Nuclear Physics

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- **27.4 Mass excess and nuclear binding energy**
- **27.5 Radioactive decay**

The Unified Atomic Mass Unit (u)

- The kilogram is an inconvenient unit for measuring atomic masses.
- A more suitable unit used is called the unified atomic mass unit. (u)
- The unified atomic mass unit is one-twelfth of the mass of a single atom of carbon-12 and from experiment, $1u = 1.66 \times 10^{-27}$ kg.
- The table below shows the atomic masses for some particles and atoms.

Particle	Mass / u
Electron (e)	0.000549
Neutron (n)	1.008665
Proton (p)	1.007276
Hydrogen atom (p+e)	1.007825
Helium atom (2p+2n+2e)	4.002603
Alpha particle (2p+2n)	4.001505

Mass & Binding Energy

- Adding masses of a proton and an electron gives the mass of a hydrogen atom.
- However, adding masses of the elements of a helium atom (2p+2e+2n) gives you an atomic mass of 4.03298u, which is higher than the measured mass of a helium atom which is 4.002603u. (based on previous table)
- The helium atom is lighter by 0.030377u.
- In general, all atoms are lighter than the sum of the masses of their protons, neutrons and electrons.
- The explanation to this discrepancy (lost in mass) is the **concept of binding energy.**
- This difference between the total mass of all its separate nucleons and the mass of the nucleus itself is called the **mass defect.**
- In order to separate a nucleus into its nucleons, work need to be done / a minimum amount of energy must be supplied to overcome the strong nuclear force which keeps the nucleons together.

Mass & Binding Energy

- This **minimum amount of energy needed to pull a nucleus apart into its separate nucleons** is known as the **binding energy** of the nucleus.
- Based on Einstein's mass-energy equation:-

  i.) the mass of a system increases when energy is supplied to it.

  ii.) the mass of a system decreases when energy is released from it.

- Thus, the energy which is used to pull the nucleus apart is then converted into matter, hence, restoring the nucleons to their original masses.
- Likewise, when nuclei were to be created, a strong nuclear force had to do work in order to bring the nucleons together. The energy to do this work came from the direct conversion of mass to energy of the nucleons.
- The nucleons has to lose some amount of mass, which will be converted to energy in order to do work to bring the nucleons together.
- This explains why the assembled nucleus is lighter than its individual nucleons.

Mass & Binding Energy

- It was Einstein who relate mass and energy numerically / quantitatively in his famous mass-energy equation:

$$E = mc^2$$

where E = energy (J)
m = mass converted (kg)
c = velocity of light (ms^{-1})

- This equation tells us that total amount of mass and energy in a system is constant. They may be conversion from one to another, but the total amount of "mass-energy" remains constant.

Mass & Binding Energy

- If we apply this equation to the "missing" mass of the helium nucleus, we can calculate its binding energy:

missing mass = 4.031882u – 4.001505u = 0.030377u
$m = 0.030377 \times (1.66 \times 10^{-27}$ kg)
$m = 5.043 \times 10^{-29}$ kg

$E = mc^2 = (5.043 \times 10^{-29})(3.00 \times 10^8)^2$
$E = 4.54 \times 10^{-12}$ J or 28.4 MeV (since 1 eV = 1.6×10^{-19} J)

- Therefore, to split each helium nucleus into its components would require an input of 28.4 MeV of energy.

Example 1

- Use the data below to determine the minimum energy required to split a nucleus of oxygen-16 ($^{16}_{8}O$) into its separate nucleons. Give your answer in Joules.

mass of proton = 1.672 623 x 10^{-27} kg

mass of neutron= 1.674 929 x 10^{-27} kg

mass of oxygen-16 nucleus = 26.551 559 x 10^{-27} kg

Example 2

- The Sun releases vast amount of energy. Its power output is 4.0×10^{26} W. Estimate by how much its mass decreases each second because of this energy loss.

- Calculate energy released if a alpha particle nucleus is formed from separate protons and neutrons. Calculate also the energy released per nucleon.

mass of proton = $1.672\ 623 \times 10^{-27}$ kg

mass of neutron= $1.674\ 929 \times 10^{-27}$ kg

mass of alpha particle = $6.644\ 661 \times 10^{-27}$ kg

Example 3

- A nucleus of Uranium-238 decays by emitting an alpha particle and changes into an isotope of Thorium.

$$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

mass of Uranium-238 = $3.952\ 830 \times 10^{-25}$ kg

mass of Thorium-234 = $3.88\ 631\ 339 \times 10^{-25}$ kg

mass of alpha particle = $6.644\ 661 \times 10^{-27}$ kg

- What is the mass changed in this process? Is mass increased or decreased?
- What is the amount of energy release d?
- If 1 mole of Uranium-238 is used, what is the amount of energy released?

Example 4

- A nucleus of Beryllium $^{10}_{4}Be$ decays into an isotope of boron (B) by beta emission.

  i.) Write a nuclear decay equation for the nucleus of beryllium-10.

  ii.) Calculate the energy released in this decay.

  mass of Beryllium-10 = $1.662\ 38 \times 10^{-26}$ kg

  mass of Boron-4 = $1.662\ 19 \times 10^{-26}$ kg

  mass of electron = $9.109\ 56 \times 10^{-31}$ kg

Binding Energy & Stability

- We can now begin to see why some nuclei are more stable than others.
- If a nucleus is formed from separate nucleons, energy is released.
- In order to pull the nucleus apart, energy must be put in; in other words, work must be done against the forces holding the nucleons together.
- The more energy involved in this, the more stable is the nucleus.
- The minimum energy needed to pull a nucleus apart into its separate nucleons to infinity is known as the binding energy of the nucleus.
- In order to compare different nuclides, we need to consider the binding energy per nucleon.

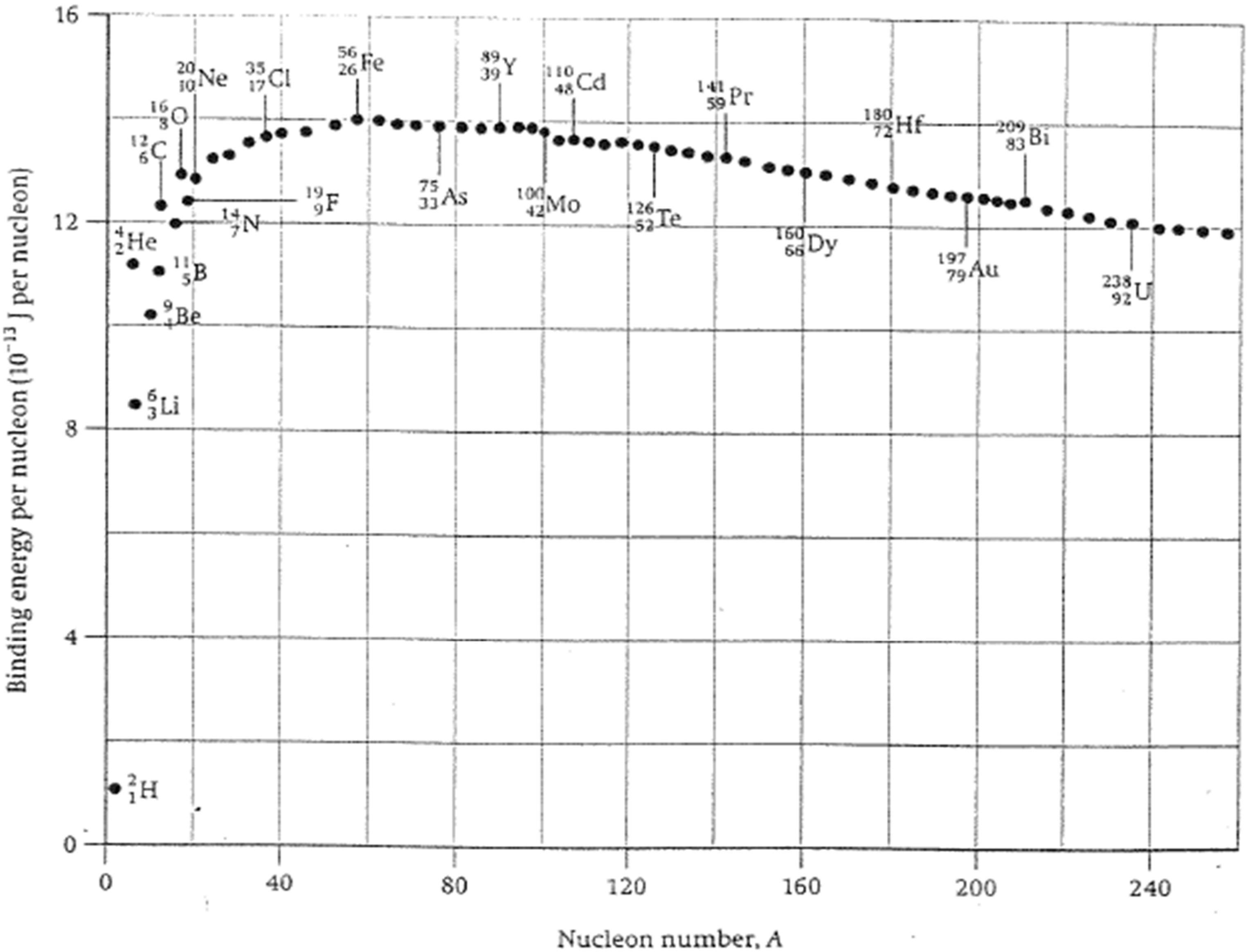
Binding Energy & Stability

- We can calculate the **binding energy per nucleon** for a nuclide as follows:
  i.) Determine the mass defect of the nucleus.
  ii.) Use Einstein's mass-energy equation to determine the binding energy.
  iii.) Divide the binding energy of the nucleus by the number of nucleons to calculate the binding energy per nucleon.

- The graph in the next slide shows the binding energy per nucleon for stable nuclei. This graph is plotted against the nucleon number A. The greater the value of the binding energy per nucleon, the more tightly bound are the nucleons that make up the nucleus.
- From the graph, we see that:
  a.) The general trend is for light nuclei to have low binding energies per nucleon.
  b.) For A > 20, there is not much variation in binding energy per nucleon.
  c.) The greatest value of binding energy per nucleon is found for $^{56}_{26}Fe$. This isotope of iron is the most stable in nature.

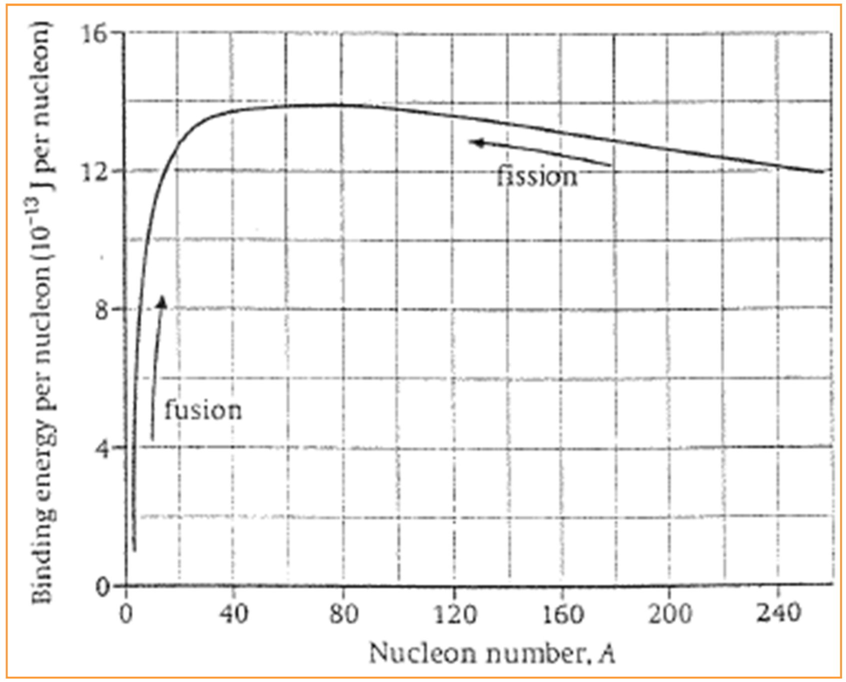
Binding Energy & Stability

Graph of Binding Energy per nucleon versus Nucleon number, A



Fission & Fusion

- We can use the binding energy per nucleon graph to help us decide which nuclear process – fission, fusion, radioactive decay are likely to occur.



Both fusion & fission are processes that tend to increase the binding energy per nucleon of the particles involved.

Fusion

- is a process whereby 2 very light nuclei join together to form a heavier nucleus.
- If 2 light nuclei fuse, the final binding energy per nucleon is greater than the original value.
- The product nucleus is more stable and energy will be released.

Fission

- Is a process in which a massive nucleus splits to form 2 smaller fragments rather than simply emitting α or β radiation.
- For uranium we have A = 235, and the typical fragments will have A = 140 & 95. The fragments have greater binding energy than original value.
- The product nucleus is more stable and energy will be released.

Example 5

- Calculate the binding energy per nucleon for the nuclide ${}^{56}_{26}\mathrm{Fe}$

  mass of neutron = 1.675×10^{-27} kg

  mass of proton = 1.673×10^{-27} kg

  mass of Fe-56 nucleus = 9.288×10^{-26} kg

- Explain why ${}^{1}_{1}\mathrm{H}$ does not appear on the graph of Binding Energy per nucleon versus Nucleon number, A.

Example 6

- The mass of ${}^{8}_{4}Be$ nucleus is 1.33×10^{-26} kg. A proton and neutron have a mass of about 1.67×10^{-27} kg. For the nucleus of ${}^{8}_{4}Be$ determine:

  i.) the mass defect in kg.

  ii.) the binding energy.

  iii.) the binding energy per nucleon.

Example 7

- Data for binding energy per nucleon is shown below.

Uranium-235 = 7.59 MeV

Xenon-142 = 8.37 MeV

Strontium-90 = 8.72 MeV

i.) Calculate the energy, in MeV, released in this fission reaction.

ii.) the mass equivalent of this energy.

Example 8

The controlled reaction between deuterium (^2_1H) and tritium (^3_1H) has involved ongoing research for many years. The reaction may be summarised as

$$^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n + Q$$

where Q = 17.7 MeV.

Binding energies per nucleon are shown

	binding energy per nucleon / MeV
^2_1H	1.12
^1_0n	–
^4_2He	7.07

i) Calculate the mass defect, in kg, of a helium ^4_2He nucleus.

ii) Determine the binding energy per nucleon, in MeV, of tritium (^3_1H).

The mathematics of Radioactive decay

- Radioactive decay is a random, spontaneous process.
- Because we cannot say when and which individual will decay, we have to start thinking about very large numbers of nuclei. (Tiny speck of radioactive material have 10^{15} nuclei.)
- Then we can talk about the average number of nuclei that we expect to decay in a particular time interval; in other words, we can find out the average decay rate.
- Although we cannot make predictions for individual nuclei, we can say that certain nuclei are more likely to decay than others. Examples, a nucleus of carbon-12 is stable, carbon-14 decays gradually over thousands of years, carbon-15 nuclei last only few seconds.

DECAY CONSTANT

- One quantity we can determine is the probability that an individual nucleus will decay in a particular time interval which we called it the decay constant, λ.
- Example, suppose we observe 1 million nuclei of a particular radio-isotope. After 1 hour, 200,000 have decayed. Then the probability that an individual nucleus will decay in 1 hour is 0.2 or 20%, since 20% of the nuclei have decayed in this time. (Of course this is only an approximate value, since we might repeat the experiment and find that only 199,000 decay. The more times we repeat the experiment, the more reliable our answer will be.)
- For example above, we have: **decay constant, $\lambda = 0.2\ h^{-1}$** ;
- Note that because we measured the probability of decay per unit time interval, λ has units of h^{-1} (or s^{-1}, day^{-1}, $year^{-1}$, etc.)

The mathematics of Radioactive decay

ACTIVITY

- The activity (A) of a radioactive source is defined as the **rate at which the nuclei decay.**
- Activity is measured in decays per second (or per hour, per day, etc.)
- An activity of one decay per second is known as one Becquerel. (1 Bq = 1 s^{-1}). ($A = \Delta N / \Delta t$)
- Clearly, the activity of a sample depends on the decay constant (λ) of the radio-isotope under consideration. The greater the decay constant (the probability that an individual nucleus decays per unit time interval), the greater is the activity of the sample.
- Beside that, it also depends on the size of the sample (N).
- In short, equation to calculate activity, A is given by: $\boxed{A = \lambda N}$

COUNT RATE

- Although we are often interested in finding the activity of a sample of radioactive material, we cannot usually measure this directly.
- This is because we cannot easily detect all of the radiation emitted. Some will escape past our detectors, and some may be absorbed within the sample itself. So our measurements give a received count rate R that is significantly lower than the activity, A.
- If we know how efficient our detecting system is, we can deduce A from R.
- If the level of background radiation is significant, then it must be subtracted to give the corrected count rate.

Example 9

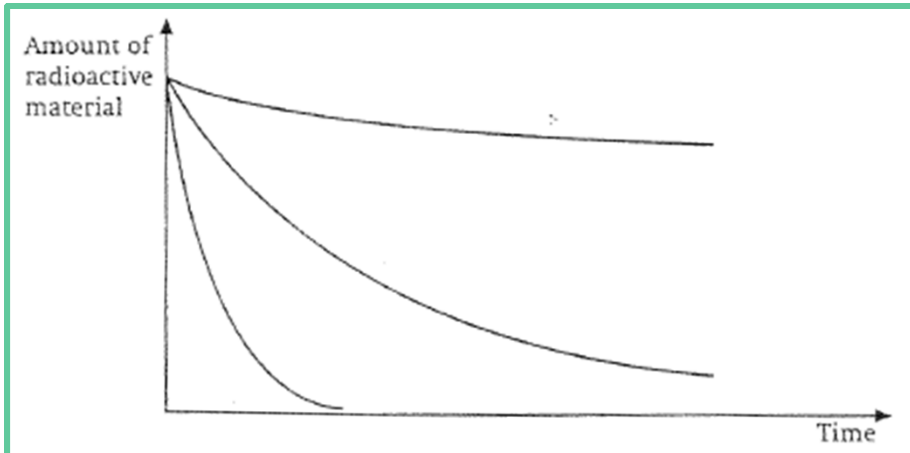
- A sample consists of 1000 undecayed nuclei of a nuclide whose decay constant is 0.20 s^{-1} . Determine the initial activity of the sample.

- A sample consists of 500 000 undecayed nuclei of a nuclide whose decay constant is 0.30 s^{-1} . Determine the initial activity of the sample.

- A small sample of radium gives a received count rate of 20 counts per minute in a detector. It is known that the counter detects only 10 %of the decays from the sample. The sample contains 1.5×10^9 undecayed nuclei. Determine the decay constant of this form of radium.

Decay Graphs & Equations

- Radioactive substances gradually diminish as time goes by. The atomic nuclei emit radiation and become different substances.
- The pattern of radioactive decay is an example of a very important pattern found in many different situations, a pattern called exponential decay. The figure below shows the decay graphs for three different radio-isotopes, each with a different rate of decay.



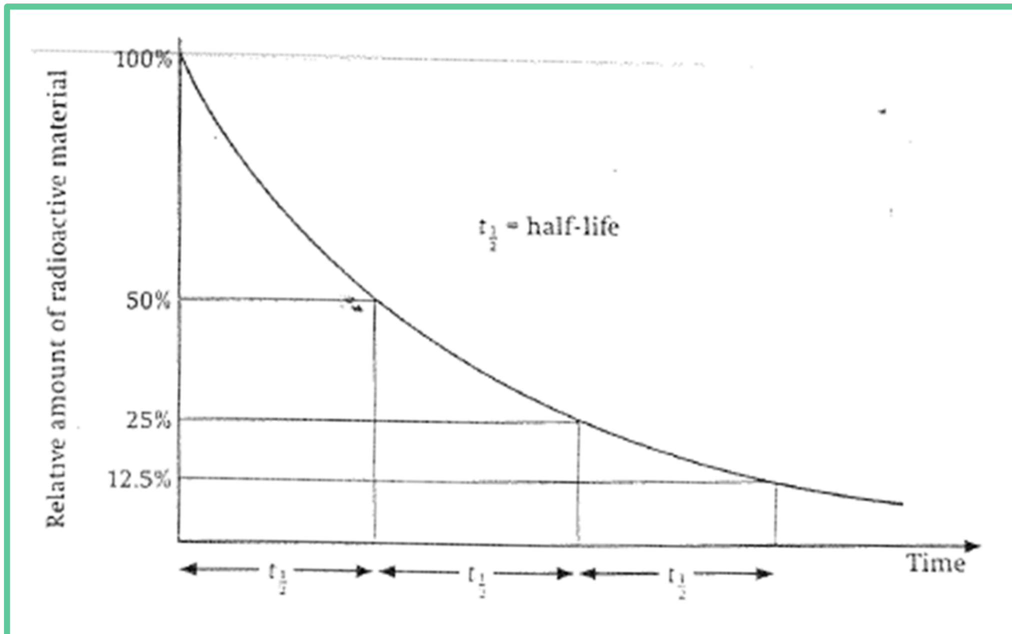
Some radioactive materials decay faster than others.

- Although the 3 graphs look different, they all have something in common – their shape.
- They are curved lines having a special property.

Decay Graphs & Equations

- In order to understand what is special about the shape of these curves, we need to know this term which called half-life. ($t_{½}$)
- The half-life ($t_{½}$) of a radio-isotope is the average time taken for half of the nuclei in a sample to decay, or time taken for the activity of a sample to decrease to half of some initial value.
- What amazes us is that it takes the same amount of time again for half of the remainder to decay, and third half-life for half of the new remainder to decay.
- That is why we have the graph we curve as below. An exponential decay pattern.

- In principle, the graph never reaches zero; it just gets closer and closer.
- But in practice, when only a few undecayed nuclei remain, it will cease to be a smooth curve and will eventually reach zero.
- We will look at the exponential equations that we can use to calculate the activity of a decaying sample.



Mathematical Decay

- We can write an equation to represent the decay graph. If we start with N_0 undecayed nuclei, then the number N that remain undecayed after time, t is given by:

$$\boxed{N = N_0 e^{-\lambda t}}$$

- In this equation, λ is the decay constant as before. If λ is in s^{-1}, then t must be in s.!

- The activity A of a sample is proportional to the number of undecayed nuclei N. Hence the activity of the sample decreases exponentially too given by:

$$\boxed{A = A_0 e^{-\lambda t}}$$

- Usually, we measure the corrected count rate R rather than the activity or the number of undecayed nuclei. Since count rate is a fraction of the activity, it too decreases exponentially with time:

$$\boxed{R = R_0 e^{-\lambda t}}$$

- In general, the exponential nature of radioactive decay can be represented by $X = X_0 e^{-\lambda t}$
  where x could represent activity (A), number of undecayed nuclei (N) or count rate (R).

Decay Constant & Half-Life

- A radio-isotope that decays rapidly has a short half-life ($t_{½}$).
- Its decay constant λ must be high, since the probability per unit time of an individual nucleus decaying must be high.
- Hence, there is a connection between half-life and decay constant.

$$\boxed{N = N_0 e^{-\lambda t}}$$

- After 1 half-life ($t = t_{½}$), the number of undecayed atoms remaining (N) has been reduced by half.

i.e. $$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

$$\ln \frac{1}{2} = -\lambda t_{\frac{1}{2}}$$

$$\boxed{t_{\frac{1}{2}} = \frac{0.693}{\lambda}}$$

Example 10

- Suppose we start an experiment with 1.0×10^{15} undecayed nuclei of an isotope for which decay constant is equal to $0.02\ s^{-1}$. Determine the number of undecayed nuclei after 20 s.

- A sample of an isotope for which $\lambda = 0.10\ s^{-1}$ contains 5.0×10^{9} undecayed nuclei at the start of an experiment. Determine

  i.) the number of undecayed nuclei after 50 s.

  ii.) Its activity after 50 s.

Example 11

- The decay constant of a particular isotope is known to be 3.0 x 10^{-4} s^{-1} . Determine how long it will take for the activity of a sample of this substance to decrease to one-eighth of its initial value.

- The isotope $^{16}_{7}N$ decays with a half-life of 7.4 s. The sample initially contains 5000 nuclei. Calculate the decay constant for this nuclei. Determine how many will remain after a time of 14.8 s and 20.0 s.