TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2012

(January 2012 Intake)

Tuesday **12 June 2012** **1.00 – 2.45 pm**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **6** printed pages.

[Turn over

1. Find the range of values of q which makes the function $q - 3qx - 2x^2$ negative for all values of x. [3]

2. Without using a calculating device, express the recurring decimal number 0.090909... as a fraction in the simplified form. [4]

3. An arithmetic series has first term a, common difference 8. The sum of the first n terms is 12000.

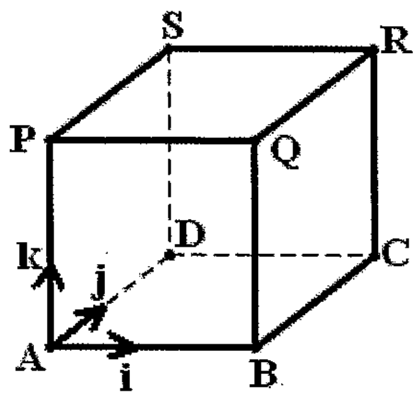
(i) Find a in terms of n. [2]

(ii) If the nth term is less than 576, find the range of possible values of n. [4]

4. **(i)** Show that the equation $3\tan\theta = 2\cos\theta$ may be written as a quadratic equation. [3]

(ii) Solve the equation of $3\tan\theta = 2\cos\theta$ for $0 \le \theta \le 2\pi$. [3]

5.



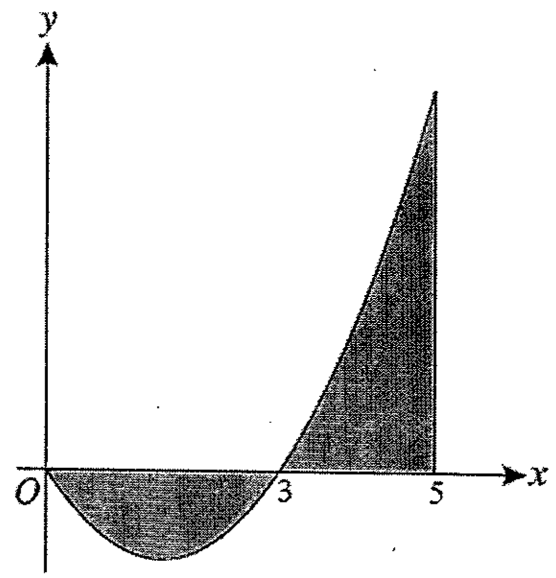
ABCDPQRS are vertices of a cube of 1 unit with base ABCD and verticals sides AP, BQ, CR and DS. Unit vectors **i**, **j** and **k** are parallel to AB, AD and AP respectively.

(i) Express the vectors $\overrightarrow{AR}$ and $\overrightarrow{SB}$ in terms of **i**, **j** and **k**. [2]

(ii) Hence, calculate the angle between vectors $\overrightarrow{AR}$ and $\overrightarrow{SB}$. [4]

[Turn over

6.



The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$. Find

(i) the total shaded area. [4]

(ii) the volume of the solid formed when the shaded region is rotated completely about the x-axis. [4]

7. **(i)** Find the binomial expansion of $(2x + 5)^3$, simplifying the terms. [3]

(ii) Hence, show that $(2x + 5)^3 - (2x - 5)^3$ can be written in the form $a + bx^2$. [2]

(iii) Hence, or otherwise, solve the equation

$$(2x + 5)^3 - (2x - 5)^3 = 110x^2 + 200x + 240$$

Leave your answers in exact form. [3]

8. A straight line l passes through the origin and has gradient of 2. A is a point on the positive x-axis and is 5 units to the origin O. B is a point on l such that the length AB is the shortest.

(i) Find the coordinates of B. [4]

Point Q is on l and point R is on AB. $P(a, 0)$ and $S(b, 0)$ are two points on the x-axis, such that $PQRS$ is a rectangle inscribed in the triangle ABO.

(ii) Find b in terms of a. [2]

(iii) Find the value of a such that the area of rectangle $PQRS$ is maximum. [4]

9. Given that $f(x) = 2x^2 - 6x + 11$.

(i) Express $f(x)$ in the form $p(x+q)^2 + r$. [2]

(ii) State, with a reason, whether $f(x)$ has an inverse. [1]

The function $g(x)$ is such that $g(x) = 2x^2 - 6x + 11$ for $x < A$, where A is a constant.

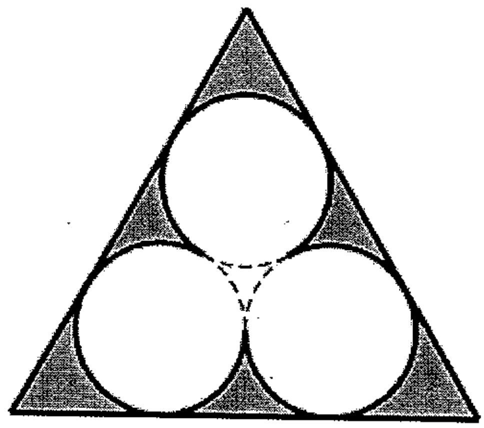
(iii) State the maximum value of A such that $g^{-1}(x)$ exists. [1]

(iv) Obtain an expression, in terms of x, for $g^{-1}(x)$. [3]

(v) State the domain and range for $g^{-1}(x)$. [2]

(vi) Sketch, in a single diagram, the graphs of $g(x)$ and $g^{-1}(x)$, making clear the relationships between two graphs. [3]

10.



A pond was constructed by digging the entire region formed by arcs of three circles, each having radius of 1 unit, which touch one another. The arcs of the pond were fenced up, and the pond was also inscribed in an equilateral triangular fences. The regions between the circular and the straight fences were to be covered with weeds (see diagram). Express, in form of $a\pi + b\sqrt{3} + c$, the value of

(i) the length of fences required. (The fences are made up of the straight edges and the arcs.) [6]

(ii) the area of the sheet of weeds (shaded region) required. [6]