QUADRATICS

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(i) A straight line passes through the point (2, 0) and has gradient m. Write down the equation of the line. [1]

(ii) Find the two values of m for which the line is a tangent to the curve $y = x^2 - 4x + 5$. For each value of m, find the coordinates of the point where the line touches the curve. [6]

(iii) Express $x^2 - 4x + 5$ in the form $(x + a)^2 + b$ and hence, or otherwise, write down the coordinates of the minimum point on the curve. [2]

(i) $y = m(x - 2)$
(ii) ± 2, (3, 2), (1, 2)
(iii) $(x - 2)^2 + 1$, (2, 1)

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The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

(i) -2, -15
(ii) 16

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Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

$m < -10, m > 2$

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A curve has equation $y = kx^2 + 1$ and a line has equation $y = kx$, where k is a non-zero constant.

(i) Find the set of values of k for which the curve and the line have no common points. [3]

(ii) State the value of k for which the line is a tangent to the curve and, for this case, find the coordinates of the point where the line touches the curve. [4]

(i) $0 < k < 4$
(ii) 4, (0.5, 2)

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Determine the set of values of k for which the line $2y = x + k$ does not intersect the curve $y = x^2 - 4x + 7$. [4]

k < 3.875

MAY/JUNE2009

Find the set of values of k for which the line $y = kx - 4$ intersects the curve $y = x^2 - 2x$ at two distinct points. [4]

k > 2, k < -6

OCTOBER/NOVEMBER2007

Determine the set of values of the constant k for which the line $y = 4x + k$ does not intersect the curve $y = x^2$. [3]

k < -4

MAY/JUNE2007

Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$. [4]

0.5

MAY/JUNE2007

Find the real roots of the equation $\dfrac{18}{x^4} + \dfrac{1}{x^2} = 4.$ **[4]**

-1.5, 1.5

MAY/JUNE2005

The equation of a curve is $y = x^2 - 3x + 4$.

(i) Show that the whole of the curve lies above the x-axis. [3]

(ii) Find the set of values of x for which $x^2 - 3x + 4$ is a decreasing function of x. [1]

The equation of a line is $y + 2x = k$, where k is a constant.

(iii) In the case where $k = 6$, find the coordinates of the points of intersection of the line and the curve. [3]

(iv) Find the value of k for which the line is a tangent to the curve. [3]

(ii) x < 1.5
(iii) (-1, 8), (2, 2)
(iv) 3.75

FUNCTIONS

MAY/JUNE2012 9709/11

The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

(ii) State the range of f in terms of k. [1]

(iii) State the smallest value of p for which f is one-one. [1]

(iv) For the value of p found in part **(iii)**, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k. [4]

(i) $(x-2)^2 - 4 + k$

(ii) $f(x) > k - 4$

(iii) 2

(iv) $2 + \sqrt{x + 4 - k},\ x > k - 4$

MAY/JUNE2012 9709/12

Functions f and g are defined by

$$f : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$

$$g : x \mapsto \frac{8}{x-3} \quad \text{for } x \in \mathbb{R},\ x \neq 3.$$

(i) Obtain expressions, in terms of x, for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $fg(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k. [5]

(i) $\frac{1}{2}(x-5),\ \frac{8}{x}+3,\ x \neq 0$

(ii) $0 < k < \frac{64}{9}$

MAY/JUNE2012 9709/12

The function f is such that $f(x) = 8 - (x-2)^2$, for $x \in \mathbb{R}$.

(i) Find the coordinates and the nature of the stationary point on the curve $y = f(x)$. [3]

The function g is such that $g(x) = 8 - (x-2)^2$, for $k \leqslant x \leqslant 4$, where k is a constant.

(ii) State the smallest value of k for which g has an inverse. [1]

For this value of k,

(iii) find an expression for $g^{-1}(x)$, [3]

(iv) sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$. [3]

(i) (2, 8), max
(ii) 2
(iii) $2 + \sqrt{8-x}$

OCTOBER/NOVEMBER2011 9709/11

Functions f and g are defined by

$$\begin{aligned} &\mathrm{f} : x \mapsto 2x^2 - 8x + 10 \quad &&\text{for } 0 \leqslant x \leqslant 2, \\ &\mathrm{g} : x \mapsto x &&\text{for } 0 \leqslant x \leqslant 10. \end{aligned}$$

(i) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{g}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(i) $2(x-2)^2 + 2$

(ii) $2 \leq f(x) \leq 10$

(iii) $2 \leq x \leq 10$

(v) $2 - \sqrt{\frac{1}{2}(x-2)}$

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The functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 3x + a,$$
$$\mathrm{g} : x \mapsto b - 2x,$$

where a and b are constants. Given that $\mathrm{ff}(2) = 10$ and $\mathrm{g}^{-1}(2) = 3$, find

(i) the values of a and b, [4]

(ii) an expression for $\mathrm{fg}(x)$. [2]

(i) -2, 8

(ii) $22 - 6x$

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Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 3 \quad \text{for } x \leqslant 0,$$
$$\mathrm{g} : x \mapsto x^2 - 6x \quad \text{for } x \leqslant 3.$$

(i) Express $\mathrm{f}^{-1}(x)$ in terms of x and solve the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(ii) On the same diagram sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]

(iii) Find the set of values of x which satisfy $\mathrm{gf}(x) \leqslant 16$. [5]

(i) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$, x = -3

(iii) $-\frac{5}{2} \leq x \leq 0$

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Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 1,$$
$$\mathrm{g} : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [2]

(ii) Hence find the value of a for which $\mathrm{fg}(a) = \mathrm{gf}(a)$. [3]

(iii) Find the value of b ($b \neq a$) for which $\mathrm{g}(b) = b$. [2]

(iv) Find and simplify an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$. [2]

The function h is defined by

$$\mathrm{h} : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $\mathrm{h}^{-1}(x)$. [2]

(i) $2x^2 - 3,\ 4x^2 + 4x - 1$
(ii) -1
(iii) 2
(iv) $\frac{1}{2(x^2-3)}$
(v) $-\sqrt{x+2}$

MAY/JUNE2011 9709/12

The function f is defined by $f : x \mapsto \dfrac{x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(i) Show that $ff(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [2]

(ii) $\frac{x+3}{2x-1}$

MAY/JUNE2011 9709/12

The function f is such that $f(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $f(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = f(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

(i) (a) $-1 \leq f(x) \leq 3$

(b) $\frac{1}{4}\pi, \frac{3}{4}\pi$

(ii) (b) Yes. Because it is a one-to-one function.

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Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate fg(2). [2]

(ii) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $\mathrm{g}'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

(i) 26

(iii) $6(x-2)^2$, since g'(x) > 0, there is no turning point, therefore it is a one-to-one function.

(iv) $\frac{x+4}{3}$, $\sqrt[3]{\frac{x-8}{2}} + 1$

OCTOBER/NOVEMBER2010 9709/11

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 3,$$
$$\mathrm{g} : x \mapsto x^2 - 2x.$$

Express $\mathrm{gf}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [5]

$4(x+1)^2 - 1$

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A function f is defined by $\mathrm{f} : x \mapsto 3 - 2\tan\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $\mathrm{f}\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression, in terms of x, for $\mathrm{f}^{-1}(x)$. [3]

(i) $f(x) \leq 3$
(ii) $3 - 2\sqrt{3}$
(iv) $2tan^{-1}(\frac{3-x}{2})$

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The function f is defined by

$$f(x) = x^2 - 4x + 7 \quad \text{for} \quad x > 2.$$

(i) Express $f(x)$ in the form $(x - a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \quad \text{for} \quad x > 2.$$

The function h is such that f = hg and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

(i) $(x-2)^2+3, f(x) > 3$
(ii) $2+\sqrt{x-3}, x > 3$
(iii) x^2+3

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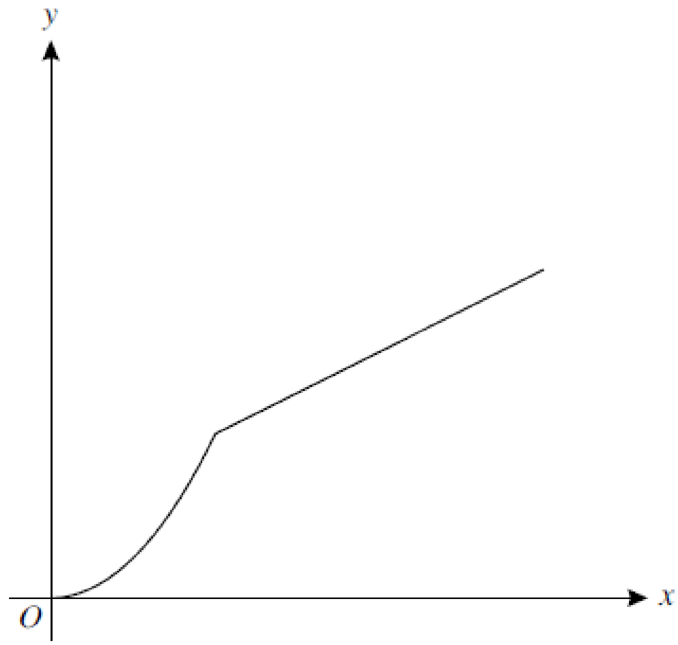


The diagram shows the function f defined for $0 \leqslant x \leqslant 6$ by

$$x \mapsto \tfrac{1}{2}x^2 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$x \mapsto \tfrac{1}{2}x + 1 \quad \text{for } 2 < x \leqslant 6.$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define $f^{-1}(x)$, giving the set of values of x for which each expression is valid. [4]

(i) 0 < f(x) < 4

(iii) $f^{-1}(x) = \sqrt{2x}, 0 < x < 2$

$f^{-1}(x) = 2x - 2, 2 < x < 4$

MAY/JUNE2010 9709/11

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x-b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

(i) $2(x-3)^2 - 11$

(ii) $f(x) \geq -11$

(iii) $-1 < x < 7$

(iv) 22

MAY/JUNE2010 9709/12

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

(i) $f(x) \leq 2$

(ii) 13

MAY/JUNE2010 9709/12

The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 2$. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [2]

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

(iv) State the largest value of A for which g has an inverse. [1]

(v) For this value of A, find the value of $g^{-1}(3)$. [2]

(i) 0.730, 2.41

(ii) k < 1, k > 7

(iv) $\frac{3}{2}\pi$

(v) 2.80

MAY/JUNE2010 9709/13

The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

(i) 4, 12
(ii) $2(x-2)^2 + 6$
(iii) $f(x) \geq 6$
(iv) 2
(v) $g^{-1}(x) = \sqrt{\frac{x-6}{2}} + 2$

MAY/JUNE2010 9709/13

The function $f : x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

(i) 4, 6
(ii) $-2 \leq f(x) \leq 10$
(iii) $4 - 3\sqrt{3}$

OCTOBER/NOVEMBER2009 9709/11

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \; x > 0,$$
$$\mathrm{g} : x \mapsto \frac{2x-1}{x+3}, \quad x \in \mathbb{R}, \; x \neq -3.$$

(i) Solve the equation $\mathrm{gf}(x) = x$. [3]

(ii) Express $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $\mathrm{g}^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(i) $\frac{1}{2}\sqrt{2}$

(ii) $f^{-1}(x) = \frac{1}{2}(x-1), g^{-1}(x) = \frac{1+3x}{2-x}$

OCTOBER/NOVEMBER2009 9709/12

The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State, with a reason, whether f has an inverse. [1]

(i) $2 \leq f(x) \leq 8$

(iii) No. it is not a one-to-one function.

MAY/JUNE2009

The function f is defined by $f : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$. [3]

(i) $2(x-3)^2 - 5$

(ii) 6

(iii) $-5 \leq f(x) \leq 13$

(iv) it is a one-to-one function

(v)$g^{-1}(x) = \sqrt{\frac{x+5}{2}} + 3$

OCTOBER/NOVEMBER2008

The function f is defined by

$$f : x \mapsto 3x - 2 \quad \text{for } x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$g : x \mapsto 6x - x^2 \quad \text{for } x \in \mathbb{R}.$$

(ii) Express $gf(x)$ in terms of x, and hence show that the maximum value of $gf(x)$ is 9. [5]

The function h is defined by

$$h : x \mapsto 6x - x^2 \quad \text{for } x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants. [2]

(iv) Express $h^{-1}(x)$ in terms of x. [3]

(ii) $gf(x) = -9x^2 + 30x - 16$
(ii) $9 - (x - 3)^2$
(iv) $h^{-1}(x) = 3 + \sqrt{9 - x}$

MAY/JUNE2008

The function f is such that $f(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(i) $f'(x) = 9(3x+2)^2$, always positive. Therefore it is an increasing function.

(ii) $f^{-1}(x) = \frac{\sqrt[3]{x+5}-2}{3}$, $x \geq 3$

MAY/JUNE2008

Functions f and g are defined by

$$f : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{2-x} \quad \text{for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of k for which the equation $fg(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $fg(x) = x$ for the values of k found in part **(i)**. [3]

(i) -7 5

(ii) -4, 8

OCTOBER/NOVEMBER2007

The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

(i) $2(x-2)^2+3$

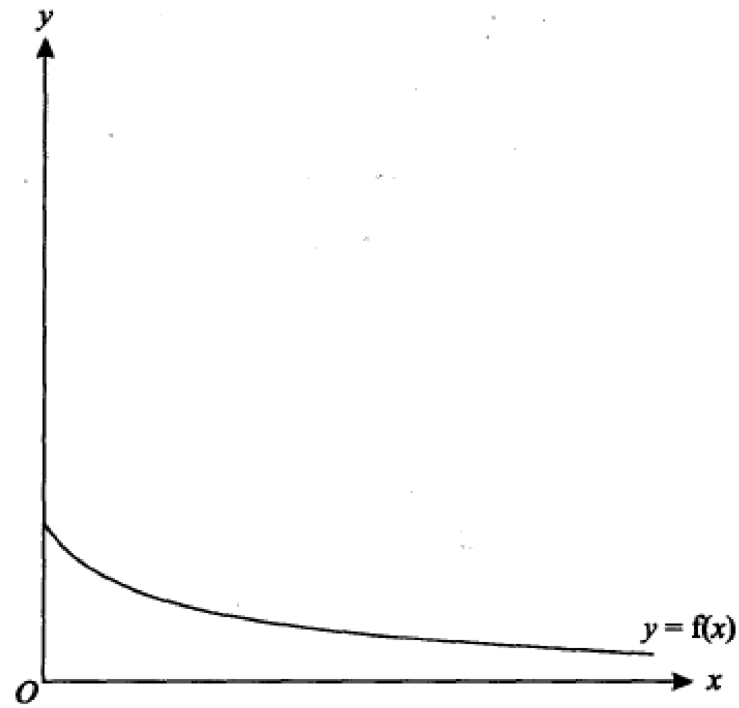
(ii) $f(x) \geq 3$

(iii) It is not a one-to-one function

(iv) 2

(v) $g^{-1}(x) = 2 - \sqrt{\frac{x-3}{2}}, g^{-1}(x) \leq 2$

MAY/JUNE2007



The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \dfrac{6}{2x+3}$ for $x \geqslant 0$.

(i) Find an expression, in terms of x, for $f'(x)$ and explain how your answer shows that f is a decreasing function. [3]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs. [2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$. [3]

(i) $f'(x) = -12(2x+3)^{-2}$, always negative → decreasing

(ii) $f^{-1}(x) = \frac{1}{2}\left(\frac{6}{x} - 3\right), 0 < x \leq 2$

(iv) 1

OCTOBER/NOVEMBER2006

The function f is defined by $f : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) > 4$. [3]

(ii) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

(iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $g(x) = 10$. [3]

(i) x < -1, x > 4

(ii) $(x - \frac{3}{2})^2 - \frac{9}{4}$

(iii) $f(x) \geq -\frac{9}{4}$

(iv) no inverse. It is not a one-to-one function

(v) 25

MAY/JUNE2006

Functions f and g are defined by

$$f : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$g : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $f(x) = g(x)$ has two equal roots and solve the equation $f(x) = g(x)$ in these cases. [6]

(ii) Solve the equation $fg(x) = 5$ when $k = 6$. [3]

(iii) Express $g^{-1}(x)$ in terms of x. [2]

(i) k = 4, root = 1, k = -8, root = -5
(ii) x = 7
(iii) $g^{-1}(x) = \frac{9-2x}{x}$

OCTOBER/NOVEMBER2005

A function f is defined by f : $x \mapsto (2x-3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

(i) Find an expression, in terms of x, for $f'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

(i) $f'(x) = 6(2x-3)^2$, always positive → increasing

(ii) $f^{-1}(x) = \frac{\sqrt[3]{x+8}+3}{2}, -7 \leq x \leq 117$

MAY/JUNE2005

A function f is defined by f : $x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by g : $x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$. [2]

(i) $1 \leq f(x) \leq 5$

(iii) $\frac{\pi}{2}$

(iv) $g^{-1}(x) = sin^{-1}(\frac{3-x}{2})$

OCTOBER/NOVEMBER2004

The function $f : x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation $ff(x) = 11$. [3]

The function $g : x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation $f(x) = g(x)$ has exactly one real solution. [3]

The function $h : x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $h^{-1}(x)$ and state the domain of h^{-1}. [4]

(i) 5
(ii) 16
(iii) $(x-3)^2 - 9$
(iv) $h^{-1}(x) = \sqrt{x+9} + 3,\ x \geq -9$

MAY/JUNE2004

The functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $\mathrm{f}(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $\mathrm{gf}(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, making clear the relationship between the graphs. [2]

(i) $x < -3$, $x > 5$

(ii) $f(x) \geq -1$. No inverse since it is not a one-to-one function.

OCTOBER/NOVEMBER2003

Functions f and g are defined by

$$f : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{4}{2 - x}, \quad x \in \mathbb{R}, \ x \neq 2.$$

(i) Find the value of x for which $fg(x) = 7$. [3]

(ii) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $f^{-1}(x) = g^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between these two graphs. [3]

(i) $\frac{4}{3}$

(ii) $f^{-1}(x) = \frac{1}{2}(x + 5), \ g^{-1}(x) = 2 - \frac{4}{x}$

MAY/JUNE2003

The equation of a curve is $y = 8x - x^2$.

(i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]

(iii) Find the set of values of x for which $y \geqslant -20$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 8x - x^2$, for $x \geqslant 4$.

(iv) State the domain and range of g^{-1}. [2]

(v) Find an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [3]

(i) $16 - (x - 4)^2$
(ii) (4, 16)
(iii) $-2 \leq x \leq 10$
(iv) $x \leq 16$, $g^{-1}(x) \geq 4$
(v) $g^{-1}(x) = 4 + \sqrt{16 - x}$

MAY/JUNE2003

The function f is defined by $\mathrm{f} : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $\mathrm{f}(2) = 1$ and $\mathrm{f}(5) = 7$.

(i) Find the values of a and b. [2]

(ii) Solve the equation $\mathrm{ff}(x) = 0$. [3]

(a) 2, -3
(b) 2.25

OCTOBER/NOVEMBER2002

(i) Express $2x^2 + 8x - 10$ in the form $a(x+b)^2 + c$. [3]

(ii) For the curve $y = 2x^2 + 8x - 10$, state the least value of y and the corresponding value of x. [2]

(iii) Find the set of values of x for which $y \geqslant 14$. [3]

Given that $\mathrm{f} : x \mapsto 2x^2 + 8x - 10$ for the domain $x \geqslant k$,

(iv) find the least value of k for which f is one-one, [1]

(v) express $\mathrm{f}^{-1}(x)$ in terms of x in this case. [3]

(i) $2(x+2)^2 - 18$
(ii) least value = -18 when x = -2
(iii) $x \leq -6, x \geq 2$
(iv) -2
(v) $f^{-1}(x) = \sqrt{\frac{x+18}{2}} - 2$

MAY/JUNE2002

The functions f and g are defined by

$$f : x \mapsto 3x + 2, \qquad x \in \mathbb{R},$$
$$g : x \mapsto \frac{6}{2x+3}, \qquad x \in \mathbb{R},\ x \neq -1.5.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 3$. [3]

(ii) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [3]

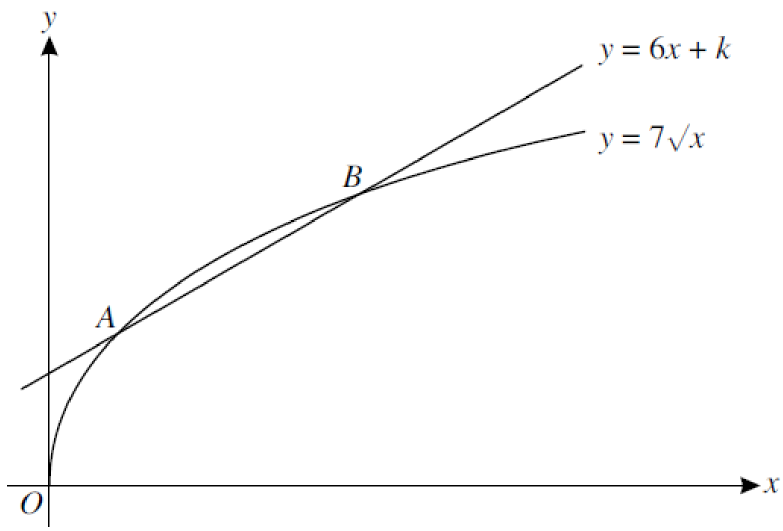
(iii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x, and solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

(i) 7.5

(iii) $f^{-1}(x) = \frac{x-2}{3},\ g^{-1}(x) = \frac{1}{2}\left(\frac{6}{x} - 3\right),\ x = 2, -4.5$

COORDINATE GEOMETRY

MAY/JUNE2012 9709/11



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

(i) $\frac{4}{9}, \frac{1}{4}$

(ii) $\frac{49}{24}$

MAY/JUNE2012 9709/11

The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$. The mid-point of AC is M and the perpendicular bisector of AC cuts the x-axis at B.

(i) Find the equation of MB and the coordinates of B. [5]

(ii) Show that AB is perpendicular to BC. [2]

(iii) Given that $ABCD$ is a square, find the coordinates of D and the length of AD. [2]

(i) $y = -2x + 6, (3, 0)$

(iii) (-1, 8), $\sqrt{40}$

MAY/JUNE2012 9709/12

The point A has coordinates $(-1, -5)$ and the point B has coordinates $(7, 1)$. The perpendicular bisector of AB meets the x-axis at C and the y-axis at D. Calculate the length of CD. [6]

2.5

MAY/JUNE2012 9709/13

The curve $y = \dfrac{10}{2x+1} - 2$ intersects the x-axis at A. The tangent to the curve at A intersects the y-axis at C.

(i) Show that the equation of AC is $5y + 4x = 8$. [5]

(ii) Find the distance AC. [2]

(ii) 2.56

MAY/JUNE2012 9709/13

The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B. Find the equation of the perpendicular bisector of the line AB. [6]

(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

(i) $y = 2x - 6$
(ii) $k < -\sqrt{48},\ k > \sqrt{48}$

OCTOBER/NOVEMBER2011 9709/11

A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the mid-point of AB. [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

(i) 6.71, $\left(-\frac{1}{2}, 5\right)$

(ii) 3, 11

OCTOBER/NOVEMBER2011 9709/12

The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

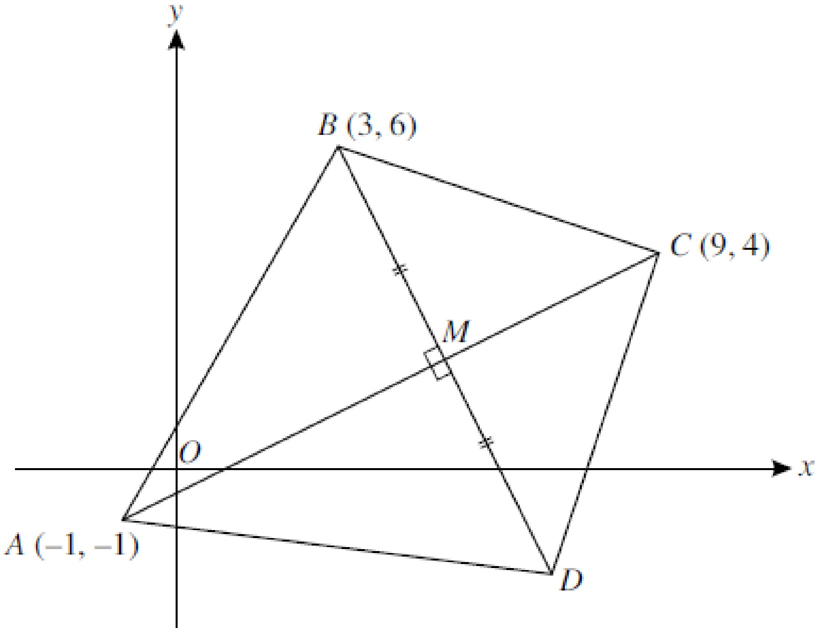
(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

(ii) Find the value of k for which the line is a tangent to the curve. [3]

(i) (2, 3), (6, 1)

(ii) 8.5

OCTOBER/NOVEMBER2011 9709/12

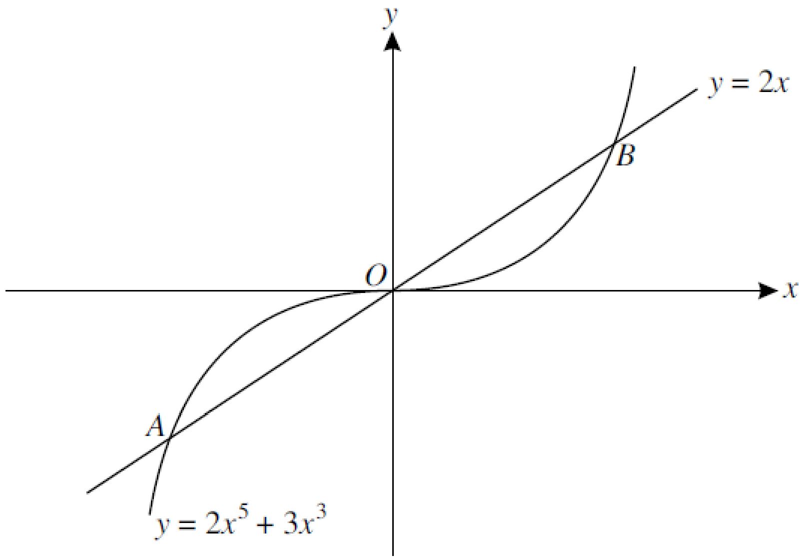


The diagram shows a quadrilateral $ABCD$ in which the point A is $(-1, -1)$, the point B is $(3, 6)$ and the point C is $(9, 4)$. The diagonals AC and BD intersect at M. Angle $BMA = 90^\circ$ and $BM = MD$. Calculate

(i) the coordinates of M and D, [7]

(ii) the ratio $AM : MC$. [2]

(i) (5, 2), (7, -2)
(ii) $\sqrt{45}$: $\sqrt{20}$ (or 3 : 2)



The diagram shows the curve $y = 2x^5 + 3x^3$ and the line $y = 2x$ intersecting at points A, O and B.

(i) Show that the x-coordinates of A and B satisfy the equation $2x^4 + 3x^2 - 2 = 0$. [2]

(ii) Solve the equation $2x^4 + 3x^2 - 2 = 0$ and hence find the coordinates of A and B, giving your answers in an exact form. [3]

(ii) $\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{2}{\sqrt{2}}\right)$

MAY/JUNE2011 9709/11

(i) Express $2x^2 - 4x + 1$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

(i) $2(x-1)^2 - 1,\ (1, -1)$
(ii) (-0.5, 3.5)
(iii) $y = -\frac{1}{5}x + \frac{17}{5}$

MAY/JUNE2011 9709/12

A curve has equation $y = \dfrac{4}{3x-4}$ and $P\,(2, 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

(i) $y = -3x + 8$
(ii) $\pm 108.4°\ (or\ \pm 71.6°)$

MAY/JUNE2011 9709/12

The line L_1 passes through the points $A\ (2,\ 5)$ and $B\ (10,\ 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]

(i) (3.6, 1.8)
(ii) 3.58

MAY/JUNE2011 9709/13

The line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

6, 3

OCTOBER/NOVEMBER2010 9709/11

The equation of a curve is $y = 3 + 4x - x^2$.

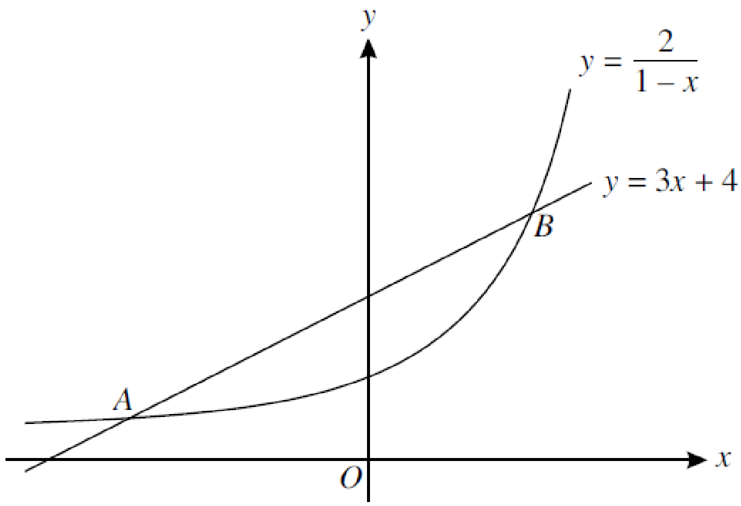
(i) Show that the equation of the normal to the curve at the point $(3, 6)$ is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B, find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

(ii) (-4.5, 2.25)
(iii) (0.5, 4.75)

OCTOBER/NOVEMBER2010 9709/12



The diagram shows part of the curve $y = \frac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B.

(i) Find the coordinates of A and B. [4]

(ii) Find the length of the line AB and the coordinates of the mid-point of AB. [3]

(i) (-1, 1), (2/3, 6)
(ii) 5.27, (-1/6, 7/2)

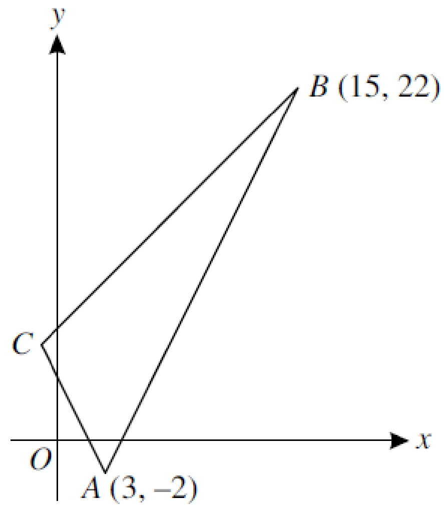
OCTOBER/NOVEMBER2010 9709/13

Points A, B and C have coordinates $(2, 5)$, $(5, -1)$ and $(8, 6)$ respectively.

(i) Find the coordinates of the mid-point of AB. [1]

(ii) Find the equation of the line through C perpendicular to AB. Give your answer in the form $ax + by + c = 0$. [3]

(i) (3.5, 2)
(ii) $x - 2y + 4 = 0$



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

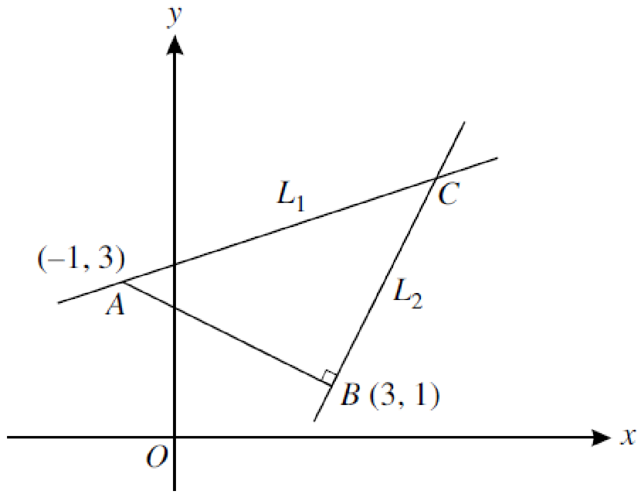
(i) Find the gradient of AB and deduce the value of m. [2]

(ii) Find the coordinates of C. [4]

The perpendicular bisector of AB meets BC at D.

(iii) Find the coordinates of D. [4]

(i) 2, m = 1
(ii) (-1, 6)
(iii) (5, 12)



In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB. The line L_2 passes through B and is perpendicular to AB. The lines L_1 and L_2 meet at C. Find the coordinates of C. [6]

(5, 5)

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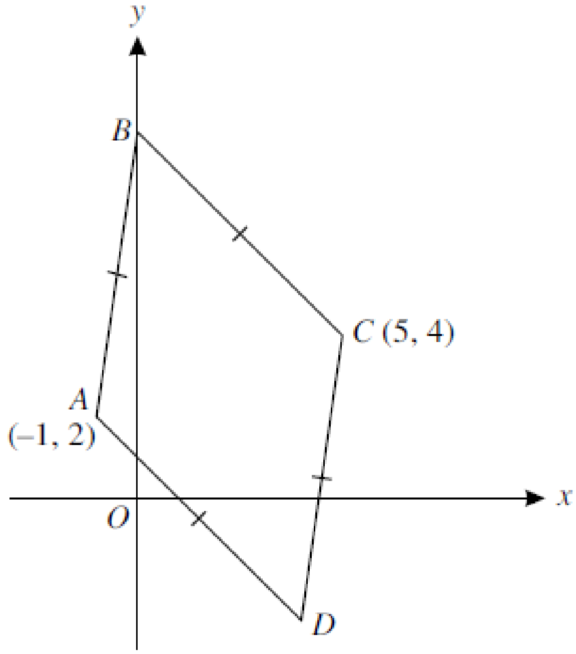


The diagram shows a rhombus $ABCD$ in which the point A is $(-1, 2)$, the point C is $(5, 4)$ and the point B lies on the y-axis. Find

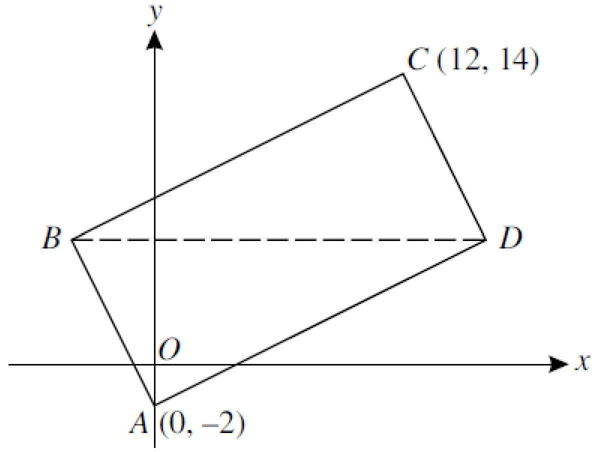
(i) the equation of the perpendicular bisector of AC, [3]

(ii) the coordinates of B and D, [3]

(iii) the area of the rhombus. [3]

(i) $y = -3x + 9$
(ii) (0, 9), (4, -3)
(iii) 40

OCTOBER/NOVEMBER2009 9709/11



The diagram shows a rectangle $ABCD$. The point A is $(0, -2)$ and C is $(12, 14)$. The diagonal BD is parallel to the x-axis.

(i) Explain why the y-coordinate of D is 6. [1]

The x-coordinate of D is h.

(ii) Express the gradients of AD and CD in terms of h. [3]

(iii) Calculate the x-coordinates of D and B. [4]

(iv) Calculate the area of the rectangle $ABCD$. [3]

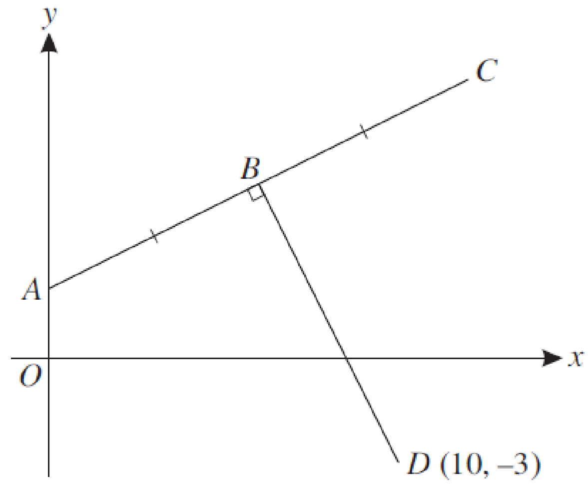
(i) y-coordinate same as the y-coordinate of the mid-oint of AC.

(ii) $m\ of\ AD = \frac{8}{h}\ or\ \frac{h-12}{8}, m\ of\ CD = \frac{8}{12-h}\ or\ \frac{-h}{8}$

(iii) 16, -4

(iv) 160

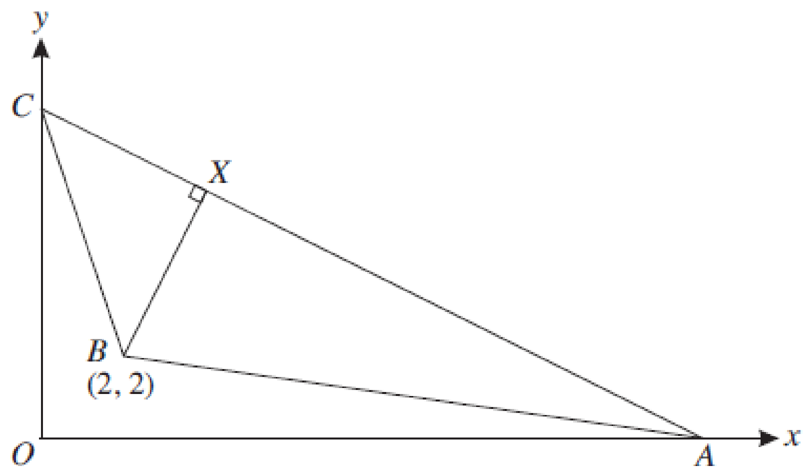
MAY/JUNE2009



The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D\,(10, -3)$ to B is perpendicular to AC. Calculate the coordinates of B and C. [7]

(6, 5), (12, 8)

MAY/JUNE2008



In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates (2, 2). The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

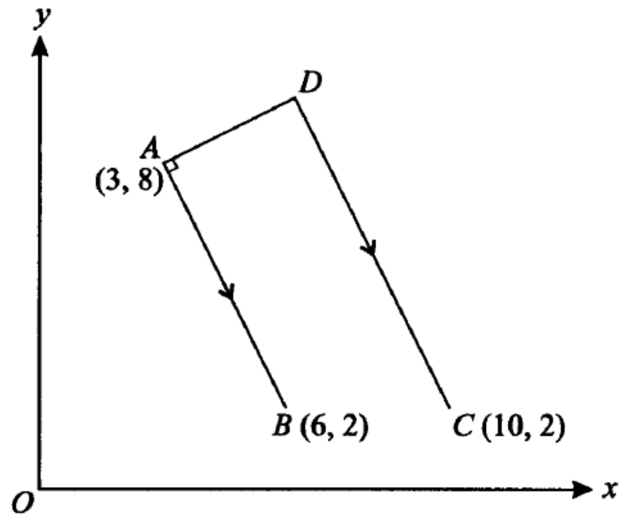
The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]

(i) (4, 6)
(ii) (6, 10)
(iii) 40.9

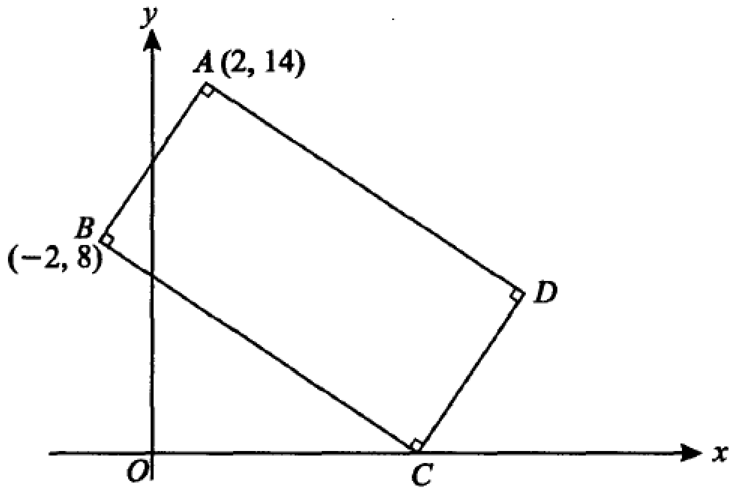
OCTOBER/NOVEMBER2007



The three points A (3, 8), B (6, 2) and C (10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]

(6.2, 9.6)

MAY/JUNE2007



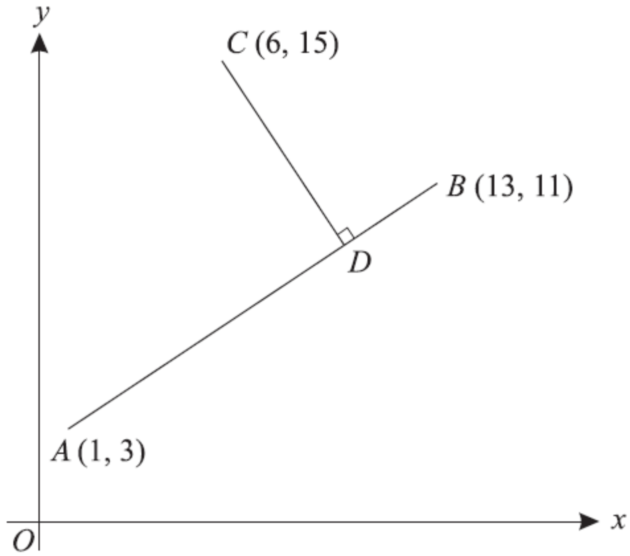
The diagram shows a rectangle $ABCD$. The point A is (2, 14), B is (−2, 8) and C lies on the x-axis. Find

(i) the equation of BC, [4]

(ii) the coordinates of C and D. [3]

(i) $3y + 2x = 20$
(ii) (10, 0), (14, 6)

OCTOBER/NOVEMBER2006



The three points $A\ (1,\ 3)$, $B\ (13,\ 11)$ and $C\ (6,\ 15)$ are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

(i) $y = -\frac{3}{2}x + 24$

(ii) (10, 9)

MAY/JUNE2006

The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points. [6]

3.75

OCTOBER/NOVEMBER2005

Three points have coordinates A (2, 6), B (8, 10) and C (6, 0). The perpendicular bisector of AB meets the line BC at D. Find

(i) the equation of the perpendicular bisector of AB in the form $ax + by = c$, [4]

(ii) the coordinates of D. [4]

(i) $2y + 3x = 31$
(ii) (7, 5)

OCTOBER/NOVEMBER2005

The equation of a curve is $xy = 12$ and the equation of a line l is $2x + y = k$, where k is a constant.

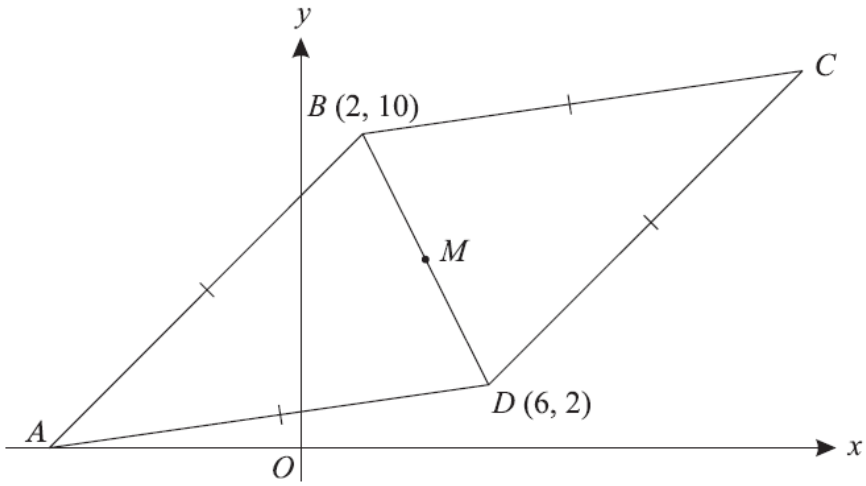
(i) In the case where $k = 11$, find the coordinates of the points of intersection of l and the curve. [3]

(ii) Find the set of values of k for which l does not intersect the curve. [4]

(iii) In the case where $k = 10$, one of the points of intersection is $P\,(2, 6)$. Find the angle, in degrees correct to 1 decimal place, between l and the tangent to the curve at P. [4]

(i) (1.5, 8), (4, 3)
(ii) $-\sqrt{96} < k < \sqrt{96}$
(iii) 8.1° or 8.2°

MAY/JUNE2005



The diagram shows a rhombus $ABCD$. The points B and D have coordinates (2, 10) and (6, 2) respectively, and A lies on the x-axis. The mid-point of BD is M. Find, by calculation, the coordinates of each of M, A and C. [6]

(4, 6), (-8, 0), (16, 12)

OCTOBER/NOVEMBER2004

The equation of a curve is $y = x^2 - 4x + 7$ and the equation of a line is $y + 3x = 9$. The curve and the line intersect at the points A and B.

(i) The mid-point of AB is M. Show that the coordinates of M are $\left(\frac{1}{2}, 7\frac{1}{2}\right)$. [4]

(ii) Find the coordinates of the point Q on the curve at which the tangent is parallel to the line $y + 3x = 9$. [3]

(iii) Find the distance MQ. [1]

(ii) (0.5, 5.25)
(iii) 2.25

MAY/JUNE2004

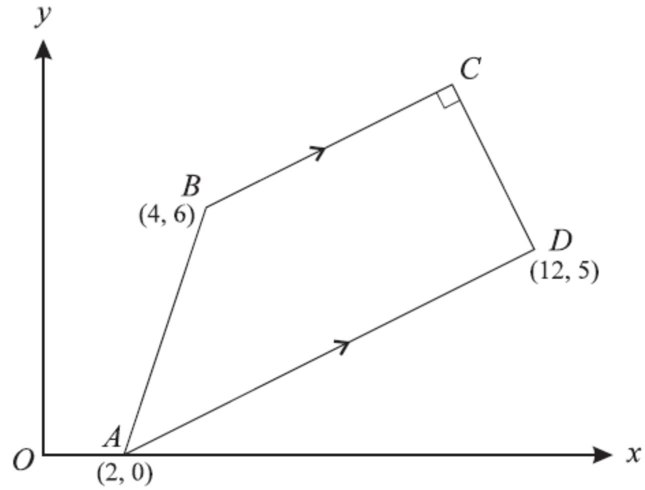
The curve $y = 9 - \dfrac{6}{x}$ and the line $y + x = 8$ intersect at two points. Find

(i) the coordinates of the two points, [4]

(ii) the equation of the perpendicular bisector of the line joining the two points. [4]

(i) (2, 6), (-3, 11)
(ii) $y = x + 9$

OCTOBER/NOVEMBER2003



The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are (2, 0), (4, 6) and (12, 5) respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

(i) $BC: y = \frac{1}{2}x + 4,\ \ CD: y = -2x + 29$

(ii) (10, 9)

OCTOBER/NOVEMBER2003

Find the coordinates of the points of intersection of the line $y + 2x = 11$ and the curve $xy = 12$. [4]

(1.5, 8), (4, 3)

MAY/JUNE2003

The line L_1 has equation $2x + y = 8$. The line L_2 passes through the point A (7, 4) and is perpendicular to L_1.

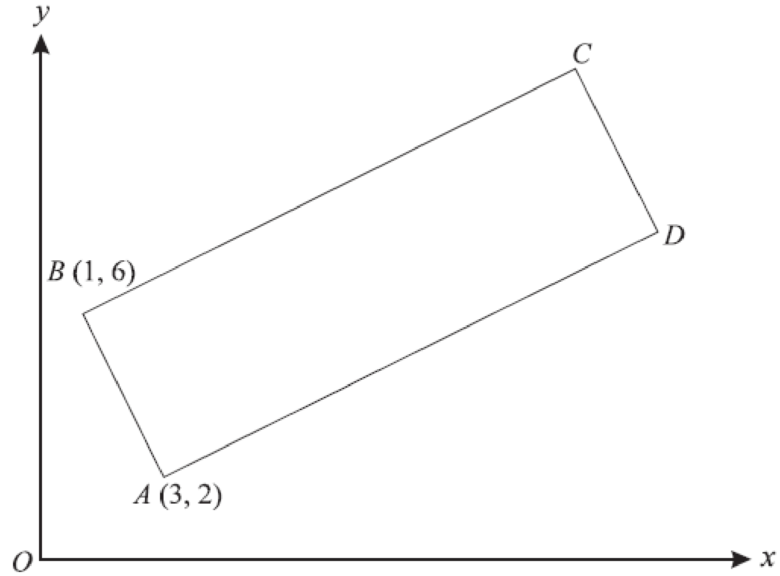
(i) Find the equation of L_2. [4]

(ii) Given that the lines L_1 and L_2 intersect at the point B, find the length of AB. [4]

(i) $y = \frac{1}{2}x + \frac{1}{4}$

(ii) 4.47

OCTOBER/NOVEMBER2002



The diagram shows a rectangle $ABCD$, where A is $(3, 2)$ and B is $(1, 6)$.

(i) Find the equation of BC. [4]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C, [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

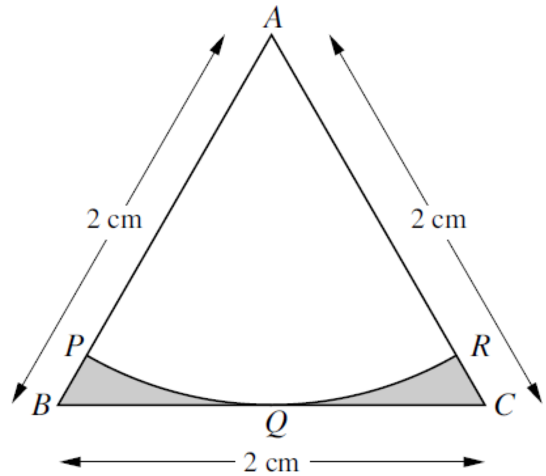
(i) $2y = x + 11$
(ii) (13, 12)
(iii) 35.8 or 35.7

MAY/JUNE2002

The line $x + 2y = 9$ intersects the curve $xy + 18 = 0$ at the points A and B. Find the coordinates of A and B. [4]

(12, -1.5), (-3, 6)

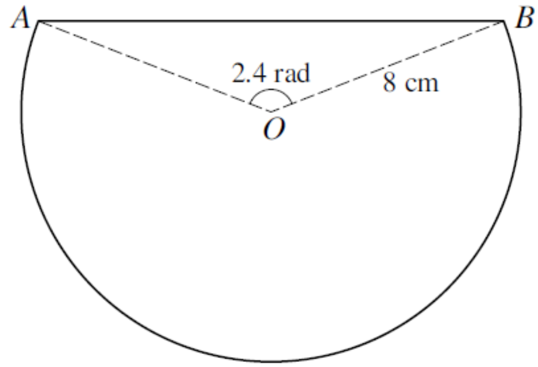
CIRCULAR MEASURE



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

$\sqrt{3} - \frac{\pi}{2}$

MAY/JUNE2012 9709/12



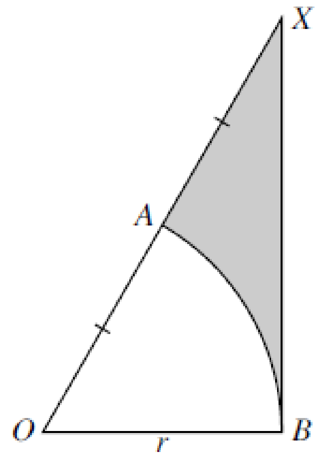
The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

(i) the length of AB, [2]

(ii) the perimeter of the plate, [3]

(iii) the area of the plate. [3]

(i) 14.9
(ii) 46.0
(iii) 146



In the diagram, AB is an arc of a circle with centre O and radius r. The line XB is a tangent to the circle at B and A is the mid-point of OX.

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

Express each of the following in terms of r, π and $\sqrt{3}$:

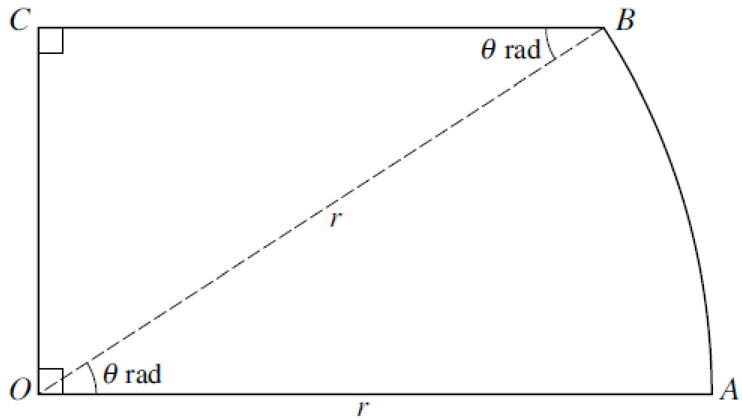
(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. [2]

(ii) $P = r + (\frac{1}{3}r\pi + r\sqrt{3})$

(iii) $\frac{1}{2}r^2\sqrt{3} - \frac{1}{6}r^2\pi$

OCTOBER/NOVEMBER2011 9709/11



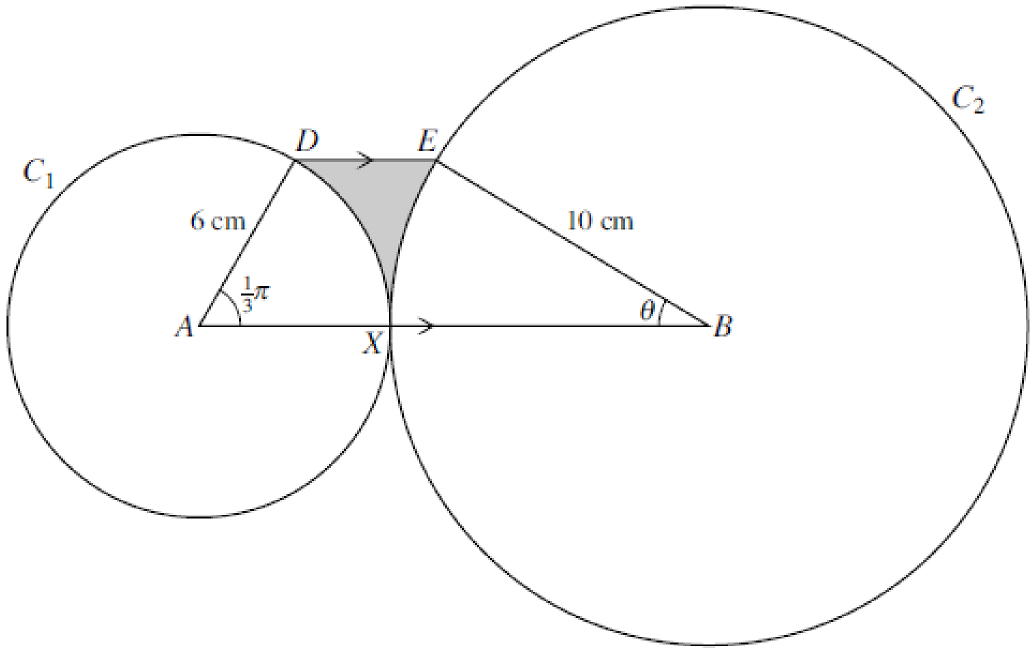
The diagram represents a metal plate $OABC$, consisting of a sector OAB of a circle with centre O and radius r, together with a triangle OCB which is right-angled at C. Angle $AOB = \theta$ radians and OC is perpendicular to OA.

(i) Find an expression in terms of r and θ for the perimeter of the plate. [3]

(ii) For the case where $r = 10$ and $\theta = \frac{1}{5}\pi$, find the area of the plate. [3]

(i) $r(1 + \theta + cos\theta + sin\theta)$
(ii) 55.2

OCTOBER/NOVEMBER2011 9709/12

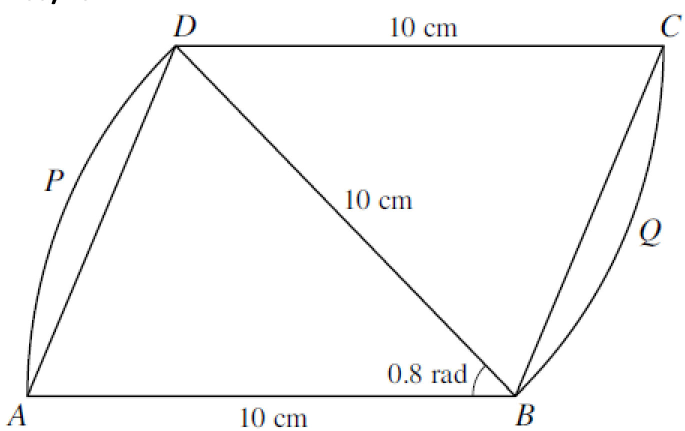


The diagram shows a circle C_1 touching a circle C_2 at a point X. Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB. Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

(i) By considering the perpendicular distances of D and E from AB, show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]

(ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

(ii) 16.20



In the diagram, $ABCD$ is a parallelogram with $AB = BD = DC = 10$ cm and angle $ABD = 0.8$ radians. APD and BQC are arcs of circles with centres B and D respectively.

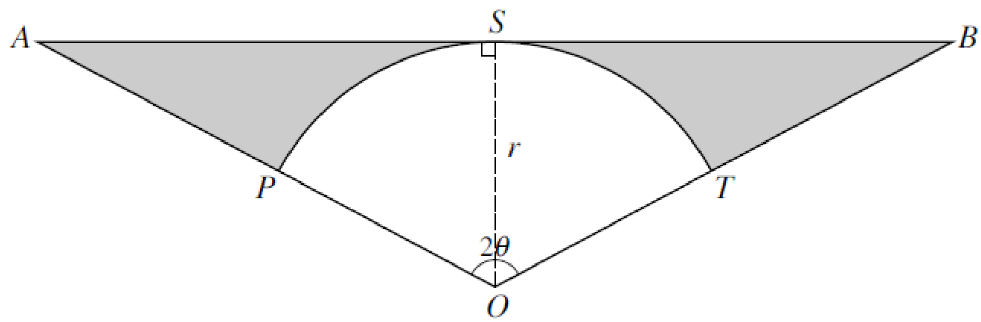
(i) Find the area of the parallelogram $ABCD$. [2]

(ii) Find the area of the complete figure $ABQCDP$. [2]

(iii) Find the perimeter of the complete figure $ABQCDP$. [2]

(i) 71.7
(ii) 80
(iii) 36

MAY/JUNE2011 9709/11



In the diagram, OAB is an isosceles triangle with $OA = OB$ and angle $AOB = 2\theta$ radians. Arc PST has centre O and radius r, and the line ASB is a tangent to the arc PST at S.

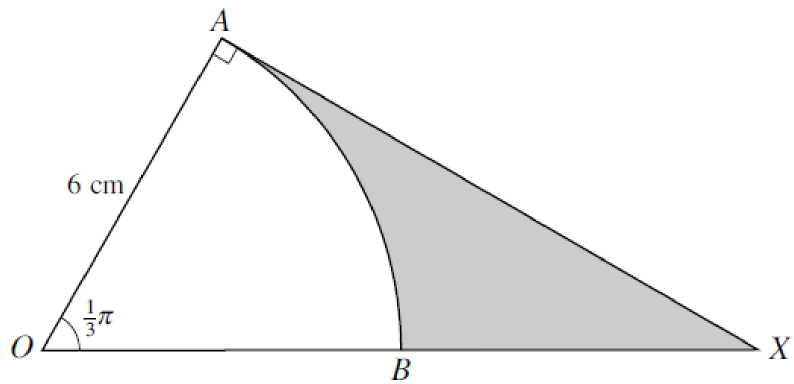
(i) Find the total area of the shaded regions in terms of r and θ. [4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and $r = 6$, find the total perimeter of the shaded regions, leaving your answer in terms of $\sqrt{3}$ and π. [5]

(i) $r^2(tan\theta - \theta)$

(ii) $12 + 12\sqrt{3} + 4\pi$

MAY/JUNE2011 9709/13



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

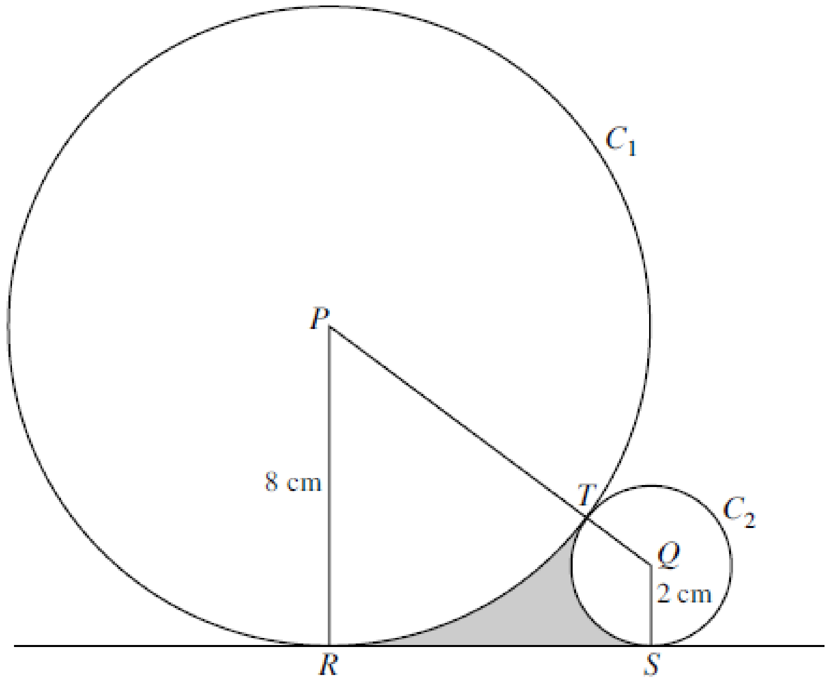
(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

(ii) $18\sqrt{3} - 6\pi$
(iii) $6\sqrt{3} + 2\pi + 6$



The diagram shows two circles, C_1 and C_2, touching at the point T. Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

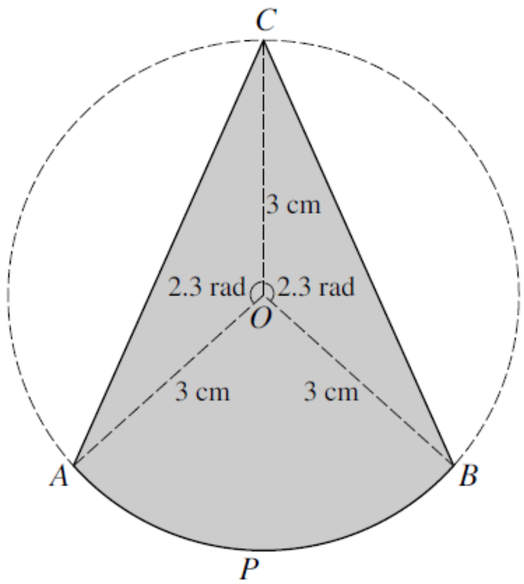
(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

(ii) 0.9273rad
(iii) 5.90

OCTOBER/NOVEMBER2010 9709/12

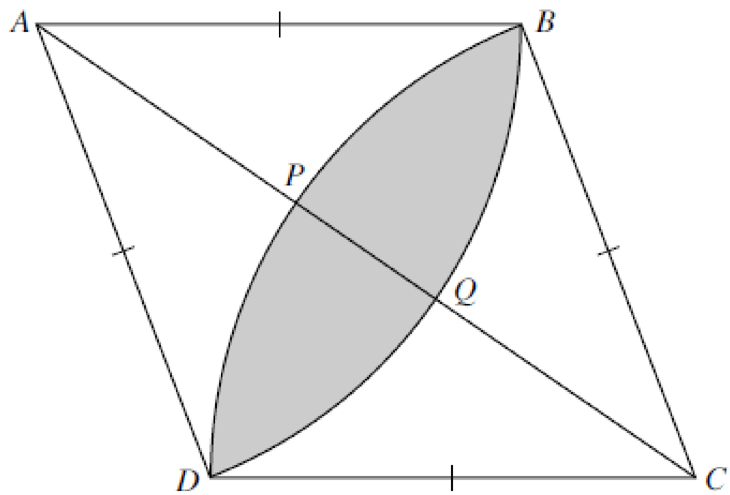


The diagram shows points A, C, B, P on the circumference of a circle with centre O and radius 3 cm. Angle AOC = angle BOC = 2.3 radians.

(i) Find angle AOB in radians, correct to 4 significant figures. [1]

(ii) Find the area of the shaded region $ACBP$, correct to 3 significant figures. [4]

(i) 1.683
(ii) 14.3

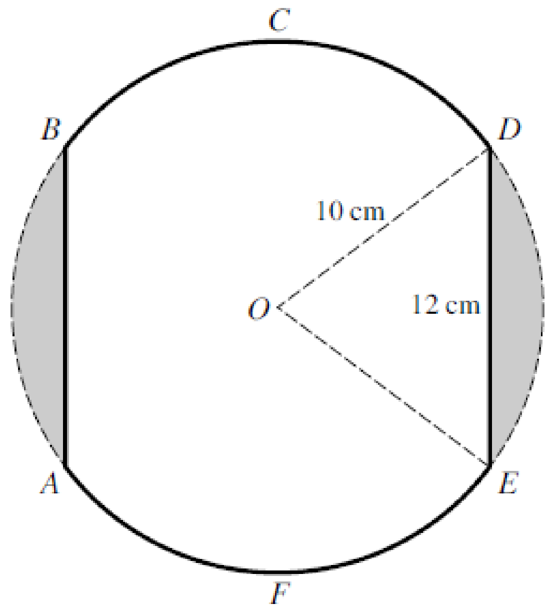


The diagram shows a rhombus $ABCD$. Points P and Q lie on the diagonal AC such that BPD is an arc of a circle with centre C and BQD is an arc of a circle with centre A. Each side of the rhombus has length 5 cm and angle $BAD = 1.2$ radians.

(i) Find the area of the shaded region $BPDQ$. [4]

(ii) Find the length of PQ. [4]

(i) 6.70
(ii) 1.75



The diagram shows a metal plate $ABCDEF$ which has been made by removing the two shaded regions from a circle of radius 10 cm and centre O. The parallel edges AB and ED are both of length 12 cm.

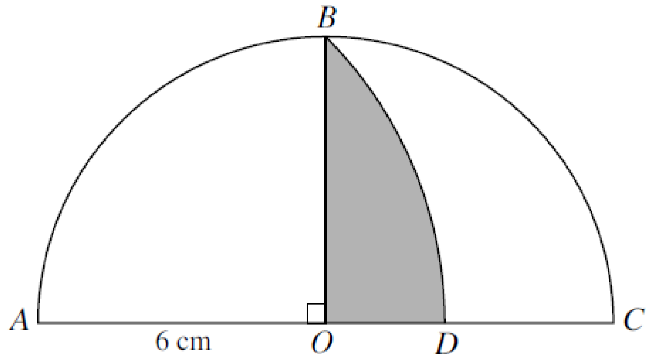
(i) Show that angle DOE is 1.287 radians, correct to 4 significant figures. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

(ii) 61.1
(iii) 281 or 282

OCTOBER/NOVEMBER2009 9709/11



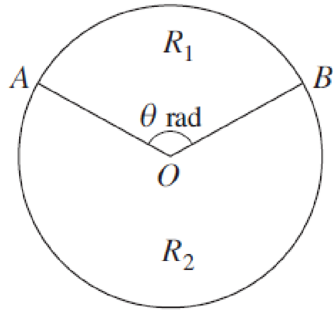
The diagram shows a semicircle ABC with centre O and radius 6 cm. The point B is such that angle BOA is 90° and BD is an arc of a circle with centre A. Find

(i) the length of the arc BD, [4]

(ii) the area of the shaded region. [3]

(i) 6.66
(ii) 10.3

MAY/JUNE2009



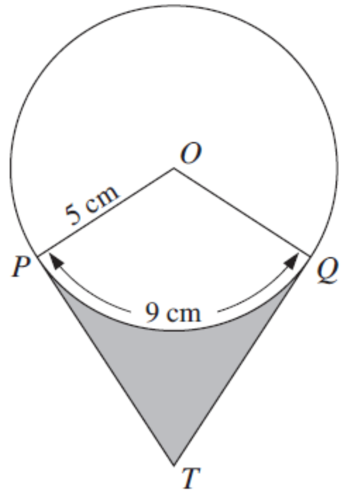
The diagram shows a circle with centre O. The circle is divided into two regions, R_1 and R_2, by the radii OA and OB, where angle $AOB = \theta$ radians. The perimeter of the region R_1 is equal to the length of the major arc AB.

(i) Show that $\theta = \pi - 1$. [3]

(ii) Given that the area of region R_1 is $30\,\text{cm}^2$, find the area of region R_2, correct to 3 significant figures. [4]

(ii) 58.0

OCTOBER/NOVEMBER2008



In the diagram, the circle has centre O and radius 5 cm. The points P and Q lie on the circle, and the arc length PQ is 9 cm. The tangents to the circle at P and Q meet at the point T. Calculate

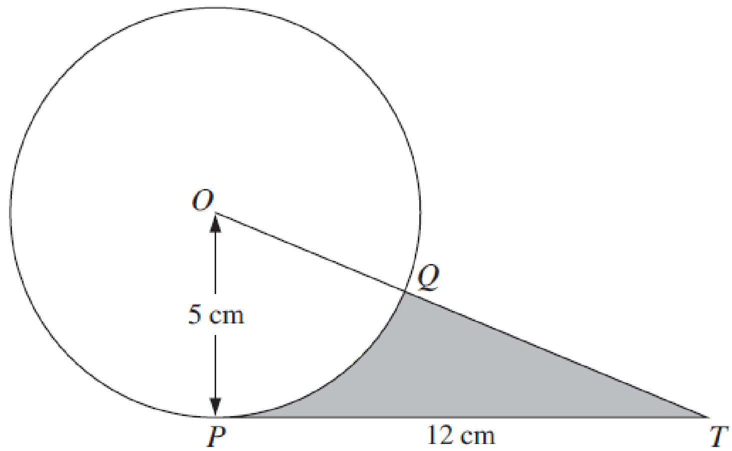
(i) angle POQ in radians, [2]

(ii) the length of PT, [3]

(iii) the area of the shaded region. [3]

(i) 1.8rad
(ii) 6.30
(iii) 9.00 (allow 8.95 to 9.05)

MAY/JUNE2008



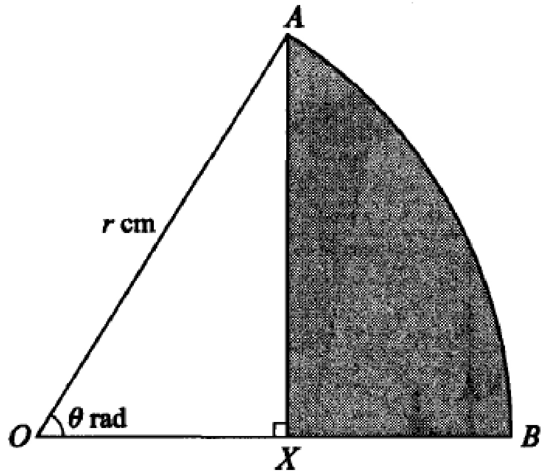
The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12$ cm. The line OT cuts the circle at the point Q.

(i) Find the perimeter of the shaded region. [4]

(ii) Find the area of the shaded region. [3]

(i) 25.9
(ii) 15.3

OCTOBER/NOVEMBER2007



In the diagram, AB is an arc of a circle, centre O and radius r cm, and angle $AOB = \theta$ radians. The point X lies on OB and AX is perpendicular to OB.

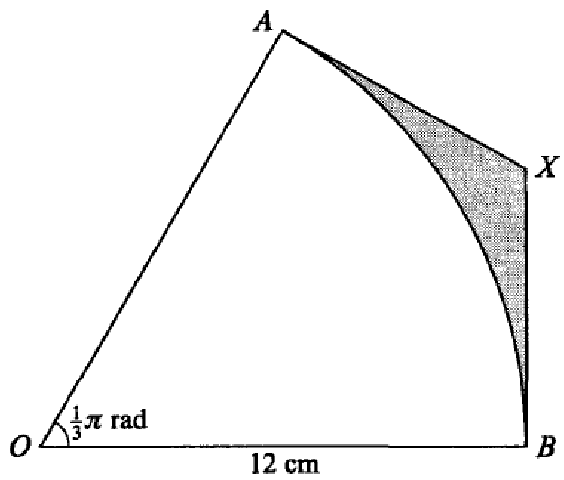
(i) Show that the area, A cm^2, of the shaded region AXB is given by

$$A = \tfrac{1}{2}r^2(\theta - \sin\theta\cos\theta). \qquad [3]$$

(ii) In the case where $r = 12$ and $\theta = \frac{1}{6}\pi$, find the perimeter of the shaded region AXB, leaving your answer in terms of $\sqrt{3}$ and π. [4]

(ii) $18 - 6\sqrt{3} + 2\pi$

MAY/JUNE2007



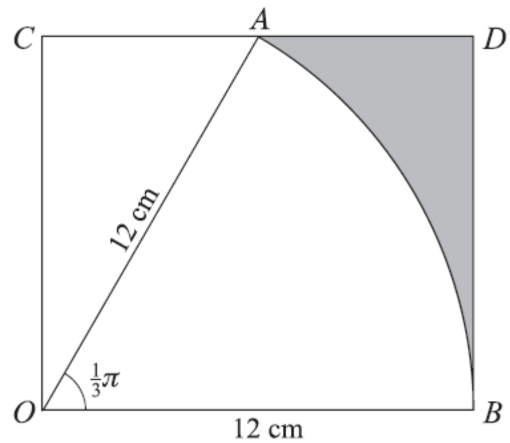
In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

(i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$. [2]

(ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [3]

(i) $4\sqrt{3}$
(ii) $48\sqrt{3} - 24\pi$

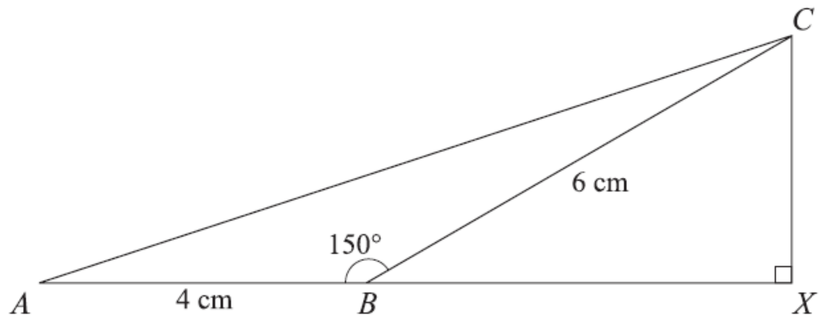
OCTOBER/NOVEMBER2006



In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b. [6]

$54\sqrt{3} - 24\pi$

MAY/JUNE2006



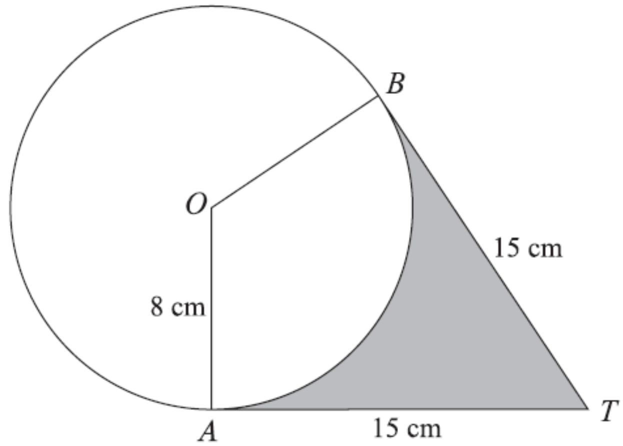
In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150^\circ$. The line CX is perpendicular to the line ABX.

(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\dfrac{3}{4+3\sqrt{3}}\right)$. [4]

(ii) Show that the exact length of AC is $\sqrt{(52+24\sqrt{3})}$ cm. [2]

(i)BX = $3\sqrt{3}$

MAY/JUNE2006



The diagram shows a circle with centre O and radius 8 cm. Points A and B lie on the circle. The tangents at A and B meet at the point T, and $AT = BT = 15$ cm.

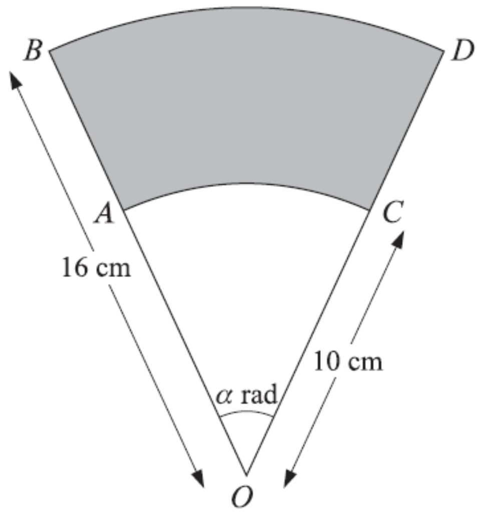
(i) Show that angle AOB is 2.16 radians, correct to 3 significant figures. [3]

(ii) Find the perimeter of the shaded region. [2]

(iii) Find the area of the shaded region. [3]

(ii) 47.3
(iii) 50.8 or 50.9

OCTOBER/NOVEMBER2005

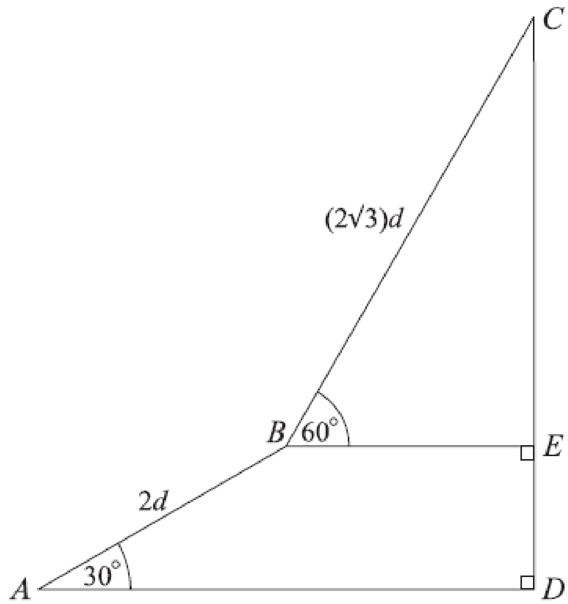


In the diagram, OAB and OCD are radii of a circle, centre O and radius 16 cm. Angle $AOC = \alpha$ radians. AC and BD are arcs of circles, centre O and radii 10 cm and 16 cm respectively.

(i) In the case where $\alpha = 0.8$, find the area of the shaded region. [2]

(ii) Find the value of α for which the perimeter of the shaded region is 28.9 cm. [3]

(i) 62.4
(ii) 0.65



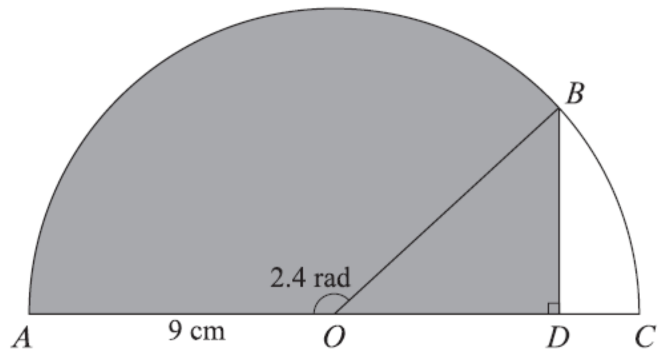
In the diagram, $ABED$ is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are $2d$ and $(2\sqrt{3})d$ respectively, and angles BAD and CBE are $30°$ and $60°$ respectively.

(i) Find the length of CD in terms of d. [2]

(ii) Show that angle $CAD = \tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$. [3]

(i) 4d

MAY/JUNE2005



In the diagram, ABC is a semicircle, centre O and radius 9 cm. The line BD is perpendicular to the diameter AC and angle $AOB = 2.4$ radians.

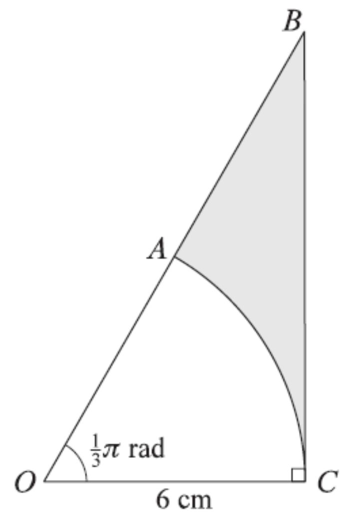
(i) Show that $BD = 6.08$ cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

(ii) 43.3
(iii) 117

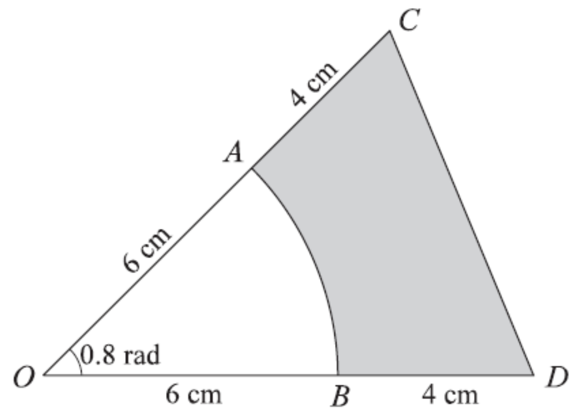
OCTOBER/NOVEMBER2004



In the diagram, AC is an arc of a circle, centre O and radius 6 cm. The line BC is perpendicular to OC and OAB is a straight line. Angle $AOC = \frac{1}{3}\pi$ radians. Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [5]

$18\sqrt{3} - 6\pi$

MAY/JUNE2004



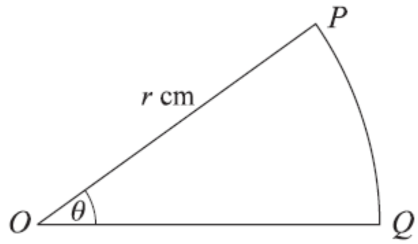
In the diagram, OCD is an isosceles triangle with $OC = OD = 10$ cm and angle $COD = 0.8$ radians. The points A and B, on OC and OD respectively, are joined by an arc of a circle with centre O and radius 6 cm. Find

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [4]

(i) 21.5
(ii) 20.6

OCTOBER/NOVEMBER2003



The diagram shows the sector OPQ of a circle with centre O and radius r cm. The angle POQ is θ radians and the perimeter of the sector is 20 cm.

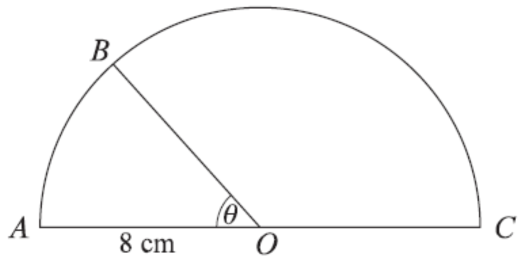
(i) Show that $\theta = \frac{20}{r} - 2$. [2]

(ii) Hence express the area of the sector in terms of r. [2]

(iii) In the case where $r = 8$, find the length of the chord PQ. [3]

(ii) $A = 10r - r^2$
(iii) 3.96 (allow 3.95)

MAY/JUNE2003



The diagram shows a semicircle ABC with centre O and radius 8 cm. Angle $AOB = \theta$ radians.

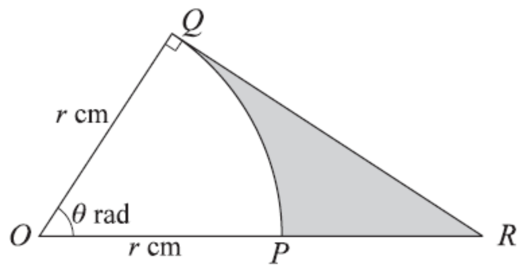
(i) In the case where $\theta = 1$, calculate the area of the sector BOC. [3]

(ii) Find the value of θ for which the perimeter of sector AOB is one half of the perimeter of sector BOC. [3]

(iii) In the case where $\theta = \frac{1}{3}\pi$, show that the exact length of the perimeter of triangle ABC is $(24 + 8\sqrt{3})$ cm. [3]

(i) 68.5
(ii) 0.381

OCTOBER/NOVEMBER2002



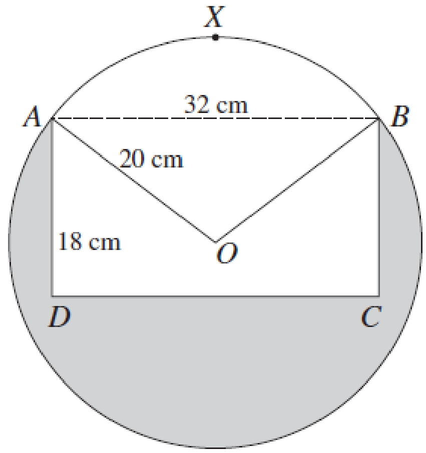
In the diagram, OPQ is a sector of a circle, centre O and radius r cm. Angle $QOP = \theta$ radians. The tangent to the circle at Q meets OP extended at R.

(i) Show that the area, A cm^2, of the shaded region is given by $A = \frac{1}{2}r^2(\tan\theta - \theta)$. [2]

(ii) In the case where $\theta = 0.8$ and $r = 15$, evaluate the length of the perimeter of the shaded region. [4]

(ii) 34.0 (accept 33.9)

MAY/JUNE2002



The diagram shows the circular cross-section of a uniform cylindrical log with centre O and radius 20 cm. The points A, X and B lie on the circumference of the cross-section and $AB = 32$ cm.

(i) Show that angle $AOB = 1.855$ radians, correct to 3 decimal places. [2]

(ii) Find the area of the sector $AXBO$. [2]

The section $AXBCD$, where $ABCD$ is a rectangle with $AD = 18$ cm, is removed.

(iii) Find the area of the new cross-section (shown shaded in the diagram). [3]

(ii) 371
(iii) 502 (accept 501)

TRIGONOMETRY

MAY/JUNE2012 9709/11

Solve the equation $\sin 2x = 2\cos 2x$, for $0° \leqslant x \leqslant 180°$. [4]

31.7, 121.7

MAY/JUNE2012 9709/12

(i) Prove the identity $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\dfrac{2}{\sin x \cos x} = 1 + 3\tan x$, for $0° \leqslant x \leqslant 180°$. [4]

(ii) 45, 116.6

MAY/JUNE2012 9709/13

(i) Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta \sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

MAY/JUNE2012 9709/13

(i) Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]

(i) 54.2, 144.2, 234.2, 324.2
(ii) 12

OCTOBER/NOVEMBER2011 9709/11

(i) Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

(ii) 4

(iii) 20

OCTOBER/NOVEMBER2011 9709/12

(i) Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0^\circ \leqslant x \leqslant 180^\circ$. [3]

(ii) Verify that $x = 30^\circ$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0^\circ \leqslant x \leqslant 180^\circ$. [2]

(iii) Hence state the set of values of x, for $0^\circ \leqslant x \leqslant 180^\circ$, for which $\sin x < \cos 2x$. [2]

(ii) 150

(iii) $0 \leq x < 30$ and $150 < x \leq 180$

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(i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70^\circ) - 8\cos(\theta + 70^\circ) - 7 = 0$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [4]

(ii) 61.8, 158.2

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(i) Show that the equation $2\tan^2\theta\sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\sin^2\theta = 1$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

(ii) 45, 135, 225, 315

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(i) Prove the identity $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

(ii) 19.5, 160.5

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(i) Prove the identity $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

(ii) 64.6, 295.4

OCTOBER/NOVEMBER2010 9709/11

(i) Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) 109.5, 250.5

OCTOBER/NOVEMBER2010 9709/12

Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x.$$ [4]

OCTOBER/NOVEMBER2010 9709/13

Solve the equation $15\sin^2 x = 13 + \cos x$ for $0° \leqslant x \leqslant 180°$. [4]

113.6, 70.5

OCTOBER/NOVEMBER2010 9709/13

(i) Sketch the curve $y = 2\sin x$ for $0 \leqslant x \leqslant 2\pi$. [1]

(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

$$2\pi \sin x = \pi - x.$$

State the equation of the straight line. [3]

(ii) 3 roots

MAY/JUNE2010 9709/11

The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, [1]

(iii) $\sin x$. [2]

(i) -k

(ii) $\frac{1}{k}$

(iii) $\frac{k}{\sqrt{1+k^2}}$

MAY/JUNE2010 9709/11

The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b. [2]

(ii) State the greatest and least values of $f(x)$. [2]

(iii) Solve the equation $f(x) + 1 = 0$. [3]

(i) $2 - 5cos^2x$

(ii) -3, 2

(iii) 0.685, 2.46

MAY/JUNE2010 9709/12

(i) Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

(ii) 143.1, 323.1

MAY/JUNE2010 9709/13

(i) Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

(ii) 120, 240

OCTOBER/NOVEMBER2009 9709/11

Solve the equation $3\tan(2x + 15°) = 4$ for $0° \leqslant x \leqslant 180°$. [4]

19.1, 109.1

OCTOBER/NOVEMBER2009 9709/11

The equation of a curve is $y = 3\cos 2x$. The equation of a line is $x + 2y = \pi$. On the same diagram, sketch the curve and the line for $0 \leqslant x \leqslant \pi$. [4]

OCTOBER/NOVEMBER2009 9709/12

(i) Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. [3]

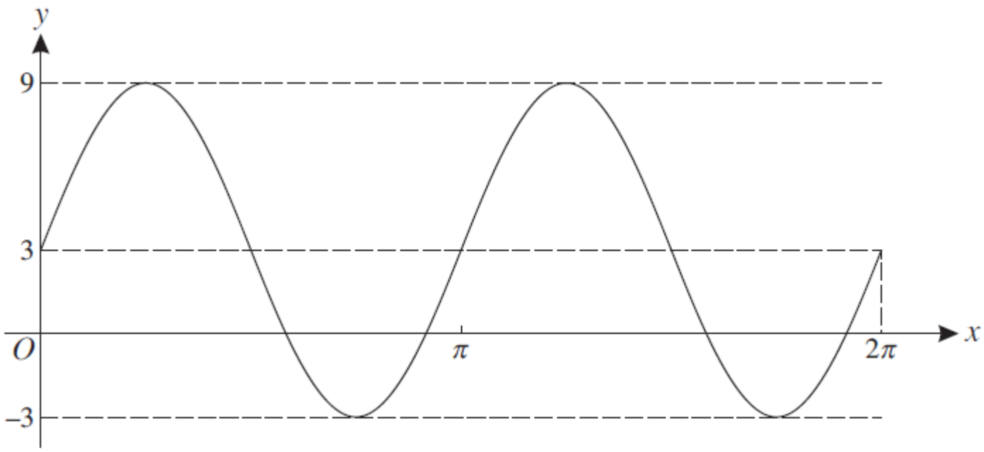
(ii) Solve the equation $(\sin x + \cos x)(1 - \sin x \cos x) = 9 \sin^3 x$ for $0° \leqslant x \leqslant 360°$. [3]

(ii) 26.6, 206.6

MAY/JUNE2009

Prove the identity $\dfrac{\sin x}{1 - \sin x} - \dfrac{\sin x}{1 + \sin x} \equiv 2 \tan^2 x$. [3]

MAY/JUNE2009



The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$.

(i) Find the values of a, b and c. [3]

(ii) Find the smallest value of x in the interval $0 \leqslant x \leqslant 2\pi$ for which $y = 0$. [3]

(i) 6, 2, 3

(ii) $\frac{7}{12}\pi$

OCTOBER/NOVEMBER2008

Prove the identity

$$\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \equiv \frac{2}{\cos x}. \qquad [4]$$

OCTOBER/NOVEMBER2008

The function f is such that $f(x) = a - b\cos x$ for $0° \leqslant x \leqslant 360°$, where a and b are positive constants. The maximum value of $f(x)$ is 10 and the minimum value is -2.

(i) Find the values of a and b. [3]

(ii) Solve the equation $f(x) = 0$. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

(i) 4, 6

(ii) 48.2, 311.8

MAY/JUNE2008

In the triangle ABC, $AB = 12$ cm, angle $BAC = 60°$ and angle $ACB = 45°$. Find the exact length of BC. [3]

$6\sqrt{6}$

MAY/JUNE2008

(i) Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

(ii) 60, 300

OCTOBER/NOVEMBER2007

(i) Show that the equation $3\sin x \tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

(ii) Hence solve the equation $3\sin x \tan x = 8$ for $0° \leqslant x \leqslant 360°$. [3]

(ii) 70.5, 289.5

MAY/JUNE2007

Prove the identity $\dfrac{1-\tan^2 x}{1+\tan^2 x} \equiv 1 - 2\sin^2 x$. [4]

MAY/JUNE2007

The function f is defined by $f(x) = a + b\cos 2x$, for $0 \leqslant x \leqslant \pi$. It is given that $f(0) = -1$ and $f(\frac{1}{2}\pi) = 7$.

(i) Find the values of a and b. **[3]**

(ii) Find the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis. **[3]**

(iii) Sketch the graph of $y = f(x)$. **[2]**

(i) 3, -4
(ii) 0.36, 2.78

OCTOBER/NOVEMBER2006

Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

(i) $\frac{21}{25}$
(ii) $\frac{4}{21}$

MAY/JUNE2006

Solve the equation

$$\sin 2x + 3\cos 2x = 0,$$

for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

54.2, 144.2

OCTOBER/NOVEMBER2005

Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0^\circ \leqslant \theta \leqslant 180^\circ$. [4]

90, 131.8

MAY/JUNE2005

(i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. [2]

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [2]

(ii) 71.6, 251.6

OCTOBER/NOVEMBER2004

(i) Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

(ii) 2 points of intersection

OCTOBER/NOVEMBER2004

The function $f : x \mapsto 5\sin^2 x + 3\cos^2 x$ is defined for the domain $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\sin^2 x$, stating the values of a and b. [2]

(ii) Hence find the values of x for which $f(x) = 7\sin x$. [3]

(iii) State the range of f. [2]

(i) $3 + 2sin^2x$

(ii) $\frac{\pi}{6}, \frac{5\pi}{6}$

(iii) $3 \leq f(x) \leq 5$

MAY/JUNE2004

(i) Show that the equation $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ can be written as a quadratic equation in $\tan\theta$. [2]

(ii) Hence, or otherwise, solve the equation in part **(i)** for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

(ii) 45, 104.0

OCTOBER/NOVEMBER2003

(i) Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

(ii) 30, 150, 210, 330

MAY/JUNE2003

Find all the values of x in the interval $0° \leqslant x \leqslant 180°$ which satisfy the equation $\sin 3x + 2\cos 3x = 0$. [4]

38.9, 98.9, 158.9

MAY/JUNE2003

(i) Sketch the graph of the curve $y = 3\sin x$, for $-\pi \leqslant x \leqslant \pi$. [2]

The straight line $y = kx$, where k is a constant, passes through the maximum point of this curve for $-\pi \leqslant x \leqslant \pi$.

(ii) Find the value of k in terms of π. [2]

(iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]

(ii) $\frac{6}{\pi}$

(iii) $(-\frac{\pi}{2}, -3)$

OCTOBER/NOVEMBER2002

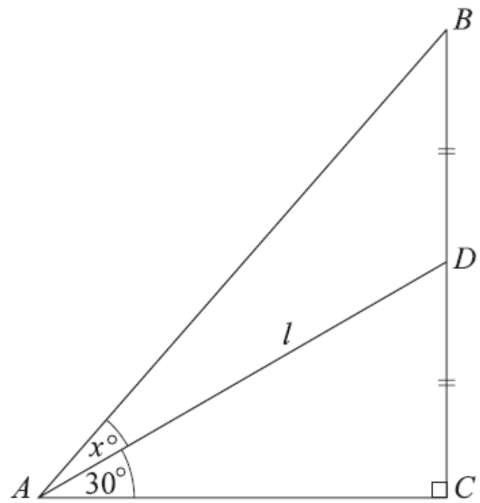
(i) Show that the equation $3\tan\theta = 2\cos\theta$ can be expressed as

$$2\sin^2\theta + 3\sin\theta - 2 = 0.$$ [3]

(ii) Hence solve the equation $3\tan\theta = 2\cos\theta$, for $0° \leqslant \theta \leqslant 360°$. [3]

(ii) 30, 150

OCTOBER/NOVEMBER2002



In the diagram, triangle ABC is right-angled and D is the mid-point of BC. Angle $DAC = 30°$ and angle $BAD = x°$. Denoting the length of AD by l,

(i) express each of AC and BC exactly in terms of l, and show that $AB = \frac{1}{2}l\sqrt{7}$, [4]

(ii) show that $x = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 30$. [2]

(i) $AC = \frac{\sqrt{3}}{2}l, BC = l$

MAY/JUNE2002

(i) Show that $\sin x \tan x$ may be written as $\dfrac{1 - \cos^2 x}{\cos x}$. [1]

(ii) Hence solve the equation $2 \sin x \tan x = 3$, for $0^\circ \leqslant x \leqslant 360^\circ$. [4]

(ii) 60, 300

MAY/JUNE2002

The function f, where $f(x) = a \sin x + b$, is defined for the domain $0 \leqslant x \leqslant 2\pi$. Given that $f\left(\frac{1}{2}\pi\right) = 2$ and that $f\left(\frac{3}{2}\pi\right) = -8$,

(i) find the values of a and b, [3]

(ii) find the values of x for which $f(x) = 0$, giving your answers in radians correct to 2 decimal places, [2]

(iii) sketch the graph of $y = f(x)$. [2]

(i) 5, -3
(ii) 0.64, 2.50

SERIES

MAY/JUNE2012 9709/11

Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$. [4]

$-560x^6$

MAY/JUNE2012 9709/11

(a) The first two terms of an arithmetic progression are 1 and $\cos^2 x$ respectively. Show that the sum of the first ten terms can be expressed in the form $a - b\sin^2 x$, where a and b are constants to be found. [3]

(b) The first two terms of a geometric progression are 1 and $\frac{1}{3}\tan^2\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(i) Find the set of values of θ for which the progression is convergent. [2]

(ii) Find the exact value of the sum to infinity when $\theta = \frac{1}{6}\pi$. [2]

(a) $10 - 45sin^2x$

(b) (i) $0 < \theta < \frac{\pi}{3}$ (ii) $\frac{9}{8}$

MAY/JUNE2012 9709/12

The coefficient of x^3 in the expansion of $(a + x)^5 + (2 - x)^6$ is 90. Find the value of the positive constant a. [5]

5

MAY/JUNE2012 9709/12

(a) In an arithmetic progression, the sum of the first n terms, denoted by S_n, is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

(b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. [5]

(a) 2

(b) 27

MAY/JUNE2012 9709/13

The first three terms in the expansion of $(1-2x)^2(1+ax)^6$, in ascending powers of x, are $1-x+bx^2$. Find the values of the constants a and b. [6]

$\frac{1}{2}, -4\frac{1}{4}$

MAY/JUNE2012 9709/13

The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the nth term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of n. [5]

(i) $\frac{3}{4}$

(ii) $\frac{3}{2}$, 21

OCTOBER/NOVEMBER2011 9709/11

Find the term independent of x in the expansion of $\left(2x + \dfrac{1}{x^2}\right)^6$. [3]

240

OCTOBER/NOVEMBER2011 9709/11

(a) The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]

(b) A geometric progression has first term 1 and common ratio r. A second geometric progression has first term 4 and common ratio $\frac{1}{4}r$. The two progressions have the same sum to infinity, S. Find the values of r and S. [3]

(a) 29

(b) $\frac{4}{5}$, 5

OCTOBER/NOVEMBER2011 9709/12

(a) An arithmetic progression contains 25 terms and the first term is -15. The sum of all the terms in the progression is 525. Calculate

(i) the common difference of the progression, [2]

(ii) the last term in the progression, [2]

(iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

(a) (i) 3
(ii) 57
(iii) 570
(b) (i) 6516
(ii) 56827

OCTOBER/NOVEMBER2011 9709/12

(i) Find the first 3 terms in the expansion of $(2 - y)^5$ in ascending powers of y. [2]

(ii) Use the result in part **(i)** to find the coefficient of x^2 in the expansion of $\left(2 - (2x - x^2)\right)^5$. [3]

(i) $32 - 80y + 80y^2$
(ii) 400

OCTOBER/NOVEMBER2011 9709/13

The coefficient of x^2 in the expansion of $\left(k + \frac{1}{3}x\right)^5$ is 30. Find the value of the constant k. [3]

3

OCTOBER/NOVEMBER2011 9709/13

The first and second terms of a progression are 4 and 8 respectively. Find the sum of the first 10 terms given that the progression is

(i) an arithmetic progression, [2]

(ii) a geometric progression. [2]

(i) 220
(ii) 4092

MAY/JUNE2011 9709/11

Find the coefficient of x in the expansion of $\left(x + \dfrac{2}{x^2}\right)^7$. [3]

MAY/JUNE2011 9709/11

A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

(i) 41000

(ii) 22100

MAY/JUNE2011 9709/12

(i) Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]

(ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k. [2]

(i) $\frac{135}{4}x^2, -\frac{540}{8}x^3$

(ii) 1

MAY/JUNE2011 9709/12

(a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k, [3]

(ii) the sum to infinity of the progression. [2]

(a) 12.1
(b) (i) 12
(ii) 81

MAY/JUNE2011 9709/13

The coefficient of x^3 in the expansion of $(a + x)^5 + (1 - 2x)^6$, where a is positive, is 90. Find the value of a. [5]

5

MAY/JUNE2011 9709/13

(a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

(a) 45

OCTOBER/NOVEMBER2010 9709/11

In the expansion of $(1 + ax)^6$, where a is a constant, the coefficient of x is -30. Find the coefficient of x^3. [4]

-2500

OCTOBER/NOVEMBER2010 9709/11

(a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

(a) 12, 1.5
(b) 64

OCTOBER/NOVEMBER2010 9709/12

(i) Find the first 3 terms in the expansion, in ascending powers of x, of $(1 - 2x^2)^8$. [2]

(ii) Find the coefficient of x^4 in the expansion of $(2 - x^2)(1 - 2x^2)^8$. [2]

(i) $1 - 16x^2 + 112x^4$
(ii) 240

OCTOBER/NOVEMBER2010 9709/12

(a) The first and second terms of an arithmetic progression are 161 and 154 respectively. The sum of the first m terms is zero. Find the value of m. [3]

(b) A geometric progression, in which all the terms are positive, has common ratio r. The sum of the first n terms is less than 90% of the sum to infinity. Show that $r^n > 0.1$. [3]

(a) 47

OCTOBER/NOVEMBER2010 9709/13

Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^9$. [3]

-84

OCTOBER/NOVEMBER2010 9709/13

(a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]

(b) An arithmetic progression has third term 90 and fifth term 80.

(i) Find the first term and the common difference. [2]

(ii) Find the value of m given that the sum of the first m terms is equal to the sum of the first $(m+1)$ terms. [2]

(iii) Find the value of n given that the sum of the first n terms is zero. [2]

(a) 95
(b) (i) 100, -5
(ii) 20
(iii) 41

MAY/JUNE2010 9709/11

(i) Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in descending powers of x. [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$. [2]

(i) $32x^5 - 240x^3 + 720x$
(ii) 240

MAY/JUNE2010 9709/11

The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference. [4]

The nth term of the progression is 46.

(ii) Find the value of n. [2]

(i) a = 10, d = 1.5
(ii) 25

MAY/JUNE2010 9709/12

(i) Find the first 3 terms in the expansion of $(1 + ax)^5$ in ascending powers of x. [2]

(ii) Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a. [2]

(iii) For this value of a, find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$. [3]

(i) $1 + 5ax + 10a^2x^2$
(ii) 0.4
(iii) -2.4

MAY/JUNE2010 9709/12

(a) Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]

(b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

(i) 8200
(ii) (a) 45
(b) 27

MAY/JUNE2010 9709/13

The first term of a geometric progression is 12 and the second term is -6. Find

(i) the tenth term of the progression, [3]

(ii) the sum to infinity. [2]

(i) $-\frac{3}{128}$
(ii) 8

MAY/JUNE2010 9709/13

(i) Find the first three terms, in descending powers of x, in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1 + x^2)\left(x - \frac{2}{x}\right)^6$. [2]

(i) $x^6 - 12x^4 + 60x^2$
(ii) 48

OCTOBER/NOVEMBER2009 9709/11

(i) Find the first 3 terms in the expansion of $(2 - x)^6$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x + ax^2)(2 - x)^6$ is 48, find the value of the constant a. [3]

(i) $64 - 192x + 240x^2$
(ii) 3

OCTOBER/NOVEMBER2009 9709/11

The first term of an arithmetic progression is 8 and the common difference is d, where $d \neq 0$. The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term, respectively, of a geometric progression whose common ratio is r.

(i) Write down two equations connecting d and r. Hence show that $r = \frac{3}{4}$ and find the value of d. [6]

(ii) Find the sum to infinity of the geometric progression. [2]

(iii) Find the sum of the first 8 terms of the arithmetic progression. [2]

(i) $8 + 4d = 8r, 8 + 7d = 8r^2, d = -\frac{1}{2}$

(ii) 32

(iii) 50

OCTOBER/NOVEMBER2009 9709/12

(i) Find, in terms of the non-zero constant k, the first 4 terms in the expansion of $(k + x)^8$ in ascending powers of x. [3]

(ii) Given that the coefficients of x^2 and x^3 in this expansion are equal, find the value of k. [2]

(i) $k^8 + 8k^7x + 28k^6x^2 + 56k^5x^3$

(ii) 2

OCTOBER/NOVEMBER2009 9709/12

A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases:

(i) the progression is arithmetic, [3]

(ii) the progression is geometric with a positive common ratio. [3]

(i) 117
(ii) 128

MAY/JUNE2009

(i) Find the first 3 terms in the expansion of $(2 + 3x)^5$ in ascending powers of x. [3]

(ii) Hence find the value of the constant a for which there is no term in x^2 in the expansion of $(1 + ax)(2 + 3x)^5$. [2]

(i) $32 + 240x + 720x^2$
(ii) -3

MAY/JUNE2009

(a) Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5^3 and 0.5^5. [3]

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression. [4]

(a) $\frac{2}{3}$

(b) 5150

OCTOBER/NOVEMBER2008

Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^6$. [3]

3.75

OCTOBER/NOVEMBER2008

The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n. [4]

8

MAY/JUNE2008

(i) Find the first 3 terms in the expansion, in ascending powers of x, of $(2 + x^2)^5$. [3]

(ii) Hence find the coefficient of x^4 in the expansion of $(1 + x^2)^2(2 + x^2)^5$. [3]

(i) $32 + 80x^2 + 80x^4$
(ii) 272

MAY/JUNE2008

The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

(i) $\frac{2}{3}$

(ii) 243

(iii) 270

OCTOBER/NOVEMBER2007

(i) Find the first three terms in the expansion of $(2+u)^5$ in ascending powers of u. [3]

(ii) Use the substitution $u = x + x^2$ in your answer to part (i) to find the coefficient of x^2 in the expansion of $(2 + x + x^2)^5$. [2]

(i) $32 + 80u + 80u^2$

(ii) 160

OCTOBER/NOVEMBER2007

The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. [1]

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

(ii) Show that $3a = 8d$. [3]

(iii) Find the common ratio of the geometric progression. [2]

(i) $a + 4d, a + 14d$
(iii) 2.5

MAY/JUNE2007

The second term of a geometric progression is 3 and the sum to infinity is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

(i) 6
(ii) -450

OCTOBER/NOVEMBER2006

Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

60

OCTOBER/NOVEMBER2006

(a) Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

(a) 10836
(b) (i) 96
(ii) 432

MAY/JUNE2006

Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was $5000. Find

(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

(i) 8144
(ii) 71034

MAY/JUNE2006

The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b. [5]

5, -0.5, 20

OCTOBER/NOVEMBER2005

A small trading company made a profit of \$250 000 in the year 2000. The company considered two different plans, plan A and plan B, for increasing its profits.

Under plan A, the annual profit would increase each year by 5% of its value in the preceding year. Find, for plan A,

(i) the profit for the year 2008, [3]

(ii) the total profit for the 10 years 2000 to 2009 inclusive. [2]

Under plan B, the annual profit would increase each year by a constant amount \$$D$.

(iii) Find the value of D for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for both plans. [3]

(i) 369000
(ii) 3140000
(iii) 14300

MAY/JUNE2005

(i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1+kx)(2-x)^6$. [2]

(i) $64-192x+240x^2$
(ii) 1.25

MAY/JUNE2005

A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression. [6]

175, 205

OCTOBER/NOVEMBER2004

Find the coefficient of x in the expansion of $\left(3x - \dfrac{2}{x}\right)^5$. [4]

1080

OCTOBER/NOVEMBER2004

Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ..., [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

(i) 239
(ii) 3280

MAY/JUNE2004

A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, [2]

(ii) the sum of the first ten terms. [2]

(i) $\frac{3}{4}$

(ii) 242

MAY/JUNE2004

Find the coefficient of x^3 in the expansion of

(i) $(1 + 2x)^6$, [3]

(ii) $(1 - 3x)(1 + 2x)^6$. [3]

(i) 160

(ii) -20

OCTOBER/NOVEMBER2003

(a) A debt of $3726 is repaid by weekly payments which are in arithmetic progression. The first payment is $60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

(b) Find the sum to infinity of the geometric progression whose first term is 6 and whose second term is 4. [3]

(a) 61.50
(b) 18

MAY/JUNE2003

Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [3]

-40

MAY/JUNE2003

In an arithmetic progression, the 1st term is -10, the 15th term is 11 and the last term is 41. Find the sum of all the terms in the progression. [5]

542.5

OCTOBER/NOVEMBER2002

Find the value of the term which is independent of x in the expansion of $\left(x+\frac{3}{x}\right)^4$. [3]

54

OCTOBER/NOVEMBER2002

A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression, [3]

(ii) the sum to infinity of the progression. [2]

(i) 27.0, $\frac{2}{3}$

(ii) 81.0

MAY/JUNE2002

A progression has a first term of 12 and a fifth term of 18.

(i) Find the sum of the first 25 terms if the progression is arithmetic. [3]

(ii) Find the 13th term if the progression is geometric. [4]

(i) 750

(ii) 40.5 or 40.6

VECTORS

MAY/JUNE2012 9709/11

Two vectors **u** and **v** are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.

(i) Find the values of p for which **u** is perpendicular to **v**. [3]

(ii) For the case where $p = 1$, find the angle between the directions of **u** and **v**. [4]

(i) -4

(ii) 30.0°

MAY/JUNE2012 9709/12

(i) Find the angle between the vectors $3\mathbf{i} - 4\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. [4]

The vector $\overrightarrow{OA}$ has a magnitude of 15 units and is in the same direction as the vector $3\mathbf{i} - 4\mathbf{k}$. The vector $\overrightarrow{OB}$ has a magnitude of 14 units and is in the same direction as the vector $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

(ii) Express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of **i**, **j** and **k**. [3]

(iii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(i) 31°

(ii) $9i - 12k,\ 4i + 6j - 12k$

(iii) $\frac{1}{\sqrt{61}}\begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix}$

MAY/JUNE2012 9709/11

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of the constant p for which angle $BOC = 90°$. [2]

(i) $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$

(ii) 5

OCTOBER/NOVEMBER2011 9709/11

Relative to an origin O, the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA}.\overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60°$. [4]

(i) $25 + p^2$

(iii) ± 3.87

OCTOBER/NOVEMBER2011 9709/12

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where p is a constant.

(i) Find the value of p for which angle AOB is $90°$. [3]

(ii) In the case where $p = 4$, find the vector which has magnitude 28 and is in the same direction as $\overrightarrow{AB}$. [4]

(i) -8.5

(ii) $-12i + 24j + 8k$

OCTOBER/NOVEMBER2011 9709/13

Relative to an origin O, the position vectors of points A and B are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

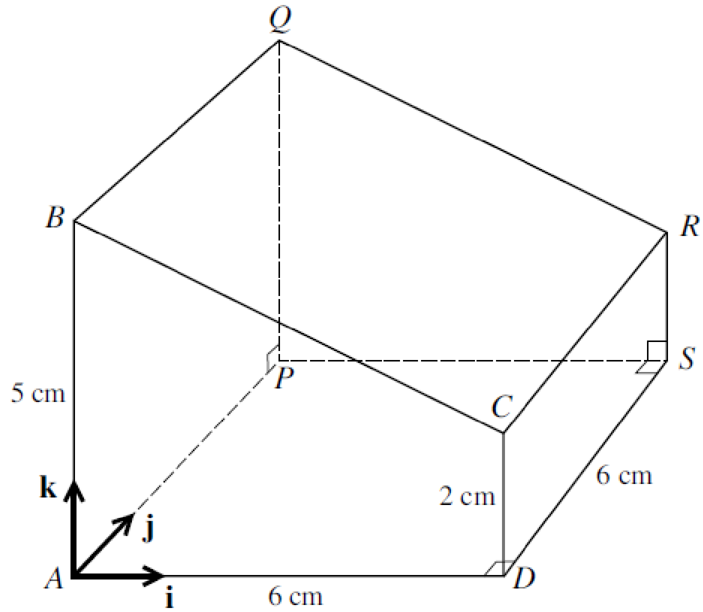
(i) Use a scalar product to find angle BOA. [4]

The point C is the mid-point of AB. The point D is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

(ii) Find $\overrightarrow{DC}$. [4]

(i) 71.4 or 71.5

(ii) $-6i + 5j + 4k$



The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors **i**, **j** and **k** are parallel to AD, AP and AB respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to calculate angle PCQ. [4]

(i) $-6i + 6j - 2k, -6i + 6j + 3k$
(ii) 32.7°

MAY/JUNE2011 9709/12

Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find angle ABC. [6]

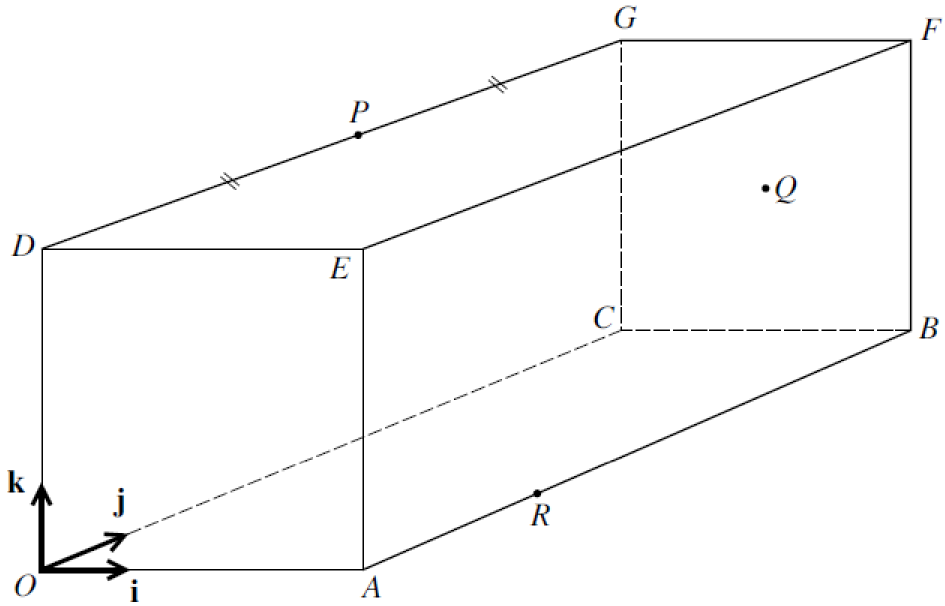
The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D. [2]

(i) 112.4°

(ii) $\begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}$

MAY/JUNE2011 9709/13



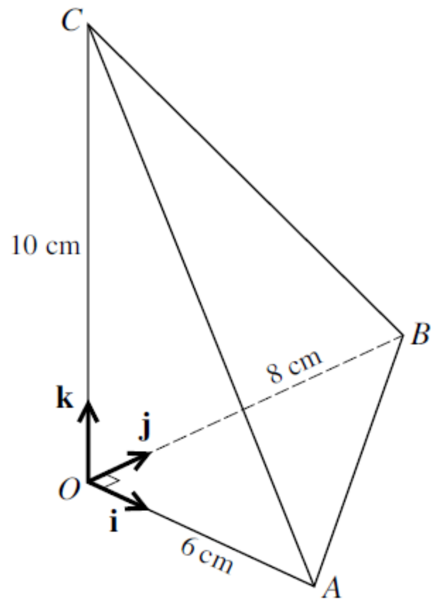
In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle RQP. [4]

(i) $3i + 6j - 3k, -3i + 8j + 3k$
(ii) $63.2°$

OCTOBER/NOVEMBER2010 9709/11

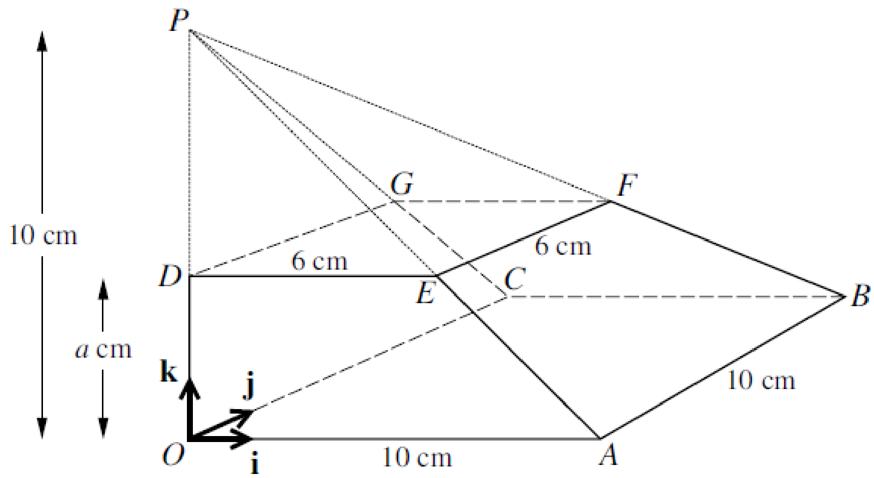


The diagram shows a pyramid $OABC$ with a horizontal base OAB where $OA = 6$ cm, $OB = 8$ cm and angle $AOB = 90°$. The point C is vertically above O and $OC = 10$ cm. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OC as shown.

Use a scalar product to find angle ACB. [6]

48.0°

OCTOBER/NOVEMBER2010 9709/12



The diagram shows a pyramid $OABCP$ in which the horizontal base $OABC$ is a square of side 10 cm and the vertex P is 10 cm vertically above O. The points D, E, F, G lie on OP, AP, BP, CP respectively and $DEFG$ is a horizontal square of side 6 cm. The height of $DEFG$ above the base is a cm. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

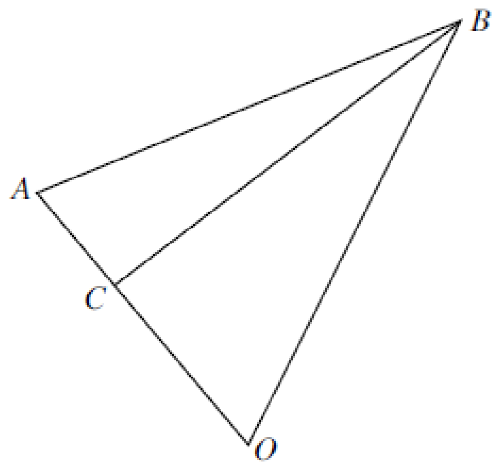
(i) Show that $a = 4$. [2]

(ii) Express the vector $\overrightarrow{BG}$ in terms of **i**, **j** and **k**. [2]

(iii) Use a scalar product to find angle GBA. [4]

(ii) $-10i - 4j + 4k$

(iii) 69.6°



The diagram shows triangle OAB, in which the position vectors of A and B with respect to O are given by

$$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = -3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$

C is a point on OA such that $\overrightarrow{OC} = p\overrightarrow{OA}$, where p is a constant.

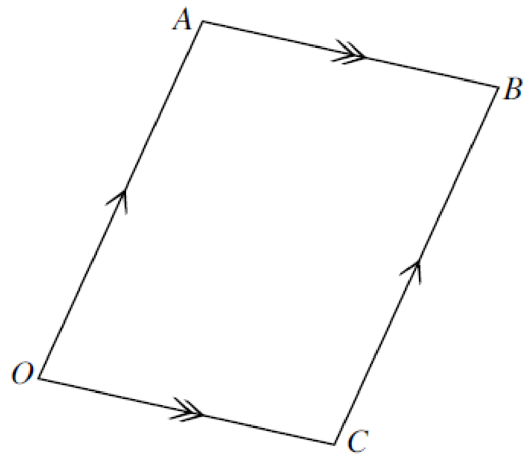
(i) Find angle AOB. [4]

(ii) Find $\overrightarrow{BC}$ in terms of p and vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

(iii) Find the value of p given that BC is perpendicular to OA. [4]

(i) 66.6^{o}
(ii) $3i - 2j + 4k + p(2i + j - 3k)$
(iii) 0.571

MAY/JUNE2010 9709/11



The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find

(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]

(ii) the acute angle between the diagonals of the parallelogram, [5]

(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]

(i) $\frac{1}{6}(4i + 2j + 4k)$
(ii) 74.2°
(iii) 15.4

MAY/JUNE2010 9709/12

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ p \end{pmatrix}.$$

(i) Find the value of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [2]

(ii) Find the values of p for which the magnitude of $\overrightarrow{AB}$ is 7. [4]

(i) 5
(ii) 4, -2

MAY/JUNE2010 9709/13

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}, \quad \overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}.$$

(i) Use a scalar product to find angle ABC. [6]

(ii) Find the perimeter of triangle ABC, giving your answer correct to 2 decimal places. [2]

(i) 63.6°
(ii) 18.32

OCTOBER/NOVEMBER2009 9709/11

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.$$

(i) Find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and has magnitude 30. [3]

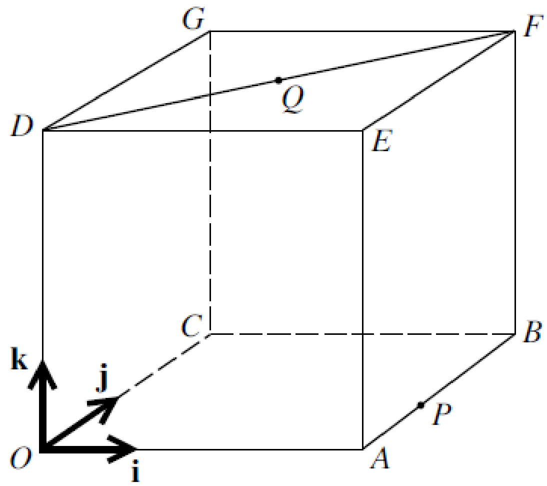
(iii) Find the value of the constant p for which $\overrightarrow{OA} + p\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OC}$. [3]

(i) 160.5°

(ii) $\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}$

(iii) 0.5

OCTOBER/NOVEMBER2009 9709/12



In the diagram, $OABCDEFG$ is a cube in which each side has length 6. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$ and the point Q is the mid-point of DF.

(i) Express each of the vectors $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Find the angle OQP. [4]

(i) $3i + 3j + 6k, -3i + j + 6k$
(ii) 53.0°

MAY/JUNE2009

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}.$$

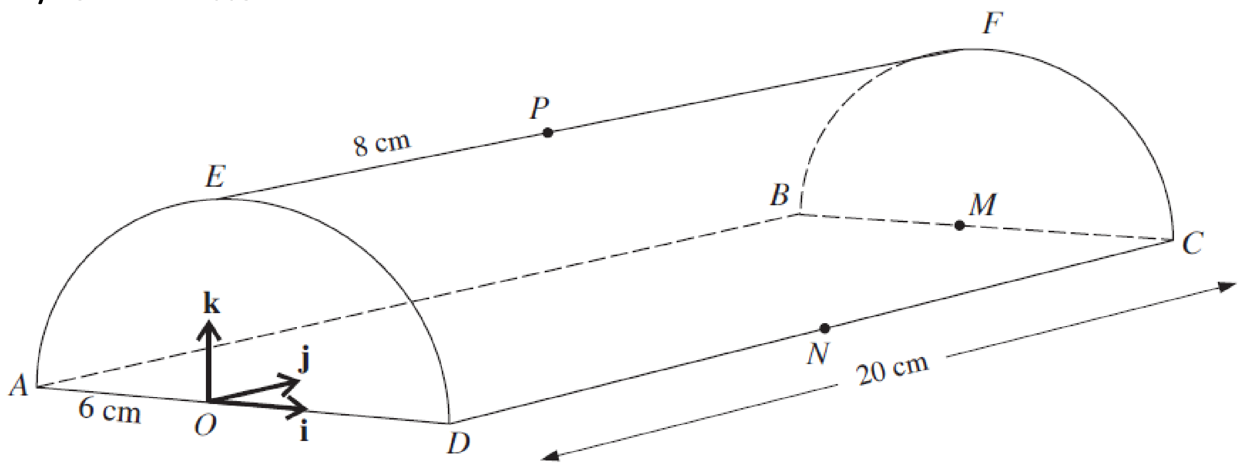
(i) Find the value of $\overrightarrow{OA}.\overrightarrow{OB}$ and hence state whether angle AOB is acute, obtuse or a right angle. [3]

(ii) The point X is such that $\overrightarrow{AX} = \frac{2}{5}\overrightarrow{AB}$. Find the unit vector in the direction of OX. [4]

(i) -6, obtuse angle

(ii) $\frac{1}{6}(4i - 4j + 2k)$

OCTOBER/NOVEMBER2008



The diagram shows a semicircular prism with a horizontal rectangular base $ABCD$. The vertical ends AED and BFC are semicircles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the mid-point of BC is M and the mid-point of DC is N. The points E and F are the highest points of the semicircular ends of the prism. The point P lies on EF such that $EP = 8$ cm.

Unit vectors **i**, **j** and **k** are parallel to OD, OM and OE respectively.

(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to calculate angle APN. [4]

(i) $-6i - 8j - 6k, 6i + 2j - 6k$
(ii) 99°

MAY/JUNE2008

Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

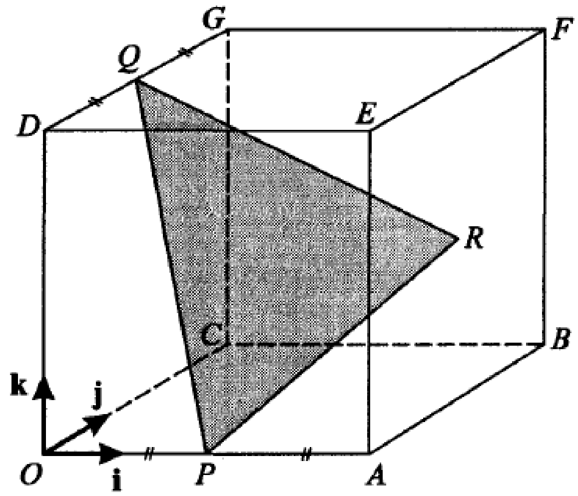
(i) Find the value of p for which OA and OB are perpendicular. [2]

(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree. [3]

(iii) Express the vector $\overrightarrow{AB}$ is terms of p and hence find the values of p for which the length of AB is 3.5 units. [4]

(i) -2
(ii) 40^{o}
(iii) $i - 3j + (p-2)k$, p = 0.5, 3.5

OCTOBER/NOVEMBER2007



The diagram shows a cube $OABCDEFG$ in which the length of each side is 4 units. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.

(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle QPR. [4]

(iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

(i) $2i + 2j + 2k, -2i + 2j + 4k$
(ii) 61.9°
(iii) 12.8

MAY/JUNE2007

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

(i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$. **[4]**

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

(ii) Find the values of m, n and k. **[4]**

(i) $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$

(ii) -2, 3, -8

OCTOBER/NOVEMBER2006

The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

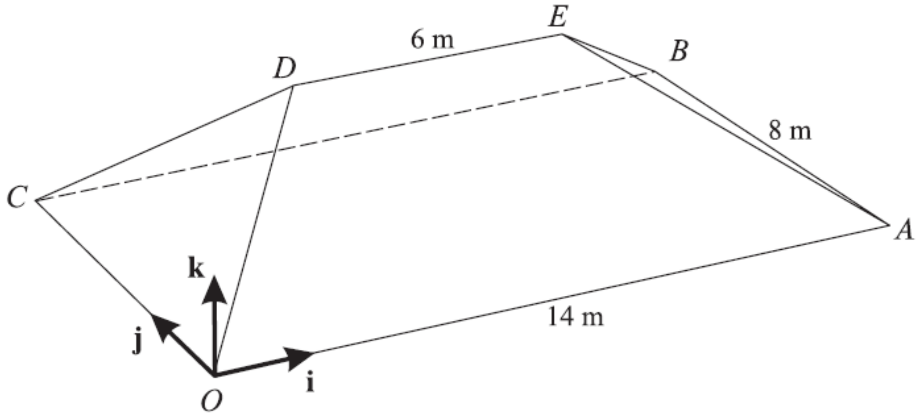
(i) Calculate angle AOB. [3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(i) 36.7°

(ii) $\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k$

MAY/JUNE2006



The diagram shows the roof of a house. The base of the roof, $OABC$, is rectangular and horizontal with $OA = CB = 14$ m and $OC = AB = 8$ m. The top of the roof DE is 5 m above the base and $DE = 6$ m. The sloping edges OD, CD, AE and BE are all equal in length.

Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards.

(i) Express the vector $\overrightarrow{OD}$ in terms of **i**, **j** and **k**, and find its magnitude. [4]

(ii) Use a scalar product to find angle DOB. [4]

(i) $4i + 4k + 5k$
(ii) 43.7°

OCTOBER/NOVEMBER2005

Relative to an origin O, the position vectors of points P and Q are given by

$$\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix},$$

where q is a constant.

(i) In the case where $q = 3$, use a scalar product to show that $\cos POQ = \frac{1}{7}$. [3]

(ii) Find the values of q for which the length of $\overrightarrow{PQ}$ is 6 units. [4]

(ii) -3, 5

MAY/JUNE2005

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree. [4]

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(iii) The point C is such that $\overrightarrow{OC} = 6\mathbf{j} + p\mathbf{k}$, where p is a constant. Given that the lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are equal, find the possible values of p. [4]

(i) 99^{o}

(ii) $\frac{1}{7}(2i - 6j + 3k)$

(iii) -7, 5

OCTOBER/NOVEMBER2004

The points A and B have position vectors $\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $-5\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ respectively, relative to an origin O.

(i) Use a scalar product to calculate angle AOB, giving your answer in radians correct to 3 significant figures. [4]

(ii) The point C is such that $\overrightarrow{AB} = 2\overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(i) 0.907

(ii) $\frac{1}{12}(-8i + 4j + 8k)$

MAY/JUNE2004

Relative to an origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix},$$

where p and q are constants. Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

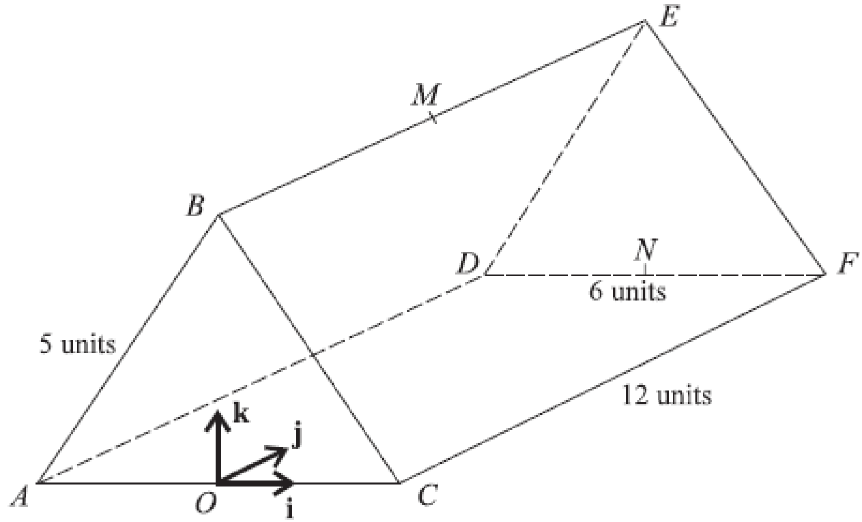
(ii) the value of p for which angle $AOC = 90°$, [3]

(iii) the values of q for which the length of $\overrightarrow{AD}$ is 7 units. [4]

(i) $\frac{1}{6}(2i - 4j + 4k)$

(ii) 10

(iii) -7, 5



The diagram shows a triangular prism with a horizontal rectangular base $ADFC$, where $CF = 12$ units and $DF = 6$ units. The vertical ends ABC and DEF are isosceles triangles with $AB = BC = 5$ units. The mid-points of BE and DF are M and N respectively. The origin O is at the mid-point of AC.

Unit vectors **i**, **j** and **k** are parallel to OC, ON and OB respectively.

(i) Find the length of OB. [1]

(ii) Express each of the vectors $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

(iii) Evaluate $\overrightarrow{MC}.\overrightarrow{MN}$ and hence find angle CMN, giving your answer correct to the nearest degree. [4]

(i) 4
(ii) $3i - 6j - 4k,\ 6j - 4k$
(iii) -20, 111°

MAY/JUNE2003

The points A, B, C and D have position vectors $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $2\mathbf{j} + 7\mathbf{k}$ and $-2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$ respectively.

(i) Use a scalar product to show that BA and BC are perpendicular. [4]

(ii) Show that BC and AD are parallel and find the ratio of the length of BC to the length of AD. [4]

(ii) 2 : 5

OCTOBER/NOVEMBER2002

Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} p \\ p \\ p+1 \end{pmatrix}$, find

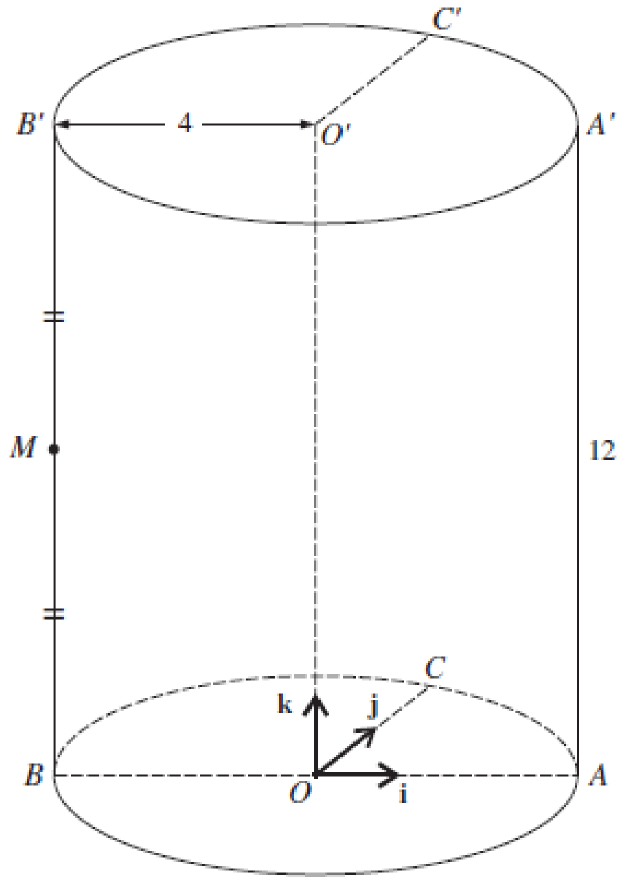
(i) the angle between the directions of $\mathbf{a}$ and $\mathbf{b}$, [4]

(ii) the value of p for which $\mathbf{b}$ and $\mathbf{c}$ are perpendicular. [3]

(i) 103.8^{o}

(ii) $-\frac{3}{11}$

MAY/JUNE2002



The diagram shows a solid cylinder standing on a horizontal circular base, centre O and radius 4 units. The line BA is a diameter and the radius OC is at 90° to OA. Points O', A', B' and C' lie on the upper surface of the cylinder such that OO', AA', BB' and CC' are all vertical and of length 12 units. The mid-point of BB' is M.

Unit vectors **i**, **j** and **k** are parallel to OA, OC and OO' respectively.

(i) Express each of the vectors $\overrightarrow{MO}$ and $\overrightarrow{MC'}$ in terms of **i**, **j** and **k**. [3]

(ii) Hence find the angle OMC'. [4]

(i) $4i-6k, 4i+4j+6k$
(ii) 109.7° (allow 109.6°)

DIFFERENTIATION

MAY/JUNE2012 9709/11

A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius, r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of k and the rate at which the mass is increasing on this day. [5]

0.0032, 0.096

MAY/JUNE2012 9709/11

It is given that a curve has equation $y = f(x)$, where $f(x) = x^3 - 2x^2 + x$.

(i) Find the set of values of x for which the gradient of the curve is less than 5. [4]

(ii) Find the values of $f(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

(i) $-\frac{2}{3} < x < 2$

(ii) max at $\left(\frac{1}{3}, \frac{4}{27}\right)$, min at (1, 0)

MAY/JUNE2012 9709/12

The equation of a curve is $y = 4\sqrt{x} + \dfrac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [3]

(ii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

(i) $\frac{2}{\sqrt{x}} - \frac{1}{x^{1.5}}$

(ii) 0.105

MAY/JUNE2012 9709/13

A curve is such that $\dfrac{d^2y}{dx^2} = -4x$. The curve has a maximum point at $(2, 12)$.

(i) Find the equation of the curve. [6]

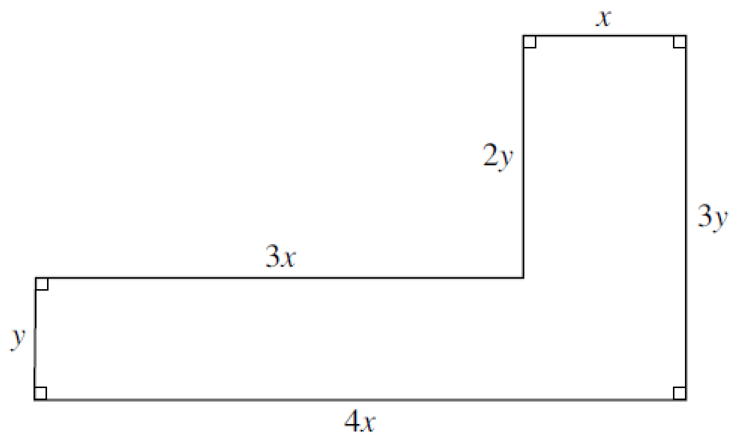
A point P moves along the curve in such a way that the x-coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the y-coordinate is changing when $x = 3$, stating whether the y-coordinate is increasing or decreasing. [2]

(i) $y = -\frac{2x^3}{3} + 8x + \frac{4}{3}$

(ii) decreasing at 0.5 units per second

OCTOBER/NOVEMBER2011 9709/11



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

(i) Find an expression for y in terms of x. [1]

(ii) Given that the area of the garden is A m^2, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

(i) $y = \frac{1}{6(48-8x)}$
(iii) 72

OCTOBER/NOVEMBER2011 9709/11

A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]

OCTOBER/NOVEMBER2011 9709/13

A curve $y = f(x)$ has a stationary point at $P(3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where k is a constant.

(i) Show that $k = -2$ and hence find the x-coordinate of the other stationary point, Q. [4]

(ii) Find $f''(x)$ and determine the nature of each of the stationary points P and Q. [2]

(iii) Find $f(x)$. [4]

(i) -2 (or 3)
(ii) f''(x) = x – 2, min at P, max at Q
(iii) $\frac{2}{3}x^3 - x^2 - 12x + 17$

MAY/JUNE2011 9709/11

The volume of a spherical balloon is increasing at a constant rate of $50\,\text{cm}^3$ per second. Find the rate of increase of the radius when the radius is $10\,\text{cm}$. [Volume of a sphere $= \frac{4}{3}\pi r^3$.] [4]

0.0398

MAY/JUNE2011 9709/11

The variables x, y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \dfrac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

(ii) 120, minimum

MAY/JUNE2011 9709/13

Differentiate $\dfrac{2x^3+5}{x}$ with respect to x. [3]

$4x-\dfrac{5}{x^2}$

OCTOBER/NOVEMBER2010 9709/12

The length, x metres, of a Green Anaconda snake which is t years old is given approximately by the formula

$$x = 0.7\sqrt{(2t-1)},$$

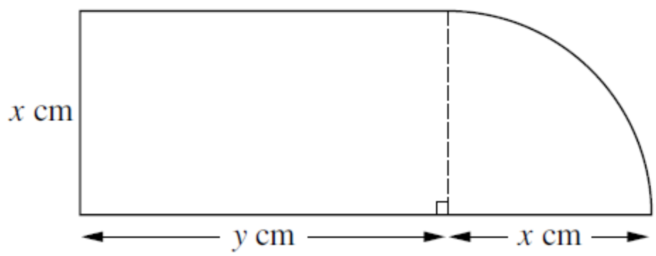
where $1 \leqslant t \leqslant 10$. Using this formula, find

(i) $\dfrac{dx}{dt}$, [2]

(ii) the rate of growth of a Green Anaconda snake which is 5 years old. [2]

(i) $0.7(2t-1)^{-0.5}$
(ii) 0.233

OCTOBER/NOVEMBER2010 9709/11



The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

(i) Express y in terms of x. [2]

(ii) Show that the area of the plate, A cm^2, is given by $A = 30x - x^2$. [2]

Given that x can vary,

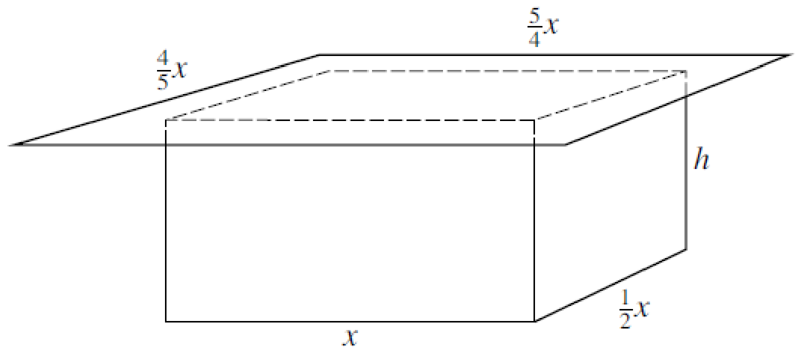
(iii) find the value of x at which A is stationary, [2]

(iv) find this stationary value of A, and determine whether it is a maximum or a minimum value. [2]

(i) $y = 30 - x - \frac{\pi x}{4}$

(iii) 15

(iv) 675, maximum



The diagram shows an open rectangular tank of height h metres covered with a lid. The base of the tank has sides of length x metres and $\frac{1}{2}x$ metres and the lid is a rectangle with sides of length $\frac{5}{4}x$ metres and $\frac{4}{5}x$ metres. When full the tank holds $4\,\text{m}^3$ of water. The material from which the tank is made is of negligible thickness. The external surface area of the tank together with the area of the top of the lid is $A\,\text{m}^2$.

(i) Express h in terms of x and hence show that $A = \frac{3}{2}x^2 + \dfrac{24}{x}$. [5]

(ii) Given that x can vary, find the value of x for which A is a minimum, showing clearly that A is a minimum and not a maximum. [5]

(ii) x = 2

OCTOBER/NOVEMBER2010 9709/13

A curve has equation $y = \dfrac{1}{x-3} + x$.

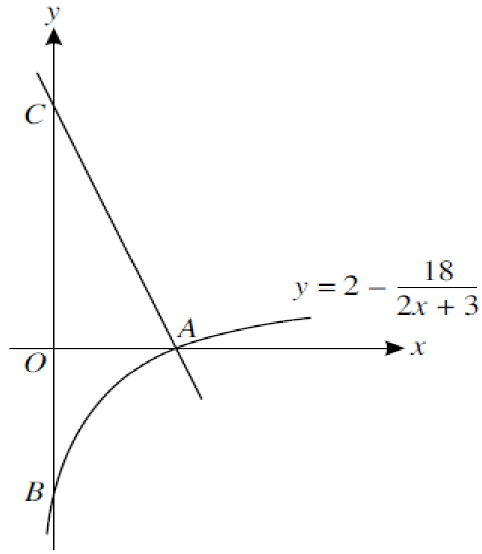
(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [2]

(ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

(i) $-\frac{1}{(x-3)^2} + 1, \frac{2}{(x-3)^3}$

(ii) (4, 5) is minimum, (2, 1) is maximum

MAY/JUNE2010 9709/11



The diagram shows part of the curve $y = 2 - \frac{18}{2x + 3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC. [2]

(ii) 10.75

MAY/JUNE2010 9709/12

A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm^2.

(i) Express h in terms of x and show that the volume, $V\text{ cm}^3$, of the block is given by

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

(ii) 64
(iii) maximum

MAY/JUNE2010 9709/12

The equation of a curve is $y = \frac{1}{6}(2x-3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y-axis. [3]

(iii) Find the set of values of x for which $\frac{1}{6}(2x-3)^3 - 4x$ is an increasing function of x. [3]

(i) $(2x-3)^2 - 4$
(ii) $2y + 9 = 10x$
(iii) x < 0.5, x > 2.5

OCTOBER/NOVEMBER2009 9709/11

The equation of a curve is $y = x^4 + 4x + 9$.

Find the coordinates of the stationary point on the curve and determine its nature. [4]

(-1, 6), minimum

OCTOBER/NOVEMBER2009 9709/11

The equation of a curve is $y = \dfrac{12}{x^2 + 3}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

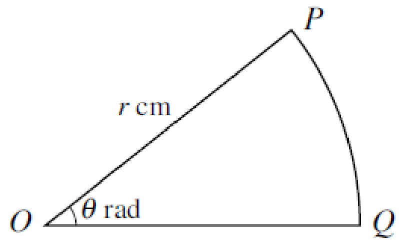
(ii) Find the equation of the normal to the curve at the point $P\,(1, 3)$. [3]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

(i) $-\frac{24x}{(x^2+3)^2}$

(ii) $y = \frac{2}{3}x + \frac{7}{3}$

(iii) -0.018



A piece of wire of length 50 cm is bent to form the perimeter of a sector POQ of a circle. The radius of the circle is r cm and the angle POQ is θ radians (see diagram).

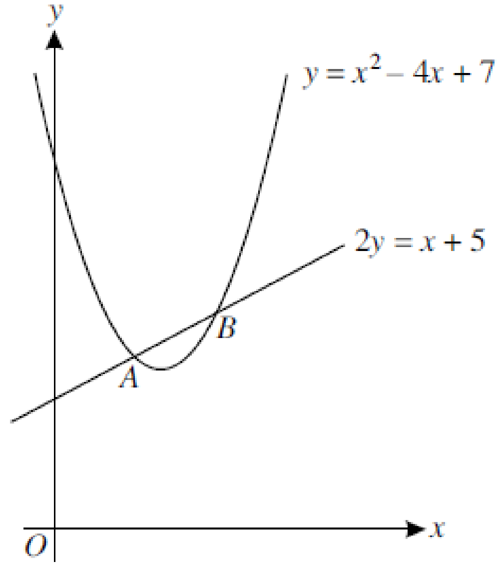
(i) Express θ in terms of r and show that the area, A cm^2, of the sector is given by

$$A = 25r - r^2.$$ [4]

(ii) Given that r can vary, find the stationary value of A and determine its nature. [4]

(ii) 156.25, maximum

OCTOBER/NOVEMBER2009 9709/12



(i) The diagram shows the line $2y = x + 5$ and the curve $y = x^2 - 4x + 7$, which intersect at the points A and B. Find

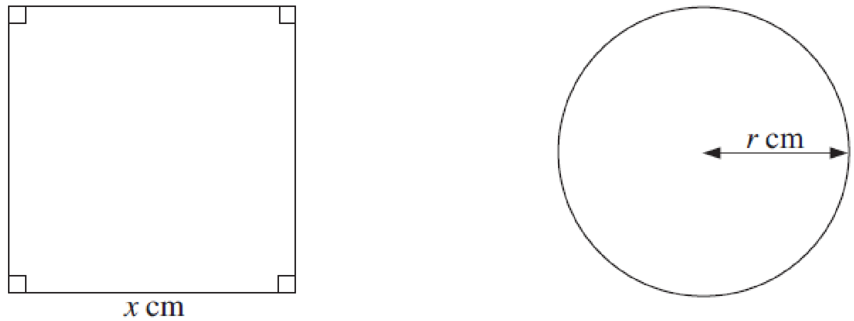
(a) the x-coordinates of A and B, [3]

(b) the equation of the tangent to the curve at B, [3]

(c) the acute angle, in degrees correct to 1 decimal place, between this tangent and the line $2y = x + 5$. [3]

(a) 3, 1.5
(ii) $y = 2x - 2$
(ii) 37°

OCTOBER/NOVEMBER2008



A wire, 80 cm long, is cut into two pieces. One piece is bent to form a square of side x cm and the other piece is bent to form a circle of radius r cm (see diagram). The total area of the square and the circle is A cm^2.

(i) Show that $A = \dfrac{(\pi + 4)x^2 - 160x + 1600}{\pi}$. [4]

(ii) Given that x and r can vary, find the value of x for which A has a stationary value. [4]

(ii) 11.2

OCTOBER/NOVEMBER2008

The equation of a curve is $y = 5 - \dfrac{8}{x}$.

(i) Show that the equation of the normal to the curve at the point $P\,(2,\ 1)$ is $2y + x = 4$. [4]

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. [3]

(iii) Find the length of PQ. [2]

(ii) (-8, 6)
(iii) 11.2

MAY/JUNE2008

The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

(i) 1.5, 2

OCTOBER/NOVEMBER2007

The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

(i) $6(2x-3)^2-6,\ 24(2x-3)$

(ii) x = 2 → minimum, x = 1 → maximum

MAY/JUNE2007

The equation of a curve is $y = 2x + \dfrac{8}{x^2}$.

(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

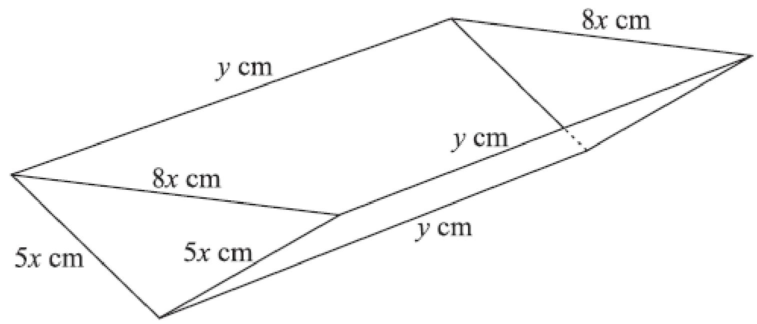
(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. [3]

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. [3]

(i) $2 - \frac{16}{x^3}, \frac{48}{x^4}$

(ii) (2, 6) minimum

OCTOBER/NOVEMBER2006



The diagram shows an open container constructed out of $200\,\text{cm}^2$ of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

(ii) Show that the volume, $V\,\text{cm}^3$, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

(iii) $1\frac{2}{3}$
(iv) maximum

OCTOBER/NOVEMBER2006

The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(i) $\frac{1}{3}$

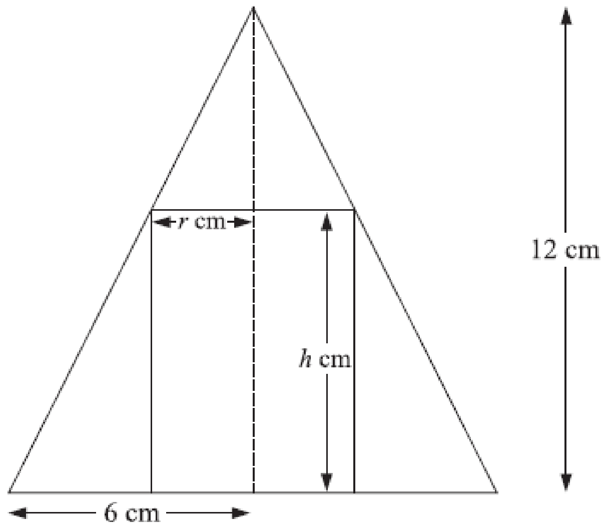
(ii) 0.015

MAY/JUNE2006

A curve has equation $y = \dfrac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k. [3]

12

OCTOBER/NOVEMBER2005



The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]

(ii) 64π

MAY/JUNE2005

Find the gradient of the curve $y = \dfrac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

$-\dfrac{8}{3}$

OCTOBER/NOVEMBER2004

A curve has equation $y = x^2 + \dfrac{2}{x}$.

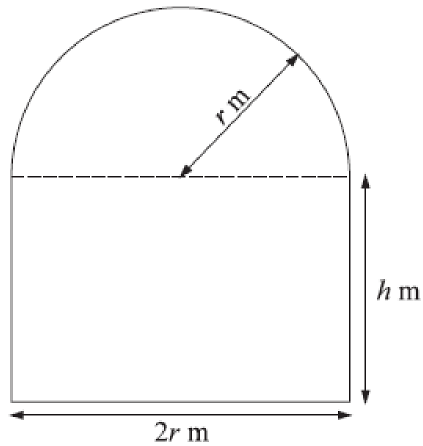
(i) Write down expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(i) $2x - \frac{2}{x^2},\ 2 + \frac{4}{x^3}$

(ii) (1, 3) minimum

MAY/JUNE2004



The diagram shows a glass window consisting of a rectangle of height h m and width $2r$ m and a semicircle of radius r m. The perimeter of the window is 8 m.

(i) Express h in terms of r. [2]

(ii) Show that the area of the window, A m^2, is given by

$$A = 8r - 2r^2 - \tfrac{1}{2}\pi r^2.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value is a maximum or a minimum. [2]

(i) $h = 4 - r - \frac{1}{2}\pi r$

(iii) 1.12

(iv) maximum

OCTOBER/NOVEMBER2003

A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is 72 cm^3.

(i) Express y in terms of x and show that the total surface area, $A\text{ cm}^2$, of the block is given by

$$A = 4x^2 + \frac{216}{x}. \qquad [3]$$

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

(ii) 3
(iii) 108, minimum

MAY/JUNE2003

Differentiate $4x + \dfrac{6}{x^2}$ with respect to x. [2]

$4 - \dfrac{12}{x^3}$

MAY/JUNE2003

The equation of a curve is $y = \sqrt{(5x + 4)}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of x has the constant value 0.03 units per second. Find the rate of increase of y at the instant when $x = 1$. [2]

(i) $\frac{5}{6}$

(ii) 0.025

OCTOBER/NOVEMBER2002

A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]

(i) $3x^2 + 6x - 9$
(ii) -3, 1
(iii) -27, 5

MAY/JUNE2002

A hollow circular cylinder, open at one end, is constructed of thin sheet metal. The total external surface area of the cylinder is 192π cm^2. The cylinder has a radius of r cm and a height of h cm.

(i) Express h in terms of r and show that the volume, V cm^3, of the cylinder is given by

$$V = \tfrac{1}{2}\pi(192r - r^3).$$ [4]

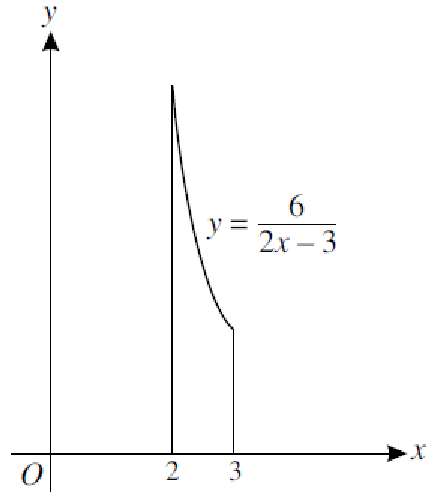
Given that r can vary,

(ii) find the value of r for which V has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

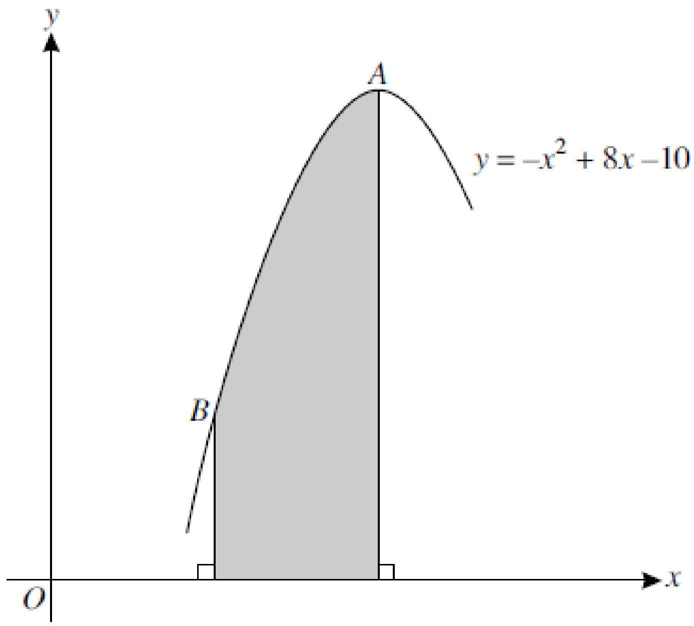
(ii) 8
(iii) 512π, maximum

INTEGRATION



The diagram shows the region enclosed by the curve $y = \frac{6}{2x-3}$, the x-axis and the lines $x = 2$ and $x = 3$. Find, in terms of π, the volume obtained when this region is rotated through 360° about the x-axis. [4]

12π



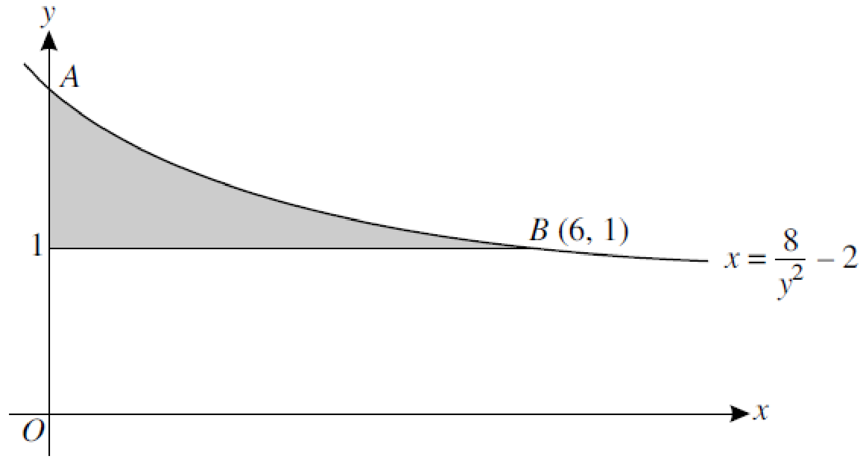
The diagram shows part of the curve $y = -x^2 + 8x - 10$ which passes through the points A and B. The curve has a maximum point at A and the gradient of the line BA is 2.

(i) Find the coordinates of A and B. [7]

(ii) Find $\int y\,dx$ and hence evaluate the area of the shaded region. [4]

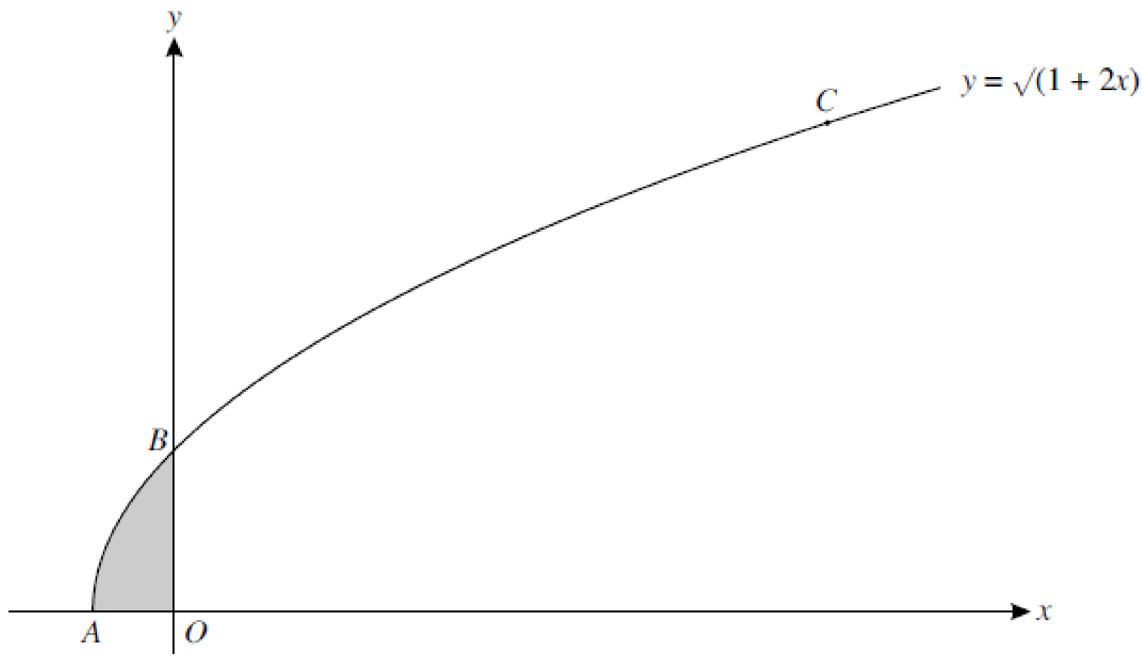
(i) (4, 6), (2, 2)
(ii) $9\frac{1}{3}$

MAY/JUNE2012 9709/13



The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y-axis at the point A. The point $B\,(6,\ 1)$ lies on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the ***y*-axis**. [6]

$6\frac{2}{3}\pi$



The diagram shows the curve $y = \sqrt{(1 + 2x)}$ meeting the x-axis at A and the y-axis at B. The y-coordinate of the point C on the curve is 3.

(i) Find the coordinates of B and C. [2]

(ii) Find the equation of the normal to the curve at C. [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [5]

(i) (0, 1), (4, 3)
(ii) y = -3x + 15
(iii) $\frac{2}{15}\pi$

OCTOBER/NOVEMBER2011 9709/11

A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geqslant -4$.

(i) State the value of x for which $f(x)$ has a stationary value. [1]

(ii) Find an expression for $f(x)$ in terms of x. [4]

(i) 3

(ii) $x^2 - 6x + 5$

OCTOBER/NOVEMBER2011 9709/12

A curve is such that $\frac{dy}{dx} = 5 - \frac{8}{x^2}$. The line $3y + x = 17$ is the normal to the curve at the point P on the curve. Given that the x-coordinate of P is positive, find

(i) the coordinates of P, [4]

(ii) the equation of the curve. [4]

(i) (2, 5)

(ii) $y = 5x + \frac{8}{x} - 9$

OCTOBER/NOVEMBER2011 9709/12

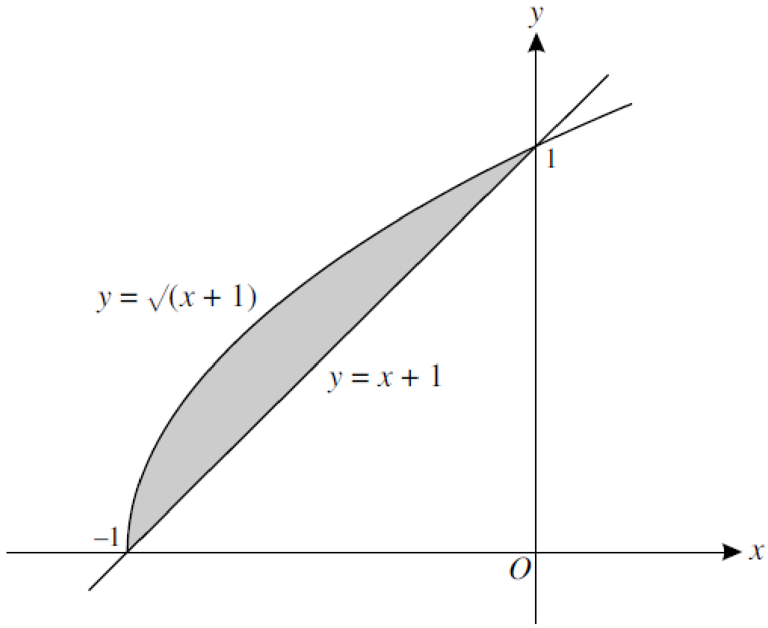
The equation of a curve is $y = \sqrt{(8x - x^2)}$. Find

(i) an expression for $\frac{dy}{dx}$, and the coordinates of the stationary point on the curve, [4]

(ii) the volume obtained when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. [4]

(i) (4, 4)
(ii) $\frac{256}{3}\pi$

OCTOBER/NOVEMBER2011 9709/13



The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{(x + 1)}$, meeting at $(-1, 0)$ and $(0, 1)$.

(i) Find the area of the shaded region. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [7]

(i) $\frac{1}{6}$

(ii) $\frac{1}{5}\pi$

MAY/JUNE2011 9709/11

(i) Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

(ii) 6.4π

MAY/JUNE2011 9709/11

A curve is such that $\frac{dy}{dx} = \frac{3}{(1 + 2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

(i) $y = -\frac{3}{2(1+2x)} + 1$

(ii) x > 1, x < -2

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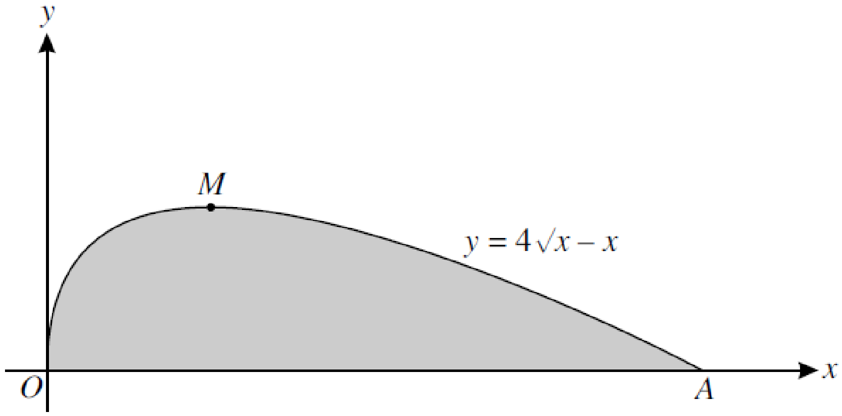


The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x-axis at O and A.

(i) Find the coordinates of A and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [6]

(i) (16, 0), (4, 4)
(ii) 136.5π

MAY/JUNE2011 9709/12

Find $\int \left(x^3 + \frac{1}{x^3}\right) dx.$ [3]

$\frac{x^4}{4} - \frac{x^2}{2} + c$

MAY/JUNE2011 9709/13

Find $\int (3x-2)^5 \, dx$ and hence find the value of $\int_0^1 (3x-2)^5 \, dx.$ [4]

$\frac{(3x-2)^6}{18} + c$, -3.5

MAY/JUNE2011 9709/13

A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ and $P\,(9,\ 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\frac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1} k$ with the positive x-axis. Find the value of k. [2]

(i) $y = 4\sqrt{x} - x + 2$
(ii) (4, 6)
(iii) $-\frac{1}{x^{\frac{3}{2}}}$, maximum
(iv) 3

OCTOBER/NOVEMBER2010 9709/11

Find $\int \left(x + \frac{1}{x}\right)^2 \mathrm{d}x.$ [3]

$\frac{x^3}{3} - \frac{1}{x} + 2x + c$

OCTOBER/NOVEMBER2010 9709/11

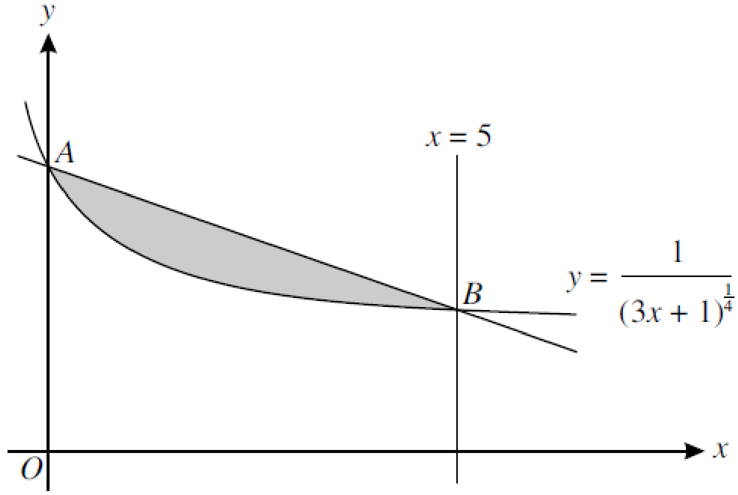
The equation of a curve is $y = \frac{9}{2-x}$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through 360° about the x-axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

(i) $\frac{9}{(x-2)^2}, \frac{dy}{dx} \neq 0$, no turning points.
(ii) 40.5π
(iii) k < −8 , k > 4



The diagram shows part of the curve $y = \dfrac{1}{(3x+1)^{\frac{1}{4}}}$. The curve cuts the y-axis at A and the line $x = 5$ at B.

(i) Show that the equation of the line AB is $y = -\frac{1}{10}x + 1$. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [9]

(ii) $\frac{11}{12}\pi$

OCTOBER/NOVEMBER2010 9709/13

A curve has equation $y = f(x)$. It is given that $f'(x) = 3x^2 + 2x - 5$.

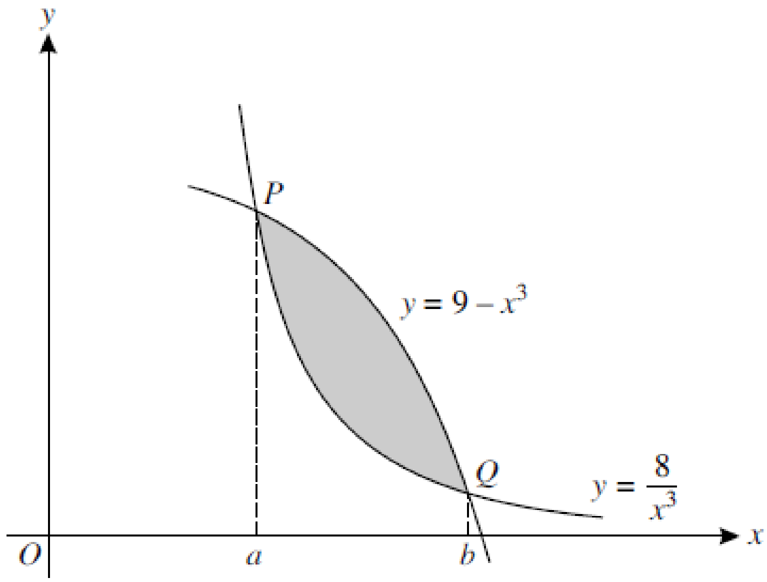
(i) Find the set of values of x for which f is an increasing function. [3]

(ii) Given that the curve passes through $(1, 3)$, find $f(x)$. [4]

(i) $x < -5/3$, $x > 1$

(ii) $f(x) = x^3 + x^2 - 5x + 6$

OCTOBER/NOVEMBER2010 9709/13



The diagram shows parts of the curves $y = 9 - x^3$ and $y = \dfrac{8}{x^3}$ and their points of intersection P and Q. The x-coordinates of P and Q are a and b respectively.

(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b. [4]

(ii) Find the area of the shaded region between the two curves. [5]

(iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of c. [4]

(i) a = 1, b = 2
(ii) 2.25
(iii) $\sqrt{2}$

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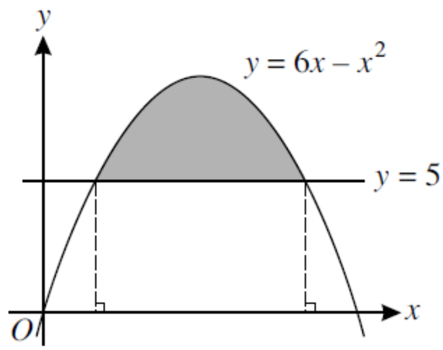


The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

$10\frac{2}{3}$

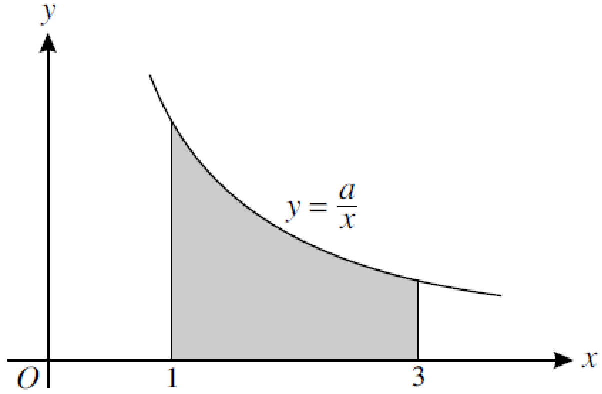
MAY/JUNE2010 9709/11

A curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point $(9, 2)$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

(i) $y = 2x^{\frac{3}{2}} - 6x + 2$
(ii) x = 4, minimum



The diagram shows part of the curve $y = \frac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x-axis is 24π, find the value of a. [4]

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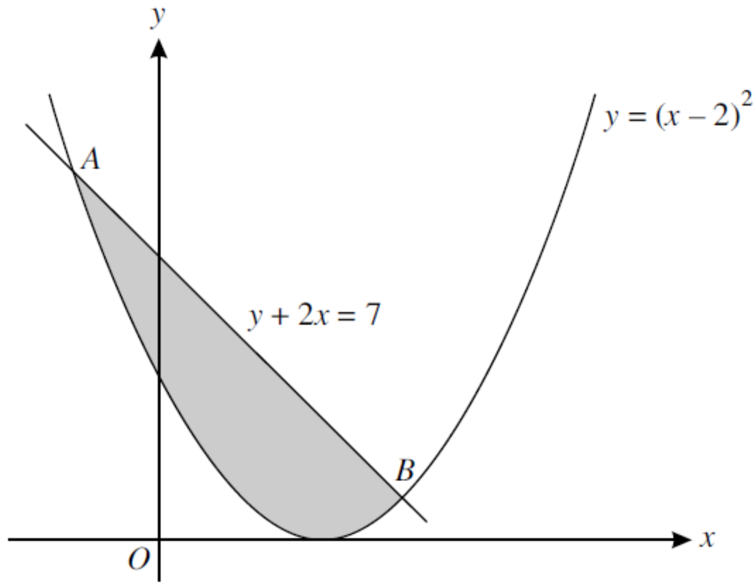

The diagram shows the curve $y = (x-2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region. [8]

$10\frac{2}{3}$

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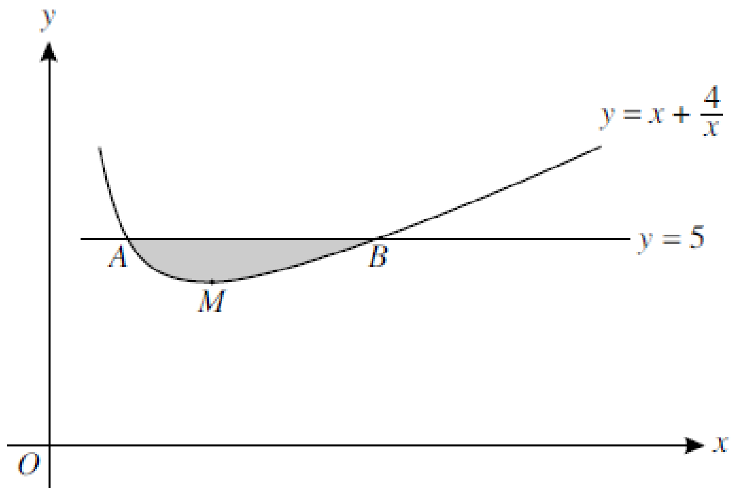


The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M. The line $y = 5$ intersects the curve at the points A and B.

(i) Find the coordinates of A, B and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

(i) (1, 5), (4, 5), (2, 4)
(ii) 18π

MAY/JUNE2010 9709/13

The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(3x-2)}}$. Given that the curve passes through the point $P(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

(i) $y = -\frac{1}{3}x + \frac{35}{3}$

(ii) $y = 4\sqrt{3x-2} + 3$

OCTOBER/NOVEMBER2009 9709/11

The equation of a curve is $y = x^4 + 4x + 9$.

Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]

11.2

OCTOBER/NOVEMBER2009 9709/11

A curve is such that $\frac{dy}{dx} = k - 2x$, where k is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 3$ are perpendicular, find the value of k. [4]

(ii) Given also that the curve passes through the point (4, 9), find the equation of the curve. [3]

(i) 5

(ii) $y = 5x - x^2 + 5$

OCTOBER/NOVEMBER2009 9709/12

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$. Given that the curve passes through the point (4, 6), find the equation of the curve. [4]

$y = 6\sqrt{x} - \frac{x^2}{2} + 2$

OCTOBER/NOVEMBER2009 9709/12

The function f is such that $f(x) = \frac{3}{2x+5}$ for $x \in \mathbb{R}$, $x \neq -2.5$.

(i) Obtain an expression for $f'(x)$ and explain why f is a decreasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$. [2]

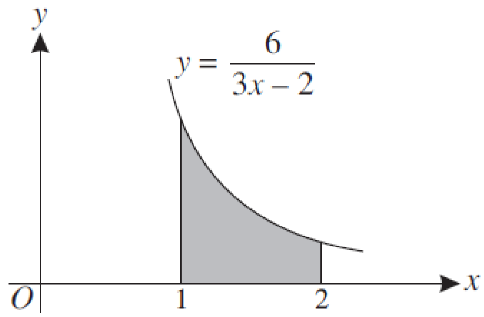
(iii) A curve has the equation $y = f(x)$. Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 2$ is rotated through 360° about the x-axis. [4]

(i) $f'(x) = -6(2x+5)^{-2}$, always negative → decreasing

(ii) $f^{-1}(x) = \frac{3-5x}{2x}$

(iii) 1.26

MAY/JUNE2009



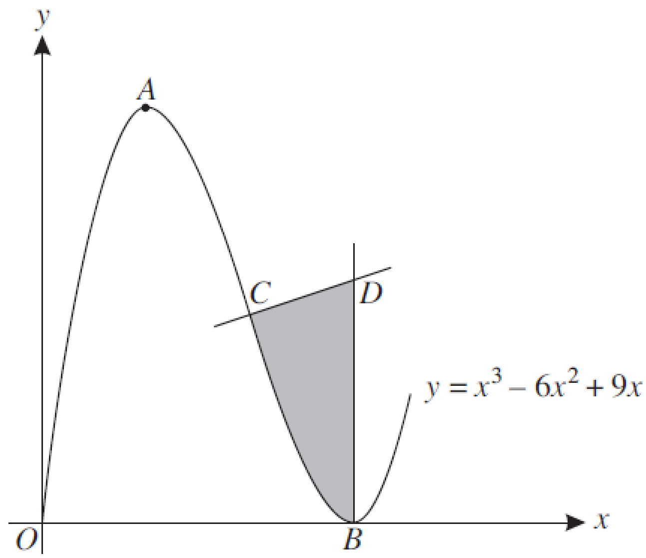
The diagram shows part of the curve $y = \dfrac{6}{3x-2}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [5]

(i) -1.125
(ii) 9π

MAY/JUNE2009



The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at $C\,(2,\ 2)$ meets the normal to the curve at B at the point D.

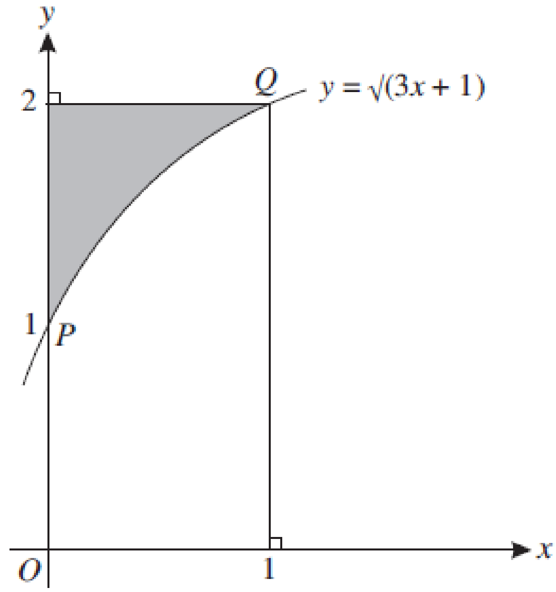
(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

(iii) Find the area of the shaded region. [5]

(i) (1, 4), (3, 0)
(ii) $3y = x + 4$
(iii) $1\frac{5}{12}$

OCTOBER/NOVEMBER2008



The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P\,(0, 1)$ and $Q\,(1, 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

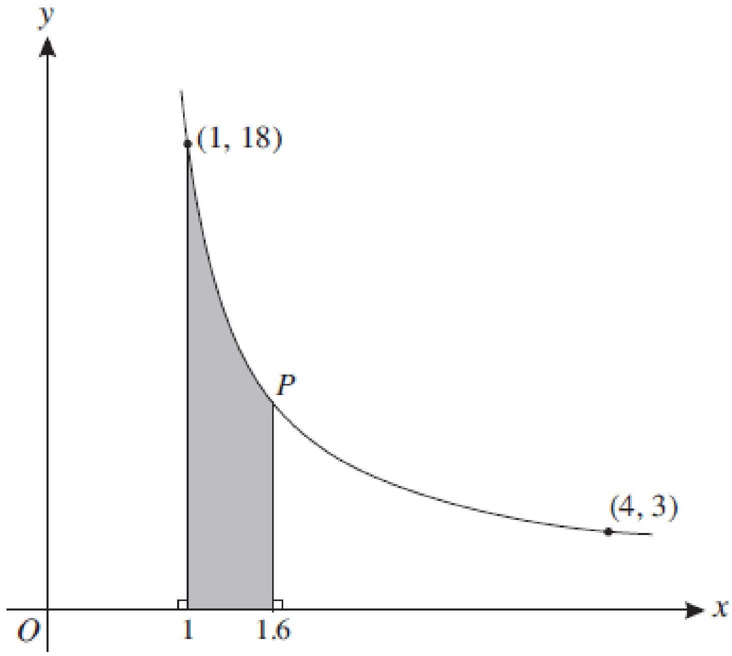
(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Tangents are drawn to the curve at the points P and Q.

(iii) Find the acute angle, in degrees correct to 1 decimal place, between the two tangents. [4]

(i) $\frac{4}{9}$
(ii) 1.5π
(iii) 19.4°

MAY/JUNE2008



The diagram shows a curve for which $\frac{dy}{dx} = -\frac{k}{x^3}$, where k is a constant. The curve passes through the points (1, 18) and (4, 3).

(i) Show, by integration, that the equation of the curve is $y = \frac{16}{x^2} + 2$. [4]

The point P lies on the curve and has x-coordinate 1.6.

(ii) Find the area of the shaded region. [4]

(ii) 7.2

OCTOBER/NOVEMBER2007

Find the area of the region enclosed by the curve $y = 2\sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$. [4]

9.33

OCTOBER/NOVEMBER2007

A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

(i) the equation of the curve, [3]

(ii) the equation of the normal to the curve at P, [3]

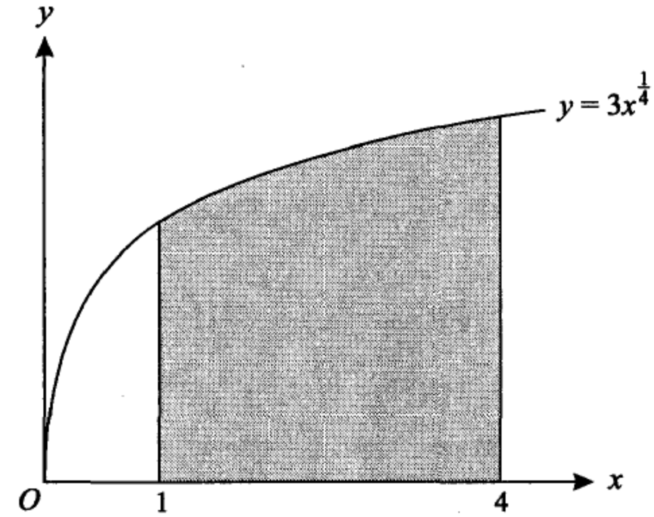
(iii) the coordinates of Q. [3]

(i) $y = 4x - \frac{1}{2}x^2 + 3$

(ii) $y = -\frac{1}{2}x + 10$

(iii) (7, 6.5)

MAY/JUNE2007



The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]

42π

MAY/JUNE2007

The equation of a curve is $y = 2x + \frac{8}{x^2}$.

Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [3]

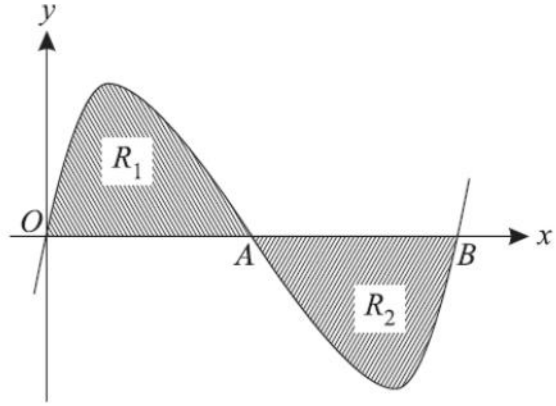
7

OCTOBER/NOVEMBER2006

The equation of a curve is $y = \frac{6}{5 - 2x}$.

The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

OCTOBER/NOVEMBER2006



The diagram shows the curve $y = x(x-1)(x-2)$, which crosses the x-axis at the points $O\,(0, 0)$, $A\,(1, 0)$ and $B\,(2, 0)$.

(i) The tangents to the curve at the points A and B meet at the point C. Find the x-coordinate of C. [5]

(ii) Show by integration that the area of the shaded region R_1 is the same as the area of the shaded region R_2. [4]

(i) $\frac{5}{3}$

MAY/JUNE2006

A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{(6-2x)}}$, and $P(1, 8)$ is a point on the curve.

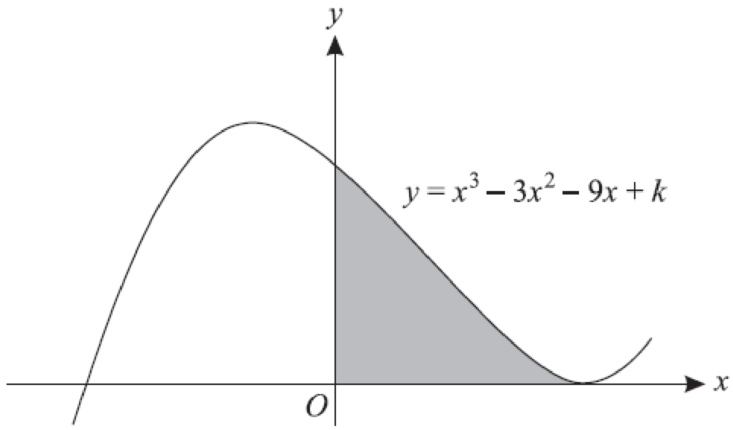
(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

(i) (8.5, 4.25)

(ii) $y = -4(6-2x)^{\frac{1}{2}} + 16$

MAY/JUNE2006



The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.

(i) Find the value of k. [4]

(ii) Find the coordinates of the maximum point of the curve. [1]

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x. [1]

(iv) Find the area of the shaded region. [4]

(i) 27
(ii) (-1, 32)
(iii) -1 < x < 3
(iv) 33.75

OCTOBER/NOVEMBER2005

A curve is such that $\frac{dy}{dx} = \frac{16}{x^3}$, and (1, 4) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A line with gradient $-\frac{1}{2}$ is a normal to the curve. Find the equation of this normal, giving your answer in the form $ax + by = c$. [4]

(iii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [4]

(i) $y = -\frac{8}{x^2} + 12$

(ii) $2y + x = 22$

(iii) 8

MAY/JUNE2005

A curve is such that $\frac{dy}{dx} = 2x^2 - 5$. Given that the point (3, 8) lies on the curve, find the equation of the curve. [4]

$y = \frac{2x^3}{3} - 5x + 5$

MAY/JUNE2005

A curve has equation $y = \frac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$. [4]

(i) 14.4
(ii) 8

OCTOBER/NOVEMBER2004

A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(4x-3)}}$ and $P\,(3, 3)$ is a point on the curve.

(i) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$. [3]

(ii) Find the equation of the curve. [4]

(i) $x + 2y = 9$
(ii) $y = 3(4x - 3) - 6$

OCTOBER/NOVEMBER2004

A curve has equation $y = x^2 + \frac{2}{x}$.

Find the volume of the solid formed when the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated completely about the x-axis. [6]

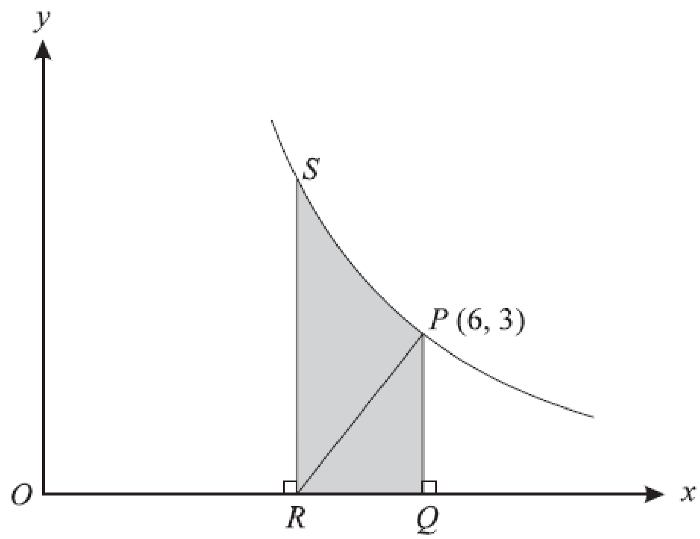
44.6

MAY/JUNE2004

Evaluate $\int_0^1 \sqrt{(3x + 1)}\, dx$. [4]

1.56

MAY/JUNE2004



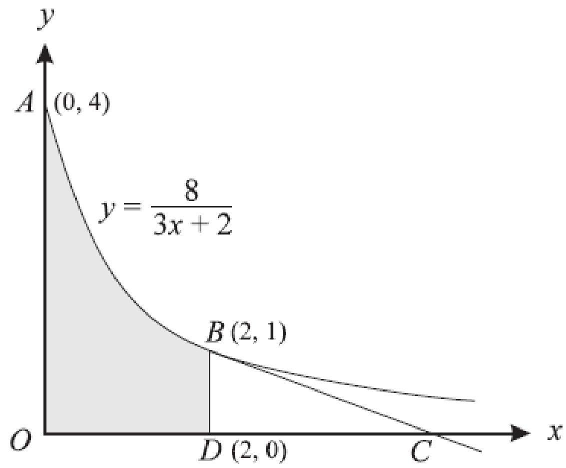
The diagram shows part of the graph of $y = \dfrac{18}{x}$ and the normal to the curve at P (6, 3). This normal meets the x-axis at R. The point Q on the x-axis and the point S on the curve are such that PQ and SR are parallel to the y-axis.

(i) Find the equation of the normal at P and show that R is the point $(4\frac{1}{2}, 0)$. [5]

(ii) Show that the volume of the solid obtained when the shaded region $PQRS$ is rotated through $360°$ about the x-axis is 18π. [4]

(i) $y = 2x - 9$
(ii) 18π

OCTOBER/NOVEMBER2003



The diagram shows points A (0, 4) and B (2, 1) on the curve $y = \dfrac{8}{3x+2}$. The tangent to the curve at B crosses the x-axis at C. The point D has coordinates (2, 0).

(i) Find the equation of the tangent to the curve at B and hence show that the area of triangle BDC is $\frac{4}{3}$. [6]

(ii) Show that the volume of the solid formed when the shaded region $ODBA$ is rotated completely about the x-axis is 8π. [5]

(i) $y = -\frac{3}{8}x + \frac{7}{4}$

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A curve is such that $\frac{dy}{dx} = 3x^2 - 4x + 1$. The curve passes through the point (1, 5).

(i) Find the equation of the curve. [3]

(ii) Find the set of values of x for which the gradient of the curve is positive. [3]

(i) $y = x^3 - 2x^2 + x + 5$

(ii) $x < \frac{1}{3}, x > 1$

MAY/JUNE2003

Find $\int \left(4x + \frac{6}{x^2}\right) dx$. [3]

$2x^2 - \frac{6}{x} + c$

MAY/JUNE2003

The equation of a curve is $y = \sqrt{(5x + 4)}$.

Find the area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. [5]

2.53

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The gradient at any point (x, y) on a curve is $\sqrt{(1 + 2x)}$. The curve passes through the point $(4, 11)$. Find

(i) the equation of the curve, [4]

(ii) the point at which the curve intersects the y-axis. [2]

(i) $y = \frac{(1+2x)^{\frac{3}{2}}}{3} + 2$

(ii) $(0, \frac{7}{3})$

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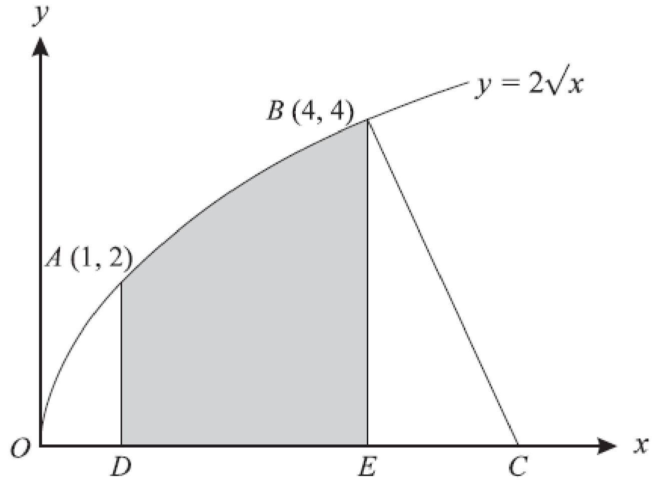

The diagram shows the points $A\,(1,\ 2)$ and $B\,(4,\ 4)$ on the curve $y = 2\sqrt{x}$. The line BC is the normal to the curve at B, and C lies on the x-axis. Lines AD and BE are perpendicular to the x-axis.

(i) Find the equation of the normal BC. [4]

(ii) Find the area of the shaded region. [4]

(i) $y = -2x + 12$

(ii) $\frac{28}{3}$

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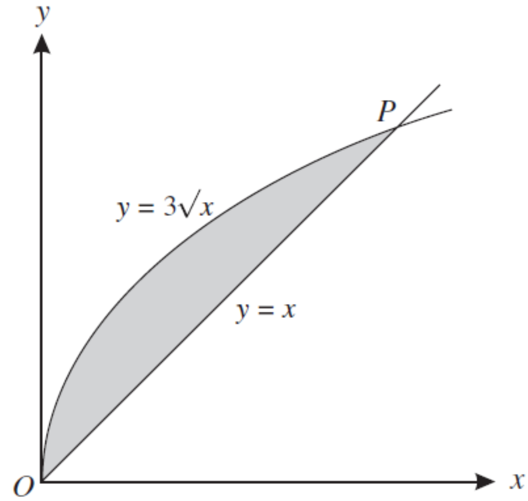

The diagram shows the curve $y = 3\sqrt{x}$ and the line $y = x$ intersecting at O and P. Find

(i) the coordinates of P, [1]

(ii) the area of the shaded region. [5]

(i) (9, 9)
(ii) 13.5

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A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$ and $P(1, 5)$ is a point on the curve.

(i) The normal to the curve at P crosses the x-axis at Q. Find the coordinates of Q. [4]

(ii) Find the equation of the curve. [4]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of increase of the y-coordinate when $x = 1$. [3]

(i) $(\frac{23}{3}, 0)$

(ii) $y = -\frac{6}{2x+1} + 7$

(iii) 0.4