UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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1	$6C4 \times [2(x)]^4 \times \left[\frac{1}{(x^2)}\right]^2$	B2		B1 for 2/3 terms correct
	240	B1	[3]	Identified as answer. Allow $240x^0$
2	$\frac{\delta y}{\delta x}$ $9x^2$ $12x + 4$	M1A1		
	$(3x$ $2)^2 \geq 0$	A1	[3]	
3	**(i)** Correct cosine curve for at least 1 oscillation	B1		Range $1 \rightarrow 1$. Ignore labels on θ axis
	Exactly 2 complete oscillations in $[0, 2\pi]$	B1		
	Line y $\frac{1}{2}$ correct	B1	[3]	
	(ii) 4	B1√	[1]	Ft *their* graph. Accept $30°$, $150°$, $210°$, $330°$
	(iii) 20	B1√	[1]	Or $5\times$ *their* part **(ii)**
4	**(i)** 3	B1	[1]	
	(ii) $f(x)$ x^2 $6x(+c)$	M1A1		Dependent on c present
	Subst $(3,\ 4)$	M1		cao
	c $5 \rightarrow f(x)$ x^2 $6x + 5$	A1	[4]	
5	**(i)** **Arc** AB $r\theta$	M1		
	OC $r\sin\theta$ or BC $r\cos\theta$	M1		oe eg BC $r\sin\frac{\theta}{\tan\theta}$ etc
	$r(1 + \theta + \cos\theta + \sin\theta)$ correctly derived	A1	[3]	OC & BC reversed loses M1A1
	(ii) Sector OAB $\frac{1}{2}\times 10^2 \times \frac{\pi}{5}$ $(\ 31.42)$	M1		oe Δ in terms of π and 10
	ΔOCB $\frac{1}{2\left(10\cos\frac{\pi}{5}\right)\left(10\sin\frac{\pi}{5}\right)}$ $(\ 23.78)$	M1		Allow OC & BC reversed (ie max 4/6)
	Total area 55.2	A1	[3]	
6	**(a)** $a + 5d$ 23	B1		Solution of 2 linear equations
	$5(2a + 9d)$ 200	B1		
	Attempt solution, expect d 6 a 7	M1		
	29	A1	[4]	

	(b) $\frac{1}{1\quad r}(\quad)\frac{4}{1\quad\frac{1}{4}r}$	M1		Use of S_∞ formula twice
	$r\quad\frac{4}{5}$ oe $S\quad 5$	A1A1	[3]	
7	**(i)** $y\quad\frac{1}{6(48\quad 8x)}$ oe	B1	[1]	
	(ii) $A\quad 4xy+2xy$ or $3xy+3xy\quad 6xy$	M1		
	$A\quad x(48\quad 8x)\quad 48x\quad 8x^2$	A1	[2]	AG
	(iii) $\frac{\delta A}{\delta x}\quad 48\quad 16x$	B1		
	$A\quad 72$ cao	M1A1		Attempt to solve derivative $\quad 0$ Expect $x\quad 3$
	$\frac{\delta^2 A}{\delta x^2}\quad 16\quad(<0)\Rightarrow$ Maximum	B1	[4]	www Accept other complete methods
8	**(i)** $(4i+7j\quad pk).(8i\quad j\quad pk)\quad 25+p^2$	M1A1	[2]	$x_1x_2+y_1y_2+z_1z_2$ (Not $25+(\quad p)^2$)
	(ii) $25+p^2\quad 0\Rightarrow$ no real solutions	B1√	[1]	Ft provided equation has no real solutions
	(iii) $\cos 60\quad\frac{OA.OB}{\lvert OA\rvert\lvert OB\rvert}$ used	M1		$OA.OB$ must be scalar
	$\lvert OA\rvert\quad\sqrt{65+p^2}$ or $\lvert OB\rvert\quad\sqrt{65+p^2}$	M1		Not $\sqrt{65\quad p^2}$ unless follows $\sqrt{65+(\quad p)^2}$
	$\frac{25+p^2}{65+p^2}\quad\frac{1}{2}$ or $\frac{his\ scalar\ (i)}{65+p^2}\quad\frac{1}{2}$	A1√		Scalar product $\quad 25+p^2$ can score here if not scored in part **(i)**
	$p\quad\pm 3.87$ or $\pm\sqrt{15}$	A1	[4]	
9	**(i)** $x^2+3x+4\quad 2x+6\Rightarrow x^2+x\quad 2(\quad 0)$	M1		3-term simplification
	$(x\quad 1)(x+2)\quad 0\rightarrow(1,8),(\quad 2,2)$	DM1A1		DM1 for attempted solution for x
	$AB\quad\sqrt{3^2+6^2}\quad 6.71$ or $\sqrt{45}$ or $3\sqrt{5}$	B1		cao ($\sqrt{45}$ from wrong points scores B0)
	$\left(\quad\frac{1}{2},5\right)$	B1√	[5]	Ft *their* coordinates

	(ii) $x^2 + (3\ \ k)x + 2k\ \ \ 6(\ \ 0)$	M1		Simplified to 3-term quadratic
	$(3\ \ k)^2\ \ \ 4(2k\ \ \ 6)\ \ \ 0$	DM1		Apply $b^2\ \ \ 4ac\ \ \ 0$ as function of k only
	$(3\ \ k)(11\ \ \ k)\ \ \ 0$	DM1		Attempt factorisation or use formula Both correct
	$k\ \ \ 3$ or 11	A1	[4]	NB Alternative methods for **(ii)** possible
10	**(i)** B $(0,1)$ C $(4,3)$	B1, B1	[2]	If B0B0 then SCB1 for both $y\ \ \ 1$ & $x\ \ \ 4$
	(ii) $\frac{\delta y}{\delta x}\ \ \ \frac{1}{2}\times 2(1+2x)^{\frac{1}{2}}$	M1A1		$^{\frac{1}{2}}$ required & at least one of $\frac{1}{2}\times 2$ for M1
	Grad. of normal $\ \ \ 3$	B1		
	$y\ \ 3\ \ \ 3(x\ \ \ 4)$ or $y\ \ \ 3x+15$ oe	B1√	[4]	Ft only from *their* C
	(iii) $y^2\ \ \ 1+2x \Rightarrow x\ \ \ \frac{1}{2(y^2\ \ \ 1)}$ SOI	B1		$\int x^2 \delta y$, square $\frac{1}{2}(y^2\ \ \ 1)$ & attempt int^n
	$(\pi)\times\frac{1}{4}\times\int(y^4\ \ \ 2y^2+1)\delta y$	M1		
	$(\pi)\times\frac{1}{4}\left[\frac{y^5}{5}\ \ \ \frac{2y^3}{3}+y\right]$	A1		Apply limits $0 \rightarrow$ *their* 1 (from *their B*)
	$(\pi)\times\frac{1}{4}\left[\frac{1}{5}\ \ \ \frac{2}{3}+1\right]$	DM1		cao SCB1 for $\int y^2 \delta x \rightarrow \frac{\pi}{4}$ (scores 1/5)
	$\frac{2}{15}\pi$	A1	[5]	
11	**(i)** $2(x\ \ \ 2)^2+2$	B 1, B1, B1	[3]	For 2, $\ \ 2$, 2
	(ii) $2 \le f(x) \le 10$ oe	B1	[1]	Allow $<$ etc. Ignore notation
	(iii) $2 \le x \le 10$	B1√	[1]	Ft from part **(ii)**. Ignore notation
	(iv) $f(x)$: ≈ half parabola from $(0,10)$ to $(2,2)$	B1		Or from int with y axis to int with *their* $y\ \ \ x$
	$g(x)$: line through 0 at $\approx 45°$	B1		
	$f^{\ 1}(x)$: reflection of *their* $f(x)$ in $g(x)$	B1√		
	Everything totally correct	B1	[4]	

	(v) $(x \quad 2)^2 \quad \frac{1}{2}(y \quad 2)$	M1		Allow +√ or √. Dep on final ans as f^n of x
	$x \quad 2 \pm \sqrt{\frac{1}{2}(y \quad 2)}$	M1		
	$f^{\ 1}(x) \quad 2 \quad \sqrt{\frac{1}{2}(x \quad 2)}$	A1	[3]	cao

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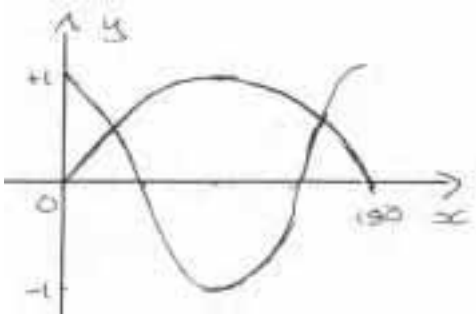
1	**(i)** $(2 \quad y)^5 = 32 \quad 80y + 80y^2$ **(ii)** $(2 \quad (2x \quad x^2))^5$ "y" = "$2x \quad x^2$" $\rightarrow 80 + 320 = 400$	B2,1 [2] M1 M1 A1 [3]	−1 for each error. Accept 2^5. Allow for $y = 2x + x^2$ Needs to consider exactly 2 terms. CO – accept $400x^2$, accept full expansion if $400x^2$ is part of it.
2	$f: x \mapsto 3x + a$, $g: x \mapsto b \quad 2x$ **(i)** $f^2(x) = 3(3x + a) + a$ $f^2(2) = 18 + 4a = 10 \rightarrow a = -2$ $g^{-1}(x) = \frac{b \quad x}{2} \rightarrow \frac{b \quad 2}{2} = 3 \quad b = 8$ or $g(3) = 2 \rightarrow b - 6 = 2 \quad b = 8$ **(ii)** $fg(x) = 3(b \quad 2x) + a$ $= 22 - 6x$	 B1 B1 M1 A1 [4] M1 A1√ [2]	 Must be correct – unsimplified ok co Correct method leading to a value for b co Must be fg not gf. √ on a and b $(3b + a - 6x)$ must be two term answer.
3	$\overrightarrow{OA} \quad 5\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \overrightarrow{OB} \quad 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k}$ **(i)** $\overrightarrow{OA}.\overrightarrow{OB} = 10 + 7 + 2p$ $= 0 \rightarrow p = -8½$ **(ii)** $\mathbf{AB} = -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ Modulus = √(9+36+4) Magnitude 28 → 28 ×unit vector → $-12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}$.	M1 DM1 A1 [3] B1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ ....=0 co co (accept negative) For modulus Scales by ×28 ÷ modulus. Co – could leave as "4 × …".
4	**(i)** $y^2 + 2x \quad 13$, $2y + x \quad 8$ $\rightarrow y^2 \quad 4y + 3 \quad 0$, $x^2 \quad 8x + 12 \quad 0$ → (2, 3) and (6, 1) **(ii)** Removes $x \rightarrow y^2 + 2(k \quad 2y) \quad 13$ Uses $b^2 \quad 4ac$ on "quadratic = 0) → $k = 8½$ or $\frac{dy}{dx} = -½ = \frac{1}{y} \rightarrow y=2, x=4½, k=8½$	M1 A1 DM1 A1 [4] M1 DM1 A1 [3]	Complete elimination of x or y co (allow multiples) – needs 3 terms Solution of quadratic = 0 Needs all 4 coordinates. Complete elimination of x or y. Use of discriminant =0, <0 or >0 Co (M1 equating m of line and curve M1 x to y A1 for k)

5	**(i)** 	B1 B1 B1 [3]	y = sin$x$ (0,0). (π,0) + curve y = cos2x One full cycle. y = cos2x starts and finishes at (0, 1) and oscillates between −1 and +1. Do not penalise graphs from 0 to 360.
	(ii) Evidence of sin 30 = cos 60 = 0.5 Other root is 150°	B1 B1 [2]	co co
	(iii) $0 \leq x < 30$ and $150 < x \leq 180$ (x < 30 or x > 150 ok)	B1 B1√ [2]	Condone < or ≤ throughout
6	**(i)** D to $AX = 6\sin\frac{\pi}{3} = 6\sqrt{3}\div 2$ E to $AX = 10\sin\theta$ Equate these → $\theta = \sin^{-1}\frac{3\sqrt{3}}{10}$.	B1 B1 B1 [3]	co Needs $-\sqrt{3}\div 2$ not just $3\sqrt{3}$. co Correct method. ag. Use of decimals loses this B mark.
	(ii) Arc $DX = 6.\frac{1}{3}\pi = 2\pi$ Arc $EX = 10\times 0.5464 = 5.464$ Horizontal steps = $6\cos\frac{1}{3}\pi$ and $10\cos\theta$ $DE = 10 + 6 - 6\cos\frac{1}{3}\pi - 10\cos\theta$ Perimeter = arc DX + arc BX + DE → 16.20	B1 M1 M1 M1 A1 [5]	co Use of $s=r\theta$ radians. Attempt at both steps needed Full method for DE. Co – must be exactly 16.20, not more or less places.
7	$\frac{dy}{dx} \quad 5 \quad \frac{8}{x^2}$, Normal $3y + x \quad 17$ **(i)** Gradient of line = −⅓ $\frac{dy}{dx} \quad 3 \rightarrow \quad x = 2,\ y = 5$	B1 M1 DM1 A1 [4]	co Use of $m_1m_2 = -1$ DM1 solution. A1 co.
	(ii) $y \quad 5x + 8x^{-1}(+c)$ Uses (2, 5) → $c = -9$	B1 B1 M1 A1 [4]	co.co. doesn't need +c. Use of +c following integration. co.

8 $y = \sqrt{8x - x^2}$		
(i) $\frac{dy}{dx} = \frac{1}{2}(8x - x^2)^{\frac{1}{2}} \times (8 - 2x)$	B1	B1 for everything but ×(8-2x)
	B1	B1 for × (8−2x), even if B0
$= 0$ when $x = 4$.	M1	Sets to 0 + attempt at solution.
→ (4, 4)	A1 [4]	Co – A0 if fortuitous because of B0 earlier.
(ii) $y = 0$ when $x = 0$ or 8	B1	Anywhere
$\text{Vol} = \pi \int (8x - x^2)dx$		
$= \pi \left[4x^2 - \frac{x^3}{3}\right]$	B2,1	−1 for each error (not including π)
$\rightarrow \frac{256\pi}{3}$	B1 [4]	co
9 **(i)** Gradient of $AC = ½$	B1	co
Gradient of $BD = -2$	M1	Use of $m_1 m_2 = -1$ with AC
Eqn of BD is $y - 6 = -2(x - 3)$	M1	Correct formula for straight line
Eqn of AC is $y + 1 = \frac{1}{2}(x + 1)$	M1	Solution.
Sim eqns → $M(5, 2)$	A1	co
Vector move – or midpoint back		
→ $D(7, -2)$	M1 A1√ [7]	Correct method. √ on M.
(ii) Ratio of $AM : MC = \sqrt{45} : \sqrt{20}$	M1	Correct distance formula.
or Vector step → 3 : 2	A1 [2]	Looks at the two x or y steps. Must be numerical, 1.5 ok, not as roots
10 **(a)** $a = -15$, $n = 25$		
(i) Use of S_n → $d = 3$.	M1 A1 [2]	Must be correct formula. co
(ii) Last term = $a + 24d$	M1	Must be $a + 24d$
→ 57	A1√	√ for his d.
(or $525 = ½ \times 25 \times (-15 + l) \rightarrow l = 57$)	[2]	
(iii) Positive terms are 3,6, ...57		
Either $a = 0$ or 3, $n = 19$ or 20	M1	Correct use of formula for S_n.
Use of S_{19} or S_{20}		
→ 570	A1 [2]	co
(b) $r = 1.05$	B1	In either part **(i)** or **(ii)**.
(i) 11^{th} term = ar^{10} = \$6516 or \$6520	B1 [2]	co
(ii) $S_{11} = \frac{4000 \times (1.05^{11} - 1)}{.05}$	M1	Correct sum formula with their r.
= \$56800 or (56827)	A1 [2]	co

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1	$k^2 \times \left(\frac{1}{3(x)}\right)^2$ ×10 (or correct factorials) $10 \times k^2 \times \frac{1}{9}$ $30 \Rightarrow k$ 3	B2 B1 [3]	B1 for 2/3 terms correct cao
2	**(i)** 5[8 + 9 × 4] 220 **(ii)** $\frac{4\left(2^{10} \quad 1\right)}{2 \quad 1}$ 4092	M1 A1 [2] M1 A1 [2]	Use correct formula with a=4, d=4 Use correct formula with a=4, r=2 or ½ 4090 without 4092 A0
3	**(i)** $2x^5 + 3x^2 = 2x \Rightarrow 2x^5 + 3x^2 - 2x = 0$ $[x(2x]^4 + 3x^2 - 2) = 0$ $2x^4 + 3x^2 - 2 = 0$ **(ii)** $(x^2 + 2)(2x^2 - 1) = 0$ x = ± $1/\sqrt{2}$ only $\left(1/\sqrt{2}, 2/\sqrt{2}\right), \left(1/\sqrt{2}, 2/\sqrt{2}\right)$	M1 A1 [2] M1 A1 A1 [3]	First line essential **AG** Factorising needed for A1 Reasonable attempt at solving a quadratic in x^2 For a correct pair of solutions, either 2 x's or 1 x and 1 y SC (±0.707, ±1.41) AWRT B1
4	**(i)** $10^2 \sin 0.8 = 71.7$ **(ii)** sector(s) = (2) × ½ × 10^2 × 0.8 = (2) × 40 Total area = 80 **(iii)** arc(s) = (2) × 10 × 0.8 16+20 = 36	M1A1 [2] M1 A1 [2] M1 A1 [2]	Completely correct method for a triangle Correct formula used for a sector Correct formula used for an arc
5	**(i)** $3\cos^2 x + 8\cos x + 4 = 0$ $(3\cos x + 2)(\cos x + 2) = 0$ cosx = $\frac{2}{3}$ **(ii)** cos(θ + 70) = $\frac{2}{3}$, θ = 61.8 θ + 70 = 131.8 (or 228.2) θ = 158.2	M1 M1 A1 [3] M1 A1 M1 A1 [4]	Use of $c^2 + s^2 = 1$ Factorising, formula or completing the square needed **AG** Ignore cos$x$ = –2 also offered SC B1 if –2/3 and –2 seen

6	**(i)**	Scalar product = 15 8+3 $10 = \|\mathbf{OA}\|\,\|\mathbf{OB}\| \cos\theta$ $\|\mathbf{OA}\| = \sqrt{26}$, $\|\mathbf{OB}\| = \sqrt{38}$ Angle *BOA* = 71.4 or 71.5 or 1.25 radians	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct magnitude for either Linking everything correctly cao
	(ii)	$\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$ or $\mathbf{b}+\frac{1}{2}(\mathbf{a}-\mathbf{b})$ or $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ $-2\mathbf{b}$ + *their* **c** oe $-6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$	M1 M1 A2,1,0 [4]	
7	**(i)**	$y = m(x - 2)$ oe	B1 [1]	Accept $y = mx + c$, $c = -2m$
	(ii)	$x^2 - 4x + 5 = mx - 2m \Rightarrow x^2 - x(4 + m) + 5 + 2m = 0$ $(4 + m)^2 - 4(5 + 2m) = 0 \Rightarrow m^2 - 4 = 0$ $m = \pm 2$ $m = 2 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow x = 3$ $m = -2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$ (3, 2), (1, 2)	M1 DM1 A1 DM1 A1 A1 [6]	Apply $b^2 - 4ac$ Substitute their m and attempt to solve for x Allow for a pair of x values or 1 *x* and 1 y.
	OR	$\mathbf{m=2^x-4}$ $\mathbf{y=m^x-2m}$, $\mathbf{y= x2 - 4x + 5}$	M1 M1 M1 A1 A1 A1	**Eliminating 2 variables from 3 equations.** **Obtaining a quadratic in x or y.** **Solving their quadratic correctly.** A pair of x values or 1 *x* and 1 y.. m=2,–2 also needed for final mark.
	(iii)	$(x - 2)^2 + 1$, (2, 1)	B1,B1 [2]	

8	**(i)** $f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0$ $k = -2$ $(x - 3)(x + 2) = 0$ $x = -2$, (Allow also = 3)	M1 A1 M1 A1 [4]	AG
	(ii) $f''(x) = 4x \quad 2$ $f''(3) > 0$ hence min at P $f''(\ 2) < 0$ hence max at Q	B1 B1 [2]	3 min, –2 max independent of f″(x) Accept anywhere in question
	(iii) $f(x) \quad \frac{2}{3}x^3 \quad x^2 \quad 12x \ (+c)$ Sub (3, 10) $\rightarrow -10 = 18 - 9 - 36 + c$ $c = 17$	B2,1,0 M1 A1 [4]	Dependent on c present Condone y = , or equation =
9	**(i)** $f^{-1}(x) \quad \frac{1}{2}x \quad \frac{3}{2}$ $2x + 3 \quad \frac{1}{2}x \quad \frac{3}{2} \Rightarrow x \quad 3$	B1 M1A1 [3]	
	(ii) 2 lines approximately correct, reflected in $y=x$ & meeting at (3, 3)	B3,2,1,0 [3]	Can be implied by graph or in writing. Ignore lines extended
	(iii) $gf(x) = (2x + 3)^2 - 6(2x + 3)$ $4x^2 - 9$ $4x^2 - 9 \le 16 \Rightarrow x^2 \le \frac{25}{4}$ $\frac{5}{2} \le x \le 0$	M1 A1 M1 A1A1 [5]	Solving any quadratic to do with f and g ≤16, to x = Condone < and >

10	**(i)** $\int(x+1)^{\frac{1}{2}} \quad (x+1)$ or $\int(y^2 \quad 1) \quad (y \quad 1)$	M1	Dealing with line as a triangle or integral with correct limits.
	$\frac{2}{3}(x+1)^{\frac{3}{2}} \quad \frac{1}{2}x^2 \quad x$ or $\frac{1}{3}y^2 \quad \frac{1}{2}y^2$	M1A1	Attempt at integral of curve.
	$\frac{2}{3} \quad \left(0 \quad \frac{1}{2}+1\right)$ or $\frac{1}{3} \quad \frac{1}{2}$	DM1	Applying limits $-1 \rightarrow 0$ or $0 \rightarrow 1$ to curve
	$\frac{1}{6}$	A1 [5]	π included loses last mark.
	(ii) $V_1 \quad (\pi)\int(y^2 \quad 1)^2 \quad (\pi)\int y^4 \quad 2y^2+1$	M1	
	$(\pi)\left[\frac{y^5}{5} \quad \frac{2y^2}{3}+y\right]$	A1	Attempt at $\int x^2 \quad dy$ for curve
	$(\pi)\left[\frac{1}{5} \quad \frac{2}{3}+1\right]$	DM1	Apply limits $0 \rightarrow 1$
	$V_1 \quad \frac{8}{15(\pi)}$ or 0.533(π) (AWRT)	A1	
	or $(\pi)\left[y\uparrow 3/3 \quad y\uparrow 2+y\right]$	M1	
	$V_2 \quad \frac{1}{3}\pi$	A1	Vol of cone or attempt to $\int x^2 dy$ for line
	Volume $= \frac{8}{15}\pi \quad \frac{1}{3}\pi = \frac{1}{5}\pi$ (or 0.628)	A1 [7]	
	OR $(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)$	M1	
	$(\pi)\int y^4 \quad 3y^2+2y$	M1	
	$(\pi)\left[y\uparrow 5/5 \quad y\uparrow 3+y\uparrow 2\right]$	A1,A1,A1	Attempt to $\int x^2 dy$
	$(\pi)\left[\frac{1}{5} \quad 1+1\right]$	DM1	Attempt to $\int (x_1^2 - x_2^2)$
	$\frac{1}{5}\pi$	A1	Apply limits 0→1 dependent on first M1

$\int_{-1}^{0} x+1 \quad \int_{-1}^{0} (x+1))^2$ $\left[\frac{x^2}{2}+x\right] \quad \left[\frac{x+1^3}{3}\right]$	M1	SC MR integrating about x axis
SC $=\left[(0) \quad \left(\frac{1}{2} \quad 1\right)\right] \quad \left[\frac{1}{3} \quad 0\right]$	M1	Use of –1,0 as limits
$\frac{1}{2} \quad \frac{1}{3} = \frac{1}{6}\pi \quad (0.524)$	A1	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

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9709/21 Paper 2, maximum raw mark 50

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 **EITHER** State or imply non-modular inequality $(4 - 5x)^2 < 3^2$, or corresponding equation or pair of linear equations — M1

Obtain critical values $\frac{1}{5}$ and $\frac{7}{5}$ — A1

State correct answer $\frac{1}{5} < x < \frac{7}{5}$ — A1

OR State one critical value, e.g. $x = \frac{1}{5}$, by solving a linear equation (or inequality) or from a graphical method or by inspection — B1

State the other critical value correctly — B1

State correct answer $\frac{1}{5} < x < \frac{7}{5}$ — B1 [3]

2 Integrate and obtain term of the form $k\ln(4x + 1)$ — M1

State correct term $\frac{1}{2}\ln(4x+1)$ — A1

Substitute limits correctly — M1

Use law for the logarithm of a quotient or a power — M1

Obtain given answer correctly — A1 [5]

3 Obtain derivative of the form $k \sec^2 2x$, where $k = 1$ or $k = \frac{1}{2}$ — M1

Obtain correct derivative $\sec^2 2x$ — A1

Use correct method for solving $\sec^2 2x = 4$ — M1

Obtain answer $x = \frac{1}{6}\pi$ (or 0.524 radians) — A1

Obtain answer $x = \frac{1}{3}\pi$ (or 1.05 radians) and no others in range — A1 [5]

4 Carry out recognizable solution method for quadratic in 3^x — M1

Obtain $3^x = 5$ and $3^x = 2$ — A1

Use logarithmic method to solve an equation of the form $3^x = k$, where $k > 0$ — M1

State answer 1.46 — A1

State answer 0.631 — A1 [5]

5 **(i)** Substitute $x = \frac{1}{2}$ and equate to 10 — M1

Obtain answer $a = -16$ — A1

Either show that f(3) = 0 or divide by $(x - 3)$ obtaining a remainder of zero — B1 [3]

(ii) At any stage state that $x = 3$ is a solution — B1

Attempt division by $(x - 3)$ reaching a partial quotient of $4x^2 + kx$ — M1

Obtain quadratic factor $4x^2 - 4x - 3$ — A1

Obtain solutions $x = \frac{3}{2}$ and $x = -\frac{1}{2}$ — A1

S.C. M1A1√ if value of 'a' incorrect [4]

6 **(i)** Consider sign of $x^3 - 2x^2 + 5x - 3$ at $x = 0.7$ and $x = 0.8$ M1
Complete the argument correctly with appropriate calculations A1 [2]

(ii) Rearrange equation to given equation or *vice versa* B1
State $a = 2$ and $b = 5$ B1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.74 A1
Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.735, 0.745) B1 [3]

7 **(i)** Use product rule to differentiate y M1
Obtain correct derivative in any form in t for y A1
Use $\frac{dy}{dx}$ $\frac{dy}{dt} \div \frac{dx}{dt}$ M1
Obtain given answer correctly A1 [4]

(ii) State $t = 0$ M1
State that $\frac{dy}{dx}$ 0 and make correct conclusion A1 [2]

(iii) Substitute $t = -2$ into equation for x or y M1
Obtain $(e^{6}, 4e^{2} + 3)$ A1 [2]

8 **(i)** Make relevant use of the $\cos(A + B)$ formula M1*
Make relevant use of the $\cos 2A$ and $\sin 2A$ formulae M1*
Obtain a correct expression in terms of $\cos x$ and $\sin x$ A1
Use $\sin^2 x = 1 - \cos^2 x$ to obtain an expression in terms of $\cos x$ M1(dep*)
Obtain given answer correctly A1 [5]

(ii) Replace integrand by $\frac{1}{2}\cos 3x + \frac{1}{2}\cos x$, or equivalent B1
Integrate, obtaining $\frac{1}{6}\sin 3x + \frac{1}{2}\sin x$, or equivalent B1 + B1√
Use limits correctly M1
Obtain given answer A1 [5]

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Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values –8 and 0 A1
State correct answer $x < -8$ or $x > 0$ A1

OR Obtain one critical value, e.g. $x = -8$, by solving a linear equation (or inequality) or from a graphical method or by inspection B1
Obtain the other critical value similarly B2
State correct answer $x < -8$ or $x > 0$ B1 [4]

2 Use law for the logarithm of a product, a quotient or a power M1*
Obtain $(x+1)\log 4 = (2x-3)\log 5$, or equivalent A1
Solve for x M1(dep*)
Obtain answer $x = 3.39$ A1 [4]

3 **(i)** Obtain correct derivative B1
Obtain $x = 2$ only B1 [2]

(ii) State or imply correct ordinates 0.61370..., 0.80277..., 1.22741..., 1.78112... B1
Use correct formula, or equivalent, correctly with $h = 1$ and four ordinates Ml
Obtain answer 3.23 with no errors seen Al [3]

(iii) Justify statement that the trapezium rule gives an over-estimate B1 [1]

4 State at least one correct integral B1
Use limits correctly to obtain an equation in e^{2k}, e^{4k} M1
Carry out recognizable solution method for quadratic in e^{2k} M1
Obtain $e^{2k} = 1$ and $e^{2k} = 3$ A1
Use logarithmic method to solve an equation of the form $e^{\lambda a} = b$, where $b > 0$ M1
Obtain answer $k = \frac{1}{2}\ln 3$ A1 [6]

5 **(i)** Make a recognisable sketch of a relevant graph, e.g. $y = \sin x$ or $y = \frac{1}{x}$ B1
Sketch a second relevant graph and justify the given statement B1 [2]

(ii) Consider sign of $\frac{1}{x} \quad \sin x$ at $x = 1.1$ and $x = 1.2$, or equivalent M1
Complete the argument correctly with appropriate calculations A1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.11 A1
Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.105, 1.115) B1 [3]

6 **(i)** State $\frac{dx}{dt}$ $4\sin\theta\cos\theta$ or equivalent (nothing for $\frac{dy}{dx}$ $4\sec^2\theta$) B1

Use $\frac{dy}{dx}$ $\frac{dy}{d\theta}\div\frac{dx}{d\theta}$ M1

Obtain given answer correctly A1 [3]

(ii) Substitute θ $\frac{\pi}{4}$ in $\frac{dy}{dx}$ and both parametric equations M1

Obtain $\frac{dy}{dx}$ 4 and coordinates (2, 4) A1

Form equation of tangent at their point M1

State equation of tangent in correct form $y = 4x - 4$ A1 [4]

7 **(i)** Substitute $x = -2$, equate to zero and obtain a correct equation in any form B1

Substitute $x = -1$ and equate to 12 M1

Obtain a correct equation in any form A1

Solve a relevant pair of equations for a or b M1

Obtain $a = 2$ and $b = 6$ A1 [5]

(ii) Attempt division by $x + 2$ and reach a partial quotient of $2x^2 - 7x$ M1

Obtain quotient $2x^2 - 7x + 3$ A1

Obtain linear factors $2x - 1$ and $x - 3$ A1

[Condone omission of repetition that $x + 2$ is a factor.)

[If linear factors $2x - 1$, $x - 3$ obtained by remainder theorem or inspection, award B2 + B1.]

S.C. M1A1√ if a, b not both correct [3]

8 **(i)** State $R = \sqrt{34}$ B1

Use trig formula to find α M1

Obtain $\alpha = 30.96^\circ$ with no errors seen A1 [3]

(ii) Carry out evaluation of $\cos^{-1}\left(\frac{\pm 4}{R}\right)$ ($\approx 46.6861^\circ$ or 313.3139°) M1

Obtain answer 15 .7° A1

Carry out correct method for second answer M1

Obtain answer 282.3° or 282.4° and no others in the range A1 [4]

(iii) State $3\sqrt{34}$ $(= -3R)$ B1√ [1]

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1 Obtain derivative of the form $\frac{k}{5x+1}$, where $k = 1$, 5 or $\frac{1}{5}$ M1

Obtain correct derivative $\frac{5}{5x+1}$ A1

Substitute $x = 4$ into expression for derivative and obtain $\frac{5}{21}$ A1√ [3]

2 **EITHER** State or imply non-modular inequality $(2x - 3)^2 \leqslant (3x)^2$, or corresponding equation or pair of linear equations M1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1

Obtain critical values –3 and $\frac{3}{5}$ A1

State correct answer $x \leqslant -3$ or $x \geqslant \frac{3}{5}$ A1

OR State one critical value, e.g. $x = -3$, by solving a linear equation (or inequality) or from a graphical method or by inspection B1

State the other critical value correctly B2

State correct answer $x \leqslant -3$ or $x \geqslant \frac{3}{5}$ B1 [4]

3 Use $2\ln(x + 3) = \ln(x + 3)^2$ M1

Use law for addition or subtraction of logarithms M1

Obtain correct quadratic expression in x A1

Make reasonable solution attempt at a 3-term quadratic M1

State $x = 9$ and no other solutions (condone $x = -1$ not deleted) A1 [5]

4 **(i)** State correct expression $\frac{1}{2} + \frac{1}{2}\cos 2x$, or equivalent B1 [1]

(ii) Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly M1

State correct integral $\frac{1}{2}x + \frac{1}{4}\sin 2x$, or equivalent A1

Obtain correct integral (for $\sin 2x$ term) of $\frac{1}{2}\cos 2x$ B1

Attempt to substitute limits, using exact values M1

Obtain given answer correctly A1 [5]

5 Use trig identity correctly to obtain a quadratic in $\tan 2\theta$ M1

Solve the quadratic correctly M1

Obtain $\tan 2\theta = 1$ or $-\frac{4}{5}$ A1

Obtain one correct answer A1

Carry out correct method for second answer from either root M1

Obtain remaining 3 answers from 22.5°, 112.5°, 70.7°, 160.7° and no others in the range A1

[Ignore answers outside the given range] [6]

6 **(i)** Substitute $x = 1$ or $x = -2$ and equate to zero — M1
Obtain a correct equation in any form with powers of x values calculated — A1
Obtain a second correct equation in any form — A1
Solve a relevant pair of equations for a or for b — M1
Obtain $a = 3$ and $b = -5$ — A1 [5]

(ii) Attempt division by $x^2 + x - 2$, or equivalent, and reach a partial quotient of $x^2 + kx$ — M1
Obtain partial quotient $x^2 + 2x$ — A1
Obtain $x^2 + 2x - 1$ with no errors seen — A1
S.C. M1A1√ if 'a' and/or 'b' incorrect — [3]

7 **(i)** At any stage, state the correct derivative of $e^{\frac{1}{2}x}$ — B1
Use product rule — M1
Obtain correct derivative in any form — A1
Equate derivative to 3 and obtain given equation correctly — A1 [4]

(ii) Consider sign of $2 + 6e^{\frac{1}{2}x} - x$, or equivalent — M1
Complete the argument correctly with appropriate calculations — A1 [2]

(iii) Use the iterative formula correctly at least once — M1
Obtain final answer 3.21 — A1
Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (3.205, 3.215) — B1 [3]

8 **(i)** State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent — B1

Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1

Obtain given answer correctly — A1 [3]

(ii) Equate gradient expression to –1 and rearrange — M1
Obtain $y = 2x$ — A1
Substitute into original equation to obtain an equation in x^2 (or y^2) — M1
Obtain $2x^2 - 3x - 2 = 0$ (or $y^2 - 3y - 4 = 0$) — A1
Correct method to solve their quadratic equation — M1
State answers $(-\frac{1}{2}, -1)$ and $(2, 4)$ — A1 [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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1 Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent — B1
Solve a 3-term quadratic for e^x or for u — M1
Obtain simplified solution $e^x = 3$ or $u = 3$ — A1
Obtain final answer $x = 1.10$ and no other — A1 [4]

2 *EITHER*: Use chain rule — M1
obtain $\frac{dx}{dt}$ $6\sin t \cos t$, or equivalent — A1
obtain $\frac{dy}{dt}$ $6\cos^2 t \sin t$, or equivalent — A1
Use $\frac{dy}{dx}$ $\frac{dy}{dt} \div \frac{dx}{dt}$ — M1
Obtain final answer $\frac{dy}{dx}$ $\cos t$ — A1
OR: Express y in terms of x and use chain rule — M1
Obtain $\frac{dy}{dx}$ $k(2 \quad \frac{x}{3})^{\frac{1}{2}}$, or equivalent — A1
Obtain $\frac{dy}{dx}$ $(2 \quad \frac{x}{3})^{\frac{1}{2}}$, or equivalent — A1
Express derivative in terms of t — M1
Obtain final answer $\frac{dy}{dx}$ $\cos t$ — A1 [5]

3 **(i)** *EITHER*: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ — M1
Obtain quotient $x^2 + 4x + 3$ — A1
Equate remainder of form lx to zero and solve for a, or equivalent — M1
Obtain answer $a = 1$ — A1
OR: Substitute a complex zero of $x^2 - x + 1$ in p(x) and equate to zero — M1
Obtain a correct equation in a in any unsimplified form — A1
Expand terms, use $i^2 = -1$ and solve for a — M1
Obtain answer $a = 1$ — A1 [4]
[SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B. The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]

(ii) State answer, e.g. $x = -3$ — B1
State answer, e.g. $x = -1$ and no others — B1 [2]

4 Separate variables and attempt integration of at least one side — M1
Obtain term $\ln(x + 1)$ — A1
Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ — M1
Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ — A1
Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ — M1
Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta \quad \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) — A1√
Rearrange and obtain $x \quad \sqrt{(2 \sin 2\theta)} \quad 1$, or simple equivalent — A1 [7]

5 **(i)** Make recognisable sketch of a relevant graph over the given interval B1
Sketch the other relevant graph and justify the given statement B1 [2]

(ii) Consider the sign of sec $x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent M1
Complete the argument with correct calculated values A1 [2]

(iii) Convert the given equation to sec $x = 3 - \frac{1}{2}x^2$ or work *vice versa* B1 [1]

(iv) Use a correct iterative formula correctly at least once M1
Obtain final answer 1.13 A1
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135) A1 [3]
[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]

6 **(i)** State or imply $R = \sqrt{10}$ B1
Use trig formulae to find α M1
Obtain $\alpha = 71.57°$ with no errors seen A1 [3]
[Do not allow radians in this part. If the only trig error is a sign error in $\cos(x - \alpha)$ give M1A0]

(ii) Evaluate $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. (50.7684…°) (Allow 50.7° here) B1√
Carry out an appropriate method to find a value of 2θ in $0° < 2\theta < 180°$ M1
Obtain an answer for θ in the given range, e.g. $\theta = 61.2°$ A1
Use an appropriate method to find another value of 2θ in the above range M1
Obtain second angle, e.g. $\theta = 10.4°$, and no others in the given range A1 [5]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan\theta$, $\sin 2\theta$, $\cos 2\theta$,or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]

7 **(i)** Use a correct method to express $\overrightarrow{OP}$ in terms of λ M1
Obtain the given answer A1 [2]

(ii) *EITHER*: Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of λ M1
Using the correct method for the moduli, divide scalar products by products of moduli and express cos AOP = cos BOP in terms of λ, or in terms of λ and OP M1*
OR1: Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ M1
Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express cos AOP = cos BOP in terms of λ, or λ and OP M1*
Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} \quad \dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$ A1
Solve for λ M1(dep*)
Obtain $\lambda = \frac{3}{8}$ A1 [5]
[SR: The M1* can also be earned by equating cos AOP or cos BOP to a sound attempt at cos $\frac{1}{2}$ AOB and obtaining an equation in λ. The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]
[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for OP to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]

(iii) Verify the given statement correctly B1 [1]

8 **(i)** Use any relevant method to determine a constant M1
Obtain one of the values $A = 3$, $B = 4$, $C = 0$ A1
Obtain a second value A1
Obtain the third value A1 [4]

(ii) Integrate and obtain term $-3\ln(2-x)$ B1√
Integrate and obtain term $k\ln(4+x^2)$ M1
Obtain term $2\ln(4+x^2)$ A1√
Substitute correct limits correctly in a complete integral of the form $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$ M1
Obtain given answer following full and correct working A1 [5]

9 **(i)** Use product rule M1
Obtain correct derivative in any form A1
Equate derivative to zero and solve for x M1
Obtain answer $x = e^{\frac{1}{2}}$, or equivalent A1
Obtain answer $y = -\frac{1}{2}e^{1}$, or equivalent A1 [5]

(ii) Attempt integration by parts reaching $kx^3 \ln x \pm k\int x^3.\frac{1}{x}dx$ M1*
Obtain $\frac{1}{3}x^3 \ln x \quad \frac{1}{3}\int x^2\,dx$, or equivalent A1
Integrate again and obtain $\frac{1}{3}x^3 \ln x \quad \frac{1}{9}x^3$, or equivalent A1
Use limits $x = 1$ and $x = e$, having integrated twice M1(dep*)
Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent A1 [5]
[SR: An attempt reaching $ax^2(x \ln x - x) + b\int 2x(x \ln x \quad x)\,dx$ scores M1. Then give the first A1 for $I = x^2(x \ln x - x) - 2I + \int 2x^2\,dx$, or equivalent.]

10 **(a)** *EITHER*: Square $x + iy$ and equate real and imaginary parts to 1 and $2\sqrt{6}$ respectively M1*
Obtain $x^2 - y^2 = 1$ and $2xy = 2\sqrt{6}$ A1
Eliminate one variable and find an equation in the other M1(dep*)
Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent A1
Obtain answers $\pm(\sqrt{3} \quad i\sqrt{2})$ A1 [5]

OR: Denoting $1 \quad 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$ and find values of R and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$ M1*
Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta)$, and $\cos\theta \quad \frac{1}{5}$ or $\sin\theta \quad \frac{2\sqrt{6}}{5}$ or $\tan\theta \quad 2\sqrt{6}$ A1
Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$ M1(dep*)
Obtain $\cos\frac{1}{2}\theta \quad \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta \quad \pm\sqrt{\frac{2}{5}}$, or equivalent A1
Obtain answers $\pm(\sqrt{3} \quad i\sqrt{2})$, or equivalent A1
[Condone omission of ± except in the final answers.]

(b) Show point representing 3i on a sketch of an Argand diagram B1
Show a circle with centre at the point representing 3i and radius 2 B1√
Shade the interior of the circle B1√
Carry out a complete method for finding the greatest value of arg z M1
Obtain answer 131.8° or 2.30 (or 2.3) radians A1 [5]
[The f.t. is on solutions where the centre is at the point representing –3i.]

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9709/32 Paper 3, maximum raw mark 75

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4 Separate variables and attempt integration of at least one side — M1
Obtain term $\ln(x + 1)$ — A1
Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$ — M1
Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ — A1
Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ — M1
Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta \quad \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$) — A1√
Rearrange and obtain $x \quad \sqrt{(2\sin 2\theta)} \quad 1$, or simple equivalent — A1 [7]

5 **(i)** Make recognisable sketch of a relevant graph over the given interval — B1
Sketch the other relevant graph and justify the given statement — B1 [2]

(ii) Consider the sign of sec $x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent — M1
Complete the argument with correct calculated values — A1 [2]

(iii) Convert the given equation to sec $x = 3 - \frac{1}{2}x^2$ or work *vice versa* — B1 [1]

(iv) Use a correct iterative formula correctly at least once — M1
Obtain final answer 1.13 — A1
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135) — A1 [3]
[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]

6 **(i)** State or imply $R = \sqrt{10}$ — B1
Use trig formulae to find α — M1
Obtain $\alpha = 71.57°$ with no errors seen — A1 [3]
[Do not allow radians in this part. If the only trig error is a sign error in $\cos(x - \alpha)$ give M1A0]

(ii) Evaluate $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. (50.7684…°) (Allow 50.7° here) — B1√
Carry out an appropriate method to find a value of 2θ in $0° < 2\theta < 180°$ — M1
Obtain an answer for θ in the given range, e.g. $\theta = 61.2°$ — A1
Use an appropriate method to find another value of 2θ in the above range — M1
Obtain second angle, e.g. $\theta = 10.4°$, and no others in the given range — A1 [5]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan\theta$, $\sin 2\theta$, $\cos 2\theta$,or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]

7 **(i)** Use a correct method to express $\overrightarrow{OP}$ in terms of λ M1
Obtain the given answer A1 [2]

(ii) *EITHER*: Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of λ M1
Using the correct method for the moduli, divide scalar products by products of moduli and express cos AOP = cos BOP in terms of λ, or in terms of λ and OP M1*
OR1: Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ M1
Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express cos AOP = cos BOP in terms of λ, or λ and OP M1*
Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} \quad \dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$ A1
Solve for λ M1(dep*)
Obtain $\lambda = \frac{3}{8}$ A1 [5]
[SR: The M1* can also be earned by equating cos AOP or cos BOP to a sound attempt at cos $\frac{1}{2}$ AOB and obtaining an equation in λ. The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]
[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for OP to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]

(iii) Verify the given statement correctly B1 [1]

8 **(i)** Use any relevant method to determine a constant M1
Obtain one of the values $A = 3$, $B = 4$, $C = 0$ A1
Obtain a second value A1
Obtain the third value A1 [4]

(ii) Integrate and obtain term $-3\ln(2-x)$ B1√
Integrate and obtain term $k\ln(4+x^2)$ M1
Obtain term $2\ln(4+x^2)$ A1√
Substitute correct limits correctly in a complete integral of the form $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$ M1
Obtain given answer following full and correct working A1 [5]

9 **(i)** Use product rule M1
Obtain correct derivative in any form A1
Equate derivative to zero and solve for x M1
Obtain answer $x = e^{\frac{1}{2}}$, or equivalent A1
Obtain answer $y = -\frac{1}{2}e^{1}$, or equivalent A1 [5]

(ii) Attempt integration by parts reaching $kx^3 \ln x \pm k\int x^3.\frac{1}{x}dx$ M1*
Obtain $\frac{1}{3}x^3 \ln x \quad \frac{1}{3}\int x^2\,dx$, or equivalent A1
Integrate again and obtain $\frac{1}{3}x^3 \ln x \quad \frac{1}{9}x^3$, or equivalent A1
Use limits $x = 1$ and $x = e$, having integrated twice M1(dep*)
Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent A1 [5]
[SR: An attempt reaching $ax^2(x \ln x - x) + b\int 2x(x \ln x \quad x)\,dx$ scores M1. Then give the first A1 for $I = x^2(x \ln x - x) - 2I + \int 2x^2\,dx$, or equivalent.]

10 **(a)** *EITHER*: Square $x + iy$ and equate real and imaginary parts to 1 and $2\sqrt{6}$ respectively M1*
Obtain $x^2 - y^2 = 1$ and $2xy = 2\sqrt{6}$ A1
Eliminate one variable and find an equation in the other M1(dep*)
Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent A1
Obtain answers $\pm(\sqrt{3} \quad i\sqrt{2})$ A1 [5]
OR: Denoting $1 \quad 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$ and find values of R and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$ M1*
Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta)$, and $\cos\theta \quad \frac{1}{5}$ or $\sin\theta \quad \frac{2\sqrt{6}}{5}$ or $\tan\theta \quad 2\sqrt{6}$ A1
Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$ M1(dep*)
Obtain $\cos\frac{1}{2}\theta \quad \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta \quad \pm\sqrt{\frac{2}{5}}$, or equivalent A1
Obtain answers $\pm(\sqrt{3} \quad i\sqrt{2})$, or equivalent A1
[Condone omission of ± except in the final answers.]

(b) Show point representing 3i on a sketch of an Argand diagram B1
Show a circle with centre at the point representing 3i and radius 2 B1√
Shade the interior of the circle B1√
Carry out a complete method for finding the greatest value of arg z M1
Obtain answer 131.8° or 2.30 (or 2.3) radians A1 [5]
[The f.t. is on solutions where the centre is at the point representing –3i.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

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UNIVERSITY of CAMBRIDGE
International Examinations

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**Either** Obtain correct unsimplified version of x or x^2 term in expansion of $(2 + x)^2$ or $(1 + \frac{1}{2}x)^2$	M1	
	Correct first term 4 from correct work	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	
	Or Differentiate and evaluate f(0) and f′ (0) where f″ $(x) = k(2+x)^3$	M1	
	State correct first term 4	B1	
	Obtain $-4x$	A1	
	Obtain $+ 3x^2$	A1	[4]
2	Use correct quotient or product rule or equivalent	M1	
	Obtain $\frac{(1+e^{2x}).2e^{2x} \quad e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = \ln 3$ into attempt at first derivative and show use of relevant logarithm property at least once in a correct context	M1	
	Confirm given answer $\frac{9}{50}$ legitimately	A1	[4]
3	**(i)** State or imply $R = 17$	B1	
	Use correct trigonometric formula to find α	M1	
	Obtain 61.93° with no errors seen	A1	[3]
	(ii) Evaluate $\cos^{-1}\frac{12}{R}$ $(= 45.099)$	M1	
	Obtain answer 107.0°	A1	
	Carry out correct method for second answer	M1	
	Obtain answer 16.8° and no others between 0° and 360°	A1	[4]

4	(i)	Separate variables and attempt integration on both sides	M1*	
		Obtain $2N^{0.5}$ on left-hand side or equivalent	A1	
		Obtain $-60e^{0.02t}$ on right-hand side or equivalent	A1	
		Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{0.02t}$	DM1*	
		Obtain $2N^{0.5} = -60e^{0.02t} + 80$ or equivalent	A1	
		Conclude with $N = (40 - 30e^{0.02t})^2$ or equivalent	A1	[6]
	(ii)	State number approaches 1600 or equivalent, following expression of form $(c + de^{0.02t})^n$	B1√	[1]
5	(i)	**Either** Use integration by parts and reach an expression $kx^2 \ln x \pm n\int x^2 . \frac{1}{x} dx$	M1	
		Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Or Use Integration by parts and reach an expression $kx(x\ln x - x) \pm m\int x\ln x - x dx$	M1	
		Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Substitute limits correctly and equate to 22, having integrated twice	DM1*	
		Rearrange and confirm given equation $a = \sqrt{\frac{87}{2\ln a \quad 1}}$	A1	[5]
	(ii)	Use iterative process correctly at least once	M1	
		Obtain final answer 5.86	A1	
		Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)	A1	
		(6 → 5.8030 → 5.8795 → 5.8491 → 5.8611 → 5.8564)		[3]

6	**(i)**	Use correct method for finding modulus of their w^2 or w^3 or both	M1	
		Obtain $\lvert w^2 \rvert = 2$ and $\lvert w^3 \rvert = 2\sqrt{2}$ or equivalent	A1	
		Use correct method for finding argument of their w^2 or w^3 or both	M1	
		Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$	A1ft	[4]
	(ii)	Obtain centre $-\frac{1}{2} - \frac{1}{2}i$ (their w^2)	B1ft	
		Calculate the diameter or radius using $\lvert w - w^2 \rvert$ w21 or right-angled triangle or cosine rule or equivalent	M1	
		Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent	A1	
		Obtain $\lvert z + \frac{1}{2} + \frac{1}{2}i \rvert = \frac{1}{2}\sqrt{10}$ or equivalent	A1ft	[4]
7	**(i)**	Substitute $x = \frac{1}{2}$ and equate to zero		
		or divide by $(2x - 1)$, reach $\frac{a}{2}x^2 + kx + ...$and equate remainder to zero		
		or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c		
		or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B	M1	
		Obtain $a = 2$	A1	
		Attempt to find quadratic factor by division or inspection or equivalent	M1	
		Obtain $(2x - 1)(x^2 + 2)$	A1cwo	[4]
	(ii)	State or imply form $\frac{A}{2x - 1} + \frac{Bx + C}{x^2 + 2}$, following factors from part **(i)**	B1√	
		Use relevant method to find a constant	M1	
		Obtain $A = -4$, following factors from part **(i)**	A1√	
		Obtain $B = 2$	A1	
		Obtain $C = 5$	A1	

8	**(i)**	Differentiate y to obtain $3\sin^2 t \cos t - 3\cos^2 t \sin t$ o.e.	B1	
		Use $\frac{dy}{dx}$ $\frac{dy}{dt} / \frac{dt}{dx}$	M1	
		Obtain given result $-3\sin t \cos t$	A1cwo	[3]
	(ii)	Identify parameter at origin as $t = \frac{3}{4}\pi$	B1	
		Use $t = \frac{3}{4}\pi$ to obtain $\frac{3}{2}$	B1	[2]
	(iii)	Rewrite equation as equation in one trig variable e.g. $\sin 2t = \frac{2}{3}$, $9\sin^4 x - 9\sin^2 x + 1 = 0$, $\tan^2 x + 3\tan x + 1 = 0$	B1	
		Find at least one value of t from equation of form $\sin 2t = k$ o.e.	M1	
		Obtain 1.9	A1	
		Obtain 2.8 and no others	A1	[4]

9	**(i)**	Calculate scalar product of direction of l and normal to p	M1	
		Obtain $4 \times 2 + 3 \times (-2) + (-2) \times 1 = 0$ and conclude accordingly	A1	[2]
	(ii)	Substitute $(a, 1, 4)$ in equation of p and solve for a	M1	
		Obtain $a = 4$	A1	[2]
	(iii)	**Either**		
		Attempt use of formula for perpendicular distance using $(a, 1, 4)$	M1	
		Obtain at least $\frac{2a \quad 2+4 \quad 10}{\sqrt{4+4+1}} \quad 6$	A1	
		Obtain $a = 13$	A1	
		Attempt solution of $\frac{2a \quad 8}{3} \quad 6$	M1	
		Obtain $a = -5$	A1	
		Or		
		Form equation of parallel plane and substitute $(a, 1, 4)$	M1	
		Obtain $\frac{2a+2}{3} \quad \frac{10}{3} = 6$	A1	
		Obtain $a = 13$	A1	
		Solve $\frac{2a+2}{3} \quad \frac{10}{3} \quad 6$	M1	
		Obtain $a = -5$	A1	
		Or		
		State a vector from a pt on the plane to $(a, 1, 4)$ *e.g.* $\begin{pmatrix} a \quad 5 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} a \\ 1 \\ 6 \end{pmatrix}$	B1	
		Calculate the component of this vector in the direction of the unit normal and equate to 6 : $\frac{1}{3}\begin{pmatrix} a \quad 5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad 6$	M1	
		Obtain $a = 13$	A1	
		Solve $\frac{1}{3}\begin{pmatrix} a \quad 5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad 6$	M1	
		Obtain $a = -5$	A1	

		Or		
		State or imply perpendicular line $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$	B1	
		Substitute components for p and solve for μ	M1	
		Obtain $\mu \quad \frac{8 \quad 2a}{9}$	A1	
		Equate distance between $(a, 1, 4)$ and foot of perpendicular to ±6	M1	
		Obtain $\frac{3(8 \quad 2a)}{9}$ ±6 or equivalent and hence –5 and 13	A1	[5]
10	**(i)**	State or imply $\frac{du}{dx}$ $\sec^2 x$	B1	
		Express integrand in terms of u and du	M1	
		Integrate to obtain $\frac{u^{n+1}}{n+1}$ or equivalent	A1	
		Substitute correct limits correctly to confirm given result $\frac{1}{n+1}$	A1	[4]
	(ii)	**(a)** Use $\sec^2 x = 1 + \tan^2 x$ twice	M1	
		Obtain integrand $\tan^4 x + \tan^2 x$	A1	
		Apply result from part **(i)** to obtain $\frac{1}{3}$	A1	[3]
		Or Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)**	M1	
		Obtain $\int u^2 \mathrm{d}u$	A1	
		Apply limits correctly and obtain $\frac{1}{3}$	A1	
		(b) Arrange, perhaps implied, integrand to $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$	B1	
		Attempt application of result from part **(i)** at least twice	M1	
		Obtain $\frac{1}{8} + \frac{4}{6} + \frac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent	A1	[3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/41 Paper 4, maximum raw mark 50

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1	d = 2 × 8	B1		
	[25 × 16cos20]	M1		For using WD = Fdcosα
	Work done is 376 J	A1	3	
2		M1		For applying Newton's second law to either particle (3 terms)
	0.65g – T = 0.65a and T – 0.35g = 0.35a	A1		Accept (0.65 – 0.35)g = (0.65 + 0.35)a as an alternative to one of these equations
		M1		For solving for T
	Tension in the string is 4.55 N	A1		
	Magnitude of resultant is 9.1 N	B1ft	5	
3	**(i)** **(a)** [2 × 12cos40 – 15cos50]	M1		For resolving in direction *AB*
	Component is 8.74 N	A1		
	(b) Component is 11.5 N	B1	3	
	(ii) Magnitude is 14.4 N or direction is 52.7° (or 0.920^c) anticlockwise from **i** dir'n	M1		For using $R^2 = X^2 + Y^2$ or $\tan\theta = Y/X$
		A1		
	Direction is 52.7° (or 0.920^c) anticlockwise from **i** dir'n or magnitude is 14.4 N	B1	3	
4	**(i)** $1.76 = 0.8u + 0.32a$	M1		For using $s = ut + \frac{1}{2}at^2$ for *AB*
		A1		
	$[1.76 + 2.16 = (0.8 + 0.6)u + \frac{1}{2}(0.8 + 0.6)^2a$ or $2.16 = (u + 0.8a)0.6 + \frac{1}{2}0.6^2a]$	M1		For using $s = ut + \frac{1}{2}at^2$ for *AC* or $v = u + at$ for *AB* and $s = ut + \frac{1}{2}at^2$ for *BC*
	$3.92 = 1.4u + 0.98a$ or $2.16 = 0.6u + 0.66a$	A1		
	$u = 1.4$ and $a = 2$	M1		For solving for *u* and *a*
		A1	6	
	(ii) $[2 = 10\sin\theta]$	M1		For using $a = g\sin\theta$
	$\theta = 11.5$	A1	2	
5	**(i)** $F = 12\cos\alpha$	B1		
		M1		For resolving forces vertically
	$R_1 = 2g + 12\sin\alpha$	A1		
	$[12 \times 0.8 \leq \mu(2g + 12 \times 0.6)]$	M1		For using $F_1 \leq \mu R$
	$\mu \geq 9.6/27.2 = 6/17$	A1	5	AG

	(ii)	$12\cos\alpha > \mu R_2$	B1		
		$R_2 = 2g - 12 \times 0.6$	B1		
		$\mu < 9.6/12.8 = 3/4$	B1	3	
6	**(i)**	PE gain = $1200g \times 45$	B1		
		WD = $1200g \times 45 + 360\,000$	M1		For WD by car's engine = PE gain + WD against resistance
		Work done is 900 000 J or 900 kJ	A1	3	
	(ii)	WD against resistance = 360 × sin5/sin1 (kJ) or {360000 ÷ (45/sin5°)} × (45/sin1°) (J) or 697.24... × 2578.44... (J) or 1798 (kJ)	B1		
		KE gain = 1660 + 540 – 1798	B1ft		Accept 1660 + 540 – 1800
		[$402000 = \frac{1}{2}1200(v^2 - 225)$]	M1		For using KE gain = ½ m($v^2 - 15^2$)
		Speed is 29.9 ms ^{1}	A1	4	AG
	(iii)	$\frac{P_B}{P_C}\left(\frac{DF_B}{DF_C}\right) \times \frac{v_B}{v_C} = 1.5 \times 15/29.9$	M1		For using P = Fv
			A1		
		Ratio is 0.75	A1	3	
7	**(i)**	$v(100) = 0.16 \times 1000 - 0.016 \times 10000 = 0$	B1	1	AG
	(ii)	$a = 1.5 \times 0.16t^{\frac{1}{2}} \quad 0.032t$	M1		For using $a = dv/dt$
			A1		
		[$t^{\frac{2}{3}} = 0.24/0.032 \rightarrow t = 56.25 \rightarrow$ $v_{max} = 0.16 \times 421.875 - 0.016 \times 3164.0625$]	M1		For solving a = 0 and subst into $v(t)$
		Maximum speed is 16.9 ms ^{1} (or $16\frac{7}{8}$ ms ^{1})	A1	4	
	(iii)	$s = 2/5 \times 0.16t^{\frac{5}{2}} - 0.016t^3/3$	M1		For using $s = \int v\,dt$
			A1		
		Distance is 1070 m	A1	3	
	(iv)	$\frac{1}{3}t^{\frac{1}{2}}(0.192 \quad 0.016\sqrt{t}) = 0$	M1		For attempting to solve $s(t) = 0$
		Value of t is 144	A1	2	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/42 Paper 4, maximum raw mark 50

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International Examinations

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1	**(i)**	F = 720/12	B1		
		[F – R = 75 × 0.16]	M1		For use of Newton's second law
		R = 48	A1	3	
	(ii)	[720/v > 48]	M1		For using P/v – R = ma and $a > 0 \rightarrow P/v > R$
		v < 15 i.e. speed is less than 15 ms ^{1}	A1	2	
2	**(i)**	F = 0.2 × 6g cos8	B1		
		[6g sin8 – F = 6a]	M1		For use of Newton's second law
		Deceleration is 0.589 ms ^{2}	A1	3	Accept a = –0.589
	(ii)		M1		For use of $0 = u^2 + 2as$
		Distance is 7.64 m	A1	2	
3			M1		For using $v = \int a\,dt$
		$v = (0.8/0.25)\, t^{0.25} + (C)$	A1		
		C = 1.8	B1		
			M1		For using $s = \int v\,dt$
		$s = (3.2/1.25)t^{1.25} + 1.8t + (K)$	A1ft		ft only from an incorrect non-zero value of C
		Distance is 111 m	A1	6	
4	**(i)**	For triangle of forces with 60º shown correctly, or			
		$C\cos\phi = 4\cos30$ and $C\sin\phi = 10 – 4\sin30$, or F = 4cos30 and R = 10 – 4sin30	B1		
		$[C^2 = 4^2 + 10^2 – 2 \times 4 \times 10\cos60$ or $C^2 = (4\cos30)^2 + (10 – 4\sin30)^2]$	M1		For using cosine rule or for using $C^2 = (C\cos\phi)^2 + (C\sin\phi)^2$ or $C^2 = F^2 + R^2$
		C = 8.72	A1	3	
	(ii)	$[\mu = 4\cos30/(10 – 4\sin30)]$	M1		For using $\mu = F/R = C\cos\phi/C\sin\phi$
		Coefficient is 0.433 (accept 0.43)	A1	2	
4	Alternative Method				
	(i)	For obtaining $\phi = 66.6º$ or $\tan\phi = 4 \div \sqrt{3}$ from $4 \div \sin(90º + \phi) = 10 \div \sin(150º – \phi)$	B1		
		For using C N and (4 N or 10 N) in Lami's theorem to find C $[C \div \sin120º = (4 \div \sin156.6º$ or $10 \div \sin83.4º)]$	M1		
		C = 8.72	A1	3	
	(ii)	$[\mu = \sqrt{3} \div 4$ or $\mu = \cos66.6º \div \sin66.6º]$	M1		For using $\mu = F/R = C\cos\phi / C\sin\phi$
		Coefficient is 0.433 (accept 0.43)	A1	2	

5	(i)		M1		For applying Newton's second law to A or to B
		$0.9g - T = 0.9a$ or $T - 0.6g = 0.6a$	A1		
		$T - 0.6g = 0.6a$ or $0.9g - T = 0.9a$ or $(0.9 - 0.6)g = (0.9 + 0.6)a$	B1		
		Acceleration is 2 ms ^{2} and tension is 7.2 N	A1	4	
	(ii)		M1		For using $0 = u - gt$
		$u = 3$	A1		
		$[3^2 = 2 \times 2\ h]$ $[\frac{1}{2}(0.9 + 0.6)3^2 = (0.9 - 0.6)gh]$	M1		For using $v^2 = 0^2 + 2ah$ with $v_{taut} = u_{slack}$ or for using KE gain = PE loss while the string is in tension
		Height is 2.25 m	A1	4	
6	(i)	KE loss = $\frac{1}{2}\ 16000(15^2 - 12^2)$	B1		
		PE gain = $16000g(AB/20)$	B1		
			M1		For using WD by DF = PE gain + WD against resistance – KE loss
		$1200 = 0.8g(AB) + 1.24(AB) - 648$	A1		
		Distance AB is 200m	A1	5	
	(ii)	Distance BD is 300m	B1	1	
	(iii)	WD against resistance =			
		$1240(BC) + 1860(300 - BC)$	B1ft		ft distance BD
			M1		For using KE loss = PE gain + WD against res'ce – WD by DF
		$\frac{1}{2}\ 16000(12^2 - 7^2) =$ $2400000 + (558000 - 620BC) - 7200 \times 300$	A1		
		Distance BC is 61.3 m	A1	4	

Alternative for Q6 part **(iii)**.

For BC $16000a = 7200 - 1240 - 8000$ and for CD $16000a = 7200 - 1860 - 8000$	B1
For using $v^2 = u^2 + 2as$ for both BC and CD	M1
$v_C^2 = 144 - 2 \times 0.1275(BC)$ and $49 = v_c^2 - 2 \times 0.16625(300 - BC)$	A1
For eliminating v_c^2 and obtaining BC = 61.3 m	A1

SR for candidates who assume that the acceleration is constant in part **(i)**, although there is no justification for the assumption (max. 3/5)

For appropriate use of Newton's second law and $v^2 = u^2 + 2as$ $[1200000 \div AB - 1240 - 160000/20 = 16000a$ and $a = (12^2 - 15^2)/2(AB)]$	M1
For eliminating a and attempting to solve for AB	M1
Distance AB is 200m	A1

7	(i)	(a)	[2 × ½ (1 + 9)400]	M1		For using area property for distance
			Approximation is 4000 m	A1	2	
		(b)		M1		For using the gradient property for acceleration
			Accelerations are 0.02 ms ^{2} and – 0.02 ms ^{2}	A1	2	Accept deceleration is 0.02 ms ^{2}
	(ii)	(a)		M1		For using a = dv/dt and attempting to solve a= 0.02 or a = –0.02.
			$0.04 – 0.0001t = \pm 0.02$	A1ft		
			Values of t are 200 and 600	A1	3	
		(b)	$v_1 – v = 0.02t + 1 – 0.04t + 0.00005t^2$	B1		
			$v_1 – v = [0.00005t^2 – 0.02t + 2 – 1]$ $= 0.00005(t^2 – 400t + 40000) – 1$ $= 0.00005(t – 200)^2 – 1$	B1	2	AG
		(c)	For using $(v_1 – v)_{min}$ occurs when			
			$t = 200 \rightarrow –1 \leq v_1 – v$	B1		
			For using $(v_1 – v)_{max}$ occurs when t = 0 and when t = 400 $\rightarrow v_1 – v \leq 1$	B1	2	

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1	**(i)**	M1		For using the gradient property for acceleration or v = u + at
	Acceleration is 0.02 ms^{-2}	A1		
	Acceleration is – 0.21 ms^{-2}	A1	3	
	(ii) [½ (1.5 + 2.1) ×30 + ½ 2.1 × 10 – ½ 2.2 × 20]	M1		For using the area property for displacement
	Distance AB is 42.5 m	A1	2	
	(iii) Total distance walked is 86.5 m	B1ft	1	ft error in '64.5'or '22.0' or both
2		M1		For resolving in **i** and **j** directions.
	X = 31 + 26cosα, Y = 58 – 26sinα	A1		
	X = 55, Y = 48	A1		May be implied
		dM1		For using R = $(X^2 + Y^2)^{½}$ or tan θ = Y/X
	Resultant is 73N or Direction is at 41.1º to **i** direction	A1		
	Direction is at 41.1º to **i** direction or Resultant is 73N	B1	6	
Alternative solution for Q2				
	[tan θ_{12} = 58/31, R_{12}^2= 31^2 + 58^2]	M1		For finding an angle and the hypotenuse of a right angled Δ whose other sides are 31 & 58
	θ_{12} = 61.9º and R_{12} = 65.76	A1		
	[Incl. angle = (180 – θ_{12}– α)º, $R^2 = 26^2 + R_{12}^2 – 2 \times 26R_{12}$cos (incl. angle)]	M1		For finding the included angle between sides R_{12} and 26 and using the cosine rule to find R
	Incl. angle = 95.5º, Resultant is 73 N	A1		
	[sin β = 26sin95.5/73; θ = 61.9 – β]	M1		For using the sine rule in the triangle to find the angle opposite 26 and subtracting this from θ_{12}
	Direction is at 41.1º to **i** direction	A1		
3		M1		For using Newton's second law
	0.9g – 7.2 = 0.9a (a = 2)	A1		
	[$v^2 = 2 \times (0.9g – 7.2)/0.9 \times 2$] (v = $\sqrt{8}$)	M1		For using $v^2 = (0^2) + 2ah$
	$u_{slack} = v_{taut} = 2\sqrt{g \quad 8}$	B1ft		ft incorrect equation for a
	[distance = 4 – 32/g]	M1		For using $(0^2) = u^2 – 2gh$ and distance = 2h
	Distance is 0.8 m	A1	6	

4	**(i)**	$0.8g \times 4$	B1		For finding PE at A For using ½ $mv_C^2 = PE_A$ or
		[½ $0.8v^2 = 32$]	M1		½ $mv_B^2 = PE_A$ and $v_C = v_B$
		Speed at C = 8.94 ms^{-1}	A1	3	
	(ii)	[Either $F = 0.3(0.8g)$ and $-2.4 = 0.8a$ or $F = 0.3(0.8g)$ and $WD = 2.4 \times 5$]	M1		For using $F = \mu mg$ and either Newton's 2nd law to find a or WD = F × BC
		[$v^2 =$ ans**(i)**$^2 - 2 \times 3 \times 5$ or ½ $0.8v^2 = 32 - 12$]	M1		For using either $v^2 = u^2 + 2as$ or ½ mv^2 = PE loss – WD by F
		Speed at C = 7.07 ms^{-1}	A1	3	
5	**(i)**		M1		For using $s = \int v dt$
		Displacement is $2t^3 - kt^4/4$	A1	2	
	(ii)	$t = 6/k$	B1	1	
	(iii)	[$2 \times 216/k^3 - k \times 1296/4k^4 = 108$ $\rightarrow 2 \times 216 - 1296/4 = 108k^3$]	dM1		For substituting for t in displacement and equating to 108
		$k = 1$	A1	2	
	(iv)	$dv/dt = 12t - 3kt^2$	B1		
		$= 0$ when $t = (0), 4$	B1		
		maximum value is 32	B1	3	
6			M1		For resolving forces horizontally
		$R = T\cos 30$	A1		
			M1		For resolving forces vertically (either case)
		$F = T\sin 30 - 2g$	A1		(preventing upwards motion)
		$-F = T\sin 30 - 2g$	A1		(preventing downwards motion)
			M1		For using $F = \mu R$ (either case) and attempting to solve for T
		$T = 2g/(\sin 30 \pm 0.24\cos 30)$ either case	A1		
		$T = 28.3$ and $T = 68.5$	A1	8	

7	**(i)**	DF = 30000/v or			
		WD by DF = 30000 × 100	B1		
		DF = R = 750 (v = 40) or WD by DF = WD by R = 750 × AB	B1		
		Distance AB is 4000 m	B1	3	
	(ii)	–750 = 600 a (a = – 1.25)	B1		
		$20^2 = 40^2 + 2(-1.25)BC$	M1		For using $v^2 = u^2 + 2as$
		Distance BC = 480 m	A1	3	
	Alternative for **(ii)**				
			M1		For using 'Loss of energy = WD against resistance'
		½ 600(40^2– 20^2) = 750(BC)	A1		
		Distance BC = 480 m	A1		
	(iii)	WD by engine = 30000 × 14	B1		
		Gain in KE = ½ 600 (30^2 – 20^2)	B1		
		[750 × CD = 420 000 – 150 000]	M1		For using 750 × CD = WD by engine – gain in KE
		Distance CD is 360 m	A1	4	

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GCE Advanced Subsidiary Level and GCE Advanced Level

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**(i)**	$9 \times 0.4 = 0.6 \times T\sin 30$	M1		Moments about A
		$T = 12N$	A1	[2]	
	(ii)		M1		For resolving horizontally and vertically
		$\mu = (9 - 12\sin 30)/(12\cos 30)$	M1		For using $F = \mu R$
		$\mu = 0.289$	A1	[3]	
2	**(i)**	$x = (v\cos 60)0.6$ and $y = (v\sin 60)0.6 - g0.6^2/2$	M1		Finds both coordinates in terms of t = 0.6
			DM1		Relates coordinates and 45º angle
		$\tan 45 = [(v\sin 60)0.6 - g0.6^2/2]/[(v\cos 60)0.6]$	A1		$(v\sin 60)0.6 - g0.6^2/2 = (v\cos 60)0.6$
		$v = 8.2(0)$ ms $^{-1}$ AG	A1	[4]	
	(ii)		M1		Relates velocity components and 45º
		$8.2\sin 60 - gt = 8.2\cos 60$	A1		$\tan 45 = (8.2\sin 60 - gt)/(8.2\cos 60)$
		$T = 0.3(00)$ s	A1	[3]	
3	**(i)**	$0.25g = 20e/0.4$	M1		Uses $T = \lambda x/L$
		OP ($= 0.05 + 0.4$) $= 0.45$ m	A1	[2]	
	(ii)	$20 \times 0.05^2/(2 \times 0.4) + 0.25v^2/2$ $= 0.25g \times 0.45$	M1 A1		
		$v = 2.92$ ms $^{-1}$	A1	[3]	
	(iii)	$20(d - 0.4)^2/(2 \times 0.4) = 0.25gd$	M1		Hence $d^2 - (0.8 + 0.1)d + 0.16 = 0$
		$d = [0.9 \pm \sqrt{(0.9^2 - 4 \times 0.16)}]/2$	M1		Solves a 3 term quadratic equation
		$d = 0.656$	A1	[3]	Ignore d = 0.244 if seen
4	**(i)**	$\tan\theta = 0.7/(2.4/4)$	M1		
		$\theta = 49.4º$	A1	[2]	
	(ii)	$h/2 = 2.4/4$	M1		
		$h = 1.2$	A1	[2]	

	(iii)		M1	Table of values idea, accept w = 1
		$4wVG = w \times 2.4 \times 3/4 + 3w(2.4 + h/2)$	A1	
			M1	Centre of mass above common circumference
		$VG = [\sqrt{(0.7^2 + 2.4^2)}]/\cos\alpha$	A1	$\cos\alpha = 2.4/2.5 = 0.96$
		$h = 0.944$	A1 [5]	
5	**(i)**	$0.05dv/dt = 0.05g – 0.01v$	M1	Uses Newton's Second Law
		$dv/dt = 10 – 0.2v$ AG	A1	
		$\int dv/(10 – 0.2v) = \int dt$	M1	
		$–\ln(10 – 0.2v)/0.2 = t (+ c)$	A1	
		$t = 0, v = 0$, hence $c = –5\ln10$	M1	–4.60517…
		$\ln(10 – 0.2v)/10 = 0.2t$, $1 – 0.02v = e^{0.2t}$ $v = 50 – 50e^{0.2t}$	A1 [6]	
	(ii)	$dx/dt = 50 – 50e^{0.2t}$		
		$x = \int (50 – 50e^{0.2t})dt$	M1	
		$x = 50t + 50e^{0.2t}/0.2 (+c)$	A1	
		$h = [50t + 50e^{0.2t}/0.2]_2^0$	M1	Or uses h = 0, t = 0 to evaluate c = (–250) and then finds h(2)
		$h = 17.6$	A1 [4]	
6	**(i)**	$\theta = \sin^{-1}(0.2/0.7) = 16.6^\circ$ with the vertical	B1	73.4º with the horizontal
		$T\cos\theta = 0.3g$	M1	T = 3.13 Resolves vertically
		$T + T\sin\theta = 0.3\omega^2 \times 0.2$	M1	Uses Newton's Second Law radially
		$\omega = 8.19$	A1	
		KE ($= 0.3 \times (8.19 \times 0.2)^2/2$) = 0.402 J	A1 [5]	Accept 0.403 J

(ii)	$(0.9 - AB)/AB = ½$	M1	$\alpha = \tan^{1}0.5 = 26.565^{o}$ or $BC/(0.9–BC) = ½$
	$AB = 0.6$ m	A1	$BC = 0.3$ m
	$T\cos\alpha - T\sin\alpha = 0.3g$	M1	Resolves vertically
	$T = 6.71$	A1	
	$T\cos\alpha + T\sin\alpha = 0.3\omega^2 \times 0.6\sin\alpha$	M1	$0.3\omega^2 \times 0.3\cos\alpha$ Uses Newton's Second Law radially
	$\omega = 10.6$	A1 [6]	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/52 Paper 5, maximum raw mark 50

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GCE Advanced Subsidiary Level and GCE Advanced Level

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9709 MATHEMATICS

9709/53 Paper 5, maximum raw mark 50

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1	$17\sin 50 - 2g$	B1		Vertical component of velocity
	$v^2 = (17\sin 50 - 2g)^2 + (17\cos 50)^2$	M1		Pythagoras with 2 perpendicular components
	$v = 13(.0)$ ms ^{1}	A1	[3]	
2	**(i)** 0.212	B1	[1]	From $(0.6/2)\cos 45$
	(ii) **(a)** $0.3\cos 45 \times (2 \times 7) = (2 \times 06\sin 45) \times F$	M1		Moments about A
	$F = 3.5$	A1	[2]	
	(ii) **(b)** $0.3\cos 45 \times (2 \times 7) = 0.6F$	M1		Or Ans **(i)**/cos45
	$F = 4.95$	A1	[2]	
3	**(i)** $x = (25\cos 45)t$	B1		
	$y = (25\sin 45)t - gt^2/2$	B1		
	$y = x(25\sin 45)/(25\cos 45) - g[x/(25\cos 45)^2]/2$	M1		Eliminates t between 2 simultaneous equations
	$y = x - 0.016x^2$	A1	[4]	
	(ii) $2.4 = x - 0.016x^2$	M1		Creates and attempts to solve a quadratic equation ($x = 2.5, 60$)
	Distance = 57.5 m	A1	[2]	
4	**(i)** $0.4\delta v/\delta t = 0.2v^2$	M1		Newton's Second Law with $a = \delta v/\delta t$
	$\int v^{-2}\delta v \quad 0.5\int \delta t$	A1		
	$-v^{-1} = -0.5t\ (+c)$			
	$t = 0$, $v = 8$, hence $c = -0.125$	M1		
	$v = 1/(0.125 + 0.5t) = 8/(1 + 4t)$ *AG*	A1	[4]	
	(ii) $\delta x/\delta t = 8/(1 + 4t)$	M1*		
	$x = 8\int \delta t / (1 + 4t)$			
	$x = \frac{8}{4}\ln(1 + 4t)\ (+c)$	A1		Accept $c = 0$ assumed
	$t = 1.5$, $x = \frac{8}{4}*\ln(1 + 4 \times 1.5)$	D* M1		Or limits used $\frac{8}{4}[\ln(1 + 4t)]_0^{1.5}$
	$OP = 3.89$ m	A1		4

5	**(i)**	$0.2\omega^2 \times 1.2 = 6$	M1		Uses radial acceleration on R, 1 force
		$\omega = 5$	A1	[2]	
	(ii)	$m\omega^2 \times 2 \times 0.4 = 10 - 6$	M1 A1		Uses radial acceleration on Q, 2 forces
		$m = 0.2$ kg	A1	[3]	
	(iii)	$0.2 \times (5 \times 1.2)^2/2 = M(5 \times 0.4)^2/2$	M1		
		$M = 1.8$ kg	A1		
		$1.8 \times 5^2 \times 0.4 = T - 10$	DM1		
		$T = 28$ N	A1	[4]	
6	**(i)**		M1		Table of moments idea
		$\pi 0.6^2 \times 0.6 \times 0.3 - 2\pi 0.6^3/3 \times 3 \times 0.6/8$	A1		Correct elements
		$= (\pi 0.6^3 + 2\pi 0.6^3/3)d$	A1		Correct composite
		$d = 0.09$ m	A1	[4]	
	(ii)		M1		Table of moments idea (about O)
			A1		Correct elements
		$\frac{2}{3}\pi 0.6^3 \times \frac{3}{8} \times 0.6 - \pi \times 0.6^3 \times 0.3$ $+ 0.48A \times 0.36 = 0$	A1		
		$A = 3\pi/16$ m^2	A1	[4]	
		OR	M1		Table of moments idea (about O)
			A1		Correct elements
		$[\frac{2}{3}\pi \times 0.6^3 + \pi \times 0.6^3] \times 0.09 = 0.48A \times 0.36$	A1		
		$A = 3\pi/16$	A1		
	(iii)	Increase in length $[= 2 \times (0.6 - 0.48)] = 0.24$m	B1	[1]	Remove cylinder with centre of mass at O

7	**(i)**	$0.8g\sin30 = 20e/0.4$	M1	
		$e = 0.08$ m	A1 [2]	
	(ii)		M1	Conservation of KE, PE, EE
		$0.8v^2/2 + 20 \times 0.08^2/(2 \times 0.4)$	A1	Correct start terms, signs accurate
		$= 0.8g(0.4 + 0.08)\sin30$	A1	Correct final term, sign accurate
		$v = 2.1(0)$ ms ^{1}	A1 [4]	
	(iii)		M1*	
		$0.8gd\sin30 = 20(d - 0.4)^2/(2 \times 0.4)$	A1	$4d = 25(d - 0.4)^2$
		$25d^2 - 24d + 4 = 0$	D* M1	Obtains and solves a 3 term quadratic equation.
		$d = 0.745$ m	A1 [4]	

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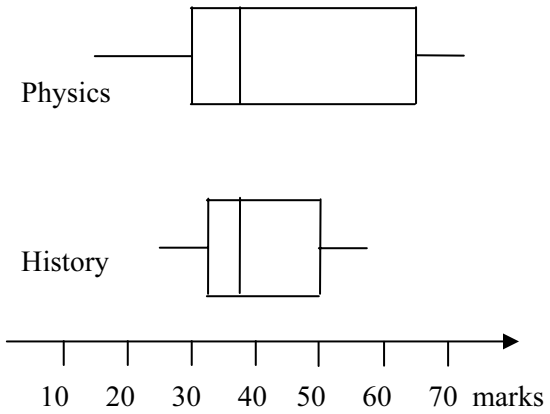
1	$\mu = 250 \times 0.86 = 215$	B1		250 × 0.86 and 250 × 0.86 × 0.14 seen o.e
	$\sigma^2 = 250 \times 0.86 \times 0.14 = 30.1$	M1		Standardising, with or without cc, must have sq rt in denom
	$\mathrm{P}(X > 210) = 1 - \Phi\left(\frac{210.5 \quad 215}{\sqrt{30.1}}\right)$	M1		Continuity correction 210.5 or 209.5 only
	$= \Phi(0.820)$	M1		Correct region (> 0.5) ft their mean
	$= 0.794$	A1	[5]	Correct answer
2	**(i)** $133/n + 25 = 28.325$	M1		Equation involving 133, 25 and 28.325
	$n = 40$	A1		Correct answer for n
	$3762/40 - 3.325^2 = 82.99$	M1		Using coded mean in variance formula
	standard deviation = 9.11	A1	[4]	Correct answer
	(ii) $82.99 = \Sigma x^2/40 - 28.325^2$	M1		Using uncoded material in variance formula
	$\Sigma x^2 = (82.99 + 28.325^2) \times 40$			
	$= 35412$ (35400)	A1		Correct answer
	OR			
	$\Sigma(x - 25)^2 = \Sigma x^2 - 50\Sigma x + 40 \times 25^2$	M1		Expanding and substituting for Σx
	$\Sigma x^2 = 3762 + 50 \times 1133 + 25000$			
	$= 35412$	A1	[2]	Correct answer
3	**(i)** $\mathrm{P}(X = 1) = \mathrm{P(GBBB)}\ 4 \times {}_1\mathrm{C}_1$	M1		Considering values of X of 1, 2, 3, 4
	$= 5/8 \times 3/7 \times 2/6 \times 1/5 \times 4 = 1/14$	M1		Attempting to find the probability of at least 2 values of X
	$\mathrm{P}(X = 2) = \mathrm{P(GGBB)} \times {}_4\mathrm{C}_2 = 3/7$			
	$\mathrm{P}(X = 3) = \mathrm{P(GGGB)} \times {}_4\mathrm{C}_3 = 3/7$	A1		One correct probability
	$\mathrm{P}(X = 4) = \mathrm{P(GGGG)} \times {}_4\mathrm{C}_4 = 1/14$	A1		All correct
	OR			
	$\mathrm{P}(1) = {}_5\mathrm{C}_1 / {}_8\mathrm{C}_4 = 1/14$	M1		Considering values of X of 1, 2, 3, 4
	$\mathrm{P}(2) = {}_3\mathrm{C}_2 \times {}_5\mathrm{C}_2 / {}_8\mathrm{C}_4 = 3/7$	M1		Dividing by ${}_8\mathrm{C}_4$
	$\mathrm{P}(3) = {}_3\mathrm{C}_1 \times {}_5\mathrm{C}_3 / {}_8\mathrm{C}_4 = 3/7$	A1		One correct probability
	$\mathrm{P}(4) = {}_5\mathrm{C}_4 / {}_8\mathrm{C}_4 = 1/14$	A1	[4]	All correct
	(ii) $\mathrm{Var}(X) = 1/14 + 12/7 + 27/7 + 16/14 - (5/2)^2$	M1		Using a variance formula correctly with mean^2 subtracted numerically, no extra division
	$= 15/28$ (0.536)	A1	[2]	Correct final answer

4	**(i)**	History: lowest 27, highest 57, LQ = 33 med = 39 UQ = 50  	M1 [1]	Attempt to find history quartiles and median by putting in order or stem and leaf (can be implied if the answer is reasonable) Correct history median and quartiles Uniform scale and labels Correct history graph ft their quartiles line not through box Correct physics graph
	(ii)	Physics marks are more spread out than History marks	B1 [1]	Any sensible comment
5	**(i)**	$z = 1.882$ or 1.881	B1	± 1.882 or ± 1.881 seen
		$1.882 = (32 - 20) / \sigma$	M1	Equation using their z (must be a z-value) 32, 20 and s
		$\sigma = 6.38$	A1 [3]	Correct answer
	(ii)	$P(x > 13) = P\left(z > \frac{13 \quad 20}{6.376}\right)$	M1	Standardising
		$= P(z > -1.0978)$	M1	Correct area > 0.5
		$= 0.864$	A1 [3]	Correct answer
	(iii)	P(at least 2) = 1 – P(0, 1)	M1	Using 0.03 and 0.97 or 0.06 and 0.94 in a binomial expression powers summing to 7
		$= 1 - (0.97)^7 - (0.03)(0.97)^6{}_7C_1$	M1	Correct unsimplified binomial expansion
		$= 0.0171$	A1 [3]	Correct answer
6	**(a)**	**(i)** $\frac{12!}{2!3!2!} = 19958400$ (20,000,000)	M1 A1 [2]	Dividing by 2! 3! 2! Correct answer
		(ii) $\frac{4!}{2!} \times \frac{9!}{2!3!} = 362880$	B1 B1 B1 [3]	4! seen multiplied 9! or 9 × 8! seen multiplied Correct final answer
	(b)	**(i)** $3876 \times 4!$	M1	Multiplying by 4!
		$= 93024$	A1 [2]	Correct answer
		(ii) $(3!)^4 \times 4!$	M1	3! or 6 or 4! seen
		$= 31104$	A1 [2]	Correct final answer

7	**(i)**	P(2, N2, 2) = 1/4 × 1 × 1/7 = 1/28	M1		Considering at least two options of 2s and 8s
		P(8, 8, N8) = 1/4 × 2/5 × 3/7 = 3/70	M1		Considering three options for the 8s
		P(8, N8, 8) = 1/4 × 3/5 × 4/7 = 3/35	M1		Summing their options if more than 3 in total
		P(N8, 8, 8) = 3/4 × 2/5 × 4/7 = 6/35	B1		One option correct
		Σ = 47/140 (0.336)	A1	[5]	Correct answer
	(ii)	P(2, 2 given same) = $\frac{1/28}{47/140}$	M1		1/28 in numerator of a fraction
		= 5/47 (0.106)	A1	[2]	Correct answer
	(iii)	P(X) = 47/140	M1		Attempt to compare P(A and B) with P(A) × P(B) or using conditional probabilities
		P(Y) = 1/4			
		P(X and Y) = 1/28 ≠ 47/140 × 1/4	A1		Legitimate correct answer
		Not independent		[2]	

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GCE Advanced Subsidiary Level and GCE Advanced Level

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9709/62 Paper 6, maximum raw mark 50

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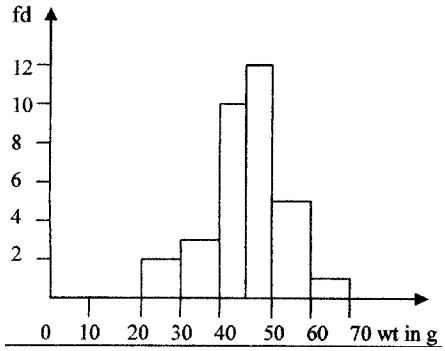
1	$x = 59.4$	B1		
		M1		Correct method (can be implied by correct answer)
	$\sigma = 7.68$	A1	[3]	Correct answer
2	**(i)** each in 2 ways = 2^{12}	M1		2^{12} seen
	= 4096	A1	[2]	Correct answer
	(ii) $\frac{12!}{7!5!}$	B1	[1]	
	= 792			
3	**(a)** *G R L*			
	11 7 7 = 15C11 × 10C7 × 8C7 = 1310400	M1		Multiplying 3 combinations
	13 6 6 = 15C13 × 10C6 × 8C6 = 617400	A1		One of 1310400, 617400, 14112 seen
	15 5 5 = 15C15 × 10C5 × 8C5 = 14112	M1		Adding 3 options
	Total = 1941912 (1940000)	A1	[4]	Correct answer
	(b) e.g. * E * R * E (GG) N * A * E * gives 6 ways for G	B1		7! / 3! Or 7!/3!3! seen oe
	$\frac{7!}{3!} \times 6$ or 8!/3! – 2 × 7!/3!	B1		Multiplying by 6 (gaps) oe
	= 5040 ways.	B1	[3]	Correct final answer
4	**(i)** 45 – 50 g	B1	[1]	
	(ii) LQ in 40 – 45			
	UQ in 50 – 60	M1		Considering groups containing LQ and UQ (can be implied)
	Smallest IQ range could be 5			
	Largest IQ range could be 20	A1	[2]	Correct answer
	(iii) 50	B1	[1]	
	(iv) freqs 0, 20, 30, 50, 60, 50, 10			
	fd 0, 2, 3, 10, 12, 5, 1	M1		Attempt at frequencies and fd
		B1		Correct labels and scales with a histogram-type shape
		B1		Correct bar widths starting at 20
		A1	[4]	Correct heights of bars
5	**(i)** $4p + p + 3p = 1$ so P(blue) = 1/8 AG	B1	[1]	Must show something
	(ii) P(*R*) = ½, P(*B*) = 1/8, P(*G*) = 3/8	M1	[3]	Multiplying P (*R*, *B*, *G*) together
	P(all different) = ½ × 1/8 × 3/8 × 3!	M1		Mult by 3!
	=9/64 (0.141)	A1		Correct answer

	(iii)	mean = 136 × 1/8 = 17, var = 14.875 $P(<20) = P\left(z < \frac{19.5 \quad 17}{\sqrt{14.875}}\right)$ = Φ(0.648) = 0.742	B1 M1 M1 M1 A1	[5]	Unsimplified mean and variance correct Standardising, need sq rt Cont correction 19.5 or 20.5 Correct area, > 0.5 legit Correct answer
6	**(i)**	P(0, 1, 2) $=(0.85)^6 + (0.15)(0.85)^5{}_6C_1 +$ $(0.15)^2(0.85)^4{}_6C_2$ = 0.953	B1 M1 A1	[3]	0.15 and 0.85 seen Any binomial expression Σpowers = 6, Σ p = 1 Correct answer
	(ii)	P(*D*) = 0.6 × 0.1 + 0.4 × 0.55 = 0.28 $P(B\|D) = \frac{P(B \cap D)}{P(D)}$ 0.06/0.28 = 0.2143 P(> 1) = 1 – P(0) $= 1 – (0.7857)^5$ = 1 – 0.7078 = 0.701	M1 A1 M1 √A1 M1 A1	 [6]	Attempt to find P(*D*) 0.28 seen Using cond prob formula to find P(*B*|*D*) Correct unsimplified answer Binomial expression 1 –P(0) or 1 –P(0, 1) Σ p = 1 Correct answer accept 0.700
7	**(i)**	$z_1 = \frac{12 \quad 8}{\sqrt{24}} = 0.816$ $\Phi_1(0.816) = 0.7926$ $z_2 = \frac{7 \quad 8}{\sqrt{24}} = –0.204$ $\Phi_2(–0.204) = 1 – 0.5808$ Prob = 0.7926 – (1 – 0.5808) = 0.373	M1 M1 A1	 [3]	Standardising any one, no sq rt no cc Correct area $\Phi_1 + \Phi_2 – 1$ Correct answer
	(ii)	$z = \frac{0 \quad \mu}{2\mu} = –0.5$ P(z < –0.5) = 1 – 0.6915 = 0.309 or 30.9%	M1 A1	 [2]	Standardising, no cc no sq rt, one variable Correct answer oe
	(iii)	$z = \frac{3\mu \quad \mu}{2\mu} = 1$ P(z > 1) = 1 – 0.8413 = 0.1587 70 × 0.1587 = 11.1	M1 M1 A1	 [3]	Standardising and eliminating μ Subt from 1 and multiplying by 70 Correct answer accept 11 or 12
	(iv)	z = 1.45 $1.45 = \frac{6 \quad \mu}{2\mu}$ μ = 1.54	B1 M1 A1	 [3]	± 1.45 seen Solving for μ with 6, 2 μ , μ and their z Correct answer

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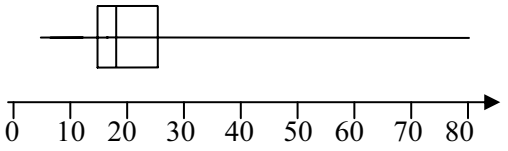
1	**(i)**	$z = 0.38$	B1	$\pm 0.38\,(0)$ seen or implied
		$\pm\dfrac{25-\mu}{\mu/3} \quad 0.38$	M1	Standardising attempt resulting in z = some μ/σ/both, no continuity correction
			M1	Substituting to eliminate μ or σ and attempt to solve linear equation
		$\mu = 22.2,\ \sigma = 7.40$	A1 [4]	Both correct
	(ii)	$P(4) = {}^6C_4(0.352)^4(0.648)^2$	M1	${}^6C_r \times (p)^r \times (1-p)^{6-r}$, $r = 2$ or 4
		$= 0.0967$	A1 [2]	Correct answer
2	**(i)**	$P(F) = \dfrac{12}{30}\ (0.4)$	B1	$\dfrac{12}{30}$ or $\dfrac{16}{30}$ or $\dfrac{5}{30}$ seen
		or $P(W) = \dfrac{16}{30}\ (0.533)$	M1	Valid attempt to find P(F or W)
		or $P(M \cap W') = \dfrac{5}{30}\ (0.167)$		
		$(F \text{ or } W) = \dfrac{13}{30}+\dfrac{3}{30}+\dfrac{9}{30}$	A1	Correct unsimplified expression
		or $1 - \dfrac{5}{30}$ or $\dfrac{12}{30}+\dfrac{16}{30}-\dfrac{3}{30}$		
		$= \dfrac{5}{6}\ (0.833)$	A1 [4]	Correct answer
	(ii)	$P(M) = 18/30\ (0.6)$, $P(W) = 16/30\ (0.533)$, $P(M) \times P(W) = 8/25\ (0.32)$	M1	Valid attempt to find P(M), P(W) and P(M) × P(W)
		P(M and W) = 13/30 (0.433) $\neq$ 8/25 (0.32) not independent	A1	P(M and W) = 13/30 $\neq$ 8/25 and correct conclusion
		OR		
		$P(M \mid W) = \dfrac{P(M \text{ and } W)}{P(W)} = \dfrac{13/30}{16/30} = \dfrac{13}{16}\ (0.813)$	M1	Valid attempt to find P(M and W), P(W) and P(M and W) ÷ P(W)
		$\neq \dfrac{18}{30} = P(M)$, not independent	A1	$\dfrac{13}{16} \neq \dfrac{18}{30} = P(M)$
		OR		
		$P(W \mid M) = \dfrac{P(M \text{ and } W)}{P(W)} = \dfrac{13/30}{18/30} = \dfrac{13}{18}$	M1	Valid attempt to find P(M and W), P(M) and P(M and W) ÷ P(M)
		$\neq \dfrac{16}{30} = P(W)$, not independent	A1 [2]	$\dfrac{13}{16} \neq \dfrac{18}{30} = P(M)$

3	**(i)**	P(3m) = 4/5 (0.8) P(5m) = 1/5 (0.2)	B1	P(3m) = 4/5 or P(5m) = 1/5 seen or implied
		E(X) = 17/5 (3.4)	B1	Correct E(X)
			M1	Subtract their mean^2 numerically from Σx^2p, no extra dividing
		Var(X) = 16/25 (0.64)	A1 [4]	Correct answer
	(ii)	P(3, 5) + P(5, 3) = 0.8 × 0.2 +0.2 × 0.8	M1	Summing two 2-factor terms
		= 8/25 (0.32)	A1√ [2]	Correct answer, ft on 2 × p × (1 − p), their p
	(iii)	P(11) = P(3, 3, 5) + P(3, 5, 3) + P(5, 3, 3)	M1	Mult 2 probs for 3 with 1 prob for 5
		= (4/5 × 4/5 × 1/5) × 3	M1	Multiplying probs for 11 by 3 or summing 3 options
		= 48/125 (0.384)	A1 [3]	Correct final answer
4	**(i)**	3! × 4! × 8! × 3!	M1	Multiplying 3 factorials together
			M1	Multiplying by 3!
		= 34 836 480 (34 800 000)	A1 [3]	Correct answer
	(ii)	$^3C_2 \times {}^4C_2 \times {}^8C_2$	M1	Multiplying (only) 3 combinations together
		= 504	A1 [2]	Correct answer
	(iii)	Fr Fa H		
		3 1 2 = $^8C_3 \times {}^3C_1 \times {}^4C_2 = 1008$	M1	Multiplying 3 combinations, only
		3 2 1 = $^8C_3 \times {}^3C_2 \times {}^4C_1 = 672$	M1	Summing 3 options
		4 1 1 = $^8C_4 \times {}^3C_1 \times {}^4C_1 = 840$	A1	3 correct combination answers
		total ways = 2520	A1 [4]	Correct answer
5	**(i)**	LQ = 15, Median = 18, UQ = 26	B1	LQ = 15, Median = 18, and UQ = 26
			B1	Linear scale and labels
			B1√	Quartiles and median box, ft on their values, but M − LQ < UQ − M
			B1√	Whiskers from 5 to LQ and UQ to 80, ft on their values
			[4]	
	(ii)	most (3/4) are earning less than 26K, not many earning high salaries, etc	B1 [1]	Any sensible answer
	(iii)	**(a)** IQ range = 11	B1	IQR = 11
		high outlier is above 26 + 1.5 × 11	M1	Their UQ + 1.5 × their IQ range
		= 42500 euros	A1 [3]	Correct answer
		(b) Low outlier is below 15 – 1.5 × 11 = –1.5	B1√ [1]	Correct reason, must involve subtraction, ft on their LQ and 1.5 × their IQR

6	**(i)**	$P(O \text{ given } +) = \frac{0.37}{0.83}(0.4458)$	B1 A1 M1	0.83 seen or implied Attempt to find P(O given +) using conditional probability fraction Binomial term ${}^9C_r p^r(1-p)^{9-r}$, $r \neq 0$ or 9
		$P(0, 1, 2) = (0.4458)^0(0.5542)^9 + {}^9C_1(0.4458)^1(0.5542)^8 + {}^9C_2(0.4458)^2(0.5542)^7$	M1 A1	Binomial expression P(0, 1, 2) or P(0, 1, 2, 3) powers summing to 9 any $0 < p < 1$ Correct unsimplified expression
		$= 0.156$	A1 [6]	Correct final answer
	(ii)	$\mu = 150 \times 0.35 = 52.5,$	B1	150×0.35 (52.5) and $150 \times 0.35 \times 0.65$ (34.125) seen
		$\sigma^2 = 150 \times 0.35 \times 0.65 = 34.125$	M1 M1	Standardising, using sd not variance Using continuity correction, 59.5 or 60.5
		$P(> 60.5) = P\left(z > \pm \frac{60.5 \quad 52.5}{\sqrt{34.125}}\right)$	M1	correct area (< 0.5, for mean < their 60)
		$= 1 - \Phi(1.369)$		
		$= 0.0854$ or 0.0855	A1 [5]	correct value

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

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9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

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1	**(i)** Mean = 2.6	B1		
	Var = 4 × 1.3	M1		M1 for either 4 ×, or for Var(X) = 1.3 implied
	= 5.2	A1	[3]	
	(ii) Var ≠ mean or $2X$ does not take all integer values	B1	[1]	X and X are not independent oe
2	H_0: P(correct) = $^1/_5$ H_1: P(correct) > $^1/_5$ B(100, $^1/_5$) ≈ N(20, 16)	B1		Accept p Accept H_o: $\mu = 20$ H_1: $\mu > 20$
	$\frac{26.5 \quad 20}{4} = 1.625$	M1 A1		Allow wrong or no cc or denom = 16 For ± 1.625
		A1		
	comp z = 1.645	M1		Valid comparison of z or areas (0.0521 > 0.05)
	Claim not justified	A1ft	[5]	In context. No contradictions. Ft their z.
3	Var(Tot) = $0.02^2 + 0.03^2 + 0.01^2 = 0.0014$	B1		
	Mean(Tot) = 0.37 Tot ~ N(0.37, 0.0014)	B1		
	$\frac{0.30 \quad 0.37}{\sqrt{'0.0014'}}$ (= –1.871)	M1		Allow without √. No cc
	Φ("–1.871") = 1 – Φ("1.871")	M1		
	= 0.0306 or 0.0307	A1	[5]	Correct area
4	**(i)** Est(μ) = 331(.125)	B1		
	$\text{Est}(\sigma^2) = \frac{8}{7}\left(\frac{"877179"}{8} \quad "331.125"^2\right)$	M1		Allow their Σx^2
	= 4.125 or 4.13	A1	[3]	
	(ii) z = 2.326	B1		
	$331 \pm z \times \sqrt{\frac{4.2}{50}}$	M1		Allow incorrect z ($\neq$ 1, 0), not a prob
	= 330 to 332 (3 sfs)	A1	[3]	Ignore brackets, if given. CWO
	(iii) No, because 333 is not within CI	B1ft	[1]	

5	**(i)**	±1.645 used	B1	
		$\dfrac{\bar{x} - 22}{\frac{3.5}{\sqrt{12}}} > 1.645$	M1	
		$\bar{x} > 23.66(20)$ $\bar{x} > 23.7$ **AG**	A1 [3]	Accept '=' (standardising using 23.7 scores M1A0) or $\bar{x} = 23.66(20)$
	(ii)	$P(\bar{x} < 23.7 \mid \mu = 25.8)$	M1	For attempt type II error and standardising
		$\dfrac{23.662 - 25.8}{\frac{3.5}{\sqrt{12}}}$ 2.116	A1	$\dfrac{23.7 - 25.8}{\frac{3.5}{\sqrt{12}}}$ 2.078
		Φ('–2.116') = 1 – Φ('2.116') (= 1 – 0.9828)	M1	Φ("–2.078") = 1 – Φ(–2.078) (= 1 – 0.9812)
		= 0.0172 (3 sfs)	A1 [4]	= 0.0188
6	**(i)**	Customers arrive independently or randomly	B1 [1]	In context. Allow "singly"
	(ii)	$e^{-6} \times \dfrac{6^5}{5!}$	M1	Poisson P(5), allow any mean
		= 0.161 (3 sfs)	A1 [2]	
	(iii)	$\lambda = 2.4$	B1	
		$e^{-2}\left(1 + 2.4 + \dfrac{2.4^2}{2!}\right)$	M1	Poisson P(0, 1, 2), allow their mean allow one end error
		= 0.570 (3 sfs)	A1 [3]	
	(iv)	N(24, 24)	B1	Stated or implied
		$\dfrac{295 - 24}{\sqrt{24}}$ (= 1.123)	M1	Allow with wrong or no cc and/or no √ Correct area
		Φ("1.123")	M1	
		= 0.869 (3 sfs)	A1 [4]	

7 (i) (a)	X or 5	B1 [1]	
(b)	V or 3	B1	Should mention values or prob Not just graph or spread eg not "More spread"
	Higher and lower values more likely or there are more higher and lower values or more prob at both extremes	B1dep [2]	
(ii)	$\frac{2+1}{2} \times 0.5$ or $\int_0^{0.5} (2 \quad 2x)\mathrm{d}x$	M1	('or' method requires linear function and correct limits)
	$= 0.75$	A1 [2]	CWO
(iii) (a)	$\int_0^1 ax^n \mathrm{d}x = 1$	M1	Attempt integ of correct form = 1 (ignore limits)
	$\left[\frac{ax^{n+1}}{n+1}\right]_0^1$ 1	A1	Correct integrand & limits
	$\frac{a}{n+1}$ 1	A1	No errors seen
	$(a = n + 1$ **AG**)	[3]	
(b)	$\int_0^1 ax^{n+1} \mathrm{d}x$ $\frac{5}{6}$ oe	M1*	Integral of form $\int xf(x)\mathrm{d}x$ $\frac{5}{6}$, ignore limits
	$\left[\frac{ax^{n+2}}{n+2}\right]_0^1$ $\frac{5}{6}$ oe	A1	Correct integrand & limits
	$\frac{a}{n+2}$ $\frac{5}{6}$ $(6a = 5n + 10)$	M1dep	Attempt to use $a = n + 1$ within 2nd equ to get an equ in n (or a)
	$a = 5, n = 4$	A1 [4]	

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1	$50 = a + b \times 54$	B1		
	$100 = b^2 \times 144$ or $10 = b \times 12$	B1		
	$b = \frac{5}{6}$ oe	M1		Solving two simultaneous equations
	$a = 5$	A1	[4]	Both correct
2	$2 \times z \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.157$	M1 M1		For $\sqrt{(pq/n)}$ in equation For equation of the form $2 \times z \times \mathrm{f}(n) = 0.157$
	$z = 2.326$	B1		
	$n = 4 \times 2.326^2 \times 0.35 \times 0.65 \div 0.157^2$ (=199.738…)	M1		Rearrange to form n = ... from a correct equation in n, but allow any z and/or factor of "2" errors
	$n = 200$	A1	[5]	cao
3	**(i)** Number all members	B1		
	Explain the selection of 3-digit random numbers	B1		
	Omit repeats OR omit nos. over 750 (until have 8 nos.)	B1	[3]	
	(ii) Est $(\mu) = 20$	B1		
	Est $(\sigma^2) = \frac{8}{7}\left(\frac{3636}{8} \quad 20^2\right)$	M1		$1/7 \times (3636 - 160^2/8)$
	$= \frac{436}{7}$ or 62.3 (3 sfs)	A1	[3]	$(7.89...)^2$ M1A1, but 7.89... only M1A0
	(iii) Amounts spent last week in café by all club members	B1	[1]	
4	**(i)** $\int_0^1 ke^{\ x} \mathrm{d}x = 1$	M1		Int = 1, ignore limits
	$\left[ke^{\ x} \right]_0^1 = 1$	A1		Correct integral & limits, & = 1
	$(= -ke^{\ 1} - (-ke^0))$			
	$= k \times \frac{e \quad 1}{e} = 1$ or $k(e-1) = e$			
	$k = \frac{e}{e \quad 1}$ **AG**	A1	[3]	Correctly obtained, no errors seen

	(ii) $\frac{e}{e-1}\int_0^1 xe^{-x}\,dx$	M1	Attempt $\int xf(x)dx$, ignore limits
	$=\frac{e}{e-1}\left(\left[x(-e^{-x})\right]_0^1 - \int_0^1(-e^{-x})dx\right)$	M1*	Attempt integration by parts the correct way round, ignore limits
	$=\frac{e}{e-1}\left(\left[-xe^{-x}\right]_0^1 - \left[e^{-x}\right]_0^1\right)$	M1dep*	Attempt second integral of the form $\pm\int e^{-x}dx$, ignore limits
	$\left(=\frac{e}{e-1}(-e^{-1}-0-(e^{-1}-1))\right)$		
	$=\frac{e}{e-1}\left(1-\frac{2}{e}\right)$ or $\frac{e-2}{e-1}$ oe	A1 [4]	Accept k instead of $\frac{e}{e-1}$ throughout except ans
5	**(i)** Assume pop sd same (105)	B1	
	H_0: Pop mean = 1150 H_1: Pop mean < 1150	B1	Allow "μ" but not just "mean"
	$\frac{\frac{21800}{20}-1150}{\frac{105}{\sqrt{20}}}$	M1	Allow $\div\frac{105}{20}$. (Accept "totals" method)
	$=\pm 2.556$ or 2.56	A1	Or 0.0053 if prob/area comparison used
	Compare with $z = \pm 2.326$ (for a clear 2 tail test compare with ± 2.576)	M1	Correct comparison of z or prob/area consistent with their test
	Evidence that mean distance decreased	A1ft [6]	In context. Allow mean dist decreased ft their z and/or clear 2 tail test
	(ii) 0.01	B1	
	Concluding there has been a decrease when there has not.	B1 [2]	In context

6	**(i)**	E(Tot) = 2 × 36 + 55	(= 127)	B1		(Or ±13)
		Var(Tot) = $2 \times 3.5^2 + 5.2^2$	(= 51.54)	B1		
		$\frac{140 - 127}{\sqrt{"51.54"}}$	(= 1.811)	M1		For standardising with their mean and var. Allow without √ or with false cc, but their mean and variance must involve parameters from both given distributions
		Φ("1.811")				
		= 0.965 (3 sfs)		A1	[4]	
	(ii)	E(*RM*) = 36 + 1.5 × 55	(= 118.5)	B1		(Or ±18.5)
		Var(*RM*) = $3.5^2 + 1.5^2 \times 5.2^2$	(= 73.09)	B1		
		$\frac{100 - 1185}{\sqrt{73.09}}$	(= –2.164)	M1		For standardising with their mean and var. Allow without √ or with false cc, but their mean and variance must involve parameters from both given distributions
		1 – Φ("–2.164") = Φ ("2.164")				
		0.985 (3 sfs)		A1	[4]	
7	**(i)**	**(a)** $1 - e^{1.2}(1 + 1.2)$	(= 0.3374)	M1		M1 for Poisson either P(0 or 1) or P(0) with λ = 1.2 or 2.4 or 1.4 or 2.8, accept one end error
		$1 - e^{1.4}$	(= 0.7534)	A1		Both expressions fully correct
		$(1 - e^{1.2}(1 + 1.2)) \times (1 - e^{1.4})$		M1		Their Poisson P(0 or 1) × P(0)
		= 0.254 (3 sfs)		A1	[4]	
	(i)	**(b)** $\lambda = 2.6$ seen		B1		
		$1 - e^{2.6}(1 + 2.6 + 2.6^2 \div 2)$		M1		Poisson 1 – P(0, 1, 2), allow 1 – P(0, 1, 2, 3), with attempt at combined λ for M and W. Accept combination method: at least 4 correct terms and "1 –" M1; all terms correct B1
		= 0.482 (3 sfs)		A1	[3]	

(ii) N(52, 52)		B1	Seen or implied
$\frac{60.5-52}{\sqrt{52}}$	(= 1.179)	M1	Standardising with N(λ, λ) with $\lambda = 10 \times 5.2$ or 10×2.6 Allow with wrong or no cc or no √
$1 - \Phi$("1.179")		M1	Their correct area
(= 1 – 0.8808)			
= 0.119 (3 sfs)		A1 [4]	