TOPIC 11

IDEAL GASES

- Equation of state ($pV = nRT$)
- kinetic theory of gases
- Pressure of a gas
- Kinetic energy of a molecule

What is Ideal gases?

- Specifically, an ideal gas is one that obeys perfectly the equation of state (pV = nRT) of a gas, and that fulfills all assumptions of the kinetic theory of gases.

- In reality, there are no gases can fit this definition perfectly.

- But we assume that gases are ideal to <u>simplify calculations .</u>

The 3 Gas Laws

- When discussing thermal behaviour of a gas, there are 3 inter-dependent quantities which need to be considered.

- Experiments in the 17^{th} and 18^{th} centuries showed that these 3 quantities, which are volume (V), pressure (P), and temperature (T) of a given gas are all related.

- Physicists then came out with 3 gas laws which relate these 3 variables. They are Boyle’s law, Charles’ law, Pressure law.

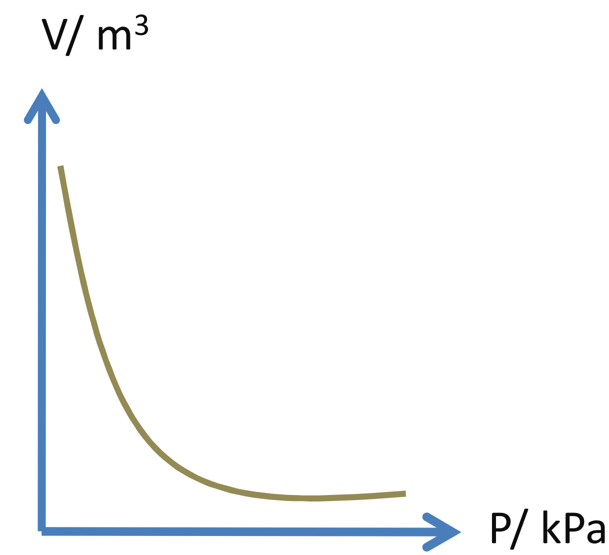
Boyle's law

- The pressure of a fixed mass of gas is inversely proportional to its volume if the temperature is constant.
- In mathematical form, this can be stated as:

$P \,\alpha\, 1/V$ (where T = constant)

$PV = \text{constant}$

$$P_1V_1 = P_2V_2$$



- Here, P_1 and V_1 represent the pressure and volume of the gas **before** a change, and P_2 and V_2 represent the pressure and volume of the gas **after** a change.

Example 1

- The pressure of a bubble under the sea surface is 120 cmHg. When the bubble rises to the surface of the sea, its volume becomes 25 cm^3. Assuming that atmospheric pressure is 76 cm Hg and there is no change in temperature, what is the original volume of the bubble?

Example 2

- A cylindrical flask of cross-sectional area A is fitted with an airtight piston that is free to slide up and down. Contained within the flask is an ideal gas. Initially, the pressure applied by the piston is 130 kPa and the height of the piston above the base of the flask is 25 cm. When additional mass is added to the piston, the pressure increases to 170 kPa. Assuming the system is always at the temperature 290 K, find the new height of the piston.

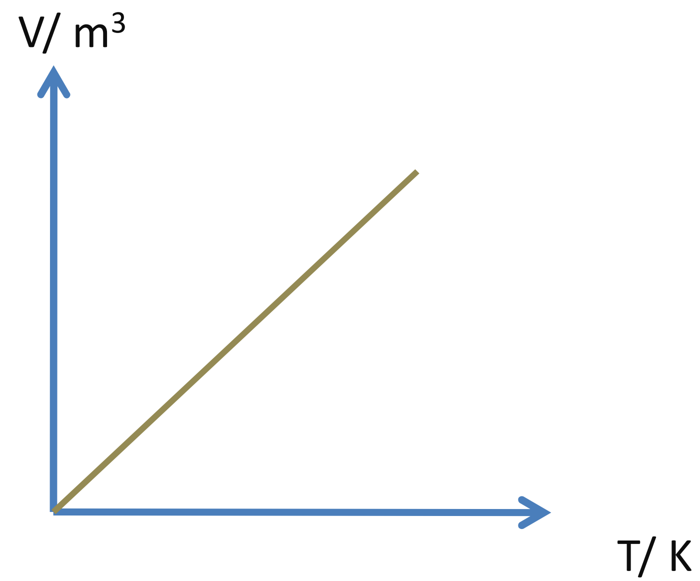
Charles' law

- The volume of a fixed mass of gas is directly proportional to its absolute temperature if the pressure is constant.

$V \propto T$ (P = constant)

$\frac{V}{T}$ = constant

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



- Here, V_1 and T_1 represent the volume and temperature of the gas **before** a change, and V_2 and T_2 represent the volume and temperature of the gas **after** a change.

Example 3

- An ideal gas is defined as one for which, at constant pressure, the volume of the gas is proportional to the absolute temperature. Calculate the absolute temperature T when an ideal gas has volume 0.00783 m^3, assuming that the same mass of the ideal gas has volume 0.00308 m^3 when at the same pressure and at temperature 273 K

Example 4

- A cylinder contains 200 cm^3 of gas at a temperature of 300 K. The gas is heated until its temperature increases by 30 K. If the piston of the cylinder expands under constant pressure, what is the final volume of the gas?

Example 5

- A quantity of gas is heated from 300 K to 360 K at constant pressure. Find the percentage increase in the volume of the gas.

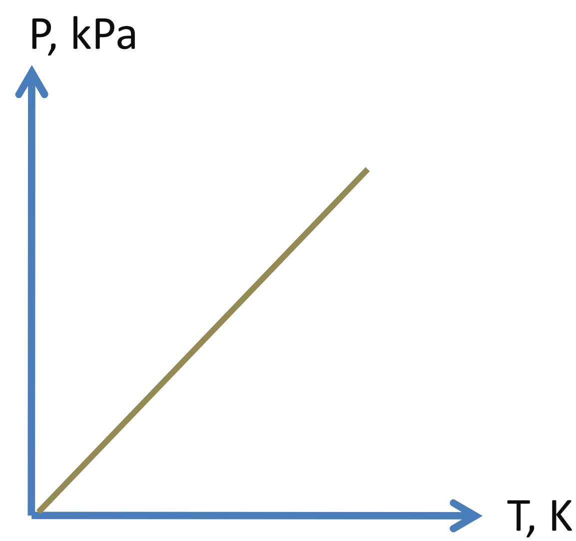
Pressure law

- The pressure of a fixed mass of gas is directly proportional to its absolute temperature if the volume is constant.

$P \propto T$ (V = constant)

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$



- Here, P_1 and T_1 represent the pressure and temperature of the gas **before** a change, and P_2 and T_2 represent the pressure and temperature of the gas **after** a change.

Example 6

- A fixed mass of gas in an enclosed metal container has a pressure of 2.5×10^5 Pa. If the gas is heated from 27 °C to 87 °C, calculate the final pressure of the gas.

Combined Gas Equation

- In practice, real gases obey the 3 gas laws only at moderate pressures and at temperatures.
- Only an ideal gas obeys all the 3 gas laws perfectly and is given by combining the 3 gas laws expressions, for a fixed mass of gas to give us the equation as below:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\boxed{\frac{PV}{T} = constant}$$

- This is called the combined gas equation.
- Very useful in solving problems in which volume, temperature and pressure vary simultaneously.
- It does not matter what units are use for pressure and volume provided both sides' units are same, but <u>temperature must be in Kelvin!!</u>

Example 7

- A fixed mass of gas, in passing through a jet engine, has its pressure increased from 3.0×10^5 Pa to 1.3×10^6 Pa, while its temperature rises from 80 °C to 1500 °C. By what factor does the volume of the gas change?

Ideal Gas Equation

- So far we have only seen how p, V & T are related.
- Physicists later discovered through experiments that number of moles of a given gas is directly proportional to the volume of a gas.
- So, if we consider *n* moles of an ideal gas, we can write the complete equation for the ideal gas in the following form:-

$$pV = nRT$$

- This equation is called the **ideal gas equation.** It relates all 4 variables quantities discussed. (p in **Pa**, V in **m^3**, T in **K** & n in **moles**)
- R is known as the constant of proportionality called the molar gas constant. ($R = 8.31\ Jmol^{-1}\ K^{-1}$)
- It does not matter whether we are considering light gas like hydrogen or heavier gas like carbon dioxide. As long as it is behaving as an ideal gas, we can use the above equation with the same constant, R.

Example 8

- Calculate the volume of 2 moles neon gas behaving as an ideal gas at a temperature of 350 K and a pressure of 1.2×10^5 Pa.

- A 5.0×10^{-2} m^3 vessel contains 3 moles of a gas at 40 ºC. Find the gas pressure.

Example 9

- The condition called standard temperature and pressure (STP) for a gas is defined to be a temperature of $0^{o}C = 273.15$ K and pressure of 1 atm = 1.013×10^5 Pa. if you want to keep a mole of ideal gas at STP, how big a container you need?

- A car tyre contains 0.020 m^3 of air at 27 °C and at a pressure of 3.0×10^5 Pa. Calculate the mass of air in that tyre.(molar mass of air = 28.8 g mol^{-1})

Ideal gas equation in other form

- Now we need to be able to distinguish between the number of moles, the number of molecules, and the Avogadro constant.
- The Avogadro constant is the number of molecules in 1 mole which has a fixed value of 6.02×10^{23} number molecules.
- The number of moles for a given amount of gas can be calculated as below:

$$Number\,of\,moles\,(n) = \frac{total\ number\ of\ molecules\ (N)}{number\ of\ molecules\ in\ 1\ mol\ (N_A)}$$

OR

$$Number\,of\,moles\,(n) = \frac{total\ mass\ of\ a\ given\ gas (m)}{Molar\ mass\ of\ the\ given\ gas\ (M)}$$

- Molar mass is the mass of 1 mole.
- By substituting $n = N/N_A$ into $pV = nRT$, we obtain $pV = (N/N_A)RT$.
- Since R and N_A is a constant, thus R/N_A will give us another constant called the Boltzmann's constant, given by the symbol k. **In short, $k = R/N_A$**
- So we can write our ideal gas equation in another form which is **$pV = NkT$** where $k = 1.38 \times 10^{-23}\ JK^{-1}$

Example 10

- Calculate the volume of 5.0 mol of an ideal gas at a pressure of 1.0×10^5 Pa and a temperature of 200 °C.

- A sample of gas contains 3.0×10^{24} atoms. Calculate the volume of the gas at a temperature of 300 K and a pressure of 120kPa.

Example 11

- A 0.1 m^3 cylinder contains 0.5 kg nitrogen at a pressure of 5.0 x 10^5 Pa. Determine the temperature of the gas. (molar mass of nitrogen = 28 $gmol^{-1}$)

- At what temperature would a 1.0 kg of oxygen occupy 1.0 m^3 at a pressure of 1.0 x 10^5 Pa? (molar mass of oxygen = 32 $gmol^{-1}$)

The Kinetic Theory of Gas

- During the same time period when physicists came up with the ideal gas equation, they has also developed an explanation of how gas exerts pressure to its surrounding.
- This explanation is based on the **Kinetic Theory of Gas.**
- The basic idea of this theory was that ***gases consist of atoms or molecules moving about at great speed and that the gas exerts a pressure on the walls of its container because of the continued impacts of the molecules with the walls and the reaction of the walls as the many molecules strike and rebound.***
- The ***value of the pressure is the total rate of the momentum change of the molecules (or also known as force) per unit area of the wall***.

The Kinetic Theory of Gas

- In order to calculate what is the pressure exerted by the gas in a given container, physicists had made several assumptions of the Kinetic Theory of Gas, which are as below:

  1.) The volume of molecules of a gas is negligible compared to the volume of the gas or vessel.

  2.) Molecules continuously move in random directions.

  3.) Collision between molecules, and between molecules and walls are perfectly elastic.

  4.) Time of a collision is very much lesser than the time in between collisions. Thus (change in momentum) / (time between collision) gives the average force.

  5.) The inter-molecular forces are negligible except during collisions.

  6.) Molecule moves with uniform velocity. Their mean kinetic energy is solely depending on their absolute temperature of the gas.

Derivation of the pressure exerted by gas

- We start by picturing a single molecule in a cube-shaped box of side L, with mass m and moving with velocity of c_x. (in the x-component)
- Consider the collision of the single molecule with one side of the cube.
- It rebounds elastically in the opposite direction with its velocity of $-c_x$.
- The change in momentum arises from this single collision is thus: $\mathbf{mc_x - (-mc_x) = 2mc_x}$
- If the molecule is to collide with that same wall again, it would take a time of **t** = $\mathbf{2L / c_x}$

  (2L is the distance to travel to the other end and to come back again while c_x is the speed which is constant due to elastic collision.)
- Now we can calculate what is the force that the single molecule exerts on one side of the wall by this equation. Force = change in momentum / time

  F = $2mc_x / (2L/c_x)$ = $\mathbf{mc_x^2/L}$.
- From the definition of pressure, we have **p** = F/A = $\mathbf{(mc_x^2/L)/(L^2)}$
- **Pressure = $\mathbf{(mc_x^2/L^3) = (mc_x^2/V)}$**

Derivation of the pressure exerted by gas

- The pressure equation is for a single molecule, but if the gas contains N molecules having x-components of velocity c_{x1} , c_{x2} , c_{x3} , ...c_{xN} , the total pressure on one face is given by:
- $P = (m/V)(c_{x1}^2 + c_{x2}^2 + c_{x3}^2 + ... + c_{xN}^2)$
- $P = (Nm/V) < c_x^2 >$

where $< c_x^2 > = (c_{x1}^2 + c_{x2}^2 + c_{x3}^2 + ... + c_{xN}^2) / N$

$< c_x^2 >$ = the mean square speed of the gas molecules travelling in x-direction

$\sqrt{< c^2 >}$ = the root mean square (rms) speed of the gas molecules

- Since the molecules in the cube are moving in all possible directions (all 3 coordinates-x, y, z), very few will be moving exactly parallel to x-axis. If we consider each molecule as having components of velocity c_x , c_y , c_z , then the resultant mean square speed of the gas would be

$$< c^2 > = < c_x^2 > + < c_y^2 > + < c_z^2 >$$

Derivation of the pressure exerted by gas

- Also, since there are a large number of molecules in random motion, we can assume that there will be equal numbers moving in each of the 3 coordinates.
- Hence, $<c_x^2> = <c_y^2> = <c_z^2>$
- Which will then simplify this expression $<c^2> = <c_x^2> + <c_y^2> + <c_z^2>$ to this expression $<c^2> = 3 <c_x^2>$
- Now we have $<c_x^2> = 1/3 <c^2>$

- Substituting $<c_x^2> = 1/3 <c^2>$ into the expression of $p = (Nm/V) <c_x^2>$, would then give us:

$$\boxed{p = (1/3)(Nm/V) <c^2>}$$ **; since Nm/v = ρ**

$$\boxed{p = (1/3)(\rho) <c^2>}$$

Kinetic energy of a molecule

- For ideal gas, internal energy is equal to total kinetic energy of all molecules, since P.E is zero.
- After obtaining a formula that can help us to find the pressure exerts by some amount of gas, we can also calculate the kinetic energy of the amount of gas by modifying the equation slightly.

$$p = \frac{1}{3}\rho < c^2 > \;\Rightarrow\; p = \frac{1}{3}\left(\frac{Nm}{V}\right) < c^2 > \;\Rightarrow\; 3pV = Nm < c^2 > \;\Rightarrow\; \frac{3pV}{2} = \frac{1}{2}(Nm) < c^2 >$$

Thus, the **total K.E of gas** would be:

$$\frac{1}{2}(Nm) < c^2 > = \frac{3pV}{2} = \frac{3NkT}{2} = \frac{3nRT}{2}$$

The **mean K.E of gas** would then be:

$$\frac{1}{2}(m) < c^2 > = \frac{3pV}{2N} = \frac{3kT}{2} = \frac{3RT}{2N_A}$$

Example 12

- Find the rms speed of gas molecules with the following speeds:

(a) 60 ms^{-1}, -70 ms^{-1}, 80 ms^{-1}, -50 ms^{-1}, and 60 ms^{-1}

(b) 300 ms^{-1}, 600 ms^{-1}, 400 ms^{-1}, -500 ms^{-1}, 300 ms^{-1} and -400 ms^{-1}

Example 13

- What is the root mean square speed of one oxygen molecule at 300 K? (mass of 1 oxygen molecule = 5.344×10^{-26} kg)

- Calculate the rms speed of nitrogen molecules if its density at pressure 1×10^5 Pa is 1.25 kgm^{-3}

Example 14

- Find the internal energy of 1 mole of ideal gas molecules at 300 K.

- Find the rms speed of 1 molecule of CO_2 at 30˚C. (Molar mass CO_2 = 44 g)

Example 15

- An ideal gas has a molar mass of 4 g. Total K.E. of a mass M of this gas is 375 J at 27 ˚C. Calculate i.) the K.E. at 127 ˚C. ii.) value of M.

- A vessel of 50 cm^3 contains hydrogen at 27 ˚C and pressure 1×10^5 Pa. Find the total K.E. of the hydrogen.

- Find the average KE. of a i.) hydrogen ii.) nitrogen molecule at 500 K. Which type moves faster at 500K.