TOPIC 8
Gravitational field

8.1 Gravitational Field
8.2 Force between point masses
8.3 Field of a point mass
8.4 Field near the surface of the Earth
8.5 Gravitational potential

Newton's Law of Universal Gravitation

- Any 2 bodies with masses can attract each other. The universal effect is known as **gravitation.** The force with which one body attracts the other due to their masses is known as **gravitational force.** **So, if a mass 'feels' a gravitational force then it is in a gravitational field.**
- Whenever a body has a force exerted *on* it, that force is exerted by some other body. If the force of gravity is being exerted on objects on Earth, what is the origin of that force? Newton's realization was that the force must come from the Earth. He further realized that this force must be what keeps the Moon in its orbit.

Anywhere on Earth, whether in Alaska, Australia, or Peru, the force of gravity acts downward toward the center of the Earth.

Newton's Law of Universal Gravitation

- A statement of Newton's Law of gravitation said that, **any two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation.**
- the law of universal gravitation reads:

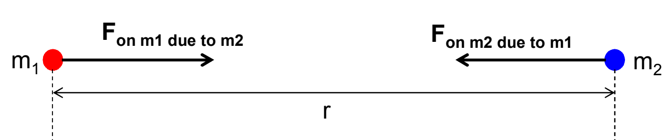
$$F = -G\frac{m_1 m_2}{r^2}$$

- Where G is called the universal gravitational constant:

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}.$$

Newton's Law of Universal Gravitation

- Newton's calculation showed that Earth's force of gravity extends into space, but weakens with distance **(the force of gravity varies inversely with the square of the distance from the center of the Earth)**
- The minus sign represents the fact that the force is attractive. The radial distance, r is measured outwards from the attracting body (earth), the force F acts in the opposite direction. (towards the earth)



Example 1

- Can you attract another person gravitationally? A 50kg person and a 70-kg person are sitting on a bench about 50 cm apart from each other. Estimate the magnitude of the gravitational force each exerts on the other.

Example 2

- Determine the force of gravitational attraction between the Earth and a 75 kg student if the student is standing at sea level, a distance of 6.4×10^6 m from the Earth's centre. [Mass of Earth: 5.98×10^{24} kg]

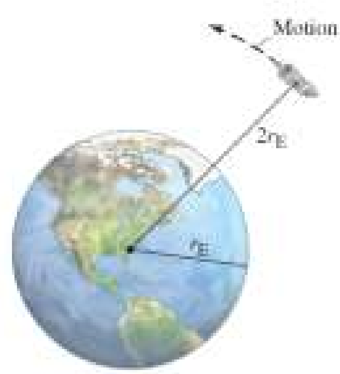
Example 3

- What gravitational force does the Earth exert on a 1.0 kg mass which is at a distance of 3.8 x 10^8 m from the centre of the Earth? The Earth may be considered to be sphere of mass 6.0 x 10^{24} kg.

Example 4

- What is the force of gravity acting on a 2000kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance r_E = 6380 km above the Earth's surface)?

The mass of the Earth is m_E = 5.98 x 10^{24} kg.



Example 5

- Find the net force on the Moon (m_M = 7.35 x 10^{22} kg) due to the gravitational attraction of both the Earth (m_E = 5.98 x 10^{24} kg) and the Sun (m_S = 1.99 x 10^{30} kg), assuming they are at right angles to each other.

(the Moon is 3.84 x 10^8 m from the Earth

and 1.50 x 10^{11} m from the Sun)

Gravitational Field

- According to the field concept, a **gravitational field** surrounds every body that has mass, and this field permeates all space; it extends throughout the universe.
- A second body at a particular location experiences a force because it is in the field of the first body, and vice versa.
- **Gravitational field strength (g)** at a point is defined as force acting per unit mass.

$$\boxed{g = \frac{force}{mass} = \frac{F}{m}}$$

- S.I unit: Nkg^{-1} or ms^{-2}
- Vector quantity

Gravitational Field Strength ‘g’

By definition of gravitational field strength, g, the force acting on the mass, m is given by

F = mg … (1)

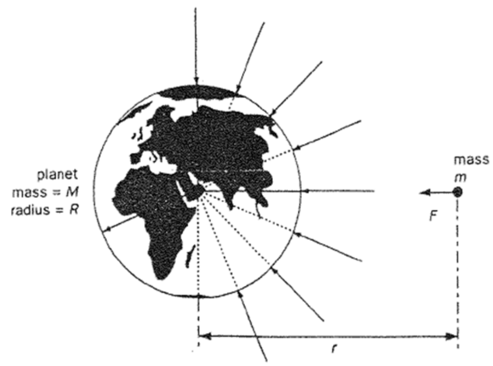
From Newton’s Law of gravitation, the force of this mass at a distance r from a planet of mass M is given by:

$F = - GMm / r^2$ … (2)

Equating (1) = (2);

$mg = - GMm / r^2$

$g = - GM / r^2$



$$g = -\frac{GM}{r^2}$$

Gravitational Field Strength ‘g’

TABLE 6–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s²)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth’s surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

Example 6

- Find the gravitational acceleration g on planet Venus, assuming the planet to be a sphere of radius 6052 km. [Mass of Venus: 4.87×10^{24} kg]

- Two 35000 kg ferries each are moving on parallel courses, 150 m apart. What is the magnitude of the acceleration of one of the ferries toward the other due to their mutual gravitational attraction?

Example 7

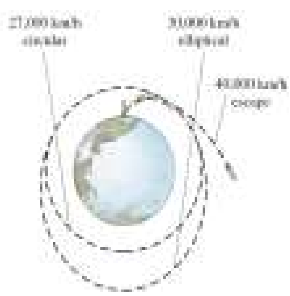
- Determine the magnitude of gravitational field strength of a point.
  [mass of Earth: 6.0×10^{24} kg; radius of Earth: 6.4×10^{6} m]

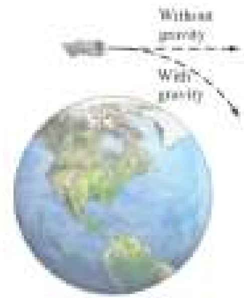
(a) 1000 km above the surface of the Earth

(b) On the surface of the Earth

Satellites and "Weightlessness"

- Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.





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Satellites and "Weightlessness"

- Any body orbiting a planet is a satellite of that planet.
- The moon is a natural satellite of the planet earth.
- Artificial satellites in earth's orbits provide communication links between ground stations, observational data on the weather, and navigation information.
- These satellites maintain their orbits due to the gravitational attraction between themselves and the earth.

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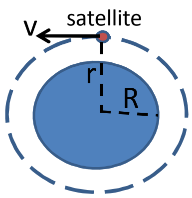
Satellite orbiting close to the surface of a planet



Tangential linear speed of satellite

» Consider a satellite *m* travelling round a planet of mass M, radius R, in a circular orbit of radius r, which is nearly equal to R.

» Tangential speed of satellite: v

Gravitational force acting on satellite: $F = \frac{GMm}{r^2}$

Centripetal force acting on satellite: $F_c = \frac{mv^2}{r}$

Satellite orbiting close to the surface of a planet

- In order for the satellite not to fall towards the earth, the centripetal force, F_c must be equal to the gravitational acting on it.
- $F = F_c$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\frac{GM}{r}} \Rightarrow v = \sqrt{g\,r}$$

Period of revolution of satellite

- We know that;

$$V = \sqrt{\frac{GM}{r}} \Rightarrow M = \frac{rv^2}{G} \quad \text{.........} \; (1)$$

- Period of satellite,

$$t = \frac{s}{v} \Rightarrow T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} \quad \text{..........} \; (2)$$

Sub (2) → (1)

$$M = \frac{rv^2}{G}$$

$$M = \frac{r}{G} \times \frac{4\pi^2 r^2}{T^2} \Rightarrow \boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{GM}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

Example 8

- A satellite travels at a height of 150 km above the surface of the Earth. Determine:

  [Radius of Earth: 6370 km; gravitational field strength: 9.8 Nkg^{-1}]

(a) The tangential linear speed of the satellite (in kms^{-1})

(a) The period of revolution of the satellite (in minutes)

Geostationary or synchronous orbit

Meaning

- A special type of orbit in which a satellite is positioned so that, as it orbits, the earth rotates below it at the same rate. The satellite remains above a fixed point on the earth's surface.
- This orbit is know as a **geostationary orbit.**
- There are about 300 satellites in such orbit.

Usefulness of the orbit

- Because the period of revolution of the satellite is 24 hours, same as the period of rotation of the Earth, the satellite always appears to be stationary to an observer on Earth.
- Used for telecommunications & satellites TV transmission.

Geostationary will exhibit the following characteristics:

- It will revolve in the same direction as the Earth
- It will rotate with the same period of rotation as that of the Earth
- It will move directly above the Earth's equator
- The centre of the geostationary orbit is at the centre of the Earth

Example 9

- For any future mission to Mars, it would be desirable to set up a system of three or four geostationary satellites around Mars to allow communication between the planet and earth. Calculate the radius of a suitable orbit around Mars.

[Mars has mass 6.4×10^{23} kg and rotational period of 24.6 hours]

Example 10

- A geosynchronous satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine
- (a) the height above the Earth's surface such a satellite must orbit, and
- (b) such a satellite's speed.
- (c) Compare to the speed of a satellite orbiting 200 km above Earth's surface.

[mass of Earth: 6.0×10^{24} kg; radius of Earth: 6.4×10^{6} m]

Example 11

- Although some international telephone signals are sent via telecommunication satellites in geostationary orbits, some are sent along cables on the earth's surface. This reduces the time delay between sending and receiving signal. Estimate the time delay for communication via a satellite, and explain why it is less significant when cables are used. Given:

  radius or geostationary orbit = 42300 km,

  radius of earth = 6400 km,

  speed of em waves in free space, $c = 3.0 \times 10^8 \text{ ms}^{-1}$

Gravitational potential energy

- Previously in our study of energy, we have seen that when a body is on the earth's surface, the system which comprises of the body and earth surface has zero gravitational potential energy. And as the body moves away from earth surface, the P.E will increase accordingly. **(P.E = mgh)**
- This study is only to be recommended whenever bodies close to the earth's surface are being considered. However, based on Newton's Law of Gravitation, we know that $g = -GM/r^2$, thus **P.E = -GMm / r.**
- Now we have another equation for P.E whereby zero gravitational potential energy can only take place when both objects are infinitely far apart. This is the true position in which **a mass has zero gravitational potential energy is when it is removed to infinity.**

Gravitational potential energy

- If a body is on the earth's surface, work has to be done on that body in order to increase the distance between the body and the planet. This **increases the gravitational potential energy stored.**
- If enough work is done, the distance between the body and the planet can be increased to infinity, whereby **energy stored will have increased to zero.**
- Having said that, we must treat the **potential energy stored in the attractive gravity field as negative at all distance less than infinity.**

Gravitational potential energy

- In short, gravitational potential energy is the work done in bringing object from infinity to the point.

- gpe, U = work done on the body = F x r
- $U = (- GMm / r^2)(r)$; $\mathbf{U = - GMm / r}$

- Remember, this means that the potential energy at infinity is zero and has a negative value at any other positions.

Gravitational potential, Ø

- The gravitational potential, Φ, at a point in a gravitational field is the work done in moving unit mass from infinity to that point.
- Also defined as the potential energy per unit mass of a small test mass placed at that point.
- $\Phi = U / m = (-GMm / r)(1/m)$
- **$\Phi = -GM / r$**
- Negative sign indicates that the potential at infinity (zero) is higher than potential at any point closer to the Earth.
- All points at a distance r in a concentric circles have the same potential, known as equipotential.

Example 12

- Determine the gravitational potential at a point 5000 km above the surface of the Moon.

  [Mass of the Moon: 7.32×10^{22} kg; radius of Moon: 1740 km]

- Potential energy of mass of 4.5 kg at a height of 5.4×10^{6} m is given by:

  [mass of Earth: 6.0×10^{24} kg; radius of Earth: 6.4×10^{6} m]

Example 13

- A rocket with mass 5.3×10^3 kg is launched from a launch pad on the equator. What is the gravitational potential energy of the rocket?

[Mass of Earth: 5.98×10^{24} kg; radius of the Earth: 6.4×10^6m]

Escape speed

Meaning

- An object on the surface of a planet is pulled by gravitational force. However, we can lift it up to a sufficiently large height so that the gravitational force on it is reduced to zero, and enabling it to escape from the planet.

- Escape speed: **minimum speed which an object must have in order to escape from the surface of the planet to go off deep into outer space and never return.**

Equation for escape speed

- An object is launched vertically upwards with escape speed v_e.
- Starting with this speed the object can reach a height where:
  i.) The final gravitational potential energy U_f of the object is zero.
  ii.) The final velocity v_f of the object is zero.

$$E_i = K_i + U_i$$
$$E_f = K_f + U_f$$
$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv_e^2 + \left(-\frac{GMm}{R}\right) = 0 + 0$$

m: mass of the object

M: mass of the planet

R: radius of the planet

Equation for escape speed

$$v_e^2 = \frac{2GM}{R} \Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

- We know that:

$$mg = \frac{GMm}{R^2} \Rightarrow GM = gR^2$$

- We may express the equation in another form:

$$\boxed{v_e = \sqrt{2gR}}$$

Example 14

- Determine the escape speed of an object on Mars.
  [m_{mars} = 6.39 x 10^{23} kg; R_{mars} = 3.33 x 10^6m]

- Calculate the escape velocity of a rocket leaving planet Saturn.
  [R_{saturn} = 60268 km; m_{saturn} = 5.69 x 10^{25} kg]