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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST 2011

(June 2010 Intake)

Thursday **18 August 2011** **12.30 pm – 2.15 pm**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

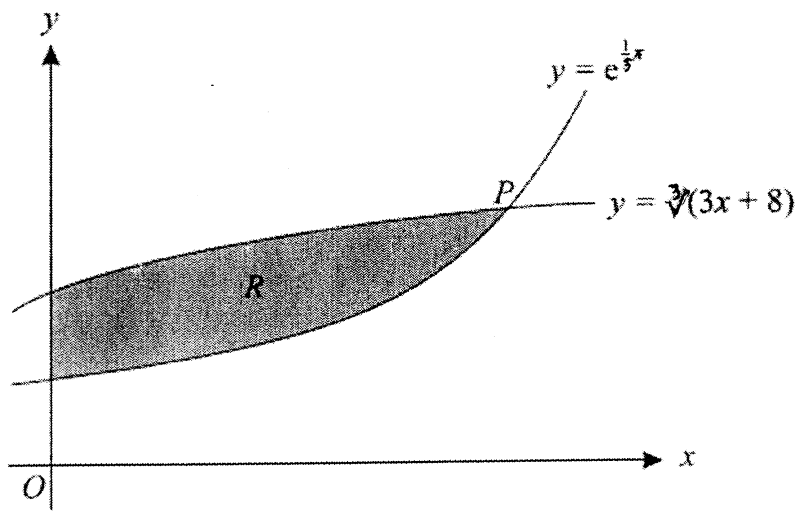
This document consists of **4** printed pages.

1 Solve the inequalities $|x + 2| \geq 2x + 1$ [4]

2 Find the exact value of *x* for the equation $2e^{x} + 2e^{-x} = 5$. [4]

3 The polynomial $x^3 + ax^2 + bx + 3$, where *a* and *b* are constants, is denoted by *p(x)*. It is given that $(x - 2)$ is a factor of *p'(x)* and when *p(x)* is divided by $(x - 2)$ it gives a remainder of -5. Find the values of *a* and *b*. [5]

4



The diagram shows part of each of the curves $y = e^{\frac{1}{5}x}$ and $y = \sqrt[3]{3x + 8}$. The curves meet, as shown in the diagram, at the point *P*. The region *R*, shaded in the diagram, is bounded by the two curves and by the *y*-axis.

(i) Show by calculation that the *x*-coordinate of *P* lies between 5.2 and 5.3. [2]

(ii) Show that the *x*-coordinate of *P* satisfies the equation $x = \frac{5}{3}\ln(3x + 8)$. [2]

(iii) Use an iterative formula, based on the equation in part (ii), to find the *x*-coordinate of *P* correct to 2 decimal places. [3]

5 **(i)** Express $3sin\theta + 2cos\theta$ in the form $Rsin(\theta + \alpha)$, where $R > 0$ and $0^o < \alpha < 90^o$. [3]

(ii) Hence solve the equation $3sin\theta + 2cos\theta = \frac{7}{2}$, giving all solutions for which $0^o < \theta < 360^o$. [4]

6 **(i)** Express $\frac{2(4x^2+1)}{(2x+1)(2x-1)}$ in partial fractions. [4]

(ii) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\, dx$ is $2 + \ln k$, giving the value of the constant k. [4]

7 A curve has parametric equations

$$x = 2\cot t\,,\quad y = 2\sin^2 t\,,\quad 0 < t \leq \frac{\pi}{2}$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of the parameter *t*. [4]

(ii) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$. [4]

8 With respect to the origin *O*, the points *A* and *B* have position vectors given by $\overrightarrow{OA} = -\boldsymbol{i} + \boldsymbol{j} + 5\boldsymbol{k}$ and $\overrightarrow{OB} = 5\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$. The point *P* lies on the line *AB* and *OP* is perpendicular to *AB*.

(i) Find a vector equation for the line *AB*. [1]

(ii) Find the position vector of *P*. [4]

(iii) Find the equation of the plane which contains *AB* and which is perpendicular to the plane *OAB*, giving your answer in the form $ax + by + cz = d$. [3]

[Turn over

9 The volume of a spherical balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(i) Find $\dfrac{dr}{dt}$ in terms of r and t. [3]

(ii) Given that V=0 when t=0, solve the differential equation $\dfrac{dV}{dt} = \dfrac{1000}{(2t+1)^2}$, to obtain V in terms of t. [4]

Hence, find the radius of the balloon and $\dfrac{dr}{dt}$ at time $t = 5$. [4]

10 **(i)** The complex numbers z and w have moduli k and $3k^2$ respectively and argument α and 4α respectively, where k is a positive constant and $-\frac{\pi}{4} < \alpha < \frac{\pi}{4}$.

Express in the form $a + bi$ in terms of k and α.

(a) z^3 [3]

(b) $\dfrac{z}{w^*}$ [3]

(ii) Describe completely, in geometrical terms, the loci given by $|z-1| = |z+i|$ and $|z-3+3i| = 2$ and sketch both loci on the same diagram. [4]

Obtain, in the form $a + bi$, the complex numbers representing the points of intersection of the loci, giving the exact values of a and b. [3]