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## FOREWORD

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This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned.**

# ADDITIONAL MATHEMATICS

## GCE Ordinary Level

Paper 4037/01

Paper 1

### General comments

Although there were some good scripts, the new style of syllabus and paper presented many candidates with considerable difficulty. There were obviously some topics, particularly “matrices” which some candidates had covered if no real depth, if at all. It was very noticeable that the more stereotyped topics from the previous syllabus, notably binomial expansions, solution of simultaneous linear and quadratic equations, coordinate geometry and topics within the calculus sections, were most accessible to the candidates.

### Comments on specific questions

#### Question 1

This question was very well answered by nearly all candidates. The standard of algebra shown in the elimination of either  $x$  or  $y$  and in solving the resulting quadratic equation was excellent. Candidates seemed comfortable either with the manipulation of fractions in eliminating  $x$  or by the squaring required to eliminate  $y$ . A lot of candidates lost the last mark through inaccurate use of decimals when a  $y$ -value of  $\frac{1}{3}$  was expressed as 0.34.

Answer:  $(3\frac{1}{3}, \frac{1}{3})$  and  $(2\frac{1}{2}, 2)$ .

#### Question 2

Attempts varied considerably. Most candidates realised the need to integrate but such attempts to integrate  $e^{kx}$  as  $e^{\frac{kx^2}{2}}$  or  $e^{kx+1}$  showed a considerable lack of understanding of the exponential function. At least a third of all attempts also ignored the use of the point  $(0, 3)$  to evaluate the constant of integration.

Answer:  $y = \frac{e^{4x}}{4} - e^{-x} + \frac{15}{4}$ .

#### Question 3

Attempts also varied considerably and there were very few correct answers. A significant number of candidates failed to realise that calculators were of no use in this type of question. Most candidates seemed to be familiar with one of the two processes needed, that is to express  $\sqrt{18}$  as  $3\sqrt{2}$  or  $\frac{4}{\sqrt{2}}$  as  $2\sqrt{2}$ , but very few seemed confident at using both. The common error of expressing  $(a + b)^2$  as  $a^2 + b^2$  also led to loss of marks in part (ii).

Answers: (i)  $-2 + 11\sqrt{2}$ ; (ii)  $55 - 8\sqrt{2}$ .

#### Question 4

This produced a large number of perfectly correct solutions and although most candidates worked by finding **PQ** first, several went directly to the answer from using **OR** = 5**OQ** - 4**OP**. Use of **PQ** = **p** - **q**, and even **p** + **q**, was seen and over a quarter of all candidates did not appreciate the term “unit vector”.

Answer:  $\frac{1}{75} \begin{pmatrix} 21 \\ 72 \end{pmatrix}$ .

#### Question 5

Part (i) was badly answered with very few candidates sketching both graphs correctly. In a large number of cases  $y = \frac{1}{4} + \sin x$  was sketched as either  $\sin x$  or as  $\frac{1}{4} \sin x$  and  $\frac{1}{2} \cos 2x$  was only shown in the range 0 to  $\pi$ . A surprising number of graphs were shown in which curves were replaced by straight lines, even at the turning points! Very few candidates realised that the value of  $k$  in part (ii) could be obtained by equating the two equations. Most candidates attempted to read a value of  $x$  from their sketches and to deduce a value for  $k$  from this. Such attempts received no credit unless an accurate graph had been drawn.

Answers: (i) Sketch; (ii)  $k = 4$ .

#### Question 6

This was also poorly answered and several candidates ignored the question altogether. In part (a), the total number of 720 was often obtained without candidates being able to cope with the number not beginning with “0”. Part (b) proved to be more successfully answered with many candidates appreciating the need to consider four different cases (though often the case with “all women” was ignored). A surprising number of candidates also realised the need to calculate such expressions as  $\binom{5}{2}$  and  $\binom{4}{2}$  but then found the sum rather than the product. Very rarely was the solution “Total – no women” seen.

Answers: (a) 600; (b) 121.

#### Question 7

This was well answered and generally a source of high marks. Candidates were able to write down the expansion unsimplified and had no real problems with the binomial coefficients. Subsequent errors with  $(-x^2)^n$  were however considerable. Often the minus sign was ignored completely, at times all terms were negative apart from the first and  $(-x^2)^n$  was in many instances taken as  $x^{2+n}$  or as  $-x^{2+n}$ . Part (ii) also suffered from the obvious error of expressing  $(1 + x^2)^2$  as  $1 + x^4$ , but it was pleasing to note that most candidates realised the need to consider more than one term in finding the coefficient of  $x^6$ .

Answers: (i)  $32 - 80x^2 + 80x^4 - 40x^6 + 10x^8 - x^{10}$ ; (ii) 40.

#### Question 8

There were very few completely correct solutions, though candidates did better on part (ii) than on part (i). The majority of attempts at the area of triangle SXY attempted to find the sides by Pythagoras’s Theorem – no progress was made! Only about a half realised the need to evaluate  $A$  by subtracting the sum of the areas of three right-angled triangles from the area of the square. In part (ii), most candidates realised the need to differentiate and to set the differential to zero. Errors in misusing the “ $\frac{1}{2}$ ” or in differentiation, particularly with  $\frac{d}{dx}(1) = 1$  and  $\frac{d}{dx}(kx^2) = 3qx$  were surprisingly common. Of those obtaining  $x = \frac{1}{2q}$ , most realised that  $QY = YR$  but many forgot to substitute this value of  $x$  into the expression for  $A$ .

Answers: (i) Proof; (ii) Proof,  $A = \frac{1}{2} - \frac{1}{8q}$ .

### Question 9

This was well answered and a source of high marks. In part (i) most candidates realised the need to use the product rule, though at least a quarter of all attempts ignored the “2” from the differential of  $\sqrt{(2x+5)}$ . Part (ii) was well answered, though a considerable number took  $\delta x$  to be  $10 - p$  or failed to substitute  $x = 10$  to obtain a numerical value for the gradient. Part (iii) also presented few problems and it was pleasing to see the number of attempts correctly using the chain rule and realising either the need to either divide by  $\frac{dy}{dx}$  or to

invert  $\frac{dy}{dx}$  in order to obtain  $\frac{dx}{dt}$ .

Answers: (i) Proof,  $k = 3$ ; (ii)  $\pm 6p$ ; (iii) 0.5 units per second.

### Question 10

There were very few correct solutions. Most candidates had obviously had little, if any, experience in manipulating matrices and were unable to set up the basic matrices needed for each part. The basic rule of compatibility of matrices for multiplication – namely that for multiplication to be possible, the number of columns of the first matrix must equal the number of rows of the second, was only sketchily known by many candidates. Many candidates failed to realise that the blanks in the given arrays had to be replaced by “0” in the matrix prior to multiplication.

$$(i) \begin{pmatrix} 50 & 75 & 100 \end{pmatrix} \begin{pmatrix} 400 & 0 & 400 & 500 & 600 \\ 300 & 0 & 0 & 300 & 600 \\ 400 & 600 & 600 & 0 & 400 \end{pmatrix}, (ii) \begin{pmatrix} 400 & 0 & 400 & 500 & 600 \\ 300 & 0 & 0 & 300 & 600 \\ 400 & 600 & 600 & 0 & 400 \end{pmatrix} \begin{pmatrix} 13 \\ 7 \\ 10 \\ 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 16500 \\ 10200 \\ 18600 \end{pmatrix}$$

$$(iii) \begin{pmatrix} 2.10 & 3.00 & 3.75 \end{pmatrix} \begin{pmatrix} 16500 \\ 10200 \\ 18600 \end{pmatrix} = 135\,000.$$

### Question 11

Attempts at this question were very variable and rarely produced high marks. Most candidates were confident in completing the square of the quadratic, though having removed the “2” many left the “ $-8x$ ” and found  $b$  to be  $-4$ . Very few candidates realised that the answers to parts (i) to (iv) proceeded directly from the completion of the square. At least a third of all attempts gave the range in part (i) as  $5 \leq f(x) \leq 15$ , using the endpoints of the domain. Even worse were the attempts that just gave a table of values for  $f(x)$  for  $0 \leq x \leq 5$ . Only a few candidates realised that a function needed to be one-one over the whole domain to have an inverse. It was obvious from the answers that many candidates did not realise that a quadratic function could have an inverse providing that the domain did not include the value of  $x$  at which the graph of the function had a stationary value. Only a handful of solutions were seen in which the value of  $k$  was given as the  $x$ -value at the stationary point or directly from the first answer as  $x = -b$ . Only a few realised in part (iv) that the inverse of a quadratic could be obtained directly once the quadratic was written in the ‘completed square’ form.

Answers:  $a = 2$ ,  $b = -2$ ,  $c = -3$ ; (i)  $-3 \leq f(x) \leq 15$ ; (ii) Not one-one; (iii)  $k = 2$ ; (iv)  $g^{-1}: x \mapsto \sqrt{\frac{x+3}{2}} + 2$ .

### Question 12 EITHER

Rather surprisingly, especially considering the stereotyped part (b), this was not selected by many candidates and marks were low. In part (a) most candidates converted  $y = ax^n$  to  $\lg y = \lg a + n \lg x$ , realised that the gradient was  $n$ , but then took this to be  $\frac{64-27}{4-2.25}$  instead of using logarithms of these numbers. The few correct solutions came from solving a pair of simultaneous equations but expressing the logarithms to an insufficient accuracy meant that the final answers were often inaccurate. The graphs drawn in part (b) were of a high standard and most realised that the  $y$ -intercept was  $\ln m_0$ . Unfortunately errors over sign, either through taking the gradient of the line as positive or by thinking that the gradient was  $+k$ , meant that full marks were rarely achieved.

Answers: (a)  $n = 1.5$ ,  $a = 8$ ,  $p = 125$ ; (b) Graph,  $k = 0.04$ ,  $m_0 = 60$ .

**Question 12 OR**

This was the more popular option and proved to be a source of high marks, even from weaker candidates. Point  $C$  was usually obtained from solving simultaneously the line equations for  $BC$  and  $CD$ , and the standard of algebra was very good. Many weaker candidates obtained the equation of  $BC$  in the form

$\frac{y-11}{x-4} = \frac{1}{2}$  and then assumed that “ $y-11=1$  and  $x-4=2$ ”. Only a handful of solutions were seen in

which a ratio method was used to find  $E$ , most preferring to solve the simultaneous equations for  $AB$  and  $CD$ . A common error was to assume that  $C$  was the mid-point of  $ED$ . Attempts at part (ii) were pleasing, though again it was rare, but not unseen, to see solutions coming from considerations of the ratio of (length)<sup>2</sup>. The

more common solution was to use Pythagoras's Theorem along with the formulae “ $\frac{1}{2}bh$ ” and “ $\frac{1}{2}(a+b)h$ ”

but many others preferred to use the matrix method for area.

Answers: (i)  $C(14, 16)$ ; (ii)  $E(9, 26)$ ; (iii) 25:39.

**Paper 4037/02**

**Paper 2**

**General comments**

The overall performance of candidates was somewhat lower than in previous years. This was to be expected, perhaps, from the change in style of the examination to one in which the candidates' choice was much more restricted. Another contributory factor was the inclusion in the syllabus of a number of new, and less familiar, topics.

**Comments on specific questions****Question 1**

Most candidates could find the adjoint matrix correctly and the idea of multiplying by the reciprocal of the determinant was generally well known. Combination of the various minus signs caused difficulty for weaker candidates. Most candidates showed they understood how to combine matrices as required by  $\mathbf{A} - 3\mathbf{A}^{-1}$ . Many candidates did not understand the identity matrix, some took it to be  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  while others took it to be

$\begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$ . Some treated  $\mathbf{I}$  as though it was 1 leading to  $k = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$ ; the same result was obtained by

some candidates arriving at  $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} = k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ .

Answer: 10.

**Question 2**

Relatively few candidates scored full marks. Some drew graphs of  $y = x + 1$  and  $y = 2x - 3$ , whilst others drew graphs for positive values of  $x$  only. Attempts at the graph of  $y = |2x - 3|$  were generally better than attempts at  $y = |x| + 1$ . Many candidates failed to understand the shape of the graph between (1, 1) and (2, 1), joining these points by means of a straight line or a curve. Attempts at  $y = |x| + 1$  frequently resulted in graphs depicting  $y = |x + 1|$  or  $|y - 1| = x$ . Some candidates produced diagrams showing the four lines  $y = x + 1$ ,  $y = -x + 1$ ,  $y = 2x - 3$  and  $y = -2x + 3$ .

Answer: (ii) 2.

### Question 3

Candidates generally showed a lack of clarity and understanding in their use of set notation. This was particularly true in part (i) where  $(H \cap P)'$  was frequently given as the answer. Part (ii) resulted in considerably more correct answers with most candidates offering  $P \subset M$ , which was accepted, rather than  $P \subseteq M$ . Incorrect answers were usually either  $P \in M$  or  $M \subset P$ . A few candidates offered perfectly correct alternative answers e.g.,  $P \cap M = P$  or  $P \cap M' = \emptyset$ . Parts (iii) and (iv) presented some language difficulties as demonstrated e.g., by the answer "Only students studying mathematics" to part (iii). In general part (iii) was answered correctly but in part (iv)  $H \cup M$  was almost always taken to indicate either "students taking History or Mathematics" or "students taking History and Mathematics".

Answers: (i)  $H \cap P = \emptyset$ ; (ii)  $P \subseteq M$ ; (iii) Students studying Mathematics only; (iv) Students studying History or Mathematics or both, but not Physics.

### Question 4

All but the weakest candidates scored reasonably well on this question. The factor  $x + 2$  was usually spotted and the quadratic factor  $x^2 - 6x + 1$  almost always followed. Many candidates took  $x + 2$  to be a solution of the given equation with the result that  $x = -2$  never appeared. Most candidates proceeded from

$x^2 - 6x + 1 = 0$  to  $x = \frac{6 \pm \sqrt{32}}{2}$  but many could not then give the answer in the required form – some gave

decimal answers and other offerings were  $\frac{6 \pm 4\sqrt{2}}{2}$ ,  $3 \pm 4\sqrt{2}$ ,  $3 \pm \sqrt{8}$ ,  $6 \pm 2\sqrt{2}$ .

Answers:  $-2$ ,  $3 \pm 2\sqrt{2}$ .

### Question 5

This proved to be the most difficult question on the paper, mainly because candidates appeared unable to handle vectors in this situation. Many candidates omitted the question completely or made feeble attempts, sometimes introducing a spurious right-angled triangle of velocities. The relatively few candidates who quickly obtained  $50\mathbf{i} - 100\mathbf{j}$  almost invariably quoted this as the speed of the plane; some then found a relevant angle but the correct bearing was very rarely obtained. The majority of those making mainly successful attempts followed the tortuous route of calculating two speeds and the difference of two angles, constructing a triangle of velocity and then applying the cosine rule to find the speed, followed by the sine rule to obtain an angle leading to the bearing. But even those who managed to perform all these calculations correctly usually gave the bearing as  $153.4^\circ$  rather than  $333.4^\circ$ . Some candidates did not appreciate the significance of the 4 hours and inevitably became confused, trying to combine distance with velocity. Others ignored the unit vectors taking, for instance, the velocity  $(250\mathbf{i} + 160\mathbf{j}) \text{ kmh}^{-1}$  to indicate a speed of 410 km.

Answers:  $112 \text{ kmh}^{-1}$ ,  $333.4^\circ$ .

### Question 6

Although the better candidates produced a large number of correct evaluations of  $k$ , usually via the quotient rule, weaker candidates often failed to do so, the usual errors being misquoting the quotient rule, applying incorrect signs to the derivative of  $\cos x$  and/or  $\sin x$ , and spurious cancellations. Many candidates ignored the "Hence" and attempted the integration of part (ii) directly, resulting in answers involving  $\ln(1 - \sin x)$  or  $(x - \cos x)^{-1}$ . Strangely, many who understood that part (ii) involved the reversal of the result from part (i)

i.e.,  $\int \frac{1}{1 - \sin x} dx = \frac{\cos x}{1 - \sin x}$  took  $\int \frac{\sqrt{2}}{1 - \sin x} dx$  to be  $\frac{1}{\sqrt{2}} \left( \frac{\cos x}{1 - \sin x} \right)$ .

Answers: (i) 1; (ii) 2.



### Question 7

Candidates generally scored well on this question. Part (i) caused little difficulty to the large majority of candidates although many found it necessary to find angle  $AOB$  in degrees and then convert to radians. Some candidates used laborious methods, finding  $OX$  and then applying the sine rule or even the cosine rule. The ideas of arc length and area of sector were almost always correct, as was part (iii). Part (ii) caused more difficulty with a fairly large percentage of candidates attempting to obtain the answer by subtracting the perimeter of the sector from the perimeter of the triangle.

Answers: (ii) 21.8m; (iii) 11.5m<sup>2</sup>.

### Question 8

Better candidates were able to obtain full marks with relative ease. Some of the weaker candidates used an incorrect trigonometrical ratio in one of the triangles, but of those who used  $\sin\theta$  and  $\tan\theta$  correctly a considerable number were unable, or ignored, the request to “express  $AB$  in terms of  $\theta$ ”. Quite a few candidates applied the sine rule to triangle  $ADB$  arriving at  $AB = \frac{5 \sin(90^\circ - \theta)}{\sin \theta}$  which was acceptable; unfortunately  $\sin(90^\circ - \theta)$  rarely, if ever, resulted in  $\cos\theta$ , almost invariably becoming  $\sin 90^\circ - \sin\theta$ . Expressing  $\tan\theta$  in terms of  $\sin\theta$  and  $\cos\theta$  led some candidates to  $6\sin\theta = \frac{5 \sin \theta}{\cos \theta}$  and hence  $\cos\theta = \frac{5}{6}$ , but those candidates arriving at  $6\sin^2\theta = 5\cos\theta$  were usually able to complete the question successfully, although there were some errors in sign, and hence factorisation, and also a few candidates who took  $\cos\theta(6\cos\theta + 5) = 6$  to imply  $\cos\theta = 6$  or  $6\cos\theta + 5 = 6$ .

Answers: (i)  $6\sin\theta$ ,  $\frac{5}{\tan \theta}$ ; (ii)  $48.2^\circ$ .

### Question 9

- (a) Few of the candidates who chose to consider the discriminant took the simpler route of eliminating  $x$  to obtain a quadratic in  $y$ , the vast majority preferring to eliminate  $y$ , obtaining  $(x + k)^2 = 4x + 8$ . A variety of errors then occurred, with  $(x + k)^2$  becoming  $x^2 + k^2$  or  $x^2 + 2k + k^2$  or, most frequently, the equation above becoming  $(x + k)^2 - 4x + 8 = 0$ . Some of the weaker candidates were unable to identify correctly the elements  $a$ ,  $b$  and  $c$  of the discriminant  $b^2 - 4ac$ . Some candidates successfully applied the calculus; implicit differentiation was occasionally seen, but it was most usual for candidates to attempt, not always correctly, to differentiate  $(4x + 8)^{1/2}$ . Differentiation of  $(4x + 8)^{1/2}$ , whether correct or not, was as far as some candidates could go in that they did not understand the need to equate their result to 1, the gradient of the tangent  $y = x + k$ .
- (b) There was a widespread failure to identify this question with the routine solution of a quadratic inequality. The small minority of candidates who recognised that  $\{x : x > 2\} \cup \{x : x < -4\}$  implied  $(x - 2)(x + 4) > 0$  almost always proceeded quickly to the correct solution. Some candidates obtained the correct answers by solving  $4 + 2a = b$  and  $16 - 4a = b$  but many eschewed the equality signs attempting to solve  $4 + 2a > b$  and  $16 - 4a > b$ , arriving at  $a > 2$ ,  $b < 8$ . Many others took note of  $x > 2$  and  $x < -4$  and substituted  $x = 3$  and  $x = -5$  (or  $-3$ ) in the equation  $x^2 + ax = b$ .

Answers: (a) 3; (b) 2, 8.

### Question 10

Part (i) produced many correct solutions but also a fair number of inept attempts e.g.,  $2x - (x - 3) = 1$  or  $10$ ,  $\frac{2x}{x-3} = 1$  and  $\frac{\lg 2x}{\lg(x-3)} = 1$  or  $10$  followed by the “cancellation” of  $\lg$ . In part (ii) most candidates appreciated that change of base was necessary but many could not profitably proceed any further. The most successful candidates were those who replaced  $4 \log_3 y$  by  $\frac{4}{\log_3 y}$  and then used a further symbol (often  $y$ ) to represent  $\log_3 y$ . The alternative, replacing  $\log_3 y$  by  $\frac{1}{\log_y 3}$ , was seen infrequently, but many candidates

changed both terms on the left-hand side of the equation to logarithms to the base 10. Candidates frequently made complications for themselves by rendering the 4 as  $\log_3 81$  or  $\log_y y^4$  or  $\lg 10000$  depending on the base chosen. The most commonly occurring error was to write what should have been  $(\log_3 y)^2$  as  $\log_3 y^2$ ; this then became  $2\log_3 y$  or was combined with  $4\log_3 y$ , i.e.,  $\log_3 y^4$  to give  $\log_3 y^4 - \log_3 y^2 = \log_3 y^2$ .

Answers: (i) 3.75; (ii) 9.

### Question 11

This question was a good source of marks for many candidates. Nearly all candidates were capable of finding  $f^{-1}$  correctly, the only error occurring when  $x = 3y - 7$  became  $x - 7 = 3y$ . Similarly, apart from the occasional arithmetic error,  $g^{-1}$  was usually correct, although candidates were quite often unable to give 0 as the value of  $x$  for which  $g^{-1}$  is not defined; alternative offers were 2, 6 or  $-6$  while some candidates failed to offer any value. Relatively few candidates had any difficulty with part (ii). Poor algebra spoiled some

attempts with  $3\left(\frac{12}{x-2}\right)$  becoming  $\frac{36}{3x-6}$  or  $\frac{36}{x-2} - 7 = x$  becoming  $36 - 7 = x(x-2)$ . A few candidates

omitted the  $x$ , thus solving  $fg(x) = 0$ , whilst some confused the order of operation and, in effect, solved  $gf(x) = x$ . One or two of the weakest candidates attempted to solve  $f(x) \times g(x) = 0$  and the solution of  $f^{-1}g^{-1}(x) = x$  was also seen. Graphs usually contained correct segments of both lines but the choice of axes was such that all the points of intersection with the axes could not be shown, the coordinates often being calculated separately. Some of the weakest candidates clearly did not appreciate that  $y = 3x - 7$  and

$y = \frac{1}{3}(x + 7)$  were linear equations with their graphical representations being straight lines. Although it was

not essential, candidates might have used the reflective property of  $f$  and  $f^{-1}$  in the line  $y = x$  as a confirmation of the correctness of their graph but knowledge of this property was rarely in evidence although some candidates attempted to make use of it despite the differing scales on their axes. Many candidates read the final phrase of part (iii) as a request for the coordinate of the point of intersection of the graphs of  $f$  and  $f^{-1}$ .

Answers: (i)  $\frac{x+7}{3}; 2 + \frac{12}{x}, x = 0$ ; (ii)  $-10, 5$ ; (iii)  $f: (2\frac{1}{3}, 0), (0, -7)$ ;  $f^{-1}: (-7, 0), (0, 2\frac{1}{3})$ .

### Question 12 EITHER

This proved to be the easier of the two options with very many of the better candidates obtaining full marks.

Finding the coordinates of  $P$  was successfully accomplished by a variety of methods, using  $x_P = -\frac{b}{2a}$ ,

completing the square  $(x-3)^2 + 1 = 0$  leading to  $y_P = 1$  when  $x_P = 3$ , and finding  $x_P$  via  $\frac{d}{dx}(x^2 - 6x + 10) = 0$ .

Some weaker candidates quickly went wrong, finding  $P$  to be  $(2, 2)$  through assuming  $\frac{d}{dx}(x^2 - 6x + 10) = -2$

at  $P$ . Continuing with this line of reasoning candidates then found the equation of  $PQ$  to be  $y = 6 - 2x$  which, on solving with the equation of the curve, led to  $(x-2)^2 = 0$  and puzzlement. Other candidates found  $P$

correctly but then assumed that  $\frac{d}{dx}(x^2 - 6x + 10) = -2$  at  $Q$ . The integration, usually of  $x^2 - 6x + 10$

although quite frequently of  $(7-2x) - (x^2 - 6x + 10)$ , was very good and was almost always correct.

Evaluation of the integral was often correct but  $\int_0^3$  was also seen with some frequency. A correct plan for

finding the shaded area was sometimes lacking, with the area between the chord  $PQ$  and the curve being treated as though it was the area beneath the curve or with the entire area bounded by the axes and the lines  $x = 3$ ,  $y = 5$  and the chord  $PQ$  regarded as a trapezium.

Answer:  $9\frac{2}{3}$  units<sup>2</sup>.



**Question 12 OR**

This was clearly the less popular alternative and with good cause in that candidates rarely answered it well. There were two main reasons for this; firstly, an inability to find the value  $T$  and, secondly, a lack of understanding of the velocity-time graph. Virtually all candidates were able to evaluate  $v_B$  as 15. Many candidates made no attempt to find  $T$ ; some took it to be the value of  $t$  obtained from  $15 = \frac{1}{225}(20 - t)^3$  and

of those who understood that the required value of  $t$  was to be obtained from  $\frac{1}{225}(20 - t)^3 = 0$  most were unable to solve this equation, frequently expanding  $(20 - t)^3$ . In part (ii) virtually all candidates understood that  $\frac{dv}{dt}$  gives the acceleration and the only commonly occurring error was the omission of the minus sign

arising from  $\frac{d}{dt}(20 - t)$ . The sketch required in part (iii) was very rarely correct in that most offerings consisted of either a straight line joining (0, 0) to (5, 15) with a straight line joining (5, 15) to some point on the time axis, or a curve representing  $\frac{1}{225}(20 - t)^3$  for  $0 \leq t \leq 20$ . In the first of these cases the distance AC was calculated as the area of a triangle and so no integration was in evidence. In the second case the integral of  $\frac{1}{225}(20 - t)^3$  was usually taken to be  $\frac{1}{900}(20 - t)^4$ , even by those candidates who had included

$\frac{d}{dt}(20 - t)$  when dealing with the differentiation of part (ii), and then evaluated from 0 to 20.

Answers: (i)  $15\text{ms}^{-1}$ , 20; (ii)  $-0.48\text{ms}^{-2}$ ; (iv)  $93\frac{3}{4}\text{m}$ .

# FURTHER MATHEMATICS

## GCE Advanced Level

Paper 9231/01

Paper 1

### General comments

The majority of candidates produced good work in response to at least half of the questions, though, in contrast, there were some who clearly were ill-prepared for this examination and so, overall, made very little progress. At the outset to this report, therefore, it should be emphasised that achievement at this level requires knowledge of a syllabus which is end on to that for A Level Mathematics and thus is not an immediate consequence of this basic knowledge, however well understood.

Clarity and legibility of working varied with Centres to a considerable extent so that it is worth emphasising at the beginning of the life of this syllabus that Examiners can only mark what can be read. In any case, it must be helpful to the candidate to work in an ordered way so that when a response runs into difficulties errors can be identified easily.

Related to the need for coherence is, of course, the absolute need for accuracy. In this respect there were many deficiencies in all but the most simple situations.

These negative effects were augmented further by rubric infringements in **Question 12** which provides 2 alternatives. Since all questions are to be attempted, this is the only part of the paper where any rubric infringement is possible, yet despite the obvious waste of time that would be involved by this strategy, there were, nonetheless, a substantial number of candidates who tried to better their lot in this way, but to no avail. It is to be hoped, therefore, that future candidates will promote their own interests by keeping strictly to what the question paper asks them to do.

Knowledge of the syllabus and understanding of the concepts involved were uneven. Thus the syllabus material covered by **Questions 1** to **5** and **Question 11** was well understood and, in consequence, a significant minority of candidates obtained most of their marks in this area. At the other extreme, particular questions for which responses frequently showed a conceptual void, were **Question 6**, involving induction,

**Question 7** which requires the determination of  $\frac{d^2y}{dx^2}$  in terms of the parameter  $t$ , **Question 8 (ii)** which

requires the determination of the area of a surface of revolution about the  $y$ -axis, **Question 9** involving the use of complex numbers to sum a trigonometric series, **Question 10** which tests the basic ideas of linear spaces, and finally **Question 12 EITHER (ii)** and **Question 12 OR (iii)** on the use of the calculus in optimisation problems.

In summary, therefore, it can be said that lack of technical expertise together with inadequate syllabus coverage, both in extent and depth, were the main reasons why many candidates did not do well in this examination.

### Comments on specific questions

#### Question 1

Almost all candidates obtained the correct characteristic equation and solved it accurately. Subsequently there followed a variety of possible eigenvectors but, almost without exception, these were correct.

*Answer:* Eigenvalues are 1, 2; eigenvectors can be any non-zero scaling of  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ .

## Question 2

For the first part, the method of integration by parts was generally perceived to be an effective way to proceed. However, there were a number of errors, usually sign errors, in the working and also some cases of omission of limits, particularly in the 'u-v' term. In contrast, it was good to see a number of correctly worked arguments based on a consideration of, for example,  $D_x [(1-x)^n \cos x]$ .

For the second part, candidates generally used the reduction formulae correctly to obtain, essentially,  $S_3$  in terms of  $\sin(1)$ . However, in the context of numerical evaluation, about half of all candidates interpreted  $\sin(1)$  as  $\sin(1^\circ)$ .

Answer:  $S_3 = 0.042886$ .

## Question 3

The majority of candidates began by writing  $(2n-1)^3 = 8n^3 - 12n^2 + 6n - 1$  and then after applying standard summation formulae, worked accurately to obtain the displayed result. Only a minority of responses proceeded along the lines of  $S_N = \sum_1^{2N} n^3 - 8 \sum_1^N n^3 = \frac{1}{4}(2N)^2(2N+1)^2 - 2(N)^2(N+1)^2$ , etc., from which, using obvious factorisations, the result follows immediately.

Most candidates began the second part of this question with a correct preliminary result such as  $\sum_{n=N+1}^{2N} n^3 = 4N^2(8N^2-1) - N^2(2N^2-1)$  and then simplified this expression without apparent difficulty. A small minority of responses showed incorrect partitions which may be symbolised as  $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N+1}$  or  $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N-1}$ .

A few candidates expanded  $(2n-1)^3$  and then, again, applied standard summation formulae, but such a complicated strategy proved to be very error prone.

Answer:  $3N^2(10N^2-1)$ .

## Question 4

This question was answered accurately by most candidates. Responses showed, almost without exception, a correct overall strategy and there were few scripts in which the correct general solution did not appear. In sharp contrast, very few candidates were able to provide a satisfactory explanation as to why, independently of the initial conditions,  $y \approx 3x + 2$  when  $x$  is large and positive. In fact, something like the argument set out below was expected.

As  $x \rightarrow \infty$ ,  $e^{-x} [A \sin(2x) + B \cos(2x)] \rightarrow 0$ , whatever the values of  $A$  and  $B$  and hence whatever the initial conditions. Thus independently of the initial conditions,  $y \approx 3x + 2$  for large positive  $x$ .

Answer: General solution:  $y = e^{-x} [A \sin(2x) + B \cos(2x)] + 3x + 2$ .

## Question 5

Responses to this question showed some suboptimal solution strategies and also many basic working errors.

In the first place, the required  $y$ -equation can be obtained expeditiously by noting that  $y = \frac{x}{2x-1} \Rightarrow x = \frac{y}{2y-1}$  and so substituting for  $x$  in the given cubic leads at once to the required result for  $y$ .

For part (i), it is then sufficient to observe that, as from the  $x$  and  $y$  cubic equations it is obvious that  $\alpha\beta\gamma = -1$  and that  $\alpha\beta\gamma/(\alpha-2)(\beta-2)(\gamma-2) = -\frac{1}{3}$ , then  $(\alpha-2)(\beta-2)(\gamma-2) = 3$ .

For part (ii), the optimal argument is also simple. Thus it is only necessary to write the following:

$$\sum \alpha(\beta - 2)(\gamma - 2) = (\alpha - 2)(\beta - 2)(\gamma - 2) \sum \alpha / (\alpha - 2) = 3 \times 3 = 9.$$

However, the majority of candidates got involved in more complicated arguments. Thus there were even some who first attempted to evaluate  $\sum \alpha / (\alpha - 2)$ ,  $\sum \alpha \beta / (\alpha - 2)(\beta - 2)$  and  $\alpha \beta \gamma / (\alpha - 2)(\beta - 2)(\gamma - 2)$  from the given x-equation, and then started all over again in an attempt to find answers for parts (i) and (ii). Such protracted arguments generated many errors.

Answers: (i) 3; (ii) 9.

### Question 6

The quality of most responses to this question was not good. Even where the central part of the induction argument was present, it was common for there to be no clear statement of the inductive hypothesis nor of an unambiguous conclusion. In fact, only a minority of candidates produced a completely satisfactory response.

In this respect something like the following was required:

Let  $P(k)$  be the statement,  $u_k < 4$  for some  $k$ .

Then  $P(k) \Rightarrow 4 - u_{k+1} = 4 - (5u_k + 4)/(u_k + 2) = (4 - u_k)/(u_k + 2) \Rightarrow u_{k+1} < 4$ , since all  $u_n$  are given to be positive. Thus  $P(k) \Rightarrow P(k+1)$ , and since also  $P(1)$  is true, for it is given that  $u_1 < 4$ , then by induction it follows that  $P(n)$  is true for all  $n \geq 1$ .

In the second part of the question, few candidates made significant progress. All that was required here was to write  $u_{n+1} - u_n = \dots = (4 - u_n)(u_n + 1)/(u_n + 2) > 0$ , and thus as  $0 < u_n < 4$  and all  $u_n$  are positive, then  $u_{n+1} > u_n$ .

In this context, one had the impression that some candidates were groping towards this kind of argument, but lacked the technical expertise to see it through.

### Question 7

This standard exercise involving the obtaining of  $\frac{d^2 y}{dx^2}$  in a parametric context showed up at least one important conceptual error. Overall the quality of responses can only be described as disappointing.

In the first part of the question, the working was generally methodologically correct and accurate. It was in the remainder of the question that many responses fell apart. The most common error was the supposition

that  $\frac{d^2 y}{dx^2} = D_t \left( \frac{dy}{dx} \right)$ . Actually from this it is possible, in this case, to obtain the required values of  $t$ , but such

arguments, which are essentially incorrect, obtained little credit. Another persistent, but less common, error

was the writing of  $\frac{d^2 y}{dx^2}$  as  $\frac{d^2 y}{dt^2} \div \frac{d^2 x}{dt^2}$ . This result, of course, did not get any credit.

Answers:  $\frac{dy}{dx} = t^4(t-3)e^{-t}$ ;  $\frac{d^2 y}{dx^2} = (t^7 - 8t^6 + 12t^5)e^{-t}$ ;  $t = 2, 6$ .

### Question 8

This question was not answered as well as might be expected and certainly one persistent cause of failure was lack of technical competence.

In part (i), most responses showed a correct integral representation of the arc length, but nearly a half of all candidates did not recognise that  $\sqrt{\left[ \frac{1}{4}(x^{1/3} - x^{-1/3})^2 + 1 \right]} = \frac{1}{2}(x^{1/3} + x^{-1/3})$  and so made no further progress.

However, most of those who did get through this stage did go on to obtain the required result.

In part (ii), it was clear that many candidates were put off by having to consider the surface area  $S$  generated by the rotation of  $C$  about the  $y$ -axis. Thus the (correct) integral representation of  $S$  by  $\pi \int_1^8 x(x^{1/3} + x^{-1/3})dx$  appeared in only a minority of scripts, though usually this was evaluated accurately.

Answers: (i)  $\frac{63}{8}$ ; (ii)  $\frac{2556\pi}{35}$ .

### Question 9

There were very few good quality responses to this question.

In the first part, a common error was the supposition that the given series is geometric with common ratio  $\frac{1}{3}\cos 2\theta$ . Thus, within the scope of this view of the question, complex numbers did not feature at all. In

those responses which did show an attempt to determine the real part of  $S_N = \frac{1-z^N}{1-z}$  where  $z = \frac{1}{3}e^{2i\theta}$ , there was much suppressed detail and erroneous working to be found. Thus although there was some appreciation of how to proceed, there were relatively few who could produce a completely accurate proof of the given result.

In the last part of this question, only a small minority of candidates comprehended that since  $3^{-N+1} \rightarrow 0$ ,  $3^{-N+2} \rightarrow 0$ , as  $N \rightarrow \infty$ , then the given infinite series is convergent. Even where such statements appeared in responses, it was not always the case that the correct sum to infinity emerged from the working.

Answer:  $S_\infty = \frac{9 - \cos 2\theta}{10 - 6\cos 2\theta}$ .

### Question 10

The majority of those candidates who produced serious work in response to this question established the linear independence of the vectors  $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$  by the use of equations and likewise for the vectors  $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ . In contrast, a minority reduced the  $4 \times 3$  matrices  $(\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3)$  and  $(\mathbf{b}_1 \ \mathbf{b}_2 \ \mathbf{b}_3)$  to the echelon form. This is extremely easy to effect yet, surprisingly, there were errors even at this very basic level of operation.

There were also those that argued that, as it is given that the three vectors  $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$  span  $V_1$ , then  $V_1$  must be of dimension 3, and likewise for  $V_2$ . Such arguments implicitly ignore the possibility of linear dependence and as such are worthless.

Only a minority appeared to comprehend that  $\dim(V_3) = 2$  and some gave 3 or even 4 vectors as a basis for this subspace. Thus again there was clear evidence of a general lack of understanding of the basic concepts of linear spaces.

About half of all candidates were able to produce 2 linearly independent vectors which belong to  $W$ , as required in part (i), but in part (ii), few could produce a satisfactory argument to show that  $W$  is not a linear space. This is most easily effected by showing closure does not hold.

A basis for  $V_3$  is  $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$ ; (i)  $\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ ; (ii)  $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \notin V_1 \cup V_2 \quad - \quad V_1 \cap V_2 = W$ .

### Question 11

Most responses showed correct methodology and accurate working to the extent that it can be said that this question was well answered by the majority of candidates.



In part (i) the vector product was used in a relevant way. Almost all failures to produce a correct result were due to accuracy errors.

The working in part (ii) was generally accurate. Most responses showed the position vector of a general point on  $l_3$  in terms of a single parameter,  $s$ . Subsequently, 3 linear equations in  $s$  and  $t$  appeared and it was good to see that, almost always, the values of  $s$  and  $t$  obtained from 2 of the equations were checked out in the third.

In part (iii) since  $l_1 \cap l_3 \equiv \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ , then the shortest distance,  $p$ , between  $l_1$  and  $l_2$  can be evaluated by using  $l_2 \cap l_3 \equiv 4\mathbf{i} - \mathbf{j} - 9\mathbf{k}$ . (This follows immediately from the working in part (ii) and, in fact, most candidates had already obtained this position vector.) Thus  $p = \sqrt{3^2 + 3^2 + 6^2} = 3\sqrt{6}$ .

Very few candidates argued in this simple way, but preferred to use the standard formula for the length of the common perpendicular between 2 skew lines. This strategy was not always implemented accurately so that arguments such as  $p = \frac{(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \cdot (3\mathbf{i} + 7\mathbf{j} + 7\mathbf{k})}{\sqrt{6}} = \dots = 3\sqrt{6}$ , often appeared in an erroneous form.

Answers: (i)  $7x - y + 4z = -7$ ; (iii)  $3\sqrt{6}$ .

### Question 12 EITHER

A minority of candidates began by attempting to resolve the given rational function of  $x$  into partial fractions without any particular objective in view. Moreover, a number of such resolutions began with the form  $B/(x - 2a) + C/(x + 2a)$ , and not with  $A + B/(x - 2a) + C/(x + 2a)$ . In fact, the derivation of the correct partial fraction form of  $y$  does not enhance prospects in parts (i) or in (iii), though it can be helpful in part (ii) if the sign of  $\frac{d^2y}{dx^2}$  is used to determine the nature of the stationary points.

In part (i) most candidates obtained, or simply wrote down, the equations of the vertical asymptotes though some missed out the horizontal asymptote altogether or gave an incorrect result, e.g., the  $x$ -axis.

In part (ii) most candidates got as far as showing  $\frac{dy}{dx} = 0 \Rightarrow x = a, 4a$  and went on to obtain the ordinates

of the stationary points. Beyond this, however, responses generally ran into difficulties, mainly on account of inaccuracies in the working. Candidates, generally, appeared not to have the technical expertise necessary

either to obtain a correct result for  $\frac{d^2y}{dx^2}$ , in any form, or to establish its sign at the stationary points of  $C$  in a

convincing way. A simple argument in this context is to observe that  $\frac{dy}{dx}$  can be written as  $(x - a)F(x)$  where

$F(x)$  is an easily identifiable rational function, actually it is  $2a^2(x - 4a)/(x^2 - 4a^2)^2$ , and thus as

$\frac{d^2y}{dx^2} = F(x) + (x - a)F'(x)$ , then at  $x = a$ ,  $\frac{d^2y}{dx^2} = F(a) = \frac{-2}{3a} < 0$ . The other stationary point can be

considered similarly by writing  $y = (x - 4a)G(x)$ , where  $G(x) = 2a^2(x - a)/(x^2 - 4a^2)^2$ . It then follows that at

$x = 4a$ ,  $\frac{d^2y}{dx^2} = G(4a) = \frac{1}{24a} > 0$ . However, very few candidates argued in this way.

In part (iii) few sketch graphs were without error and, in fact, some did not even show 3 branches. Undoubtedly failure here was due to erroneous or incomplete results obtained earlier on. No doubt, on this account, some candidates must have been baffled by the clear inconsistency between the number of asymptotes obtained in part (i) and the display on their graphic calculator. This is especially a type of question for which the intelligent use of such a calculator can materially enhance the quality of responses, but in this instance there was very little evidence of such a causal relationship. Less than half of all candidates obtained full credit here.

Answers: (i)  $x = 2a, x = -2a, y = a$ ; (ii) maximum at  $(a, 0)$ , minimum at  $(4a, \frac{3a}{4})$ .

### Question 12 OR

In part (i) although the outline of  $C$  was usually correct, there was a persistent failure to indicate the scale in terms of  $a$ . Responses to this question were expected to include a clear indication of the location of the origin, the line  $\theta = 0$  and the labelling of the extreme point  $(2a, 0)$ , yet in this respect there were many deficiencies.

In part (ii) most responses began with the integral  $\frac{k}{2} \int_0^\pi a^2(1+\cos\theta)^2 d\theta$  where, in most cases,  $k$  was either 1 (the most popular erroneous value) or 2, which is correct. Some candidates started with  $k = 1$ , but then introduced a factor of 2, without explanation, later on in the working. A few started with other correct forms such as  $\frac{1}{2} \int_{-\pi}^\pi a^2(1+\cos\theta)^2 d\theta$ .

For the integration, the working was generally accurate and complete.

In part (iii) the starting point here is to write  $y = r \sin \theta = a \sin \theta (1 + \cos \theta)$  and then to set  $\frac{dy}{d\theta} = 0$ . In this

respect, it is helpful to write  $y = a \sin \theta + (a/2) \sin(2\theta)$  so that  $\frac{dy}{d\theta} = 0 \Rightarrow \cos \theta + \cos(2\theta) = 0$  follows

immediately. The minimum of  $y$  is then easily found to occur at  $\theta = -\pi/3$ . However, only a small minority of candidates were able even to formulate  $y$  in terms of  $\theta$ , as above, and few of these went on to obtain the correct minimum value of  $y$ .

Finally there was a small minority of candidates who attempted to obtain the  $x - y$  equation of  $C$  and then by implicit differentiation go on to obtain the minimum of  $y$ . However, few of these had the necessary technical expertise to work this complicated strategy through to a successful conclusion.

Answer: (iii) Minimum value of  $y = \frac{-3\sqrt{3}}{4}$ .

Paper 9231/02

Paper 2

### General comments

The standard of the candidates was very variable, some producing excellent work while others had no real grasp of the syllabus. With the exception of the latter, almost all candidates were able to complete all the questions, suggesting that there was no undue time pressure. Although intermediate working was usually shown, some candidates simply wrote down final values of, for example, variances and correlation coefficients. Where such values were incorrect, these candidates may have needlessly lost marks for a correct method, since insufficient working was given to demonstrate their method of calculation. There appeared to be a slight preference for the Mechanics alternative over the Statistics one in the final question, though in some cases different candidates from the same Centre made differing choices.

### Comments on specific questions

#### Question 1

Finding the impulse of the force from the product of the force's magnitude and its period of action rarely presented problems, and the units were usually given correctly. Most candidates then equated the impulse to the product of the bullet's mass and the required speed, again stating the units of the result, while others first calculated the acceleration and hence the speed.

Answers: (i) 20 Ns; (ii) 250 ms<sup>-1</sup>.

## Question 2

The question states that the components of the velocity after the collision should first be found, but some candidates ignored this and tried unsuccessfully to derive the given equation directly by some invalid method. Instead they should have noted that the component  $U \cos \theta$  parallel to the cushion is unchanged, while the perpendicular component changes by a factor  $e$  to  $eU \sin \theta$ . The most convincing way of finding the lost kinetic energy is to consider the difference in the total kinetic energy before and after impact. Some candidates apparently relied on the fact that only the velocity component perpendicular to the cushion changes, and therefore found only the loss in the corresponding component of the kinetic energy. Unfortunately the majority of candidates who derived the given result in this way did not give an explicit justification, leaving open the possibility that they had simply worked backwards from the expression quoted in the question without any real understanding.

## Question 3

The simplest approach is to use the expression for the moment of inertia of a rectangular lamina given in the *List of Formulae*, substituting  $\frac{r}{2\sqrt{2}}$  in place of  $a$  and  $b$ . Many candidates failed to do this, with some using the given formula for a thin rod in association with the perpendicular axis theorem, but without adequate justification, and others purporting to achieve the given result without any valid reasoning. The second part, concerning the moment of inertia of the combined lamina, was frequently omitted. Those who attempted it successfully usually related the masses of the square and circular laminas to that of the combined body in terms of their common density, and substituted for the latter in an expression for the required moment of inertia. Although many candidates realised that the final part can be solved using conservation of energy, they often omitted one of the three contributory energy terms. An alternative valid approach which was also seen is to relate the net force and the couple acting on the particle and the lamina to their linear and rotational acceleration respectively.

Answer:  $6.52 \text{ rad s}^{-1}$ .

## Question 4

The first equation of motion was often derived successfully by applying Newton's Law perpendicular to the string, though some candidates wrongly considered the radial direction. Most stated the approximation  $\sin \theta \approx \theta$  correctly, and knew the general approach to expressing the right hand side of equation (A) in its alternative form. While most candidates realised that this expression could also be rewritten in terms of  $\phi$ , many overlooked the left hand side of the equation, while a few seemed to believe that the rearrangement is only valid if  $\phi$  is small. The period was often found correctly from  $\frac{2\pi}{\omega}$ , but common faults were to omit the length of the pendulum, or less seriously not to simplify the expression. The value  $\alpha$  about which  $\theta$  now oscillates defeated the great majority of candidates, most of whom made no attempt to find it.

Answer: (ii)  $R = \frac{25g}{24}$ ,  $\alpha = \tan^{-1} \left( \frac{7}{24} \right)$ .

## Question 5

The tension is found in both parts by taking moments for the stone about A. Although this is fairly simple for  $T_1$ , a common fault was to include the moment of the tension in only one of the sections  $DH$  or  $DK$  of the rope. The coefficient of friction  $\mu$  is as usual related to the friction  $F$  and reaction  $R$  by  $\mu \geq \frac{F}{R}$ , with  $F$  and  $R$  found by horizontal and vertical resolution of forces. Finding the correct moment equation in the second part presented significantly more difficulty, since the most relevant angles are no longer  $30^\circ$  or  $60^\circ$ , thus requiring some trigonometric effort, even though many candidates overlooked this.

Answers: (i) 177, 0.5; (ii) 131.

**Question 6**

Although most candidates knew how to find the confidence interval in principle, the majority used an incorrect tabular value instead of the  $t$ -value 2.718, with a high proportion opting instead for the  $z$ -value 2.326 or 2.054. The necessary assumption that the population is normal was frequently omitted.

Answer: [55.1, 61.2].

**Question 7**

The usual approach to this question is to use the binomial distribution  $B(5, 0.5)$  to calculate the expected values corresponding to the second row of the given table, and then calculate the corresponding  $\chi^2$  value, here 5.13. Comparison with the tabular value 11.07 leads to the conclusion that the coins are fair. While several candidates chose instead to attempt to calculate an appropriate  $z$ -value, their approach was usually invalid.

**Question 8**

Most candidates appreciated that the  $\chi^2$  test is appropriate here, and found the value of approximately 4.0 correctly. Comparison with the tabular value 9.488 leads to the conclusion of independence. The second part was by contrast very poorly done, with only a handful of candidates both identifying the problem of an expected value being less than 5, and identifying it as roughly 4.85 in the Low/Low cell.

**Question 9**

Almost all candidates knew how to find the coefficients  $a$  and  $b$ , usually by first using the given formula for  $b$ , and less often by solving the linear least squares equations explicitly. However a common fault was not to retain additional figures in the working, and the resulting rounding errors affected subsequent calculations. Substitution of  $x = 80$  gives the corresponding value of  $y$  and hence the solution in part (ii). Part (iii) was frequently, and wrongly, answered by substituting a value of either 2 or 2000 in the equation of the regression line, instead of noting that the ratio of a change in  $y$  to the corresponding change in  $x$  is  $b$ . Most candidates applied the formula for the product moment correlation coefficient  $r$ , and many also commented that their preceding answers are reliable. The correct approach to the final part is to note that the product of the two regression coefficients of  $y$  on  $x$  and  $x$  on  $y$  is  $r^2$ , since calculating the required coefficient from its formula is not making use of the previous answers as specified in the question.

Answers: (i)  $a = -1.75$ ,  $b = 1.05$ ; (ii) \$82400; (iii) \$2100; (iv) 0.996; (v) 0.944.

**Question 10**

The correct test to apply in the first part is the two-sample  $t$ -test with a common unknown population variance, but many candidates wrongly applied the paired-sample one. The former test yields a  $t$ -value of magnitude 1.13, and comparison with the tabular value 1.812 leads to the conclusion that the racing driver's claim is not justified. The corresponding method should be used for the confidence interval, and here the appropriate tabular value of  $t$  is 2.228.

Answer: [-32.6, 10.6].

**Question 11 EITHER**

The first part is readily solved by equating the kinetic and potential energies of the particle at the initial and final points, and simplifying. The following part, in which the required force  $R$  is found by summing the other two radial forces on the particle, presented no difficulty for most candidates, and they usually then equated  $R$  to zero in order to solve the resulting equation for  $\theta$ . The final two parts proved much more challenging, however, with some candidates vainly attempting to solve part (iii) by considering the vertical component of the particle's motion. The correct approach is to find the constant horizontal component of its velocity and also the distance to the vertical through  $O$ , and hence the time. The final part is concerned with the vertical motion under gravity, and is best solved by showing that in the given time the particle falls a distance 0.75 m, which equals the height above  $D$  of the point at which contact is lost with the hoop.

**Question 11 OR**

Most candidates were able to sum the first three terms of the Poisson expansion with parameter 3, and subtract their sum from unity in order to find the first probability. The second part needs only a realisation that the probability of picking up no passengers in a period of  $t$  hours equals the first term of the Poisson expansion with parameter 12. Convincing explanations of the given equation for  $P(T < t)$  were very rare, however, and many candidates made no serious attempt at this. By contrast most found the probability density function  $12e^{-12t}$  by differentiation, but while some were able to quote the values of  $E(T)$  and  $\text{Var}(T)$ , others attempted to find them by integration, often unsuccessfully. The median time is found by equating the given expression for  $P(T < t)$  to 0.5 and solving for  $t$ , and the only common fault here was to round the answer to fewer than the 3 significant figures specified in the rubric.

Answers: (i)  $0.577$ ;  $\frac{1}{12}$ ;  $\frac{1}{144}$ ;  $0.0578$  hours.



# MATHEMATICS

## GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01

Paper 1

### General comments

The majority of candidates had been well prepared for this paper which they generally found to their liking. There were some excellent responses, but also some from candidates for whom the level seemed too advanced. The standard of algebra was good throughout and most scripts were easy to mark with working shown in full. There was little evidence that candidates had been forced to rush to complete the paper in the allocated time.

### Comments on specific questions

#### Question 1

This proved to be a successful starting question with the majority of candidates correctly eliminating one variable to form a quadratic equation in the other. Apart from the occasional algebraic or arithmetic slip, most attempts were perfectly correct.

Answers: (12, -1.5) and (-3, 6).

#### Question 2

Part (i) was very well done but a surprising number of candidates failed to spot the link between the two parts. Of the rest, the setting up and subsequent solution of the quadratic in  $\cos x$  was accurately done and most candidates realised that there were two solutions in the required range.

Answers: (i) Proof; (ii)  $60^\circ$  and  $300^\circ$ .

#### Question 3

A surprising number of candidates read  $3\sqrt{x}$  as  $x^{\frac{1}{3}}$  but were able to obtain the method marks available throughout the question. The solution of " $3\sqrt{x} = x$ " also presented problems with  $(3\sqrt{x})^2 = 3x$  being a common error. The standard of integration was accurate, though inability to evaluate  $2x^{\frac{3}{2}}$  at  $x = 9$  caused further problems.

Answers: (i) (9, 9); (ii) 13.5 unit<sup>2</sup>.

#### Question 4

This was again well answered and there were many correct answers to both parts. In part (i), most candidates obtained  $d = 1.5$  but  $S_n = \frac{n}{2}(a + (n-1)d)$  and use of  $u_n$  instead of  $S_n$  were two common errors.

In part (ii), most candidates obtained  $r^4 = 1.5$  but use of decimals to insufficient accuracy or use of  $S_n$  instead of  $u_n$  led to frequent loss of marks.

Answers: (i) 750; (ii) 40.5.

**Question 5**

As in a similar 8709 question last year, the main error came in the first part with about a quarter of all candidates ignoring, or failing to cope with, the dimensions of the cylinder (height of 12 units and radius of 4 units). The ability to use the scalar product to calculate an angle was excellent but unfortunately many candidates believed that angle  $OMC'$  came from the scalar product of **OM** with **MC'** rather than with **MO** and **MC'**.

Answers: (i)  $4\mathbf{i} - 6\mathbf{k}$ ,  $4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ ; (ii)  $109.7^\circ$ .

**Question 6**

There were many completely correct solutions to this question but in part (i) it was disappointing to see so many candidates failing to realise the need to use radians and failing to realise that  $\sin(\pi/2) = 1$  and that  $\sin(3\pi/2) = -1$ . In part (ii) only a relatively small number of candidates realised that there was a second solution in the domain (i.e.  $x = \pi - 0.64$ ) and many candidates again failed to use radian measure. The sketch graphs of  $y = 5\sin x - 3$  were very variable with many candidates failing to show the maximum and minimum points at  $\pi/2$  and  $3\pi/2$  respectively.

Answers: (i)  $a = 5$  and  $b = -3$ ; (ii) 0.64, 2.50; (iii) Sketch.

**Question 7**

Part (i) was well answered with most candidates bisecting  $AB$  at  $M$  and using right-angled trigonometry to evaluate angle  $AOM$  and then doubling. Others preferred to use the cosine rule and were generally accurate. Candidates should read the question carefully and ensure that the angle is shown to be 1.855 radians and not 1.85 radians. Part (ii) was nearly always correct, though some candidates were confused as to the difference between segment and sector. Less than a half of all candidates coped with part (iii) – the most common error being to consider the unshaded area as “rectangle + sector” area rather than “rectangle + sector – triangle” area.

Answers: (i) Proof; (ii)  $371 \text{ cm}^2$ ; (iii)  $502 \text{ cm}^2$ .

**Question 8**

This caused considerable difficulty for most candidates. The formulae for surface area and volume of a cylinder were poorly learnt and many candidates failed to appreciate the implication of “open at one end”. Consequently answers were “fiddled” and it was quite common to see a correct formula for “volume” changed in order to produce the required result. Most candidates realised the need to use calculus for parts (ii) and (iii) and the standard of differentiation was good. Coping with the constant “ $\frac{1}{2}\pi$ ” caused many problems as this was often wrongly used in expanding the bracket, or omitted completely thereby causing an incorrect value for  $V$ . Surprisingly many candidates also omitted to answer the request to find the stationary value in part (iii). Use of the second derivative to differentiate between maximum and minimum points was well done.

Answers: (i)  $h = \frac{192 - r^2}{2r}$ ; (ii) 8; (iii) 1610, maximum.

**Question 9**

This question was answered badly with a lot of confusion over the equation of the curve and the equation of the tangent. The use of the formula  $m_1 m_2 = -1$  to find the gradient of the normal was accurate but many candidates failed to obtain a numerical value for this prior to finding the equation of the line. A significant number of candidates took the x-axis as  $x = 0$  instead of  $y = 0$ . There were only a small proportion of correct answers to part (ii) with candidates either integrating incorrectly, usually by omitting the “ $\frac{1}{2}$ ”, or by completely ignoring the constant of integration. Part (iii) was usually correct, though many candidates had given up before reaching it.

Answers: (i)  $7\frac{2}{3}$ ; (ii)  $y = 7 - \frac{6}{2x+1}$ ; (iii) 0.4 units per second.

**Question 10**

This was very well answered and generally a source of high marks. Apart from a few who confused gf with fg, part (i) was nearly always correct. Part (ii) presented more problems especially when candidates produced sketches with different scales on the two axes. The sketch of  $y = f(x)$  was usually correct and most candidates realised that  $f^{-1}$  was a reflection of  $f$  in the line  $y = x$ . Unfortunately because of different scales this often led to  $y = f^{-1}(x)$  having a negative gradient and being positioned in the wrong quadrant. Part (iii) was extremely well answered with inverse functions being correct and with an impressive standard of algebra used to solve the resulting quadratic equation in  $x$ .

Answers: (i)  $7\frac{1}{2}$ ; (ii) Sketch; (iii)  $f^{-1}(x) = \frac{1}{3}(x - 2)$ ,  $g^{-1}(x) = \frac{6 - 3x}{2x}$ ,  $x = 2$  or  $-4\frac{1}{2}$ .

**Paper 9709/02**

**Paper 2**

**General comments**

A wide range of ability of candidates was apparent in the responses to the paper. A significant number of candidates scored marks of 30 or more and displayed a high degree of mathematical expertise. At the same time, there were many candidates who were clearly not equal to the demands of the syllabus and of this paper, and such candidates struggled to record enough marks to produce a total in double figures.

The overwhelming majority of candidates had sufficient time to attempt all of the questions; those which were answered well included **Questions 2, 4 (i) and 7**, and those which caused widespread difficulty were **Questions 4 (iii) and 6 (b)**. Responses to **Questions 1, 3 and 6 (a)** were mixed. It was disappointing when questions apparently providing a straightforward and familiar test of basic syllabus topics were not answered with much conviction.

Candidates are advised to work through the questions sequentially, but many were unable to do so. Sometimes 2, 3 or even 4 attempts were made at a solution, and a lack of confidence seemed to lie at the root of this approach.

**Comments on specific questions****Question 1**

Although a majority of candidates made a good enough attempt to score 2 or 3 marks, few could successfully produce the final solution, often due to the belief that  $a > b$  implies  $(-a) > (-b)$ . A significant majority could only show that  $x < 1$ , and were unable to totally remove the modulus signs. Those who squared each side of the inequality were invariably more successful.

Answer:  $x < 1$ ,  $x > 7$ .

**Question 2**

- (i) This part was invariably well answered. Having set  $f(-2) = 0$ , several candidates then erred in expanding and merging the terms.
- (ii) Almost every candidate scored the first two marks, but several made no attempt to factorise the quadratic factor or used inappropriate signs.

Answers: (i)  $a = 7$ ; (ii)  $f(x) \equiv (x + 2)(x - 1)(3x + 4)$ .

### Question 3

This question produced the widest range of marks. Many excellent solutions were seen by the Examiners, but at the other extreme there were attempts based on using  $y$  as a linear function of  $x$  rather than using  $\ln y$  against  $\ln x$ . Several solutions featured the correct gradient and vertical intercept of the line without any link between those quantities and those of  $n$  and  $\ln A$ .

Answers:  $A = 9.97$ ,  $n = -0.15$ .

### Question 4

- (i) This part was generally well answered except where  $\tan x$  was set equal to  $\frac{3}{2}$  rather than to  $\frac{2}{3}$ .
- (ii) Almost no solution featured a second angle, less than zero, corresponding to  $\cos^{-1}\left(\frac{3.5}{R}\right)$ . The basic technique leading to the first solution of  $\theta = \alpha + \cos^{-1}\left(\frac{3.5}{R}\right)$  was understood by almost every candidate, though a few tried to square each side using the incorrect identity  $(a + b)^2 \equiv a^2 + b^2$ .
- (iii) Virtually no solution was stated, as requested in the question paper, using the results from part (i). Candidates preferred to use the calculus to seek a stationary point; this arduous way to earn one mark was sometimes further complicated by a failure to calculate the  $y$ -coordinate at the point.

Answers: (i)  $R = \sqrt{13}$ ,  $\alpha = 33.7^\circ$ ; (ii)  $47.6^\circ$ ,  $19.8^\circ$ ; (iii)  $(33.7, \sqrt{13})$

### Question 5

- (i) Surprisingly few candidates could differentiate the function  $2xe^{-x}$ , with many answers containing only one term. Of those candidates who successfully differentiated, many could not solve the equation  $(1 - x)e^{-x} = 0$ ; finite solutions of  $e^{-x} = 0$  were sought, or logarithms used.
- (ii) Despite the answer being given, this part also defeated many candidates. Again, logarithms were resorted to in many solutions.
- (iii) Although extremely popular, most final answers were given as 0.35, seemingly on the basis that no iterate was smaller than this number, even though values in excess of 0.355 were invariably given as the last iterate.

Answers: (i)  $(1, 2e^{-1})$ ; (iii) 0.36.

### Question 6

- (a)(i) Many candidates obtained a multiple of  $\sin 2x$  as  $\int \cos 2x \, dx$ , but a variety of other solutions of the form  $\lambda \cos 2x$ ,  $\beta \sin(2x^2)$  or  $\mu \sin x$  were seen.
- (ii) Few candidates recognised that  $\cos^2 x$  takes the form  $a + b \cos 2x$  and then successfully integrated. Other solutions featured a wide variety of false integrals such as  $\frac{\cos^3 x}{3}$  or  $\frac{\cos^3 x}{\sin x}$ .
- (b)(i) Only a handful of candidates produced 3 correct ordinates; 2 or 4 ordinates were common. Among solutions featuring the correct formula, many were spoilt by use of  $22.5$  (degrees) instead of  $\frac{\pi}{8}$  (radians) for  $h$ .

- (ii) Few correct graphs of  $y = \sec x$ ,  $0 \leq x < \frac{\pi}{2}$ , were seen. Many candidates did not attempt to stretch the curve.

Answers: (a)(ii)  $\frac{1}{8}(\pi - 2)$ ; (b)(i) 0.90, (ii) over-estimate.

### Question 7

This question proved to be the saviour of many candidates who had struggled earlier and was very successfully answered.

- (i) The only problem encountered was the use of  $\frac{dt}{dx} = 1 + \frac{t}{2}$  following the correct  $\frac{dx}{dt} = 1 + \frac{2}{t}$ .
- (ii) Even those candidates who erred in part (i) staged a full recovery and few errors were seen.
- (iii) This part was basically very well done, though conversions of  $1 - \ln \frac{1}{2}$  to  $1 + \ln 2$  were often done via use of logarithms. Almost no-one scored the final mark, due to the mistaken belief that  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right)$ . The correct form of  $\frac{d^2y}{dx^2}$  when  $x$  and  $y$  are functions of a parameter  $t$  is not expected for this paper; instead, Examiners were looking for an investigation of the sign of  $\frac{dy}{dx}$  on either side of the point where  $t = 1$ .

Answers: (i)  $\frac{2t-1}{t+2}$ ; (ii)  $3y = x + 5$ ; (iii) minimum.

Papers 9709/03 and 8719/03

Paper 3

### General comments

There was a wide variety of standard of work by candidates on this paper and a corresponding range of marks from zero to full marks. The paper appeared to be accessible to candidates who were fully prepared and no question seemed to be of unusual difficulty. However, less well prepared candidates found parts of the paper difficult and, in some cases, omitted questions such as **Question 8** (vector geometry) and **Question 9** (complex numbers), presumably because these parts of the syllabus had not been covered. Examiners noted that such candidates tended to present work poorly. Overall, the least well answered questions were **Question 5** (stationary points), **Question 8** (vector geometry) and **Question 10** (calculus). By contrast **Question 1** (trigonometric identity), **Question 2** (binomial series), **Question 6** (partial fractions) and **Question 7** (differential equation) were generally well answered. It was felt that adequately prepared candidates had sufficient time to attempt all questions.

The detailed comments that follow inevitably refer to mistakes and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts which showed very good and sometimes excellent understanding and capability over the syllabus being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the only "correct" answer.



### Comments on specific questions

#### Question 1

This was generally well answered and a wide variety of methods was seen. Candidates usually gave sufficient working to justify their arguments.

#### Question 2

This question was also well answered. The main errors arose in the handling of signs, in numerical simplification, and in replacing  $x$  by  $-3x$  in the general expansion of  $(1+x)^n$ .

Answer:  $1 + x + 2x^2 + \frac{14}{3}x^3$ .

#### Question 3

Those who embarked on long division or inspection did well on this question. However some candidates seemed to be uncomfortable with quadratic factors. Thus there were vain attempts to find real linear factors of  $x^2 + x + 2$  and use the remainder theorem. Very occasionally a correct complex zero was used to evaluate  $a$  using the remainder theorem.

Answers: 6;  $x^2 - x + 3$ .

#### Question 4

Part (i) was usually done well but part (ii) was clearly unfamiliar to some candidates.

Answers: (i) 1.26; (ii)  $x = \frac{2}{3} \left( x + \frac{1}{x^2} \right), \sqrt[3]{2}$ .

#### Question 5

Examiners were disappointed by the quality of work on this question. Errors in differentiation at the start of the problem and mistakes in obtaining an equation in one trigonometrical function were common. Candidates who obtained coordinates of the stationary points usually used the second derivative to determine their nature. Here errors in differentiation and evaluation regularly lost marks.

Answers:  $\frac{1}{6}\pi$ , maximum point;  $\frac{5}{6}\pi$ , minimum point.

#### Question 6

Examiners noted that most candidates were prepared for this question and tended to score well on it. Most

candidates set  $f(x)$  identically equal to  $\frac{A}{3x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ , but the form  $\frac{A}{3x+1} + \frac{Dx+E}{(x+1)^2}$  was

sometimes seen and was acceptable. A common error was to set out with an incomplete form of fractions. Errors in identifying the numerator of  $f(x)$  with that of the combined fractions proved costly. A thorough check of the algebraic work at this stage would have been very helpful to some candidates.

In part (ii) those candidates who worked with the second form above were not often able to integrate

$\frac{Dx+E}{(x+1)^2}$  and Examiners were disappointed to see many poor attempts at integrating terms of the form  $\frac{C}{(x+1)^2}$ .

Answer:  $\frac{-3}{3x+1} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$  or  $\frac{-3}{3x+1} + \frac{x+3}{(x+1)^2}$ .

### Question 7

There were many successful solutions to part (i). A minority of candidates merely showed that the given differential equation was satisfied at  $t = 0$ . This does not show that  $m$  satisfies the equation at all times.

In part (ii) most candidates separated variables correctly and attempted to migrate  $\frac{1}{(50 - m)^2}$ , but here, as in

**Question 6 (ii)**, Examiners were disappointed by the inability of candidates to integrate correctly.

The remaining parts were done well and a pleasing number of candidates answered part (iv) satisfactorily.

Answers: (iii)  $m = 25$ ,  $t = 90$ ; (iv)  $m$  tends to 50.

### Question 8

The first part of this question was answered well by those who attempted it. Most solutions involved the vector equation of the line and the evaluation of the parameter of the point of intersection.

The second part discriminated well. Those who knew how to proceed usually tried to find a vector perpendicular to both the line  $l$  and the plane  $p$ , and then use one of the points on  $l$  and this vector to obtain the required equation. A less common alternative was to write down a 2-parameter equation of the required plane, using a point on  $l$ , a direction vector for  $l$  and the normal to  $p$ , express  $x$ ,  $y$  and  $z$  in parametric form and then eliminate the parameters. Some excellent work was seen here, marred only by the occasional failure to present the equation of the plane in the required form.

Answers: (i)  $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$ ; (ii)  $-\frac{2}{3}x + \frac{4}{3}y + \frac{5}{3}z = 1$ .

### Question 9

Part (i) was quite well answered. Those candidates who used the polar form of  $n$  to find the modulus and argument of  $u^2$  and  $u^3$  tended to be more successful than those who calculated  $u^2$  and  $u^3$ .

In part (ii) many candidates were aware that the complex conjugate was also a root and  $1 + i\sqrt{3}$  was justified by a variety of methods.

Part (iii) was poorly done. The plotting of  $i$  and  $u$  was often omitted or simply incorrect. Correct descriptions of the circle were not matched by correct sketches, and the line associated with  $\arg u$  often failed to pass through  $u$ . Fully correct sketches were rarely seen.

Answers: (i)  $2\left(\cos \frac{1}{3}\pi + i \sin \frac{1}{3}\pi\right)$ ;  $4, \frac{2}{3}\pi$ ;  $8, \pi$ ; (ii)  $1 - i\sqrt{3}$ .

### Question 10

Most candidates answered part (i) correctly but the work on the second derivative of  $(\ln x)^2$  was disappointing. In part (iii) there were many incomplete or incorrect attempts. Some candidates failed to give an explicit statement of the integral for the area in terms of  $x$  and those who did often took the lower limit to be 0 instead of 1. The final part involves integrating by parts twice. Most candidates completed the first stage correctly but the second integration was often accompanied by errors of sign and in the substitution of limits. Here, as in **Question 4 (ii)**, it was clear that some candidates are unaware of the meaning in these contexts of the adjective 'exact'.

Answers: (i) 1; (iv)  $e - 2$ .

## Paper 9709/04

## Paper 4

**General comments**

The attempts at this paper were mixed. Many candidates demonstrated a good understanding of the topics; a few candidates scored very low marks.

There were only a few very high marks and this was because the topics represented by **Question 5 (ii)** and **Question 7 (iii)** were widely unknown. In **Question 5 (ii)** very many candidates inappropriately used constant acceleration formulae; this was also the case in **Question 6 (ii)**. Candidates should be aware of the importance of constant acceleration formulae in this syllabus, but must also be aware that these formulae should not be used in cases where the acceleration is not constant.

Whereas two topics were widely unknown, another topic was widely misunderstood. The syllabus requires candidates to be able to 'find (and use) components (of forces)'. In **Question 3 (ii)** many candidates found just the angle between the resultant force and the direction of OA, giving the answer as  $53.13^\circ$ , instead of the component in that direction; many others gave the answer simply as  $12\cos 53.13^\circ$ , without calculating its value.

In very many cases the accuracy required by the rubric was not attained because of premature approximation. Frequently occurring cases included  $4.3 \times 4 = 17.2$  in **Question 1**,  $49 \times 0.2 = 9.8$  in **Question 2 (i)**,  $10 - 10\sin(106 - 90)^\circ = 7.24$  in **Question 3 (ii)**,  $\frac{2}{3.3} = 0.606$  in **Question 4 (iii)** and  $150\,000 \times 28 = 4\,200\,000$  in **Question 6 (i)(a)**.

Some candidates gave answers to insufficient accuracy, the most common of which were 17 in **Question 1**,  $106^\circ$  in **Question 3 (i)** and 0.3 or 0.33 in **Question 4 (i)**.

**Comments on specific questions****Question 1**

This was found to be a straightforward starter question with most candidates scoring all three marks. One common mistake was to omit  $\cos 30^\circ$ ; another was to use  $\sin 30^\circ$  instead of  $\cos 30^\circ$ . Some candidates used a distance of 2 m, calculated from  $\frac{s}{10} = \frac{0 + 0.4}{2}$ .

Answer: 17.3 J.

**Question 2**

This question was well attempted with most candidates scoring 4 or 5 marks.

The most common mistake was to assume implicitly that the acceleration is zero. Thus the frictional force was equated to the component of the weight, parallel to the plane, to produce an answer of 10.4 N in part (i).

Candidates who made the assumption usually made no sense of part (ii) and scored 1 mark for the question, this being a mark for writing the component of the weight as  $5g \sin 12^\circ$ . However, some candidates did not continue with the false assumption in part (ii), and thus could score all 3 marks available in this part.

Answers: (i) 9.78 N; (ii) increasing.

**Question 3**

Very many candidates used the incorrect triangle with sides 10, 10 and 12 and with the angle  $\theta$  opposite the side of length 12. In candidates' sketches the  $\theta$  was shown to be obtuse, as in the diagram in the question paper. When calculation led to the acute angled answer of  $73.7^\circ$ , some candidates simply changed this to the corresponding obtuse angle of  $106.3^\circ$  without explanation.

Candidates who sketched the correct diagram often assigned the symbol  $\theta$  to angles other than that to which it is assigned in the question. This led to much confusion.

Some candidates treated the triangle as though it is right angled, for calculation purposes, and found an incorrect value for  $\theta$  from  $\cos \frac{\theta}{2} = \frac{10}{12}$ .

Among the candidates who used the cosine rule correctly, those who wrote  $10^2 = 12^2 + 10^2 - 2(12)(10) \cos \frac{\theta}{2}$  usually continued to the correct answer. This is also the case with candidates who introduced a symbol to represent  $180^\circ - \theta$ , usually  $\alpha$ , and wrote  $12^2 = 10^2 + 10^2 - 2(10)(10) \cos \alpha$ . However, many candidates who expanded  $\cos(180^\circ - \theta)$  made mistakes, mainly with signs, and thus failed to obtain the correct answer.

Candidates who recognised that the direction of the resultant bisects the angle  $AOB$ , and resolved in this direction obtaining  $2 \times 10 \cos \frac{\theta}{2} = 12$ , found the correct answer with an economy of effort.

Candidates who used components were roughly equally divided in initially writing  $X = 10 + 10 \cos \theta$ ,  $Y = 10 \sin \theta$  or  $X = 10 - 10 \cos(180^\circ - \theta)$ ,  $Y = 10 \sin(180^\circ - \theta)$  or  $X = 10 - 10 \sin(\theta - 90^\circ)$ ,  $Y = 10 \cos(\theta - 90^\circ)$ . Many mistakes were made including sign errors in dealing with the expansions of  $\cos/\sin [(180^\circ - \theta)/(\theta - 90^\circ)]$  and, as in the cosine rule case, candidates who assigned a separate symbol to  $180^\circ - \theta$  or to  $\theta - 90^\circ$  fared better than those who tried to expand a trigonometrical ratio of a compound angle. Other mistakes made were the omission of the mixed product term in squaring the expression for  $X$ , and dropping the coefficient 100 when applying  $\cos^2 \alpha + \sin^2 \alpha = 1$ .

Part (ii) was poorly attempted; many candidates did not seem to know what was required.

Answers: (i)  $106.3^\circ$ ; (ii)  $7.2 \text{ N}$ .

#### Question 4

This question was very well attempted and many candidates scored all 7 marks. Sometimes irrelevant inequality signs were used.

In part (i) a very common error was to write  $\frac{15}{45} = 0.3$ . Some candidates treated the given constant speed as though it is an acceleration of  $2 \text{ ms}^{-2}$ . Thus the frictional force became  $6 \text{ N}$  and  $\mu$  became  $\frac{2}{15}$ .

In part (ii) some candidates left the pulling force in the equation of motion and, because the frictional force was re-calculated from  $\mu R$  with  $\mu$  equal to  $0.3$  or  $0.33$ , a non-zero value for acceleration was often obtained.

In part (iii) a few candidates did not square the  $u$  ( $= 2$ ) in applying  $v^2 = u^2 + 2as$ . Some candidates found  $t = 0.6 \text{ s}$  without continuing to find the required distance.

Answers: (i)  $\frac{1}{3}$ ; (ii)  $\frac{10}{3} \text{ ms}^{-2}$ ; (iii)  $0.6 \text{ m}$ .

#### Question 5

Both (a) and (b) of part (i) were almost always answered correctly.

Those candidates who recognised the need to use calculus answered part (ii) very well. However there were very few such candidates; the vast majority inappropriately used constant acceleration formulae.

Answers: (i)(a)  $50 \text{ s}$ , (b)  $225 \text{ m}$ ; (ii)(a)  $0.0054$ , (b)  $13.5 \text{ ms}^{-1}$ .

**Question 6**

Part **(i)(a)** was well attempted, although a significant minority of candidates used  $h = 800$  in evaluating  $mgh$ .

Part **(b)** was very well attempted; sometimes the mark for this answer was the only one scored for the question.

Correct answers were obtained in part **(c)** by subtracting the answer in **(a)** from the answer in **(b)**, and by multiplying the resultant of the driving force and the weight component parallel to the plane by 800, in roughly equal numbers.

This part was, however, considerably less well attempted than parts **(a)** and **(b)**, very many candidates obtaining an incorrect answer by repeating the calculation in **(a)**.

Very many candidates realised that the required work done is obtained as a linear combination of work and energy terms in part **(ii)**. However many mistakes were made, the most common of which were the omission of one of the three terms, sign errors, and including the resisting force value of 900 instead of the work done value of  $900 \times 800$ .

Many candidates assumed implicitly that the acceleration is constant, and obtained this constant acceleration using  $v^2 = u^2 + 2as$ . They then used Newton's second law to find the (constant) driving force and hence the work done by this force. Candidates obtaining the correct answer for this special case scored 3 of the 5 marks available.

Answers: **(i)(a)** 4 190 000 J, **(b)** 5 600 000 J, **(c)** 1 410 000 J; **(ii)** 2 660 000 J.

**Question 7**

Almost all candidates obtained correct equations by applying Newton's second law. However, the answer for acceleration was often wrong. This was usually because of an error in subtracting  $0.15g$  from  $0.25g$ , frequently as  $0.4g$ , or as 10 following  $0.1g$ .

Part **(ii)** was poorly attempted. Many candidates found  $v = 5$  (from  $v = 0 + at$ ) and  $s = 5$  (from  $s = \frac{1}{2}(0 + v)t$ ) for the motion while the string is taut. They then used  $s = \frac{1}{2}at^2$  incorrectly, with  $s = 5$  and with either  $a = 2.5$  or  $a = g$ , to obtain  $t = 2$  (not surprisingly) or  $t = 1$  as their answer for this part.

Many candidates used a correct method for finding the time during the upward motion of A whilst the string is slack, but relatively few doubled this to obtain the total time.

Part **(iii)** was very poorly attempted. Almost every candidate showed  $v$  as having the same sign for both A and B, for the part of the motion for which the string is taut, usually with one line segment superimposed on the other.

Beyond  $t = 2$  the graphs usually petered out, or  $v$  for particle A was shown either as being constant or increasing. Occasionally  $v$  for particle A was shown to decrease uniformly from  $v = 5$  at  $t = 2$ , but in such cases the graph either terminated at  $(2.5, 0)$ , or  $v$  was shown to increase uniformly from 0 to 5 in the interval  $2.5 < t < 3$ .

Rarely was any attempt made to indicate on the graph that  $v = 0$  for the particle B, in the interval  $2 < t < 3$ .

Answers: **(i)**  $2.5 \text{ ms}^{-2}$ ; **(ii)** 1 s; **(iii)** diagram.

Papers 8719/05 and 9709/05

Paper 5

**General comments**

This paper proved to be a fair test in that any candidate with some understanding of basic mechanical ideas could make some progress in all questions with the possible exception of **Question 2** which was often ignored altogether.



All indications suggested that, with few exceptions, the majority of the candidates had sufficient time to attempt all the questions that they were capable of answering. Paradoxically it was the shorter earlier questions which created most difficulty, whilst even candidates of moderate ability scored well on **Questions 6 and 7**.

Without doubt, it was regrettable to see candidates, right across the ability range, carelessly throw marks away through neglecting to give required answers correct to 3 significant figures. For example in **Question 7** most candidates successfully found  $V = 13.29174\dots$  which was then rounded to 13.3. However, most candidates then used the value 13.3 to find the value of  $T$  and obtained the incorrect value 0.868. In subsequent calculations in the question it is essential to use the best value of  $V$  held in the calculator to obtain  $T = 0.869$ . The best values of both  $V$  and  $T$  would then be used to obtain the required angle in part (iii).

When a required answer is given in a question it serves a two-fold purpose. In the first place it boosts a candidate's confidence but, more importantly, it enables the candidate to make a fresh start with the remainder of the question if an error has been made in the first part. For example, in **Question 5** if the candidate failed to show that the tension in the string was 12.2N, then this value should have been used in part (ii) in order to gain the maximum 4 marks. A number of candidates continued to use their incorrect value for the tension in part (ii). Similarly in **Question 6 (a)(ii)** some candidates used  $g = 9.8$  or  $9.81$ , despite the instructions on page 1 of the question paper to use  $g = 10$ . Having failed to get the given differential equation, they then persisted with their version in part (b)(ii) rather than making a fresh start with the given equation.

### Comments on specific questions

#### Question 1

Although this question posed little difficulty for the more able candidates, many of the remainder failed to read the question properly. Rather than finding the gain in GPE many merely stated its value in the initial position. A number of candidates found the extension when the particle was hanging freely at rest and then gave the energy changes from the initial position to the rest position. Those candidates with a poor understanding of energy principles often maintained that the gain in GPE was equal to the loss in EPE, and often one of these terms was missing from the energy equation attempt in part (ii) of the question.

Answers: (i) 1.25J and 0.6J; (ii)  $2.94 \text{ ms}^{-1}$ .

#### Question 2

This question was often ignored due to a failure by the candidates to appreciate that it depended on the knowledge of the position of the centre of mass of the triangular prism. Many of those who did make some headway again failed to answer the question asked. In part (i) the length of the base left on the shelf was given as the answer, and in part (ii) some attempted to find the number of books to fill the space between the four books already on the shelf illustrated in the diagram. However the most frequent error was to answer part (i) correctly as 6.67cm but then in part (ii) to subtract twice this value from 100cm before dividing by 5.

Answers: (i)  $\frac{20}{3}$  (= 6.67) cm; (ii) 18.

#### Question 3

More able candidates coped well with this question but for the rest the main difficulty was finding the extension of the string. Many candidates did not even see the necessity of calculating the total length of the string in the equilibrium position. Hence some of the various incorrect attempts at the extension were  $0.14$ ,  $0.96 - 0.8 = 0.16$ ,  $1.0 - 0.96 = 0.04$  and  $0.8 - 0.5 = 0.3$ . Hooke's Law was then often applied incorrectly with, for example, a correct extension of 0.1m for half the length of the string but then using 0.8m for the total natural length of the string.

Less able candidates often showed an inability to resolve vertically with attempts such as  $W = 2T$ ,  $W = T \cos \theta$  or  $W = \frac{1}{2} T \cos \theta$ .

Answer:  $W = 1.68$ .

**Question 4**

Part (i) was well answered except for a minority who either did not know the difference between speed and angular speed, or thought that the acceleration was given by  $m\omega^2$ .

Apart from the better candidates, the rest of the question was not well answered. Even those who started with two correct equations failed to get the required answers through using prematurely approximated values of either the acceleration or  $\cos 45^\circ$ .

The main difficulty however was the understanding of what external forces were acting on  $P$ . If a diagram was drawn the force exerted by the cone was either vertical or omitted. Many thought that  $m\omega^2$  was the required force. Even those who had the force in the correct direction and had also applied Newton's Second Law of Motion correctly towards the centre of the circle were still quite capable of showing their lack of understanding of circular motion by then stating the incorrect  $R = mg\cos 45^\circ$  or  $R\cos 45^\circ = mg$ .

Answers: (i)  $13.6 \text{ ms}^{-2}$ ; (ii) Tension = 0.759N, Force = 5.00N.

**Question 5**

Able candidates produced good solutions, but the majority of the remainder did not realise that the first step was to find the centre of mass of the lamina. Of those who did, a frequent error was to divide the lamina into two rectangles but then accord them the same area  $0.05 \text{ m}^2$ . If a diagram was drawn it was not always clear about which axis moments were being taken and this may have accounted for the frequent incorrect distance 0.55m rather than 0.45m. As commented earlier, candidates who could not successfully find the tension should then have taken the value 12.2N to tackle part (ii). There seemed to be a common misapprehension that the force acting at  $A$  was vertical and so, for many, part (ii) was restricted to either resolving vertically or taking moments about  $B$ .

Answer: (ii) 11.0N.

**Question 6**

This proved to be a very popular question, probably helped by the fact that the two required differential equations were written into the question. Candidates of even moderate ability were able to obtain high marks, the failures being those candidates who omitted the constants of integration. A minor error of some of the better candidates was to have the speeds zero and  $2 \text{ ms}^{-1}$  the wrong way round when solving by using definite integrals.

Answers: (i) 8 m; (ii)(b)  $4\ln \frac{5}{3}$  (= 2.04) s.

**Question 7**

The majority of candidates knew which ideas to apply in each part of the question but the main failure lay in the lack of accuracy which has been commented on earlier.

In part (i) only the weakest candidates failed to obtain  $V$ , usually because of poor algebraic manipulation. On the other hand they usually knew how to find a value for  $T$  from their value of  $V$ . Despite the instruction to use the equation of the trajectory, some of the poorer candidates seemed to use any equation that came to hand. It was not unusual to see attempts based on the formulae for the range on a plane and its total time of flight. The methodology for finding the angle was well known, either through finding the horizontal and vertical components of velocity at  $A$  or by differentiating the equation of the trajectory. However with the first method, it should be borne in mind that using the equation  $v^2 = u^2 + 2as$  to find the vertical component at  $A$  will give the magnitude of the velocity but not whether the direction of motion at  $A$  is vertically up or down.

One popular error was to substitute the values of  $V$  and  $T$  into the equation  $s = ut + \frac{1}{2}at^2$  for the vertical component. Errors in  $V$  or  $T$  often concealed the fact that, not surprisingly, the angle came out to be  $30^\circ$ !

Answers: (i)  $13.3 \text{ ms}^{-1}$ ; (ii) 0.869 s; (iii)  $10.1^\circ$ .

## Paper 9709/06

## Paper 6

**General comments**

This paper produced a wide range of marks from 0 to 50 out of 50. Many Centres however, entered candidates who had clearly not covered the syllabus and thus a large number failed to reach the required standard. Premature approximation leading to a loss of marks was only experienced in a few scripts, most candidates realising the necessity of working with, say,  $\sqrt{21}$  instead of 4.58.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order. Clear diagrams on normal distribution questions would have helped many candidates to earn more marks, as many found the wrong area.

**Comments on specific questions****Question 1**

This was a very straightforward question for those candidates who knew the definitions of independent and mutually exclusive events. A few candidates muddled up  $P(A \cap B)$  with  $P(A \cup B)$  and some had the correct inequalities but drew the wrong conclusions from them. The mismatch in the question did not affect any candidates' work or marks. Some candidates thought 'exhaustive' was the same as 'mutually exclusive'.

*Answers:* not independent, not mutually exclusive.

**Question 2**

This question gave many candidates full marks. As usual, some plotted midpoints, but the vast majority realised it had to be upper class boundaries, although there was a good mix of 14.5, 15 and 15.5. The graphs were well done with clear labelling and almost everybody who drew a cumulative frequency curve managed to find the median and interquartile range correctly. Answers were checked for accuracy from candidates' graphs.

**Question 3**

All the good candidates had no trouble with this question. Many weaker candidates on the other hand, did not appear to understand what was required at all and could not get started, despite an example being given.

A surprising number of candidates quoted  $\text{Var}(A) = E(A)^2 - [E(A)]^2$  and then did not square  $E(A)$  when performing the subtraction.

*Answers:* (i)  $a$       1      4      9      16;      (ii) 5.33, 30.9.

$$P(A = a) \quad \frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6}$$

**Question 4**

(i) This part proved too difficult for most candidates despite being specifically mentioned on the syllabus. Many candidates thought the mean was 110 and then divided 5460 by 30 to get the variance. This did not earn them any marks.

(ii) This was a perfectly straightforward normal distribution question. Some candidates used a continuity correction, and a surprising number failed to get the correct area, i.e. they did not subtract from 1. A diagram with the required area shaded in would have told candidates at once whether the area was greater or less than 0.5.

*Answers:* (i) 108, 13.4; (ii) 0.431.

### Question 5

This question was very well done by most candidates, a pleasing reflection on the work on this relatively new topic. Even part (iii), which needed some thought, was well attempted by many candidates.

Answers: (i) 2520; (ii) 360; (iii) 1440.

### Question 6

- (i) This part, which should have taken three or four lines, took many candidates three or four pages. Once again, a diagram would have shown immediately that the mean was 3.6. As it was, candidates went round and round, often writing the same expression many times, before arriving at the answer. About half the candidates ignored the fact that the standardised value had to be negative, and just ignored the minus sign when it appeared in their answer.
- (ii) Only the best candidates attempted this part of the question. Most failed to recognise that the required probability of success was given in part (i) and attempted with mixed success to find it from scratch. They then thought that they had finished the question, and did not continue to find the binomial probabilities asked for. Others used the probability as 0.9.

Answers: (i) 3.6, 2; (ii) 0.879.

### Question 7

This question was well done by the majority of candidates and proved a good finish to the paper.

- (i) The mean and variance of a binomial were quoted and the corresponding equations solved with varying degrees of conciseness, but almost all candidates arrived at the correct answer eventually. They then performed the binomial calculation correctly, a few dropping the final mark by giving the answer correct to 2 significant figures instead of 3.
- (ii) This part also had a good response; many candidates were confident with the normal approximation, and most added a continuity correction. The final answer needed to be accurate enough to show that candidates had used the far column of the normal table.

Answers: (i) 20, 0.162; (ii) 0.837.

Papers 8719/07 and 9709/07

Paper 7

### General comments

This was a well-balanced paper which resulted in a good range of marks. Candidates did not appear to have difficulty in completing the paper in the given time. Solutions were, in general, well presented.

It was noted by Examiners that some candidates appeared unprepared for certain topics, in particular **Question 5** on Type I and Type II errors. Many candidates omitted it completely or made a very poor attempt. Questions which were particularly well attempted were **Questions 6** and **7 (i) and (ii)**.

Candidates must note that 3 significant figure accuracy is required throughout the paper. Marks were lost, for instance, in **Question 7** by writing 0.056 instead of 0.0556, thus showing either a lack of understanding of significant figures or a lack of awareness of the required accuracy.

**Comments on specific questions****Section A****Question 1**

Only a small proportion of candidates gained full marks on this question. Whilst many candidates successfully found the confidence interval, very few were then able to deduce its width.

Common errors included using the wrong z-value, and confusion between standard deviation and variance.

Answer: 1.18.

**Question 2**

This was not a particularly well attempted question. There was much confusion between two possible correct methods. Answers showed unfamiliarity with a basic formula for the confidence interval for a proportion, with candidates using  $np$  (33) in their formula instead of the proportion  $p$  (0.275).

Other common errors included using a wrong z-value and even forgetting to multiply by 1.96, despite previous correct working.

Answer:  $0.195 < p < 0.355$ .

**Question 3**

Many candidates made errors in calculating the variance of the sugar and coffee. Those that realised they were dealing with a sum rather than a multiple of independent normal variables and correctly used  $n \text{ var}(A)$  usually went on to complete the question correctly. The most common error was to use  $n^2 \text{ var}(A)$  for the sugar and coffee i.e.  $N(1500, 3600)$  rather than  $N(1500, 1200)$  for the sugar and  $N(1000, 3600)$  rather than  $N(1000, 720)$  for the coffee. Confusion between standard deviation and variance was also seen, and some candidates forgot to consider the weight of the purse or incorrectly changed the combined variance by 350.

Answer: 0.873.

**Question 4**

Part (i) was well attempted. However errors were frequent in part (ii).

The question required a two-tail test to be carried out, though some marks were still available for those who carried out a one-tailed test successfully. Some candidates were unable to set up their hypotheses, often not being precise enough.  $H_0 = 12$  for example is not acceptable, candidates must clearly state  $m = 12$ , or population mean = 12, for  $H_0$ .

A common error in calculating the z-value was to use  $\sqrt{(50.34)}$  rather than  $\sqrt{(50.34/150)}$ .

Candidates must clearly show that they are comparing their calculated z-value with the critical value (i.e. 1.645 here, or 1.282 if a one tail test was carried out). This must be clearly done with an inequality statement and/or a diagram with both values clearly shown. The conclusion must then be drawn with no contradictions.

Answers: (i) 14.2; (ii) 50.3; (iii) Reject exam board's claim.

**Question 5**

This was a poorly attempted question. Many candidates indicated that they knew what was meant by a type I and type II error but were unable to calculate this probability in the context of the question.

Part (i) was better attempted than part (ii). Some candidates lost marks by rounding too early and reaching a final answer of 0.0214 rather than 0.0215 (the given answer). In part (ii) many candidates correctly found the probability of 9 or 10 heads as 0.1493, but then either forgot to calculate 9 or 10 tails or thought it would also be 0.1493 (incorrectly assuming symmetry).

It was disappointing that such a large number of candidates omitted this question completely or were unable to make a sensible attempt.

Answers: (i) 0.0215; (ii) 0.851.



**Question 6**

This question was quite well attempted with most candidates realising that a Poisson Distribution was required with a Normal approximation for part (ii). In general marks were lost by incorrectly identifying the Poisson means, not giving 3 significant figure accuracy and omission of a continuity correction in part (iii). Many candidates used  $N(6, 6)$  rather than  $N(24, 24)$  in the last part.

Answers: (i) 0.161; (ii) 0.865; (iii) 0.179.

**Question 7**

In general parts (i) and (ii) were well attempted, though again not writing answers to 3 significant figures caused the loss of marks for some candidates. It was also noted by Examiners that weaker candidates did not realise that the mean value of  $X$  was  $E(X)$ . These candidates gave a totally incorrect answer to part (i) and only calculated  $E(X)$  in part (ii). Marks were not recoverable. Algebraic errors were quite common, particularly when removing brackets, but attempts at integration were good.

Candidates who were able to make a start on part (iii) were usually able to solve the resulting quadratic, though on occasions some very basic errors were made here. Not all realised that only one solution was valid and that this was in thousands of tonnes.

Answers: (i) 0.333; (ii) 0.0556; (iii) 859 tonnes.

**GCE Ordinary Level**

Paper 4024/01

Paper 1

**General comments**

This was a successful paper of an appropriate standard which discriminated well at all levels. Weaker candidates found enough questions to demonstrate what they knew, and the strongest found some challenges in the later parts of the paper.

There were some excellent scripts which were well presented and explained. The minority of candidates who presented untidy and muddled working stood out in contrast. At worst Examiners are unable to give credit for correct working to them, or to those who showed no working. All working should be shown in the spaces allowed on the question paper, not on loose sheets.

Examiners were pleased to note an increased confidence in algebraic manipulation this year. Areas of general weakness that were noted on this paper included the manipulation of negative numbers and many examples of poor elementary subtraction and division in **Questions 10, 14 and 24**. Geometrical transformations were generally weak, and attention needs to be given to constructions and scale drawings.

**Comments on specific questions****Question 1**

As intended, the majority of candidates worked in decimals. The first part was usually correct, though a few misplaced decimal points appeared. Very many correct answers to the second part were seen. Stronger candidates confidently wrote down the answer in most cases. Weaker candidates showed working, when errors crept in sometimes, leading to answers such as 1.15, 0.015, 0.01 or 0.005.

Answers: (a) 0.006; (b) 1.015.



**Question 2**

This question was very well answered. Although a few worked in decimals, most used fractions as intended. A few thought 'of' required subtraction, doubtless thinking 'off' was intended. Some inverted one or other of the fractions, usually reaching  $32/75$ , but many scored full marks.

Answers: (a)  $\frac{5}{12}$ ; (b)  $\frac{3}{8}$ .

**Question 3**

The first part was well done but some gave the answer "4 or -2". The second caused some difficulties. Many either ignored the word reciprocal, or did not understand it, so common answers were -3 or  $-\frac{1}{3}$  or  $\frac{1}{8}$ . A small number gave the answer  $2^3$ .

Answers: (a) 4; (b) 3.

**Question 4**

This question was quite well done, though the order of the operations in the first part was not always understood, leading to the answer of 4. Standard form was slightly less well understood this year, though the majority of the candidates reached the correct answer.

Answers: (a) 12; (b)  $3.2 \times 10^{-3}$ .

**Question 5**

Those that used tally marks usually found the correct frequencies, but some errors were found among solutions that did not show tallies. Unfortunately a few did not complete their solutions, only showing the tallies. Many candidates were unsure whether the mode was 'Blue', or 10 or both.

Answers: (a) 8, 10, 1, 6; (b) Blue.

**Question 6**

There were many good attempts at this question, but the first part was sometimes spoilt by sign errors, leading to answers such as  $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$  or  $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ .

Answers: (a)  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ ; (b)  $(3, -\frac{1}{2})$ .

**Question 7**

Many candidates scored well on this question. At least a quarter of solutions to part (b) quoted an angle ( $40^\circ$ ) rather than a bearing. A small number thought that angle  $P$  is bisected by the North line. The bearing of the third part was sometimes measured the wrong way, leading to the answer  $10^\circ$  or  $010^\circ$ .

Answers: (a)  $50^\circ$ ; (b)  $040^\circ$ ; (c)  $350^\circ$ .

**Question 8**

The majority of candidates knew how to solve these equations, but the unusual layout led to rather more sign errors than usual, often in an initial rearrangement of one of the equations.

Very few candidates seemed to check their solutions effectively.

Answers:  $x = 4$ ,  $y = -\frac{1}{2}$ .

**Question 9**

The explanation required was often too vague to be convincing. The better solutions used the Venn Diagram, showing 22, 4, 25 and 1 people. They then stated that the sum of these is greater than the number on the tour. Many did not get beyond adding 26 and 29. Many of these in effect found that at least 5 went to both events, without realising that this was the answer to the next part.

Answers to part **(b)** showed a lack of understanding by many candidates, with little thought being given to whether the answers quoted were possible. Greatest numbers were often less than least numbers, and greatest numbers greater than 26 were common.

*Answer:* **(b)** Least 5, Greatest 26.

**Question 10**

This question was well done. The pattern was usually spotted and generalised. In the third part a large number failed to complete the evaluation of the term, either leaving the answer as  $10^3 - 20$  or obtaining 9980 or, surprisingly often, 1980.

*Answers:* **(a)**  $5^3 - 10$ ; **(b)**  $n^3 - 2n$ ; **(c)** 980.

**Question 11**

The response to this question was very disappointing. The majority thought the scale factor of the enlargement was 4, but  $-4$  and  $-\frac{1}{4}$  were also popular. The second transformation was very often thought to be a stretch. When a shear was identified,  $y = 4$  was often thought to be invariant. The shear factor was rarely correct.

*Answers:* **(a)**  $\frac{1}{4}$ ; **(b)** Shear, of shear factor  $-1$ , with  $y = 0$  invariant.

**Question 12**

Although there was some confusion between  $x$  and  $y$ , there were a number of good answers to this question. Although the question emphasised that the region did not include the boundaries, this was often ignored by the candidates, so that while the inequalities were usually the right way round, too often  $\geq$  appeared in place of  $>$ . Sometimes more than three inequalities were seen, or three equations quoted.

*Answers:*  $x > -3$ ;  $y < 5$ ;  $y > x + 2$ .

**Question 13**

Many candidates scored heavily in this question. It was pleasing to observe how many candidates spotted that the answer to **(c)** could be written down at once, since it was given that  $f(4) = 21$ . Clearly some candidates were unfamiliar with the inverse function notation,  $f^{-1}(21)$ , however.

*Answers:* **(a)** 1; **(b)** 5; **(c)** 4.

**Question 14**

The main error in the first part was to express the cost of the ice cream as a fraction of the cost of the ticket, rather than the total cost. In the second part the new cost was expressed as a percentage of the old cost. This should have led to 105%, when the next step usually followed, but too often faulty division led to 15%. Also there were many who expressed the increase as a percentage of the new price.

*Answers:* **(a)** 18°; **(b)** 5%.

**Question 15**

There were some excellent constructions seen in response to this question. These candidates understood that the bisector of angle  $T$  and the perpendicular bisector of  $AB$  were needed to find  $P$ , and then they drew good circles through  $A$  and  $B$ . A common error was to draw the circle which had  $AB$  as diameter. Sadly, for some reason or other, many candidates avoided this question.

### Question 16

The correct reading from the diagram ( $30^\circ$ ) was given by many candidates, but surprisingly many measured the given diagram, usually obtaining  $33^\circ$ .

The response to the second part was very disappointing. Many made no attempt to answer the question. A small number tried to calculate an answer, but were unable to complete the solution. A few produced sketches that indicated correct ideas, but were not turned into accurate scale diagrams. The majority of the good solutions seen showed an accurate scale drawing of the triangle and added the additional vertical lines to represent the surveyor's height, though a few drew the triangle then added 1.8 metres to the height represented on the triangle. Some drawings were spoiled by incorrect angles or incorrectly scaled lengths (usually the length 1.8 metres).

Answers: (a)  $30^\circ$ ; (b) 30 to 32 m.

### Question 17

This was generally well answered by candidates. There were sometimes sign errors in the first part. A few treated the second part as a fraction simplification question, but most reached  $5x = 9$ . This sometimes led to  $x = 9 - 5 = 4$  or, more often, to  $x = 5/9$ .

Answers: (a)  $(3r - t)(6c - d)$ ; (b)  $x = 1.8$ .

### Question 18

A pleasing number of candidates scored well on this question, though the reasons were sometimes unconvincing. The majority found the first angle, but too many gave an incorrect reason, such as assuming that angle  $BPR$  is a right angle or that  $APR$  is a tangent. Examiners were looking for angles in the same segment. Although many correct values for angle  $PMA$  were seen, the reasons were again often poorly expressed. Examiners hoped to see reference to the fact that  $AB$  is a diameter of the circle and to the property of an angle at the centre of a circle.

Answers: (a)  $21^\circ$ ; (b)  $48^\circ$ .

### Question 19

The first part caused unexpected difficulty. Many candidates assumed that the winner took the longest time, so the answer 3 h 36 min was surprisingly common. The median was usually correct. A small number of candidates gave two answers to both of these parts, one in hours and the other in minutes (e.g. 2.5 h 150 min). They were given some credit. There were good answers to the last part, but some found the combination of the 24 hour clock and interpretation of the curve rather demanding.

Answers: (a) 2 h 30 min; (b) 3 h 12 min; (c) 14 12.

### Question 20

The majority of candidates scored well on this question, though many were content to leave the answer to the first part as  $V = \frac{k}{P}$ . A small number used direct variation rather than inverse.

Answers: (a)  $V = \frac{3}{P}$ ; (b)  $P = \frac{1}{3}$ ;  $V = \frac{3}{5}$  or 0.6.

### Question 21

The first part was generally well done, but the answer was often left in an unsimplified form. The second was well done by the stronger candidates, but the weaker ones rarely did anything effective after a correct first step. A small number used values of  $R$  and/or  $S$  from the first part in the second.

Answers: (a) 32.5; (b)  $V = \frac{S}{3S - R}$ .

**Question 22**

The first part was not well done. Too many merely converted 15 m to 0.015 km. The second part was much better, with many correct answers. The more able tackled the last part quite effectively, but the weaker were challenged. The small number that used the area of a trapezium reached the answer easily, but more formed an equation to compare the distance travelled in the three parts of the motion to 750 m. Some of these found the motion at constant speed ended after 40 seconds, but did not answer the question set. A common error was to use such things as 750/15.

Answers: (a) 54 km/h; (b) 30 s; (c) 10 s.

**Question 23**

There were some excellent responses to this question. These recognised that the given matrix represented a stretch of factor 2, parallel to the x-axis. This produces diamonds going as far as (2, 0), (4, 0) and (8, 0) in parts (a), (c) and (d). Weaker candidates were unable to multiply matrices accurately, so progress was limited.

A common error in (b) was  $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$ , while the answer in (e) was left as  $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}^n$  or  $\begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$  too often.

Answers: (b)  $\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ ; (e)  $\begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$ .

**Question 24**

Some candidates tackled this question with confidence, gaining good results, but it was a searching question.

In (a)(i) the fact that the base of the pyramid is a triangle, with area  $\frac{1}{2} \times 5 \times 5$ , escaped many. As in

**Question 14**, poor division ( $125 \div 6 = 2.83$ ) led to loss of marks on a number of scripts.

In (a)(ii) the use of the volumes of similar bodies implied that the sides of the second pyramid were twice those of the original. Thus the cuts pass through vertices of the cube, being the diagonals of the faces that contain  $P$ .

In (b) most candidates tried to take the volume of the prism from the whole cube, though a few tried to find the area of cross-section first. The distinction between a prism and a pyramid was not always appreciated, and the fraction used was often not  $\frac{1}{2}$ . As in **Question 10**, poor subtraction ( $1000 - 125 = 875$ ) was noted in some cases.

Answers: (a)(i)  $20.8 \text{ cm}^3$ ; (b)  $875 \text{ cm}^3$ .

Paper 4024/02

Paper 2

**General comments**

The standard of the paper proved to be appropriate, with almost all candidates able to attempt a number of questions and many able to gain high (80+) marks. One or two parts of **Section A** (in particular **Question 6 (a) and (d)**) and the final parts of all the **Section B** questions proved to be challenging, even to the strongest candidates, and were thus good discriminators.

Most candidates appeared to have sufficient time to complete the paper, although a number (rather more than in previous years) ignored the rubric and attempted all 5 questions in **Section B** – often meaning that they rushed the paper, made careless mistakes and had no time available for checking their work.

Presentation was generally good, with most candidates laying out their work clearly and organising their responses in order to make them easy to follow. In only a small number of cases was there a significant omission of working. There were again, however, a few candidates who divided their pages into two, making the recording of marks particularly difficult.

A significant number of candidates spent unreasonable amounts of time on parts of questions which were worth only one or two marks. In particular, although there was only one mark available, many candidates spent a considerable time on **Question 3 (b)**, sometimes covering over a page with various trigonometrical calculations. Candidates should be aware that if only one mark is available then very little working is required.

Although premature approximation was mentioned in some detail last year, almost all Examiners noticed even more cases this year. The rubric asks for three significant figure answers, but a large number of candidates approximated to two in the early stages of a question and inevitably lost marks when their answers were well outside the acceptable ranges. It was also worrying to see an increasing number of candidates simply writing down the first three figures, rather than 'correcting' the third, for example giving  $\sqrt{427}$  as 20.6. There was again a small number of candidates losing marks through using grads rather than degrees.

### Comments on specific questions

#### Question 1

- (a) This part was usually correct. A small number of candidates divided by 1.60 instead of multiplying.
- (b) A few candidates found £8 but forgot to add on the £3. Some found 2% of £403 or £397.  
A number of candidates found (ii) difficult. Some tried to use proportion, evaluating  $400 \times \frac{15}{11}$ , while others forgot to subtract £3 as the first step.
- (c) Candidates showed a good understanding of ratio and, when errors were made, it was usually in the arithmetic rather than the method.
- (d) This part proved to be more difficult for many candidates, even those who gained high marks on the rest of the paper. The majority based their answers on 8% of \$135. Relatively few candidates identified \$135 with 108%.

Answers: (a) \$640; (b)(i) £11, (ii) £600; (c) \$175; (d) \$10.

#### Question 2

- (a) Most candidates were able to expand the brackets correctly as  $2q^2 + 6rq - rq - 3r^2$ , but instead of combining the two middle terms, many then attempted to refactorise their expression into two brackets.
- (b) Part (i) was usually correct although the minus sign was sometimes lost. A few evaluated  $(-10)^3$  and a small number squared instead of cubed. There was less success with (ii) with many sign errors, particularly from those who tried to combine the fractions over a negative denominator.  
A few candidates left their answer as  $-2\frac{2}{4}$ .
- (c) This was generally well answered, although many left their answer as  $3(y^2-1)$ . A few dropped the 3 from their final answer and other attempts had too many 3s, for example  $3(y+3)(y-3)$ .
- (d) Average and weaker candidates found this question very difficult.  $200 + x$  was seen frequently, but expressions such as  $2120 + 5x$  were common. It was also common to see 5 multiplying the wrong expression, and the fact that this produced a negative answer did not seem to worry the candidates.

Answers: (a)  $2q^2 + 5rq - 3r^2$ ; (b)(i) -40, (ii)  $-2\frac{1}{2}$ ; (c)  $3(y-1)(y+1)$ ; (d) 28.



## Question 3

The majority of candidates gained good marks here, although many used extremely long and complicated methods. Thus although some candidates gained full marks in 4 or 5 lines, others took 2 or 3 full pages. Marks were lost through premature approximation on numerous occasions.

- (a) Part (i) was usually correct, although the sine rule was often used and those who wrote  $\sin C = \frac{2}{3} = 0.66$  lost the accuracy mark. Those who started part (ii) with  $\cos 31 = \frac{18}{AT}$  usually continued correctly although a few continued  $AT = 18 \cos 31$ .
- (b) Most candidates showed that they knew what an angle of elevation was, but some took over a page to get to their answer.

Answers: (a)(i)  $41.8^\circ$ , (ii)  $21.0^\circ$ ; (b)  $39^\circ$ .

## Question 4

- (a) A large number of candidates were able to prove the result, but many either assumed the sum to be  $720^\circ$  or left the part unanswered.
- (b) This was not very well answered, with many candidates apparently not familiar with the terms 'reflex', 'obtuse' and 'acute'. In (a) the answer was often given as  $150^\circ$ . In (b) a significant number gave an answer of  $120^\circ$ , believing  $P\hat{A}S$  to be equal to  $B\hat{A}F$  as vertically opposite angles.

Answers: (b)(i)(a)  $210^\circ$ ; (b)  $150^\circ$ ; (c)  $15^\circ$ , (ii) Equilateral.

## Question 5

- (a) Many candidates did not understand what was required in part (i). There were answers of 2D, 3D up to 6D, others wrote 'rectangle' or 'cuboid' and yet more quoted a formula which they evaluated. Despite these wrong interpretations many did, in fact, use the correct values of the length, breadth and height in the rest of the question. Part (ii) was well done, although a few did not realise that it was a closed box and others gave the surface area of the inside and outside.

In the remaining parts of the question there was some confusion over the use of the various formulae for the surface area and volume of cylinders and spheres.

- (b) Common errors resulted from the use of  $2\pi rh + \pi r^2$  or  $\pi rh + 2\pi r$  or similar. A few could not see how to find the height of the cylinder and left  $h$  in their answers.
- (c) This was very well answered, although a few only found the volume of one ball and others thought the volume of a sphere was given by  $4\pi r^2$  (in spite of the fact that the correct formula was quoted in the question).
- (d) Some candidates took the answer to (c) as the volume of the cylinder, but the majority knew what was required. Many of these, however, did not give their answer correct to three decimal places.
- (e) Many did not see the connection with the previous part and started afresh to calculate the space in each container.

Answers: (a)(i) 12 by 12 by 6, (ii)  $576 \text{ cm}^2$ ; (b)  $509 \text{ cm}^2$ ; (c)  $452 \text{ cm}^3$ ; (d) 0.785; (e) Box.

## Question 6

- (a) Relatively few candidates appeared to understand the term 'histogram'. The great majority drew columns of the correct widths, but most simply drew columns with heights as the given frequencies.
- (b) This again was not answered well, sometimes a class interval being given as an answer.



- (c) Rather surprisingly many candidates showed that they did not understand how to calculate the mean from grouped frequencies. Many opted to use the upper bounds of the intervals or even the class widths rather than the mid-points.
- (d) This proved to be one of the most difficult parts of the paper. Of those who did attempt it, many were successful with part (i), but very few achieved the correct answer in (ii). Quite a number reached  $\frac{15}{79}$ , but failed to realise that this needed to be doubled. A significant number did not read the question carefully enough and assumed that there was replacement.

Answers: (b) 15; (c) 5.64; (d)(i)  $\frac{19}{316}$ , (ii)  $\frac{30}{79}$ .

### Question 7

- (a) Candidates regularly found the length of the minor arc and although a few then subtracted from the circumference, the majority thought that was the answer. Others used  $\pi r^2$  for the circumference.
- (b)(i) It was common to see the explanation “opposite angles are supplementary” or “angle at the centre is twice the angle at the circumference”. Relatively few stated that the tangent and radius were perpendicular or that the quadrilateral was cyclic.
- (ii) This was quite well answered, although many used the sine rule rather than basic trigonometry.
- (iii) A good number calculated the arc  $ADB$  again, sometimes being successful here after being unsuccessful in part (a). Some realised that the arc  $ADB$  should be added to twice their answer to (ii), but then added on the two radii  $AC$  and  $CB$ .
- (c)(i) There were many correct answers, although both 2 and 8 appeared fairly frequently.
- (ii) This was generally well answered.
- (iii) This proved difficult, and of those who had the right idea and got as far as 16.2 many went on to give 16 minutes 2 seconds as their answer.

Answers: (a) 25.1 cm; (b)(ii) 10.4 cm, (iii) 45.9 cm; (c)(i) 4, (ii)  $30^\circ$ , (iii) 16 minutes 12 seconds.

### Question 8

- (a) Many excellent curves were seen, and hardly any incorrect plots. There were a few cases of ruled lines joining points and it was fairly common to see the curve flattened between  $x = -2$  and  $-3$  and between  $x = 2$  and  $3$ .
- (b) Many quoted 58 as the maximum value, either from their flattened curve or from the table and a few gave the  $x$  value. It was also common to see  $-1.2$  as the least value of  $x$ , candidates failing to realise that the curve intersected  $y = 50$  at a point with a lower value of  $x$ .
- (c) Most candidates drew good tangents but some had difficulty with the calculation of the gradient, quite a number forgetting that it was negative.
- (d) Most candidates drew the line carefully and many successfully found its equation. In part (iii) very few attempted to find  $a$  and  $b$  by equating  $27 - 8x$  and  $30 - 18x + x^3$ . It was much more common to see very lengthy methods involving the solution of simultaneous equations.

Answers: (b)(i)  $58 < y \leq 60$ , (ii)  $-3.6 \leq x \leq -3.4$ ; (c)  $-16 \leq \text{gradient} \leq -12$ ; (d)(ii)  $y = 27 - 8x$ , (iii)  $a = -10$ ,  $b = 3$

### Question 9

This was the most popular **Section B** question and there was a high success rate overall, particularly for the first three parts. Some marks were, however, lost as a result of premature approximation in one or more parts.

- (a) This was very well answered, although even in this part some candidates used very long methods, finding  $DC$  first.
- (b) The cosine rule was used very effectively and there were very few cases where candidates made the error of progressing from  $845 - 836\cos 60$  to  $9\cos 60$ , an error which has been common in earlier years.
- (c) Again candidates applied the formula correctly and most reached the correct result, although some found a perpendicular height first and then used  $\frac{1}{2} \text{ base} \times \text{height}$ . Some weaker candidates simply evaluated  $\frac{1}{2} \times 19 \times 22$ .
- (d) This part proved much more difficult and few used their answer to part (c). Many candidates found either  $B\hat{A}D$  or  $B\hat{D}A$  and then evaluated  $22\sin B\hat{A}D$  or  $19\sin B\hat{D}A$ . Some found the wrong perpendicular distance, often that from  $D$  to  $AB$ . Of the incorrect methods used, many assumed that the perpendicular bisected either  $A\hat{B}D$  or the side  $AD$ . It was also regularly assumed, either in this or in earlier parts, that  $A\hat{D}C$  was a right angle or that triangle  $ADB$  was isosceles.

Answers: (a) 14.9 cm; (b) 20.7 cm; (c)  $181 \text{ cm}^2$ ; (d) 17.5 cm.

### Question 10

This was a somewhat less popular question, but most of those who did attempt it gained good marks. Weaker candidates gained marks from the first two parts and from the solution of the quadratic.

- (a)(b) These were generally well done, although in a number of cases answers appeared with units inserted.
- (c)(i) The derivation of the quadratic equation was also well answered, though there were cases of carelessness which did not lead to the right equation. It was surprising that candidates were unable to spot their mistakes.
- (ii) The quadratic formula was well known and many gained all four marks. The main error came with the sign in the  $b^2 - 4ac$  part; this frequently became 1525 instead of 2525. Quite a number of candidates lost the last mark as they did not give the two answers to the required degree of accuracy.
- (iii) The evaluation of  $\frac{200}{x+5}$ , instead of  $\frac{200}{x}$  was seen occasionally and some candidates could not see the connection between the positive answer to part (ii) and the requirement of part (iii).

Answers: (a)  $\frac{200}{x}$ ; (b)  $\frac{200}{x+5}$ ; (c)(ii) 47.62 and  $-2.62$ , (iii) 4 hours 12 minutes.

### Question 11

This was the least popular question and although most candidates could tackle some parts, relatively few were able to gain high marks.

- (a)(i) Most candidates recognised that Pythagoras was required and there were many correct answers, but  $(-9)^2 = -81$  was not uncommon.
- (ii) Again, most candidates knew what was required, but there were frequent errors with the signs.
- (b)(i) On the whole this was poorly answered. Many candidates referred to the ratio of sides, others thought one pair of equal angles was sufficient and there were numerous references to SAS and AAS.

- (ii) Most candidates gained these two marks.
- (iii) Again, most candidates achieved the correct answer and gained the mark, although relatively few appreciated the property linking the areas of similar figures. The majority used the formula  $\frac{1}{2} a \times b \times \sin \theta$  in each triangle.
- (iv) This part proved very difficult for all but the strongest candidates, with many producing nonsense, with expressions such as  $\frac{p}{q}$  or  $p - 6$ .
- (v) There were a few good, clear answers, but many were meaningless.

Answers: (a)(i) 41, (ii)  $\begin{pmatrix} 12 \\ -56 \end{pmatrix}$ ; (b)(iii)  $\frac{1}{4}$ ; (iv)(a)  $3p$ , (b)  $\frac{3}{4}q$ , (c)  $p + \frac{3}{4}q$ , (d)  $3p + q$ .