TUTORIAL ANSWERS – MOTION IN A CIRCLE

1.) a.) At top of circular path, the net force acting towards the centre is tension & weight.

Hence **$T + mg = mv^2 / L$**

For string to be just taut, T = 0 ;

Thus, **$0 + mg = mv^2 / L$**

$V = \sqrt{(gL)}$

b.) At point B, total energy is equals to the kinetic energy. (Taking the position B to be zero potential energy level)

$E_{totsl} = (1/2)(m)(V)^2$

At point C, total energy is equals to kinetic energy + potential energy.

$E_{totsl} = (1/2)(m)(\sqrt{(gL)})^2 + mg(2L) = (5/2)\ mgL$

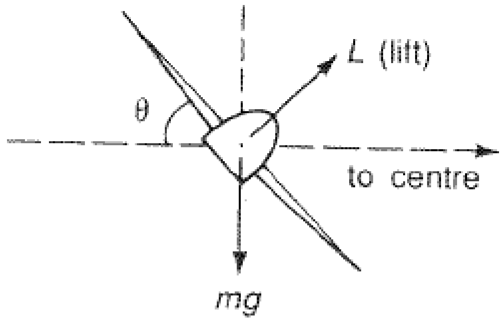
Assuming no work done against friction, total energy must be conserved.

$(1/2)(m)(V)^2 = (5/2)\ mgL$; thus, **$V = \sqrt{(5gL)}$**

2.) a.) $\omega = v / r = (0.2 \times 10^3) / (1.5 \times 10^3)$

$\omega = 0.13\ \text{rads}^{-1}$

b.)



The vertical forces $L\cos\theta$ and mg will cancel each other out. The resultant is simply horizontal component of the lift, $L\sin\theta$.

$L\sin\theta = mv^2 / r = (1.0 \times 10^4)(0.2 \times 10^3)^2 / (1.5 \times 10^3) =$ **2.67×10^5 N**

c.) The passenger is also moving in circular motion For the passenger to do this, there must be a force to provide the centripetal acceleration. In this case, the floor of the aircraft exerts a normal force, N on the passenger. The force component $N\sin\theta$ will be responsible for circular motion, meanwhile the force component of $N\cos\theta$ balances his weight.

3.) a.) a rate of change of angular displacement, i.e.

$\omega = \Delta\theta / \Delta t$, where θ is measured in radians , ω is in radians persecond

b.) (i) $v = r\omega$

(ii) For a given angular velocity, the linear velocity is directly proportional to the radius of circular motion, r. Hence by changing the cord length, the linear velocity,v can be varied while maintaining the same angular velocity, ω

(iii) Tension of the cord is necessary to provide the centripetal force giving rise to the perpendicular acceleration that changes the velocity direction in order for the stone to follow the circular path of motion.

(iv) $T = ma$; $a = v^2 / r$

$T = mv^2 / r = mv(v/r)$; $\omega = v / r$

$T = mv\omega$ (shown)

c.) (i) 1.) at highest point, centripetal acceleration, $a = v^2 / r = (12)^2 / 7$

$a = 20.6\ ms^{-2}$

2.) $R + mg = mv^2 / r$

$R = mv^2 / r - mg$

$R = (60)(20.6 - 9.81)$; **$R = 647\ N$**

(ii) 1.) $\Delta U = mg\Delta h = (60)(9.81)(14.0)$ **$= 8.24\ kJ$**

2.) $E_{total(bottom)}$ = solely based on K.E = $(1/2)(m)v_{bottom}^2$

$E_{total(top)}$ = based on K.E + P.E = $(1/2)(m)v_{top}^2 + mg\Delta h$

Neglecting frictional loss, $E_{total(bottom)} = E_{total(top)}$

$(1/2)(m)v_{bottom}^2 = (1/2)(m)v_{top}^2 + mg\Delta h$

$v_{bottom} = \sqrt{[(12)^2 + (2)(9.81)(14.0)]}$ **$= 20.5\ ms^{-1}$**

c.) (iii) To stay in contact with the track at the top of the loop. The reaction force, R must be greater or equal to zero. Thus, centripetal force needed at the top of the loop must also be greater than or equal to te weight of the passengers. Hence, the speed of the passenger at the top and speed at the bottom entry point must have certain minimum speed in order to complete the loop without passengers falling off the track.

4.) a.) $T\sin\theta = mv^2 / r$

b.) $T\cos\theta = mg$

c.) $\sin\theta = (0.40 / 0.80) = 0.5$

$\theta = 30^\circ$

$T\sin\theta = mv^2 / r$ --- (1)

$T\cos\theta = mg$ --- (2)

(1) / (2) ;

$\tan\theta = v^2 / rg$

$v^2 = rg\tan\theta = (0.4)(10)(\tan 30^\circ)$

$v = \sqrt{[(0.4)(10)(\tan 30^\circ)]}$

$v = 1.52\ ms^{-1}$

d.) T = circumference of circle / linear speed

$T = (2\pi)(0.4) / (1.52)$

Period, $T = 1.65$ s

e.) $T\cos\theta = mg$

$T = mg / \cos\theta = (0.2)(10) / (\cos 30^\circ)$

Tension, $T = 2.31$ N