UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

2554075339

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC13 11_9709_71/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Each computer made in a factory contains 1000 components. On average, 1 in 30 000 of these components is defective. Use a suitable approximate distribution to find the probability that a randomly chosen computer contains at least 1 faulty component. [4]

2 Heights of a certain species of animal are known to be normally distributed with standard deviation 0.17 m. A conservationist wishes to obtain a 99% confidence interval for the population mean, with total width less than 0.2 m. Find the smallest sample size required. [4]

3 Following a change in flight schedules, an airline pilot wished to test whether the mean distance that he flies in a week has changed. He noted the distances, x km, that he flew in 50 randomly chosen weeks and summarised the results as follows.

$$n = 50 \qquad \Sigma x = 143\,300 \qquad \Sigma x^2 = 410\,900\,000$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) In the past, the mean distance that he flew in a week was 2850 km. Test, at the 5% significance level, whether the mean distance has changed. [5]

4 The number of radioactive particles emitted per 150-minute period by some material has a Poisson distribution with mean 0.7.

(i) Find the probability that at most 2 particles will be emitted during a randomly chosen 10-hour period. [3]

(ii) Find, in minutes, the longest time period for which the probability that no particles are emitted is at least 0.99. [5]

5 The volume, in cm^3, of liquid left in a glass by people when they have finished drinking all they want is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{8}$. [2]

(ii) 20% of people leave at least d cm^3 of liquid in a glass. Find d. [3]

(iii) Find $\mathrm{E}(X)$. [3]

6 At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]

7 Kieran and Andreas are long-jumpers. They model the lengths, in metres, that they jump by the independent random variables $K \sim \text{N}(5.64,\ 0.0576)$ and $A \sim \text{N}(4.97,\ 0.0441)$ respectively. They each make a jump and measure the length. Find the probability that

(i) the sum of the lengths of their jumps is less than 11 m, [4]

(ii) Kieran jumps more than 1.2 times as far as Andreas. [6]

BLANK PAGE

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6370714324

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC13 11_9709_72/FP

UNIVERSITY of CAMBRIDGE
International Examinations

1 Each computer made in a factory contains 1000 components. On average, 1 in 30 000 of these components is defective. Use a suitable approximate distribution to find the probability that a randomly chosen computer contains at least 1 faulty component. [4]

2 Heights of a certain species of animal are known to be normally distributed with standard deviation 0.17 m. A conservationist wishes to obtain a 99% confidence interval for the population mean, with total width less than 0.2 m. Find the smallest sample size required. [4]

3 Following a change in flight schedules, an airline pilot wished to test whether the mean distance that he flies in a week has changed. He noted the distances, x km, that he flew in 50 randomly chosen weeks and summarised the results as follows.

$$n = 50 \qquad \Sigma x = 143\,300 \qquad \Sigma x^2 = 410\,900\,000$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) In the past, the mean distance that he flew in a week was 2850 km. Test, at the 5% significance level, whether the mean distance has changed. [5]

4 The number of radioactive particles emitted per 150-minute period by some material has a Poisson distribution with mean 0.7.

(i) Find the probability that at most 2 particles will be emitted during a randomly chosen 10-hour period. [3]

(ii) Find, in minutes, the longest time period for which the probability that no particles are emitted is at least 0.99. [5]

5 The volume, in cm^3, of liquid left in a glass by people when they have finished drinking all they want is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{8}$. [2]

(ii) 20% of people leave at least d cm^3 of liquid in a glass. Find d. [3]

(iii) Find E(X). [3]

6 At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]

7 Kieran and Andreas are long-jumpers. They model the lengths, in metres, that they jump by the independent random variables $K \sim \mathrm{N}(5.64, 0.0576)$ and $A \sim \mathrm{N}(4.97, 0.0441)$ respectively. They each make a jump and measure the length. Find the probability that

(i) the sum of the lengths of their jumps is less than 11 m, [4]

(ii) Kieran jumps more than 1.2 times as far as Andreas. [6]

BLANK PAGE

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5959837373

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC13 11_9709_73/RP

UNIVERSITY of CAMBRIDGE
International Examinations

1 A random sample of 80 values of a variable X is taken and these values are summarised below.

$$n = 80 \qquad \Sigma x = 150.2 \qquad \Sigma x^2 = 820.24$$

Calculate unbiased estimates of the population mean and variance of X and hence find a 95% confidence interval for the population mean of X. [6]

2 A traffic officer notes the speeds of vehicles as they pass a certain point. In the past the mean of these speeds has been $62.3\,\text{km}\,\text{h}^{-1}$ and the standard deviation has been $10.4\,\text{km}\,\text{h}^{-1}$. A speed limit is introduced, and following this, the mean of the speeds of 75 randomly chosen vehicles passing the point is found to be $59.9\,\text{km}\,\text{h}^{-1}$.

(i) Making an assumption that should be stated, test at the 2% significance level whether the mean speed has decreased since the introduction of the speed limit. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

3 The waiting time, T weeks, for a particular operation at a hospital has probability density function given by

$$\mathrm{f}(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Given that $\mathrm{E}(T) = \frac{16}{3}$, find $\mathrm{Var}(T)$. [3]

(ii) 10% of patients have to wait more than n weeks for their operation. Find the value of n, giving your answer correct to the nearest integer. [5]

4 Goals scored by Femchester United occur at random with a constant average of 1.2 goals per match. Goals scored against Femchester United occur independently and at random with a constant average of 0.9 goals per match.

(i) Find the probability that in a randomly chosen match involving Femchester,

(a) a total of 3 goals are scored, [2]

(b) a total of 3 goals are scored and Femchester wins. [3]

The manager promises the Femchester players a bonus if they score at least 35 goals in the next 25 matches.

(ii) Find the probability that the players receive the bonus. [4]

5 A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by X.

(i) Write down the value of E(X) and show that Var(X) = $\frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

(ii) Find the probability of a Type I error. [5]

(iii) With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

6 The lifetimes, in hours, of Longlive light bulbs and Enerlow light bulbs have the independent distributions N(1020, 45^2) and N(2800, 52^2) respectively.

(i) Find the probability that the total of the lifetimes of 5 randomly chosen Longlive bulbs is less than 5200 hours. [4]

(ii) Find the probability that the lifetime of a randomly chosen Enerlow bulb is at least 3 times that of a randomly chosen Longlive bulb. [6]

BLANK PAGE