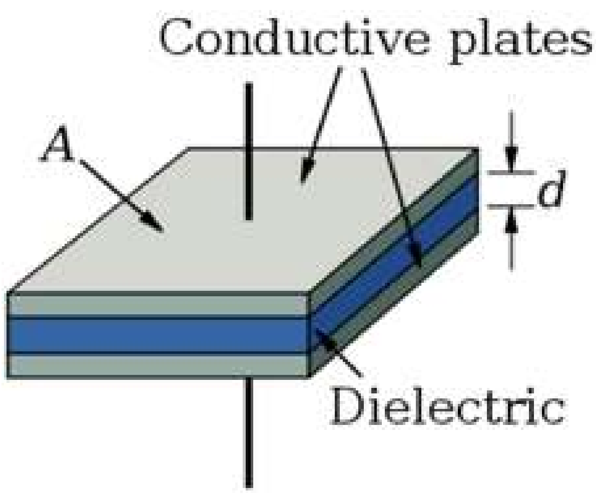
Topic 18 : Capacitance

- Sub- topics:
- 18.1 Capacitors and capacitance
- 18.2 Energy stored in a capacitor

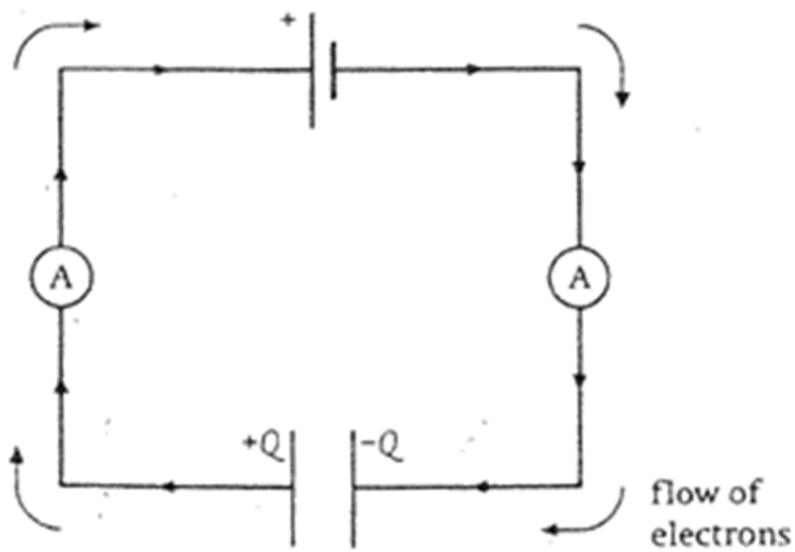
Capacitor

- Is a device that is capable of storing electrical charge.
- Very useful component used in many electrical & electronic circuits.
- Example, capacitors are used in computers, they are charged up in normal use, and then they gradually discharge if there is a power failure, so that the computer will operate long enough to save valuable data.
- All capacitor have 2 leads, connected to 2 metal plates where the charge is stored. Between the plates is an insulating material called the dielectric. (air, plastic sheets, oil)



Capacitor

- To charge up a capacitor, it must be connected to a voltage supply. This pushes electrons on to one plate causing it to be -ve charge; electrons are then repelled from the opposite plate, leaving it +ve charge.



Capacitor

- Note that there is a flow of electrons all the way round the circuit until the capacitor is charged up to the supply voltage V.
- To make the capacitor store more charge, we would have to use a higher voltage.
- If we connect the leads of the charged capacitor together, electrons flow back around the circuit and the capacitor is now discharged.
- We can observe capacitor discharging as follows:

➢ Connect the 2 leads of a capacitor to the terminals of a battery. Disconnect, and then reconnect the leads to a LED. It is best to have a protective resistor in series with the LED. The LED will glow as the capacitor discharges. This may last for some time, as only a small current will flow through the high resistance of the LED.

Capacitance

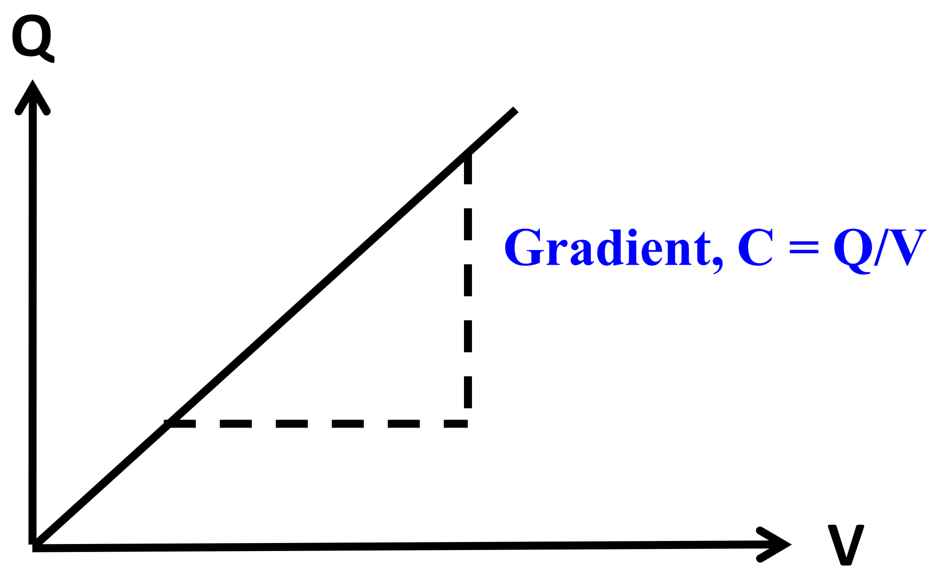
- Capacitance, C of a capacitor is defined as the charge stored per unit p.d. applied to the capacitor. (do not be confuse with the symbol for the quantity capacitance, C and the unit for charge, C (Coulomb).
- The unit of capacitance is the coulomb per volt (CV^{-1}) and this unit is given the unit name farad, F.

$$\boxed{C = \frac{Q}{V} \quad , \quad \begin{array}{l} Q = \text{charge stored} \\ V = \text{applied p.d} \end{array}}$$

- 1 farad is the capacitance of a conductor that is at a potential of one volt (1V) when it carries a charge of one Coulomb (1C).

Q-V graph - relationship between charge and potential

- If the charge and the p.d. is measured at various times of the charging process, the following Q - V graph is obtained.
- The gradient is equal to the quantity known as ***capacitance.***



Example 1

- A capacitor of capacitance 10 μF is connected to a battery of emf 12 V. What are the charges on its plate?

- Calculate the charge stored by a 220 μF capacitor charged up to 15V. Give your answers in microcoulombs.

- Calculate the average current required to charge a 50 μF capacitor to a p.d. of 10 V in a time interval of 0.01s.

Example 2

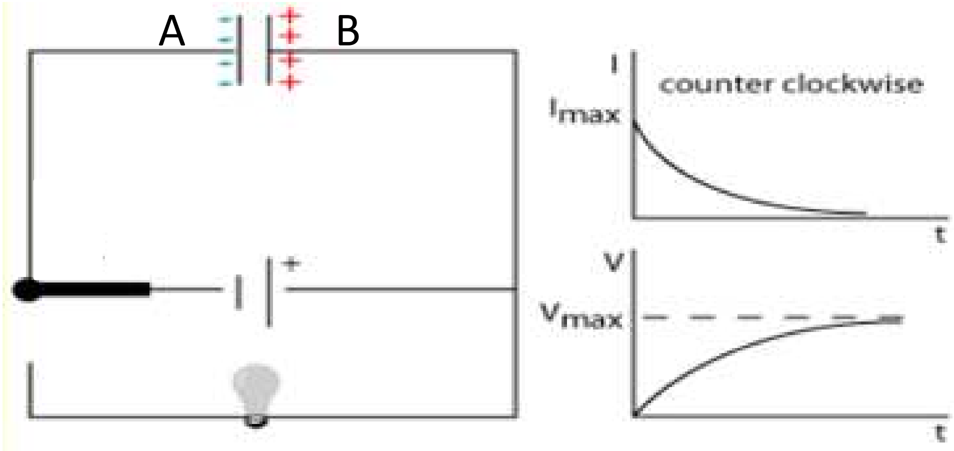
- A parallel plate capacitor of 3.5 nF is constructed using air as dielectric material. The distance between the parallel plates of the capacitor is 5.0 mm. A potential difference of 10 kV is applied across the plates, find:

  (a) The charge on each plate

  (b) The electric field strength between the plates

- A steady current of 50 μA is supplied to the plates of a capacitor and causes the potential difference between them to rise from 0 to 5 V in 20 s. What is the capacitance?

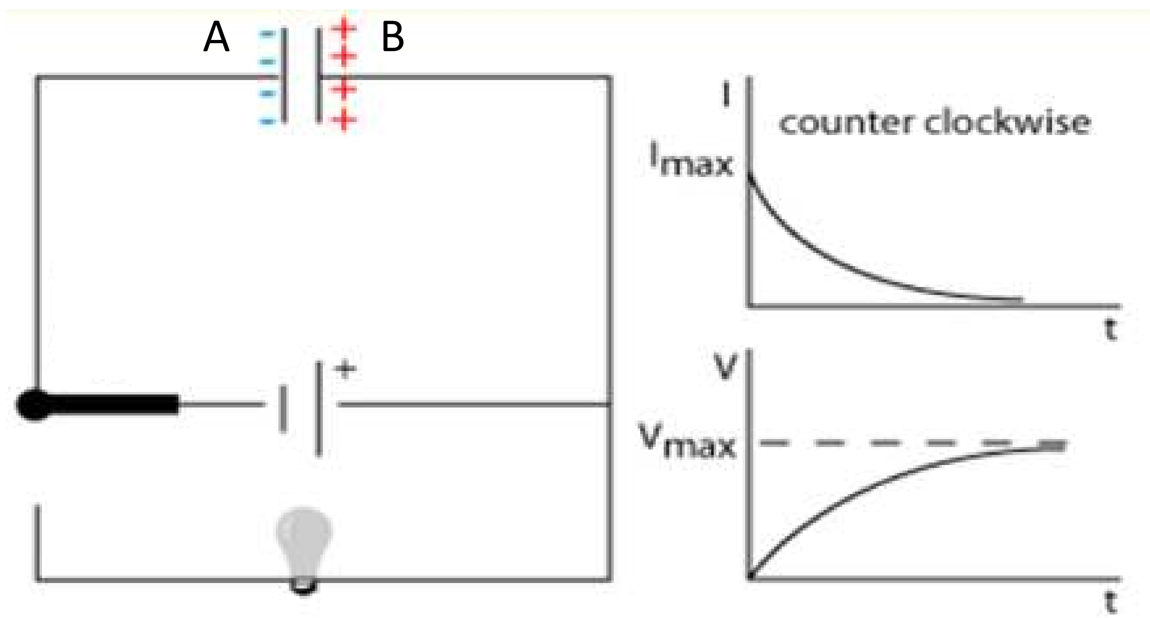
Capacitor charging process



- Electrons are 'pumped' into plate A from the negative terminal of the battery causing plate A to be negative charged.
- Due to the excessive electrons at plate A, the electrons in plate B are repelled out of plate B, causing it to be positively charged.
- In other words, when the battery is connected to a capacitor, electrons start to move out of B , travel through the battery and move into A.
- The movement of the electrons produces an electric current in the circuit.

Capacitor charging process

- As time passes, the magnitude of the current begins to decrease as shown in the figure.
- Lastly, the current becomes zero.
- When the **current still flowing** in the circuit, the **capacitor is said to be charging**. But when the **current has stopped flowing**, the **capacitor is said to have become fully charged.**
- A charged capacitor stores charge of Q Coulomb.
- **The current decreases exponentially with time.**



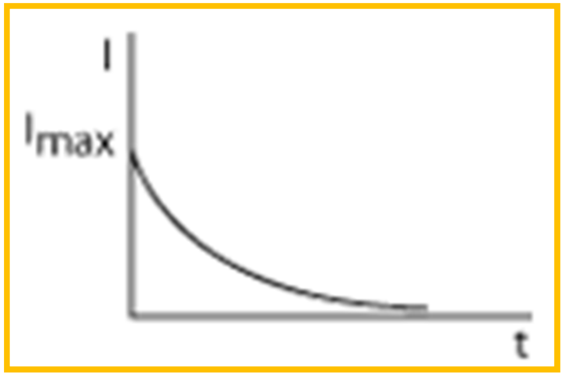
Capacitor charging process

- Initially capacitor is uncharged. Both ***A*** and ***B*** are neutral, so p.d across ***A*** and ***B*** is zero.
- When dc battery is connected to the capacitor, ***B*** starts to lose electrons while ***A*** starts acquire electrons. So, ***B*** becomes positively charged while ***A*** becomes negatively charged
- As time passes, ***B*** loses more electrons and becomes more positively charged. ***A*** acquires more electrons and becomes more negatively charged, hence, p.d across ***A*** and ***B*** will increase with time.
- At certain time, when the current stops flowing, the p.d reaches a maximum value, which is equal to the emf of the battery. (when the p.d across the capacitor becomes equal in magnitude to the emf of the battery, the p.d and emf cancel each other. Hence, the effective net voltage across capacitor is zero. This is the reason why the current stop flowing)

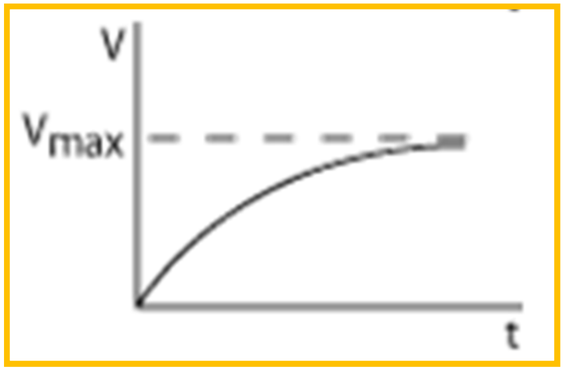
Summary - capacitor charging process

- A dc battery connected to uncharged capacitor produces a charging current in the capacitor.

- Current starts flowing in the circuit with a maximum value. The current then decreases exponentially with time and finally becomes zero. When current stops flowing, the capacitor is said to have becomes fully charged. The charged capacitor now stores certain amount of charge.

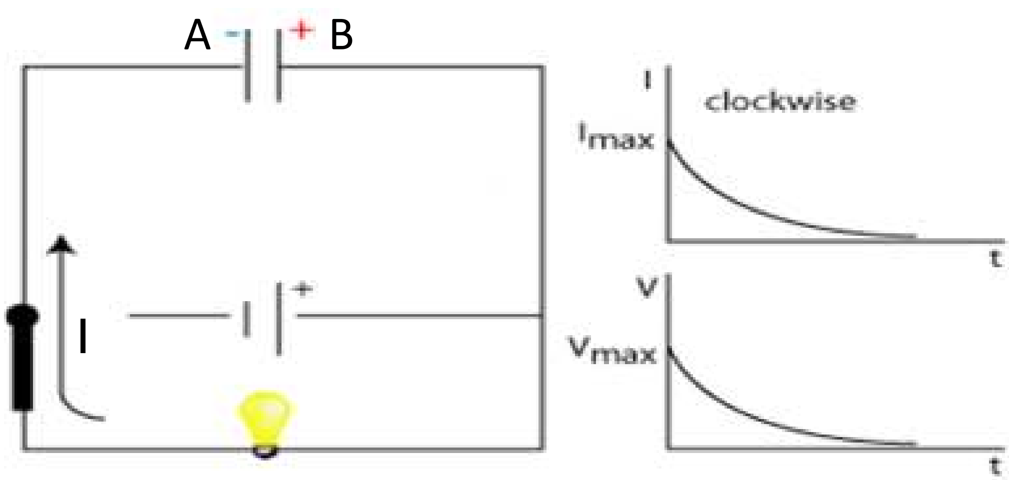


- Initially, p.d across the uncharged capacitor is zero. As the capacitor becomes charged, p.d across the capacitor increases. P,d increases exponentially with time. Finally, p.d reaches a max value equals to emf of the battery. At the same time, current stops flowing.



Capacitor discharging process

- *Suppose the dc battery is removed. Since the capacitor is charged, a p.d exists across A and B. A is negatively charged while B is positively charged.*
- *When a wire is connect electrically from A to B, free electrons immediately begin to flow out of A and begin to flow into B.*
- *The movement of electrons in the external circuit produces current. The current decreases with time.*
- *When the current stopped flowing, it means that A and B have finally become electrically neutral. (the capacitor is said to have discharged completely)*



Capacitor discharging process

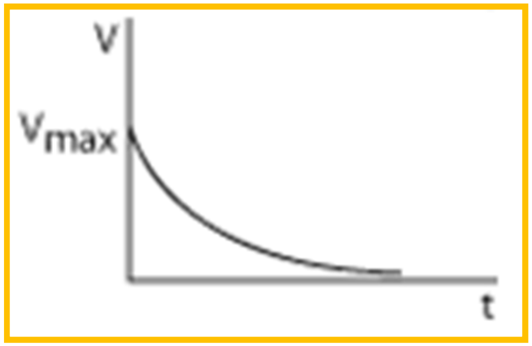
- As electrons flow out of A and into B, the amount of charge stored in each conductor begins to decrease with time. As a result, A will becomes less negative charge while B will become less positive charge.
- As time passes, A and B finally become electrically neutral and uncharged. The p.d across them becomes zero.
- p.d decreases exponentially with time.

Summary - capacitor discharging process

- A charged capacitor will discharge if we connect a conductor across the two plates

- While the capacitor discharges, a current flows in the circuit. The current decreases exponentially with time until it reaches zero. The capacitor becomes electrically neutral.

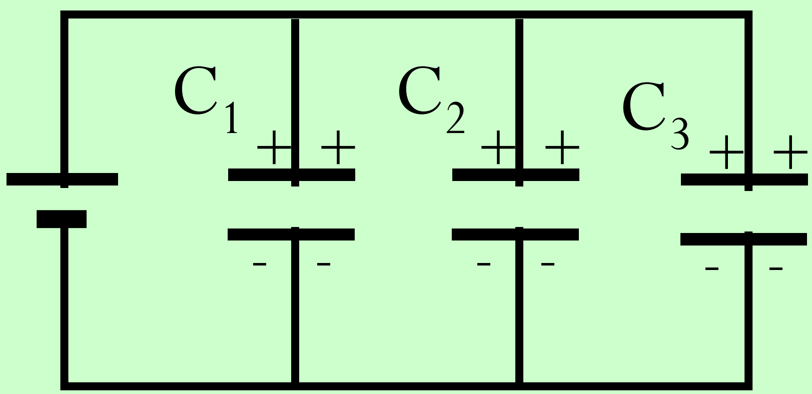
- While capacitor discharges, the p.d across it decreases. The p.d decreases exponentially with time until it reaches zero



Capacitors in Parallel

Capacitors which are all connected to the same source of potential are said to be connected in parallel. See below:

Parallel capacitors:



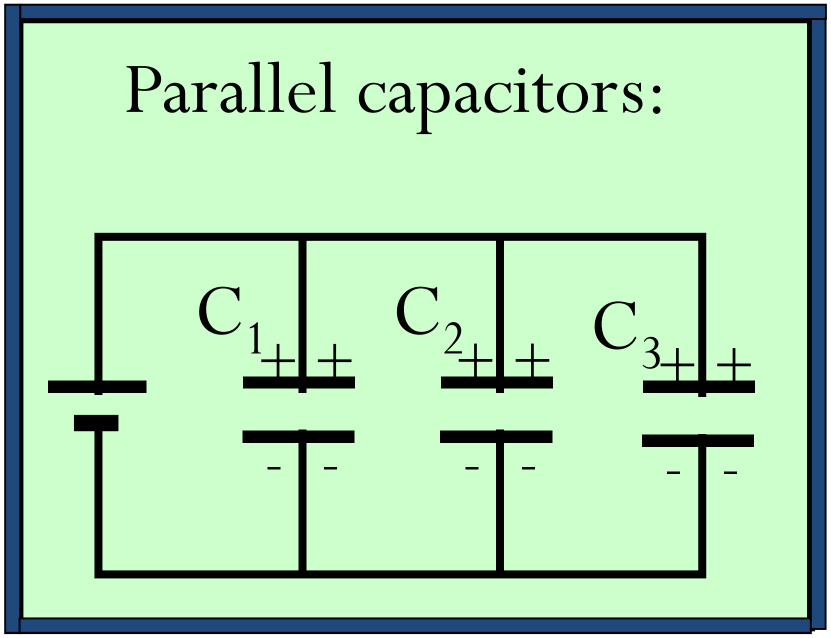
Voltages:

$$V_T = V_1 = V_2 = V_3$$

Charges:

$$Q_T = Q_1 + Q_2 + Q_3$$

Capacitors in Parallel



$$C = Q/V\ ;\ \ Q = CV$$

$$Q_T = Q_1 + Q_2 + Q_3$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$

Since voltages are equal:

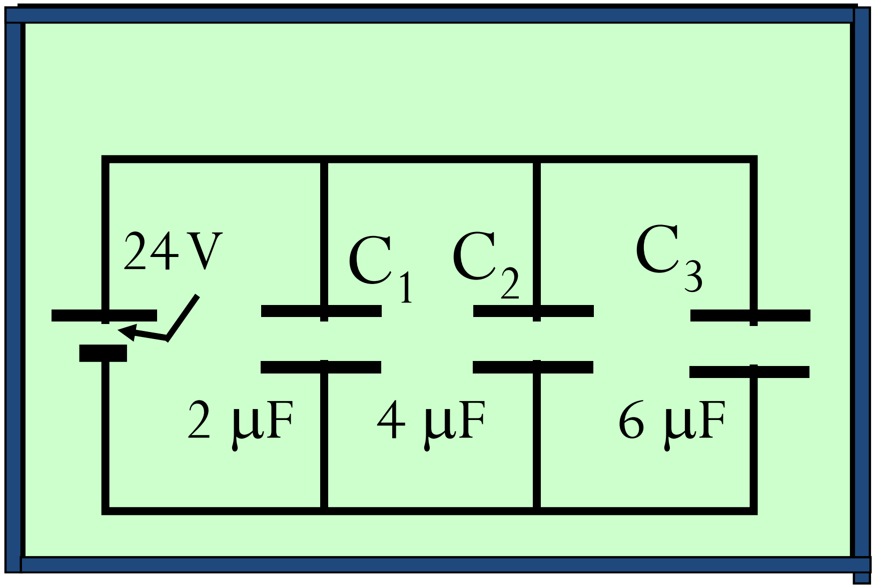
Thus,

$$C_T = C_1 + C_2 + C_3$$

The expression for capacitors in parallel is similar to that for resistors in series.

Example 3

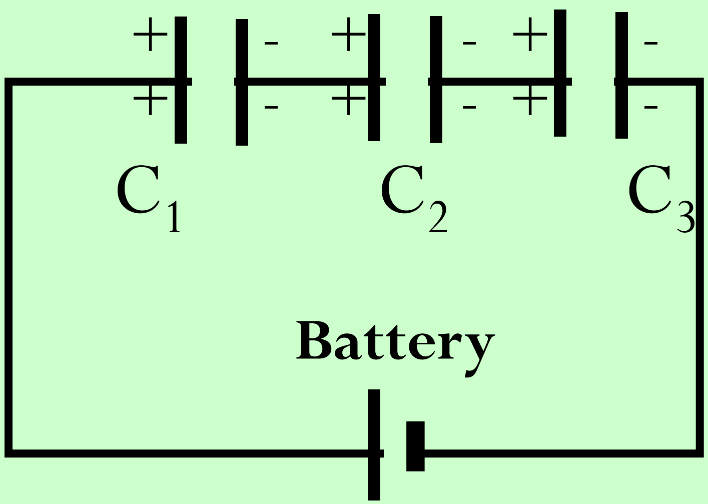
- Find the charge across each capacitor and hence the total charge Q_T.



Capacitor in Series

Capacitors connected along a single path are said to be connected in series. See circuit below:

Series capacitors:



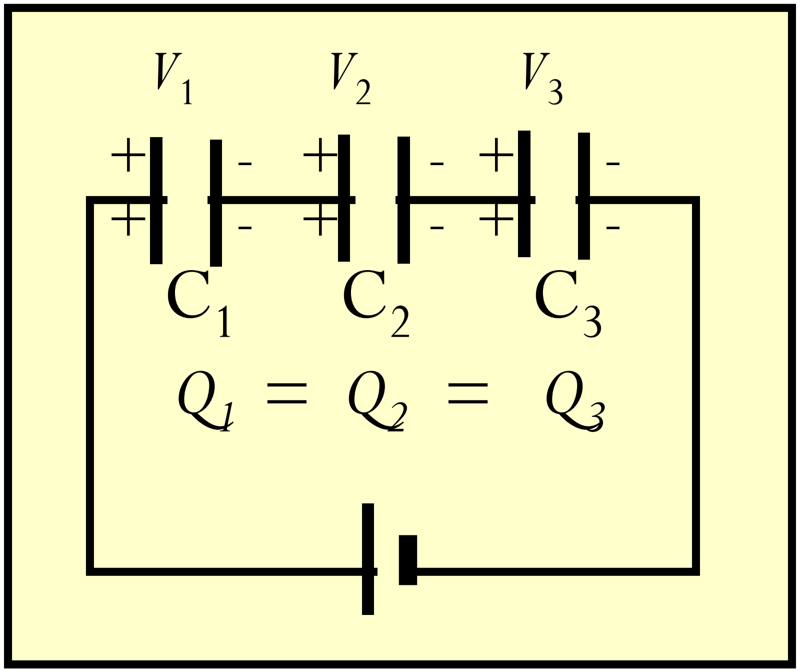
Voltages:

$$V_T = V_1 + V_2 + V_3$$

Charges:

$$Q_T = Q_1 = Q_2 = Q_3$$

Capacitors in Series



$$C = Q/V \; ; \; V = Q/C$$

$$V_T = V_1 + V_2 + V_3$$

$$Q_T/C_T = Q_1/C_1 + Q_2/C_2 + Q_3/C_3$$

Since charges are equal:

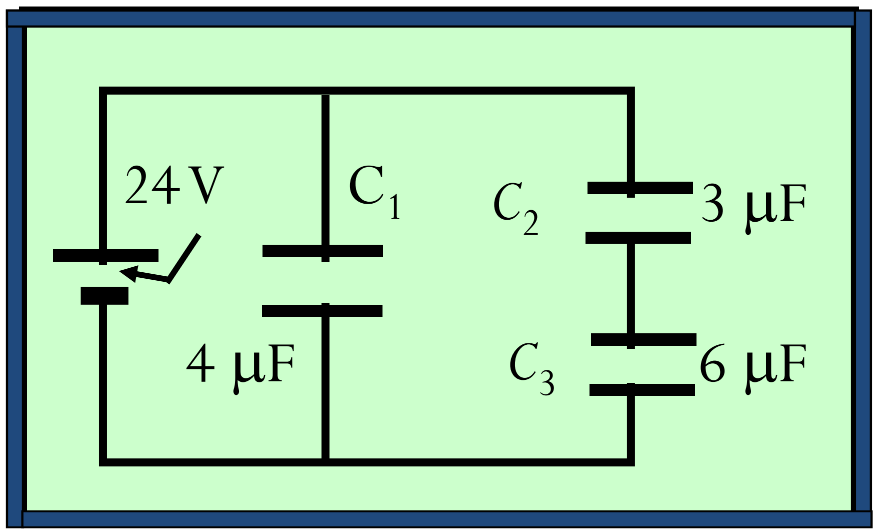
Thus,

$$1/C_T = 1/C_1 + 1/C_2 + 1/C_3$$

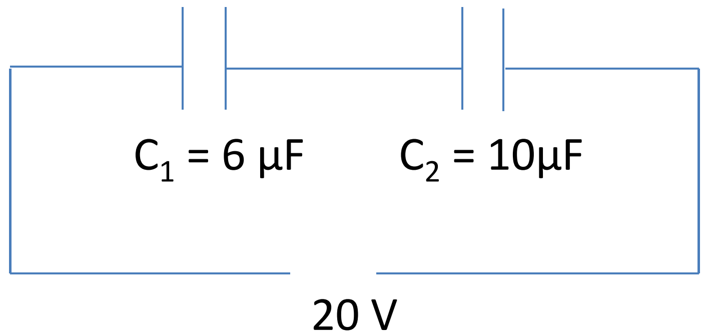
The expression for capacitors in series is similar to that for resistors in parallel.

Example 4

Find the equivalent capacitance of the circuit drawn below.



Example 5



- A constant voltage of 20 V is applied to two capacitors as shown. Determine:

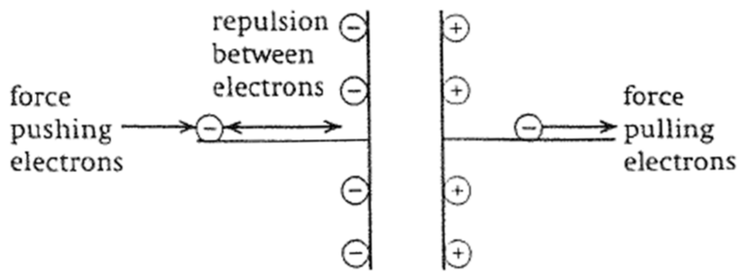
  (a) the total capacitance in the circuit

  (b) the charge stored in each capacitor

  (c) the p.d across each capacitor

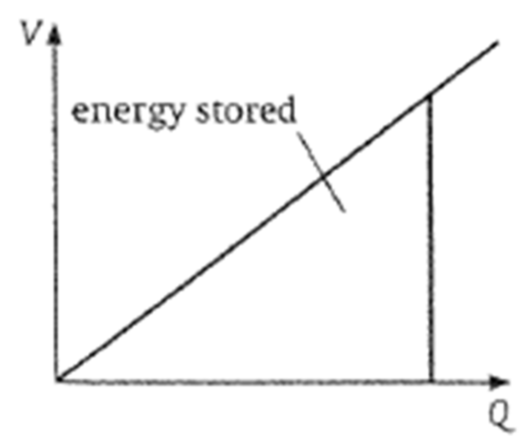
Energy stored in charged capacitor

- In order to charge up a capacitor, work must be done to push electrons on to 1 plate and off the other.



When a capacitor is charged, work must be done to push electrons against the repulsion of the existing electrons.

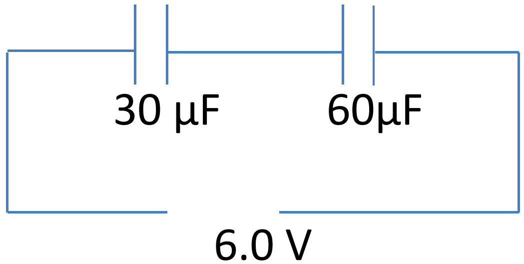
- At first there is only a small amount of negative charge on the left hand plate. Adding more electrons is relatively easy because there is not much repulsion.
- As the charge stored increases, the repulsion between the electrons on the plate and the new electrons increases, a greater amount of work must be done to increase the charge stored.



Energy stored in charged capacitor

- The previous figure shows the energy stored in a capacitor.
- We can use this graph to calculate the work done in charging up the capacitor.
- By using the area uder the graph, the work done in charging a capacitor to a particular p.d. is given by $W = \frac{1}{2} QV$
- Substituting $Q = CV$ into the equation, gives 2 further equations which are $W = \frac{1}{2} CV^2$ or $W = \frac{1}{2} Q^2/C$
- These 3 equations tell us the work done in charging up the capacitor. This is equal to the energy stored by the capacitor, since this is the amount of energy released when the capacitor is discharged.

Example 6



- Refer to the circuit above, determine:

  (a) the total capacitance of the circuit

  (b) the electric energy stored in each capacitor

Example 7

- Which store more charge, a 100 μF capacitor charged to 200 V, or a 200 μF capacitor charged to 100 V? Which stores more energy?

- A 10 000 μF capacitor is charged up to 12 V, and then connect across a lamp rated at “12 V, 36 W”.

  a.) Calculate the energy stored by the capacitor.

  b.) Estimate the time the lamp stays fully lit. Assume energy is dissipated in the lamp at a steady rate.

Sharing charge, sharing energy

- If a capacitor is charged and then connected to a second capacitor which is uncharged, what happens to the charge and energy that it stores?
- Note that, when capacitors are connected together, they are in parallel, because they have the same p.d. across them.
- Their combined capacitance is equal to the sum of their individual capacitances.
- The charge stored is now shared between the two capacitors, the total amount of charge stored must remain the same, since charge is conserved. The charge is shared between the two capacitors in proportion to their capacitances.
- Now the p.d. can be calculated from $V = Q/C$ and energy from $W = ½ CV^2$

Example 8

- Consider two 100 mF capacitors. One is charged to 10 V, disconnected from the supply, and then connected across the other. Calculate the initial energy stored and the final energy stored by the combination.