TAYLOR'S UNIVERSITY COLLEGE
Wisdom · Integrity · Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2007

(January/March 2006 Intake)

Tuesday **3 April 2007** **8.30 am – 10.15am**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 3 printed pages.

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1. Evaluate $\int_0^1 \frac{x^2}{\sqrt{4-x^2}}dx$ using the trapezium rule with 5 intervals. (4)

2. Find the equation of the normal to the curve $3x^2 - xy + 2y^2 = 8$ at the point (1,5).

(5)

3. The expression $2x^3 + ax^2 + bx + 2$ is exactly divisible by $(x+2)$ and leaves a remainder of 12 on division by $(x-2)$. Calculate the values of a and b and factorise the expression completely. (6)

4. Express $2\sin\theta - 5\cos\theta$ in the form $R\sin(\theta-\alpha)$ where $R > 0$, and α is an acute angle. Hence, solve $2\sin\theta - 5\cos\theta = 4$ for $0^o \le \theta \le 360^o$. (6)

5. Find the exact value of $\int_0^{\frac{\pi}{2}} e^{2x}\sin x\, dx$. (7)

6. The equation $x^3 - x - 3 = 0$ has one root, α .

(i) Show that α lies between 1 and 2. (2)

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x^3_n - 3, \qquad \text{(A)}$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \qquad \text{(B)}$$

Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. (5)

7. Express $\frac{32x^2 + 17x + 18}{(2-3x)(1+2x)^2}$ in partial fractions and hence obtain its series expansion in ascending powers of x, stating the terms up to and including the terms in x^3, and the values of x for which the expansion is valid. (8)

8. Find the modulus of the complex number $\frac{7+3i}{5-2i}$, and show that its argument is $\frac{1}{4}\pi$. (4)

Describe with the aid of a sketch the locus of the point in an Argand diagram representing the complex number z, where $\arg\left(\frac{z}{5-2i}\right) = \frac{1}{4}\pi$. (4)

9. The line has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. (3)

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. (6)

10. In places where very heavy rainfall occurs, 'storm drains' are used to collect rainwater and prevent flooding. A storm drain can be modelled as a rectangular tank with vertical sides, so that the volume of water in the tank is proportional to the depth of the water. During a storm, rainwater flows into the tank at a constant rate. Water drains away from the bottom of the tank at a rate which is proportional to the square root of the depth of water in the tank. At time t hours the depth of water in the tank is h metres. Explain how the information given above leads to the differential equation

$$\frac{dh}{dt} = A - B\sqrt{h} \ ,$$

where A and B are positive constants. (3)

When rainfall stops and no more water is flowing into the tank, the value of A is zero. For this case, find the value of B, given that it takes half an hour for the depth of water in the tank to fall from 1 metre to zero. (4)

In another case, $A = 2$ and $B = 1$, so that the differential equation is $\frac{dh}{dt} = 2 - \sqrt{h}$.

(i) By means of the substitution $x = 2 - \sqrt{h}$, or otherwise, find

$$\int \frac{1}{2-\sqrt{h}}\, dh .$$ (5)

(ii) Given that the tank is initially empty, find the time taken for the depth of water in the tank to reach 1 metre. (3)