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CAMBRIDGE A LEVEL PROGRAMME

A2 TRIAL EXAMINATION MARCH/APRIL 2011

(January/March 2010 Intakes)

Monday **4 April 2011** **8.30 am – 10.15 am**

MATHEMATICS 9709/33

PAPER 3 Pure Mathematics 3 (P3) 1 hour 45 minutes

Additional materials: Answer Booklet/Paper
List of formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

[Turn over

1 Solve $|x - 2a| < 2x + a$, where $a > 0$. [4]

2 Find the exact value of $\int_0^1 x^2 e^{-3x}\, dx$. [5]

3 The polynomial $ax^3 + 8x^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - 2x - 3$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, factories $p(x)$ completely. [1]

4 A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

5 (i) Express $\sqrt{6}\sin 2\theta - \sqrt{3}\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$\sqrt{6}\sin 2\theta - 5 = \sqrt{3}\cos 2\theta - 7$$

giving all solutions in the interval $-180° < \theta < 180°$. [4]

6 A population growth is modeled by the differential equation

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant. Given that the initial population is P_0 and that time is measured in days,

(i) Solve the differential equation, giving P in terms of P_0, λ and t. [4]

Given also that $\lambda = 2.5$,

(ii) Find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using this model. [3]

7 With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \alpha(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \beta(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where α and β are scalar parameters.

(i) Show that l_1 and l_2 meet and find the position vector of their point of intersection. [5]

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(ii) Show that point A lies on l_1. [1]

(iii) Find an equation for the plane containing l_1 and is perpendicular to l_2, giving your answer in the form $ax + by + cz = d$. [2]

8 **(i)** By sketching a suitable pair of graphs, show that the equation $x^2 + 20\ln x = 15$ has one real root for $x > 0$. [2]

(ii) Show by calculation that this root lies between 1 and 2. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = e^{\left(\frac{15 - x_n^2}{20}\right)}$$

converges, then it converges to the root of the equation given in part (i). [2]

(iv) Use this iterative formula, with initial value $x_1 = 2$, to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

9 **(i)** Express $\dfrac{10 - 17x + 14x^2}{(2+x)(1-2x)^2}$ in partial fraction. [5]

(ii) Hence obtain the expansion of $\dfrac{10 - 17x + 14x^2}{(2+x)(1-2x)^2}$ in ascending powers of x, up and including the term in x^3. State the range of values of x for which this expansion is valid. [6]

10 **(i)** The complex number $5 - 2i$ is denoted by a, express $\dfrac{a}{a^*}$ in the form $p + iq$ [2]

(ii) The complex number $-3 + 4i$ is denoted by b, express $-3 + 4i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Hence find b^4. [5]

(iii) Sketch an Argand diagram showing the points representing the complex numbers a and b. Show on the same diagram the locus of the complex number z such that $|z - b| = 3$. [4]

(iv) Hence, obtain the least possible value of $|z - a|$. [2]