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TAYLOR'S COLLEGE

Wisdom • Integrity • Excellence

CAMBRIDGE 'A' LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH / APRIL 2005

(January 2004 & July 2003 [2-year] Intake)

Thursday **31 March 2005** **8.30 am – 10.15 am**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet /Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

 [Turn over

1. Find the first three non-zero terms in the expansion , in ascending powers of x , of $\frac{(1+x)}{(1-x)}$. [4]

2 . Find the exact value of $\int_1^e (\ln x)^2\, dx$. [4]

3. The parametric equations of a curve are $x = a(1 + t)$, $y = a(t^2 - 1)$ where a is a constant and $t \neq 0$. Find the equation of the normal to the curve, in terms of a, at the point where $t = 1$. [5]

4. (i) Show that the equation $2^{1-x} = 4x + 1$ can be arranged in the form

$$x = \frac{1}{2}(2^{-x}) + Q$$ stating the value of the constant Q . [2]

(ii) Using the iterative formula

$$X_{n+1} = \frac{1}{2}\left(2^{-x_n}\right) + Q\ , \quad X_0 = 0.2$$

with the value of Q found in part (i), find the root of the equation up to 4 decimal places. [4]

5. Given that $f(x) \equiv 4\cos x - b\sin x \equiv \sqrt{20}\cos(x + \alpha)$, $b > 0$, $0 < \alpha < \frac{\pi}{2}$,

(i) find the value of b and the value of $\tan\alpha$. [3]

(ii) Solve the equation $f(x) = \sqrt{5}$, for $0 < x < 2\pi$, giving your answer to 3 significant figures. [3]

(iii) Find the maximum value of $\frac{1}{f(x)+5}$, $x \in \Re$. [1]

6 Evaluate $\int_0^1 \frac{1}{1+x^2}\, dx$ using

(i) the trapezium rule with 5 intervals , [4]

(ii) integration with the substitution $x = \tan\theta$. [3]

7 The rate of change per minute in the temperature (T °C) of an object is proportional to the difference between T and T_m where T_m is the room temperature.

(i) Show that $\dfrac{dT}{dt} = k(T - T_m)$ where k is a constant called the Cooling Coefficient. [1]

Copper has a Cooling Coefficient of -0.19 and a copper rod which was out in the sun cools from 65 °C to room temperature of 15 °C.

(ii) Solve the differential equation, expressing T in terms of t. [3]

(iii) At what rate is the rod's temperature decreasing when ,

(a) its temperature is 40 °C, [1]

(b) it has cooled for 2 minutes . [2]

(iv) How long would it take for the temperature to reach 20 °C. ? [2]

8 Given that $f(x) = 2x^3 + 9x^2 + 10x + 3$ and one of the roots of the equation $f(x) = 0$ is -3 ,

(i) factorise f(x) completely. [2]

(ii) Express $\dfrac{10}{f(x)}$ in partial fractions. [3]

(iii) Evaluate $\displaystyle\int_0^1 \frac{10}{f(x)}\,dx$, giving your answer in the form of ln k ,

where k is a rational number. [4]

[Turn over

9 The complex number z_1 has modulus 2 and argument $\frac{\pi}{3}$.

(i) Write z_1 in the form $a + bi$, where $a , b \in \Re$. [1]

(ii) Write down the modulus and argument of :

(a) z_1^3 [2]

(b) $\frac{1}{z_1^2}$ [2]

(iii) Given that $z_2 = 3 - i$ and $z_3 = 2 + 3i$,

(a) find $\frac{z_3}{z_2}$ in the form $c + di$ where $c , d \in \Re$ [2]

(b) Use an Argand diagram to find , in the form $a + bi$, the complex number which satisfy the following pairs of equation :

$$| z - z_2 | = 3 \text{ and } \text{argument } (z - z_2) = \frac{\pi}{3} .$$

[5]

10 The equation of the plane Π_1 is $y + z = 0$ and the equation of the line L is

$\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + t(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$. Find :

(i) the position vector of the point of intersection of L and the plane Π_1, [3]

(ii) the length of the perpendicular distance from the origin to the line L , [3]

(iii) an equation of the plane Π_2 which contains the line L and the origin, [3]

(iv) the acute angle between the planes Π_1 and Π_2, giving your answer correct to the nearest tenth of a degree. [3]