

Section 2 of Unit 05 (Mechanics 2)

Equilibrium of a Rigid Body and Hooke's Law

Recommended Prior Knowledge

Students must have studied M1 before starting this Section.

Context

Sections 1 and 2 are independent of each other, but students will generally be more comfortable with the dynamics in Section 1 rather than with the mainly statics in Section 2.

Outline

The Section considers the concept of the moment of a force and the position of the centre of mass of symmetrical, composite and other shapes. The equilibrium of a rigid body under a set of forces, including the cases of toppling and sliding on an inclined plane, is considered in detail. The Section concludes by looking at problems involving an elastic string (or spring).

Topic	Learning Outcomes	Suggested Teaching Activities	Resources	On-Line Resources
2	Equilibrium of a rigid body <ul style="list-style-type: none"> To calculate the moment of a force about a point, in two dimensional situations only (understanding the vector nature of moments is not required). 	Introduce the concept of the moment of a force. Illustrate the idea of a turning effect by means of a first school balance on which a mass of 2 units can be balanced by a mass of 1 unit, providing the smaller mass is twice as far from the point of balance as the larger mass. Illustrate the same point by letting the smallest member of the group balance an open door by pushing at the point furthest from the hinge against the largest member of the group pushing at a point much nearer the hinge. Develop from these ideas, the definition of the moment of a force about a point O as being the product of the magnitude of the force and the perpendicular distance of the line of action of the force from O . Differentiate between "clockwise" and "anticlockwise" moments. Look at problems involving parallel forces acting on a light (or heavy) rod. Introduce through this, the idea of a uniform rod (or beam).	Simple balance – the type found in primary or first schools.	www.mathsrevision.net/alevel/mechanics Revision notes on moments.

	<ul style="list-style-type: none"> ▪ To use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry. 	<p>Discuss the forces acting when a solid body hangs in equilibrium suspended by a string from a point on its surface. The students should be able to recognise that there are only two forces acting and that these must be in the same vertical line. This introduces the idea of the point at which the mass of the object must act. Introduce the concept of "centre of mass". Let the students develop the idea that the position of the centre of mass of a lamina can be found experimentally by suspending it from two different points. By drawing two separate lines and finding the point of intersection, the position of the centre of mass can be deduced. Discuss the use of symmetry in finding the position of the centre of mass of such shapes as a rectangular lamina, a circular lamina, a sphere etc.</p>	<p>OHP illustrating the method of experimentally finding the centre of mass of an irregular shape. Have available different irregular shapes (laminas) and other composite shapes.</p>	<p>www.mathsrevision.net/alevel/mechanics Revision notes on "centre of mass".</p>
	<ul style="list-style-type: none"> ▪ To use given information about the centre of mass of a triangular lamina and other simple shapes. 	<p>Discuss and list the positions of the centres of mass of all the shapes in this syllabus ie a triangular lamina, a solid hemisphere, a hemispherical shell, a circular arc, a circular sector and a solid cone or pyramid. By cutting out a piece of card in the shape of a right angled triangle of sides 12cm and 9cm and suspending the triangle from two different points, the result for a triangular lamina can be verified. Similar experiments on a semicircle or hemisphere will enable other results to be verified.</p>	<p>OHP listing the centres of mass of all the shapes in the syllabus.</p>	
	<ul style="list-style-type: none"> ▪ To determine the position of the centre of mass of a composite body by considering an equivalent system of particles (in simple cases only, e.g. a uniform L-shaped lamina). 	<p>By considering a composite body as being composed of two or more simpler shapes (for each of which the centre of mass is known), the students should be able to determine the position of more complicated shapes. Starting with a system in one dimension where several particles are fixed to a thin light rod, they should realise that the sum of the moments of all of the particles is equal to the moment of the whole mass. This will lead to the exact position of the centre of mass. The students should practice with a large number of composite shapes including the examples:</p>	<p>OHP showing the full calculation for finding the centre of mass of a composite shape.</p>	

	<ul style="list-style-type: none"> • a rectangle with a triangular hole, • a rectangle with a circular hole, • two rods welded together at right angles, • a solid hemisphere fixed to a solid circular cylinder, • a triangular lamina with a protective metal band attached to its three edges. 		
<ul style="list-style-type: none"> ▪ To use the principle that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and to use the converse of this. 	<p>Look at the problem of a rigid body in equilibrium acted on by non-parallel forces. Students should be able to work with each of the following forces – weight, contact force represented by two components (normal and frictional), tension in a string, forces at a pivot or smooth joint. Students should understand that if the body is in equilibrium then:</p> <ul style="list-style-type: none"> • the vector sum of the forces is zero (or that the forces will balance in any given direction) and • the sum of the moments of the forces about any point is also zero (clockwise moments balance the anti-clockwise moments). 	OHP illustrating these forces.	
<ul style="list-style-type: none"> ▪ To solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding (problems set will not involve complicated trigonometry). 	<p>The students should be encouraged to draw neat diagrams showing all the forces acting in each of the following situations</p> <ul style="list-style-type: none"> • A ladder, with one end on rough ground, resting against a wall which may be rough or smooth. • A rod resting with one end on the ground (rough or smooth) and supported on a peg (rough or smooth) at some point of its length. • A rod pivoted at one end and held in equilibrium at an angle to the vertical by means of a horizontal force or a force acting perpendicular to the rod – this force could be the tension in a string. • A ladder resting against a wall (rough or smooth) with a string attached to some point on the ladder. 	Neat, large diagrams on OHP illustrating each of the situations listed opposite.	

		<p>The students will need a lot of practice, particularly in finding the moment of a force (force times perpendicular distance), where an angle is involved. This is the equation that presents most difficulty to even the brightest of students.</p> <p>Introduce the situation of a solid cylinder, resting in equilibrium with its circular base on a rough horizontal plane. Discuss with the students the possible outcomes when the angle of the plane with the horizontal is slowly increased. Will the cylinder slide first or topple first? The students should appreciate that sliding starts at the instant when the frictional force becomes equal to μR (students can be shown this to be the instant when $\alpha = \tan^{-1} \mu$). The students should appreciate that toppling starts at the instant when the centre of mass is vertically above the lowest point of contact. This situation should be investigated with different shapes and with different parts of the shapes in contact with the surface (eg a rectangular block with (a) its longest side, and (b) its shortest side, parallel to the line of greatest slope of the plane.</p>	OHP illustrating the different situations of sliding and toppling.	www.mathsrevision.net/alevel/mechanics Revision notes on "toppling".
4	Hooke's law <ul style="list-style-type: none"> To use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term modulus of elasticity. 	<p>Discuss generally with students the properties of an elastic string or spring – that the length will vary depending upon the force exerted on it. Demonstrate that particles of mass m, $2m$ and $3m$, hanging one at a time from an elastic string, will produce extensions that are in the ratios 1:2:3. State Hooke's law and show that the relationship can be expressed as $T = ke$ and develop this into $T = \frac{\lambda e}{l}$ where λ is the modulus of elasticity. Illustrate to students that the units of λ are units of force (newtons).</p>	Have available elastic springs and strings.	www.mathsrevision.net/alevel/mechanics Revision notes on elastic strings.

	<ul style="list-style-type: none"> ▪ To use the formula for the elastic potential energy stored in a string or spring. ▪ To solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed. 	<p>Having developed the formula $T = \frac{\lambda e}{l}$, investigate the graph of T against e and deduce that the work needed to stretch an unstretched string by an amount e is $\frac{\lambda e^2}{2l}$.</p> <p>Discuss with students why the work done in stretching a string from a length l to a length $l+e$ is not the same as the work done in stretching it from a length $l+e$ to a length $l+2e$.</p> <p>Look at different problems involving elastic strings. In particular ensure that students can determine the extension when a particle of a given mass hangs freely in equilibrium from one end. Extend this to the problem of a mass fixed to the mid-point of an elastic string and hanging in equilibrium with both ends of the string fixed at the same level and a given distance apart.</p> <p>Look at different problems requiring the use of the conservation of energy. Students should appreciate that in a catapult, elastic potential energy is converted into kinetic energy and that in a bungee jump where the "victim" starts and finishes at rest, gravitational potential energy is converted into elastic potential energy. Discuss the application of the conservation of energy to these situations.</p> <p>Extend this study to cases in which air resistance or friction may have an influence, so that energy is not conserved (eg a particle attached to one end of an elastic string sliding down a rough plane)</p>	OHP and video if available of different situations – bungee jumping and catapults.	
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