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CAMBRIDGE A LEVEL PROGRAMME
SEMESTER ONE EXAMINATION JUNE 2011
(January 2011 Intake)

Thursday **9 June 2011** **8.30 – 10.15 am**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

[Turn over

1 The equation of a curve is $y = x^2 - 5x + 7$. Show that the curve is always positive. [3]

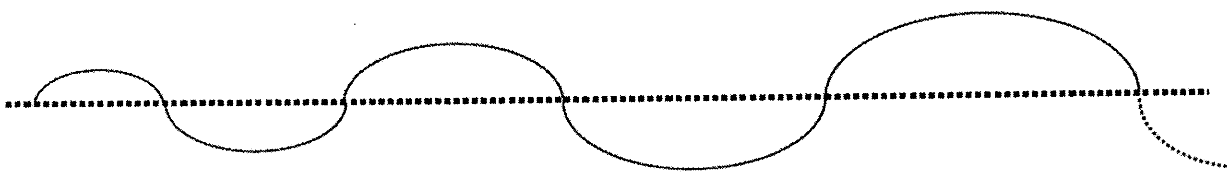
2 **(i)** Find the first three terms in the expansion of $(2 + x)^8$ in ascending powers of x, simplifying the coefficients. [3]

(ii) Determine the coefficient of y^4 in the expansion of $(2 + \frac{1}{2}y^2)^8$. [2]

3 **(i)** Show that the equation $15\cos^2\theta = 13 + \sin\theta$ can be written as a quadratic equation in $\sin\theta$. [2]

(ii) Hence solve the equation, giving all values of θ such that $0° \leq \theta \leq 360°$. [4]

4

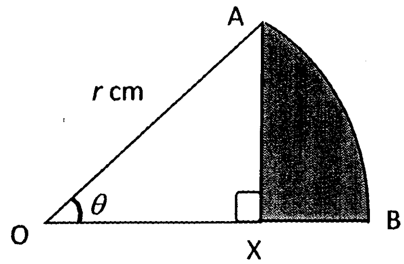


A piece of wire is bent to form arcs of semicircles, as shown in the above figure. The radius of the smallest semicircle is 3 cm and the radii of the following semicircles increase by 2 cm, in sequence.

(i) If the radius of the largest semicircle is 41cm, calculate the number of semicircles that are formed. [2]

(ii) Can the semicircle pattern in the figure be formed from a 360π cm long wire? [4]

5



In the diagram, AB is an arc of a circle, centre O and radius r cm, and angle AOB = θ radians. The point X lies on OB and AX is perpendicular to OB.

(i) Show that the area, A cm ^{2}, of the shaded region AXB is given by

$$A = \frac{1}{2}r^2(\theta - \sin\theta\cos\theta) \qquad [3]$$

(ii) In the case where $r = 18$ and $\theta = \frac{1}{6}\pi$, find the perimeter of the shaded region AXB, leaving your answer in terms of $\sqrt{3}$ and π. [3]

6 A curve is such that $\frac{dy}{dx} = p - x$, where p is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 4$ are perpendicular, find the value of p. [4]

(ii) Given also that the curve passes through the point (2, 7), find the equation of the curve. [3]

7 The points A and B have position vectors $4\mathbf{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$ and $3\mathbf{i} + 6\boldsymbol{j} + 6\boldsymbol{k}$ respectively, relative to the origin O. Find

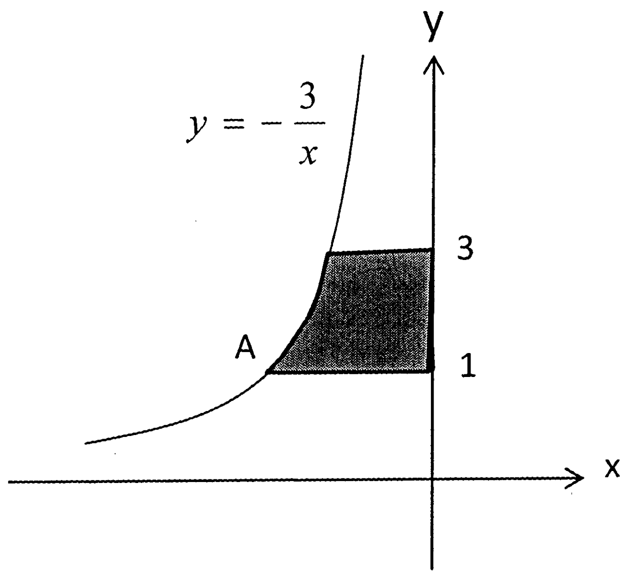
(i) the unit vector in the direction of **OA**, [2]

(ii) the length of **AB**, and [2]

(iii) the acute angle between **OA** and **OB**. [3]

[Turn over

8



The diagram shows part of the curve $y = -\frac{3}{x}$

(i) Find the equation of the normal to the curve at A. [3]

(ii) Find the volume obtained when the shaded region is rotated through 360^0 about the y-axis from $y = 1$ to $y = 3$, giving your answer in terms of π. [4]

9 Three points have coordinates A(– 2, 4), B(6, 12) and C(4, 4). The perpendicular bisector of AB meets the line BC at D. Find

(i) the equation of the perpendicular bisector of AB in the form $ax + by = c$. [4]

(ii) the coordinates of D. [4]

10 If $f(x) = x^2 - 8x + 17$, $x < A$, A is a constant.

(i) State the largest value of A for which f has an inverse. [2]

(ii) Find $f^{-1}(x)$ and its domain. [4]

(iii) Sketch $f(x)$ and $f^{-1}(x)$, making clear the relationship between the two graphs. [3]

11 The equation of a curve is $y = 6x^2 - x^3$.

(i) Find the coordinates of the two stationary points on the curve, and determine the nature of each of these stationary points. [6]

(ii) State the set of values of x for which y is a decreasing function of x. [2]

(iii) The gradient at the point M on the curve is 12. Find the equation of the tangent to the curve at M. [3]