UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

# MARK SCHEME for the October/November 2008 question paper

## 9231 FURTHER MATHEMATICS

**9231/02** Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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## Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

| | |
|---|---|
| AEF | Any Equivalent Form (of answer is equally acceptable) |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

**Penalties**

| | |
|---|---|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting. |

| Qu No | Mark Scheme Details | | | Part Mark | Total |
|---|---|---|---|---|---|
| 1 | Find MI about $A$ of $AB$, A$C$ (M1 for either): | $I_{AB} = (6m/24)(⅓3^2 + 3^2)a^2 = 3ma^2$ | M1 | | |
| | | $I_{AC} = (10m/24)(⅓5^2 + 5^2)a^2$ | | | |
| | | $= (125/9)\ ma^2$ | A1 | | |
| | Find MI about $A$ of $BC$: | $I_{BC} = (8m/24)(⅓4^2 + 6^2 + 4^2)a^2$ | | | |
| | | $= (172/9)ma^2$ | M1 A1 | | |
| | Sum to find MI about $A$ of wire: | $I = (324/9)ma^2 = 36ma^2$ | A1 | 5 | **[5]** |
| 2 | Find speeds from cons. of energy (M1 for either): | $½mv_1^2 = mga\,(1 - \cos\theta)$ | M1 A1 | | |
| | | $½mv_2^2 = mga\,(1 + \cos\theta)$ | A1 | | |
| | Find $R_1$, $R_2$ by radial resolution (M1 for either): | $R_1 = mg\cos\theta - mv_1^2/\mathrm{a}$ | M1 A1 | | |
| | | $R_2 = mg\cos\theta + mv_2^2/\mathrm{a}$ | A1 | | |
| | *EITHER:* Substitute in $R_1$, $R_2$ and combine: | $R_1 = 3mg\cos\theta - 2mg$ | | | |
| | | $R_2 = 3mg\cos\theta + 2mg$ | | | |
| | | $R_2 - R_1 = 4mg$ **A.G.** | M1 A1 | | |
| | *OR:* Combine $R_1$, $R_2$ and substitute: | $R_2 - R_1 = m(v_2^2 + v_1^2)/a$ | | | |
| | | $= 4mg$ **A.G.** | (M1 A1) | 8 | **[8]** |
| 3 | *EITHER:* Relate angular acceln. to tension for block: | $2ma\,\mathrm{d}^2\theta/\mathrm{d}t^2 = 2mg - T - mg/10$ | M1 A1 | | |
| | Relate angular acceln. to tension for disc: | $I\,\mathrm{d}^2\theta/\mathrm{d}t^2 = aT,\ I = ½\,ma^2$ | M1 A1 | | |
| | Eliminate tension $T$: | $(½ + 2)ma^2\,\mathrm{d}^2\theta/\mathrm{d}t^2 = (2 - 0{\cdot}1)mga$ | M1 | | |
| | Find $\mathrm{d}^2\theta/\mathrm{d}t^2$ : | $19g/25a$ *or* $0{\cdot}76g/a$ *or* $7{\cdot}6/a$ | A1 | | |
| | Use $(\mathrm{d}\theta/\mathrm{d}t)^2 = 2\,\mathrm{d}^2\theta/\mathrm{d}t^2\,2\pi$ ($\surd$ on $\mathrm{d}^2\theta/\mathrm{d}t^2$): | $(\mathrm{d}\theta/\mathrm{d}t)^2 = 76\pi g/25a$ A.E.F. | M1 A1$\surd$ | | |
| | Find $\mathrm{d}\theta/\mathrm{d}t$ (A.E.F.): | $\mathrm{d}\theta/\mathrm{d}t = 3{\cdot}09\surd(g/\mathrm{a})$ *or* $9{\cdot}77/\surd a$ | A1 | 9 | |
| | *OR:* Use conservation of energy for system: | $½I\,(\mathrm{d}\theta/\mathrm{d}t)^2 + ½\,2m\,(a\,\mathrm{d}\theta/\mathrm{d}t)^2$ | (M1 A1) | | |
| | | $= 2mga\theta - 0{\cdot}1\,mga\theta$ | (M1 A1) | | |
| | Put $\theta = 2\pi$ and find $\mathrm{d}\theta/\mathrm{d}t$ (A.E.F.): | $\mathrm{d}\theta/\mathrm{d}t = 3{\cdot}09\surd(g/a)$ *or* $9{\cdot}77/\surd a$ | (M1 A1) | | |
| | Differentiate energy eqn w.r.t. $t$: | $(5ma^2/4)\,2\,\mathrm{d}^2\theta/\mathrm{d}t^2 = 1{\cdot}9\,mga$ | (M1 A1) | | |
| | Find $\mathrm{d}^2\theta/\mathrm{d}t^2$ : | $19g/25a$ *or* $0{\cdot}76g/a$ *or* $7{\cdot}6/a$ | (A1) | (9) | **[9]** |

| Qu No | Mark Scheme Details | | | Part Mark | Total |
|---|---|---|---|---|---|
| 4 | **(i)** Use conservation of momentum: | $0{\cdot}1v_A + mv_B = 0{\cdot}1 \times 5 - m \times 2$ | M1 | | |
| | Find $m$: | $m = (0{\cdot}5 - 0{\cdot}1\ v_A) / (2 + v_B)$ | A1 | | |
| | Use $v_A > 0$ to find lower bound on $m$: | $v_B > 0$, $m < 0{\cdot}5/2 = 0{\cdot}25$ **A.G.** | M1 A1 | 4 | |
| | **(ii)** Use Newton's law of restitution: | $v_B - v_A = ½(2 + 5) = 7/2$ | M1 A1 | | |
| | Put $m = 0{\cdot}2$ and find one of $v_A$, $v_B$: | $2\ v_B + v_A = 1$, $v_A = -2$ *or* $v_B = 1{\cdot}5$ | M1 A1 | | |
| | Find magnitude of impulse [N s]: | $0{\cdot}1\ (5 - v_A)$ *or* $0{\cdot}2\ (1{\cdot}5 + 2) = 0{\cdot}7$ | M1 A1 | 6 | **[10]** |
| 5 | Find equation of motion at general point: | $m\ \mathrm{d}^2x/\mathrm{d}t^2 = mg\ ((a - x)/a)^{½}$ | | | |
| | | $- mg\ ((a + x)/a)^{-¼}$ | M1 | | |
| | Expand terms and approximate: | $\approx mg\ ((1 - x/2a) - (1 - x/4a))$ | M1 A1 | | |
| | Simplify to give SHM eqn: | $\mathrm{d}^2x/\mathrm{d}t^2 = -gx/4a$ | A1 | 4 | |
| | Use SHM eqn to find speed when $x = 0$: | $v_{max}{}^2 = (g/4a)\ (0{\cdot}04a)^2$ | M1 A1 | | |
| | Simplify (A.E.F.): | $v_{max} = 0{\cdot}02\ \sqrt{(ag)}$ *or* $0{\cdot}0632\ \sqrt{a}$ | A1 | | |
| | Use SHM eqn to find time when $v = ½v_{max}$: | $½a\omega = a\omega \sin \omega t$ (A.E.F.) | M1 A1 | | |
| | Substitute $\omega = \sqrt{(g/4a)}$ and simplify: | $t = \sqrt{(4a/g)} \sin^{-1} ½$ | M1 | | |
| | | $= (\pi/3)\ \sqrt{(a/g)}$ (A.E.F.) | A1 | 7 | **[11]** |
| 6 | Use standard formula for pooled estimate, e.g.: | $((128 - 15^2/5) + (980 - 36^2/10))/13$ | M1 | | |
| | Award A1 for one term in numerator, e.g.: | $5 \times 16{\cdot}6$ *or* $10 \times 85{\cdot}04$ *or* $83$ *or* $850{\cdot}4$ | | | |
| | | *or* $4 \times 20{\cdot}75$ *or* $9 \times 94{\cdot}5$ | A1 | | |
| | Calculate value of pooled estimate: | $71{\cdot}8$ | A1 | 3 | **[3]** |
| 7 | **(i)** Find sample mean: | $\bar{x} = ½(61{\cdot}21 + 64{\cdot}39) = 62{\cdot}8$ | M1 A1 | | |
| | Use confidence interval formula: | $\bar{x} \pm ts/\sqrt{n}$ for any $t$ | M1 | | |
| | Use correct tabular $t$: | $t_{24,099} = 2{\cdot}492$ | A1 | | |
| | Calculate standard deviation: | $s = 1{\cdot}59 \times 5 / 2{\cdot}492 = 3{\cdot}19$ | A1 | 5 | |
| | **(ii)** State assumption (A.E.F.): | Population has normal distribution | B1 | 1 | |
| | **(iii)** State valid reason (A.E.F.): | 72 exceeds upper limit of interval | *B1 | | |
| | State conclusion (A.E.F., dep *B1): | Yes, it does reduce pulse rate | B1 | 2 | **[8]** |

| Qu No | Mark Scheme Details | | | Part Mark | Total |
|---|---|---|---|---|---|
| 8 | **(i)** Formulate two eqns for means: | $\bar{y} + 0{\cdot}425\,\bar{x} = 1{\cdot}28$ *and* | | | |
| | | $\bar{x} + 0{\cdot}516\,\bar{y} = 1{\cdot}05$ | M1 | | |
| | Solve for means: | $\bar{x} = 0{\cdot}499$, $\bar{y} = 1{\cdot}068$ *or* $1{\cdot}07$ | M1 A1 | 3 | |
| | **(ii)** Find correlation coefficient for sample: | $r^2 = 0{\cdot}425 \times 0{\cdot}516$; $r = -\ 0{\cdot}468$ | M1; *A1 | 2 | |
| | **(iii)** State hypotheses: | $H_0$: $\rho = 0$, $H_1$: $\rho \neq 0$ | B1 | | |
| | Valid method for reaching conclusion: | $\rho \neq 0$ if $\|r\| >$ tabular value | M1 | | |
| | Use of correct tabular value: | $\rho_{25,2\cdot5\%} = 0{\cdot}396$ | *B1 | | |
| | Correct conclusion (A.E.F., dep *A1, *B1): | Coefficient does differ from zero | A1 | 4 | **[9]** |
| 9 | Integrate f($t$) to give F($t$): | $F(t) = -9/8t^2$ | M1 | | |
| | Apply limits: | $F(2{\cdot}5) - F(2) = -(9/8)\,(2{\cdot}5^{-2} - 2^{-2})$ | A1 | | |
| | Evaluate and multiply by 100: | $10{\cdot}125$ **A.G.** | A1 | 3 | |
| | State hypotheses (A.E.F.): | $H_0$: f($t$) fits data, $H_1$: doesn't fit | B1 | | |
| | Find $\chi^2$ (A1 if at least 3 terms correct): | $\chi^2 = 1{\cdot}5^2/62{\cdot}5 + 4{\cdot}875^2/21{\cdot}875$ | | | |
| | | $+ 5{\cdot}875^2/10{\cdot}125 + 2{\cdot}5^2/5{\cdot}5$ | M1 A1 | | |
| | Evaluate $\chi^2$: | $= 0{\cdot}036 + 1{\cdot}086 + 3{\cdot}409 + 1{\cdot}136$ | | | |
| | | $= 5{\cdot}67$ [$\pm\, 0{\cdot}01$] | *A1 | | |
| | Compare with consistent tab. value (to 2 dp): | $\chi_{3,\,0.9}{}^2 = 6{\cdot}251$ | *B1√ | | |
| | ($\chi_{2,\,0.9}{}^2 = 4{\cdot}605$, $\chi_{1,\,0.9}{}^2 = 2{\cdot}706$) | | | | |
| | Consistent conclusion (A1 dep *A1, *B1): | Distribution fits data (A.E.F.) | M1√ A1 | 7 | **[10]** |
| 10 | Replace $2^x$ by $e^{kx}$ to find $k$: | $f(x) = ae^{-kx}$; $k = \ln 2$ | M1; A1 | | |
| | Show $a = k$ by e.g. $\int_0^\infty f(x) = 1$: | $[-(a/k)\,e^{-kx}]_0^\infty = 1$, $a = k$ *or* $\ln 2$ | M1 A1 | 4 | |
| | State value of E($X$): | $1 / \ln 2$ *or* $1{\cdot}44$ | B1 | 1 | |
| | Find distribution fn G of $Y$: | $G(y) = P(Y \leq y) = P(X \leq k^{-1} \ln y)$ | M1 A1 | | |
| | | $= F(k^{-1} \ln y) = (a/k)(1 - e^{-\ln y})$ | M1 A1 | | |
| | | $= 1 - 1/y$ (CAO) | A1 | | |
| | Find probability density function g of $Y$: | $g(y) = 1/y^2$ (CAO) | M1 A1 | | |
| | State interval for either G or g: | $y \geq 1$ | B1 | 8 | **[13]** |

| Qu No | Mark Scheme Details | | | Part Mark | Total |
|---|---|---|---|---|---|
| **11 a** | *EITHER:* Observe or deduce when $R_B$ is maximised: | $R_{B,\,max}$ occurs when dog at $B$ | M1 | | |
| | Moments for ladder about $A$: | $6a\,R_{B,\,max} = 4aW + 8a \times \tfrac{1}{4}W$ | M1 | | |
| | *OR:* Moments when dog is $x$ hor. from wall: | $6a\,R_B = 4aW + (8a - x)\,\tfrac{1}{4}W$ | (M1) | | |
| | Deduce limit on $R_B$ : | $R_{B,\,max}$ occurs when dog at $B$ | (M1) | | |
| | Find max. value of $R_B$: | $R_{B,\,max} = W$ | A1 | | |
| | Resolve horizontally for ladder $AB$: | $F_A = R_B$ | B1 | | |
| | Resolve vertically for ladder $AB$: | $R_A = W + \tfrac{1}{4}W$ [$= 5W/4$] | B1 | | |
| | Find bound on $\mu$ from $F_A \le \mu R_A$ : | $\mu \ge F_A / R_A \ge R_{B,\,max} / (5W/4)$ | M1 | | |
| | | $\mu \ge 4/5$ **A.G.** | A1 | 7 | |
| | Find friction $F_{cube}$ along $DE$ by hor. resolution: | $F_{cube} = F_A$ *or* $R_B$ | B1 | | |
| | Find reaction $R_{cube}$ from floor by vert. resolution: | $R_{cube} = W + \tfrac{1}{4}W + kW$ | B1 | | |
| | Show that $F_{cube} \le \mu R_{cube}$ : | $\mu R_{cube} \ge W + 4kW/5 \ge W$ | | | |
| | | $= R_{B,\,max} \ge F_{cube}$ | M1 A1 | 4 | |
| | Find moments about $D$ opposing effect of $R_{cube}$ : | $2akW + a5W/4 - 4a\,F_A$ | M1 | | |
| | Find smallest value of $k$ for which moments $\ge 0$: | $(4W - 5W/4) / 2W = 11/8$ | M1 A1 | 3 | **[14]** |
| **11 b** | State hypotheses (A.E.F.): | $H_0$: $\mu_2 = \mu_1$, $H_1$: $\mu_2 > \mu_1$ | B1 | | |
| | Calculate $\Sigma (x_i - \overline{x})^2$ (M1 for either) | 8·24, 4·62[4] | | | |
| | *or* estimate variances: | 0·168 *or* 0·165, 0·0784 *or* 0·0771 | M1 A1 A1 | | |
| | Find $s^2$ (M0 if inconsistent denominators used): | $s^2 = 0{\cdot}168/50 + 0{\cdot}0784/60$ | | | |
| | | *or* $0{\cdot}165/49 + 0{\cdot}0771/59$ | | | |
| | | [= 0·00467] | *M1 | | |
| | Find value of $z$ (dep *M1): | $z = (1803{\cdot}6/60 - 1492{\cdot}0/50) / s$ | M1 | | |
| | | $= (30{\cdot}06 - 29{\cdot}84)/0{\cdot}0683 = 3{\cdot}22$ | *A1 | | |
| | **S.R.** Using pooled estimate of variance: | $s^2 = (8{\cdot}24 + 4{\cdot}62)/108 = 0{\cdot}119$ | (M0) | | |
| | | $z = 0{\cdot}22 / s\surd(1/50 + 1/60) = 3{\cdot}33$ | (B1) | | |
| | Find tabular. value (to 2 dp): | $\Phi^{-1}(0{\cdot}98) = 2{\cdot}05[4]$ | *B1 | | |
| | Compare values for conclusion (A1 dep *A1, *B1): | $\mu_2 > \mu_1$ (A.E.F., M1√ on values) | M1√ A1 | 10 | |
| | Find limiting value of $z$ (to 2 dp): | $z = (0{\cdot}22 - 0{\cdot}1)/s = 1{\cdot}756$ | M1 A1 | | |
| | Find $\Phi(z)$ and hence values of $\alpha$ (to 1 dp): | $\Phi(z) = 0{\cdot}9604$, $\alpha \ge 3{\cdot}9$ *or* $4{\cdot}0$ | M1 A1 | 4 | |
| | $s^2 = 0{\cdot}119$ gives: | $z = 1{\cdot}816$, $\alpha \ge 3{\cdot}47$ | (M1 M1) | | **[14]** |