TEST 2(Coordinate Geometry, Circular Measure & Trigonometry)

1. It is given that x and y satisfies the equation $tan^{-1}x + tan^{-1}y + tan^{-1}(xy) = \frac{7}{12}\pi$. Find the value of y when $x = 1$. [3]

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}(xy) = \frac{7}{12}\pi$

$\tan^{-1}(1) + \tan^{-1}(y) + \tan^{-1}(y) = \frac{7}{12}\pi$

$\frac{\pi}{4} + 2\tan^{-1}y = \frac{7}{12}\pi$ ——— (1)

$2\tan^{-1}y = \frac{7}{12}\pi - \frac{\pi}{4} = \frac{1}{3}\pi$ ——— (1)

$\tan^{-1}y = \frac{1}{6}\pi$, $y = \frac{1}{\sqrt{3}}$ ——— (1) $= 0.577$

2. The curves $y = 2sin^2x$ and $y = 3sinx - 1$ in the domain $0 \le x \le \frac{\pi}{2}$ intersects at M and N. Find the length, MN in the form $\frac{1}{a}\sqrt{b\pi^2 + c}$, where a, b and c are positive ~~constants~~ integers. [7]

Intersection:

$2\sin^2 x = 3\sin x - 1$

$(2\sin x - 1)(\sin x - 1) = 0$ ——— (1)

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = 1$

$x = 30°$ $\quad$ $x = 90°$ ——— (1)

at $x = 90°$ or $\pi/2$ $\quad$ at $x = 30°$ or $\pi/6$

$y = 2$ $\quad$ $y = \frac{1}{2}$.

M $(30°, \frac{1}{2})$ $\quad$ N $(90°, 2)$ ——— (2)

length $MN = \sqrt{(\frac{\pi}{2} - \frac{\pi}{6})^2 + (2 - \frac{1}{2})^2}$ ——— (1)

$= \sqrt{(\frac{2}{6}\pi)^2 + \frac{9}{4}}$

$= \sqrt{\frac{4}{36}\pi^2 + \frac{81}{36}}$ ——— (1)

$= \frac{1}{6}\sqrt{4\pi^2 + 81}$ ——— (1)

3. (a) Show that $\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. [2]

(b) Hence solve the equation $\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x$ for $-\pi \le x \le \pi$. [5]

(a) LHS $= \frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x}$

$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$ ——— (1)

$= \tan^2 x - 1$ ——— (1)

= RHS

(b) $\tan^2 x - 1 = 5 - \tan x$

$\tan^2 x + \tan x - 6 = 0$ ——— (1)

$(\tan x - 2)(\tan x + 3) = 0$

$\tan x = 2$, $\tan x = -3$ ——— (2)

$63.4°$ $\quad$ -116.3

$x = 1.11$, -2.03

$= -1.25$, 1.89 rad. ——— (2)

-71.6 $\quad$ $108.3°$

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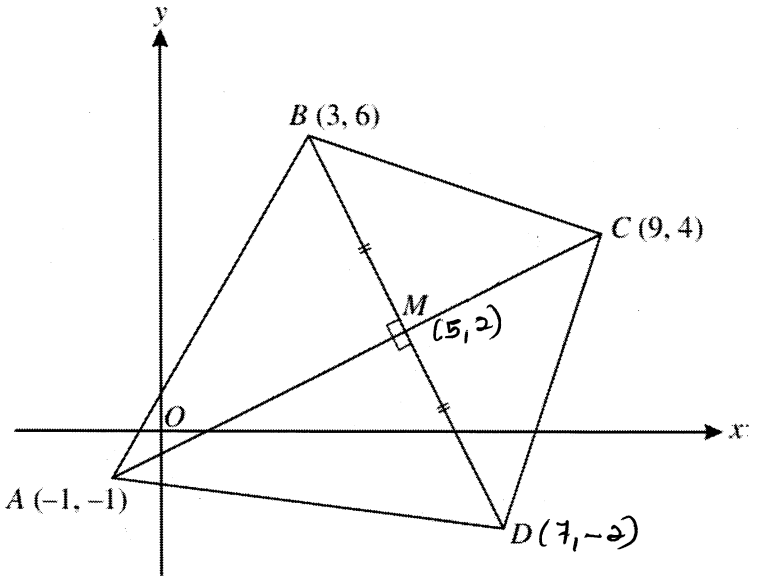

4. The diagram shows a quadrilateral ABCD in which the point A is (-1, -1), the point B is (3, 6) and the point C is (9, 4). The diagonals are AC and BD intersects at M. Angle BMA = 90° and BM = MD. Calculate

   (a) the coordinates of M and D. [7]

   (b) the ratio AM : MC. [2]

   (c) the area of the quadrilateral ABCD. [2]

(a) gradient of AC $= \frac{5}{10} = \frac{1}{2}$ —— (1)

gradient of BD $= -2$ —— (1)

equation of AC:

$y + 1 = \frac{1}{2}(x + 1)$

equation of BD

$y - 6 = -2(x - 3)$

} simultaneous equation

M (5, 2) —— (3)

M is mid-point of B(3, 6) and D(x, y)

$(5, 2) = \left(\frac{3-x}{2}, \frac{6-y}{2}\right)$ —— (1)

$x = 7, \; y = -2$

D (7, -2) —— (1)

(b) $AM = \sqrt{45} = 3\sqrt{5}$

$MC = \sqrt{20} = 2\sqrt{5}$ } —— (1)

AM : MC

3 : 2 —— (1)

(c) Area = [2 x Δ AMB] + [2 x ΔCMB] —— (1)

$= (AM)(MB) + (CM)(MB)$

Length of MB $= \sqrt{20}$

$= (\sqrt{45})(\sqrt{20}) + (\sqrt{20})(\sqrt{20})$

$= 50 \text{ units}^2$ —— (1)

[or use matrix to find area]

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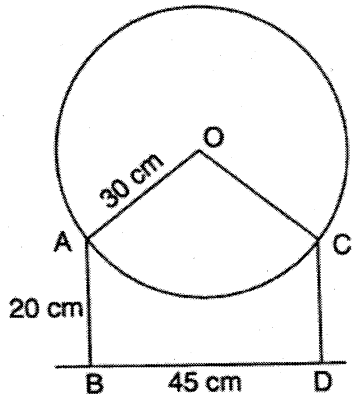


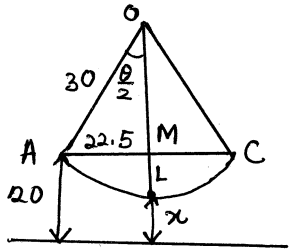
5. A circular disc, centre O and radius 30cm, rests on two vertical supports AB and CD each 20cm tall and 45cm apart. Calculate, correct to 3 significant figures

(a) angle AOC in radians, [3]

(b) the height of the lowest point of arc AC above BD. [3]

(c) the fraction of the area of the disc that lies above the level of AC. [6]

(a)



$OM^2 = 30^2 - 22.5^2$

$OM = 19.843\ cm$

$\sin\frac{\theta}{2} = \frac{22.5}{30}$, $\underline{\theta = 1.70 rad.}$ — (3)

$\theta = 1.696$

(b) L is the lowest point of AC.

$ML + x = 20$

$x = 20 - ML$ [$ML = OL - OM$

$= 30 - 19.843$

$x = 9.84\ cm$ — (1) $= 10.157$] — (2)

(10.2)

(c) Area of disc $= 900\pi$ — (1)

Area of segment ALC $= \left(\frac{1}{2} \times 30^2 \times \theta\right) - \frac{1}{2} \times 30^2 \times \sin\theta$ — (1)

$= 450(1.696 - \sin 1.696)$

Fraction of $= 316.72$ — (2)

Area above AC $= \frac{900\pi - 316.72}{900\pi} = \underline{0.888}$ — (2)