CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a quotient or product or $2 = \log_{10} 100$	M1	
		Remove logarithms and obtain $x + 9 = 100x$, or equivalent	A1	
		Obtain answer $x = \frac{1}{11}$	A1	3
2		State a correct unsimplified version of the x or x^2 or x^3 term	M1	
		State correct first two terms $1 - x$	A1	
		Obtain the next two terms $2x^2 - \frac{14}{3}x^3$	A1 + A1	4
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{3} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]		
3	(i)	Use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$	M1	
		Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in $\tan x$ in any correct form	A1	
		Reduce the equation to the given form	A1	3
	(ii)	Solve the given quadratic for $\tan x$	M1	
		Obtain a correct answer, e.g. $x = 21.6°$	A1	
		Obtain a second answer, e.g. $x = 128.4°$, and no others	A1	3
		[Ignore answers outside the given interval. Treat answers in radians as a misread (0.377, 2.24).]		
4	(i)	Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$	M1	
		Complete the argument correctly with correct calculated values	A1	2
	(ii)	State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$	B1	
		Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa*	B1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.14	A1	
		Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	3
5		Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term in the form $a\sqrt{(2x+1)}$	M1	
		Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
		Obtain term of the form $k\tan\theta$	M1	
		Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k \tan \theta$, $ak \neq 0$	M1	
		Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2}\tan\theta + \frac{1}{2}$	A1	
		Rearrange and obtain $x = \frac{1}{8}(\tan\theta + 1)^2 - \frac{1}{2}$, or equivalent	A1	7

6		Obtain correct derivative of RHS in any form	B1	
		Obtain correct derivative of LHS in any form	B1	
		Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
		Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
		By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
		Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
		Obtain $y = \frac{1}{2}$	A1	7
7	(a)	*EITHER*: Multiply numerator and denominator by $1 - 4i$, or equivalent, and use $i^2 = -1$	M1	
		Simplify numerator to $-17 - 17i$, or denominator to 17	A1	
		Obtain final answer $-1 - i$	A1	
		OR: Using $i^2 = -1$, obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = -1$ or $y = -1$, or equivalent	A1	
		Obtain final answer $-1 - i$	A1	3
	(b) (i)	Show a point representing $2 + i$ in relatively correct position	B1	
		Show a circle with centre $2 + i$ and radius 1	B1$\sqrt{}$	
		Show the perpendicular bisector of the line segment joining i and 2	B1	
		Shade the correct region	B1	4
	(ii)	State or imply that the angle between the tangents from the origin to the circle is required	M1	
		Obtain answer 0.927 radians (or 53.1°)	A1	2
8	(i)	Use a correct method for finding a constant	M1	
		Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	4
	(ii)	Integrate and obtain term $-3\ln(2-x)$	B1$\sqrt{}$	
		Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
		Obtain term $\frac{3}{2}\ln(2+x^2)$	A1$\sqrt{}$	
		Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$	M1	
		Obtain given answer after full and correct working	A1	5
9	(i)	Substitute for x and dx throughout using $u = \sin x$ and d$u = \cos x$ dx, or equivalent	M1	
		Obtain integrand e^{2u}	A1	
		Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
		Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
		Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	5

	(ii)	Use chain rule or product rule	M1	
		Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
		Equate derivative to zero and obtain a quadratic equation in sin x	M1	
		Solve a 3-term quadratic and obtain a value of x	M1	
		Obtain answer 0.896	A1	**6**
10	**(i)**	Express general point of l in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda)$	B1	
		Substitute in given equation of p and solve for λ	M1	
		Obtain final answer $-\frac{1}{2}\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or equivalent, from $\lambda = -\frac{1}{2}$	A1	**3**
	(ii)	State or imply a vector normal to the plane, e.g. $2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
		Obtain answer 23.2° (or 0.404 radians)	A1	**4**
	(iii)	*EITHER*: State $2a + 3b - 5c = 0$ or $3a - 2b + 2c = 0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 4 : 19 : 13$, or equivalent	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$, and evaluate d	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})\times(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i} -19\mathbf{j} -13\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		*OR*2: Attempt to form a 2-parameter equation with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r} = \mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})+\mu(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		OR3: Using a relevant point and relevant direction vectors, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain correct values of two cofactors	A1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	**5**