TAYLOR'S UNIVERSITY COLLEGE
Wisdom • Integrity • Excellence

CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2009

(January & March 2008 Intakes)

Thursday **2 April 2009** **8.30 am – 10.15 am**

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of 5 printed pages.

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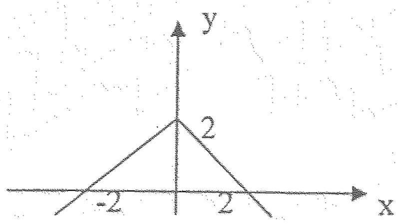
1



(i) The diagram shows the graph of $y = 2 - |x|$.

Sketch the graph of $y = |2 - |x||$. [1]

(ii) Solve the inequality $|x+1| < |x-2|$. [3]

2 Consider $f(x) = x^3 - 2x - 5$.

(i) Show that there is a root α in the interval $2 \le x \le 3$. [1]

The root α is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{(2 + \frac{5}{x_n})} \quad , \quad x_0 = 2$$

(ii) Calculate the value of α, giving your answer to 4 significant figures. [3]

3 Find the exact value of $\int_0^1 3x^2 e^x \, dx$. [5]

4 A curve is given parametrically by the equation

$$x = 4\cos t \ , \ y = 3\sin t \text{ where } \quad 0 \le t \le \frac{\pi}{2}$$

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent at the point P, where $t = p$ is $3x\cos p + 4y\sin p = 12$. [3]

5 (i) Prove that $1+\sin\theta-\cos\theta = 2\sin\ (\frac{1}{2}\theta)\left[\cos(\frac{1}{2}\theta)+\sin(\frac{1}{2}\theta)\right]$. [2]

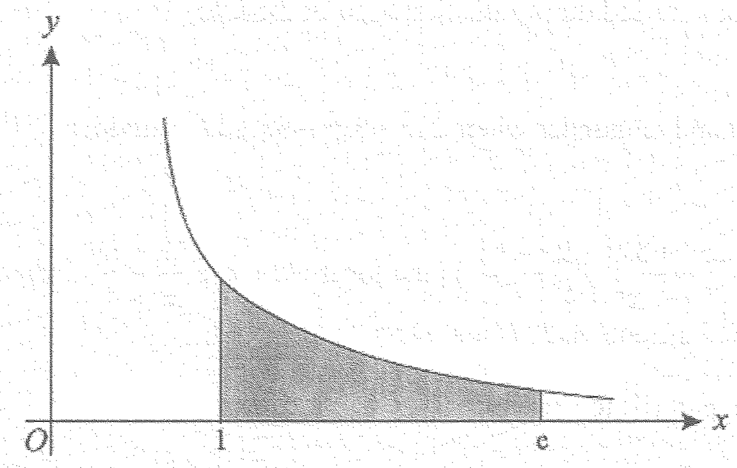
(ii) Hence, or otherwise, solve the equation

$$1+\sin\theta-\cos\theta = 0 \quad , \quad 0 \le \theta \le 2\pi$$ [4]

6 (i) Given that $y = \dfrac{4\ln x - 3}{4\ln x + 3}$

Show that $\dfrac{dy}{dx} = \dfrac{24}{x(4\ln x + 3)^2}$. [3]

(ii)



The diagram shows part of the curve with equation $y = \dfrac{2}{\sqrt{x}(4\ln x + 3)}$.

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = e$ and $y = 0$.

Find the exact volume of the solid produced when this shaded region is rotated 360^0 about the x – axis. [4]

[Turn over

7 The organizer of a sale, which lasted for 3 hours and raised a total of £ 1000, attempted to create a model to represent the relationship between s and t and £s is the amount which has been raised at time t hours after the start of the sale. In the model, s and t were taken to be continuous variables .The organizer assumed that the rate of raising money varied directly as the time remaining and inversely as the amount already raised .Show that for this model,

$$\frac{ds}{dt} = k\left(\frac{3-t}{s}\right) \quad \text{where } k \text{ is a constant}$$

[2]

Solve the differential equation and show that it can be written in the form

$$\frac{s^2}{1000^2} + \frac{(3-t)^2}{3^2} = 1.$$

[6]

Hence, find the amount raised during the first hour of the sale. [1]

8 (i) Find the quotient and remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence, express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii)Use the result in part (ii) to show that [5]

$$\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}\,dx = 14 + \frac{1}{2}\ln\left(\frac{13}{5}\right).$$

9 (i) Suppose $z = \frac{1}{2}(\cos\theta + i\sin\theta)$.

(a) State $|z|$ and find $(1-z)^*$ [3]

(b) Find the imaginary part of $\dfrac{1}{1-z}$ [4]

(ii) Given the set of the complex numbers z is such that $|z - 2i| = 1$.

Sketch $|z - 2i| = 1$ on an Argand diagram. Find the largest possible value of $\arg z$. [4]

10 Lines L_1 , L_2 and L_3 has vector equations

$$L_1 : \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k})$$
$$L_2 : \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$
$$L_3 : \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} + c\mathbf{j} + \mathbf{k})$$

(i) Calculate the angle between L_1 and L_2. [3]

(ii) Given that L_2 and L_3 intersect, find the value of c. [3]

(iii) Find the equation of the plane Π containing the point $A(5,-1,-2)$ and is perpendicular to L_2. [3]

(iv) Determine the perpendicular distance of $B(6,0,3)$ to the plane Π. [4]