Algebra

A2 Trial Examination

August 2010

1 Expand $\dfrac{1}{(3+x)^4}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$$\left[\frac{1}{81} - \frac{4}{243}x + \frac{10}{729}x^2\right]$$

4 The polynomial $x^4 + x^3 + ax^2 - 17x - 30$ is denoted by $p(x)$, where a is a constant. It is given that when $p(x)$ is divided by $(x+4)$ the remainder is 182. Find the value of a and factorise $p(x)$ completely. [4]

$$[-3\, ;f(x) = (x^2+2x+5)(x+2)(x-3)]$$

March 2010

1 The polynomial $x^3 + ax^2 + bx - 6$ is divisible by $x+3$ and $x-2$. Determine the values of a and b. [4]

$$[2, -5]$$

August 2009

2 Solve the inequality $|x+1| < |2x+1|$. [3]

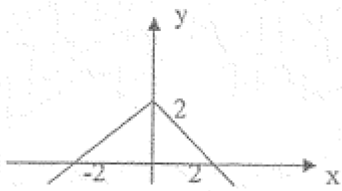
$$\left[x < -\frac{2}{3}\ or\ x > 0\right]$$

10 By using partial fractions, or otherwise, find the first three terms in the expansion of $\dfrac{1}{(1-x)^2(2+x)}$ [10]

$$\left[\frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2\right]$$

March 2009

1



(i) The diagram shows the graph of $y = 2 - |x|$.

Sketch the graph of $y = |2 - |x||$. [1]

(ii) Solve the inequality $|x+1| < |x-2|$. [3]

$$\left[x < \frac{1}{2}\right]$$

August 2008

4 (i) Sketch on the same axes the graphs of $y = |2x-6|$ and $y = |3x+1|$ [2]

(ii) Find the values of x for which $|2x-6| = |3x+1|$ [2]

(iii) Hence or otherwise find the range of values of x for which $|2x-6| > |3x+1|$. [2]

$$[1, -7; -7 < x < 1]$$

March 2008

1. Solve $|18 - 4x| < 2$. [3]

[4 < x < 5]

7. Let $f(x) = 6x^4 - 7x^3 + ax^2 + bx - 12$ where a and b are constants. If $(x-1)$ is a factor of f(x) and when f(x) is divided by $(x+1)$, the remainder is -50.

i) Find the values of a and b. [4]

ii) With these values of a and b, factorise f(x) completely. [4]

[-19, 32; $(x+2)(x-1)(2x-3)(3x-2)$**]**

10. Let $f(x) = \dfrac{3 - 5x + 3x^2}{(1-2x)(1+x^2)}$.

i) Express f(x) in partial fractions. [3]

ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected, $f(x) = 3 + x + 2x^2 + 9x^3$. [5]

iii) State the range of values of x for the expansion to be valid. [2]

$$\left[\frac{1}{1-2x} + \frac{2-x}{1+x^2}; -\frac{1}{2} < x < \frac{1}{2}\right]$$

August 2007

1. Solve the inequality $2\,|x-3| \;<\; |x-2|$. [3]

$$\left[\frac{8}{3} < x < 4\right]$$

2. Expand $\sqrt{(4+x^2)}$ in ascending powers of x up to and including the term x^4. [4]

$$\left[2+\frac{x^2}{4}-\frac{x^4}{64}\right]$$

7. Use the factor theorem to show that $x+2$ is a factor of $x^3 + ax^2 + 2ax + 8$ for all values of the constant a. [2]
Hence solve the equation
$$x^3 + 7x^2 + 14x + 8 = 0$$ [4]

$$[-2,-1,-4]$$

March 2007

3. The expression $2x^3 + ax^2 + bx + 2$ is exactly divisible by $(x+2)$ and leaves a remainder of 12 on division by $(x-2)$. Calculate the values of a and b and factorise the expression completely. (6)

$$[1;-5;(x+2)(2x-1)(x-1)]$$

7. Express $\dfrac{32x^2+17x+18}{(2-3x)(1+2x)^2}$ in partial fractions and hence obtain its series expansion in ascending powers of x, stating the terms up to and including the terms in x^3, and the values of x for which the expansion is valid. (8)

$$\left[\frac{8}{(2-3x)}+\frac{5}{(1+2x)^2};9-14x+69x^2-\frac{293}{2}x^3+\ldots;|x|<\frac{1}{2}\right]$$

August 2006

1. Find the expansion of $\dfrac{1+x^2}{\sqrt{(1+4x)}}$ in ascending powers of x up to and including the term in x^2. [3]

$$\left[1-2x+7x^2+\ldots\right]$$

4. Show that both $(x-\sqrt{3})$ and $(x+\sqrt{3})$ are factors of $f(x) = x^4 + x^3 - x^2 - 3x - 6$. [3]

Given that $f(x)$ can be expressed as two quadratic factors. Write down one quadratic factor of $f(x)$, and find the other quadratic factor of this polynomial. [3]

$$\left[x^2-3; x^2+x+2\right]$$

March 2006

1. Sketch, on the same diagram, the graphs of $y = |x-1|$ and $y = |x^2+x-2|$.
Hence, or otherwise, solve the inequality $|x^2+x-2| \geq |x-1|$. [4]

$$\left[x \leq -3 \text{ and } x \geq -1\right]$$

4. Let $p(x) = 4x^3 + 12x^2 + 5x - 6$.

(a) Calculate $p(2)$ and $p(-2)$, and state what you can deduce from your answers. [4]

(b) Hence or otherwise factorize $p(x)$ completely. [2]

$$\left[84, 0; x-2 \text{ is not a factor}, x+2 \text{ is not a factor}; (x+2)(2x+3)(2x-1)\right]$$

8. Given that $f(x) = \dfrac{x}{(x-1)(x+2)}$

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that the coefficient of x^3 in the expansion of $f(x)$ in ascending powers of x is $-\dfrac{3}{8}$. [5]

$$\left[\frac{1}{3(x-1)}+\frac{2}{3(x+2)}\right]$$

August 2005

1. The real polynomial $P(x) = (ax-b)^3$ has a remainder of -8 on division by $x-1$ and a remainder of 27 on division by $x-2$. Find the constants a and b. [3]

$$\left[5,7\right]$$

4. Express $\dfrac{5x^2-4x+12}{(x-2)(x^2+4)}$ in partial fractions. [3]

Hence obtain $\dfrac{5x^2-4x+12}{(x-2)(x^2+4)}$ as a series of ascending powers of x up to and including the term in x^3. [3]

$$\left[\frac{3}{x-2}+\frac{2x}{x^2+4}; -\frac{3}{2}-\frac{1}{4}x-\frac{3}{8}x^2-\frac{5}{16}x^3\right]$$

March 2005

1. Find the first three non-zero terms in the expansion , in ascending powers of x, of $\frac{(1+x)}{(1-x)}$.

[4]

$\left[1+2x+2x^2+\ldots\right]$