Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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1 Solve the equation $\log_{10}(x+9) = 2 + \log_{10} x$. [3]

2 Expand $(1+3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 **(i)** Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0. \qquad [3]$$

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

4 The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

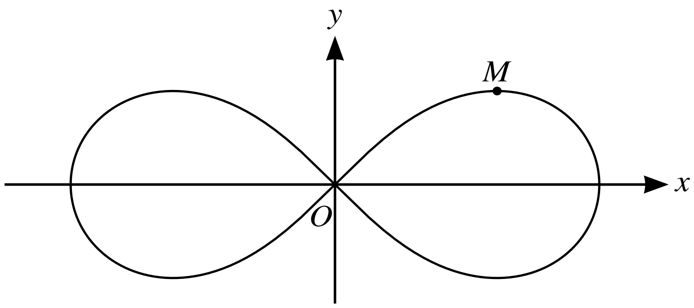
(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x+1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

6



The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M. Find the coordinates of M. [7]

7 **(a)** The complex number $\dfrac{3 - 5i}{1 + 4i}$ is denoted by u. Showing your working, express u in the form $x + iy$, where x and y are real. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 - i| \leqslant 1$ and $|z - i| \leqslant |z - 2|$. [4]

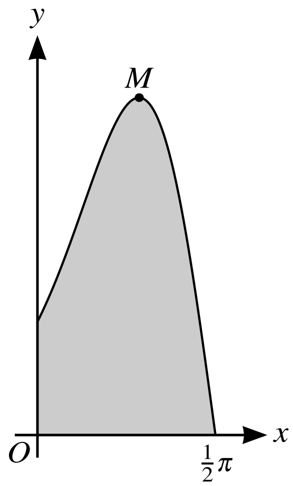
(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

8 Let $f(x) = \dfrac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^{1} f(x)\, dx = 3 \ln 3$. [5]

9



The diagram shows the curve $y = e^{2\sin x} \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]