CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS 8719/3

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 **(P3)**

MAY/JUNE SESSION 2002

1 hour 45 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

1 Prove the identity

$$\cot\theta - \tan\theta \equiv 2\cot 2\theta.$$ [3]

2 Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 The polynomial $x^4 + 4x^2 + x + a$ is denoted by $\mathrm{p}(x)$. It is given that $(x^2 + x + 2)$ is a factor of $\mathrm{p}(x)$.

Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

4 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right),$$

with initial value $x_1 = 1$, converges to α.

(i) Use this formula to find α correct to 2 decimal places, showing the result of each iteration. [3]

(ii) State an equation satisfied by α, and hence find the exact value of α. [2]

5 The equation of a curve is $y = 2\cos x + \sin 2x$. Find the x-coordinates of the stationary points on the curve for which $0 < x < \pi$, and determine the nature of each of these stationary points. [7]

6 Let $\mathrm{f}(x) = \dfrac{4x}{(3x+1)(x+1)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = 1 - \ln 2$. [5]

7 In a certain chemical process a substance is being formed, and t minutes after the start of the process there are m grams of the substance present. In the process the rate of increase of m is proportional to $(50 - m)^2$. When $t = 0$, $m = 0$ and $\frac{dm}{dt} = 5$.

(i) Show that m satisfies the differential equation

$$\frac{dm}{dt} = 0.002(50 - m)^2. \qquad [2]$$

(ii) Solve the differential equation, and show that the solution can be expressed in the form

$$m = 50 - \frac{500}{t + 10}. \qquad [5]$$

(iii) Calculate the mass of the substance when $t = 10$, and find the time taken for the mass to increase from 0 to 45 grams. [2]

(iv) State what happens to the mass of the substance as t becomes very large. [1]

8 The straight line l passes through the points A and B whose position vectors are $\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ respectively. The plane p has equation $x + 3y - 2z = 3$.

(i) Given that l intersects p, find the position vector of the point of intersection. [4]

(ii) Find the equation of the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = 1$. [6]

9 The complex number $1 + i\sqrt{3}$ is denoted by u.

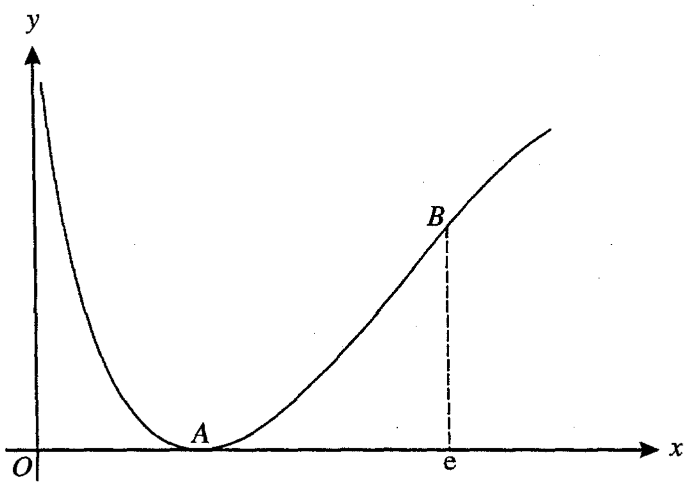
(i) Express u in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Hence, or otherwise, find the modulus and argument of u^2 and u^3. [5]

(ii) Show that u is a root of the equation $z^2 - 2z + 4 = 0$, and state the other root of this equation. [2]

(iii) Sketch an Argand diagram showing the points representing the complex numbers i and u. Shade the region whose points represent every complex number z satisfying both the inequalities

$$|z - i| \leqslant 1 \quad \text{and} \quad \arg z \geqslant \arg u. \qquad [4]$$

10



The function f is defined by $f(x) = (\ln x)^2$ for $x > 0$. The diagram shows a sketch of the graph of $y = f(x)$. The minimum point of the graph is A. The point B has x-coordinate e.

(i) State the x-coordinate of A. [1]

(ii) Show that $f''(x) = 0$ at B. [4]

(iii) Use the substitution $x = e^u$ to show that the area of the region bounded by the x-axis, the line $x = e$, and the part of the curve between A and B is given by

$$\int_0^1 u^2 e^u \, du.$$ [3]

(iv) Hence, or otherwise, find the exact value of this area. [3]