

# Forces

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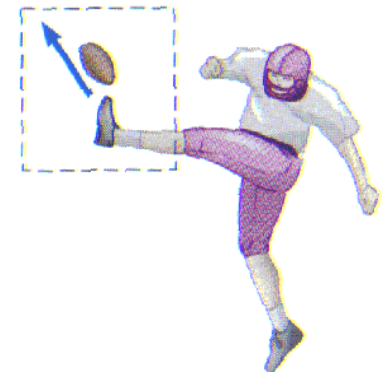
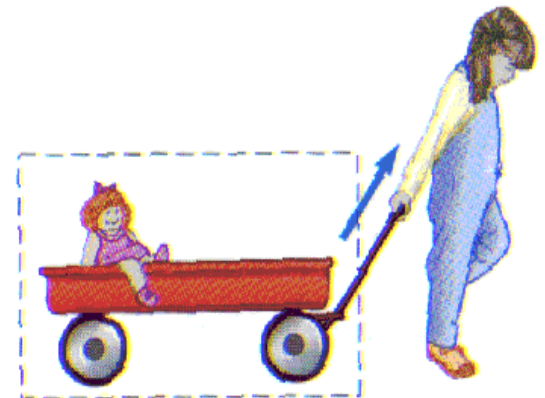
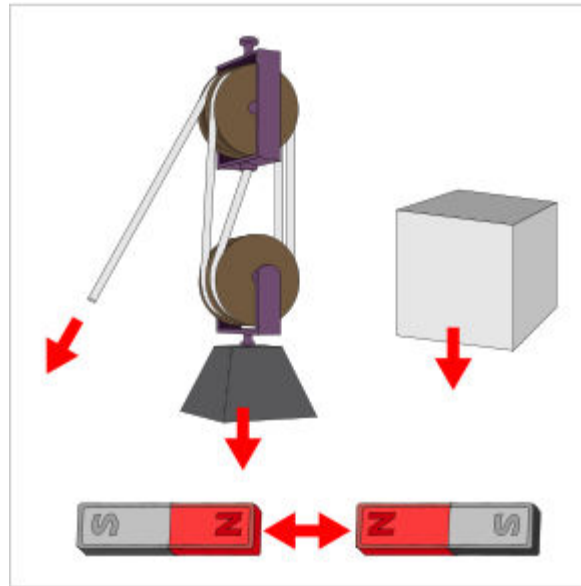
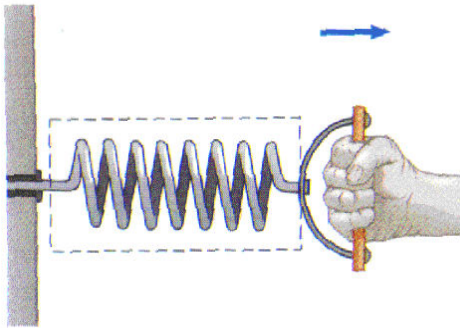
# Forces



- Some words used in physics have a meaning that is more closely defined than the word's meaning in everyday use.
- There is multitude of words in the English language that represent force. Some examples: push, pull, hit, tension, knock, effort, friction, thrust, pressure, strength, power, etc.
- In science it is essential to be careful in the use of words, so that when a word is used its meaning is clear.
- Some terms in above list is scientifically inaccurate.
- Example, power means work done per unit time, it does not mean force. Pressure means force per unit area, not force alone.
- On the other hand, some terms are simply descriptions of particular situation where forces occur, like tension, thrust, friction.

# Force

- A force is defined as *a push or pull due to the interaction between objects which produces or tends to produce motion, stops or tends to stop motion.*




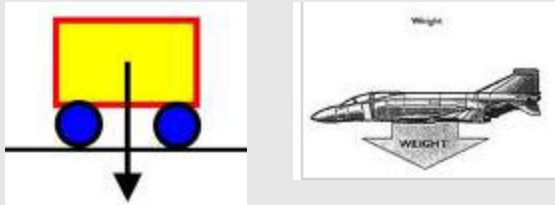
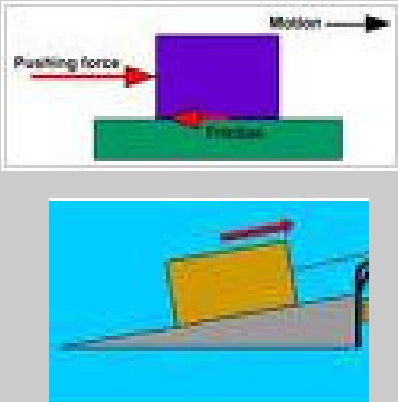
# Types of forces

- All forces between objects can be placed into two broad categories:
  - **contact force**
  - **contact-less force** - forces resulting from action-at-a-distance
- **Contact forces** are those types of forces which result when the two interacting objects are perceived to be physically contacting each other.
  - E.g: frictional force, tension force, air resistance force, spring force, normal reaction force.
- **Contact-less forces** or **action-at-a-distance** forces are those types of forces which result even when the two interacting objects are not in physical contact with each other, yet are able to exert a push or pull despite their physical separation.
  - E.g: gravitational force, electrical force, magnetic force.



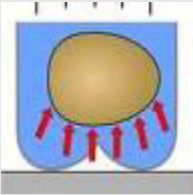

# Force is a vector

- Force is a vector quantity as the direction of a force is important.
- The net force on an object is the vector sum of all the forces on that object.
- E.g.: A car moving uphill. What are the related forces acting on it?
- If we knew the direction and magnitudes on each force, we could work out their combined effect on the car.
- Will the car accelerate uphill? Or will it slide backwards down the hill?
- The combined effect of several forces is known as resultant force / net force.
- Thus, by knowing the resultant force acting on the object, we can calculate the magnitude of its acceleration.
- Remember force and acceleration will always acts on the same direction.

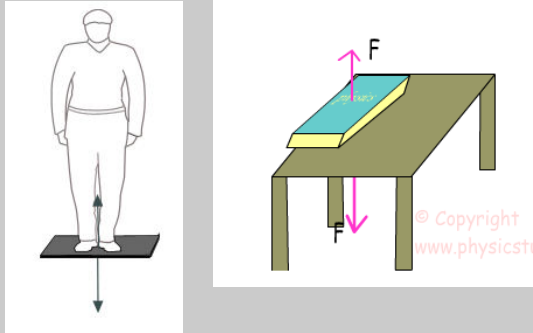
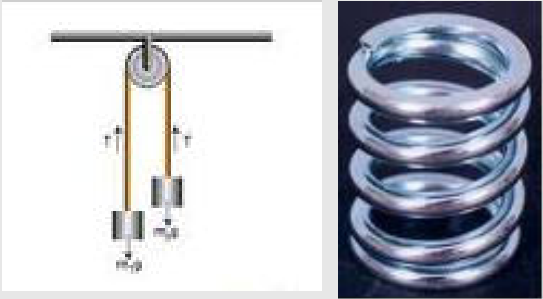
# Common Forces

Diagram	Force	Situation
	<p><b>Applied force (pushes / pulls). (<math>F</math>)</b> Object can accelerate by pushing or pulling it.</p>	<p>Pushing &amp; pulling. Force of car engine.</p>
	<p><b>Gravitation force / Weight. (<math>W</math>)</b> Force of gravity acting on the object. Usually shown by an arrow pointing vertically downwards from object's center of gravity.</p>	<p>Any object in a gravitational field.</p>
	<p><b>Friction. (<math>F_{fr}</math>)</b> Arises when 2 surfaces rub over another. If an object is sliding along the ground, friction acts in the opposite direction to its motion.</p>	<p>Pulling object along the ground. Vehicles cornering or skidding. Sliding down a slope.</p>

# Common Forces

Diagram	Force	Situation
 	<p><b>Drag / Air resistance / Viscous. (<math>F_D</math> or <math>D</math>)</b>            When object moves through air / liquid, there is friction between them. Object has to push aside the air/ liquid as it moves along. Drag opposes the motion of object.</p>	<p>Vehicle moving.            Aircraft flying.            Parachuting.            Object falling through air or water.            Ship sailing.</p>
 	<p><b>Upthrust. (<math>U</math>)</b>            Object placed in fluid / water experiences an upward force. That's why something float in water.</p> <p>Upthrust is a vertical upward force acting on a body immersed partially / wholly in a fluid due to the difference in pressure of the fluid..            Deeper you go, greater the pressure.</p>	<p>Boats and iceberg floating.            People swimming.            Divers surfacing.            Hot air balloon rising.</p>

# Common Forces

Diagram	Force	Situation
	<p><b>Contact Force. (<math>F_N</math> or <math>N</math>)</b>  Also known as normal reaction force.  Always acts at right angles to the surface which produces it.  Floor pushes straight upwards.  Wall pushes back horizontally.</p>	<p>Standing on ground.  Leaning against a wall.  Sitting on a chair.</p>
	<p><b>Tension. (<math>T</math>)</b>  Force on string and spring when it is stretched.</p> <p>If you pull on the end of a string, it tends to stretch. But tension of string will pull back against you, trying to shorten the string.</p> <p>If you stretch a spring, tension of spring will pull back and try to shorten the spring.  If you compress a spring, tension of spring will act to expand the spring.</p>	<p>Pulling a rope.  Compressing or stretching a spring.</p>



# Identify Forces

- Name these forces

1.) Upward push of water on a submerged object

Upthrust , (U)

2.) Force which wears away two surfaces as they move over one another.

Friction, ( $F_{fr}$ )

3.) Force which pulled the apple off Isaac Newton's tree.

Gravitational Force / Weight, (W )

4.) Force which stops you falling through the floor.

Contact / Normal Reaction force, ( $F_N$  or N)

5.) Force in a string which holding up a hanging light.

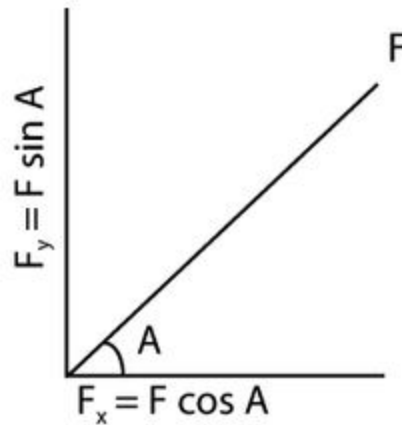
Tension, (T)

6.) Force which makes it difficult to run through shallow water.

Drag / Viscous ( $F_D$  or D)

# Components of vectors

- Force is a vector quantity.
- If we have a force,  $F$  projected at certain angle,  $A$  to the horizontal, we can resolve the force,  $F$  into its horizontal and vertical components.



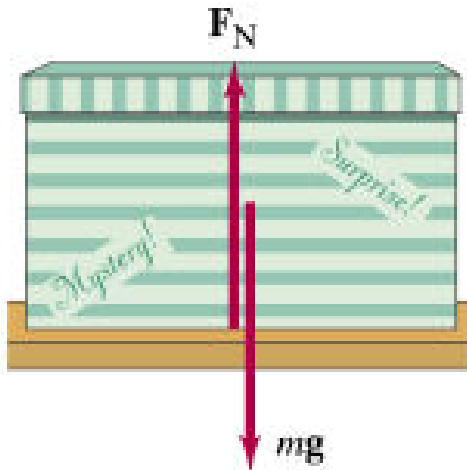
- These are:
  - 1.) Horizontal component,  $F_x = F \cos A$
  - 2.) Vertical component,  $F_y = F \sin A$

# Centre of gravity

- We have weight because of the gravitational force acting on us.
- Each part of our body – arms, legs, and head for example experiences this force.
- However, it is much simpler to picture the overall effect of gravity as acting at a single point.
- This is our **centre of gravity**.
- The **centre of gravity** of an object is defined as the point where all the weight of the object may be considered to act.
- Centre of gravity of a regular shaped object of uniform weight distribution is easy to find and is usually in the middle.
- For irregular objects, it can be found by experiment. (small holes are made around the edge on the irregular objects. A line is drawn on the object along the vertical string of the plumb line. The centre of gravity will be at the point of intersection of the lines drawn.

# Understanding “Weight & Normal Force”

- All objects dropped near the surface of the Earth would fall with the same acceleration,  $g$  if air resistance is negligible. The force that cause this fall is called the **gravitational force**.
- Applying the Newton’s 2<sup>nd</sup> Law to an object of mass  $m$  falling freely due to gravity can be written as  $F_G = mg$ . The magnitude of the force of gravity on an object,  $mg$ , is commonly called the object’s **weight**.
- From the 2<sup>nd</sup> law, the resultant force on an object that remain at rest is simply zero.
- Therefore, there must be another force on the object to balance the gravitational force.
- For an object resting on the table, the table exerts an upward force. This force that act perpendicular to the common surface of the contact is called the **normal force**.  $F_N$



# Example 1

A friend has give you a special gift, a box of mass 10.0kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table.

(a) Determine the weight of the box and the normal force exerted on it by the table.

$$W = mg = (10)(9.81) = 98.1 \text{ N}$$

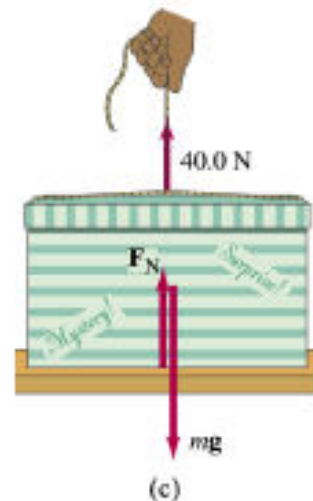
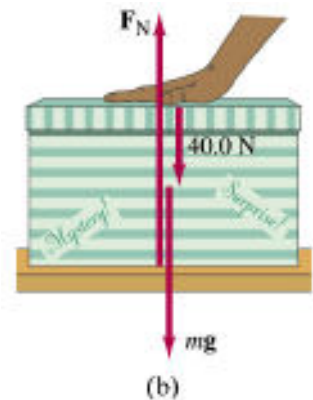
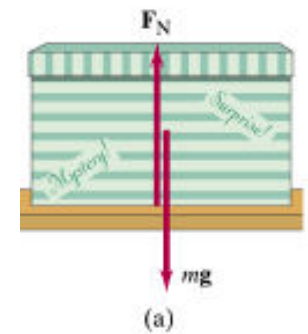
$$F_N = mg = 98.1 \text{ N}$$

(b) now your friend push down on the box with a force of 40.0 N. Again determine the normal force exerted on the box by the table.

$$F_N = mg + 40.0 = 138.1 \text{ N}$$

(c) If your friend pulls upward on the box with a force of 40.0N, what now is the normal force exerted on the box by the table?

$$F_N = mg - 40.0 = 58.1 \text{ N}$$

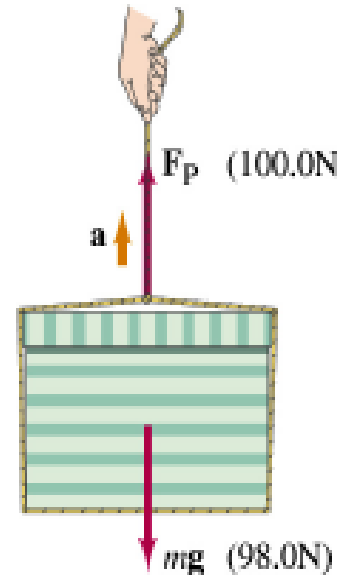


## Example 2

What happens when a person pulls upward on the box with a force equal to, or greater than the box's weight? For example, let  $F_p = 100\text{N}$ .

$F_N$  will be zero as box no longer in contact with table.

If  $F_p > mg$ , there will be resultant force, box will accelerate.



# Example 3

A 65kg woman descends in an elevator that briefly accelerates at  $0.2g$  downward. She stands on a scale that reads in kg. (a) During this acceleration, what is her weight and what does the scale read?

(b) What does the scale read when the elevator descends at a constant speed of  $2.0\text{ms}^{-1}$ ?

a.)  $W = mg = (65)(9.81) = 637.65 \text{ N}$

$$F_{\text{Net}} = ma$$

$$(mg - F_N) = ma$$

$$(637.65 - F_N) = (65)(0.2 \times 9.81)$$

$$F_N = 510.12 \text{ N}$$

b.)  $F_{\text{Net}} = ma$

$$(mg - F_N) = ma$$

$$(637.65 - F_N) = 0$$

$$F_N = 637.65 \text{ N}$$



# Example 4

Mary pulls the 10kg box with the attached cord, along the smooth surface of the table. The magnitude of the force exerted by Mary is  $F_P = 40\text{N}$ , and it is exerted at  $30^\circ$  angle. Calculate

(a) the acceleration of the box

(b) the magnitude of the upward force  $F_N$  exerted by the table on the box.

Assume that the friction can be neglected.

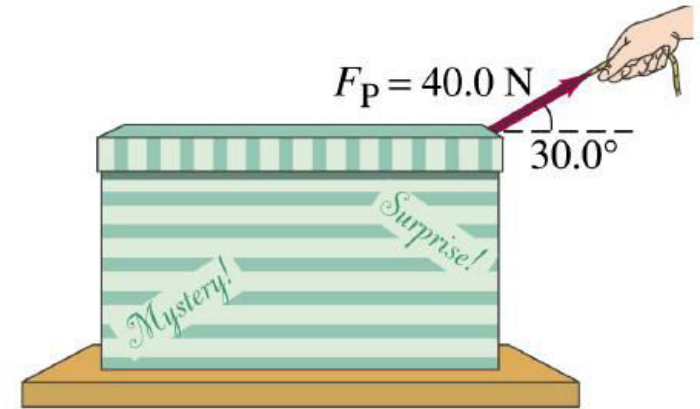
$$\text{a.) } F_{p(\text{horizontal})} = 40.0 \cos 30^\circ = 34.64 \text{ N}$$

$$F_{\text{Net}} = ma$$

$$34.64 = (10)a$$

$$a = 3.46 \text{ ms}^{-2}$$

$$\begin{aligned} \text{b.) } F_N &= mg - F_{p(\text{vertical})} \\ &= (10)(9.81) - (40.0 \sin 30^\circ) \\ &= 78.1 \text{ N} \end{aligned}$$





# Example 5

A box of mass 10 kg is placed on a smooth (frictionless) incline that makes an angle  $30^\circ$  with the horizontal.

a.) Determine the weight.

$$W = mg = (10)(9.81) = 98.1 \text{ N}$$

b.) Determine the component of weight perpendicular to the incline.

$$mg \cos 30^\circ = 85 \text{ N}$$

c.) Determine the normal force.

$$F_N = mg \cos 30^\circ = 85 \text{ N}$$

d.) Determine the component of weight parallel to the incline.

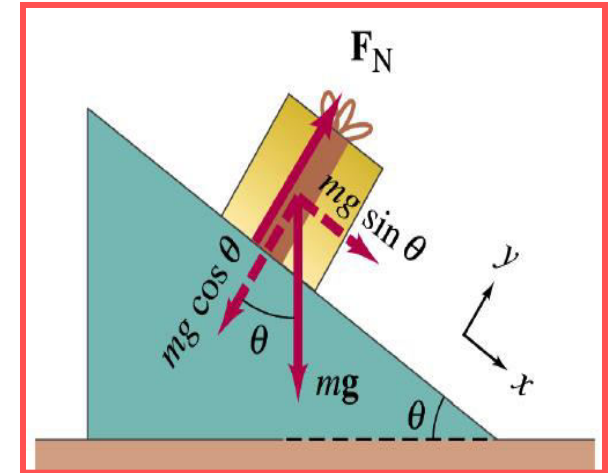
$$mg \sin 30^\circ = 49.1 \text{ N}$$

e.) Determine the resultant force and the acceleration.

$$F_{\text{Net}} = ma$$
$$(mg \sin 30^\circ) = (10)a ; a = 4.91 \text{ ms}^{-2}$$

f.) If the incline is rough and has a maximum frictional force of 20 N, determine its resultant force and acceleration.

$$F_{\text{Net}} = ma$$
$$(mg \sin 30^\circ - 20) = (10)a ; a = 2.91 \text{ ms}^{-2}$$



# Example 6

- Calculate the horizontal and vertical components of a force of 50 N which is acting at  $40^\circ$  to the horizontal. (ans: 38.30 N, 32.14 N)
- A body of weight 100 N rests on a plane which is inclined at  $30^\circ$  to the horizontal. Calculate the components of the weight parallel and perpendicular to the plane. (ans: parallel is 50.00 N, perpendicular is 86.60 N)
- A body of mass 5.0 kg is pulled up a smooth plane inclined at  $30^\circ$  to the horizontal by a force of 40 N acting parallel to the plane. Calculate the acceleration of the body and the force exerted on it by the plane. (assume  $g = 10 \text{ m/s}^2$ ) (ans:  $3 \text{ ms}^{-2}$ , 43.30 N)

# Example 7

A boy of mass 40 kg is on a waterslide which has an inclined plane at  $30^\circ$  to the horizontal. The frictional force up the slope is 120 N. Calculate the boy's acceleration down the slope.

(Take the acceleration of free fall  $g$  to be  $9.81 \text{ ms}^{-2}$ ) (ans:  $1.91 \text{ ms}^{-2}$ )

# Example 8

A boy of mass 40 kg is on a waterslide which is an inclined plane at  $25^\circ$  to the horizontal.

Calculate the boy's acceleration down the slope:

(Take the acceleration of free fall  $g$  to be  $9.81 \text{ ms}^{-2}$ )

a.) when there is no friction and the only force acting on the child is his weight.

( ans:  $4.15 \text{ ms}^{-2}$  )

b.) if a frictional force of 80 N acts up the slope.

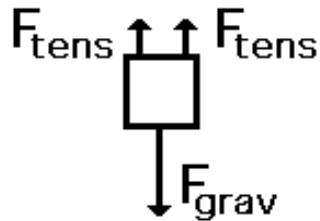
( ans:  $2.15 \text{ ms}^{-2}$  )

# Free Body Diagrams

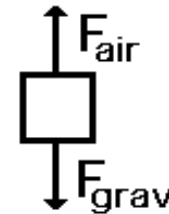
- So far we have only seen how forces acting on a single object.
- What happens if they are 2 or more objects acting on a given system?
- How do we analyse the forces acting on each body and the perhaps the acceleration as well?
- We can do so by applying the idea of “free-body diagrams”.
- Free body diagrams are diagrams used to **show the relative magnitude and direction of all forces acting ON an object in a given situation.**

# Examples of Free Body Diagram

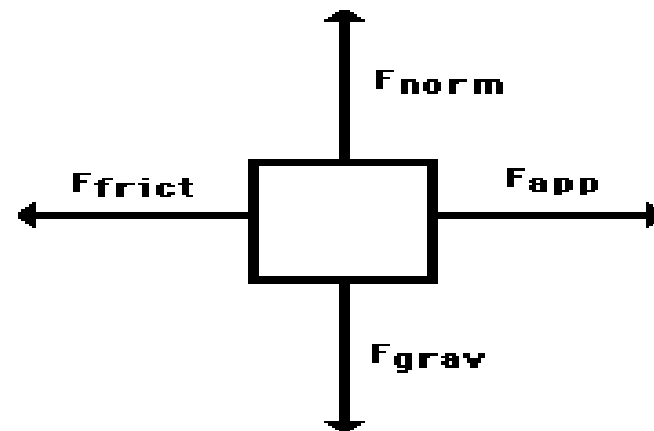
- A box is suspended motionless from the ceiling by two ropes. A free-body diagram for this situation looks like this.



- A skydiver is descending with a constant velocity. Consider air resistance. The forces acting upon the skydiver are shown as below



- A car which is moving with a constant speed on a rough ground. A free-body diagram for this situation looks like this.



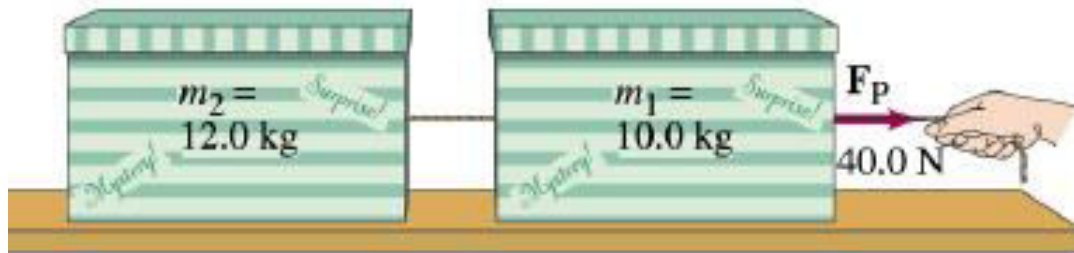
# Free body Diagram – Problem Solving

- How to start your mind thinking and getting involved in the problem at hand.
- 1.) Draw a sketch of the situation.
  - 2.) Consider only one object (at a time), draw a **free body diagram** for that body, showing ALL forces acting ON that object only, including the unknown force that you have to solve for.
  - 3.) *DO NOT show any forces that the body exerts on other bodies.*  
Label each force as to its source (gravitational, friction, etc).
  - 4.) Only forces acting *ON the given object* can be included in the  $\sum F = ma$  for *that particular object*.

## Example 9

Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes had masses of 12kg and 10kg. A horizontal force  $F_p$  of 40N is applied to the 10kg box. Find

- (a) the acceleration of each box
- (b) the tension in the cord connecting the boxes.



a.)  $F_{\text{Net}} = ma$   
 $40 = (22)a ; a = 1.82 \text{ ms}^{-2}$

b.) Consider  $m_2$

$$F_{\text{Net}} = ma$$
$$T = (12)(1.82) = 21.8 \text{ N}$$

OR

b.) Consider  $m_1$

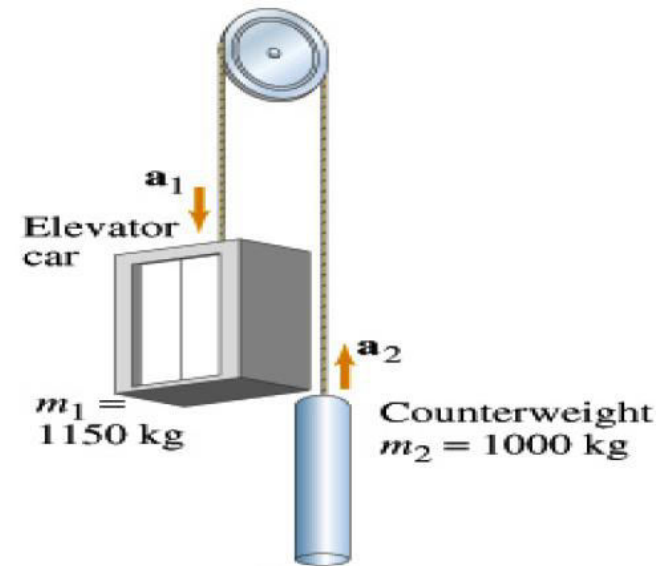
$$F_{\text{Net}} = ma$$
$$(F_p - T) = (10)(1.82)$$
$$(40.0 - T) = 18.2$$
$$T = 21.8 \text{ N}$$



# Example 10

Consider an elevator ( $m_E$ ) and its counterweight ( $m_c$ ) as shown in below. Assume that the tension  $F_T$  in the cable has the same magnitude on both side of the pulley. Calculate the

- (a) the acceleration of the elevator
- (b) the tension in the cable.

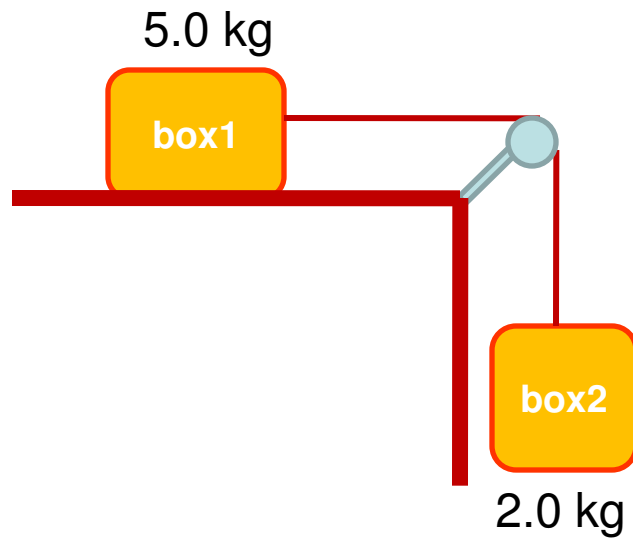


$$a = 0.68 \text{ m/s}^2$$

$$T = 10,500 \text{ N}$$

# Example 11

- 2 boxes are connected by a cord running over a pulley. The friction on box 1 = 9.8 N. Assuming mass of cord as well as friction at pulley is small and negligible.
  - Find the acceleration of the system. (assuming the cable doesn't stretch)
  - The tension of the cable.



$$a = 1.4 \text{ m/s}^2$$

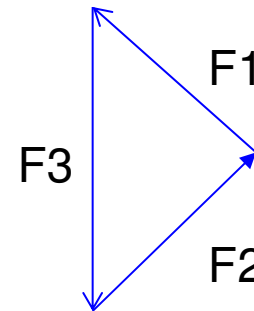
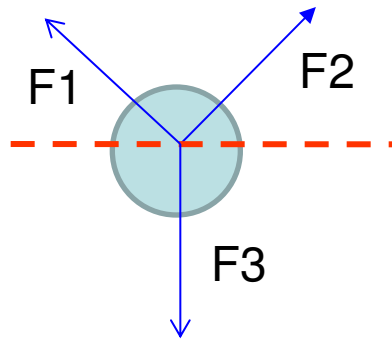
$$T = 17 \text{ N}$$

# System in Equilibrium

- Two conditions for system in equilibrium:
  - 1.) **Resultant forces in any direction must be equal to zero.**  
(Equilibrium of Forces)
  - 2.) **Resultant moments about any point must be equal to zero.**  
(Equilibrium of Moments)
- Why?
  - The forces on it must balance, otherwise they would cause translational motion (from one place to another)
  - The moments must balance, otherwise they would cause rotational motion (turning)

# 1<sup>st</sup> Condition: Equilibrium of Forces

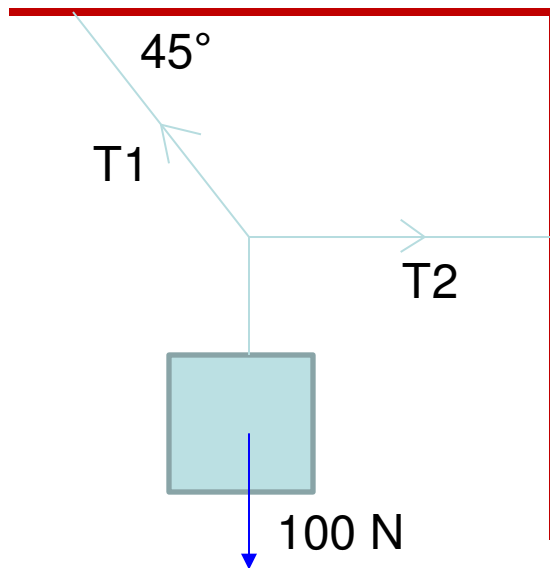
- Equilibrium of forces means the resultant of all the forces acting on the system is ZERO in any direction. (  $F_{resultant} = 0$  )
- In other words, a body is said to be in translational equilibrium if the resultant of all the forces acting on the body is zero.



- How do we determine then?
  - a.) **Mathematically, resolve all vectors to horizontal and vertical components.**  
If the resultant force horizontally is zero & resultant force vertically is also zero, then we conclude that system is in translation equilibrium.
  - b.) **By drawing a vector addition diagram.**  
For an equilibrium of forces the vector addition of the forces must yield a closed polygon regardless of how many forces are there. (the vectors must all point in one direction!)  
To do this accurately, it would required proper scale and correct angle.

# Example 12

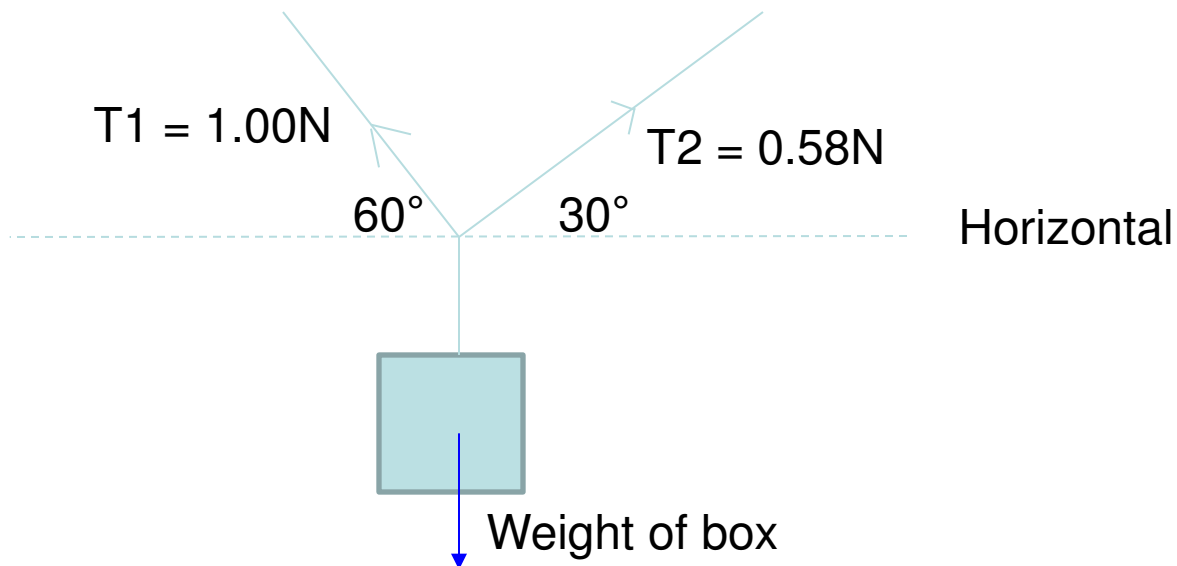
- The below system is in equilibrium. Find the tension of T1 & T2 in the cords supporting the object of weight 100N.



- Vertical component :  $T1 (\sin 45) = 100\text{N}$  ( ans: 141.4 N )
- Horizontal component :  $T1 (\cos 45) = T2$  ( ans: 100.0 N )

# Example 13

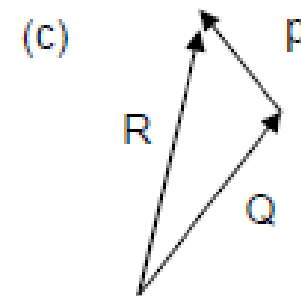
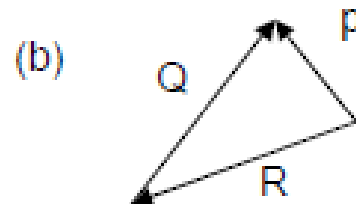
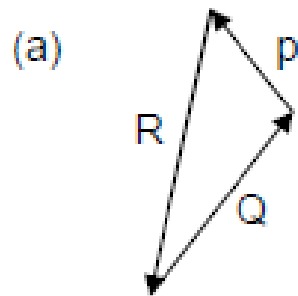
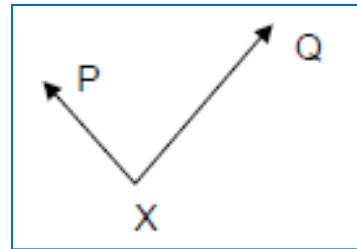
The free body diagram below shows 3 forces that act on a box hanging at rest from 2 strings.



- Calculate horizontal & vertical component of each string tension.
- Calculate the weight of the box.
- Draw a vector diagram of the forces on the box. Is it a closed polygon?
- Use your vector diagram to calculate the weight of the box.

# Example 14

2 forces  $P$  and  $Q$  act on a point  $X$  as shown as below, which of the following vector  $R$  make the system in the state of equilibrium?



# Moment of a force

- The 2<sup>nd</sup> condition for a system to be in equilibrium is that the resultant moment about any point must be zero.
- In other words, the resultant rotational motion must be zero. (  $\text{Moment}_{\text{resultant}} = 0$  )
- But first of all, let us learn what moment is and the equation related to it.
- Force can make things accelerate. They can also make things turn around.
- **The turning effect cause by a force is the quantity we called moment.**
- Example of a spanner turning a nut.
- Moment of force depends on 2 quantities:
  - 1.) the magnitude of force
  - 2.) the perpendicular distance of the force from the pivot

Moment = force x perpendicular distance of the pivot from the line of action of the force

- Pivot is basically the axis / centre point of which the mechanism rotates or turn.



# Moment of a force

- Unit of moment is Newton metre (Nm).
- It is a vector quantity.



Axis of rotation

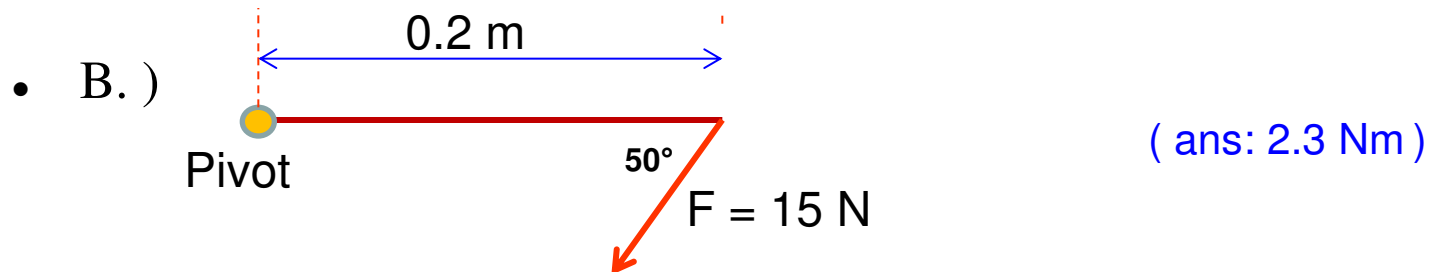
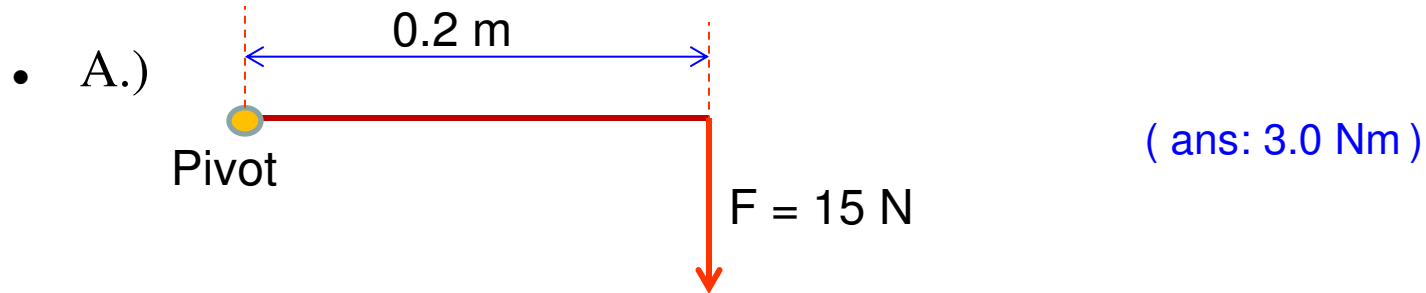


Axis of rotation

Diagram	Type of Moments	Sign Convention (usually)
	Anti-clockwise	Positive (+)
	Clockwise	Negative (-)

# Moment of a force

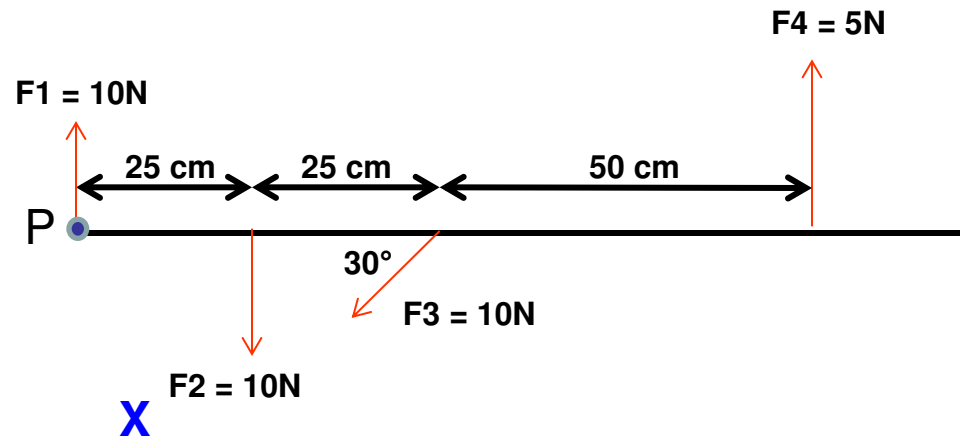
- Calculate the moment of force respectively..



# Principle of moments

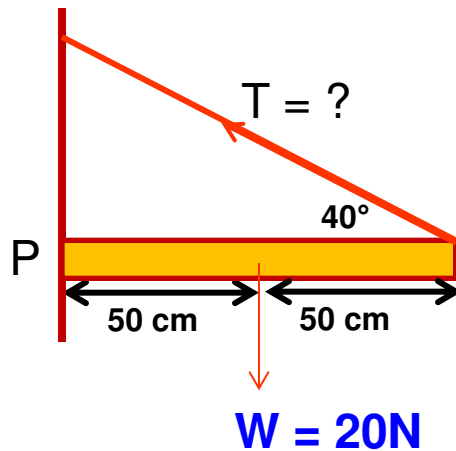
- We can use the idea of moment of a force to solve 2 sorts of problem.
  - 1.) Check whether object will remain balanced or start to rotate.
  - 2.) Calculate the unknown force or distance if knowing the object is balanced.
- To solve problems above, we need to know the principle of moments.
- Principle of moments states that *for any object that is in equilibrium, the sum of the clockwise moments **equals** the sum of the anticlockwise moments about any point provided by the force acting on the object.*
- This is the 2<sup>nd</sup> condition for system in equilibrium.

# Example 15



- Calculate the moment of each force about point P. ( $F_1$ ,  $F_2$ ,  $F_3$ ,  $F_4$ )
- State whether each moment is clockwise or anticlockwise.
- Are the moment of forces balanced ?
- $F_1 = 0\text{ Nm}$ ,  $F_2 = 2.5\text{ Nm (cw)}$ ,  $F_3 = 2.5\text{ Nm (cw)}$ ,  $F_4 = 5\text{ Nm (acw)}$
- Yes, moments of forces are balanced.

# Example 16



- Calculate  $T$ .  
sum of CW moment of forces =  $20 \times 0.5 = 10\text{Nm}$   
sum of ACW moment of force =  $T \times (1\sin 40)$   
 $T \times (1\sin 40) = 10$  ;  **$T = 15.6\text{N}$**

# A Special Pair of Forces called COUPLE

- **A couple** is defined as *a pair of equal and opposite forces which are parallel and whose lines of action do not coincide.*
- Acting on an object, a couple produces **rotation**.



- Figure above shows the forces needed to turn a car's steering wheel. The 2 forces of 20N up and 20N down will have a turning effect on the wheel. The wheel is not in equilibrium because the pair of forces cause it to rotate.
- A pair of force like this is known as a couple.

# Couple

- To form a couple, the 2 forces must be *equal in magnitude*, *parallel but opposite direction*, and *separated by some distance (d)*
- Since a couple would cause rotation, thus the turning effect caused by it is known as **torque**.
- Generally, the torque of a couple = **one of the forces x perpendicular distance between forces**
- Take note that:
  - 1.) Moment of a single force depends on the point or pivot about which the moment acts. Further the force is from the pivot, greater the moment.
  - 2.) The torque of a couple only depends on the perpendicular distance between the 2 forces. It does not depend on the point which it acts.

## Example 17 (Moment & Couple)

- Find the moment of a 10 N force about a point O in which its perpendicular distance from the point of action of the force is 4.0 m

Moment about O = **force x perpendicular distance of the pivot from the line of action of the force**

$$= 4 \times 10$$

$$= 40 \text{ Nm}$$

- Find the torque due to a 20 N couple about an axis through O which is 2.0 m from one of the forces and 4.0 m from the other. The lines of action of the couple are perpendicular to the distance between the forces.

Torque = **one of the forces x perpendicular distance between forces**

$$= 20 \times 6$$

$$= 120 \text{ Nm}$$



# Example 18

How much force  $F_m$  must the biceps muscle exert when a 5.0kg ball is held in the hand (a) with the arm horizontal as in figure? (b) when the arm is at  $30^\circ$  angle?

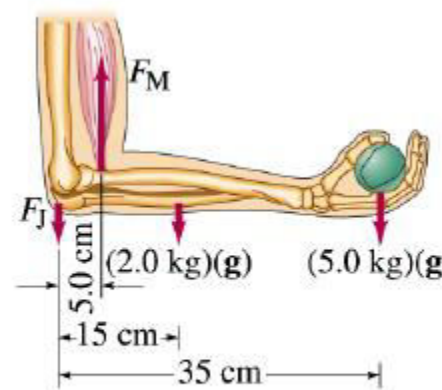
The biceps muscle is connected to the forearm by a tendon attached 5.0cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0kg and the center of gravity is as shown.

(a)

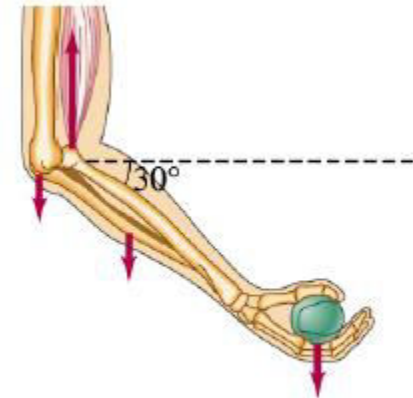
**CW moment = Anti CW moment**

$$(2 \times 9.81)(0.15) + (5 \times 9.81)(0.35) = F_M (0.05)$$

$$F_M = 402.21 \text{ N}$$



(a)



(b)

(b)

**CW moment = Anti CW moment**

$$(2 \times 9.81)(0.15)(\cos 30^\circ) + (5 \times 9.81)(0.35)(\cos 30^\circ) = F_M (0.05)(\cos 30^\circ)$$

$$F_M = 402.21 \text{ N}$$