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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION DECEMBER 2010

(June 2010 Intake)

Monday | **6 December 2010** | **1.00 – 2.45 pm**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages.

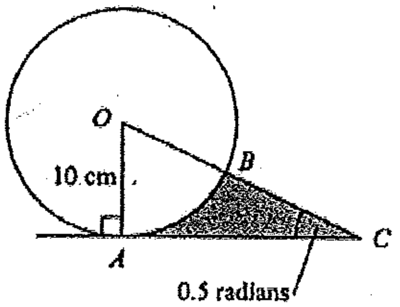
1 (i) Expand $(1 + 2x)^3$. [2]

(ii) Hence, find the term in x^2 in the expansion of $(2 - x + x^2)(1 + 2x)^3$. [2]

2 (a) Find the range of values of x for which $x(8-x) \le 15$. [2]

(b) Find the value of m for which the straight line $y = mx + 12$ is a tangent to the curve $x^2 + xy + 9 = 0$. [3]

3

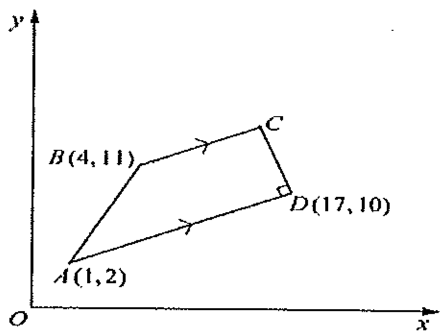


The diagram shows a circle, centre O, of radius 10 cm. The line AC is perpendicular to the radius OA, and the line OC intersects the circle at B. Given that angle OCA is 0.5 radians, calculate

(i) the area of the shaded region. [3]

(ii) the perimeter of the shaded region. [3]

4



The diagram, which is not drawn to scale, shows a trapezium $ABCD$ in which BC is parallel to AD. The side AD is perpendicular to DC. Point A is (1, 2), B is (4, 11) and D is (17, 10). Find the

(i) coordinates of C. [4]

(ii) point P which is the foot of the perpendicular from the point B to the line AD. [3]

5 (a) Show that $\cos\theta\left(\frac{1}{1-\sin\theta}-\frac{1}{1+\sin\theta}\right)$ can be written in the form $k\tan\theta$ where k is an integer. [3]

(b) Solve the equation $2 + 4\cos^2 2x^{\circ} = 5$ giving all solutions in the interval $-180^{\circ} \le x \le 180^{\circ}$. [4]

6 Express $2x^2 - 8x + 5$ in the form $a(x + b)^2 + c$ where a, b and c are integers.
The function f is defined by $f : x \rightarrow 2x^2 - 8x + 5$ for the domain $0 \le x \le 5$. [2]

(i) Find the range of f. [1]

(ii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \rightarrow 2x^2 - 8x + 5$ for the domain $x \ge k$.

(iii) Find the smallest value of k for which g has an inverse. [1]

(iv) For this value of k, find an expression for g^{-1}. [3]

7 Relative to the origin O, points A, B and C have position vectors $7\mathbf{i} + 6\mathbf{j} - \mathbf{k}$, $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ respectively. The point P is given by $\overrightarrow{AB} = 2\overrightarrow{BP}$.
Find the

(i) position vector of P [3]

(ii) angle of APC [3]

(iii)area of the triangle APC [2]

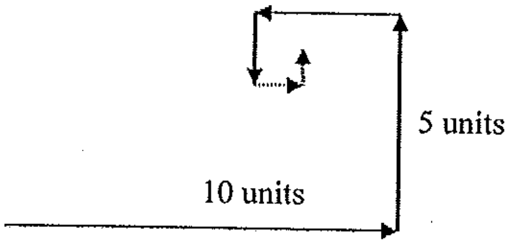
[Turn over

8 (a) Find

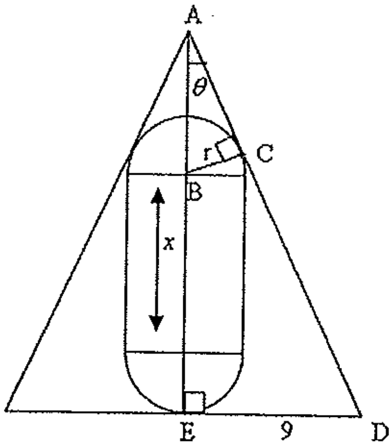
$$\left(-\frac{1}{2}+(m-1)d\right)+\left(-\frac{1}{2}+md\right)+\left(-\frac{1}{2}+(m+1)d\right)+\ldots\ldots+\left(-\frac{1}{2}+(m+n)d\right),$$

where m, n and d are constants, leaving your answer in terms of m, n and d. [4]

(b) An ant of negligible size walks a distance of 10 units from the origin in the x-y plane along the x-axis. It then turns left and goes up 5 units from its current point. If the ant continues turning left and going half the distance it had previously walked, repeating the pattern, find the coordinates of the point where the ant will eventually end up. [5]



9 A capsule made of two similar hemispheres of radius r cm, one at each end of a cylinder, is inscribed into a fixed cone of height 12 cm and base radius 9 cm. The two figures have a common axis of symmetry and the cross-section is shown below.



(i) Show that the height of the cylinder of the capsule, x cm, is related to the radius of the hemisphere through the equation, $x = 12 - \frac{8}{3}r$. [3]

(ii) Hence show that the volume of the capsule $V = -\frac{4}{3}\pi r^3 + 12\pi r^2$. [3]

(iii) Given that, when $r = 2$ cm, x is decreasing at a rate of 0.1 cms^{-1}, find the rate of change of the volume of the capsule at this instant. [4]

10

(i) Sketch, on the same coordinate axes, the line $y = \frac{1}{2}x$ and the curve $y = \sqrt{x}$.
Find the coordinates of the points of intersection. [4]

(ii) Find the area of the region bounded by the line $y = \frac{1}{2}x$ and the curve $y = \sqrt{x}$. [3]

(iii) Find the volume of the solid formed when the region is rotated through 360^0 about the y-axis. [4]