Topic 5 – Trigonometry

Trigonometry *(from Greek trigonon "triangle" + metron "measure")*

- sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians);
- use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles, e.g. $\cos 150° = -\frac{1}{2}\sqrt{3}$;
- use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations;
- use the identities $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ and $\sin^2\theta + \cos^2\theta = 1$;
- find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).
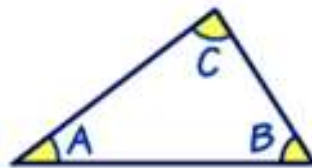
The three most common **functions** in trigonometry are Sine, Cosine and Tangent You will use them a lot!
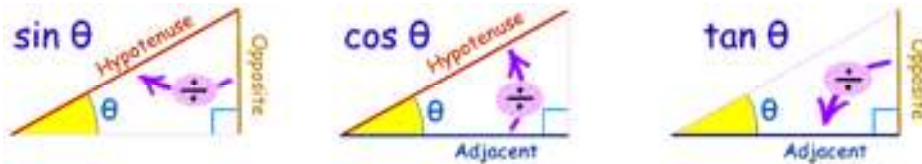
Example 1
A right-angled triangle ABC has $\tan\theta = 2$, find $\sin\theta$ and $\cos\theta$.
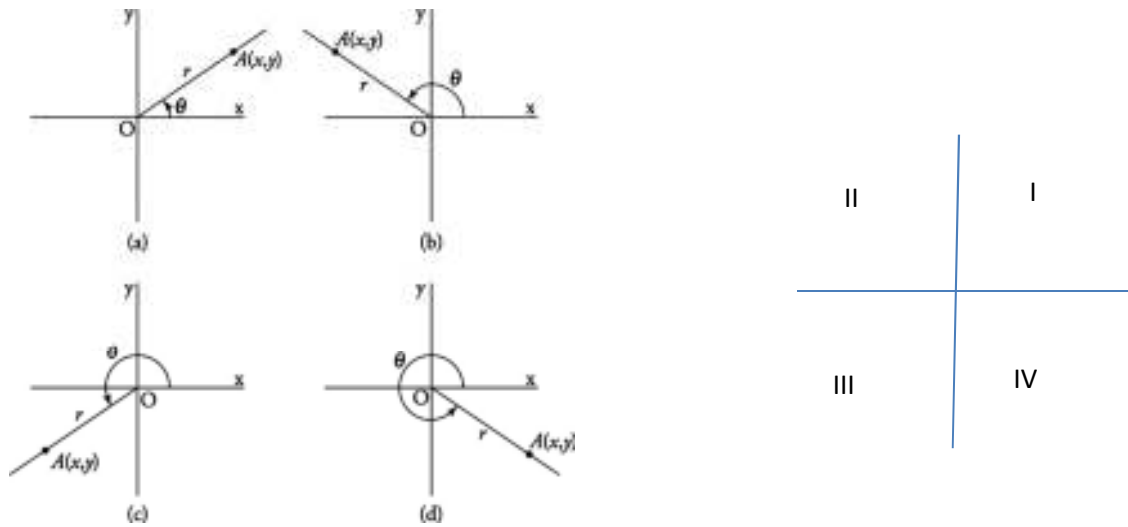
Triangles Contain 180°

In a triangle, the three angles always add to 180°:   **A + B + C = 180°**
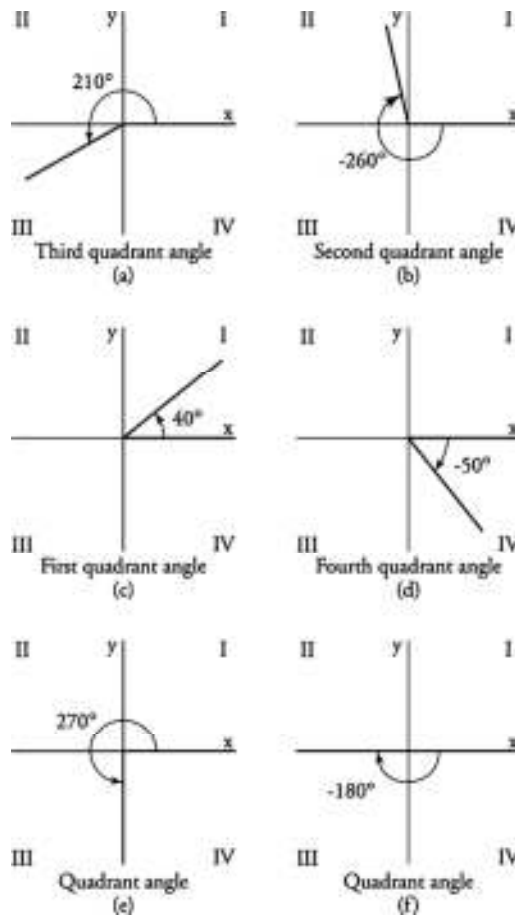
General Angles

- Names

- Quadrants

(a)   (b)   (c)   (d)

| | |
II	I
III	IV

- Positive and Negative angles

II   y   I
    210°
                        x
III   IV
Third quadrant angle
(a)

II   y   I
    -260°
                        x
III   IV
Second quadrant angle
(b)

II   y   I
            40°
                        x
III   IV
First quadrant angle
(c)

II   y   I
                -50°
                        x
III   IV
Fourth quadrant angle
(d)

II   y   I
    270°
                        x
III   IV
Quadrant angle
(e)

II   y   I
    -180°
                        x
III   IV
Quadrant angle
(f)

For any angle θ,   cos(-θ) = cosθ
                    sin(-θ) = -sinθ
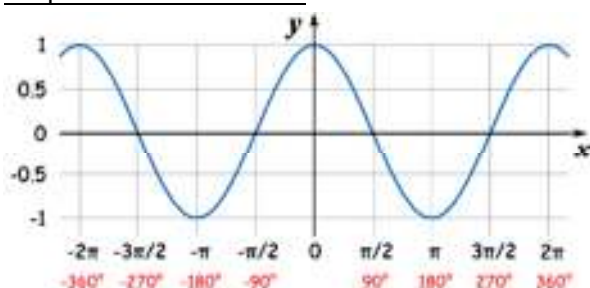   tan(-θ) = -tanθ

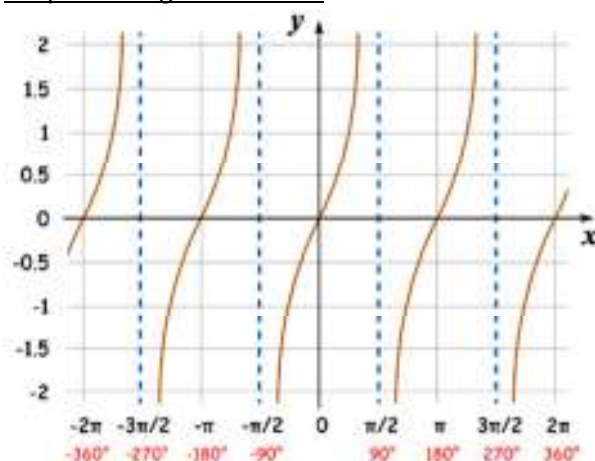**Graphs of the Sine, Cosine and Tangent Functions**

Graph of Sine Function
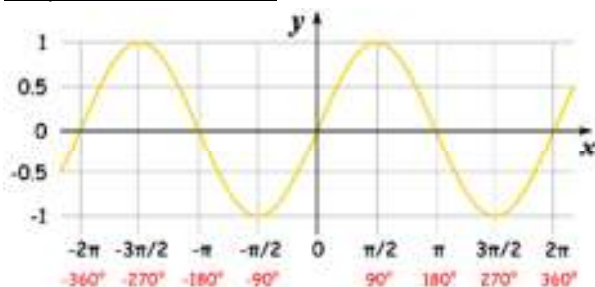
Graph of Cosine Function

Graph of Tangent Function

Exercise 10A – Pure Mathematics 1(page 141)   Question 5 and 6

Minimum and Maximum of a Trigonometric Function

Example 2
If $0° \leq \theta \leq 360°$, find the minimum and maximum values of
   a) $7\sin \theta - 3$ b) $5\cos \theta + 2$ c) $4 - 3\sin \theta$
Example 3
If $0° \leq \theta \leq 360°$, find the range of values of $3 - 2\cos \theta$.

Exercise 10A – Pure Mathematics 1(page 141)   Question 2
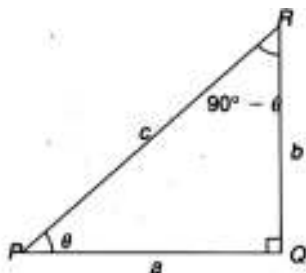
Trigonometric Ratios of Complementary Angles

In the triangle PQR, angle P and angle R are complementary angles as

$$\angle P + \angle R = 90°.$$

Example 3

By referring to the right-angled triangle below, show that $\sin(90° - \theta) = \cos\theta.$ Hence deduce the value of $\dfrac{\cos 20°}{\cos 20° + \sin 70°}.$

Note: $\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$

Solving Equations Involving Trigonometric Functions

Simple Identities

1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta},\ \cos\theta \neq 0$
2. $\sin^2\theta + \cos^2\theta = 1$

Example 4

Given that $\cos\theta = \dfrac{3}{5}$, find the possible values of
(a) $\sin\theta$, (b) $\tan\theta$.

Example 5

Given $x = 2\sin\theta$ and $y = \cos\theta + 1$, show that $x^2 + 4(y - 1)^2 = 4$.

Example 6

Find all the angles between $0°$ and $360°$ which satisfy the equation
$$3\cos x + 2\sin x = 0.$$

Example 7

Find all the angles between $0°$ and $360°$ inclusive for which
(a) $2\sin x \cos x = \sin x$, (b) $\cos^2 y - \cos y = 2$.

Example 8

Find all the angles between $0°$ and $360°$ which satisfy the equation
$\sin y = 4 \tan y$.

Example 9

Find all the angles between $0°$ and $360°$ which satisfy the equation
$2 \cos^2 y - 1 = \sin y$.

Example 10

Find all the angles between $0°$ and $360°$ which satisfy the equation
(a) $\cos (x + 30°) = -0.3$,
(b) $\sin 2x = 0.866$,
(c) $\tan (2x - 50°) = -0.7$.