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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION AUGUST/SEPTEMBER 2010

(June 2009 Intake)

Wednesday **1 September 2010** **12.30 pm – 2.15 pm**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

[Turn over

1 Expand $\frac{1}{(3+x)^4}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve $4^x + 15 = 2^{x+3}$, giving the values of x to 3 significant figures. [4]

3 Find the gradient of the curve with equation

$$x^2 + 2xy - y^3 = 1$$

at the point (2, -1). [4]

4 The polynomial $x^4 + x^3 + ax^2 - 17x - 30$ is denoted by $p(x)$, where a is a constant. It is given that when $p(x)$ is divided by $(x+4)$ the remainder is 182. Find the value of a and factorise $p(x)$ completely. [4]

5 Given that $\frac{dy}{dx} = e^{x-y}$ and $y = 1$ when $x = 0$, find the exact value of y when $x = 1$. [5]

6 (i) By sketching a suitable pair of graphs, show that the equation $e^x = x + 5$ has one negative root and one positive root. [2]

(ii) Use the iterative formula

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 5}{e^{x_n} - 1}$$

with initial value $x_0 = 1.2$ to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The parametric equations of a curve are

$$x = a\sin\theta, \qquad y = a\theta\cos\theta$$

where a is a positive constant and $0 < \theta < \frac{\pi}{2}$. Find $\frac{dy}{dx}$ in terms of θ, and hence show that the gradient of the curve is zero where $\tan\theta = \frac{1}{\theta}$. [6]

8 Show that $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx = \frac{1}{8}\pi^2 - \frac{1}{2}$. [6]

9 (a) (i) Express $4\cos\theta - 6\sin\theta$ in the form of $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the value of α correct to 2 decimal places. [2]

(ii) Hence, solve the equation

$$4\cos\theta - 6\sin\theta = 5$$

giving all values of θ such that $0° < \theta < 360°$. [3]

(b) If $\operatorname{cosec}\gamma = \sin\beta - \cos\beta$, show that $\tan^2\gamma = -\operatorname{cosec} 2\beta$. [3]

10 (i) Express $\frac{x+4}{(x+1)^2(x+2)}$ in partial fractions. [5]

(ii) The finite region R is bounded by the curve $y = \frac{x+4}{(x+1)^2(x+2)}$, the x-axis, the y-axis and the line $x = 3$. Find the area R, giving your answer in the form $p + \ln q$, where p and q are rational numbers to be found. [4]

11 (a) A complex number z satisfies $|z|=|z-u|$, where u is denoted by $-2+2i$.

(i) Sketch the locus of the point which represents z in an Argand diagram. [2]

(ii) Find the least value of $|z|$. [2]

(iii) Express u in the form $r(\cos\theta+i\sin\theta)$, where $r>0$ and $-\pi<\theta\leq\pi$. [2]

(b) The complex number w is such that $ww^*+2w=3+4i$, where w^* is the complex conjugate of w. Find w in the form $a+bi$, where a and b are real. [4]

12 The lines l_1 and l_2 have vector equations

$$\mathbf{r}=-\mathbf{i}+2\mathbf{j}-4\mathbf{k}+s(-2\mathbf{i}+\mathbf{j}+3\mathbf{k}) \quad \text{and} \quad \mathbf{r}=-\mathbf{j}+7\mathbf{k}+t(-\mathbf{i}+\mathbf{j}-\mathbf{k})$$

where s and t are variable parameters.

(i) Show that the lines intersect and are perpendicular to each other. [4]

(ii) Find a vector equation of the straight line l_3 in terms of λ, which passes through the point of intersection of l_1 and l_2 and the point with position vector $4\mathbf{i}+\lambda\mathbf{j}-\mathbf{k}$, where λ is a real number. [2]

(iii) The line l_3 makes an angle θ with the plane containing l_1 and l_2. Find $\sin\theta$ in terms of λ. [4]