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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION DECEMBER 2011

(June 2011 Intake)

Wednesday **7 Dec 2011** **8.30 – 10.15 am**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

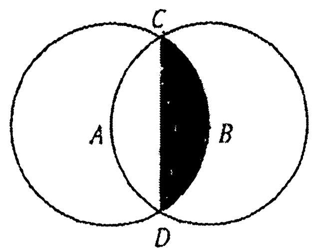
This document consists of **4** printed pages.

1 Solve the equation $3\tan^2 x - 2\sqrt{3}\tan x = 3$, for $0° \leq x \leq 360°$. [4]

2 (i) Find the first three terms, in ascending powers of x, of the binomial expansion of $(2 + kx)^7$ where k is a non-zero constant. Give each term in its simplest form. [4]

(ii) Given that the coefficient of x^2 is 6 times the coefficient of x, find the value of k. [2]

3



The diagram shows two circles, with centres A and B, intersecting at C and D in such a way that the centre of each lies on the circumference of the other. The radius of each circle is 1 unit.

(i) Find the angle CAD and calculate the area of the shaded region (bounded by the arc CBD and the straight line CD). [3]

(ii) Hence, show that the area of the region common to the interiors of the two circles is approximately 39% of the area of one circle. [4]

4 The curve $y = 1 + \frac{1}{x}$ and the line $4x - y + 1 = 0$ intersect at two points.

(i) Find the coordinates of the two points. [4]

(ii) Find the equation of the perpendicular bisector of the line joining the two points. [4]

5 (i) Express $9x^2 + 12x + 7$ in the form $(ax + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(ii) Find the set of values of x for which $9x^2 + 12x + 7 > 19$. [4]

(iii) Find the set of values taken by $\frac{1}{9x^2+12x+7}$ for all real values of x. [2]

6 A trading company made a profit of £50,000 in 2006 (Year 1). A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$. The model therefore predicts that in 2007 (Year 2) a profit of £50,000r will be made.

(i) Write down an expression for the predicted profit in Year n. [1]

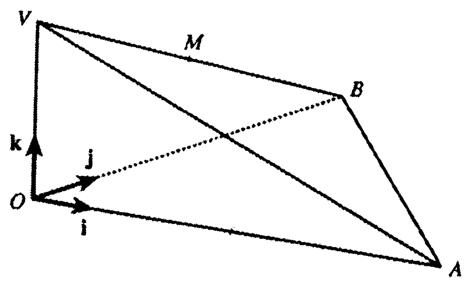
(ii) The model predicts that in Year n, the profit made will exceed £200,000.
Show that $n > \frac{\log 4}{\log r} + 1$. [3]

Using the model with $r = 1.09$,

(iii) find the year in which the profit made will first exceed £200,000. [2]

(iv) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10,000. [3]

7



The diagram shows a triangular pyramid $OABV$, whose base is the right angled triangle OAB and whose vertical height is OV. The perpendicular unit vectors **i**, **j** and **k** are directed along OA, OB and OV as shown, and the position vectors of A, B and V are given by

$$\overrightarrow{OA} = 10\boldsymbol{i}, \quad \overrightarrow{OB} = 8\boldsymbol{j}, \quad \overrightarrow{OV} = 6\boldsymbol{k}.$$

(i) The point M is the midpoint of VB. Find the position vector of M and the length of OM. [3]

(ii) The point P lies on OA, and has position vector $p\boldsymbol{i}$. Show that the value of the scalar product $\overrightarrow{VB}.\overrightarrow{MP}$ is -14. [3]

(iii) For the case where P is at the midpoint of OA, find angle PMB, giving your answer correct to the nearest degree. [3]

[Turn over

8 The functions are defined by

$$f(x) = 4x^2 - 1, \qquad x \geq A, \quad \text{where } A \text{ is a constant.}$$
$$g(x) = \sqrt{x}, \qquad x \geq 0.$$

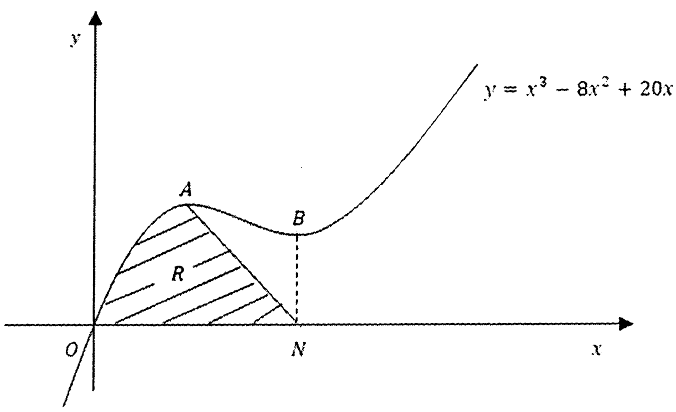
(i) Find $fg(x)$ and $gf(x)$. [2]

(ii) State the smallest value of A for f to have an inverse. [1]

(iii) For these value of A, sketch on the same axes, the graphs of $y = f(x)$ and its inverse. [3]

(iv) Solve the equation for $f(x) = gf(x)$. [4]

9



The diagram above shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(i) Find the x coordinates of A and B. [4]

(ii) Hence, verify that A is a maximum point. [2]

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown above, is bounded by the curve, the x-axis and the line from A to N.

(iii) Find the exact area of R. [7]