TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2010

(January/March 2009 Intake)

Tuesday **30 March 2010** **8.30 am – 10.15 am**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages

1 The polynomial $x^3 + ax^2 + bx - 6$ is divisible by $x+3$ and $x-2$. Determine the values of a and b. [4]

2 Find , correct to two significant figures, the value of x such that
$\dfrac{e^x - 2e^{-x}}{2e^x + e^{-x}} = \dfrac{1}{5}$. [4]

3 Show that the equation $\ln x + x - 3 = 0$ has a root in between 1 and e. Hence, use the iterative formula $x_{n+1} = 3 - \ln x_n$ with initial value $x_1 = 1$ to find the root, correct to two decimal places. [4]

4 The gradient of the curve $y = f(x)$ is inversely proportional to the square root of x and the curve passes through (0, 3) and (4, 23). Find the equation of the curve. [5]

5 Show that $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx = \dfrac{1}{5}(2e^{\pi} + 1)$ [7]

6 (a) Given that $5\sin\theta + 12\cos\theta = R\sin(\theta + \alpha)$, find the values of R and α for which $R > 0$ and $0^0 < \alpha < 90^0$. [3]

(b) Find all values of θ between 0^0 and 360^0 satisfying: $5\sin\theta + 12\cos\theta = 4$. [3]

(c) Find the greatest value of the expression $\dfrac{1}{5\sin\theta + 12\cos\theta + 20}$, stating the smallest positive value of θ at which this occurs. [2]

7 A curve is defined by the parametric equations $x = 2t^3$ and $y = 2t^2$ where $t \neq 0$.

(i) Prove that the equation of the tangent at the point with parameter t is $2x - 3ty + 2t^3 = 0$. [4]

(ii) The tangent at the point where $t = 2$ meets the curve again at the point where $t =$ u. Find the value of u. [4]

8 Given that $\dfrac{x^2-2x-9}{(2x-1)(x^2+3)} \equiv \dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+3}$

(i) Determine the values of A, B and C. [4]

(ii) Hence, evaluate $\int_1^2 \dfrac{x^2-2x-9}{(2x-1)(x^2+3)}\,dx$, giving your answer correct to two decimal places. [5]

9 The position vectors of three points A, B, C on a plane ski-slope are $\mathbf{a}=4\mathbf{i}+2\mathbf{j}-\mathbf{k}, \mathbf{b}=-2\mathbf{i}+26\mathbf{j}+11\mathbf{k}, \mathbf{c}=16\mathbf{i}+17\mathbf{j}+2\mathbf{k},$ where the units are in metres.

(i) Show that the vector $2\mathbf{i}-3\mathbf{j}+7\mathbf{k}$ is perpendicular to $\overrightarrow{AB}$ and also perpendicular to $\overrightarrow{AC}$. Hence find the equation of the plane of the ski-slope. [4]

The track for an overhead railway lies along the straight edge DEF, where D and E have position vectors

$\mathbf{d}=130\mathbf{i}-40\mathbf{j}+20\mathbf{k}$ and $\mathbf{e}=90\mathbf{i}-20\mathbf{j}+15\mathbf{k},$

and F is a point on the ski-slope.

(ii) Find the equation of the straight line DE. [3]

(iii) Find the position vector of the point F. [3]

(iv) Show that $\overrightarrow{DF}=15(-8\mathbf{i}+4\mathbf{j}-\mathbf{k})$ and hence find the length of the track. [3]

10 (a) Solve the equation $\left(\dfrac{3-i}{2+i}\right)z=\dfrac{5+5i}{-1+2i}$, giving your answer in the form $a+ib$. [4]

(b) The roots of the equation $z^2+6z+13=0$ are denoted by z_1 and z_2, where $\arg(z_1)>0$. Find z_1 and z_2, and show these roots on a sketch of an Argand diagram. [3]

Find the modulus and argument of (z_1+1). [2]

On the same diagram, sketch the loci given by $|z+1|=2\sqrt{2}$ and $\arg(z+1)=\dfrac{\pi}{4}$. Hence, find in the form of $a+ib$, the complex number satisfying the above two loci. [4]