REPRESENTATION OF DATA

MAY/JUNE2012 9709/61

The lengths of the diagonals in metres of the 9 most popular flat screen TVs and the 9 most popular conventional TVs are shown below.

Flat screen :	0.85	0.94	0.91	0.96	1.04	0.89	1.07	0.92	0.76
Conventional :	0.69	0.65	0.85	0.77	0.74	0.67	0.71	0.86	0.75

(i) Represent this information on a back-to-back stem-and-leaf diagram. [4]

(ii) Find the median and the interquartile range of the lengths of the diagonals of the 9 conventional TVs. [3]

(iii) Find the mean and standard deviation of the lengths of the diagonals of the 9 flat screen TVs. [2]

(ii) 0.74, 0.13
(iii) 0.927, 0.0882

MAY/JUNE2012 9709/62

The ages, x years, of 150 cars are summarised by $\Sigma x = 645$ and $\Sigma x^2 = 8287.5$. Find $\Sigma(x - \bar{x})^2$, where $\bar{x}$ denotes the mean of x. [4]

5514

MAY/JUNE2012 9709/62

The back-to-back stem-and-leaf diagram shows the values taken by two variables A and B.

	A		B	
(3)	3 1 0	15	1 3 3 5	(4)
(2)	4 1	16	2 2 3 4 4 5 7 7 7 8	(10)
(3)	8 3 3	17	0 1 3 3 3 4 6 6 7 9 9	(11)
(12)	9 8 8 6 5 5 4 3 2 1 1 0	18	2 4 7	(3)
(8)	9 9 8 8 6 5 4 2	19	1 5	(2)
(5)	9 8 7 1 0	20	4	(1)

Key: 4 | 16 | 7 means $A = 0.164$ and $B = 0.167$.

(i) Find the median and the interquartile range for variable A. [3]

(ii) You are given that, for variable B, the median is 0.171, the upper quartile is 0.179 and the lower quartile is 0.164. Draw box-and-whisker plots for A and B in a single diagram on graph paper. [3]

(i) 0.186, 0.019

MAY/JUNE2012 9709/63

Ashfaq and Kuljit have done a school statistics project on the prices of a particular model of headphones for MP3 players. Ashfaq collected prices from 21 shops. Kuljit used the internet to collect prices from 163 websites.

(i) Name a suitable statistical diagram for Ashfaq to represent his data, together with a reason for choosing this particular diagram. [2]

(ii) Name a suitable statistical diagram for Kuljit to represent her data, together with a reason for choosing this particular diagram. [2]

(i) stem and leaf or bpx-and-whisker – shows shape/spread/range
(ii) histogram or box-and-whisker – shows shape/spread
or cumulative frequency graph – shows shape/large number of prices

MAY/JUNE2012 9709/63

The heights, x cm, of a group of young children are summarised by

$$\Sigma(x - 100) = 72, \qquad \Sigma(x - 100)^2 = 499.2.$$

The mean height is 104.8 cm.

(i) Find the number of children in the group. [2]

(ii) Find $\Sigma(x - 104.8)^2$. [3]

(i) 15
(ii) 153.6

OCTOBER/NOVEMBER2011 9709/61

The values, x, in a particular set of data are summarised by

$$\Sigma(x - 25) = 133, \qquad \Sigma(x - 25)^2 = 3762.$$

The mean, $\bar{x}$, is 28.325.

(i) Find the standard deviation of x. [4]

(ii) Find Σx^2. [2]

(i) 9.11
(ii) 35412

OCTOBER/NOVEMBER2011 9709/61

The marks of the pupils in a certain class in a History examination are as follows.

28 33 55 38 42 39 27 48 51 37 57 49 33

The marks of the pupils in a Physics examination are summarised as follows.

Lower quartile: 28, Median: 39, Upper quartile: 67.
The lowest mark was 17 and the highest mark was 74.

(i) Draw box-and-whisker plots in a single diagram on graph paper to illustrate the marks for History and Physics. [5]

(ii) State one difference, which can be seen from the diagram, between the marks for History and Physics. [1]

(ii) physics marks are more spread out than history marks.

OCTOBER/NOVEMBER2011 9709/62

The following are the times, in minutes, taken by 11 runners to complete a 10 km run.

48.3 55.2 59.9 67.7 60.5 75.6 62.5 57.4 53.4 49.2 64.1

Find the mean and standard deviation of these times. [3]

59.4, 7.68

OCTOBER/NOVEMBER2011 9709/62

The weights of 220 sausages are summarised in the following table.

Weight (grams)	<20	<30	<40	<45	<50	<60	<70
Cumulative frequency	0	20	50	100	160	210	220

(i) State which interval the median weight lies in. [1]

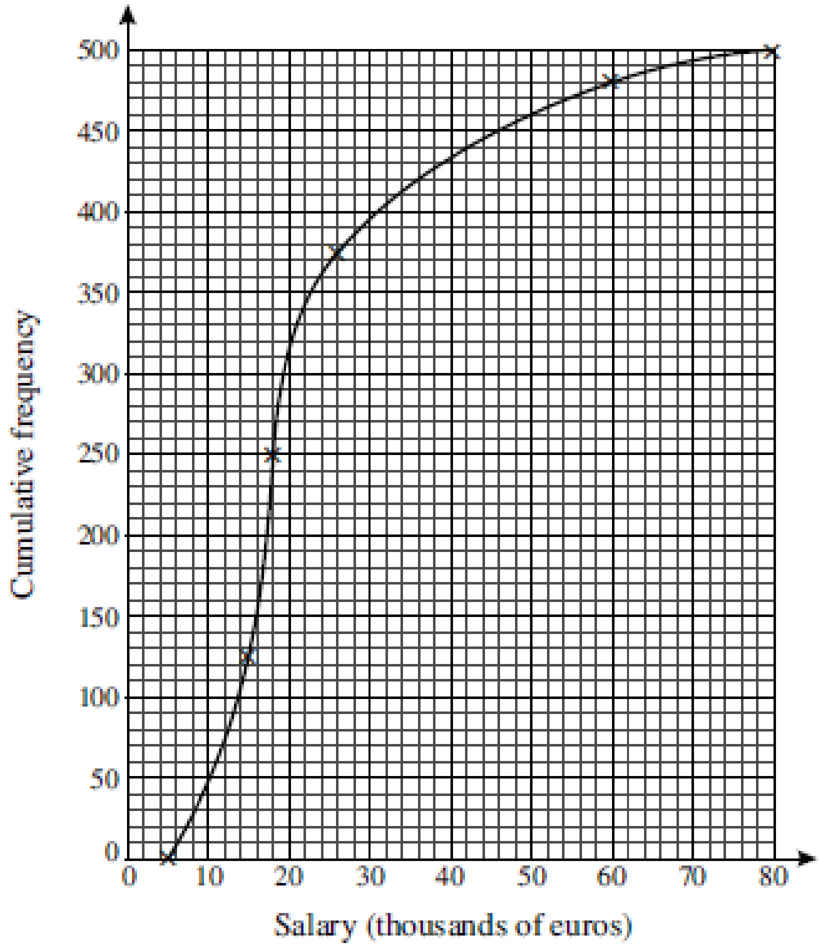
(ii) Find the smallest possible value and the largest possible value for the interquartile range. [2]

(iii) State how many sausages weighed between 50 g and 60 g. [1]

(iv) On graph paper, draw a histogram to represent the weights of the sausages. [4]

(i) 45-50
(ii) 5, 20
(iii) 50

OCTOBER/NOVEMBER2011 9709/63



The cumulative frequency graph shows the annual salaries, in thousands of euros, of a random sample of 500 adults with jobs, in France. It has been plotted using grouped data. You may assume that the lowest salary is 5000 euros and the highest salary is 80 000 euros.

(i) On graph paper, draw a box-and-whisker plot to illustrate these salaries. [4]

(ii) Comment on the salaries of the people in this sample. [1]

(iii) An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(a) How high must a salary be in order to be classified as an outlier? [3]

(b) Show that none of the salaries is low enough to be classified as an outlier. [1]

(ii) most (3/4) are earning less than 26K, not many earning high salaries, etc.
(iii) (a) 42500 euros
(b) low outlier is below -1.5.

MAY/JUNE2011 9709/61

There are 5000 schools in a certain country. The cumulative frequency table shows the number of pupils in a school and the corresponding number of schools.

Number of pupils in a school	$\leqslant 100$	$\leqslant 150$	$\leqslant 200$	$\leqslant 250$	$\leqslant 350$	$\leqslant 450$	$\leqslant 600$
Cumulative frequency	200	800	1600	2100	4100	4700	5000

(i) Draw a cumulative frequency graph with a scale of 2 cm to 100 pupils on the horizontal axis and a scale of 2 cm to 1000 schools on the vertical axis. Use your graph to estimate the median number of pupils in a school. [3]

(ii) 80% of the schools have more than n pupils. Estimate the value of n correct to the nearest ten. [2]

(iii) Find how many schools have between 201 and 250 (inclusive) pupils. [1]

(iv) Calculate an estimate of the mean number of pupils per school. [4]

(i) 270
(ii) 160
(iii) 500
(iv) 268

MAY/JUNE2011 9709/62

A sample of 36 data values, x, gave $\Sigma(x - 45) = -148$ and $\Sigma(x - 45)^2 = 3089$.

(i) Find the mean and standard deviation of the 36 values. [3]

(ii) One extra data value of 29 was added to the sample. Find the standard deviation of all 37 values. [4]

(i) 40.9, 8.30
(ii) 8.41

MAY/JUNE2011 9709/62

A hotel has 90 rooms. The table summarises information about the number of rooms occupied each day for a period of 200 days.

Number of rooms occupied	1 – 20	21 – 40	41 – 50	51 – 60	61 – 70	71 – 90
Frequency	10	32	62	50	28	18

(i) Draw a cumulative frequency graph on graph paper to illustrate this information. [4]

(ii) Estimate the number of days when over 30 rooms were occupied. [2]

(iii) On 75% of the days at most n rooms were occupied. Estimate the value of n. [2]

(ii) 180 (accept 174-180)
(iii) 59 (accept 58-60)

MAY/JUNE2011 9709/63

Red Street Garage has 9 used cars for sale. Fairwheel Garage has 15 used cars for sale. The mean age of the cars in Red Street Garage is 3.6 years and the standard deviation is 1.925 years. In Fairwheel Garage, $\Sigma x = 64$ and $\Sigma x^2 = 352$, where x is the age of a car in years.

(i) Find the mean age of all 24 cars. [2]

(ii) Find the standard deviation of the ages of all 24 cars. [4]

(i) 4.02
(ii) 2.19

MAY/JUNE2011 9709/63

The following cumulative frequency table shows the examination marks for 300 candidates in country A and 300 candidates in country B.

Mark	<10	<20	<35	<50	<70	<100
Cumulative frequency, A	25	68	159	234	260	300
Cumulative frequency, B	10	46	72	144	198	300

(i) Without drawing a graph, show that the median for country B is higher than the median for country A. [2]

(ii) Find the number of candidates in country A who scored between 20 and 34 marks inclusive. [1]

(iii) Calculate an estimate of the mean mark for candidates in country A. [4]

(ii) 91

(iii) 37.6

OCTOBER/NOVEMBER2010 9709/61

Anita made observations of the maximum temperature, t °C, on 50 days. Her results are summarised by $\Sigma t = 910$ and $\Sigma(t - \bar{t})^2 = 876$, where $\bar{t}$ denotes the mean of the 50 observations. Calculate $\bar{t}$ and the standard deviation of the observations. [3]

18.2, 4.19

OCTOBER/NOVEMBER2010 9709/61

The weights in grams of a number of stones, measured correct to the nearest gram, are represented in the following table.

Weight (grams)	1 – 10	11 – 20	21 – 25	26 – 30	31 – 50	51 – 70
Frequency	$2x$	$4x$	$3x$	$5x$	$4x$	x

A histogram is drawn with a scale of 1 cm to 1 unit on the vertical axis, which represents frequency density. The 1 – 10 rectangle has height 3 cm.

(i) Calculate the value of x and the height of the 51 – 70 rectangle. [4]

(ii) Calculate an estimate of the mean weight of the stones. [3]

(i) 15, 0.75
(ii) 26.6

OCTOBER/NOVEMBER2010 9709/62

Esme noted the test marks, x, of 16 people in a class. She found that $\Sigma x = 824$ and that the standard deviation of x was 6.5.

(i) Calculate $\Sigma(x - 50)$ and $\Sigma(x - 50)^2$. [3]

(ii) One person did the test later and her mark was 72. Calculate the new mean and standard deviation of the marks of all 17 people. [3]

(i) 24, 712
(ii) 52.7, 7.94

OCTOBER/NOVEMBER2010 9709/62

The weights in kilograms of 11 bags of sugar and 7 bags of flour are as follows.

Sugar:	1.961	1.983	2.008	2.014	1.968	1.994	2.011	2.017	1.977	1.984	1.989
Flour:	1.945	1.962	1.949	1.977	1.964	1.941	1.953				

(i) Represent this information on a back-to-back stem-and-leaf diagram with sugar on the left-hand side. [4]

(ii) Find the median and interquartile range of the weights of the bags of sugar. [3]

(ii) 1.989, 0.034

OCTOBER/NOVEMBER2010 9709/63

Name the distribution and suggest suitable numerical parameters that you could use to model the weights in kilograms of female 18-year-old students. [2]

Normal distribution, mean = 60, variance = 90

OCTOBER/NOVEMBER2010 9709/63

Delip measured the speeds, x km per hour, of 70 cars on a road where the speed limit is 60 km per hour. His results are summarised by $\Sigma(x - 60) = 245$.

(i) Calculate the mean speed of these 70 cars. [2]

His friend Sachim used values of $(x - 50)$ to calculate the mean.

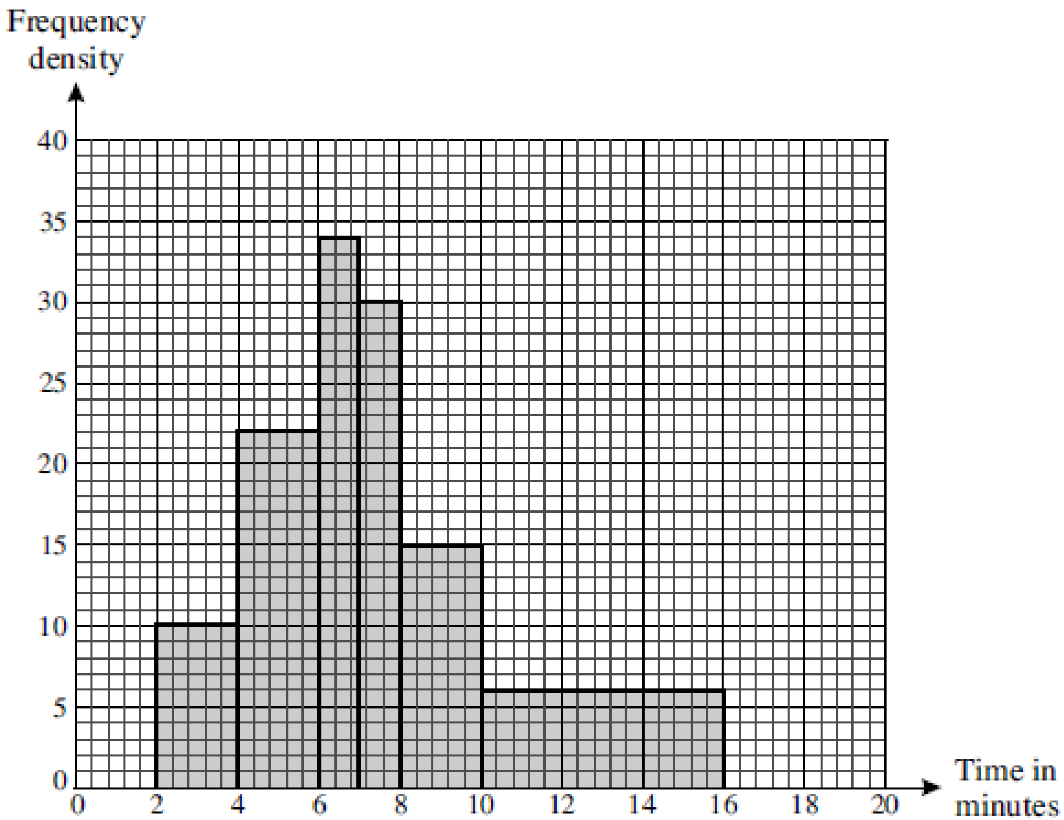
(ii) Find $\Sigma(x - 50)$. [2]

(iii) The standard deviation of the speeds is 10.6 km per hour. Calculate $\Sigma(x - 50)^2$. [2]

(i) 63.5
(ii) 945
(iii) 20623

OCTOBER/NOVEMBER2010 9709/63

The following histogram illustrates the distribution of times, in minutes, that some students spent taking a shower.



(i) Copy and complete the following frequency table for the data. [3]

Time (t minutes)	$2 < t \leqslant 4$	$4 < t \leqslant 6$	$6 < t \leqslant 7$	$7 < t \leqslant 8$	$8 < t \leqslant 10$	$10 < t \leqslant 16$
Frequency						

(ii) Calculate an estimate of the mean time to take a shower. [2]

(iii) Two of these students are chosen at random. Find the probability that exactly one takes between 7 and 10 minutes to take a shower. [3]

(ii) 7.55
(iii) 0.429

MAY/JUNE2010 9709/61

The numbers of people travelling on a certain bus at different times of the day are as follows.

17	5	2	23	16	31	8
22	14	25	35	17	27	12
6	23	19	21	23	8	26

(i) Draw a stem-and-leaf diagram to illustrate the information given above. [3]

(ii) Find the median, the lower quartile, the upper quartile and the interquartile range. [3]

(iii) State, in this case, which of the median and mode is preferable as a measure of central tendency, and why. [1]

(ii) 19, 10, 24, 14
(iii) median because mode could be any number which is duplicated more than twice.

MAY/JUNE2010 9709/62

The times in minutes for seven students to become proficient at a new computer game were measured. The results are shown below.

15 10 48 10 19 14 16

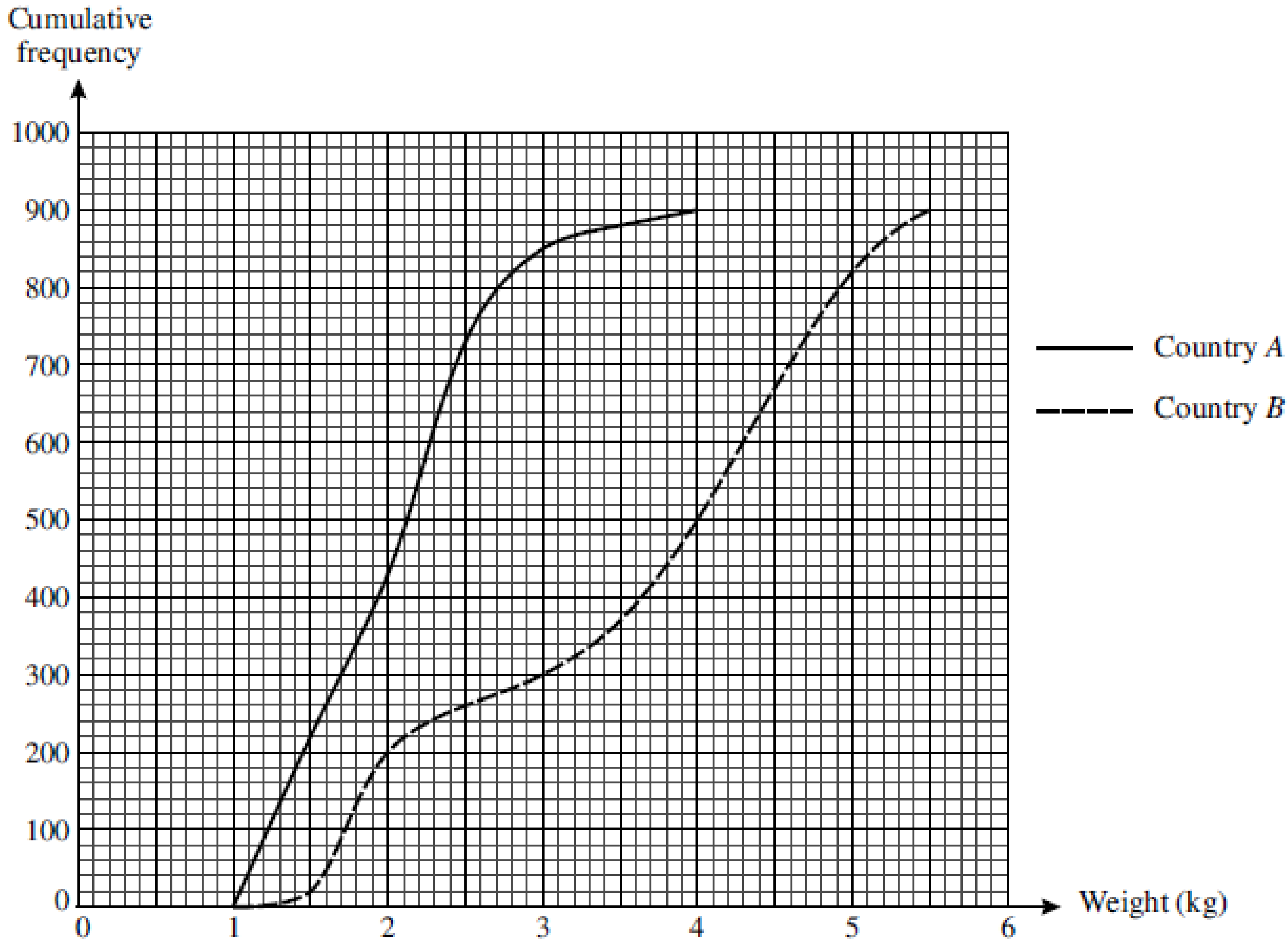
(i) Find the mean and standard deviation of these times. [2]

(ii) State which of the mean, median or mode you consider would be most appropriate to use as a measure of central tendency to represent the data in this case. [1]

(iii) For each of the two measures of average you did not choose in part (ii), give a reason why you consider it inappropriate. [2]

(i) 18.9, 12.3
(ii) median
(iii) mode inappropriate because it is 10 and this is the lowest value.
mean inappropriate because it is affected by the outlier (48).

MAY/JUNE2010 9709/62



The birth weights of random samples of 900 babies born in country *A* and 900 babies born in country *B* are illustrated in the cumulative frequency graphs. Use suitable data from these graphs to compare the central tendency and spread of the birth weights of the two sets of babies. [6]

median A = 2.0-2.1, median B = 3.8-3.9, country B has heavier babies on average.
IQR A = 0.9 , IQR B = 2.3, Country B has greater spread of weights.

MAY/JUNE2010 9709/63

The heights, x cm, of a group of 82 children are summarised as follows.

$$\Sigma(x - 130) = -287, \qquad \text{standard deviation of } x = 6.9.$$

(i) Find the mean height. [2]

(ii) Find $\Sigma(x - 130)^2$. [2]

(i) 126.5

(ii) 4908.5

MAY/JUNE2010 9709/63

The lengths of some insects of the same type from two countries, X and Y, were measured. The stem-and-leaf diagram shows the results.

	Country X		Country Y	
(10)	9 7 6 6 6 4 4 4 3 2	80		
(18)	8 8 8 7 7 6 6 5 5 5 4 4 3 3 3 2 2 0	81	1 1 2 2 3 3 3 5 5 6 7 8 9	(13)
(16)	9 9 9 8 8 7 7 6 5 5 3 2 2 1 0 0	82	0 0 1 2 3 3 3 q 4 5 6 6 7 8 8	(15)
(16)	8 7 6 5 5 5 3 3 2 2 2 1 1 1 0 0	83	0 1 2 2 4 4 4 4 5 5 6 6 7 7 7 8 9	(17)
(11)	8 7 6 5 5 4 4 3 3 1 1	84	0 0 1 2 4 4 5 5 6 6 7 7 7 8 9	(15)
		85	1 2 r 3 3 5 5 6 6 7 8 8	(12)
		86	0 1 2 2 3 5 5 5 8 9 9	(11)

Key: 5 | 81 | 3 means an insect from country X has length 0.815 cm and an insect from country Y has length 0.813 cm.

(i) Find the median and interquartile range of the lengths of the insects from country X. [2]

(ii) The interquartile range of the lengths of the insects from country Y is 0.028 cm. Find the values of q and r. [2]

(iii) Represent the data by means of a pair of box-and-whisker plots in a single diagram on graph paper. [4]

(iv) Compare the lengths of the insects from the two countries. [2]

(i) 0.825, 0.019
(ii) 4, 2
(iv) Y has longer insects on average. Y has larger range.

OCTOBER/NOVEMBER2009 9709/61

A library has many identical shelves. All the shelves are full and the numbers of books on each shelf in a certain section are summarised by the following stem-and-leaf diagram.

3	3 6 9 9	(4)
4	6 7	(2)
5	0 1 2 2	(4)
6	0 0 1 1 2 3 4 4 4 4 4 5 5 6 6 6 7 8 8 9	(20)
7	1 1 3 3 3 5 6 6 7 8 9 9	(12)
8	0 2 4 5 5 6 8	(7)
9	0 0 1 2 4 4 4 4 5 5 6 7 7 8 8 9 9 9	(18)

Key: 3 | 6 represents 36 books

(i) Find the number of shelves in this section of the library. [1]

(ii) Draw a box-and-whisker plot to represent the data. [5]

In another section all the shelves are full and the numbers of books on each shelf are summarised by the following stem-and-leaf diagram.

2	1 2 2 2 3 3 4 5 6 6 6 7 9	(13)
3	0 1 1 2 3 4 4 5 6 6 7 7 7 8 8	(15)
4	2 2 3 5 7 7 8 9	(8)

Key: 3 | 6 represents 36 books

(iii) There are fewer books in this section than in the previous section. State one other difference between the books in this section and the books in the previous section. [1]

(i) 67

(iii) books are fatter/wider, or standard deviation/IQR of the number of books per shelf s less.

OCTOBER/NOVEMBER2009 9709/62

The following table gives the marks, out of 75, in a pure mathematics examination taken by 234 students.

Marks	1–20	21–30	31–40	41–50	51–60	61–75
Frequency	40	34	56	54	29	21

(i) Draw a histogram on graph paper to represent these results. [5]

(ii) Calculate estimates of the mean mark and the standard deviation. [4]

(ii) 37.5, 16.9

MAY/JUNE2009

During January the numbers of people entering a store during the first hour after opening were as follows.

Time after opening, x minutes	Frequency	Cumulative frequency
$0 < x \leqslant 10$	210	210
$10 < x \leqslant 20$	134	344
$20 < x \leqslant 30$	78	422
$30 < x \leqslant 40$	72	a
$40 < x \leqslant 60$	b	540

(i) Find the values of a and b. [2]

(ii) Draw a cumulative frequency graph to represent this information. Take a scale of 2 cm for 10 minutes on the horizontal axis and 2 cm for 50 people on the vertical axis. [4]

(iii) Use your graph to estimate the median time after opening that people entered the store. [2]

(iv) Calculate estimates of the mean, m minutes, and standard deviation, s minutes, of the time after opening that people entered the store. [4]

(v) Use your graph to estimate the number of people entering the store between $(m - \frac{1}{2}s)$ and $(m + \frac{1}{2}s)$ minutes after opening. [2]

(i) 494, 46
(iii) 13.5-14.6
(iv) 18.2, 14.2
(v) 155-170

OCTOBER/NOVEMBER2008

Rachel measured the lengths in millimetres of some of the leaves on a tree. Her results are recorded below.

32	35	45	37	38	44	33	39	36	45

Find the mean and standard deviation of the lengths of these leaves. [3]

38.4, 4.57

OCTOBER/NOVEMBER2008

The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

115	120	158	132	125
104	142	160	145	104
162	117	109	124	134

(i) Draw a stem-and-leaf diagram to represent the data. [3]

(ii) Find the median and the quartiles. [2]

(iii) On graph paper, using a scale of 2 cm to represent 10 beats per minute, draw a box-and-whisker plot of the data. [3]

(ii) 125, Q1 = 115, Q3 = 145

MAY/JUNE2008

The stem-and-leaf diagram below represents data collected for the number of hits on an internet site on each day in March 2007. There is one missing value, denoted by x.

0	0 1 5 6	(4)
1	1 3 5 6 6 8	(6)
2	1 1 2 3 4 4 4 8 9	(9)
3	1 2 2 2 x 8 9	(7)
4	2 5 6 7 9	(5)

Key: 1 | 5 represents 15 hits

(i) Find the median and lower quartile for the number of hits each day. [2]

(ii) The interquartile range is 19. Find the value of x. [2]

(i) 24, 16
(ii) 5

MAY/JUNE2008

As part of a data collection exercise, members of a certain school year group were asked how long they spent on their Mathematics homework during one particular week. The times are given to the nearest 0.1 hour. The results are displayed in the following table.

Time spent (t hours)	$0.1 \leqslant t \leqslant 0.5$	$0.6 \leqslant t \leqslant 1.0$	$1.1 \leqslant t \leqslant 2.0$	$2.1 \leqslant t \leqslant 3.0$	$3.1 \leqslant t \leqslant 4.5$
Frequency	11	15	18	30	21

(i) Draw, on graph paper, a histogram to illustrate this information. [5]

(ii) Calculate an estimate of the mean time spent on their Mathematics homework by members of this year group. [3]

(ii) 2.1

OCTOBER/NOVEMBER2007

A summary of 24 observations of x gave the following information:

$$\Sigma(x-a) = -73.2 \quad \text{and} \quad \Sigma(x-a)^2 = 2115.$$

The mean of these values of x is 8.95.

(i) Find the value of the constant a. [2]

(ii) Find the standard deviation of these values of x. [2]

(i) 12
(ii) 8.88

OCTOBER/NOVEMBER2007

The arrival times of 204 trains were noted and the number of minutes, t, that each train was late was recorded. The results are summarised in the table.

Number of minutes late (t)	$-2 \leqslant t < 0$	$0 \leqslant t < 2$	$2 \leqslant t < 4$	$4 \leqslant t < 6$	$6 \leqslant t < 10$
Number of trains	43	51	69	22	19

(i) Explain what $-2 \leqslant t < 0$ means about the arrival times of trains. [1]

(ii) Draw a cumulative frequency graph, and from it estimate the median and the interquartile range of the number of minutes late of these trains. [7]

(i) some trains were up to 2minutes early.
(ii) median = 2.1-2.4, IQR = 3.2-3.6

MAY/JUNE2007

The length of time, t minutes, taken to do the crossword in a certain newspaper was observed on 12 occasions. The results are summarised below.

$$\Sigma(t-35) = -15 \qquad \Sigma(t-35)^2 = 82.23$$

Calculate the mean and standard deviation of these times taken to do the crossword. [4]

33.75, 2.3

MAY/JUNE2007

The lengths of time in minutes to swim a certain distance by the members of a class of twelve 9-year-olds and by the members of a class of eight 16-year-olds are shown below.

9-year-olds:	13.0	16.1	16.0	14.4	15.9	15.1	14.2	13.7	16.7	16.4	15.0	13.2
16-year-olds:	14.8	13.0	11.4	11.7	16.5	13.7	12.8	12.9				

(i) Draw a back-to-back stem-and-leaf diagram to represent the information above. [4]

(ii) A new pupil joined the 16-year-old class and swam the distance. The mean time for the class of nine pupils was now 13.6 minutes. Find the new pupil's time to swim the distance. [3]

(b) 15.6

OCTOBER/NOVEMBER2006

The weights of 30 children in a class, to the nearest kilogram, were as follows.

50	45	61	53	55	47	52	49	46	51
60	52	54	47	57	59	42	46	51	53
56	48	50	51	44	52	49	58	55	45

Construct a grouped frequency table for these data such that there are five equal class intervals with the first class having a lower boundary of 41.5 kg and the fifth class having an upper boundary of 61.5 kg. [4]

OCTOBER/NOVEMBER2006

In a survey, people were asked how long they took to travel to and from work, on average. The median time was 3 hours 36 minutes, the upper quartile was 4 hours 42 minutes and the interquartile range was 3 hours 48 minutes. The longest time taken was 5 hours 12 minutes and the shortest time was 30 minutes.

(i) Find the lower quartile. [2]

(ii) Represent the information by a box-and-whisker plot, using a scale of 2 cm to represent 60 minutes. [4]

(i) 54

MAY/JUNE2006

The salaries, in thousands of dollars, of 11 people, chosen at random in a certain office, were found to be:

40, 42, 45, 41, 352, 40, 50, 48, 51, 49, 47.

Choose and calculate an appropriate measure of central tendency (mean, mode or median) to summarise these salaries. Explain briefly why the other measures are not suitable. [3]

Median 47000. Data have outlier, are skew etc.

MAY/JUNE2006

32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? [1]

(ii) How many teams play in exactly 2 matches? [1]

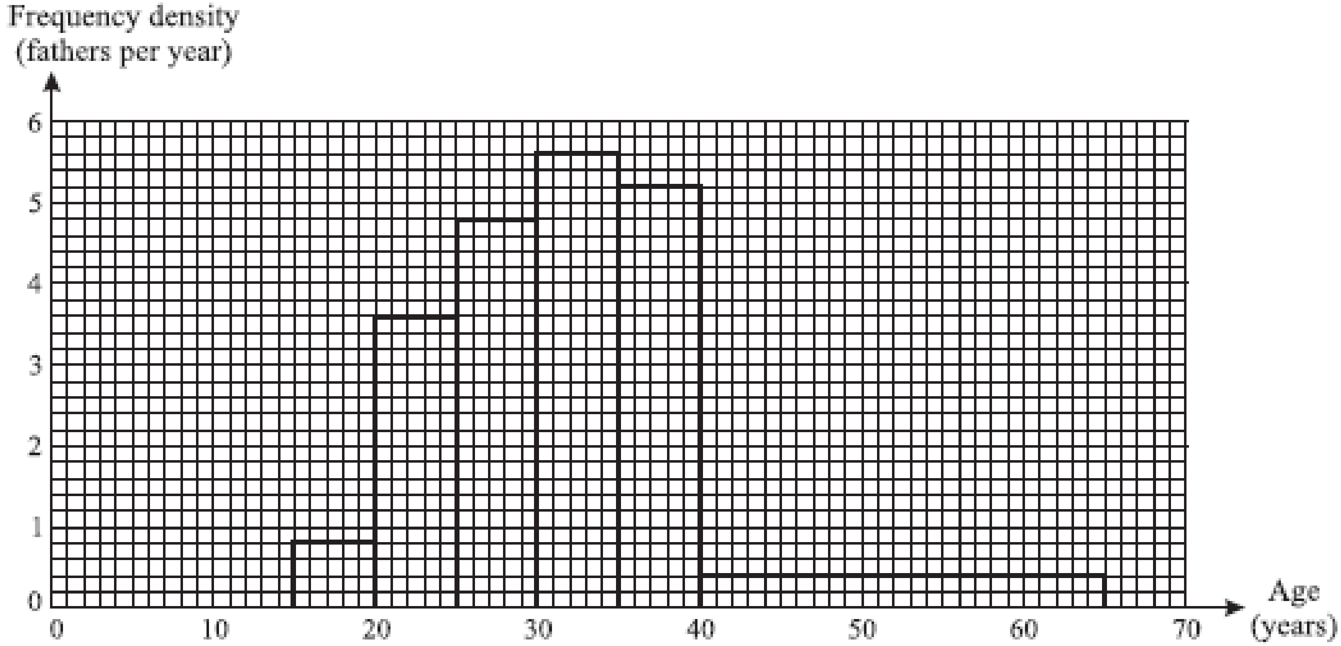
(iii) Draw up a frequency table for the numbers of matches which the teams play. [3]

(iv) Calculate the mean and variance of the numbers of matches which the teams play. [4]

(i) 16
(ii) 8
(iv) 1.94, 1.43

MAY/JUNE2006

Each father in a random sample of fathers was asked how old he was when his first child was born. The following histogram represents the information.



(i) What is the modal age group? [1]

(ii) How many fathers were between 25 and 30 years old when their first child was born? [2]

(iii) How many fathers were in the sample? [2]

(iv) Find the probability that a father, chosen at random from the group, was between 25 and 30 years old when his first child was born, given that he was older than 25 years. [2]

(i) 30-35
(ii) 24
(iii) 110
(iv) 0.273

OCTOBER/NOVEMBER2005

A study of the ages of car drivers in a certain country produced the results shown in the table.

Percentage of drivers in each age group

	Young	Middle-aged	Elderly
Males	40	35	25
Females	20	70	10

Illustrate these results diagrammatically. [4]

2 bar charts

OCTOBER/NOVEMBER2005

A group of 10 married couples and 3 single men found that the mean age $\bar{x}_w$ of the 10 women was 41.2 years and the standard deviation of the women's ages was 15.1 years. For the 13 men, the mean age $\bar{x}_m$ was 46.3 years and the standard deviation was 12.7 years.

(i) Find the mean age of the whole group of 23 people. [2]

(ii) The individual women's ages are denoted by x_w and the individual men's ages by x_m. By first finding Σx_w^2 and Σx_m^2, find the standard deviation for the whole group. [5]

(i) 44.1
(ii) 19254.5, 29964.74, 14.0

MAY/JUNE2005

The following table shows the results of a survey to find the average daily time, in minutes, that a group of schoolchildren spent in internet chat rooms.

Time per day (t minutes)	Frequency
$0 \leqslant t < 10$	2
$10 \leqslant t < 20$	f
$20 \leqslant t < 40$	11
$40 \leqslant t < 80$	4

The mean time was calculated to be 27.5 minutes.

(i) Form an equation involving f and hence show that the total number of children in the survey was 26. [4]

(ii) Find the standard deviation of these times. [2]

(ii) 16.1

MAY/JUNE2005

The following back-to-back stem-and-leaf diagram shows the cholesterol count for a group of 45 people who exercise daily and for another group of 63 who do not exercise. The figures in brackets show the number of people corresponding to each set of leaves.

	People who exercise		People who do not exercise	
(9)	9 8 7 6 4 3 2 2 1	3	1 5 7 7	(4)
(12)	9 8 8 8 7 6 6 5 3 3 2 2	4	2 3 4 4 5 8	(6)
(9)	8 7 7 7 6 5 3 3 1	5	1 2 2 2 3 4 4 5 6 7 8 8 9	(13)
(7)	6 6 6 6 4 3 2	6	1 2 3 3 3 4 5 5 5 7 7 8 9 9	(14)
(3)	8 4 1	7	2 4 5 5 6 6 7 8 8	(9)
(4)	9 5 5 2	8	1 3 3 4 6 7 9 9 9	(9)
(1)	4	9	1 4 5 5 8	(5)
(0)		10	3 3 6	(3)

Key: 2 | 8 | 1 represents a cholesterol count of 8.2 in the group who exercise and 8.1 in the group who do not exercise.

(i) Give one useful feature of a stem-and-leaf diagram. [1]

(ii) Find the median and the quartiles of the cholesterol count for the group who do not exercise. [3]

You are given that the lower quartile, median and upper quartile of the cholesterol count for the group who exercise are 4.25, 5.3 and 6.6 respectively.

(iii) On a single diagram on graph paper, draw two box-and-whisker plots to illustrate the data. [4]

(i) shows all the data
(ii) median = 6.5, Q1 = 5.4, Q3 = 8.3

OCTOBER/NOVEMBER2004

The lengths of cars travelling on a car ferry are noted. The data are summarised in the following table.

Length of car (x metres)	Frequency	Frequency density
$2.80 \leqslant x < 3.00$	17	85
$3.00 \leqslant x < 3.10$	24	240
$3.10 \leqslant x < 3.20$	19	190
$3.20 \leqslant x < 3.40$	8	a

(i) Find the value of a. [1]

(ii) Draw a histogram on graph paper to represent the data. [3]

(iii) Find the probability that a randomly chosen car on the ferry is less than 3.20 m in length. [2]

(i) 40
(iii) 0.882

OCTOBER/NOVEMBER2004

The ages, x years, of 18 people attending an evening class are summarised by the following totals: $\Sigma x = 745, \Sigma x^2 = 33\,951$.

(i) Calculate the mean and standard deviation of the ages of this group of people. [3]

(ii) One person leaves the group and the mean age of the remaining 17 people is exactly 41 years. Find the age of the person who left and the standard deviation of the ages of the remaining 17 people. [4]

(i) 41.4, 13.2
(ii) 48, 13.4

MAY/JUNE2004

Two cricket teams kept records of the number of runs scored by their teams in 8 matches. The scores are shown in the following table.

Team A	150	220	77	30	298	118	160	57
Team B	166	142	170	93	111	130	148	86

(i) Find the mean and standard deviation of the scores for team A. [2]

The mean and standard deviation for team B are 130.75 and 29.63 respectively.

(ii) State with a reason which team has the more consistent scores. [2]

(i) 139 (138.75), 83.1
(ii) team B. smaller standard deviation

MAY/JUNE2004

In a recent survey, 640 people were asked about the length of time each week that they spent watching television. The median time was found to be 20 hours, and the lower and upper quartiles were 15 hours and 35 hours respectively. The least amount of time that anyone spent was 3 hours, and the greatest amount was 60 hours.

(i) On graph paper, show these results using a fully labelled cumulative frequency graph. [3]

(ii) Use your graph to estimate how many people watched more than 50 hours of television each week. [2]

60-70 for straight lines, 40-70 for curve

OCTOBER/NOVEMBER2003

A computer can generate random numbers which are either 0 or 2. On a particular occasion, it generates a set of numbers which consists of 23 zeros and 17 twos. Find the mean and variance of this set of 40 numbers. [4]

0.850, 0.978

OCTOBER/NOVEMBER2003

The floor areas, x m^2, of 20 factories are as follows.

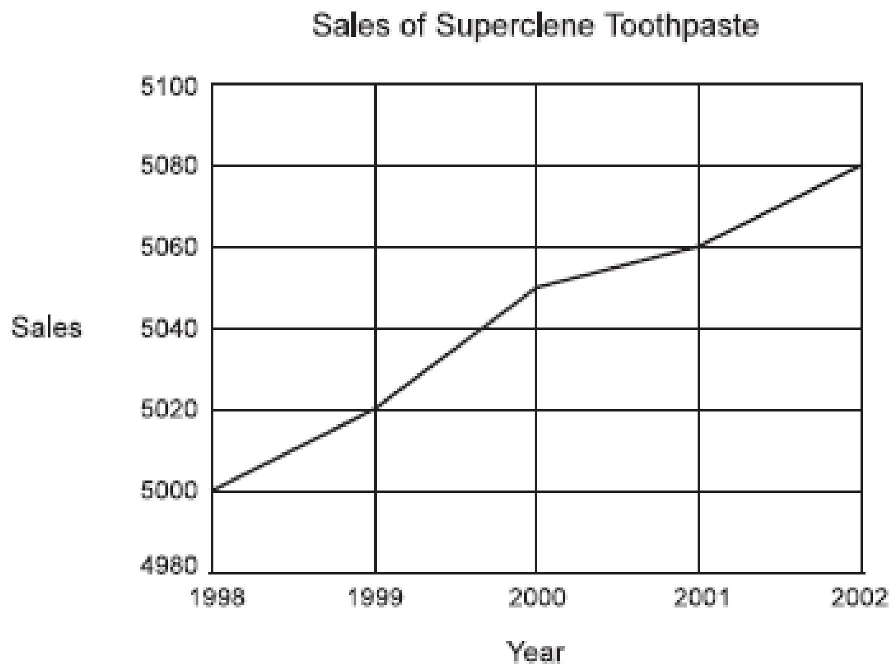
150	350	450	578	595	644	722	798	802	904
1000	1330	1533	1561	1778	1960	2167	2330	2433	3231

Represent these data by a histogram on graph paper, using intervals

$0 \leqslant x < 500,\ 500 \leqslant x < 1000,\ 1000 \leqslant x < 2000,\ 2000 \leqslant x < 3000,\ 3000 \leqslant x < 4000.$ [4]

MAY/JUNE2003

(i)



The diagram represents the sales of Superclene toothpaste over the last few years. Give a reason why it is misleading. [1]

(ii) The following data represent the daily ticket sales at a small theatre during three weeks.

52, 73, 34, 85, 62, 79, 89, 50, 45, 83, 84, 91, 85, 84, 87, 44, 86, 41, 35, 73, 86.

(a) Construct a stem-and-leaf diagram to illustrate the data. [3]

(b) Use your diagram to find the median of the data. [1]

(i) False zero
(ii) (b) 79

MAY/JUNE2003

A random sample of 97 people who own mobile phones was used to collect data on the amount of time they spent per day on their phones. The results are displayed in the table below.

Time spent per day (t minutes)	$0 \leqslant t < 5$	$5 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 70$
Number of people	11	20	32	18	10	6

(i) Calculate estimates of the mean and standard deviation of the time spent per day on these mobile phones. [5]

(ii) On graph paper, draw a fully labelled histogram to represent the data. [4]

(i) 18.4, 13.3

OCTOBER/NOVEMBER2002

The weights in kilograms of two groups of 17-year-old males from country P and country Q are displayed in the following back-to-back stem-and-leaf diagram. In the third row of the diagram, … 4 | 7 | 1 … denotes weights of 74 kg for a male in country P and 71 kg for a male in country Q.

Country P		Country Q
	5	1 5
	6	2 3 4 8
9 8 7 6 4	7	1 3 4 5 6 7 7 8 8 9
8 8 6 6 5 3	8	2 3 6 7 7 8 8
9 7 7 6 5 5 5 4 2	9	0 2 2 4
5 4 4 3 1	10	4 5

(i) Find the median and quartile weights for country Q. [3]

(ii) You are given that the lower quartile, median and upper quartile for country P are 84, 94 and 98 kg respectively. On a single diagram on graph paper, draw two box-and-whisker plots of the data. [4]

(iii) Make two comments on the weights of the two groups. [2]

(i) median = 78, Q1 = 72, Q3 = 88
(iii) people heavier in P than in Q. weights more spread out in Q.

MAY/JUNE2002

The manager of a company noted the times spent in 80 meetings. The results were as follows.

Time (t minutes)	$0 < t \leqslant 15$	$15 < t \leqslant 30$	$30 < t \leqslant 60$	$60 < t \leqslant 90$	$90 < t \leqslant 120$
Number of meetings	4	7	24	38	7

Draw a cumulative frequency graph and use this to estimate the median time and the interquartile range. [6]

MAY/JUNE2002

(i) In a spot check of the speeds $x\,\mathrm{km\,h^{-1}}$ of 30 cars on a motorway, the data were summarised by $\Sigma(x - 110) = -47.2$ and $\Sigma(x - 110)^2 = 5460$. Calculate the mean and standard deviation of these speeds. [4]

(ii) On another day the mean speed of cars on the motorway was found to be $107.6\,\mathrm{km\,h^{-1}}$ and the standard deviation was $13.8\,\mathrm{km\,h^{-1}}$. Assuming these speeds follow a normal distribution and that the speed limit is $110\,\mathrm{km\,h^{-1}}$, find what proportion of cars exceed the speed limit. [3]

(i) 108, 13.4
(ii) 0.431

PERMUTATIONS & COMBINATIONS

MAY/JUNE2012 9709/61

(a) Seven friends together with their respective partners all meet up for a meal. To commemorate the occasion they arrange for a photograph to be taken of all 14 of them standing in a line.

(i) How many different arrangements are there if each friend is standing next to his or her partner? [3]

(ii) How many different arrangements are there if the 7 friends all stand together and the 7 partners all stand together? [2]

(b) A group of 9 people consists of 2 boys, 3 girls and 4 adults. In how many ways can a team of 4 be chosen if

(i) both boys are in the team, [1]

(ii) the adults are either all in the team or all not in the team, [2]

(iii) at least 2 girls are in the team? [2]

(a) (i) 645120

(ii) 50803200

(b) (i) 21

(ii) 6

(iii) 51

MAY/JUNE2012 9709/62

An English examination consists of 8 questions in Part A and 3 questions in Part B. Candidates must choose 6 questions. The order in which questions are chosen does not matter. Find the number of ways in which the 6 questions can be chosen in each of the following cases.

(i) There are no restrictions on which questions can be chosen. [1]

(ii) Candidates must choose at least 4 questions from Part A. [3]

(iii) Candidates must either choose both question 1 and question 2 in Part A, or choose neither of these questions. [3]

(i) 462
(ii) 406
(iii) 210

MAY/JUNE2012 9709/63

(i) In how many ways can all 9 letters of the word TELEPHONE be arranged in a line if the letters P and L must be at the ends? [2]

How many different selections of 4 letters can be made from the 9 letters of the word TELEPHONE if

(ii) there are no Es, [1]

(iii) there is exactly 1 E, [2]

(iv) there are no restrictions? [4]

(i) 1680
(ii) 15
(iii) 20
(iv) 56

OCTOBER/NOVEMBER2011 9709/61

(a) Find the number of different ways in which the 12 letters of the word STRAWBERRIES can be arranged

(i) if there are no restrictions, [2]

(ii) if the 4 vowels A, E, E, I must all be together. [3]

(b) **(i)** 4 astronauts are chosen from a certain number of candidates. If order of choosing is not taken into account, the number of ways the astronauts can be chosen is 3876. How many ways are there if order of choosing is taken into account? [2]

(ii) 4 astronauts are chosen to go on a mission. Each of these astronauts can take 3 personal possessions with him. How many different ways can these 12 possessions be arranged in a row if each astronaut's possessions are kept together? [2]

(a) (i) 19958400
(ii) 362880
(b) (i) 93024
(ii) 31104

OCTOBER/NOVEMBER2011 9709/62

Twelve coins are tossed and placed in a line. Each coin can show either a head or a tail.

(i) Find the number of different arrangements of heads and tails which can be obtained. [2]

(ii) Find the number of different arrangements which contain 7 heads and 5 tails. [1]

(i) 4096
(ii) 792

OCTOBER/NOVEMBER2011 9709/62

(a) Geoff wishes to plant 25 flowers in a flower-bed. He can choose from 15 different geraniums, 10 different roses and 8 different lilies. He wants to have at least 11 geraniums and also to have the same number of roses and lilies. Find the number of different selections of flowers he can make. [4]

(b) Find the number of different ways in which the 9 letters of the word GREENGAGE can be arranged if exactly two of the Gs are next to each other. [3]

(a) 1941912
(b) 5040

OCTOBER/NOVEMBER2011 9709/63

Mary saves her digital images on her computer in three separate folders named 'Family', 'Holiday' and 'Friends'. Her family folder contains 3 images, her holiday folder contains 4 images and her friends folder contains 8 images. All the images are different.

(i) Find in how many ways she can arrange these 15 images in a row across her computer screen if she keeps the images from each folder together. [3]

(ii) Find the number of different ways in which Mary can choose 6 of these images if there are 2 from each folder. [2]

(iii) Find the number of different ways in which Mary can choose 6 of these images if there are at least 3 images from the friends folder and at least 1 image from each of the other two folders. [4]

(i) 34836480
(ii) 504
(iii) 2520

MAY/JUNE2011 9709/61

A cricket team of 11 players is to be chosen from 21 players consisting of 10 batsmen, 9 bowlers and 2 wicketkeepers. The team must include at least 5 batsmen, at least 4 bowlers and at least 1 wicketkeeper.

(i) Find the number of different ways in which the team can be chosen. [4]

Each player in the team is given a present. The presents consist of 5 identical pens, 4 identical diaries and 2 identical notebooks.

(ii) Find the number of different arrangements of the presents if they are all displayed in a row. [1]

(iii) 10 of these 11 presents are chosen and arranged in a row. Find the number of different arrangements that are possible. [3]

(i) 148176
(ii) 6930
(iii) 6930

MAY/JUNE2011 9709/62

(i) Find the number of different ways that the 9 letters of the word HAPPINESS can be arranged in a line. [1]

(ii) The 9 letters of the word HAPPINESS are arranged in random order in a line. Find the probability that the 3 vowels (A, E, I) are not all next to each other. [4]

(iii) Find the number of different selections of 4 letters from the 9 letters of the word HAPPINESS which contain no Ps and either one or two Ss. [3]

(i) 90720
(ii) 0.917
(iii) 20

MAY/JUNE2011 9709/63

Fahad has 4 different coloured pairs of shoes (white, red, blue and black), 3 different coloured pairs of jeans (blue, black and brown) and 7 different coloured tee shirts (red, orange, yellow, blue, green, white and purple).

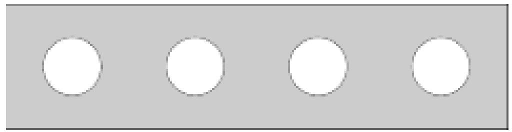
(i) Fahad chooses an outfit consisting of one pair of shoes, one pair of jeans and one tee shirt. How many different outfits can he choose? [1]

(ii) How many different ways can Fahad arrange his 3 jeans and 7 tee shirts in a row if the two blue items are not next to each other? [2]

Fahad also has 9 different books about sport. When he goes on holiday he chooses at least one of these books to take with him.

(iii) How many different selections are there if he can take any number of books ranging from just one of them to all of them? [3]

(i) 84
(ii) 2903040
(iii) 511



Pegs are to be placed in the four holes shown, one in each hole. The pegs come in different colours and pegs of the same colour are identical. Calculate how many different arrangements of coloured pegs in the four holes can be made using

(i) 6 pegs, all of different colours, [1]

(ii) 4 pegs consisting of 2 blue pegs, 1 orange peg and 1 yellow peg. [1]

Beryl has 12 pegs consisting of 2 red, 2 blue, 2 green, 2 orange, 2 yellow and 2 black pegs. Calculate how many different arrangements of coloured pegs in the 4 holes Beryl can make using

(iii) 4 different colours, [1]

(iv) 3 different colours, [3]

(v) any of her 12 pegs. [3]

(i) 360
(ii) 12
(iii) 360
(iv) 720
(v) 1170

OCTOBER/NOVEMBER2010 9709/62

A committee of 6 people, which must contain at least 4 men and at least 1 woman, is to be chosen from 10 men and 9 women.

(i) Find the number of possible committees that can be chosen. [3]

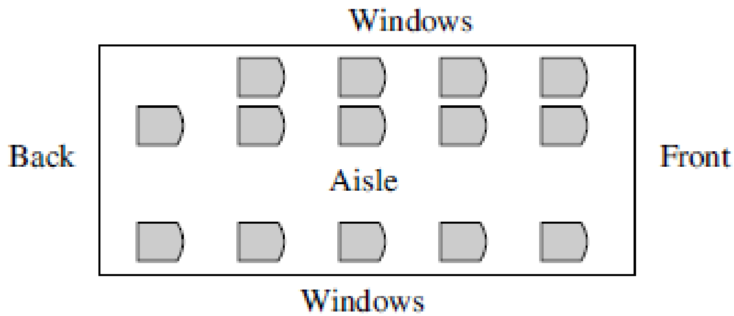
(ii) Find the probability that one particular man, Albert, and one particular woman, Tracey, are both on the committee. [2]

(iii) Find the number of possible committees that include either Albert or Tracey but not both. [3]

(iv) The committee that is chosen consists of 4 men and 2 women. They queue up randomly in a line for refreshments. Find the probability that the women are not next to each other in the queue. [3]

(i) 9828
(ii) 0.0812
(iii) 4494
(iv) 0.667

OCTOBER/NOVEMBER2010 9709/63



A small aeroplane has 14 seats for passengers. The seats are arranged in 4 rows of 3 seats and a back row of 2 seats (see diagram). 12 passengers board the aeroplane.

(i) How many possible seating arrangements are there for the 12 passengers? Give your answer correct to 3 significant figures. [2]

These 12 passengers consist of 2 married couples (Mr and Mrs Lin and Mr and Mrs Brown), 5 students and 3 business people.

(ii) The 3 business people sit in the front row. The 5 students each sit at a window seat. Mr and Mrs Lin sit in the same row on the same side of the aisle. Mr and Mrs Brown sit in another row on the same side of the aisle. How many possible seating arrangements are there? [4]

(iii) If, instead, the 12 passengers are seated randomly, find the probability that Mrs Lin sits directly behind a student and Mrs Brown sits in the front row. [4]

(i) 4.36×10^{10}
(ii) 17280
(iii) 0.0687

MAY/JUNE2010 9709/61

(i) Find the number of different ways that a set of 10 different mugs can be shared between Lucy and Monica if each receives an odd number of mugs. [3]

(ii) Another set consists of 6 plastic mugs each of a different design and 3 china mugs each of a different design. Find in how many ways these 9 mugs can be arranged in a row if the china mugs are all separated from each other. [3]

(iii) Another set consists of 3 identical red mugs, 4 identical blue mugs and 7 identical yellow mugs. These 14 mugs are placed in a row. Find how many different arrangements of the colours are possible if the red mugs are kept together. [3]

(i) 512
(ii) 151200
(iii) 3960

MAY/JUNE2010 9709/62

Nine cards, each of a different colour, are to be arranged in a line.

(i) How many different arrangements of the 9 cards are possible? [1]

The 9 cards include a pink card and a green card.

(ii) How many different arrangements do not have the pink card next to the green card? [3]

Consider all possible choices of 3 cards from the 9 cards with the 3 cards being arranged in a line.

(iii) How many different arrangements in total of 3 cards are possible? [2]

(iv) How many of the arrangements of 3 cards in part **(iii)** contain the pink card? [2]

(v) How many of the arrangements of 3 cards in part **(iii)** do not have the pink card next to the green card? [2]

(i) 362880
(ii) 282240
(iii) 504
(iv) 168
(v) 476

MAY/JUNE2010 9709/63

Three identical cans of cola, 2 identical cans of green tea and 2 identical cans of orange juice are arranged in a row. Calculate the number of arrangements if

(i) the first and last cans in the row are the same type of drink, [3]

(ii) the 3 cans of cola are all next to each other and the 2 cans of green tea are not next to each other. [5]

(i) 50
(ii) 18

OCTOBER/NOVEMBER2009 9709/61

(a) Find how many numbers between 5000 and 6000 can be formed from the digits 1, 2, 3, 4, 5 and 6

(i) if no digits are repeated, [2]

(ii) if repeated digits are allowed. [2]

(b) Find the number of ways of choosing a school team of 5 pupils from 6 boys and 8 girls

(i) if there are more girls than boys in the team, [4]

(ii) if three of the boys are cousins and are either all in the team or all not in the team. [3]

(a) (i) 60
(ii) 216
(b) (i) 1316
(ii) 517

OCTOBER/NOVEMBER2009 9709/62

(a) **(i)** Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and 6 with no digit being repeated. [1]

(ii) Find how many different odd numbers greater than 500 can be made using some or all of the digits 1, 3, 5 and 6 with no digit being repeated. [4]

(b) Six cards numbered 1, 2, 3, 4, 5, 6 are arranged randomly in a line. Find the probability that the cards numbered 4 and 5 are **not** next to each other. [3]

(a) (i) 24
(ii) 28
(b) 2/3

MAY/JUNE2009

A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopranos, 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen? [2]

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next to each other? [3]

(iii) The 4 tenors and 4 basses in the group stand in a single line with all the tenors next to each other and all the basses next to each other. How many possible arrangements are there if three of the tenors refuse to stand next to any of the basses? [3]

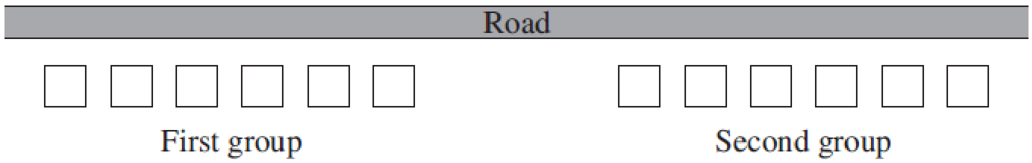
(i) 33033000
(ii) 86400
(iii) 288

OCTOBER/NOVEMBER2008

A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style A, 2 houses in style B, 3 houses in style C, 4 houses in style D and 1 house in style E.

(i) Find the number of possible arrangements of these 12 houses. [2]

(ii)



The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles A and D are in the first group and all the houses in styles B, C and E are in the second group. [3]

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style B and exactly one house in style C. Find the number of ways in which these four houses can be selected. [2]

(i) 831600
(ii) 900
(iii) 126

MAY/JUNE2008

Issam has 11 different CDs, of which 6 are pop music, 3 are jazz and 2 are classical.

(i) How many different arrangements of all 11 CDs on a shelf are there if the jazz CDs are all next to each other? [3]

(ii) Issam makes a selection of 2 pop music CDs, 2 jazz CDs and 1 classical CD. How many different possible selections can be made? [3]

(i) 2177280
(ii) 90

OCTOBER/NOVEMBER2007

The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made? [2]

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit? [4]

(i) 120
(ii) 48

MAY/JUNE2007

(i) Find the number of ways in which all twelve letters of the word REFRIGERATOR can be arranged

(a) if there are no restrictions, [2]

(b) if the Rs must all be together. [2]

(ii) How many different selections of four letters from the twelve letters of the word REFRIGERATOR contain no Rs and two Es? [3]

(i) (a) 9979200

(b) 181440

(ii) 15

OCTOBER/NOVEMBER2006

Six men and three women are standing in a supermarket queue.

(i) How many possible arrangements are there if there are no restrictions on order? [2]

(ii) How many possible arrangements are there if no two of the women are standing next to each other? [4]

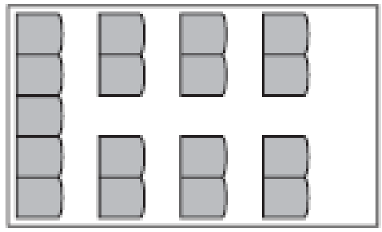
(iii) Three of the people in the queue are chosen to take part in a customer survey. How many different choices are possible if at least one woman must be included? [3]

(i) 362880

(ii) 151200

(iii) 64

MAY/JUNE2006



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.

(i) How many possible seating arrangements are there for the 11 passengers? [2]

(ii) How many possible seating arrangements are there if 5 particular people sit in the back row? [3]

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

(iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 married couples and 1 other person, who may or may not be married? [3]

(i) 4.94×10^{11}
(ii) 79833600
(iii) 21

OCTOBER/NOVEMBER2005

A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces? [2]

(ii) How many different arrangements are there if the 4 empty spaces are next to each other? [3]

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to each other. [2]

(i) 259459200

(ii) 3628800

(iii) 0.986

MAY/JUNE2005

(a) A football team consists of 3 players who play in a defence position, 3 players who play in a midfield position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and one forward player, [2]

(ii) if exactly one forward player must be included, together with any two others. [2]

(b) Find how many different arrangements there are of the nine letters in the words GOLD MEDAL

(i) if there are no restrictions on the order of the letters, [2]

(ii) if the two letters D come first and the two letters L come last. [2]

(a) (i) 15

(ii) 45

(b) (i) 90720

(ii) 120

OCTOBER/NOVEMBER2004

The word ARGENTINA includes the four consonants R, G, N, T and the three vowels A, E, I.

(i) Find the number of different arrangements using all nine letters. [2]

(ii) How many of these arrangements have a consonant at the beginning, then a vowel, then another consonant, and so on alternately? [3]

(i) 90720
(ii) 720

MAY/JUNE2004

(a) The menu for a meal in a restaurant is as follows.

Starter Course
Melon
or
Soup
or
Smoked Salmon
Main Course
Chicken
or
Steak
or
Lamb Cutlets
or
Vegetable Curry
or
Fish
Dessert Course
Cheesecake
or
Ice Cream
or
Apple Pie
All the main courses are served with salad and either new potatoes or french fries.

(i) How many different three-course meals are there? [2]

(ii) How many different choices are there if customers may choose only two of the three courses? [3]

(b) In how many ways can a group of 14 people eating at the restaurant be divided between three tables seating 5, 5 and 4? [3]

(a) (i) 90

(ii) 69

(b) 252252

OCTOBER/NOVEMBER2003

(a) A collection of 18 books contains one Harry Potter book. Linda is going to choose 6 of these books to take on holiday.

(i) In how many ways can she choose 6 books? [1]

(ii) How many of these choices will include the Harry Potter book? [2]

(b) In how many ways can 5 boys and 3 girls stand in a straight line

(i) if there are no restrictions, [1]

(ii) if the boys stand next to each other? [4]

(a) (i) 18564
(ii) 6188
(b) (i) 40320
(ii) 2880

MAY/JUNE2003

A committee of 5 people is to be chosen from 6 men and 4 women. In how many ways can this be done

(i) if there must be 3 men and 2 women on the committee, [2]

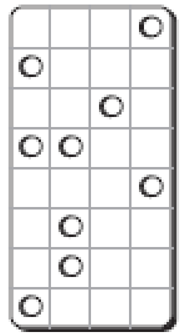
(ii) if there must be more men than women on the committee, [3]

(iii) if there must be 3 men and 2 women, and one particular woman refuses to be on the committee with one particular man? [3]

(i) 120
(ii) 186
(iii) 90

OCTOBER/NOVEMBER2002

In a certain hotel, the lock on the door to each room can be opened by inserting a key card. The key card can be inserted only one way round. The card has a pattern of holes punched in it. The card has 4 columns, and each column can have either 1 hole, 2 holes, 3 holes or 4 holes punched in it. Each column has 8 different positions for the holes. The diagram illustrates one particular key card with 3 holes punched in the first column, 3 in the second, 1 in the third and 2 in the fourth.



(i) Show that the number of different ways in which a column could have exactly 2 holes is 28. [1]

(ii) Find how many different patterns of holes can be punched in a column. [4]

(iii) How many different possible key cards are there? [2]

(ii) 162
(iii) 688747536

MAY/JUNE2002

The digits of the number 1223678 can be rearranged to give many different 7-digit numbers. Find how many different 7-digit numbers can be made if

(i) there are no restrictions on the order of the digits, [2]

(ii) the digits 1, 3, 7 (in any order) are next to each other, [3]

(iii) these 7-digit numbers are even. [3]

(i) 2520
(ii) 360
(iii) 1440

PROBABILITY

MAY/JUNE2012 9709/61

Maria has 3 pre-set stations on her radio. When she switches her radio on, there is a probability of 0.3 that it will be set to station 1, a probability of 0.45 that it will be set to station 2 and a probability of 0.25 that it will be set to station 3. On station 1 the probability that the presenter is male is 0.1, on station 2 the probability that the presenter is male is 0.85 and on station 3 the probability that the presenter is male is p. When Maria switches on the radio, the probability that it is set to station 3 and the presenter is male is 0.075.

(i) Show that the value of p is 0.3. [1]

(ii) Given that Maria switches on and hears a male presenter, find the probability that the radio was set to station 2. [4]

(ii) 0.785

MAY/JUNE2012 9709/62

A box of biscuits contains 30 biscuits, some of which are wrapped in gold foil and some of which are unwrapped. Some of the biscuits are chocolate-covered. 12 biscuits are wrapped in gold foil, and of these biscuits, 7 are chocolate-covered. There are 17 chocolate-covered biscuits in total.

(i) Copy and complete the table below to show the number of biscuits in each category. [2]

	Wrapped in gold foil	Unwrapped	Total
Chocolate-covered			
Not chocolate-covered			
Total			30

A biscuit is selected at random from the box.

(ii) Find the probability that the biscuit is wrapped in gold foil. [1]

The biscuit is returned to the box. An unwrapped biscuit is then selected at random from the box.

(iii) Find the probability that the biscuit is chocolate-covered. [1]

The biscuit is returned to the box. A biscuit is then selected at random from the box.

(iv) Find the probability that the biscuit is unwrapped, given that it is chocolate-covered. [1]

The biscuit is returned to the box. Nasir then takes 4 biscuits without replacement from the box.

(v) Find the probability that he takes exactly 2 wrapped biscuits. [4]

(ii) 0.4
(iii) 0.556
(iv) 0.588
(v) 0.368

MAY/JUNE2012 9709/63

Suzanne has 20 pairs of shoes, some of which have designer labels. She has 6 pairs of high-heeled shoes, of which 2 pairs have designer labels. She has 4 pairs of low-heeled shoes, of which 1 pair has designer labels. The rest of her shoes are pairs of sports shoes. Suzanne has 8 pairs of shoes with designer labels in total.

(i) Copy and complete the table below to show the number of pairs in each category. [2]

	Designer labels	No designer labels	Total
High-heeled shoes			
Low-heeled shoes			
Sports shoes			
Total			20

Suzanne chooses 1 pair of shoes at random to wear.

(ii) Find the probability that she wears the pair of low-heeled shoes with designer labels. [1]

(iii) Find the probability that she wears a pair of sports shoes. [1]

(iv) Find the probability that she wears a pair of high-heeled shoes, given that she wears a pair of shoes with designer labels. [1]

(v) State with a reason whether the events 'Suzanne wears a pair of shoes with designer labels' and 'Suzanne wears a pair of sports shoes' are independent. [2]

Suzanne chooses 1 pair of shoes at random each day.

(vi) Find the probability that Suzanne wears a pair of shoes with designer labels on at most 4 days out of the next 7 days. [3]

(ii) 0.05
(iii) 0.5
(iv) 0.25
(v) not independent
(vi) 0.904

OCTOBER/NOVEMBER2011 9709/61

Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

(i) Find the probability that exactly two of the selected balls have the same number. [5]

(ii) Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2. [2]

(iii) Event X is 'exactly two of the selected balls have the same number'. Event Y is 'the ball selected from bag A has number 2'. Showing your working, determine whether events X and Y are independent or not. [2]

(i) 0.336
(ii) 0.106
(iii) not independent

OCTOBER/NOVEMBER2011 9709/63

In a group of 30 teenagers, 13 of the 18 males watch 'Kops are Kids' on television and 3 of the 12 females watch 'Kops are Kids'.

(i) Find the probability that a person chosen at random from the group is either female or watches 'Kops are Kids' or both. [4]

(ii) Showing your working, determine whether the events 'the person chosen is male' and 'the person chosen watches Kops are Kids' are independent or not. [2]

(i) 0.833
(ii) not independent

MAY/JUNE2011 9709/61

When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

0.921

MAY/JUNE2011 9709/63

Tim throws a fair die twice and notes the number on each throw.

(i) Tim calculates his final score as follows. If the number on the second throw is a 5 he multiplies the two numbers together, and if the number on the second throw is not a 5 he adds the two numbers together. Find the probability that his final score is

(a) 12, [1]

(b) 5. [3]

(ii) Events A, B, C are defined as follows.

A: the number on the second throw is 5

B: the sum of the numbers is 6

C: the product of the numbers is even

By calculation find which pairs, if any, of the events A, B and C are independent. [5]

(i) (a) 1/36

(b) 5/36

(ii) None are independent

OCTOBER/NOVEMBER2010 9709/61

Three friends, Rick, Brenda and Ali, go to a football match but forget to say which entrance to the ground they will meet at. There are four entrances, A, B, C and D. Each friend chooses an entrance independently.

- The probability that Rick chooses entrance A is $\frac{1}{3}$. The probabilities that he chooses entrances B, C or D are all equal.
- Brenda is equally likely to choose any of the four entrances.
- The probability that Ali chooses entrance C is $\frac{2}{7}$ and the probability that he chooses entrance D is $\frac{3}{5}$. The probabilities that he chooses the other two entrances are equal.

(i) Find the probability that at least 2 friends will choose entrance B. [4]

(ii) Find the probability that the three friends will all choose the same entrance. [4]

(i) 0.0762
(ii) 0.0571

OCTOBER/NOVEMBER2010 9709/62

A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

- If the spinner lands on an **even-numbered** side, Raj **multiplies** the two numbers showing on the dice to get his score.
- If the spinner lands on an **odd-numbered** side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side. [6]

8/11

OCTOBER/NOVEMBER2010 9709/63

It was found that 68% of the passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all probabilities as decimals. [2]

(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey. [3]

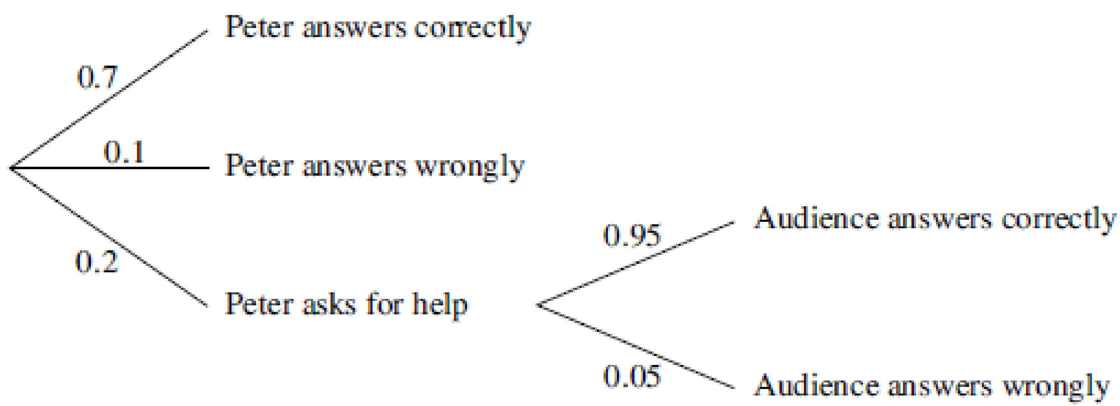
(ii) 0.842

MAY/JUNE2010 9709/61

In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.



(i) Show that the probability that the first question is answered correctly is 0.89. [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

(ii) Find the probability that the first two questions are both answered correctly. [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]

(ii) 0.781
(iii) 0.372

MAY/JUNE2010 9709/62

Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.

R : both of the numbers are greater than 8.

(i) Find P(Q). [2]

(ii) Find P(R). [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

(i) 6/144
(ii) 1/9
(iii) Yes
(iv) No

MAY/JUNE2010 9709/63

A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

0.176

OCTOBER/NOVEMBER2009 9709/62

Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

(i) Draw a fully labelled tree diagram to show this information. [2]

(ii) Given that Maria did **not** have jam for breakfast, find the probability that she had toast. [4]

(ii) 0.654

MAY/JUNE2009

At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

$$\text{elephant ride } \frac{6}{10}, \quad \text{camel ride } \frac{7}{10}, \quad \text{jungle tractor ride } \frac{8}{10}.$$

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

(i) P(E) = ¼, P(C) = ¼, P(JT) = ½
(iii) 0.725
(iv) 0.273

OCTOBER/NOVEMBER2008

There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]

(ii) 0.224

(iii) 0.392

(iv) 0.633

MAY/JUNE2008

In country A 30% of people who drink tea have sugar in it. In country B 65% of people who drink tea have sugar in it. There are 3 million people in country A who drink tea and 12 million people in country B who drink tea. A person is chosen at random from these 15 million people.

(i) Find the probability that the person chosen is from country A. [1]

(ii) Find the probability that the person chosen does not have sugar in their tea. [2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country B. [2]

(i) 0.2

(ii) 0.42

(iii) 0.667

MAY/JUNE2007

Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]

(i) 0.8
(ii) 0.625

OCTOBER/NOVEMBER2006

Two fair dice are thrown.

(i) Event A is 'the scores differ by 3 or more'. Find the probability of event A. [3]

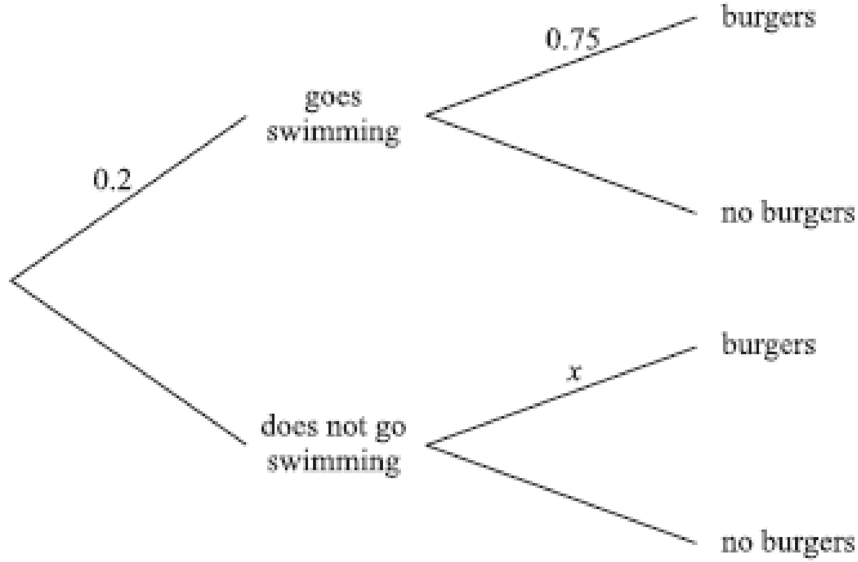
(ii) Event B is 'the product of the scores is greater than 8'. Find the probability of event B. [2]

(iii) State with a reason whether events A and B are mutually exclusive. [2]

(i) 1/3
(ii) 20/36
(iii) Not mutually exclusive

MAY/JUNE2006

The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

(i) 0.438
(ii) 0.3

OCTOBER/NOVEMBER2005

Boxes of sweets contain toffees and chocolates. Box A contains 6 toffees and 4 chocolates, box B contains 5 toffees and 3 chocolates, and box C contains 3 toffees and 7 chocolates. One of the boxes is chosen at random and two sweets are taken out, one after the other, and eaten.

(i) Find the probability that they are both toffees. [3]

(ii) Given that they are both toffees, find the probability that they both came from box A. [3]

(i) 0.252
(ii) 0.440

MAY/JUNE2005

Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find $P(M)$. [2]

(ii) Find $P(M \text{ and } E)$. [1]

(iii) Are M and E independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

(i) 0.482
(ii) 0.322
(iii) not independent
(iv) 0.635

OCTOBER/NOVEMBER2004

When Andrea needs a taxi, she rings one of three taxi companies, A, B or C. 50% of her calls are to taxi company A, 30% to B and 20% to C. A taxi from company A arrives late 4% of the time, a taxi from company B arrives late 6% of the time and a taxi from company C arrives late 17% of the time.

(i) Find the probability that, when Andrea rings for a taxi, it arrives late. [3]

(ii) Given that Andrea's taxi arrives late, find the conditional probability that she rang company B. [3]

(i) 0.072
(ii) 0.25

MAY/JUNE2004

When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

(ii) 0.247
(iii) 0.263

OCTOBER/NOVEMBER2003

In a certain country 54% of the population is male. It is known that 5% of the males are colour-blind and 2% of the females are colour-blind. A person is chosen at random and found to be colour-blind. By drawing a tree diagram, or otherwise, find the probability that this person is male. [6]

0.746

MAY/JUNE2003

The people living in 3 houses are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C$, $1P$, $2G$	$2C$, $2P$, $3G$	$1C$, $1G$

(i) All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent. [2]

(ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent. [3]

(iii) Given that the person chosen by the method in part **(ii)** is a grandparent, calculate the probability that there is also a parent living in the house. [4]

(i) 3/8
(ii) 0.405
(iii) 10/17

OCTOBER/NOVEMBER2002

Ivan throws three fair dice.

(i) List all the possible scores on the three dice which give a total score of 5, and hence show that the probability of Ivan obtaining a total score of 5 is $\frac{1}{36}$. [3]

(ii) Find the probability of Ivan obtaining a total score of 7. [3]

(i) 122, 212, 221, 113, 131, 311

(ii) 5/72

OCTOBER/NOVEMBER2002

Rachel and Anna play each other at badminton. Each game results in either a win for Rachel or a win for Anna. The probability of Rachel winning the first game is 0.6. If Rachel wins a particular game, the probability of her winning the next game is 0.7, but if she loses, the probability of her winning the next game is 0.4. By using a tree diagram, or otherwise,

(i) find the conditional probability that Rachel wins the first game, given that she loses the second, [5]

(ii) find the probability that Rachel wins 2 games and loses 1 game out of the first three games they play. [4]

(i) 0.429

(ii) 0.31

MAY/JUNE2002

Events A and B are such that $P(A) = 0.3$, $P(B) = 0.8$ and $P(A \text{ and } B) = 0.4$. State, giving a reason in each case, whether events A and B are

(i) independent, [2]

(ii) mutually exclusive. [2]

(i) not independent
(ii) not mutually exclusive

DISCRETE RANDOM VARIABLE

MAY/JUNE2012 9709/61

A spinner has 5 sides, numbered 1, 2, 3, 4 and 5. When the spinner is spun, the score is the number of the side on which it lands. The score is denoted by the random variable X, which has the probability distribution shown in the table.

x	1	2	3	4	5
$P(X = x)$	0.3	0.15	$3p$	$2p$	0.05

(i) Find the value of p. [1]

A second spinner has 3 sides, numbered 1, 2 and 3. The score when this spinner is spun is denoted by the random variable Y. It is given that $P(Y = 1) = 0.3$, $P(Y = 2) = 0.5$ and $P(Y = 3) = 0.2$.

(ii) Find the probability that, when both spinners are spun together,

(a) the sum of the scores is 4, [3]

(b) the product of the scores is less than 8. [3]

(i) 0.1
(ii) (a) 0.225
(b) 0.765

MAY/JUNE2012 9709/61

In a certain mountainous region in winter, the probability of more than 20 cm of snow falling on any particular day is 0.21.

(i) Find the probability that, in any 7-day period in winter, fewer than 5 days have more than 20 cm of snow falling. [3]

(ii) For 4 randomly chosen 7-day periods in winter, find the probability that exactly 3 of these periods will have at least 1 day with more than 20 cm of snow falling. [4]

(i) 0.994

(ii) 0.405

MAY/JUNE2012 9709/62

The random variable X has the probability distribution shown in the table.

x	2	4	6
$P(X = x)$	0.5	0.4	0.1

Two independent values of X are chosen at random. The random variable Y takes the value 0 if the two values of X are the same. Otherwise the value of Y is the larger value of X minus the smaller value of X.

(i) Draw up the probability distribution table for Y. [4]

(ii) Find the expected value of Y. [1]

(ii) 1.36

MAY/JUNE2012 9709/62

In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

(i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

(ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3]

(i) 0.993
(ii) 22

MAY/JUNE2012 9709/63

The six faces of a fair die are numbered 1, 1, 1, 2, 3, 3. The score for a throw of the die, denoted by the random variable W, is the number on the top face after the die has landed.

(i) Find the mean and standard deviation of W. [3]

(ii) The die is thrown twice and the random variable X is the sum of the two scores. Draw up a probability distribution table for X. [4]

(iii) The die is thrown n times. The random variable Y is the number of times that the score is 3. Given that $\mathrm{E}(Y) = 8$, find $\mathrm{Var}(Y)$. [3]

(i) 1.83, 0.898
(iii) 5.33

OCTOBER/NOVEMBER2011 9709/61

A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable X is the number of girls in the team.

(i) Draw up a probability distribution table for X. [4]

(ii) Given that $E(X) = \frac{5}{2}$, calculate $Var(X)$. [2]

(ii) 0.536

OCTOBER/NOVEMBER2011 9709/62

There are a large number of students in Luttley College. 60% of the students are boys. Students can choose exactly one of Games, Drama or Music on Friday afternoons. It is found that 75% of the boys choose Games, 10% of the boys choose Drama and the remainder of the boys choose Music. Of the girls, 30% choose Games, 55% choose Drama and the remainder choose Music.

(i) 6 boys are chosen at random. Find the probability that fewer than 3 of them choose Music. [3]

(ii) 5 Drama students are chosen at random. Find the probability that at least 1 of them is a boy. [6]

(i) 0.953
(ii) 0.701

OCTOBER/NOVEMBER2011 9709/63

A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

(i) 3.4, 0.64
(ii) 0.32
(iii) 0.384

MAY/JUNE2011 9709/61

Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits. [4]

0.655

MAY/JUNE2011 9709/61

The possible values of the random variable X are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of X being 0 is $\frac{1}{10}$. The probabilities for all the other values of X are equal. Calculate

(i) $P(X < 2)$, [2]

(ii) the variance of X, [3]

(iii) the value of a for which $P(-a \leqslant X \leqslant 2a) = \frac{17}{35}$. [1]

(i) 0.486
(ii) 5.33
(iii) 1

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(a) **(i)** Find the probability of getting at least one 3 when 9 fair dice are thrown. [2]

(ii) When n fair dice are thrown, the probability of getting at least one 3 is greater than 0.9. Find the smallest possible value of n. [4]

(b) A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the first person to draw a yellow ball. Julie draws the first ball. Find the probability that Ronnie wins the game. [4]

(a) (i)0.806
(ii) 13
(b) 0.393

MAY/JUNE2011 9709/62

A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

0.109

MAY/JUNE2011 9709/62

Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the nth choice of cards. [2]

(i) 0.08
(iii) 2.56
(iv) 0.08
(v) $(0.2)^{n-1} \times 0.4$

MAY/JUNE2011 9709/63

The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

(i) 10
(ii) 11
(iii) 0.115

OCTOBER/NOVEMBER2010 9709/61

Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X.

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $\mathrm{Var}(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

(i) 1/9
(iii) 4.02 (accept 4.025)
(iv) 0.0741
(v) 1/6

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The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

x	1	4	5	7	9
$P(X = x)$	$4p$	$5p^2$	$1.5p$	$2.5p$	$1.5p$

Find p. [3]

0.1

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(i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

(iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

(i) constant/given probability, independent trials, fixed/given no. of trials, only two outcomes.
(ii) 0.196
(ii) 0.480

OCTOBER/NOVEMBER2010 9709/63

In a probability distribution the random variable X takes the value x with probability kx, where x takes values 1, 2, 3, 4, 5 only.

(i) Draw up a probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $\mathrm{E}(X)$. [2]

(i) 0.0667
(ii) 3.67

MAY/JUNE2010 9709/61

The probability distribution of the discrete random variable X is shown in the table below.

x	-3	-1	0	4
$\mathrm{P}(X = x)$	a	b	0.15	0.4

Given that $\mathrm{E}(X) = 0.75$, find the values of a and b. [4]

0.2, 0.25

MAY/JUNE2010 9709/61

The numbers of rides taken by two students, Fei and Graeme, at a fairground are shown in the following table.

	Roller coaster	Water slide	Revolving drum
Fei	4	2	0
Graeme	1	3	6

(i) The mean cost of Fei's rides is \$2.50 and the standard deviation of the costs of Fei's rides is \$0. Explain how you can tell that the roller coaster and the water slide each cost \$2.50 per ride. [2]

(ii) The mean cost of Graeme's rides is \$3.76. Find the standard deviation of the costs of Graeme's rides. [5]

(i) sd = 0, so all rides must cost the same i.e. the mean.
(ii) 1.03

MAY/JUNE2010 9709/62

A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

(ii) Show that $\mathrm{E}(X) = \frac{8}{7}$ and calculate $\mathrm{Var}(X)$. [3]

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

(ii) 0.408
(iii) 0.156

MAY/JUNE2010 9709/63

Christa takes her dog for a walk every day. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is a probability of 0.75 that the dog will bark.

(i) Find the probability that they go to the park on more than 5 of the next 7 days. [2]

(ii) Find the probability that the dog barks on any particular day. [2]

(iii) Find the variance of the number of times they go to the park in 30 days. [1]

(i) 0.159
(ii) 0.51
(iii) 7.2

MAY/JUNE2010 9709/63

Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits.

(i) Show that $P(X = 2) = \frac{3}{7}$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $E(X)$ and $Var(X)$. [3]

(iv) Given that $X = 2$, find the probability that the digit chosen from set A was 2. [2]

(iii) 13/7, 2.78
(iv) 0.4

OCTOBER/NOVEMBER2009 9709/61

The mean number of defective batteries in packs of 20 is 1.6. Use a binomial distribution to calculate the probability that a randomly chosen pack of 20 will have more than 2 defective batteries. [5]

0.212

OCTOBER/NOVEMBER2009 9709/61

The probability distribution of the random variable X is shown in the following table.

x	-2	-1	0	1	2	3
$\mathrm{P}(X = x)$	0.08	p	0.12	0.16	q	0.22

The mean of X is 1.05.

(i) Write down two equations involving p and q and hence find the values of p and q. [4]

(ii) Find the variance of X. [2]

(i) 0.15, 0.27
(ii) 2.59

OCTOBER/NOVEMBER2009 9709/62

Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times. [4]

37.5

OCTOBER/NOVEMBER2009 9709/62

In a particular discrete probability distribution the random variable X takes the value $\frac{120}{r}$ with probability $\frac{r}{45}$, where r takes all integer values from 1 to 9 inclusive.

(i) Show that $P(X = 40) = \frac{1}{15}$. [2]

(ii) Construct the probability distribution table for X. [3]

(iii) Which is the modal value of X? [1]

(iv) Find the probability that X lies between 18 and 100. [2]

(iii) 13.3
(iv) 0.444

MAY/JUNE2009

Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that $P(X = 2) = \frac{5}{16}$. [2]

(ii) The table below shows the probability distribution of X.

x	2	3	4	5	6	7
$P(X = x)$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Calculate $E(X)$ and $Var(X)$. [4]

(ii) 3.75, 2.19

OCTOBER/NOVEMBER2008

A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

(i) Find the probability of obtaining at least 7 odd numbers in 8 throws of the die. [4]

The die is thrown twice. Let X be the sum of the two scores. The following table shows the possible values of X.

		Second throw					
		1	3	5	5	6	6
	1	2	4	6	6	7	7
	3	4	6	8	8	9	9
First	5	6	8	10	10	11	11
throw	5	6	8	10	10	11	11
	6	7	9	11	11	12	12
	6	7	9	11	11	12	12

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Calculate $E(X)$. [2]

(iv) Find the probability that X is greater than $E(X)$. [2]

(i) 0.195
(iii) 8.67
(iv) 0.556

MAY/JUNE2008

Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X. [4]

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

(iii) 0.938

OCTOBER/NOVEMBER2007

The random variable X takes the values -2, 0 and 4 only. It is given that $P(X = -2) = 2p$, $P(X = 0) = p$ and $P(X = 4) = 3p$.

(i) Find p. [2]

(ii) Find $E(X)$ and $Var(X)$. [4]

(i) 0.167
(ii) 1.33, 7.56

OCTOBER/NOVEMBER2007

Box A contains 5 red paper clips and 1 white paper clip. Box B contains 7 red paper clips and 2 white paper clips. One paper clip is taken at random from box A and transferred to box B. One paper clip is then taken at random from box B.

(i) Find the probability of taking both a white paper clip from box A and a red paper clip from box B. [2]

(ii) Find the probability that the paper clip taken from box B is red. [2]

(iii) Find the probability that the paper clip taken from box A was red, given that the paper clip taken from box B is red. [2]

(iv) The random variable X denotes the number of times that a red paper clip is taken. Draw up a table to show the probability distribution of X. [4]

(i) 0.117
(ii) 0.783
(iii) 0.851

MAY/JUNE2007

A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. **[3]**

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. **[2]**

(iii) The number of **green** peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. **[5]**

(i) 0.273
(ii) 12/55

OCTOBER/NOVEMBER2006

The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.26	q	$3q$	0.05	0.09

(i) Find the value of q. [2]

(ii) Find $E(X)$ and $Var(X)$. [3]

(i) 0.15
(b) 1.56, 1.41

OCTOBER/NOVEMBER2005

A box contains 300 discs of different colours. There are 100 pink discs, 100 blue discs and 100 orange discs. The discs of each colour are numbered from 0 to 99. Five discs are selected at random, one at a time, with replacement. Find

(i) the probability that no orange discs are selected, [1]

(ii) the probability that exactly 2 discs with numbers ending in a 6 are selected, [3]

(iii) the probability that exactly 2 orange discs with numbers ending in a 6 are selected, [2]

(iv) the mean and variance of the number of pink discs selected. [2]

(i) 0.132
(ii) 0.0729
(iii) 0.0100
(iv) 5/3, 10/9

OCTOBER/NOVEMBER2005

In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

(iii) Draw up a probability distribution table for Mario's profit. [3]

(iv) Calculate his expected profit. [2]

(i) 2
(iv) 1.05

MAY/JUNE2005

A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places. [5]

OCTOBER/NOVEMBER2004

A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

(i) By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable L denotes the largest of the three numbers drawn.

(ii) Find the probability that L is 4. [1]

(iii) Draw up a table to show the probability distribution of L. [3]

(iv) Calculate the expectation and variance of L. [3]

(i) 123, 124, 125, 134, 135, 145, 234, 235, 245, 345, 0.4
(ii) 0.3
(iv) 4.5, 0.45

MAY/JUNE2004

Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X. [3]

x	1	2	3	4	5	6
$P(X = x)$						

(ii) Find $E(X)$. [2]

(ii) 91/36

OCTOBER/NOVEMBER2003

Single cards, chosen at random, are given away with bars of chocolate. Each card shows a picture of one of 20 different football players. Richard needs just one picture to complete his collection. He buys 5 bars of chocolate and looks at all the pictures. Find the probability that

(i) Richard does not complete his collection, [2]

(ii) he has the required picture exactly once, [2]

(iii) he completes his collection with the third picture he looks at. [2]

(i) 0.774
(ii) 0.204
(iii) 0.0451

OCTOBER/NOVEMBER2003

A discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X=x)$	$3c$	$4c$	$5c$	$6c$

(i) Find the value of the constant c. [2]

(ii) Find $E(X)$ and $Var(X)$. [4]

(iii) Find $P(X > E(X))$. [2]

(i) 0.0556
(ii) 2.78, 1.17
(iii) 0.611

MAY/JUNE2003

A box contains 10 pens of which 3 are new. A random sample of two pens is taken.

(i) Show that the probability of getting exactly one new pen in the sample is $\frac{7}{15}$. [2]

(ii) Construct a probability distribution table for the number of new pens in the sample. [3]

(iii) Calculate the expected number of new pens in the sample. [1]

(iii) 3/5

OCTOBER/NOVEMBER2002

The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b. [1]

(ii) Given that $E(X) = 4$, find a and b. [3]

(i) a + b = 0.45
(ii) 0.15, 0.3

MAY/JUNE2002

A fair cubical die with faces numbered 1, 1, 1, 2, 3, 4 is thrown and the score noted. The area A of a square of side equal to the score is calculated, so, for example, when the score on the die is 3, the value of A is 9.

(i) Draw up a table to show the probability distribution of A. [3]

(ii) Find $E(A)$ and $Var(A)$. [4]

(ii) 5.33, 30.9

NORMAL DISTRIBUTION

MAY/JUNE2012 9709/61

It is given that $X \sim \mathrm{N}(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

0.729

MAY/JUNE2012 9709/61

The lengths of body feathers of a particular species of bird are modelled by a normal distribution. A researcher measures the lengths of a random sample of 600 body feathers from birds of this species and finds that 63 are less than 6 cm long and 155 are more than 12 cm long.

(i) Find estimates of the mean and standard deviation of the lengths of body feathers of birds of this species. [5]

(ii) In a random sample of 1000 body feathers from birds of this species, how many would the researcher expect to find with lengths more than 1 standard deviation from the mean? [4]

(i) 9.9, 3.15 or 3.16
(ii) 317

The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

(i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

(ii) Find the mean and standard deviation of the times to play this symphony. [5]

(iii) Assuming that the times to play the two symphonies are independent of each other, find the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

(i) 0.433 or 0.434
(ii) 30.0, 2.77
(iii) 0.0266

MAY/JUNE2012 9709/63

The lengths, in cm, of trout in a fish farm are normally distributed. 96% of the lengths are less than 34.1 cm and 70% of the lengths are more than 26.7 cm.

(i) Find the mean and the standard deviation of the lengths of the trout. [5]

In another fish farm, the lengths of salmon, X cm, are normally distributed with mean 32.9 cm and standard deviation 2.4 cm.

(ii) Find the probability that a randomly chosen salmon is 34 cm long, correct to the nearest centimetre. [3]

(iii) Find the value of t such that $P(31.8 < X < t) = 0.5$. [4]

(i) 28.4, 3.25
(ii) 0.149
(iii) 35.1

OCTOBER/NOVEMBER2011 9709/61

When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate. [5]

0.794

OCTOBER/NOVEMBER2011 9709/61

The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

(i) Find the standard deviation of the weights of the letters. [3]

(ii) Find the probability that a randomly chosen letter weighs more than 13 g. [3]

(iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean. [3]

(i) 6.38
(ii) 0.864
(iii) 0.0171

OCTOBER/NOVEMBER2011 9709/62

A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time. [3]

(iii) The spinner is spun 136 times. Use a suitable approximation to find the probability that it lands on the blue side fewer than 20 times. [5]

(ii) 0.141
(iii) 0.742

OCTOBER/NOVEMBER2011 9709/62

(i) In a certain country, the daily minimum temperature, in °C, in winter has the distribution N(8, 24). Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C. [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

(ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]

(iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]

(iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ. [3]

(i) 0.373
(ii) 0.309
(iii) 11.1
(iv) 1.54

OCTOBER/NOVEMBER2011 9709/63

The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that $P(X < 25) = 0.648$.

(i) Find the values of μ and σ. [4]

(ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25. [2]

(i) 22.2, 7.40
(ii) 0.0967

OCTOBER/NOVEMBER2011 9709/63

Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or −. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A−, 2% are B−, 1% are AB− and 7% are O−.

(i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+. [6]

(ii) A random sample of 150 people in the UK is taken. Find the probability that more than 60 people are group A+. [5]

(i) 0.156
(ii) 0.0854 (or 0.0855)

MAY/JUNE2011 9709/61

(a) The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

(b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

(a) 0.545, 0.693
(b) 3.09

MAY/JUNE2011 9709/62

In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

(i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

(ii) Give a reason why the use of a normal approximation is justified. [1]

(i) 0.590
(ii) np and nq both >5

MAY/JUNE2011 9709/62

The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

(i) Find the value of μ. [3]

(ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

(i) 21.0
(ii) 0.746

MAY/JUNE2011 9709/63

The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

(i) Find μ. [3]

(ii) Find $P(10 < X < 20)$. [3]

(iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

(i) 13.9
(ii) 0.829 or 0.830
(iii) 0.962

OCTOBER/NOVEMBER2010 9709/61

On average, 2 apples out of 15 are classified as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]

0.807

OCTOBER/NOVEMBER2010 9709/61

The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

(i) For a random sample of 350 students, find the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

(ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

(i) 335 or 336
(ii) 0.994

OCTOBER/NOVEMBER2010 9709/62

The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.

(i) For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]

(ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

(i) P(Zotoc) = 0.985, P(Ganmor) = 0.988
(ii) 4.97

OCTOBER/NOVEMBER2010 9709/63

The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

(i) Find the standard deviation of the times spent by people visiting this dentist. [3]

(ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]

(iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]

(iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes. [5]

(i) 2.23
(ii) 0.654
(iii) 0.112
(iv) 0.2495 (0.249 or 0.250)

MAY/JUNE2010 9709/61

The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2. It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ. [5]

6.70, 23.0

MAY/JUNE2010 9709/61

In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

(i) 0.311
(ii) $np < 5$, so not possible
(iii) 0.181

MAY/JUNE2010 9709/62

The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

(i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

(ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]

(i) 0.8538
(ii) 0.215

MAY/JUNE2010 9709/62

The random variable X is normally distributed with mean μ and standard deviation σ.

(i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]

(ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ. [3]

(i) 0.952
(ii) -1.57σ

MAY/JUNE2010 9709/63

The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

(i) Find the mean and standard deviation of the heights the children can jump. [5]

(ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]

(iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

(i) 6.29, 132
(ii) 0.978
(iii) 0.805

OCTOBER/NOVEMBER2009 9709/61

The times for a certain car journey have a normal distribution with mean 100 minutes and standard deviation 7 minutes. Journey times are classified as follows:

'short' (the shortest 33% of times),
'long' (the longest 33% of times),
'standard' (the remaining 34% of times).

(i) Find the probability that a randomly chosen car journey takes between 85 and 100 minutes. [3]

(ii) Find the least and greatest times for 'standard' journeys. [4]

(i) 0.484
(ii) 96.9, 103.1

OCTOBER/NOVEMBER2009 9709/61

A box contains 4 pears and 7 oranges. Three fruits are taken out at random and eaten. Find the probability that

(i) 2 pears and 1 orange are eaten, in any order, [3]

(ii) the third fruit eaten is an orange, [3]

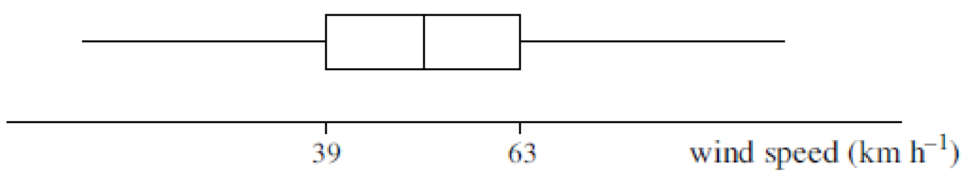
(iii) the first fruit eaten was a pear, given that the third fruit eaten is an orange. [3]

There are 121 similar boxes in a warehouse. One fruit is taken at random from each box.

(iv) Using a suitable approximation, find the probability that fewer than 39 are pears. [5]

(i) 0.255
(ii) 0.636
(iii) 0.4
(iv) 0.149

OCTOBER/NOVEMBER2009 9709/62



Measurements of wind speed on a certain island were taken over a period of one year. A box-and-whisker plot of the data obtained is displayed above, and the values of the quartiles are as shown. It is suggested that wind speed can be modelled approximately by a normal distribution with mean μ km h^{-1} and standard deviation σ km h^{-1}.

(i) Estimate the value of μ. [1]

(ii) Estimate the value of σ. [3]

(i) 51
(ii) 17.8

OCTOBER/NOVEMBER2009 9709/62

The weights, X grams, of bars of soap are normally distributed with mean 125 grams and standard deviation 4.2 grams.

(i) Find the probability that a randomly chosen bar of soap weighs more than 128 grams. [3]

(ii) Find the value of k such that $P(k < X < 128) = 0.7465$. [4]

(iii) Five bars of soap are chosen at random. Find the probability that more than two of the bars each weigh more than 128 grams. [4]

(i) 0.238
(ii) 116
(iii) 0.0910

MAY/JUNE2009

The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ. [3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

(i) 997
(ii) 0.140

MAY/JUNE2009

On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. [3]

(ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars. [5]

(i) 0.748
(ii) 0.887

OCTOBER/NOVEMBER2008

On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys. [5]

0.652

OCTOBER/NOVEMBER2008

(i) The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0 °C. [3]

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0 °C is 0.8888. Find the value of μ. [3]

(i) 0.0276
(ii) 7.72

MAY/JUNE2008

In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

(i) 8.75
(ii) 0.546

MAY/JUNE2008

A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

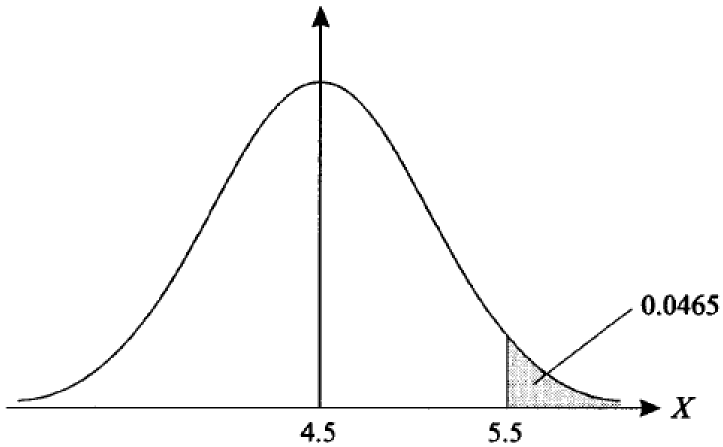
(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]

(iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times. [5]

(i) 0.00563
(ii) 0.526
(iii) 0.956

OCTOBER/NOVEMBER2007



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

(i) Find the standard deviation of X. [3]

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

(i) 0.595
(ii) 0.573

OCTOBER/NOVEMBER2007

On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]

(ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. [5]

(iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8. [2]

(i) 0.298 (allow 0.2985)
(ii) 0.118
(iii) 13

MAY/JUNE2007

(a) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation. [4]

(b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]

(a) 7.24 or 7.23
(b) 546 (accept 547) or 533 (or 534)

MAY/JUNE2007

The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

(i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]

(ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

(i) 0.365
(ii) 0.576

OCTOBER/NOVEMBER2006

(i) Give an example of a variable in real life which could be modelled by a normal distribution. [1]

(ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, find the value of μ. [3]

(iii) If 300 observations are taken at random from the distribution in part **(ii)**, estimate how many of these would be greater than 22.0. [4]

(i) heights, weights, times etc
(ii) 12.9
(iii) 7

OCTOBER/NOVEMBER2006

A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, [2]

(ii) more than 17 small bands. [3]

An office pack contains 150 elastic bands.

(iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. [6]

(i) 0.117
(ii) 0.0060
(iii) 0.556

MAY/JUNE2006

The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.

(i) Find the standard deviation. [3]

(ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected. [3]

(iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]

(i) 7.29
(ii) 0.136
(iii) 0.370

MAY/JUNE2006

A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

(ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. [5]

(i) 0.126
(ii) 0.281

OCTOBER/NOVEMBER2005

In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

(i) Find the mean lifetime, giving your answer to the nearest hour. [3]

(ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

(iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

(i) 5080
(ii) 0.0273
(iii) 0.730

MAY/JUNE2005

It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]

0.677

MAY/JUNE2005

Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

(i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]

(ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

(i) 0.00428

(ii) limits between 1.71 and 2.09

OCTOBER/NOVEMBER2004

The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.

(i) Find the value of μ. [3]

(ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days. [4]

(i) 48.6
(ii) 0.00438 or 0.00449

OCTOBER/NOVEMBER2004

(i) State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

(iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

(i) constant p, independent trials, fixed number of trials, only two outcomes
(ii) 0.419
(iii) 0.0782 or 0.0783

MAY/JUNE2004

Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classified as small.

(i) Find the proportion of melons which are classified as small. [3]

(ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classified as large. [5]

(i) 0.2025
(ii) 481

MAY/JUNE2004

A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]

(iii) A random sample of 1600 tapes is taken. Use a suitable approximation to find the probability that there are at least 290 damaged tapes. [5]

(i) 0.398
(ii) 9
(iii) 0.972

OCTOBER/NOVEMBER2003

In a normal distribution, 69% of the distribution is less than 28 and 90% is less than 35. Find the mean and standard deviation of the distribution. [6]

23.6, 8.91 (accept 8.89 to 8.92)

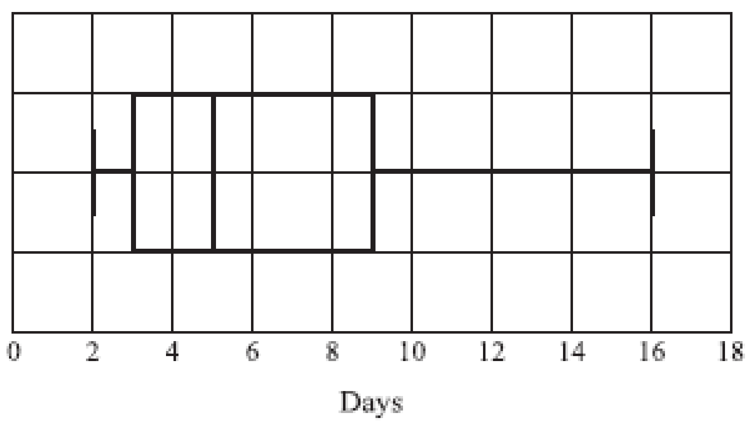
OCTOBER/NOVEMBER2003

The length of time a person undergoing a routine operation stays in hospital can be modelled by a normal distribution with mean 7.8 days and standard deviation 2.8 days.

(i) Calculate the proportion of people who spend between 7.8 days and 11.0 days in hospital. [4]

(ii) Calculate the probability that, of 3 people selected at random, exactly 2 spend longer than 11.0 days in hospital. [2]

(iii) A health worker plotted a box-and-whisker plot of the times that 100 patients, chosen randomly, stayed in hospital. The result is shown below.



State with a reason whether or not this agrees with the model used in parts (i) and (ii). [2]

(i) 0.3735
(ii) 0.0419
(iii) Not symmetrical so not normal. Does not agree with the hospital's figures.

MAY/JUNE2003

(i) The height of sunflowers follows a normal distribution with mean 112 cm and standard deviation 17.2 cm. Find the probability that the height of a randomly chosen sunflower is greater than 120 cm. [3]

(ii) When a new fertiliser is used, the height of sunflowers follows a normal distribution with mean 115 cm. Given that 80% of the heights are now greater than 103 cm, find the standard deviation. [3]

(i) 0.321

(ii) 14.3

MAY/JUNE2003

Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.

(i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]

(ii) Use a suitable approximation to calculate the probability that Kamal's hens lay fewer than 20 eggs on any particular day. [5]

(i) 0.0829

(ii) 0.275

OCTOBER/NOVEMBER2002

The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.

(i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]

(ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

(i) 0.334
(ii) 49.9 or 49.8

OCTOBER/NOVEMBER2002

(i) A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]

(ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]

(iii) A large box of randomly chosen biscuits contains 120 biscuits. Using a suitable approximation, find the probability that it contains fewer than 35 chocolate biscuits. [5]

(i) 0.0584
(ii) 0.307
(iii) 0.829

MAY/JUNE2002

(i) In a normal distribution with mean μ and standard deviation σ, $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$. Write down the value of μ, and calculate the value of σ. [4]

(ii) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2.8. [4]

(i) 3.6, 2
(ii) 0.879

MAY/JUNE2002

(i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

(ii) Another garden shop sells polyanthus plants in boxes of 100. The shop's advertisement states that the probability of any polyanthus plant producing a pink flower is 0.3. Use a suitable approximation to find the probability that a box contains fewer than 35 plants which produce pink flowers. [4]

(i) (a) 20

(b) 0.162

(ii) 0.837