TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

AS TRIAL EXAMINATION MARCH/APRIL 2010

(June 2009 Intake)

Monday **5 April 2010** **1.30 pm – 3.15 pm**

MATHEMATICS 9709/13

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together

The number of marks is given in brackets [ ] at the end of each question or part question
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers

This document consists of 5 printed pages.

[Turn over

1 Find the set of values of x for which $8x - x^2 \geq -20$. [3]

2 Find the coefficient of the term which is independent of x in the expansion of $\left(2x^3 - \frac{3}{x}\right)^{12}$. [4]

3 Given that one root of the equation $px^2 + 3x + q = 0$ is three times the other root. Show that $16pq = 27$. [4]

4 A solar eclipse can be modeled by two overlapping circles each with radius r. Show that when the distance between the centers of the circles is $\frac{3}{2}r$, the area of overlapping is approximately 14.4% of the area of one of the circles. [5]

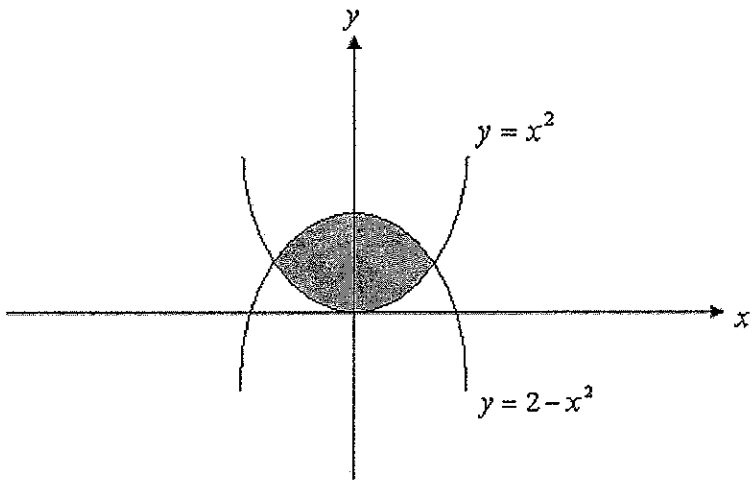
5 $OABC$ is a rectangle with position vectors **a** and **b**. The vectors **a** and **b** are given by $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $s\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ respectively.

(i) Find the value of s. [3]

(ii) Find the obtuse angle between the diagonals. [3]

6 The curves $y = 2\sin^2 x$ and $y = 3\sin x - 1$ in the domain $0 \leq x \leq \frac{\pi}{2}$ intersect at A and B. Show that $AB = \frac{1}{6}\sqrt{4\pi^2 + 81}$. [6]

7

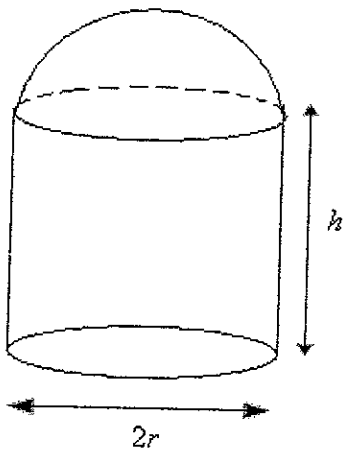


The diagram shows the graphs of $y = x^2$ and $y = 2 - x^2$. Calculate

(i) the area of the shaded region. [4]

(ii) the volume of the solid generated when the shaded region is rotated completely about the y-axis. [4]

8



A closed container consists of a cylinder of radius r cm and height h cm surmounted by a hemisphere of radius r cm at one end as shown in the diagram. The volume of the container is 45π cm^3.

(i) Express h in terms of r and hence show that the total surface area of the container is given by

$$L = \frac{5}{3}\pi r^2 + \frac{90\pi}{r}.$$ [3]

[The surface area A and volume V of a sphere of radius r are given by the formulae $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$.]

Given that r can vary,

(ii) find the stationary value of L, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

9 **(i)** Find the equation of line through A(-2, 1) perpendicular to the line joining B(1, 7) and C(3, 1). [3]

(ii) Find the point of intersection of the line BC with the line obtained in part **(i)**. [3]

(iii) Hence, find the perpendicular distance from A to BC. [1]

(iv) Use the result obtained in part **(iii)** to find the area of the triangle ABC. [2]

10 The function $g(x) = x^2 - 6x$ is defined for the domain $x \geq k$.

(i) Express $x^2 - 6x$ in the form $(x-p)^2 - q$, where p and q are constants. [1]

(ii) Find the least value of k for which g is one-to-one. [1]

(iii) Find an expression for $g^{-1}(x)$ and state its domain and range. [5]

(iv) Sketch, in a single diagram, the graph of $y = g(x)$ and $y = g^{-1}(x)$, making clear the relationship between the two graphs. [3]

11 **(i)** In a geometric progression, the sum of the first five terms is 44 and the sum of the next five terms is $-\frac{11}{8}$. Find the sum to infinity of this series. [6]

(ii) The sum, S_n of the first n terms of a series is given by

$$S_n = \frac{1}{4}(3n^2 + 13n).$$

Show that the series is an arithmetic progression and write down the first three terms of the series. [6]