CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

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| **MARK SCHEME** |
| **MAXIMUM MARK : 75** |
| **SYLLABUS/COMPONENT :9709 /3, 8719 /3**<br>**MATHEMATICS**<br>**(Pure 3)** |

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

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|---|---|---|---|
| 1 | *EITHER*: Express LHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ | M1 | |
| | Make sufficient relevant use of double-angle formula(e) | M1 | |
| | Complete proof of the result | A1 | |
| | *OR*: Express RHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ | M1 | |
| | Express RHS as the difference (or sum) of two fractions | M1 | |
| | Complete proof of the result | A1 | 3 |
| | [SR: an attempt ending with $\frac{1-\tan^2\theta}{\tan\theta} = \cot\theta - \tan\theta$ earns M1 B1 only.] | | |
| 2 | *EITHER*: Show correct (unsimplified) version of the $x$ or the $x^2$ or the $x^3$ term | M1 | |
| | Obtain correct first two terms $1 + x$ | A1 | |
| | Obtain correct quadratic term $2x^2$ | A1 | |
| | Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) | A1 | |
| | [The M mark may be implied by correct simplified terms, if no working is shown. It is not earned by unexpanded binomial coefficients involving $-\frac{1}{3}$, e.g. ${}^{-\frac{1}{3}}C_1$ or $\binom{-\frac{1}{3}}{2}$.] | | |
| | [An attempt to divide 1 by the expansion of $(1-3x)^{\frac{1}{3}}$ earns M1 if the expansion has a correct (unsimplified) $x$, $x^2$, or $x^3$ term and if the partial quotient contains a term in $x$. The remaining A marks are awarded as above.] | | |
| | *OR*: Differentiate and calculate f(0), f′(0), where $f'(x) = k(1-3x)^{-\frac{1}{3}-1}$ | M1 | |
| | Obtain correct first two terms $1 + x$ | A1 | |
| | Obtain correct quadratic term $2x^2$ | A1 | |
| | Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) | A1 | 4 |
| 3 | Attempt to find $a$ and/or quadratic factor by division or by inspection | M1 | |
| | Obtain partial quotient or factor $x^2 - x$ | A1 | |
| | State answer $a = 6$ | B1 | |
| | State or imply the other factor is $x^2 - x + 3$ | A1 | 4 |
| | [The M1 is earned if division has produced a partial quotient $x^2 + bx$, or if inspection has an unknown factor $x^2 + bx + c$ and has reached an equation in $b$ and/or $c$.] | | |
| | [SR: a correct division with unresolved constant remainder can earn M1A1B0A1.] | | |
| | [NB: successive division by a pair of incorrect linear factors, e.g. $x - 1$ and $x + 2$ or $x + 1$ and $x + 2$, can earn M1A0 or M1A1(if their product is of the form $x^2 + x + k$).] | | |

4 (i) Use the formula correctly at least once — M1
State $\alpha = 1.26$ as final answer — A1
Show sufficient iterations to justify $\alpha = 1.26$ to 2d.p., or show there is a sign change in the interval (1.255, 1.265) — A1 — 3

(ii) State any suitable equation in one unknown e.g. $x = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$ — B1
State exact value of $\alpha$ (or $x$) is $\sqrt[3]{2}$ or $2^{\frac{1}{3}}$ — B1 — 2

5 Obtain derivative $\pm 2\sin x + k\cos 2x$ or $\pm 2\sin x + k(\cos^2 x \pm \sin^2 x)$ — M1
Equate derivative to zero and use trig formula to obtain an equation involving only one trig function — M1
Obtain a correct equation of this type e.g. $2\sin^2 x + \sin x - 1 = 0$ or $\cos 2x = \cos\left(\frac{1}{2}\pi - x\right)$ — A1
Obtain value $x = \frac{1}{6}\pi$ (allow 0.524 radians or 30°) — A1
Show by any method that the corresponding point is a maximum point — A1
Obtain second value $x = \frac{5}{6}\pi$ (allow 2.62 radians or 150°), and no others in range — A1✓
Determine that it corresponds to a minimum point — A1 — 7

6 (i) State or imply $f(x) \equiv \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$ — B1
State or obtain $A = -3$ — B1
State or obtain $B = 2$ — B1
Use any relevant method to find $C$ — M1
Obtain $C = 1$ — A1 — 5
[Special case: allow the form $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$ and apply the above scheme ($A = -3$, $D = 1$, $E = 3$).]
[SR: if f($x$) is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of $C$, or $E$, or $B$ in the above, and M1 for finding one coefficient.]

(ii) Integrate and obtain terms $-\ln(3x+1) - \frac{2}{(x+1)} + \ln(x+1)$ — B1 + B1 + B1✓
Use limits correctly — M1
Obtain the given answer correctly — A1 — 5

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| 7 | (i) State that $\frac{dm}{dt} = k(50 - m)^2$ | B1 | |
| | Justify $k = 0.002$ | B1 | 2 |
| | (ii) Separate variables and attempt to integrate $\frac{1}{(50-m)^2}$ | M1 | |
| | Obtain $\pm\frac{1}{(50-m)}$ and $0.002t$, or equivalent | A1 | |
| | Evaluate a constant or use limits $t = 0$, $m = 0$ | M1 | |
| | Obtain any correct form of solution e.g. $\frac{1}{(50-m)} = 0.002t + \frac{1}{50}$ | A1 | |
| | Obtain given answer correctly | A1 | 5 |
| | (iii) Obtain answer $m = 25$ when $t = 10$ | B1 | |
| | Obtain answer $t = 90$ when $m = 45$ | B1 | 2 |
| | (iv) State that $m$ approaches 50 | B1 | 1 |
| 8 | (i) State or imply a simplified direction vector of $l$ is $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, or equivalent | B1 | |
| | State equation of $l$ is $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{2}$, or equivalent | B1✓ | |
| | Substitute in equation of $p$ and solve for $\lambda$, or one of $x$, $y$, or $z$ | M1 | |
| | Obtain point of intersection $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$ | A1 | 4 |
| | [Any notation is acceptable.] | | |
| | (ii) State or imply a normal vector of $p$ is $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ | B1 | |
| | *EITHER*: Use scalar product to obtain $a + 3b - 2c = 0$ | M1 | |
| | Use points on $l$ to obtain two equations in $a$, $b$, $c$ e.g. $a + c = 1$, $4a - b + 3c = 1$ | B1✓ | |
| | Solve simultaneous equations, obtaining one unknown | M1 | |
| | Obtain one correct unknown e.g. $a = -\frac{2}{3}$ | A1 | |
| | Obtain the other unknowns e.g. $b = \frac{4}{3}$, $c = \frac{5}{3}$ | A1 | |
| | *OR*: Use scalar product to obtain $a + 3b - 2c = 0$ | M1 | |
| | Use scalar product to obtain $3a - b + 2c = 0$ | B1✓ | |
| | Solve simultaneous equations to obtain one ratio e.g. $a : b$ | M1 | |
| | Obtain $a : b : c = 2 : -4 : -5$, or equivalent | A1 | |
| | Obtain $a = -\frac{2}{3}$, $b = \frac{4}{3}$, $c = \frac{5}{3}$ | A1 | |
| | [NB: candidates may transfer from the *EITHER* to *OR* scheme by subtracting the two "point" equations, or transfer from *OR* to *EITHER* by finding one of the "point" equations.] | | |
| | *OR*: Calculate the vector product $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})\times(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ | M1 | |
| | Obtain answer $-4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$, or equivalent | A1✓ | |
| | Substitute in $-4x + 8y + 10z = d$ to find $d$, or equivalent | M1 | |
| | Obtain $d = 6$, or equivalent | A1 | |
| | Obtain $a = -\frac{2}{3}$, $b = \frac{4}{3}$, $c = \frac{5}{3}$ | A1 | |
| | *OR*: State or imply a correct equation of the plane e.g. $\mathbf{r} = \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \mathbf{i} + \mathbf{k}$ | M1 | |
| | State 3 equations in $x$, $y$, $z$, $\lambda$, and $\mu$, e.g. $x = 3\lambda + \mu + 1$, $y = -\lambda + 3\mu$, $z = 2\lambda - 2\mu + 1$ | A1✓ | |
| | Eliminate $\lambda$ and $\mu$ | M1 | |
| | Obtain equation $-4x + 8y + 10z = 6$, or equivalent | A1 | |
| | Obtain $a = -\frac{2}{3}$, $b = \frac{4}{3}$, $c = \frac{5}{3}$ | A1 | 6 |
| | [SR: condone the use of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ for $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ in the *EITHER* scheme and the first *OR* scheme.] | | |

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| 9 | (i) State or imply that $r = 2$ | B1 | |
| | State or imply that $\theta = \frac{1}{3}\pi$ (allow 1.05 radians or 60°) | B1 | |
| | Obtain modulus 4, and argument $\frac{2}{3}\pi$ of $u^2$ (allow $2^2$; 2.09 or 2.10 radians or 120°) | B1 + B1✓ | |
| | Obtain modulus 8 and argument $\pi$ of $u^3$ (allow $2^3$; 3.14 or 3.15 radians or 180°) | B1✓ | 5 |
| | [Follow through on wrong $r$ and $\theta$.] | | |
| | [SR: if $u^2$ and $u^3$ are only given in polar form, give B1✓ for $u^2$ and B1✓ for $u^3$.] | | |
| | (ii) *EITHER*: Deduce that $u^2 - 2u + 4 = 0$ from $u^3 + 8 = 0$ | | |
| | *OR*: Verify that $u^2 - 2u + 4 = 0$ by calculation | B1 | |
| | State that the other root is $1 - i\sqrt{3}$, or equivalent | B1 | 2 |
| | [NB: stating that the roots are $1 \pm i\sqrt{3}$ is sufficient for both B marks.] | | |
| | (iii) Show both points correctly on an Argand diagram | B1 | |
| | Show the correct relevant circle | B1 | |
| | Show line (segment) correctly | B1 | |
| | Shade the correct region | B1 | 4 |
| | [SR: allow work on separate diagrams to be eligible for the first three B marks.] | | |
| 10 | (i) State at any stage that the $x$-coordinate of $A$ is equal to 1, or that $A$ is the point (1,0) | B1 | 1 |
| | (ii) State $f'(x) = 2\dfrac{\ln x}{x}$, or equivalent | B1 | |
| | Use product or quotient rule for the next differentiation | M1 | |
| | Obtain $2.\dfrac{1}{x}.\dfrac{1}{x} + 2\ln x.\left(\dfrac{-1}{x^2}\right)$, or any equivalent correct unsimplified form | A1 | |
| | Verify that $f''(e) = 0$ | A1 | 4 |
| | (iii) State or imply area is $\displaystyle\int_1^e (\ln x)^2\,dx$ | B1 | |
| | Use $\dfrac{dx}{du} = e^u$, or equivalent, in substituting for $x$ throughout | M1 | |
| | Obtain given answer correctly (allow change of limits to be done mentally) | A1 | 3 |
| | (iv) Attempt the first integration by parts, going the correct way | M1 | |
| | Obtain $(u^2 - 2u \pm 2)e^u$, or equivalent, after two applications of the rule | A1 | |
| | Obtain exact answer in terms of e, in any correct form, e.g. $(e - 2e + 2e) - 2$, or $e - 2$ | A1 | 3 |

[The substitution in (iii) may be done in reverse i.e. starting with the $u$ integral and obtaining the $x$ integral. The M1A1 scheme applies, but only an explicit statement will earn the B1.]
[The M1A1A1 in (iv) applies to those working in terms of $x$ and obtaining $x((\ln x)^2 - 2\ln x \pm 2)$, or equivalent.]