

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

## MARK SCHEME for the October/November 2006 question paper

### 9709 MATHEMATICS

9709/01

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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### Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\checkmark$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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<p>1. <math>\left(x + \frac{2}{x}\right)^6</math> Term in <math>x^2</math> has  <math>{}^6C_2</math> - needs factorials or 15.  <math>\times (x)^4 \times (2/x)^2</math>  <math>\rightarrow 60</math> ( needs selecting)          (first 2 marks can be obtained from expansion only)</p>	<p>B1          B1          B1          [3]</p>	<p>Binomial coeff.          Needs <math>2^2 (2x)^2 = 2x^2</math> gets B1. Needs () co.</p>
<p>2. <math>x = \sin^{-1} \frac{2}{3} \rightarrow \sin x = \frac{2}{3}</math>          (i) <math>\cos^2 x = 1 - \sin^2 x = \frac{21}{25}</math>          (ii) <math>\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{4}{21}</math></p>	<p>M1 A1 [2]          M1 A1 [2]</p>	<p>Formula only-use of 90° triangle ok          co – loses if decimals blatantly used          Formula only – or use of triangle ok          Correct from his answer to (i).</p>
<p>3. <math>OC = 6\sqrt{3}</math> and <math>AC = 6</math>          Sector area = <math>\frac{1}{2} r^2 \theta</math> [= <math>24\pi</math>]          Area of rectangle = <math>12 \times 6\sqrt{3}</math>          Area of triangle = <math>\frac{1}{2} \times 6 \times 6\sqrt{3}</math>  <math>\rightarrow 54\sqrt{3} - 24\pi</math></p>	<p>B1 B1          M1          M1          M1          A1          [6]</p>	<p>Wherever these come.(must have <math>\sqrt{3}</math>)          Use of correct formula with radians.          Use of base <math>\times</math> height (not for <math>12 \times 12</math>)          Use of <math>\frac{1}{2}</math> base <math>\times</math> height (needs trig)          co. Ok without stating <math>a=54, b=24</math>.</p>
<p>4. <math>\mathbf{a} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}</math>          (i) <math>\mathbf{a} \cdot \mathbf{b} = 3 + 12 + 12 = 27</math>  <math>\mathbf{a} \cdot \mathbf{b} = \sqrt{54} \times \sqrt{21} \cos \theta</math>  <math>\rightarrow \theta = 36.7^\circ</math> or 0.641 radians          (ii)          Vector <math>\mathbf{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}</math>          Vector <math>\mathbf{OC} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}</math>          Unit vector = Vector <math>\mathbf{OC} \div 9</math>  <math>= \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}</math></p>	<p>M1          M1          A1 [3]          M1          M1          A1 [4]</p>	<p>Ok to work throughout with column vectors or with <math>\mathbf{i}, \mathbf{j}, \mathbf{k}</math>.          Use of <math>x_1x_2 + y_1y_2 + z_1z_2</math>          Use of <math>\sqrt{\phantom{x}} \sqrt{\phantom{x}} \cos \theta</math>          In either degrees or in radians.          For use of <math>\mathbf{b} - \mathbf{a}</math>.          For <math>\mathbf{a} + 3(\mathbf{b} - \mathbf{a})</math> or equivalent          For division by Modulus of <math>\mathbf{OC}</math>.          Co.</p>

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<p>5. (i) <math>m</math> of <math>AB = 8/12</math>  <math>m</math> of perpendicular <math>= -12/8</math>  eqn of <math>CD</math> <math>y - 15 = -\frac{3}{2}(x - 6)</math></p> <p>(ii) eqn of <math>AB</math> <math>y - 3 = \frac{2}{3}(x - 1)</math>  Sim eqns <math>2y + 3x = 48</math> and <math>3y = 2x + 7</math>  <math>\rightarrow D(10, 9)</math></p>	<p>M1  M1 A1  [3]</p> <p>M1 A1√  DM1  A1  [4]</p>	<p>Use of <math>m_1 m_2 = -1</math> and <math>y</math>-step/<math>x</math>-step  Correct form of eqn of line. co.</p> <p>Could be given in (i)  Needs both M marks from (i).  co.</p>
<p>6. (a) <math>a = 105</math>  Either <math>l = 399</math> or <math>d = 7</math>  <math>n = 43</math>  <math>\rightarrow 10\ 836</math></p> <p>(b) <math>r^2 = 64/144 \rightarrow r = \frac{2}{3}</math></p> <p>(i) Either <math>x = ar \rightarrow x = 96</math>  or <math>\frac{144}{x} = \frac{x}{64} \rightarrow x = 96</math></p> <p>(ii) Use of <math>S_{\infty} = \frac{a}{1-r}</math>  <math>\rightarrow 432</math></p>	<p>B1  B1  B1  B1  [4]</p> <p>M1</p> <p>M1 A1</p> <p>M1  A1  [5]</p>	<p>co  co  co  co</p> <p>award in either part</p> <p>either method ok</p> <p>Used with his <math>a</math> and <math>r</math></p> <p>Co  (nb do not penalise if <math>r</math> and <math>l</math> or <math>x</math> negative as well as positive.)</p>
<p>7. (i) <math>y = x^3 - 3x^2 + 2x</math>  <math>\frac{dy}{dx} = 3x^2 - 6x + 2</math>  At <math>A(1,0)</math>, <math>m = -1 \rightarrow y = -1(x - 1)</math>  At <math>B(2,0)</math>, <math>m = 2 \rightarrow y = 2(x - 2)</math>  Sim equations <math>\rightarrow x = \frac{2}{3}</math></p> <p>(ii) <math>R_1 = \int_0^1 (x^3 - 3x^2 + 2x) dx</math>  <math>= \left[ \frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} \right] = \frac{1}{4}</math>  <math>R_2 = [ ]^2 - [ ]^1 = -\frac{1}{4}</math></p>	<p>M1 A1</p> <p>M1 once</p> <p>M1 A1  [5]</p> <p>M1</p> <p>A1√  DM1</p> <p>A1  [4]</p>	<p>Attempt at differentiation. co.</p> <p>Correct form of eqn of tangent – not normal.</p> <p>Solution of Sim Eqns – even if normals.</p> <p>Attempt at integration.</p> <p>Integration correct for his cubic  Correct use of limits once.</p> <p>Both correct (allow <math>\pm</math> in either/both cases) (0 to 2 <math>\rightarrow</math> 0 + reason ok)  Ignore errors over <math>\pm</math></p>



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<p>8. <math>y = \frac{6}{5-2x}</math></p> <p>(i) <math>\frac{dy}{dx} = -6(5-2x)^{-2} \times (-2)</math>  <math>= \frac{12}{(5-2x)^2} \rightarrow \frac{4}{3}</math></p> <p>(ii) Use of chain rule.  <math>\frac{dx}{dt} = 0.02 \div (i) = 0.015</math></p> <p>(iii) <math>V = \pi \int \left( \frac{36}{(5-2x)^2} \right) dx</math>  <math>= 36\pi \left[ \frac{(5-2x)^{-1}}{-1} \right] + (-2)</math>  <math>[ ]^1 - [ ]^0 = \frac{12\pi}{5}</math></p>	<p>B1 M1 A1 [3]</p> <p>M1 A1✓ [2]</p> <p>M1 A1 M1 DM1 A1 [5]</p>	<p>B1 for <math>\frac{dy}{dx} = -6(5-2x)^{-2}</math>, M1 for <math>(\times -2)</math></p> <p>Co.</p> <p>M1 for dividing 0.02 by answer to (i).</p> <p>Attempt at integration of <math>y^2</math> (ignore <math>\pi</math>)  For <math>36\pi \left[ \frac{(5-2x)^{-1}}{-1} \right]</math>, M1 for <math>\div (-2)</math>.</p> <p>DM1 for correct use of limits- not earned if <math>[ ]^0</math> ignored or put to 0.</p>
<p>9. (i) Height = <math>3x</math>,  <math>10xy + \frac{1}{2} \cdot 8x \cdot 3x \cdot 2 = 200</math>  <math>\rightarrow y = \frac{200 - 24x^2}{10x}</math></p> <p>(ii) <math>V = \frac{1}{2} \cdot 8x \cdot 3xy = 240x - 28.8x^3</math></p> <p>(iii) <math>\frac{dV}{dx} = 240 - 86.4x^2</math>  <math>= 0</math> when <math>x = 1\frac{2}{3}</math>.</p> <p>(iv) <math>\frac{d^2V}{dx^2} = -172.8x</math>  <math>\rightarrow -ve \rightarrow</math> Maximum</p>	<p>B1 M1 A1 [3]</p> <p>M1 A1 [2]</p> <p>M1 DM1 A1 [3]</p> <p>M1 A1✓ [2]</p>	<p>Anywhere in the question.  Linking 200 with <math>4/5</math> areas. Allow slight slip in formulae (particularly with <math>\frac{1}{2}</math>)  co. Answer given.</p> <p>M1 for statement "<math>V = \text{area} \times y</math>", co ag.</p> <p>Attempt at differentiating.</p> <p>Sets to 0 and attempts to solve, co.</p> <p>Looks at sign of 2<sup>nd</sup> differential.  Correct deduction from correct diff.  Ignore inclusion of <math>-5/3</math>.</p>
<p>10. (i) <math>x^2 - 3x - 4 \rightarrow -1</math> and <math>4</math>  <math>\rightarrow x &lt; -1</math> and <math>x &gt; 4</math></p> <p>(ii) <math>x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}</math></p> <p>(iii) <math>f(x)</math> (or <math>y</math>) <math>\geq -\frac{9}{4}</math></p> <p>(iv) No inverse – not 1 : 1.</p> <p>(v) Quadratic in <math>\sqrt{x}</math>.  Solution <math>\rightarrow \sqrt{x} = 5</math> or <math>-2</math>  <math>\rightarrow x = 25</math></p>	<p>M1 A1 A1 [3]</p> <p>B1 B1 [2]</p> <p>B1✓ [1]</p> <p>B1 [1]</p> <p>M1 DM1 A1 [3]</p>	<p>Solving Quadratic = 0. Correct values.  co. – allow <math>\leq</math> and/or <math>\geq</math>. <math>4 &lt; x &lt; -1</math> ok</p> <p>B1 for <math>\frac{3}{2}</math>. B1 for <math>\frac{9}{4}</math>.</p> <p>✓ for <math>f(x) \geq -b</math>.</p> <p>Independent of previous working.</p> <p>Recognition of "Quadratic in <math>\sqrt{x}</math>"  Method of solution.  co. Loses this mark if other answers given. Nb ans only full marks.</p>