TOPIC 14: OSCILLATIONS

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14.2 ENERGY IN SIMPLE HARMONIC MOTION

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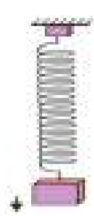
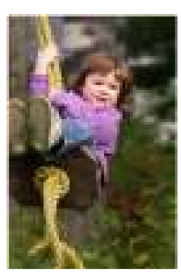
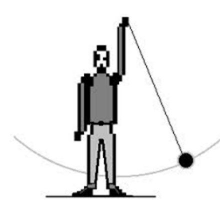
VIBRATIONS / OSCILLATIONS

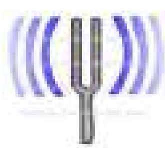
- Oscillations and vibrations are everywhere.
- Examples: object at the end of spring, vibration on guitar / piano, a child on the swing, pendulum,

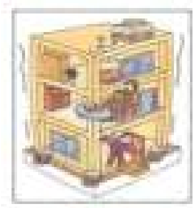


VIBRATIONS / OSCILLATIONS

- More examples: tuning fork, a plastic ruler held at one end and struck gently on the other, car oscillate up & down when they hit a bump, buildings and bridges vibrate when vehicles pass or the wind is fierce.



VIBRATIONS / OSCILLATIONS

- All these oscillations may seem very different from one another. This chapter we will see the characteristics that are shared by these oscillations.
- General we classified oscillations into 2:

  1.) Free oscillations

  2.) Forced oscillations

FREE OSCILLATIONS

- The easiest oscillations to understand is the free oscillations.
- When you pluck a guitar string, it vibrates for some time after you released it. It will vibrate at a particular frequency (number of vibrations per unit time).
- This is called its natural frequency of vibration, and it gives rise to the particular note you hear. Change the length of the string changes the natural frequency.
- Similarly, the prongs of a tuning fork have a natural frequency of vibration, which you can observe when you strike it.
- Every oscillator has a natural frequency of vibration, the frequency which it vibrates freely after an initial disturbance.

FORCED OSCILLATIONS

- On the other hand, many objects are forced to vibrate.
- If you sit on a bus, you may notice that the vibrations from the engine are transmitted to your body causing you to vibrate with same frequency.
- These are NOT free vibrations of your body, they are forced vibrations. Their frequency is NOT the natural frequency of vibration of your body, but the forcing frequency of the bus on you.
- Likewise, you can force the metre ruler to oscillate by waving it up and down; however, its natural frequency of vibration will be much greater than this, as you will discover if you hold one end down on the bench and twang the other end.

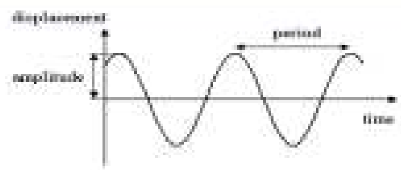
OBSERVING OSCILLATIONS

- Many oscillations are too rapid or too small for us to observe. Our eyes cannot respond rapidly enough if the frequency is more than about 5 Hz. (anything faster than this will appears blur.)

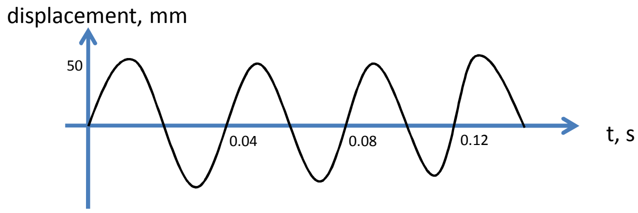
- In order to see the general characteristics of oscillating systems, we need to find suitable systems that oscillate slowly. Ex: pendulum, mass-spring system.

DESCRIBING OSCILLATIONS

- The oscillations of simple pendulum & mass-spring system shows same pattern of movement.
- It moves faster when it passed through the centre of oscillation and slower towards the end of the oscillation.
- In fact at extreme position, it stops momentarily, reverses its direction and accelerates back towards the centre again.
- When we describe the characteristics of the oscillations of objects, we must be familiar with terms like displacement, amplitude, frequency , angular frequency, period, & phase difference at least.



Example 1



- Deduce, from the numerical values given on the graph, the values for the shm of

(a) The period

(b) The frequency

(c) The angular frequency

(d) The amplitude of the oscillation

SIMPLE HARMONIC MOTION

- There are many situations where we can observed a special kind of motion which is called simple harmonic motion. (SHM)
- Some are more obvious than others. Ex: plucking a guitar string, pendulum.
- Some less obvious like:
  - single tone sound wave travels through air whereby air molecules vibrate with shm
  - AC current flows in a wire, electrons vibrate with shm
  - Bonds between atoms vibrate with shm.

SIMPLE HARMONIC MOTION

- By definition, S.H.M is a motion in which the *acceleration of the body is directly proportional to the displacement* from a fixed point, and is always *directed towards that point. Acceleration & displacement is always in opposite direction.*

- In other words, the body is oscillating equal distances either side of some fixed point. The further the body is from this point, the greater its acceleration back towards that point.

- The resultant force exerted on the body must increase with distance form this point and always be acting in a direction towards that fixed point, so as to restore the body to its undisturbed position.

- In this undisturbed position, the resultant force on the body will be zero – it will be in equilibrium.

SIMPLE HARMONIC MOTION

- Three requirements for shm:

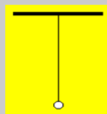
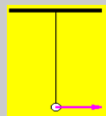
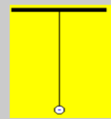
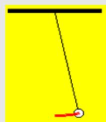
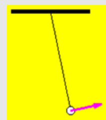
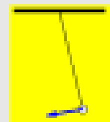
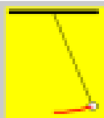
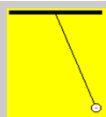
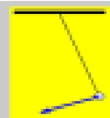
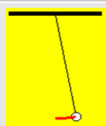
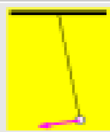
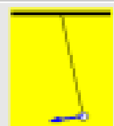
1.) there must be a **position where the mass is in equilibrium**.
(when mass is undisturbed, it must be in equilibrium.)

2.) the mass that **oscillates**.
(mass / object must be pulled to one side of its equilibrium position to oscillates.)

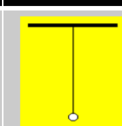
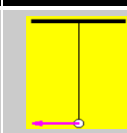
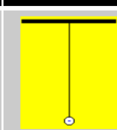
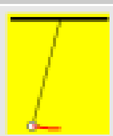
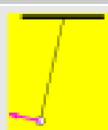
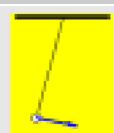
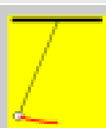
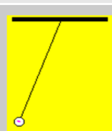
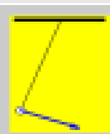
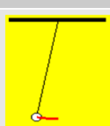
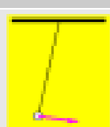
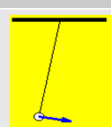
3.) a **restoring force** that acts to **return the mass to its equilibrium position**.
(the restoring force is directly proportional to the displacement of the mass from its equilibrium position and always directed towards that point.)

http://www.walter-fendt.de/ph14e/pendulum.htm

SIMPLE HARMONIC MOTION

TIME	DISPLACEMENT, x	VELOCITY, v	ACCELERATION, a
t = 0	 ‘0’ zero	 ‘+’ maximum	 ‘0’ zero
t = T/8	 ‘+’ increasing	 ‘+’ decreasing	 ‘-’ increasing
t = 2T/8 or T/4	 ‘+’ maximum	 ‘0’ zero	 ‘-’ maximum
t = 3T/8	 ‘+’ decreasing	 ‘-’ increasing	 ‘-’ decreasing

SIMPLE HARMONIC MOTION

TIME	DISPLACEMENT, x	VELOCITY, v	ACCELERATION, a
t = 4T/8 or T/2	 ‘0’ zero	 ‘-’ maximum	 ‘0’ zero
t = 5T/8	 ‘-’ increasing	 ‘-’ decreasing	 ‘+’ increasing
t = 6T/8 or 3/4	 ‘-’ maximum	 ‘0’ zero	 ‘+’ maximum
t = 7T/8	 ‘-’ decreasing	 ‘+’ increasing	 ‘+’ decreasing

EQUATIONS FOR SHM

- Displacement

- Displacement is generally represented by:

$x = x_0 sinwt$; where $w = 2\pi f$,
x_0 = amplitude ,
x = displacement at time t

EQUATIONS FOR SHM

- Velocity
- Velocity is a rate of change of displacement.
- Thus differentiating your displacement equation will give the equation for velocity. It is represented by:

$v = wx_0 coswt$

Squaring the velocity will give us: $v^2 = w^2x_0^2cos^2wt$

$v^2 = w^2x_0^2 (1 - sin^2wt)$; $sin^2wt + cos^2wt = 1$

$v^2 = w^2(x_0^2 - x_0^2 sin^2wt)$; $x = x_0 sinwt$

$v^2 = w^2(x_0^2 - x^2)$

$v = \pm w\sqrt{(x_0^2 - x^2)}$; x_0 = amplitude,
x = displacement at time t
v = velocity at time t

$v_{max} = \pm wx_0$; when $x = 0$ (displacement = 0)

EQUATIONS FOR SHM

- Acceleration
- Acceleration is a rate of change of velocity.
- Thus differentiating your velocity equation will give the equation for acceleration. It is represented by:

$a = -w^2x_0 sinwt$; (do take note of the negative sign!)

Since $x = x_0 sinwt$,

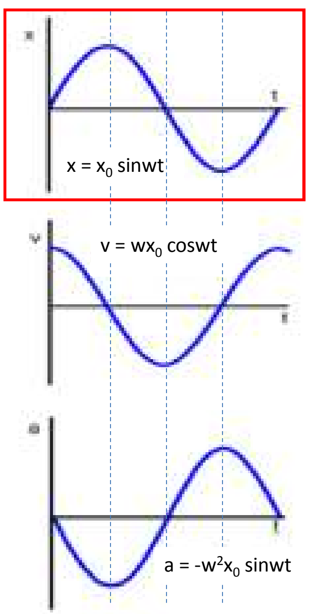
$a = -w^2x$; x = *displacement at time t*

a = *acceleration at time t*

$a_{max} = -w^2x_0$; x_0 = *amplitude*

- The equation of $a_{max} = -w^2x_0$ is in fact the defining equation for the definition of S.H.M.
- S.H.M is a motion in which the *acceleration of the body is directly proportional to the displacement* from a fixed point, and is always *directed towards that point. Acceleration & displacement is always in opposite direction.*

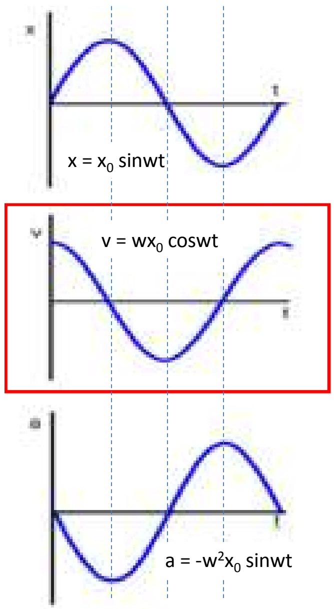
GRAPHICAL REPRESENTATION



Displacement - time graph

- Displacement of the oscillating mass varies according to a smooth curve.
- Mathematically it is a sine curve, its variation is described s sinusoidal.
- This graph allows us to determine amplitude and period of oscillations.
- When t = 0, x = 0. This means we chosen to consider the motion to start when mass is at the midpoint of its oscillation and is moving to the right.
- We could have chosen any other point in the cycle as the starting point, but it is conventional to start as shown here.

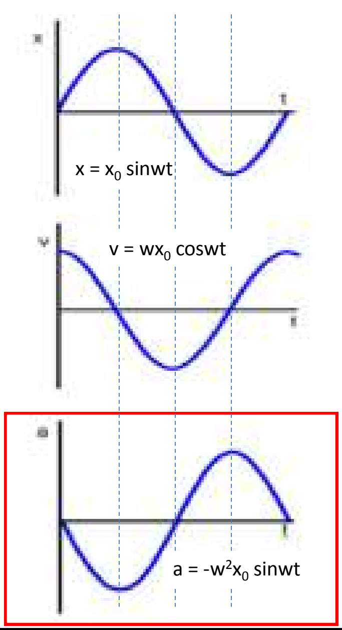
GRAPHICAL REPRESENTATION



Velocity - time graph

- Again, we have smooth curve.
- The shape of the curve is same as for the x – t graph, but it starts at a different point in the cycle.
- When t = 0, the mass is at midpoint of its oscillation, and this is where it is moving fastest.
- Hence, the velocity has its maximum value at this point (midpoint / equilibrium position).
- Its value is positive since it is moving towards the right.

GRAPHICAL REPRESENTATION

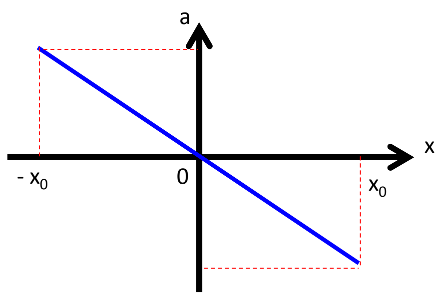


Acceleration - time graph

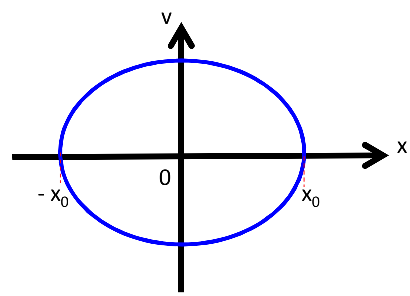
- At the start, mass is at midpoint, where it is in equilibrium. Thus no resultant force. (restoring force = 0)
- Since there is no resultant force on it, its acceleration = 0.
- As it moves to the right, the restoring force acts towards the left, giving it a negative acceleration.
- The acceleration has its greatest value when the mass is displaced furthest from the equilibrium position.
- Notice that whenever displacement is positive, acceleration is negative & vice versa. Hence the graph is 'upside down' version of displacement graph. **($a \propto -x$)**

GRAPHICAL REPRESENTATION

acceleration – displacement graph



velocity – displacement graph



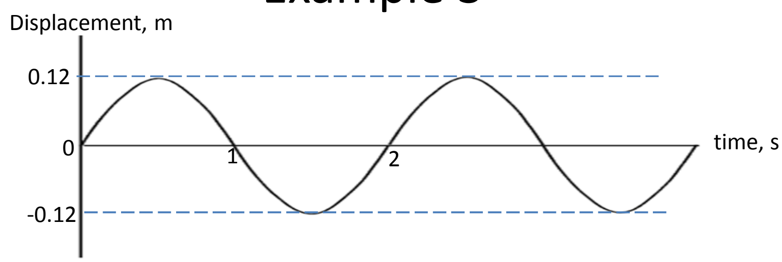
Example 2

- A body vibrates in SHM in a vertical direction with an amplitude of 50 mm and a periodic time of 4.0 s. Assuming that the displacement is zero at t = 0 and body moves upwards once is released. Calculate the displacement and direction after

(i) 2.5 s

(ii) 5.0 s

Example 3



- Determine:

(a) The amplitude

(b) The period

(c) The frequency

(d) The angular frequency

(e) The magnitude of acceleration

(i) when the displacement is zero

(ii) when the displacement is at its maximum

(f) The maximum velocity of the pendulum bob

Example 4

- A block of mass 0.75 kg is fastened to an unstrained horizontal spring whose spring constant is 82 N/m. the block is given a displacement of +0.12 m where the + sign indicates that the displacement is along +x axis and then released from rest.

(a) Find the magnitude of the restoring force that the spring exerts on the block just before the block is released

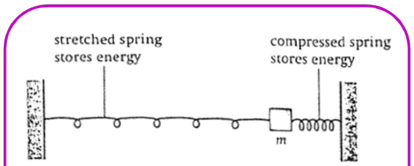
(b) Find the angular frequency

(c) Find the period of the motion

(d) What is the maximum speed of the block?

(e) Determine the magnitude of the maximum acceleration of the block

ENERGY CHANGES IN S.H.M

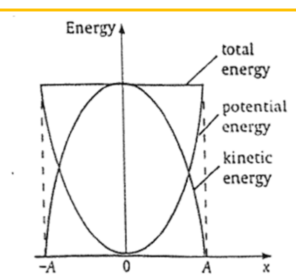


The elastic potential energy stored in the springs is converted to kinetic energy when the mass is released.

- During S.H.M. there is a constant interchange of energy between 2 forms; potential & kinetic.
- We can see this by considering the mass-spring system.
- When the mass is pulled to one side, 1 spring is compressed and the other is stretched. The springs store elastic potential energy.
- When the mass is released, it moves back towards the central position, accelerating as it goes. It has increasing kinetic energy.
- The potential energy stored in the springs decreases and the kinetic energy of the mass increases by a corresponding amount.
- Once the mass has passed the midpoint of its oscillation, its kinetic energy decreases and the energy is transferred back to the springs.
- Provided the oscillations are undamped (no loss of energy), the total energy in the system remains constant.

ENERGY CHANGES IN S.H.M

- We can represent this energy in two ways:
- <u>Energy – displacement graph</u>

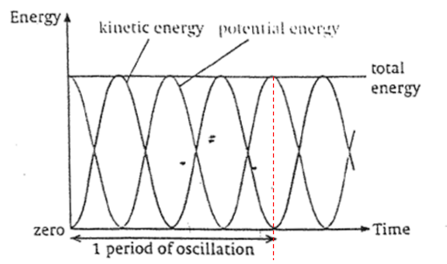


The kinetic energy is maximum at zero displacement; the potential energy is maximum at maximum displacement (A and $-A$).

- Potential energy is maximum when displacement is maximum. (positive or negative)
- Kinetic energy is maximum when displacement is zero. Total energy remains constant throughout.

ENERGY CHANGES IN S.H.M

- Energy – time graph



The kinetic energy and potential energy of an oscillator vary periodically, but the total energy remains constant if the system is undamped.

- Both KE & PE go through 2 complete cycles during 1 period of the oscillation.
- This is because KE is maximum when the mass is passing through the midpoint to the left and to the right, PE is maximum at both ends of the oscillation.

EQUATION FOR KINETIC ENERGY

- For kinetic energy

$$v = w\sqrt{x_o^2 - x^2}$$

$$E_k = \frac{1}{2} mv^2$$

$$\boxed{E_k = \frac{1}{2} mw^2 (x_o^2 - x^2)}$$

The maximum kinetic energy occurs when $x = 0$ as it is at the centre of the oscillation that the velocity is greatest, so the

$$\boxed{\text{Maximum kinetic energy } = \frac{1}{2} mw^2 x_o^2}$$

EQUATION FOR POTENTIAL ENERGY

- For potential energy
- If the potential energy is taken to be zero at the centre of the oscillation, then the maximum KE must also be the total energy of the oscillation. Since the total energy is constant, any loss of the KE must be converted to be potential energy. The value of the potential energy at any displacement is given by:

$$E_p = \text{total energy} - E_k$$

$$= \frac{1}{2} mw^2 x_o^2 - \frac{1}{2} mw^2 (x_o^2 - x^2)$$

$$\boxed{E_p = \frac{1}{2} mw^2 x^2}$$

Example 5

- An object of mass 0.30 kg execute linear SHM on a smooth surface at frequency 10 Hz with amplitude 5.0 cm. Neglect all resistance forces. Determine:

(a) The energy of the system

(b) The potential energy and kinetic energy when the displacement of the object is 3.0 cm

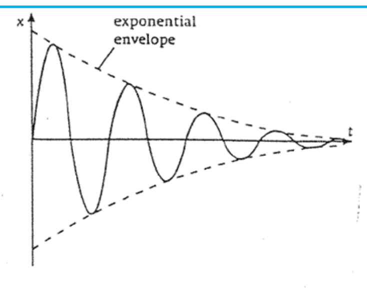
Example 6

- A 0.5 kg object connected to a light spring with a spring constant of 20 N/m oscillates on a frictionless horizontal surface

(a) Calculate the total energy of the system and the maximum speed of the object if the amplitude of the motion is 3.0 cm

(b) What is the velocity of the object when the displacement is 2.0 cm?

DAMPED OSCILLATIONS



Damped oscillations.

Amplitude decreases exponentially with time.

Notice that **frequency does not change as amplitude decreases.**

- In principle, oscillations can go on forever and ever. In practice, however, they do not.
- They die out, either rapidly or gradually due to friction.
- For example, a child on a swing knows that the amplitude of her swinging will decline until eventually she will come to rest unless she can put some energy into the swinging to keep it going. This happen because of friction (friction of swing and frame, friction with air.)
- The amplitude of oscillations decrease as the friction transfer energy away from her to the surroundings.

DAMPED OSCILLATIONS

- Damping is an effect that tends to reduce the amplitude of oscillations in an oscillatory system due to the loss of energy.
- Damping can be very useful if we want to get rid of vibrations.
- For example, car has springs which make the ride much more comfortable for us when the car goes over the bump.
- However we wouldn't want to spend every car journey vibrating up and down.
- So springs are damped by the shock absorbers, and we return rapidly to a smooth ride after every bump.
- Again, if lab ammeter is not critically damped, the meter needle will suffer from oscillations at the final reading it is suppose to register when a current is flowing through it.

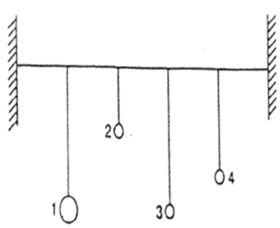
RESONANCE

- Force oscillation is motion produced when a system is acted upon by an external vibrating force.
- Resonance is a phenomenon in which a system responds at maximum amplitude to an external driving force. This occurs only when the driving force frequency is equal to or every close to the natural undamped frequency of the driven system.
- At resonance, there is maximum transfer of energy from the driving system into the driven system.
- With no damping, the amplitude and energy of the system will increase continuously.

RESONANCE

- For resonance to occur, we must have a system that is capable of oscillating freely. We must also have some way in which the system is forced to oscillate.
- When the forcing frequency matches that of the natural frequency of the system, the amplitude of oscillations grow dramatically.
- If the driving frequency does not quite match the natural frequency, the amplitude of the oscillations will increase, but not the same extent as when resonance is achieved.

DEMONSTRATING FORCED OSCILLATIONS & RESONANCE

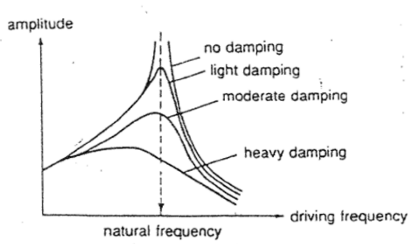


Barton Pendulum Experiment

- In this Barton's pendulum experiment, several lengths of pendulum are hang from a horizontal string.
- Each has its own natural frequency of oscillation.
- The driver, pendulum 1 has a larger mass at one of the end, and its length is equal to that of pendulum 3.
- This driver, pendulum 1 is set to swing, causing others to gradually swing too.
- It is observed that the pendulum with length matches that of the driver pendulum builds up a large amplitude so that it is resonating.
- Pendulum 2 & 4 has different natural frequency to that of the driver, thus has little effect.

RESONANCE (sharpness of the response)

- The **sharpness of the response** depends on the **damping forces** involved.
- Theoretically, with no damping, the amplitude of oscillation of the driven system should tend to be infinitely great when the driving frequency is equal to the natural frequency of oscillation of the driven system.
- In practice, this is no so, as damping is always present.
- With **light damping**, the **amplitude of oscillation of the driven system is high, within a very narrow range** of driving frequencies close to the natural frequency of the system. The response is then said to be sharp.
- With **heavier damping**, the maximum **response is spread out** over a wide range of frequencies.



Adding damper has 2 effects on the system:
1.) the amplitude of the peak oscillation decreases.
2.) the frequency at which maximum response occurs also decreases.

More heavy damping, the more decrease in amplitude and the resonant frequency.

RESONANCE (GOOD & BAD)

- **Example of constructive resonance:**
  1.) Radio and television tuning circuitry.
  (resonate at frequency of desired station)
- 2.) microwave cooking.
  (microwaves resonate with the $f_{natural}$ of water molecules)
- 3.) Magnetic Resonance Imaging. (MRI)
  (different atomic nuclei resonate with particular radio frequency.
  By analyzing the absorption of radio waves, a computer-generated image can be produced.)

- **Example of destructive resonance:**
  1.) Earthquakes.
  (building vibrate in resonance with the seismic waves)
  2.) Bridges collapse.
  (resonance due to strong gust of wind)

Example 7

- A particle oscillating in SHM with a period of 2 s, and an amplitude of 5 cm. What is its speed and acceleration at:

  a.) the centre of the oscillation;

  b.) the amplitude position;

  c.) a displacement of 2 cm ?

  d.) Calculate the PE and KE at displacement of 2 cm