UNIT 2 Coordinate Geometry and Calculus (P1)

Recommended Prior Knowledge. Students will need an understanding and proficiency in the algebraic techniques from either O Level Mathematics or IGCSE Mathematics and should have covered at least the work in Topic 1 (Quadratic functions) in Unit 1 (Algebra). Most students should have met the basic concepts of working with lengths in coordinate geometry.

Context This Unit ideally, though not necessarily, follows on from Unit 1 (Algebra), though the work in Topic 1 in Unit 1 must have been covered. This is probably the most important unit in the course, and is needed for the Pure Mathematics in A2 (P3) and also the Mechanics or Statistics in A2 (M2 or S2). It is an essential component for any further study in mathematics, engineering or the sciences.

Outline. The Unit begins by looking at the concept of "steepness" (gradient) of a straight line and its connection with the equation of that line. It proceeds to look at the gradient (varying) of a curve and examines the basics of differential calculus. This leads to fundamental work on differentiation and integration and looks at the use of the calculus in various situations, amongst which are problems on maxima and minima and on the area and volume of rotation of a curve about an axis.

Topic	Learning Outcomes	Suggested Teaching Activities	Resources	On-Line Resources
3	**Coordinate Geometry** • Find the length and mid-point of a line segment, given the coordinates of the end-points. • Find the gradient of a line segment, given the coordinates of the end-points.	Begin by looking at the calculation of the length of a line segment and the coordinates of the mid-point , given the coordinates of the two end-points. This should be familiar to most students. Look at the concept of "steepness", relating this to gradient and to a simple definition of (*y*-step/*x*-step). This may also be familiar to some students. Look at the link with road signs (20% etc). Discuss positive and negative gradients and look at the numerical value of the gradients of lines parallel to the x-axis and of lines parallel to the *y*-axis.	Lots of 2mm graph paper. The CUP text Pure Maths 1	www.bbc.co.uk/education/asguru/maths/ →Methods →Coordinate Geometry Straight Line Graphs. Parallel and Perpendicular.

	• Understand and use the relationship between the gradients of parallel and perpendicular lines	Discuss the properties of gradients both of lines that are parallel and of lines that are perpendicular. Encourage the students to deduce these relationships for themselves by accurate drawings, on a sheet of graph paper, of lines that are both parallel and perpendicular.		
	• Interpret and use linear equations, particularly the form $y = mx + c$ and $y - y_1 = m(x - x_1)$. • Find the equation of a straight line from either (a) 2 given points or (b) a given point and its gradient.	Let students draw several lines from given equations and use their graphs in a general discussion on the relationship between gradient and the algebraic form of a line. Similar discussion should lead to the formation and development of the equations; $y = mx + c,\ y - y_1 = m(x - x_1)$ and $\frac{x}{a} + \frac{y}{b} = 1$. Students should be able to form the algebraic equations of lines from a variety of given situations; in particular from two given points or from a given point and a given gradient. The work on parallel and perpendicular lines fits in appropriately at this stage. Students should be encouraged to use all of the work covered so far on making more complicated calculations from problems on parallelograms, squares, isosceles triangles etc.	Lots of graph paper.	As above
	• Understand the relationship between a graph and its associated algebraic equation. • Use the relationship between points of intersection of graphs and	The work above should lead naturally to an understanding of the relationship between a graph and its associated algebraic equation. Students should appreciate the use of simultaneous equations to find the point at which two lines meet, or to link up with Unit 1 and find the two points where a line and a curve intersect. General discussion should then follow about the general situation of a line meeting a curve. Students should	As above	As above

	solutions of equations (including, in simple cases, the correspondence between a line being a tangent to a curve and a repeated root of an equation).	appreciate that, in the case for which the curve is that of a quadratic function, there are three situations that occur geometrically: 2 points of intersection, the line being a tangent to the curve and no intersection. General discussion should follow to relating these three situations with the solution of the quadratic equation resulting from the solution of the simultaneous equations i.e. whether $b^2 - 4ac$ is greater than, less than or equal to 0.		
8	**Differentiation** • Understand the idea of the gradient of a curve, • Use the derivative of x^n (for any rational *n*), together with constant	General discussion can now follow on looking at the gradient of a curve. Students will realise that this is variable and should be able to form a general feeling of "gradient increasing" and "gradient decreasing". They should also come up with the idea that the gradient of a curve can only be obtained from the gradient of the tangent. The derivation of the result "$y = x^2 \Rightarrow m = 2x$" should be obtained by the students themselves. This can be done by drawing an accurate graph and measuring the gradients at say x=0,1, 2, 0.5, -1 etc . Students should also be encouraged to realise that the gradient of the tangent at a point *P* is the same as the limit value of the gradient of the chord *PQ* where *Q* is a point on the curve close to *P* and as *Q* approaches *P*. Using a calculator with *P*(2,4) and *Q*(2.1, 2.1^2) on the curve $y=x^2$ and then with *Q*(2.01, 2.01^2) etc soon produces the result that $m \to 4$. Similar results at x=3 etc should produce the formula $m=2x$. Students should be encouraged to deduce similar results for $y=x^3$, $y=x^{-1}$, $y=2x^2$ etc. This exercise makes an excellent piece of coursework to be covered for homework. General discussion follows in which	Graph paper for drawing different graphs and measuring the gradients of the tangents. OHP slides of graphs and of calculations showing results for the chord *PQ* are useful.	www.bbc.co.uk/education/asguru/maths/ →Methods →Differentiation →The derivative →First principles →Gradient →Polynomials

	multiples, sums, differences of functions. • Use the notations f´x), f´´(x), $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ • (the technique of differentiation from first principles is not required). • Differentiation of composite functions using the chain rule.	the general rules of differentiation of $y=x^n$ should be deduced. This is the most appropriate place to introduce the notation within the syllabus, particularly that of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. If time, and the ability of the class is appropriate, you can introduce students to differentiation from first principles (This is not however in the syllabus!). Discuss the problem of differentiating composite functions. Show them that differentiating $(1+2x)^2$ by expanding the brackets is equivalent to $2(1+2x)\times 2$. Lots of practice on composite functions is recommended.	Any A level textbook will contain this work.	
	• Apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change (including related rates of change). • Locate stationary points and use information about stationary points in sketching graphs (the ability to distinguish between maximum and minimum points is required, but identification of points of inflexion is not included).	Students should be encouraged with lots of practice in basic differentiation, in particular with use of variables other than y and x. The general idea of rate of change should be discussed, along with problems on related rates of change. All of the above can now be linked to the work on coordinate geometry in finding the equations of tangents and normals. Introduce students to the problem of finding the minimum point on a quadratic curve. Show students that at a minimum point the gradient is increasing and therefore $\frac{d^2y}{d\mathbf{x}^2}$ is positive; likewise at a maximum point the gradient is		www.bbc.co.uk/education/asguru/maths/ →Methods →Differentiation →Maxima and Minima →Stationary Points

		decreasing and $\frac{d^2y}{dx^2}$ is negative. Link this work with the work in Unit 1 on completing the square to find maximum and minimum points on a quadratic curve. Give the students lots of practice working with both methods. Students should have practice on finding the coordinates and nature of stationary points on other curves such as cubic equations and such curves as $y = 9x + \frac{4}{x}$. Show the students how this work can be used to solve everyday problems on maximum and minimum values, particularly with area and volume,		
9	**Integration** • Understand integration as the reverse of differentiation. • Integrate x ^{n} (for any rational *n* except −1), together with constant multiples, sums and differences. • Solve problems involving the evaluation of a constant of integration, e.g. to find the equation of a curve through (1, -2) for which $\frac{dy}{dx}$ = 2*x*+1. • Integrate (*ax*+*b*)^n (for any rational *n* except −1), together with constant multiples, sums and differences.	Encourage students to work backwards from a given gradient to the equation of the curve. Encourage them to see the need for a constant of integration. General discussion should follow on finding the general rule for integrating x^n. Encourage students to deduce the value of *n* for which this fails. Students will need a lot of practice on both basic integration and the finding of the equation of a curve from its gradient and a point. Repeat the exercise above with students working backwards from a gradient of the type (*ax*+*b*)^n. n.b. Students at this stage tend to become confused over the gradient of a tangent (constant) and the gradient of a curve. On being given a formula for the gradient, there is a		vwww.bbc.co.uk/education/asguru/maths/ →Methods →Integration →Looking at the Need for Integration →Defining the integral →Notation.

		natural tendency to substitute a value of *x* and then to use the equation *y=mx+c* as the equation of the curve. Similarly, when students are asked to find the tangent to a curve, there is a tendency for the "*m*" in "*y=mx+c"* to be a function of *x*. It is worth spending time emphasising the faults in this thinking!		
9 cont	• Evaluate definite integrals . • Use definite integration to find the area of a region bounded by a curve and lines parallel to the axes, or between two curves. • Evaluate definite integrals .(including simple cases of "improper" integrals, such as $\int_0^1 x^{-\frac{1}{2}}\mathrm{d}x$ and $\int_0^\infty x^{-2}\mathrm{d}x$). • Use definite integration to find a volume of revolution about one of the axes.	General discussion of definite integration and in particular its link with finding the area under a graph. Show students how the technique works for a given linear equation (such as *y=3x* for *x* from 2 to 4) in which an exact value can be checked by other methods. Students will need a lot of practice. Emphasise that a lower limit of 0 cannot be ignored. Using geometrical situations, show students that it is possible to give a finite value to a seemingly improper integral, such as the two suggested in the syllabus. Discuss with students the case of "area below the *x*-axis being negative" and show how calculation of area is not automatically a definite integral if the curve crosses the axis between the two limits. Discuss and show students the method of finding an area lying between the curve and the *y*-axis. Look at the volume of rotation of a curve about an axis. Try and let students sketch the resulting shape for a curve such as $y=x^2$. Encourage them to find for themselves that there is no connection between the solids obtained by rotating about the two axes.	Examples on OHP if appropriate.	
