20 Marking scheme: Worksheet (A2)

The period of an oscillator is the time for one complete oscillation. [1] The frequency of an oscillator is the number of oscillations completed per unit time (or per second). [1] The gradient of a displacement against time graph is equal to velocity. [1] The magnitude of the velocity (speed) is a maximum at 0 s or 0.4 s or 0.8 s. [1] **b** For s.h.m., acceleration ∞ –displacement. The magnitude of the acceleration is maximum when the displacement is equal to the amplitude of the motion. [1] The magnitude of the acceleration is a maximum at 0.2 s or 0.6 s or 1.0 s. [1] **3 a** $T = \frac{13.2}{12}$ [1] T = 1.1 s[1] **b** $f = \frac{1}{T} = \frac{1}{1.1}$ [1] $f = 0.909 \approx 0.91 \text{ Hz}$ [1] 4 a Amplitude = 0.10 m[1] **b** Period = 4.0×10^{-2} s [1] $\mathbf{c} \quad f = \frac{1}{T} = \frac{1}{0.04}$ [1] f = 25 Hz[1] $\mathbf{d} \quad \omega = 2\pi f = 2\pi \times 25$ [1] $\omega = 157 \text{ rad s}^{-1} \approx 160 \text{ rad s}^{-1}$ [1] e Maximum speed = $\omega A = 157 \times 0.10$ [1] maximum speed = $15.7 \text{ m s}^{-1} \approx 16 \text{ m s}^{-1}$ [1] 5 a Phase difference = $2\pi \times \left(\frac{t}{T}\right)$ where T is the period and t is the time lag between the motions of the two objects. phase difference = $2\pi \times \left(\frac{t}{T}\right) = 2\pi \times \left(\frac{2.5}{10}\right)$ [1] phase difference = $\frac{\pi}{2} \approx 1.6$ rad [1] **b** Phase difference = $2\pi \times \left(\frac{t}{T}\right) = 2\pi \times \left(\frac{5.0}{10}\right)$ [1] phase difference = $\pi \approx 3.1$ rad [1] **6 a** A = 16 cm [1] **b** $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{2.8}$ [1] $\omega = 2.24 \text{ rads}^{-1} \approx 2.2 \text{ rads}^{-1}$ [1] c $a = (2\pi f)^2 x$ (magnitude only) [1] For maximum acceleration, the displacement x must be 16 cm. $a = \left(2\pi \times \frac{1}{2.8}\right)^2 \times 16 \times 10^{-2}$ [1] $a = 0.806 \text{ m s}^{-2} \approx 0.81 \text{ m s}^{-2}$ [1] **d** Maximum speed = $\omega A = 2.24 \times 0.16$ [1]

maximum speed = $0.358 \text{ m s}^{-1} \approx 0.36 \text{ m s}^{-1}$

[1]

7 **a**
$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{2.0}$$
 [1]

$$\omega = 3.14 \text{ rad s}^{-1} \approx 3.1 \text{ rad s}^{-1}$$
 [1]

b
$$a = -(2\pi f)^2 x$$
 or $a = -\omega^2 x$ [1]

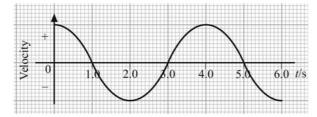
$$a = 3.14^{2} \times 3.0 \times 10^{-2}$$
 [1]
 $a \approx 0.30 \text{ m s}^{-2}$

$$\mathbf{c} \quad x = A \cos (2\pi f t) = A \cos (\omega t) \tag{1}$$

$$x = 3.0 \times 10^{-2} \cos (3.14 \times 6.7)$$
 [1]

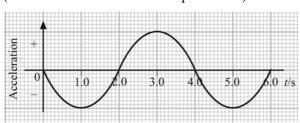
$$x \approx -1.7 \times 10^{-2} \,\mathrm{m}$$
 [1]

8 a Gradient of x–t graph = velocity



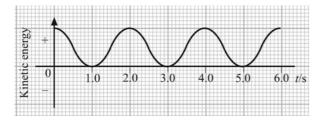
[2]

b Gradient of v-t graph = acceleration (for s.h.m. acceleration ∞ -displacement)



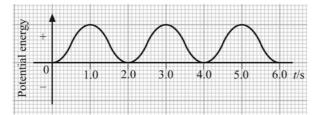
[2]

c Kinetic energy = $\frac{1}{2}mv^2 \propto v^2$



[2]

d Potential energy = total energy – kinetic energy



[2]

9 a
$$a = -(2\pi f)^2 x$$
 [1]

Therefore
$$(2\pi f)^2 = 6.4 \times 10^5$$
 [1]

$$f = \frac{\sqrt{6.4 \times 10^5}}{2\pi} = 127 \text{ Hz} \approx 130 \text{ Hz}$$
 [1]

b F = ma

Acceleration is maximum at maximum displacement, so magnitude of maximum force is given by:

$$F = ma = 0.700 \times (6.4 \times 10^5 \times 0.08)$$
 [1]

$$F = 3.58 \times 10^4 \,\mathrm{N} \approx 3.6 \times 10^4 \,\mathrm{N}$$

10 a According to Hooke's law,
$$F = -kx$$
 [1]

(The minus sign shows that the force is directed towards the equilibrium position.)

From Newton's second law:
$$F = ma$$
 [1]

Equating, we have:
$$ma = -kx$$
 [1]

Hence: $a = -\left(\frac{k}{m}\right)x$

b For s.h.m. we have
$$a = -(2\pi f)^2 x$$
 [1]

Hence
$$(2\pi f)^2 = \frac{k}{m}$$
 [1]

Therefore $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$$\mathbf{c} \quad f = \frac{1}{T} = \frac{1}{0.4} = 2.5 \,\text{Hz}$$
 [1]

$$2.5 = \frac{1}{2\pi} \sqrt{\frac{k}{850}}$$
 [1]

$$k = (2\pi \times 2.5)^2 \times 850 \approx 2.1 \times 10^5 \text{ N m}^{-1}$$
 [1]