

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/21

Paper 2 (Pure Mathematics), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	21

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	21

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	21

- 1 Either State or imply non-modular inequality $(x+1)^2 < (3x+5)^2$, or
corresponding equation or pair of linear equations M1
Make reasonable solution attempt at a 3-term quadratic, or solve
two linear equations M1
Obtain critical values -2 and $-\frac{3}{2}$ A1
State correct answer $x < -2$ or $x > -\frac{3}{2}$ A1
- Or Obtain one critical value, e.g. $x = -2$, by solving a linear equation (or inequality) B1
or from a graphical method or by inspection B2
Obtain the other critical value similarly B1
State correct answer $x < -2$ or $x > -\frac{3}{2}$ [4]

- 2 (i) Consider sign of $x^4 + 2x - 9$ at $x = 1.5$ and $x = 1.6$ M1
Complete the argument correctly with appropriate calculations A1 [2]
($f(1.5) = -0.9375$, $f(1.6) = 0.7536$)
- (ii) Rearrange $x^4 + 2x - 9 = 0$ to given equation or *vice versa* B1 [1]
- (iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.56 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1 [3]

$x_0 = 1.5$	$x_0 = 1.55$	$x_0 = 1.6$
1.5874	1.5614	1.5362
1.5424	1.5556	1.5685
1.5653		1.5520
1.5536		1.5604
1.5595		1.5561
1.5565		

or show there is a sign change in the interval (1.555, 1.565)

- 3 Obtain derivative $e^{2x} - 5e^x + 4$ B1
Equate derivative to zero and carry out recognisable solution method for a quadratic in e^x M1
Obtain $e^x = 1$ or $e^x = 4$ A1
Obtain $x = 0$ and $x = \ln 4$ A1
Use an appropriate method for determining nature of at least one stationary point M1
 $\left(\frac{d^2y}{dx^2} = 2e^{2x} - 5e^x, \text{ when } x = 0, \frac{d^2y}{dx^2} = -(3), x = \ln 4, \frac{d^2y}{dx^2} = +(12) \right)$
Conclude maximum at $x = 0$ and minimum at $x = \ln 4$ (no errors seen) A1 [6]
- 4 (i) Substitute $x = 3$ and equate to 14 ($9a + 3b + 35 = 14$) M1
Substitute $x = -2$ and equate to 24 ($4a - 2b = 24$) M1
Obtain a correct equation in any form A1
Solve a relevant pair of equations for a or for b M1
Obtain $a = 1$ and $b = -10$ A1 [5]

Page 5	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	21

- (ii) Attempt division by $x^2 + 2x - 8$ and reach a partial quotient of $x - k$ M1
Obtain quotient $x - 1$ with no errors seen (can be done by observation) A1
Correct solution method for quadratic e.g. factorisation M1
All solutions $x = 1$, $x = 2$ and $x = -4$ given and no others CWO A1 [4]
- 5 (i) State $\frac{dx}{d\theta} = -2 \sin 2\theta + \sin \theta$ or $\frac{dy}{d\theta} = 8 \sin \theta \cos \theta$ B1
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ M1
Use $\sin 2\theta = 2 \sin \theta \cos \theta$ M1
Obtain given answer correctly A1 [4]
- (ii) Equate derivative to -4 and solve for $\cos \theta$ M1
Obtain $\cos \theta = \frac{1}{2}$ A1
Obtain $x = -1$ A1
Obtain $y = 3$ A1 [4]
- 6 (a) (i) Attempt to divide by e^{2x} and attempt to integrate 2 terms M1
Integrate a term of form ke^{-2x} correctly A1✓
Fully correct integral $x - 3e^{-2x} (+c)$ A1 [3]
- (ii) State correct expression $\frac{1}{2} \cos 2x + \frac{1}{2}$ or equivalent B1
Integrate an expression of the form $a + b \cos 2x$, where $ab \neq 0$, correctly M1
State correct integral $\frac{3 \sin 2x}{4} + \frac{3x}{2} (+c)$ A1 [3]
- (b) State or imply correct ordinates 5.46143..., 4.78941..., 4.32808... B1
Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates M1
Obtain answer 4.84 with no errors seen A1 [3]
- 7 (i) State $R = \sqrt{10}$ B1
Use trig formula to find α M1
Obtain $\alpha = 18.43^\circ$ with no errors seen A1 [3]
- (ii) Carry out evaluation of $\cos^{-1}\left(\frac{2}{R}\right) (\approx 50.77^\circ)$ M1
Carry out correct method for one correct answer M1
Obtain one correct answer e.g. 34.6° A1
Carry out correct method for a further answer M1
Obtain remaining 3 answers 163.8° , 214.6° , 343.8° and no others in the range A1 [5]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/22

Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	22

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	22

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	22

1	(i) State indefinite integral of the form $k \ln(4x - 1)$, where $k = 2, 4$, or $\frac{1}{2}$	M1	
	State correct integral $\frac{1}{2} \ln(4x - 1)$	A1	[2]
	(ii) Substitute limits correctly	M1	
	Use law for the logarithm of a power or a quotient	M1	
	Obtain $\ln 3$ correctly	A1	[3]
2	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate (numerator) of derivative to zero and solve for x	DM1	
	Obtain $x = \frac{1}{3}$	A1	
	Obtain $y = \frac{3}{2}$	A1	[5]
3	Use trig identity correctly to obtain a quadratic in $\csc \theta$ or $\sin \theta$	M1	
	Solve the quadratic correctly	M1	
	Obtain $\sin \theta = \frac{1}{4}$ or $-\frac{2}{3}$	A1	
	Obtain one correct answer	A1	
	Carry out correct method for second answer from either root	DM1	
	Obtain remaining 3 answers from 14.5, 165.5, 221.8, 318.2 and no others in the range	A1	
	[Ignore answers outside the given range]		[6]
4	(i) Substitute $x = 3$ or $x = -2$ and equate to zero	M1	
	Obtain a correct equation in any form	A1	
	Obtain a second correct equation in any form	A1	
	Solve a relevant pair of equations for a or for b	M1	
	Obtain $a = 4$ and $b = -3$	A1	[5]
	(ii) Attempt division by $x + 2$ (or $x - 3$) and obtain partial quotient of $ax^2 + kx$	M1	
	Obtain linear factors $4x + 1$, $x + 2$ and $x - 3$	A1	
	[If linear factor $4x + 1$ obtained by remainder theorem or inspection, award B2]		
	[If linear factor $4x + 1$ obtained by division by $x^2 - x - 6$, award M1 A1]		[2]
	Alternative Method:		
	Attempt to form identity $(x^2 - x - 6)(rx + s) \equiv ax^3 + bx^3 - 25x - 6$	M1	
	Attempt to equate like terms	M1	
	Leads to $s = 1$ B1, $r = 4$ A1, $b = -3$ A1, $a = 4$	A1	
	Obtain linear factors $4x + 1$, $x + 2$ and $x - 3$	A1	

Page 5	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	22

- 5 (i) State $\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{dy}{dt} = \frac{3}{t}$ B1
- Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
- Use $y = 6$ to find t M1
- Obtain $t = e^2$ A1
- Obtain $\frac{dy}{dx} = \frac{6}{e}$ A1 [5]
- (ii) Obtain x and form equation of the tangent at their point M1
- Obtain correct equation for tangent $\left(y - 6 = \frac{6}{e}(x - (1 + e))\right)$ A1
- Show that tangent passes through $(1, 0)$ by substitution A1 [3]
- 6 (a) Expand brackets and use $\sin^2 x + \cos^2 x = 1$ M1
- Obtain $1 - \sin 2x$ A1
- Integrate and obtain term of form $\pm k \cos 2x$, where $k = \frac{1}{2}, 1$ or 2 M1
- State correct integral $x + \frac{\cos 2x}{2} (+c)$ A1 [4]
- (b) (i) State or imply correct ordinates $1.4142\dots, 1.0823\dots, 1$ B1
- Use correct formula, or equivalent, correctly with $h = \frac{\pi}{8}$ and three ordinates M1
- Obtain answer 0.899 with no errors seen A1 [3]
- (ii) Make a recognisable sketch of $y = \operatorname{cosec} x$ for $0 < x \leq \frac{1}{2}\pi$ B1
- Justify statement that the trapezium rule gives an over-estimate B1 [2]

Page 6	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	22

- 7 (i) Integrate to obtain terms $4x^2$ and $\frac{1}{2}e^x$ B1 + B1
Substitute limits correctly M1
Obtain correct equation in any form $4a^2 + \frac{1}{2}e^a - \frac{1}{2} = \frac{1}{2}$ A1
Rearrange to given answer correctly A1 [5]

- (ii) Consider sign of $\sqrt{\frac{2-e^a}{8}} - a$, or equivalent M1
Complete the argument correctly with appropriate calculations A1 [2]
($f(0.2) = 0.112$, $f(0.3) = -0.015$)

- (iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.29 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1

$x_0 = 0.2$	$x_0 = 0.25$	$x_0 = 0.3$
0.3120	0.2992	0.2851
0.2815	0.2853	0.2894
0.2905	0.2894	
0.2879		

or show there is a sign change in the interval (0.285, 0.295) [3]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/23

Paper 2 (Pure Mathematics), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	23

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	23

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	23

- 1 Either State or imply non-modular inequality $(x+1)^2 < (3x+5)^2$, or
corresponding equation or pair of linear equations M1
Make reasonable solution attempt at a 3-term quadratic, or solve
two linear equations M1
Obtain critical values -2 and $-\frac{3}{2}$ A1
State correct answer $x < -2$ or $x > -\frac{3}{2}$ A1
- Or Obtain one critical value, e.g. $x = -2$, by solving a linear equation (or inequality)
or from a graphical method or by inspection B1
Obtain the other critical value similarly B2
State correct answer $x < -2$ or $x > -\frac{3}{2}$ B1 [4]

- 2 (i) Consider sign of $x^4 + 2x - 9$ at $x = 1.5$ and $x = 1.6$ M1
Complete the argument correctly with appropriate calculations
($f(1.5) = -0.9375$, $f(1.6) = 0.7536$) A1 [2]
- (ii) Rearrange $x^4 + 2x - 9 = 0$ to given equation or *vice versa* B1 [1]
- (iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.56 A1
Show sufficient iterations to justify its accuracy to 2 d.p. B1 [3]

$x_0 = 1.5$	$x_0 = 1.55$	$x_0 = 1.6$
1.5874	1.5614	1.5362
1.5424	1.5556	1.5685
1.5653		1.5520
1.5536		1.5604
1.5595		1.5561
1.5565		

or show there is a sign change in the interval (1.555, 1.565)

- 3 Obtain derivative $e^{2x} - 5e^x + 4$ B1
Equate derivative to zero and carry out recognisable solution method for a quadratic in e^x M1
Obtain $e^x = 1$ or $e^x = 4$ A1
Obtain $x = 0$ and $x = \ln 4$ A1
Use an appropriate method for determining nature of at least one stationary point M1
 $\left(\frac{d^2y}{dx^2} = 2e^{2x} - 5e^x, \text{ when } x = 0, \frac{d^2y}{dx^2} = -(3), x = \ln 4, \frac{d^2y}{dx^2} = +(12) \right)$
Conclude maximum at $x = 0$ and minimum at $x = \ln 4$ (no errors seen) A1 [6]
- 4 (i) Substitute $x = 3$ and equate to 14 ($9a + 3b + 35 = 14$) M1
Substitute $x = -2$ and equate to 24 ($4a - 2b = 24$) M1
Obtain a correct equation in any form A1
Solve a relevant pair of equations for a or for b M1
Obtain $a = 1$ and $b = -10$ A1 [5]

Page 5	Mark Scheme	Syllabus	Paper
	GCE AS LEVEL – October/November 2013	9709	23

	(ii)	Attempt division by $x^2 + 2x - 8$ and reach a partial quotient of $x - k$ Obtain quotient $x - 1$ with no errors seen (can be done by observation) Correct solution method for quadratic e.g. factorisation All solutions $x = 1, x = 2$ and $x = -4$ given and no others CWO	M1 A1 M1 A1	[4]
5	(i)	State $\frac{dx}{d\theta} = -2\sin 2\theta + \sin \theta$ or $\frac{dy}{d\theta} = 8\sin \theta \cos \theta$ Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ Use $\sin 2\theta = 2\sin \theta \cos \theta$ Obtain given answer correctly	B1 M1 M1 A1	[4]
	(ii)	Equate derivative to -4 and solve for $\cos \theta$ Obtain $\cos \theta = \frac{1}{2}$ Obtain $x = -1$ Obtain $y = 3$	M1 A1 A1 A1	[4]
6	(a) (i)	Attempt to divide by e^{2x} and attempt to integrate 2 terms Integrate a term of form ke^{-2x} correctly Fully correct integral $x - 3e^{-2x} (+c)$	M1 A1 A1	[3]
	(ii)	State correct expression $\frac{1}{2}\cos 2x + \frac{1}{2}$ or equivalent Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly State correct integral $\frac{3\sin 2x}{4} + \frac{3x}{2} (+c)$	B1 M1 A1	[3]
	(b)	State or imply correct ordinates 5.46143..., 4.78941..., 4.32808... Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates Obtain answer 4.84 with no errors seen	B1 M1 A1	[3]
7	(i)	State $R = \sqrt{10}$ Use trig formula to find α Obtain $\alpha = 18.43^\circ$ with no errors seen	B1 M1 A1	[3]
	(ii)	Carry out evaluation of $\cos^{-1}\left(\frac{2}{R}\right) (\approx 50.77^\circ)$ Carry out correct method for one correct answer Obtain one correct answer e.g. 34.6° Carry out correct method for a further answer Obtain remaining 3 answers $163.8^\circ, 214.6^\circ, 343.8^\circ$ and no others in the range	M1 M1 A1 M1 A1	[5]