Work, Energy & Power

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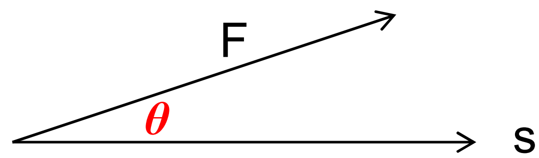
Work

- When a force acts upon an object to cause a displacement of the object, it is said that **work** was done upon the object.
- There are three key *ingredients* to work - force, displacement, and cause. In order for a force to qualify as having done *work* on an object, there must be a displacement and the force must *cause* the displacement.
- The direction of the force is parallel to the displacement.
- There are several good examples of work that can be observed in everyday life - a horse pulling a plow through the field, a father pushing a grocery cart down the aisle of a grocery store, a weightlifter lifting a barbell above his head, a car slowing down due to red lights, an expanding gas & etc.
- In each case described here there is a force exerted upon an object to cause that object to be displaced.

Work

- The work done is defined as the *product on force and the distance moved in the direction of the force.*
- In general, the equation form for work done is $W = Fs \cos \theta$

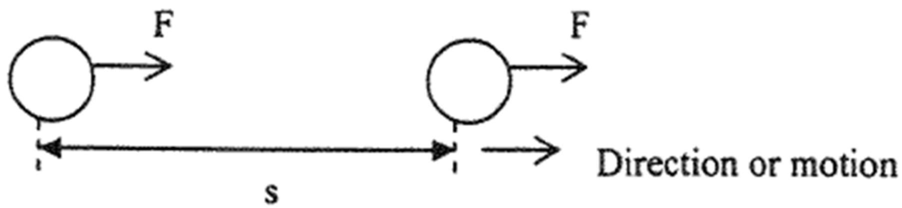


- where **F** is the force, **s** is the displacement, and **(θ)** is defined as the angle between the force and the displacement vector.

- The standard metric unit is the **Joule** (abbreviated **J**).
- 1 Joule is equivalent to 1 Newton of force causing a displacement of 1 meter.

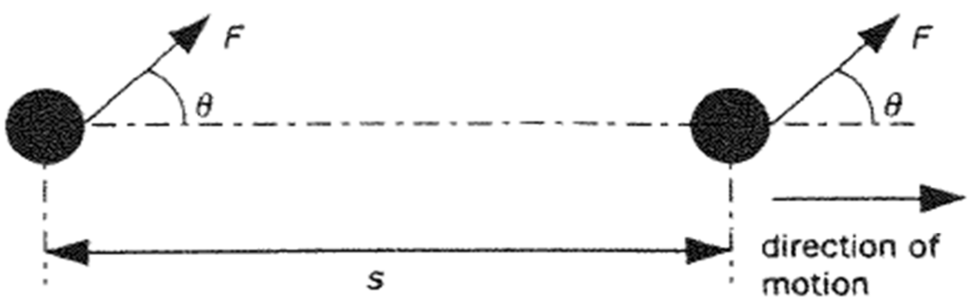
Work done by constant force

1) Force and displacement are in the same direction.



When a constant force (F) moves a distance (s) along its line of action, the work done (W) is given by W= Fs.

2) Force and motion are in different direction



Work done by a force at an angle to the direction of motion

The component of force along the direction of motion is $F \cos \theta$.

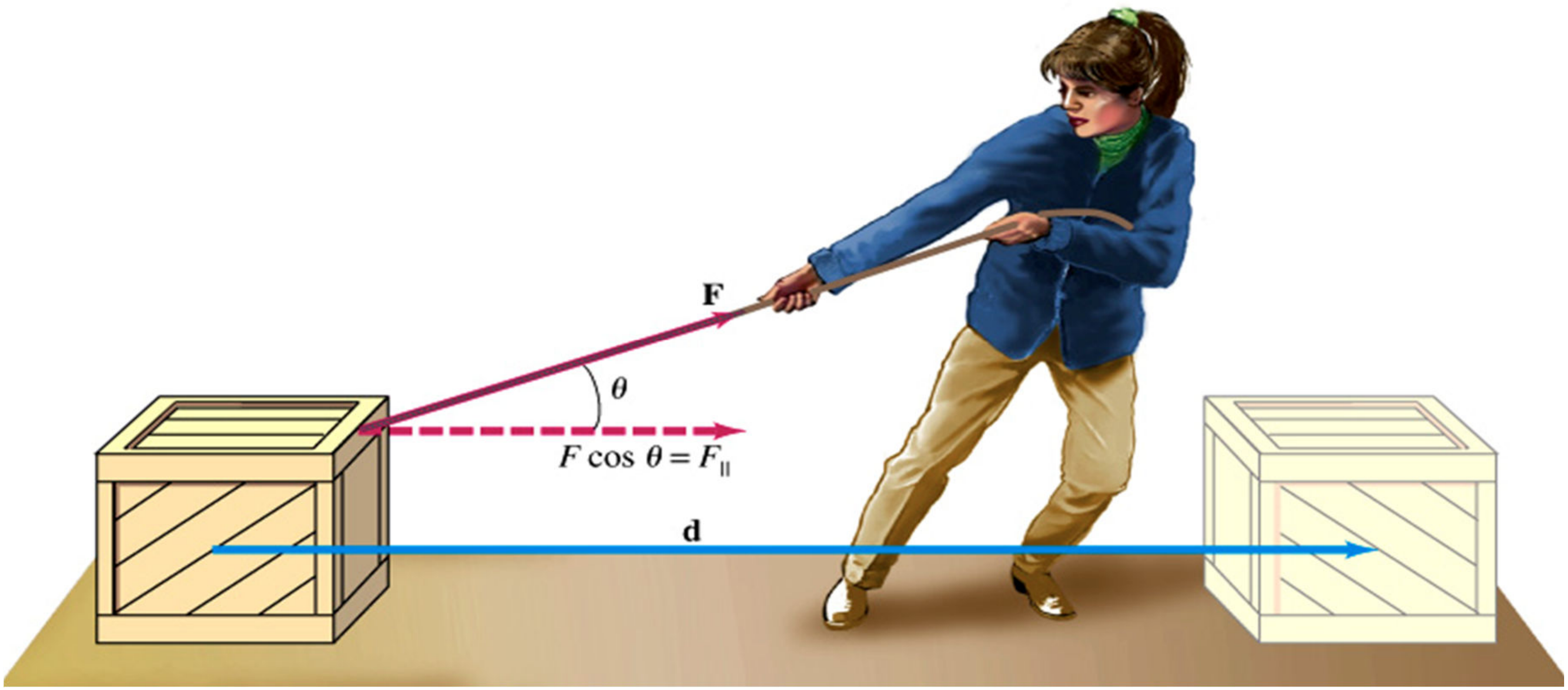
$\therefore$ work done, $W = Fs \cos \theta$

Work done by constant force

Note: If F is perpendicular to direction of motion, then $\theta = 90°$, i.e. $\cos\theta = 0$,
$\therefore Fs\cos\theta = 0$

E.g. when a body moves horizontally along the earth's surface, no work is done by the gravitational force (weight) which acts vertically, although there may well be other forces doing work.

Work



$$W = F\, d \cos\theta$$

- If $\theta = 0$ and $\cos\theta = 1$, work $W = Fd$.
- If $\theta = 90^0$ and $\cos\theta = 0$, no work is done.
- Unit of work is **Joule (J)**

  1 J = N·m

Work

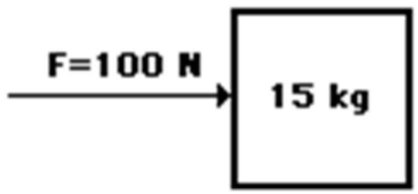
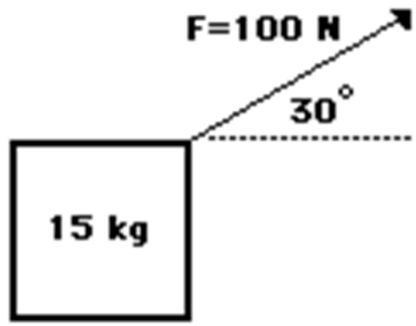
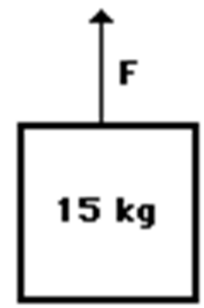
Read the following five statements and determine whether or not they represent examples of work:

(a) A teacher applies a force to a wall and becomes exhausted.
(b) A book falls off a table and free falls to the ground.
(c) A waiter carries a tray full of meals above his head by one arm straight across the room at constant speed. (Careful!)
(d) A rocket accelerates through space.

Work in Summary

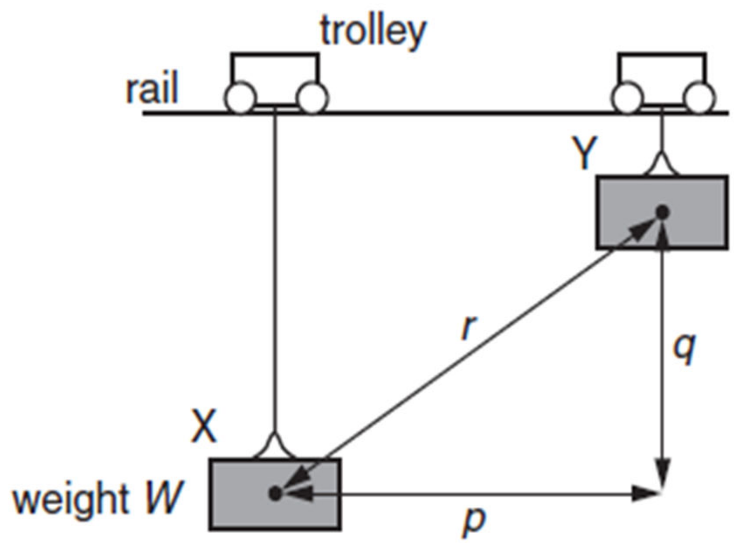
- In summary, work is done when a force acts upon an object to cause a displacement.
- Three quantities must be known in order to calculate the amount of work. Those three quantities are:
  1.) force,
  2.) displacement
  3.) angle between the force and the displacement.
- The area under a force-displacement graph is equal to the work done by the force

Calculate Work Done

Diagram A	Diagram B	Diagram C
		
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.	A 100 N force is applied at an angle of 30º to the horizontal to move a 15 kg object at a constant speed for a horizontal distance of 5 m.	An upward force is applied to lift a 15 kg object to a height of 5 meters at constant speed.

Example 1

A weight W hangs from a trolley that runs along a rail. The trolley moves horizontally through a distance p and simultaneously raises the weight through a height q.

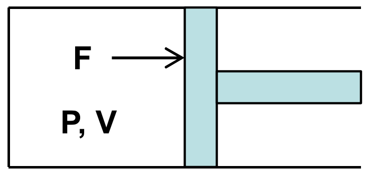


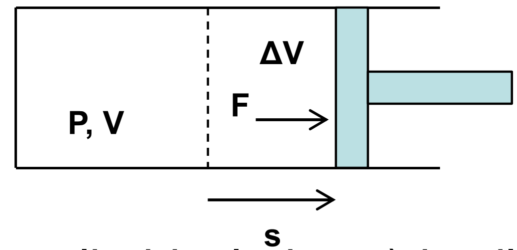
As a result, the weight moves through a distance r from X to Y. It starts and finishes at rest.

How much work is done on the weight during this process?

A Wp **B** $W(p + q)$ **C** Wq **D** Wr

Work Done by Gas





- If gas expands in its container (for this case a cylindrical piston) it will push the piston outwards. Thus, work is done on its surroundings.
- What is the work done by gas?
- Work done by gas in expanding the volume of the cylinder from V to ($V + \Delta V$) :

$W = Fs$

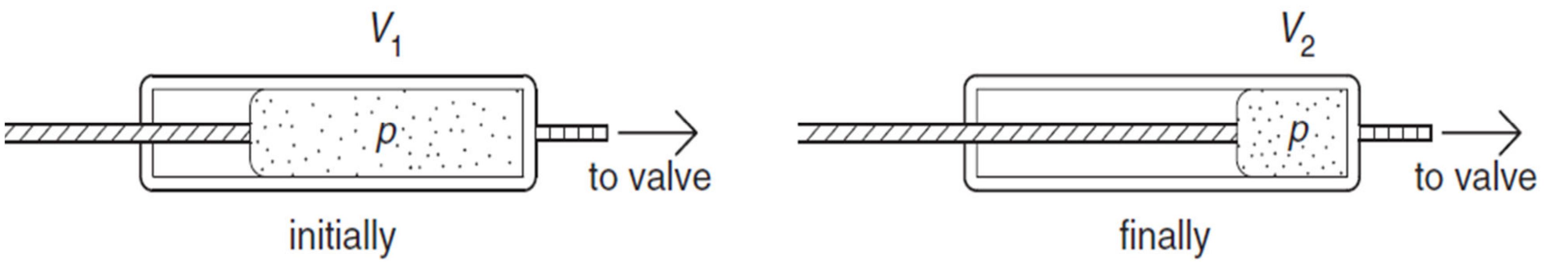
$= (P \times A)s$; where A is the cross sectional area of the piston.

$= P \times (As)$

$= P \times (\Delta V)$; **$W = P\Delta V$** (for this case we are assuming pressure is constant)

Example 2

Air in a bicycle pump is forced through a valve at a constant pressure p. In one stroke of the pump the volume of air in the pump chamber is reduced from V_1 to V_2.



What is the work done on this air in one stroke of the pump?

A $\dfrac{p(V_1 + V_2)}{2}$

B $p(V_1 + V_2)$

C $p(V_1 - V_2)$

D pV_1

Types of energy

Type of Energy	Some common examples
Chemical	Fuels such as oil, wood, coal, food, electric cells.
Nuclear	Atomic bombs, nuclear reactors.
Electrical	Power tools, current in the electric drill, electrical appliances.
Kinetic	All objects in motion.
Gravitational Potential	Object that was raised, waterfall.
Elastic Potential	Compressed or stretched springs, stretched elastic band of a catapult.
Internal	Sum of the random potential & kinetic energies of all the molecules in a body.

Consequence of Work Done

- Work done by a body A on another body B results in the transfer of energy from A to B.

- But in order for object A to do work, object A must first have energy because **energy is the ability to do work**.

- Therefore, when object A do work on object B, there will be transfer of energy from A to B as a consequence of the work done by A.

- For example, gravity does work on a falling stone and results in the release of the gravitational potential energy and a gain in the kinetic energy for the stone if air resistance is negligible.

- But if the object experiencing a force does not move then there is no work done e.g. holding a dumbbell above your head at rest.

ENERGY
Its Conversion & Conservation

Conservation of Energy

The principle of conservation of energy states that energy can neither be created nor destroyed. It can be transformed from 1 form to another and transfer from 1 body to another but the total amount remains constant.

For example, a closed system which initially possesses 100 J of chemical energy may at some point of time have the chemical energy transformed into 100 J of kinetic and heat energy.

ENERGY
Its Conversion & Conservation

<u>Conversion of Energy</u>

- Energy can be converted from one form to another.
- For example of work done by:

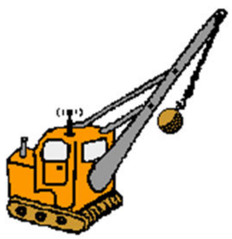
1.) Lighting up of a bulb.
(electrical energy - heat and light energy)

2.) A cyclist going up the hill and down.
(chemical energy of body –
gravitational potential energy – kinetic energy)

3.) A hammer knocking a nail
(gravitational potential energy – kinetic energy –
work done in driving the nail into the wood, heat & sound)

Potential energy

- An object can store energy as the result of its position.
- For example, the heavy ball of a demolition machine is storing energy when it is held at an elevated position. This stored energy of position is referred to as potential energy.



- Similarly, a bow, when not drawn there is no energy stored in the bow. Yet when its position is altered from its usual equilibrium position (drawn the bow), the bow is able to store energy by virtue of its position.
- This stored energy of certain position is referred to as potential energy.
- **Potential energy** is the energy stored by an object due to its position in a force field or due to its configuration.

Potential energy

- The 3 most general types of potential energy are:
  1.) Gravitational Potential Energy
  2.) Elastic Potential Energy
  3.) Electric Potential Energy

- Gravitational potential energy is the potential energy of an object due to its position in the gravitational field. ($U = mgh$)

- Electrical potential energy is the potential energy of an object due to its position in the electrical field. ($U = Q_1 Q_2 / 4\pi\varepsilon_0 r$)

- Elastic potential energy is the potential energy of an object due to the inter atomic charge interaction in the atoms. ($U = ½ kx^2$)

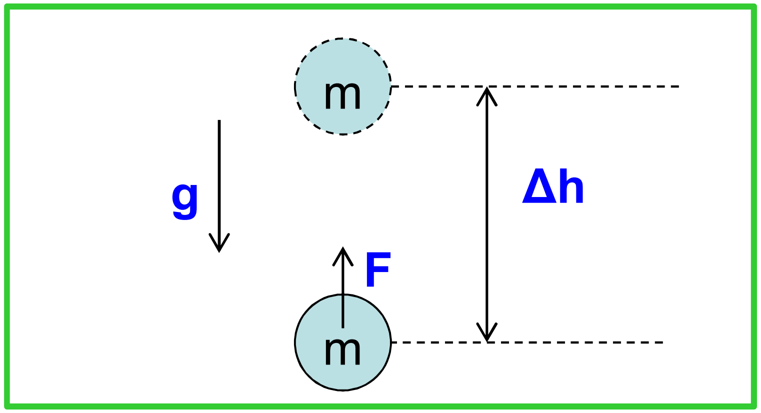
Gravitational Potential Energy

- Gravitational potential energy is the energy stored in an object as the result of its vertical position or height in the earth gravitational field.
- The gravitational potential energy of the massive ball of a demolition machine is dependent on two variables - the **mass of the ball** and the **height to which it is raised.**
- There is a direct relation between gravitational potential energy and the mass of an object. *More massive objects have greater gravitational potential energy.*
- There is also a direct relation between gravitational potential energy and the height of an object. *The higher that an object is elevated, the greater the gravitational potential energy.*
- These relationships are expressed by the following equation:

$$E_p = mg\Delta h$$

Gravitational Potential Energy

- Derivation of $\mathbf{E_p = mg\Delta h}$



- Consider an object of mass 'm' being lifted vertically with constant velocity, through a vertical distance, 'Δh'.
- Thus, work done by external force, $W = F\Delta h$.
- For constant velocity, $F = mg$.

- Work done in raising the object through vertical distance Δh:

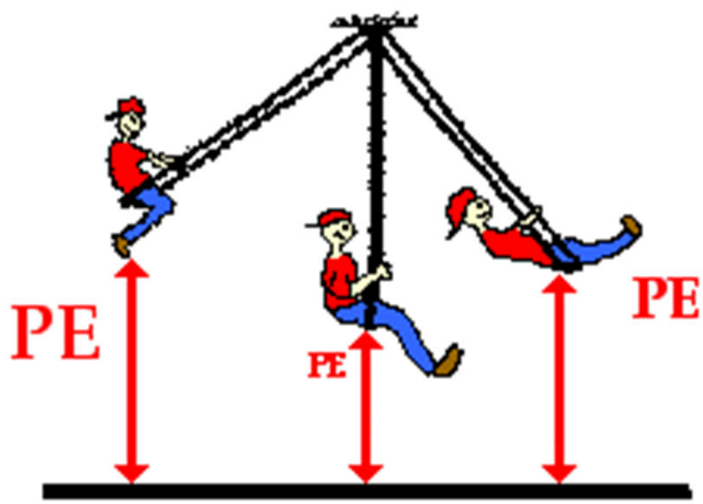
$$\mathbf{W = mg\Delta h}$$

- **The work done in this case became the gain in gravitational potential energy.!**

Gravitational Potential Energy

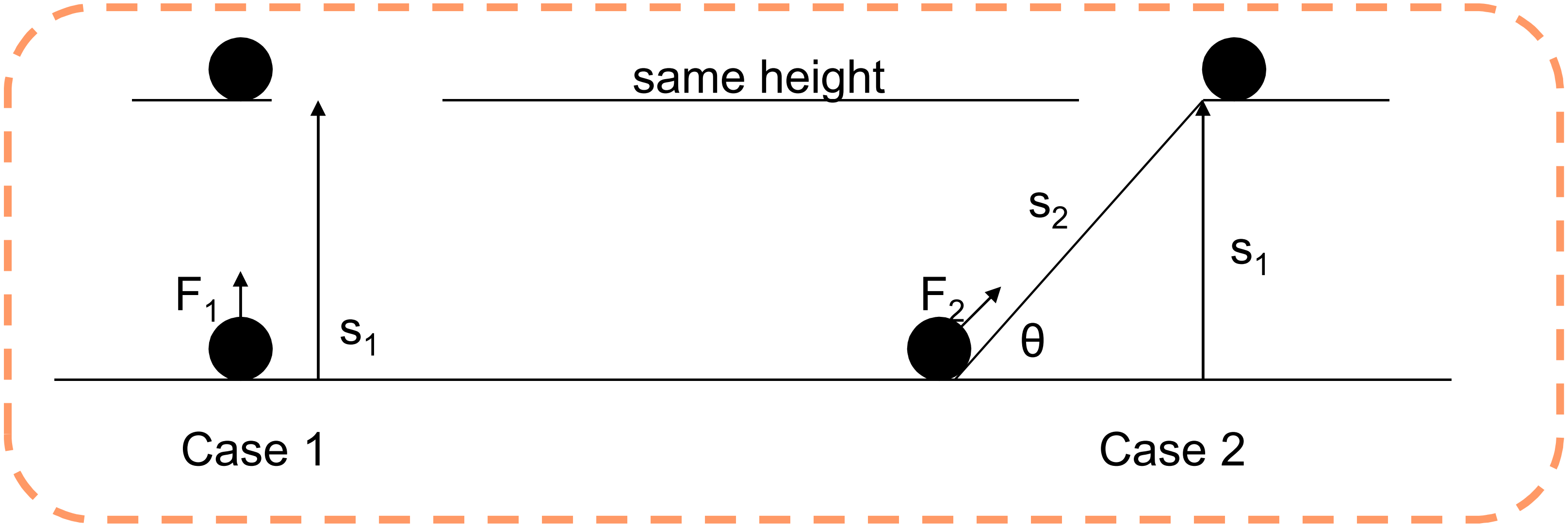
- To determine the gravitational potential energy of an object, ***a zero height position* must first be arbitrarily assigned.** Typically, the ground is considered to be a position of zero height.

- Since the gravitational potential energy of an object is directly proportional to its vertical height, a *doubling* of the height will result in a *doubling* of the gravitational potential energy. A *tripling* of the height will result in a *tripling* of the gravitational potential energy.



Gravitational Potential Energy

- Imagine a situation where a ball is raised vertically, first by lifting it up vertically, and then by pulling it up a frictionless ramp at an angle.
- The gain in E_p is the same in both cases and so by the law of conservation of energy, the work done must also be the same.
- $\mathbf{F_1 s_1 = mgh = F_2 s_2}$
- But as $\mathbf{s_2 > s_1}$, it follows that $\mathbf{F_2 < F_1}$
- Hence by using a ramp, the displacement is increased, but the force needed to raise the ball is reduced. The ramp is a simple form of ***machine.***



Kinetic Energy

- **Kinetic energy** is the energy of motion.
- An object that has motion whether it is vertical or horizontal motion has kinetic energy.
- The amount of kinetic energy that an object has depends upon two variables: the mass (m) of the object and the speed (v) of the object. The following equation is used to represent the kinetic energy (**E_k**) of an object.

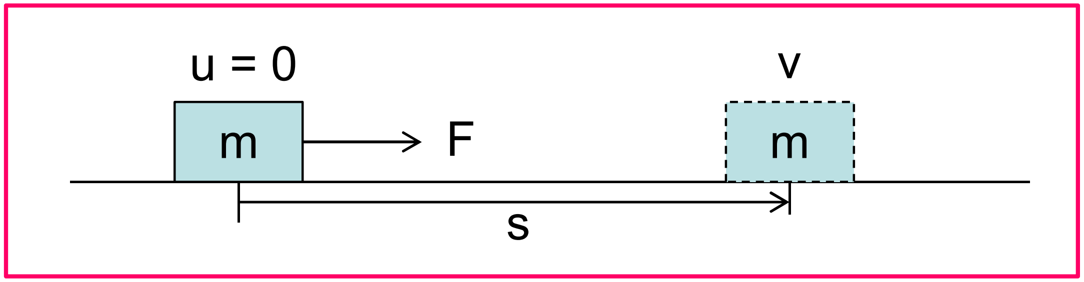
$$\boxed{E_k = \frac{1}{2}(mv^2)}$$

- This equation reveals that the kinetic energy of an object is directly proportional to the square of its speed.

- That means that for a twofold increase in speed, the kinetic energy will increase by a factor of four. For a threefold increase in speed, the kinetic energy will increase by a factor of nine. And for a fourfold increase in speed, the kinetic energy will increase by a factor of sixteen.

Kinetic Energy

- Derivation of $\mathbf{E_k = ½ (mv^2)}$



- Consider a car being accelerated from rest ($u = 0$) to velocity v. To give it acceleration a, it is pushed by a force F for a distance s.
- Apply kinematics equation of $\mathbf{v}^2 = \mathbf{u}^2 + 2as$; *whereby $u = 0$*

  $\mathbf{v}^2 = 2as \Rightarrow a = \mathbf{v}^2/2s$

- Work done is $W = Fs$; *since there is acceleration, the force will be $F = ma$*

  $W = (ma)s$; *substitute $a = \mathbf{v}^2/2s$ into the equation*

  $W = (m)(\mathbf{v}^2/2s)s$

  $W = ½ (m\mathbf{v}^2)$

- Since work is converted into kinetic energy, thus after distance s, the K.E is:

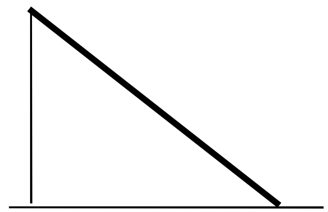
$$\boxed{\mathbf{E_k = ½ (mv^2)}}$$

Example 3

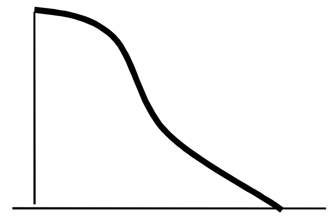
1. Calculate the increase in kinetic energy of a car of mass 800 kg when it accelerates from 20 m/s to 30 m/s
2. A body of mass 50 kg moves from a height of 20 m to 50 m. Calculate the change in the body's E_p .
3. If a ball of mass 0.5 kg is travelling at a velocity of 4 m/s, what is the kinetic energy? What is the E_k if the ball travels at double the speed?
4. A stone has a mass of 4 kg. It is at a height of 8 m above the ground. What is its kinetic energy as it passes a point 6 m above the ground? Air resistance is negligible.
5. A platform diver had a kinetic energy of 12 000 J just prior to hitting the surface of the water. If her mass is 40 kg, then what is her speed prior to hitting the water surface?
6. During take off an airplane's speed changes from zero to 40 m s^{-1} while traveling 1000 m along a runway. If its mass is 20,000 kg, find the driving force which needs to be applied to it?

Example 4

Figure (a) shows a smooth straight slide and figure (b) shows a smooth curved slide. Both slides have the same vertical height.



(a)



(b)

A boy of mass *m* slides down from the top of the straight slide and another boy of mass 2*m* slides down from the curved slide. The speed at which the first boy reaches the bottom of the straight slide is v_s and the speed at which the second boy reaches the bottom of the curved slide is v_c. Which of the following is true?

A $v_c = v_s$ **B** $v_c = 2v_s$ **C** $v_c > 2v_s$ **D** $2v_c = v_s$

Relationship between Ep & Ek with work done by external forces

Work Done by External Force

- Whenever work is done upon an object by an external force, there will be a change in the K.E / P.E / both of the object.
- Eg of external forces: friction, air resistance, normal, applied force.

- Mathematically, we can write the expression like this:
- $K.E_{initial} + P.E_{initial} + Work_{ext} = K.E_{final} + P.E_{final}$

- An important note that should be made about the above equation is that the work done by external forces can be a positive or negative value.
- The work done will be positive if the force is acting in the direction of the displacement.
- The work done will be negative if the force is acting opposite to the direction of the displacement.

Relationship between Ep & Ek with work done by internal forces

Work Done by Internal Force

- Whenever work is done upon an object by an internal force, there will be NO change in the total (K.E + P.E) of the object.
- However, some amount of K.E may be converted to P.E and vice versa, but the total (K.E + P.E) is still the same. (conserved)
- Eg of internal forces: gravitational, electric, magnetic force.

- Mathematically, we can write the expression like this:
- $K.E_{initial} + P.E_{initial} = K.E_{final} + P.E_{final}$

Example of relationship between Ep & Ek with work done by external & internal force

Case 1: Work done by INTERNAL force (when air resistance negligible)

($K.E_{initial} + P.E_{initial} = K.E_{final} + P.E_{final}$)

- If an object falls due to the pull of gravity, it speeds up continuously.
- Its g.p.e decreases and its k.e increases.
- For this case, energy is being transformed from potential energy to kinetic energy.
- Assuming no energy lost due to heat.

Loss in P.E = Gain in K.E.

Case 2: Work done by EXTERNAL force (when air resistance NOT negligible)

($K.E_{initial} + P.E_{initial} + Work_{ext} = K.E_{final} + P.E_{final}$)

- If an object falls due to the pull of gravity, its g.p.e continue to decrease while its k.e will remain constant if air resistance is present.
- For this case, energy lost from potential energy is used to do work against the air resistance.
- **Loss in P.E = Work done against air resistance.**

Example 5

- Consider a weightlifter who applies an upwards force of 1000 N to a barbell to displace it upwards a given distance of 0.25 m at a constant speed. If the barbell begins with 1500 J of potential energy, calculate the final potential energy.

- Now consider a car that is skidding from a high speed to a lower speed. The force of friction between the tires and the road exerts a leftward force of 8000 N on the rightward moving car over a given distance of 30 m. If the car begins with 320 000 J of K.E, calculate what is the K.E after 30 m.

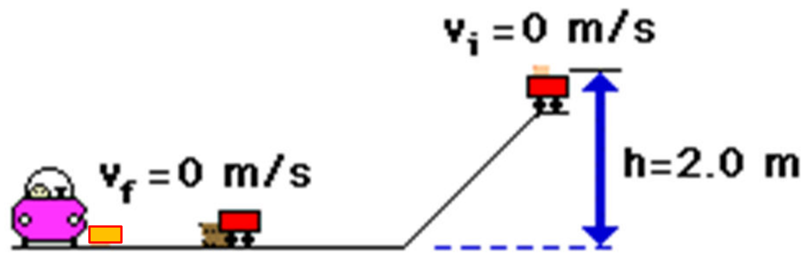
Example 6

- A 1000-kg car traveling with a speed of 25 ms^{-1} skids to a stop. The car experiences a 8000 N frictional force. Determine the stopping distance of the car.

- At the end of the Shock-Wave roller coaster ride, the 6000 kg train of cars (includes passengers) is slowed from a speed of 20 ms^{-1} to a speed of 5 ms^{-1} over a distance of 20 m. Determine the braking force required to slow the train of cars by this amount.

Example 7

- A shopping cart full of groceries is sitting at the top of a 2.0 m hill. The cart begins to roll until it hits a stump at the bottom of the hill. Upon impact, a 0.25 kg can of peaches flies horizontally out of the shopping cart and hits a parked car with an average force of 500 N. How deep a dent is made in the car (i.e., over what distance does the 500 N force act upon the can of peaches before bringing it to a stop)?



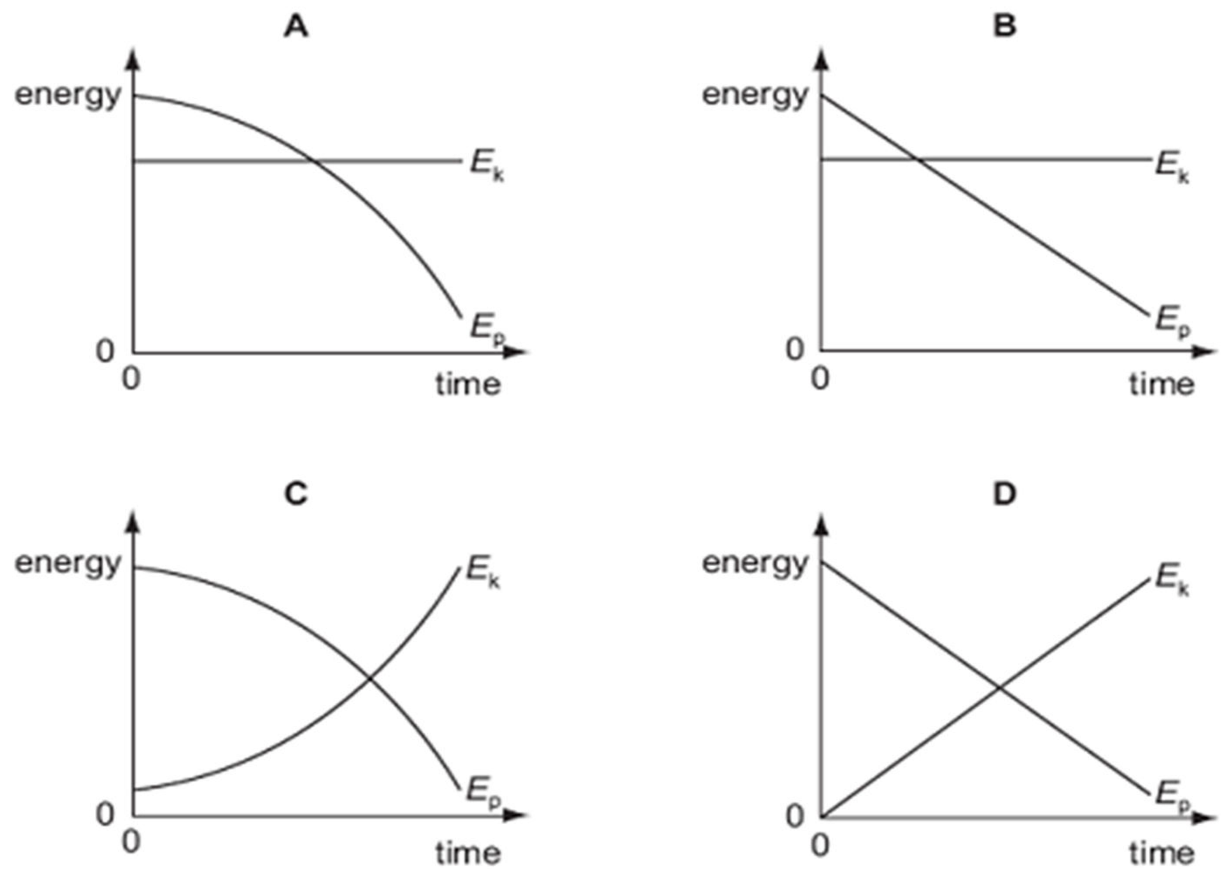
Example 8

- A horizontal force of 100 N is used to push a box across a horizontal floor. What is the gain in K.E. of the box after moving through a distance of 8.0 m?
  A.) Frictional force is zero B.) Frictional force is 30 N

- A constant force of 11 kN parallel to an inclined plane, moved a body of weight 25 kN through a distance of 35 m along the inclined plane at constant speed. The body gains 15 m in height. How much of the work done is dissipated as heat?

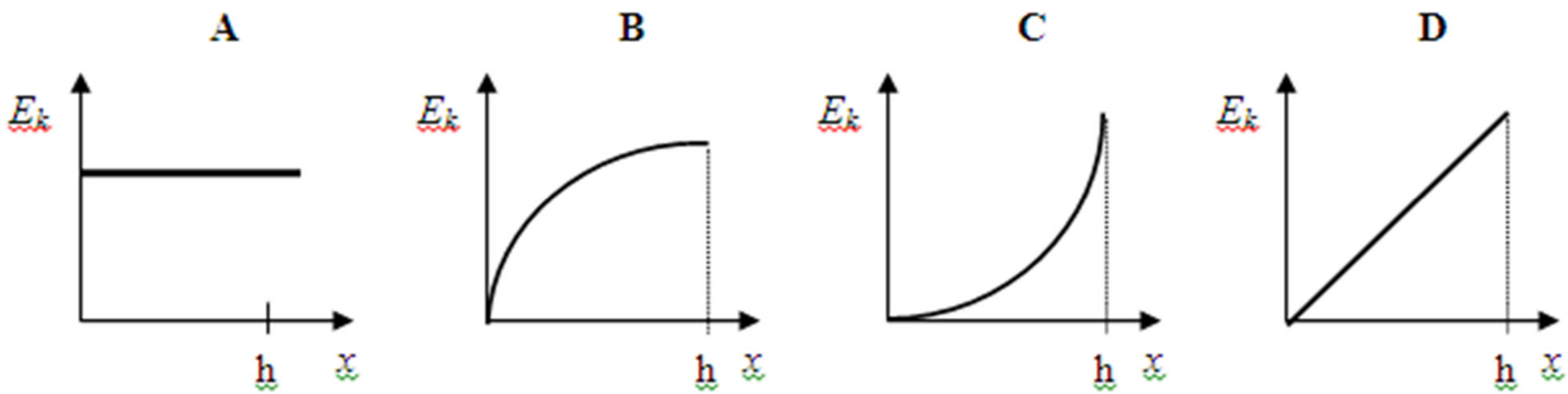
Example 9

A steel ball is falling at constant speed in air.
Which graph shows the variation with time of the gravitational potential energy E_P and the kinetic energy E_K of the ball?



Example 10

A small steel sphere is released from rest at a height, h above the ground level. Which of the following graphs most closely represents the variation of kinetic energy of the sphere, E_k with the distance it moves, x? (Ignore the air resistance)



Example 11

A ball is thrown vertically upwards.

Neglecting air resistance, which statement is correct?

A The kinetic energy of the ball is greatest at the greatest height attained.

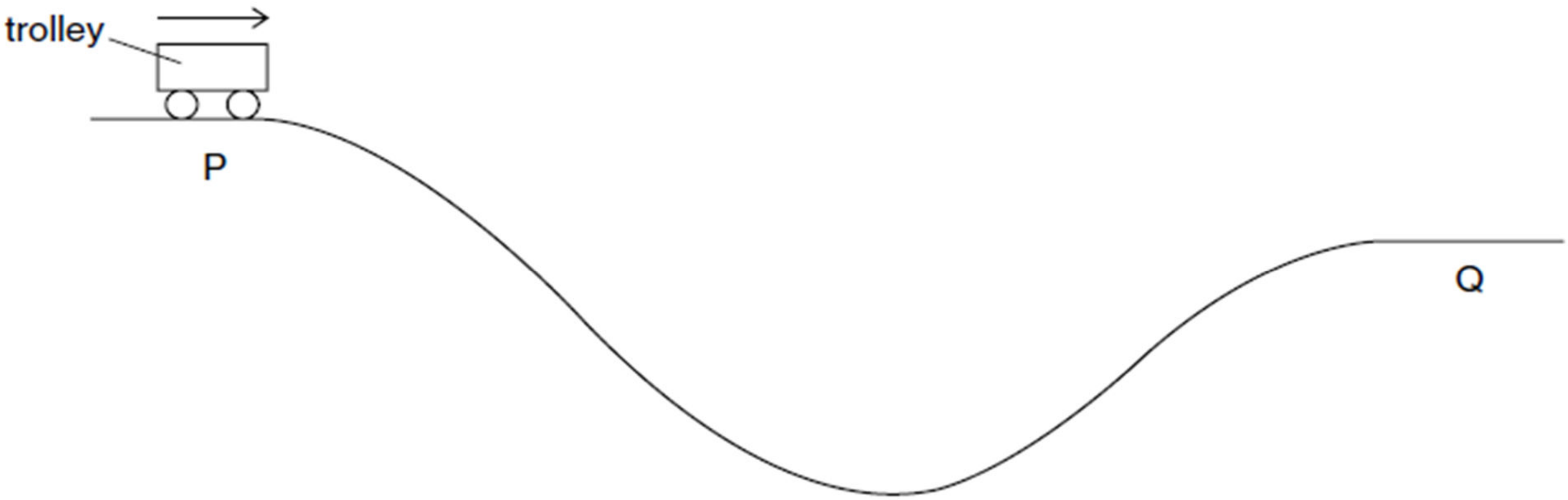
B By the principle of conservation of energy, the total energy of the ball is constant throughout its motion.

C By the principle of conservation of momentum, the momentum of the ball is constant throughout its motion.

D The potential energy of the ball increases uniformly with time during the ascent.

Example 12

A trolley runs from P to Q along a track. At Q its potential energy is 50 kJ less than at P.



At P, the kinetic energy of the trolley is 5 kJ. Between P and Q the work the trolley does against friction is 10 kJ.

What is the kinetic energy of the trolley at Q?

A 35 kJ

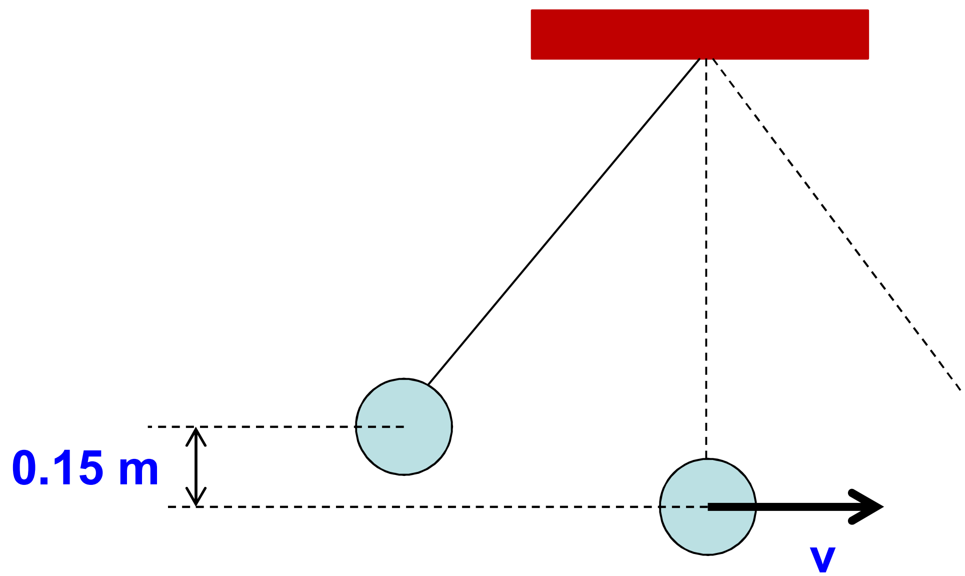
B 45 kJ

C 55 kJ

D 65 kJ

Example 13

- A pendulum consist of a copper sphere of mass 5.0 kg hanging from a long string. The sphere is pulled to the side so that it is 0.15 m above its lowest position. It is then released. How fast will it be moving when it passes through the lowest point along its path?



Power

- Power ***P*** is defined as the ***rate at which work is done*** *or* ***rate of change of energy.***

- And since the expression for work is the product of force and displacement, the expression for power can be rewritten as (force x displacement)/time.

- Since the expression for velocity is displacement/time, the expression for power can be rewritten once more as the product of force and velocity. This is shown on the right.

- The unit of Power is **J/s or Watt (W).**

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \cdot \text{Displacement}}{\text{Time}}$$

$$\text{Power} = \text{Force} \cdot \frac{\text{Displacement}}{\text{Time}}$$

$$\boxed{\text{Power} = \text{Force} \cdot \text{Velocity}}$$

Example 14

- Let's assume the weight of an athlete is 48 kg, running up a flight of stairs from ground to the top of steps of height 50 m in 2 min. What is the power exerted by the athlete.

- A hoist operated by an electric motor has a mass of 500 kg. It raises a load of 300 kg vertically at a steady speed of 0.2 m/s. Frictional resistance can be taken to be constant at 1200 N. What is the power required?

Example 15

To travel at a constant speed, a car engine provides 24 kW of useful power. The driving force on the car is 600 N.

At what speed does it travel?

A $2.5\,\mathrm{m\,s^{-1}}$

B $4.0\,\mathrm{m\,s^{-1}}$

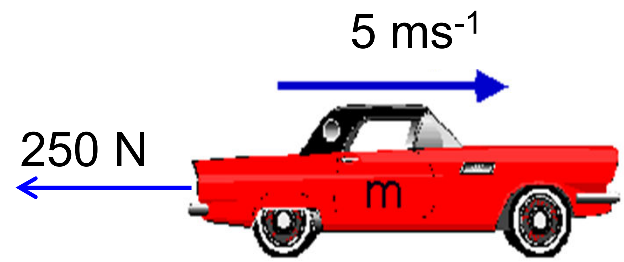
C $25\,\mathrm{m\,s^{-1}}$

D $40\,\mathrm{m\,s^{-1}}$

- Let's assume a car is travelling at a steady velocity of 30 ms^{-1} along a level road. If the total of the frictional forces acting on the car are 700 N, what is the output power of the car?

Example 16

- A car of mass 500kg is travelling along a horizontal road. The engine of the car is working at a constant rate of 5 kW. The total resistance to motion is constant and is 250N. What is the acceleration of the car when its speed is 5ms^{-1}?



Example 17

- A car of mass 900 kg has an engine with power output of 42 kW. It can achieve a maximum speed of 120 km/h along the level.

  a.) What is the frictional force?

  b.) If the maximum power and the resistance remained the same what would be the maximum speed the car could achieve up an incline of 10° along the slope?

Efficiency

- The ***efficiency*** of a practical device is the ***ratio of the useful output energy to the source input energy***, usually given as a percentage or ratio.

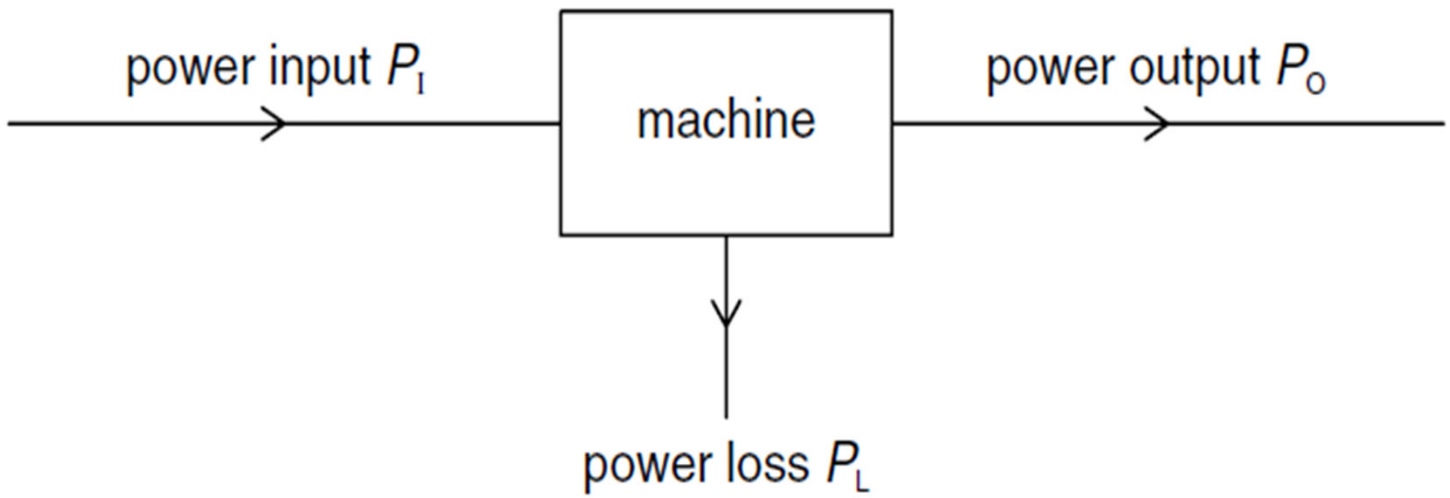
- Efficiency % = (output energy)/(input energy) x 100

- For example, if some boiled water gains 4200 J of internal energy from an input of 10000 J of electrical energy, the efficiency is 42% and the loss is 58%

- Many energy transfers are inefficient. That is, only part of the energy is transferred to where it is wanted. The rest is wasted, and appears in some form that is not wanted (such as heat).

Example 18

Power is transferred through a machine as shown.



What is the efficiency of the machine?

A $\dfrac{P_I}{P_O + P_L}$ **B** $\dfrac{P_L}{P_I}$ **C** $\dfrac{P_L}{P_O}$ **D** $\dfrac{P_O}{P_I}$

Example 19

- A crane lifts a 100 kg block of concrete through a vertical height of 16 m in 20 s. If the power supplied to the motor driving the crane is 1 kW, what is the efficiency of the motor?

Example 20

An electric motor is used to lift an elevator of mass 500 kg through a vertical distance of 200 m in 3 min. It has an overall efficiency of 75 %.

Calculate:

a.) the total electrical power required (take $g = 9.81\ ms^{-2}$)