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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2012

(January and March 2011 Intakes)

Monday **2 April 2012** **8.30 am – 10.15 am**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

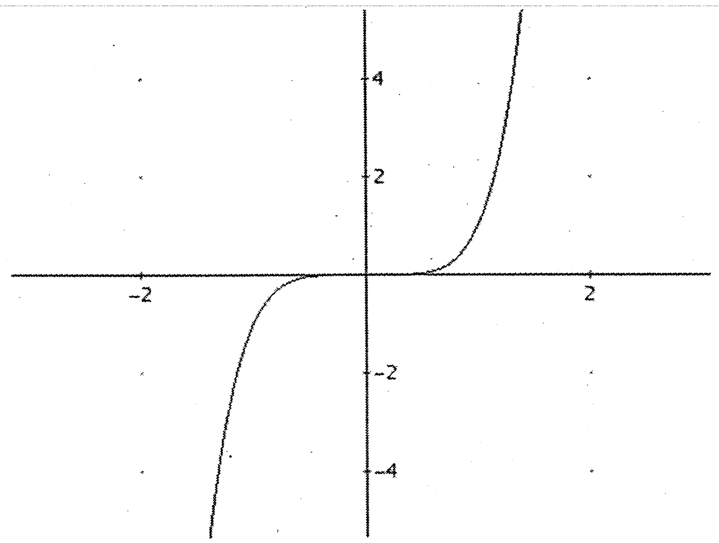
1 If $a^2 + b^2 = 14ab$, using $(a+b)^2 = a^2 + b^2 + 2ab$ or otherwise, prove that

$$\log\left(\frac{a+b}{4}\right) = \frac{1}{2}(\log a + \log b).$$ [3]

2 If $\frac{b}{a}$ is small enough for powers of $\frac{b}{a}$ higher than the third to be neglected show that

$$(a+b)^{\frac{1}{2}} - (a-b)^{\frac{1}{2}} = a^{\frac{1}{2}}\left(\frac{b}{a} + \frac{b^3}{8a^3}\right).$$ [5]

3 **(i)** Given below is the graph of $y = x^5$. Copy the graph and sketch $y = a - bx$ on the same diagram, where a and b are positive constants. Show that the equation $x^5 = a - bx$ has exactly one root. [1]



(ii) Determine the integer N such that the equation $x^5 + 2x - 53 = 0$ has a root in the interval $N < x < N+1$. N=2 [1]

(iii) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the root correct to 3 decimal places. $x \approx 2.175$ (3 d.p.) [3]

4 **(i)** Express $2\tan^2\theta - \frac{1}{\cos\theta}$ in the form of a quadratic in terms of $\sec\theta$. [2]

$2\sec^2\theta - \sec\theta - 2$

(ii) Hence solve, for $0^\circ < \theta < 360^\circ$, $2\tan^2\theta - \frac{1}{\cos\theta} = 4$. [4]

$131.8^\circ, 228.2^\circ, 60^\circ, 300^\circ$

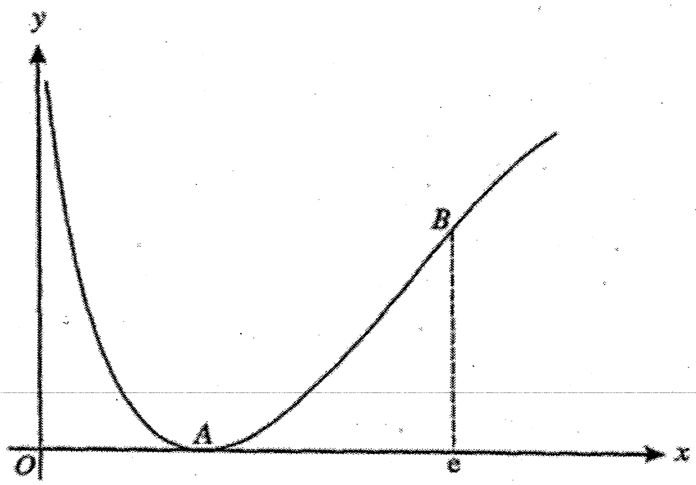
5 Given that $y = 0$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^3 + 1}{y^2},$$

$y = \sqrt[3]{e^{3x} - 1}$

obtaining an expression for y in terms of x. [6]

6



The function f is defined by $f(x) = (\ln x)^2$ for $x > 0$. The diagram shows a sketch of the graph of $y = f(x)$. The minimum point of the graph is A.

(i) State the x-coordinate of A. $x = 1$ [1]

(ii) Use the substitution $x = e^u$ to show that the area of the region bounded by the x-axis, the line $x = e$, and the part of the curve between A and B is given by

$$\int_0^1 u^2 e^u \, du.$$ [3]

(iii) Hence, or otherwise, find the exact value of this area. [3]

$e - 2$

7 The equation of a curve is $y = 2\sin x + \cos 2x$. Find the x-coordinates of the stationary points on the curve for which $0 < x < \pi$, and determine the nature of each of these stationary points. [9]

$x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5}{6}\pi$

~~(max.)~~ (min.) (max.) (max.)

[Turn over

8 If $f(x) = ax^3 + (a+b)x^2 + (a+2b)x + 1$ is exactly divisible by $(x+1)$, express b in terms of a, and find the quotient when the division is carried out, expressing the coefficients in terms of a only. $b = 1 - a$ [5]

Prove that in this case the equation $f(x) = 0$ has only one real root if $a^2 - 6a + 1 < 0$. [2]

Show that this inequality implies that $3 - 2\sqrt{2} < a < 3 + 2\sqrt{2}$. [3]

9 Relative to the origin, the position vectors of points A and B are $4\mathbf{i} - 11\mathbf{j} + 4\mathbf{k}$ and $7\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ respectively. $l : \underset{\sim}{r} = \begin{pmatrix} 7 \\ 1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$

(i) Find a vector equation for the line l passing through points A and B. [2]

(ii) Find the position vector of the point P on l such that OP is perpendicular to l. Hence find the perpendicular distance from O to l. $\vec{OP} = \begin{pmatrix} 7+t \\ 1+4t \\ 7+t \end{pmatrix}$, 9 units [5]

(iii) The planes π_1 and π_2 have equations $3x - y - z = 2$ and $x + 5y + z = 14$ respectively. Find the line of intersection of π_1 and π_2. [4]

10 **(i)** The complex numbers z satisfies $|z + 4 - 4i| = 3$.

(a) Describe, with the aid of a sketch, the locus of the points which represents z in an Argand diagram. [2]

(b) Find the least possible value of $|z + 2i|$. [2]

(ii) If $z = \cos\theta + i\sin\theta$ where $0 < \theta < \frac{\pi}{2}$, prove that $\frac{1+z}{1-z} = i\cot\frac{\theta}{2}$. [4]

(iii) In an Argand diagram, the points O, A, Z, P and Q represent the complex numbers 0, 1, z, $1 + z$, and $1 - z$ respectively. Show these points on an Argand diagram. Hence, explained why $\angle POQ = \frac{\pi}{2}$. [5]

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A2 TRIAL EXAMINATION MARCH/APRIL 2012

(January and March 2011 Intakes)

Friday **6 April 2012** **8.30 am – 9.45 am**

MATHEMATICS 9709/43

PAPER 4 Mechanics 1 (M1) 1 hour 15 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use 10 ms $^{-2}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
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[Turn over

1 A particle is projected vertically upwards from a point O with an initial speed 20 m s^{-1}. At the same instant another particle is released from rest at a point 50 m vertically above O. Find the height above O at which the particles meet. [5]

18.75m

2

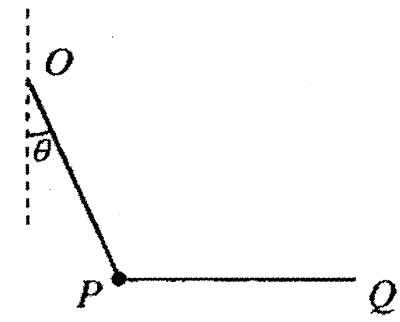


Fig. 1

A light inextensible string has one end attached to a fixed point O. To the other end is attached a particle of mass 0.5 kg. A light inextensible string PQ is attached to P. The end Q is held so that P hangs in equilibrium with PQ horizontal (see Fig. 1). The tension in PQ is 1.5 N. Find the inclination θ of the string OP to the vertical, and find also the tension in OP. 16.7°, 5.22N [5]

3 A light inextensible string passes over a smooth light fixed pulley and masses of 3kg and 7 kg are attached to its ends. The system is held at rest with the string taut, and then released.

4ms^{-2}, 42N

(i) Find the acceleration of each mass and the tension in the string. [4]

After the 7kg mass has descended a distance of one metre, it strikes an inelastic horizontal table. (Assume throughout that the 3kg mass does not reach the pulley.)

(ii) Find the time during which the 7kg mass is at rest on the table. [3]

$\frac{1}{5}\sqrt{8}$ s

4 A car of mass 1500 kg is travelling on a horizontal straight road and passes through a point A with speed 30 m s^{-1}. The power of the car's engine is 21 kW and the resistance to the car's motion is 1000 N.

(i) Find the deceleration of the car at A. $-0.2ms^{-2}$ [3]

(ii) Show that the speed of the car does not fall below $21ms^{-1}$ while the car continues to move with the engine exerting a constant power of 21 kW. [4]

5 Two particles P and Q, with mass km and m respectively, are connected by a light inextensible string passing over a smooth pulley at the top of a double inclined plane. Particle P is on the rough plane inclined at an angle α, where the coefficient of friction between the particle and the plane is μ and the particle Q is on the smooth plane inclined at an angle θ. Given that $\theta = tan^{-1}\left(\frac{4}{3}\right)$ and $\alpha = tan^{-1}\left(\frac{3}{4}\right)$, find the value of μ in terms of k if the particle Q moves down the plane with an acceleration $\frac{1}{2}ms^{-2}$. $\mu = \frac{15-13k}{16k}$

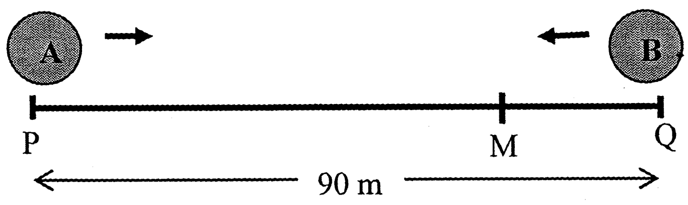
Given that $0 < \mu < 1$, show that $\frac{15}{29} < k < \frac{15}{13}$. [7]

6 A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 24 kW. The resistance to the car's motion is constant and equal to 400 N. The car passes through the points A and B with speeds 15 m s^{-1} and 25 m s^{-1} respectively. The car takes 30 s to travel from A to B.

(i) Find the acceleration of the car at A. $1ms^{-2}$ [3]

(ii) By considering work and energy, find the distance AB. 1200m [6]

7 The following diagram shows the positions and directions of motion of two objects, A and B, moving in a straight line passing two fixed points, P and Q, respectively. Object A passes the fixed point P and object B passes the fixed point Q simultaneously. The distance PQ is 90 m.



The velocity of A, v_A m s^{-1}, is given $v_A = 10 + 8t - 2t^2$, where t is the time, in seconds, after it passes P while B travels with a constant velocity of -3 m s^{-1}.
Object A stops instantaneously at point M.
(Assume that the positive direction of motion is towards the right.)

Find

(i) the maximum velocity, in m s^{-1} of A, 18 ms^{-1} [3]

66 $^2/_3$ m

(ii) the distance, in m, of M from P, $8\frac{1}{3}$ m [4]

(iii) the distance, in m, between A and B when A is at the point M. [3]