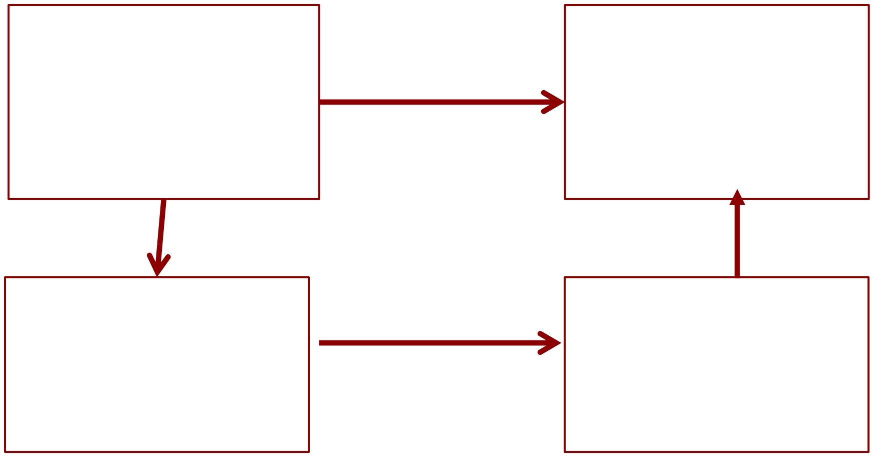
Gravitational Field



Newton's Law of Universal Gravitation

Every particle in the universe attracts every other particle with a force, which is directly proportional to the product of their *masses* and inversely proportional to the *square of the distance* between their centres.

Example on Forces

Calculate the gravitational force of the Earth on an astronaut who weighs 780 N on the Earth's surface, when the astronaut is in a spacecraft orbiting the Earth at a height of 200 km. The radius of the Earth is 6400 km. [733N]

Solution

$F \propto 1/ r^2$

$F = k/ r^2$

$F r^2 = k$

$780 \times 6400 = W \times (6400+200)$

$W = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Example

Calculate the gravitational force between the Earth and the Moon.

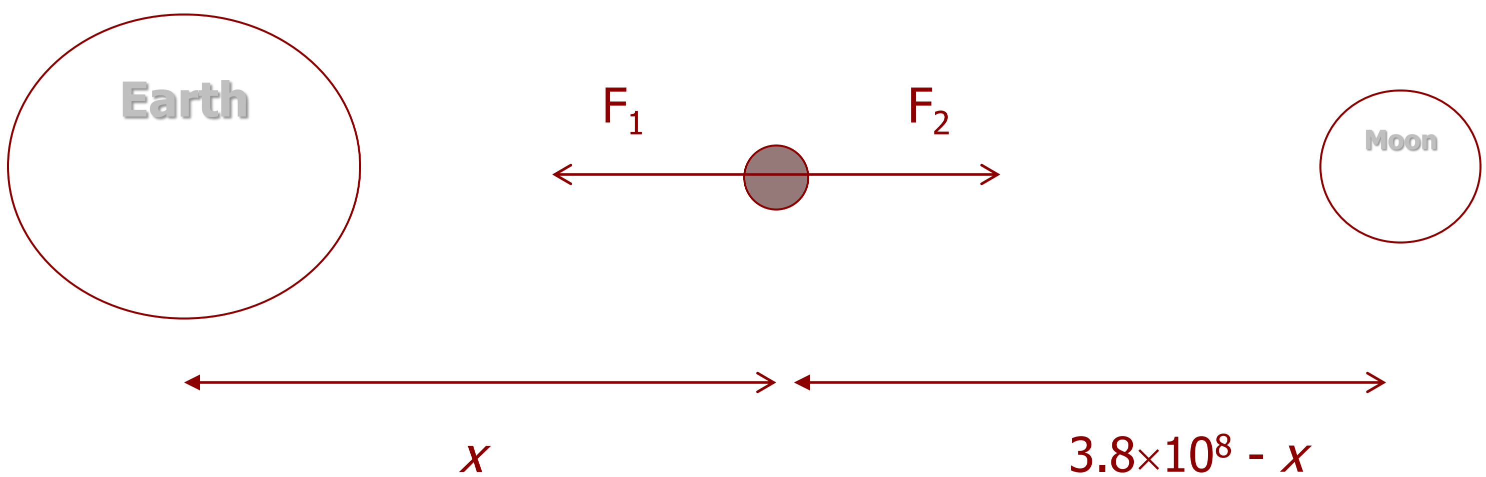
(Given: $M_E = 6\times10^{24}$ kg; Mm = 7.4×10^{22} kg; radius of the Earth $R_E = 6.4\times10^{6}$ m; distance between the centre of Earth and the moon = 3.8×10^{8} m)

$$F = \frac{GMm}{r^2} = \frac{6.67x10^{-11}(6x10^{24})(7.4x10^{22})}{(3.8x10^{8})^{\,2}} = \ldots..N$$

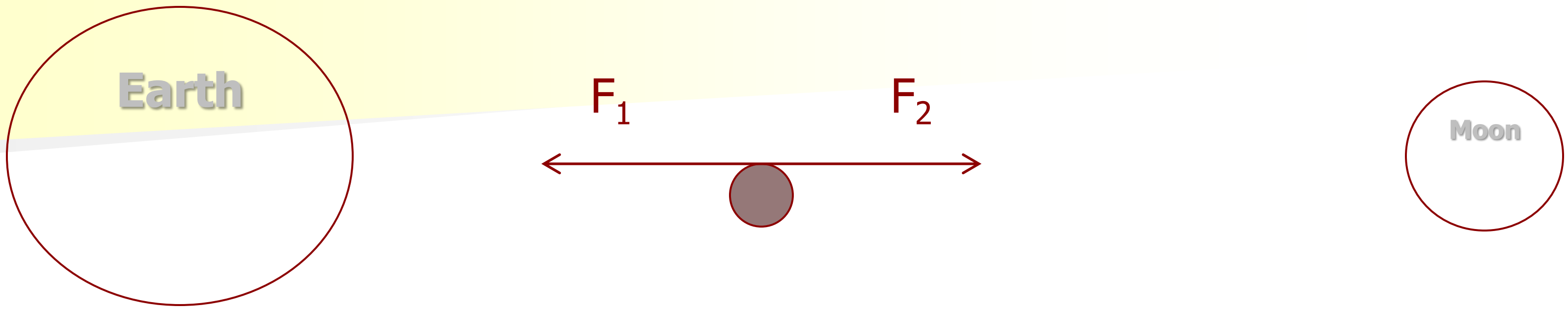
Example

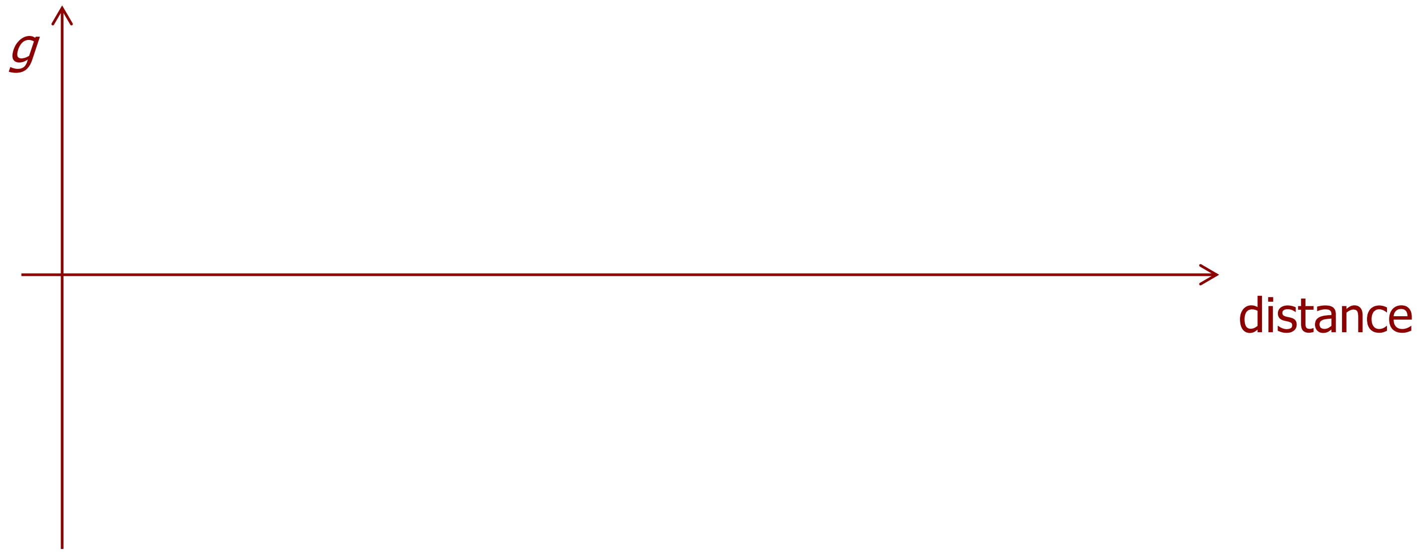
A space capsule travels between the Earth and the moon. By considering only the gravity due to the Earth and the moon, calculate the distance from the centre of the Earth where it is under zero gravity. [3.42×10^{8} m]

(Given: $M_E = 6\times10^{24}$ kg; Mm = 7.4×10^{22} kg; radius of the Earth $R_E = 6.4\times10^{6}$ m; distance between the centre of Earth and the moon = 3.8×10^{8} m)



Solution





Examples on strength

Gravitational field strength - gravitational force per unit mass

$$g = \frac{GM}{r^2}$$

Where M is the mass that produces grav field and r is the distance measured from its centre outwardly

Mass of the Earth $M_E = 6 \times 10^{24}$ kg;

radius $R_E = 6.4 \times 10^6$ m

a) Calculate the value of *g* near the Earth's surface?

Give two assumptions you make for this calculation.

b) Why is the value calculated in (a) different from the empirical value, i.e. 9.81 ms^{-2}?

c) Why is the acceleration due to gravity g at the equator greater that that at the poles?

Solution

a) g = ……….

two assumptions

- Earth is assumed to be perfectly spherical
- density of the Earth is uniform

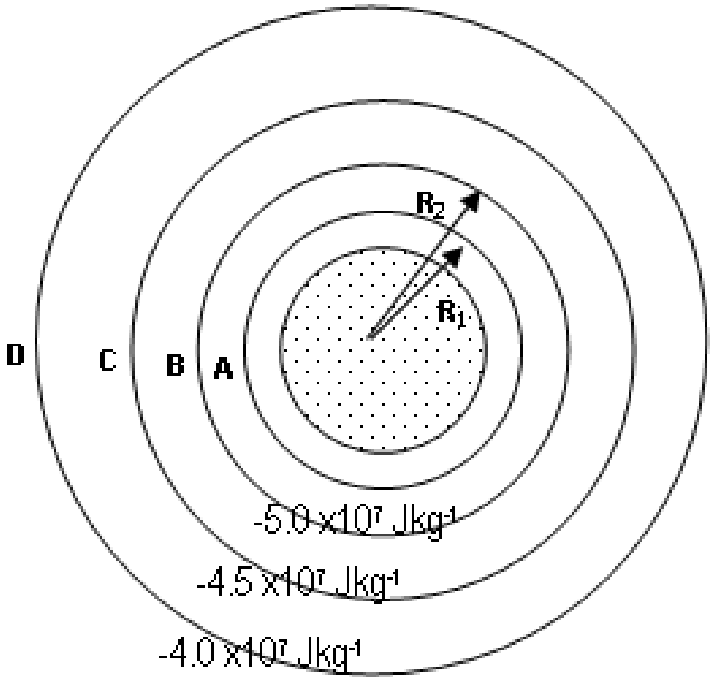
b) - the earth is not perfectly spherical but it is flattened

- the mass of the earth is not uniformly distributed

c) Due to the non-spherical shape of the earth, the distance from the centre of the earth to the poles is shorter, since g is inversely proportional to distance squared hence, g is larger at the poles.

Potential

Potential at a point is defined as the work done to bring a unit mass from the infinity to that point.



The change of GPE, ΔE_p or the minimum work done W

W = ΔGPE

= **mass $\times$ potential difference** $\Delta\phi$

= mass $\times$ $(\phi_2 - \phi_1)$

Work done to bring a mass of 5 kg from B to D

= mass $\times$ $(\phi_D - \phi_B)$

= 5 kg x [- 4 – (-5)] x 10^{-7} Jkg^{-1}

= 5 x 10^{-7} J

GPE

$$E = \frac{-GMm}{r}$$

Mass of the Earth $M_E = 6 \times 10^{24}$ kg;

radius $R_E = 6.4 \times 10^6$ m

1) Determine the GPE of a 50 kg mass on the surface of the Earth.

$$E = \frac{-GMm}{r} = \frac{-6.67x10^{-11}(6x10^{24})(50)}{6.4x10^6} = \text{.......................} \text{ J}$$

2) Determine the GPE of a 50 kg mass at a point of 100 km above the surface of the Earth

$$E = \frac{-GMm}{r} = \frac{-6.67x10^{-11}(6x10^{24})(50)}{6.4x10^6+100x10^3} = \text{..........................} \text{ J}$$

3) Determine the loss in GPE when a 50 kg mass falls from at a point of 100 km above the surface of the Earth to the surface.

Launching / Returning

Launching from r_1 to r_2

Loss in KE = Gain in GPE

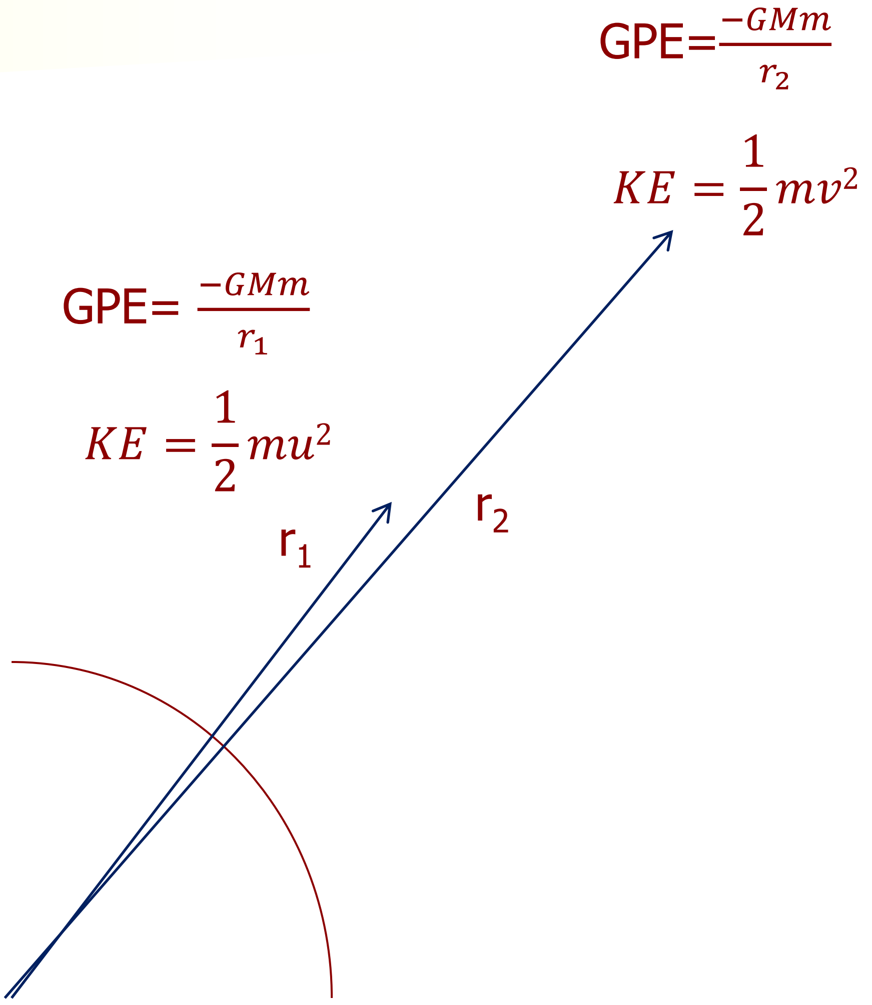
$$\left|\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right| = \frac{-GMm}{r_2} - \left(\frac{-GMm}{r_1}\right)$$

Launching from surface to infinity

$$\left|0 - \frac{1}{2}mu^2\right| = 0 - \left(\frac{-GMm}{R}\right)$$

$$u = \sqrt{\frac{2GM}{R}}$$

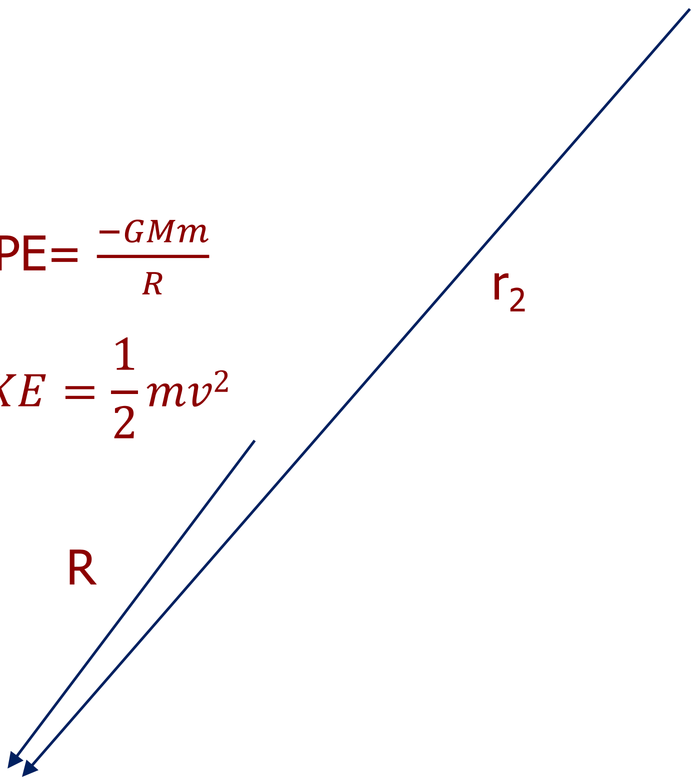
Escape velocity - minimum speed required for an aircraft, e.g. rocket to escape completely from a planet's gravitational field to infinity and never to return, without any further input of energy.



Launching / Returning

GPE=0

$KE = 0$



GPE= $\frac{-GMm}{R}$

$KE = \frac{1}{2}mv^2$

Returning from r_2 to r_1

Gain in KE = Loss in GPE

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \left|\frac{-GMm}{r_1} - \left(\frac{-GMm}{r_2}\right)\right|$$

Returning from infinity to surface

$$\frac{1}{2}mv^2 - 0 = \left|\frac{-GMm}{R} - 0\right|$$

Example

Under the influence of the Sun's gravitational pull, a piece of rock at infinity is at rest relative to the Sun. If it falls towards the Sun along a straight radial line, at what speed the rock hits the Sun?

Radius of Sun, $R_S = 7\times10^8$ m

Mass of Sun, $M_S = 2\times10^{30}$ kg

[Ans: 6.17×10^5 ms^{-1}]

Returning from infinity to surface

$$\frac{1}{2}mv^2 - 0 = \left|\frac{-GMm}{R} - 0\right|$$

$$\frac{1}{2}mv^2 = 0 - \left(\frac{-GMm}{R}\right)$$

$$V = \sqrt{\frac{2GM}{R}} = \ldots\ldots\ldots\ldots\ldots\ldots$$

Questions

A rocket has a mass of 1000 kg and the planet has a surface potential equal to –100 $MJkg^{-1}$.

1. Assume the fuel is used quickly to boost the rocket to high speed, calculate the GPE of the rocket that must be increased for it to escape completely.

[10^{11}J]

2. If the rocket is only given 40 000 MJ of KE from the fuel, determine gravitational potential at the point.

[- 60$MJkg^{-1}$]

3. Determine the max. distance from the surface it can reach. (radius of the planet =$2x10^6$ m)

[1.33×10^6 m]

Escape velocity, $V_{esc.} = \sqrt{\frac{2GM}{R}}$

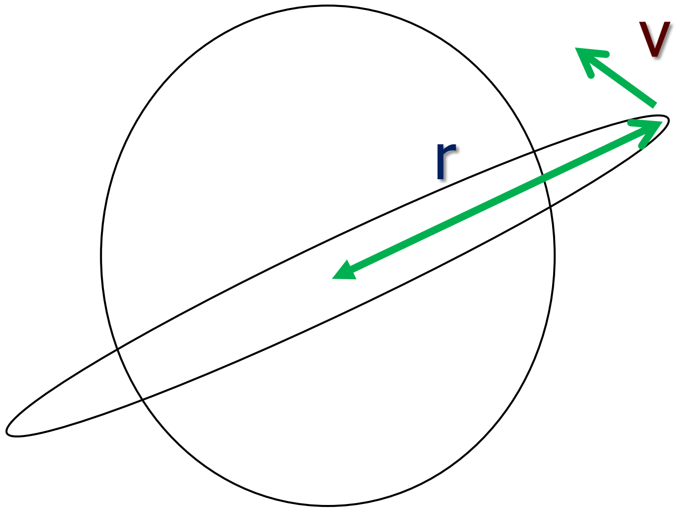
Based on the information given below, determine the escape velocity from the atmosphere of these planets.[11 kms^{-1}, 617 kms^{-1}, 2.4 kms^{-1}]

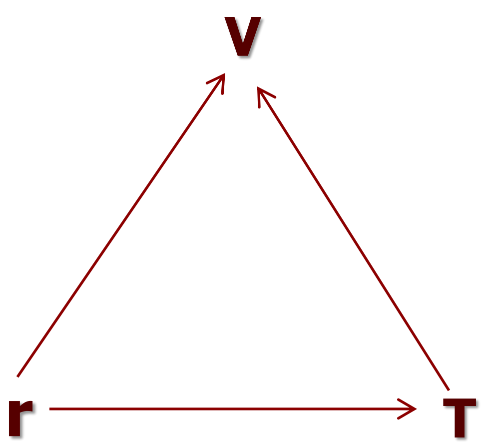
Planets	Radius/m	Mass/kg	$V_{esc.}$
Earth	6.4×10^{6}	6×10^{24}	
Sun	7×10^{8}	2.0×10^{30}	
Moon	1.74×10^{6}	7.4×10^{22}	

Orbiting motion

Gravitational force provides centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad (1) \qquad \text{or} \qquad \frac{GMm}{r^2} = mrw^2 \quad (2)$$





Examples

(Given: $M_E = 6 \times 10^{24}$ kg; $M_m = 7.4 \times 10^{22}$ kg; radius of the Earth $R_E = 6.4 \times 10^6$ m; radius of the moon $R_m = 1.74 \times 10^6$ m; distance between the centre of Earth and moon = 3.8×10^8 m)

a) Determine the speed of the Hubble space telescope orbiting at a height of 596 km above the Earth's surface. [7.56×10^3 ms^{-1}]

$$v=\sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\,(6\times10^{24})}{596000+6.4\times10^{6}}}$$

b) The Earth is orbiting the sun at a distance of 1.5×10^{11}m in one year's time. Estimate the mass of the sun. (one year = 365.25 days) [2×10^{30} kg]

$$T^2 = \frac{4\pi^2}{GM}r^3 \qquad M = \frac{4\pi^2}{GT^2}r^3 = \frac{4\pi^2}{G(365.25\text{x}24\text{x}3600)^2}(1.5\times10^{11})^3 = \cdots$$

c) For a satellite which is close to the Earth's surface, its radius of orbit may be assumed to be equal to the Earth's radius. Estimate the speed and the period of the satellite. [$7.91 \times 10^3 ms^{-1}$; 85 min]

Geosynchronous/Geostationary orbit.

- A satellite at **geosynchronous/geostationary orbit**.
  - has an orbital period that is equal to the rotational period of the Earth (i.e. 24hrs) or same angular velocity,
  - orbits in the same plane as the equator,
  - in the same direction as the Earth.

therefore it always appears to be above the same point on the Earth's surface.

Examples

1) For a geostationary satellite orbits around the Earth, determine:

   i) the orbital radius, r

   ii) the height above the surface

   iii) the orbital speed, v

   (given $M_E = 6 \times 10^{24}$ kg; radius of the Earth $R_E = 6.4 \times 10^{6}$ m)

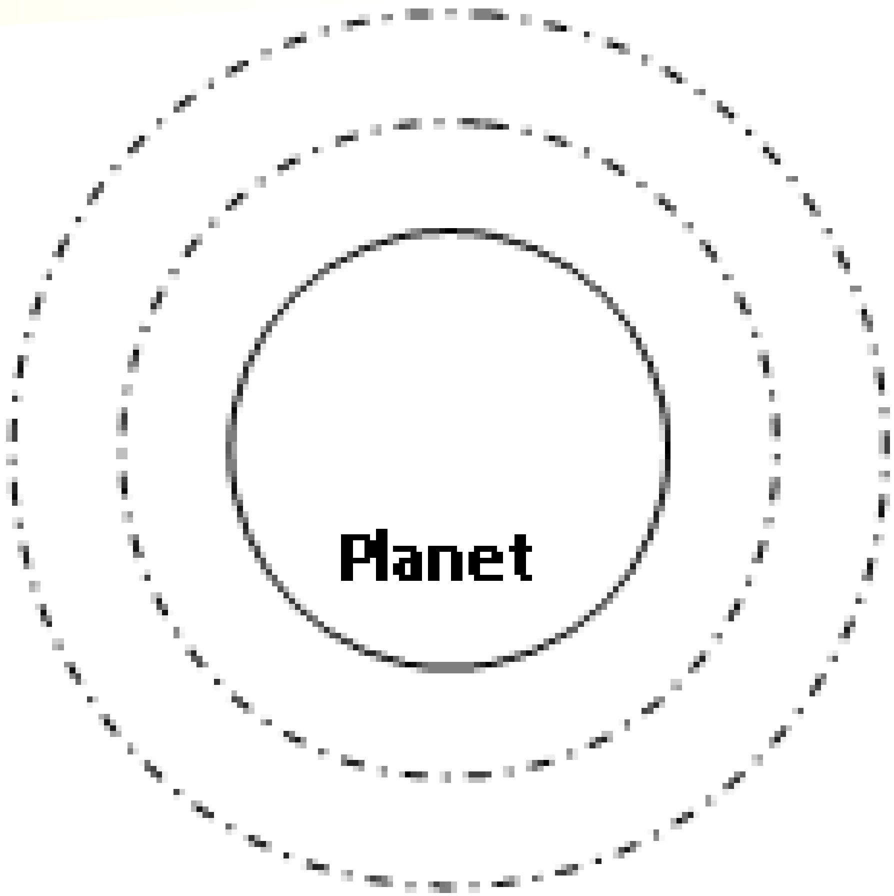
2) Determine whether the moon is a synchronous satellite of the Earth.

   (Given: $M_E = 6 \times 10^{24}$ kg; $M_m = 7.4 \times 10^{22}$ kg; radius of the Earth $R_E = 6.4 \times 10^{6}$ m; radius of the moon $R_m = 1.74 \times 10^{6}$ m; distance between the centre of Earth and moon $= 3.8 \times 10^{8}$ m)

Example

A planet with few equipotential lines around is shown beside.

Sketch the gravitational field strength of the planet. [2]



Example

A satellite of mass 5000 kg orbits in a circle at an altitude of 200 km above the Earth.

Determine the

i) KE,

ii) GPE,

iii) total energy of this satellite when it is orbiting

(given $M_E = 6 \times 10^{24}$ kg; radius of the Earth $R_E = 6.4 \times 10^{6}$ m)

Revision Questions

1. State Newton's law of gravitation. [2]
2. Define gravitational potential.[2]
3. Why is the gravitational potential energy of a mass a negative number? [3]
4. For an orbiting satellite, Show that orbital period squared, $T^2 \propto r^3$, where *r* is orbital radius. [3]
5. Planet A is orbiting the Sun with a period of 700 days. Determine the ratio of the radius of planet A's orbit around the Sun to the radius of Earth's orbit around the Sun. (Earth rotates around the Sun in 365 days.)
6. State one difference and one similarity between electric force and gravitational force.[2]