TAYLOR'S UNIVERSITY COLLEGE
Wisdom · Integrity · Excellence

CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION NOV/DEC 2009

(June 2009 Intake)

Monday **30 November 2009** **1.00 pm – 2.45 pm**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

[Turn over

1 Find the non-zero value of b if the coefficient of x^2 in the expansion of $(2b+3x)^6$ is equal to the coefficient of x^5 in the expansion of $(5+bx)^8$. [3]

2 Let $\mathbf{p} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{q} = 12\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the three unit vectors parallel to x, y and z axes respectively.

Evaluate $\mathbf{p} \cdot \mathbf{q}$ and hence find the cosine of angle between $\mathbf{p}$ and $\mathbf{q}$. [4]

3 In an arithmetic progression, the tenth term is six times larger than the second term and the fifth term is 23. Find

(i) the common difference, [2]

(ii) the sum of first twelve terms. [2]

4 A company that manufactures cat food wishes to pack the food in the closed cylindrical tins. If each tin is to have a volume of $432\pi\ \text{cm}^3$,

(i) find the radius, r and the height, h of the cylindrical tin if the surface area, S is to be a minimum. [4]

(ii) find the minimum surface area. [2]

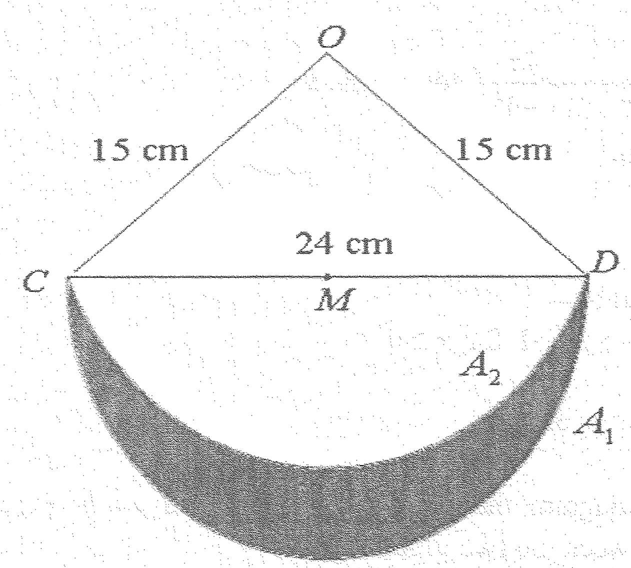
5 T is the tangent to the curve $y = x^2 + 7x - 5$ at (2, 13) and N is the normal to the curve $y = 2x^2 - 3x + 5$ at (1, 4). Find

(i) the equation of the tangent, T, [2]

(ii) the equation of the normal, N, [2]

(iii) the coordinates of the point of intersection of T and N. [2]

6 In the diagram, OCD is a triangle, $OC = OD = 15$ cm and $CD = 24$ cm. M is the midpoint of CD. The arc A_1 forms a semicircle with diameter CD, and A_2 is arc of a circle with centre O and radius 15 cm.



Calculate,

(i) the area of the triangle OCD, [2]

(ii) the angle COD, in radians, [2]

(iii) the area of the shaded region bounded by arcs A_1 and A_2, giving your answer to 2 decimal places. [3]

7 Sketch the curve $y = 2\sin x$ and $y = 5\cos x$ for $0° \leq x \leq 360°$. [3]

(i) Solve the equation $2\sin x = 5\cos x$. [3]

(ii) Find the range of values of x in this interval for which $2\sin x \geq 5\cos x$. [2]

8 P is the point of intersection of the lines $3x - y = 7$ and $x + 4y + 2 = 0$. Find

(i) the equation of the line perpendicular to $2x + 3y = 40$ which passes through P, [4]

(ii) the perpendicular distance of P from the line $2x + 3y = 40$. [4]

9 The functions f and fg are defined by

$$f : x \to x^2 - 1 \text{ for } x \in \mathbb{R},$$

and

$$fg : x \to \frac{4x}{(x-1)^2} \text{ for } x \in \mathbb{R},\ x \neq 1.$$

Define, in a similar form,

(i) the function g. [4]

The function h is defined by

$$h : x \to x^2 - 1 \text{ for } x \geq 0.$$

(ii) express $h^{-1}(x)$ in terms of x. [2]

(iii) Sketch on a single diagram, the graphs of $y = h(x)$ and $y = h^{-1}(x)$, making clear the relationship between the two graphs. [2]

10 Function f is defined by $f : x \to x^2 + 2x + 2,\ x \in \mathbb{R}$.

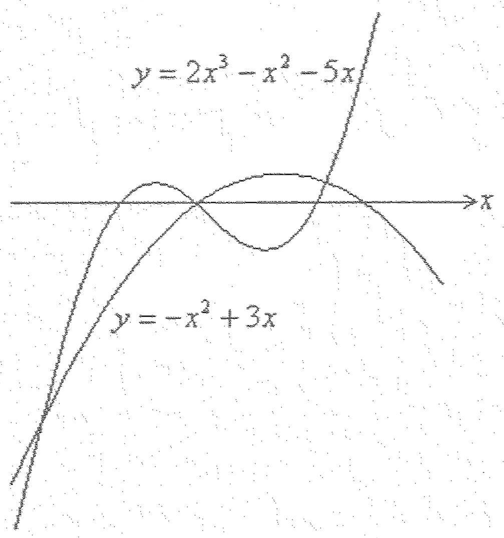
(i) Without using any sketch, prove that $f(x) > 0$ for all values of x. [2]

(ii) Solve the equation $f(x) = 5$ and hence, or otherwise, deduce the solution to the equation

$$1 + 2\sqrt{x} - 3x = 0$$ [6]

(iii) Find the set of values of x for which $f(x) \geq 5$. [2]

11 (a)

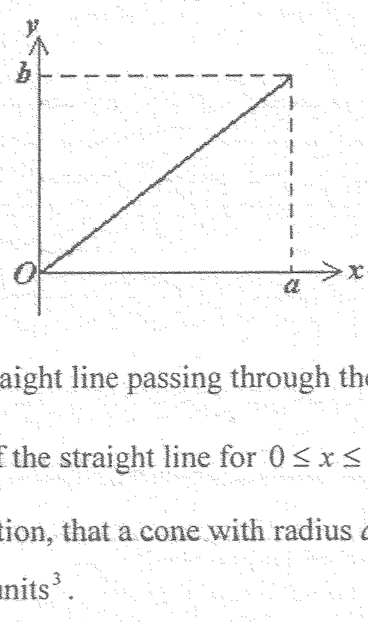


The diagram above shows curves $y = 2x^3 - x^2 - 5x$ and $y = -x^2 + 3x$ sketched on the same coordinate system.

(i) Show that the x-coordinate of the points of intersection between the curves are -2, 0 and 2 respectively. [2]

(ii) Show that the area of the region bounded by the curves is 16 units^2. [4]

(b)



The diagram shows a straight line passing through the origin and the point (a, b).

(i) State the equation of the straight line for $0 \le x \le a$. [1]

(ii) Show, using integration, that a cone with radius a units and height b units has a volume of $\frac{1}{3}\pi a^2 b$ units^3. [4]