UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/23 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Use conservation of momentum, e.g.:	$3mv_A + 6mv_B = 3mu$	M1		
	Use Newton's law of restitution (A1 if both correct):	$v_A - v_B = -eu$	M1 A1		
	Solve for v_A and v_B (A.E.F.; A0 if any dirn. error):	$v_A = ⅓(1-2e)u,\ v_B = ⅓(1+e)u$	M1 A1		
	Find loss of kinetic energy by substituting for v_A, v_B:	$½m(3u^2 - 3v_A{}^2 - 6v_B{}^2)$			
		$= mu^2\{3/2 - (1-4e+4e^2)/6$			
		$- (1+2e+e^2)/3\}$			
	(some working needed to earn A1;	$= mu^2\{3/2 - ½(1+2e^2)\}$			
	ignore any earlier dirn. error)	$= mu^2(1-e^2)$ **A.G.**	M1 A1		
				7	**[7]**
2	Find AP (or BP) in terms of x and a:	$AP = \sqrt{(4a^2 + x^2)}$	B1		
	Find T by Hooke's Law for one string [or both]:	$T = 2mg(AP - a) / a$	B1		
	Find eqn of motion at general point:	$m\mathrm{d}^2x/\mathrm{d}t^2 = -2Tx/AP$	M1		
	Combine to find $\mathrm{d}^2x/\mathrm{d}t^2$:	$\mathrm{d}^2x/\mathrm{d}t^2 = -(4gx/a)(1 - a/AP)$	M1		
	A.G.	$= -(4gx/a)\{1 - ½(1 + x^2/4a^2)^{-½}\}$	A1	5	
	M.R. Vertical motion loses only this A1				
	Motion along AB loses all marks				
	Neglect $(x/a)^2$:	$\mathrm{d}^2x/\mathrm{d}t^2 = -(4gx/a)\,½ = -2gx/a$	B1		
	State period of motion using $2\pi/\omega$ (A.E.F.):	$2\pi\sqrt{(a/2g)}$ *or* $\pi\sqrt{(2a/g)}$	B1	2	**[7]**
3	Apply $v^2 = \omega^2(A^2 - x^2)$ at both points:	$4^2 = \omega^2(A^2 - 6^2),\ 3^2 = \omega^2(A^2 - 8^2)$	M1		
	Combine to find A:	$7A^2 = 32^2 - 18^2,\quad A = 10$ **A.G.**	M1 A1		
	Find period T using ω:	$\omega = ½,\ T = 2\pi/\omega = 4\pi$ *or* 12·6 [s]	B1	4	
	EITHER: Use $x = A \sin \omega t$ to find time from O to A:	$t_{OA} = \omega^{-1}\sin^{-1}(6/10)$ [= 1·287]	M1		
	Use $x = A \sin \omega t$ to find time from O to B:	$t_{OB} = \omega^{-1}\sin^{-1}(8/10)$ [= 1·855]	M1		
	Combine to find time from A to B:	$t_{OA} + t_{OB} = 3{\cdot}14$ *or* π [s]	M1 A1		
	OR: Use $x = A \cos \omega t$ to find time to A:	$t_A = \omega^{-1}\cos^{-1}(6/10)$ [= 1·855]	(M1)		
	Use $x = A \cos \omega t$ to find time to B:	$t_B = \omega^{-1}\cos^{-1}(-8/10)$ [= 4·996]	(M1)		
	Combine to find time from A to B:	$t_B - t_A = 3{\cdot}14$ *or* π [s]	(M1 A1)	4	**[8]**
4	Use conservation of energy (B0 for $v^2 = \ldots$):	$½mv^2 = ½mu^2 + mga(1 - \cos\theta)$	B1		
	Equate radial forces:	$R = mg\cos\theta - mv^2/a$	B1		
	Eliminate v to find R:	$R = mg(3\cos\theta - 2) - mu^2/a$ **A.G.**	M1 A1	4	
(i)	Find $\cos\alpha$ when $R = 0$:	$\cos\alpha = ⅓(¼ + 2) = ¾$	M1 A1	2	
(ii)	Find v^2 at this point:	$v_1{}^2 = ¼\,ga + ½\,ga = ¾\,ga$	B1		
	Find vertical comp. of v_1 at plane:	$v_2{}^2 = v_1{}^2\sin^2\alpha + 2ga(1 + \cos\alpha)$	M1 A1		
	(using u in place of v_1 can earn M1 A0)	$= (21/64 + 7/2)\,ga$	A1		
	(A.E.F.)	$v_2 = \sqrt{(245ga/64)}$ *or* $(7/8)\sqrt{(5ga)}$			
		or $1{\cdot}96\sqrt{(ga)}$ *or* $6{\cdot}19\sqrt{a}$	A1	5	**[11]**

Question Number	Mark Scheme Details			Part Mark	Total
5	Find MI of rod about A:	$I_{rod} = (4/3)m(3a)^2 = 12ma^2$	B1		
	Find MI of circular lamina about A:	$I_{lamina} = 2ma^2 + 64ma^2 = 66ma^2$	M1 A1		
	Find MI of rod plus lamina about A:	$I = I_{rod} + I_{lamina} = 78ma^2$ **A.G.**	A1	4	
(i)	Find new MI including particle about A:	$I' = I + (2m)(6a)^2 = 150ma^2$	B1		
	Use eqn of circular motion to find $\mathrm{d}^2\theta/\mathrm{d}t^2$:	$I'\mathrm{d}^2\theta/\mathrm{d}t^2 = [–]\ mg(3 + 8 + 12)a \sin\theta$	M1 A1		
	(using I can earn B0 M1 A1 A0)	$\mathrm{d}^2\theta/\mathrm{d}t^2 = –(23g/150a)\sin\theta$	A1	4	
(ii)	Use energy when AB vertical to find ω:	$½I'\omega^2 = ½I'\omega_0^2 + (3 + 8 + 12)mga$	M1 A1		
	(or by integration)	$\omega^2 = 9g/25a + 46mga/150ma^2$			
		$\omega = \sqrt{(2g/3a)}$ *or* $0{\cdot}816\sqrt{(g/a)}$			
	(using I can earn M1 A1 A0)	*or* $2{\cdot}58/\sqrt{a}$ (A.E.F.)	A1	3	**[11]**
6	Estimate population variance:	$s^2 = 0{\cdot}912 / 9$			
	(allow biased here: $0{\cdot}0912$ *or* $0{\cdot}3020^2$)	$= 0{\cdot}1013$ *or* $38/375$ *or* $0{\cdot}3183^2$	M1		
	State hypotheses (B0 for $\bar{x}$ …):	H_0: $\mu = 8{\cdot}05$, H_1: $\mu > 8{\cdot}05$	B1		
	Calculate value of t (to 3 sf):	$t = (\bar{x} – 8{\cdot}05)/(s/\sqrt{10}) = 2{\cdot}03$	M1 A1		
	State or use correct tabular t value:	$t_{9,\ 0.95} = 1{\cdot}83[3]$	B1		
	(or can compare $\bar{x}$ with $8{\cdot}05 + 0{\cdot}184[5] = 8{\cdot}23$)				
	Valid method for reaching conclusion:	Reject H_0 if $t >$ tabular value	M1		
	Conclusion (A.E.F., needs correct values):	$2{\cdot}03 > 1{\cdot}83$ so mean is greater	A1	7	**[7]**
7 (i)	State or find λ:	$\lambda = 1/(\text{standard deviation}) = 1/8$	M1 A1	2	
(ii)	Find F(10) – F(5):	$e^{–5\lambda} – e^{–10\lambda} = e^{–0{\cdot}625} – e^{–1{\cdot}25} = 0{\cdot}249$	M1 A1	2	
(iii)	Formulate eqn for median m of T:	$1 – e^{–\lambda m} = ½$	M1		
	Find value of m:	$m = –8 \ln ½ = 5{\cdot}55$ (*or* $5{\cdot}54$)	M1 A1	3	**[7]**
8	Find expected values (to 1 d.p.):	24·48 30·72 16·8			
	(lose A1 if rounded to integers)	48·96 61·44 33·6			
		28·56 35·84 19·6	M1 A1		
	State (at least) null hypothesis (A.E.F.):	H_0: Grade independent of town	B1		
	Calculate value of χ^2 to 1 d.p.:	$\chi^2 = 0{\cdot}009 + 0{\cdot}350 + 0{\cdot}467$			
		$+ 1{\cdot}669 + 0{\cdot}034 + 1{\cdot}719$			
		$+ 2{\cdot}567 + 0{\cdot}094 + 5{\cdot}518$			
		$= 12{\cdot}4[3]$	M1 A1		
	Compare with consistent tabular value (to 1 d.p.):	$\chi_{4,\ 0.975}{}^2 = 11{\cdot}1[4]$	B1		
	Conclusion consistent with values (A.E.F):	Grade not independent of town	A1 √	7	
	State town with max. contribn. (√ on exp. values):	Town C	B1 √		
	State valid comment (√ on exp. values), e.g.:	Signal poorer than expected in C	B1 √	2	**[9]**
9	Integrate f(x) to find F(x) for $–a \le x \le a$ (A.E.F.):	$F(x) = x/2a + c = (x + a)/2a$	M1 A1		
	(Finding $F(x) = x/2a$ earns M1 A0)				
	State F(x) for other intervals:	$F(x) = 0\ (x < –a)$, $F(x) = 1\ (x > a)$	B1	3	
	Relate dist. fn. G(y) of Y to X for central interval:	$G(y) = P(Y < y) = P(e^X < y)$			
	(working may be omitted; ignore other intervals)	$= P(X < \ln y) = F(\ln y)$			
		$= (\ln y + a)/2a$; $(e^{–a} \le y \le e^a)$	M1 A1; A1	3	
	Find k from $P(Y \ge k) = ¼$ with $a = 4$:	$1 – G(k) = ¼$, $(\ln k)/2a + ½ = ¾$	M1 A1		
	(Using $G(k) = ¼$ can earn M1)	$\ln k = 2$, $k = e^2$ *or* $7{\cdot}39$	A1	3	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
10	Estimate population variance using P's sample:	$s_P^2 = (1224 - 240^2/50) / 49$			
	(allow use of biased: $\sigma_{P,50}^2 = 1{\cdot}44$ *or* $1{\cdot}2^2$)	$= 1{\cdot}469$ *or* $72/49$ *or* $1{\cdot}212^2$	M1 A1		
	Estimate population variance using Q's sample:	$s_Q^2 = (754 - 168^2/40) / 39$			
	(allow use of biased: $\sigma_{Q,40}^2 = 1{\cdot}21$ *or* $1{\cdot}1^2$)	$= 1{\cdot}241$ *or* $242/195$ *or* $1{\cdot}114^2$	A1		
	Estimate population variance for combined sample:	$s^2 = s_P^2/50 + s_Q^2/40$			
	(allow use of $\sigma_{P,50}^2$, $\sigma_{Q,40}^2$: $0{\cdot}05905$ *or* $0{\cdot}243^2$)	$= 0{\cdot}0604$ *or* $0{\cdot}246^2$	M1		
	Find confidence interval:	$(240/50 - 168/40) \pm zs$	M1		
	Use $z = 1{\cdot}96$ and evaluate correct to 3 d.p.:	$0{\cdot}6 \pm 0{\cdot}482$ *or* $[0{\cdot}118, 1{\cdot}082]$	A1		
	(allow use of $0{\cdot}05905$ instead of $0{\cdot}0604$:	$0{\cdot}6 \pm 0{\cdot}476$ *or* $[0{\cdot}124, 1{\cdot}076]$)		6	
	State hypotheses:	$H_0: \mu_P = \mu_Q$, $H_1: \mu_P > \mu_Q$	B1		
	Calculate value of z (to 2 d.p.):	$z = (240/50 - 168/40)/s = 2{\cdot}44$	M1 *A1		
		(allow $0{\cdot}6/0{\cdot}243 = 2{\cdot}47$)			
	State or use correct tabular z value:	$z_{0.99} = 2{\cdot}326$ [*or* $2{\cdot}33$]	*B1		
	(or can compare $0{\cdot}6$ with $0{\cdot}572$ or $0{\cdot}56[5]$)				
	Valid method for reaching conclusion:	Reject H_0 if $z >$ tabular value	M1		
	Correct conclusion (AEF, dep *A1, *B1):	μ_P is greater than μ_Q	A1	6	
	S.R. Allow (implicit) assumption of equal variances, but deduct A1 if not explicit:				
	Pooled estimate of common variance s^2:	$(50\sigma_{P,50}^2 + 40\sigma_{Q,40}^2)/88 = 1{\cdot}368$			
	Confidence interval:	$0{\cdot}6 \pm 1{\cdot}96s\sqrt{(1/50+1/40)}$			
		$= 0{\cdot}6 \pm 0{\cdot}486$ *or* $[0{\cdot}114, 1{\cdot}086]$			
	Value of z (to 2 dp):	$0{\cdot}6/s\sqrt{(1/50+1/40)} = 2{\cdot}42$			**[12]**
11 (a)	*EITHER:*				
	Take moments about A for rod to find R_C:	$R_C\, 3a = kW\, 4a \cos\theta + W\, 2a \cos\theta$	M1 A1		
		$R_C = (2k + 1)(2W/5)$	A1		
	Resolve horizontal forces on rod to find F_A:	$F_A = R_C \sin\theta = (2k + 1)(8W/25)$	M1 A1		
	Resolve vertical forces on rod to find R_A:	$R_A = W + kW - R_C \cos\theta$			
		$= (13k + 19)(W/25)$	M1 A1		
	OR: Take moments about C for rod:	$F_A\, 3a \sin\theta - R_A\, 3a \cos\theta$			
		$= kWa \cos\theta - Wa \cos\theta$	(M1 A1)		
		$4F_A - 3R_A = (k - 1)W$	(A1)		
	Resolve forces parallel to rod:	$F_A \cos\theta + R_A \sin\theta$			
		$= kW \sin\theta + W \sin\theta$	(M1)		
		$3F_A + 4R_A = 4(k + 1)W$	(A1)		
	Combine to find F_A, R_A:	$F_A = (2k + 1)(8W/25)$ *and*			
		$R_A = (13k + 19)(W/25)$	(M1 A1)		
	Use $F_A \le \mu R_A$ to find $\mu_{\min}$:	$\mu_{\min} = 8(2k + 1)/(13k + 19)$ **A.G.**	M1 A1	9	
	(No use of inequality loses A1)				
	Find bound on k (allow use of equality):	$160k + 80 \le$ (*or* $=$) $117k + 171$	M1		
		Bound $= 91/43$ *or* $2{\cdot}12$	A1		
	Correct use of inequality	$k \le 91/43$ *or* $2{\cdot}12$	A1	3	**[12]**

Question Number	Mark Scheme Details			Part Mark	Total
11 (b)	Find correlation coefficient r:	$r^2 = (-0{\cdot}5)(-1{\cdot}2) = 0{\cdot}6$ *or* $0{\cdot}775^2$	M1 A1		
		$r = -0{\cdot}775$	*A1	3	
	State both hypotheses (B0 for r …):	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1		
	State or use correct tabular one-tail r value:	$r_{5,\,2.5\%} = 0{\cdot}878$	*B1		
	Valid method for reaching conclusion:	Accept H_0 if $\lvert r\rvert <$ tabular value	M1		
	Correct conclusion (AEF, dep *A1, *B1):	Coeff. does not differ from zero	A1	4	
	EITHER: Find two indep. eqns for p, q, e.g.:	$14 + q = (-0{\cdot}5)(13 + p) + 5 \times 5$			
	(M1 for either)	*or* $(14 + q)/5 = (-0{\cdot}5)(13 + p)/5 + 5$	M1 A1		
		$13 + p = (-1{\cdot}2)(14 + q) + 5 \times 7{\cdot}6$			
		or $(13 + p)/5 = (-1{\cdot}2)(14 + q)/5 + 7{\cdot}6$	A1		
	Combine to find p, q:	$p = 7$, $q = 1$	A1, A1		
	OR: Combine regression lines to find $\bar{x}$, $\bar{y}$:	$\bar{x} = 4$; $\bar{y} = 3$	(M1 A1; A1)		
	Use sample data to find p, q:	$p = 7$, $q = 1$	(A1, A1)	5	**[12]**