TAYLOR'S UNIVERSITY COLLEGE
Wisdom · Integrity · Excellence

CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2009

(June 2008 Intake)

Wednesday **9 September 2009** **1.30 pm – 3.15 pm**

MATHEMATICS 9709/32

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

[Turn over

1 An engineer estimated the area of the vertical cross-section of water flowing under a bridge. For her model she measured the depth of water at 4 m intervals from one end of the bridge to the other end. Her results are given in the table.

Distance from one end (m)	0	4	8	12	16	20	24
Depth (m)	1.2	2.3	3.8	4.9	3.2	1.9	0.6

She used the trapezium rule to estimate the area of the cross-section.

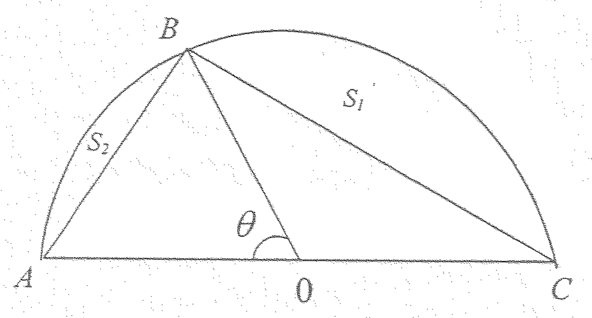
Calculate the estimate she obtained. [2]

2 Solve the inequality $|x+1| < |2x+1|$. [3]

3 Use integration by parts to find $\int 4xe^{-2x}\,dx$. [4]

4 Solve the differential equation $\frac{dy}{dx} = \sqrt{y}\,sec^2 3x$ given that $y = 1$ when $x = 0$, expressing your answer in the form $y = f(x)$. [5]

5



The diagram shows a semicircle ABC on AC as diameter. The mid-point of AC is 0, and angle $AOB = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$. The area of the segment S_1 bounded by the chord BC is twice the area of the segment S_2 bounded by the chord AB.

Show that $3\theta = \pi + sin\,\theta$. [3]

Use an iterative method, based on rearrangement $\theta = \frac{1}{3}(\pi + sin\,\theta)$, together with a suitable starting value, to find θ correct to 3 decimal places. [2]

You should show the value of each approximation that you calculate.

6 Two planes have vector equations

$$\mathbf{r}.(2\mathbf{i}-3\mathbf{j}-\mathbf{k})=14 \text{ and } \mathbf{r}.(11\mathbf{i}+\mathbf{j}-2\mathbf{k})=42$$

(i) Find the acute angle between these two planes. [2]

(ii) Determine a vector equation for the line of intersection of these two planes. [5]

7 The rate, in cm^3s^{-1}, at which oil is leaking from an engine sump at any time t seconds is proportional to the volume of oil, V cm^3, in the sump at that instant.

At time $t = 0$, $V = A$.

(i) By performing and integrating a differential equation, show that

$$V = Ae^{-kt},$$ [4]

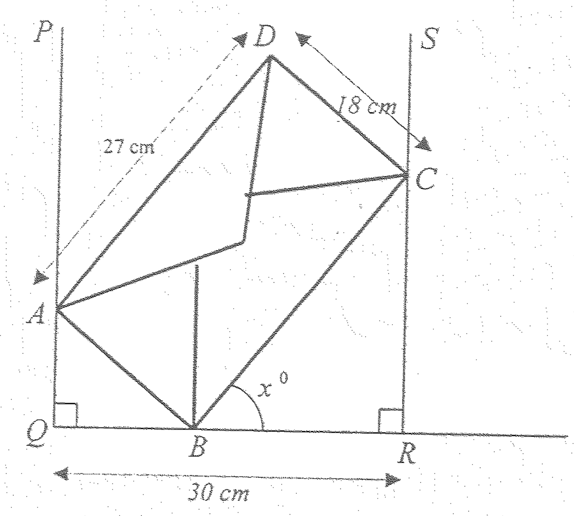
Where k is a positive constant.

(ii) Sketch a graph to show the relation between V and t. [2]

Given further that $V = \frac{1}{2}A \quad at \quad t = T$,

(iii) Show that $kT = \ln 2$. [2]

8



The figure shows the rectangular cross-section $PQRS$ of a letter rack. A rectangular envelope $ABCD$ rests in the vertical plane $PQRS$ inside the letter rack. QR is horizontal. $QR = 30\,cm$, $AD = 27\,cm$ and $CD = 18\,cm$. The bottom edge, BC, of the envelope, makes an angle x^0 with the base QR of the rack.

(i) Prove that $9\cos x^0 + 6\sin x^0 = 10$. [3]

(ii) Express $9\cos x^0 + 6\sin x^0$ in the form $R\cos(x^0 - \alpha^0)$, where $R > 0$ and $0 < \alpha < 90$, giving the values of R and α to 2 decimal places. [3]

(iii) Hence, or otherwise, find x, giving your answer to the nearest tenth of a degree. [3]

9 The number of bacteria present in a culture is modeled by $y = y_0 e^{kt}$, where $k > 0$ and t is the number of hours after 12 noon.

At 1.00 p.m., the number of bacteria present has doubled.

(i) According to the formula, how many bacteria are present at 12 noon? [1]

(ii) Find the value of k. [2]

(iii) At what time will the number of bacteria have increased ten-fold? [3]
Give the answer to the nearest minute.

(iv) The rate of growth when $t = 5$ is cy_0. Find c. [2]

(v) Show that the rate of increase of the number of bacteria is proportional to the number of bacteria present at that time. [1]

10 By using partial fractions, or otherwise, find the first three terms in the expansion of

$$\frac{1}{(1-x)^2(2+x)}$$ [10]

11 **(a)** A complex number z satisfies $|z-3-4i|=2$. Describe in geometrical terms, with the aid of a sketch, the locus of the point which represents z in an Argand diagram. [2]

Find

(i) the greatest value of $|z|$, [2]

(ii) the difference between the greatest and least values of arg z. [2]

(b) Given that $z=1+i\sqrt{3}$,

(i) find $|z|$ and arg z. [2]

(ii) Hence, or otherwise, show that $z^5-16z^*=0$. [5]