TAYLOR'S UNIVERSITY COLLEGE
Wisdom · Integrity · Excellence

CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION DECEMBER 2007

(July 2007 Intake)

Monday **3 December 2007** **1.00 pm – 2.45 pm**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

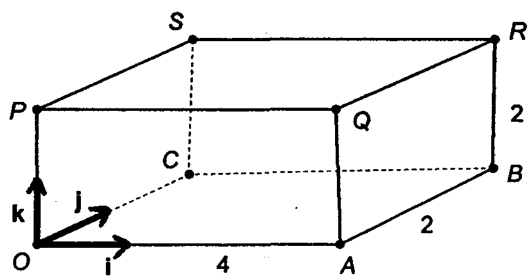
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

1 Expand $(1 + 2y)^{12}$ in ascending powers of y up to and including the term in y^3. Simplify the coefficients. [2]

2 The distance between the centers of two coplanar circles, each with a radius of 5cm, is 6cm. Calculate the common area shared by the two circles, giving your answer in cm^2 correct to two significant figures. [4]

3



The diagram shows a cuboid $OABCPQRS$ in which the length of OA is 4 units and the length of AB and BR are each 2 units. Unit vectors **i**, **j**, **k** are taken along the edges OA, OC, OP respectively. Calculate the angle between the directions of $\overrightarrow{OB}$ and $\overrightarrow{SA}$. [5]

4 **(a)** Solve the inequality $x^2 < 3x + 4$. [2]

(b) By means of substitution or otherwise, find two values of x such that

$$2(4^x) + 4^{-x} = 3.$$ [4]

5 Find all values of x for which $0° < x < 360°$ that satisfy the equation

(a) $\sin\left(\frac{1}{2}x\right) = \frac{1}{4}$. [3]

(b) $1 - \cos^2 x = 3\sin x - 4\sin^3 x$. [4]

6 (a) The function *f* and *g* are defined by:

$$f : x \mapsto 3x - 1, \quad x \in \Re$$
$$g : x \mapsto x^2 + 1, \quad x \in \Re$$

(i) Find range of *g*. [1]
(ii) Determine the values of *x* for which $gf(x) = fg(x)$. [3]

(b) The function $h : x \mapsto x^2 + 3x, \quad x \in \Re, \quad x \geq q$, is one-to-one. Find the least value of *q* and the inverse function of *h*, h^{-1}. [5]

7 The straight line *l* has equation $2y - x + 7 = 0$. The straight line *l'* passes through the point *P*(-1, 6) and is perpendicular to *l*.

(a) Find the equation of *l'*, giving your answer in the form $ax + by + c = 0$. [3]

(b) Find the coordinate of the point of intersection of *l* and *l'*. [2]

(c) Show that the perpendicular distance from *P* to *l* is $4\sqrt{5}$. [2]

(d) It is given that the points *Q*(-7, -7) and *R*(9, 1) lie on *l*. Find the exact area of the triangle *PQR*. [3]

8 (a) Find $\int_{-1}^{0} \frac{1}{(2x+1)^2} dx$. [3]

(b) Find the area bounded by the curves $y = \sqrt{8x}$ and $y = x^2$. Find also the volume of revolution generated when the area is rotated through 360° about the *x* axis. [7]

9 (a) Find the sum of the positive integers less than 200 which are multiples of 4 or 7. Hence, or otherwise, find the sum of the positive integers less than 200 which are not multiples of 4 or 7. [5]

(b) In a geometric progression, the sum of the first five terms is 44 and the sum of the next five terms is $-\frac{11}{8}$. Find the common ratio and first term of this series.

[5]

[Turn over

10 (a) Two parallel sides of a rectangle respectively lengthen at a rate of 2 cm per second, while the other two parallel sides shorten such that the area of the rectangle is always 50 cm^2. If, at time t, the length of each lengthening side is x, the length of each shortening side is y, and the perimeter of the rectangle is p, show that $\frac{dp}{dx} = 2 - \frac{100}{x^2}$ and $\frac{dp}{dt} = 4\left(1 - \frac{50}{x^2}\right)$. [5]

Find the rate of change in the perimeter when

(i) $x = 5$ cm. [1]

(ii) $y = 5$ cm. [2]

(b) Given that the gradient of the curve $y = 3x - \frac{8}{x^2}$ at the point (a, b) is 5. Find the values of a and b. [4]