TAYLOR'S UNIVERSITY COLLEGE

Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2007

(June 2006 Intake)

Wednesday **5 September 2007** **1.30 pm – 3.15 pm**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

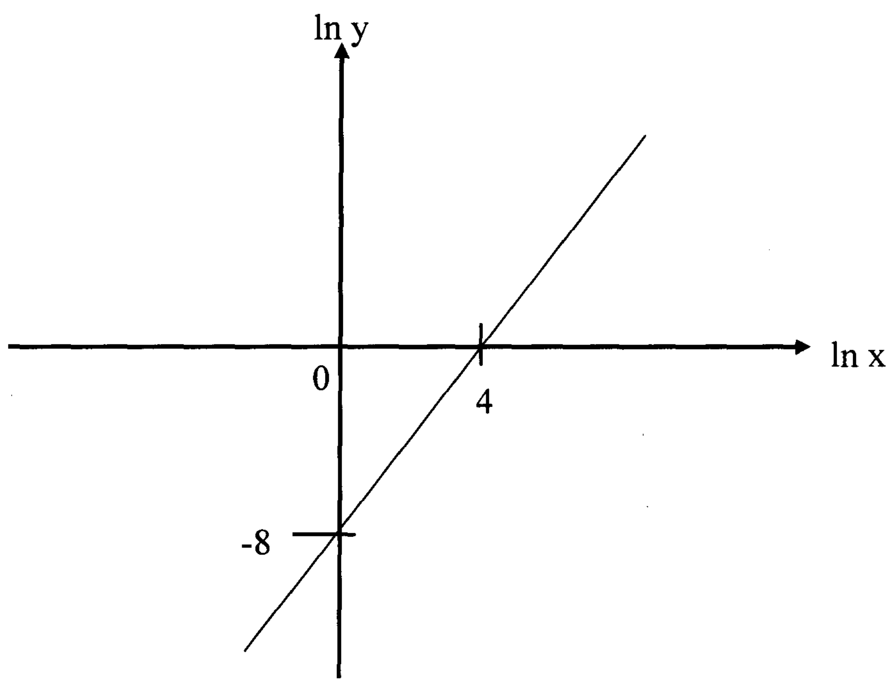
1. Solve the inequality $2\,|x-3| \;<\; |x-2|$. [3]

2. Expand $\sqrt{(4+x^2)}$ in ascending powers of x up to and including the term x^4. [4]

3. Use the trapezium rule, with 4 intervals, to estimate the value of $\int_1^3 \ln x \, dx$, giving your answer correct to 4 significant figures. [4]

4. Given that $x = t\ln t$ and $y = (\ln t)(2+\ln t)$, show that $\frac{dy}{dx} = \frac{2}{t}$. [4]

5. It is believed that the relationship between the 2 quantities x and y is of the form $y = kx^n$, where k and n are constants. A plot of $\ln y$ against $\ln x$ produced a straight line as shown below. Find the exact values of k and n. [5]



6. Write $3\cos 2\theta - \sin 2\theta$ in the form $R\cos(2\theta + \alpha)$. Hence, solve for θ the equation $3\cos 2\theta - \sin 2\theta = 2$, giving all solutions between 0^0 and 360^0, correct to the nearest degree. [5]

7. Use the factor theorem to show that $x + 2$ is a factor of $x^3 + ax^2 + 2ax + 8$ for all values of the constant a. [2]
Hence solve the equation

$$x^3 + 7x^2 + 14x + 8 = 0$$ [4]

8. (i) Express $\dfrac{1}{y^2 - y}$ in partial fractions. [3]

(ii) Given the differential equation $\dfrac{1}{2}\left(\dfrac{dy}{dx}\right) + xy = xy^2$ and that when $x = 0$, $y = 2$, find y in terms of x. [5]

9. Using integration by parts show that $\displaystyle\int_0^{\frac{\pi}{3}} x\sin 6x\,dx = -\frac{\pi}{18}$. [5]

Hence, show that $\displaystyle\int_0^{\frac{\pi}{3}} x^2\cos 6x\,dx = \frac{\pi}{54}$. [3]

10. Given that the curve $y = xe^x - 3e^x - 2x$ has only one turning point, show that the x-coordinate of this turning point lies between $x = 2$ and $x = 3$. [4]

(i) Show that, if a sequence of values given by the iterative formula $x_{n+1} = 2 + 2e^{-x_n}$ converges, then it converges to the x-coordinate of this turning point. [2]

(ii) Use this iterative formula, with initial value $x_0 = 2$, to determine the x-coordinate of this turning point correct to 3 decimal places, showing the result of each iteration. [2]

[Turn over

11. The complex number $\dfrac{2}{-1+i}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex number z which satisfy both the inequalities $|z| < 2$ and $-\dfrac{\pi}{4} \leq \arg(z - u^2) \leq 0$.

[4]

12. The line l has equation $\mathbf{r} = 7\boldsymbol{i} + 3\boldsymbol{k} + t(5\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k})$, and the plane π has equation $\mathbf{r} \cdot (\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}) = 1$. Find the coordinates of the point A where l meets π. The point B has coordinates $(7, 0, 3)$, and C is the foot of the perpendicular from B to π. Find the coordinates of C. Find a vector equation for the line AC, and calculate the angle BAC, giving your answer to the nearest degree. [10]