TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2011

(March 2011 Intake)

Thursday **9 June 2011** **8.15 – 10.15 am**

MATHEMATICS 9709/1,6

Paper 1 and Paper 6 2 hours

Additional materials: Answer Paper
Graph Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

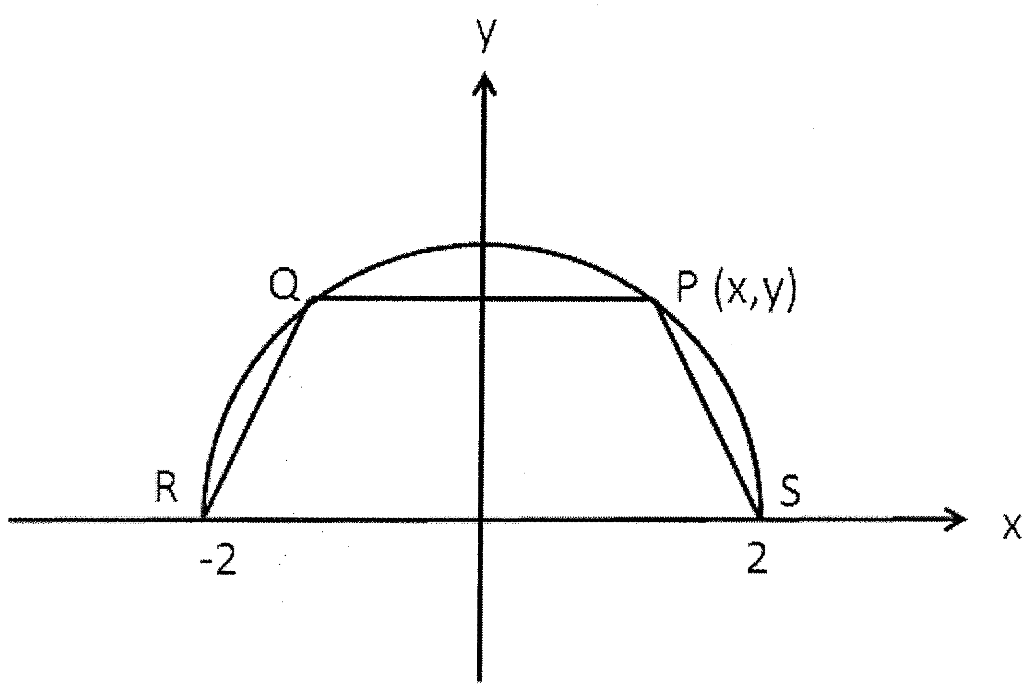
This document consists of **6** printed pages.

Section A: Pure Mathematics (P1)

1 Find the value of m for which the straight line $y = mx + 4$ is the tangent to the curve $y = x^2 + 5$. [3]

2 Find the term independent of x in the expansion of $(x - \frac{2}{x^2})^6$. [3]

3



An isosceles trapezium $PQRS$ is inscribed between the parabola $y = 4 - x^2$ and the x-axis. $P\,(x, y)$ is a variable point on the parabola $(0 < x < 2)$ and the base of the trapezium RS lies along the x-axis.

(i) Show that the area of the trapezium is given by $A(x) = (4 - x^2)(x + 2)$. [3]

(ii) Hence find the value of x such that area is maximum. [3]

4 If $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{k}$, find

(i) $3\mathbf{a} - 2\mathbf{b}$. [1]

(ii) $|3\mathbf{a} - 2\mathbf{b}|$. [1]

(iii) $-3\mathbf{a} \cdot (\mathbf{b} - \mathbf{a})$. [2]

(iv) angle between $-3\boldsymbol{a}$ and $(\boldsymbol{b} - \boldsymbol{a})$. [3]

5

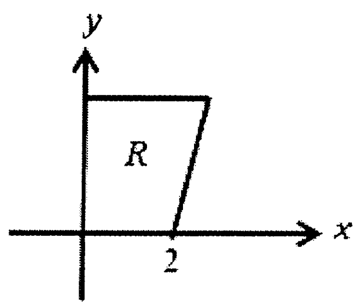


Figure 1

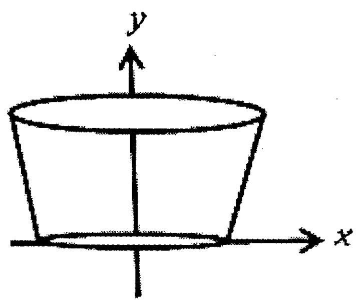


Figure 2

The region R in figure 1 is bounded by the line $y = 8(x - 2)$, the axes and the line $y = 4$. Volume of a whisky glass is formed when R is rotated through 360° about the y-axis. (see figure 2).

(i) A whisky taster holds the glass upright, and pours in whisky to a depth of 2 cm. He then adds water to a further depth of 2 cm. Find the volume of whisky in the glass. [3]

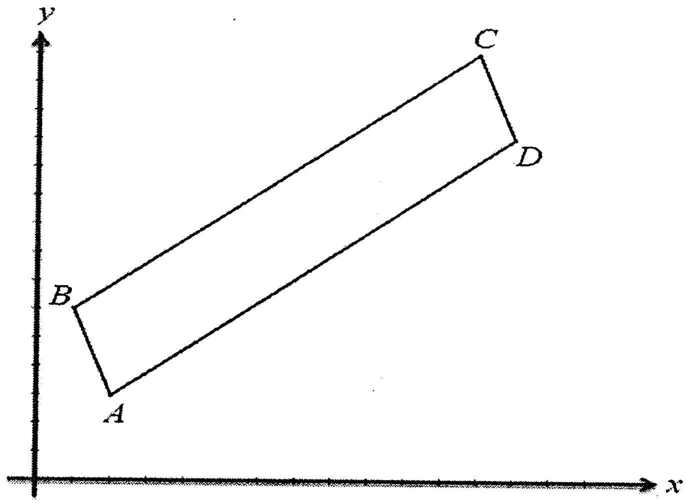
(ii) He then poured in the water first then the whisky, each to a depth of 2 cm. Show that the glass would have contained approximately 25 % more whisky than by the first method. [4]

6 **(a)** Prove the identity $(1 + \sin\theta + \cos\theta)^2 = 2(1 + \sin\theta)(1 + \cos\theta)$. [3]

(b) Solve the equation $\tan^2\theta = 2\cos^2\theta$, for $-\pi \le \theta \le \pi$. [5]

[Turn over

7



A rectangle $ABCD$ has points A (3, 2) and B (1, 6).

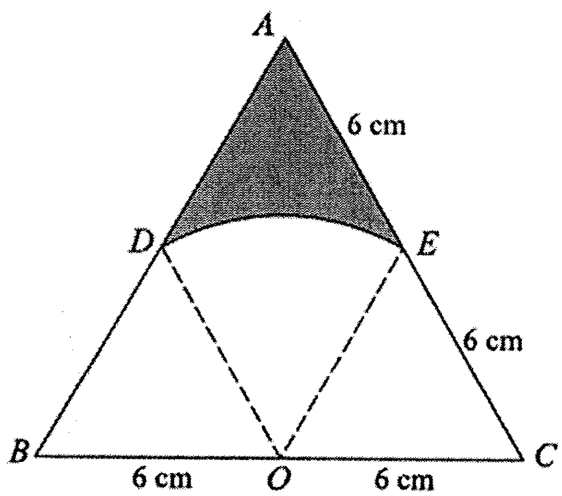
(i) Find the equation of BC. [3]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C. [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

8



The diagram shows an equilateral triangle ABC with sides of length 12 cm. The mid-point of BC is O, and a circular arc with centre O joins D and E, the mid-points of AB and AC.

(i) Find the length of the arc DE, and find the area of the sector ODE. [4]

(ii) Find the exact area of the shaded region in terms of $a\sqrt{3} - b\pi$, where a and b are positive integers. [4]

9 **(a)** A building has a large sloping tiled roof. The tiles are arranged in horizontal rows. The numbers of tiles in the rows form an arithmetic sequence. In the top row there are 120 tiles. Below that, each row has 8 tiles more than the row above it. The bottom row has 280 tiles. Calculate

(i) the number of rows of tiles on the roof. [2]

(ii) the number of tiles on the roof. [2]

(b) A geometric sequence u_1 , u_2, u_3...... is defined by u_n=2 $(4)^n$

(i) Write down the values of u_1, u_2, and verify that u_3 =128. [2]

(ii) Show that the sum of the first 20 terms is $\frac{8}{3}(4^{20} - 1)$. [3]

10 The function f is defined by $f: x \to 2x^2 - 8x + 11$ for $\overset{-2}{\cancel{2}} \leq x \leq A$.

i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry. [1]

iii) When A has this value, find the range of $f(x)$. [2]

The function g is defined by $g: x \to 2x^2 - 8x + 11$ for $x \leq 2$.

iv) Obtain the expression for $g^{-1}(x)$ [3]

v) Sketch in a single diagram, the graph of $y = g(x)$ and $y = g^{-1}(x)$, making clear relationship between the two graphs. [3]

Section B : Statistics (S1)

11 100 students are tested to determine their intelligence quotient, (IQ) and the results were as follows:

IQ	45-	55-	65-	75-	85-	95-	105-	115-	125-134
No of students	1	1	2	6	21	29	24	12	4

All IQ's are given to the nearest integer.

(i) Construct a grouped frequency table for these data such that the first class having a lower boundary of 44.5kg and upper boundary of 54.5 kg. Hence draw a cumulative frequency graph. [4]

(ii) Calculate the mean (m) and the standard deviation (s). [3]

(iii) From the graph estimate how many students have IQ's within 1 standard deviation on either side of the mean. ($m \pm s$). [2]