UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

* 3 3 6 3 7 2 0 4 3 4 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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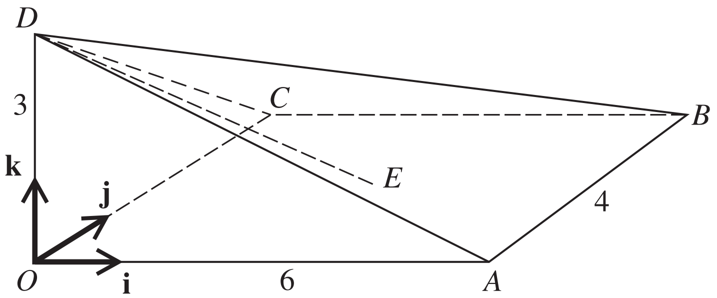
1 **(i)** Find the first three terms when $(2 + 3x)^6$ is expanded in ascending powers of x. [3]

(ii) In the expansion of $(1 + ax)(2 + 3x)^6$, the coefficient of x^2 is zero. Find the value of a. [2]

2 A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = \dfrac{1}{\sqrt{(x+6)}} + \dfrac{6}{x^2}$ and that $\mathrm{f}(3) = 1$. Find $\mathrm{f}(x)$. [5]

3



The diagram shows a pyramid $OABCD$ in which the vertical edge OD is 3 units in length. The point E is the centre of the horizontal rectangular base $OABC$. The sides OA and AB have lengths of 6 units and 4 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Express each of the vectors $\overrightarrow{DB}$ and $\overrightarrow{DE}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to find angle BDE. [4]

4 **(i)** Solve the equation $4\sin^2 x + 8\cos x - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Hence find the solution of the equation $4\sin^2\left(\tfrac{1}{2}\theta\right) + 8\cos\left(\tfrac{1}{2}\theta\right) - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [2]

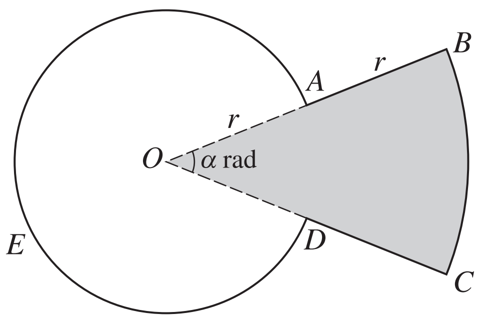
5 The function f is defined by

$$\mathrm{f} : x \mapsto x^2 + 1 \quad \text{for} \quad x \geqslant 0.$$

(i) Define in a similar way the inverse function f^{-1}. [3]

(ii) Solve the equation $\mathrm{ff}(x) = \tfrac{185}{16}$. [3]

6



The diagram shows a metal plate made by fixing together two pieces, $OABCD$ (shaded) and $OAED$ (unshaded). The piece $OABCD$ is a minor sector of a circle with centre O and radius $2r$. The piece $OAED$ is a major sector of a circle with centre O and radius r. Angle AOD is α radians. Simplifying your answers where possible, find, in terms of α, π and r,

(i) the perimeter of the metal plate, [3]

(ii) the area of the metal plate. [3]

It is now given that the shaded and unshaded pieces are equal in area.

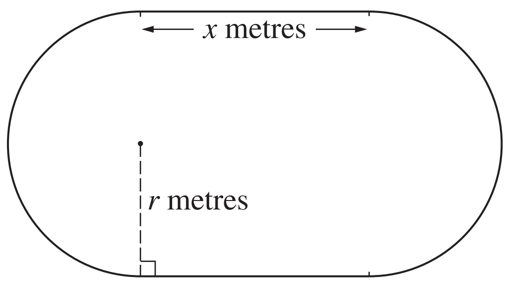
(iii) Find α in terms of π. [2]

7 The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

(i) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

(ii) A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]

8



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

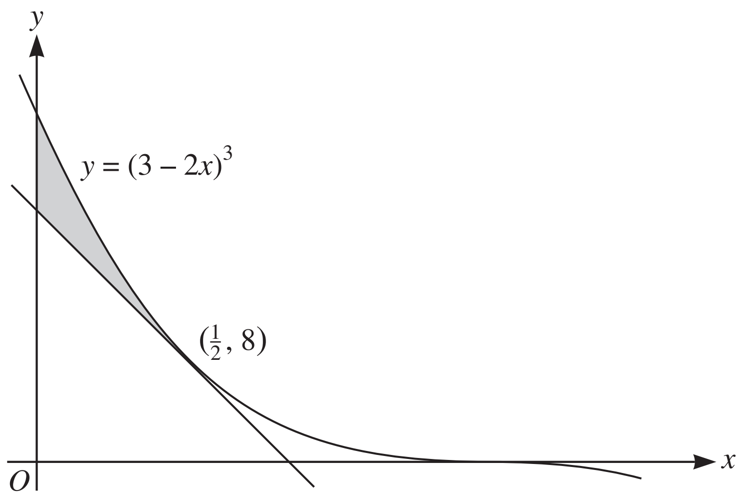
(i) Show that the area, $A\,\text{m}^2$, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]

(ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

9 **(a)** In an arithmetic progression the sum of the first ten terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term. [5]

(b) A geometric progression has first term a, common ratio r and sum to infinity 6. A second geometric progression has first term $2a$, common ratio r^2 and sum to infinity 7. Find the values of a and r. [5]

10



The diagram shows the curve $y = (3 - 2x)^3$ and the tangent to the curve at the point $\left(\frac{1}{2}, 8\right)$.

(i) Find the equation of this tangent, giving your answer in the form $y = mx + c$. [5]

(ii) Find the area of the shaded region. [6]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Given that $\cos x = p$, where x is an acute angle in degrees, find, in terms of p,

(i) $\sin x$, [1]

(ii) $\tan x$, [1]

(iii) $\tan(90° - x)$. [1]

2

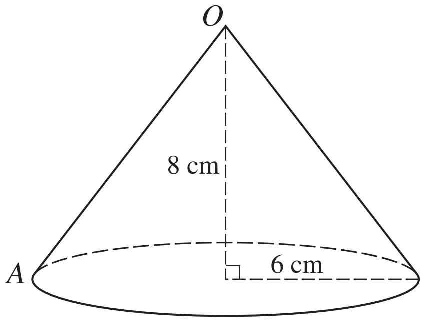


Fig. 1

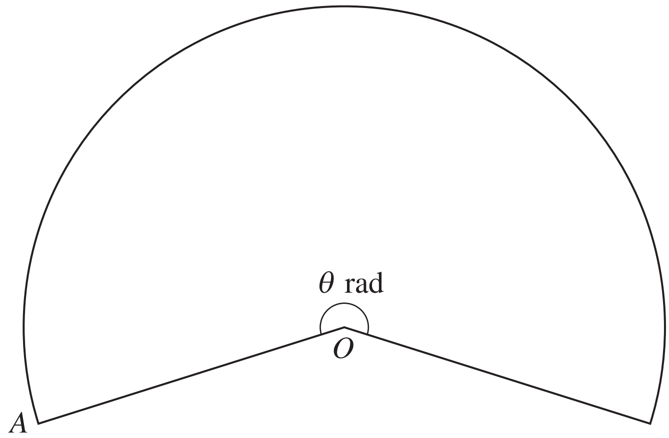


Fig. 2

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is θ radians. Calculate

(i) the value of θ, [4]

(ii) the area of paper needed to make the cone. [2]

3 The equation of a curve is $y = \dfrac{2}{\sqrt{(5x-6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find $\displaystyle\int \frac{2}{\sqrt{(5x-6)}}\,dx$ and hence evaluate $\displaystyle\int_2^3 \frac{2}{\sqrt{(5x-6)}}\,dx$. [4]

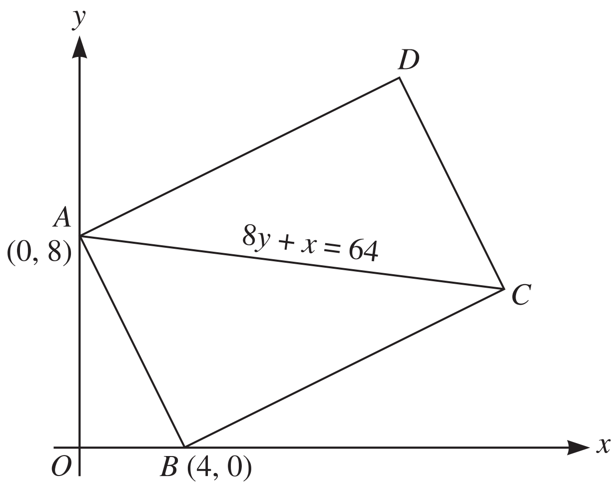
4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} + p\mathbf{k}.$$

(i) In the case where $p = 6$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

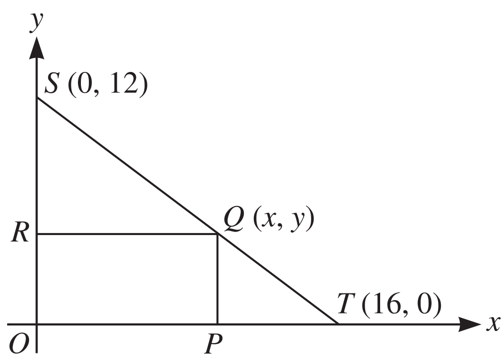
(ii) Find the values of p for which angle $AOB = \cos^{-1}\left(\frac{1}{5}\right)$. [4]

5



The diagram shows a rectangle $ABCD$ in which point A is $(0, 8)$ and point B is $(4, 0)$. The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D. [7]

6



In the diagram, S is the point $(0, 12)$ and T is the point $(16, 0)$. The point Q lies on ST, between S and T, and has coordinates (x, y). The points P and R lie on the x-axis and y-axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A, of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

7 **(a)** An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the nth mile takes 9 minutes, find the value of n. [2]

(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon. [2]

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [4]

[Questions 8, 9 and 10 are printed on the next page.]

8 A function f is defined by $\mathrm{f} : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $\mathrm{f}(x) = 0$. [3]

(ii) Find the range of f. [2]

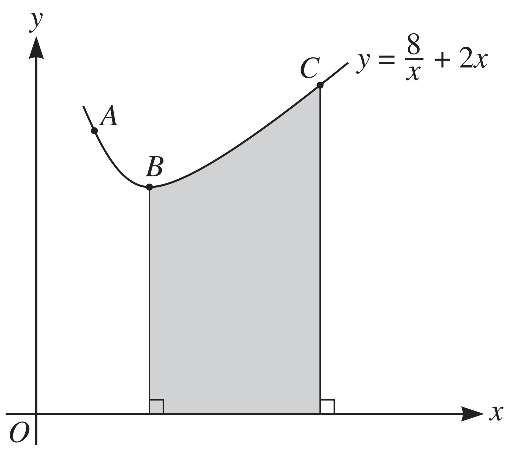
(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

A function g is defined by $\mathrm{g} : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant k$.

(iv) State the maximum value of k for which g has an inverse. [1]

(v) Obtain an expression for $\mathrm{g}^{-1}(x)$. [2]

9



The diagram shows part of the curve $y = \frac{8}{x} + 2x$ and three points A, B and C on the curve with x-coordinates 1, 2 and 5 respectively.

(i) A point P moves along the curve in such a way that its x-coordinate increases at a constant rate of 0.04 units per second. Find the rate at which the y-coordinate of P is changing as P passes through A. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

10 A curve has equation $y = 2x^2 - 3x$.

(i) Find the set of values of x for which $y > 9$. [3]

(ii) Express $2x^2 - 3x$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and state the coordinates of the vertex of the curve. [4]

The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 2x^2 - 3x \quad \text{and} \quad \mathrm{g}(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $\mathrm{gf}(x) = 0$ has equal roots. [3]

MATHEMATICS **9709/13**

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This document consists of **4** printed pages.

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[Turn over

1 Solve the inequality $x^2 - x - 2 > 0$. [3]

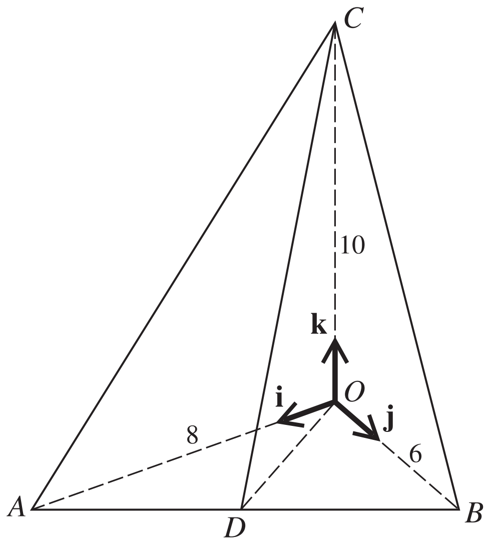
2 A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = x^{-\frac{3}{2}} + 1$ and that $\mathrm{f}(4) = 5$. Find $\mathrm{f}(x)$. [4]

3 The point A has coordinates $(3, 1)$ and the point B has coordinates $(-21, 11)$. The point C is the mid-point of AB.

(i) Find the equation of the line through A that is perpendicular to $y = 2x - 7$. [2]

(ii) Find the distance AC. [3]

4



The diagram shows a pyramid $OABC$ in which the edge OC is vertical. The horizontal base OAB is a triangle, right-angled at O, and D is the mid-point of AB. The edges OA, OB and OC have lengths of 8 units, 6 units and 10 units respectively. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

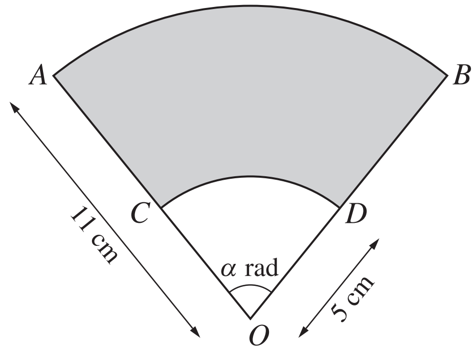
(i) Express each of the vectors $\overrightarrow{OD}$ and $\overrightarrow{CD}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to find angle ODC. [4]

5 **(a)** In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio. [2]

(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference. [4]

6



The diagram shows sector OAB with centre O and radius 11 cm. Angle $AOB = \alpha$ radians. Points C and D lie on OA and OB respectively. Arc CD has centre O and radius 5 cm.

(i) The area of the shaded region $ABDC$ is equal to k times the area of the unshaded region OCD. Find k. [3]

(ii) The perimeter of the shaded region $ABDC$ is equal to twice the perimeter of the unshaded region OCD. Find the exact value of α. [4]

7 **(a)** Find the possible values of x for which $\sin^{-1}(x^2 - 1) = \frac{1}{3}\pi$, giving your answers correct to 3 decimal places. [3]

(b) Solve the equation $\sin(2\theta + \frac{1}{3}\pi) = \frac{1}{2}$ for $0 \leqslant \theta \leqslant \pi$, giving θ in terms of π in your answers. [4]

8 **(i)** Find the coefficient of x^8 in the expansion of $(x + 3x^2)^4$. [1]

(ii) Find the coefficient of x^8 in the expansion of $(x + 3x^2)^5$. [3]

(iii) Hence find the coefficient of x^8 in the expansion of $\left[1 + (x + 3x^2)\right]^5$. [4]

9 A curve has equation $y = \dfrac{k^2}{x+2} + x$, where k is a positive constant. Find, in terms of k, the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

10 The function f is defined by $\mathrm{f} : x \mapsto x^2 + 4x$ for $x \geqslant c$, where c is a constant. It is given that f is a one-one function.

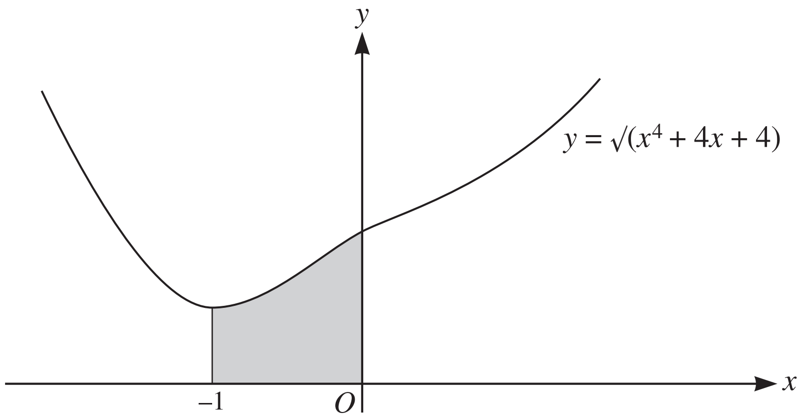
(i) State the range of f in terms of c and find the smallest possible value of c. [3]

The function g is defined by $\mathrm{g} : x \mapsto ax + b$ for $x \geqslant 0$, where a and b are positive constants. It is given that, when $c = 0$, $\mathrm{gf}(1) = 11$ and $\mathrm{fg}(1) = 21$.

(ii) Write down two equations in a and b and solve them to find the values of a and b. [6]

[Question 11 is printed on the next page.]

11



The diagram shows the curve $y = \sqrt{(x^4 + 4x + 4)}$.

(i) Find the equation of the tangent to the curve at the point (0, 2). [4]

(ii) Show that the x-coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x-coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x-axis. Find the volume of revolution. [4]