MATHEMATICS

GCE Advanced Subsidiary Level

Paper 8709/01
Paper 1

General comments

The response to this paper was pleasing. There were many excellent scripts and the presentation was generally good. Candidates seemed to have sufficient time to answer all the questions and there was little evidence of later questions being rushed. Much time was lost, however, on **Question 3**, by candidates who interpreted the word 'sketch' as 'accurate graph'. Many candidates also need to read the rubric which requests that answers to angles should be given correct to 1 decimal place, unless otherwise requested.

Comments on specific questions

Question 1

The majority of candidates elected to form a quadratic equation in x by eliminating y from the given equations. Only about half of these recognised the need to set $b^2 - 4ac$ to 0, and many others failed to set the quadratic to 0 before applying $b^2 - 4ac = 0$. Many candidates preferred to equate the gradient of the line with the differential of $x^2 - 6x + 14$ and then to obtain x, y and finally k. This method was usually successful, but setting the gradient to either 0 or to 2 were common errors.

Answer: $k = 10$.

Question 2

(i) Most candidates realised the need to take 2 out of the expression, but errors such as '$2x^2 - 12x + 11 = 2(x^2 - 12x + 5.5)$ leading to $2(x - 6)^2 + \ldots$' were common.

(ii) This was badly answered with many candidates either substituting $x = 0$ or obtaining a table of values. The answer $-7 \le f(x) \le 11$ was common, as was $f(x) \ge 11$. Many candidates failed to realise that there was a link with part **(i)**, (i.e. $f(x) \ge c$) and preferred to use calculus to find the minimum point (3, −7). Even then it was not automatic to state $f(x) \ge -7$.

Answers: **(i)** $2(x - 3)^2 - 7$; **(ii)** $f(x) \ge -7$.

Question 3

(i) Most candidates correctly drew the graph of $y = \cos x$, although 'V' shapes were common. Very few drew the graph of $y = \cos 3x$ correctly, most thinking either that the graph lay between −3 and +3 or that the graph of $y = \cos x$ had the same shape as $y = \cos 3x$ between 0 and 2π. Many candidates wasted considerable time by ignoring the instruction 'sketch' and instead drawing accurate graphs.

(ii) Only a handful of candidates realised that for f to have an inverse it needed to be 1 : 1 and that this only occurred for $0 \le x \le \pi$, leading to $k = \pi$.

Answers: **(i)** Sketch; **(ii)** $k = \pi$.

Question 4

Candidates should realise that a request for an answer in terms of π and $\sqrt{3}$ means that use of a calculator will lead to loss of marks. Many candidates failed to realise that angle $POQ = \frac{1}{3}\pi$ or that $\cos 30° = \frac{1}{2}\sqrt{3}$.

Use of $s = r\theta$ with θ in degrees occurred occasionally and a common error was to assume that the radius of the arc *PXQ* was *PS* rather than *PO*. The most common error however was to express $\cos 30° = \frac{6}{OS}$ as $OS = 6\cos 30°$.

Answer: $4\pi + 8\sqrt{3}$.

Question 5

This was well answered and a half of all attempts were completely correct. Occasionally the surface area was given as $3x^3$ and there were solutions in which areas of only 3 or 5 faces were considered. Differentiation was usually correct. In part **(ii)**, although the chain rule was usually correctly quoted for the variables concerned, there were misunderstandings over $\frac{dA}{dt} = 0.14$ and often the rate of decrease of x was given as $\frac{dt}{dx}$ rather than $\frac{dx}{dt}$.

Answers: **(i)** $A = 14x^2$ and $\frac{dA}{dx} = 28x$; **(ii)** 0.0025.

Question 6

At least a half of all attempts were completely correct. There were occasional errors in the calculation of the gradient of *AB*, but most candidates realised that the gradient of the line L_2 was $-1 \div \frac{1}{2}$. There were a few misunderstandings of the relationship between L_1 and L_2, particularly from weaker candidates who automatically involved the mid-point of the line *AB*. The solution of the simultaneous equations was very well done.

Answer: (4, 6).

Question 7

(i) Although a few candidates took '$(2s + c)^2 = 4s^2 + c^2$ ' and '$(2c - s)^2 = 4c^2 \pm s^2$ ', most correctly cancelled the $4\cos\theta\sin\theta$ and reduced the expression to $5s^2 + 5c^2$ and from there to 5. Others however divided throughout by 5 and stated the answer as 1.

(ii) Most realised the need to collect terms and to express $7\sin\theta = 4\cos\theta$ as $\tan\theta = \frac{4}{7}$. Despite having the formula on the formula sheet, many expressed $\tan\theta$ as $\cos\theta \div \sin\theta$, or were unable to manipulate the expressions correctly. Candidates need to read questions closely, for many omitted to find the corresponding values of θ. It was very rare for candidates to go wrong over which quadrants to use.

Answers: **(i)** $a^2 + b^2 = 5$; **(ii)** $\tan\theta = \frac{4}{7}$, $\theta = 29.7°$ or $209.7°$.

Question 8

At least a quarter of all candidates failed to read the question carefully and took the progression as being arithmetic. Of the other attempts, many were completely correct and it was surprising to see some weaker candidates scoring highly on this question. Some wrote down all the amounts for the first 10 years of operation, and others incorrectly took the nth term as ar^n. In part **(ii)** common errors were to take the 20th term rather than the sum of 20 terms and the formula for sum of 20 terms was often taken as $S_n = \frac{1}{2}n(1 - r^{n-1})/(1 - r)$. It was pleasing that in part **(iii)** the majority of candidates realised the need to find the sum to infinity.

Answers: **(i)** 775kg; **(ii)** 17 600kg; **(iii)** 20 000kg.

Question 9

(i) About a half of all candidates took the equation of the curve to be the equation of the tangent and a straight line with gradient 21 was depressingly common. Surprisingly enough, many of these candidates then realised the need to integrate the given expression for $\frac{dy}{dx}$ and produced this in part **(ii)**. The integration was generally good, though common errors were to fail to cope with the negative power, to ignore the integral of −3 or to fail to realise the need to include and evaluate the constant of integration.

(ii) Most realised the need to set $\frac{dy}{dx}$ to 0 to find the stationary point and despite a few algebraic slips, most obtained $x = 2$.

Answers: **(i)** $y = -\frac{12}{x^2} - 3x + 31$; **(ii)** (2, 22).

Question 10

There were many excellent responses to this question and the candidates' ability to calculate 'scalar product' was impressive. Unfortunately, too many marks were lost in the first part through failure to obtain correct expressions for the vectors $\overrightarrow{MN}$ and $\overrightarrow{MD}$. A surprising number of candidates totally ignored the dimensions of the problem (i.e. OA = 6cm, OC = 8cm and OB = 16cm) and gave their answers with coefficients of **i**, **j** and **k** as ±1 or $\pm\frac{1}{2}$. Others were able to cope with $\overrightarrow{MD}$ but struggled with the fact that N was the mid-point of AC. Even then, all the method marks available for part **(ii)** were usually obtained.

Answers: **(i)** $\overrightarrow{MN} = -3\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{MD} = -6\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}$; **(ii)** −14, 97º.

Question 11

Many candidates scored highly on the question, but for weaker candidates, failure to recognise the need for 'function of a function' led to loss of marks. Surprisingly, several candidates took the gradient of the tangent to be the gradient of the normal and calculated $-1 \div \frac{dy}{dx}$, but otherwise part **(i)** was usually correct. The follow through mark for part **(ii)** was nearly always obtained, though a few candidates put $y = 0$ rather than $x = 0$.

In part **(iii)**, the integration was surprisingly better than the differentiation in part **(i)** and more candidates included the '$\frac{1}{8}$' than the '8' in part **(i)**. Most candidates worked about the x-axis and realised the need to subtract areas. Use of limits was generally correct, though 0 to 5 was often used instead of 0 to 3. The most common error was to automatically assume that the value of any expression at 0 is 0. Candidates choosing to work about the y-axis fared badly because it was very rarely realised that the limits for the line were different from the limits for the curve.

Answers: **(i)** $5y = 4x + 13$; **(ii)** (0, 2.6); **(iii)** $\frac{16}{15}$ or 1.07 unit^2 .

Paper 8709/02

Paper 2

General comments

Few high marks were scored by candidates, largely due to candidates' weaknesses in certain areas of the syllabus, such as integration and iteration techniques. There was a general tendency to use degrees, rather than radians, for angles.

Conversely, there were certain key techniques, such as differentiation of functions, that were commonly the source of many marks for candidates.

Work was clear and neat, with no evidence of candidates running out of time at the end of the paper.

Comments on specific questions

Question 1

Many candidates scored full marks and recognised that each value of $\tan x$ gave rise to two values of x. Weaker solutions, that scored no marks, showed no use of the condition $\sec^2 x = 1 + \tan^2 x$ and intractable equations involving both $\cos x$ and $\sin x$, with neither of these having been eliminated, were common.

Answer: $x = 135^\circ$ or 315°, 56.3° or 236.3°.

Question 2

(i) As the answer was given, weaker candidates struggled to produce it via errors such as $u^2 = 2^{x+1}$ and $4^x = 2.2^x = 2u$. However, most candidates obtained the given result correctly.

(ii) Many candidates obtained the correct result $u = 1 + \sqrt{13} \approx 4.6055$, but then failed to solve for $x = \ln u \div \ln 2$.

Answer: **(ii)** $x = 2.20$.

Question 3

(i) Most candidates correctly sketched the line $2y = x + 1$, but very few obtained a plot of $2y = |x - 4|$, and most graphs showed portions of that line *below* the x-axis.

(ii) There were a pleasing number of fully correct solutions, sometimes based on 'trial and error', or simply by quoting the answer. Weaker solutions involved squaring one or both of the equations of the two lines, but with only *one* side being squared, e.g. $2y = (x - 4)^2$.

Answer: **(ii)** $x = 1.5$, $y = 1.25$.

Question 4

Although many candidates noted that $\ln y = \ln A + n\ln x$, wrong answers were often based on false variants, such as $\ln y = (An)\ln x$. There was a marked tendency to substitute the values (1, 2.4) and (4, 0.6) into the original equation $y = Ax^n$ rather than into the $\ln y$ versus $\ln x$ relation.

Answer: $n = -0.6$, $A = e^3 = 20.1$

Question 5

(a) This part was very well done, with a sound grasp of all the essential ideas.

(b) There were many excellent solutions, and this question was admirably handled. Only occasionally was the derivative of *xy* given as $x\frac{dy}{dx}$ only.

Answers: **(a)** $\frac{4}{9}$, **(b)** $y + 4x = 14$.

Question 6

(i) Although this part presented little or no problems, candidates struggled thereafter.

(ii) Almost all candidates expressed $u_2 = \tan^{-1}2$ in **degrees**, without noting the vast disparity between this value and that of $u_1 = 1$.

(iii) Almost no one could cope with this part, failing to spot that, as $n \to \infty$, u_n and u_{n+1} tend to the same limiting value, this latter being the required value x_m.

Answer: **(ii)** 1.08.

Question 7

(i) There were many sign errors, for example, $\cos^2 x \equiv \frac{1}{2}(1 - \cos 2x)$, and often a factor '2' was omitted.

(ii) Candidates failed to remove brackets and use the twin results of **(i)**. Many candidates believed that $\int (2\sin x + 3\cos x)^2 dx = \int [\phi(x)]^2 dx = \frac{\phi^3}{3}, \frac{\phi^3}{3\phi'}$ or $2\phi'$.

Answer: **(ii)** $\frac{(13\pi + 34)}{8} \approx 9.36$.

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Paper 4

General comments

The paper was generally well attempted. However candidates often failed to obtain answers correct to three significant figures, even when correct methods were used. This is a problem which Centres are urged to address. Most inaccuracy arises from premature approximation.

There is clearly a problem too, with terminology. Familiarity with and understanding of terms germane to the syllabus are expected. In very many cases candidates attempted to find forces when work done by forces was required, and coefficient of friction when frictional force was required.

Inappropriate use was made of $F = \mu R$ in both **Question 5** and **Question 7**.

Comments on specific questions

Question 1

Most candidates answered this question correctly.

A few candidates used sine instead of cosine; some did not use the given angle, obtaining the incorrect answer of 2400 J.

Mis-reading 30 N and 10° as 10 N and 30° was fairly common.

Answer: 2360 J.

Question 2

Most candidates scored the two marks in part **(i)** of the question. However mistakes were frequently made in part **(ii)**, the most common of which were:

Showing the maximum speed as 12.5 ms^{-1} (from 1500/120) despite having the correct answer in **(i)**.
Having a positive slope for the constant speed stage.
Having a positive slope for the deceleration stage.
Failing to terminate the deceleration stage on the *t*-axis, even in cases where the slope is negative.
Showing a slope for the acceleration stage which is as steep, or steeper, than that for the deceleration stage.

Answers: **(i)** 25 ms^{-1}; **(ii)** Sketch.

Question 3

This question was well attempted.

Many candidates failed to score the last two marks because they gave the wrong direction (downwards instead of upwards) for the frictional force.

Many candidates lost the final mark because of lack of accuracy.

Answer: $P = 0.768$.

Question 4

Most candidates recognised the need to integrate the given $v(t)$ in part **(i)**, although some obtained $s = 4t^2 - 0.04t^4$ using the inappropriate constant speed formula 'speed = distance /time', and some obtained $s = 2t^2 - 0.06t^4$ using the inappropriate constant acceleration formula $s = ut + ½ at^2$ via $s = ½ (4 - 0.12t^2)t^2$.

Some candidates who did integrate found problems in dealing with the constant of integration. Some found it to be equal to 100 leading to the equation $2t^2 - 0.01t^4 = 0$ and some left the constant as '*c*' and were thus unable to solve $2t^2 - 0.01t^4 + c = 100$.

Most candidates recognised that setting $s(t) = 100$ leads to a quadratic equation in t^2, but attempts to solve the quadratic were generally rather poor. A common approach was to say $t^2(2 - 0.01t^2) = 100 \Rightarrow t^2 = 100$ or $2 - 0.01t^2 = 100 \Rightarrow t^2 = 100$ or $t^2 = -9800$, or a variation of this erroneous method.

Frequently candidates used a basically correct method for the quadratic, but called the repeated root t instead of t^2.

Yet another erroneous approach was to use the quadratic formula, but with $a = 2$ and $b = -0.01$, instead of the other way round.

In part **(ii)** most candidates recognised the need to differentiate $v(t)$ and most did this correctly.

Answers: **(i)** $t = 10$; **(ii)** slowing down.

Question 5

Almost all candidates answered part **(i)** correctly, but in part **(ii)** most candidates used a circular argument in which the required result was implicitly assumed.

In part **(iii)** most candidates recognised the need to resolve forces horizontally and vertically on *B*. In some cases the weight of *B* was omitted when resolving vertically, but generally candidates obtained equations with the correct numbers of terms.

A number of candidates failed to realise the need also to resolve forces vertically on *R* in order to be able to quantify the tension occurring in their equations.

A common error in resolving forces on *B* vertically was to have the wrong sign for the tension term, giving the normal component as 0.5 N in cases which were otherwise correct.

Many candidates incorrectly used μR instead of F for the frictional component, giving an answer for μ instead of an answer for F.

Answers: **(i)** $T_{AR} = T_{BR}$; **(iii)** 5.5 N, 4.33 N.

Question 6

Almost all candidates obtained the correct answer in part **(i)**.

Candidates also correctly found the speed of *P* at the instant that *Q* strikes the ground. However it is doubtful that many candidates were correctly motivated to find this speed, judging from the absence of correct work, from almost all candidates, beyond this stage. Very few candidates appreciated that *P* is moving under gravity whilst the string is slack.

A few candidates did obtain the distance travelled upwards by *P*, under gravity, but many such candidates failed to double this distance to obtain the required answer.

Answers: **(i)** 1.11 ms^{-2}; **(ii)** 0.316 s; **(iii)** 1 m.

Question 7

In part **(i)** many candidates obtained the gain in gravitational potential energy correctly, but asserted incorrectly that this is the work done against the resistance to motion. Others obtained the work done by the car's engine correctly, but asserted that this is the required answer.

Among the candidates who had all three terms present in the work/energy equation, many made sign errors.

Many candidates used the formula 'WD = $Fd\cos\alpha$ inappropriately (= $1800 \times 500 \cos 6^{\circ}$).

Many candidates who found a quantity of work of some sort, divided this quantity by 500 to obtain an answer for the resistance, believing this to be what was required for the answer.

A few candidates approached part **(i)** by first resolving forces on the car to find the magnitude of the resistance, and then multiplying by the distance travelled. Such candidates were generally successful. However almost all of the candidates who set out to find the magnitude of the resistance asserted that this was the answer sought, demonstrating an absence of understanding of work done.

In part **(ii)** many candidates obtained the gain in kinetic energy correctly, but asserted incorrectly that this is the work done by the car's engine.

Very few candidates had all four terms present in the work/energy equation. It was very common for one or both of the gain in gravitational potential energy and the work done by the resistance to be omitted.

Much the most common approach in part **(ii)** was to assume, implicitly and wrongly, that the acceleration is constant. In this case a special ruling allowed candidates to score up to 2 marks. Very few achieved this however; rarely were all four terms present in the equation of motion, and multiplication of the driving force by the distance was often omitted.

In both parts **(i)** and **(ii)** it was very common to see μR written for the resistance to motion, and sometimes a value for 'coefficient of friction' was found.

Although a candidate's (erroneous) method for finding the work done by the car's engine in part **(ii)** implies that the driving force is constant (usually 2360 N), this was often followed in part **(iii)** by actual values (usually 9440 and 2360) in the ratio 4:1 at the top and bottom of the hill. Where such cases led to the correct answer of 10:1 for the required ratio, using relatively correct working, Examiners allowed 1 mark out of 3 to be scored.

Some candidates scored all three marks in part **(iii)**, and in a few cases these were the only marks scored in the question.

Answers: **(i)** 273 000 J; **(ii)** 1 180 000 J; **(iii)** 10:1.

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Paper 6

General comments

The paper produced a wide range of marks. All questions were well attempted, with almost everyone finishing in the required time. It was pleasing to see that most candidates worked to at least 4 significant figures and corrected to 3 at the end.

Comments on specific questions

Question 1

There was, alas, about one fifth of candidates who could not do the standard deviation, squaring Σx to find $\Sigma(x^2)$. An accuracy mark was also lost by those who worked to 3 significant figures throughout, instead of 4 significant figures and correcting to 3 at the end. Candidates who just wrote down the correct answers from their calculator gained full marks, while other candidates received no marks for just incorrect answers.

Answers: 13.1, 2.76.

Question 2

This, the first serious question on permutations and combinations in the new syllabus, showed that most candidates had covered the topic and had an idea how to start. However, many candidates failed to associate the words *in order*, which were written in italics to bring attention to them, with the associated permutations, and wrote ${}_{10}C_6$ instead of ${}_{10}P_6$. The second part was well attempted.

Answers: **(a)** 151 200; **(b)** 144.

Question 3

This was a straightforward question, which gained full marks for many candidates.

Answers: **(i)** 0.82; **(ii)** 0.293.

Question 4

(i) The histogram had a variety of frequency densities, ranging from frequency/class width, frequency/midpoint, class width/frequency, cumulative frequency/class width, and many other combinations. The good point is that nearly all candidates realised that some adjustment had to be made. Axes were labelled and headings were clear.

(ii) This part was the worst answered part of the whole paper. Many candidates did not attempt it, some found the probability of one church having less than 61 people, and then multiplied it by 3, or just left it as a single probability. A few candidates used combinations. Both cubing and combinations were acceptable.

Answer: **(ii)** 0.171 or 0.172.

Question 5

The normal distribution does not offer many varieties of approach, either finding a probability as in part **(i)** or finding an x as in part **(ii)**. The second part proved too difficult for many candidates. Many could not read the tables backwards, and many seemed to use the body of the tables rather than the critical values part at the foot of the page. This resulted in a slightly different z-value, which was not penalised this time, but could be in future. Part **(iii)** involved appreciating that a probability is just that, and an associated number can be obtained from $P(S) = \dfrac{n(S)}{n(E)}$. Candidates who used a continuity correction gained one mark only in each part.

Answers: **(i)** 0.117; **(ii)** 20.4; **(iii)** 23.

Question 6

This question was well done by the majority of candidates. Some candidates did not appear to understand what 'fewer' meant, and some even found P(0) + P(1) + P(2) …up to P(9)! They were not penalised except in terms of time. Part **(ii)** also produced many good solutions. There was the usual number of wrong and absent continuity corrections and muddles with standard deviation and variance of course, but overall it was a pleasingly answered question.

Answers: **(i)** 0.849; **(ii)** 0.0519.

Question 7

Most candidates managed part **(i)** and this showed them how to approach part **(ii)**, with mixed success, but provided they had some probabilities in their distribution table, credit was given for applying their knowledge of mean and variance. A small percentage of candidates failed to recognise that 'mean' was the same as E(*X*), and divided their E(*X*) by the number of different values of *X* that they had.

Answers: **(ii)**

X	0	1	2	3
Prob	0.167	0.5	0.3	0.0333

(iii) 1.2, 0.56.