

(a) $OB = 7 \sin \theta + 3 \Rightarrow OB^2 = 49 \sin^2 \theta + 42 \sin \theta + 9$
 $OE = 7 \cos \theta + 4 \Rightarrow OE^2 = 49 \cos^2 \theta + 56 \cos \theta + 16$

$$\Rightarrow DE^2 = OB^2 + OE^2$$

$$DE^2 = 74 + 42 \sin \theta + 56 \cos \theta$$

$$42 \sin \theta + 56 \cos \theta = R \cos(\theta - \alpha)$$

$$R = \sqrt{42^2 + 56^2} = 70$$

$$\alpha = \tan^{-1} \frac{42}{56} = 36.87^\circ$$

(b) (i)

C_1 touches OX , $OB = 4$

$$OB = 7 \sin \theta + 3$$

$$4 = 7 \sin \theta + 3 \Rightarrow \theta = 8.2^\circ$$

C_2 touches OY , $OE = 3$

$$OE = 7 \cos \theta + 4$$

$$3 = 7 \cos \theta + 4 \Rightarrow \theta = 98.2^\circ$$

$$\therefore 8.2^\circ \leq \theta \leq 98.2^\circ$$

(c) $DE^2 = 74 + 70 \cos(\theta - 36.87^\circ)$ $-28.7^\circ \leq \theta - 36.87^\circ \leq 61.3^\circ$

$$\max DE = \sqrt{74 + 70(1)} = 12$$

$$\text{when } \cos(\theta - 36.87^\circ) = 1 \Rightarrow \theta = 36.87^\circ$$

$$\min DE = \sqrt{74 + 70 \cos(98.2 - 36.87)^\circ} = 10.37$$

$$\text{when } \theta = 98.2^\circ$$