UNIVERSITY *of* CAMBRIDGE
International Examinations

SYLLABUS

Cambridge International A Level
Further Mathematics

9231

For examination in June and November 2014

Contents

1. Introduction

1.1 Why choose Cambridge?

University of Cambridge International Examinations is the world's largest provider of international education programmes and qualifications for 5 to 19 year olds. We are part of the University of Cambridge, trusted for excellence in education. Our qualifications are recognised by the world's universities and employers.

Recognition

A Cambridge International AS or A Level is recognised around the world by schools, universities and employers. The qualifications are accepted as proof of academic ability for entry to universities worldwide, though some courses do require specific subjects.

Cambridge International A Levels typically take two years to complete and offer a flexible course of study that gives students the freedom to select subjects that are right for them. Cambridge International AS Levels often represent the first half of an A Level course but may also be taken as a freestanding qualification. They are accepted in all UK universities and carry half the weighting of an A Level. University course credit and advanced standing is often available for Cambridge International A/AS Levels in countries such as the USA and Canada.

Learn more at **www.cie.org.uk/recognition**

Excellence in education

We understand education. We work with over 9000 schools in over 160 countries who offer our programmes and qualifications. Understanding learners' needs around the world means listening carefully to our community of schools, and we are pleased that 98 % of Cambridge schools say they would recommend us to other schools.

Our mission is to provide excellence in education, and our vision is that Cambridge learners become confident, responsible, innovative and engaged.

Cambridge programmes and qualifications help Cambridge learners to become:

- **confident** in working with information and ideas – their own and those of others
- **responsible** for themselves, responsive to and respectful of others
- **innovative** and equipped for new and future challenges
- **engaged** intellectually and socially, ready to make a difference.

Support in the classroom

We provide a world-class support service for Cambridge teachers and exams officers. We offer a wide range of teacher materials to Cambridge schools, plus teacher training (online and face-to-face), expert advice and learner-support materials. Exams officers can trust in reliable, efficient administration of exams entry and excellent, personal support from our customer services. Learn more at **www.cie.org.uk/teachers**

Not-for-profit, part of the University of Cambridge

We are a part of Cambridge Assessment, a department of the University of Cambridge and a not-for-profit organisation.

We invest constantly in research and development to improve our programmes and qualifications.

1.2 Why choose Cambridge International AS and A Level?

Cambridge International AS and A Levels have a proven reputation for preparing students well for university, employment and life. They help develop the in-depth subject knowledge and understanding which are so important to universities and employers.

You can offer almost any combination of 55 subjects. Students can specialise or study a range of subjects, ensuring breadth. Giving students the power to choose helps motivate them throughout their studies.

Cambridge International AS and A Level gives you building blocks to build an individualised curriculum that develops your learners' knowledge, understanding and skills in:

- in-depth subject content
- independent thinking
- applying knowledge and understanding to new as well as familiar situations
- handling and evaluating different types of information sources
- thinking logically and presenting ordered and coherent arguments
- making judgements, recommendations and decisions
- presenting reasoned explanations, understanding implications and communicating them clearly and logically
- working and communicating in English.

The syllabuses are international in outlook, but retain a local relevance. They have been created specifically for an international student body with content to suit a wide variety of schools and avoid cultural bias.

1.3 Why choose Cambridge International A Level Further Mathematics?

Cambridge International A Level Further Mathematics is accepted by universities and employers as proof of mathematical knowledge and understanding. Successful candidates gain lifelong skills, including:

- a deeper understanding of mathematical principles;
- the further development of mathematical skills including the use of applications of mathematics in the context of everyday situations and in other subjects that they may be studying;
- the ability to analyse problems logically, recognising when and how a situation may be represented mathematically;
- the use of mathematics as a means of communication;
- a solid foundation for further study.

1.4 Cambridge AICE (Advanced International Certificate of Education) Diploma

Cambridge AICE (Advanced International Certificate of Education) Diploma is the group award of Cambridge International AS and A Level.

Cambridge AICE Diploma involves the selection of subjects from three curriculum groups – Mathematics and Science; Languages; Arts and Humanities.

A Cambridge International A Level counts as a double-credit qualification and a Cambridge International AS Level as a single-credit qualification within the Cambridge AICE Diploma award framework.

To be considered for an AICE Diploma, a candidate must earn the equivalent of six credits by passing a combination of examinations at either double credit or single credit, with at least one course coming from each of the three curriculum areas.

The AICE Diploma is comprised of examinations administered in May/June and October/November series each year.

Further Mathematics (9231) falls into Group 1, Mathematics and Sciences.

Learn more about the AICE Diploma at **http://www.cie.org.uk/qualifications/academic/uppersec/aice**

1.5 How can I find out more?

If you are already a Cambridge school

You can make entries for this qualification through your usual channels. If you have any questions, please contact us at **international@cie.org.uk**

If you are not yet a Cambridge school

Learn about the benefits of becoming a Cambridge school at **www.cie.org.uk/startcambridge**.
Email us at **international@cie.org.uk** to find out how your organisation can become a Cambridge school.

2. Assessment at a glance

All candidates take two papers.

Paper 1	**3 hours**
There are about 11 questions of different marks and lengths on Pure Mathematics. Candidates should answer **all** questions except for the final question (worth 12–14 marks) which will offer two alternatives, only one of which must be answered. 100 marks weighted at 50% of total	

Paper 2	**3 hours**
There are 4 or 5 questions of different marks and lengths on Mechanics (worth a total of 43 or 44 marks) followed by 4 or 5 questions of different marks and lengths on Statistics (worth a total of 43 or 44 marks) and one final question worth 12 or 14 marks. The final question consists of two alternatives, one on Mechanics and one on Statistics. Candidates should answer **all** questions except for the last question where only one of the alternatives must be answered. 100 marks weighted at 50% of total	

Electronic Calculators
Candidates should have a calculator with standard 'scientific' functions for use in the examination. Graphic calculators will be permitted but candidates obtaining results solely from graphic calculators without supporting working or reasoning will not receive credit. Computers, and calculators capable of algebraic manipulation, are not permitted. All the regulations in the *Cambridge Handbook* apply with the exception that, for examinations on this syllabus only, graphic calculators are permitted.

Mathematical Instruments
Apart from the usual mathematical instruments, candidates may use flexicurves in this examination.

Mathematical Notation
Attention is drawn to the list of mathematical notation at the end of this booklet.

Examiners' Reports (SR(I) booklets)
Reports on the June examinations are distributed to Caribbean Centres in November/December and reports on the November examinations are distributed to other International Centres in April/May.

Availability

This syllabus is examined in the May/June examination series and the October/November examination series.

This syllabus is available to private candidates.

Centres in the UK that receive government funding are advised to consult the Cambridge website **www.cie.org.uk** for the latest information before beginning to teach this syllabus.

Combining this with other syllabuses

Candidates can combine this syllabus in an examination series with any other Cambridge syllabus, except:

- syllabuses with the same title at the same level

3. Syllabus aims and objectives

3.1 Aims

The aims for Advanced Level Mathematics 9709 apply, with appropriate emphasis.

The aims are to enable candidates to:

- develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment;
- develop an understanding of mathematical principles and an appreciation of mathematics as a logical and coherent subject;
- acquire a range of mathematical skills, particularly those which will enable them to use applications of mathematics in the context of everyday situations and of other subjects they may be studying;
- develop the ability to analyse problems logically, recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem;
- use mathematics as a means of communication with emphasis on the use of clear expression;
- acquire the mathematical background necessary for further study in this or related subjects.

3.2 Assessment objectives

The assessment objectives for Advanced Level Mathematics 9709 apply, with appropriate emphasis.

The abilities assessed in the examinations cover a single area: **technique with application**. The examination will test the ability of candidates to:

- understand relevant mathematical concepts, terminology and notation;
- recall accurately and use successfully appropriate manipulative techniques;
- recognise the appropriate mathematical procedure for a given situation;
- apply combinations of mathematical skills and techniques in solving problems;
- present mathematical work, and communicate conclusions, in a clear and logical way.

4. Curriculum content

4.1 Paper 1

Knowledge of the syllabus for Pure Mathematics (units P1 and P3) in Mathematics 9709 is assumed, and candidates may need to apply such knowledge in answering questions.

Theme or topic	Curriculum objectives
1. Polynomials and rational functions	*Candidates should be able to:* • recall and use the relations between the roots and coefficients of polynomial equations, for equations of degree 2, 3, 4 only; • use a given simple substitution to obtain an equation whose roots are related in a simple way to those of the original equation; • sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes).
2. Polar coordinates	• understand the relations between cartesian and polar coordinates (using the convention $r \geqslant 0$), and convert equations of curves from cartesian to polar form and *vice versa*; • sketch simple polar curves, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as symmetry, the form of the curve at the pole and least/greatest values of r); • recall the formula $\int_{\alpha}^{\beta} r^2\,\mathrm{d}\theta$ for the area of a sector, and use this formula in simple cases.

3. Summation of series	• use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums; • use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions; • recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.
4. Mathematical induction	• use the method of mathematical induction to establish a given result (questions set may involve divisibility tests and inequalities, for example); • recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases e.g. find the nth derivative of xe^x.

5. Differentiation and integration	• obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between y and x is defined implicitly or parametrically;
	• derive and use reduction formulae for the evaluation of definite integrals in simple cases;
	• use integration to find:
	  • mean values and centroids of two- and three-dimensional figures (where equations are expressed in cartesian coordinates, including the use of a parameter), using strips, discs or shells as appropriate,
	  • arc lengths (for curves with equations in cartesian coordinates, including the use of a parameter, or in polar coordinates),
	  • surface areas of revolution about one of the axes (for curves with equations in cartesian coordinates, including the use of a parameter, but not for curves with equations in polar coordinates).

6. Differential equations	• recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary function and a particular integral; • find the complementary function for a second order linear differential equation with constant coefficients; • recall the form of, and find, a particular integral for a second order linear differential equation in the cases where a polynomial or e^{bx} or $a \cos px + b \sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral; • use a substitution to reduce a given differential equation to a second order linear equation with constant coefficients; • use initial conditions to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.

7. Complex numbers	• understand de Moivre's theorem, for a positive integral exponent, in terms of the geometrical effect of multiplication of complex numbers;
	• prove de Moivre's theorem for a positive integral exponent;
	• use de Moivre's theorem for positive integral exponent to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle;
	• use de Moivre's theorem, for a positive or negative rational exponent:
	    • in expressing powers of $\sin \theta$ and $\cos \theta$ in terms of multiple angles,
	    • in the summation of series,
	    • in finding and using the nth roots of unity.

8. Vectors	• use the equation of a plane in any of the forms $ax + by + cz = d$ or $\mathbf{r.n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, and convert equations of planes from one form to another as necessary in solving problems;
	• recall that the vector product $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\|\mathbf{a}\|\,\|\mathbf{b}\| \sin\theta\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector, or in component form as $(a_2b_3 - a_3b_2)\,\mathbf{i} + (a_3b_1 - a_1b_3)\,\mathbf{j} + (a_1b_2 - a_2b_1)\,\mathbf{k}$;
	• use equations of lines and planes, together with scalar and vector products where appropriate, to solve problems concerning distances, angles and intersections, including:
	  • determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists,
	  • finding the perpendicular distance from a point to a plane,
	  • finding the angle between a line and a plane, and the angle between two planes,
	  • finding an equation for the line of intersection of two planes,
	  • calculating the shortest distance between two skew lines,
	  • finding an equation for the common perpendicular to two skew lines.

9. Matrices and linear spaces	• recall and use the axioms of a linear (vector) space (restricted to spaces of finite dimension over the field of real numbers only); • understand the idea of linear independence, and determine whether a given set of vectors is dependent or independent; • understand the idea of the subspace spanned by a given set of vectors; • recall that a basis for a space is a linearly independent set of vectors that spans the space, and determine a basis in simple cases; • recall that the dimension of a space is the number of vectors in a basis; • understand the use of matrices to represent linear transformations from $\mathbb{R}^n \to \mathbb{R}^m$;

	• understand the terms 'column space', 'row space', 'range space' and 'null space', and determine the dimensions of, and bases for, these spaces in simple cases; • determine the rank of a square matrix, and use (without proof) the relation between the rank, the dimension of the null space and the order of the matrix; • use methods associated with matrices and linear spaces in the context of the solution of a set of linear equations; • evaluate the determinant of a square matrix and find the inverse of a non-singular matrix (2×2 and 3×3 matrices only), and recall that the columns (or rows) of a square matrix are independent if and only if the determinant is non-zero; • understand the terms 'eigenvalue' and 'eigenvector', as applied to square matrices; • find eigenvalues and eigenvectors of 2×2 and 3×3 matrices (restricted to cases where the eigenvalues are real and distinct); • express a matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use this expression, e.g. in calculating powers of matrices.

4.2 Paper 2

Knowledge of the syllabuses for Mechanics (units M1 and M2) and Probability and Statistics (units S1 and S2) in Mathematics 9709 is assumed. Candidates may need to apply such knowledge in answering questions; harder questions on those units may also be set.

Theme or topic	Curriculum objectives
	Candidates should be able to:
MECHANICS (Sections 1 to 5)	
1. Momentum and impulse	• recall and use the definition of linear momentum, and show understanding of its vector nature (in one dimension only); • recall Newton's experimental law and the definition of the coefficient of restitution, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms 'perfectly elastic' ($e = 1$) and 'inelastic' ($e = 0$); • use conservation of linear momentum and/or Newton's experimental law to solve problems that may be modelled as the direct impact of two smooth spheres or the direct or oblique impact of a smooth sphere with a fixed surface; • recall and use the definition of the impulse of a constant force, and relate the impulse acting on a particle to the change of momentum of the particle (in one dimension only).
2. Circular motion	• recall and use the radial and transverse components of acceleration for a particle moving in a circle with variable speed; • solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy (including finding the tension in a string or a normal contact force, locating points at which these are zero, and conditions for complete circular motion).

3. Equilibrium of a rigid body under coplanar forces	• understand and use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the centre of mass by considerations of symmetry in suitable cases; • calculate the moment of a force about a point in 2 dimensional situations only (understanding of the vector nature of moments is not required); • recall that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this; • use Newton's third law in situations involving the contact of rigid bodies in equilibrium; • solve problems involving the equilibrium of rigid bodies under the action of coplanar forces (problems set will not involve complicated trigonometry).
4. Rotation of a rigid body	• understand and use the definition of the moment of inertia of a system of particles about a fixed axis as $\sum mr^2$ and the additive property of moment of inertia for a rigid body composed of several parts (the use of integration to find moments of inertia will not be required); • use the parallel and perpendicular axes theorems (proofs of these theorems will not be required); • recall and use the equation of angular motion $C = I\ddot{\theta}$ for the motion of a rigid body about a fixed axis (simple cases only, where the moment C arises from constant forces such as weights or the tension in a string wrapped around the circumference of a flywheel; knowledge of couples is not included and problems will not involve consideration or calculation of forces acting at the axis of rotation); • recall and use the formula $\frac{1}{2}I\omega^2$ for the kinetic energy of a rigid body rotating about a fixed axis; • use conservation of energy in solving problems concerning mechanical systems where rotation of a rigid body about a fixed axis is involved.

5. Simple harmonic motion	• recall a definition of SHM and understand the concepts of period and amplitude; • use standard SHM formulae in the course of solving problems; • set up the differential equation of motion in problems leading to SHM, recall and use appropriate forms of solution, and identify the period and amplitude of the motion; • recognise situations where an exact equation of motion may be approximated by an SHM equation, carry out necessary approximations (e.g. small angle approximations or binomial approximations) and appreciate the conditions necessary for such approximations to be useful.
STATISTICS (Sections 6 to 9)	
6. Further work on distributions	• use the definition of the distribution function as a probability to deduce the form of a distribution function in simple cases, e.g. to find the distribution function for Y, where $Y = X^3$ and X has a given distribution; • understand conditions under which a geometric distribution or negative exponential distribution may be a suitable probability model; • recall and use the formula for the calculation of geometric or negative exponential probabilities; • recall and use the means and variances of a geometric distribution and negative exponential distribution.

7. Inference using normal and *t*-distributions	• formulate hypotheses and apply a hypothesis test concerning the population mean using a small sample drawn from a normal population of unknown variance, using a t-test;
	• calculate a pooled estimate of a population variance from two samples (calculations based on either raw or summarised data may be required);
	• formulate hypotheses concerning the difference of population means, and apply, as appropriate:
	    • a 2-sample t-test,
	    • a paired sample t-test,
	    • a test using a normal distribution
	    • (the ability to select the test appropriate to the circumstances of a problem is expected);
	• determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a t-distribution;
	• determine a confidence interval for a difference of population means, using a t-distribution, or a normal distribution, as appropriate.

8. χ^2-tests	• fit a theoretical distribution, as prescribed by a given hypothesis, to given data (questions will not involve lengthy calculations); • use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding goodness of fit analysis (classes should be combined so that each expected frequency is at least 5); • use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a contingency table (Yates' correction is not required, but classes should be combined so that the expected frequency in each cell is at least 5).
9. Bivariate data	• understand the concept of least squares, regression lines and correlation in the context of a scatter diagram; • calculate, both from simple raw data and from summarised data, the equations of regression lines and the product moment correlation coefficient, and appreciate the distinction between the regression line of y on x and that of x on y; • recall and use the facts that both regression lines pass through the mean centre $(\bar{x}, \bar{y})$ and that the product moment correlation coefficient r and the regression coefficients b_1, b_2 are related by $r^2 = b_1 b_2$; • select and use, in the context of a problem, the appropriate regression line to estimate a value, and understand the uncertainties associated with such estimations; • relate, in simple terms, the value of the product moment correlation coefficient to the appearance of the scatter diagram, with particular reference to the interpretation of cases where the value of the product moment correlation coefficient is close to +1, –1 or 0; • carry out a hypothesis test based on the product moment correlation coefficient.

5. Mathematical notation

The list which follows summarises the notation used in the Cambridge's Mathematics examinations. Although primarily directed towards Advanced/HSC (Principal) level, the list also applies, where relevant, to examinations at Cambridge O Level/S.C.

1 Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$\{x :...\}$	the set of all x such that …
$n(A)$	the number of elements in set A
$\emptyset$	the empty set
$\mathscr{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, ..., n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y\,R\,x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq$	is less than or equal to, is not greater than
$>$	is greater than
$\geq$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b,\ ab,\ a.b$	a multiplied by b
$a \div b,\ \frac{a}{b},\ a / b$	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbb{Z}^+$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a

Δx, δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y\,\mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\dfrac{\partial V}{\partial x}$	the partial derivative of V with respect to x
$\dot{x}$, $\ddot{x}$, ...	the first, second, ... derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$, $\log_{\mathrm{e}} x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6 Circular and hyperbolic functions

$\left.\begin{array}{l}\sin,\ \cos,\ \tan,\\ \operatorname{cosec},\ \sec,\ \cot\end{array}\right\}$	the circular functions
$\left.\begin{array}{l}\sin^{-1},\ \cos^{-1},\ \tan^{-1},\\ \operatorname{cosec}^{-1},\ \sec^{-1},\ \cot^{-1}\end{array}\right\}$	the inverse circular functions
$\left.\begin{array}{l}\sinh,\ \cosh,\ \tanh,\\ \operatorname{cosech},\ \operatorname{sech},\ \coth\end{array}\right\}$	the hyperbolic functions
$\left.\begin{array}{l}\sinh^{-1},\ \cosh^{-1},\ \tanh^{-1},\\ \operatorname{cosech}^{-1},\ \operatorname{sech}^{-1},\ \coth^{-1}\end{array}\right\}$	the inverse hyperbolic functions

7 Complex numbers

i	square root of -1
z	a complex number, $z = x + \mathrm{i}\,y = r(\cos\theta + \mathrm{i}\sin\theta)$
$\operatorname{Re} z$	the real part of z, $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of z, $\operatorname{Im} z = y$
$\lvert z\rvert$	the modulus of z, $\lvert z\rvert = \sqrt{x^2 + y^2}$
$\arg z$	the argument of z, $\arg z = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z, $x - \mathrm{i}\,y$

8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$

9 Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and statistics

A, B, C, etc.	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A \vert B)$	probability of the event A conditional on the event B
X, Y, R, etc.	random variables
x, y, r, etc.	values of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x), \ldots$	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x), \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \le x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}(\mathrm{g}(X))$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \ldots$
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$, m	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\mathrm{Cov}(X, Y)$	covariance of X and Y

6. Resource list

These titles represent some of the texts available in the UK at the time of printing this booklet. Teachers are encouraged to choose texts for class use which they feel will be of interest to their students and will support their own teaching style. ISBN numbers are provided wherever possible.

A LEVEL MATHEMATICS (9709) AND A LEVEL FURTHER MATHEMATICS (9231)

Endorsed Textbooks for Cambridge International A Level Mathematics (9709)

The following textbooks are endorsed by Cambridge for use with the 9709 syllabus. Please contact Cambridge University Press for further information.

Author	Title	Publisher	ISBN
Neill & Quadling	*Pure Mathematics 1*	Cambridge University Press	0 521 53011 3
Neill & Quadling	*Pure Mathematics 2 & 3*	Cambridge University Press	0 521 53012 1
Quadling	*Mechanics 1*	Cambridge University Press	0 521 53015 6
Quadling	*Mechanics 2*	Cambridge University Press	0 521 53016 4
Dobbs & Miller	*Statistics 1*	Cambridge University Press	0 521 53013 X
Dobbs & Miller	*Statistics 2*	Cambridge University Press	0 521 53014 8

Suggested Books

Pure Mathematics

Author	Title	Publisher	ISBN
Backhouse, Houldsworth & Horrill	*Pure Mathematics 1*	Longman, 1985	0 582 35386 6
Backhouse, Houldsworth & Horrill	*Pure Mathematics 2*	Longman, 1985	0 582 35387 4
Backhouse, Houldsworth, Horrill & Wood	*Essential Pure Mathematics*	Longman, 1991	0582 066581
Bostock & Chandler	*Core Maths for Advanced Level*	Nelson Thornes, 2000	0 7487 5509 8
Butcher & Megeny	*Access to Advanced Level Maths* (short introductory course)	Nelson Thornes, 1997	0 7487 2999 2
Emanuel, Wood & Crawshaw	*Pure Mathematics 1*	Longman, 2001	0 582 40550 5
Emanuel, Wood & Crawshaw	*Pure Mathematics 2*	Longman, 2001	0 582 40549 1

Hunt	*Graded Exercises in Pure Mathematics* (Practice questions)	Cambridge University Press, 2001	0 521 63753 8
Martin, Brown, Rigby & Riley	*Complete Advanced Level Mathematics : Pure Mathematics: Core Text*	Nelson Thornes, 2000	0 7487 3558 5
Morley	*Practice for Advanced Mathematics – Pure Mathematics* (Practice questions)	Hodder & Stoughton Educational, 1999	0 340 701676
Sadler & Thorning	*Understanding Pure Mathematics*	Oxford University Press, 1987	019 914243 2
Smedley & Wiseman	*Introducing Pure Mathematics*	Oxford University Press, 2001	0 19 914803 1
SMP	*Mathematics for AS and A Level – Pure Mathematics*	Cambridge University Press, 1997	0 521 56617 7
Solomon	*Advanced Level Mathematics : Pure Mathematics*	John Murray, 1995	0 7195 5344 X

Further Pure Mathematics

Author	Title	Publisher	ISBN
Gaulter & Gaulter	*Further Pure Mathematics*	Oxford University Press, 2001	0 19 914735 3

Integrated Courses

Author	Title	Publisher	ISBN
Berry, Fentern, Francis & Graham	*Discovering Advanced Mathematics – AS Mathematics*	Collins Educational, 2000	0 00 322502 X
Berry, Fentern, Francis & Graham	*Discovering Advanced Mathematics – A2 Mathematics*	Collins Educational, 2001	0 00 322503 8

Mechanics

Author	Title	Publisher	ISBN
Adams, Haighton, Trim	*Complete Advanced Level Mathematics : Mechanics : Core Text*	Nelson Thornes, 2000	0 7487 3559 3
Bostock & Chandler	*Mechanics for Cambridge International A Level*	Nelson Thornes, 1996	07487 2596 2
Jefferson & Beadsworth	*Introducing Mechanics*	Oxford University Press, 2000	0 19 914710 8
Kitchen & Wake	*Graded Exercises in Mechanics* (Practice questions)	Cambridge University Press, 2001	0 521 64686 3
Nunn & Simmons	*Practice for Advanced Mathematics* (Practice questions)	Hodder & Stoughton Educational, 1998	0 340 70166 8
Sadler & Thorning	*Understanding Mechanics*	Oxford University Press, 1996	019 914675 6
SMP	*Mathematics for Cambridge International AS and A Level – Mechanics*	Cambridge University Press, 1997	0 521 56615 0
Solomon	*Advanced Level Mathematics : Mechanics*	John Murray, 1995	07195 7082 4
Young	*Maths in Perspective 2: Mechanics*	Hodder & Stoughton Educational, 1989	07131 78221

Further Mechanics

Author	Title	Publisher	ISBN
Jefferson & Beadworth	*Further Mechanics*	Oxford University Press, 2001	0 19 914738 8
	Advanced Modular Mathematics – Mechanics 3 & 4	Collins Educational, 1995	0 00 322401 5

Statistics

Author	Title	Publisher	ISBN
Clarke & Cooke	*A Basic Course in Statistics*	Hodder & Stoughton Educational, 1998	0 340 71995 8
Crawshaw & Chambers	*A Concise Course in Advanced Level Statistics*	Nelson Thornes, 2001	0 7487 5475X
Crawshaw & Chambers	*A-Level Statistics Study Guide*	Nelson Thornes, 1997	0 7487 2997 6
McGill, McLennan, Migliorini	*Complete Advanced Level Mathematics : Statistics : Core Text*	Nelson Thornes, 2000	07487 3560 7
Norris	*Graded Exercises in Statistics* (Practice questions)	Cambridge University Press, 2000	0 521 65399 1
Rees	*Foundations of Statistics*	Chapman & Hall, 1987	0 412 28560 6
Smith	*Practice for Advanced Mathematics: Statistics* (Practice questions)	Hodder & Stoughton Educational, 1998	0 340 70165X
SMP	*Mathematics for AS and A Level – Statistics*	Cambridge University Press, 1997	0 521 56616 9
Solomon	*Advanced Level Mathematics: Statistics*	John Murray, 1996	0 7195 7088 3
Upton & Cook	*Introducing Statistics*	Oxford University Press, 2001	0 19 914801 5
Upton & Cook	*Understanding Statistics*	Oxford University Press, 1997	0 19 914391 9

Further Statistics

Author	Title	Publisher	ISBN
	Advanced Modular Mathematics – Statistics 3 & 4	Collins Educational, 1997	0 00 322416 3

Resources are also listed on Cambridge's public website at **www.cie.org.uk**. Please visit this site on a regular basis as the Resource lists are updated through the year.

Access to teachers' email discussion groups and regularly updated resource lists may be found on the Cambridge Teacher Support website at **http://teachers.cie.org.uk**. This website is available to teachers at registered Cambridge Centres.

7. Additional information

7.1 Guided learning hours

Cambridge International A Level syllabuses are designed on the assumption that candidates have about 360 guided learning hours per subject over the duration of the course. ('Guided learning hours' include direct teaching and any other supervised or directed study time. They do not include private study by the candidate.)

However, these figures are for guidance only, and the number of hours required may vary according to local curricular practice and the candidates' prior experience of the subject.

7.2 Recommended prior learning

Knowledge of the syllabus for Pure Mathematics (units P1 and P3) in Mathematics 9709 is assumed for Paper 1, and candidates may need to apply such knowledge in answering questions.

Knowledge of the syllabus for Mechanics units (M1 and M2) and Probability and Statistics units (S1 and S2) in Mathematics 9709 is assumed for Paper 2. Candidates may need to apply such knowledge in answering questions; harder questions on those units may also be set.

7.3 Progression

Cambridge International A Level Further Mathematics provides a suitable foundation for the study of Mathematics or related courses in higher education.

7.4 Component codes

Because of local variations, in some cases component codes will be different in instructions about making entries for examinations and timetables from those printed in this syllabus, but the component names will be unchanged to make identification straightforward.

7.5 Grading and reporting

Cambridge International A Level results are shown by one of the grades A*, A, B, C, D or E indicating the standard achieved, Grade A* being the highest and Grade E the lowest. 'Ungraded' indicates that the candidate has failed to reach the standard required for a pass at either Cambridge International AS Level or A Level. 'Ungraded' will be reported on the statement of results but not on the certificate.

If a candidate takes a Cambridge International A Level and fails to achieve grade E or higher, a Cambridge International AS Level grade will be awarded if both of the following apply:

- the components taken for the Cambridge International A Level by the candidate in that series included all the components making up a Cambridge International AS Level
- the candidate's performance on these components was sufficient to merit the award of a Cambridge International AS Level grade.

For languages other than English, Cambridge also reports separate speaking endorsement grades (Distinction, Merit and Pass), for candidates who satisfy the conditions stated in the syllabus.

Percentage uniform marks are also provided on each candidate's statement of results to supplement their grade for a syllabus. They are determined in this way:

- A candidate who obtains…
  … the minimum mark necessary for a Grade A* obtains a percentage uniform mark of 90%.
  … the minimum mark necessary for a Grade A obtains a percentage uniform mark of 80%.
  … the minimum mark necessary for a Grade B obtains a percentage uniform mark of 70%.
  … the minimum mark necessary for a Grade C obtains a percentage uniform mark of 60%.
  … the minimum mark necessary for a Grade D obtains a percentage uniform mark of 50%.
  … the minimum mark necessary for a Grade E obtains a percentage uniform mark of 40%.
  … no marks receives a percentage uniform mark of 0%.

Candidates whose mark is none of the above receive a percentage mark in between those stated according to the position of their mark in relation to the grade 'thresholds' (i.e. the minimum mark for obtaining a grade). For example, a candidate whose mark is halfway between the minimum for a Grade C and the minimum for a Grade D (and whose grade is therefore D) receives a percentage uniform mark of 55%.

The percentage uniform mark is stated at syllabus level only. It is not the same as the 'raw' mark obtained by the candidate, since it depends on the position of the grade thresholds (which may vary from one series to another and from one subject to another) and it has been turned into a percentage.

7.6 Access

Reasonable adjustments are made for disabled candidates in order to enable them to access the assessments and to demonstrate what they know and what they can do. For this reason, very few candidates will have a complete barrier to the assessment. Information on reasonable adjustments is found in the *Cambridge Handbook* which can be downloaded from the website **www.cie.org.uk**

Candidates who are unable to access part of the assessment, even after exploring all possibilities through reasonable adjustments, may still be able to receive an award based on the parts of the assessment they have taken.

7.7 Resources

Copies of syllabuses, the most recent question papers and Principal Examiners' reports for teachers are on the Syllabus and Support Materials CD-ROM, which we send to all Cambridge International Schools. They are also on our public website – go to **www.cie.org.uk/alevel**. Click the Subjects tab and choose your subject. For resources, click 'Resource List'.

You can use the 'Filter by' list to show all resources or only resources categorised as 'Endorsed by Cambridge'. Endorsed resources are written to align closely with the syllabus they support. They have been through a detailed quality-assurance process. As new resources are published, we review them against the syllabus and publish their details on the relevant resource list section of the website.

Additional syllabus-specific support is available from our secure Teacher Support website **http://teachers.cie.org.uk** which is available to teachers at registered Cambridge schools. It provides past question papers and examiner reports on previous examinations, as well as any extra resources such as schemes of work or examples of candidate responses. You can also find a range of subject communities on the Teacher Support website, where Cambridge teachers can share their own materials and join discussion groups.

University of Cambridge International Examinations
1 Hills Road, Cambridge, CB1 2EU, United Kingdom
Tel: +44 (0)1223 553554 Fax: +44 (0)1223 553558
Email: international@cie.org.uk   www.cie.org.uk