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TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2005

(July 2004 Intake)

Tuesday **30 August 2005** **1.00 pm – 2.45 pm**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 5 printed pages.

[Turn over

6. (i) Express $10\,i$ in the form $r(\cos\theta + i\sin\theta)$, where r > 0 and $0^0 < \theta < 180^0$.

[1]

(ii) The complex number $5 - 2i$ is denoted by a .Express $\frac{a}{a^*}$ in the form $p + iq$.

[2]

(iii) In each of the following cases, show by a clear drawing in an Argand diagram, the set of points representing z, given that

(a) $|z-1| = |z+i|$ (b) $Re(z+2) = 3$. [4]

7. The area of a circle of radius r metres is $A\ m^2$

(i) Find $\frac{dr}{dA}$ in terms of r . [1]

(ii) The area increases with time t seconds in such a way that

$\frac{dA}{dt} = \frac{2}{(t+1)^3}$. Find an expression, in terms of r and t, for $\frac{dr}{dt}$. [2]

(iii) Solve the differential equation $\frac{dA}{dt} = \frac{2}{(t+1)^3}$ to obtain A in terms of t,

given that $A = 0$ when $t = 0$. [3]

(iv) Show that, when t = 1, $\frac{dr}{dt} = 0.081$, correct to 2 significant figures. [3]

[Turn over

1. The real polynomial $P(x) = (ax-b)^3$ has a remainder of -8 on division by $x-1$ and a remainder of 27 on division by $x-2$. Find the constants a and b. [3]

2. (i) Show that the iterative formula $x_{n+1} = \dfrac{2x_n^{\ 3} + 5}{3x_n^{\ 2} + 2}$ can be rearranged into the form $x^3 + ax + b = 0$. [1]

   (ii) With these values of a and b, use an iteration based on this rearrangement with initial approximation $x_0 = 1$ to find a solution to the equation .Leave your answer correct to three decimal places [3]

3. Given the equation of a curve is $2x^2 - 3xy + y^2 = 5$, find the equation of the tangent to the curve at the point (4,3). [5]

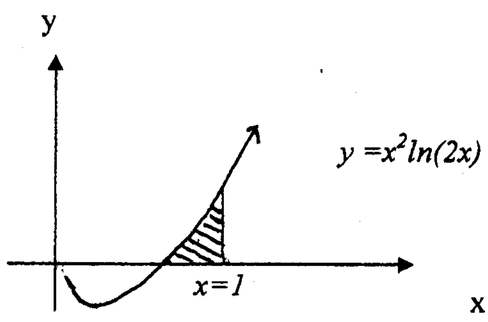
4. Express $\dfrac{5x^2 - 4x + 12}{(x-2)(x^2+4)}$ in partial fractions. [3]

   Hence obtain $\dfrac{5x^2 - 4x + 12}{(x-2)(x^2+4)}$ as a series of ascending powers of x up to and including the term in x^3 . [3]

5. By means of the substitution $x = \tan\theta$ or otherwise, find the exact value of

   $\displaystyle\int_0^{\infty} \frac{x^2}{(1+x^2)^2}\,dx$. [7]

8. The sketch below shows the curve with equation $y = x^2 \ln(2x)$



(i) Find the exact coordinates of the stationary point. [5]

(ii) Find the x-coordinate of the point where the curve cuts the x- axis.
Hence calculate the area of the shaded region. [5]

9. (i) Show that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$. [2]

(ii) Find all solutions of the equation $2\operatorname{cosec}2\theta = 3\tan\theta + 1$ for $-\pi \le \theta \le \pi$. [6]

(iii) Express $3\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0^0 < \alpha < 90^0$. Hence or otherwise, solve the equation $3\cos\theta - \sin\theta = -1$ for $-180^0 \le \theta \le 180^0$. [4]

$-4+6$

$0+3$

10. Two lines l_1 and l_2 have equations given by

$$\vec{r} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \text{ and } \vec{r} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \text{ respectively.}$$

$\lambda =$

$\mu =$

$(2,3,4)$.

(i) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection. [5]

(ii) Show that the vector equation of the plane π_1 which contains l_2 and is perpendicular to the plane $-2x+4y+7z=36$ is $\vec{r} \cdot \begin{pmatrix} 7 \\ -14 \\ 10 \end{pmatrix} = 12$. [4]

(iii) Find the angle between π_1 and l_1 . [3]