UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0212815821

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

JC11 06_9709_11/2R

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International Examinations

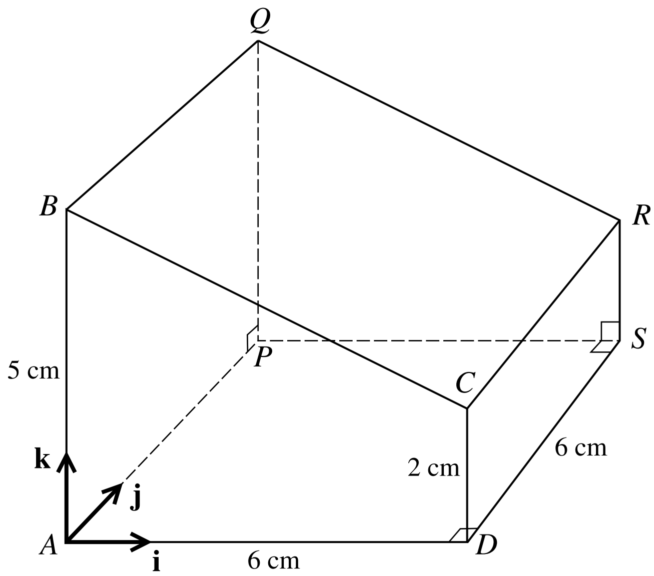
1 Find the coefficient of x in the expansion of $\left(x + \dfrac{2}{x^2}\right)^7$. [3]

2 The volume of a spherical balloon is increasing at a constant rate of $50\,\text{cm}^3$ per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere $= \frac{4}{3}\pi r^3$.] [4]

3 **(i)** Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through $360°$ about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

4



The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to AD, AP and AB respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to calculate angle PCQ. [4]

5 **(i)** Show that the equation $2\tan^2\theta\sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

6 The variables x, y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \frac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

7 A curve is such that $\frac{dy}{dx} = \frac{3}{(1 + 2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

8 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

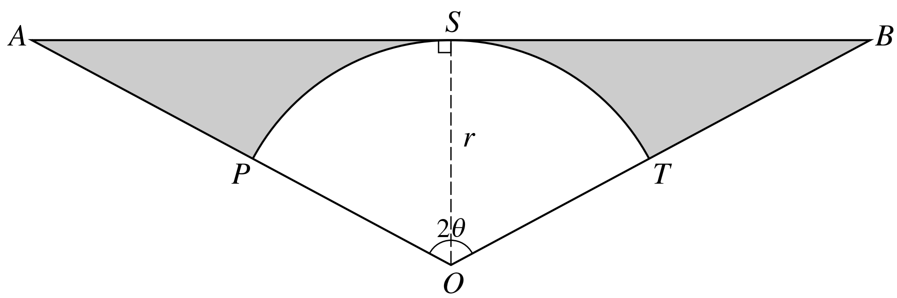
Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

9



In the diagram, OAB is an isosceles triangle with $OA = OB$ and angle $AOB = 2\theta$ radians. Arc PST has centre O and radius r, and the line ASB is a tangent to the arc PST at S.

(i) Find the total area of the shaded regions in terms of r and θ. [4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and $r = 6$, find the total perimeter of the shaded regions, leaving your answer in terms of $\sqrt{3}$ and π. [5]

[Questions 10 and 11 are printed on the next page.]

10 **(i)** Express $2x^2 - 4x + 1$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

11 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 1,$$
$$\mathrm{g} : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [2]

(ii) Hence find the value of a for which $\mathrm{fg}(a) = \mathrm{gf}(a)$. [3]

(iii) Find the value of b $(b \neq a)$ for which $\mathrm{g}(b) = b$. [2]

(iv) Find and simplify an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$. [2]

The function h is defined by

$$\mathrm{h} : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $\mathrm{h}^{-1}(x)$. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_12/2R

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1 Find $\displaystyle\int \left(x^3 + \frac{1}{x^3}\right) \mathrm{d}x$. [3]

2 **(i)** Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]

(ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k. [2]

3 The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

4 A curve has equation $y = \dfrac{4}{3x - 4}$ and $P\,(2, 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

5 **(i)** Prove the identity $\dfrac{\cos\theta}{\tan\theta(1 - \sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1 - \sin\theta)} = 4$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

6 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{x + 3}{2x - 1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(i) Show that $\mathrm{ff}(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $\mathrm{f}^{-1}(x)$. [2]

7 The line L_1 passes through the points $A\,(2, 5)$ and $B\,(10, 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]

8 Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find angle ABC. [6]

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D. [2]

9 The function f is such that $\mathrm{f}(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $\mathrm{f}(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = \mathrm{f}(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

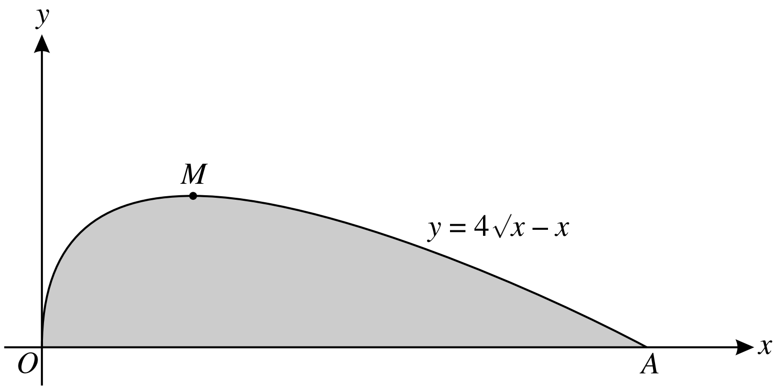
10 **(a)** A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k, [3]

(ii) the sum to infinity of the progression. [2]

11



The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x-axis at O and A.

(i) Find the coordinates of A and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [6]

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Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_13/2R

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1 The coefficient of x^3 in the expansion of $(a + x)^5 + (1 - 2x)^6$, where a is positive, is 90. Find the value of a. [5]

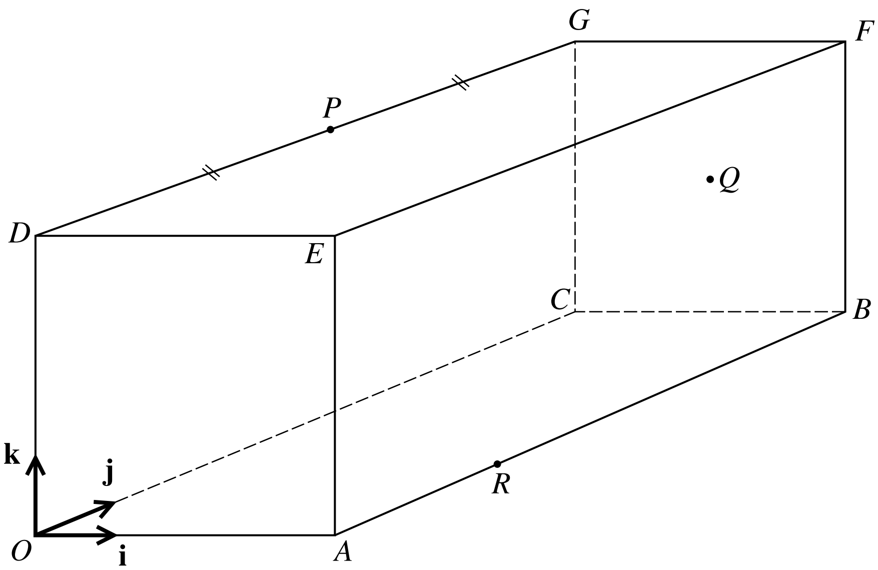
2 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

3 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

4 **(a)** Differentiate $\frac{2x^3 + 5}{x}$ with respect to x. [3]

(b) Find $\int (3x - 2)^5 \, dx$ and hence find the value of $\int_0^1 (3x - 2)^5 \, dx$. [4]

5



In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

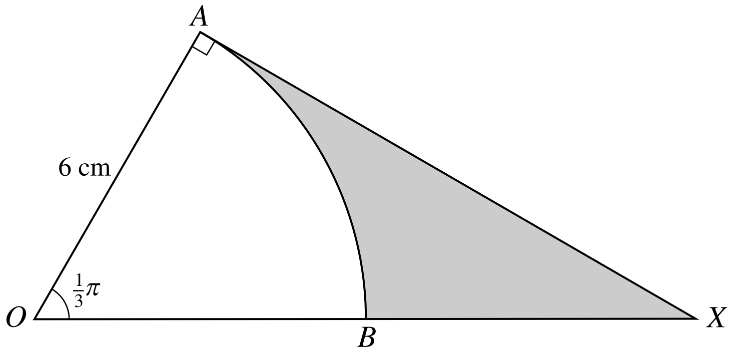
(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle RQP. [4]

6 **(a)** A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

7



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

8 **(i)** Prove the identity $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

9 A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{x}} - 1$ and $P\,(9, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\dfrac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1}k$ with the positive x-axis. Find the value of k. [2]

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate fg(2). [2]

(ii) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $\mathrm{g}'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

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General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_21/RP

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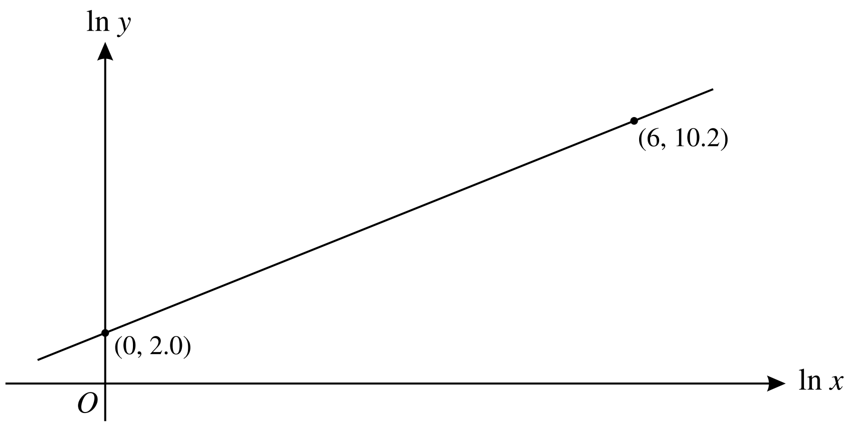
1 Solve the equation $|3x + 4| = |2x + 5|$. [3]

2 A curve has parametric equations

$$x = 3t + \sin 2t, \quad y = 4 + 2\cos 2t.$$

Find the exact gradient of the curve at the point for which $t = \frac{1}{6}\pi$. [4]

3



The variables x and y satisfy the equation $y = Kx^m$, where K and m are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0, 2.0) and (6, 10.2), as shown in the diagram. Find the values of K and m, correct to 2 decimal places. [5]

4 The polynomial $f(x)$ is defined by

$$f(x) = 3x^3 + ax^2 + ax + a,$$

where a is a constant.

(i) Given that $(x + 2)$ is a factor of $f(x)$, find the value of a. [2]

(ii) When a has the value found in part **(i)**, find the quotient when $f(x)$ is divided by $(x + 2)$. [3]

5 Find the value of $\frac{dy}{dx}$ when $x = 4$ in each of the following cases:

(i) $y = x\ln(x - 3)$, [4]

(ii) $y = \frac{x - 1}{x + 1}$. [3]

6 **(a)** Find $\int 4e^x(3 + e^{2x})\,dx$. [4]

(b) Show that $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} (3 + 2\tan^2\theta)\,d\theta = \frac{1}{2}(8 + \pi)$. [4]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 14 - x^2$$

has exactly two real roots. [3]

(ii) Show by calculation that the positive root lies between 1.2 and 1.3. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{2}\ln(14 - x^2).$$ [1]

(iv) Use an iteration process based on the equation in part **(iii)**, with a suitable starting value, to find the root correct to 2 decimal places. Give the result of each step of the process to 4 decimal places. [3]

8 **(i)** Express $4\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Solve the equation $4\sin\theta - 6\cos\theta = 3$ for $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Find the greatest and least possible values of $(4\sin\theta - 6\cos\theta)^2 + 8$ as θ varies. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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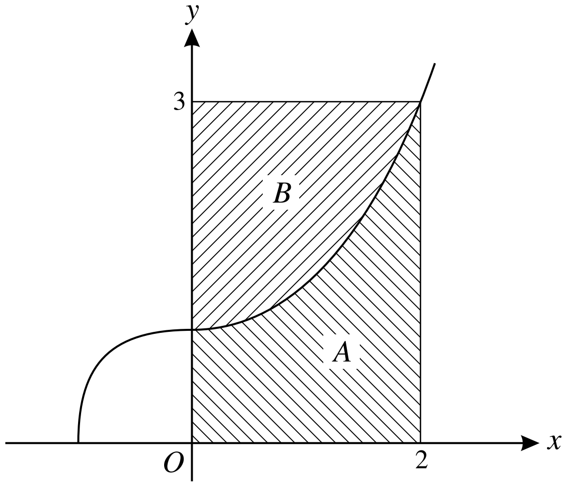
1 Use logarithms to solve the equation $3^x = 2^{x+2}$, giving your answer correct to 3 significant figures. [4]

2



The diagram shows the curve $y = \sqrt{(1 + x^3)}$. Region A is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. Region B is bounded by the curve and the lines $x = 0$ and $y = 3$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of region A. Give your answer correct to 2 decimal places. [3]

(ii) Deduce an approximation to the area of region B and explain why this approximation under-estimates the true area of region B. [2]

3 The sequence $x_1, x_2, x_3, \ldots$ defined by

$$x_1 = 1, \quad x_{n+1} = \tfrac{1}{2}\sqrt[3]{(x_n^2 + 6)}$$

converges to the value α.

(i) Find the value of α correct to 3 decimal places. Show your working, giving each calculated value of the sequence to 5 decimal places. [3]

(ii) Find, in the form $ax^3 + bx^2 + c = 0$, an equation of which α is a root. [2]

4 **(a)** Find the value of $\displaystyle\int_0^{\frac{2}{3}\pi} \sin\left(\tfrac{1}{2}x\right)\,dx$. [3]

(b) Find $\displaystyle\int e^{-x}(1 + e^x)\,dx$. [3]

5 A curve has equation $x^2 + 2y^2 + 5x + 6y = 10$. Find the equation of the tangent to the curve at the point $(2, -1)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

6 The curve $y = 4x^2 \ln x$ has one stationary point.

(i) Find the coordinates of this stationary point, giving your answers correct to 3 decimal places. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

7 The cubic polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + ax^2 + bx + 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that, when $\mathrm{p}(x)$ is divided by $(x + 1)$, the remainder is 24.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

8 **(i)** Prove that $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) \equiv 4\cos 2\theta$. [3]

(ii) Hence

(a) solve for $0^\circ \leqslant \theta \leqslant 180^\circ$ the equation $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) = 3$, [4]

(b) find the exact value of $\operatorname{cosec}^2 15^\circ - \sec^2 15^\circ$. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_23/FP

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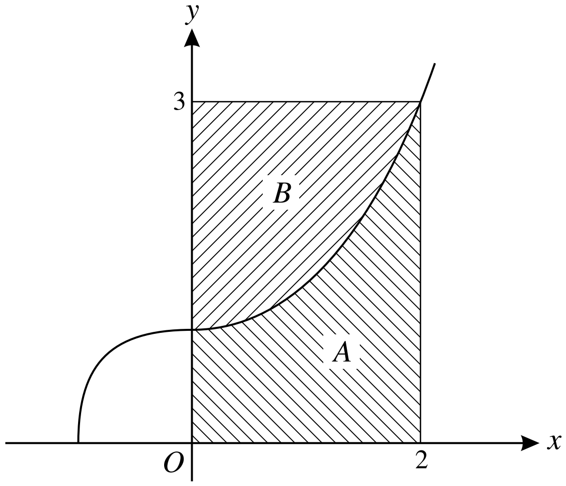
1 Use logarithms to solve the equation $3^x = 2^{x+2}$, giving your answer correct to 3 significant figures. [4]

2



The diagram shows the curve $y = \sqrt{(1 + x^3)}$. Region A is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. Region B is bounded by the curve and the lines $x = 0$ and $y = 3$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of region A. Give your answer correct to 2 decimal places. [3]

(ii) Deduce an approximation to the area of region B and explain why this approximation under-estimates the true area of region B. [2]

3 The sequence $x_1, x_2, x_3, \ldots$ defined by

$$x_1 = 1, \quad x_{n+1} = \tfrac{1}{2}\sqrt[3]{(x_n^2 + 6)}$$

converges to the value α.

(i) Find the value of α correct to 3 decimal places. Show your working, giving each calculated value of the sequence to 5 decimal places. [3]

(ii) Find, in the form $ax^3 + bx^2 + c = 0$, an equation of which α is a root. [2]

4 **(a)** Find the value of $\displaystyle\int_0^{\frac{2}{3}\pi} \sin\left(\tfrac{1}{2}x\right)\,dx$. [3]

(b) Find $\displaystyle\int e^{-x}(1 + e^x)\,dx$. [3]

5 A curve has equation $x^2 + 2y^2 + 5x + 6y = 10$. Find the equation of the tangent to the curve at the point $(2, -1)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

6 The curve $y = 4x^2 \ln x$ has one stationary point.

(i) Find the coordinates of this stationary point, giving your answers correct to 3 decimal places. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

7 The cubic polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + ax^2 + bx + 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that, when $\mathrm{p}(x)$ is divided by $(x + 1)$, the remainder is 24.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

8 **(i)** Prove that $\sin^2 2\theta(\operatorname{cosec}^2 \theta - \sec^2 \theta) \equiv 4\cos 2\theta$. [3]

(ii) Hence

(a) solve for $0° \leqslant \theta \leqslant 180°$ the equation $\sin^2 2\theta(\operatorname{cosec}^2 \theta - \sec^2 \theta) = 3$, [4]

(b) find the exact value of $\operatorname{cosec}^2 15° - \sec^2 15°$. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4001209312

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

JC11 06_9709_31/FP

UNIVERSITY of CAMBRIDGE
International Examinations

[Turn over

1 Expand $\sqrt[3]{(1-6x)}$ in ascending powers of x up to and including the term in x^3, simplifying the coefficients. [4]

2 Find $\dfrac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \dfrac{\tan x}{x}$. [2]

3 Points A and B have coordinates $(-1, 2, 5)$ and $(2, -2, 11)$ respectively. The plane p passes through B and is perpendicular to AB.

(i) Find an equation of p, giving your answer in the form $ax + by + cz = d$. [3]

(ii) Find the acute angle between p and the y-axis. [4]

4 The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + 25x^2 - 4x - 12.$$

(i) Show that $f(-2) = 0$ and factorise $f(x)$ completely. [4]

(ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures. [3]

5 The curve with equation

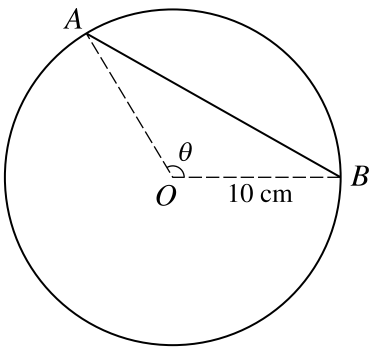
$$6e^{2x} + ke^y + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

6



The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio 1 : 4 and it is required to find the length of AB. The angle AOB is θ radians.

(i) Show that $\theta = \frac{2}{5}\pi + \sin\theta$. [3]

(ii) Showing all your working, use an iterative formula, based on the equation in part **(i)**, with an initial value of 2.1, to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

7 The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2 + 1)\,dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x - 2)\ln x\,dx$. [3]

(ii) Hence find the exact value of I. [5]

8 The complex number u is defined by $u = \frac{6 - 3\mathrm{i}}{1 + 2\mathrm{i}}$.

(i) Showing all your working, find the modulus of u and show that the argument of u is $-\frac{1}{2}\pi$. [4]

(ii) For complex numbers z satisfying $\arg(z - u) = \frac{1}{4}\pi$, find the least possible value of $|z|$. [3]

(iii) For complex numbers z satisfying $|z - (1 + \mathrm{i})u| = 1$, find the greatest possible value of $|z|$. [3]

9 **(i)** Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4\theta\,d\theta$. [3]

[Question 10 is printed on the next page.]

10 The number of birds of a certain species in a forested region is recorded over several years. At time t years, the number of birds is N, where N is treated as a continuous variable. The variation in the number of birds is modelled by

$$\frac{dN}{dt} = \frac{N(1800 - N)}{3600}.$$

It is given that $N = 300$ when $t = 0$.

(i) Find an expression for N in terms of t. [9]

(ii) According to the model, how many birds will there be after a long time? [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1540228500

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The total number of marks for this paper is 75.
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JC11 06_9709_32/RP

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International Examinations

1 Solve the inequality $|x| < |5 + 2x|$. [3]

2 **(i)** Show that the equation

$$\log_2(x + 5) = 5 - \log_2 x$$

can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation

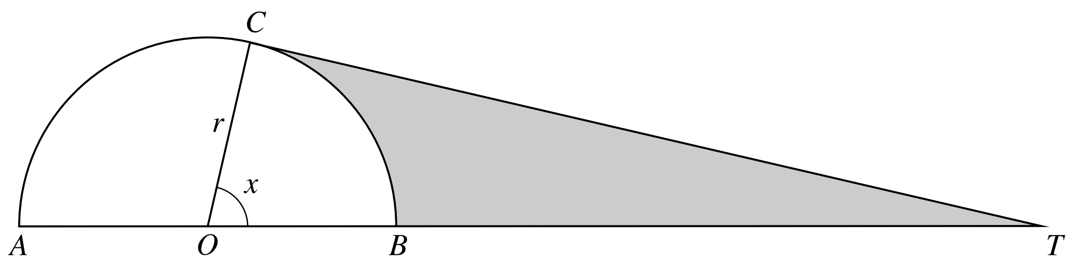
$$\log_2(x + 5) = 5 - \log_2 x.$$ [2]

3 Solve the equation

$$\cos\theta + 4\cos 2\theta = 3,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

4



The diagram shows a semicircle ACB with centre O and radius r. The tangent at C meets AB produced at T. The angle BOC is x radians. The area of the shaded region is equal to the area of the semicircle.

(i) Show that x satisfies the equation

$$\tan x = x + \pi.$$ [3]

(ii) Use the iterative formula $x_{n+1} = \tan^{-1}(x_n + \pi)$ to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The parametric equations of a curve are

$$x = \ln(\tan t), \quad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

6 A certain curve is such that its gradient at a point (x, y) is proportional to xy. At the point $(1, 2)$ the gradient is 4.

(i) By setting up and solving a differential equation, show that the equation of the curve is $y = 2e^{x^2-1}$. [7]

(ii) State the gradient of the curve at the point $(-1, 2)$ and sketch the curve. [2]

7 **(a)** The complex number u is defined by $u = \dfrac{5}{a + 2i}$, where the constant a is real.

(i) Express u in the form $x + iy$, where x and y are real. [2]

(ii) Find the value of a for which $\arg(u^*) = \frac{3}{4}\pi$, where u^* denotes the complex conjugate of u. [3]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z| < |z - 2 - 2i|$. [4]

8 **(i)** Express $\dfrac{5x - x^2}{(1 + x)(2 + x^2)}$ in partial fractions. [5]

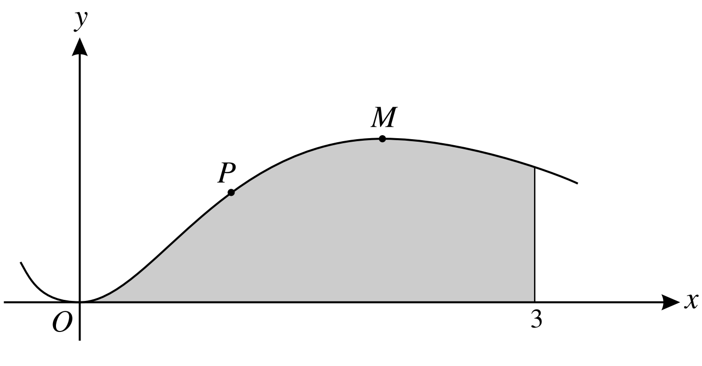
(ii) Hence obtain the expansion of $\dfrac{5x - x^2}{(1 + x)(2 + x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

9 Two planes have equations $x + 2y - 2z = 7$ and $2x + y + 3z = 5$.

(i) Calculate the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

10



The diagram shows the curve $y = x^2 e^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 3$ is equal to $2 - \dfrac{17}{e^3}$. [5]

(ii) Find the x-coordinate of the maximum point M on the curve. [4]

(iii) Find the x-coordinate of the point P at which the tangent to the curve passes through the origin. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3311907393

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
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This document consists of **3** printed pages and **1** blank page.

JC11 06_9709_33/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Use logarithms to solve the equation $5^{2x-1} = 2(3^x)$, giving your answer correct to 3 significant figures. [4]

2 The curve $y = \dfrac{\ln x}{x^3}$ has one stationary point. Find the x-coordinate of this point. [4]

3 Show that $\displaystyle\int_0^1 (1-x)e^{-\frac{1}{2}x}\,dx = 4e^{-\frac{1}{2}} - 2$. [5]

4 **(i)** Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2\theta) = k(1 - 3\tan^2\theta).$$ [4]

(ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(i)** Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

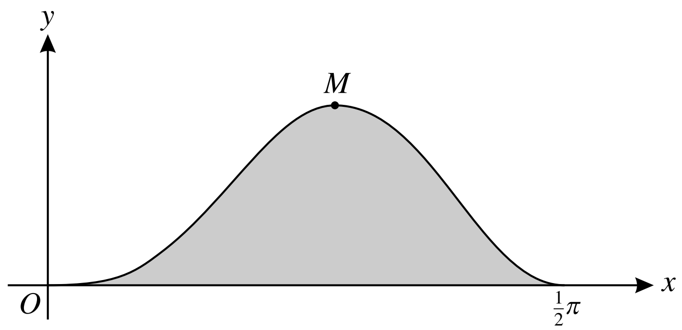
giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

8



The diagram shows the curve $y = 5\sin^3 x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [5]

(ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [5]

9 In a chemical reaction, a compound X is formed from two compounds Y and Z. The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.01(10 - x)(20 - x).$$ [1]

(ii) Solve this differential equation and obtain an expression for x in terms of t. [9]

(iii) State what happens to the value of x when t becomes large. [1]

10 With respect to the origin O, the lines l and m have vector equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

(i) Prove that l and m do not intersect. [4]

(ii) Calculate the acute angle between the directions of l and m. [3]

(iii) Find the equation of the plane which is parallel to l and contains m, giving your answer in the form $ax + by + cz = d$. [5]

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General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/41**

Paper 4 Mechanics 1 **(M1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3847556196

READ THESE INSTRUCTIONS FIRST

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Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
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The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 A car of mass 700 kg is travelling along a straight horizontal road. The resistance to motion is constant and equal to 600 N.

(i) Find the driving force of the car's engine at an instant when the acceleration is $2\,\text{m}\,\text{s}^{-2}$. [2]

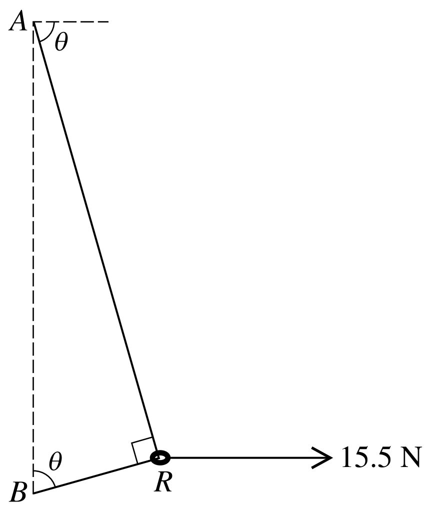
(ii) Given that the car's speed at this instant is $15\,\text{m}\,\text{s}^{-1}$, find the rate at which the car's engine is working. [2]

2 A load of mass 1250 kg is raised by a crane from rest on horizontal ground, to rest at a height of 1.54 m above the ground. The work done against the resistance to motion is 5750 J.

(i) Find the work done by the crane. [3]

(ii) Assuming the power output of the crane is constant and equal to 1.25 kW, find the time taken to raise the load. [2]

3



A small smooth ring R of weight 8.5 N is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B, with A vertically above B. A horizontal force of magnitude 15.5 N acts on R so that the ring is in equilibrium with angle $ARB = 90°$. The part AR of the string makes an angle θ with the horizontal and the part BR makes an angle θ with the vertical (see diagram). The tension in the string is T N. Show that $T \sin \theta = 12$ and $T \cos \theta = 3.5$ and hence find θ. [6]

4 A block of mass 11 kg is at rest on a rough plane inclined at 30° to the horizontal. A force acts on the block in a direction up the plane parallel to a line of greatest slope. When the magnitude of the force is $2X$ N the block is on the point of sliding down the plane, and when the magnitude of the force is $9X$ N the block is on the point of sliding up the plane. Find

(i) the value of X, [3]

(ii) the coefficient of friction between the block and the plane. [4]

5 A train starts from rest at a station A and travels in a straight line to station B, where it comes to rest. The train moves with constant acceleration $0.025\,\mathrm{m\,s^{-2}}$ for the first 600 s, with constant speed for the next 2600 s, and finally with constant deceleration $0.0375\,\mathrm{m\,s^{-2}}$.

(i) Find the total time taken for the train to travel from A to B. [4]

(ii) Sketch the velocity-time graph for the journey and find the distance AB. [3]

(iii) The speed of the train t seconds after leaving A is $7.5\,\mathrm{m\,s^{-1}}$. State the possible values of t. [1]

6 A particle travels in a straight line from a point P to a point Q. Its velocity t seconds after leaving P is $v\,\mathrm{m\,s^{-1}}$, where $v = 4t - \frac{1}{16}t^3$. The distance PQ is 64 m.

(i) Find the time taken for the particle to travel from P to Q. [5]

(ii) Find the set of values of t for which the acceleration of the particle is positive. [4]

7 Loads A and B, of masses 1.2 kg and 2.0 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. A is held at rest and B hangs freely, with both straight parts of the string vertical. A is released and starts to move upwards. It does not reach the pulley in the subsequent motion.

(i) Find the acceleration of A and the tension in the string. [4]

(ii) Find, for the first 1.5 metres of A's motion,

(a) A's gain in potential energy,

(b) the work done on A by the tension in the string,

(c) A's gain in kinetic energy.

[3]

B hits the floor 1.6 seconds after A is released. B comes to rest without rebounding and the string becomes slack.

(iii) Find the time from the instant the string becomes slack until it becomes taut again. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/42**

Paper 4 Mechanics 1 **(M1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_42/2R

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1 A load is pulled along horizontal ground for a distance of 76 m, using a rope. The rope is inclined at 5° above the horizontal and the tension in the rope is 65 N.

(i) Find the work done by the tension. [2]

At an instant during the motion the velocity of the load is $1.5\,\mathrm{m\,s^{-1}}$.

(ii) Find the rate of working of the tension at this instant. [2]

2 An object of mass 8 kg slides down a line of greatest slope of an inclined plane. Its initial speed at the top of the plane is $3\,\mathrm{m\,s^{-1}}$ and its speed at the bottom of the plane is $8\,\mathrm{m\,s^{-1}}$. The work done against the resistance to motion of the object is 120 J. Find the height of the top of the plane above the level of the bottom. [4]

3

The velocity-time graph shown models the motion of a parachutist falling vertically. There are four stages in the motion:

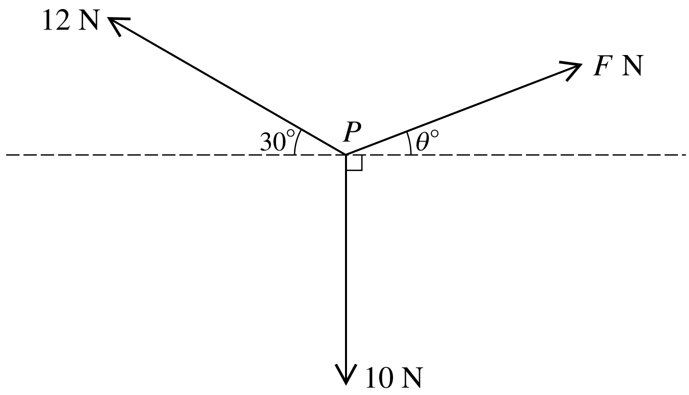
- falling freely with the parachute closed,
- decelerating at a constant rate with the parachute open,
- falling with constant speed with the parachute open,
- coming to rest instantaneously on hitting the ground.

(i) Show that the total distance fallen is 1048 m. [2]

The weight of the parachutist is 850 N.

(ii) Find the upward force on the parachutist due to the parachute, during the second stage. [5]

4



The three coplanar forces shown in the diagram act at a point P and are in equilibrium.

(i) Find the values of F and θ. [6]

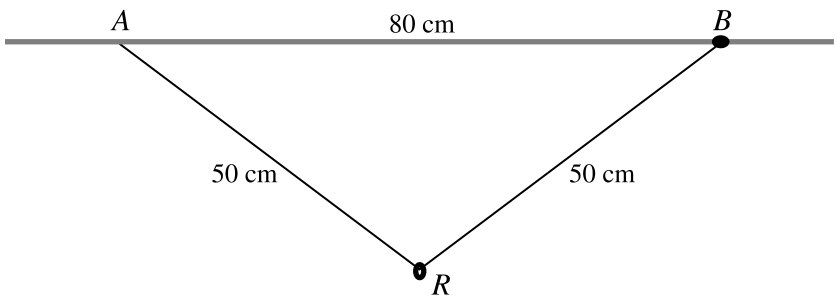
(ii) State the magnitude and direction of the resultant force at P when the force of magnitude 12 N is removed. [2]

5 Two particles P and Q are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of P and Q are $12\,\mathrm{m\,s^{-1}}$ and $7\,\mathrm{m\,s^{-1}}$ respectively and the heights of P and Q above the ground, t seconds after projection, are h_P m and h_Q m respectively. Each particle comes to rest on returning to the ground.

(i) Find the set of values of t for which the particles are travelling in opposite directions. [3]

(ii) At a certain instant, P and Q are above the ground and $3h_P = 8h_Q$. Find the velocities of P and Q at this instant. [5]

6



A small smooth ring R, of mass 0.6 kg, is threaded on a light inextensible string of length 100 cm. One end of the string is attached to a fixed point A. A small bead B of mass 0.4 kg is attached to the other end of the string, and is threaded on a fixed rough horizontal rod which passes through A. The system is in equilibrium with B at a distance of 80 cm from A (see diagram).

(i) Find the tension in the string. [3]

(ii) Find the frictional and normal components of the contact force acting on B. [4]

(iii) Given that the equilibrium is limiting, find the coefficient of friction between the bead and the rod. [2]

7 A walker travels along a straight road passing through the points A and B on the road with speeds $0.9\,\mathrm{m\,s^{-1}}$ and $1.3\,\mathrm{m\,s^{-1}}$ respectively. The walker's acceleration between A and B is constant and equal to $0.004\,\mathrm{m\,s^{-2}}$.

(i) Find the time taken by the walker to travel from A to B, and find the distance AB. [3]

A cyclist leaves A at the same instant as the walker. She starts from rest and travels along the straight road, passing through B at the same instant as the walker. At time t s after leaving A the cyclist's speed is $kt^3\,\mathrm{m\,s^{-1}}$, where k is a constant.

(ii) Show that when $t = 64.05$ the speed of the walker and the speed of the cyclist are the same, correct to 3 significant figures. [5]

(ii) Find the cyclist's acceleration at the instant she passes through B. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/43**

Paper 4 Mechanics 1 **(M1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 7 5 6 4 9 6 3 7 9 4 *

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This document consists of **4** printed pages.

JC11 06_9709_43/2R

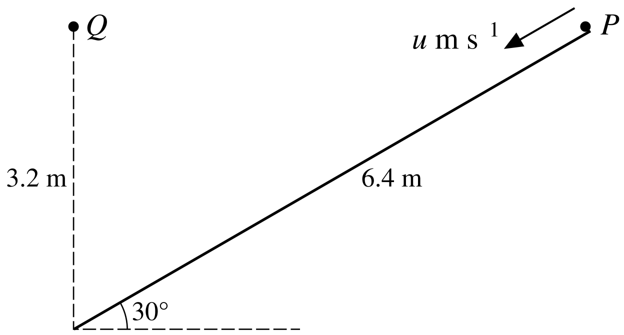
UNIVERSITY of CAMBRIDGE
International Examinations

[Turn over

1 A block is pulled for a distance of 50 m along a horizontal floor, by a rope that is inclined at an angle of $\alpha°$ to the floor. The tension in the rope is 180 N and the work done by the tension is 8200 J. Find the value of α. [3]

2 A car of mass 1250 kg is travelling along a straight horizontal road with its engine working at a constant rate of P W. The resistance to the car's motion is constant and equal to R N. When the speed of the car is $19\,\mathrm{m\,s^{-1}}$ its acceleration is $0.6\,\mathrm{m\,s^{-2}}$, and when the speed of the car is $30\,\mathrm{m\,s^{-1}}$ its acceleration is $0.16\,\mathrm{m\,s^{-2}}$. Find the values of P and R. [6]

3

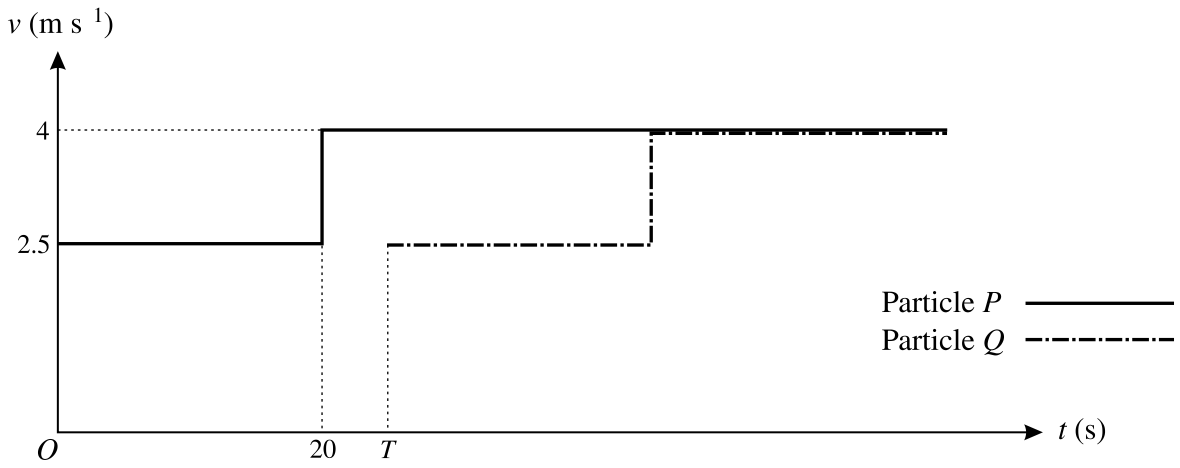


A particle P is projected from the top of a smooth ramp with speed $u\,\mathrm{m\,s^{-1}}$, and travels down a line of greatest slope. The ramp has length 6.4 m and is inclined at 30° to the horizontal. Another particle Q is released from rest at a point 3.2 m vertically above the bottom of the ramp, at the same instant that P is projected (see diagram). Given that P and Q reach the bottom of the ramp simultaneously, find

(i) the value of u, [4]

(ii) the speed with which P reaches the bottom of the ramp. [2]

4



The diagram shows the velocity-time graphs for the motion of two particles P and Q, which travel in the same direction along a straight line. P and Q both start at the same point X on the line, but Q starts to move T s later than P. Each particle moves with speed $2.5\,\mathrm{m\,s^{-1}}$ for the first 20 s of its motion. The speed of each particle changes instantaneously to $4\,\mathrm{m\,s^{-1}}$ after it has been moving for 20 s and the particle continues at this speed.

(i) Make a rough copy of the diagram and shade the region whose area represents the displacement of P from X at the instant when Q starts. [1]

It is given that P has travelled 70 m at the instant when Q starts.

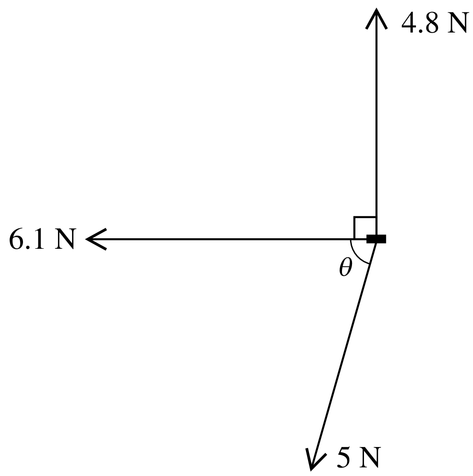
(ii) Find the value of T. [2]

(iii) Find the distance between P and Q when Q's speed reaches $4\,\mathrm{m\,s^{-1}}$. [2]

(iv) Sketch a single diagram showing the displacement-time graphs for both P and Q, with values shown on the t-axis at which the speed of either particle changes. [2]

[Questions 5, 6 and 7 are printed on the next page.]

5



A small block of mass 1.25 kg is on a horizontal surface. Three horizontal forces, with magnitudes and directions as shown in the diagram, are applied to the block. The angle θ is such that $\cos\theta = 0.28$ and $\sin\theta = 0.96$. A horizontal frictional force also acts on the block, and the block is in equilibrium.

(i) Show that the magnitude of the frictional force is 7.5 N and state the direction of this force. [4]

(ii) Given that the block is in limiting equilibrium, find the coefficient of friction between the block and the surface. [2]

The force of magnitude 6.1 N is now replaced by a force of magnitude 8.6 N acting in the same direction, and the block begins to move.

(iii) Find the magnitude and direction of the acceleration of the block. [3]

6 A lorry of mass 15 000 kg climbs a hill of length 500 m at a constant speed. The hill is inclined at 2.5° to the horizontal. The resistance to the lorry's motion is constant and equal to 800 N.

(i) Find the work done by the lorry's driving force. [4]

On its return journey the lorry reaches the top of the hill with speed $20\,\mathrm{m\,s^{-1}}$ and continues down the hill with a constant driving force of 2000 N. The resistance to the lorry's motion is again constant and equal to 800 N.

(ii) Find the speed of the lorry when it reaches the bottom of the hill. [5]

7 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is $a\,\mathrm{m\,s^{-2}}$, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the initial speed of the particle is zero. [4]

(ii) Find the maximum speed of the particle. [2]

(iii) Find the distance AB. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
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MATHEMATICS **9709/51**

Paper 5 Mechanics 2 **(M2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_51/2R

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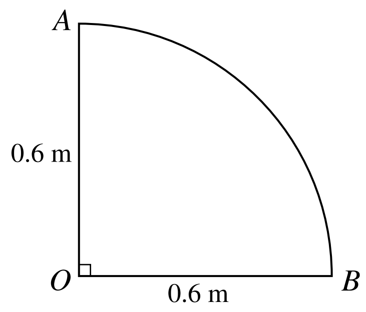
[Turn over

1 A particle is projected with speed $15\,\mathrm{m\,s^{-1}}$ at an angle of 40° above the horizontal from a point on horizontal ground. Calculate the time taken for the particle to hit the ground. [2]

2



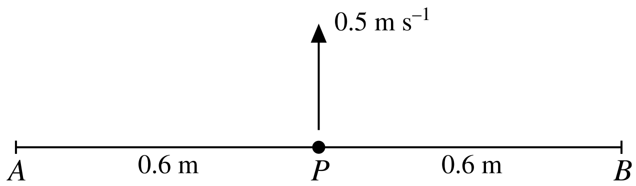
AOB is a uniform lamina in the shape of a quadrant of a circle with centre O and radius $0.6\,\mathrm{m}$ (see diagram).

(i) Calculate the distance of the centre of mass of the lamina from A. [3]

The lamina is freely suspended at A and hangs in equilibrium.

(ii) Find the angle between the vertical and the side AO of the lamina. [3]

3



A light elastic string of natural length $1.2\,\mathrm{m}$ and modulus of elasticity $24\,\mathrm{N}$ is attached to fixed points A and B on a smooth horizontal surface, where $AB = 1.2\,\mathrm{m}$. A particle P is attached to the mid-point of the string. P is projected with speed $0.5\,\mathrm{m\,s^{-1}}$ along the surface in a direction perpendicular to AB (see diagram). P comes to instantaneous rest at a distance $0.25\,\mathrm{m}$ from AB.

(i) Show that the mass of P is $0.8\,\mathrm{kg}$. [3]

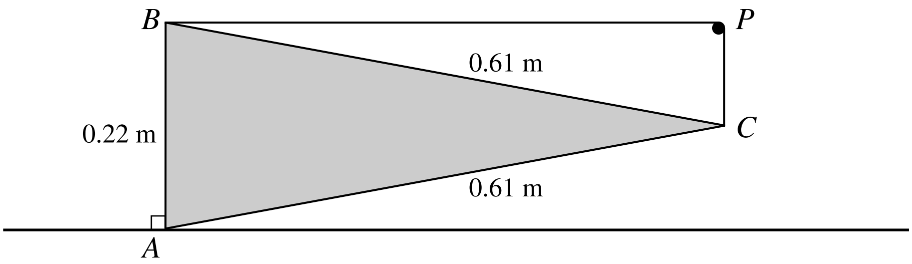
(ii) Calculate the greatest deceleration of P. [3]

4 A particle P starts from rest at a point O and travels in a straight line. The acceleration of P is $(15 - 6x)\,\mathrm{m\,s^{-2}}$, where $x\,\mathrm{m}$ is the displacement of P from O.

(i) Find the value of x for which P reaches its maximum velocity, and calculate this maximum velocity. [5]

(ii) Calculate the acceleration of P when it is at instantaneous rest and $x > 0$. [2]

5



ABC is a uniform triangular lamina of weight 19 N, with $AB = 0.22$ m and $AC = BC = 0.61$ m. The plane of the lamina is vertical. A rests on a rough horizontal surface, and AB is vertical. The equilibrium of the lamina is maintained by a light elastic string of natural length 0.7 m which passes over a small smooth peg P and is attached to B and C. The portion of the string attached to B is horizontal, and the portion of the string attached to C is vertical (see diagram).

(i) Show that the tension in the string is 10 N. [3]

(ii) Calculate the modulus of elasticity of the string. [2]

(iii) Find the magnitude and direction of the force exerted by the surface on the lamina at A. [3]

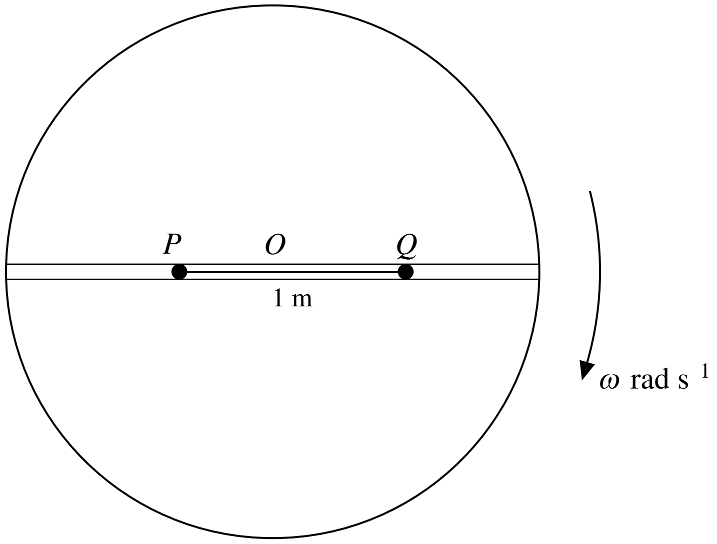
6 A particle P is projected from a point O on horizontal ground. 0.4 s after the instant of projection, P is 5 m above the ground and a horizontal distance of 12 m from O.

(i) Calculate the initial speed and the angle of projection of P. [6]

(ii) Find the direction of motion of the particle 0.4 s after the instant of projection. [3]

[Question 7 is printed on the next page.]

7



A narrow groove is cut along a diameter in the surface of a horizontal disc with centre O. Particles P and Q, of masses 0.2 kg and 0.3 kg respectively, lie in the groove, and the coefficient of friction between each of the particles and the groove is μ. The particles are attached to opposite ends of a light inextensible string of length 1 m. The disc rotates with angular velocity ω rad s^{-1} about a vertical axis passing through O and the particles move in horizontal circles (see diagram).

(i) Given that $\mu = 0.36$ and that both P and Q move in the same horizontal circle of radius 0.5 m, calculate the greatest possible value of ω and the corresponding tension in the string. [6]

(ii) Given instead that $\mu = 0$ and that the tension in the string is 0.48 N, calculate

(a) the radius of the circle in which P moves and the radius of the circle in which Q moves, [3]

(b) the speeds of the particles. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/52**

Paper 5 Mechanics 2 **(M2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_52/2R

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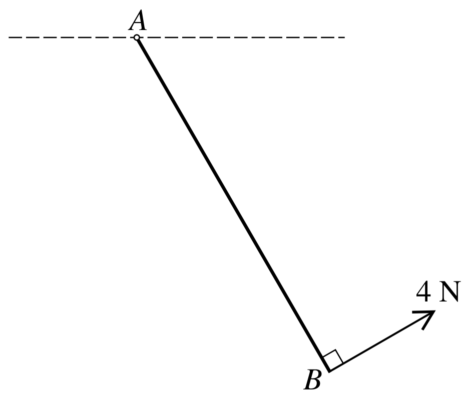
1



A uniform rod AB of weight 16 N is freely hinged at A to a fixed point. A force of magnitude 4 N acting perpendicular to the rod is applied at B (see diagram). Given that the rod is in equilibrium,

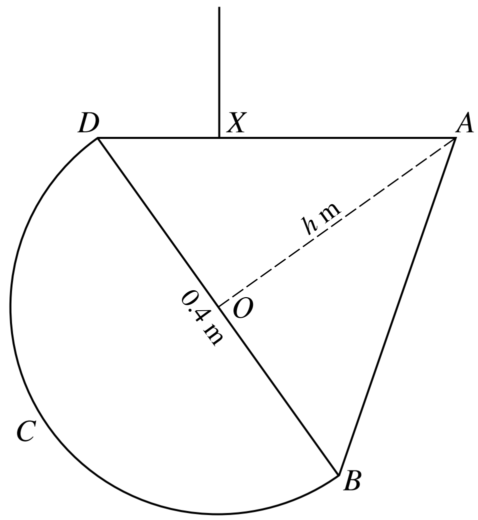
(i) calculate the angle the rod makes with the horizontal, [2]

(ii) find the magnitude and direction of the force exerted on the rod at A. [4]

2 A uniform lamina $ABCD$ consists of a semicircle BCD with centre O and diameter 0.4 m, and an isosceles triangle ABD with base BD = 0.4 m and perpendicular height h m. The centre of mass of the lamina is at O.

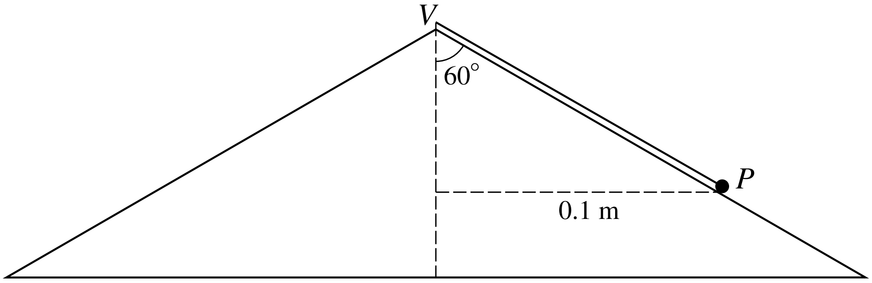
(i) Find the value of h. [4]

(ii)



The lamina is suspended from a vertical string attached to a point X on the side AD of the triangle (see diagram). Given the lamina is in equilibrium with AD horizontal, calculate XD. [3]

3



A particle P of mass 0.5 kg is attached to the vertex V of a fixed solid cone by a light inextensible string. P lies on the smooth curved surface of the cone and moves in a horizontal circle of radius 0.1 m with centre on the axis of the cone. The cone has semi-vertical angle 60° (see diagram).

(i) Calculate the speed of P, given that the tension in the string and the contact force between the cone and P have the same magnitude. [4]

(ii) Calculate the greatest angular speed at which P can move on the surface of the cone. [4]

4 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 12 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.24 kg. P is projected vertically upwards with speed 3 m s^{-1} from a position 0.8 m vertically below O.

(i) Calculate the speed of the particle when it is moving upwards with zero acceleration. [5]

(ii) Show that the particle moves 0.6 m while it is moving upwards with constant acceleration. [4]

5 A particle P of mass 0.4 kg moves in a straight line on a horizontal surface and has velocity $v\text{ m s}^{-1}$ at time t s. A horizontal force of magnitude $k\sqrt{v}$ N opposes the motion of P. When $t = 0$, $v = 9$ and when $t = 2$, $v = 4$.

(i) Express $\frac{\mathrm{d}v}{\mathrm{d}t}$ in terms of k and v, and hence show that $v = \frac{1}{4}(t - 6)^2$. [5]

(ii) Find the distance travelled by P in the first 3 seconds of its motion. [4]

6 A particle P is projected with speed 26 m s^{-1} at an angle of 30° below the horizontal, from a point O which is 80 m above horizontal ground.

(i) Calculate the distance from O of the particle 2.3 s after projection. [4]

(ii) Find the horizontal distance travelled by P before it reaches the ground. [3]

(iii) Calculate the speed and direction of motion of P immediately before it reaches the ground. [4]

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MATHEMATICS **9709/53**

Paper 5 Mechanics 2 **(M2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7500019585

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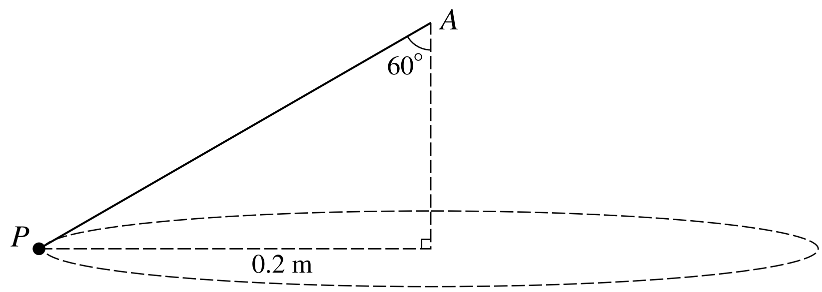
1



A particle P of mass 0.4 kg is attached to a fixed point A by a light inextensible string. The string is inclined at 60° to the vertical. P moves with constant speed in a horizontal circle of radius 0.2 m. The centre of the circle is vertically below A (see diagram).

(i) Show that the tension in the string is 8 N. [2]

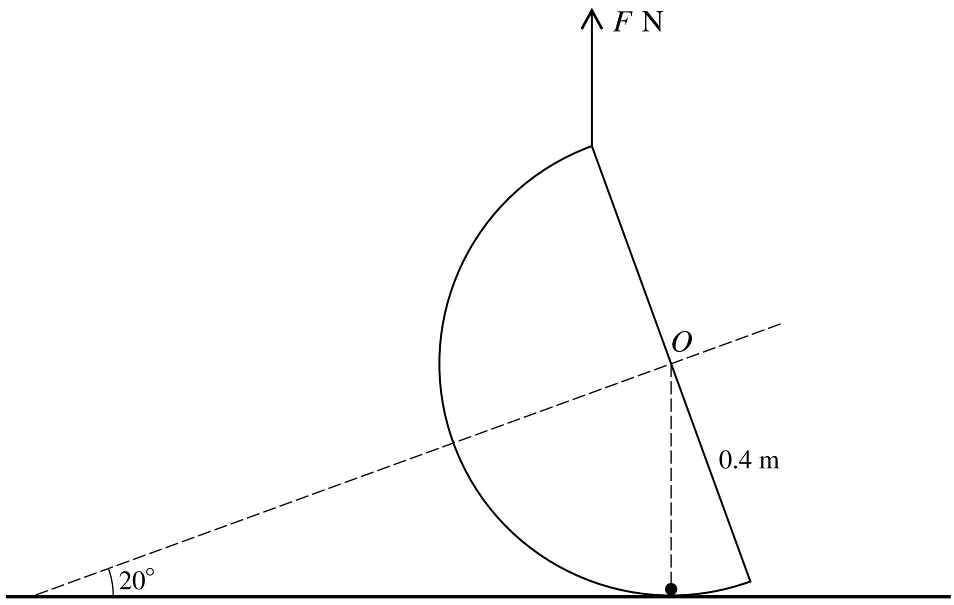
(ii) Calculate the speed of the particle. [2]

2 A stone is thrown with speed $15\,\mathrm{m\,s^{-1}}$ horizontally from the top of a vertical cliff 20 m above the sea. Calculate

(i) the distance from the foot of the cliff to the point where the stone enters the sea, [3]

(ii) the speed of the stone when it enters the sea. [3]

3



A smooth hemispherical shell, with centre O, weight 12 N and radius 0.4 m, rests on a horizontal plane. A particle of weight W N lies at rest on the inner surface of the hemisphere vertically below O. A force of magnitude F N acting vertically upwards is applied to the highest point of the hemisphere, which is in equilibrium with its axis of symmetry inclined at 20° to the horizontal (see diagram).

(i) Show, by taking moments about O, that $F = 16.48$ correct to 4 significant figures. [3]

(ii) Find the normal contact force exerted by the plane on the hemisphere in terms of W. Hence find the least possible value of W. [3]

4 The ends of a light elastic string of natural length 0.8 m and modulus of elasticity λ N are attached to fixed points A and B which are 1.2 m apart at the same horizontal level. A particle of mass 0.3 kg is attached to the centre of the string, and released from rest at the mid-point of AB. The particle descends 0.32 m vertically before coming to instantaneous rest.

(i) Calculate λ. [4]

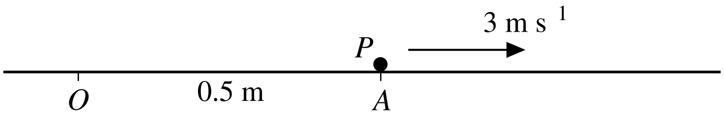
(ii) Calculate the speed of the particle when it is 0.25 m below AB. [4]

5 One end of a light elastic string of natural length 0.3 m and modulus of elasticity 6 N is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.2 kg, which moves on the plane in a circular path with centre O. The angular speed of P is ω rad s^{-1}.

(i) For the case $\omega = 5$, calculate the extension of the string. [4]

(ii) Express the extension of the string in terms of ω, and hence find the set of possible value of ω. [4]

6



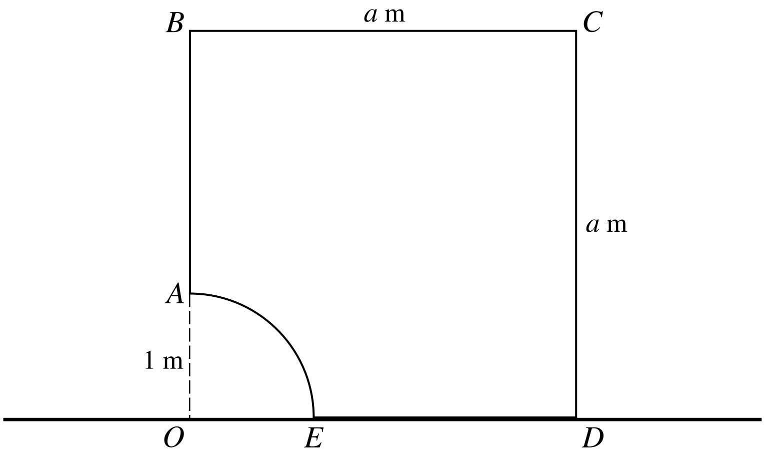
O and A are fixed points on a horizontal surface, with $OA = 0.5$ m. A particle P of mass 0.2 kg is projected horizontally with speed 3 m s^{-1} from A in the direction OA and moves in a straight line (see diagram). At time t s after projection, the velocity of P is v m s^{-1} and its displacement from O is x m. The coefficient of friction between the surface and P is 0.5, and a force of magnitude $\frac{0.4}{x^2}$ N acts on P in the direction PO.

(i) Show that, while the particle is in motion, $v\frac{dv}{dx} = -\left(5 + \frac{2}{x^2}\right)$. [2]

(ii) Calculate the distance travelled by P before it comes to rest, and show that P does not subsequently move. [7]

[Question 7 is printed on the next page.]

7



ABCDE is the cross-section through the centre of mass of a uniform prism resting in equilibrium with *DE* on a horizontal surface. The cross-section has the shape of a square *OBCD* with sides of length a m, from which a quadrant *OAE* of a circle of radius 1 m has been removed (see diagram).

(i) Find the distance of the centre of mass of the prism from O, giving the answer in terms of a, π and $\sqrt{2}$. [5]

(ii) Hence show that

$$3a^2(2 - a) < \tfrac{3}{2}\pi - 2,$$

and verify that this inequality is satisfied by $a = 1.68$ but not by $a = 1.67$. [4]

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MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits. [4]

2 When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

3 The possible values of the random variable X are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of X being 0 is $\frac{1}{10}$. The probabilities for all the other values of X are equal. Calculate

(i) $P(X < 2)$, [2]

(ii) the variance of X, [3]

(iii) the value of a for which $P(-a \leqslant X \leqslant 2a) = \frac{17}{35}$. [1]

4 A cricket team of 11 players is to be chosen from 21 players consisting of 10 batsmen, 9 bowlers and 2 wicketkeepers. The team must include at least 5 batsmen, at least 4 bowlers and at least 1 wicketkeeper.

(i) Find the number of different ways in which the team can be chosen. [4]

Each player in the team is given a present. The presents consist of 5 identical pens, 4 identical diaries and 2 identical notebooks.

(ii) Find the number of different arrangements of the presents if they are all displayed in a row. [1]

(iii) 10 of these 11 presents are chosen and arranged in a row. Find the number of different arrangements that are possible. [3]

5 **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

(b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

6 There are 5000 schools in a certain country. The cumulative frequency table shows the number of pupils in a school and the corresponding number of schools.

Number of pupils in a school	$\leqslant 100$	$\leqslant 150$	$\leqslant 200$	$\leqslant 250$	$\leqslant 350$	$\leqslant 450$	$\leqslant 600$
Cumulative frequency	200	800	1600	2100	4100	4700	5000

(i) Draw a cumulative frequency graph with a scale of 2 cm to 100 pupils on the horizontal axis and a scale of 2 cm to 1000 schools on the vertical axis. Use your graph to estimate the median number of pupils in a school. [3]

(ii) 80% of the schools have more than n pupils. Estimate the value of n correct to the nearest ten. [2]

(iii) Find how many schools have between 201 and 250 (inclusive) pupils. [1]

(iv) Calculate an estimate of the mean number of pupils per school. [4]

7 **(a)** **(i)** Find the probability of getting at least one 3 when 9 fair dice are thrown. [2]

(ii) When n fair dice are thrown, the probability of getting at least one 3 is greater than 0.9. Find the smallest possible value of n. [4]

(b) A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the first person to draw a yellow ball. Julie draws the first ball. Find the probability that Ronnie wins the game. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

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JC11 06_9709_62/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

2 In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

(i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

(ii) Give a reason why the use of a normal approximation is justified. [1]

3 A sample of 36 data values, x, gave $\Sigma(x - 45) = -148$ and $\Sigma(x - 45)^2 = 3089$.

(i) Find the mean and standard deviation of the 36 values. [3]

(ii) One extra data value of 29 was added to the sample. Find the standard deviation of all 37 values. [4]

4 (i) Find the number of different ways that the 9 letters of the word HAPPINESS can be arranged in a line. [1]

(ii) The 9 letters of the word HAPPINESS are arranged in random order in a line. Find the probability that the 3 vowels (A, E, I) are not all next to each other. [4]

(iii) Find the number of different selections of 4 letters from the 9 letters of the word HAPPINESS which contain no Ps and either one or two Ss. [3]

5 A hotel has 90 rooms. The table summarises information about the number of rooms occupied each day for a period of 200 days.

Number of rooms occupied	1 – 20	21 – 40	41 – 50	51 – 60	61 – 70	71 – 90
Frequency	10	32	62	50	28	18

(i) Draw a cumulative frequency graph on graph paper to illustrate this information. [4]

(ii) Estimate the number of days when over 30 rooms were occupied. [2]

(iii) On 75% of the days at most n rooms were occupied. Estimate the value of n. [2]

6 The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

(i) Find the value of μ. [3]

(ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

7 Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the nth choice of cards. [2]

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General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_63/2R

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1 Red Street Garage has 9 used cars for sale. Fairwheel Garage has 15 used cars for sale. The mean age of the cars in Red Street Garage is 3.6 years and the standard deviation is 1.925 years. In Fairwheel Garage, $\Sigma x = 64$ and $\Sigma x^2 = 352$, where x is the age of a car in years.

(i) Find the mean age of all 24 cars. [2]

(ii) Find the standard deviation of the ages of all 24 cars. [4]

2 Fahad has 4 different coloured pairs of shoes (white, red, blue and black), 3 different coloured pairs of jeans (blue, black and brown) and 7 different coloured tee shirts (red, orange, yellow, blue, green, white and purple).

(i) Fahad chooses an outfit consisting of one pair of shoes, one pair of jeans and one tee shirt. How many different outfits can he choose? [1]

(ii) How many different ways can Fahad arrange his 3 jeans and 7 tee shirts in a row if the two blue items are not next to each other? [2]

Fahad also has 9 different books about sport. When he goes on holiday he chooses at least one of these books to take with him.

(iii) How many different selections are there if he can take any number of books ranging from just one of them to all of them? [3]

3 The following cumulative frequency table shows the examination marks for 300 candidates in country A and 300 candidates in country B.

Mark	<10	<20	<35	<50	<70	<100
Cumulative frequency, A	25	68	159	234	260	300
Cumulative frequency, B	10	46	72	144	198	300

(i) Without drawing a graph, show that the median for country B is higher than the median for country A. [2]

(ii) Find the number of candidates in country A who scored between 20 and 34 marks inclusive. [1]

(iii) Calculate an estimate of the mean mark for candidates in country A. [4]

4 Tim throws a fair die twice and notes the number on each throw.

(i) Tim calculates his final score as follows. If the number on the second throw is a 5 he multiplies the two numbers together, and if the number on the second throw is not a 5 he adds the two numbers together. Find the probability that his final score is

(a) 12, [1]

(b) 5. [3]

(ii) Events A, B, C are defined as follows.

A: the number on the second throw is 5

B: the sum of the numbers is 6

C: the product of the numbers is even

By calculation find which pairs, if any, of the events A, B and C are independent. [5]

5 The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $\mathrm{P}(X > 20) = 0.04$.

(i) Find μ. [3]

(ii) Find $\mathrm{P}(10 < X < 20)$. [3]

(iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

6 The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_71/RP

UNIVERSITY of CAMBRIDGE International Examinations

1 On average, 2 people in every 10 000 in the UK have a particular gene. A random sample of 6000 people in the UK is chosen. The random variable X denotes the number of people in the sample who have the gene. Use an approximating distribution to calculate the probability that there will be more than 2 people in the sample who have the gene. [4]

2 **(a)** The time taken by a worker to complete a task was recorded for a random sample of 50 workers. The sample mean was 41.2 minutes and an unbiased estimate of the population variance was 32.6 minutes^2. Find a 95% confidence interval for the mean time taken to complete the task. [3]

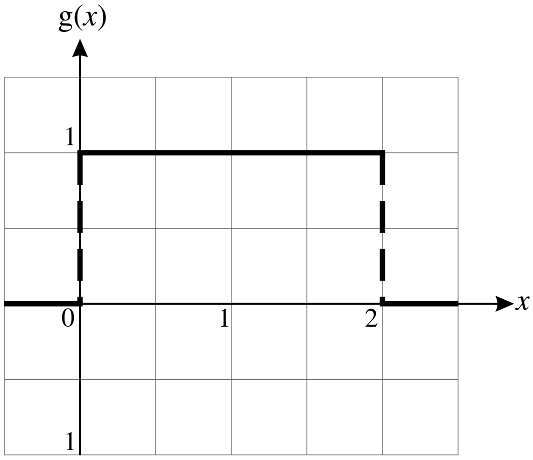
(b) The probability that an α% confidence interval includes only values that are lower than the population mean is $\frac{1}{16}$. Find the value of α. [2]

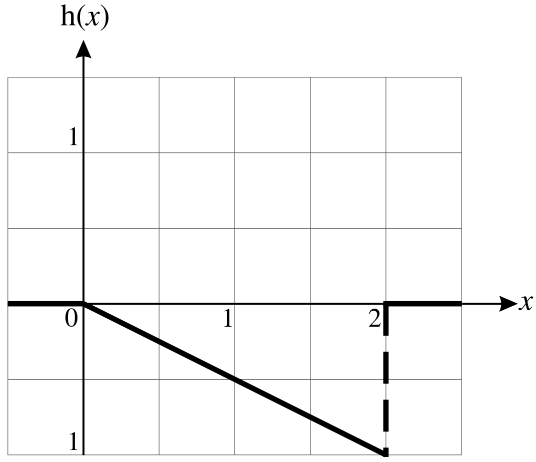
3 Past experience has shown that the heights of a certain variety of rose bush have been normally distributed with mean 85.0 cm. A new fertiliser is used and it is hoped that this will increase the heights. In order to test whether this is the case, a botanist records the heights, x cm, of a large random sample of n rose bushes and calculates that $\bar{x} = 85.7$ and $s = 4.8$, where $\bar{x}$ is the sample mean and s^2 is an unbiased estimate of the population variance. The botanist then carries out an appropriate hypothesis test.

(i) The test statistic, z, has a value of 1.786 correct to 3 decimal places. Calculate the value of n. [3]

(ii) Using this value of the test statistic, carry out the test at the 5% significance level. [3]

4 **(a)**





The diagrams show the graphs of two functions, g and h. For each of the functions g and h, give a reason why it cannot be a probability density function. [2]

(b) The distance, in kilometres, travelled in a given time by a cyclist is represented by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{30}{x^2} & 10 \leqslant x \leqslant 15, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $E(X) = 30 \ln 1.5$. [3]

(ii) Find the median of X. Find also the probability that X lies between the median and the mean. [5]

5 Cans of drink are packed in boxes, each containing 4 cans. The weights of these cans are normally distributed with mean 510 g and standard deviation 14 g. The weights of the boxes, when empty, are independently normally distributed with mean 200 g and standard deviation 8 g.

(i) Find the probability that the total weight of a full box of cans is between 2200 g and 2300 g. [6]

(ii) Two cans of drink are chosen at random. Find the probability that they differ in weight by more than 20 g. [5]

6 The number of injuries per month at a certain factory has a Poisson distribution. In the past the mean was 2.1 injuries per month. New safety procedures are put in place and the management wishes to use the next 3 months to test, at the 2% significance level, whether there are now fewer injuries than before, on average.

(i) Find the critical region for the test. [5]

(ii) Find the probability of a Type I error. [1]

(iii) During the next 3 months there are a total of 3 injuries. Carry out the test. [3]

(iv) Assuming that the mean remains 2.1, calculate an estimate of the probability that there will be fewer than 20 injuries during the next 12 months. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_72/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The weights of bags of fuel have mean 3.2 kg and standard deviation 0.04 kg. The total weight of a random sample of three bags is denoted by T kg. Find the mean and standard deviation of T. [4]

2 X is a random variable having the distribution $\mathrm{B}(12, \frac{1}{4})$. A random sample of 60 values of X is taken. Find the probability that the sample mean is less than 2.8. [5]

3 The number of goals scored per match by Everly Rovers is represented by the random variable X which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. [2]

Assume now that $X \sim \mathrm{Po}(1.8)$.

(ii) Find $\mathrm{P}(2 < X < 6)$. [2]

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [3]

4 A doctor wishes to investigate the mean fat content in low-fat burgers. He takes a random sample of 15 burgers and sends them to a laboratory where the mass, in grams, of fat in each burger is determined. The results are as follows.

9 7 8 9 6 11 7 9 8 9 8 10 7 9 9

Assume that the mass, in grams, of fat in low-fat burgers is normally distributed with mean μ and that the population standard deviation is 1.3.

(i) Calculate a 99% confidence interval for μ. [4]

(ii) Explain whether it was necessary to use the Central Limit theorem in the calculation in part **(i)**. [2]

(iii) The manufacturer claims that the mean mass of fat in burgers of this type is 8 g. Use your answer to part **(i)** to comment on this claim. [2]

5 The number of adult customers arriving in a shop during a 5-minute period is modelled by a random variable with distribution Po(6). The number of child customers arriving in the same shop during a 10-minute period is modelled by an independent random variable with distribution Po(4.5).

(i) Find the probability that during a randomly chosen 2-minute period, the total number of adult and child customers who arrive in the shop is less than 3. [3]

(ii) During a sale, the manager claims that more adult customers are arriving than usual. In a randomly selected 30-minute period during the sale, 49 adult customers arrive. Test the manager's claim at the 2.5% significance level. [6]

6 Jeevan thinks that a six-sided die is biased in favour of six. In order to test this, Jeevan throws the die 10 times. If the die shows a six on at least 4 throws out of 10, she will conclude that she is correct.

(i) State appropriate null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) Explain what is meant by a Type II error in this situation. [1]

(iv) If the die is actually biased so that the probability of throwing a six is $\frac{1}{2}$, calculate the probability of a Type II error. [3]

7 A random variable X has probability density function given by

$$f(x) = \begin{cases} k(1-x) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Find $P(X > \frac{1}{2})$. [1]

(iii) Find the mean of X. [3]

(iv) Find a such that $P(X < a) = \frac{1}{4}$. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 06_9709_73/RP

UNIVERSITY of CAMBRIDGE
International Examinations

1 A hotel kitchen has two dish-washing machines. The numbers of breakdowns per year by the two machines have independent Poisson distributions with means 0.7 and 1.0. Find the probability that the total number of breakdowns by the two machines during the next two years will be less than 3. [4]

2 In a random sample of 70 bars of Luxcleanse soap, 18 were found to be undersized.

(i) Calculate an approximate 90% confidence interval for the proportion of all bars of Luxcleanse soap that are undersized. [4]

(ii) Give a reason why your interval is only approximate. [1]

3 At an election in 2010, 15% of voters in Bratfield voted for the Renewal Party. One year later, a researcher asked 30 randomly selected voters in Bratfield whether they would vote for the Renewal Party if there were an election next week. 2 of these 30 voters said that they would.

(i) Use a binomial distribution to test, at the 4% significance level, the null hypothesis that there has been no change in the support for the Renewal Party in Bratfield against the alternative hypothesis that there has been a decrease in support since the 2010 election. [4]

(ii) **(a)** Explain why the conclusion in part **(i)** cannot involve a Type I error. [1]

(b) State the circumstances in which the conclusion in part **(i)** would involve a Type II error. [1]

4 On average, 1 in 2500 people have a particular gene.

(i) Use a suitable approximation to find the probability that, in a random sample of 10 000 people, more than 3 people have this gene. [4]

(ii) The probability that, in a random sample of n people, none of them has the gene is less than 0.01. Find the smallest possible value of n. [3]

5 Each drink from a coffee machine contains X cm^3 of coffee and Y cm^3 of milk, where X and Y are independent variables with $X \sim \mathrm{N}(184, 15^2)$ and $Y \sim \mathrm{N}(50, 8^2)$. If the total volume of the drink is less than 200 cm^3 the customer receives the drink without charge.

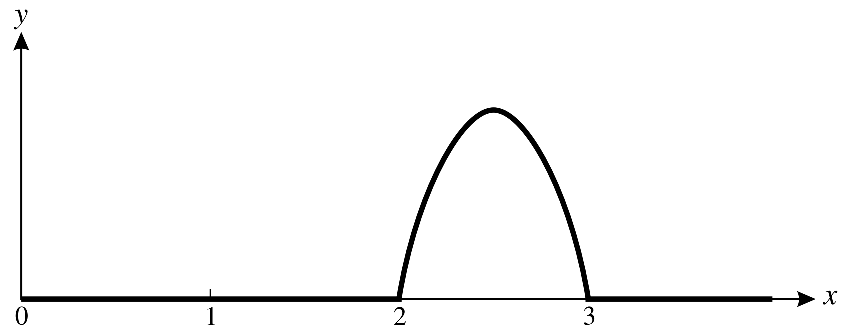
(i) Find the percentage of drinks which customers receive without charge. [4]

(ii) Find the probability that, in a randomly chosen drink, the volume of coffee is more than 4 times the volume of milk. [5]

6 The distance travelled, in kilometres, by a Grippo brake pad before it needs to be replaced is modelled by $10\,000X$, where X is a random variable having the probability density function

$$\mathrm{f}(x) = \begin{cases} -k(x^2 - 5x + 6) & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of $y = \mathrm{f}(x)$ is shown in the diagram.



(i) Show that $k = 6$. [2]

(ii) State the value of $\mathrm{E}(X)$ and find $\mathrm{Var}(X)$. [4]

(iii) Sami fits four new Grippo brake pads on his car. Find the probability that at least one of these brake pads will need to be replaced after travelling less than 22 000 km. [3]

7 Previous records have shown that the number of cars entering Bampor on any day has mean 352 and variance 121.

(i) Find the probability that the mean number of cars entering Bampor during a random sample of 200 days is more than 354. [4]

(ii) State, with a reason, whether it was necessary to assume that the number of cars entering Bampor on any day has a normal distribution in order to find the probability in part **(i)**. [2]

(iii) It is thought that the population mean may recently have changed. The number of cars entering Bampor during the day was recorded for each of a random sample of 50 days and the sample mean was found to be 356. Assuming that the variance is unchanged, test at the 5% significance level whether the population mean is still 352. [4]

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