

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question paper

**9709 MATHEMATICS
8719 HIGHER MATHEMATICS**

9709/03, 8719/03

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Grade thresholds for Syllabus 9709/8719 (Mathematics and Higher Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 3	75	61	55	27

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
 - When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.
- The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
 - For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

June 2005

GCE AS AND A LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

**MATHEMATICS AND HIGHER MATHEMATICS
PAPER 3**

Page 1	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2005	9709/8719	3

- 1 EITHER:** Obtain correct unsimplified version of the x or x^2 or x^3 term **M1**
 State correct first two terms $1 - 2x$ **A1**
 Obtain next two terms $6x^2 - 20x^3$ **A1 + A1**
 [The M mark is not earned by versions with unexpanded binomial coefficients, e.g. $\left(-\frac{1}{2}\right)$.]
- OR:** Differentiate expression and evaluate $f(0)$ and $f'(0)$,
 where $f'(x) = k(1 + 4x)^{-\frac{3}{2}}$ **M1**
 State correct first two terms $1 - 2x$ **A1**
 Obtain next two terms $6x^2 - 20x^3$ **A1 + A1 4**
- 2 (i)** Show or imply correct decimal ordinates 1.2755..., 1, 0.8223... **B1**
 Use correct formula, or equivalent, with $h = 0.6$ and three ordinates **M1**
 Obtain correct answer 1.23 with no errors seen **A1 3**
 [SR: if the area is calculated with one interval, or three or more, give **D1** for a correct answer.]
- (ii)** Give an adequate justification, e.g. one trapezium over-estimates area and the other under-estimates, **or** errors cancel out **B1 1**
- 3 (i)** Use quadratic formula, or the method of completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$ **M1**
 Obtain a root, e.g. $2 + i$ **A1**
 Obtain the other root $-2 + i$ **A1 3**
 [Roots given as $\pm 2 + i$ earn **A1 + A1**.]
- (ii)** Obtain modulus $\sqrt{5}$ (or 2.24) of both roots **B1√**
 Obtain argument of $2 + i$ as 26.6° or 0.464 radians
 (allow ± 1 in final figure) **B1√**
 Obtain argument of $-2 + i$ as 153.4° or 2.68 radians
 (allow ± 1 in final figure) **B1√ 3**
 [SR: in applying the follow through to the roots obtained in **(i)**, if both roots are real or pure imaginary, the mark for the moduli is not available and only **B1√** is given if both arguments are correct; also if one of the two roots is real or pure imaginary and the other is neither then **B1√** is given if both moduli are correct and **B1√** if both arguments are correct.]
- (iii)** Show both roots on an Argand diagram in relatively correct positions **B1√ 1**
 [This follow through is only available if at least one of the two roots is of the form $x + iy$ where $xy \neq 0$.]

Page 2	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2005	9709/8719	3

- 4 (i)** State or imply $dx = \sec^2 \theta d\theta$ **or** $\frac{dx}{d\theta} = \sec^2 \theta$ **B1**
Substitute for x and dx throughout the integral **M1**
Obtain integral in terms of θ in any correct form **A1**
Reduce to the given form correctly **A1** **4**
- (ii)** State integral $\frac{1}{2} \sin 2\theta$ **B1**
Use limits $\theta = 0$ and $\theta = \frac{1}{4}\pi$ correctly in integral of the form $k \sin 2\theta$ **M1**
Obtain answer $\frac{1}{2}$ or 0.5 **A1** **3**
- 5 (i)**
- EITHER:** Attempt division by $x^2 - x + 3$ reaching a partial quotient $x^2 + x$ **B1**
Complete division and equate constant remainder to zero **M1**
Obtain answer $a = -6$ **A1**
- OR:** Commence inspection and reach unknown factor of $x^2 + x + c$ **B1**
Obtain $3c = a$ and an equation in c **M1**
Obtain answer $a = -6$ **A1**
- State or obtain factor $x^2 + x - 2$ **B1**
State or obtain factors $x + 2$ and $x - 1$ **B1 + B1** **6**
- (ii)** State that $x^2 + x - 2 = 0$, has two (real) roots **B1**
Show that $x^2 - x + 3 = 0$, has no (real) roots **B1** **2**
- 6 (i)**
- EITHER:** Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ **B1**
Use double angle formulae to express LHS in terms of $\cos \theta$ (and maybe $\sin \theta$) **M1**
Obtain any correct expression in terms of $\cos \theta$ alone **A1**
Reduce correctly to the given form **A1**
- OR:** Use double angle formula to express RHS in terms of $\cos 2\theta$ **M1**
Express $\cos^2 2\theta$ in terms of $\cos 4\theta$ **B1**
Obtain any correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ **A1**
Reduce correctly to the given form **A1** **4**
- (ii)** Using the identity, carry out method for calculating one root **M1**
Obtain answer 27.2° (or 0.475 radians) or 27.3° (or 0.476 radians) **A1**
Obtain a second answer, e.g. 332.8° (or 5.81 radians) **A1√**
Obtain remaining answers, e.g. 152.8° and 207.2° (or 2.67 and 3.62 radians) and no others in range **A1√** **4**

Page 3	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2005	9709/8719	3

- 7 (i) Make recognisable sketch of a relevant graph over the given range,
e.g. $y = \operatorname{cosec} x$ **B1**
- Sketch the other relevant graph, e.g. $y = \frac{1}{2}x + 1$, and justify the given statement **B1 2**
- (ii) Consider sign of $\operatorname{cosec} x - \frac{1}{2}x - 1$ at $x = 0.5$ and $x = 1$, or equivalent **M1**
- Complete the argument correctly with appropriate calculations **A1 2**
- (iii) Rearrange $\operatorname{cosec} x = \frac{1}{2}x + 1$ in the given form, or *vice versa* **B1 1**
- (iv) Use the iterative formula correctly at least once **M1**
Obtain final answer $x = 0.80$ **A1**
Show sufficient iterations to at least 3 d.p. to justify its accuracy to 2 d.p.,
or show there is a sign change in the interval (0.795, 0.805) **A1 3**
- 8 (i) Attempt to express integrand in partial fractions,
e.g. obtain A or B in $\frac{A}{y} + \frac{B}{4-y}$ **M1**
- Obtain $\frac{1}{4}(\frac{1}{y} + \frac{1}{4-y})$, or equivalent **A1**
- Integrate and obtain $\frac{1}{4}\ln y - \frac{1}{4}\ln(4-y)$, or equivalent **A1√ + A1√ 4**
- (ii) Separate variables correctly, integrate $\frac{A}{y} + \frac{B}{4-y}$ and obtain further
term x , or equivalent **M1***
Use $y = 1$ and $x = 0$ to evaluate a constant, or as limits **M1(dep*)**
Obtain answer in any correct form **A1**
Obtain final answer $y = 4/(3e^{-4x} + 1)$, or equivalent **A1 4**
- (iii) State that y approaches 4 as x becomes very large **B1 1**
- 9 (i) Use quotient or product rule **M1**
Obtain derivative in any correct form **A1**
Equate derivative to zero and solve for x or x^2 **M1**
Obtain $x = 1$ correctly **A1 4**
[Differentiating $(x^2 + 1)y = x$ using the product rule can also earn the first **M1A1**.]
[SR: if the quotient rule is misused, with a 'reversed' numerator or v instead of v^2 in the denominator, award **M0A0** but allow the following **M1A1**.]
- (ii) Obtain indefinite integral of the form $k \ln(x^2 + 1)$, where $k = \frac{1}{2}, 1$ or 2 **M1***
Use limits $x = 0$ and $x = p$ correctly, or equivalent **M1(dep*)**
Obtain answer $\frac{1}{2} \ln(p^2 + 1)$ **A1 3**
[Also accept $-\ln \cos \theta$ or $\ln \cos \theta$, where $x = \tan \theta$, for the first **M1***.]
- (iii) Equate to 1 and convert equation to the form $p^2 + 1 = \exp(1/k)$ **M1**
Obtain answer $p = 2.53$ **A1 2**

Page 4	Mark Scheme	Syllabus	Paper
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- 10 (i)** State or imply a direction vector for AB is $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent **B1**
- EITHER:** State equation of AB is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent **B1√**
 Equate at least two pairs of components of AB and l and solve for s or for t **M1**
 Obtain correct answer for s or for t , e.g. $s = 0$ or $t = -2$; $s = -\frac{5}{3}$ or $t = -\frac{1}{3}$ or $s = 5$ or $t = 3$ **A1**
- Verify that all three pairs of equations are not satisfied and that the lines fail to intersect **A1**
- OR:** State a Cartesian equation for AB , e.g. $\frac{x-2}{-1} = \frac{y-2}{2} = \frac{z-1}{2}$, and for l ,
 e.g. $\frac{x-4}{1} = \frac{y+2}{2} = \frac{z-2}{1}$ **B1√**
 Solve a pair of equations, e.g. in x and y , for one unknown **M1**
 Obtain one unknown, e.g. $x = 4$ or $y = -2$ **A1**
 Obtain corresponding remaining values, e.g. of z , and show lines do not intersect **A1**
- OR:** Form a relevant triple scalar product,
 e.g. $(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}) \cdot ((-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + \mathbf{k}))$ **B1√**
 Attempt to use correct method of evaluation **M1**
 Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant **A1**
 Obtain correct non-zero value, e.g. -20 , and state that the lines do not intersect **A1 5**
- (ii)**
- EITHER:** Obtain a vector parallel to the plane and not parallel to l , e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
 Use scalar product to obtain an equation in a , b and c , e.g. $a + 2b + c = 0$ **B1**
 Form a second relevant equation, e.g. $2a - 4b + c = 0$ and solve for one ratio, e.g. $a : b$ **M1**
 Obtain final answer $a : b : c = 6 : 1 : -8$ **A1**
 Use coordinates of a relevant point and values of a , b and c in general equation and find d **M1**
 Obtain answer $6x + y - 8z = 6$, or equivalent **A1**
- OR:** Obtain a vector parallel to the plane and not parallel to l , e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
 Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \times (2\mathbf{i} - 4\mathbf{j} + \mathbf{k})$ **M1**
 Obtain two correct components of the product **A1**
 Obtain correct answer, e.g. $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$ **A1**
 Substitute coordinates of a relevant point in $6x + y - 8z = d$, or equivalent, to find d **M1**
 Obtain answer $6x + y - 8z = 6$, or equivalent **A1**
- OR:** Obtain a vector parallel to the plane and not parallel to l , e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
 Obtain a second relevant vector parallel to the plane and correctly form a 2-parameter equation for the plane,
 e.g. $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ **M1**
 State 3 correct equations in x , y , z , λ and μ **A1**
 Eliminate λ and μ **M1**

Page 5	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2005	9709/8719	3

Obtain equation in any correct form **A1**
 Obtain answer $6x + y - 8z = 6$, or equivalent **A1**

OR: Using the coordinates of A and two points on l , state three simultaneous equations in a , b , c and d , e.g. $2a + 2b + c = d$, $4a - 2b + 2c = d$ and $5a + 3c = d$ **B1**
 Solve and find one ratio, e.g. $a:b$ **M1**
 State one correct ratio **A1**
 Obtain a ratio of three unknowns, e.g. $a:b:c = 6:1:-8$, or equivalent **A1**
 Either use coordinates of a relevant point and found ratio to find fourth unknown, e.g. d , **or** find the ratio of all four unknowns **M1**
 Obtain answer $6x + y - 8z = 6$, or equivalent **A1 6**