UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 3 3 1 3 6 7 1 8 3 4 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

1 Find $2^2 + 4^2 + \ldots + (2n)^2$. [2]

Hence find $1^2 - 2^2 + 3^2 - 4^2 + \ldots - (2n)^2$, simplifying your answer. [3]

2 Let $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$. Prove by mathematical induction that, for every positive integer n,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}.$$ [5]

3 Find a cubic equation with roots α, β and γ, given that

$$\alpha + \beta + \gamma = -6, \qquad \alpha^2 + \beta^2 + \gamma^2 = 38, \qquad \alpha\beta\gamma = 30.$$ [3]

Hence find the numerical values of the roots. [3]

4 The curve C has equation

$$2xy^2 + 3x^2y = 1.$$

Show that, at the point $A\,(-1,\ 1)$ on C, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -4$. [3]

Find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at A. [5]

5 Let

$$I_n = \int_0^{\frac{1}{4}\pi} \tan^n x \,\mathrm{d}x,$$

where $n \geqslant 0$. Use the fact that $\tan^2 x = \sec^2 x - 1$ to show that, for $n \geqslant 2$,

$$I_n = \frac{1}{n-1} - I_{n-2}.$$ [4]

Show that $I_8 = \frac{1}{7} - \frac{1}{5} + \frac{1}{3} - 1 + \frac{1}{4}\pi$. [4]

6 The curves C_1 and C_2 have polar equations

$$C_1: \quad r = a,$$
$$C_2: \quad r = 2a\cos 2\theta, \text{ for } 0 \leqslant \theta \leqslant \tfrac{1}{4}\pi,$$

where a is a positive constant. Sketch C_1 and C_2 on the same diagram. [3]

The curves C_1 and C_2 intersect at the point with polar coordinates (a, β). State the value of β. [1]

Show that the area of the region bounded by the initial line, the arc of C_1 from $\theta = 0$ to $\theta = \beta$, and the arc of C_2 from $\theta = \beta$ to $\theta = \frac{1}{4}\pi$ is

$$a^2\left(\tfrac{1}{6}\pi - \tfrac{1}{8}\sqrt{3}\right).$$ [4]

7 A curve C has parametric equations $x = e^t \cos t$, $y = e^t \sin t$, for $0 \leqslant t \leqslant \pi$. Find the arc length of C. [4]

Find the area of the surface generated when C is rotated through 2π radians about the x-axis. [7]

8 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10\sin t. \qquad [6]$$

Find the particular solution, given that $x = 5$ and $\frac{dx}{dt} = 2$ when $t = 0$. [4]

State an approximate solution for large positive values of t. [1]

9 The curve C with equation

$$y = \frac{ax^2 + bx + c}{x - 1},$$

where a, b and c are constants, has two asymptotes. It is given that $y = 2x - 5$ is one of these asymptotes.

(i) State the equation of the other asymptote. [1]

(ii) Find the value of a and show that $b = -7$. [3]

(iii) Given also that C has a turning point when $x = 2$, find the value of c. [3]

(iv) Find the set of values of k for which the line $y = k$ does not intersect C. [4]

10 The lines l_1 and l_2 have equations

$$l_1: \ \mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}) \quad \text{and} \quad l_2: \ \mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \mu(4\mathbf{i} + 6\mathbf{j} + \mathbf{k}).$$

Find a cartesian equation of the plane Π containing l_1 and l_2. [4]

Find the position vector of the foot of the perpendicular from the point with position vector $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$ to Π. [4]

The line l_3 has equation $\mathbf{r} = \mathbf{i} + 10\mathbf{j} + 3\mathbf{k} + v(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$. Find the shortest distance between l_1 and l_3. [5]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

A 3×3 matrix $\mathbf{A}$ has eigenvalues -1, 1, 2, with corresponding eigenvectors

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$

respectively. Find

(i) the matrix $\mathbf{A}$,

(ii) $\mathbf{A}^{2n}$, where n is a positive integer.

[14]

OR

Determine the rank of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 2 & -1 & -4 & 3 \\ 3 & -3 & -2 & 2 \\ 5 & -4 & -6 & 5 \end{pmatrix}.$$ [3]

Show that if

$$\mathbf{Ax} = p\begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \end{pmatrix} + q\begin{pmatrix} -1 \\ -1 \\ -3 \\ -4 \end{pmatrix} + r\begin{pmatrix} -1 \\ -4 \\ -2 \\ -6 \end{pmatrix},$$

where p, q and r are given real numbers, then

$$\mathbf{x} = \begin{pmatrix} p + \lambda \\ q + \lambda \\ r + \lambda \\ \lambda \end{pmatrix},$$

where λ is real. [4]

Find the values of p, q and r such that

$$p\begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \end{pmatrix} + q\begin{pmatrix} -1 \\ -1 \\ -3 \\ -4 \end{pmatrix} + r\begin{pmatrix} -1 \\ -4 \\ -2 \\ -6 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 8 \\ 15 \end{pmatrix}.$$ [3]

Find the solution $\mathbf{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$ of the equation $\mathbf{Ax} = \begin{pmatrix} 3 \\ 7 \\ 8 \\ 15 \end{pmatrix}$ for which $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \frac{11}{4}$. [4]