UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7874282319

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC13 06_9709_21/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Solve the equation $|2^x - 7| = 1$, giving answers correct to 2 decimal places where appropriate. [5]

2 Solve the equation $\ln(3 - 2x) - 2\ln x = \ln 5$. [5]

3 **(i)** Show that $12\sin^2 x\cos^2 x \equiv \frac{3}{2}(1 - \cos 4x)$. [3]

(ii) Hence show that

$$\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} 12\sin^2 x\cos^2 x\,\mathrm{d}x = \frac{\pi}{8} + \frac{3\sqrt{3}}{16}.$$ [3]

4 The polynomial $ax^3 - 5x^2 + bx + 9$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$, and that when $\mathrm{p}(x)$ is divided by $(x + 1)$ the remainder is 8.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

5 The parametric equations of a curve are

$$x = \mathrm{e}^{2t}, \quad y = 4t\mathrm{e}^{t}.$$

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)}{\mathrm{e}^t}$. [4]

(ii) Find the equation of the normal to the curve at the point where $t = 0$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 4x - 2,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.9$. [2]

(iii) Show that this root also satisfies the equation

$$x = \frac{1 + 2\tan x}{4\tan x}.$$ [1]

(iv) Use the iterative formula $x_{n+1} = \frac{1 + 2\tan x_n}{4\tan x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(i)** Express $5\sin 2\theta + 2\cos 2\theta$ in the form $R\sin(2\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$5\sin 2\theta + 2\cos 2\theta = 4,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$, [5]

(iii) determine the least value of $\dfrac{1}{(10\sin 2\theta + 4\cos 2\theta)^2}$ as θ varies. [2]

BLANK PAGE