30 Marking scheme: Worksheet (A2)

1 a change in energy = change in mass × (speed of light)^2 or $\Delta E = \Delta mc^2$ [1]

b i $\Delta E = \Delta mc^2$

$\Delta E = 0.001 \times (3.0 \times 10^8)^2$ [1]

$\Delta E = 9.0 \times 10^{13}$ J [1]

ii $\Delta E = \Delta mc^2$

$\Delta E = 9.1 \times 10^{-31} \times (3.0 \times 10^8)^2 = 8.19 \times 10^{-14}$ J [1]

$\Delta E \approx 8.2 \times 10^{-14}$ J [1]

2 a i Mass = $\dfrac{6.65 \times 10^{-27}}{1.66 \times 10^{-27}} = 4.01$ u [1]

ii Mass = $\dfrac{2.16 \times 10^{-26}}{1.66 \times 10^{-27}} = 13.01$ u [1]

b i Mass = $1.01 \times 1.66 \times 10^{-27} = 1.68 \times 10^{-27}$ kg [1]

ii Mass = $234.99 \times 1.66 \times 10^{-27} = 3.90 \times 10^{-25}$ kg [1]

3 In all nuclear reactions the following quantities are conserved:

- charge (or proton number)
- nucleon number
- mass–energy
- momentum.

Any **three** of the above. [3]

4 a The nucleons within the nucleus are held tightly together by the strong nuclear force. [1]

b The binding energy of a nucleus is the **minimum** energy required to separate the nucleus into its constituent protons and neutrons. [1]

c binding energy per nucleon = $\dfrac{\text{binding energy}}{\text{number of nucleons}}$

binding energy per nucleon = $\dfrac{128}{16}$ [1]

binding energy per nucleon = 8.0 MeV [1]

5 a The half-life of a radioactive isotope is the **mean** time taken for the number of nuclei of the isotope to decrease to half the initial number. [1]

b i 20 minutes is 1 half-life, so number of nuclei left = $\dfrac{N_0}{2}$ [1]

ii 1.0 hour is 3 half-lives. [1]

Number of nuclei left = $\left(\dfrac{1}{2}\right)^3 N_0 = \dfrac{N_0}{8}$ [1]

6 a i Activity is equal to the number of emissions (or decays of nuclei) per second.
Hence, there are 540 α-particles emitted in 1 second. [1]

ii Number of α-particles emitted in 1 h = $540 \times 3600 \approx 1.9 \times 10^6$ [1]

b Energy released in 1 s = number of α-particles emitted in 1 s × energy of each α-particle [1]

energy released in 1 s = $540 \times 8.6 \times 10^{-14}$ [1]

energy released in 1 s = 4.64×10^{-11} J $\approx 4.6 \times 10^{-11}$ J [1]

c Rate of emission of energy = energy released per second

rate of emission of energy = 4.6×10^{-11} J s^{-1} = 4.6×10^{-11} W [1]

7 a The nuclide $^{56}_{26}\text{Fe}$ is the most stable. [1]

It has the maximum value for the binding energy per nucleon. [1]

b Binding energy = binding energy per nucleon × number of nucleons

binding energy ≈ $12.3 \times 10^{-13} \times 12$ [1]

binding energy ≈ 1.5×10^{-11} J [1]

c From the graph, the binding energies per nucleon of $^{2}_{1}\text{H}$ and $^{4}_{2}\text{He}$ are approximately 1.0×10^{-13} J and 11.2×10^{-13} J. [1]

energy released = difference in binding energy per nucleon × number of nucleons [1]

energy released = $[11.2 \times 10^{-13} - 1.0 \times 10^{-13}] \times 4$ [1]

energy released = 4.08×10^{-12} J ≈ 4.1×10^{-12} J [1]

d High temperatures (~10^8 K) and pressures. [2]

8 $92\ ^{1}_{1}\text{proton} + 143\ ^{1}_{0}\text{neutron} \rightarrow\ ^{235}_{92}\text{uranium}$ [1]

mass defect = [(143 × 1.009) + (92 × 1.007)]u – (234.992)u [1]

mass defect = 1.939 u = $1.939 \times 1.66 \times 10^{-27}$ kg [1]

mass defect = 3.219×10^{-27} kg [1]

binding energy = mass defect × (speed of light)^2 [1]

binding energy = $3.219 \times 10^{-27} \times (3.0 \times 10^8)^2 = 2.897 \times 10^{-10}$ J [1]

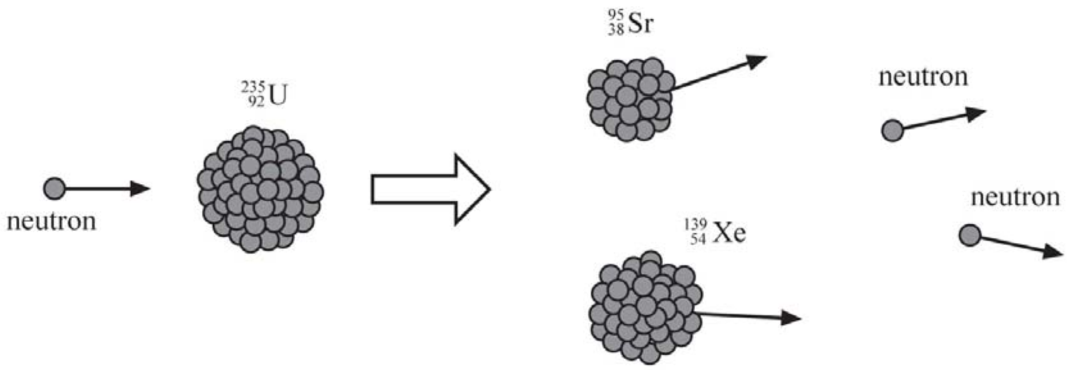
$$\text{binding energy per nucleon} = \frac{\text{binding energy}}{\text{number of nucleons}}$$

$$\text{binding energy per nucleon} = \frac{2.897 \times 10^{-10}}{235} = 1.233 \times 10^{-12} \approx 1.2 \times 10^{-12}\ \text{J}$$ [1]

9 a Fission is the splitting of a heavy nucleus like $^{235}_{92}\text{U}$ into two approximately equal fragments.

The splitting occurs when the heavy nucleus absorbs a neutron. [1]

b i All particles identified on the diagram. [1]



ii In the reaction above, there is a decrease in the mass of the particles. [1]

According to $\Delta E = \Delta mc^2$, a decrease in mass implies that energy is released in the process. [1]

iii The change in mass is Δm given by:

$\Delta m = [1.575 \times 10^{-25} + 2.306 \times 10^{-25} + 2(1.675 \times 10^{-27})] - [3.902 \times 10^{-25} + 1.675 \times 10^{-27}]$ [1]

$\Delta m = -4.250 \times 10^{-28}$ kg [1]

(The minus sign means a decrease in mass and hence energy is released in this reaction.)

$\Delta E = \Delta mc^2$ [1]

$\Delta E = 4.250 \times 10^{-28} \times (3.0 \times 10^8)^2$ [1]

$\Delta E = 3.83 \times 10^{-11}$ J ≈ 3.8×10^{-11} J [1]

10 Binding energy of ‘reactant’ = 236 × 7.59 = 1791 MeV (binding energy of neutron = 0) [1]
Total binding energy of ‘products’ = (146 × 8.41) + (87 × 8.59) ≈ 1975 MeV [1]
Therefore energy released = 1975 – 1791 = 184 MeV [1]

11 a $t_{1/2} = \dfrac{0.693}{\lambda}$ so $\lambda = \dfrac{0.693}{t_{1/2}}$ [1]

$\lambda = \dfrac{0.693}{56}$ [1]

$\lambda = 1.238 \times 10^{-2}\ \text{s}^{-1} \approx 1.2 \times 10^{-2}\ \text{s}^{-1}$ [1]

b $A = \lambda N$ [1]

$A = \dfrac{0.693}{56} \times 6.0 \times 10^{10}$ [1]

$A \approx 7.4 \times 10^{8}$ Bq [1]

12 a $A = \lambda N$ so $\lambda = \dfrac{A}{N}$ [1]

$\lambda = \dfrac{5.0 \times 10^{9}}{8.0 \times 10^{14}}$ [1]

$\lambda = 6.25 \times 10^{-6}\ \text{s}^{-1} \approx 6.3 \times 10^{-6}\ \text{s}^{-1}$ [1]

b $t_{1/2} = \dfrac{0.693}{\lambda}$ [1]

$t_{1/2} = \dfrac{0.693}{6.25 \times 10^{-6}}$ [1]

$t_{1/2} = 1.11 \times 10^{5}\ \text{s} \approx 1.1 \times 10^{5}\ \text{s}$ [1]

c $N = N_0\, e^{-\lambda t}$ [1]

$N = 8.0 \times 10^{14}\, e^{-(6.25 \times 10^{-6} \times 40 \times 3600)}$ [1]

$N = 3.25 \times 10^{14} \approx 3.3 \times 10^{14}$ [1]

13 a The decay constant is the probability that an individual nucleus will decay per unit time. [1]

b i $t_{1/2} = \dfrac{0.693}{\lambda}$ so $\lambda = \dfrac{0.693}{t_{1/2}}$ [1]

$\lambda = \dfrac{0.693}{18 \times 24 \times 3600}$ [1]

$\lambda = 4.46 \times 10^{-7}\ \text{s}^{-1} \approx 4.5 \times 10^{-7}\ \text{s}^{-1}$ [1]

ii $A = \lambda N$ [1]

$A = 4.46 \times 10^{-7} \times 4.0 \times 10^{12}$ [1]

$A = 1.78 \times 10^{6}$ Bq ≈ 1.8×10^{6} Bq [1]

iii 36 days is equal to 2 half-lives. [1]

activity = $\left(\dfrac{1}{2}\right)^{2} \times 1.78 \times 10^{6} = 4.45 \times 10^{5}$ Bq ≈ 4.5×10^{5} Bq [1]

14 a number of nuclei = number of moles × N_A

number of nuclei = $\dfrac{1.0 \times 10^{-6}}{226} \times 6.02 \times 10^{23}$ [1]

number of nuclei = $2.66 \times 10^{15} \approx 2.7 \times 10^{15}$ [1]

b $A = \lambda N$ [1]

$A = \left(\dfrac{0.693}{t_{1/2}}\right) \times N = \dfrac{0.693 \times 2.66 \times 10^{15}}{1600 \times 365 \times 24 \times 3600}$ [1]

$A \approx 3.7 \times 10^{4}$ Bq [1]

15 According to Einstein's equation: $\Delta E = \Delta mc^2$ [1]

In this case, ΔE is the energy of two photons and Δm is the mass of two protons. [1]

Hence:

$2 \times \dfrac{hc}{\lambda} = (2 \times m_p)\, c^2$ [1]

$\lambda = \dfrac{hc}{m_p c^2} = \dfrac{h}{m_p c} = \dfrac{6.63 \times 10^{-34}}{1.7 \times 10^{-27} \times 3.0 \times 10^8}$ [1]

$\lambda = 1.3 \times 10^{-15}$ m [1]

16 For **fusion**, we have:

energy released per kg = number of 'pairs' of ^2_1H in 1 kg × 4.08×10^{-12} J (from **7 c**) [1]

energy per kg = $\left(\dfrac{1}{2} \times \dfrac{1000}{2} \times 6.02 \times 10^{23}\right) \times 4.08 \times 10^{-12}$ [1]

energy per kg = 6.14×10^{14} J ≈ 6.1×10^{14} J [1]

For **fission**, we have:

energy released per kg = number of nuclei in 1 kg × 3.83×10^{-11} J (from **9 b**) [1]

energy per kg = $\left(\dfrac{1000}{235} \times 6.02 \times 10^{23}\right) \times 3.83 \times 10^{-11}$ [1]

energy per kg = 9.8×10^{13} J [1]

There is less energy released per fusion than per fission. However, there are many more nuclei per kg for fusion. Hence fusion produces more energy per kg than fission. [1]

17 $N = N_0\, e^{-\lambda t}$ and $\lambda = \dfrac{0.693}{t_{1/2}}$ [1]

fraction left = $\dfrac{N}{N_0} = e^{-\lambda t} = e^{-(0.693t/t_{1/2})}$ [1]

fraction left = $e^{-(0.693 \times 5.0 \times 10^9 / 4.5 \times 10^9)}$ [1]

fraction left = 0.463 ≈ 0.46 [1]