Topic 5 – Trigonometry (Textbook)

Exercise 10A

(2) Find the maximum value and the minimum value of each of the following functions. In each case, give the least positive values of x at which they occur.

(a) $2 + \sin x°$

(b) $7 - 4\cos x°$

(c) $5 + 8\cos 2x°$

(d) $\dfrac{8}{3 - \sin x°}$

(e) $9 + \sin(4x - 20)°$

(f) $\dfrac{30}{11 - 5\cos\left(\frac{1}{2}x - 45\right)°}$

5 Without using a calculator, write down the exact values of the following.

(a) $\sin 135°$

(b) $\cos 120°$

(c) $\sin(-30)°$

(d) $\tan 240°$

(e) $\cos 225°$

(f) $\tan(-330)°$

(g) $\cos 900°$

(h) $\tan 510°$

(i) $\sin 225°$

(j) $\cos 630°$

(k) $\tan 405°$

(l) $\sin(-315)°$

(m) $\sin 210°$

(n) $\tan 675°$

(o) $\cos(-120)°$

(p) $\sin 1260°$

6 Without using a calculator, write down the smallest positive angle which satisfies the following equations.

(a) $\cos\theta° = \frac{1}{2}$

(b) $\sin\phi° = -\frac{1}{2}\sqrt{3}$

(c) $\tan\theta° = -\sqrt{3}$

(d) $\cos\theta° = \frac{1}{2}\sqrt{3}$

(e) $\tan\theta° = \frac{1}{3}\sqrt{3}$

(f) $\tan\phi° = -1$

(g) $\sin\theta° = -\frac{1}{2}$

(h) $\cos\theta° = 0$

Exercise 10C

1 Find, correct to 1 decimal place, the two smallest positive values of θ which satisfy each of the following equations.

(a) $\sin\theta° = 0.1$

(b) $\sin\theta° = -0.84$

(c) $\sin\theta° = 0.951$

(d) $\cos\theta° = 0.8$

(e) $\cos\theta° = -0.84$

(f) $\cos\theta° = \sqrt{\dfrac{2}{3}}$

(g) $\tan\theta° = 4$

(h) $\tan\theta° = -0.32$

(i) $\tan\theta° = 0.11$

(j) $\sin(180 + \theta)° = 0.4$

(k) $\cos(90 - \theta)° = -0.571$

(l) $\tan(90 - \theta)° = -3$

(m) $\sin(2\theta + 60)° = 0.3584$

(n) $\sin(30 - \theta)° = 0.5$

(o) $\cos(3\theta - 120)° = 0$

2 Find all values of θ in the interval $-180 \leqslant \theta \leqslant 180$ which satisfy each of the following equations, giving your answers correct to 1 decimal place where appropriate.

(a) $\sin \theta° = 0.8$

(b) $\cos \theta° = 0.25$

(c) $\tan \theta° = 2$

(d) $\sin \theta° = -0.67$

(e) $\cos \theta° = -0.12$

(f) $4 \tan \theta° + 3 = 0$

(g) $4 \sin \theta° = 5 \cos \theta°$

(h) $2 \sin \theta° = \dfrac{1}{\sin \theta°}$

(i) $2 \sin \theta° = \tan \theta°$

3 Find all the solutions in the interval $0 < \theta \leqslant 360$ of each of the following equations.

(a) $\cos 2\theta° = \frac{1}{3}$

(b) $\tan 3\theta° = 2$

(c) $\sin 2\theta° = -0.6$

(d) $\cos 4\theta° = -\frac{1}{4}$

(e) $\tan 2\theta° = 0.4$

(f) $\sin 3\theta° = -0.42$

4 Find the roots in the interval $-180 \leqslant x \leqslant 180$ of each of the following equations.

(a) $\cos 3x° = \frac{2}{3}$

(b) $\tan 2x° = -3$

(c) $\sin 3x° = -0.2$

(d) $\cos 2x° = 0.246$

(e) $\tan 5x° = 0.8$

(f) $\sin 2x° = -0.39$

7 Find, to 1 decimal place, all values of z in the interval $-180 \leqslant z \leqslant 180$ satisfying

(a) $\sin z° = -0.16$.

(b) $\cos z°(1 + \sin z°) = 0$,

(c) $(1 - \tan z°)\sin z° = 0$,

(d) $\sin 2z° = 0.23$,

(e) $\cos(45 - z)° = 0.832$,

(f) $\tan(3z - 17)° = 3$.

8 Find all values of θ in the interval $0 \leqslant \theta \leqslant 360$ for which

(a) $\sin 2\theta° = \cos 36°$,

(b) $\cos 5\theta° = \sin 70°$,

(c) $\tan 3\theta° = \tan 60°$.

9 Find all values of θ in the interval $0 \leqslant \theta \leqslant 180$ for which $2 \sin \theta° \cos \theta° = \frac{1}{2} \tan \theta°$.

Exercise 10D

2 (a) Given that angle A is obtuse and that $\sin A° = \frac{5}{14}\sqrt{3}$, find the exact value of $\cos A°$.

(b) Given that $180 < B < 360$ and that $\tan B° = -\frac{21}{20}$, find the exact value of $\cos B°$.

(c) Find all possible values of $\sin C°$ for which $\cos C° = \frac{1}{2}$.

(d) Find the values of D for which $-180 < D < 180$ and $\tan D° = 5 \sin D°$.

3 Use $\tan \theta° \equiv \dfrac{\sin \theta°}{\cos \theta°}$, $\cos \theta° \neq 0$, and $\cos^2 \theta° + \sin^2 \theta° \equiv 1$ to establish the following.

(a) $\dfrac{1}{\sin \theta°} - \dfrac{1}{\tan \theta°} \equiv \dfrac{1 - \cos \theta°}{\sin \theta°}$

(b) $\dfrac{\sin^2 \theta°}{1 - \cos \theta°} \equiv 1 + \cos \theta°$

(c) $\dfrac{1}{\cos \theta°} + \tan \theta° \equiv \dfrac{\cos \theta°}{1 - \sin \theta°}$

(d) $\dfrac{\tan \theta° \sin \theta°}{1 - \cos \theta°} \equiv 1 + \dfrac{1}{\cos \theta°}$

4 Solve the following equations for θ, giving all the roots in the interval $0 \leqslant \theta \leqslant 360$ correct to the nearest 0.1.

(a) $4 \sin^2 \theta° - 1 = 0$

(b) $\sin^2 \theta° + 2 \cos^2 \theta° = 2$

(c) $10 \sin^2 \theta° - 5 \cos^2 \theta° + 2 = 4 \sin \theta°$

(d) $4 \sin^2 \theta° \cos \theta° = \tan^2 \theta°$

5 Find all values of θ, $-180 < \theta < 180$, for which $2 \tan \theta° - 3 = \dfrac{2}{\tan \theta°}$.

Miscellaneous exercise 10

10 Prove the following identities.

(a) $\dfrac{1}{\sin \theta°} - \sin \theta° \equiv \dfrac{\cos \theta°}{\tan \theta°}$

(b) $\dfrac{1 - \sin \theta°}{\cos \theta°} \equiv \dfrac{\cos \theta°}{1 + \sin \theta°}$

(c) $\dfrac{1}{\tan \theta°} + \tan \theta° \equiv \dfrac{1}{\sin \theta° \cos \theta°}$

(d) $\dfrac{1 - 2 \sin^2 \theta°}{\cos \theta° + \sin \theta°} \equiv \cos \theta° - \sin \theta°$

11 For each of the following functions, determine the maximum and minimum values of y and the least positive values of x at which these occur.

(a) $y = 1 + \cos 2x°$

(b) $y = 5 - 4 \sin(x + 30)°$

(c) $y = 29 - 20 \sin(3x - 45)°$

(d) $y = 8 - 3 \cos^2 x°$

(e) $y = \dfrac{12}{3 + \cos x°}$

(f) $y = \dfrac{60}{1 + \sin^2(2x - 15)°}$

12 Solve the following equations for θ, giving solutions in the interval $0 \leqslant \theta \leqslant 360$.

(a) $\sin \theta° = \tan \theta°$

(b) $2 - 2 \cos^2 \theta° = \sin \theta°$

(c) $\tan^2 \theta° - 2 \tan \theta° = 1$

(d) $\sin 2\theta° - \sqrt{3} \cos 2\theta° = 0$