TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2006

(January 2005 & March 2005 Intake)

Thursday **30 March 2006** **8.30 am – 10.15 am**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

[Turn over

1. Sketch, on the same diagram, the graphs of $y = |x - 1|$ and $y = |x^2 + x - 2|$.
Hence, or otherwise, solve the inequality $|x^2 + x - 2| \geq |x - 1|$. [4]

2. Find the gradient function of the curve with equation $x + \ln y = xy^3$. [4]

3. The population figures for country P are as follow:-

Year	Population
2000	12×10^6
2005	13×10^6

Short-term population growth has been found to be described well by the model $N(t) = ab^t$, where t is the number of years beginning from 2000.

(i) By taking year 2000 as $t = 0$, estimate the constants a and b for country P. [3]

The population model of country Q is given as $N(t) = 6 \times 10^6 \times 1.089^t$.

(ii) Find the year when the population of country Q will be equal to the population of country P. [2]

4. Let $p(x) = 4x^3 + 12x^2 + 5x - 6$.

(a) Calculate $p(2)$ and $p(-2)$, and state what you can deduce from your answers. [4]

(b) Hence or otherwise factorize $p(x)$ completely. [2]

5. (i) Prove that $\tan^2(45° + \theta) = \dfrac{1 + \sin 2\theta}{1 - \sin 2\theta}$ and deduce that $\tan 60° = \sqrt{3}$. [4]

(ii) The coordinates of any point on a certain curve are given by $x = \sin \theta$, $y = \cos 2\theta$ where θ is a parameter. Find the gradient of tangent when $\theta = \dfrac{\pi}{2}$. [2]

6. The rate of increase in sales S (in thousands of units) of a product is proportional to the current level of sales and inversely proportional to the square of the time t. This is described by the differential equation $\frac{dS}{dt} = \frac{kS}{t^2}$, where t is the time in years. The saturation point for the market is 50,000 units. That is, the limit of S as $t \to \infty$ is 50. After 1 year, 10,000 units have been sold. Show that $k = \ln 5$. [6]

7. (a) By sketching the curves with equations $y = 4 - x^2$ and $y = e^x$, show that the equation $x^2 + e^x - 4 = 0$ has one negative root and one positive root. [3]

(b) Use the iterative formula $x_{n+1} = -\sqrt{(4 - e^{x_n})}$ with $x_0 = -2$ to find an approximation to the negative root of the equation, correct to three decimal places. [3]

8. Given that $f(x) = \frac{x}{(x-1)(x+2)}$

(i) Express f(x) in partial fractions. [4]

(ii) Show that the coefficient of x^3 in the expansion of f(x) in ascending powers of x is $-\frac{3}{8}$. [5]

9. The equation of a curve is given as $y = x^2e^{2x}$.

(i) Find the exact value of $\frac{dy}{dx}$ at the point $x = 2$. [4]

(ii) Use integration by parts to find the exact area between the graph of y and the x-axis from $x = 0$ to $x = 2$. [5]

[Turn over

10. It is given that $\alpha = 1 + \sqrt{3}\,i$ is a root of the cubic equation $3z^3 - 4z^2 + 8z + 8 = 0$.

(a) Write down another complex root β, and hence find the real root of the equation. [3]

(b) Find the modulus and argument of each of the complex numbers α, β, $\alpha\beta$ and $\frac{\alpha}{\beta}$. [4]

(c) Sketch the locus of points in the Argand diagram representing the complex numbers z for which $|z - \alpha| = \sqrt{3}$. [2]

11. (a) The points A and B have position vectors $\mathbf{a} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ respectively relative to a fixed point O. The line L_1 has vector equation $\mathbf{r} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Write down a vector equation for the line L_2 which passes through the points A and B. [1]

(ii) Show that the lines L_1 and L_2 intersect and state the position vector of the point of intersection. [3]

(iii) Calculate the acute angle between the lines L_1 and L_2. [2]

(b) Two planes have vector equations

$$\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} - \mathbf{k}) = 14 \quad \text{and } \mathbf{r}.(11\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 42 .$$

(i) Find the acute angle between these two planes. [2]

(ii) Determine a vector equation for the line of intersection of these two planes. [3]