

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE A/AS Level

MARK SCHEME for the November 2005 question paper

9709 MATHEMATICS

9709/01

Paper 1 maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

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| AEF | Any Equivalent Form (of answer is equally acceptable) |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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| <p>1 (i) $3 \sin^2 \theta - 2 \cos \theta - 3 = 0$ Use of $s^2 + c^2 = 1$ $3 \cos^2 \theta + 2 \cos \theta = 0$</p> <p>$\cos \theta = 0, \theta = 90^\circ$</p> <p>or $\cos \theta = -2/3, \theta = 131.8^\circ$</p> | <p>M1 A1</p> <p>B1</p> <p>A1</p> <p>[4]</p> | <p>Use of $s^2 + c^2 = 1$ to eliminate sine. Correct equation</p> <p>Co.</p> <p>Co. (to 1 d.p or more – there must be only this answer in the range 0 to 180))</p> |
| <p>2 $\alpha = 0.8$, radii 10cm and 16 cm</p> <p>(i) Area = $\frac{1}{2} \cdot 16^2 \cdot 0.8 - \frac{1}{2} \cdot 10^2 \cdot 0.8$ $\rightarrow 62.4 \text{ cm}^2$</p> <p>(ii) Arcs are 10α and 16α $12 + 10\alpha + 16\alpha = 28.9$ $\rightarrow \alpha = 0.65$</p> | <p>M1 A1</p> <p>[2]</p> <p>M1 DM1 A1</p> <p>[3]</p> | <p>Use of $\frac{1}{2}r^2\theta$ once.. Co.</p> <p>Use of $s=r\theta$ once. Forming an eqn, including the 6 + 6. co</p> |
| <p>3 (i) $2d \sin 30 + 2d\sqrt{3} \sin 60$ $= 2d \cdot \frac{1}{2} + 2d\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 4d$</p> <p>(ii) $\tan \theta = \frac{\text{ans to (i)}}{2d \cos 30 + 2\sqrt{3}d \cos 60}$ $\tan \theta = \frac{2}{\sqrt{3}}$</p> | <p>M1 A1</p> <p>[2]</p> <p>M1 DM1</p> <p>A1</p> <p>[3]</p> | <p>For one - allow even if decimals used Co – exact answer only.</p> <p>Use of $\tan = \text{opp/adj}$ in correct triangle For horizontal step attempted</p> <p>Co. Lost if decimals used.</p> |

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| <p>4 $\vec{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\vec{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix}$</p> <p>(i) $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix}$ with $q = 3$, $= -4 + 3 + 3 = 2$</p> <p>$= \sqrt{14} \cdot \sqrt{14} \cos \theta = 2$, $\cos \theta = \frac{1}{7}$</p> <p>(ii) $\vec{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 4 \\ -2 \\ q-1 \end{pmatrix}$</p> <p>$16 + 4 + (q-1)^2 = 36$ $\rightarrow q = 5$ or $q = -3$</p> | <p>M1</p> <p>M1 A1 [3]</p> <p>M1</p> <p>M1 A1 A1 [4]</p> | <p>Use of $a_1a_2+b_1b_2+c_1c_2$.</p> <p>Dot product linked with moduli and cos. co</p> <p>Allow for p-q or q-p</p> <p>Use of modulus and Pythagoras Co (for both)</p> |
| <p>5.</p> <p>(i) Similar triangles or trig (tan = opp/hyp)</p> <p>$\frac{6}{12} = \frac{r}{12-h} \rightarrow h = 12 - 2r$</p> <p>$\rightarrow V = \pi r^2 h \rightarrow V = 12\pi r^2 - 2\pi r^3$</p> <p>(ii) $dV/dr = 24\pi r - 6\pi r^2$</p> <p>$= 0$ when $r = 4 \rightarrow V = 64\pi$ (or 201)</p> | <p>M1</p> <p>A1</p> <p>M1 [3]</p> <p>M1 A1</p> <p>M1 A1 [4]</p> | <p>Valid method leading to h in terms of r</p> <p>Co</p> <p>Use of vol formula with some $h=f(r)$</p> <p>Attempt at differentiation, (2 or π or 2π missing, loses this A1 only)</p> <p>Setting his differential to 0. co.</p> |
| <p>6. (i) GP with $a = 250000$ $r = 1.05$ Year 2008 is the 9th term $ar^8 = 250000 \times 1.05^8 = 369\,000$</p> <p>(ii) $S_{10} = 250000(1.05^{10} - 1) \div 0.05$ $= 3\,140\,000$</p> <p>(iii) AP $S_{10} = 5(500\,000 + 9D)$ $=$ answer to (ii) $\rightarrow D = 14\,300$</p> | <p>B1</p> <p>M1 A1✓ [3]</p> <p>M1 A1 [2]</p> <p>M1 DM1 A1 [3]</p> | <p>For any use of $r=1.05$ $(25000 + 0.05 \times 25000)$ gets B1) Use of ar^{n-1} with $r=8$ or 9 Answer rounding to 369 000. ft on r.</p> <p>Use of correct S_n formula – for 10 only Co – must round to the correct answer (adds 10 numbers correctly M1 A1)</p> <p>Correct S_n formula. Forming + soln Co.</p> |

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| <p>7.</p> <p>(i) $M(5,8)$ gradient of $AB = \frac{2}{3}$, $\text{Perp} = -\frac{3}{2}$ Equation $y - 8 = -\frac{3}{2}(x - 5)$ $\rightarrow 2y + 3x = 31$ (or locus method M1A1M1A1)</p> <p>(ii) BC. $y = 5(x-6)$ $y = 5x - 30$ Sim Eqns $\rightarrow (7,5)$</p> | <p>B1 M1</p> <p>M1 A1</p> <p>[4]</p> <p>M1A1 DM1A1</p> <p>[4]</p> | <p>Co. Use of ystep/x-step + $m_1m_2=-1$ for AB</p> <p>Use of $y-k=m(x-h)$ not $(y+k)$ etc</p> <p>Use of $y-k=m(x-h)$ not $(y+k)$ etc. co Correct attempt at soln of BC with his answer to (i).</p> |
| <p>8. $f: x \mapsto (2x-3)^3 - 8$</p> <p>(i) $f'(x) = 3(2x-3)^2 \times 2$ Since $()^2$ is +ve, $f'(x)$ +ve for all x Therefore an increasing function. (or t.p. at $(1.5, -8)$ M1. Compares with y values at 2, or 4 + conclusion A1)</p> <p>(ii) $y = (2x-3)^3 - 8$, $2x-3 = \sqrt[3]{(y+8)}$ $\rightarrow f^{-1}(x) = \frac{\sqrt[3]{(x+8)} + 3}{2}$ Domain $-7 \leq x \leq 117$</p> | <p>B1B1</p> <p>B1 B1</p> <p>[4]</p> <p>M1 DM1 A1</p> <p>B1</p> <p>[4]</p> | <p>B1 for answer without $\times 2$. B1 for $\times 2$.</p> <p>Realising 'increasing' \rightarrow +ve gradient) Stating $()^2$ +ve for <u>all</u> x. All complete.</p> <p>Attempt to make x subject.</p> <p>Order of operations correct.</p> <p>Co – needs x not y.</p> <p>Co</p> |
| <p>9.</p> <p>(i) $2x^2 + 12 = 11x$ or $y^2 - 11y + 24 = 0$ Solution $\rightarrow (1\frac{1}{2}, 8)$ and $(4, 3)$ Guesswork B1 for one, B3 for both.</p> <p>(ii) $2x^2 - kx + 12 = 0$ Use of $b^2 - 4ac$ $k^2 < 96$ $-\sqrt{96} < k < \sqrt{96}$ or $k < \sqrt{96}$</p> <p>(iii) gradient of $2x + y = k = -2$ $dy/dx = -12/x^2 (= -3)$ Use of tangent for an angle Difference = 8.1° or 8.2°</p> | <p>M1</p> <p>DM1 A1</p> <p>[3]</p> <p>M1 A1 DM1 A1</p> <p>[4]</p> <p>B1 B1 M1 A1</p> <p>[4]</p> | <p>Elimination of one variable completely</p> <p>Correct method for soln of quadratic=0 co</p> <p>Used on quadratic=0. Allow =0, >0 etc For $k^2 - 96$ Definite use of $b^2 - 4ac < 0$ Co. (condone inclusion of \leq)</p> <p>Anywhere For differentiation only – unsimplified Used with either line or tangent Co</p> |

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| <p>10. $\frac{dy}{dx} = \frac{16}{x^3}$, through (1,4)</p> <p>(i) $y = \frac{16x^{-2}}{-2} + c$</p> <p>Use of (1,4) $\rightarrow y = \frac{-8}{x^2} + 12$</p> <p>(ii) Normal has $m = -\frac{1}{2}$, Perpendicular = 2</p> <p>$\frac{16}{x^3} = 2 \quad x = 2$</p> <p>$y = 10$</p> <p>Eqn of normal $y - 10 = -\frac{1}{2}(x - 2)$ ie $2y + x = 22$</p> <p>(iii) $A = \int (\frac{-8}{x^2} + 12) dx$</p> <p>$= \frac{8}{x} + 12x$</p> <p>$= [] \text{ at } 2 - [] \text{ at } 1$</p> <p>$\rightarrow 8$</p> | <p>M1 A1</p> <p>M1A1 [4]</p> <p>M1</p> <p>DM1</p> <p>M1 A1 [4]</p> <p>M1</p> <p>A1√</p> <p>M1</p> <p>A1 [4]</p> | <p>Attempt to \int. Allow A1 unsimplified. (ignore +c for these marks)</p> <p>Using coordinates in an integrated expression. Co.</p> <p>Use of $m_1 m_2 = -1$</p> <p>Link with $dy/dx \rightarrow$ value for x</p> <p>Using calculated x and y to find equation. Allow for correct 3 term linear equation</p> <p>Attempt at integration of y from (i) even if linear</p> <p>Allow ft, providing original has two terms, one of which is a -ve power</p> <p>Correct use of correct limits in some integrated expression.</p> |
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