TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2006

(June 2005 Intake)

Wednesday **30 August 2006** **8.30 am – 10.15 am**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

 [Turn over

1. Find the expansion of $\dfrac{1+x^2}{\sqrt{(1+4x)}}$ in ascending powers of x up to and including the term in x^2. [3]

2. Use integration by parts to show that the exact value of $\int_1^e (\ln x)^2 dx$ is $e-2$. [4]

3. It is given that $f(x) = 10\cos^2 x - 8\sin x\cos x + 4\sin^2 x$.

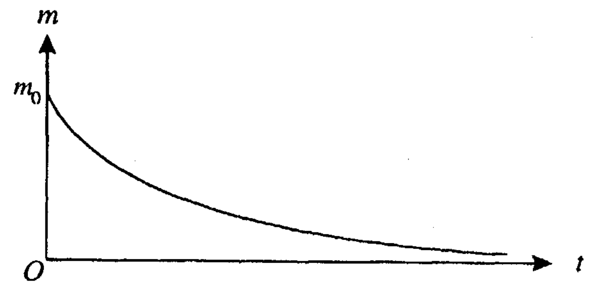
   Express $f(x)$ in the form $a\cos 2x + b\sin 2x + c$, where a, b and c are constants. [3]

   Hence, by rewriting $f(x)$ in the form $R\cos(2x+\alpha) + c$, or otherwise, show that the greatest and least values of $f(x)$ are 12 and 2 respectively. [3]

4. Show that both $(x-\sqrt{3})$ and $(x+\sqrt{3})$ are factors of $f(x) = x^4 + x^3 - x^2 - 3x - 6$. [3]

   Given that $f(x)$ can be expressed as two quadratic factors. Write down one quadratic factor of $f(x)$, and find the other quadratic factor of this polynomial. [3]

5. During a chemical reaction, the mass m of one of the chemicals involved decreases at a rate which is proportional to m. Express this information as a differential equation involving m and the time t.
   When $t = 0$ the mass of the chemical is m_0. Show by integration that the solution of the differential equation is $m = m_0e^{-kt}$, where k is a positive constant. [4]



A sketch of the graph of $m = m_0e^{-kt}$ is shown in the diagram. The mass M of another chemical involved in the reaction varies in such a way that $m + M = A$, where A is a constant.

   (i) Describe what happens to the value of M as t becomes large. [1]

   (ii) Show that $\dfrac{dM}{dt} = k(A-M)$. [2]

6. Given that $\dfrac{x^2+4}{x^2+2x}$ can be written as $A+\dfrac{B}{x}+\dfrac{C}{x+2}$.

Find the values of A, B and C. [5]

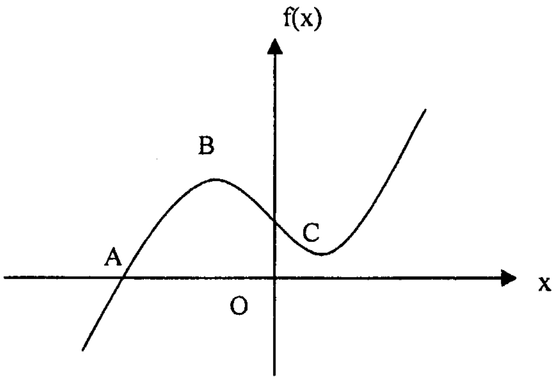
Hence evaluate $\displaystyle\int_1^4 \frac{x^2+4}{x^2+2x}\,dx.$ [4]

7. The curve $y = x\sqrt{(1+x)}$ cuts the x-axis at two points.

(i) Find the coordinates of these two points of intersection. [3]

(ii) Use the substitution $u = 1 + x$ to find the area enclosed by the curve and the x-axis. [6]

8. The diagram below shows the graph of $f(x) = x^3 - 2x + 3\cos x$.



(i) Show that point A is located between $x = -2$ and $x = -1$. [2]

(ii) Find the equation satisfied by the x-coordinates of the two turning points, B and C. [2]

(iii) Show that your equation in (ii) can also be solved numerically using the iterative formula

$$x_{n+1} = \sqrt{\frac{2}{3} + \sin x_n}$$ [2]

(iv) Taking $x_0 = 1$ as a first approximation, find the coordinates of C, correct to 3 decimal places. [3]

[Turn over

9. (a) Find the argument of the complex number $\dfrac{5+i}{2+3i}$. [3]

(b) Given that w and z are two complex numbers such that
$|w-6+4i| = 5$ and $|z+2-4i| \le 4$.

(i) Sketch and label the loci of w and z. [2]

(ii) Obtain the exact least possible value of $|w-z|$. [1]

(c) Find, in the form a + bi, where a and b are real and exact, the product of
$(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6})^2$ and $2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$. [4]

10. The position vectors **a, b, c, d**, relative to the origin O, of the points A, B, C and D are given by **a** = 2**i**, **b** = 2**i** + 4**j**, **c** = 4**j** and **d** = **i** + 2**j** + 2**k**.

(i) Obtain a unit vector normal to the plane Π_1, the plane containing B, C and D. [4]

(ii) Calculate the angle between the planes Π_1 and Π_2, the plane containing A, B and D. [4]

(iii) Find the position vector of K, the foot of the perpendicular from C to the plane Π_2, and hence find the distance of C from this plane. [4]