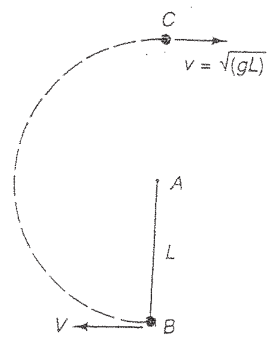
Motion in a Circle

Q1

A particle is suspended from a point A by an inextensible string of length L. It is projected from B with a velocity V, perpendicular to AB, which is just sufficient for it to reach the point C.

(a) Show that, if the string is just to be taut when the particle reaches C, its speed there, v, is $\sqrt{(gL)}$.
(b) Find the speed, V, with which the particle should be projected from B.



Q2

An aircraft of mass 1.0×10^4 kg is traveling at a constant speed of $0.2\ \mathrm{km\ s^{-1}}$ in a horizontal circle of radius 1.5 km.

(a) What is the angular velocity of the aircraft?
(b) Show on a sketch the forces acting on the aircraft in the vertical plane containing the aircraft and the centre of the circle. Find the magnitude and direction of their resultant.
(c) Explain why a force is exerted on a passenger by the aircraft. In what direction does this force act?

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Section A

Answer **four** questions from this section.

1 (a) State what is meant by *angular velocity*. [2]

(b) A stone is tied to one end of a cord and then made to rotate in a horizontal circle about a point C with the cord horizontal, as shown in Fig. 1.1.

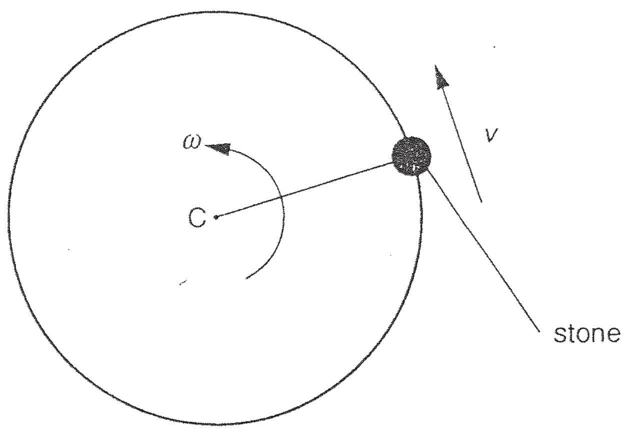

Fig. 1.1

The stone has speed v and angular velocity ω about C.

(i) Write down a relation between the speed v, the length r of the cord and the angular velocity ω.

(ii) Explain how v can be made to vary when ω is constant.

(iii) Explain why there needs to be a tension in the cord to maintain the circular motion.

(iv) Write down an expression for the acceleration of the stone in terms of v and r. Hence, if the stone has mass m, show that the tension T in the cord is given by

$$T = mv\omega.$$ [8]

(c) On one particular ride in an amusement park, passengers 'loop-the-loop' in a vertical circle, as illustrated in Fig. 1.2.

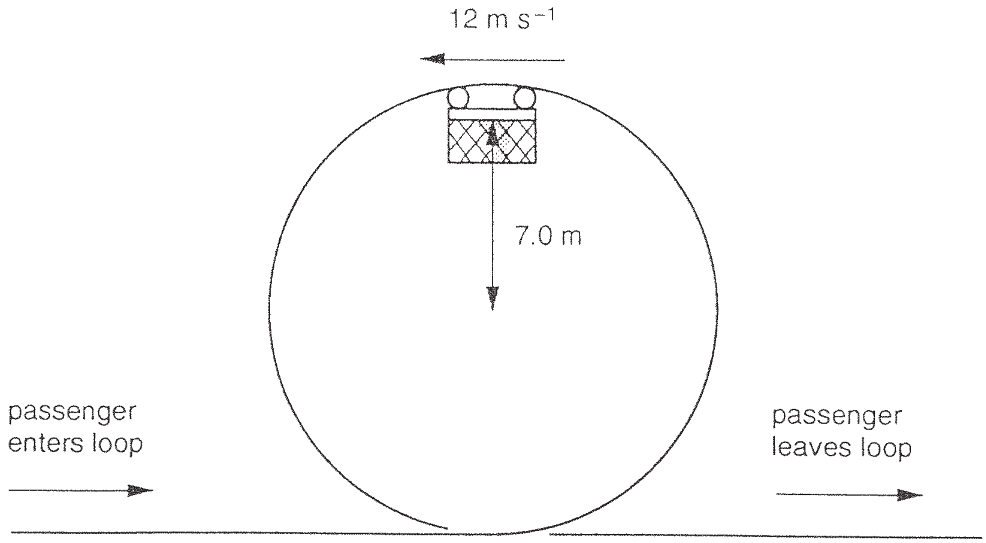

Fig. 1.2

The loop has a radius of 7.0 m and a passenger, mass 60 kg, is travelling at 12 $m\,s^{-1}$ when at the highest point of the loop. Assume that frictional forces may be neglected.

(i) Calculate, for the passenger when at the highest point,

(1) the centripetal acceleration,

(2) the force the seat exerts on the passenger.

(ii) The passenger now moves round and descends to the bottom of the loop. Calculate

(1) the change in potential energy of the passenger in moving from the top of the loop to the bottom,

(2) the speed of the passenger on leaving the loop.

(iii) Operators of this ride must ensure that the speed at which the passengers enter the loop is above a certain minimum value. Suggest a reason for this. [10]

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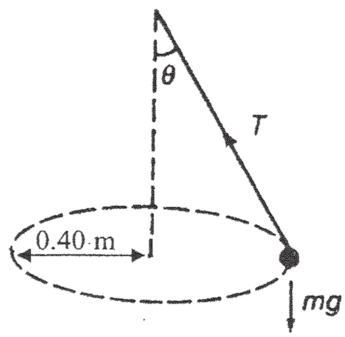
1. A conical pendulum consists of a small bob of mass 0.20 kg attached to an inextensible string of length 0.80 m. The bob rotates in a horizontal circle of radius 0.40 m, of which the centre is vertically below the point of suspension. (Assume $g = 10\ ms^{-2}$)



a) Write down an expression that relates the string tension, T with the linear speed, v of the bob.

.... [1]

b) Write down an expression that shows equilibrium of forces in the vertical direction.

....[1]

c) Calculate the linear speed of the bob in ms^{-1}.

linear speed = ... [3]

d) Calculate the period of rotation of the bob.

period = ... [2]

e) Find the tension in the string.

tension = ... [2]

2. The mass of the Earth is 5.98×10^{24} kg and its mean radius is 6.37×10^{6} m.

(a) Use Newton's law of gravitation to calculate the gravitational force acting on a 1.00 kg mass on the surface of the Earth. Assume that the Earth acts as a point mass.

gravitational force = ... [3]