UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **October/November 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

2930091484

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Find the cartesian equation corresponding to the polar equation $r = (\sqrt{2})\sec(\theta - \frac{1}{4}\pi)$. [3]

Sketch the the graph of $r = (\sqrt{2})\sec(\theta - \frac{1}{4}\pi)$, for $-\frac{1}{4}\pi < \theta < \frac{3}{4}\pi$, indicating clearly the polar coordinates of the intersection with the initial line. [2]

2 The curve C has equation $y = 2x^{\frac{1}{2}}$ for $0 \leqslant x \leqslant 4$. Find

(i) the mean value of y with respect to x for $0 \leqslant x \leqslant 4$, [3]

(ii) the y-coordinate of the centroid of the region enclosed by C, the line $x = 4$ and the x-axis. [3]

3 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 26t^2 + 3t + 13.$$ [6]

4 Let $f(r) = r(r+1)(r+2)$. Show that

$$f(r) - f(r-1) = 3r(r+1).$$ [1]

Hence show that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. [2]

Using the standard result for $\sum_{r=1}^{n} r$, deduce that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [2]

Find the sum of the series

$$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + \ldots + 2(n-1)^2 + n^2,$$

where n is odd. [3]

5 Let I_n denote $\int_0^\infty x^n e^{-2x}\, dx$. Show that $I_n = \frac{1}{2}nI_{n-1}$, for $n \geqslant 1$. [2]

Prove by mathematical induction that, for all positive integers n, $I_n = \frac{n!}{2^{n+1}}$. [6]

6 Use de Moivre's theorem to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$ [3]

Without using a calculator, verify that $\cos 4\theta = -\cos 3\theta$ for each of the values $\theta = \frac{1}{7}\pi, \frac{3}{7}\pi, \frac{5}{7}\pi, \pi$. [2]

Using the result $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, show that the roots of the equation

$$8c^4 + 4c^3 - 8c^2 - 3c + 1 = 0$$

are $\cos\frac{1}{7}\pi, \cos\frac{3}{7}\pi, \cos\frac{5}{7}\pi, -1$. [2]

Deduce that $\cos\frac{1}{7}\pi + \cos\frac{3}{7}\pi + \cos\frac{5}{7}\pi = \frac{1}{2}$. [2]

7 The curve C has equation

$$y = \lambda x + \frac{x}{x-2},$$

where λ is a non-zero constant. Find the equations of the asymptotes of C. [3]

Show that C has no turning points if $\lambda < 0$. [3]

Sketch C in the case $\lambda = -1$, stating the coordinates of the intersections with the axes. [3]

8 The curve C has parametric equations

$$x = \tfrac{1}{3}t^3 - \ln t, \quad y = \tfrac{4}{3}t^{\frac{3}{2}},$$

for $1 \leqslant t \leqslant 3$. Find the arc length of C. [6]

Find also the area of the surface generated when C is rotated through 2π radians about the x-axis. [4]

9 The plane Π has equation

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}).$$

The line l, which does not lie in Π, has equation

$$\mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + 12\mathbf{k} + t(8\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}).$$

Show that l is parallel to Π. [4]

Find the position vector of the point at which the line with equation $\mathbf{r} = 5\mathbf{i} - 4\mathbf{j} + 7\mathbf{k} + s(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ meets Π. [4]

Find the perpendicular distance from the point with position vector $9\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$ to l. [4]

10 Write down the eigenvalues of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & -16 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix}.$$ [1]

Find corresponding eigenvectors. [4]

Let n be a positive integer. Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{A}^n = \mathbf{PDP}^{-1}.$$ [2]

Find $\mathbf{P}^{-1}$ and $\mathbf{A}^n$. [5]

Hence find $\lim_{n\to\infty}\left(3^{-n}\mathbf{A}^n\right)$. [1]

11 Answer only **one** of the following two alternatives.

EITHER

The roots of the equation $x^4 - 3x^2 + 5x - 2 = 0$ are $\alpha, \beta, \gamma, \delta$, and $\alpha^n + \beta^n + \gamma^n + \delta^n$ is denoted by S_n. Show that

$$S_{n+4} - 3S_{n+2} + 5S_{n+1} - 2S_n = 0. \qquad [2]$$

Find the values of

(i) S_2 and S_4, [3]

(ii) S_3 and S_5. [6]

Hence find the value of

$$\alpha^2(\beta^3 + \gamma^3 + \delta^3) + \beta^2(\gamma^3 + \delta^3 + \alpha^3) + \gamma^2(\delta^3 + \alpha^3 + \beta^3) + \delta^2(\alpha^3 + \beta^3 + \gamma^3). \qquad [3]$$

OR

The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -1 & 4 \\ 3 & 4 & 6 & 1 \\ -1 & 2 & 8 & -7 \end{pmatrix}.$$

The range space of T is R. In any order,

(i) show that the dimension of R is 2,

(ii) find a basis for R and obtain a cartesian equation for R,

(iii) find a basis for the null space of T.

[9]

The vector $\begin{pmatrix} 8 \\ 7 \\ k \end{pmatrix}$ belongs to R. Find the value of k and, with this value of k, find the general solution of

$$\mathbf{Mx} = \begin{pmatrix} 8 \\ 7 \\ k \end{pmatrix}. \qquad [5]$$