



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Advanced Level

**MATHEMATICS**

**9709/31**

Paper 3 Pure Mathematics 3 (P3)

**October/November 2013**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.



- 1 The equation of a curve is  $y = \frac{1+x}{1+2x}$  for  $x > -\frac{1}{2}$ . Show that the gradient of the curve is always negative. [3]

- 2 Solve the equation  $2|3^x - 1| = 3^x$ , giving your answers correct to 3 significant figures. [4]

- 3 Find the exact value of  $\int_1^4 \frac{\ln x}{\sqrt{x}} dx$ . [5]

- 4 The parametric equations of a curve are

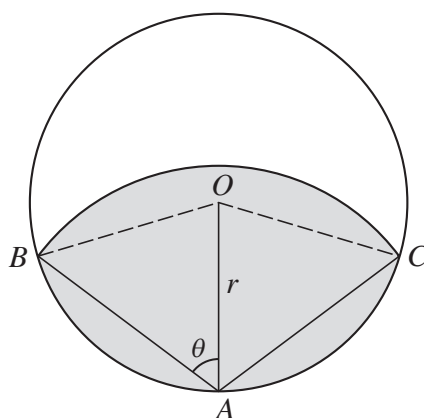
$$x = e^{-t} \cos t, \quad y = e^{-t} \sin t.$$

Show that  $\frac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$ . [6]

- 5 (i) Prove that  $\cot \theta + \tan \theta \equiv 2 \operatorname{cosec} 2\theta$ . [3]

- (ii) Hence show that  $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta d\theta = \frac{1}{2} \ln 3$ . [4]

6



In the diagram,  $A$  is a point on the circumference of a circle with centre  $O$  and radius  $r$ . A circular arc with centre  $A$  meets the circumference at  $B$  and  $C$ . The angle  $OAB$  is  $\theta$  radians. The shaded region is bounded by the circumference of the circle and the arc with centre  $A$  joining  $B$  and  $C$ . The area of the shaded region is equal to half the area of the circle.

- (i) Show that  $\cos 2\theta = \frac{2 \sin 2\theta - \pi}{4\theta}$ . [5]

- (ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2} \cos^{-1} \left( \frac{2 \sin 2\theta_n - \pi}{4\theta_n} \right),$$

with initial value  $\theta_1 = 1$ , to determine  $\theta$  correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let  $f(x) = \frac{2x^2 - 7x - 1}{(x - 2)(x^2 + 3)}$ .

(i) Express  $f(x)$  in partial fractions. [5]

(ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

8 Throughout this question the use of a calculator is not permitted.

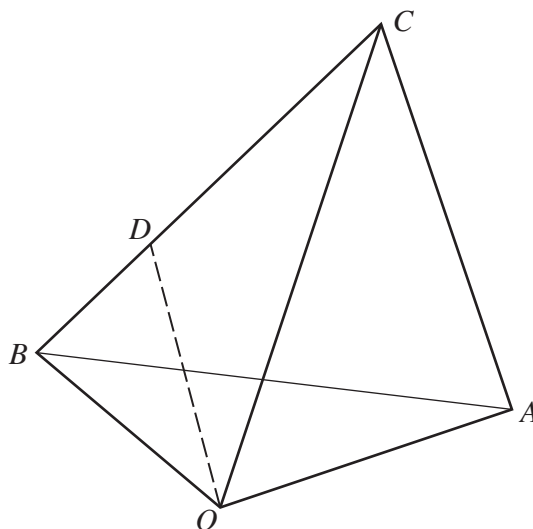
(a) The complex numbers  $u$  and  $v$  satisfy the equations

$$u + 2v = 2i \quad \text{and} \quad iu + v = 3.$$

Solve the equations for  $u$  and  $v$ , giving both answers in the form  $x + iy$ , where  $x$  and  $y$  are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers  $z$  satisfying  $|z + i| = 1$  and the locus representing complex numbers  $w$  satisfying  $\arg(w - 2) = \frac{3}{4}\pi$ . Find the least value of  $|z - w|$  for points on these loci. [5]

9



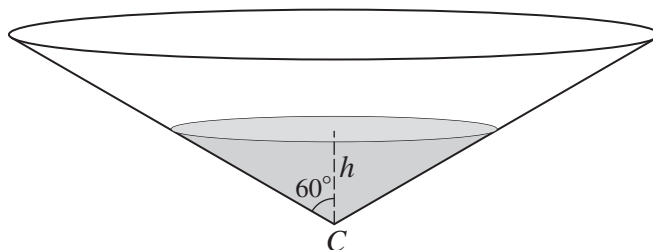
The diagram shows three points  $A$ ,  $B$  and  $C$  whose position vectors with respect to the origin  $O$  are given by  $\vec{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ ,  $\vec{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$  and  $\vec{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ . The point  $D$  lies on  $BC$ , between  $B$  and  $C$ , and is such that  $CD = 2DB$ .

(i) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ . [6]

(ii) Find the position vector of  $D$ . [1]

(iii) Show that the length of the perpendicular from  $A$  to  $OD$  is  $\frac{1}{3}\sqrt{65}$ . [4]

[Question 10 is printed on the next page.]



A tank containing water is in the form of a cone with vertex  $C$ . The axis is vertical and the semi-vertical angle is  $60^\circ$ , as shown in the diagram. At time  $t = 0$ , the tank is full and the depth of water is  $H$ . At this instant, a tap at  $C$  is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to  $\sqrt{h}$ , where  $h$  is the depth of water at time  $t$ . The tank becomes empty when  $t = 60$ .

- (i) Show that  $h$  and  $t$  satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where  $A$  is a positive constant.

[4]

- (ii) Solve the differential equation given in part (i) and obtain an expression for  $t$  in terms of  $h$  and  $H$ .

[6]

- (iii) Find the time at which the depth reaches  $\frac{1}{2}H$ .

[1]

[The volume  $V$  of a cone of vertical height  $h$  and base radius  $r$  is given by  $V = \frac{1}{3}\pi r^2 h$ .]



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Advanced Level

---

**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3 (P3)

**October/November 2013**

**1 hour 45 minutes**

Additional Materials:      Answer Booklet/Paper  
   Graph Paper  
   List of Formulae (MF9)

---

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

---

This document consists of **4** printed pages.



- 1 The equation of a curve is  $y = \frac{1+x}{1+2x}$  for  $x > -\frac{1}{2}$ . Show that the gradient of the curve is always negative. [3]

- 2 Solve the equation  $2|3^x - 1| = 3^x$ , giving your answers correct to 3 significant figures. [4]

- 3 Find the exact value of  $\int_1^4 \frac{\ln x}{\sqrt{x}} dx$ . [5]

- 4 The parametric equations of a curve are

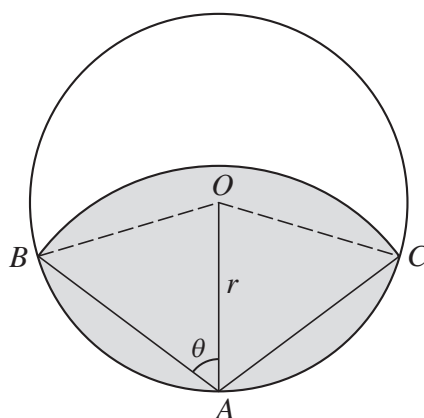
$$x = e^{-t} \cos t, \quad y = e^{-t} \sin t.$$

Show that  $\frac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$ . [6]

- 5 (i) Prove that  $\cot \theta + \tan \theta \equiv 2 \operatorname{cosec} 2\theta$ . [3]

- (ii) Hence show that  $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta d\theta = \frac{1}{2} \ln 3$ . [4]

6



In the diagram,  $A$  is a point on the circumference of a circle with centre  $O$  and radius  $r$ . A circular arc with centre  $A$  meets the circumference at  $B$  and  $C$ . The angle  $OAB$  is  $\theta$  radians. The shaded region is bounded by the circumference of the circle and the arc with centre  $A$  joining  $B$  and  $C$ . The area of the shaded region is equal to half the area of the circle.

- (i) Show that  $\cos 2\theta = \frac{2 \sin 2\theta - \pi}{4\theta}$ . [5]

- (ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2} \cos^{-1} \left( \frac{2 \sin 2\theta_n - \pi}{4\theta_n} \right),$$

with initial value  $\theta_1 = 1$ , to determine  $\theta$  correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let  $f(x) = \frac{2x^2 - 7x - 1}{(x - 2)(x^2 + 3)}$ .

(i) Express  $f(x)$  in partial fractions. [5]

(ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

8 Throughout this question the use of a calculator is not permitted.

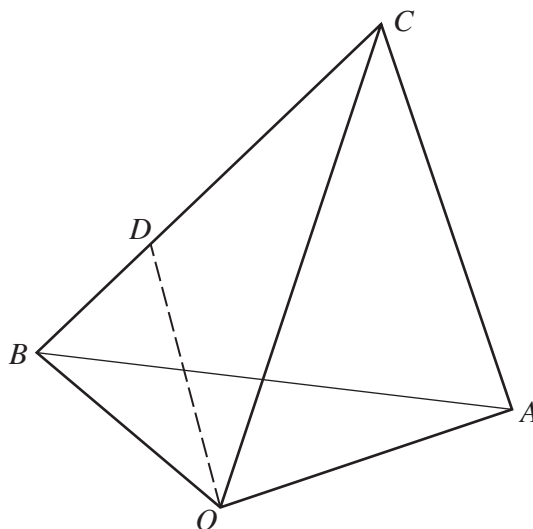
(a) The complex numbers  $u$  and  $v$  satisfy the equations

$$u + 2v = 2i \quad \text{and} \quad iu + v = 3.$$

Solve the equations for  $u$  and  $v$ , giving both answers in the form  $x + iy$ , where  $x$  and  $y$  are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers  $z$  satisfying  $|z + i| = 1$  and the locus representing complex numbers  $w$  satisfying  $\arg(w - 2) = \frac{3}{4}\pi$ . Find the least value of  $|z - w|$  for points on these loci. [5]

9



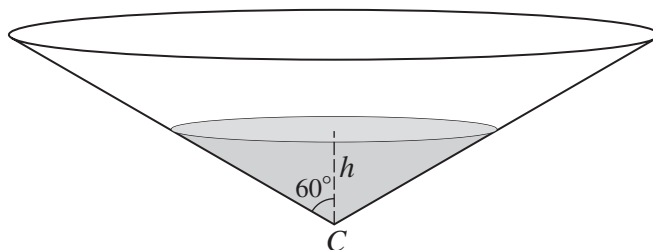
The diagram shows three points  $A$ ,  $B$  and  $C$  whose position vectors with respect to the origin  $O$  are given by  $\vec{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ ,  $\vec{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$  and  $\vec{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ . The point  $D$  lies on  $BC$ , between  $B$  and  $C$ , and is such that  $CD = 2DB$ .

(i) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ . [6]

(ii) Find the position vector of  $D$ . [1]

(iii) Show that the length of the perpendicular from  $A$  to  $OD$  is  $\frac{1}{3}\sqrt{65}$ . [4]

[Question 10 is printed on the next page.]



A tank containing water is in the form of a cone with vertex  $C$ . The axis is vertical and the semi-vertical angle is  $60^\circ$ , as shown in the diagram. At time  $t = 0$ , the tank is full and the depth of water is  $H$ . At this instant, a tap at  $C$  is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to  $\sqrt{h}$ , where  $h$  is the depth of water at time  $t$ . The tank becomes empty when  $t = 60$ .

- (i) Show that  $h$  and  $t$  satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where  $A$  is a positive constant.

[4]

- (ii) Solve the differential equation given in part (i) and obtain an expression for  $t$  in terms of  $h$  and  $H$ .

[6]

- (iii) Find the time at which the depth reaches  $\frac{1}{2}H$ .

[1]

[The volume  $V$  of a cone of vertical height  $h$  and base radius  $r$  is given by  $V = \frac{1}{3}\pi r^2 h$ .]





UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Advanced Level

**MATHEMATICS**

**9709/33**

Paper 3 Pure Mathematics 3 (**P3**)

**October/November 2013**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 Given that  $2 \ln(x + 4) - \ln x = \ln(x + a)$ , express  $x$  in terms of  $a$ . [4]
- 2 Use the substitution  $u = 3x + 1$  to find  $\int \frac{3x}{3x+1} dx$ . [4]
- 3 The polynomial  $f(x)$  is defined by
- $$f(x) = x^3 + ax^2 - ax + 14,$$
- where  $a$  is a constant. It is given that  $(x + 2)$  is a factor of  $f(x)$ .
- (i) Find the value of  $a$ . [2]
- (ii) Show that, when  $a$  has this value, the equation  $f(x) = 0$  has only one real root. [3]
- 4 A curve has equation  $3e^{2x}y + e^xy^3 = 14$ . Find the gradient of the curve at the point  $(0, 2)$ . [5]
- 5 It is given that  $\int_0^p 4xe^{-\frac{1}{2}x} dx = 9$ , where  $p$  is a positive constant.
- (i) Show that  $p = 2 \ln\left(\frac{8p+16}{7}\right)$ . [5]
- (ii) Use an iterative process based on the equation in part (i) to find the value of  $p$  correct to 3 significant figures. Use a starting value of 3.5 and give the result of each iteration to 5 significant figures. [3]
- 6 Two planes have equations  $3x - y + 2z = 9$  and  $x + y - 4z = -1$ .
- (i) Find the acute angle between the planes. [3]
- (ii) Find a vector equation of the line of intersection of the planes. [6]
- 7 (i) Given that  $\sec \theta + 2 \operatorname{cosec} \theta = 3 \operatorname{cosec} 2\theta$ , show that  $2 \sin \theta + 4 \cos \theta = 3$ . [3]
- (ii) Express  $2 \sin \theta + 4 \cos \theta$  in the form  $R \sin(\theta + \alpha)$  where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the value of  $\alpha$  correct to 2 decimal places. [3]
- (iii) Hence solve the equation  $\sec \theta + 2 \operatorname{cosec} \theta = 3 \operatorname{cosec} 2\theta$  for  $0^\circ < \theta < 360^\circ$ . [4]
- 8 (i) Express  $\frac{7x^2 + 8}{(1+x)^2(2-3x)}$  in partial fractions. [5]
- (ii) Hence expand  $\frac{7x^2 + 8}{(1+x)^2(2-3x)}$  in ascending powers of  $x$  up to and including the term in  $x^2$ , simplifying the coefficients. [5]

- 9 (a) Without using a calculator, use the formula for the solution of a quadratic equation to solve

$$(2 - i)z^2 + 2z + 2 + i = 0.$$

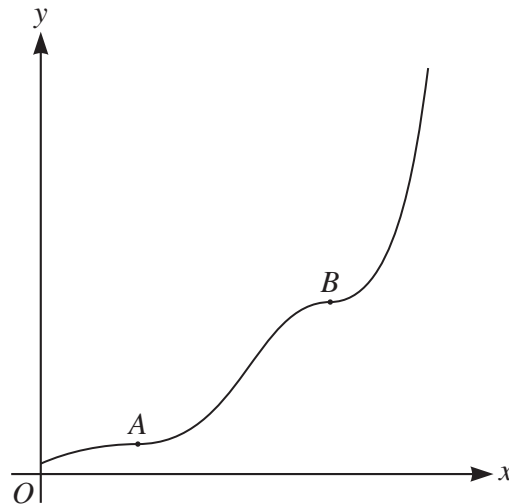
Give your answers in the form  $a + bi$ .

[5]

- (b) The complex number  $w$  is defined by  $w = 2e^{\frac{1}{4}\pi i}$ . In an Argand diagram, the points  $A$ ,  $B$  and  $C$  represent the complex numbers  $w$ ,  $w^3$  and  $w^*$  respectively (where  $w^*$  denotes the complex conjugate of  $w$ ). Draw the Argand diagram showing the points  $A$ ,  $B$  and  $C$ , and calculate the area of triangle  $ABC$ .

[5]

10



A particular solution of the differential equation

$$3y^2 \frac{dy}{dx} = 4(y^3 + 1) \cos^2 x$$

is such that  $y = 2$  when  $x = 0$ . The diagram shows a sketch of the graph of this solution for  $0 \leq x \leq 2\pi$ ; the graph has stationary points at  $A$  and  $B$ . Find the  $y$ -coordinates of  $A$  and  $B$ , giving each coordinate correct to 1 decimal place.

[10]

**BLANK PAGE**

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.