UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5512628315

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC11 11_9709_11/2R

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Find the term independent of x in the expansion of $\left(2x + \dfrac{1}{x^2}\right)^6$. [3]

2 A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]

3 **(i)** Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

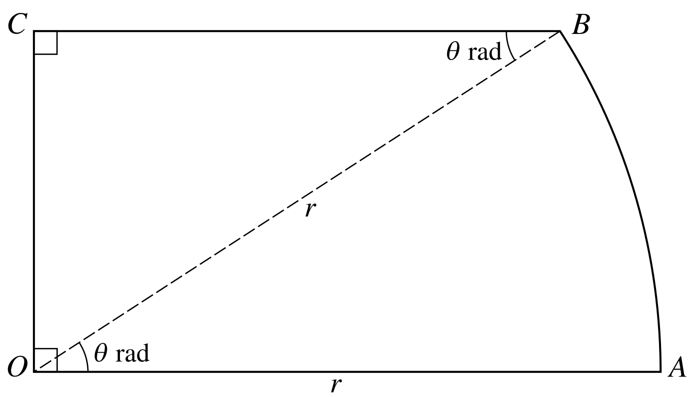
(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

4 A function f is defined for $x \in \mathbb{R}$ and is such that $\mathrm{f}'(x) = 2x - 6$. The range of the function is given by $\mathrm{f}(x) \geqslant -4$.

(i) State the value of x for which $\mathrm{f}(x)$ has a stationary value. [1]

(ii) Find an expression for $\mathrm{f}(x)$ in terms of x. [4]

5



The diagram represents a metal plate $OABC$, consisting of a sector OAB of a circle with centre O and radius r, together with a triangle OCB which is right-angled at C. Angle $AOB = \theta$ radians and OC is perpendicular to OA.

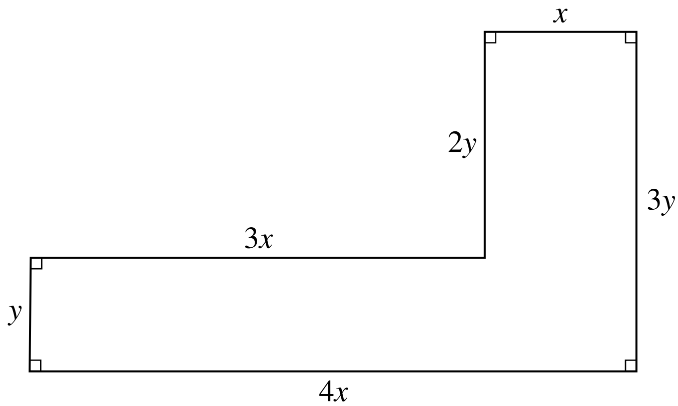
(i) Find an expression in terms of r and θ for the perimeter of the plate. [3]

(ii) For the case where $r = 10$ and $\theta = \frac{1}{5}\pi$, find the area of the plate. [3]

6 **(a)** The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]

(b) A geometric progression has first term 1 and common ratio r. A second geometric progression has first term 4 and common ratio $\frac{1}{4}r$. The two progressions have the same sum to infinity, S. Find the values of r and S. [3]

7



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

(i) Find an expression for y in terms of x. [1]

(ii) Given that the area of the garden is A m^2, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

8 Relative to an origin O, the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA} \cdot \overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60°$. [4]

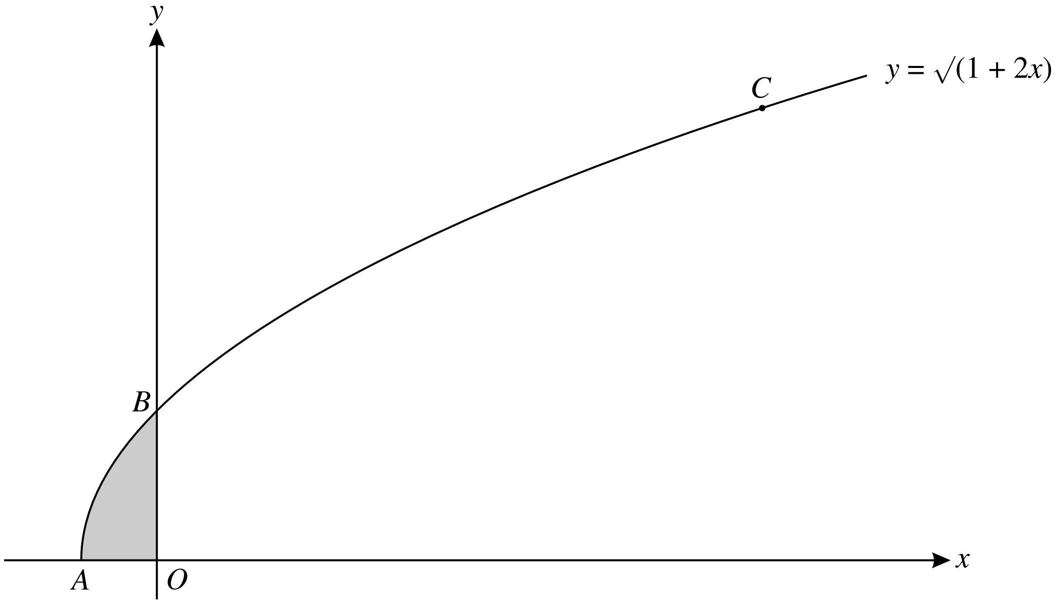
9 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the mid-point of AB. [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

[Questions 10 and 11 are printed on the next page.]

10



The diagram shows the curve $y = \sqrt{(1 + 2x)}$ meeting the x-axis at A and the y-axis at B. The y-coordinate of the point C on the curve is 3.

(i) Find the coordinates of B and C. [2]

(ii) Find the equation of the normal to the curve at C. [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [5]

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$\mathrm{g} : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{g}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1691972400

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JC11 11_9709_12/2R

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 **(i)** Find the first 3 terms in the expansion of $(2 - y)^5$ in ascending powers of y. [2]

(ii) Use the result in part **(i)** to find the coefficient of x^2 in the expansion of $\left(2 - (2x - x^2)\right)^5$. [3]

2 The functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 3x + a,$$
$$\mathrm{g} : x \mapsto b - 2x,$$

where a and b are constants. Given that $\mathrm{ff}(2) = 10$ and $\mathrm{g}^{-1}(2) = 3$, find

(i) the values of a and b, [4]

(ii) an expression for $\mathrm{fg}(x)$. [2]

3 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where p is a constant.

(i) Find the value of p for which angle AOB is $90°$. [3]

(ii) In the case where $p = 4$, find the vector which has magnitude 28 and is in the same direction as $\overrightarrow{AB}$. [4]

4 The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

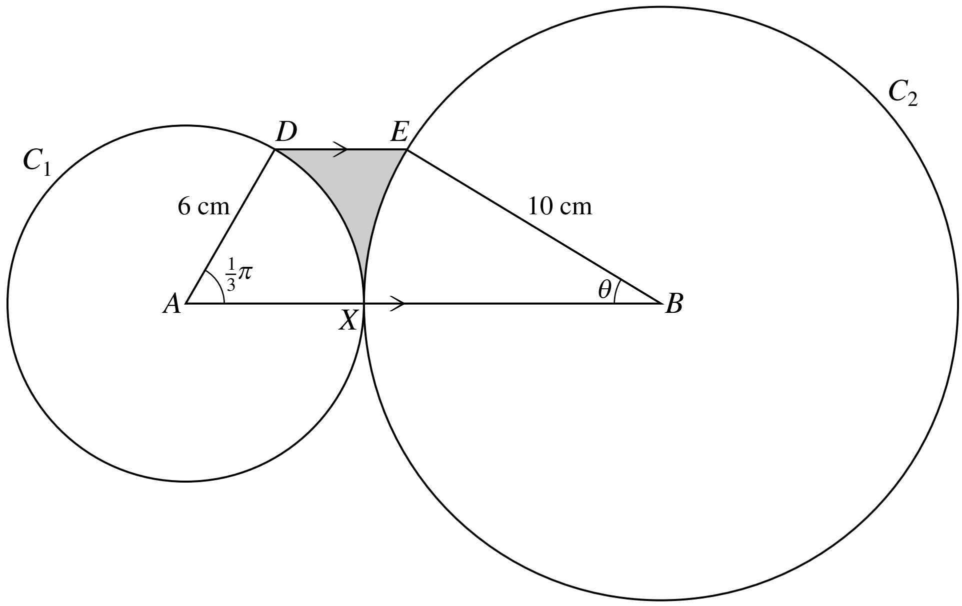
(ii) Find the value of k for which the line is a tangent to the curve. [3]

5 **(i)** Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0° \leqslant x \leqslant 180°$. [3]

(ii) Verify that $x = 30°$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0° \leqslant x \leqslant 180°$. [2]

(iii) Hence state the set of values of x, for $0° \leqslant x \leqslant 180°$, for which $\sin x < \cos 2x$. [2]

6



The diagram shows a circle C_1 touching a circle C_2 at a point X. Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB. Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

(i) By considering the perpendicular distances of D and E from AB, show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]

(ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

7 A curve is such that $\frac{dy}{dx} = 5 - \frac{8}{x^2}$. The line $3y + x = 17$ is the normal to the curve at the point P on the curve. Given that the x-coordinate of P is positive, find

(i) the coordinates of P, [4]

(ii) the equation of the curve. [4]

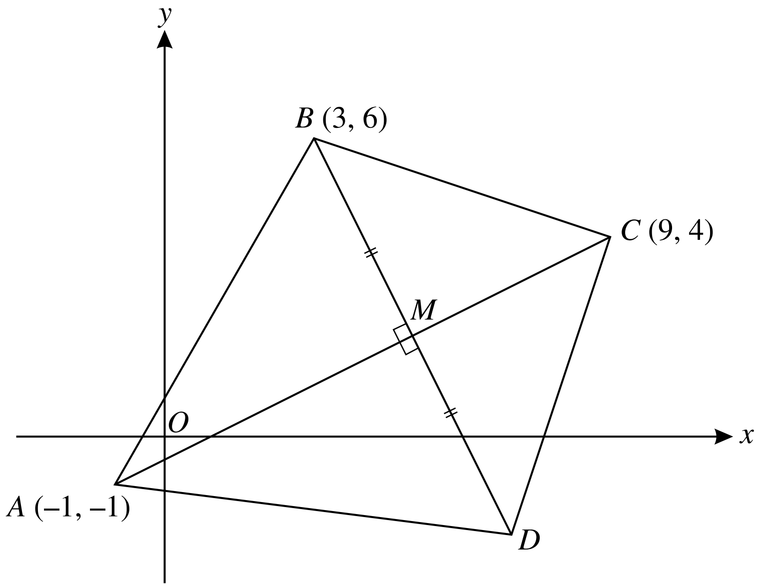
8 The equation of a curve is $y = \sqrt{(8x - x^2)}$. Find

(i) an expression for $\frac{dy}{dx}$, and the coordinates of the stationary point on the curve, [4]

(ii) the volume obtained when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. [4]

[Questions 9 and 10 are printed on the next page.]

9



The diagram shows a quadrilateral $ABCD$ in which the point A is $(-1, -1)$, the point B is $(3, 6)$ and the point C is $(9, 4)$. The diagonals AC and BD intersect at M. Angle $BMA = 90°$ and $BM = MD$. Calculate

(i) the coordinates of M and D, [7]

(ii) the ratio $AM : MC$. [2]

10 **(a)** An arithmetic progression contains 25 terms and the first term is -15. The sum of all the terms in the progression is 525. Calculate

(i) the common difference of the progression, [2]

(ii) the last term in the progression, [2]

(iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 11_9709_13/2R

UNIVERSITY *of* CAMBRIDGE
International Examinations

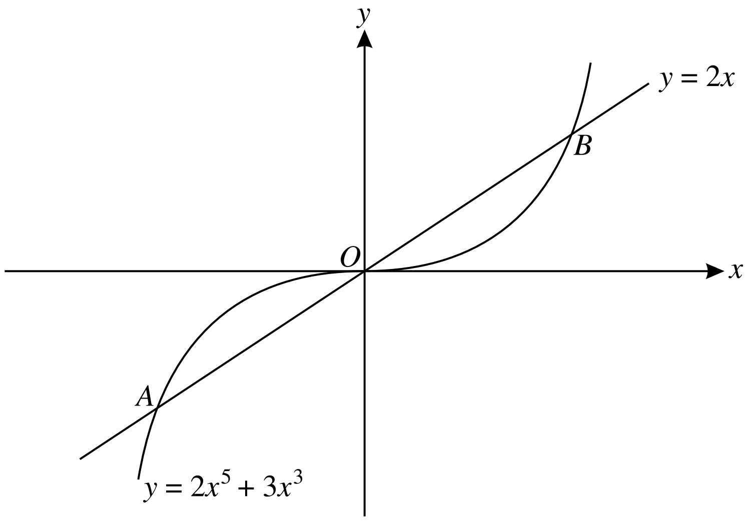
1 The coefficient of x^2 in the expansion of $\left(k + \frac{1}{3}x\right)^5$ is 30. Find the value of the constant k. [3]

2 The first and second terms of a progression are 4 and 8 respectively. Find the sum of the first 10 terms given that the progression is

(i) an arithmetic progression, [2]

(ii) a geometric progression. [2]

3

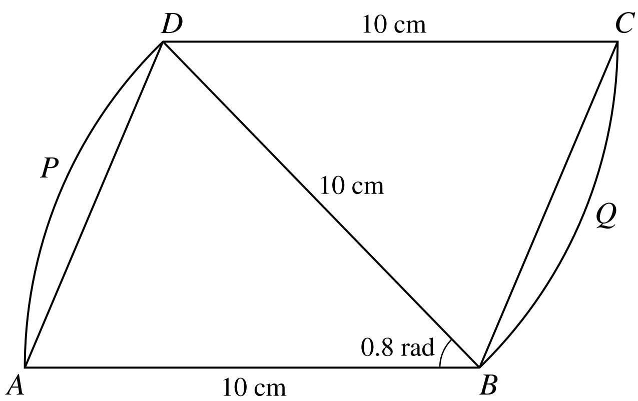


The diagram shows the curve $y = 2x^5 + 3x^3$ and the line $y = 2x$ intersecting at points A, O and B.

(i) Show that the x-coordinates of A and B satisfy the equation $2x^4 + 3x^2 - 2 = 0$. [2]

(ii) Solve the equation $2x^4 + 3x^2 - 2 = 0$ and hence find the coordinates of A and B, giving your answers in an exact form. [3]

4



In the diagram, $ABCD$ is a parallelogram with $AB = BD = DC = 10$ cm and angle $ABD = 0.8$ radians. APD and BQC are arcs of circles with centres B and D respectively.

(i) Find the area of the parallelogram $ABCD$. [2]

(ii) Find the area of the complete figure $ABQCDP$. [2]

(iii) Find the perimeter of the complete figure $ABQCDP$. [2]

5 **(i)** Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70^\circ) - 8\cos(\theta + 70^\circ) - 7 = 0$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [4]

6 Relative to an origin O, the position vectors of points A and B are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

(i) Use a scalar product to find angle BOA. [4]

The point C is the mid-point of AB. The point D is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

(ii) Find $\overrightarrow{DC}$. [4]

7 **(i)** A straight line passes through the point (2, 0) and has gradient m. Write down the equation of the line. [1]

(ii) Find the two values of m for which the line is a tangent to the curve $y = x^2 - 4x + 5$. For each value of m, find the coordinates of the point where the line touches the curve. [6]

(iii) Express $x^2 - 4x + 5$ in the form $(x + a)^2 + b$ and hence, or otherwise, write down the coordinates of the minimum point on the curve. [2]

8 A curve $y = \mathrm{f}(x)$ has a stationary point at $P\,(3, -10)$. It is given that $\mathrm{f}'(x) = 2x^2 + kx - 12$, where k is a constant.

(i) Show that $k = -2$ and hence find the x-coordinate of the other stationary point, Q. [4]

(ii) Find $\mathrm{f}''(x)$ and determine the nature of each of the stationary points P and Q. [2]

(iii) Find $\mathrm{f}(x)$. [4]

9 Functions f and g are defined by

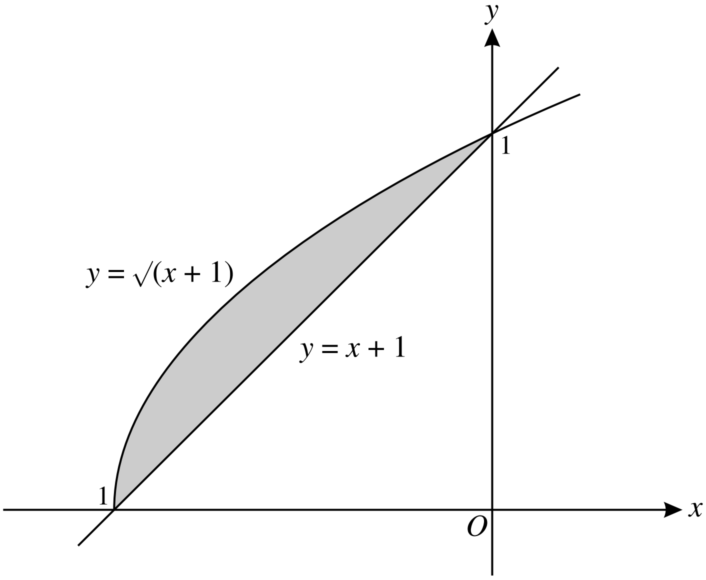
$$\mathrm{f} : x \mapsto 2x + 3 \quad \text{for } x \leqslant 0,$$
$$\mathrm{g} : x \mapsto x^2 - 6x \quad \text{for } x \leqslant 3.$$

(i) Express $\mathrm{f}^{-1}(x)$ in terms of x and solve the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(ii) On the same diagram sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]

(iii) Find the set of values of x which satisfy $\mathrm{gf}(x) \leqslant 16$. [5]

10



The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{(x + 1)}$, meeting at $(-1, 0)$ and $(0, 1)$.

(i) Find the area of the shaded region. [5]

(ii) Find the volume obtained when the shaded region is rotated through $360°$ about the ***y*-axis**. [7]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

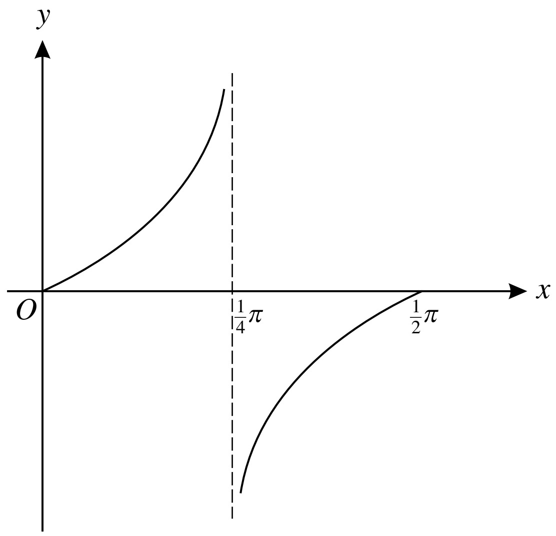
JC11 11_9709_21/RP

UNIVERSITY of CAMBRIDGE International Examinations

1 Solve the inequality $|4 - 5x| < 3$. [3]

2 Show that $\displaystyle\int_2^6 \frac{2}{4x+1}\,dx = \ln\tfrac{5}{3}$. [5]

3



The diagram shows the part of the curve $y = \frac{1}{2}\tan 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. Find the x-coordinates of the points on this part of the curve at which the gradient is 4. [5]

4 Solve the equation $3^{2x} - 7(3^x) + 10 = 0$, giving your answers correct to 3 significant figures. [5]

5 The polynomial $4x^3 + ax^2 + 9x + 9$, where a is a constant, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(2x - 1)$ the remainder is 10.

(i) Find the value of a and hence verify that $(x - 3)$ is a factor of $p(x)$. [3]

(ii) When a has this value, solve the equation $p(x) = 0$. [4]

6 **(i)** Verify by calculation that the cubic equation

$$x^3 - 2x^2 + 5x - 3 = 0$$

has a root that lies between $x = 0.7$ and $x = 0.8$. [2]

(ii) Show that this root also satisfies an equation of the form

$$x = \frac{ax^2 + 3}{x^2 + b},$$

where the values of a and b are to be found. [2]

(iii) With these values of a and b, use the iterative formula

$$x_{n+1} = \frac{ax_n^2 + 3}{x_n^2 + b}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The parametric equations of a curve are

$$x = e^{3t}, \quad y = t^2 e^t + 3.$$

(i) Show that $\frac{dy}{dx} = \frac{t(t+2)}{3e^{2t}}$. [4]

(ii) Show that the tangent to the curve at the point (1, 3) is parallel to the x-axis. [2]

(iii) Find the exact coordinates of the other point on the curve at which the tangent is parallel to the x-axis. [2]

8 **(i)** By first expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{6}\pi} (2\cos^3 x - \cos x)\, dx = \frac{5}{12}.$$ [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 11_9709_22/RP

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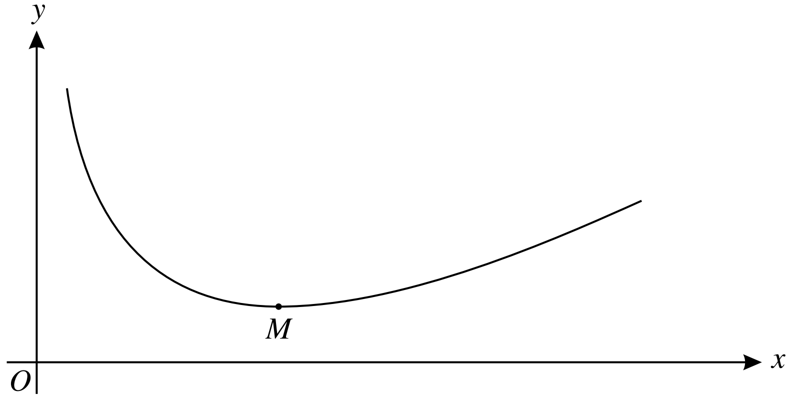
International Examinations

[Turn over

1 Solve the inequality $|x + 2| > |\frac{1}{2}x - 2|$. [4]

2 Use logarithms to solve the equation $4^{x+1} = 5^{2x-3}$, giving your answer correct to 3 significant figures. [4]

3



The diagram shows the curve $y = x - 2\ln x$ and its minimum point M.

(i) Find the x-coordinate of M. [2]

(ii) Use the trapezium rule with three intervals to estimate the value of

$$\int_2^5 (x - 2\ln x)\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

4 Find the exact value of the positive constant k for which

$$\int_0^k e^{4x}\,dx = \int_0^{2k} e^x\,dx.$$ [6]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\frac{1}{x} = \sin x,$$

where x is in radians, has only one root for $0 < x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 1.1$ and $x = 1.2$. [2]

(iii) Use the iterative formula $x_{n+1} = \dfrac{1}{\sin x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 The parametric equations of a curve are

$$x = 1 + 2\sin^2\theta, \quad y = 4\tan\theta.$$

(i) Show that $\dfrac{dy}{dx} = \dfrac{1}{\sin\theta\cos^3\theta}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$, giving your answer in the form $y = mx + c$. [4]

7 The polynomial $ax^3 - 3x^2 - 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

8 **(i)** Express $5\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\cos\theta - 3\sin\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Write down the least value of $15\cos\theta - 9\sin\theta$ as θ varies. [1]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9442318850

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This document consists of **3** printed pages and **1** blank page.

JC11 11_9709_23/RP

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Find the gradient of the curve $y = \ln(5x + 1)$ at the point where $x = 4$. [3]

2 Solve the inequality $|2x - 3| \leqslant |3x|$. [4]

3 Solve the equation $2\ln(x + 3) - \ln x = \ln(2x - 2)$. [5]

4 **(i)** Express $\cos^2 x$ in terms of $\cos 2x$. [1]

(ii) Hence show that

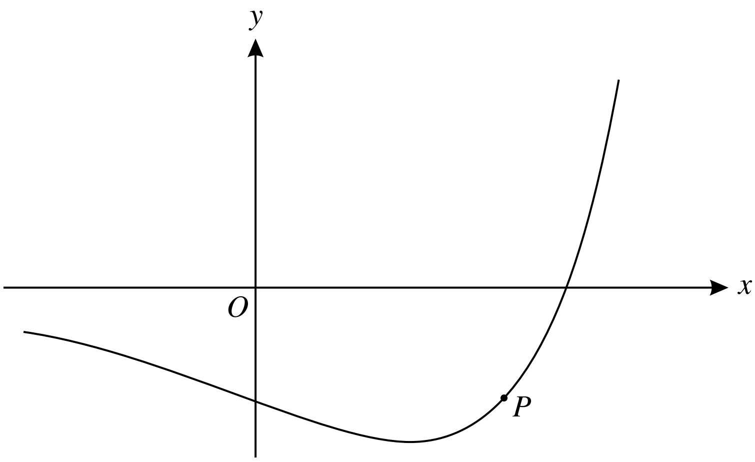
$$\int_0^{\frac{1}{6}\pi} (\cos^2 x + \sin 2x)\,dx = \tfrac{1}{8}\sqrt{3} + \tfrac{1}{12}\pi + \tfrac{1}{4}.$$ [5]

5 Solve the equation $5\sec^2 2\theta = \tan 2\theta + 9$, giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [6]

6 **(i)** The polynomial $x^4 + ax^3 - x^2 + bx + 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 1)$ and $(x + 2)$ are factors of $p(x)$. Find the values of a and b. [5]

(ii) When a and b have these values, find the quotient when $p(x)$ is divided by $x^2 + x - 2$. [3]

7



The diagram shows the curve $y = (x - 4)e^{\frac{1}{2}x}$. The curve has a gradient of 3 at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$x = 2 + 6e^{-\frac{1}{2}x}.$$ [4]

(ii) Verify that the equation in part **(i)** has a root between $x = 3.1$ and $x = 3.3$. [2]

(iii) Use the iterative formula $x_{n+1} = 2 + 6e^{-\frac{1}{2}x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 The equation of a curve is $2x^2 - 3x - 3y + y^2 = 6$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x - 3}{3 - 2y}$. [3]

(ii) Find the coordinates of the two points on the curve at which the gradient is -1. [6]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9424243601

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 Using the substitution $u = e^x$, or otherwise, solve the equation

$$e^x = 1 + 6e^{-x},$$

giving your answer correct to 3 significant figures. [4]

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \qquad y = 2\cos^3 t.$$

Find $\dfrac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0^\circ < \theta < 90^\circ$. [5]

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda\overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

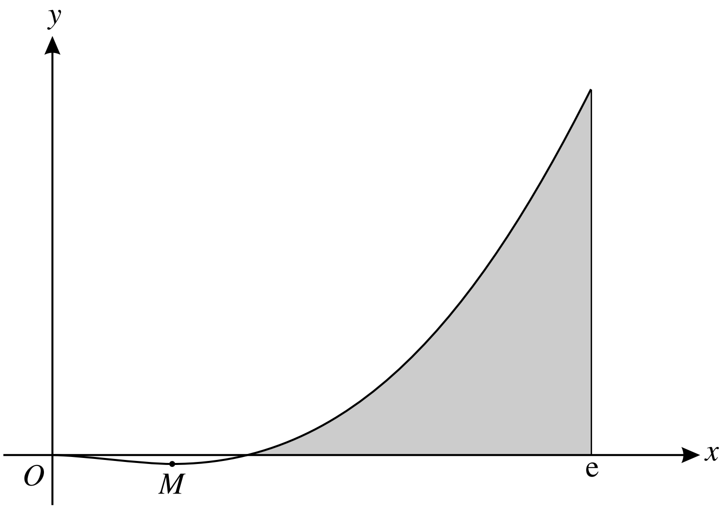
(iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

8 Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\tfrac{25}{2}\right)$. [5]

9



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0775793313

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1 Using the substitution $u = e^x$, or otherwise, solve the equation

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giving your answer correct to 3 significant figures. [4]

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0° < \theta < 90°$. [5]

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda\overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

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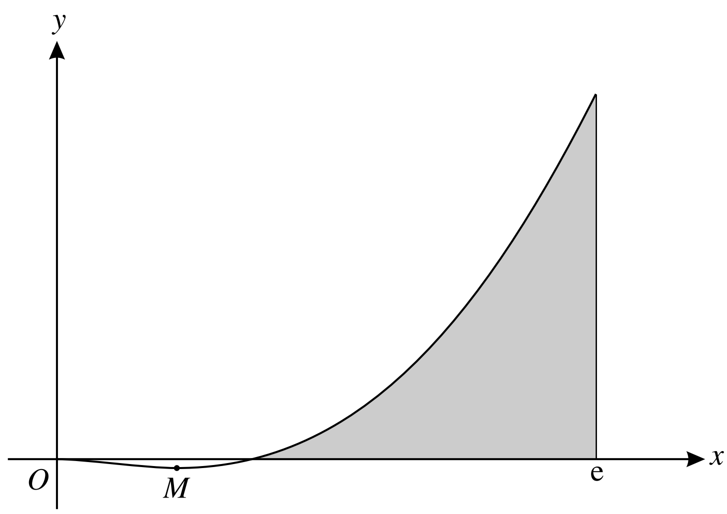
8 Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\tfrac{25}{2}\right)$. [5]

9



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4082927740

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1 Expand $\dfrac{16}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The equation of a curve is $y = \dfrac{e^{2x}}{1+e^{2x}}$. Show that the gradient of the curve at the point for which $x = \ln 3$ is $\frac{9}{50}$. [4]

3 **(i)** Express $8\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $8\cos\theta + 15\sin\theta = 12$, giving all solutions in the interval $0° < \theta < 360°$. [4]

4 During an experiment, the number of organisms present at time t days is denoted by N, where N is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for N in terms of t. [6]

(ii) State what happens to the number of organisms present after a long time. [1]

5 It is given that $\int_1^a x\ln x\,dx = 22$, where a is a constant greater than 1.

(i) Show that $a = \sqrt{\left(\dfrac{87}{2\ln a - 1}\right)}$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

6 The complex number w is defined by $w = -1 + i$.

(i) Find the modulus and argument of w^2 and w^3, showing your working. [4]

(ii) The points in an Argand diagram representing w and w^2 are the ends of a diameter of a circle. Find the equation of the circle, giving your answer in the form $|z - (a + bi)| = k$. [4]

7 The polynomial $\mathrm{p}(x)$ is defined by

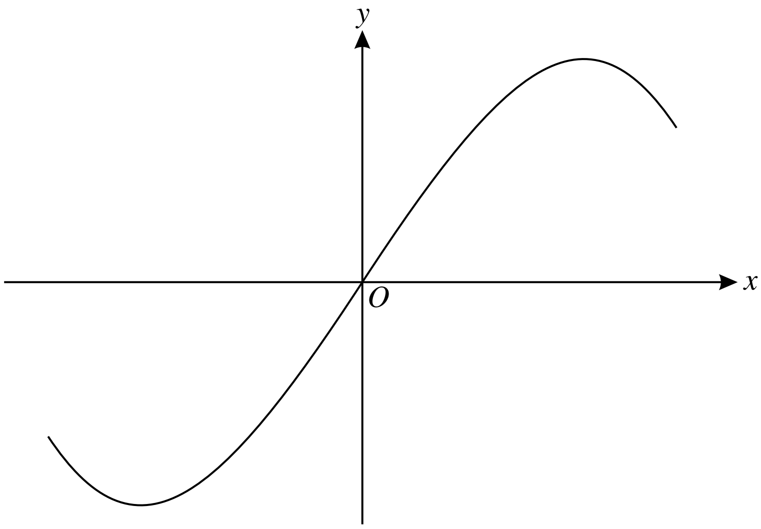
$$\mathrm{p}(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a and hence factorise $\mathrm{p}(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\dfrac{8x - 13}{\mathrm{p}(x)}$ in partial fractions. [5]

8



The diagram shows the curve with parametric equations

$$x = \sin t + \cos t, \quad y = \sin^3 t + \cos^3 t,$$

for $\frac{1}{4}\pi < t < \frac{5}{4}\pi$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3 \sin t \cos t$. [3]

(ii) Find the gradient of the curve at the origin. [2]

(iii) Find the values of t for which the gradient of the curve is 1, giving your answers correct to 2 significant figures. [4]

9 The line l has equation $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$, where a is a constant. The plane p has equation $2x - 2y + z = 10$.

(i) Given that l does not lie in p, show that l is parallel to p. [2]

(ii) Find the value of a for which l lies in p. [2]

(iii) It is now given that the distance between l and p is 6. Find the possible values of a. [5]

10 **(i)** Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2} x + \tan^n x)\,\mathrm{d}x = \frac{1}{n+1}.$$ [4]

(ii) Hence find the exact value of

(a) $\displaystyle\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\,\mathrm{d}x,$ [3]

(b) $\displaystyle\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\,\mathrm{d}x.$ [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/41**

Paper 4 Mechanics 1 **(M1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5593676829

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This document consists of **4** printed pages.

JC11 11_9709_41/2R

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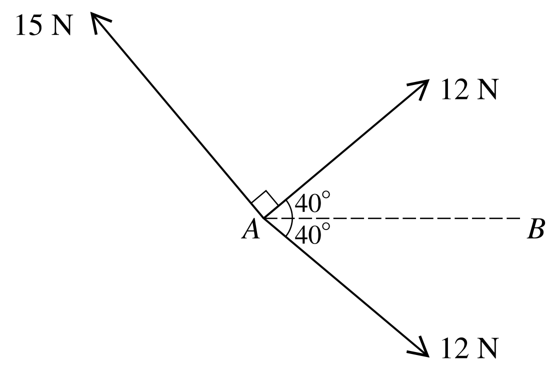
[Turn over

1 One end of a light inextensible string is attached to a block. The string is used to pull the block along a horizontal surface with a speed of $2\,\mathrm{m\,s^{-1}}$. The string makes an angle of 20° with the horizontal and the tension in the string is 25 N. Find the work done by the tension in a period of 8 seconds. [3]

2 Particles A of mass 0.65 kg and B of mass 0.35 kg are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. B is held at rest with the string taut and both of its straight parts vertical. The system is released from rest and the particles move vertically. Find the tension in the string and the magnitude of the resultant force exerted on the pulley by the string. [5]

3



Three coplanar forces of magnitudes 15 N, 12 N and 12 N act at a point A in directions as shown in the diagram.

(i) Find the component of the resultant of the three forces

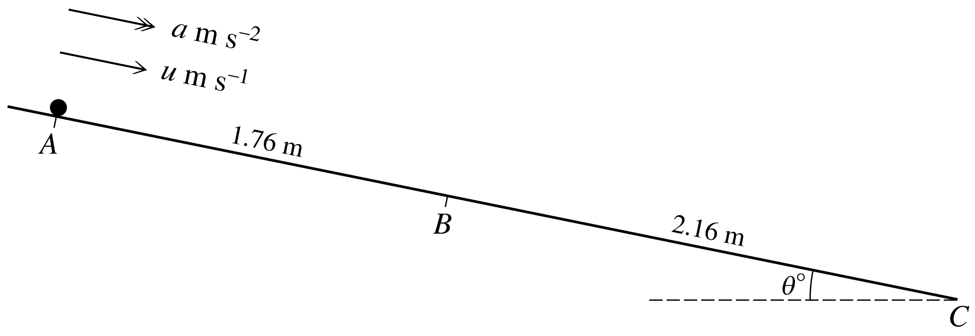
(a) in the direction of AB,

(b) perpendicular to AB.

[3]

(ii) Hence find the magnitude and direction of the resultant of the three forces. [3]

4



A, B and C are three points on a line of greatest slope of a smooth plane inclined at an angle of θ° to the horizontal. A is higher than B and B is higher than C, and the distances AB and BC are 1.76 m and 2.16 m respectively. A particle slides down the plane with constant acceleration $a\,\mathrm{m\,s^{-2}}$. The speed of the particle at A is $u\,\mathrm{m\,s^{-1}}$ (see diagram). The particle takes 0.8 s to travel from A to B and takes 1.4 s to travel from A to C. Find

(i) the values of u and a, [6]

(ii) the value of θ. [2]

5

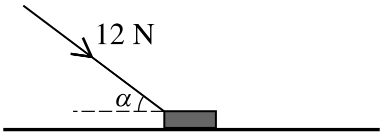


Fig. 1

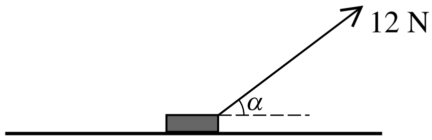


Fig. 2

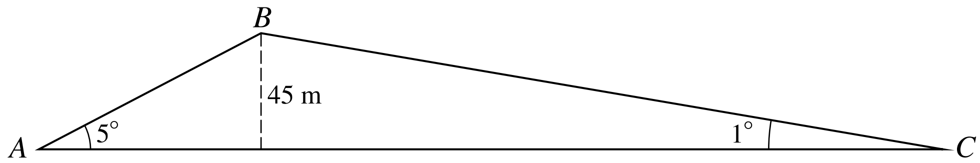
A block of mass 2 kg is at rest on a horizontal floor. The coefficient of friction between the block and the floor is μ. A force of magnitude 12 N acts on the block at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. When the applied force acts downwards as in Fig. 1 the block remains at rest.

(i) Show that $\mu \geqslant \frac{6}{17}$. [5]

When the applied force acts upwards as in Fig. 2 the block slides along the floor.

(ii) Find another inequality for μ. [3]

6



AB and *BC* are straight roads inclined at 5° to the horizontal and 1° to the horizontal respectively. *A* and *C* are at the same horizontal level and *B* is 45 m above the level of *A* and *C* (see diagram, which is not to scale). A car of mass 1200 kg travels from *A* to *C* passing through *B*.

(i) For the motion from *A* to *B*, the speed of the car is constant and the work done against the resistance to motion is 360 kJ. Find the work done by the car's engine from *A* to *B*. [3]

The resistance to motion is constant throughout the whole journey.

(ii) For the motion from *B* to *C* the work done by the driving force is 1660 kJ. Given that the speed of the car at *B* is 15 m s^{-1}, show that its speed at *C* is 29.9 m s^{-1}, correct to 3 significant figures. [4]

(iii) The car's driving force immediately after leaving *B* is 1.5 times the driving force immediately before reaching *C*. Find, correct to 2 significant figures, the ratio of the power developed by the car's engine immediately after leaving *B* to the power developed immediately before reaching *C*. [3]

[Question 7 is printed on the next page.]

7 A particle P starts from a point O and moves along a straight line. P's velocity t s after leaving O is $v\,\mathrm{m\,s^{-1}}$, where

$$v = 0.16t^{\frac{3}{2}} - 0.016t^2.$$

P comes to rest instantaneously at the point A.

(i) Verify that the value of t when P is at A is 100. [1]

(ii) Find the maximum speed of P in the interval $0 < t < 100$. [4]

(iii) Find the distance OA. [3]

(iv) Find the value of t when P passes through O on returning from A. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/42**

Paper 4 Mechanics 1 **(M1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4519530340

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This document consists of **4** printed pages.

1 A racing cyclist, whose mass with his cycle is 75 kg, works at a rate of 720 W while moving on a straight horizontal road. The resistance to the cyclist's motion is constant and equal to R N.

(i) Given that the cyclist is accelerating at 0.16 m s^{-2} at an instant when his speed is 12 m s^{-1}, find the value of R. [3]

(ii) Given that the cyclist's acceleration is positive, show that his speed is less than 15 m s^{-1}. [2]

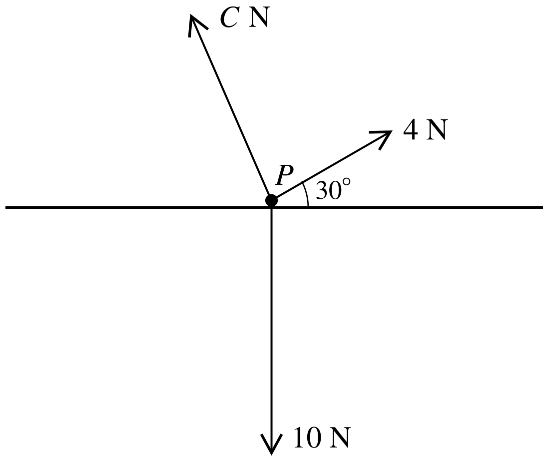
2 A block of mass 6 kg is sliding down a line of greatest slope of a plane inclined at 8° to the horizontal. The coefficient of friction between the block and the plane is 0.2.

(i) Find the deceleration of the block. [3]

(ii) Given that the initial speed of the block is 3 m s^{-1}, find how far the block travels. [2]

3 A particle P moves in a straight line. It starts from a point O on the line with velocity 1.8 m s^{-1}. The acceleration of P at time t s after leaving O is $0.8t^{-0.75}$ m s^{-2}. Find the displacement of P from O when $t = 16$. [6]

4



A particle P has weight 10 N and is in limiting equilibrium on a rough horizontal table. The forces shown in the diagram represent the weight of P, an applied force of magnitude 4 N acting on P in a direction at 30° above the horizontal, and the contact force exerted on P by the table (the resultant of the frictional and normal components) of magnitude C N.

(i) Find the value of C. [3]

(ii) Find the coefficient of friction between P and the table. [2]

5 Particles A and B, of masses 0.9 kg and 0.6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley. The system is released from rest with the string taut, with its straight parts vertical and with the particles at the same height above the horizontal floor. In the subsequent motion, B does not reach the pulley.

(i) Find the acceleration of A and the tension in the string during the motion before A hits the floor. [4]

After A hits the floor, B continues to move vertically upwards for a further 0.3 s.

(ii) Find the height of the particles above the floor at the instant that they started to move. [4]

6 A lorry of mass 16 000 kg climbs a straight hill $ABCD$ which makes an angle θ with the horizontal, where $\sin\theta = \frac{1}{20}$. For the motion from A to B, the work done by the driving force of the lorry is 1200 kJ and the resistance to motion is constant and equal to 1240 N. The speed of the lorry is $15\,\mathrm{m\,s^{-1}}$ at A and $12\,\mathrm{m\,s^{-1}}$ at B.

(i) Find the distance AB. [5]

For the motion from B to D the gain in potential energy of the lorry is 2400 kJ.

(ii) Find the distance BD. [1]

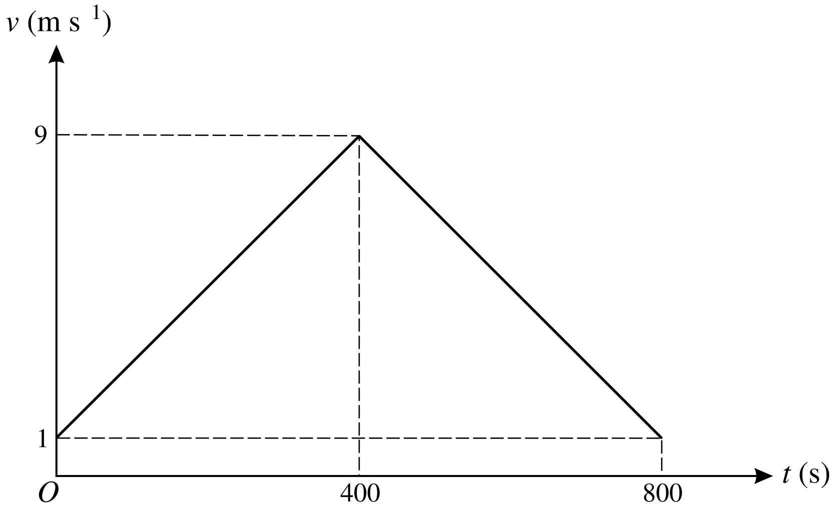
For the motion from B to D the driving force of the lorry is constant and equal to 7200 N. From B to C the resistance to motion is constant and equal to 1240 N and from C to D the resistance to motion is constant and equal to 1860 N.

(iii) Given that the speed of the lorry at D is $7\,\mathrm{m\,s^{-1}}$, find the distance BC. [4]

[Question 7 is printed on the next page.]

7 A tractor travels in a straight line from a point A to a point B. The velocity of the tractor is $v \text{ m s}^{-1}$ at time t s after leaving A.

(i)



The diagram shows an approximate velocity-time graph for the motion of the tractor. The graph consists of two straight line segments. Use the graph to find an approximation for

(a) the distance AB, [2]

(b) the acceleration of the tractor for $0 < t < 400$ and for $400 < t < 800$. [2]

(ii) The actual velocity of the tractor is given by $v = 0.04t - 0.000\,05t^2$ for $0 \leqslant t \leqslant 800$.

(a) Find the values of t for which the actual acceleration of the tractor is given correctly by the approximate velocity-time graph in part **(i)**. [3]

For the interval $0 \leqslant t \leqslant 400$, the approximate velocity of the tractor in part **(i)** is denoted by $v_1 \text{ m s}^{-1}$.

(b) Express v_1 in terms of t and hence show that $v_1 - v = 0.000\,05(t - 200)^2 - 1$. [2]

(c) Deduce that $-1 \leqslant v_1 - v \leqslant 1$. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/43**

Paper 4 Mechanics 1 **(M1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7905752214

READ THESE INSTRUCTIONS FIRST

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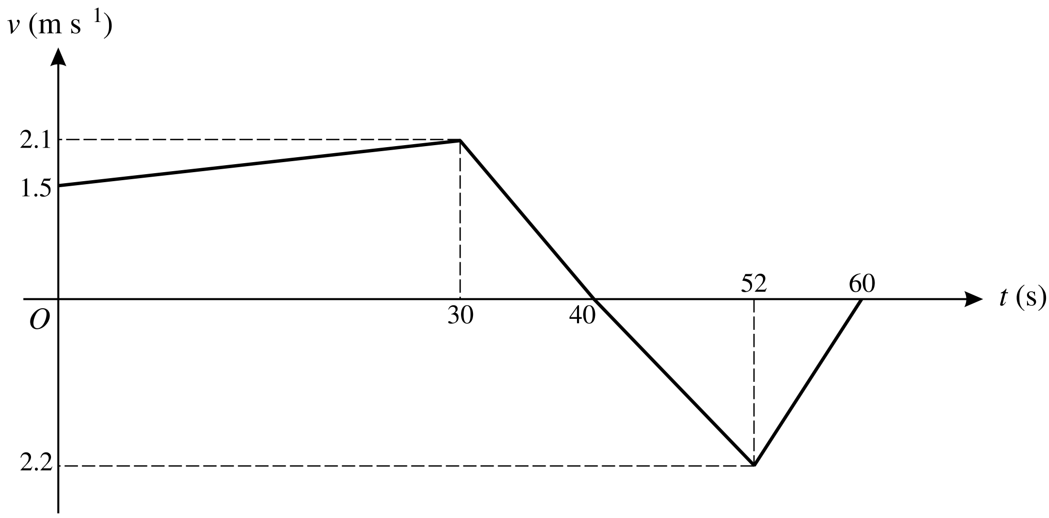
JC11 11_9709_43/2R

UNIVERSITY *of* CAMBRIDGE
International Examinations

1



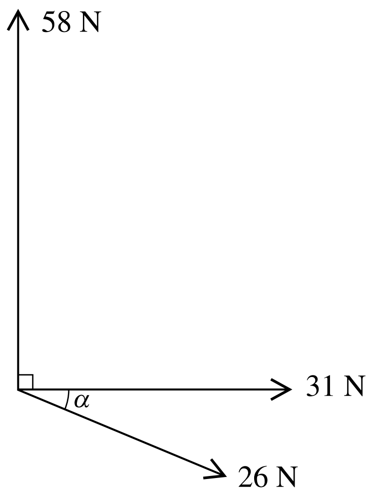
A woman walks in a straight line. The woman's velocity t seconds after passing through a fixed point A on the line is $v\,\mathrm{m\,s^{-1}}$. The graph of v against t consists of 4 straight line segments (see diagram). The woman is at the point B when $t = 60$. Find

(i) the woman's acceleration for $0 < t < 30$ and for $30 < t < 40$, [3]

(ii) the distance AB, [2]

(iii) the total distance walked by the woman. [1]

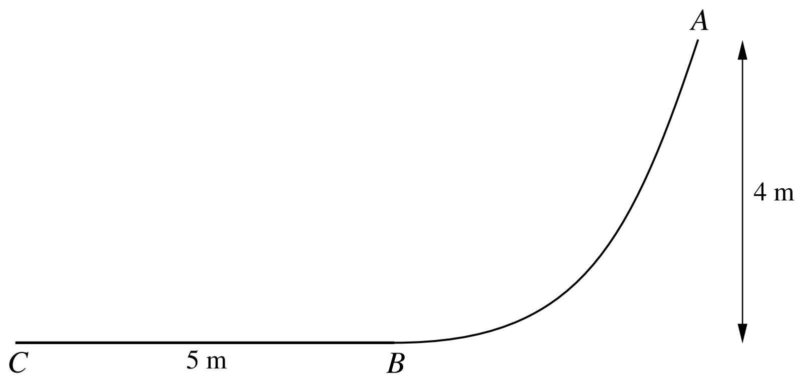
2



Coplanar forces of magnitudes 58 N, 31 N and 26 N act at a point in the directions shown in the diagram. Given that $\tan\alpha = \frac{5}{12}$, find the magnitude and direction of the resultant of the three forces. [6]

3 Particles P and Q are attached to opposite ends of a light inextensible string which passes over a fixed smooth pulley. The system is released from rest with the string taut, with its straight parts vertical, and with both particles at a height of 2 m above horizontal ground. P moves vertically downwards and does not rebound when it hits the ground. At the instant that P hits the ground, Q is at the point X, from where it continues to move vertically upwards without reaching the pulley. Given that P has mass 0.9 kg and that the tension in the string is 7.2 N while P is moving, find the total distance travelled by Q from the instant it first reaches X until it returns to X. [6]

4



ABC is a vertical cross-section of a surface. The part of the surface containing AB is smooth and A is 4 m higher than B. The part of the surface containing BC is horizontal and the distance BC is 5 m (see diagram). A particle of mass 0.8 kg is released from rest at A and slides along ABC. Find the speed of the particle at C in each of the following cases.

(i) The horizontal part of the surface is smooth. [3]

(ii) The coefficient of friction between the particle and the horizontal part of the surface is 0.3. [3]

5 A particle P moves in a straight line. It starts from rest at A and comes to rest instantaneously at B. The velocity of P at time t seconds after leaving A is $v\,\mathrm{m\,s^{-1}}$, where $v = 6t^2 - kt^3$ and k is a constant.

(i) Find an expression for the displacement of P from A in terms of t and k. [2]

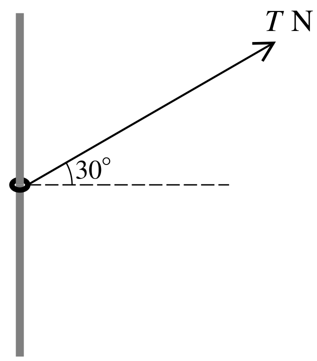
(ii) Find an expression for t in terms of k when P is at B. [1]

Given that the distance AB is 108 m, find

(iii) the value of k, [2]

(iv) the maximum value of v when the particle is moving from A towards B. [3]

6



The diagram shows a ring of mass 2 kg threaded on a fixed rough vertical rod. A light string is attached to the ring and is pulled upwards at an angle of 30° to the horizontal. The tension in the string is T N. The coefficient of friction between the ring and the rod is 0.24. Find the two values of T for which the ring is in limiting equilibrium. [8]

7 A car of mass 600 kg travels along a straight horizontal road starting from a point A. The resistance to motion of the car is 750 N.

(i) The car travels from A to B at constant speed in 100 s. The power supplied by the car's engine is constant and equal to 30 kW. Find the distance AB. [3]

(ii) The car's engine is switched off at B and the car's speed decreases until the car reaches C with a speed of $20\,\text{m}\,\text{s}^{-1}$. Find the distance BC. [3]

(iii) The car's engine is switched on at C and the power it supplies is constant and equal to 30 kW. The car takes 14 s to travel from C to D and reaches D with a speed of $30\,\text{m}\,\text{s}^{-1}$. Find the distance CD. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/51**

Paper 5 Mechanics 2 **(M2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3469964593

READ THESE INSTRUCTIONS FIRST

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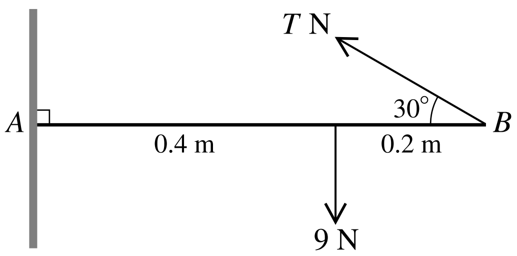
JC11 11_9709_51/3R

UNIVERSITY *of* CAMBRIDGE
International Examinations

1

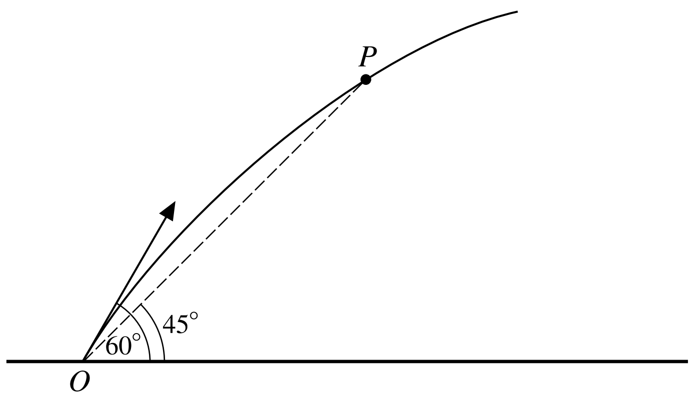


A non-uniform rod AB, of length 0.6 m and weight 9 N, has its centre of mass 0.4 m from A. The end A of the rod is in contact with a rough vertical wall. The rod is held in equilibrium, perpendicular to the wall, by means of a light string attached to B. The string is inclined at 30° to the horizontal. The tension in the string is T N (see diagram).

(i) Calculate T. [2]

(ii) Find the least possible value of the coefficient of friction at A. [3]

2



A particle P is projected from a point O at an angle of 60° above horizontal ground. At an instant 0.6 s after projection, the angle of elevation of P from O is 45° (see diagram).

(i) Show that the speed of projection of P is $8.20\,\mathrm{m\,s^{-1}}$, correct to 3 significant figures. [4]

(ii) Calculate the time after projection when the direction of motion of P is 45° above the horizontal. [3]

3 One end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.25 kg. P hangs in equilibrium below O.

(i) Calculate the distance OP. [2]

The particle P is raised, and is released from rest at O.

(ii) Calculate the speed of P when it passes through the equilibrium position. [3]

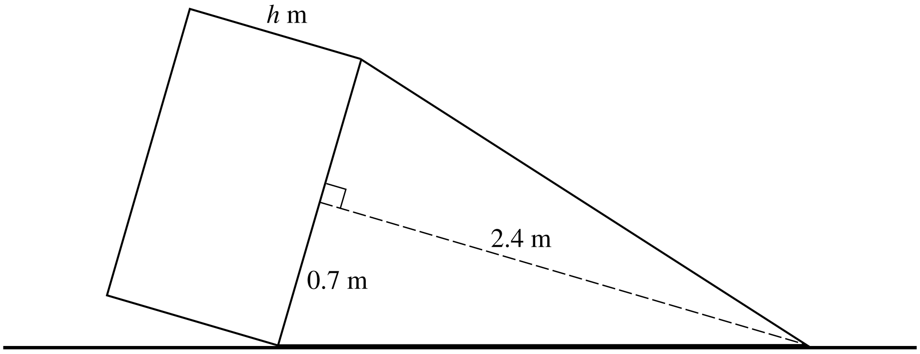
(iii) Calculate the greatest value of the distance OP in the subsequent motion. [3]

4 A uniform solid cylinder has radius 0.7 m and height h m. A uniform solid cone has base radius 0.7 m and height 2.4 m. The cylinder and the cone both rest in equilibrium each with a circular face in contact with a horizontal plane. The plane is now tilted so that its inclination to the horizontal, $\theta°$, is increased gradually until the cone is about to topple.

(i) Find the value of θ at which the cone is about to topple. [2]

(ii) Given that the cylinder does not topple, find the greatest possible value of h. [2]

The plane is returned to a horizontal position, and the cone is fixed to one end of the cylinder so that the plane faces coincide. It is given that the weight of the cylinder is three times the weight of the cone. The curved surface of the cone is placed on the horizontal plane (see diagram).



(iii) Given that the solid immediately topples, find the least possible value of h. [5]

5 A ball of mass 0.05 kg is released from rest at a height h m above the ground. At time t s after its release, the downward velocity of the ball is $v\,\text{m s}^{-1}$. Air resistance opposes the motion of the ball with a force of magnitude $0.01v$ N.

(i) Show that $\dfrac{dv}{dt} = 10 - 0.2v$. Hence find v in terms of t. [6]

(ii) Given that the ball reaches the ground when $t = 2$, calculate h. [4]

6 A smooth bead B of mass 0.3 kg is threaded on a light inextensible string of length 0.9 m. One end of the string is attached to a fixed point A, and the other end of the string is attached to a fixed point C which is vertically below A. The tension in the string is T N, and the bead rotates with angular speed $\omega\,\text{rad s}^{-1}$ in a horizontal circle about the vertical axis through A and C.

(i) Given that B moves in a circle with centre C and radius 0.2 m, calculate ω, and hence find the kinetic energy of B. [5]

(ii) Given instead that angle $ABC = 90°$, and that AB makes an angle $\tan^{-1}\left(\frac{1}{2}\right)$ with the vertical, calculate T and ω. [6]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/52**

Paper 5 Mechanics 2 **(M2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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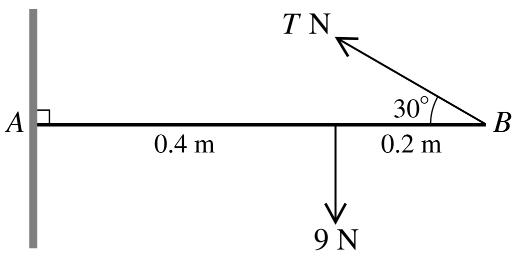
JC11 11_9709_52/RP

UNIVERSITY *of* CAMBRIDGE International Examinations

1

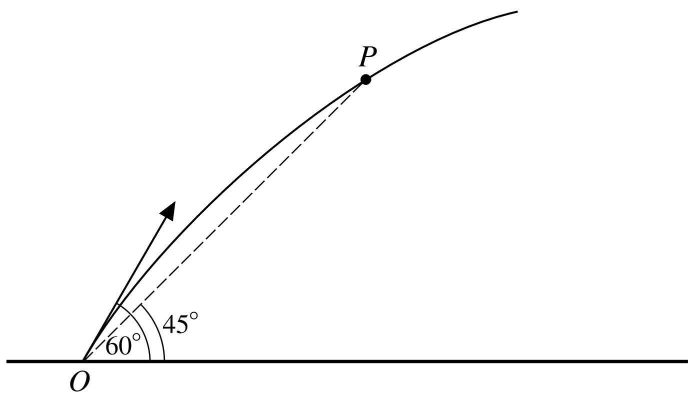


A non-uniform rod AB, of length 0.6 m and weight 9 N, has its centre of mass 0.4 m from A. The end A of the rod is in contact with a rough vertical wall. The rod is held in equilibrium, perpendicular to the wall, by means of a light string attached to B. The string is inclined at 30° to the horizontal. The tension in the string is T N (see diagram).

(i) Calculate T. [2]

(ii) Find the least possible value of the coefficient of friction at A. [3]

2



A particle P is projected from a point O at an angle of 60° above horizontal ground. At an instant 0.6 s after projection, the angle of elevation of P from O is 45° (see diagram).

(i) Show that the speed of projection of P is 8.20 m s^{-1}, correct to 3 significant figures. [4]

(ii) Calculate the time after projection when the direction of motion of P is 45° above the horizontal. [3]

3 One end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N is attached to a fixed point O. The other end of the string is attached to a particle P of mass 0.25 kg. P hangs in equilibrium below O.

(i) Calculate the distance OP. [2]

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(ii) Calculate the speed of P when it passes through the equilibrium position. [3]

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(i) Find the value of θ at which the cone is about to topple. [2]

(ii) Given that the cylinder does not topple, find the greatest possible value of h. [2]

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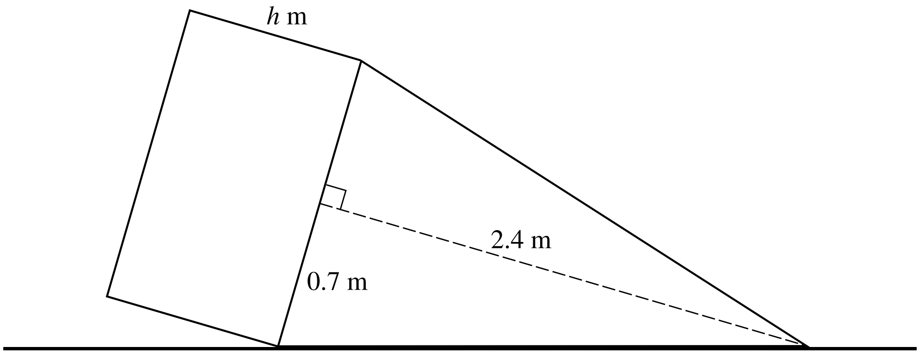

(iii) Given that the solid immediately topples, find the least possible value of h. [5]

5 A ball of mass 0.05 kg is released from rest at a height h m above the ground. At time t s after its release, the downward velocity of the ball is $v\,\text{m}\,\text{s}^{-1}$. Air resistance opposes the motion of the ball with a force of magnitude $0.01v$ N.

(i) Show that $\frac{dv}{dt} = 10 - 0.2v$. Hence find v in terms of t. [6]

(ii) Given that the ball reaches the ground when $t = 2$, calculate h. [4]

6 A smooth bead B of mass 0.3 kg is threaded on a light inextensible string of length 0.9 m. One end of the string is attached to a fixed point A, and the other end of the string is attached to a fixed point C which is vertically below A. The tension in the string is T N, and the bead rotates with angular speed $\omega\,\text{rad}\,\text{s}^{-1}$ in a horizontal circle about the vertical axis through A and C.

(i) Given that B moves in a circle with centre C and radius 0.2 m, calculate ω, and hence find the kinetic energy of B. [5]

(ii) Given instead that angle $ABC = 90°$, and that AB makes an angle $\tan^{-1}\left(\frac{1}{2}\right)$ with the vertical, calculate T and ω. [6]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/53**

Paper 5 Mechanics 2 **(M2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8795650684

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1 A particle is projected with speed $17\,\text{m}\,\text{s}^{-1}$ at an angle of $50°$ above the horizontal from a point on horizontal ground. Calculate the speed of the particle 2 s after the instant of projection. [3]

2 An object is made from two identical uniform rods AB and BC each of length 0.6 m and weight 7 N. The rods are rigidly joined to each other at B and angle $ABC = 90°$.

(i) Calculate the distance of the centre of mass of the object from B. [1]

The object is freely suspended at A and a force of magnitude F N is applied to the rod BC at C. The object is in equilibrium with AB inclined at $45°$ to the horizontal.

(ii) (a)

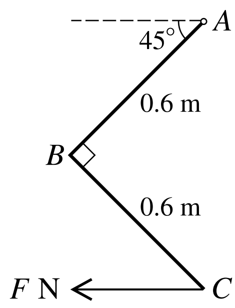


Fig. 1

Calculate F given that the force acts horizontally as shown in Fig. 1. [2]

(b)

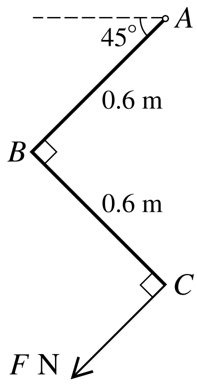


Fig. 2

Calculate F given instead that the force acts perpendicular to the rod as shown in Fig. 2. [2]

3 A particle P is projected with speed $25\,\text{m}\,\text{s}^{-1}$ at an angle of $45°$ above the horizontal from a point O on horizontal ground. At time t s after projection the horizontal and vertically upward displacements of P from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the path of P is $y = x - 0.016x^2$. [4]

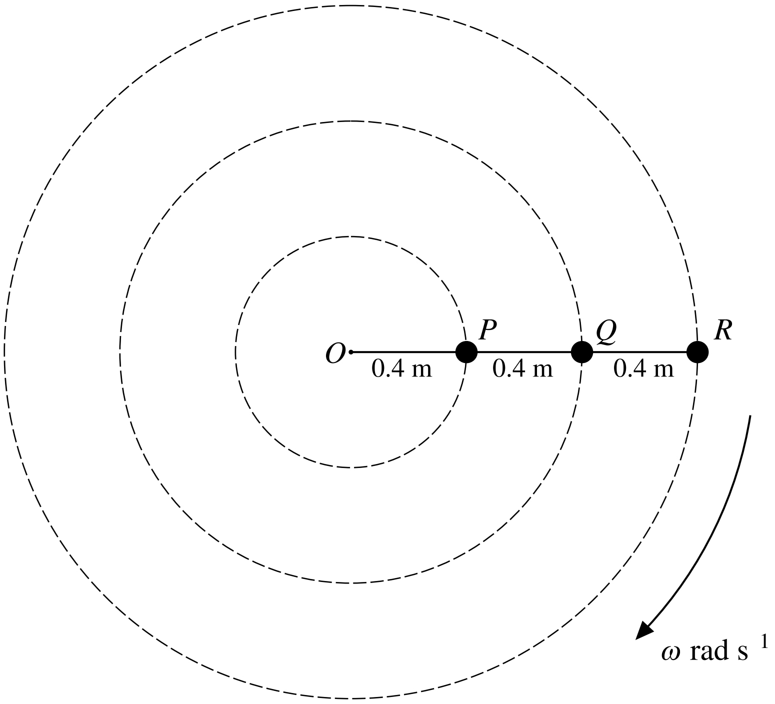
(ii) Calculate the horizontal distance between the two positions at which P is 2.4 m above the ground. [2]

4 A particle P of mass 0.4 kg is projected horizontally with velocity 8 m s^{-1} from a point O on a smooth horizontal surface. The motion of P is opposed by a resisting force of magnitude $0.2v^2$ N, where v m s^{-1} is the velocity of P at time t s after projection.

(i) Show that $v = \dfrac{8}{1 + 4t}$. [4]

(ii) Calculate the distance OP when $t = 1.5$. [4]

5



One end of a light inextensible string of length 1.2 m is attached to a fixed point O on a smooth horizontal surface. Particles P, Q and R are attached to the string so that $OP = PQ = QR = 0.4$ m. The particles rotate in horizontal circles about O with constant angular speed ω rad s^{-1} and with O, P, Q and R in a straight line (see diagram). R has mass 0.2 kg, and the tensions in the parts of the string attached to Q are 6 N and 10 N.

(i) Show that $\omega = 5$. [2]

(ii) Calculate the mass of Q. [3]

(iii) Given that the kinetic energy of P is equal to the kinetic energy of R, calculate the tension in the part of the string attached to O. [4]

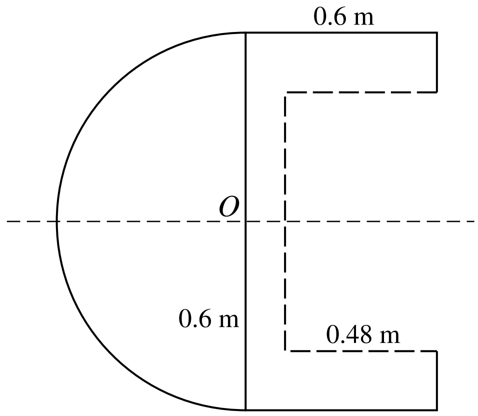
[Questions 6 and 7 are printed on the next page.]

6 A uniform solid consists of a hemisphere with centre O and radius 0.6 m joined to a cylinder of radius 0.6 m and height 0.6 m. The plane face of the hemisphere coincides with one of the plane faces of the cylinder.

(i) Calculate the distance of the centre of mass of the solid from O. [4]

[The volume of a hemisphere of radius r is $\frac{2}{3}\pi r^3$.]

(ii)



A cylindrical hole, of length 0.48 m, starting at the plane face of the solid, is made along the axis of symmetry (see diagram). The resulting solid has its centre of mass at O. Show that the area of the cross-section of the hole is $\frac{3}{16}\pi\ \mathrm{m}^2$. [4]

(iii) It is possible to increase the length of the cylindrical hole so that the solid still has its centre of mass at O. State the increase in the length of the hole. [1]

7 One end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N is attached to a particle P of mass 0.8 kg. The other end of the string is attached to a fixed point O at the top of a smooth plane inclined at 30° to the horizontal. The particle rests in equilibrium on the plane.

(i) Calculate the extension of the string. [2]

P is projected from its equilibrium position up the plane along a line of greatest slope. In the subsequent motion P just reaches O, and later just reaches the foot of the plane. Calculate

(ii) the speed of projection of P, [4]

(iii) the length of the line of greatest slope of the plane. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8893649739

READ THESE INSTRUCTIONS FIRST

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JC11 11_9709_61/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate. [5]

2 The values, x, in a particular set of data are summarised by

$$\Sigma(x-25) = 133, \qquad \Sigma(x-25)^2 = 3762.$$

The mean, $\bar{x}$, is 28.325.

(i) Find the standard deviation of x. [4]

(ii) Find Σx^2. [2]

3 A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable X is the number of girls in the team.

(i) Draw up a probability distribution table for X. [4]

(ii) Given that $\mathrm{E}(X) = \frac{5}{2}$, calculate $\mathrm{Var}(X)$. [2]

4 The marks of the pupils in a certain class in a History examination are as follows.

28 33 55 38 42 39 27 48 51 37 57 49 33

The marks of the pupils in a Physics examination are summarised as follows.

Lower quartile: 28, Median: 39, Upper quartile: 67.
The lowest mark was 17 and the highest mark was 74.

(i) Draw box-and-whisker plots in a single diagram on graph paper to illustrate the marks for History and Physics. [5]

(ii) State one difference, which can be seen from the diagram, between the marks for History and Physics. [1]

5 The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

(i) Find the standard deviation of the weights of the letters. [3]

(ii) Find the probability that a randomly chosen letter weighs more than 13 g. [3]

(iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean. [3]

6 **(a)** Find the number of different ways in which the 12 letters of the word STRAWBERRIES can be arranged

(i) if there are no restrictions, [2]

(ii) if the 4 vowels A, E, E, I must all be together. [3]

(b) **(i)** 4 astronauts are chosen from a certain number of candidates. If order of choosing is not taken into account, the number of ways the astronauts can be chosen is 3876. How many ways are there if order of choosing is taken into account? [2]

(ii) 4 astronauts are chosen to go on a mission. Each of these astronauts can take 3 personal possessions with him. How many different ways can these 12 possessions be arranged in a row if each astronaut's possessions are kept together? [2]

7 Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

(i) Find the probability that exactly two of the selected balls have the same number. [5]

(ii) Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2. [2]

(iii) Event X is 'exactly two of the selected balls have the same number'. Event Y is 'the ball selected from bag A has number 2'. Showing your working, determine whether events X and Y are independent or not. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC11 11_9709_62/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The following are the times, in minutes, taken by 11 runners to complete a 10 km run.

48.3 55.2 59.9 67.7 60.5 75.6 62.5 57.4 53.4 49.2 64.1

Find the mean and standard deviation of these times. [3]

2 Twelve coins are tossed and placed in a line. Each coin can show either a head or a tail.

(i) Find the number of different arrangements of heads and tails which can be obtained. [2]

(ii) Find the number of different arrangements which contain 7 heads and 5 tails. [1]

3 **(a)** Geoff wishes to plant 25 flowers in a flower-bed. He can choose from 15 different geraniums, 10 different roses and 8 different lilies. He wants to have at least 11 geraniums and also to have the same number of roses and lilies. Find the number of different selections of flowers he can make. [4]

(b) Find the number of different ways in which the 9 letters of the word GREENGAGE can be arranged if exactly two of the Gs are next to each other. [3]

4 The weights of 220 sausages are summarised in the following table.

Weight (grams)	<20	<30	<40	<45	<50	<60	<70
Cumulative frequency	0	20	50	100	160	210	220

(i) State which interval the median weight lies in. [1]

(ii) Find the smallest possible value and the largest possible value for the interquartile range. [2]

(iii) State how many sausages weighed between 50 g and 60 g. [1]

(iv) On graph paper, draw a histogram to represent the weights of the sausages. [4]

5 A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time. [3]

(iii) The spinner is spun 136 times. Use a suitable approximation to find the probability that it lands on the blue side fewer than 20 times. [5]

6 There are a large number of students in Luttley College. 60% of the students are boys. Students can choose exactly one of Games, Drama or Music on Friday afternoons. It is found that 75% of the boys choose Games, 10% of the boys choose Drama and the remainder of the boys choose Music. Of the girls, 30% choose Games, 55% choose Drama and the remainder choose Music.

(i) 6 boys are chosen at random. Find the probability that fewer than 3 of them choose Music. [3]

(ii) 5 Drama students are chosen at random. Find the probability that at least 1 of them is a boy. [6]

7 **(i)** In a certain country, the daily minimum temperature, in °C, in winter has the distribution $N(8, 24)$. Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C. [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

(ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]

(iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]

(iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that $P(X < 25) = 0.648$.

(i) Find the values of μ and σ. [4]

(ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25. [2]

2 In a group of 30 teenagers, 13 of the 18 males watch ‘Kops are Kids’ on television and 3 of the 12 females watch ‘Kops are Kids’.

(i) Find the probability that a person chosen at random from the group is either female or watches ‘Kops are Kids’ or both. [4]

(ii) Showing your working, determine whether the events ‘the person chosen is male’ and ‘the person chosen watches Kops are Kids’ are independent or not. [2]

3 A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

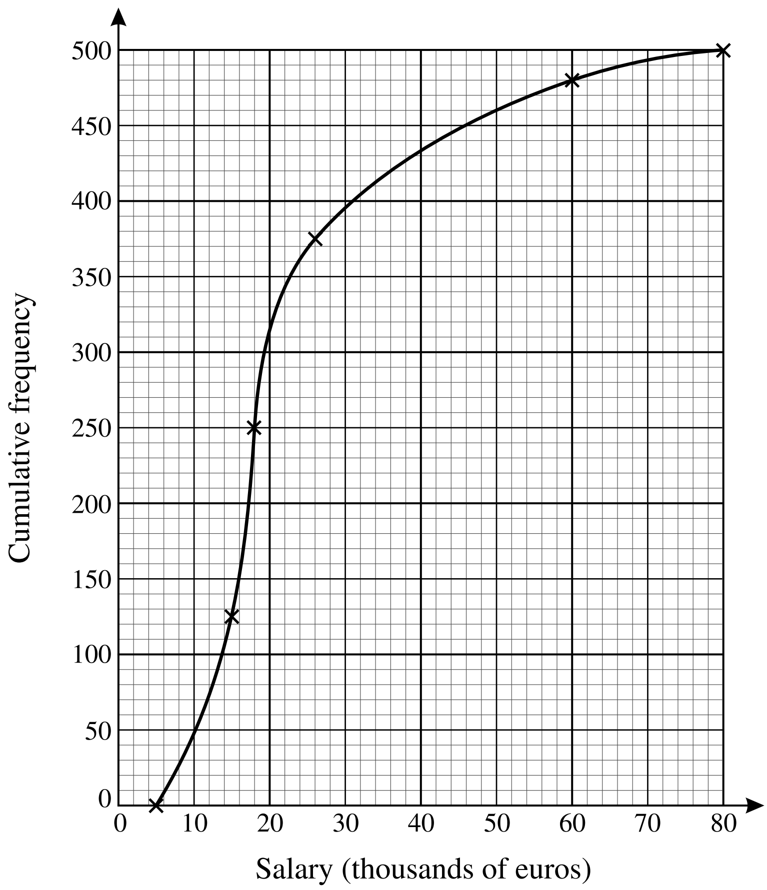
4 Mary saves her digital images on her computer in three separate folders named ‘Family’, ‘Holiday’ and ‘Friends’. Her family folder contains 3 images, her holiday folder contains 4 images and her friends folder contains 8 images. All the images are different.

(i) Find in how many ways she can arrange these 15 images in a row across her computer screen if she keeps the images from each folder together. [3]

(ii) Find the number of different ways in which Mary can choose 6 of these images if there are 2 from each folder. [2]

(iii) Find the number of different ways in which Mary can choose 6 of these images if there are at least 3 images from the friends folder and at least 1 image from each of the other two folders. [4]

5



The cumulative frequency graph shows the annual salaries, in thousands of euros, of a random sample of 500 adults with jobs, in France. It has been plotted using grouped data. You may assume that the lowest salary is 5000 euros and the highest salary is 80 000 euros.

(i) On graph paper, draw a box-and-whisker plot to illustrate these salaries. [4]

(ii) Comment on the salaries of the people in this sample. [1]

(iii) An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(a) How high must a salary be in order to be classified as an outlier? [3]

(b) Show that none of the salaries is low enough to be classified as an outlier. [1]

6 Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or –. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A–, 2% are B–, 1% are AB– and 7% are O–.

(i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+. [6]

(ii) A random sample of 150 people in the UK is taken. Find the probability that more than 60 people are group A+. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The random variable X has the distribution $\mathrm{Po}(1.3)$. The random variable Y is defined by $Y = 2X$.

(i) Find the mean and variance of Y. [3]

(ii) Give a reason why the variable Y does not have a Poisson distribution. [1]

2 An engineering test consists of 100 multiple-choice questions. Each question has 5 suggested answers, only one of which is correct. Ashok knows nothing about engineering, but he claims that his general knowledge enables him to get more questions correct than just by guessing. Ashok actually gets 27 answers correct. Use a suitable approximating distribution to test at the 5% significance level whether his claim is justified. [5]

3 Three coats of paint are sprayed onto a surface. The thicknesses, in millimetres, of the three coats have independent distributions $\mathrm{N}(0.13,\ 0.02^2)$, $\mathrm{N}(0.14,\ 0.03^2)$ and $\mathrm{N}(0.10,\ 0.01^2)$. Find the probability that, at a randomly chosen place on the surface, the total thickness of the three coats of paint is less than 0.30 millimetres. [5]

4 The volumes of juice in bottles of Apricola are normally distributed. In a random sample of 8 bottles, the volumes of juice, in millilitres, were found to be as follows.

332 334 330 328 331 332 329 333

(i) Find unbiased estimates of the population mean and variance. [3]

A random sample of 50 bottles of Apricola gave unbiased estimates of 331 millilitres and 4.20 millilitres^2 for the population mean and variance respectively.

(ii) Use this sample of size 50 to calculate a 98% confidence interval for the population mean. [3]

(iii) The manufacturer claims that the mean volume of juice in all bottles is 333 millilitres. State, with a reason, whether your answer to part **(ii)** supports this claim. [1]

5 The management of a factory thinks that the mean time required to complete a particular task is 22 minutes. The times, in minutes, taken by employees to complete this task have a normal distribution with mean μ and standard deviation 3.5. An employee claims that 22 minutes is not long enough for the task. In order to investigate this claim, the times for a random sample of 12 employees are used to test the null hypothesis $\mu = 22$ against the alternative hypothesis $\mu > 22$ at the 5% significance level.

(i) Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} > 23.7$ (correct to 3 significant figures), where $\bar{x}$ is the sample mean. [3]

(ii) Find the probability of a Type II error given that the actual mean time is 25.8 minutes. [4]

6 Customers arrive at an enquiry desk at a constant average rate of 1 every 5 minutes.

(i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. [2]

(iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. [3]

(iv) Find an estimate of the probability that fewer than 30 customers will arrive during a randomly chosen 2-hour period. [4]

7

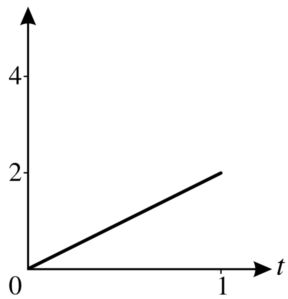

Fig. 1

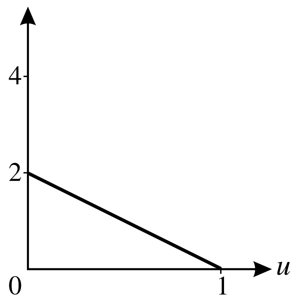

Fig. 2

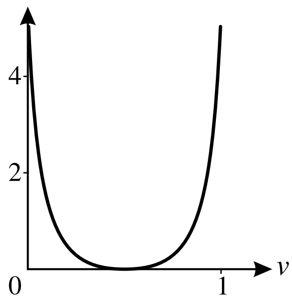

Fig. 3

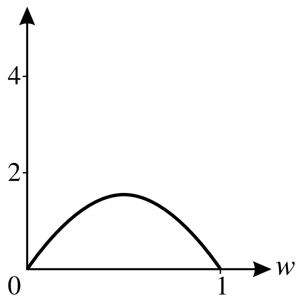

Fig. 4

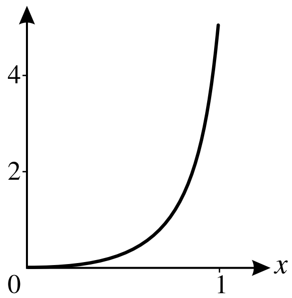

Fig. 5

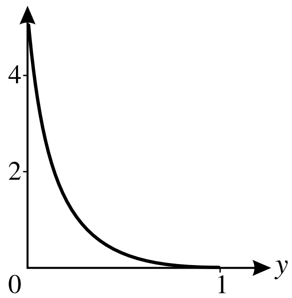

Fig. 6

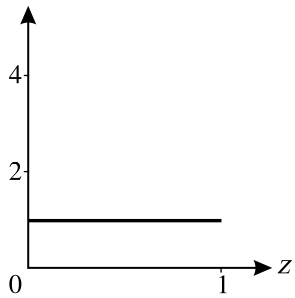

Fig. 7

Each of the random variables T, U, V, W, X, Y and Z takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

(i) (a) Which of these variables has the largest median? [1]

(b) Which of these variables has the largest standard deviation? Explain your answer. [2]

(ii) Use Fig. 2 to find $P(U < 0.5)$. [2]

(iii) The probability density function of X is given by

$$f(x) = \begin{cases} ax^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and n are positive constants.

(a) Show that $a = n + 1$. [3]

(b) Given that $E(X) = \frac{5}{6}$, find a and n. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6109556378

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JC11 11_9709_72/FP

UNIVERSITY of CAMBRIDGE International Examinations

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7

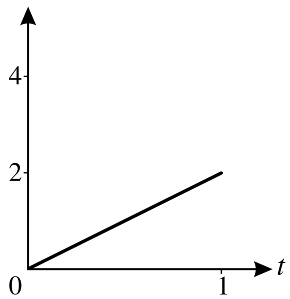

Fig. 1

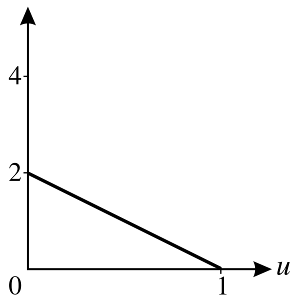

Fig. 2

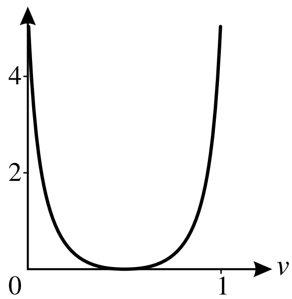

Fig. 3

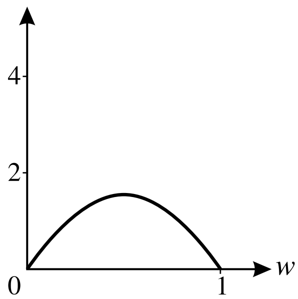

Fig. 4

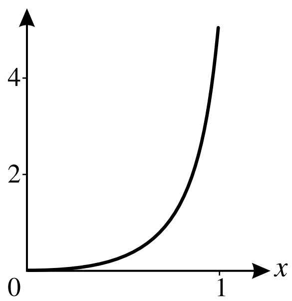

Fig. 5

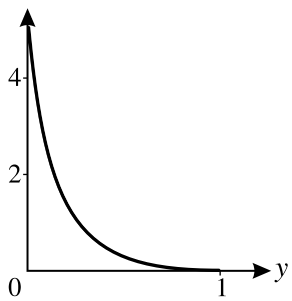

Fig. 6

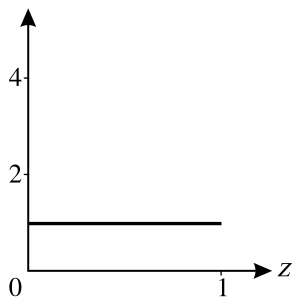

Fig. 7

Each of the random variables T, U, V, W, X, Y and Z takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

(i) (a) Which of these variables has the largest median? [1]

(b) Which of these variables has the largest standard deviation? Explain your answer. [2]

(ii) Use Fig. 2 to find $P(U < 0.5)$. [2]

(iii) The probability density function of X is given by

$$f(x) = \begin{cases} ax^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and n are positive constants.

(a) Show that $a = n + 1$. [3]

(b) Given that $E(X) = \frac{5}{6}$, find a and n. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics **(S2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5565682908

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC11 11_9709_73/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Test scores, X, have mean 54 and variance 144. The scores are scaled using the formula $Y = a + bX$, where a and b are constants and $b > 0$. The scaled scores, Y, have mean 50 and variance 100. Find the values of a and b. [4]

2 35% of a random sample of n students walk to college. This result is used to construct an approximate 98% confidence interval for the population proportion of students who walk to college. Given that the width of this confidence interval is 0.157, correct to 3 significant figures, find n. [5]

3 Jack has to choose a random sample of 8 people from the 750 members of a sports club.

(i) Explain fully how he can use random numbers to choose the sample. [3]

Jack asks each person in the sample how much they spent last week in the club café. The results, in dollars, were as follows.

15 25 30 8 12 18 27 25

(ii) Find unbiased estimates of the population mean and variance. [3]

(iii) Explain briefly what is meant by 'population' in this question. [1]

4 The random variable X has probability density function given by

$$f(x) = \begin{cases} ke^{-x} & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \dfrac{e}{e - 1}$. [3]

(ii) Find $E(X)$ in terms of e. [4]

5 Records show that the distance driven by a bus driver in a week is normally distributed with mean 1150 km and standard deviation 105 km. New driving regulations are introduced and in the next 20 weeks he drives a total of 21 800 km.

(i) Stating any assumption(s), test, at the 1% significance level, whether his mean weekly driving distance has decreased. [6]

(ii) A similar test at the 1% significance level was carried out using the data from another 20 weeks. State the probability of a Type I error and describe what is meant by a Type I error in this context. [2]

6 Ranjit goes to mathematics lectures and physics lectures. The length, in minutes, of a mathematics lecture is modelled by the variable X with distribution $N(36, 3.5^2)$. The length, in minutes, of a physics lecture is modelled by the independent variable Y with distribution $N(55, 5.2^2)$.

(i) Find the probability that the total length of two mathematics lectures and one physics lecture is less than 140 minutes. [4]

(ii) Ranjit calculates how long he will need to spend revising the content of each lecture as follows. Each minute of a mathematics lecture requires 1 minute of revision and each minute of a physics lecture requires $1\frac{1}{2}$ minutes of revision. Find the probability that the total revision time required for one mathematics lecture and one physics lecture is more than 100 minutes. [4]

7 The numbers of men and women who visit a clinic each hour are independent Poisson variables with means 2.4 and 2.8 respectively.

(i) Find the probability that, in a half-hour period,

(a) 2 or more men and 1 or more women will visit the clinic, [4]

(b) a total of 3 or more people will visit the clinic. [3]

(ii) Find the probability that, in a 10-hour period, a total of more than 60 people will visit the clinic. [4]

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