both the equations $3^a + 2^{b+1} = 7$, $9^a + 4^{b+1} = 37$.

4. (a) Given that $y = 3(4)^{x+2}$, find, without using tables or calculators,
   (i) the value of y when $x = -\frac{1}{2}$,
   (ii) the value of x when $y = 96$.
   (b) Solve the following equations, giving your answers correct to three significant figures.
   (i) $\ln x^3 + 2\ln x^2 - 5\ln x + \ln\sqrt{x} = 5$
   (ii) $3^x = 5 \times 2^{x+1}$

5. (a) Without using tables or calculators, evaluate $\dfrac{\log_5 9 + 2\log_5 6 - 4\log_5 3}{\log_5 40 - \log_5 4 - 1}$.
   (b) The curve $y = ab^x$ passes through the points $(0, 5)$ and $\left(\frac{2}{3}, \frac{5}{4}\right)$. Find the value of a and of b.

6. (a) Given that $4x^2 - 6x + 9 = A(x - 1)(2x + 1) + B(x - 1) + C$ for all values of x, find the values of A, B and C.
   (b) Solve the cubic equation $2x^3 + 36 = 11x^2 - 3x$.

7. (a) The expression $px^3 - 5x^2 + qx + 10$ has factor $2x - 1$ but leaves a remainder of -20 when divided by $x + 2$. Find the values of p and q and factorise the expression completely.
   (b) The quadratic equation $x^2 + ax + b = 2$ has roots -1 and 4. Find
   (i) the value of a and of b,
   (ii) the range of values of c for which the equation $x^2 + ax + b = c$ has real roots.

Answers

Revision Exercise 1

1. (a) $(x + 1)^2 + 25$ (b) $-2 < x \leqslant \frac{1}{2}$

2. (a) $k \leqslant 1$ or $k \geqslant 8$, $k \neq 0$; 7   (b) $2p^3 = q^2 - q^3$

3. $x = 6$, $y = \frac{1}{2}$ or $x = 1$, $y = 3$; $a = 1.63$, $b = -1$ or $a = 0$, $b = 1.58$

4. (a) (i) 24   (ii) $\frac{1}{2}$ (b) (i) 7.39   (ii) 5.68

5. (a) 2   (b) $a = 5$, $b = \frac{1}{8}$

6. (a) $A = 2$, $B = -4$, $C = 7$ (b) $-\frac{3}{2}$, 3, 4

7. (a) $p = 6$, $q = -19$; $(2x - 1)(x - 2)(3x + 5)$

   (b) (i) $a = -3$, $b = -2$ (ii) $c \geqslant -\frac{17}{4}$

Revision Exercise 2

1. $x = -3$, $y = -\frac{3}{2}$ or $x = 7$, $y = \frac{9}{2}$ 2. $\frac{2a + 5b}{3}$, $\frac{80}{81}$

3. (a) $\frac{11}{24}$ (b) (i) 20   (ii) 0.829   (iii) 0, 1

4. (a) $\frac{1}{2}$, -3 (b) $a = 2$, $b = -2$

5. (a) (i) -6 (ii) 69   (iii) Does not exist   (iv) -15

   (b) -2, $-\frac{3}{4}$, 2; 0.693   6. (a) $-1 < x < 1$ or $2 < x < 4$

   (b) $(2x - 5)(x + 1)$ (c) $-2(x - 2)^2 + 17$; 17