24 Marking scheme: Worksheet (A2)

1 **a** $Q = VC = 9.0 \times 30 \times 10^{-6}$ [1]
$Q = 2.7 \times 10^{-4}$ C (270 μC) [1]

b number of excess electrons $= \dfrac{Q}{e} = \dfrac{2.7 \times 10^{-4}}{1.6 \times 10^{-19}}$ [1]
number $= 1.69 \times 10^{15} \approx 1.7 \times 10^{15}$ [1]

2 **a** **i** The charge is directly proportional to the voltage across the capacitor.
Hence doubling the voltage will double the charge. [1]
charge = 2 × 150 = 300 nC [1]

ii Since $Q \propto V$ for a given capacitor, increasing the voltage by a factor of three will increase the charge by the same factor. [1]
charge = 3 × 150 = 450 nC [1]

b $C = \dfrac{Q}{V} = \dfrac{150 \times 10^{-9}}{3.0}$ [1]
$C = 5.0 \times 10^{-8}$ F [1]

3 **a** $E = \dfrac{1}{2}V^2C = \dfrac{1}{2} \times 9.0^2 \times 1000 \times 10^{-6}$ [1]
$E = 4.05 \times 10^{-2}\ \text{J} \approx 4.1 \times 10^{-2}$ J [1]

b For a given capacitor, energy stored $\propto$ voltage^2. [1]
energy $= 2^2 \times 4.05 \times 10^{-2} \approx 0.16$ J [1]

4 **a** $C_{total} = C_1 + C_2$ [1]
$C_{total} = 20 + 40 = 60$ nF [1]

b $\dfrac{1}{C_{total}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ [1]
$\dfrac{1}{C_{total}} = \dfrac{1}{100} + \dfrac{1}{500} = 0.012\ \mu\text{F}^{-1}$ [1]
$C_{total} = \dfrac{1}{0.012} \approx 83\ \mu\text{F}$ [1]

c $\dfrac{1}{C_{total}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$ [1]
$\dfrac{1}{C_{total}} = \dfrac{1}{10} + \dfrac{1}{50} + \dfrac{1}{100} = 0.13\ \mu\text{F}^{-1}$ [1]
$C_{total} = \dfrac{1}{0.13} \approx 7.7\ \mu\text{F}$ [1]

d Total capacitance of the two capacitors in parallel = 50 + 50 = 100 μF. [1]
$\dfrac{1}{C_{total}} = \dfrac{1}{50} + \dfrac{1}{100} = 0.03\ \mu\text{F}^{-1}$ [1]
$C_{total} = \dfrac{1}{0.03} \approx 33\ \mu\text{F}$ [1]

e Total capacitance of the two capacitors in series is 83 μF (from **b**). [1]
$C_{total} = 83 + 50 = 133\ \mu\text{F} \approx 130\ \mu\text{F}$ [1]

5 **a** $C_{total} = C_1 + C_2$ [1]
$C_{total} = 100 + 500 = 600\ \mu\text{F}$ [1]

b The potential difference across parallel components is the same and equal to 1.5 V. [1]

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c $Q = VC = 1.5 \times 600 \times 10^{-6}$ [1]
$Q = 9.0 \times 10^{-4}$ C (900 μC) [1]

d $E = \frac{1}{2}QV = \frac{1}{2} \times 9.0 \times 10^{-4} \times 1.5$ [1]
$E = 6.75 \times 10^{-4}$ J $\approx 6.8 \times 10^{-4}$ J [1]

6 a $E = \frac{1}{2}V^2C = \frac{1}{2} \times 32^2 \times 10\,000 \times 10^{-6}$ [1]
$E = 5.12$ J ≈ 5.1 J [1]

b $P = \frac{E}{t} = \frac{5.12}{0.300}$ [1]
$P \approx 17$ W [1]

7 a $Q = VC = 12 \times 1000 \times 10^{-6}$ [1]
$Q = 1.2 \times 10^{-2}$ C (12 mC) [1]

b i $C_{\text{total}} = C_1 + C_2$ [1]
$C_{\text{total}} = 1000 + 500 = 1500$ μF [1]

ii $V = \frac{Q}{C}$ (The charge Q is conserved and C is the total capacitance.) [1]

$V = \frac{1.2 \times 10^{-2}}{1500 \times 10^{-6}} = 8.0$ V [1]

8 a $\Delta Q = I\Delta t = \frac{I}{f} = \frac{225 \times 10^{-3}}{50} = 4.5 \times 10^{-3}$ C [1]

b $C = \frac{Q}{V} = \frac{4.5 \times 10^{-3}}{9.0}$ [1]
$= 5.0 \times 10^{-4}$ F $= 500$ μF [1]

c i The capacitors are in **parallel**, so the total capacitance = $2C$ and charge stored = $2Q$;
current $= 2I = 2 \times 225 = 450$ mA [1]

ii The capacitors are in **series**, so the total capacitance = $\frac{1}{2}C$ and charge stored = $\frac{1}{2}Q$;

current = $\frac{1}{2}I = \frac{1}{2} \times 225 = 113$ mA [1]

9 a $Q = CV = 200 \times 10^{-6} \times 200 = 0.040$ C [1]

b $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 200 \times 10^{-6} \times 200^2 = 4.0$ J [1]

c The two capacitors now make a pair of capacitors in parallel of total capacitance
$= 100$ μF $+ 200$ μF $= 300$ μF [1]

The charge is conserved, therefore $V = \frac{Q}{C} = \frac{0.040}{300 \times 10^{-6}}$ [1]
$= 133$ V [1]

d $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 300 \times 10^{-6} \times 133^2$ [1]
$= 2.67$ J [1]

e The energy is lost as the connected wires are heated as the current passes through them. [1]

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10 The capacitors are in **parallel**, so the total capacitance = $3C$. [1]

The total charge Q remains constant. [1]

The energy stored by a capacitor is given by $E = \dfrac{Q^2}{2C}$. [1]

$E_{\text{initial}} = \dfrac{Q^2}{2C}$ and $E_{\text{final}} = \dfrac{Q^2}{2(3C)}$ [1]

Fraction of energy stored = $\dfrac{E_{\text{final}}}{E_{\text{initial}}} = \dfrac{Q^2/2(3C)}{Q^2/2C} = \dfrac{1}{3}$ [1]

Fraction of energy 'lost' as heat in resistor = $1 - \dfrac{1}{3} = \dfrac{2}{3}$. [1]

The resistance governs how long it takes for the capacitor to discharge. The final voltage across each capacitor is independent of the resistance. Hence, the energy lost as heat is independent of the actual resistance of the resistor. [1]

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