UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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1	${}^7C_2\, x^5 \left(\frac{2}{x^2}\right)^2$ SOI and leading to final answer	B2	B1 for 2/3 parts correct leading to ans.
	84 or $84x$ as final answer	B1 [3]	If no answer; $84x$ seen scores B2, else ${}^7C_2\, x^5 \left(\frac{2}{x^2}\right)^2$ scores SCB1 only
2	$\left(\frac{dv}{dr}\right) 4\pi r^2$	M1	
	$4\pi \times 10^2$	A1	SOI at any point
	$\frac{dr}{dt} \quad \frac{\frac{dv}{dt}}{\frac{dv}{dr}}$ **OE** used	M1	Correct link between differentials with $\frac{dr}{dt}$ finally as subject
	$\frac{50}{4\pi \times 10^2} \quad \frac{1}{8\pi}$ or 0.0398	A1 [4]	Allow $\frac{50}{400\pi}$. Non-calculus methods $\frac{0}{4}$
3	**(i)** Correct shape – touching positive x-axis	B1 [1]	Ignore intersections with axes
	(ii) $(\pi)\int (x \quad 2)^4 dx$	M1	Use $(\pi)\int y^2 dx$ & attempt integrate but expansion before integn needs 5 terms
	$(\pi)\left[\frac{(x \quad 2)^5}{5}\right]$	A1	
	$(\pi)[0 \quad (\ 32)/5)]$	M1	Use of limits 0, 2 on *their* $(\pi)\int y^2 dx$
	$\frac{32\pi}{5}$ or 6.4π	A1 [4]	cao Rotation about y-axis max 1/5
4	**(i)** $\overrightarrow{CP} = 6\mathbf{i} + 6\mathbf{j} \quad 2\mathbf{k}$	B1	
	$\overrightarrow{CQ} = -6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$	B1 [2]	
	(ii) Scalar product $= 36 + 36 \quad 6$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$66 = \lvert\overrightarrow{CP}\rvert \lvert\overrightarrow{CQ}\rvert \cos\theta$	M1	Linking everything correctly
	$\lvert\overrightarrow{CP}\rvert = \sqrt{76}$, $\lvert\overrightarrow{CQ}\rvert = \sqrt{81}$	M1	Correct magnitude for either
	Angle $PCQ = 32.7°$ (or 0.571 rad)	A1 [4]	cao 147.3° converted to 32.7° gets A0

5	**(i)**	$\frac{2\sin^2\theta\sin^2\theta}{1\quad\sin^2\theta} = 1$ $2\sin^4\theta + \sin^2\theta \quad 1 \quad 0$ **AG**	M1 A1 [2]	Equation as function of sin θ
	(ii)	$(2\sin^2\theta \quad 1)(\sin^2\theta + 1) \quad 0$ $\sin\theta \quad \frac{(\pm)1}{\sqrt{2}}$ $\theta = 45°, 135°$ $\theta = 225°, 315°$	M1 A1 A1 A1 [4]	Or use formula on quadratic in $\sin^2\theta$ Provided no excess solutions in range
6	**(i)**	$z \quad 3x + 2\left(\frac{600}{x}\right)$ or $x\frac{(z \quad 3x)}{2} \quad 600$ OE $\rightarrow$ **AG**	B1 [1]	
	(ii)	$\frac{dz}{dx} \quad 3 \quad \frac{1200}{x^2}$ or $\frac{dz}{dy} \quad 2 \quad \frac{1800}{y^2}$ $= 0 \rightarrow x = 20$ or $= 0 \rightarrow y = 30$ $z \quad 60 + \frac{120}{20} \quad 120$ $\frac{d^2z}{dx^2} \quad \frac{2400}{x^3}$ $> 0 \Rightarrow$ minimum	B1 M1A1 A1√ B1√ B1 [6]	Set to 0 & attempt to solve. Allow ±20 Ft from *their x* provided positive Or other valid method Dep. on $\frac{d^2z}{dx^2} \quad \frac{k}{x^3}$ $(k > 0)$ or other valid method.
7	**(i)**	$\frac{3(1+2x)^{\quad 1}}{1} + (c)$ $y \quad \frac{3(1+2x)^{\quad 1}}{2} + (c)$ Sub (1, (1/2)) $\frac{1}{2} \quad \frac{3}{6} + c \Rightarrow c \quad 1$	B1 B1(indep) M1 A1 [4]	 Division by 2 $y =$ necessary Dependent on c present Use of $y = mx + c$ etc. gets 0/4
	(ii)	$(1 + 2x)^2(>)9$ or $4x^2 + 4x - 8(>)0$ **OE** 1, 2 $x > 1, x < -2$ ISW	M1 A1 A1 [3]	
8	**(i)**	1000, 2000, 3000... or 50, 100, 150... $\frac{40}{2(1000+40000)}$ or $\frac{40}{2(2000+39000)}$ × 5% of attempt at valid sum 41000	M1 M1 M1 A1 [4]	Recognise series, correct *a/d* (or 3 terms) Correct use of formula Can be awarded in either **(i)** or **(ii)** cao
	(ii)	1000, 1000 × 1.1, 1000 × 1.1^2 + ... or with $a = 50$ $\frac{1000(1.1^{40} \quad 1)}{1.1 \quad 1}$ 22100	M1 M1 A1 [3]	Recognise series, correct *a/r* (or 3 terms) Correct use of formula. Allow e.g. $r = 0.1$ Or answers rounding to this

9	**(i)** $AS \quad r\tan\theta$ Area $OAB \quad r^2\tan\theta$ or $(OAS) \quad \frac{1}{2}r^2\tan\theta$ Area of sector $= \frac{1}{2}r^2 \times 2\theta \; (\quad r^2\theta)$ Shaded area $= r^2(\tan\theta \quad \theta)$ **OE**	M1 A1 B1 A1 [4]	Or $(AB) = 2r\tan\theta$ or $(AO) \quad \frac{r}{\cos\theta}$ Or $OAB \quad \frac{1}{2}\frac{r^2}{\cos 2\theta}\sin 2\theta$ Or area sector $(OPS) \quad \frac{1}{2}r^2\theta$ Allow e.g. $r^2\tan\theta \quad \frac{1}{2}r^2 2\theta$
	(ii) $\cos\frac{\pi}{3} \quad \frac{6}{OA} \Rightarrow OA \quad 12$ $AP = 6$ $AS \quad 6\tan\frac{\pi}{3} (\Rightarrow AB \quad 12\sqrt{3})$ Arc $(PST) \quad 12\frac{\pi}{3}$ Perimeter $= 12 + 12\sqrt{3} + 4\pi$	M1 A1 B1 B1 A1 [5]	Or arc $(PS) \quad 6\frac{\pi}{3}$ or arc $(ST) \quad 6\frac{\pi}{3}$ Allow unsimplified 4π
10	**(i)** $2(x-1)^2 - 1 \quad OR \quad a = 2, b = -1, c = -1$ $A = (1, -1)$	B1, B1, B1 B1√ [4]	Allow alt. method for final mark
	(ii) $2x^2 \quad 5x \quad 3 \quad 0 \Rightarrow (2x+1)(x \quad 3) \quad 0$ **OE** in y $x \quad \frac{1}{2}, \quad y \quad 3\frac{1}{2}$	M1, M1 A1 [3]	Complete elim & simplify, attempt soln. Additional (3, 7) not penalised
	(iii) Mid-point of $AP = (2, 3)$	B1√	Follow through on *their A*
	Gradient of line $= \frac{\frac{1}{2}}{\frac{5}{2}} \quad \frac{1}{5}$	B1	
	Equation is $y \quad 3 \quad \frac{1}{5}(x \quad 2)$ **OE**	B1 [3]	Or $y \quad 3\frac{1}{2} \quad \frac{1}{5(x+\frac{1}{2})}$
11	**(i)** $\text{fg}(x) = 2x^2 - 3, \quad \text{gf}(x) = 4x^2 + 4x - 1$	B1, B1 [2]	fg & gf clearly transposed gets B0B0
	(ii) $2a^2 \quad 3 \quad 4a^2 + 4a \quad 1 \Rightarrow 2a^2 + 4a + 2 \quad 0$ $(a+1)^2 = 0$ $a = -1$	M1 M1 A1 [3]	Dep. quadratic. Allow x for all 3 marks Allow marks in **(ii)** if transposed in **(i)**
	(iii) $b^2 - b - 2 = 0 \rightarrow (b+1)(b-2) = 0$ $b = 2$ Allow $b = -1$ in addition	M1 A1 [2]	Allow in terms of x for M1 only Correct answer without working B2
	(iv) $\text{f}^{-1}(x) \quad \frac{1}{2}(x \quad 1)$ $\text{f}^{-1}\text{g}(x) \quad \frac{1}{2}(x^2 \quad 3)$	B1 B1√ [2]	Must be simplified. Ft from *their* f^{-1}
	(v) $x \quad (\pm)\sqrt{y+2}$ $\text{h}^{-1}(x) \quad \sqrt{x+2}$	M1 A1 [2]	

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1	$\int\left(x^3+\frac{1}{x^3}\right)\mathrm{d}x = \frac{x^4}{4}+\frac{x^{-2}}{-2}+c$	3 × B1 [3]	Allow unsimplified, 1 mark for each term, including "c"
2	$(1+\frac{3}{2}x)^6$		
	(i) Term in x^2 $\quad {}^6C_2\times\left(\frac{\pm 3x}{2}\right)^2=\frac{135x^2}{4}$	M1 A1	For either unsimplified term co
	Term in x^3 $\quad {}^6C_3\times\left(\frac{\pm 3x}{2}\right)^3=\frac{540x^3}{8}$	A1 [3]	co (omission or error with "–" can still gain 2 out of 3)
	(ii) Term in $x^3 = \frac{270x^3}{4}+\frac{135kx^3}{2}$	M1	considers exactly 2 terms in x^3
	$\rightarrow \quad k=1.$	A1 [2]	co
3	**(i)** $x^2+px+q = (x+3)(x-5)$	M1	Must be $(x+3)$ and $(x-5)$.
	$\rightarrow \quad p=-2,\ q=-15.$ (any other method ok)	A1 [2]	co
	(ii) $x^2+px+q+r = 0$		
	Use of "b^2-4ac" Uses a, b and c correctly $r=16$	M1 DM1 A1	Any use of "b^2-4ac" c must include both q and r. co
	or $=(x+k)^2 \rightarrow 2k=p$ (M1) $\quad k^2=q+r$ (M1) $\rightarrow \quad k=-1 \quad \rightarrow \quad r=16$ (A1)	[3]	
4	$y = \frac{4}{3x-4}$		
	(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = -4(3x-4)^{-2}\times 3$	B1 B1	Correct without ×3. For ×3.
	If $x=2$, $m=-3$ Eqn of tangent $y-2=-3(x-2)$	M1 A1 [4]	Correct line eqn. co (for normal M0A0)
	(ii) $\tan\theta=\pm(-3)$ $\rightarrow \quad \theta=\pm 108.4^\circ$ (or $\pm 71.6^\circ$)	M1 A1√	Correct link with (± his gradient) co (accept acute or obtuse) or -71.6° or radians
	or scalar product, $\tan\theta = y$-step ÷ x-step **or** use of $\tan(A-B)$ M1A1 for each	[2]	

5	**(i)** $\dfrac{\cos\theta}{\tan\theta(1 \quad \sin\theta)} \equiv \dfrac{\cos^2\theta}{\sin\theta(1 \quad \sin\theta)}$ $= \dfrac{1 \quad \sin^2\theta}{\sin\theta(1 \quad \sin\theta)}$ $= \dfrac{1+\sin\theta}{\sin\theta} = \dfrac{1}{\sin\theta} + 1$	M1 M1 A1 [3]	Use of t = s ÷ c Replaces $\cos^2\theta$ with $1 - \sin^2\theta$ to form f(sinθ). AG. Ensure all ok. Must show difference of 2 squares.
	(ii) $\dfrac{\cos\theta}{\tan\theta(1 \quad \sin\theta)} = 4 \quad \rightarrow \dfrac{1}{\sin\theta} + 1 = 4$ $\rightarrow \sin\theta = ⅓ \rightarrow \theta = 19.5^\circ, 160.5^\circ$	M1 A1 A1√ [3]	Linking up to obtain $\sin\theta = k$. co. √ 180° − 1st answer providing there are no other solutions in the range 0° to 360°.
6	**(i)** $f(x) = \dfrac{x+3}{2x \quad 1}$ $ff(x) = \dfrac{\dfrac{x+3}{2x \quad 1} + 3}{\dfrac{2(x+3)}{2x \quad 1} \quad 1} = \dfrac{7x}{7} = x$	B1 M1 A1 [3]	Replacing "x" twice - must be correct Correct algebra – clearing $(2x - 1)$ AG – all correct.
	(ii) $y = \dfrac{x+3}{2x \quad 1}$ $\rightarrow \quad 2xy \quad y \quad x+3$ $\rightarrow \quad x(2y \quad 1) \quad y+3$ $\rightarrow \quad f^{1}(x) = \dfrac{x+3}{2x \quad 1}$ **or** since $ff(x) = x$, $f^{1}(x) = f(x) = \dfrac{x+3}{2x \quad 1}$ (M1, A1)	M1 A1 [2]	Attempt to make x the subject and complete method co
7	**(i)** (2, 5) to (10, 9) gradient = ½ Equation of $L_2 \quad y \quad \frac{1}{2}x$. Gradient of perpendicular = − 2 Eqn of Perp $y \quad 5 \quad 2(x \quad 2)$ Sim Eqns $\rightarrow$ $C(3.6, 1.8)$	B1 B1√ M1 M1 A1 [5]	co √ on gradient of L_1 Use of $m_1m_2 = -1$ Correct form of line eqn co
	(ii) $d^2 = 1.6^2 + 3.2^2 \rightarrow d = 3.58$	M1 A1 [2]	Correct method for AC co (accept with √5 in answer)

8	**(i)** $\overrightarrow{BA}.\overrightarrow{BC}$ or $\overrightarrow{AB}.\overrightarrow{CB}$ $\overrightarrow{BA} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$ $\overrightarrow{BA}.\overrightarrow{BC} = -8$ $= 3 \times 7 \times \cos\theta$ $\rightarrow \quad \theta = 112.4^\circ$ or 1.96 radians **(ii)** $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$ $= \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}$	B1 M1 M1 M1 M1 A1 [6] M1 A1√ [2]	Correct two vectors for angle ABC. Correct method for one of the sides. Correct use for **any** pair of vectors. Correct method for moduli. All linked correctly. co (67.6° usually gets 4/6) Correct method. (allow for **d** = **a** + **b** – **c** or for **d** = **a** + **c** – **b** or for **d** = **b** + **c** – **a**) A1√ for his $\overrightarrow{BC}$.
9	**(i)** **(a)** $f(x) = 3 - 4\cos^2 x$. One limit is -1 Other limit is 3 **(b)** $3 - 4\cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2}$ $\rightarrow \quad \cos x = \pm\frac{1}{\sqrt{2}}$ $\rightarrow \quad x = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$ **(ii)** **(a)** **(b)** f has an inverse since it is 1:1 or increasing or no turning points.	B1 B1 [2] M1 A1 A1√ [3] B1 B1 [2] B1 [1]	co irrespective of inequalities co irrespective of inequalities Makes cos x the subject. co (radians). √ for "π − (1st answer)" ("exact" means that decimal answers only earn A0 A1√) Joins $(0, -1)$ to $(\pi, 7)$, providing increasing function Not a line, flattens at extremities-needs inflexion. co independent of part (**i**)

10	**(a)** $a + 5d \quad 4a$ or $\frac{(a+4a)}{2} \times 6$	B1	co
	$\frac{6}{2}(2a+5d)$ or $\frac{(a+4a)}{2} \times 6 = 360$	M1 A1	Correct left-hand side. All correct.
	Sim Eqns $a = 24^\circ$ or $\frac{2\pi}{15}$ rads	A1	Either answer.
	Arc length = 5θ Perimeter = 12.1.	M1 A1 [6]	Correct use of arc length with θ in rads. co
	(b) **(i)** $\frac{k+6}{2k+3} \quad \frac{k}{k+6}$	M1 A1	Correct eqn for k.
	$\rightarrow k^2 \quad 9k \quad 36 \quad 0 \rightarrow k = 12$ (NB stating a, ar, ar^2 as f(k) gets M1)	A1 [3]	Co condone inclusion of $k = -3$.
	(ii) $r = ⅔, \quad a = 27$ $\rightarrow S_\infty = 27 \div ⅓ = 81.$	M1 A1 [2]	Correct formula for S_∞ must have $-1 \leq r \leq 1$. co.
11	$y \quad 4\sqrt{x} \quad x.$		
	(i) At A, $4\sqrt{x} \quad x = 0 \rightarrow A(16, 0)$	B1	co – independent of working.
	$\frac{dy}{dx} \quad 2x^{\frac{1}{2}} \quad 1$ $= 0$ when $x = 4 \rightarrow (4, 4)$	B1 B1 M1 A1 [5]	B1 for each part. Sets to 0 and solves his eqn. co
	(ii) Vol = $\pi \int y^2 \, dx =$		
	$\pi \int (16x + x^2 \quad 8x^{\frac{3}{2}}) \, dx$	M1	Use of correct formula + attempt at integration
	$\pi[8x^2 + \frac{x^3}{3} \quad 8\frac{x^{\frac{5}{2}}}{\frac{5}{2}}]$ Limits 0 to 16 $\rightarrow$ 136.5π. (or 137π)	A3,2,1 DM1 A1 [6]	One mark for each term – unsimplified Correct use of his limits. co – (429 ok)

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1	$(a+x)^5 + (1 \quad 2x)^6$ Coeff of x^3 in $1^{st} = 10 \times a^2$ Coeff of x^3 in $2^{nd} = 20 \times (-2)^3$ $\rightarrow \quad 10a^2 - 160 = 90$ $\rightarrow \quad a = 5$	B1 B1 + B1 M1 A1 [5]	co co Forming an equation for a + solution co (condone ±)
2	$y = mx + 4 \qquad y = 3x^2 - 4x + 7$ Equate $\rightarrow 3x^2 - (4 + m)x + 3 = 0$ Uses $b^2 - 4ac \rightarrow (4 + m)^2 - 36$ Solution of quadratic $m = 2$ or -10 Set of values $m > 2$ or $m < -10$	M1 M1 DM1 A1 A1 [5]	Eliminates y (or x) completely Any use of $b^2 - 4ac$ Method shown. Correct end-values co
3	$\frac{x}{a} + \frac{y}{b} \quad 1$ $P(a, 0)$ and $Q(0, b)$ Distance $\rightarrow \sqrt{(a^2 + b^2)} \quad \sqrt{45}$ Gradients $\rightarrow \frac{a}{b} \quad \frac{1}{2}$ Solution of sim eqns $\rightarrow a = 6, b = 3$	 M1 A1 M1 A1 A1 [5]	 M1 even if sign(s) incorrect. Correct values a and b (both)
4	**(a)** $y \quad \frac{2x^3 + 5}{x} \quad 2x^2 + \frac{5}{x}$ $d/dx = 4x \quad \frac{5}{x^2}$ or $4x \quad 5x^{-2}$	M1 A1 + A1 [3]	Knows to divide numerator by x co
	(b) $\int (3x \quad 2)^5 dx \quad \frac{(3x \quad 2)^6}{6} \div 3 \ (+c)$ $\int_0^1 (3x \quad 2)^5 dx \quad \left[\frac{(3x \quad 2)^6}{18}\right]$ Limits used correctly $\rightarrow -3½$	B1 B1 M1 A1 [4]	B1 without "÷ 3". B1 for "÷ 3". (ignore $(+c)$) Uses limits after integration. co
5	**(i)** $\overrightarrow{PQ} = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{RQ} = -3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$	B2,1 B1 [3]	Allow B2,1 for either one, B1 for the other.
	(ii) $\overrightarrow{PQ} . \overrightarrow{RQ} = -9 + 48 - 9 = 30$ $= \sqrt{54}\ \sqrt{82} \cos RQP$ $\rightarrow RQP = 63.2°$	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct use of modulus All linked correctly co

6	**(a)** $ar^2 = 20$	B1	co
	$\frac{a}{1-r} = 3a$	B1	co
	Soln of equations → $(r = ⅔)$ $a = 45$	M1 A1 [4]	Complete method to find a. co
	(b) $a + 7d = 3(a + 2d)$ → $2a = d$ $S_8 = 4(2a + 7d) = 32d$ or $64a$ $S_4 = 2(2a + 3d) = 8d$ or $16a$	M1 A1 M1 A1 [4]	Use of $a + (n-1)d$ co correct use of S_n formula once. ag
7	**(i)** $AX = 6\tan\frac{\pi}{3} = 6\sqrt{3}$	B1 [1]	ag
	(ii) Area of triangle = ½ × 6 × $6\sqrt{3}$	M1	Use of ½bh
	Area of sector = ½ $6^2 \times \frac{\pi}{3}$	M1	Use of ½$r^2\theta$
	Area shaded = $18\sqrt{3} - 6\pi$	A1 [3]	co
	(iii) Arc $AB = 6 \times \frac{\pi}{3} = 2\pi$	M1	Use of $r\theta$
	$OX = 6 \div \cos\frac{\pi}{3} = 12$, $BX = 6$	B1	Use of trig to find (OX and then) BX.
	Perimeter = $6\sqrt{3} + 2\pi + 6$	M1 A1 [4]	
8	**(i)** $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$		
	$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \quad \frac{(1-\cos\theta)^2}{\sin^2\theta}$	M1	Use of tan = sin/cos
	$\frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} \quad \frac{1-\cos\theta}{1+\cos\theta}$	M1 A1 [3]	Use of $\sin^2 + \cos^2 = 1$. All correct. (NB ag. – ensure cancelling has been done)
	(ii) $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \quad \frac{2}{5}$		
	$\frac{1-\cos\theta}{1+\cos\theta} \quad \frac{2}{5}$	M1	Uses part **(i)** to obtain an eqn in $\cos\theta$
	$\cos\theta \quad \frac{3}{7}$	A1	co
	$\theta = 64.6°$ or $295.4°$	A1 A1 √ [4]	co. √ for 360 – "1st answer".

9	$\frac{dy}{dx} \quad \frac{2}{\sqrt{x}} \quad 1 \quad P(9, 5)$		
	(i) $y \quad 4\sqrt{x} \quad x\,(+c)$ Uses (9, 5) in an integrated expression $\rightarrow c = 2$	B1 B1 M1 A1 [4]	Ignore + c. Substitution of point after integration. co.
	(ii) $\frac{dy}{dx} \quad 0 \rightarrow x = 4, y = 6$	M1 A1 A1 [3]	Attempt to solve $dy/dx = 0$. x correct. y correct.
	(iii) $\frac{d^2y}{d^2x} \quad x^{\frac{3}{2}} \rightarrow$ –ve $\rightarrow$ Max	B1 B1√ [2]	co. √ for correct deduction.
	(iv) $\frac{dy}{dx} \quad \frac{1}{3}$ Perpendicular $m = 3$ $\tan\theta = 3$ Angle is $\tan^{-1}3$ $k = 3$	M1 A1 [2]	Use of $m_1m_2 = -1$ Needs $k = 3$

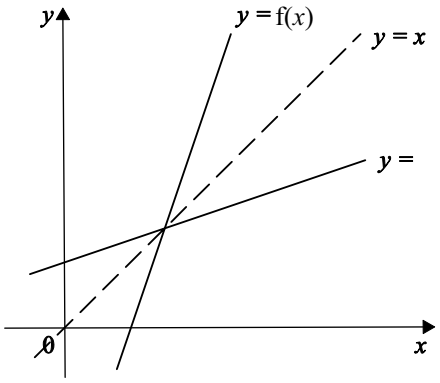
10 $f: x \mapsto 3x \quad 4 \qquad g: x \mapsto 2(x \quad 1)^3 + 8$

(i) fg(2) = f(10) = 26

$f^{-1}(x)$

M1 A1 [2] Must use g first, then f. co

(ii)



B1 $y = f(x)$ correct in 1st, 4th quadrants.
B1 $y = f^{-1}(x)$ correct in 1st, 2nd quadrants.
B1 $y = x$ marked, or quoted.
[3]

(iii) $g'(x) = 6(x - 1)^2$
$g'(x) > \rightarrow$ no turning points
$\rightarrow$ g is 1 : 1, g has an inverse.

B1 co
B1√ allow only for incorrect "6"
B1√ following from incorrect "6"
[3]

(iv) $f^{-1}(x) \quad \frac{x+4}{3}$

Attempt at making x
Order correct. – 8, ÷ 2, $\sqrt[3]{\ }$, + 1

$g^{-1}(x) \quad \sqrt[3]{\frac{x \quad 8}{2}} + 1$

B1 co
M1 May change x and y first.
M1 Must all be correct, but allow for + 8, – 1
A1 co as function of x, not y.
[4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

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for the guidance of teachers

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9709/21 Paper 2, maximum raw mark 50

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1 *EITHER* Attempt to square both sides obtaining three terms on each side M1
Attempt solution of three-term quadratic equation M1
Obtain $5x + 4x \quad 9 \quad 0$ and hence $\frac{9}{5}$ and 1 A1

OR Obtain value 1 from graphical method, inspection or linear equation B1
Obtain value $\frac{9}{5}$ similarly B2 [3]

2 State $\frac{dx}{dt} \quad 3 + 2\cos 2t$ or $\frac{dy}{dt} \quad 4\sin 2t$ (or both) B1

Use $\frac{dy}{dx} \quad \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Obtain or imply $\frac{4\sin 2t}{3 + 2\cos 2t}$ A1

Substitute $\frac{1}{6}\pi$ to obtain $\frac{1}{2}\sqrt{3}$ or exact equivalent A1 [4]

3 State or imply that $\ln y = \ln K + m \ln x$ B1
Equate intercept on axis for $\ln y$ to $\ln K$ M1
Obtain 7.39 for K A1
Attempt calculation of gradient of line M1
Obtain 1.37 for m A1 [5]

4 **(i)** Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero M1
Obtain $a = 8$ A1 [2]

(ii) Attempt to find quotient by division or inspection or use of identity M1
Obtain at least $3x^2 + 2x$ A1
Obtain $3x^2 + 2x + 4$ with no errors seen A1 [3]

5 **(i)** Differentiate $\ln(x - 3)$ to obtain $\frac{1}{x \quad 3}$ B1
Attempt to use product rule M1
Obtain $\ln(x \quad 3) + \frac{x}{x \quad 3}$ or equivalent A1
Substitute 4 to obtain 4 A1 [4]

(ii) Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $\frac{(x+1) \quad (x \quad 1)}{(x+1)^2}$ A1
Substitute 4 to obtain $\frac{2}{25}$ A1 [3]

6 **(a)** Rewrite integrand as $12e^{x} + 4e^{3x}$ B1
Integrate to obtain $12e^{x}$ … B1
Integrate to obtain … $+\frac{4}{3}e^{3x}$ B1
Include … + c B1 [4]

(b) Use identity $\tan^2\theta = \sec^2\theta - 1$ B1
Integrate to obtain $2\tan\theta + \theta$ or equivalent B1
Use limits correctly for integral of form $a\tan\theta + b\theta$ M1
Confirm given answer $\frac{1}{2}(8+\pi)$ A1 [4]

7 **(i)** Draw correct sketch of $y = e^{2x}$ B1
Draw correct sketch of $y = 14 - x^2$ B1
Indicate two real roots only from correct sketches B1 [3]

(ii) Consider sign of $e^{2x} + x^2 - 14$ for 1.2 and 1.3 or equivalent M1
Justify conclusion with correct calculations (f(1.2) = –1.54, f(1.3) = 1.15) A1 [2]

(iii) Confirm given answer $x \quad \frac{1}{2}\ln(14 \quad x^2)$ B1 [1]

(iv) Use the iteration process correctly at least once M1
Obtain final answer 1.26 A1
Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (1.255, 1.256) A1 [3]
[1.2 → 1.2653 → 1.2588 → 1.2595 ;
1.25 → 1.2604 → 1.2593 → 1.2594 ;
1.3 → 1.2522 → 1.2598 → 1.2594]

8 **(i)** State or imply $R \quad \sqrt{52}$ or $2\sqrt{13}$ B1
Use appropriate formula to find α M1
Obtain 56.31° A1 [3]

(ii) Attempt to find at least one value of θ – α M1
Obtain one correct value 80.9° of θ A1
Carry out correct method to find second answer M1
Obtain 211.7° and no others in range A1 [4]

(iii) Obtain 60, following their value of R B1 √
Obtain 8. Allow quoted solution B1 [2]

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1 Attempt use of power law for logarithms M1*
Obtain $x\log 3 = x\log 2 + 2\log 2$ or equivalent A1
Attempt solution for x of linear equation M1 dep*
Obtain 3.42 A1 [4]

2 **(i)** Show or imply correct ordinates 1, $\sqrt{2}$ or 1.414, 3 B1
Use correct formula, or equivalent, with $h = 1$ M1
Obtain 3.41 A1 [3]

(ii) Obtain 6 – 3.41 and hence 2.59, following their answer to **(i)** provided less than 6 B1√
Refer, in some form, to two line segments replacing curve and conclude with clear justification of given result that answer is an under-estimate. B1 [2]

3 **(i)** Use the iteration process correctly at least once M1
Obtain at least two correct iterates to 5 decimal places A1
Conclude $\alpha = 0.952$ A1 [3]
[1 → 0.95647 → 0.95257 → 0.95223 → 0.95220]

(ii) State or imply equation is $x = \frac{1}{2}\sqrt[3]{x^2 + 6}$ B1
Obtain $8x^3 - x^2 - 6 = 0$ B1 [2]

4 **(a)** Obtain integral form of $k\cos\frac{1}{2}x$ M1
Obtain correct $2\cos\frac{1}{2}x$ A1
Use limits correctly to obtain 1 A1 [3]

(b) Rewrite integrand as $e^x + 1$ B1
Integrate to obtain $-e^x \ldots$ B1
Integrate to obtain $\ldots + x + c$ B1 [3]

5 Obtain $4y\frac{dy}{dx}$ as derivative of $2y^2$ B1
Differentiate LHS term by term to obtain expression including at least one $\frac{dy}{dx}$ M1
Obtain $2x + 4y\frac{dy}{dx} + 5 + 6\frac{dy}{dx}$ A1
Substitute 2 and –1 to attempt value of $\frac{dy}{dx}$ M1
Obtain $\frac{9}{2}$ A1
Obtain equation $9x + 2y - 16 = 0$ or equivalent of required form A1 [6]

6	**(i)**	Attempt differentiation using product rule	M1	
		Obtain $8x\ln x + 4x$ (a.c.f.)	A1	
		Equate first derivative to zero and attempt solution	M1	
		Obtain 0.607	A1	
		Obtain –0.736 following their x-coordinate	A1√	[5]
	(ii)	Use an appropriate method for determining nature of stationary point	M1	
		Conclude point is a minimum (with no errors seen, second derivative = 8)	A1	[2]
7	**(i)**	Substitute $x = -2$ and equate to zero	M1	
		Substitute $x = -1$ and equate to 24	M1	
		Obtain $4a - 2b = 38$ and $a - b = 20$ or equivalents	A1	
		Attempt solution of two linear simultaneous equations (dependent on M1 M1)	M1	
		Obtain $a = -1$ and $b = -21$	A1	[5]
	(ii)	Attempt to find quadratic factor by division, inspection or use of identity	M1	
		Obtain $6x^2 - 13x + 5$	A1√	
		Conclude $(x+2)(2x \quad 1)(3x \quad 5)$	A1	[3]
8	**(i)**	Use $\text{cosec}\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$	B1	
		Attempt to simplify left-hand side	M1	
		Confirm given right-hand side $4\cos 2\theta$ with no errors seen	A1	[3]
	(ii)	**(a)** State or imply $\cos 2\theta = \frac{3}{4}$	B1	
		Attempt correct process to find at least one angle	M1	
		Obtain 20.7°	A1	
		Obtain 159.3° and no others in range	A1	[4]
		(b) Recognise as $\frac{4\cos 30^\circ}{\sin^2 30^\circ}$	B1	
		Obtain $8\sqrt{3}$	B1	[2]

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 Attempt use of power law for logarithms M1*
Obtain $x\log 3 = x\log 2 + 2\log 2$ or equivalent A1
Attempt solution for x of linear equation M1 dep*
Obtain 3.42 A1 [4]

2 **(i)** Show or imply correct ordinates 1, $\sqrt{2}$ or 1.414, 3 B1
Use correct formula, or equivalent, with $h = 1$ M1
Obtain 3.41 A1 [3]

(ii) Obtain 6 – 3.41 and hence 2.59, following their answer to **(i)** provided less than 6 B1√
Refer, in some form, to two line segments replacing curve and conclude with clear justification of given result that answer is an under-estimate. B1 [2]

3 **(i)** Use the iteration process correctly at least once M1
Obtain at least two correct iterates to 5 decimal places A1
Conclude $\alpha = 0.952$ A1 [3]
[1 → 0.95647 → 0.95257 → 0.95223 → 0.95220]

(ii) State or imply equation is $x = \frac{1}{2}\sqrt[3]{x^2 + 6}$ B1
Obtain $8x^3 - x^2 - 6 = 0$ B1 [2]

4 **(a)** Obtain integral form of $k\cos\frac{1}{2}x$ M1
Obtain correct $2\cos\frac{1}{2}x$ A1
Use limits correctly to obtain 1 A1 [3]

(b) Rewrite integrand as $e^x + 1$ B1
Integrate to obtain $-e^x \ldots$ B1
Integrate to obtain $\ldots + x + c$ B1 [3]

5 Obtain $4y\frac{dy}{dx}$ as derivative of $2y^2$ B1
Differentiate LHS term by term to obtain expression including at least one $\frac{dy}{dx}$ M1
Obtain $2x + 4y\frac{dy}{dx} + 5 + 6\frac{dy}{dx}$ A1
Substitute 2 and –1 to attempt value of $\frac{dy}{dx}$ M1
Obtain $\frac{9}{2}$ A1
Obtain equation $9x + 2y - 16 = 0$ or equivalent of required form A1 [6]

6	**(i)**	Attempt differentiation using product rule	M1	
		Obtain $8x\ln x + 4x$ (a.c.f.)	A1	
		Equate first derivative to zero and attempt solution	M1	
		Obtain 0.607	A1	
		Obtain –0.736 following their x-coordinate	A1√	[5]
	(ii)	Use an appropriate method for determining nature of stationary point	M1	
		Conclude point is a minimum (with no errors seen, second derivative = 8)	A1	[2]
7	**(i)**	Substitute $x = -2$ and equate to zero	M1	
		Substitute $x = -1$ and equate to 24	M1	
		Obtain $4a - 2b = 38$ and $a - b = 20$ or equivalents	A1	
		Attempt solution of two linear simultaneous equations (dependent on M1 M1)	M1	
		Obtain $a = -1$ and $b = -21$	A1	[5]
	(ii)	Attempt to find quadratic factor by division, inspection or use of identity	M1	
		Obtain $6x^2 - 13x + 5$	A1√	
		Conclude $(x+2)(2x \quad 1)(3x \quad 5)$	A1	[3]
8	**(i)**	Use $\text{cosec}\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$	B1	
		Attempt to simplify left-hand side	M1	
		Confirm given right-hand side $4\cos 2\theta$ with no errors seen	A1	[3]
	(ii)	**(a)** State or imply $\cos 2\theta = \frac{3}{4}$	B1	
		Attempt correct process to find at least one angle	M1	
		Obtain 20.7°	A1	
		Obtain 159.3° and no others in range	A1	[4]
		(b) Recognise as $\frac{4\cos 30^\circ}{\sin^2 30^\circ}$	B1	
		Obtain $8\sqrt{3}$	B1	[2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Either: Obtain $1+\frac{1}{3}kx$, where $k \neq \pm 6$ or ± 1 M1

Obtain $1 - 2x$ A1

Obtain $-4x^2$ A1

Obtain $\frac{40}{3}x^3$ or equivalent A1

Or: Differentiate expression to obtain form $k(1 - 6x)^{\frac{2}{3}}$ and evaluate f(0) and f′(0) M1

Obtain $f'(x) = -2(1 - 6x)^{\frac{2}{3}}$ and hence the correct first two terms $1 - 2x$ A1

Obtain $f''(x) = -8(1 - 6x)^{\frac{5}{3}}$ and hence $-4x^2$ A1

Obtain $f'''(x) = -80(1 - 6x)^{\frac{8}{3}}$ and hence $\frac{40}{3}x^3$ or equivalent A1 [4]

2 **(i)** Obtain $\frac{k\cos 2x}{1+\sin 2x}$ for any non-zero constant k M1

Obtain $\frac{2\cos 2x}{1+\sin 2x}$ A1 [2]

(ii) Use correct quotient or product rule M1

Obtain $\frac{x\sec^2 x - \tan x}{x^2}$ or equivalent A1 [2]

3 **(i)** Obtain $\pm\begin{pmatrix}3\\4\\6\end{pmatrix}$ as normal to plane B1

Form equation of p as $3x - 4y + 6z = k$ or $-3x + 4y - 6z = k$ and use relevant point to find k M1

Obtain $3x - 4y + 6z = 80$ or $-3x + 4y - 6z = -80$ A1 [3]

(ii) State the direction vector $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or equivalent B1

Carry out correct process for finding scalar product of two relevant vectors M1

Use correct complete process with moduli and scalar product and evaluate $\sin^{-1}$ or $\cos^{-1}$ of result M1

Obtain 30.8° or 0.538 radians A1 [4]

4 **(i)** Verify that $-96 + 100 + 8 - 12 = 0$ B1

Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity M1

Obtain $12x^2 + x - 6$ A1

State $(x + 2)(4x + 3)(3x - 2)$ A1 [4]

[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]

(ii) State $3^y = \frac{2}{3}$ and no other value B1

Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ M1

Obtain -0.369 and no other value A1 [3]

5 **(i)** Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$ B1

Obtain given result $58 + 2k = c$ **AG** B1 [2]

(ii) Differentiate left-hand side term by term, reaching $ae^{2x} + be^y \frac{dy}{dx} + ce^{2y} \frac{dy}{dx}$ M1

Obtain $12e^{2x} + ke^y \frac{dy}{dx} + 2e^{2y} \frac{dy}{dx}$ A1

Substitute $(\ln 3, \ln 2)$ in an attempt involving implicit differentiation at least once, where RHS = 0 M1

Obtain $108 - 12k - 48 = 0$ or equivalent A1

Obtain $k = 5$ and $c = 68$ A1 [5]

6 **(i)** State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$ B1

Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent M1

Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$ A1 [3]

(ii) Use iterative formula correctly at least once M1

Obtain value for θ of 2.11 A1

Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115) A1

Use correct trigonometry to find an expression for the length of AB M1

e.g. $20\sin 1.055$ or $\sqrt{200 \quad 200\cos 2.11}$

Hence 17.4 A1 [5]

[$2.1 \rightarrow 2.1198 \rightarrow 2.1097 \rightarrow 2.1149 \rightarrow 2.1122$]

7 **(i)** State or imply $dx = 2t\,dt$ or equivalent — B1
Express the integral in terms of x and dx — M1
Obtain given answer $\int_1^5 (2x - 2)\ln x\,dx$, including change of limits **AG** — A1 [3]

(ii) Attempt integration by parts obtaining $(ax^2 + bx)\ln x \pm \int (ax^2 + bx)\frac{1}{x}dx$ or equivalent — M1
Obtain $(x^2 - 2x)\ln x - \int (x^2 - 2x)\frac{1}{x}dx$ or equivalent — A1
Obtain $(x^2 - 2x)\ln x - \frac{1}{2}x^2 + 2x$ — A1
Use limits correctly having integrated twice — M1
Obtain $15\ln 5 - 4$ or exact equivalent — A1 [5]
[Equivalent for M1 is $(2x - 2)(ax\ln x + bx) - \int (ax\ln x + bx)\,2dx$]

8 **(i)** Either: Multiply numerator and denominator by $(1 - 2i)$, or equivalent — M1
Obtain $-3i$ — A1
State modulus is 3 — A1
Refer to u being on negative imaginary axis or equivalent and confirm argument as $\frac{1}{2}\pi$ — A1

Or: Using correct processes, divide moduli of numerator and denominator — M1
Obtain 3 — A1
Subtract argument of denominator from argument of numerator — M1
Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or $-0.464 - 1.107$ and hence $\frac{1}{2}\pi$ or -1.57 — A1 [4]

(ii) Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction — B1
Use correct trigonometry to find required value — M1
Obtain $\frac{3}{2}\sqrt{2}$ or equivalent — A1 [3]

(iii) Show, or imply, locus is a circle with centre $(1 + i)u$ and radius 1 — M1
Use correct method to find distance from origin to furthest point of circle — M1
Obtain $3\sqrt{2} + 1$ or equivalent — A1 [3]

9 **(i)** Express $\cos 4\theta$ as $2\cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2\sin^2 2\theta$ B1

Express $\cos 4\theta$ in terms of $\cos\theta$ M1

Obtain $8\cos^4\theta - 8\cos^2\theta + 1$ A1

Use $\cos 2\theta = 2\cos^2\theta - 1$ to obtain given answer $8\cos^4\theta - 3$ **AG** A1 [4]

(ii) **(a)** State or imply $\cos^4\theta = \frac{1}{2}$ B1

Obtain 0.572 B1

Obtain –0.572 B1 [3]

(b) Integrate and obtain form $k_1\theta + k_2\sin 4\theta + k_3\sin 2\theta$ M1

Obtain $\frac{3}{8}\theta + \frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta$ A1

Obtain $\frac{3}{32}\pi + \frac{1}{4}$ following completely correct work A1 [3]

10 **(i)** Separate variables correctly and integrate of at least one side M1

Carry out an attempt to find A and B such that $\frac{1}{N(1800 - N)} \equiv \frac{A}{N} + \frac{B}{1800 - N}$, or equivalent M1

Obtain $\frac{2}{N} + \frac{2}{1800 - N}$ or equivalent A1

Integrates to produce two terms involving natural logarithms M1

Obtain $2\ln N - 2\ln(1800 - N) = t$ or equivalent A1

Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a\ln N$, $b\ln(1800)$ and ct M1

Obtain $2\ln N - 2\ln(1800 - N) = t - 2\ln 5$ or equivalent A1

Use laws of logarithms to remove logarithms M1

Obtain $N = \frac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent A1 [9]

(ii) State or imply that N approaches 1800 B1 [1]

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1 *EITHER*: State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5+2x)$ M1

Obtain critical values –5 and $\frac{5}{3}$ only A1

Obtain final answer $x < -5$, $x > \frac{5}{3}$ A1

OR: State one critical value e.g. –5, by solving a linear equation or inequality, or from a graphical method, or by inspection B1

State the other critical value, e.g. $\frac{5}{3}$, and no other B1

Obtain final answer $x < -5$, $x > \frac{5}{3}$ B1 [3]

[Do not condone ≤ or ≥.]

2 **(i)** Use law for the logarithm of a product or quotient M1

Use $\log_2 32 = 5$ or $2^5 = 32$ M1

Obtain $x^2 + 5x - 32 = 0$, or horizontal equivalent A1 [3]

(ii) Solve a 3-term quadratic equation M1

Obtain answer $x = 3.68$ only, or exact equivalent, e.g. $\frac{\sqrt{153} - 5}{2}$ A1 [2]

3 Use correct trig formula (or formulae) and obtain an equation in $\cos\theta$ M1

Obtain $8\cos^2\theta + \cos\theta - 7 = 0$, or equivalent A1

Solve a 3-term quadratic in $\cos\theta$ and reach $\theta = \cos^{-1}(a)$ M1

Obtain answer 29.0° A1

Obtain answer 180° and no others A1 [5]

[Ignore answers outside the given interval. Treat answers in radians (0.505 and 3.14 or π) as a misread.]

[SR: The answer 180° found by inspection can earn B1.]

4 **(i)** State or imply $CT = r\tan x$ or $OT = r\sec x$, or equivalent B1

Using correct area formulae, form an equation in r and x M1

Obtain the given answer correctly A1 [3]

(ii) Use the iterative formula correctly at least once M1

Obtain the final answer 1.35 A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p. , or show there is a sign change in the interval (1.345, 1.355) A1 [3]

5 **(i)** *EITHER*: State $\frac{dx}{dt}$ $\sec^2 t / \tan t$, or equivalent — B1

State $\frac{dy}{dt}$ $2\sin t \cos t$, or equivalent — B1

Use $\frac{dy}{dx}$ $\frac{dy}{dt} \div \frac{dx}{dt}$ — M1

Obtain correct answer in any form, e.g. $2\sin^2 t \cos^2 t$ — A1

OR: Obtain $y = e^{2x} / (1 + e^{2x})$, or equivalent — B1

Use correct quotient or product rule — M1

Obtain correct derivative in any form, e.g. $2e^{2x} / (1 + e^{2x})^2$ — A1

Obtain correct derivative in terms of t in any form, e.g. $(2\tan^2 t) / (1 + \tan^2 t)^2$ — A1 [4]

(ii) State or imply t $\frac{1}{4}\pi$ when $x = 0$ — B1

Form the equation of the tangent at $x = 0$ — M1

Obtain correct answer in any horizontal form, e.g. y $\frac{1}{2}x + \frac{1}{2}$ — A1 [3]

[SR: If the *OR* method is used in part **(i)**, give B1 for stating or implying y $\frac{1}{2}$ or $\frac{dy}{dx}$ $\frac{1}{2}$ when $x = 0$.]

6 **(i)** Show that the differential equation is $\frac{dy}{dx}$ $2xy$ — B1

Separate variables correctly and attempt integration of both sides — M1

Obtain term $\ln y$, or equivalent — A1

Obtain term x^2, or equivalent — A1

Evaluate a constant, or use limits $x = 1$, $y = 2$, in a solution containing terms $a\ln y$ and bx^2 — M1

Obtain correct solution in any form — A1

Obtain the given answer correctly — A1 [7]

(ii) State that the gradient at $(-1, 2)$ is -4 — B1

Show the sketch of curve with correct concavity, positive y-intercept and axis of symmetry $x = 0$ — B1 [2]

[SR: A solution with $k \neq 2$, or not evaluated, can earn B0M1A1A1M1A1A0 in part **(i)**.]

[SR: If given answer is assumed valid, give B1 if $\frac{dy}{dx}$ is shown correctly to be equal to $2xy$, is stated to be proportional to xy, and shown to be equal to 4 at $(1, 2)$.]

7 **(a)** **(i)** *EITHER*: Multiply numerator and denominator by $a - 2i$, or equivalent — M1

Obtain final answer $\frac{5a}{a^2+4} \quad \frac{10i}{a^2+4}$, or equivalent — A1

OR: Obtain two equations in x and y, solve for x or for y — M1

Obtain final answer $x \quad \frac{5a}{a^2+4}$ and $y \quad \frac{10}{a^2+4}$, or equivalent — A1 [2]

(ii) Either state $\arg(u) = \frac{3}{4}\pi$, or express u^* in terms of a (f.t. on u) — B1√

Use correct method to form an equation in a, e.g. $5a = -10$ — M1

Obtain $a = -2$ correctly — A1 [3]

(b) Show a point representing $2 + 2i$ in relatively correct position in an Argand diagram — B1

Show the circle with centre at the origin and radius 2 — B1

Show the perpendicular bisector of the line segment from the origin to the point representing $2 + 2i$ — B1√

Shade the correct region — B1 [4]

[SR: Give the first B1 and the B1√ for obtaining $y = 2 - x$, or equivalent, and sketching the attempt.]

8 **(i)** State or imply partial fractions are of the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$ — B1

Use a relevant method to determine a constant — M1

Obtain one of the values $A = -2$, $B = 1$, $C = 4$ — A1

Obtain a second value — A1

Obtain the third value — A1 [5]

(ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$, $\left(1+\frac{1}{2}x^2\right)^{-1}$ or $(2+x^2)^{-1}$ in ascending powers of x — M1

Obtain correct unsimplified expansion up to the term in x^3 of each partial fraction — A1√ + A1√

Multiply out fully by $Bx + C$, where $BC \neq 0$ — M1

Obtain final answer $\frac{5}{2}x \quad 3x^2 + \frac{7}{4}x^3$, or equivalent — A1 [5]

[Symbolic binomial coefficients, e.g. $\binom{1}{1}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10.]

[In the case of an attempt to expand $(5x - x^2)(1 + x)^{-1}(2 + x^2)^{-1}$, give M1A1A1 for the expansions, M1 for the multiplying out fully, and A1 for the final answer.]

[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining f(0) = 0 and f '(0) = $\frac{5}{2}$, A1√ for f "(0) = –6, and A1 for f "'(0) = $\frac{21}{2}$ and the final answer (the f.t. is on A, B, C if used).]

[For the identity $5x \quad x^2 \equiv \left(2 + 2x + x^2 + x^3\right)\left(a + bx + cx^2 + dx^3\right)$ give M1A1; then M1A1 for using a relevant method to obtain two of $a = 0$, $b = \frac{5}{2}$, $c = -3$ and $d = \frac{7}{4}$; then A1 for the final answer in series form.]

9 **(i)** State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ B1
Carry out correct process for evaluating the scalar product of the two normals M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1
Obtain the final answer 79.7° (or 1.39 radians) A1 [4]

(ii) *EITHER*: Carry out a method for finding a point on the line M1
Obtain such a point, e.g. (1, 3, 0) A1
EITHER: State two correct equations for the direction vector (a, b, c) of the line, e.g. $a + 2b - 2c = 0$ and $2a + b + 3c = 0$ B1
Solve for one ratio, e.g. $a : b$ M1
Obtain $a : b : c = 8 : -7 : -3$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR1: Obtain a second point on the line, e.g. $\left(0, \frac{31}{8}, \frac{3}{8}\right)$ A1
Subtract position vectors to find a direction vector M1
Obtain $\mathbf{i} \quad \frac{7}{8}\mathbf{j} \quad \frac{3}{8}\mathbf{k}$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} \quad \frac{7}{8}\mathbf{j} \quad \frac{3}{8}\mathbf{k})$ A1√

OR2: Attempt to calculate the vector product of two normals M1
Obtain two correct components A1
Obtain $8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$, or equivalent A1
State a correct final answer, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR3: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $x = (31 - 8y) / 7$ A1
Express the first variable in terms of a third M1
Obtain a correct simplified expression, e.g. $x = (3 - 8z) / 3$ A1
Form a vector equation of the line M1
State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$ A1√

OR4: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $y = (31 - 7x) / 7$ A1
Express the third variable in terms of the second M1
Obtain a correct simplified expression, e.g. $z = (3 - 3x) / 8$ A1
Form a vector equation of the line M1
State a correct final answer, e.g. $\mathbf{r} = \frac{31}{8}\mathbf{j} + \frac{3}{8}\mathbf{k} + \lambda(-8\mathbf{i} + 7\mathbf{j} + 3\mathbf{k})$ A1√ [6]

[The f.t. is dependent on all M marks having been earned.]

10 **(i)** Attempt integration by parts and reach $\pm x^2e^{-x} \pm \int 2xe^{-x}dx$ M1*

Obtain $x^2e^{-x} + \int 2xe^{-x}dx$, or equivalent A1

Integrate and obtain $-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$, or equivalent A1

Use limits $x = 0$ and $x = 3$, having integrated by parts twice M1(dep*)

Obtain the given answer correctly A1 [5]

(ii) Use correct product or quotient rule M1

Obtain correct derivative in any form A1

Equate derivative to zero and solve for non-zero x M1

Obtain $x = 2$ with no errors send A1 [4]

(iii) Carry out a complete method for finding the x-coordinate of P M1

Obtain answer $x = 1$ A1 [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

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International Examinations

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Use law for the logarithm of a product, power or quotient M1*
Obtain a correct linear equation, e.g. $(2x \quad 1)\ln 5 \quad \ln 2 + x \ln 3$ A1
Solve a linear equation for x M1(dep*)
Obtain answer $x = 1.09$ A1 [4]

[SR: Reduce equation to the form $a^x \quad b$ M1*, obtain $\left(\frac{25}{3}\right)^x \quad 10$ Al, use correct method to calculate value of x M1(dep*), obtain answer 1.09 A1.]

2 Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $\frac{3\ln x}{x^4} + \frac{1}{x^4}$ A1
Equate derivative to zero and solve for x an equation of the form $\ln x \quad a$, where $a > 0$ M1
Obtain answer $\exp(\frac{1}{3})$, or 1.40, from correct work A1 [4]

3 Attempt integration by parts and reach $k(1 \quad x)e^{\frac{1}{2}x} \pm k\int e^{\frac{1}{2}x}dx$, or equivalent M1
Obtain $2(1 \quad x)e^{\frac{1}{2}x} \quad 2\int e^{\frac{1}{2}x}dx$, or equivalent A1
Integrate and obtain $2(1 \quad x)e^{\frac{1}{2}x} + 4e^{\frac{1}{2}x}$, or equivalent A1
Use limits $x = 0$ and $x = 1$, having integrated twice M1
Obtain the given answer correctly A1 [5]

4 **(i)** Use tan($A \pm B$) formula correctly at least once and obtain an equation in tanθ M1
Obtain a correct horizontal equation in any form A1
Use tan60° = $\sqrt{3}$ throughout M1
Obtain the given equation correctly A1 [4]

(ii) Set $k \quad 3\sqrt{3}$ and obtain $\tan^2\theta = \frac{1}{11}$ B1
Obtain answer 16.8° B1√
Obtain answer 163.2° B1√ [3]
[Ignore answers outside the given interval. Treat answers in radians (0.293 and 2.85) as a misread.]

5 **(i)** Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g. $\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2}$ 2 0 — B1

Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12 — M1

Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$ — A1

Solve for a or for b — M1

Obtain $a = 2$ and $b = -3$ — A1 [5]

(ii) Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$ — M1

Obtain quadratic factor $x^2 - x + 2$ — A1 [2]

[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B, or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C.]

6 **(i)** Make recognisable sketch of a relevant graph over the given range — B1

Sketch the other relevant graph and justify the given statement — B1 [2]

(ii) Consider the sign of $\cot x \left(1 + x^2\right)$ at $x = 0.5$ and $x = 0.8$, or equivalent — M1

Complete the argument with correct calculated values — A1 [2]

(iii) Use the iterative formula correctly at least once with $0.5 \leq x_n \leq 0.8$ — M1

Obtain final answer 0.62 — A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625) — A1 [3]

7 **(i)** Use the quadratic formula, completing the square, or the substitution $z = x + iy$ to find a root and use $i^2 = -1$ — M1

Obtain final answers $\sqrt{3} \pm i$, or equivalent — A1 [2]

(ii) State that the modulus of both roots is 2 — B1√

State that the argument of $\sqrt{3} + i$ is 150° or $\frac{5}{6}\pi$ (2.62) radians — B1√

State that the argument of $\sqrt{3}$ i is –150° (or 210°) or $-\frac{5}{6}\pi$ (–2.62) radians or $\frac{7}{6}\pi$ (3.67) radians — B1√ [3]

(iii) Carry out an attempt to find the sixth power of a root — M1

Verify that one of the roots satisfies $z^6 = -64$ — A1

Verify that the other root satisfies the equation — A1 [3]

8 **(i)** Use product and chain rule — M1

Obtain correct derivative in any form, e.g. $15\sin^2 x\cos^3 x \quad 10\sin^4 x\cos x$ — A1

Equate derivative to zero and obtain a relevant equation in one trigonometric function — M1

Obtain $2\tan^2 x \quad 3$, $5\cos^2 x \quad 2$, or $5\sin^2 x \quad 3$ — A1

Obtain answer $x = 0.886$ radians — A1 [5]

(ii) State or imply $\mathrm{d}u \quad \sin x\,\mathrm{d}x$, or $\dfrac{\mathrm{d}u}{\mathrm{d}x} \quad \sin x$, or equivalent — B1

Express integral in terms of u and $\mathrm{d}u$ — M1

Obtain $\pm\int 5\left(u^2 \quad u^4\right)\mathrm{d}u$, or equivalent — A1

Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$) — M1

Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen — A1 [5]

9 **(i)** State or imply $\dfrac{\mathrm{d}x}{\mathrm{d}t} \quad k(10 \quad x)(20 \quad x)$ and show $k = 0.01$ — B1 [1]

(ii) Separate variables correctly and attempt integration of at least one side — M1

Carry out an attempt to find A and B such that $\dfrac{1}{(10 \quad x)(20 \quad x)} \equiv \dfrac{A}{10 \quad x} + \dfrac{B}{20 \quad x}$, or equivalent — M1

Obtain $A = \frac{1}{10}$ and $B = \quad \frac{1}{10}$, or equivalent — A1

Integrate and obtain $\frac{1}{10}\ln(10 \quad x) + \frac{1}{10}\ln(20 \quad x)$, or equivalent — A1√

Integrate and obtain term $0.01t$, or equivalent — A1

Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a\ln(10 \quad x)$, $b\ln(20 \quad x)$ and ct — M1

Obtain answer in any form, e.g. $\frac{1}{10}\ln(10 \quad x) + \frac{1}{10}\ln(20 \quad x) \quad 0.01t + \frac{1}{10}\ln 2$ — A1√

Use laws of logarithms to correctly remove logarithms — M1

Rearrange and obtain $x \quad 20(\exp(0.1t) \quad 1)/(2\exp(0.1t) \quad 1)$, or equivalent — A1 [9]

(iii) State that x approaches 10 — B1 [1]

10 (i) *EITHER*: Express general point of l or m in component form, e.g. $(2 + \lambda, -\lambda, 1 + 2\lambda)$ or $(\mu, 2 + 2\mu, 6 - 2\mu)$ B1

Equate at least two pairs of components and solve for λ or for μ M1

Obtain correct answer for λ or μ (possible answers for λ are -2, $\frac{1}{4}$, 7 and for μ are 0, $2\frac{1}{4}$, $4\frac{1}{2}$) A1

Verify that all three component equations are not satisfied A1

OR: State a relevant scalar triple product, e.g. $(2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) \cdot ((\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}))$ B1

Attempt to use the correct method of evaluation M1

Obtain at least two correct simplified terms of the three terms of the expansion of the triple product or of the corresponding determinant, e.g. $-4, -8, -15$ A1

Obtain correct non-zero value, e.g. -27, and state that the lines do not intersect A1 [4]

(ii) Carry out the correct process for evaluating scalar product of direction vectors for l and m M1

Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1

Obtain answer 47.1° or 0.822 radians A1 [3]

(iii) *EITHER*: Use scalar product to obtain $a - b + 2c = 0$ B1

Obtain $a + 2b - 2c = 0$, or equivalent, from a scalar product, or by subtracting two point equations obtained from points on m, and solve for one ratio, e.g. $a : b$ M1*

Obtain $a : b : c = -2 : 4 : 3$, or equivalent A1

Substitute coordinates of a point on m and values for a, b and c in general equation and evaluate d M1(dep*)

Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1

OR1: Attempt to calculate vector product of direction vectors of l and m M1*

Obtain two correct components A1

Obtain $-2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or equivalent A1

Form a plane equation and use coordinates of a relevant point to evaluate d M1(dep*)

Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1

OR2: Form a two-parameter plane equation using relevant vectors M1*

State a correct equation e.g. $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mathbf{s}(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mathbf{t}(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ A1

State three correct equations in x, y, z, s and t A1

Eliminate s and t M1(dep*)

Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1 [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

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for the guidance of teachers

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9709/41 Paper 4, maximum raw mark 50

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1	**(i)**	[DF – 600 = 700 × 2]	M1		For using Newton's second law (3 terms needed)
		Driving force is 2000 N	A1	**[2]**	
	(ii)	[P = 2000 × 15]	M1		For using P = Fv
		Rate of working is 30000 W (or 30 kW)	A1ft	**[2]**	
2	**(i)**	Gain in PE = 1250g × 1.54 (= 19250 J)	B1		
		[WD = 1250g × 1.54 + 5750]	M1		For using WD by crane = Gain in PE + WD against resistance
		Work done is 25000 J (or 25 kJ)	A1	**[3]**	
	(ii)	[1250 = 25000 / T]	M1		for using P = Δ(WD) / Δt
		Time is 20 s	A1ft	**[2]**	ft Ans**(i)** ÷ 1250
3			M1		For resolving forces horizontally or vertically (3 terms needed)
		T$\cos\theta$ + T$\sin\theta$ = 15.5	A1		AEF
		–T$\cos\theta$ + T$\sin\theta$ = 8.5	A1		AEF
			DM1		For solving for T$\sin\theta$ and T$\cos\theta$
		T$\sin\theta$ = 12 and T$\cos\theta$ = 3.5	A1		AG
		θ = 73.7° (or 1.29°)	B1	**[6]**	
4	**(i)**		M1		For resolving forces parallel to the plane (either case) – 3 terms needed
		2X + F = 11gsin30° and 9X – F = 11gsin30°	A1		
		X = 10	A1	**[3]**	
	(ii)	F = 35	B1		May be implied.
		R = 11gcos30°	B1		
			DM1		For using μ = F/R
		Coefficient is 0.367	A1ft	**[4]**	
5	**(i)**	v(600) = 0.025 × 600	B1		
			M1		For using 0 = v(600 + 2600) – 0.0375t_3 and v(600 + 2600) = v(600)
		0 = 15 – 0.0375t_3	A1		
		Total time is 3600 s	A1	**[4]**	
	(ii)	For correct graph	M1		Shape only
		[d = ½ (2600 + 3600) × 15 or d = ½ 0.025 × 600^2 + 2600 × 15 + ½ 0.0375 × 400^2]	A1ft		For method of finding distance
		Distance is 46500	A1ft	**[3]**	
	(iii)	Values of t are 300 and 3400	B1	**[1]**	

6	**(i)**		M1		For using s = $\int v dt$
		$s = 2t^2 - t^4/64$ (+ C)	A1		
		$[t^4 - 128t^2 + 64^2 = 0]$	M1		For attempting to solve s(t) = 64
		$(t^2 - 64)^2 = 0$	A1		
		Time taken is 8 s	A1	**[5]**	
	(ii)		M1		For using a = dv/dt
		$a = 4 - 3t^2/16$	A1		
		a is positive for $0 < t < \frac{8}{\sqrt{3}}$ or $0 < t < 4.62$	B2	**[4]**	SR: Allow B1 for $t < \frac{8}{\sqrt{3}}$ SR: B1 for $0 \leq t \leq \frac{8}{\sqrt{3}}$ or 4.62
7	**(i)**		M1		For applying Newton's second law to A or to B
		T – 12 = 1.2a and 20 –T = 2a	A1		Accept (2 – 1.2)g = (2.0 + 1.2)a as an alternative for one of these equations
		Acceleration is 2.5 ms^{-2}	B1		
		Tension is 15 N	A1	**[4]**	
	(ii)	**(a)** PE gain = 12 × 1.5 = 18 J	B1		
		(b) WD on A = 15 × 1.5 = 22.5J	B1		
		(c) Gain in KE = ans**(b)** – ans**(a)** = 4.5 J	B1ft	**[3]**	alt: KE = ½ 1.2(2 × 2.5 × 1.5) = 4.5J
	(iii)	v = 1.6 × 2.5	B1ft		
			M1		For using v = u – gt
		t = 0.4 s	A1		May be implied
		Total time taken is 0.8 s	A1	**[4]**	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/42 Paper 4, maximum raw mark 50

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International Examinations

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**(i)**	[WD = 65 × 76cos5°]	M1		For using WD = Tdcos α
		Work done is 4920 J	A1	**[2]**	
	(ii)	[P = 65 cos5° × 1.5]	M1		For using P = Tvcos α
		Rate of working is 97.1 W	A1ft	**[2]**	ft for the value of ans**(i)** × 1.5÷76 SR for candidates who assume without justification that the speed is constant (max 1/2) t = 76 ÷ 1.5 = 50.6…s rate = WD/t = 4960÷50.6.. = 97.1W B1
2			M1		For using 'loss of PE = gain in KE + WD against resistance'
		PE loss = ½ 8($8^2 - 3^2$) + 120 (= 340 J)	A1		
		[340 = 8gh]	DM1		For using PE = mgh
		Height is 4.25 m	A1	**[4]**	
					SR for candidates who assume without justification that the resistance to motion is constant, usually implicitly by using constant acceleration formulae (max 3/4) For using Newton's second law with 3 terms, $v^2 - u^2 = 2as$ and h = s sinα M1 For attempting to eliminate α, a and s from the equations (80sinα – 120/s = 8a 64 – 9 = 2as, h = s sinα) M1 80s sinα – 120 = 4(64 – 9) → 80h – 120 = 220 → h = 4.25 A1
3	**(i)**	[½ 5 × 50 + ½ 7(8 + 50) + 90 × 8]	M1		For using the area property for distance or s = ½ (u + v)t
		Distance is 1048 m	A1	**[2]**	AG
	(ii)		M1		For use of the gradient property for acceleration (deceleration)
		a = (8 – 50)/(12 – 5) or d = (50 – 8)/(12 – 5)	A1		
			M1		For using Newton's second law (3 terms)
		850 – F = 85a (or –85d)	A1		
		Upward force is 1360 N	A1	**[5]**	

4	**(i)**	M1		For resolving forces in the **i** and **j** directions
	$F\cos\theta = 12\cos30°$ (= 10.932)	A1		
	$F\sin\theta = 10 - 12\sin30°$ (= 4)	A1		
		M1		For using $F^2 = X^2 + Y^2$ or $\tan\theta = Y/X$
	$F = 11.1$ or $\theta = 21.1$ (accept 21.0)	A1		
	$\theta = 21.1$ (accept 21.0) or $F = 11.1$	B1	**[6]**	
				SR for candidates who consistently have cos for sin and vice versa (max 4/6) M1 as above (resolving) A1 for $F\sin\theta = 12\sin30°$ and $F\cos\theta = 10 - 12\cos30°$ M1 as above $F^2 = ...$ & $\tan\theta = ...$ A1 for $F = 6.01$ and $\theta = 93.7$
	(ii) Magnitude is 12N	B1		
	Direction is 30° clockwise from +ve 'x' axis	B1	**[2]**	
	alternative for 4(i)			
	For triangle of forces with sides 12, F and 10 and at least one of the angles $(90° - \theta)$ or 60° or $(\theta + 30°)$	B1		
		M1		For use of cosine rule (with θ absent) or use of sine rule (with F absent) and use of $\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$
	$F^2 = 12^2 + 10^2 - 2 \times 12 \times 10\cos60°$ or $(12\cos30°)\sin\theta = (10 - 12\sin30°)\cos\theta$	A1		
	$F = 11.1$ or $\theta = 21.1$ (accept 21.0)	A1		
		M1		For correct method for θ or F
	$\theta = 21.1$ (accept 21.0) or $F = 11.1$	A1	**[6]**	
	second alternative for 4(i)			
	For using Lami's theorem with 12 N and 10 N	M1		
	$12/\sin(90 + \theta) = 10/\sin(150 - \theta)$	A1		
	$12/\cos\theta = 20 \div (\cos\theta + 3^{½}\sin\theta)$ $\rightarrow 12 \times 3^{½}\sin\theta = 8\cos\theta$ $\rightarrow \tan\theta = 2 \div (3 \times 3^{½})$ $\rightarrow \theta = 21.1$	A1		
	For using Lami's theorem with F N and (12 N or 10 N)	M1		
	$F/\sin120° = 12/\sin111.1°$ (or $10/\sin128.9°$)	A1		
	$F = 11.1$	A1	**[6]**	

	Alternative for 4(ii)				
		For X = 11.1cos21.1° and Y = 11.1sin21.1° – 10, $R^2 = X^2 + Y^2$ and tanΦ = Y/X	M1		
		Magnitude 12 N and direction 30° clockwise from +ve x-axis	A1	**[2]**	
5	**(i)**		M1		For using 0 = u – gt to find times at maximum heights.
		Times to max. height are 1.2s and 0.7s	A1		
		Range of values is 0.7 < t < 1.2	A1	**[3]**	
	(ii)		M1		For using $h = ut – ½ gt^2$ and attempting to solve $3h_A = 8h_B$ for t
		$36t – 1.5gt^2 = 56t – 4gt^2$	A1		
		t = 8/g	A1		
			M1		For using v = u – gt
		Velocities are 4m ^{1} and –1ms ^{1}	A1	**[5]**	
	Alternative for part 5(ii)				
		For using $3h_P = 8h_Q \rightarrow 3(v_P^2 – 144) \div (–20) = 8(v_Q^2 – 49) \div (–20) \rightarrow 3v_P^2 – 8v_Q^2 = 40$	B1		
		For using $v_P = 12 – 10t$ and $v_Q = 7 – 10t$ $\rightarrow$ $v_P – v_Q = 5$	B1		
		For eliminating v_Q (or v_P) and solving for v_P (or v_Q).	M1		
		$v_P^2 – 16v_P + 48 = 0 \rightarrow v_P = 4$ (or 4, 12)	A1		
		Upward velocities are 4 ms ^{1} and –1 ms ^{1}	A1	**[5]**	
6	**(i)**		M1		For resolving forces on R vertically
		$2T \cos\alpha = 0.6g$	A1		Where α = ½ angle ARB
		Tension is 5N	A1	**[3]**	
	(ii)	[$F = T \sin\alpha$]	M1		For resolving forces on B horizontally
		Frictional component is 4N	A1		
		[$N = 0.4g + T \cos\alpha$]	M1		For resolving forces on B vertically
		Normal component is 7 N	A1	**[4]**	
	(iii)		M1		For using $\mu = F/N$
		Coefficient is 4/7 or 0.571	A1ft	**[2]**	ft conditional on both M1 marks scored in **(ii)**; ft F and/or N

	Alternative for Q6(i)/(ii)				
	(i)	For finding the relevant angles and using Lami's theorem	M1		
		$6/\sin 106.26° = T/\sin 126.87°$	A1		
		Tension is 5N	A1	**[3]**	
	(ii)	$F/\sin 126.87° = 5/\sin 90°$	B1		
		Frictional component is 4N	B1		
		$(R - 4)/\sin 143.13° = 5/\sin 90°$	B1		
		Normal component is 7 N	B1	**[4]**	
7	**(i)**	$[1.3 = 0.9 + 0.004T,$ $1.3^2 = 0.9^2 + 2 \times 0.004S]$	M1		For using $v = u + at$ or $v^2 = u^2 + 2as$
		Time is 100 s (or distance is 110 m)	A1		
		Distance is 110 m (or time is 100 s)	B1	**[3]**	
	(ii)	$\int kt^3 dt = ¼ kt^4$	B1		
		$[k(¼ 100^4 - 0) = 110]$	M1		For using limits 0 to T and equating definite integral to S
		$k = 4.4 \times 10^{-6}$	A1		
		$[v_W = 0.9 + 0.004 \times 64.05,$ $v_C = 4.4 \times 10^{-6} \times 64.05^3]$	M1		For attempting to find the speed of the walker and of the cyclist.
		Both are equal to 1.16 ms^{-1} correct to 3 sf.	A1	**[5]**	
	(iii)	Acceleration $= 3kt^2$	B1		
		Acceleration at B is 0.132 ms^{-2}	B1	**[2]**	

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GCE Advanced Subsidiary Level and GCE Advanced Level

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1		M1		For using WD = Fdcos α
	$8200 = 180 \times 50 \cos\alpha$	A1		
	$\alpha = 24.3$	A1	**[3]**	
2		M1		For using DF = P/v
		M1		For using Newton's second law when v = 19 or when v = 30
	$P/19 - R = 1250 \times 0.6$ and $P/30 - R = 1250 \times 0.16$	A1		
	$[19R + 19 \times 1250 \times 0.6 = 30R + 30 \times 1250 \times 0.16]$	M1		For attempting to eliminate P or R
	$R = 750$ or $P = 28500$	A1		
	$P = 28500$ or $R = 750$	B1ft	**[6]**	ft wrong answer for R or P substituted into a **correct** linear equation.
3	**(i)** $a_P = g\sin30^o$	B1		
	$3.2 = \frac{1}{2} gt_q^2$	B1		
	$[6.4 = u(0.8) + \frac{1}{2} 5 \times (0.8)^2]$	M1		For applying $s = ut + \frac{1}{2} at^2$ to P
	$u = 6$	A1	**[4]**	
	(ii) $[v = 6 + 5 \times 0.8$ or $v^2 = 36 + 2\times5\times6.4]$	M1		For using $v = u + at$ or $v^2 = u^2 + 2as$ for P
	Speed of P is 10 ms $^{-1}$	A1	**[2]**	
	Alternative for Parts (i) and (ii) when a is not used:			
	<u>Part (i)</u>			
	$3.2 = \frac{1}{2} gt_q^2$	B1		
	For using KE gain = PE loss to obtain an equation in u and v			
	$[\frac{1}{2} (v^2 - u^2) = 6.4g\sin30^o]$	M1		
	For using $s = \frac{1}{2} (u + v)t$ to obtain a second equation in u and v			
	$[6.4 = \frac{1}{2} (u + v) \times 0.8]$	DM1		
	$u = 6$	A1	**[4]**	
	<u>Part (ii)</u>			
	Substitutes for u to find v	M1		
	Speed is 10 ms $^{-1}$	A1	**[2]**	

4	**(i)**	For correct shading composite figure consisting of 2 rectangles: 1st has boundaries t = 0 & t = 20, v = 0 and v = 2.5; 2nd has boundaries t = 20 & t = T, v = 0 and v = 4	B1	**[1]**	
	(ii)	[50 + 4(T – 20) = 70 or 4T – 30 = 70]	M1		For attempt to find equation in T
		T = 25	A1	**[2]**	
	(iii)	[Distance = 70 + (4 – 2.5)20 or 50 + 4[(T – 20) + 20] – 50]	M1		For identifying and using area representing required distance
		Distance between P and Q is 100 m	A1ft	**[2]**	ft 4T
	(iv)	For 2 straight line segments representing P, 1st with +ve slope and 2nd with steeper slope, t = 20 indicated appropriately	B1		
		For Q, 1st & 2nd segments parallel to P's and displaced to the right, t = 25 and t = 45 indicated appropriately	B1ft	**[2]**	ft T and T + 20
5	**(i)**		M1		For resolving forces in the x direction or the y direction
		$F_x - 6.1 - 5 \times 0.28 = 0$ and $F_y + 4.8 - 5 \times 0.96 = 0$	A1		
		Frictional force acts parallel to x axis and to the right	A1		
		$F_y = 0 \rightarrow F = F_x$ $\rightarrow$ Frictional force has magnitude 7.5 N	A1	**[4]**	AG
	(ii)	[$\mu = 7.5/(1.25 \times 10)$]	M1		For using $F = \mu R$ and $R = mg$
		Coefficient is 0.6	A1	**[2]**	
	(iii)	[$7.5 - 8.6 - 1.4 = 1.25a \rightarrow a = -2$]	M1		For applying Newton's second law
		Magnitude of acceleration is 2 ms^{-2}	A1		
		Direction of acceleration is parallel to x axis and to the left	B1	**[3]**	

6	**(i)**	Gain in PE = $15000g \times 500\sin 2.5^o$ J	B1		
		WD against the resistance = 800×500 J	B1		
		[3271454 + 400000]	M1		For using WD by driving force = Gain in PE + WD against resistance
		Work done is 3670000 J or 3670 kJ	A1	**[4]**	
					Alternatively, For resolving forces up the plane M1 Driving Force = $800 + 15000g\sin 2.5^o$ A1 For using WD = Driving Force × 500 M1 Work done is 3670000J A1
	(ii)	Work done by DF = 2000×500 J Gain in KE = ½ $15000(v^2 - 20^2)$	B1 B1		
			M1		For using Gain in KE = Loss in PE – WD against resistance + WD by driving force
		½ $15000(v^2 - 20^2) = 3271454 - 400000 + 1000000$	A1		
		Speed of the lorry is 30.3 ms ^{1}	A1	**[5]**	
					Alternatively, For applying Newton's second law M1 $2000 + 15000g\sin 2.5 - 800 = 15000a$ A1 For using $v^2 = u^2 + 2as$ M1 $v^2 = 20^2 + 2 \times 0.5162 \times 500$ A1 Speed is 30.3 ms ^{1} A1
7	**(i)**		M1		For using $v(t) = \int a\,dt$
		$v = \frac{1}{160}t^3 \quad \frac{1}{3200}t^4 \quad (+ C_1)$	A1		
		[$0 = 8000/160 - 160000/3200 + C_1$ $\rightarrow C_1 = 0$]	M1		For using v(20) = 0
		Initial speed is zero	A1	**[4]**	AG
	(ii)	[$t^2/800(15 - t) = 0$]	M1		For solving a = 0
		$v_{max} = v(15) = 5.27$ ms ^{1}	A1	**[2]**	
	(iii)		M1		For using $s(t) = \int v\,dt$
		$s = \frac{1}{640}t^4 \quad \frac{1}{16000}t^5 \quad (+ C_2)$	A1ft		
		[250 – 200]	M1		For using limits 0 and 20 (or equivalent)
		Distance AB is 50 m	A1	**[4]**	

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GCE Advanced Subsidiary Level and GCE Advanced Level

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9709/51 Paper 5, maximum raw mark 50

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$0 = (15\sin 40°)t - gt^2/2$ $t = 1.93$	M1 A1 [2]	Accept quoting the formula $T = 2V\sin\theta/g$ for the time of flight
2	**(i)** $x = 2 \times 0.6\sin(\pi/4)/(3\pi/4)$ [= 0.36(0)] $d^2 = 0.6^2 + 0.36^2 - 2 \times 0.6 \times 0.36\cos(\pi/4)$ $d = 0.429$	B1 M1 A1 [3]	Centre of mass from O
	(ii) $\sin\alpha/0.36 = \sin(\pi/4)/0.429$ $\alpha = 36.4°$ or 0.635^c	M1 A1 A1 [3]	
3	**(i)** EE gain $= 2 \times 24[\sqrt{(0.6^2 + 0.25^2)} - 0.6]^2/(2 \times 0.6)$ $m \times 0.5^2/2 = 0.1$ $m = 0.8$ (kg) **AG**	B1 M1 A1 [3]	EE gain = 0.1 KE loss = EE gain
	(ii) $T = 24 \times (0.65 - 0.6)/0.6$ (= 2) $2 \times 2 \times 0.25/0.65 = 0.8a$ $a = 1.92$	B1 M1 A1 [3]	Newton's Second Law with attempt to resolve 2T
4	**(i)** $a = 0$ when $x = 2.5$ $v\,dv/dx = 15 - 6x$ $\int v\,dv = \int(15 - 6x)\,dx$ $v^2/2 = [15x \quad 3x^2]$ (+ c) $v = 6.12$	B1 M1 A1 M1 A1 [5]	For use of limits 0 and 2.5 or evaluating c(=0)
	(ii) Solves $15x - 3x^2 = 0$ $a\ (= 15 - 6 \times 5) = -15 ms^{-2}$	M1 A1 [2]	$x = 5$. Accept assumption c = 0.

5	**(i)** $19 \times 0.6/3 + T \times 0.22 = T \times 0.6$ $T = 10$ AG	M1 A1 A1 [3]	Moments about A, 3 terms
	(ii) $10 = \lambda(0.11 + 0.6 - 0.7)/0.7$ $\lambda = 700$	M1 A1 [2]	
	(iii) $F^2 = 10^2 + (19 - 10)^2$ $F = 13.5$ $\alpha = \tan^{-1}(9/10) = 42.(0)^\circ$ (with horizontal)	M1 A1 B1 [3]	Or for a $= \tan^{-1}(10/9) = 48^\circ$ (with vertical)
6	**(i)** $5 = 0.4(V\sin\alpha) - g \times 0.4^2/2$ $V\sin\alpha = 14.5$ $0.4(V\cos\alpha) = 12$ hence $V\cos\alpha = 30$ $V = \sqrt{(30^2 + 14.5^2)}$ $V = 33.3$ $\alpha = 25.8^\circ$	M1 A1 B1 M1 A1 B1 [6]	α is the angle of projection Or $\tan\alpha = 14.5/30$ $\alpha = 25.8^\circ$ V = 33.3
	(ii) $v = 14.5 - 0.4g$ $\tan\theta = (14.5 - 0.4g)/30$ $\theta = \tan^{-1}0.35 = 19.3°$ with the horizontal **OR** $dy/dx = x\tan\alpha - gx^2\sec^2\alpha/(2V^2)$ $\tan\theta = \tan25.8° - 10 \times 12\sec^2 25.8°/33.3^2$ $\theta = 19.3°$ with the horizontal	B1 M1 A1 [3] M1 M1 A1	$v = \sqrt{(14.5^2 - 2g \times 5)}$ $\tan\theta = \sqrt{(14.5^2 - 2g \times 5)} / 30$ For differentiating the trajectory equation For attempting to substitute x, α and v
7	**(i)** $0.3\omega^2 \times 0.5 = T + 0.36 \times 0.3g$ $0.2\omega^2 \times 0.5 = T - 0.36 \times 0.2g$ $0.1\omega^2 \times 0.5 = 0.36 \times 0.5g$ $\omega = 6$ $T = 0.3 \times 6^2 \times 0.5 - 0.36 \times 0.3 \times 10$ $T = 4.32$	M1 A1 M1 A1 M1 A1 [6]	Newton's Second Law, 3 terms Both correct
	(ii) **(a)** $0.2\omega^2 r = 0.3\omega^2(1 - r)$ $r = 0.6$ $r_P = 0.6$ m and $r_Q = 0.4$ m	M1 A1 A1ft [3]	$0.3\omega^2 R = 0.2\omega^2(1 - R)$ $R = 0.4$
	(ii) **(b)** $0.48 = 0.2v_P^2/0.6$ or $0.48 = 0.3v_Q^2/0.4$ $v_P = 1.2$ $v_Q = 0.8$	M1 A1 A1 [3]	Newton's Second Law radially

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

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1	(i)	$16L\cos\theta = 4 \times 2L$ $\theta = 60^\circ$ or $\pi/3^c$ or 1.05^c	M1 A1 [2]	Moments about A, accept L = 1
	(ii)	$X = 4\sin60^\circ$ and $Y = 16 - 4\cos60^\circ$ $= \sqrt{[(4\sin60^\circ)^2 + (16 - 4\cos60^\circ)^2]}$ = 14.4 N $\alpha = 76.1^\circ$	B1 M1 A1ft B1 [4]	 $\tan\alpha = (16 - 4\cos60^\circ)/(4\sin60^\circ)$ ft cv(X,Y). $\alpha = 76.1^\circ$ R = 14.4 N
2	(i)	C of M semi-circle $= 4 \times 0.2/(3\pi)$ $\frac{\pi 0.2^2}{2} \times 4 \times \frac{0.2}{3\pi} = \frac{0.4h}{2} \times \frac{h}{3}$ = 0.283	B1 M1 A1 A1 [4]	(0.08488…) Moments about a relevant point.
	(ii)	$\tan\theta = 0.283/0.2$ $\cos\theta = XD/0.2$ (= 0.5774) XD = 0.115 m **OR** $\tan\alpha = 0.2/0.283$ $\sin\alpha = XD/0.2$ (= 0.5774) XD = 0.115 m	M1 M1 A1 M1 M1 A1 [3]	tanADO = h/0.2 , ADO = 54.75° For candidates ADO tanDAO = 0.2/h, DAO = 35.25 For candidate's DAO
3	(i)	$R\cos30^\circ + T\cos60^\circ = 0.5g$ $F = 0.5g/(\cos30^\circ + \cos60^\circ)$ $T\sin60^\circ - R\sin30^\circ = 0.5v^2/0.1$ $v = 0.518$ ms $^{-1}$	M1 A1 M1 A1 [4]	or with R = T = F F = 3.660… = R = T Newton's Second Law with radial acceleration
	(ii)	R = 0 $T\cos60^\circ = 0.5g$ $T\sin60^\circ = 0.5 \times \omega^2 \times 0.1$ $\omega = 13.2$ rads $^{-1}$ **OR** R = 0 $mv^2\sin30^\circ/r$ or $mr\omega^2\sin30^\circ$ $= mg\cos30^\circ$ $\omega = 13.2$ rad s $^{-1}$	B1 M1 M1 A1 B1 M1 M1 A1 [4]	Could be implied T = 10 N Newton's Second Law with radial acceleration Could be implied

4	**(i)** $0.24g = 12(x)/0.5$ $x = 0.1$	M1 A1	Finds position for equilibrium
	EITHER $\frac{1}{2} \times 0.24 \times 3^2 + 12 \times (0.8 - 0.5)^2/(2 \times 0.5) =$ $0.24v^2/2 + 12 \times 0.1^2/(2 \times 0.5)$ $+ 0.24g(0.8 - 0.5 - 0.1)$ $v = 3.61$ ms $^{-1}$	M1 A1 A1	Energy balance, initial to equilibrium positions
	OR $0.24v dv/dx = mg - 12x/0.5$ $0.24v^2/2 = 2.4x - 12x^2$ $(+ c)$ $v = 3, x = 0.3,$ $c = 1.44$ $x = 0.1$, $v = 3.61$ ms $^{-1}$	M1 A1 A1 [5]	Using Newton's Second Law Or uses limits
	(ii) $0.24 \times 3^2/2 + 12 \times (0.8 - 0.5)^2/(2 \times 0.5) =$ $0.24g(0.8 + x)$ $x = 0.1$m $s = (0.5 + 0.1) = 0.6$ m	M1 A1 A1 A1	Initial KE + initial EE = Final PE
	OR $\frac{1}{2} \times 12 \times 0.3^2/0.5 + \frac{1}{2} \times 0.24 \times 3^2$ $= \frac{1}{2} \times 0.24v^2 + 0.24 \times 10 \times 0.3$ $v = \sqrt{12}$ *Either* $0 = 12 - 2 \times 10s$ $s = 0.6$ *Or* $\frac{1}{2} \times 0.24 \times 12 = 0.24 \times 10s$ $s = 0.6$	M1 A1 M1 A1 M1 A1	Initial EE + Initial KE = (KE + PE) at equilibrium position Using $v^2 = u^2 + 2as$ Using KE at equilibrium position = Final PE
	OR $\frac{1}{2} \times 12 \times 0.3^2/0.5 + \frac{1}{2} \times 0.24 \times 3^2$ $= 0.24 \times 10y$ $y = 0.9$ $s = 0.9 - 0.3 = 0.6$	M1 A1 A1 A1 [4]	Initial EE + Initial KE = Final PE where y is the distance above the start

5	**(i)** $dv/dt = -2.5k\sqrt{v}$ $\int v^{0.5}\,dv = -2.5k\int dt$ $v^{0.5}/0.5 = -2.5kt\ (+c)$ $t = 0, v = 9$ hence $c = 6$ and $t = 2, v = 4$ hence $k = 0.4$ $v = (6-t)^2/4 = (t-6)^2/4$ AG	B1 M1 A1 M1 A1 [5]	$0.4dv/dt = -k\sqrt{v}$ LHS $= 0.8\sqrt{v}$ $\sqrt{v} = (6-t)/2$ Uses correct limits
	(ii) $x = \int (t-6)^2/4dt$ $x = (t-6)^3/(3\times 4)\ (+c)$ $t = 0, x = 0$ hence $c = 18$ $x(3) = 18 - (3-6)^3/12$ $x(3) = 15.75$ **OR** $\int v^{\frac{1}{2}}\,dv = \int -dx$ $\frac{2}{3}v^{\frac{3}{2}} = -x\ (+c)$ $x = 18 - \frac{2}{3}v^{\frac{3}{2}}$ $x = 15.75$	M1 A1 M1 A1 M1 A1 M1 A1 [4]	$\int (6-t)^2/4dt$ $-(6-t)^3/(3\times 4)\ (+c)$ Or uses limits 0, 3 Accept 15.7 or 15.8 From $mvdv/dx = -k\sqrt{v}$ Using $v = 9$, $x = 0$ so $c = 18$ Put $t = 3$ to find $v = 2.25$
6	**(i)** $x = (26\cos 30^\circ)\times 2.3$ $y = (26\sin 30^\circ)\times 2.3 + g\times 2.3^2/2$ $d^2 = 51.8^2 + 56.35^2$ $d = 76.5$ m	B1 B1 M1 A1 [4]	$= 51.788..$ $= 56.35$
	(ii) $80 = (26\sin 30^\circ)t + 10t^2/2$ $t = 2.91$s [or $(42.06-13)/10$] $x = (2.906\times 26\cos 30^\circ) = 65.4$ m **OR** $80 = x\tan 30^\circ + 10x^2/(2\times 26^2\times \cos^2 30^\circ)$ $x = 65.4$	M1 A1 A1 M1 M1 A1 [3]	or $v^2 = (26\sin 30^\circ)^2 + 2\times 10\times 80$ with $v = 42.06$ $= 26\sin 30^\circ + 10t$ solved for t Uses trajectory equation Attempts to solve the quadratic equation
	(iii) $v^2 = (26\sin 30^\circ)^2 + 2g\times 80$ $V^2 = (26\sin 30^\circ)^2 + 2g\times 80 + (26\cos 30^\circ)^2$ $V = 47.7\ \text{ms}^{-1}$ $\alpha = \tan^{-1}[(42.06)/(26\cos 30^\circ)] = 61.8^\circ$	B1 M1 A1 A1 [4]	$v = 42.06$. Accept $v = 26\sin 30^\circ + 10\times 2.91$ or award correct method to find α Below horizontal (1.08)

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1	**(i)**	$T\sin30° = 0.4g$ $T = 8N$	M1 A1 [2]	Resolves vertically
	(ii)	$T\cos30° = 0.4v^2 / 0.2$ ($= 0.4\omega^2 \times 0.2$) $v = 1.86$ ms $^{-1}$	M1 A1ft [2]	Newton's Second Law radially ft only on T from part **(i)**
2	**(i)**	$20 = gt^2 / 2$ $\quad (t = 2)$ $x = 15 \times 2$ $x = 30$	M1 DM1 A1 [3]	$y = -gx^2/(2 \times 15^2)$ use of trajectory equation $-20 = -10x^2 / (2 \times 15^2)$
	(ii)	$v = (g \times 2) = 20$ $v = \sqrt{(15^2 + 20^2)}$ $v = 25$	B1 M1 A1 [3]	
3	**(i)**	$F \times 0.4\sin20° = 12 \times (0.4 / 2)\cos20°$ $F = 16.48$ AG	M1 A1 A1 [3]	Moments about O
	(ii)	$R = -16.48 + 12 + W$ $-16.48 + 12 + W = 0$ $W = 4.48$	B1 M1 A1 [3]	Equates forces vertically Works with R = 0
4	**(i)**	$e = \sqrt{(0.6^2 + 0.32^2)} - 0.4$ ($= 0.28$) $0.3g \times 0.32 = 2[\lambda(0.28^2 - 0.2^2)] / (2 \times 0.4)$ $\lambda = 10$	B1 M1, A1 A1 [4]	Extension of half string = 0.28 m PE loss = EE gain
	(ii)	$e = \sqrt{(0.6^2 + 0.25^2)} - 0.4$ $0.3g \times 0.25 = 0.3v^2 / 2 +$ $2[10(0.25^2 - 0.2^2) / (2 \times 0.4)]$ $v = 1.12$	B1 M1 A1ft A1 [4]	Extension of half string = 0.25 m PE loss = KE gain + EE gain N.B. 0.25 is extension of half string ft on candidates λ only
5	**(i)**	$T = 6e / 0.3$ $0.2 \times 5^2 (0.3 + e) = 6e / 0.3$ $e = 0.1$	B1 M1, A1 A1 [4]	Newton's Second Law radially
	(ii)	$0.2\omega^2 (0.3 + e) = 6e / 0.3$ $e = 0.06\omega^2/(20 - 0.2\omega^2)$ $20 - 0.2\omega^2 > 0$ $(0 <) \omega < 10$	M1 A1 M1 A1 [4]	Newton's Second Law radially Other forms acceptable Uses denominator > 0 Disregard lower limit

6	**(i)**	$0.2a = -0.2g0.5 - 0.4/x^2$ $v\,dv/dx = -(5 + 2x^{-2})$ AG	M1 A1 [2]	Uses Newton's Second law
	(ii)	$\int v\,dv = -\int(5 + 2x^{-2})dx$ $v^2/2 = -5x + 2/x\ (+ c)$ $3^2/2 = -5 \times 0.5 + 2/0.5 + c$ $x = 1$ Travels $(= 1 - 0.5) = 0.5$m F towards O (0.4) less than maximum friction (= 1)	M1 A1 M1 A1 A1 M1, A1 [7]	Separates variables and integrates Hence c = 3, or $[v^2/2]_3^0 = [-5x + 2/x]_{0.5}^x$ From $0 = -5x + 2/x = 3$ Compares $0.5 \times 0.2g$ and $0.4/1^2$
7	**(i)**	$OG_{quadrant} = 2\sin(\pi/4) / (3\pi/4)$ $a^2(a\sqrt{2}/2) = \pi/4[2\sin(\pi/4) / (3\pi/4)] + (a^2 - \pi/4)x$ $x = 2\sqrt{2}(3a^3 - 2) / (12a^2 - 3\pi)$	B1 M1 A2 A1 [5]	$8 / (3\sqrt{2}\ \pi)$ –1 each error, min zero There must be 3 moment terms Other forms acceptable
	(ii)	$x\cos45° > 1$ $(6a^3 - 4) / (12a^2 - 3\pi) > 1$ $3a^2(2 - a) < 3\pi/2 - 2$ AG True when a = 1.68, not when a = 1.67 AG	B1 M1 A1 B1 [4]	RHS = 2.712.. compared with LHS = 2.709.. and 2.76.. respectively

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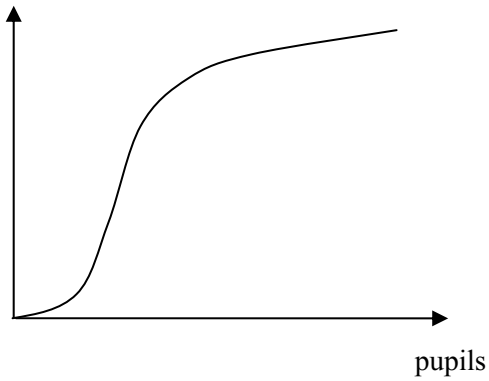
2	P(pencil case | find) = $\frac{P(\text{pencilcase and find})}{P(\text{find})} = \frac{0.7 \times 1}{0.7 + 0.3 \times 0.2}$ = 0.921	M1 A1 A1 A1 **[4]**	Attempt to use cond prob formula, must be quotient Correct num of a fraction Correct denominator Correct answer
3	**(i)** P(any other number) = 9/70 $P(X < 2) = 27/70 + 1/10$ = 34/70 (17/35) (0.486)	B1 B1ft **[2]**	9/70 Seen Ft their probs if < 1
	(ii) $E(X) = 108/70$ (54/35) (1.543) $\text{Var}(X) = ((-2)^2 + \ldots + 5^2) \times 9 / 70 - (54/35)^2$ = 5.33	M1 M1 A1 **[3]**	Valid attempt at $E(X)$ (needn't be accurate) Using a variance formula correctly with mean^2 subtracted numerically, no extra division Correct final answer
	(iii) $a = 1$	B1 **[1]**	
4	**(i)** Options 5 bat 5 bl 1 Wk in $^{10}C_5 \times {}^9C_5 \times {}^2C_1 = 63504$ ways or 5 bat 4 bl 2 Wk in $^{10}C_5 \times {}^9C_4 \times {}^2C_2 = 31752$ ways or 6 bat 4 bl 1 Wk in $^{10}C_6 \times {}^9C_4 \times {}^2C_1 = 52920$ ways Total = 148176 (148000)	M1 M1 A1 A1 **[4]**	Multiplying three combinations together Summing more than one sensible option Two options correct unsimplified Correct final answer
	(ii) $\frac{11!}{5!4!2!} = 6930$	B1 **[1]**	Correct answer evaluated
	(iii) Omit a pen $\frac{10!}{4!4!2!} = 3150$ Omit a diary $\frac{10!}{5!3!2!} = 2520$ Omit a notebook $\frac{10!}{5!4!} = 1260$ Total = 6930	M1 B1 A1 **[3]**	Summing three options One option correct Correct final answer

5 **(a)**	$z > \dfrac{2\mu - \mu}{\sigma} = \dfrac{\mu}{\sigma} = \dfrac{7\sigma^2}{3\sigma}$	M1	Standardising attempt resulting in $z >$ some μ/σ
	$\dfrac{7\sigma}{3}$ 1.272	M1 B1	Substituting to eliminate μ or σ 1.272 seen
	$\sigma = 0.545$ $\mu = 0.693$	A1 **[4]**	Both answers correct
	(b) $P(X < a + 33) = 0.75$ $z = 0.674$	M1 A1	Using 0.75 oe ± 0.674 seen
	$\dfrac{a+33-33}{\sqrt{21}}$ 0.674	M1	Standardising, no cc, must have sq rt
	$a = 3.09$	A1 **[4]**	Correct answer
6 **(i)**		M1 M1 (Indpt)	Sensible attempt at graph using u.c.b. 2500 seen in median attempt on a CF graph Can be implied
	Median 270	A1 **[3]**	Correct answer + or – 5
	(ii) 20% less than 160	M1 A1 **[2]**	Using 20% Correct answer + or – 5
	(iii) 2100 – 1600 = 500	B1 **[1]**	
	(iv) (50.5 × 200 + 125.5 × 600 + 175.5 × 800 + 225.5 × 500 + 300.5 × 2000 + 400.5 × 600 + 525.5 × 300) / 5000 = 268	M1 M1 A1 A1 **[4]**	Using an attempt at mid-points Using an attempt at frequencies Correct mid-points or frequencies Correct answer only
7 **(a)**	**(i)** P(at least one 3) = 1 – P(no 3s) $= 1 - (5/6)^9$ = 0.806	M1 A1 **[2]**	Using 1 – none Correct answer
	(ii) P(at least 1 three) = $1 - (5/6)^n$ $1 - (5/6)^n > 0.9$ $n > 12.6$ $n = 13$	B1 M1 M1 A1 **[4]**	 Equation or inequality involving n and 0.9 Solving attempt of sensible equation, can be trial Correct answer
	(b) P(R wins his 1st ball) = P(GY) = 15/56 (0.268) P(R wins 2nd ball) = P(GGGY) = 3/28 P(R wins 3rd ball) = P(GGGGGY) $\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}$ = 1/56 P(R wins) = 11/28 (0.393)	M1 M1 M1 A1 **[4]**	Using P(GY) Attempt to find P(GGGY) or P(GGGGGY) Adding three options Correct answer

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/62 Paper 6, maximum raw mark 50

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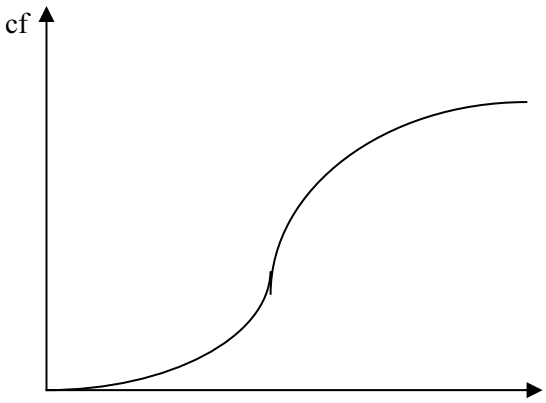
1	$20p = 4.8 \quad p = 0.24$ or $4.8/20$ $P(0, 1, 2) = (0.76)^{20} + {}^{20}C_1(0.24)^1(0.76)^{19}$ $+ {}^{20}C_2(0.25)^2(0.76)^{18}$ $= 0.109$ SR max 3 out of 4	B1 M1 A1 A1 **[4]** B1 M1 A1	Correct value for p Summing 2 or 3 binomial probs o.e., any p, $n = 5$ or 20 Correct unsimplified answer Correct answer As above Using N(4.8, 3.648) with cc 2.5 or 3.5 0.114 seen
2	**(i)** $np = 24$, $npq = 4.8$ $z = \pm\left(\frac{24.5 \quad 24}{\sqrt{4.8}}\right) = 0.228$ Prob = 0.590	B1 M1 M1 A1 **[4]**	24 and 4.8 or $\sqrt{4.8}$ seen can be unsimplified Standardising, need sq rt, cc not necessary Continuity correction 24.5 or 25.5 used Correct answer must be from 24.5
	(ii) np and nq both > 5.	B1 **[1]**	Need both
3	**(i)** Mean = $45 - 148/36 = 40.9$ or $1472/36$ *EITHER* Var = $3089/36 - (-148/36)^2 = 68.9$ sd = 8.30 *OR* $\Sigma x^2 = 3089 - 36 \times 45^2 + 90 \times 1472 = 62669$ Var = $\left(\frac{62669}{36} \quad \left(\frac{1472}{36}\right)^2\right)$ sd = 8.30	B1 M1 A1 **[3]** M1 A1	Correct answer $3089/36 - (\pm$ their coded mean$)^2$ Correct answer Expanding $\Sigma(x - 45)^2$ with at least 2 terms correct and solving, then substituting their Σx^2 in correct variance formula with their mean^2 subt numerically Correct answer
	(ii) New $\Sigma(x - 45) = -148 - 16 = -164$ New $\Sigma(x - 45)^2 = 3089 + 16^2 = 3345$ New sd = $\sqrt{3345/37 \quad (\ 164/37)^2}$ = 8.41 *OR* $\Sigma x = 36 \times 45 \quad 148 = 1472$ New $\Sigma x = 1472 + 29 = 1501$ $\Sigma x^2 = 3089 - 36 \times 45^2 + 90 \times 1472 = 62669$ New $\Sigma x^2 = 62669 + 29^2$ (= 63510) New sd = $\sqrt{63510/37 \quad (1501/37)^2}$ = 8.41	M1 M1 M1 A1 **[4]** M1 M1 M1 A1	Adding their coded new value to – 148 Adding their (coded value)^2 to 3089 Subst in coded var formula, can have one of 29 and one of –16 here Correct answer Finding Σx and adding 29 Finding Σx^2 and adding 29^2, at least 2 terms of 3089, 36×45^2, 90×1472 Subst their values in correct var formula Correct answer

4 (i)	90720	B1 **[1]**	Not 9!/2!2!
(ii)	3 vowels together $= 3! \times 7!/2!2! = 7560$	B1 B1	3! oe seen multiplied by integer oe 7 or 6! seen multiplied as a num
	Prob(not together) $= \frac{90720 \quad 7560}{90720} = \frac{83160}{90720}$	M1	Subt from their **(i)** or dividing by their **(i)** or 1 – prob
	$= 0.917$ (=11/12)	A1 **[4]**	Correct answer from correct working
(iii)	One S in ${}^5C_3$ ways = 10 SS in ${}^5C_2$ ways = 10 Total = 20	M1 M1 A1 **[3]**	${}^5C_3$ seen added ${}^5C_2$ seen added Correct answer
	OR ${}^6C_3$	M1	${}^6C_3 \times 2$ or $\div 2$ or $\times 1$ seen
		M1	${}^6C_3$ only
	$= 20$	A1	Correct answer
5 (i)	cf  Number of rooms	M1 M1 M1 A1 **[4]**	Attempt at cf table (up to 200) Linear scale minimum 0 to 200 and 20 to 80, and labels Attempt to plot points at (20.5, 10), (40.5, 42), (50.5, 104), (60.5, 154), (70.5, 182), (90.5, 200), accept (20, 10), (40, 42) or (21, 10), (41, 42) etc All points correct and joined up, allow (0, 0) or (0.5, 0)
(ii)	Line on graph up from 30	M1	Line or mark seen, can be implied if matches graph and in range
	$200 - 20 = 180$	A1 **[2]**	Accept 174 – 180 if reading from graph
	OR using lin int $10 + \frac{(30 \quad 20.5)}{20} \times 32 = 25.2$	M1	Can have 20 or 20.5
	$= 174.8$	A1	Accept decimals, 174 – 175 if using lin int
(iii)	Line on graph across from 150	M1	Line or mark seen, can be implied if matches graph and in range. 150 seen and line between 140 and 160
	59 rooms	A1 **[2]**	Accept 58 – 60
	OR lin int $50.5 + 46/50 \times 10$ $= 59$ or 60	M1 A1	Can have 50 or 50.5 Must be integer

6 **(i)**	$z = -1.282$	B1	± 1.282 or ± 1.281 seen
	$P(x < 20) = P\left(z < \frac{20 - \mu}{0.8}\right)$	M1	Standardising, no cc, must have 0.8, must be a z-value
	$-1.282 = \frac{20 - \mu}{0.8}$		
	$\mu = 21.0$ cm (21.0256)	A1 **[3]**	Correct answer
(ii)	$P(21.5 < x < 22.5)$		
	$= P\left(\frac{21.5 - 21.03}{0.8}\right) < z < \left(\frac{22.5 - 21.03}{0.8}\right)$	M1	2 attempts at standardising with their mean, must have 0.8 oe
	$= \Phi(1.8375) - \Phi(0.5875)$	M1	Subtracting 2 Φs ft their mean
	$= 0.9670 - 0.7217$		
	$= 0.2453$	A1	Needn't be entirely accurate, rounding to 0.24 or 0.25
	$P(< 2) = P(0) + P(1)$		
	$= (0.7547)^4 + (0.2453)^1(0.7547)^3\ {}^4C_1$	M1	Binomial term with ${}^4C_r p^r (1-p)^{4-r}$ seen $r \neq 0$, any $p < 1$
		M1	Bin expression for P(0) + P(1), any $p < 1$
	$= 0.746$	A1 **[6]**	Accept 3sf rounding to 0.75
7 **(i)**	$P(6) = P(3, 9) + P(9, 3) = 2/25 = 0.08$ AG	B1 **[1]**	Accept 2/25 seen
(ii)			

x	0	1	2	3	4	5	6
Prob	0.2	0.24	0.08	0.08	0.16	0.16	0.08

	M1	Values 0 – 6 seen could be in list
	A1 **[2]**	All correct
(iii) Mean = $\Sigma xp = 2.56$ (64/25)	B1 **[1]**	
(iv) $P(4, 5, 6) = 0.4(10/25)$ or $0.16 + 0.16 + 0.08$	B1 ft	ft their P(4, 5, 6) providing $p < 1$
$=$ P(draw) $\times$ 0.4	M1	Multiplying by their P(draw) providing $p < 1$
$= 0.2 \times 0.4 = 0.08$ (2/25)	A1ft **[3]**	Correct answer
(v) P(J wins on nth go)	M1	Mult by any p^n or p^{n-1}, $p < 1$
$= (0.2)^{n-1} \times 0.4$ oe	A1ft **[2]**	ft their probs

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1 **(i)**	$(3.6\times 9+64)\ /\ 24$ $= 4.02$ years	M1 A1 **[2]**	Mult by 9, adding 64 then dividing by 24 Correct answer
(ii)	$\frac{\Sigma x_A^{\,2}}{9}\quad 3.6^2 = 1.925^2$	M1	Attempt to find $\Sigma x_A^{\,2}$ using correct variance formula
	$\Sigma x_A^{\,2} = 150$	A1	Correct $\Sigma x_A^{\,2}$
	$\frac{150.0+352}{24}\quad 4.017^2 = 4.780$	M1	Using 352 + their 150 in correct variance formula
	sd = 2.19	A1 **[4]**	Correct answer
2 **(i)**	$4\times 3\times 7$ $= 84$	B1 **[1]**	Correct answer
(ii)	$10! - 9!\times 2$ $= 2903040$ (2900000)	B1 B1 **[2]**	$10! - k\times 9!$ seen oe Correct answer
	OR $8!\times 9\times 8$ $= 2903040$ (2900000)	B1 B1	$8!\times 9\times l$ seen oe Correct answer
(iii)	${}^9C_1 + {}^9C_2 + \ldots + {}^9C_9$	M1 M1	Using combinations Adding 9 combinations
	$= 511$	A1 **[3]**	Correct answer
	OR $2^9 - 1$	M1 M1	2^9 seen Subtracting 1
	$= 511$	A1	Correct answer
3 **(i)**	$\text{median}_A < 35$ or $20 \le \text{median}_A < 35$ or $\text{median}_A = 33.0/33.1/33.5/33.6$ or $\text{median}_B \ge 50$ or $50 \le \text{median}_B < 70$ or $\text{median}_B = 51.7/51.9/52.2/52.4$ $\text{median}_B > \text{median}_A$	B1 B1 **[2]**	Correct numerical statement re median_A or median_B Correct numerical statement re other median and a conclusion
	OR A has 66 cand 50 < mark < 100, so $\text{med}_A < 50$ or *B* has 156 cand 50 < mark < 100, so $\text{med}_B > 50$ $\text{median}_B > \text{median}_A$	B1 B1	As before As before
(ii)	$159 - 68 = 91$	B1 **[1]**	Correct final answer
(iii)	$\text{mean} = \begin{pmatrix} 4.5\times 25 + 14.5\times 43 + 27\times 91 \\ +\ldots.. + 84.5\times 40 \end{pmatrix} / 300$	M1 M1 M1	Using an attempt at mid-points, not end points or class widths Using an attempt at frequencies, not cum freqs Sum of 6 prods, correct freqs, divided by 300
	$= 11270 / 300 = 37.6$	A1 **[4]**	Correct answer

4 **(i)** **(a)**	P(final score is 12) = P(6, 6) = 1/36	B1 **[1]**	Correct answer
(b)	P[(1,5) + (1,4) + (2,3) + (3,2) + (4,1)]	M1	Considering P(1, 5)
		M1	Considering P[(1,4) + (2,3) + (3,2) + (4,1)]
	= 5/36	A1 **[3]**	Correct answer
(ii)	P(A) = 1/6 P(B) = P[(1,5) + (2,4) + (3,3) + (4, 2) + (5,1)] = 5/36	B1	Any two of P(A), P(B) and P(C) correct
	P(C) = 1 – P(O, O) = 3/4	B1	Third probability correct
	P(A and B) = P(1 and 5) = 1/36 ≠ P(A) × P(B) P(A and C) = P[(2,5) + (4,5) + (6,5)] = 3/36 ≠ P(A) × P(C) P(B and C) = P[(2,4) + (4,2)] = 2/36 ≠ P(B) × P(C)	M1	Numerical attempt to compare P(X and Y) with P(X) × P(Y), must be three positive probs
	None are independent.	A1√	One correct comparison and conclusion, ft their probabilities
		A1 **[5]**	Correct conclusion(s) following legitimate working
5 **(i)**	$z = \pm 1.751$	B1	Correct z
	$\pm \frac{20 \quad \mu}{\mu/4} \quad 1.751$	M1	Standardising no cc, no sqrt, must be a z-value
	$\mu = 13.9$	A1 **[3]**	Correct answer
(ii)	$P(X < 10) = P(z < \pm \frac{10 \quad 13.91}{13.91/4})$	M1	Standardising attempt with 10, their μ and their $\mu/4$, no cc, no sqrt
	= P(z < −1.124) = 1− 0.8694 = 0.131	M1	"$\Phi_1 + \Phi_2 - 1$", ft their mean
	P(10 < X < 20) = 0.96 – 0.131 = 0.829 or 0.830	A1 **[3]**	Correct answer
(iii)	$\mu = 250 \times 0.96 = 240$ $\sigma^2 = 250 \times 0.96 \times 0.04 = 9.6$	B1	240 and 9.6 or sq rt 9.6 seen unsimplified
	$P(\geq 235) = 1 - \Phi\left(\pm \frac{234.5 \quad 240}{\sqrt{9.6}}\right)$	M1	Standardising, with or without cc, must have sq rt in denom
		M1	Continuity correction 234.5 or 235.5 only
	= Φ (1.775)	M1	Correct region > 0.5, ft their mean
	= 0.962	A1 **[5]**	Correct answer

6 **(i)**	$(0.75)^n < 0.06$	M1*	Equation or inequality with 0.75^n and 0.06 or 0.94 seen
	$n > 9.78$	M1dep*	Attempt at solving by trial and error (can be implied) or using logarithms correctly
	$n = 10$	A1 **[3]**	Correct answer
(ii)	$E(X) = 14 \times 0.75$ or 10.5 Try $P(10) = {}^{14}C_{10}(0.75)^{10}(0.25)^4 = 0.220$	M1	Evaluating binomial probability for an integer value directly above or below their mean
	$P(11) = {}^{14}C_{11}(0.75)^{11}(0.25)^3 = 0.240$	M1	Evaluating the other binomial probability
	(mode is) 11	A1 **[3]**	Correct answer
	OR	M1	Evaluating binomial $P(n)$ and $P(n + 1)$
		M1	Evaluating binomial P(10), P(11) and P(12)
		A1	Correct answer
(iii)	$P(> 11)$ $= {}^{14}C_{12}(0.75)^{12}(0.25)^2 + {}^{14}C_{13}(0.75)^{13}(0.25)^1 + (0.75)^{14}$	M1	A binomial term of the form ${}^{14}C_n p^n(1-p)^{14-n}$ seen, $n \neq 0$ or 14
		M1	Summing binomial P(12, 13, 14) or P(11, 12, 13, 14,)
	$= 0.281$	A1	Correct answer 0.280 – 0.282
	$P(3) = {}^5C_3\,(0.2811)^3(0.7189)^2$	M1	A binomial term of the form ${}^5C_3 p^3(1-p)^2$ seen, any p
	$= 0.115$	A1 **[5]**	Correct answer

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

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9709/71 Paper 7, maximum raw mark 50

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1	Poisson $\lambda = 1.2$ $1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$ $= 0.121$	B1 B1 M1 A1 [4]	 1.2 seen 1 – Poisson P(0, 1, 2, 3) attempted, any λ, allow 1 end error SC: using Bin, ans 0.120: B1
2	**(a)** $41.2 \pm z \times \sqrt{\frac{32.6}{50}}$ $z = 1.96$ [39.6, 42.8] (3 sfs)	M1 B1 A1 [3]	 Allow any brackets or none, or < or "to" etc
	(b) $2 \times \frac{1}{16}$ or $\frac{1}{8}$ or 0.125 or 12.5% $\alpha = 87.5\%$	M1 A1 [2]	or 0.875
3	**(i)** $\frac{85.7-85}{\frac{4.8}{\sqrt{n}}}$ (= 1.786) $n = (\frac{1.786 \times 4.8}{0.7})^2$ $= 150$	M1 A1 A1 [3]	 Correct equation in n
	(ii) $H_0: \mu = 85.0$ $H_1: \mu > 85.0$ $z = 1.645$ Evidence that μ increased	B1 M1 A1f [3]	 Comparison 1.786 and 1.645 Allow 1.96 if $H_1: \mu \neq 85.0$ Correct conc. No contradictions. ft H_1
4	**(a)** g: Area ≠ 1 or > 1 h: pdf cannot be neg	B1 B1 [2]	
	(b) **(i)** $\int_{10}^{15} \frac{30}{x} dx$ $= [30 \ln x]_{10}^{15}$ $= 30(\ln 15 - \ln 10)$ $(= 30 \ln 1.5$ **AG**$)$	M1 A1 A1 [3]	Attempt integ $xf(x)$, ignore limits Correct integrand and limits or $30\ln(^{15}/_{10})$
	(ii) $\int_{10}^{m} \frac{30}{x^2} dx = 0.5$ $[30x^{-1}]_{10}^{m} = 0.5$ $\frac{30}{m} - (\frac{30}{10}) = 0.5$ $m = 12$ $\int_{'12'}^{30\ln 1.5} \frac{30}{x^2} dx$ $= 0.0337$ (3 sfs)	M1 A1 A1 M1 A1 [5]	Integ f(x) = 0.5, limits 10 to unknown Correct integrand, limits and = 0.5

5	**(i)**	$W \sim N(2240, 848)$ $\frac{2200-2240}{\sqrt{848}}$ (= –1.374) Φ("–1.374") = 1– Φ("1.374") (= 0.0847) $\frac{2300-2240}{\sqrt{848}}$ (= 2.060) Φ("2.060") (= 0.9803) Φ("2.060") – (1 – Φ("1.374")) = 0.896 (3 sfs)	B2 M1A1 M1 A1 [6]	B1 each parameter Standardise either value and evaluate correctly Correct combination of Φ's
	(ii)	$X_1 - X_2 \sim N(0, 392)$ $\frac{20-0}{\sqrt{392}}$ (= 1.010) (Φ("1.010" = 0.8438) P(X > 20) = 1 – Φ("1.010") (= 0.1562) 2 × P(X > 20) = 0.312 (3 sfs)	B1 M1 A1 M1 A1 [5]	May be implied
6	**(i)**	mean = 6.3 P($X \leq 1$) = e $^{6.3}$(1 + 6.3) = 0.0134 P($X \leq 2$) = e $^{6.3}$(1 + 6.3 + $\frac{6.3^2}{2}$) = 0.0498 CR is $X \leq 1$	B1 M1 M1A1 A1 [5]	B1 for 6.3 Allow incorrect λ in both probs A1 for both values
	(ii)	P(Type I error) = P($X \leq 1$) = 0.0134	B1 [1]	
	(iii)	H_0: $\lambda = 6.3$ H_1: $\lambda < 6.3$ 3 not in CR No evidence mean no. of injuries has decreased	B1 M1 A1 [3]	Can be scored in (i). Accept λ = 2.1(per month) or P($X \leq 3$) = 0.126 > 0.02 Correct conclusion
	(iv)	N(25.2, 25.2) $\frac{19.5 \quad 25.2}{\sqrt{25.2}}$ (= –1.135) Φ("–1.135") = 1 – Φ("1.135") = 0.128 (3 sfs)	B2 M1 M1 A1 [5]	B1 for N & μ = 25.2. B1 for σ^2 = 25.2 May be implied Allow with wrong or no cc or no √ Correct area

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for the guidance of teachers

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1	E(T) = 9.6 Var(wt of one bag) = 0.0016 Var(T) = 3 × 0.0016 sd of T = √(3 × 0.0016) = 0.0693	B1 M1 M1 A1 [4]	May be impl. by Var(T) = 0.0048 or 0.0144
		[Total: 4]	
2	$\bar{X}$ ~N(3, $\frac{9}{4}/60$) $\frac{2.8 \quad 3}{\sqrt{\frac{9}{4}/60}}$ (= –1.033) Φ("–1.033") = 1 – Φ("1.033") = 0.151	B2 M1 M1 A1 [5]	B1 for N & μ = 3; (oe) B1 for $^{9/4}/_{60}$ or $^{3}/_{80}$ or 0.0375 (oe) (oe working with totals or proportions) With or without c.c. With cc of $^{1}/_{120}$, Φ(1.076) 1 Φ(1.076) 0.141
		[Total: 5]	
3 (i)	Constant average rate of goals scored Goals random Goals indep	B1 B1 [2]	Any two given in context (SR score B1 for any two not in context) Not Goals scored singly (because this is inherent in the context so it's not a condition)
(ii)	$e^{1.8}(\frac{1.8^3}{3!}+\frac{1.8^4}{4!}+\frac{1.8^5}{5!})$ = 0.259	M1 A1 [2]	Poisson probs, λ = 1.8. Allow 2, 6 included
(iii)	$1 - e^{1.8}$ $(1 - e^{1.8})^{10}$ = 0.164	M1 M1 A1 [3]	Any λ. Allow end errors.
		[Total: 7]	
4 (i)	$\bar{x}$ = 8.4 $8.4 \pm z\frac{1.3}{\sqrt{15}}$ z = 2.576 [7.54, 9.26]	B1 M1 B1 A1 [4]	 Accept 2.574 to 2.579 or equiv. Accept 7.53. Accept 9.27
(ii)	No because pop normal so $\bar{X}$ normally distr	B1 B1 [2]	SR If 'Yes' or no conclusion, but 2 correct statements score B1
(iii)	8 within CI Claim justified	B1√ B1√ [2]	ft **(i)**
		[Total: 8]	

5	**(i)**	Po(3.3) $e^{-3.3}(1 + 3.3 + \frac{3.3^2}{2})$ = 0.359	B1 M1 A1 [3]	seen or implied Poisson P(0) + P(1) + P(2). Allow + P(3) Allow wrong λ. Accept equiv method.
	(ii)	X~Po(36) X~N(36, 36) $\frac{48.5 - 36}{\sqrt{36}}$ = 2.08(3) comp with 1.96 Evidence to support claim	B1 B1 M1 A1 M1 A1√ [6]	 Allow with no or wrong cc or no √ 2.08(3) or 0.0186/0.0187 if area comparison Valid comparison Correct conclusion (ft their z)
			[Total: 9]	
6	**(i)**	H_0: P(6) = $\frac{1}{6}$ H_1: P(6) > $\frac{1}{6}$	B1 [1]	Condone undefined p
	(ii)	$\left(\frac{5}{6}\right)^{10} + 10 \times \left(\frac{5}{6}\right)^{9} \times \frac{1}{6} + \binom{10}{2} \times \left(\frac{5}{6}\right)^{8} \times \frac{1}{6}^{2} + \binom{10}{3} \times \left(\frac{5}{6}\right)^{7} \times \left(\frac{1}{6}\right)^{3}$ $1 - (\left(\frac{5}{6}\right)^{10} + 10 \times \left(\frac{5}{6}\right)^{9} \times \frac{1}{6} + \binom{10}{2} \times \left(\frac{5}{6}\right)^{8} \times \left(\frac{1}{6}\right)^{2}$ $+ \binom{10}{3} \times \left(\frac{5}{6}\right)^{7} \times \left(\frac{1}{6}\right)^{3})$ = 0.0697 (3 sfs)	M1 M1 A1 [3]	(1 –) P(0,1,2,3) o.e. using B(10,1/6) allow end errors Attempt at fully correct expression for 1 – P(0,1,2,3) o.e. Accept 0.0698
	(iii)	Die biased towards a six but result < 4 so no evidence of bias	B1 [1]	or equiv. Must be in context
	(iv)	P(0, 1, 2 or 3 sixes) $(\left(\frac{1}{2}\right)^{10} + 10 \times \left(\frac{1}{2}\right)^{9} \times \frac{1}{2} + \binom{10}{2} \times \left(\frac{1}{2}\right)^{8} \times \left(\frac{1}{2}\right)^{2} + \binom{10}{3} \times \left(\frac{1}{2}\right)^{7} \times \left(\frac{1}{2}\right)^{3})$ = 0.172 or 11/64	B1 M1 A1 [3]	Stated or attempted. Can be implied Attempt at P(0,1,2,3) with p = 1/2, allow end errors.
			[Total: 8]	

7 (i)	$\int_1^1 k(1 \quad x)\mathrm{d}x = 1$ $(k[x \quad \frac{x^2}{2}]^1{}_1 = 1)$ $2k = 1$ $(k = \frac{1}{2}$ **AG)**	M1 A1 [2]	Attempt integ f(x) = 1 with correct limits
(ii)	$(\int_{0.5}^1 \frac{1}{2}(1 \quad x)\mathrm{d}x = \frac{1}{2}[x \quad \frac{x^2}{2}]_{0.5}^1)$ $= \frac{1}{16}$ or 0.0625	 B1 [1]	
(iii)	$\int_1^1 \frac{1}{2}(x \quad x^2)\mathrm{d}x$ $= \frac{1}{2}[\frac{x^2}{2} - \frac{x^3}{3}]^1{}_1$ $= -\frac{1}{3}$ or –0.333	M1 A1 A1 [3]	$\int xf(x)\mathrm{d}x$ ignore limits Correct integrand and limits
(iv)	$\int_1^a \frac{1}{2}(1 \quad x)\mathrm{d}x = 0.25$ $(\frac{1}{2}[x \quad \frac{x^2}{2}]^a{}_1 = 0.25)$ $(\frac{1}{2}(a - \frac{a^2}{2} - (-1 - \frac{1}{2}) = 0.25)$ $a^2 - 2a - 2 = 0$ $a = 1 - \sqrt{3}$ or –0.732	M1 A1 A1 [3]	Correct limits (or integral from a to 1 = 0.75) any correct QE with "= 0"(or in completed square form $(a-1)^2 = 3$) Not $a = 1 \pm \sqrt{3}$; Not –0.732 or 2.732
	[Total: 9]		

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1		$(0.7 + 1.0) \times 2$ $= 3.4$ $e^{3.4}(1 + 3.4 + 3.4^2 \div 2)$ $= 0.34(0)$ Alternative Method By Combinations	M1 A1 M1 A1 M2 A1 A1 [4]	Attempt combined mean Poisson P(0, 1, 2), any λ (Allow one end error) At least 4 correct $\lambda = 1.4$, $\lambda = 2$ All 6 correct combinations Correct answer
		[Total: 4]		
2	**(i)**	$\frac{\frac{18}{70}\times(1-\frac{18}{70})}{70}$ (= 0.00272886..) $z = 1.645$ $\frac{18}{70} \pm z \times \sqrt{"0.00272886"}$ 0.171 to 0.343	M1 B1 M1 A1 [4]	 Seen
	(ii)	Var (or sd) estimated or N ~ B used	B1 [1]	
		[Total: 5]		
3	**(i)**	$0.85^{30} + 30 \times 0.85^{29} \times 0.15 + {}^{30}C_2 \times 0.85^{28} \times 0.15^2$ $= 0.151$ > 0.04 No evidence decrease or Accept no decrease	M1 A1 M1 A1√ [4]	Allow just $0.85^{30} + 30 \times 0.85^{29} \times 0.15$ (Or critical region $X = 0$, or $X = 2$ Not in CR) Comp with 0.04 (can be implied by diagram) Correct Conclusion (ft) Use of P($X = 2$) only: max M0A0M1A1
	(ii)	**(a)** Not rejected Ho **(b)** Has been decrease or π (or p) < 0.15	B1 [1] B1 [1]	Both independent marks Must be in context
		[Total: 6]		
4	**(i)**	Po(4) $1 - e^{4}(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!})$ $= 1 - 0.43347..$ $= 0.567$ or 0.566	M1 A1 M1 A1 [4]	Use of Poisson, any mean Correct mean Allow one end error SC1: $\frac{3.5\ 4}{\sqrt{3.9984}}$ B1 SC2: Correct Bin method M1 ans 0.567 or 0.566 A1
	(ii)	$\lambda = {}^{n}/_{2500}$ $e^{\frac{n}{2500}} < 0.01$ $\frac{n}{2500} < \ln 0.01$ $n > 11512.9\ldots$ Smallest $n = 11513$	$(\frac{2499}{2500})^n$ $(\frac{2499}{2500})^n < 0.01$ $n \times \ln(\frac{2499}{2500}) < \ln 0.01$ $n > 11510.6\ldots$ Smallest $n = 11511$	B1 M1 A1 [3]	 Correct exp'n < 0.01. Allow ' =' Allow by trial
		[Total: 7]		

5	**(i)**	$E(T) = 234$, $Var(T) = 15^2 + 8^2 = 289$ $\frac{200 - 234}{\sqrt{"289"}}$ (= –2.000) Φ("–2.000") = 1 – Φ ("2.000") 1 – 0.9772 2.28%	B1 M1 M1 A1 [4]	
	(ii)	Require $P(D > 0)$ where $D = X - 4Y$ $E(D)$ (= 184 – 4 × 50) = –16 $Var(D)$ (= $15^2 + 4^2 \times 8^2$) = 1249 $\frac{0 - (-16)}{\sqrt{"1249"}}$ (= 0.453) 1 – Φ ("0.453") (= 1 – 0.6747) = 0.325	 B1 B1 M1 M1 A1 [5]	 For –16 or +16 or ± (184 – 4 × 50) For 1249 or $15^2 + 4^2 \times 8^2$
		[Total: 9]		
6	**(i)**	$k\int_2^3 (x^2 \quad 5x + 6)dx = 1$ $(-k(\frac{3^3}{3} \quad 5 \times \frac{3^2}{2} + 6 \times 3 \quad [\frac{2^3}{3} \quad 5 \times \frac{2^2}{2} + 6 \times 2]) = 1)$ $-k \times (-\frac{1}{6}) = 1$ or $k \times \frac{1}{6} = 1$ ($k = 6$ **AG**)	M1 A1 [2]	Integ = 1; ignore limits Correctly obtain $-\frac{1}{6}$ or $\frac{1}{6}$ CWO No rounded decimals	$6\int_2^3 (x^2 \quad 5x + 6)dx$ ignore limits Correctly obtain 1
	(ii)	$E(X) = 2.5$ $6\int_2^3 (x^4 \quad 5x^3 + 6x^2)dx$ (= –6 × (–1.05)) – "2.5"^{2} = 0.05	B1 M1* Dep M1* A1 [4]	Condone 25000 Integ $x^2 f(x)$; ignore limits Subtr μ^2, ISW
	(iii)	$6\int_2^{2.2} (x^2 \quad 5x + 6)dx$ (= 0.104) $1 - (1 - "0.104")^4$ = 0.355/0.356	M1 M1 A1 [3]	Integ with limits 2, 2.2 or 2.2, 3 Or equivalent
		[Total: 9]		

7	**(i)**	Var($\overline{X}$) = $\frac{121}{200}$ or SD of $\overline{X}$ = $\frac{11}{\sqrt{200}}$	
		$(\pm)\frac{354-352}{\frac{11}{\sqrt{200}}}$ (= ± 2.571)	M1 Or with cc attempted. Allow no √ Must include 200 or √ 200
			A1 2.57(1) or correct expression
		1 – Φ("2.571") (= 1 – 0.9949) = 0.0051	M1 A1 [4]
	(ii)	(No)	"No" must be seen or implied, but gains no marks by itself
		n is large, $\overline{X}$ (appr) norm distr or CLT applies	B1 $n \geq 30$ B1 [2] (SR Both statements correct, but wrong or no conclusion scores B1)
	(iii)	H_0: Pop mean = 352 H_1: Pop mean ≠ 352	B1 Allow 'μ' but not just 'mean'
		$\pm\frac{356-352}{\frac{11}{\sqrt{50}}}$ ± (= 2.57(1))	M1 Must have √ 50 A1 Correct statement or 2.57(1)
		Comp with z = ±1.96 (signs consistent) Evidence that pop mean has changed	B1√ [4] Correct comparison, and correct conclusion, follow through one tail test
		[Total: 10]	