TOPIC 7

CIRCULAR MOTION

7.1 Kinematics of uniform circular motion

7.2 Centripetal acceleration

7.3 Centripetal force

Circular motion

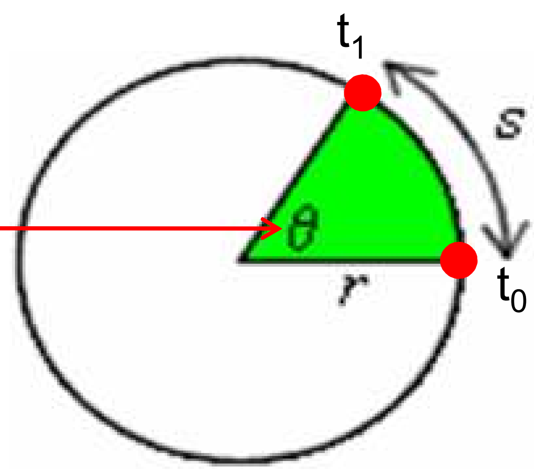
- Objects sometimes move in a curve or a circular path about some point, such as bicycles or cars turning round corners or electron orbiting around the nucleus. The objects are said to describe circular motion.
- In short, circular motion is motion following a circular path.

Angular Displacement

- Angular displacement is the term used in a circular motion, in reference to the angle through which the object has moved from its starting position. It is a vector quantity.

- **Angular displacement, $\theta = s / r$** ; s = length of the arc
- Unit is in radian (rad) r = radius of the circle

•This **small angle,θ is known** as **angular displacement** undergone by the particle



Defining the radian

- One radian is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.
- If an object moves all the way round the circumference of the circle, it basically moves a distance of $2\pi r$.
- We can then calculate the angular displacement in radians:

$$\theta = \text{circumference} / \text{radius}$$

$$\theta = 2\pi r / r = 2\pi.$$

- Hence, a complete circle contains 2π radians. But we can also say that the object has moved through 360˚. Hence, $360^\circ = 2\pi$ rad.
- Similarly, $180^\circ = \pi$ rad, $90^\circ = \pi/2$ rad, and so on.

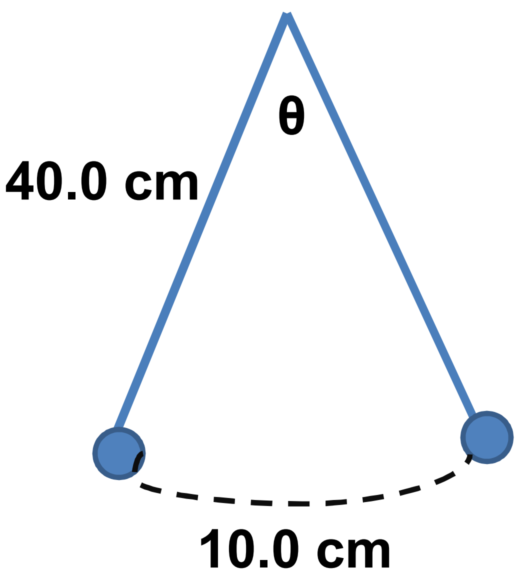
Defining the radian

- Basically, 1 rad = $360^{\circ}/2\pi = 57.3^{\circ}$

- To convert from degrees to radians, multiply by $2\pi/360^{\circ}$ or $\pi/180^{\circ}$

- To convert from radians to degrees, multiply by $360^{\circ}/2\pi$ or $180^{\circ}/\pi$

Example 1



- A simple pendulum of length 40.0 cm swings through an arc of 10.0 cm as shown. Find the angle in radians and in degree.

Angular Velocity

- When we studied linear motion, we immediately asked how fast an object is moving.
- Likewise, we can now ask how fast something is rotating.
- We call this the angular velocity and use the Greek letter ω (omega) to represent it.
- Angular velocity is a vector quantity and unit is in: rad s^{-1}.
- Define as the rate of change of angular displacement.

$$\omega = \frac{d\theta}{dt}$$

- We know that for an object to complete 1 revolution / 1 circle, the angular displacement would be 2π. Thus by knowing the period, which is the time to complete 1 circle, we can find its angular velocity:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Example 2

- An object undergoes circular motion with uniform angular speed at 100 r.p.m. Determine:

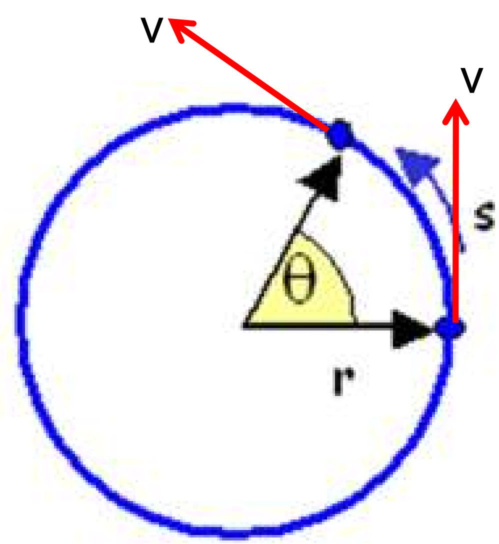
(a) The period

(b) The frequency of revolution

Relation between angular velocity and tangential linear velocity

- Consider a particle moving in a circle of radius *r* with angular frequency ω.
- The particle also has a linear velocity *v*, whose direction is always tangential to the circle.
- We have $\Delta s = r\Delta\theta$, divide both term with time, t:
- $\Delta s/ \Delta t = r(\Delta\theta/ \Delta t)$
- $v = r\,\omega$; $v = ms^{-1}$

  $\omega = rad\ s^{-1}$



Example 3

- The radius of the circular orbit of an electron in a hydrogen atom is 5.0×10^{-11} m. If the period of motion is 1.5×10^{-16} s, calculate:

  a.) The frequency of the rotation

  b.) The angular velocity

  c.) The linear speed

Example 4

- A car moves with tangential linear velocity of 50 km/h in a circle of radius 300 m. Determine the angular velocity of the car as it goes round the circle.

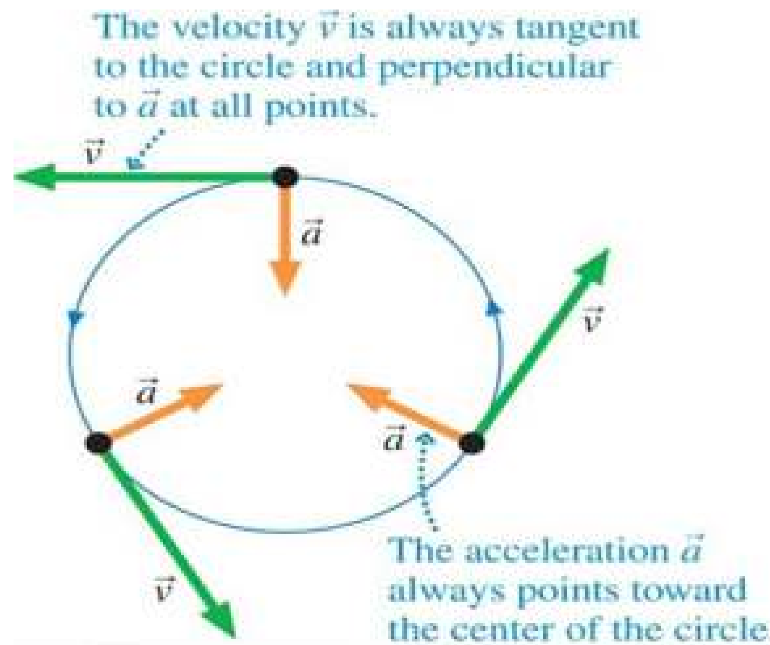
Centripetal acceleration & force

•When an object travels in a circle, its direction of motion continuously changes

•Velocity is a vector quantity

•As long as the direction changes, the velocity will have to change even though the magnitude may not have changed.

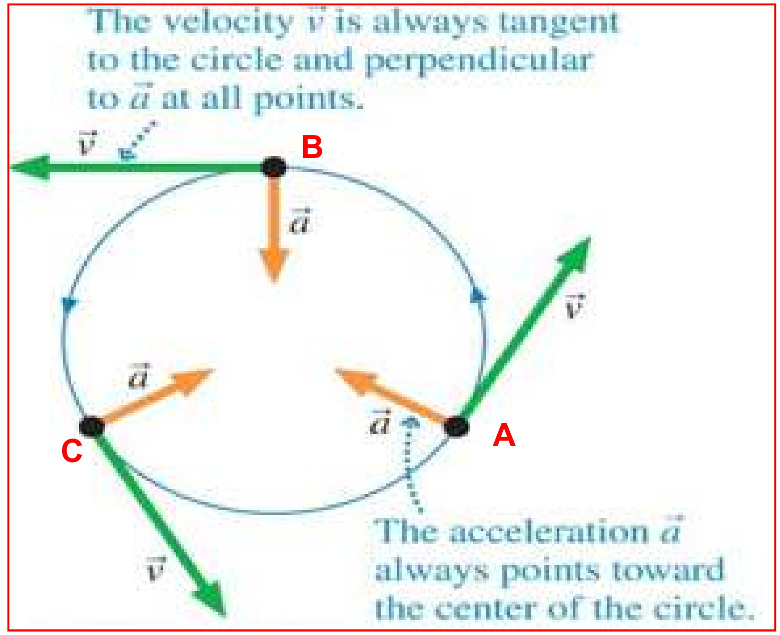
•Hence, the velocity of the object must change with time as it travels round a circle.



- A change in velocity produces acceleration
- This means that **an object travelling in a circle must experience acceleration, owing to the fact that its direction changes continuously.**

Centripetal acceleration & force

- Consider a small stone tied on the end of a string and whirled in a horizontal circle at constant speed as shown in the figure.
- The stone's velocity at any point on its path is always directed along the tangent to the circle, i.e. at right angle to the string.
- Since the speed is constant, the magnitude of the velocity at A, B, C etc is constant ($v_A=v_B=v_C$) but clearly the direction of the velocity is continuously changing.



- Velocity is a vector quantity.
- Therefore we can say that the velocity of the stone is continuously changing and the stone is accelerating.
- For such an acceleration to happen, there must be a resultant force.
- Since magnitude of the velocity is unaffected, the force cannot have any component parallel to the direction of the velocity.
- It is the tension in the string which causes the **acceleration** and **this tension (force) is always directed towards the centre of the circle.**

Centripetal acceleration & force

Conclusion:

- The resultant force must act on a body so as to make it follow a circular path as in this case the tension of the string is called the **centripetal force.**
- **The centripetal force acts at 90˚ to the direction of motion of the body,** and is therefore always **directed towards the centre of the circular path.**
- The centripetal force gives the body an **acceleration towards the centre of the circular path,** called the **centripetal acceleration.**
- If this centripetal force were to stop acting, the particle would continue in a straight line tangent to the circle at the point at which the force stopped acting.

Centripetal acceleration & force

- The equation for centripetal acceleration, a is given by:

  $\mathbf{a = v^2 / r}$

- Also, since $v = r\omega$;

  Thus, $a = (r\omega)^2 / r$

  $\mathbf{a = r\omega^2}$

- Since $F = ma$;

  Thus, centripetal force, $\mathbf{F_c = m(v^2 / r)}$ or $\mathbf{F_c = mr\omega^2}$

Example 5

- An object of mass 100 g is tied to a string of length 1.00 m and is made to travel in a horizontal circle. If the object travels through 5 rounds per seconds, determine the tension in the string.

Example 6

- An object of mass 2.00 kg rotates at constant speed in a horizontal circle of radius 5.00 m. The time for one complete revolution is 3.00s. What is the magnitude of the resultant force acting on that object?

Example7

- An object of mass 100 g is placed 40 cm from the centre of a horizontal turn-table. If the maximum frictional force between the object and the turn-table is 0.50 N, what will be the angular velocity of the turn-table when the object is about the slide?

Example 8

- A man stands on the Earth's equator. Find:

  a.) his angular velocity

  b.) his linear speed

  c.) his acceleration

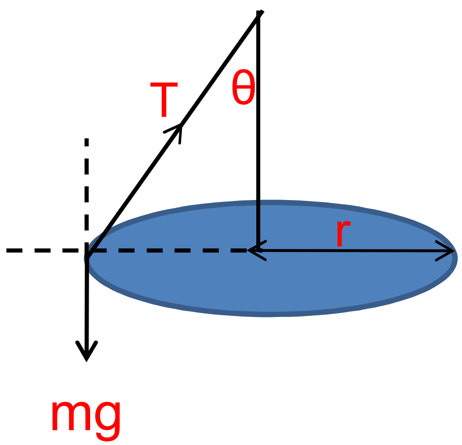
  due to the rotation of the Earth about its axis.

  [1 day = 8.6×10^4 s; radius of Earth = 6.4×10^6 m]

Example 9

- An object of mass 2.0 kg rotates at constant speed in a horizontal circle of radius 5.00 m. The time for one complete revolution is 3.00 s. What is the magnitude of the resultant force acting on the object?

Conical pendulum



- The bob of a simple pendulum can be made to travel in a horizontal circle.
- The pendulum moving in this way is known as conical pendulum.
- If we neglect air resistance, then the bob is exerted by 2 forces:

  its weight and the tension of the string.

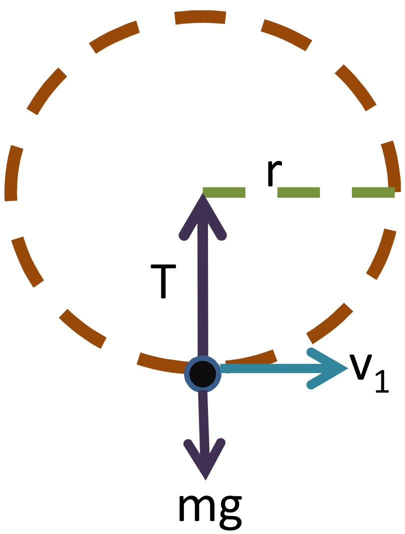
For vertical equilibrium, $\mathbf{T\cos\theta = mg}$ **--- (1)**

The forces towards the centre of a circular path is $\mathbf{T\sin\theta}$**.**

Hence for circular motion, $\mathbf{T\sin\theta = mv^2/r}$ **--- (2)**

(2) / (1) : $\mathbf{\tan\theta = v^2/gr}$

Motion in a vertical circle



- Figure beside shows a small mass "m" attached to the end of a light inextensible string and whirled in a vertical circle radius, r at a constant speed, v.

- The forces acting on the mass at all times are the weight (mg) and the tension (T) in the string.

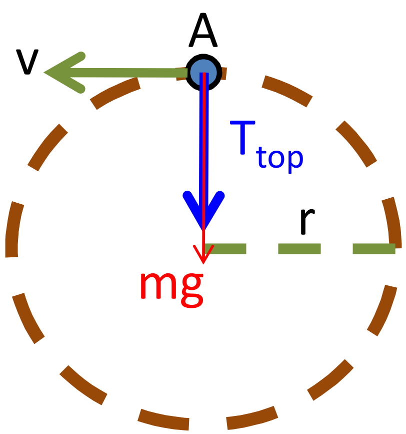
- The 2 position of interest are:

  1.) At the top of the circle.

  2.) At the bottom of the circle.

Motion in a vertical circle

At the top of the circle



- At the highest point A, the bob has tangential velocity v. The following forces exert on the bob:

  1.) The weight of the bob, mg, pointing downwards

  2.) The tension T_{top} in the string, pointing downwards

- Centripetal force acting on the bob:

  $$T_{top} + mg = mv^2 / r$$

  $$T_{top} = (mv^2 / r) - mg \quad \text{-- (1)}$$

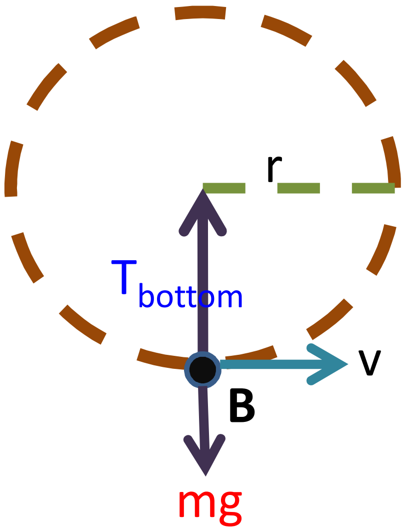
- The string will start to slack when $T_{top} = 0$

  $$0 = (mv^2 / r) - mg$$

  $$v = \sqrt{(gr)}$$

- Any speed lower than √ (gr) will cause string to slack.

Motion in a vertical circle



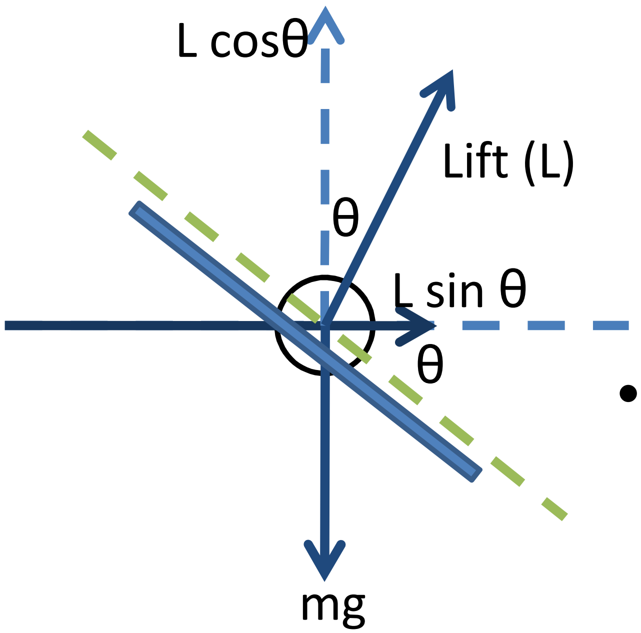
- **<u>At the bottom of the circle</u>**

- At lowest point B, the bob, of mass m, has tangential velocity *v*. The following forces exerted on the bob:
  1.) The weight of the bob, mg, pointing downwards
  2.) the tension T_{bottom} in the string, pointing upwards.

- Centripetal force acting on the bob:
  $$T_{bottom} - mg = mv^2 / r$$
  $$T_{bottom} = (mv^2 / r) + mg \quad \text{-- (2)}$$

- Comparing equation (1) & (2), we can see that tension in the string is maximum at the bottom and minimum at the top.

Banking of an airplane



- Centripetal force required to turn an airplane is contributed by horizontal component of the lift force, L.

$$L\ sin\theta = \frac{mv^2}{r} \quad ....(1)$$

- The weight is supported by vertical component of the lift force, L

$$L\ cos\theta = mg \quad ......(2)$$

- (1) /(2) $$\tan\theta = \frac{v^2}{rg}$$

Example 10

- A pail, which contains water, travels in a vertical circle of radius 1.0 m. Determine the minimum linear speed which the pail must have when it reaches the highest point of the circle so that the water does not spill out.

Example 11

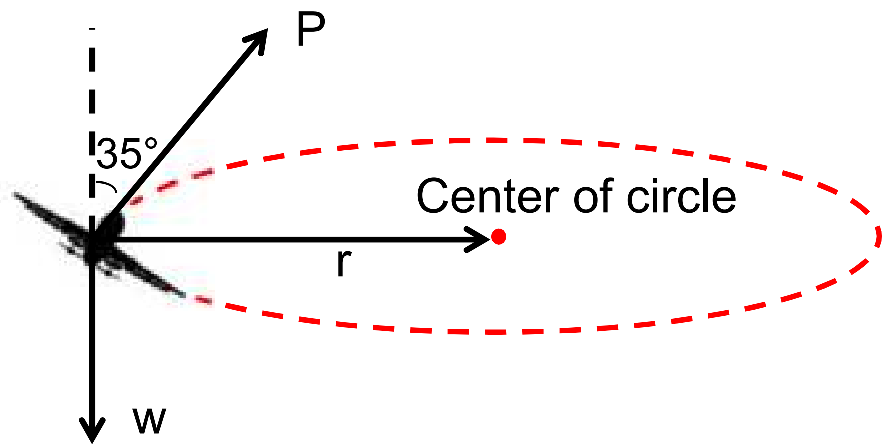
- An airplane is travelling in a circle of radius 10 km at a constant tangential linear speed of 200 m/s. Determine the angle at which the wings of the airplane must make with the horizontal.

Example 12

- A cyclist travels round a flat road of radius 10 m at speed 5 m/s. Determine the angle at which the bicycle must be inclined with the vertical so as to be in a stable position, assuming the bicycle does not skid.

Example 13

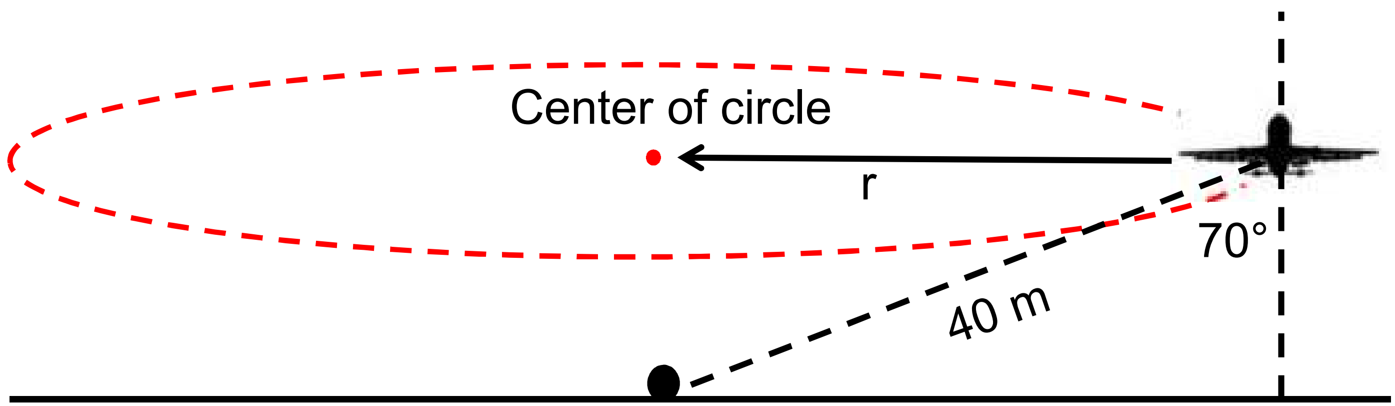
- An aircraft files with its wings tilted as shown in figure below in order to fly in a horizontal circle of radius, r. The aircraft has mass 4.00 x 10^4 kg and has a constant speed 250 ms^{-1} .With the aircraft flying in this way, 2 forces acting on the aircraft in the vertical plane are the force P acting at an angle 35° to the vertical and the weight ,w



a.) State the vertical component of P for horizontal flight.
b.) Calculate P.
c.) Calculate the horizontal component of P.
d.) Determine the acceleration of the aircraft towards the centre of the circle.
e.) Calculate the radius r of the path of the aircraft's flight.

Example 14

- A model airplane of mass 0.50 kg is attached to one end of the light inextensible cord of length 40m and the other end firmly fixed to the ground. When the plane is airborne, the cord is taut and makes an angle of 70° to the vertical. The plan flies with constant speed of 30 ms^{-1} in a horizontal circle without tilting.



a.) Draw the free body diagram of all forces acting on the plane.
b.) Find angular velocity of the plane.
c.) Find the tension of the cord.
d.) Find the lift of the plane.