Derivative of Trigo Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}\sin(ax+b) = a\cos(ax+b)$$

$$\frac{d}{dx}\cos(ax+b) = -a\sin(ax+b)$$

$$\frac{d}{dx}\tan(ax+b) = a\sec^2(ax+b)$$

KFC

Example: Find

$$(i)\frac{d}{dx}\sin 4x = (ii)\frac{d}{dx}\cos 6x =$$

Find the gradient function of the curve $y = \tan 3x$.

Find the gradient of the curve $y = \cos(3x+1)$ when $x = \pi$.

Show that
$$y = \sin(x-4) \Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = 1$$

If
$$y = \tan(7x - 2)$$
, show that $\frac{dy}{dx} - 7y^2 = 7$.

$$\frac{d}{dx}[\sin f(x)] = \cos f(x) \times \frac{d}{dx} f(x)$$

$$\frac{d}{dx}[\cos f(x)] = -\sin f(x) \times \frac{d}{dx} f(x)$$

$$\frac{d}{dx}[\tan f(x)] = \sec^2 f(x) \times \frac{d}{dx} f(x)$$

KFC

Find
$$\frac{d}{dx}\sin(x^2+1)$$
.

Show that the tangent of the curve

$$y = \cos(x^3 + x^2 + 1)$$
at the origin is parallel to the $x - axis$.

Show that the curve $y = \tan(4x^2 + 4)$ has only one turning point for all values of x.

In general,

$$\frac{d}{dx}\left[\sin^n f(x)\right] = n\sin^{n-1} f(x) \times \frac{d}{dx}\sin f(x)$$

$$\frac{d}{dx}\left[\cos^n f(x)\right] = n\cos^{n-1} f(x) \times \frac{d}{dx}\cos f(x)$$

$$\frac{d}{dx}\left[\tan^n f(x)\right] = n\tan^{n-1} f(x) \times \frac{d}{dx}\tan f(x)$$

Find
$$\frac{d}{dx} \sin^2 4x$$
.

Given that $f(x) = \cos^5(x+1)$. Find the values of x such that f'(x) = 0 for $0 \le x \le \pi$.

Find the x – coordinates of the turning points of the curve

$$y = \tan^4(x^2 + x)$$
 in the domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Determine the

nature of each turning point.

Find the equation of the tangent to the curve

$$y = \sec^3 x$$
 when $x = \frac{\pi}{4}$.

Find the equation of the normal to the curve

$$y = \operatorname{cosec}^4 3x$$
 when $x = \frac{\pi}{4}$.

Show that

$$\frac{d}{dx} \left[\cot^5 (x^3 + 1) \right] = -15x^2 \cot^6 (x^3 + 1) \sec^2 (x^3 + 1)$$

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Find the equation of the tangent and normal where

$$x = \frac{1}{3}\pi$$
 on the curve $y = \cos 3x$.

Find the equation of the tangent and normal where

$$x = \frac{1}{2}\pi$$
 on the curve $y = 3\sin^2 2x$.

The gross national product (GNP) of a country, \$ P billion, at t years after the year 2000 is given by $P = 1 + 0.02t + 0.05 \sin 0.6t$. At what rate is the GNP changing in the year (a) 2000 (b) 2005?

P, Q and R are the points on the graph of $y = \cos x$ for which

$$x = 0$$
, $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$ respectively.

- (a) Find the point S where the normal at Q meets the y axis.
- (b) Compare the distances SP, SQ and SR. Hence, use a sketch

to show how the curve $y = \cos x$, over $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, is

related to the circle with centre S and radius SQ.

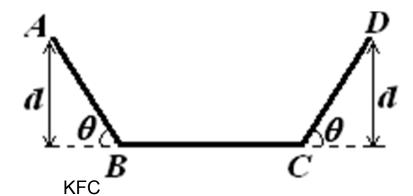
Given that $f(x) = \sin x + \sin 2x$.

- (a) Find, in terms of π , the x intercepts of the curve y = f(x) for $0 \le x \le 2\pi$.
- (b) Show that $f(\pi \theta) = \sin \theta \sin 2\theta$, and that $f(\pi \theta) + f(\pi + \theta) = 0, \forall \theta$.
- (d) Show that the greatest value of f, for $0 \le x \le 2\pi$,

occurs when
$$\cos x = \frac{-1 + \sqrt{33}}{8}$$
.

A length of channel of given depth d is to be made from a rectangular sheet of metal of width 2a. The metal is to be bent in such a way that the cross-section is shown in the figure.

- (a) Show that $BC = 2(a d \csc \theta)$
- (b) Show that area $ABCD = 2ad + d^2(\cot\theta 2\csc\theta)$.
- (c) Show that MAX area of ABCD is $d(2a-d\sqrt{3})$
- (d) Show that $2d \le a\sqrt{3}$, considering BC.



Trigo Derivative