TOPIC 4: DISCRETE RANDOM VARIABLE

In this chapter you should

- understand what a discrete random is.
- know the properties of a discrete random variable.
- be able to construct distribution table for a discrete random variable.
- How to calculate expectation, E(X) and variance, Var(X).

Discrete random variable – is a variable which can take individual values each with a given probability. The values of the variables are usually the outcome of an experiment.

Some examples of discrete random variables:

	Possible values
• The score when you throw an ordinary fair cubical die.	
• The number of heads when you toss a fair coin 3 times.	
• Your profit in dollars when you play a game with an entry fees of \$1 and prizes of \$5 and \$10.	
• The number of times you toss a coin until a tail occurs.	

Example 1

A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. [3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

(iii) The number of green peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. [5]

Example 2

A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places. [5]

Probability distribution

- Gives the probability of each possible value of the variable.
- For a d.r.v, the sum of the probabilities is 1.

$\sum P(X=x)=1$ *or* $\sum p_i = 1$ for i = 1, 2, 3,…,n

Example 3

A box contains 3 faulty fuses and 2 good fuses. You take two fuses from the box. What is the probability that you take no faulty fuses, one faulty fuse and two faulty fuses.

The random variable is X, denoted as the number of faulty fuses.
$x =$ 0, 1, 2

x	0	1	2
$P(X=x)$	$\frac{2}{5}\times\frac{1}{4}=\frac{2}{20}$	$2\times\frac{2}{5}\times\frac{3}{4}=\frac{3}{5}$	$\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}$

Probability distribution table

Expectation of X, E(X)

- E(X) is read as 'E of X'
- It is the mean or average of the typical value of X.

$E(X)=\sum xP(X=x)$ or $E(X)=\sum x_i p_i$ for i = 1, 2, …,n

Variance of X, Var(X)

The variance of X may be written as $Var(X)=E(X^2)-[E(X)]^2$

Example 4

The random variable X takes the values −2, 0 and 4 only. It is given that $P(X=-2)=2p$, $P(X=0)=p$ and $P(X=4)=3p$.

(i) Find p. [2]

(ii) Find E(X) and Var(X). [4]

Example 5

The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b.

(ii) Given that $E(X) = 4$, find a and b.

Example 6

A discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	$3c$	$4c$	$5c$	$6c$

(i) Find the value of the constant c.

(ii) Find $E(X)$ and $Var(X)$.

(iii) Find $P(X > E(X))$.

The Expectation of a Simple Function

If $E(X) = \sum xP(X = x)$

then

if g(X) is any function of the discrete random variable X, then

$$E(g(x)) = \sum g(x)P(X = x)$$

Note:

1. $E(10X) = \sum 10xP(X = x)$
2. $E(X^2) = \sum x^2 P(X = x)$
3. $E(\frac{1}{X}) = \sum \frac{1}{x} P(X = x)$
4. $E(X - 4) = \sum (x - 4)P(X = x)$

In general

1. $E(a) = a$
2. $E(aX) = aE(X)$
3. $E(aX + b) = aE(X) + b$ where a and b are any constants.

The Variance of a Simple Function

The variance of X may be written as $Var(X) = E(X^2) - [E(X)]^2$

In general

1. $Var(a) = 0$
2. $Var(aX) = a^2 Var(X)$
3. $Var(aX + b) = a^2 Var(X)$

Uniform and Symmetrical Probability Distribution

Example 4:

x	1	2	3	4	5
P(X=x)	0.1	0.2	0.4	0.2	0.1

Example 5:

x	0	1	2	3	4	5	6	7	8	9
P(X=x)	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Note:

The expectation for a uniform or symmetrical distribution is always at the centre of the distribution.

Exam Based Questions – Discrete Random Variable

Question 1

When a certain type of cell is subjected to radiation, the cell may die, survive as a single cell or divide into two cells with probabilities $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ respectively. Two cells are independently subjected to radiation. The random variable X represents the total numbers of cells in existence after this experiment.

(a) Find the probability distribution of X. **[3]**

(b) Evaluate E(X). **[2]**

(c) Find Var(X). **[2]**

Question 2

6 A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

(i) By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable L denotes the largest of the three numbers drawn.

(ii) Find the probability that L is 4. [1]

(iii) Draw up a table to show the probability distribution of L. [3]

(iv) Calculate the expectation and variance of L. [3]

THE BINOMIAL DISTRIBUTION $X - Bin(n, p)$

Conditions:

1. a finite number, **n**, trials are carried out,
2. the trials are independent,
3. the outcome of each trial is deemed either a success or a failure,
4. the probability, **p**, of a successful outcome is the same for each trial.
5. the probability of failure, **q = 1 – p**.
6. binomial distribution is a **subset** of discrete random variable.

   X – the number of successful outcomes in **n** trials.

   x = 0, 1, 2, …,n
7. the probability density function for a binomial distribution is

$$P(X = x) = {}^{n}C_{r}\, p^{x} q^{n-x}$$

8. the expected value of X, **E(X) = np**
9. the variance of X, **Var(X) = npq**

Questions:

1. The random variable X is distributed B(7, 0.2). Find, correct to three decimal places.
   a) $P(X = 3)$
   b) $P(1 < X \leq 4)$
   c) $P(X > 1)$

2. The random variable X is B(4, 0.8). Construct the probability distribution for X and find the expectation and standard deviation.

3. The probability it will be a fine day is 0.4. Find the expected number of fine days in a week and also the variance.

4. X is B(n, p) with mean 5 and standard deviation 2. Find the value s of n and p.

5. A batch of capsules of a certain drug contains 2% of damaged capsules. A bottle contains 42 of these capsules. Calculate the mean and standard deviation of the number of damaged capsules in bottle, assuming that each capsule was randomly selected for inclusion in the bottle.