TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2012

(January and March 2011 Intakes)

Monday **2 April 2012** **8.30 am – 10.15 am**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

[Turn over

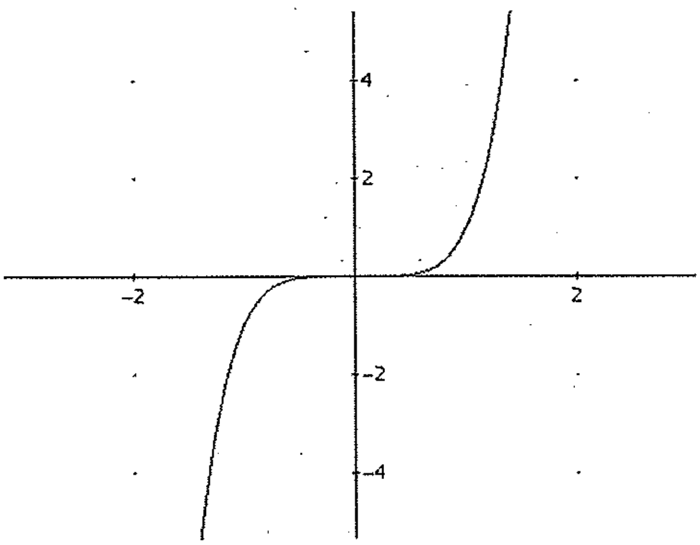
1 If $a^2 + b^2 = 14ab$, using $(a+b)^2 = a^2 + b^2 + 2ab$ or otherwise, prove that

$$\log\left(\frac{a+b}{4}\right) = \frac{1}{2}(\log a + \log b).$$ [3]

2 If $\frac{b}{a}$ is small enough for powers of $\frac{b}{a}$ higher than the third to be neglected show that

$$(a+b)^{\frac{1}{2}} - (a-b)^{\frac{1}{2}} = a^{\frac{1}{2}}\left(\frac{b}{a} + \frac{b^3}{8a^3}\right).$$ [5]

3 **(i)** Given below is the graph of $y = x^5$. Copy the graph and sketch $y = a - bx$ on the same diagram, where a and b are positive constants. Show that the equation $x^5 = a - bx$ has exactly one root. [1]



(ii) Determine the integer N such that the equation $x^5 + 2x - 53 = 0$ has a root in the interval $N < x < N+1$. [1]

(iii) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the root correct to 3 decimal places. [3]

4 **(i)** Express $2\tan^2\theta - \frac{1}{\cos\theta}$ in the form of a quadratic in terms of $\sec\theta$. [2]

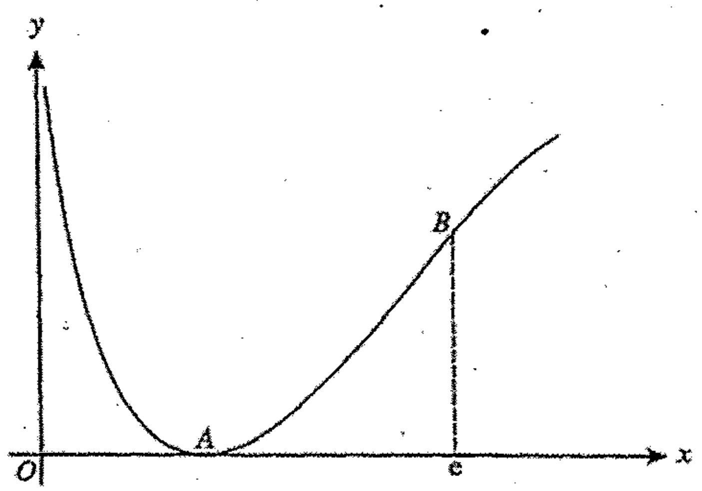
(ii) Hence solve, for $0^\circ < \theta < 360^\circ$, $2\tan^2\theta - \frac{1}{\cos\theta} = 4$. [4]

5 Given that $y = 0$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^3 + 1}{y^2},$$

obtaining an expression for y in terms of x. [6]

6



The function f is defined by $f(x) = (\ln x)^2$ for $x > 0$. The diagram shows a sketch of the graph of $y = f(x)$. The minimum point of the graph is A.

(i) State the x-coordinate of A. [1]

(ii) Use the substitution $x = e^u$ to show that the area of the region bounded by the x-axis, the line $x = e$, and the part of the curve between A and B is given by

$$\int_0^1 u^2 e^u \, du.$$ [3]

(iii) Hence, or otherwise, find the exact value of this area. [3]

7 The equation of a curve is $y = 2\sin x + \cos 2x$. Find the x-coordinates of the stationary points on the curve for which $0 < x < \pi$, and determine the nature of each of these stationary points. [9]

[Turn over

8 If $f(x) = ax^3 + (a+b)x^2 + (a+2b)x + 1$ is exactly divisible by $(x+1)$, express b in terms of a, and find the quotient when the division is carried out, expressing the coefficients in terms of a only. [5]

Prove that in this case the equation $f(x) = 0$ has only one real root if $a^2 - 6a + 1 < 0$. [2]

Show that this inequality implies that $3 - 2\sqrt{2} < a < 3 + 2\sqrt{2}$. [3]

9 Relative to the origin, the position vectors of points A and B are $4\mathbf{i} - 11\mathbf{j} + 4\mathbf{k}$ and $7\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ respectively.

(i) Find a vector equation for the line l passing through points A and B. [2]

(ii) Find the position vector of the point P on l such that OP is perpendicular to l. Hence find the perpendicular distance from O to l. [5]

(iii) The planes π_1 and π_2 have equations $3x - y - z = 2$ and $x + 5y + z = 14$ respectively. Find the line of intersection of π_1 and π_2. [4]

10 **(i)** The complex numbers z satisfies $|z + 4 - 4i| = 3$.

(a) Describe, with the aid of a sketch, the locus of the points which represents z in an Argand diagram. [2]

(b) Find the least possible value of $|z + 2i|$. [2]

(ii) If $z = \cos\theta + i\sin\theta$ where $0 < \theta < \frac{\pi}{2}$, prove that $\frac{1+z}{1-z} = i\cot\frac{\theta}{2}$. [4]

(iii) In an Argand diagram, the points O, A, Z, P and Q represent the complex numbers 0, 1, z, $1 + z$, and $1 - z$ respectively. Show these points on an Argand diagram. Hence, explained why $\angle POQ = \frac{\pi}{2}$. [5]