UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

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[Turn over

1 Given that $x = 4\left(3^{-y}\right)$, express y in terms of x. [3]

2 Solve the inequality $2x > |x - 1|$. [4]

3 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 1 - \cos 2\theta.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan\theta$. [5]

4 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

  (ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

5 In a certain industrial process, a substance is being produced in a container. The mass of the substance in the container t minutes after the start of the process is x grams. At any time, the rate of formation of the substance is proportional to its mass. Also, throughout the process, the substance is removed from the container at a constant rate of 25 grams per minute. When $t = 0$, $x = 1000$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 75$.

  (i) Show that x and t satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.1(x - 250).$$ [2]

  (ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + \mathrm{e}^x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

  (ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

  (iii) Show that this root also satisfies the equation

$$x = \tan^{-1}\left(\frac{2}{1 + \mathrm{e}^x}\right).$$ [1]

  (iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + \mathrm{e}^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The complex number $2 + i$ is denoted by u. Its complex conjugate is denoted by u^*.
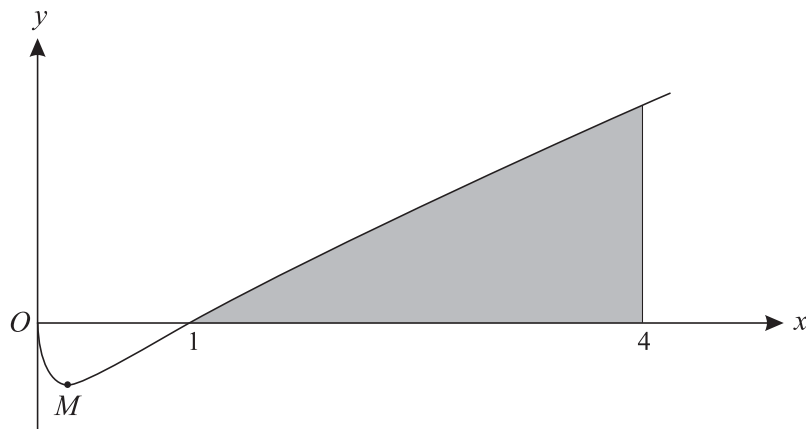
   (i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

   (ii) Express $\dfrac{u}{u^*}$ in the form $x + iy$, where x and y are real. [3]

   (iii) By considering the argument of $\dfrac{u}{u^*}$, or otherwise, prove that

   $$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

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The diagram shows a sketch of the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point $(1, 0)$.

   (i) Find the exact value of the x-coordinate of M. [4]

   (ii) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 4$. Give your answer correct to 2 decimal places. [5]

9 **(i)** Express $\dfrac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

   (ii) Hence, given that $|x| < 1$, obtain the expansion of $\dfrac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

[Turn over

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

   (i) State a vector equation for the line l. [1]

   (ii) Find the position vector of N and show that $BN = 3$. [6]

   (iii) Find the equation of the plane containing A, B and N, giving your answer in the form $ax + by + cz = d$. [5]