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CAMBRIDGE A LEVEL PROGRAMME

AS TRIAL EXAMINATION AUGUST 2011

(January and March 2011 Intakes)

Monday **22 August 2011** **12.30 pm – 2.15 pm**

MATHEMATICS 9709/13

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **6** printed pages.

1 Find the term independent of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^{10}$. [3]

2 Show that, for all real values of a, the equation $x^2+2ax+\left(2a^2+a+1\right)=0$ does not have real roots. [4]

3 Prove that if $\cos\theta \neq 0$, then $(1-\cos\theta)\left(1+\frac{1}{\cos\theta}\right)\equiv \sin\theta\tan\theta$. Hence, or otherwise, solve the equation $(1-\cos\theta)\left(1+\frac{1}{\cos\theta}\right)=\sin\theta$, $0° \leq \theta \leq 360°$. [5]

4 The line $x-2y=8$ intersects the curve $xy=24$ at points A and B.

(i) Find the coordinates of points A and B. [3]

(ii) Find the equation of the perpendicular bisector of the line segment joining points A and B. [4]

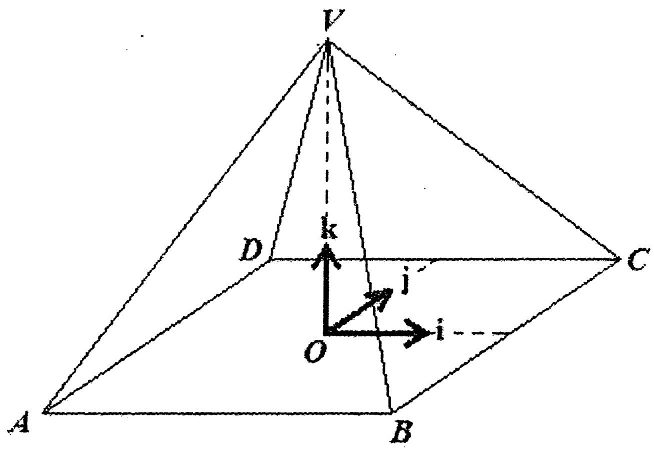
5 The equation of a curve is $y=\frac{4}{x^2}-x$.

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Determine whether the curve is increasing or decreasing at the point on the curve when $x=1$. [1]

(iii) Find the equation of the normal to the curve at point $P(2,-1)$. [3]

6



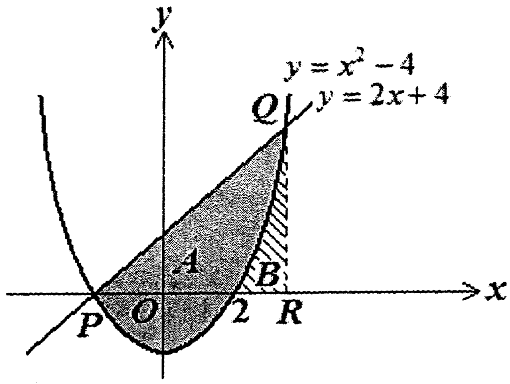
$ABCDV$ is a right pyramid with a rectangular base $ABCD$ and vertex V. The centre of the rectangle $ABCD$ is O and the length of AB is 1 unit BC λ units and VA 2 units. Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be the unit vectors parallel to AB, BC and OV respectively (see diagram).

(i) Show that the length of OV is $\frac{1}{2}\sqrt{15-\lambda^2}$ units. [2]

(ii) Express $\mathbf{BV}$ and $\mathbf{AC}$ in the form $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$. [2]

(iii) Find the angle between $\mathbf{BV}$ and $\mathbf{AC}$ if $\lambda = 2$. [4]

7



The straight line $y = 2x + 4$ intersects the curve $y = x^2 - 4$ at points P and Q. R is a point on the x-axis vertically below Q. The enclosed region bounded by the straight line and the curve is denoted as A. The enclosed region bounded by the curve, vertical line QR and the x-axis is denoted as B (see diagram).

(i) Find the x-coordinates of P and Q, [2]

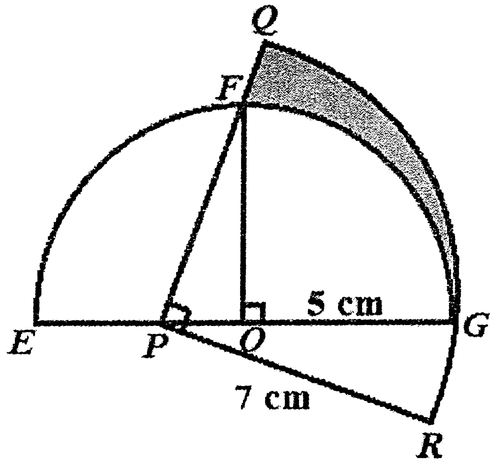
(ii) Find the area of the region A, [4]

(iii) Find, in terms of π, the volume of the solid of revolution generated by rotating the region B through $360°$ about the x-axis. [3]

8 **(i)** An infinite geometric series is given by $S = 1 + (x^2 - 1) + (x^2 - 1)^2 + \ldots$.
Find the set of values of x if S is convergent, and the value of S if $x = \frac{1}{2}$. [5]

(ii) Find the sum of all the integers in between 100 to 200 inclusive which are not divisible by 4. [5]

9



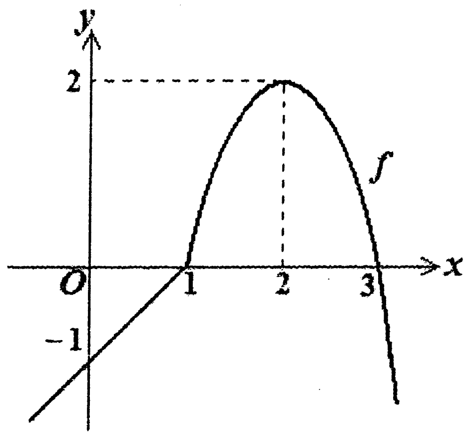
$OEFG$ is a semicircle centred at O with radius 5 cm, EG the diameter and F the midpoint of arc EG. PQR is a quadrant centred at P with radius 7 cm. P is located on EG in such a way that QFP is on a straight line and G is located on arc QR of the quadrant PQR (see diagram). Calculate

(i) the area of sector PRG, [3]

(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. [4]

10



The diagram shows the function f defined for all real values of x by

$$x \mapsto x-1 \qquad \text{for } x<1,$$
$$x \mapsto -2(x-2)^2+2 \quad \text{for } x \geq 1.$$

(i) State the range of f. [1]

(ii) Explain why f^{-1} does not exist. [1]

The function g is defined by the same function as f but with domain $x \leq k$ such that g^{-1} exists.

(iii) State the largest value of k. [1]

(iv) Based on the diagram, sketch $y=g(x)$ and $y=g^{-1}(x)$ on the same axes, making clear the relationship between the two graphs. [3]

(v) Obtain expressions to define g^{-1}, giving the set of values for which each expression is valid. [5]