Chap. 4 Dynamics

Lim WH July 2012

Learning outcome

(a) state each of Newton's laws of motion

(d) define linear momentum as the product of mass and velocity

(e) define force as rate of change of momentum

A Cow was walking. NEWTON stopped it. It stopped.

He found his first law,

"An object continues to move unless an external force is applied".

He gave a FORCE by kicking the Cow, it gave a Sound "MA"!

He formulated the 2nd law, F=MA!!

After sometime the Cow gave a kick to Newton, then he formulated the 3rd Law.

i.e,

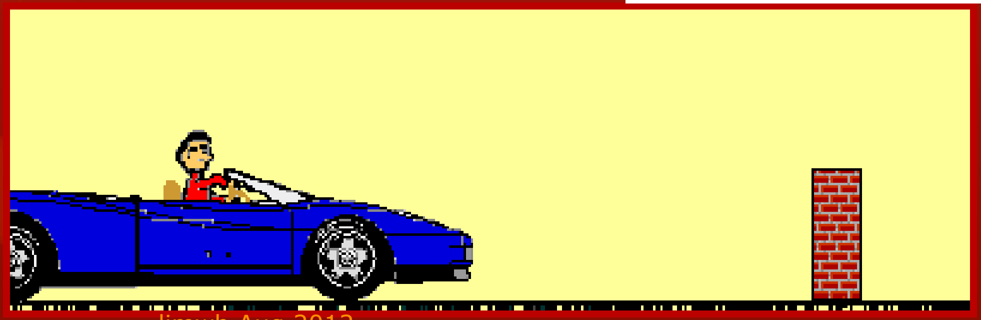
'EVERY ACTION HAS AN EQUAL & OPPOSITE REACTION'..:P

Newton's First Law

Every object remains stationary or moving at a constant speed in a straight line if no resultant force acts on.

- It implies the concept of inertia.

Give examples.



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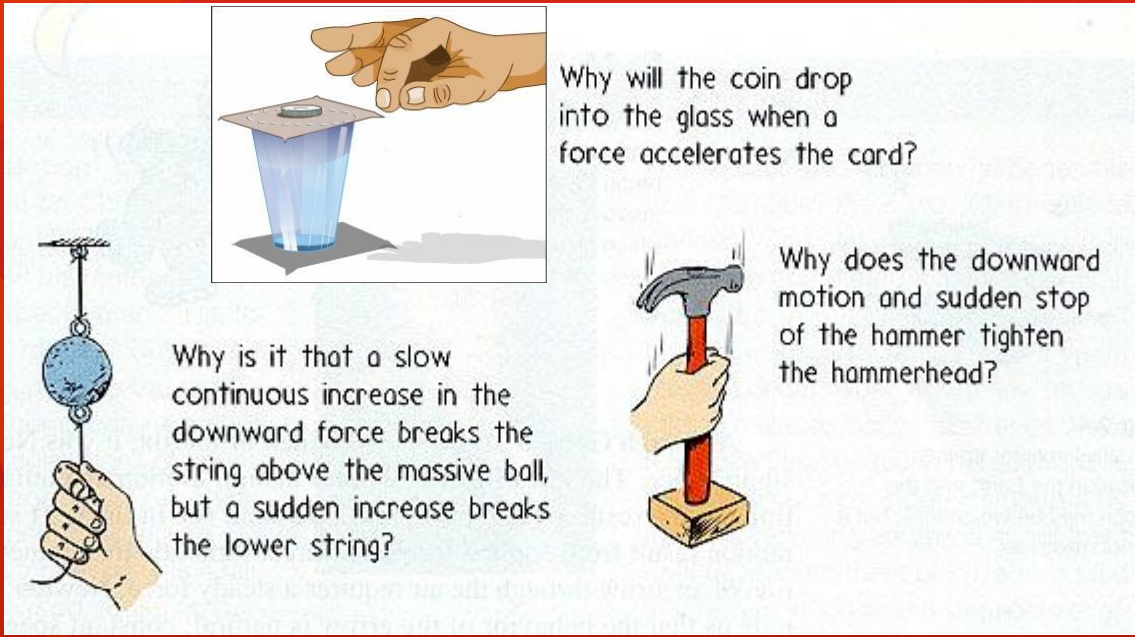
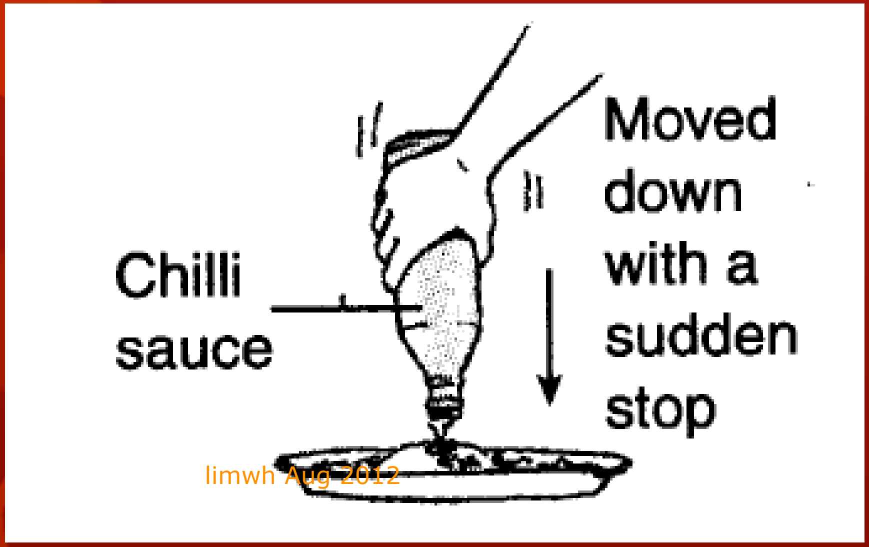
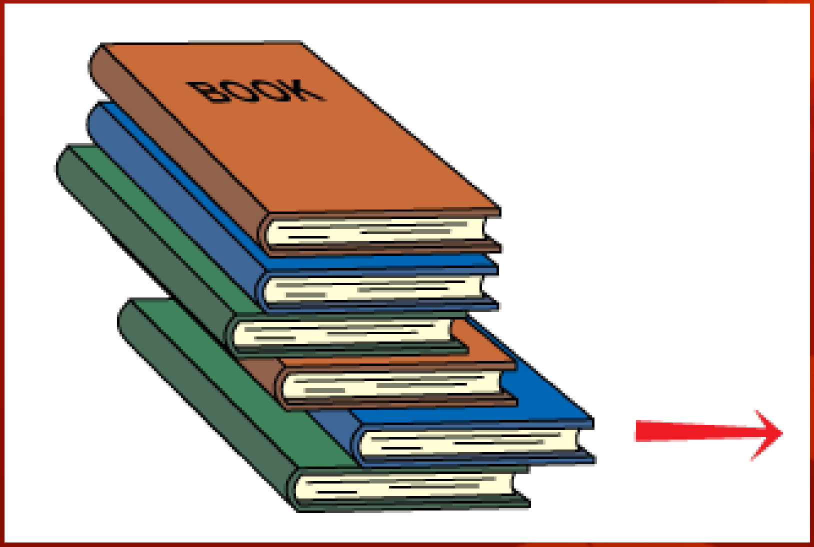


More examples





Momentum

Momentum of an object is defined as the product of its mass and its velocity.

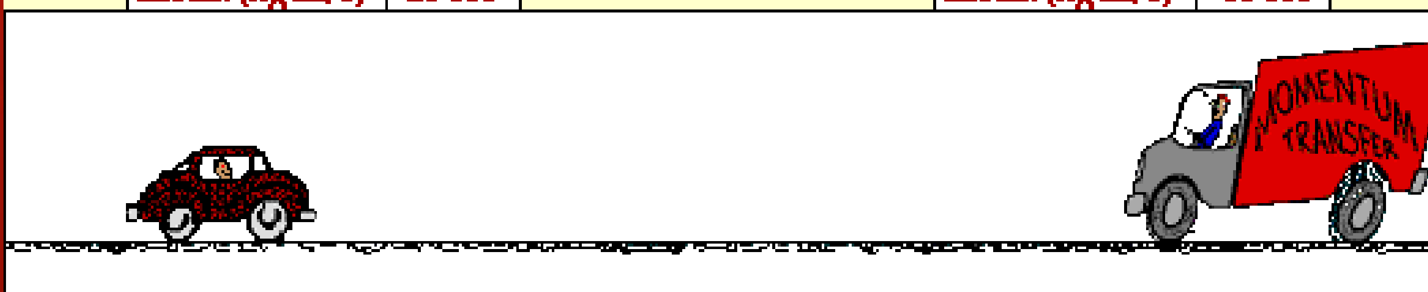
The larger the momentum, the larger the…??

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	-20.0
mom. (kg m/s)	-60 000



Newton's Second Law

The rate of change of momentum of an object is **directly** proportional to the resultant force acting on the object and is in the same direction of the resultant force.

$$F = \frac{mv - mu}{t}$$

- Thus, force is defined as the rate of change of momentum.

Graph of F vs. time

- The area under the graph = <F> t = change in momentum

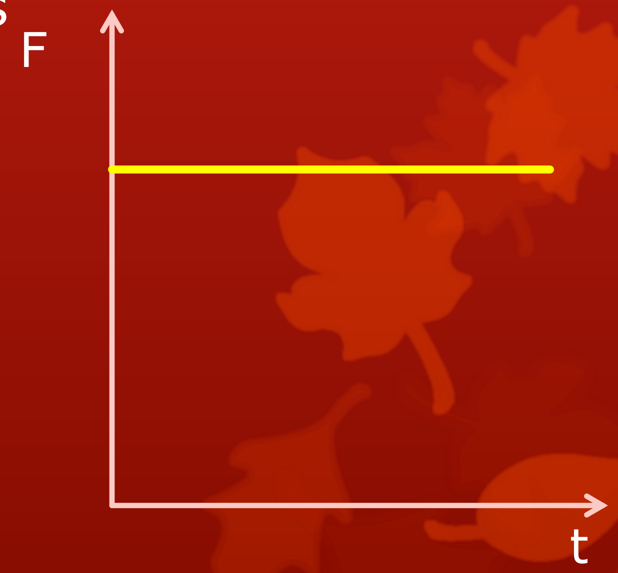
Example 1 : A constant force of 100 N acts on a mass of 2 kg which is at rest for 3 seconds.

The Change in momentum of the mass

= m(v-u) = <F>t

2 (v-0) = 300

Thus, its final velocity, v = 150 ms^{-1}



Graph of F vs. time

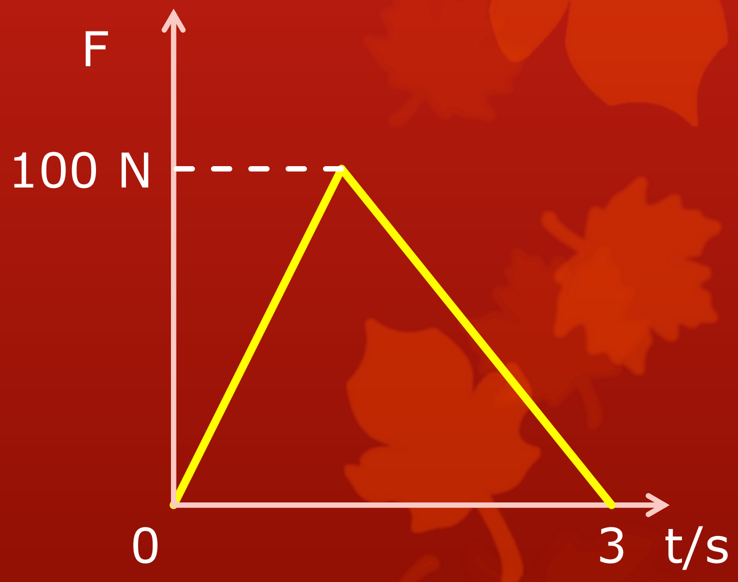
Example 2 : A force, varying as shown in the figure below, acts on a mass of 2 kg which is at rest for 3 seconds.

The Change in momentum of the mass

= m(v-u) = <F>t = area

2 (v-0) = ½ x 3 x 100

Thus, its final velocity, v = 75 ms^{-1}



F=ma

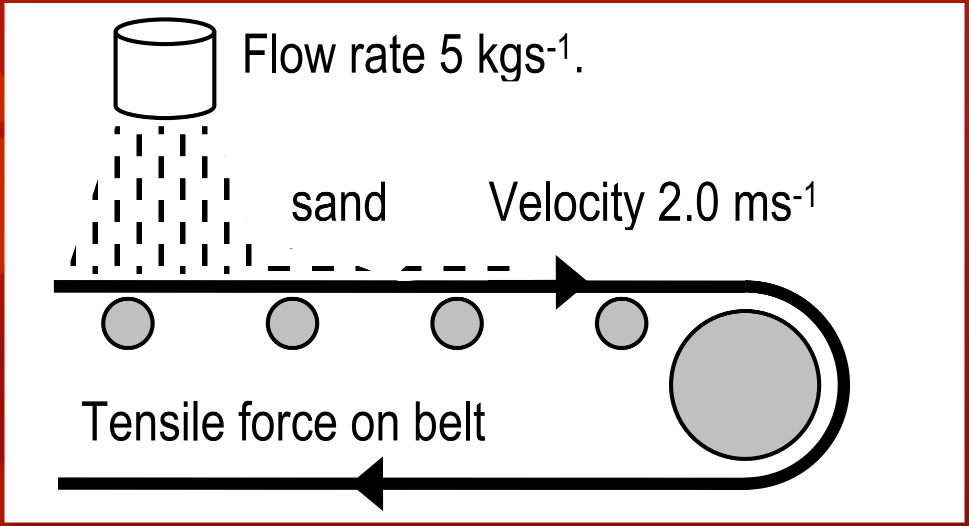
According to Newton's 2nd law,
$F \propto d(mv)/dt$
Hence, $F = m\, dv/dt + v\, dm/dt$
when there is no change of the mass m, $dm/dt = 0$
thus, $F = m\, dv/dt = ma$
$F = v\, dm/dt$ is applied on continuous flow of mass at a constant velocity. For instance,

sand falls on a conveyer belt, water ejected from a hose on a wall.

continuous flow of mass

Figure below shows sand falling onto a horizontal conveyor belt at a constant rate of 5 kgs^{-1}. If the conveyor belt is moving at velocity of 2.0 ms^{-1}.

Calculate the horizontal force needed to drive the belt.



$F = d(mv)/dt$

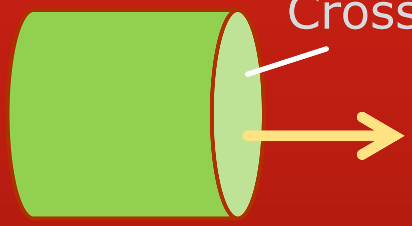
$= v\, dm/dt$

$= 2 \times 5$

$= 10$ N

continuous flow of mass – water ejected from a hose

Cross-sectional area, A



Speed of flow, v

$m = \rho V$

Mass flow rate , $dm/dt = d(\rho V)/dt$

$= \rho \; d(V)/dt$

$= \rho \; d(AL)/dt$

$= \rho A \; d(L)/dt$

$= \rho A \; v$

$Kgm^{-3} \times m^{2} \times ms^{-1}$

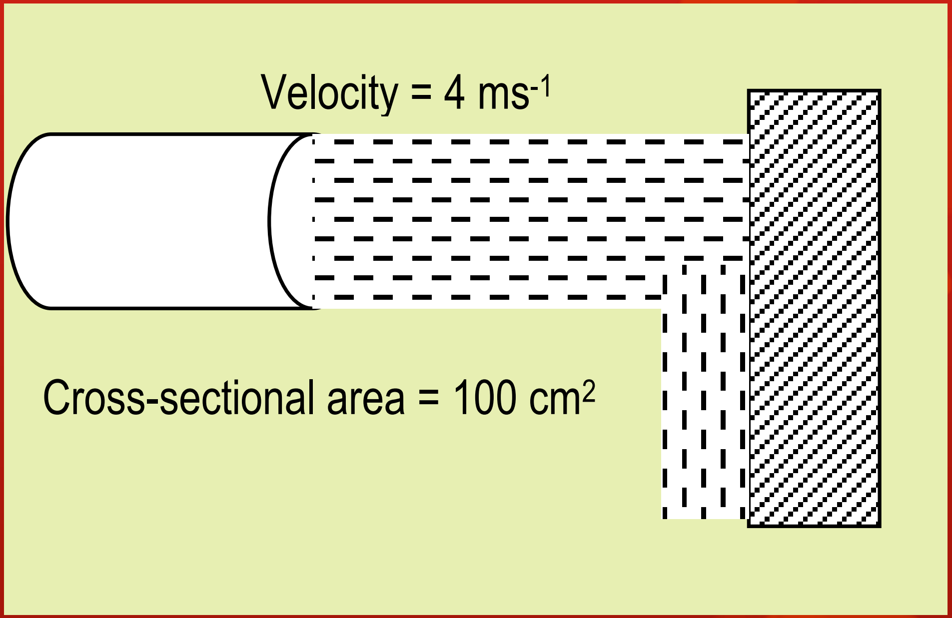
$Kgm^{-3} \times m^{3} s^{-1}$

$Kg\, s^{-1}$

Try this !

A hosepipe ejects a jet of water horizontally at a speed of 4 ms^{-1} through an area of 100 cm^2, as shown below. Assuming the water strikes the wall perpendicularly and that the water escapes parallel to the wall after the impact, calculate the force exerted on the wall.

(Density of water 1000 kgm^{-3})

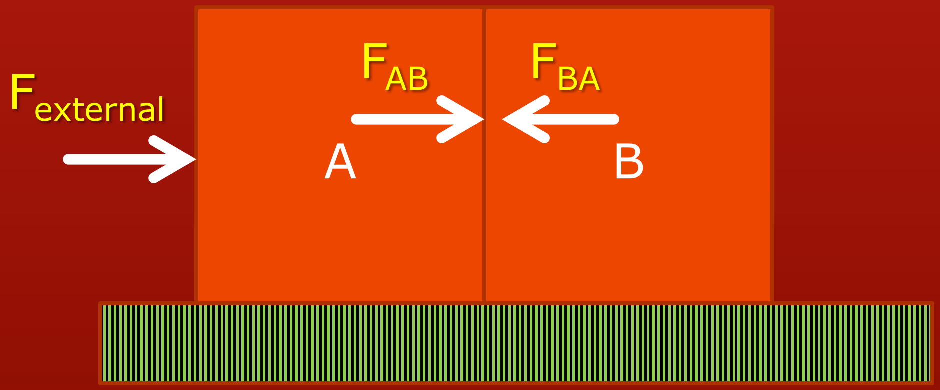


Newton’s Third Law

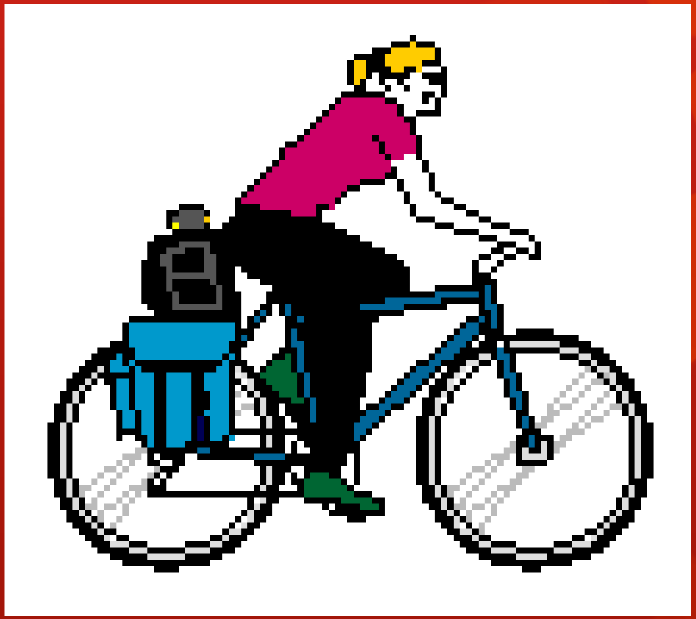
When two objects interact, they exert equal and opposite forces on one another.

That means, A exerts the equal magnitude of force on B but in opposite direction.

Action and reaction act on TWO different objects



More examples



Gravitational force exerted by the Earth on the moon

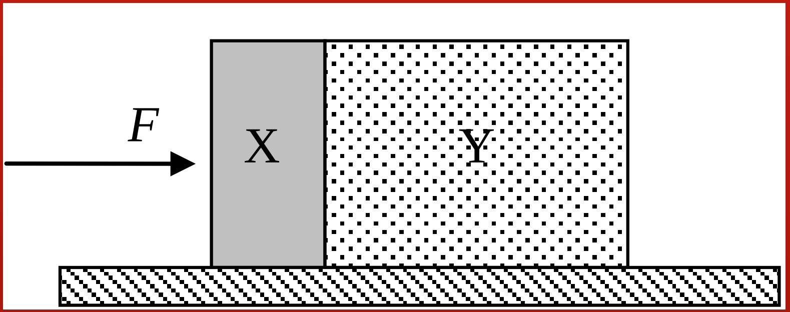
Gravitational force exerted by the Moon on the Earth

force exerted by the foot on the pedal

force exerted by the pedal on the foot

Try this !

Two blocks X and Y of masses m and 3m respectively, are accelerated along a smooth horizontal surface by a force *F* applied to block X as shown:

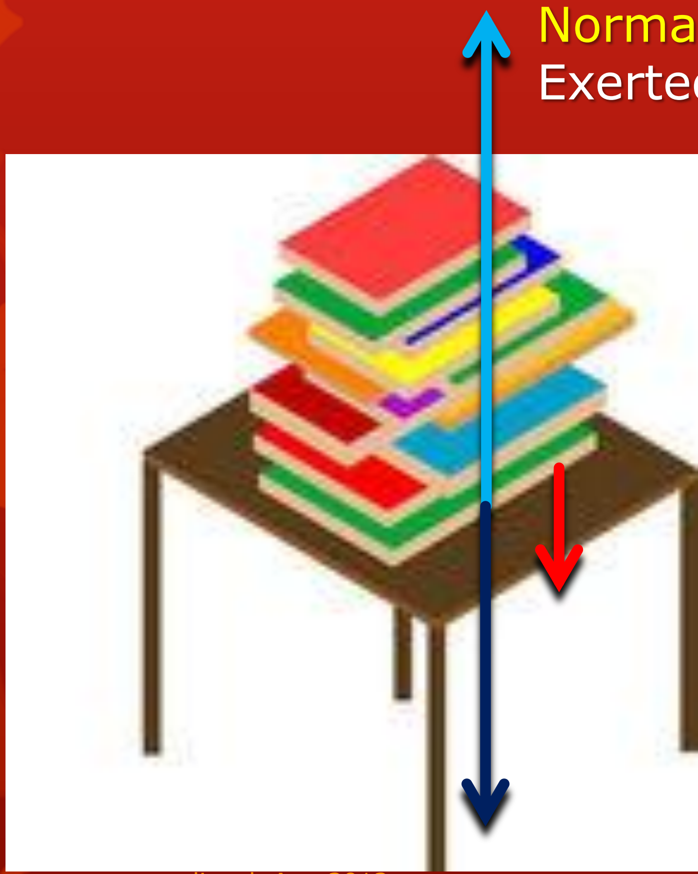


What is the magnitude of the force exerted by the block Y on the block X during acceleration?

A F   B $\frac{F}{2}$ C $\frac{2F}{3}$ D $\frac{3F}{4}$

Are normal reaction N and weight W action & reaction?

Normal reaction force N
Exerted by the table on the books



Equilibrium : N=F=W

Accelerates upwards: N=F>W

Force F
Exerted by the books on the table

weight W
Exerted by the Earth on the books

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Are normal reaction N and weight W action & reaction?

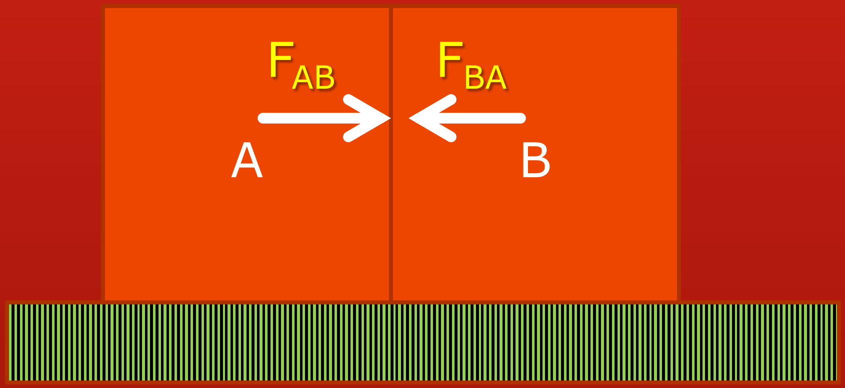
No, they are not action and reaction because

They act on the same object, i.e. book.

Their values are not always the same, for instance when the system accelerates upwards

$N=F > W$

Is the resultant force between F_{AB} and F_{BA} zero?



Not applicable to determine the resultant force as these two forces are acting on TWO different objects!

Action & reaction always act on two objects whether they move at the same speed or accelerates

Learning outcome

(b) show an understanding that mass is the property of a body that resists change in motion

(c) describe and use the concept of weight as the effect of a gravitational field on a mass

Mass & Inertia

- **Mass** is the property of a body which resists change in motion, i.e. a measure of **inertia**. It doesn't vary from place to place. It is a measure of the quantity of matter of an object.

- **Inertia** is the resistance of an object to a change in its state of motion, i.e. when it is moving it resists stopping; when it is at rest, it resists moving. The larger the mass of an object, the higher the inertia it has.

Distinguish between mass and weight

	Mass	Weight
Definition	the property of a body which resists change in motion	Gravitational force exerted by the earth on a mass
Scalar or vector?	Scalar	Vector
Unit	kg	N
Does it vary?	no	Yes. Depends gravitational field strength
Instrument	Triple beam balance, top pan balance	Spring balance(Newton meter)

Learning outcome

- *(f)* recall and solve problems using the relationship $F = ma$, appreciating that acceleration and force are always in the same direction

acceleration and force are always in the same direction

A body is moving up

Velocity +*v*

Velocity decreases thus, a -ve

Acceleration, a

Gravitational force, F

A body is moving down

Velocity +*v*

Velocity increases thus, a +ve

Acceleration, a

Gravitational force, F

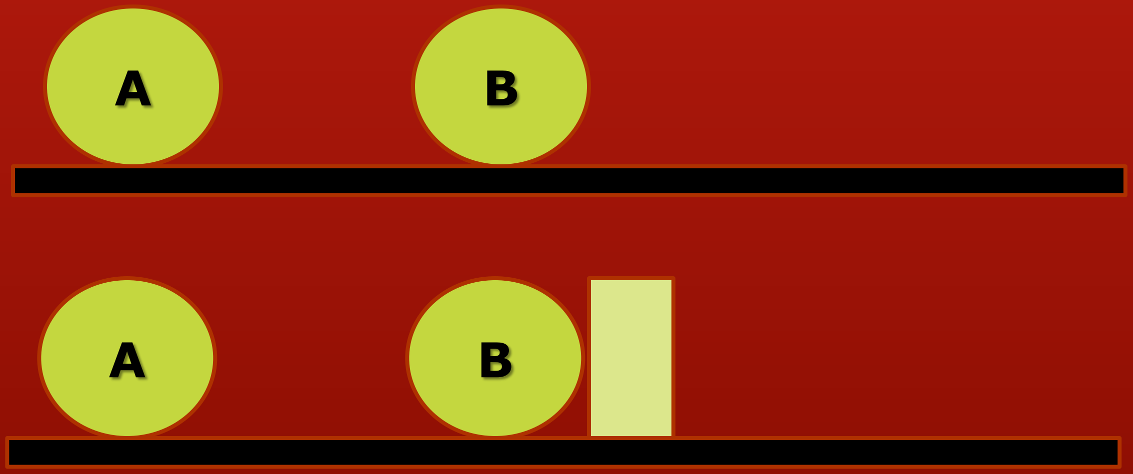
Learning outcomes

- *(g)* state the principle of conservation of momentum
- *(h)* apply the principle of conservation of momentum to solve simple problems including elastic and inelastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required)

principle of conservation of momentum

When bodies in a system interact, the total momentum of the system is conserved provided no external resultant forces acts on the system.

Force causes a change in momentum



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Total momentum = Total momentum

before collision after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$-m_1(v_1 - u_1) = m_2(v_2 - u_2)$$

$$-\Delta p_1 = \Delta p_2$$

change in momentum = change in momentum of object 1 of object 2

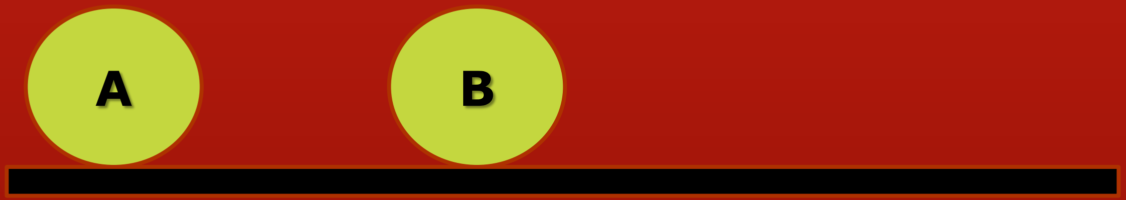
but it's in the opposite direction

A moves with 10 ms^{-1}, approaching B which is at rest.
After collision, A stops.
mass of A = 5 kg
mass of B = 5 kg

Example 1

Velocity of B after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$5 \times 10 + 0 = 0 + 5 \times v_2$$
$$v_2 = 10$$



change in momentum of A = change in momentum of B
= 50 $kgms^{-1}$
but it's in the opposite direction

A loses momentum, B gains momentum

A moves with 10 ms^{-1}, approaching B which is at rest.
After collision, A moves forward with 4.286 ms^{-1}.
mass of A = 5 kg
mass of B = 2 kg

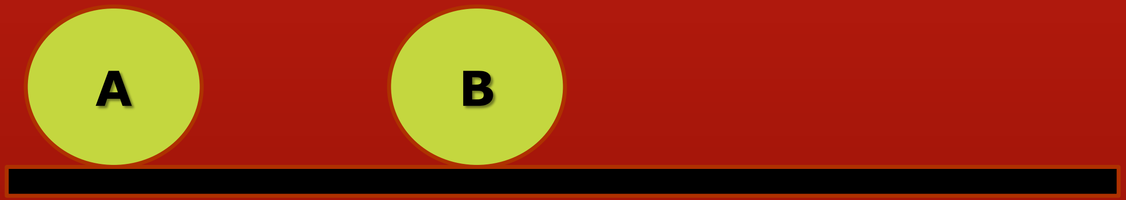
Example 2

Velocity of B after collision

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$5 \times 10 + 0 = 5 \times 4.286 + 2 \times v_2$

$v_2 = 14.285$



change in momentum of A = change in momentum of B
= 28.57 $kgms^{-1}$
but it's in the opposite direction

A loses momentum, B gains momentum

A moves with 10 ms^{-1}, approaching B which moves with $2ms^{-1}$. After collision, they move together.

mass of A = 5 kg

mass of B = 3 kg

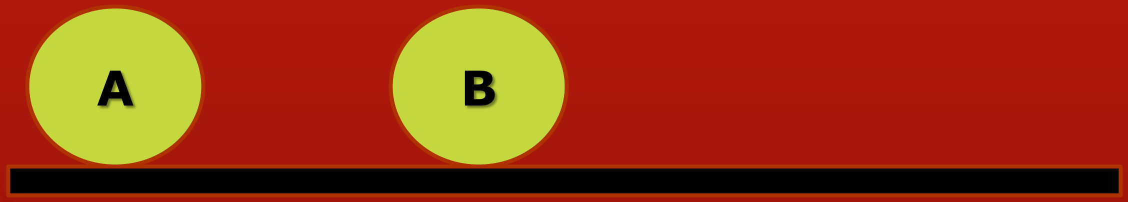
Example 3

Velocity when moving together

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$5 \times 10 + 3 \times 2 = (5+3)\,v$$

$$v = 7\ ms^{-1}$$



change in momentum of A = change in momentum of B = 15 $kgms^{-1}$

but it's in the opposite direction

A loses momentum, B gains momentum

A moves with 10 ms^{-1}, approaching B which moves with 8 ms^{-1} in the opposite direction.
After collision, B moves backward with 14.5 ms^{-1}.
mass of A = 5 kg
mass of B = 3 kg

Example 4

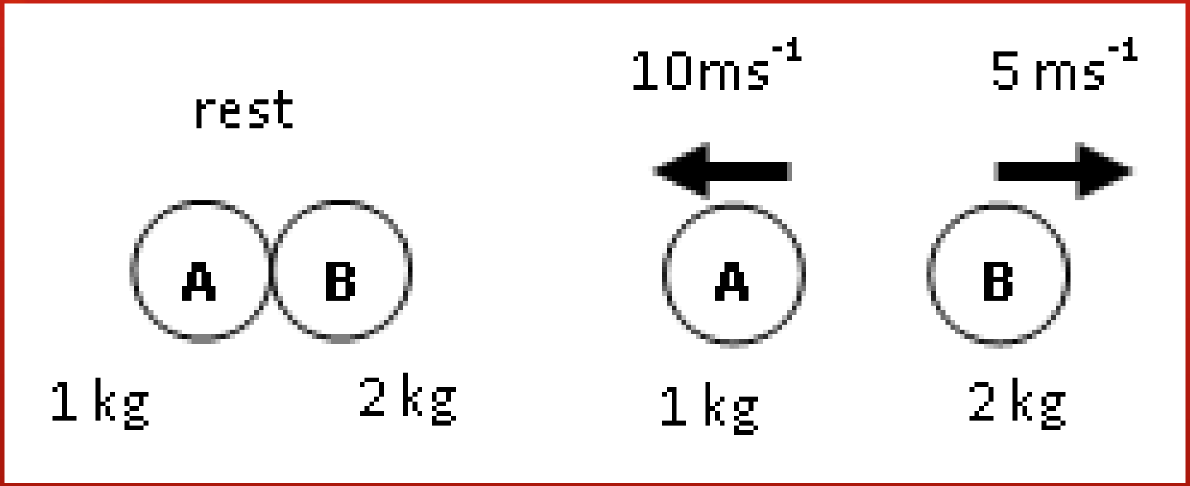
Velocity of A after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$5 \times 10 + 3 \times (-8) = 5 \times v_1 + 3 \times 14.5$$
$$v_1 = -3.5 \ ms^{-1}$$

A: from 50 to -17.5
B: from -24 to 43.5
change in momentum of A = change in momentum of B
= 67.5 $kgms^{-1}$
but it's in the opposite direction

A loses momentum, B gains momentum

Explosion



Total momentum = Total momentum

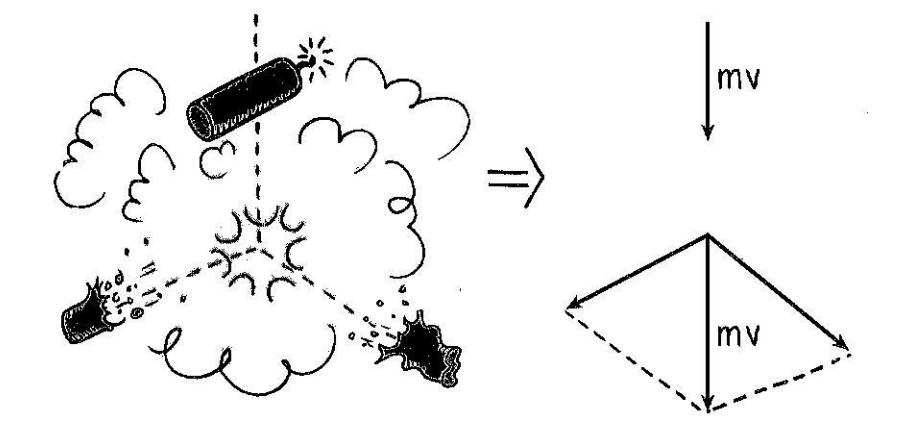
Before explosion **After explosion**

$$0 = m_1 v_1 + m_2(-v_2)$$

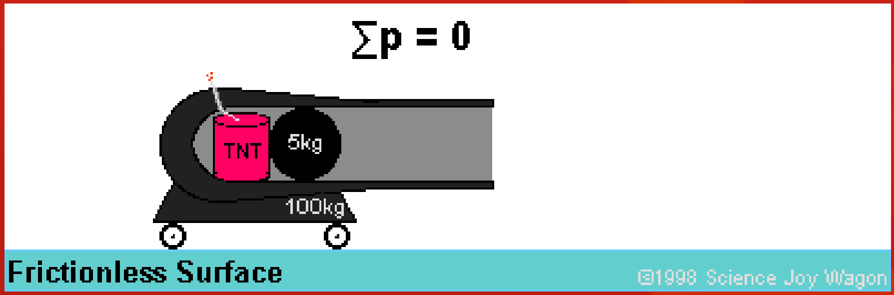
$$m_1 v_1 = m_2 v_2$$

(-ve sign indicates the opposite direction of one of the colliding bodies)

Examples of explosion



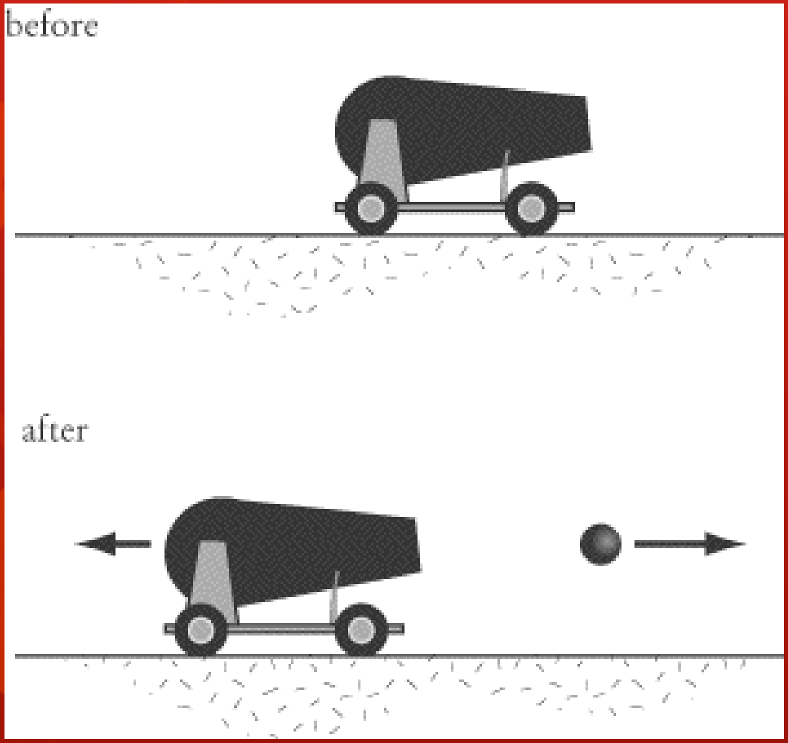
Initial momentum is directed downward, so it the final momentum.



Initial momentum is zero.
After explosion, the ball and cannon move in the opposite direction, producing a zero final momentum.

Initial momentum is zero.
When the guy moves forwards, the boat moves backwards, in the opposite direction to produce a zero final momentum.

Example 1



A 5 kg cannon ball is fired out of the cannon with a velocity of 150 ms^{-1}. Both are at rest in the beginning.

What is the velocity of the cannon which is 50 kg?

$0 = m_1v_1 + m_2 v_2$

$m_1v_1 = -m_2 v_2$

$5 \times 150 = -50 \times V$

$v = -15\ ms^{-1}$

OR you've already known that the cannon moves backwards.

$0 = m_1v_1 + m_2 (-v_2)$

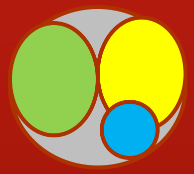
$m_1v_1 = m_2 v_2$

$5 \times 150 = 65 \times V$

$v = 15\ ms^{-1}$

Example 2

A parent nucleus, which is at rest, undergoes fission. Two daughter nuclei are produced but they don't move in exactly opposite direction. What can we tell about the process?



Learning outcomes

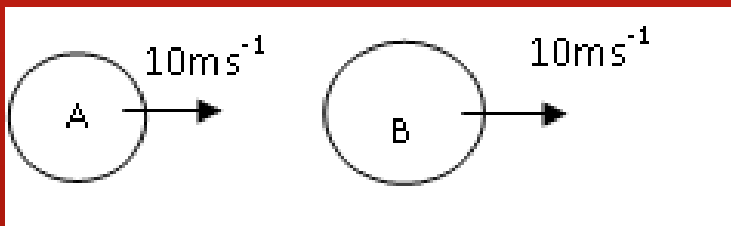
- *(i)* recognise that, for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation
- *(j)* show an understanding that, while momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place

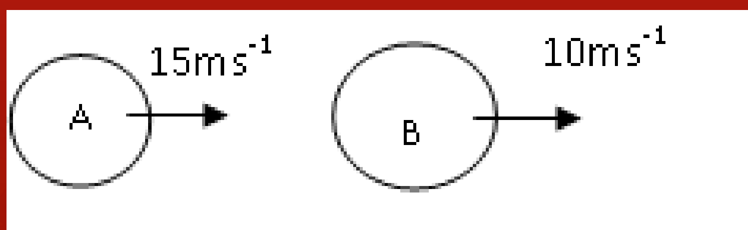
Elastic vs Inelastic collisions

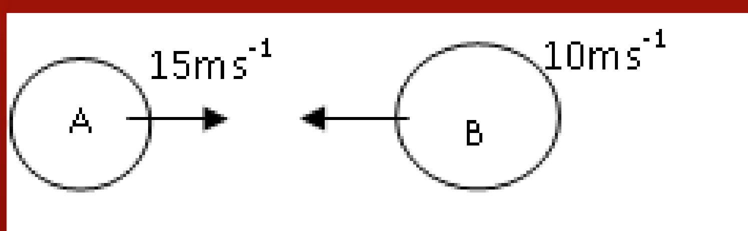
	Perfectly elastic collision	Perfectly inelastic collision
Total momentum of the system		
Total KE of the system		
Total energy of the system		
Relative speed of approach = relative speed of separation $u_1 - u_2 = v_2 - v_1$		

Relative speed of approach & relative speed of separation

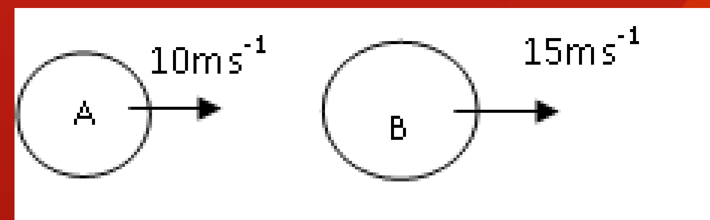
Relative speed of approach (use B as the reference)

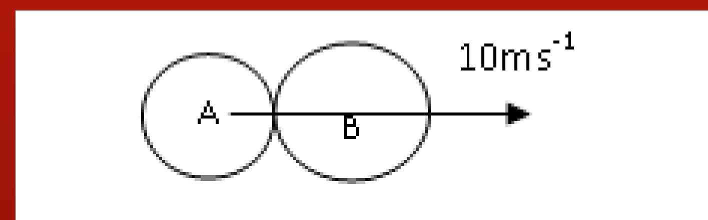


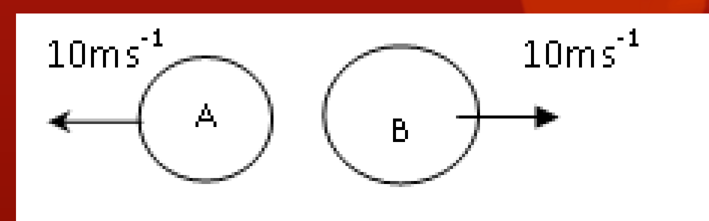




Relative speed of separation (use A as the reference)

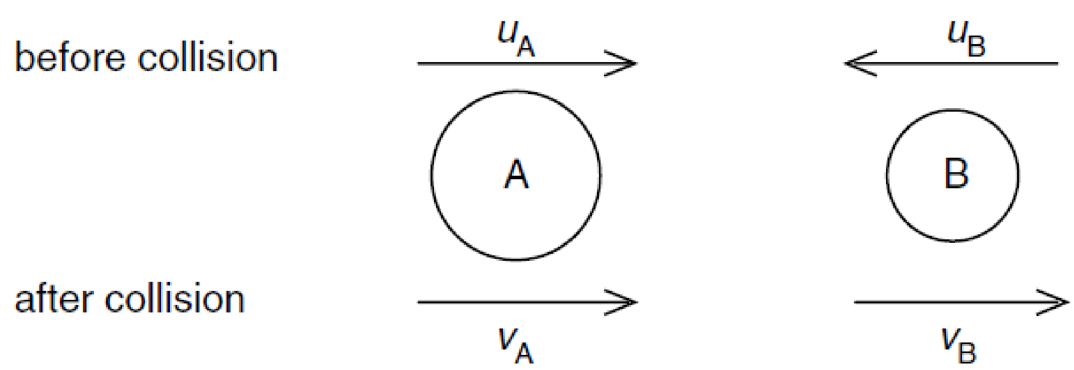






example

Two spheres A and B approach each other along the same straight line with speeds u_A and u_B. The spheres collide and move off with speeds v_A and v_B, both in the same direction as the initial direction of sphere A, as shown below.



Which equation applies to an elastic collision?

A $u_A + u_B = v_B - v_A$

B $u_A - u_B = v_B - v_A$

C $u_A - u_B = v_B + v_A$

D $u_A + u_B = v_B + v_A$

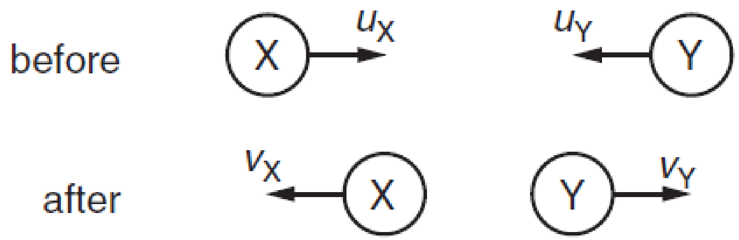
$$u_A - u_B = v_B - v_A$$
$$u_A - (-u_B) = v_B - v_A$$

$$m_A u_A + m_B u_B = m_B v_B + m_A v_A$$
$$m_A u_A + m_B(-u_B) = m_B v_B + m_A v_A$$

Try this !

Two balls X and Y approach each other along the same straight line and collide elastically.

Their speeds are u_X and u_Y respectively. After the collision they move apart with speeds v_X and v_Y respectively. Their directions are shown on the diagram.



Which of the following equations is correct?

A $u_X + u_Y = v_X + v_Y$

B $u_X + u_Y = v_X - v_Y$

C $u_X - u_Y = v_X + v_Y$

D $u_X - u_Y = v_X - v_Y$

$$\mathbf{u_x - u_y = v_y - v_x}$$

$$\mathbf{u_x - (-u_y) = v_y - (-v_x)}$$

$$\mathbf{m_x u_x + m_y u_y = m_x v_x + m_y v_y}$$

$$\mathbf{m_x u_x + m_y (-u_y) = m_x (-v_x) + m_y v_y}$$

Review examples 1-4 to determine:

(1) Total KE before and after collision

(2) Relative speed of approach & relative speed of separation

And hence, determine if the collision is elastic or inelastic

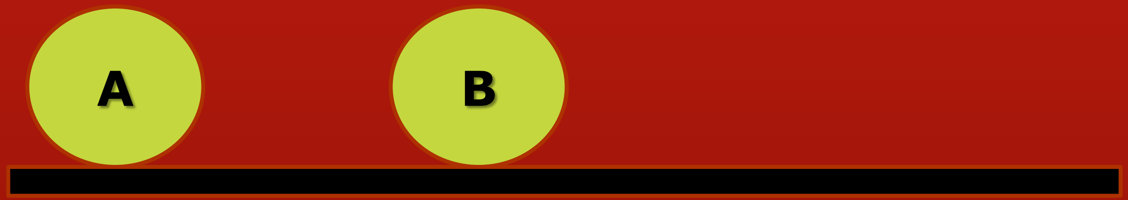
A moves with 10 ms^{-1}, approaching B which is at rest.
After collision, A stops.

mass of A = 5 kg
mass of B = 5 kg

Example 1

Velocity of B after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$5 \times 10 + 0 = 0 + 5 \times v_2$$
$$v_2 = 10$$



(1) Total KE before =

(2) total KE after =

(3) Relative speed of approach =

(4) relative speed of separation =

A moves with 10 ms^{-1}, approaching B which is at rest.
After collision, A moves forward with 4.286 ms^{-1}.
mass of A = 5 kg
mass of B = 2 kg

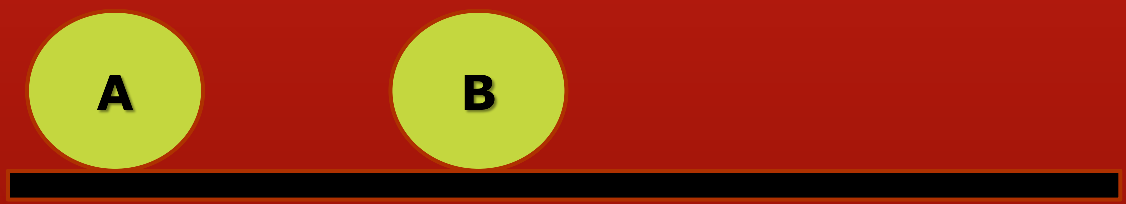
Example 2

Velocity of B after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$5 \times 10 + 0 = 5 \times 4.286 + 2 \times v_2$$

$$v_2 = 14.286$$



(1)Total KE before =

(2) total KE after =

(3)Relative speed of approach =

(4) relative speed of separation =

A moves with 10 ms^{-1}, approaching B which moves with 2ms^{-1}. After collision, they move together.

mass of A = 5 kg

mass of B = 3 kg

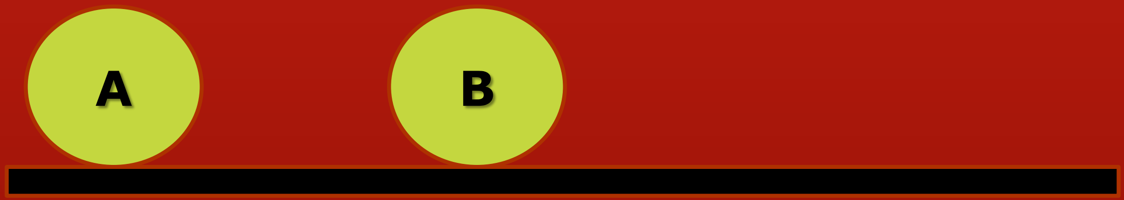
Example 3

Velocity when moving together

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$5 \times 10 + 3 \times 2 = (5+3)\,v$$

$$v = 7\ ms^{-1}$$



(1)Total KE before =

(2) total KE after =

(3)Relative speed of approach =

(4) relative speed of separation =

Example 4

A, moving with 10 ms^{-1}, has a head-on collision with B which moves with 8 ms^{-1} in the opposite direction.
After collision, B moves forward with 14.5 ms^{-1}.
mass of A = 5 kg
mass of B = 3 kg

Velocity of A after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$5 \times 10 + 3 \times (-8) = 5 \times v_1 + 3 \times 14.5$$
$$v_1 = -3.5 \ ms^{-1}$$

(1)Total KE before =

(2) total KE after =

(3)Relative speed of approach =

(4) relative speed of separation =

A, moving with 10 ms^{-1}, has a head-on collision with B which moves with 8 ms^{-1} in the opposite direction.
After collision, B moves forward with 10 ms^{-1}.
mass of A = 5 kg
mass of B = 3 kg

Example 5

Velocity of A after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$5 \times 10 + 3 \times (-8) = 5 \times v_1 + 3 \times 10$$
$$v_1 = -0.8\ ms^{-1}$$

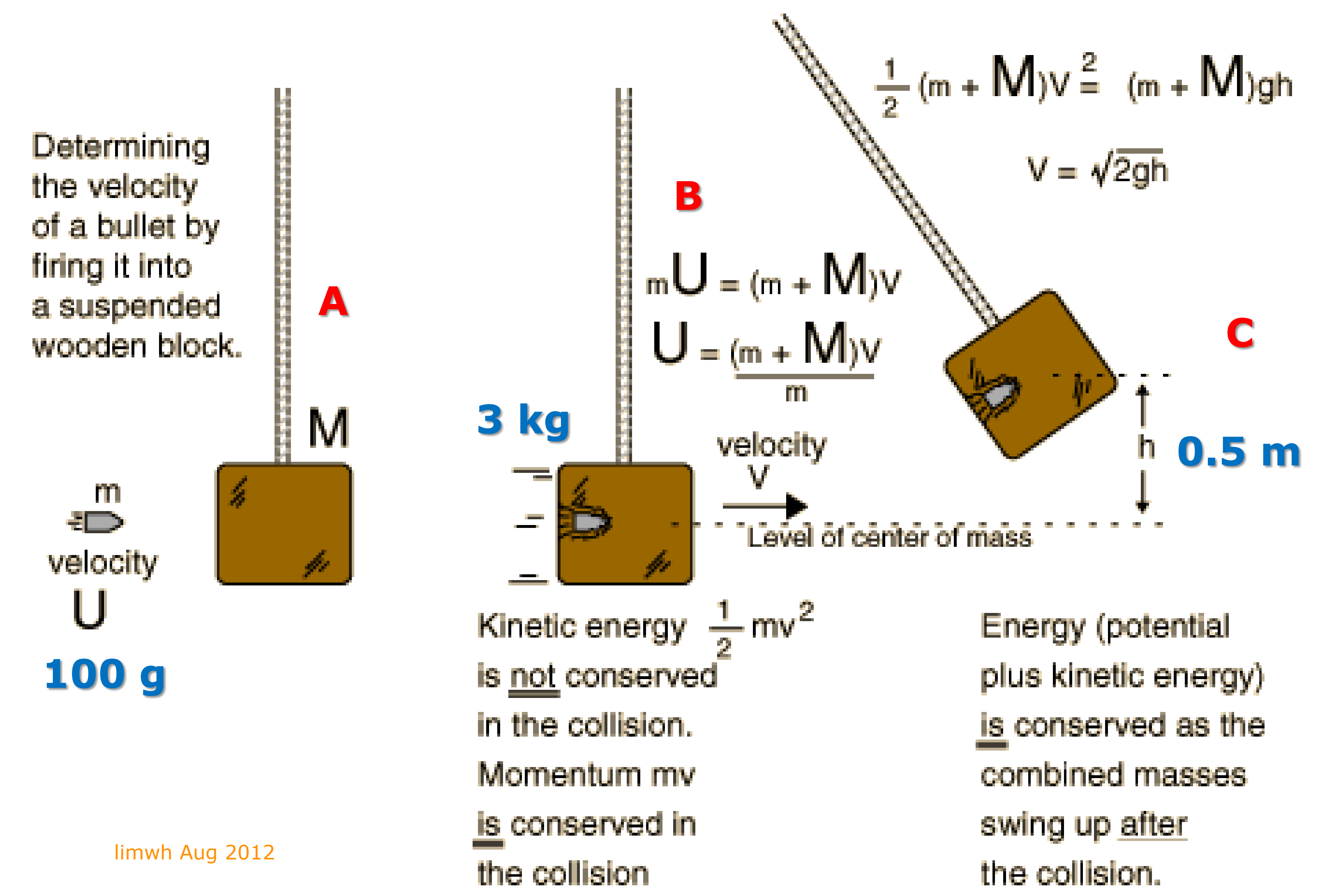
(1)Total KE before =

(2) total KE after =

(3)Relative speed of approach =

(4) relative speed of separation =

Problem solving: Ballistic Pendulum



limwh Aug 2012

(i) Determine the initial speed of the bullet u_B

- During the collision (B):

  $m\,u_B \quad = \quad (m+M)\,v$

  $100 \times u_B = (100+3000)\,v$ ---(1)

- Right after collision, they start to move up together to a height of h, (B) to (C):

  $\frac{1}{2}(m+M)\,v^2 = (m+M)g\,h$

  $\frac{1}{2}v^2 = 9.81 \times 0.5$ ---(2)

  $V = 3.13209$

From eq (1) : $u_B = 97\ ms^{-1}$

(ii) Assumptions you made for the calculations above.

- During the collision (B):

  no external resultant force acts on the bullet and block, for instance, no friction at the pivot.

- Right after collision, they start to move up together to a height of h, (B) to (C):

  air resistance is negligible such that all KE right after collision is converted into GPE

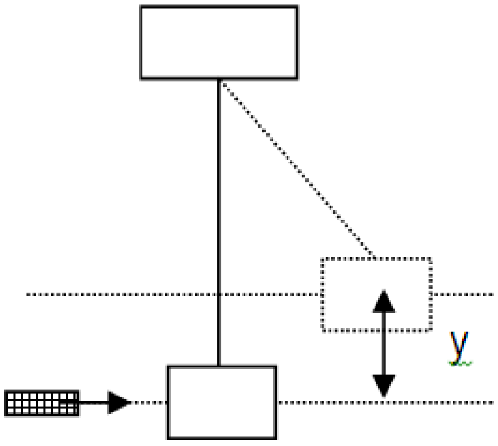
(iii) State, with a reason, if it is an elastic collision.

Inelastic collision because the relative speed of approach =97 is not equal to the relative speed of separation, which is zero.

Try this !

A bullet of mass 10 g traveling horizontally at a speed of 200 ms^{-1} embeds itself in a block of wood of mass 990 g, which is suspended by string so that it can swing freely.

a. Is the collision elastic or inelastic?
b. Determine the initial velocity at which the bullet moves together with the block.
c. Calculate the kinetic energy possessed by the bullet just before it embeds into the wood.
d. Determine the vertical height y by which the block rises after the impact.
e. Calculate the potential energy of the block at the height of y
f. From question (c) and (e), determine the loss of energy
g. Explain what has happened to the loss of energy.



[**Ans**: 2 ms^{-1}, 200J, 0.204m, 2J, 198J]

Questions ?

The End