Pure Mathematics 1

Tutorial 6 Vectors

1

The position vectors of A and B relative to the origin O are $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j}$ respectively.

(i) Given that C lies between A and B such that AC = 2CB, find the position vector of C. [3]

(ii) Calculate the angle AOC to the nearest degree. [2]

$\left[\frac{5}{3}\boldsymbol{i} + \frac{5}{3}\boldsymbol{j} + \boldsymbol{k}\,;\, 42\right]$

2

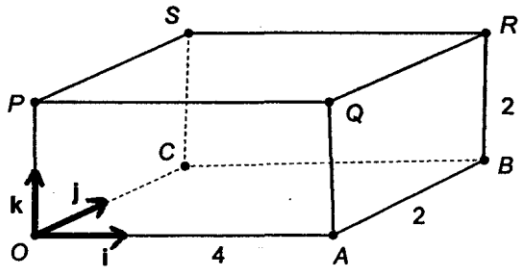
The position vectors of A and B, relative to the origin O, are $\underline{i} + 2\underline{j} + 3\underline{k}$ and $3\underline{i} + 4\underline{j} + 5\underline{k}$ respectively, the point C is given by $\overrightarrow{OC} = 2\overrightarrow{OA}$ Find

a) the length of OC [2]

b) cos ∠ AOB , in surd form. [3]

$\left[2\sqrt{14}\,;\, \frac{13\sqrt{7}}{35}\right]$

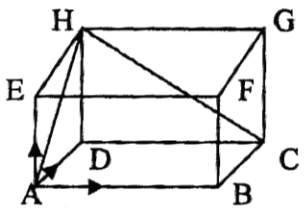
3



The diagram shows a cuboid *OABCPQRS* in which the length of *OA* is 4 units and the length of *AB* and *BR* are each 2 units. Unit vectors **i**, **j**, **k** are taken along the edges *OA*, *OC*, *OP* respectively. Calculate the angle between the directions of $\overrightarrow{OB}$ and $\overrightarrow{SA}$. [5]

[56.8]

4



The diagram above shows a box with vectors $\overrightarrow{AB} = 5i$, $\overrightarrow{AD} = 2j$ and $\overrightarrow{AE} = 3k$.

i) Find the vectors $\overrightarrow{HA}$ and $\overrightarrow{HC}$. [2]

ii) Hence find the angle AHC. [3]

$[-2\mathbf{j}-3\mathbf{k}, 5\mathbf{i}-3\mathbf{k}; 64.7]$

5

The points A and B have position vectors given respectively by

$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OB} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. The point P on $\overrightarrow{AB}$ is such that

$AP : PB = \lambda : 1 - \lambda$.

(i) Show that $\overrightarrow{OP} = (1 + \lambda)\mathbf{i} + (2 - 5\lambda)\mathbf{j} + (-2 + 8\lambda)\mathbf{k}$. [3]

(ii) Find the value of λ for which $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{AB}$ [3]

[5/18]

6

The points A, B and C have position vectors $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ and $2\mathbf{j} + 7\mathbf{k}$ respectively.

a) Show that BA and BC are perpendicular. [3]

b) Calculate the angle BAC. [3]

[52.6]

7

Three points A, B, C have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively, given by

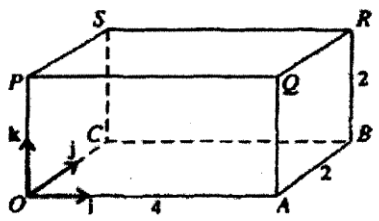
$\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = -3\mathbf{i} + 13\mathbf{j} + 7\mathbf{k}$.

a) Express $\mathbf{b} - \mathbf{a}$ and $\mathbf{c} - \mathbf{b}$ as column vectors, and hence describe precisely the position of B in relation to the points A and C. [3]

b) Calculate the angle between the directions of OB and AB, where O is the origin, giving your answer correct to the nearest degree. [3]

$\left[\begin{pmatrix}-2\\5\\2\end{pmatrix}$, B divides AC in the ratio 1: 2 ; 41$\right]$

8



The diagram shows a cuboid OABCPQRS in which the length of OA is 4 units and the lengths of AB and BR are each 2 units. Unit vectors **i**, **j**, **k** are taken along the edges OA, OC, OP respectively.

a) Express the vectors $\overrightarrow{OR}$ and $\overrightarrow{AS}$ in terms of **i**, **j** and **k**. [3]

b) Find the angle OBS. [4]

$\left[\begin{pmatrix}4\\2\\2\end{pmatrix}, \begin{pmatrix}-4\\2\\2\end{pmatrix}; 36.9\right]$

9

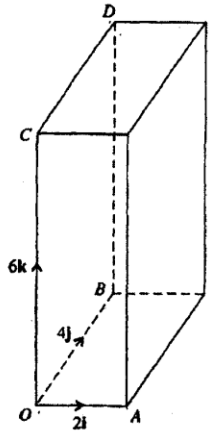
The position vectors of A and B are $2\mathbf{i}-3\mathbf{j}$ and $t\mathbf{i}+2\mathbf{j}$ respectively.

i) Find the value of t for which $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular. [3]

ii) If $t = 4$, find the angle AOB to the nearest degree. [4]

[3 ; 83]

10



The three edges of a rectangular box that have common vertex O are OA, OB and OC. The position vectors of A, B, C relative to *O* are $2i$, $4j$, $6k$ respectively. The vertex opposite to A is D.

(i) Find the position vector of the mid-point M of BD. [2]

(ii) The point P inside the box has position vector $i + 3j + 3k$. Find the angle CPA. [5]

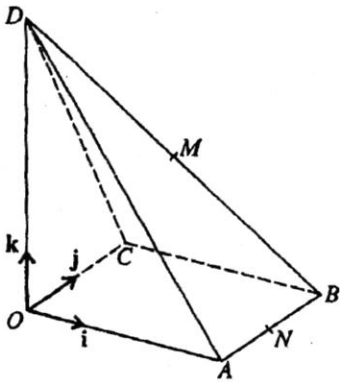
$[4\boldsymbol{j} + 3\boldsymbol{k}\,;\, 93.0]$

11 .a) The points P and Q have position vectors $\mathbf{a} + \mathbf{b}$ and $3\mathbf{a} - 2\mathbf{b}$ respectively relative to an origin O. Given that OPQR is a parallelogram, express the vectors **PQ** and **PR** in terms of **a** and **b**. [3]

b) By evaluating two scalar products, show that if OPQR is a square, then $|\mathbf{a}|^2 = 2\,|\mathbf{b}|^2$. [5]

$[\,2\mathbf{a} - 3\mathbf{b}, \mathbf{a} - 4\mathbf{b}\,]$

12



The diagram shows a pyramid DOABC. Taking unit vectors **i**,**j** and **k** as shown, the position vectors are given by

$\overrightarrow{OA} = 4\mathbf{i}, \quad \overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{OC} = 2\mathbf{j}, \quad \overrightarrow{OD} = 6\mathbf{k}$.

The mid-points of BD and AB are M and N respectively.

(i) Find the vector $\overrightarrow{MN}$ and the angle between the directions of $\overrightarrow{MN}$ and $\overrightarrow{OB}$. [5]

(ii) The point P lying on OD has position vector $p\mathbf{k}$ and is such that the angle PMN is a right angle. Find the value of p. [3]

$\left[\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix},\ 60.3;\ 5/3\right]$

13 . Two small insects A and B are crawling on the walls of a room, with A starting from ceiling. The floor is horizontal and forms the x-y plane, and the z-axis is vertically upwards. Relative to the origin, the position vector of the insects at time t seconds $(0 \le t \le 10)$ are

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + (4 - \frac{1}{10}\mathbf{t})\mathbf{k}$$

$$\overrightarrow{OB} = (\frac{1}{5}\mathbf{t} + 1)\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

where the unit of distance is the meter and **i**, **j** ,**k** are the unit vectors along the x, y and z axis respectively.

(i) Write down the height of the room. [1]
(ii) Show that the insect move in such a way that angle BOA=90°. [3]
(iii) Write down the expression of the vector $\overrightarrow{AB}$, hence find the distance between the two insects in terms of t. [4]

$$\left[4m\,;\ \frac{1}{5}t\boldsymbol{i} - 6\boldsymbol{j} + \left(\frac{1}{10}t - 2\right)\boldsymbol{k}\,;\ \sqrt{\frac{1}{20}t^2 - \frac{2}{5}t + 40}\right]$$

14 Relative to an origin O, points A and B have position vectors $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}$ respectively. Find the lengths of OA and OB. Hence or otherwise, find the exact area of the triangle AOB. [8]

$$\left[3\,;7\,;\ \frac{\sqrt{41}}{2}\right]$$

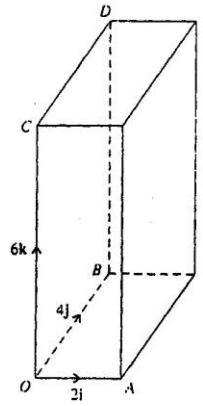
15 A and B have position vectors $\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} k \\ 10 \end{pmatrix}$ respectively relative to an origin O.

(i) If $\mathbf{c} \cdot \mathbf{a} = 0$, find a possible vector **c** and hence a unit vector perpendicular to **a**. [4]

(ii) Find the value of k if $\angle AOB = \cos^{-1}(\frac{2}{\sqrt{5}})$. [5]

$$\left[\begin{pmatrix} -3 \\ 4 \end{pmatrix} \text{ or any multiple}\,;\ \frac{1}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix}\,;5\,,\ 55\right]$$

16



The three edges of a rectangular box that have a common vertex O are OA, OB, OC. The position vectors A, B and C relative to O are $2\mathbf{i}$, $4\mathbf{j}$ and $6\mathbf{k}$ respectively. The vertex opposite to A is D (see diagram).

(i) Find the position vector of the mid-point M of BD. [2]

(ii) Find a unit vector in the direction of the vector $\overrightarrow{OM}$. [2]

(iii) The point P inside the box has position vector $\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$. Find the angle CPA. [5]

$$\left[\begin{pmatrix}0\\4\\3\end{pmatrix};\frac{1}{5}\begin{pmatrix}0\\4\\3\end{pmatrix};93.0\right]$$

17

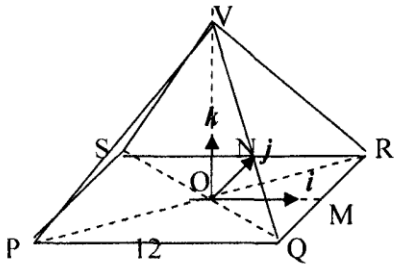


Diagram above shows a right pyramid with a square base PQRS and centre O. Given VQ = 10 cm and PQ = QR = 12 cm, and that $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors along the directions of $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{OV}$ respectively with M, N being the mid points of RQ and SR.

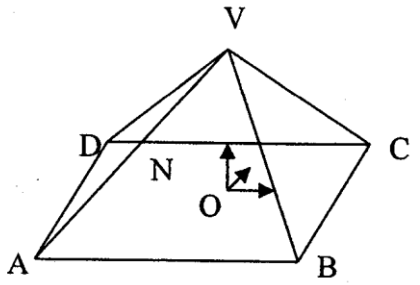
(i) Show that OV = $2\sqrt{7}$ cm [2 marks]

(ii) Find $\overrightarrow{RP}$, $\overrightarrow{RV}$ in terms of $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ [3 marks]

(iii) Evaluate $\overrightarrow{RP} \bullet \overrightarrow{RV}$. Hence calculate the angle PRV [4 marks]

$$[-12\mathbf{i}-12\mathbf{j};-6\mathbf{i}-6\mathbf{j}+2\sqrt{7}\mathbf{k};\ 144;\ 31.9]$$

18 In the diagram, O is the centre of the square base ABCD of a right pyramid, vertex V. Unit vectors **i, j, k** are parallel to AB, AD and OV respectively. The length of AB is 4 units and the length of OV is $2h$ units ($h>0$). The point O is taken as the origin for position vectors.



(i) Write down the expression of the vector $\overrightarrow{VB}$. [2]

(ii) Given the position vector $\overrightarrow{OX}$ is $\frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{1}{2}h\mathbf{k}$, find the exact value of h if $\overrightarrow{OX}$ is perpendicular to $\overrightarrow{VB}$. [3]

(iii) Find the position vector of the midpoint N of $\overrightarrow{AV}$ if the length of OV is 2 units. [4]

$$\left[\begin{pmatrix}2\\-2\\-2h\end{pmatrix};\ \sqrt{2};\ \begin{pmatrix}-1\\-1\\1\end{pmatrix}\right]$$

19 Vectors **r** and **s** are given by

$\mathbf{r} = m\mathbf{i} + (2m-1)\mathbf{j} - \mathbf{k}$, $\mathbf{s} = (1-m)\mathbf{i} + 3m\mathbf{j} + (4m-1)\mathbf{k}$, where m is a scalar.

a) Find the values of m for which **r** and **s** are perpendicular. [4]

b) When m = 2, **r** and **s** are the position vectors of the points A and B respectively, referred to an origin O.

i) Find **AB.** [3]

ii) Calculate the angle BAO, giving your answer to the nearest degree. [4]

$$\left[\mathbf{1/5,\ 1};\ \begin{pmatrix}-3\\3\\8\end{pmatrix};82\right]$$