Coordinate Geometry

P1/3/1: Length, mid-point and gradient

P1/3/2: Equation of a straight line

P1/3/3: Gradient of perpendicular and parallel lines

P1/3/4: Discussion and evaluation

P1/3/5: Intersection of graphs and solution of equations

P1/3/6: Discussion and evaluation

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P1/3/1

Length, mid-point and gradient

Learning Outcome

Students should be able to:

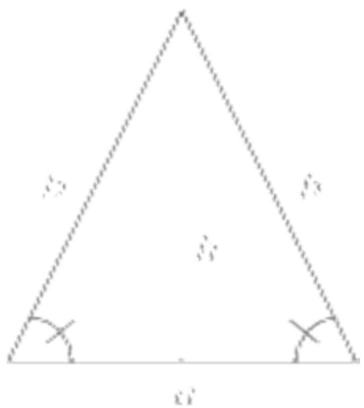
- find the distance between 2-points
- find the mid-point of a line segment
- find the gradient of a line segment

Length, mid-point and gradient

The distance between the points (x_1 , y_1) and (x_2 , y_2) or the length of the line segment joining them is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Additional notes for length, mid-point and gradient



An isosceles triangle has both two equal sides and two equal angles.

A triangle with all sides equal is called an **equilateral triangle**

Length, mid-point and gradient

The mid-point of the line segment joining (x_1, y_1) and (x_2, y_2) has coordinates

$$\left(\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2)\right)$$

Length, mid-point and gradient

The gradient of the line joining (x_1 , y_1) to (x_2 , y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Two lines are parallel if they have the same gradient.

Example 1:

$(-2,-3)$ and $(1,-6)$

For the line segment joining this pair of points,
Find

(i) the length of the line segment
(ii) the coordinates of the mid-point.
(iii) the gradient of the line.

Example 2:

Show that the points $(1,-2), (6,-1), (9,3), \text{ and } (4,2)$ are vertices of a parallelogram.

Practice Exercise

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Exercise 1A (Page 6)

Q1(i), 3, 5(e), 8, 10(g), 13

P1/3/2
Equation of a straight line

P1/3/3
Gradient of perpendicular and parallel lines

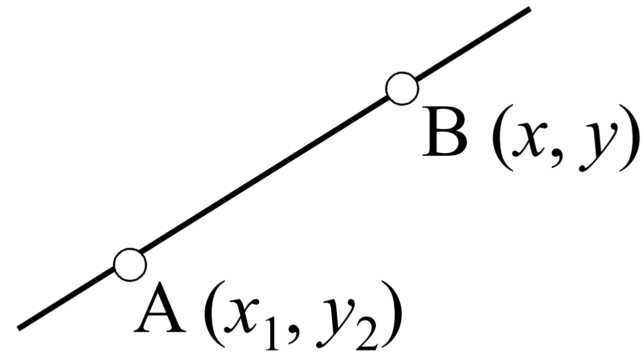
Learning Outcome

Students should be able to:

- find the equation of a line through a given point and gradient
- find the equation of a line joining two points.
- deduce from their gradient whether two lines are parallel or perpendicular

Equation of a straight line

The gradient AB is $m = \dfrac{y - y_1}{x - x_1}$



The equation of the line through (x_1 , y_1) with gradient m is

$$y - y_1 = m(x - x_1)$$

Equation of a straight line

Equation of a straight line can be written as

$$\boxed{y = mx + c}$$

The point $(0, c)$ lies on the y-axis (y-intercept)

If $m = 0$, $y = c$ (parallel to x-axis)

If m is undefined, $x = k$ (parallel to y-axis)

Example 3:

Find the equation of the line joining the point (2, -1) and (-4, 20).

Gradient of perpendicular & parallel lines

Two lines with gradients m_1 and m_2 are parallel if

$$\boxed{m_1 = m_2}$$

Two lines with gradients m_1 and m_2 are perpendicular if

$$\boxed{m_1 \times m_2 = -1}$$

Example 4:

Find the equation of the line through (1, 3) which is parallel to

$$\boxed{2x + 7y = 5}$$

Given your answer in the form $ax + by = c$.

Example 5:

Find the equation of the line through (-2, 5) which is perpendicular to the line $\boxed{y = 3x + 1}$.

Find also the point of intersection of the two lines.

Example 6:

Show that the points (0, -5), (-1, 2), (4, 7) and (5, 0) form a rhombus.

Example 7:

A line through a vertex of a triangle which is perpendicular to the opposite side is called an altitude. Find the equation of the altitude through the vertex A of the triangle ABC where A is the point (2,3), B is the (1, -7) and c is (4, -1).

Practice Exercise

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Exercise 1B (Page 11)

Q8, 9

Exercise 1C (Page 14)

Q4, 6

P1/3/5

Intersection of graphs and solution of equations

Learning Outcome

Students should be able to:

- find the point of intersection of graphs
- use relationships between points of intersection of graphs and solutions of equations.

Example 8:

Find the equation of the perpendicular bisector of the line joining (2, -5) and (-4, 3).

Example 9:

The points A(1, 2), B(3, 5), C(6, 6) and D form a parallelogram. Find the coordinates of the mid-point of AC. Use your answer to find the coordinates of D.

Example 10:

The point P is the foot of the perpendicular from the point A(0, 3) to the line $y = 3x$.

(a) Find the equation of the line AP.

(b) Find the coordinates of the point P.

(c) Find the perpendicular distance of A from the line $y = 3x$.

Example 11:

The line $3x - 4y = 8$ meets the y-axis at A. The point C has coordinates (-2, 9). The line through C perpendicular to $3x - 4y = 8$ meets it at B. Calculate the area of the triangle ABC.

Practice Exercise

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Exercise 1B (Page 12)

Q11(f)

Miscellaneous Exercise 1 (Page 15)

Q6, 15, 16, 17