TAYLOR'S UNIVERSITY COLLEGE

Wisdom - Integrity - Excellence

CAMBRIDGE A LEVEL PROGRAMME

# A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2008

**(July 2007 Intake)**

**Wednesday** **3 September 2008** **1.30 pm – 3.15 pm**

## MATHEMATICS 9709/3

**PAPER 3 Pure Mathematics 3 (P3)** 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.



**[Turn over**

1 Solve $3^{2x} - 3^{x+1} = 10$ giving the value of $x$ to 3 significant figures. [3]

2 A curve $C$ is given parametrically by equations

$$x = 2 + t$$
$$y = 1 - t^2.$$

Show that the normal at the point with parameter $t$ has equation

$$x - 2ty = 2t^3 - t + 2.$$ [4]

3 The expression $12\cos\theta - 5\sin\theta$ is denoted by $f(\theta)$. Find the value of $R$ and $\alpha$, where $R$ is positive and the angle $\alpha$ is acute, such that for all values of $\theta$

$$f(\theta) = R\cos(\theta + \alpha)$$

giving the value of $\alpha$ in degrees to the nearest $0.1°$. [3]

Hence, calculate the values of $\theta$, where $0° < \theta < 360°$, such that

$$12\cos\theta - 5\sin\theta + 4 = 0$$

giving your answers to the nearest degree. [3]

4 (i) Sketch on the same axes the graphs of $y = |2x-6|$ and $y = |3x+1|$ [2]

(ii) Find the values of $x$ for which $|2x-6| = |3x+1|$ [2]

(iii) Hence or otherwise find the range of values of $x$ for which $|2x-6| > |3x+1|$. [2]

5 Show that $\int_0^2 xe^{2x}\,dx = \frac{1}{4}(3e^4 + 1)$ [4]

Hence, find the exact value of

$$\int_0^2 x^2e^{2x}\,dx$$ [3]

6 (i) Find the modulus and argument of each of the two complex numbers $z$ satisfying the equation $\dfrac{1+z^2}{1-z^2}=i$ [4]

(ii) Sketch in an Argand diagram the set of points satisfying both

$|z|<|z-1|$ and $-\dfrac{\pi}{4}<\arg z<\dfrac{\pi}{4}$. [3]

7 (i) Given that $I=\displaystyle\int_{-1}^{1}\frac{1}{1+e^{-x}}\,dx$, show that the estimate of $I$ obtained by using the trapezium rule with three ordinates is 1(to the nearest integer). [3]

(ii) By means of substitution $u=e^x$, show that the estimate obtained in (i) is a good estimate. [4]

8 The surface area of a pond is 10000 $m^2$ and a part of it is covered by algae. At any instant, the area of algae is increasing at a rate which is directly proportional to its area at that instant.

(i) Obtain a differential equation that relates the area of algae $x$ $m^2$, with time $t$ (in days). [2]

Given that when $t=0$, $x=100$ and when $t=7$, $x=1000$,

(ii) Show that $\ln\left(\dfrac{x}{100}\right)=\dfrac{t}{7}\ln(10)$ [5]

By giving your answer correct to 3 significant figures,

(iii) Find the area of the pond **not** covered by algae when $t=10.5$ days. [2]

(iv) Find value of $t$ when the algae covers half the surface of the pond. [2]

[Turn Over

**9** The points $A$, $B$ and $C$ have position vectors, with respect to the origin, given by $\overrightarrow{OA} = (5i - j - 3k)$, $\overrightarrow{OB} = (-4i + 4j - k)$ and $\overrightarrow{OC} = (5i - 2j + 11k)$,

Find (i) a vector equation for the line $BC$, [2]

(ii) a vector equation for the plane $OAB$, [3]

(iii) the cosine of the acute angle between the lines $OA$ and $OB$ [2]

Obtain, the Cartesian equation for $\Pi$, the plane which passes through $A$ and is perpendicular to $BC$. [3]

**10** (i) Find the constants $A$, $B$ and $C$ in the identity $\dfrac{5x^2 + 4x - 20}{(x+2)(x^2+4)} \equiv \dfrac{A}{(x+2)} + \dfrac{Bx + C}{(x^2+4)}$ [3]

(ii) Find $\displaystyle\int_0^2 \frac{x}{x^2+4}\,dx$ [2]

(iii) Use the substitute $x = 2\tan\theta$ to find $\displaystyle\int_0^2 \frac{1}{x^2+4}\,dx$, [4]

(iv) Use the answers to parts (i), (ii) and (iii) above to show that

$\displaystyle\int_0^2 \frac{5x^2 + 4x - 20}{(x+2)(x^2+4)}\,dx = a\ln 2 - b\pi$, where $a$ and $b$ are positive integers.

Find $a$ and $b$. [5]