Topic : P309 DIFFERENTIATION

Subtopic:

- *Derivative of Products*
- *Derivative of Rational Functions*
- *Derivative of Parametric Equations*
- *Derivative of Implicit Functions*

Derivative of Products

(Product Rule)

Let u and v be functions of x.

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Example : Find

$$\text{(i)} \frac{d}{dx}\left[x(x+3)^4\right] = \qquad \text{(ii)} \frac{d}{dx}\left[x^2\sqrt{(x-1)}\right] =$$

Example:

Find the coordinates of the turning point of the curve $y = x \ln x$. Determine its nature.

Example:

Show that the tangent to the curve $y = x\sin 3x$ at the origin is parallel to the $x-$axis.

Example:

Find the equation of the tangent and normal to the curve $y = e^x \cos 2x$ when $x = 0$.

Example:

Show that the $x-$coordinate of the turning point of the curve $y = \sin x \ln x$ satisfies the equation $x \ln x + \tan x = 0, x > 0$.

Homework

Please attempt all the questions in the following slides.

Questions are to be discussed on the next day of the instruction.

Example : Find

$$\text{(a)}\ \frac{d}{dx}(x\cos x) \qquad\qquad \text{(b)}\ \frac{d}{dx}\left(x\sin^2 x\right)$$

Example:

Differentiate $y = \sin^4 2x \cos^3 5x$.

Example:

Find the equation of the normal to the curve $y = x\ln(2x-1)$ at the point on the curve with $x = 1$.

Example:

The volume, V, of a solid is given by $V = x^2\sqrt{8-x}$.

Find the maximum value of V and the value of x at which it occurs.

Example:

A function f is defined by $f(x) = e^x \cos x, 0 \leq x \leq 2\pi$.

(a) Solve the inequality $f'(x) < 0$.

(b) What does (a) tell you about the shape of the curve $y = f(x)$.

Example:

Show that the curve $y = \dfrac{x}{\sqrt{2x^2+1}}$ has no turning points.

Example:

Find the value of x for which x^2e^{-ax} has its maximum value, where a is a positive constant. Denoting this by c, and the maximum value by M, deduce that, if $x > c$,

$$xe^{-ax} < \frac{M}{x}.$$ Hence, show that $xe^{-ax} \to 0$ as $x \to \infty$.