



CAMBRIDGE A LEVEL PROGRAMME
AS TRIAL EXAMINATION AUGUST/SEPTEMBER 2008
(January 2008 / March 2008 Intake)

Tuesday

2 September 2008

1.30 pm – 3.15 pm

MATHEMATICS

9709/1

PAPER 1 Pure Mathematics 1 (P1)

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

- 1 Find the term which is independent of x in the expansion of $(x - \frac{1}{3x^2})^6$. [3]

- 2 Solve the following equations for $-180^\circ \leq x \leq 180^\circ$.

$$\frac{1}{2} \sin^2 x = 1 - 2 \cos x. \quad [5]$$

- 3 A spherical balloon is being inflated in such a way that its volume is increasing at a constant rate of $50 \text{ cm}^3 \text{ s}^{-1}$. At time t seconds, the radius of the balloon is r cm.

(i) Find $\frac{dr}{dt}$ when $r = 25$. [4]

- (ii) Find the rate of increase of the surface area of the balloon when its radius is 20 cm. [2]

- 4 The points $(1, 0)$, $(0, 2)$, $(2, -1)$ are denoted by A , B and C , respectively.

(i) Show that the equation of the line L through C perpendicular to AB is $2y - x + 4 = 0$. [2]

- (ii) The line AB produced meets the line L at the point D and the line DC produced meets the x -axis at the point E . Find the coordinates of point D and point E . [4]

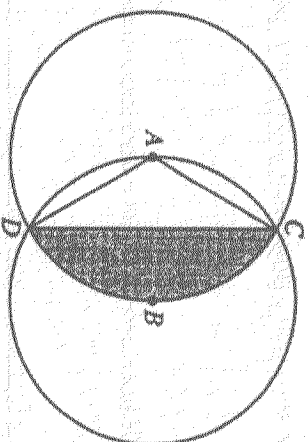
- 5 A quadratic function is defined by $f(x) = x^2 + kx + 9$, where k is a constant. It is given that the equation $f(x) = 0$ has two distinct real roots. Find the set of values that k can take. [2]

For the case where $k = -4\sqrt{3}$,

- (i) Express $f(x)$ in the form $(x + a)^2 + b$, stating the values of a and b , and hence write down the least value taken by $f(x)$, [3]

- (ii) Solve the equation $f(x) = 0$, expressing your answer in terms of surds, simplified as far as possible. [2]

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The diagram shows two circles, with centres A and B , intersecting at C and D in such a way that the centre of each lies on the circumference of the other. The radius of each circle is 1 unit. Write down the size of angle CAD and calculate the area of the shaded region (bounded by the arc CBD and the straight line CD). Hence show that the area of the region common to the interior of the two circles is approximately 39% of the area of one circle. [8]

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(i) The first four terms u_1, u_2, u_3, u_4 of an arithmetic progression are such that

$$u_4 - u_2 = 15 \quad \text{and} \quad 4u_3 = 9u_1.$$

Find the value of u_1 .

[3]

(ii) The first four terms v_1, v_2, v_3, v_4 of a geometric progression are such that

$$v_4 - v_2 = 15 \quad \text{and} \quad 4v_3 = 9v_1.$$

Find the values of v_1 .

[5]

8 The functions f and g are defined as follows

$$f : x \rightarrow x^2 + 4x, \quad x \geq -2$$

$$g : x \rightarrow x + 6, \quad x \in \mathcal{R}$$

(i) Show that the equation $gf(x) = 0$ has no real roots.

[2]

(ii) State the domain of f^{-1} .

[1]

(iii) Find an expression in terms of x for $f^{-1}(x)$.

[2]

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]
[Turn over]

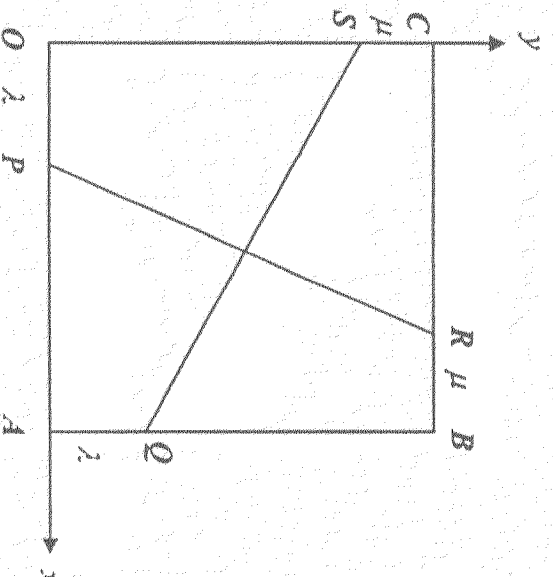
- 9 Calculate the coordinates of the stationary point on the curve with equation $y = x^{\frac{1}{2}} - x$. Show that the stationary point is a maximum point. [3]

Find the exact value of

- (i) the area of the region R between the curve and the x -axis from $x = 0$ to $x = 1$. [3]

- (ii) the volume of the solid formed when the region R is rotated completely about the x -axis, giving your answer as a multiple of π . [4]

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The unit square $OABC$ has vertices $O(0, 0)$, $A(1, 0)$, $B(1, 1)$ and $C(0, 1)$. Points P , Q , R , S are taken on the sides OA , AB , BC , CO respectively so that $OP = AQ = \lambda$ and $BR = CS = \mu$, where λ and μ each lie between 0 and 1, shown in the diagram. Take $\overrightarrow{OA} = \mathbf{i}$ and $\overrightarrow{OC} = \mathbf{j}$.

- (i) Write down \overrightarrow{PR} and \overrightarrow{QS} in terms of \mathbf{i} and \mathbf{j} . [2]

- (ii) Show that \overrightarrow{PR} and \overrightarrow{QS} are perpendicular. [2]

The point T is taken so that $\overrightarrow{OT} = \overrightarrow{OQ} + \overrightarrow{SR}$.

- (iii) Show that $\overrightarrow{RT} = \mathbf{i} + (a-1)\mathbf{j}$ and $\overrightarrow{PT} = (2-a)\mathbf{i} + a\mathbf{j}$, where $a = \lambda + \mu$. [4]

- (iv) Hence, or otherwise, show that angle $RTP = 45^\circ$. [3]

- (v) Given that P , Q and T are collinear, show that $\lambda = \mu$. [3]