TAYLOR'S COLLEGE

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CAMBRIDGE A LEVEL PROGRAMME

SEMESTER ONE EXAMINATION JUNE 2010

(January 2010 Intake)

Thursday **10 June 2010** **8.00 am – 9.45 am**

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 (P1) 1 hour 45 minutes

Additional materials: Answer Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

1 Find the set of values of q for which

$$f(x) \equiv x^2 - 2qx + q + 2$$

is greater than zero for all real values of x. [4]

2 Expand $(1-2x-x^2)^6$ in ascending power of x, up to and including the term in x^3. [3]

Hence by substituting $x = 0.1$ use the binomial expansion to evaluate $(0.79)^6$. [2]

3 Prove the identity $1+\dfrac{\tan\theta\sin\theta}{\cos\theta} \equiv \dfrac{1}{\cos^2\theta}$. [2]

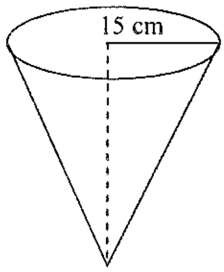
Hence solve the equation $3+\dfrac{3\tan\theta\sin\theta}{\cos\theta}-5=0$, for $0° \le \theta \le 360°$. [4]

4 The curve $y=\dfrac{1}{4}x^2-1$ and the line $2y = x+10$ intersect at the points A and B, and O is the origin.

(i) Calculate the coordinates of A and B. [3]

(ii) Hence show that OA and OB are perpendicular. [3]

5



The diagram shows a hollow right circular cone with a base radius of 15 cm and a height of 50 cm. It contains water which leaks through a small hole in the vertex at the rate of 60 cm^3s^{-1}. Find the exact rate at which the water level is falling when the height of the water in the cone is 25 cm from the vertex. [6]

6 Points A, B and C have coordinates $(4,\ 0,\ 4)$ $(0,\ 6,\ 6)$ and $(0,\ 0,\ c)$ respectively.

The point O is the origin, and the mid-point of AB is M.

(i) Given that $c = 5$, calculate the angle OMC. [5]

(ii) Find the value of c for which angle OMC is a right angle. [2]

7 **(a)** In an arithmetic progression, the first term is a and the common difference is d. Given that the sum of the first 8 terms is half the sum of the next four terms, express a in terms of d. [4]

(b) The first three terms of a geometric progression are $2p + q$, $6p + q$ and $14p + q$ where $p \neq 0$. Find

(i) q in terms of p. [4]

(ii) the common ratio of this progression. [2]

8 Function f, g, and h are defined by

$f : x \mapsto x + p$ for $x \in R$, where p is a constant,

$g : x \mapsto \dfrac{64}{4 - x}$ for $x \in R, x \neq 4$

$h : x \mapsto 12 + 8x - x^2$ for $x \leq 4$

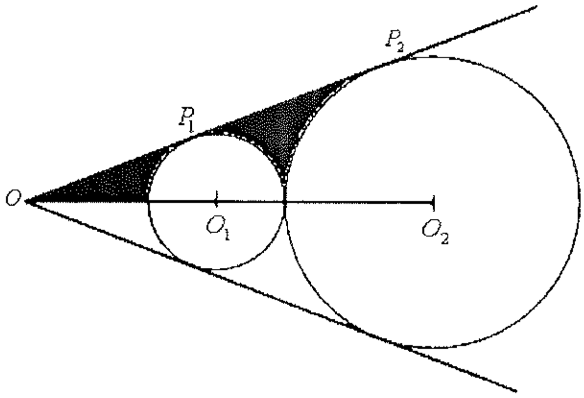
(i) Find the values of p for which equation $f(x) = g(x)$ has two equal roots. [4]

(ii) Solve the equation $fg(x) = -20$ for the positive value of p in part (i). [2]

(iii) Express $12 + 8x - x^2$ in the form $a - (x + b)^2$. [1]

(iv) Sketch, in a single diagram, the graph of $y = h(x)$ and $y = h^{-1}(x)$, making clear the relationship between the two graphs. [3]

[Turn over



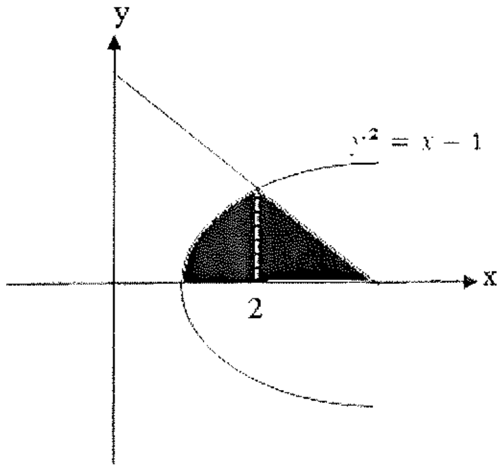
9 In the diagram, OP_1P_2 is a tangent to two circles with centres O_1 and O_2. Given that $OP_1 = 15$ cm and the radius of the circle centre O_1 is 8 cm.

(i) Find the radius of circle centre O_2. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

10 (a) Find $\int \sqrt{x-1}\, dx$. [2]



(b) The diagram shows the curve $y^2 = x - 1$ and the normal to the curve at the point $(2, 1)$.

(i) Find the equation of the normal at the point $(2, 1)$. [3]

(ii) Calculate the area of the shaded region. [6]