Chap 3 Kinematics

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Learning outcomes

(a) Define displacement, speed, velocity and acceleration

(b) use graphical methods to represent displacement, speed, velocity and acceleration

(c) find displacement from the area under a velocity-time graph

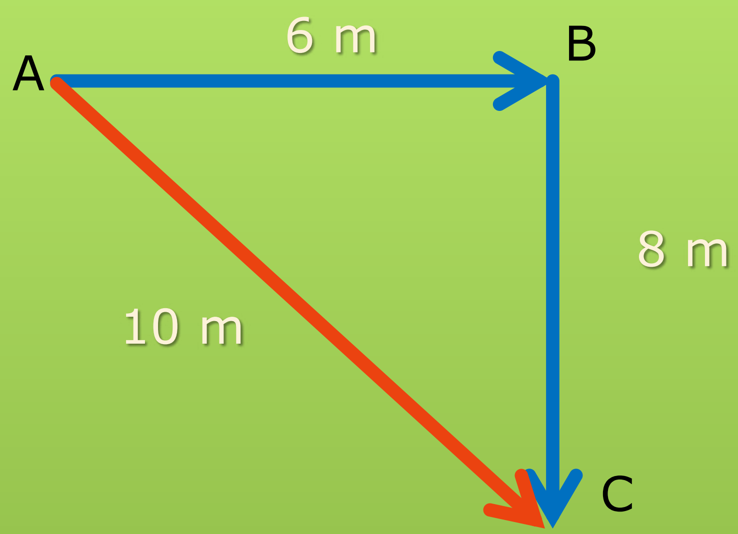
(d) use the slope of a displacement-time graph to find the velocity

(e) use the slope of a velocity-time graph to find the acceleration

(a) Definitions : displacement, speed, velocity and acceleration

1. **Distance** – **total path travelled of an object from the starting point to the finishing point.**
2. **Displacement** – **shortest distance travelled by an object along a straight line in a specific direction with respect to a reference point.**
3. **Speed** – **distance travelled per unit time**
4. **Velocity** – **the rate of change of displacement**
5. **Average speed** – **total distance travelled/ total time taken (include the time during which the object is at rest)**
6. **Average velocity** – **displacement / total time taken (include the time during which the object is at rest)**
7. **Acceleration** – **the rate of change of velocity**

Distance vs. displacement

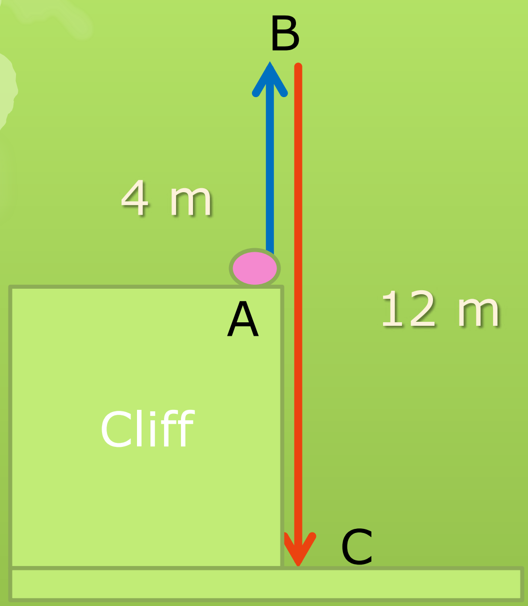


An object moves from A to B then to C.

The distance it travels is 6+8 = 14 m

Displacement from the starting point is 10 m.

Distance vs. displacement



A mass is thrown up vertically and reaches the base of the cliff, i.e. C. Taking upwards as positive,

The distance it travels is 4+12 = 16 m

Displacement from the starting point, i.e. A, is -8 m.

Instantaneous Speed vs. velocity

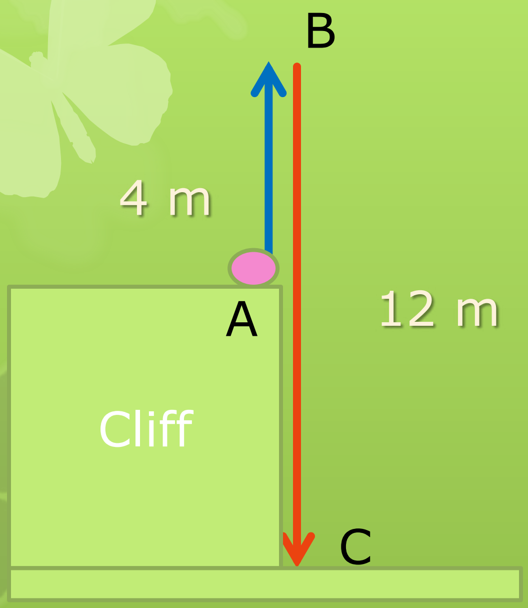
Speed = change in distance travelled/ elapsed time

Velocity = change in displacement/ elapsed time

The instantaneous speed is the magnitude of the instantaneous velocity.

Velocity may have a negative sign if the change in displacement is negative (i.e. object moves in the opposite direction) but its speed is always positive in values.

Average Speed vs. Average velocity



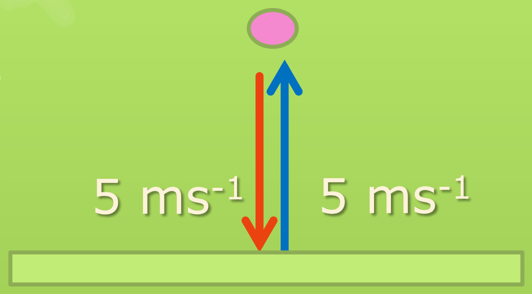
A mass is thrown up vertically and reaches the base of the cliff after 4 seconds. Determine for the whole motion,

The **average speed** is **16 /4 = 4 m s $^{-1}$**

The **average velocity** is (-)8 **/4 =** (-) **2 m s $^{-1}$**

-ve sign shows the mass is moving in the opposite direction (of course according to the chosen positive direction of the predefined frame).

Speed vs. velocity



A mass is dropped. It hits the ground with 5 m s^{-1} and rebounds with 5ms^{-1} . Taking downwards as positive,

The change in **speed** is **0 m s^{-1}**

The magnitude of change in **velocity** is **10 m s^{-1}**

Acceleration

A mass is moving in a circle with a constant speed.

After completing a revolution,

the average speed is not zero but

the average velocity is zero.

Its instantaneous speed is constant

Its instantaneous velocity keeps changing as its direction is changing.

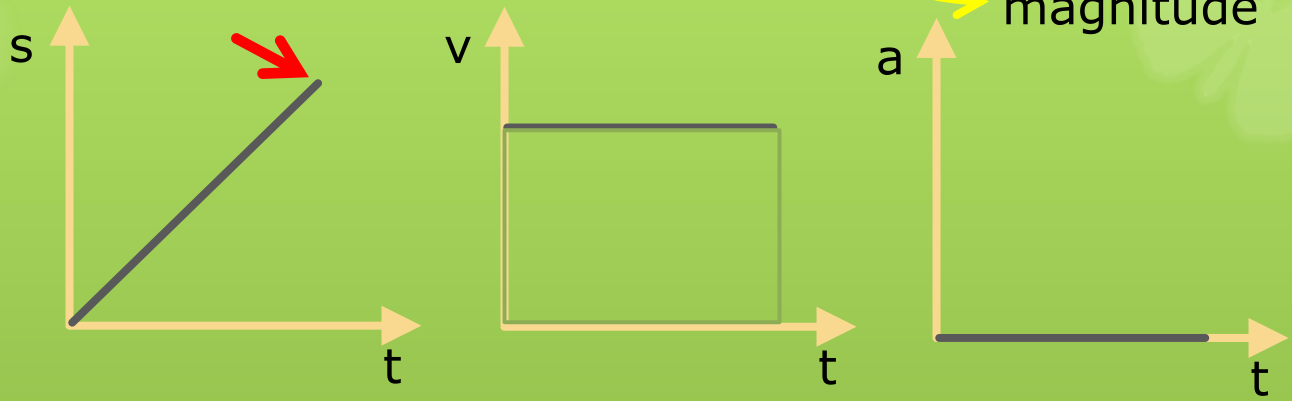
Thus, the mass is accelerating.

Acceleration = the change in velocity per unit time

(b) Graphical Methods

(1) A mass is moving with a constant velocity

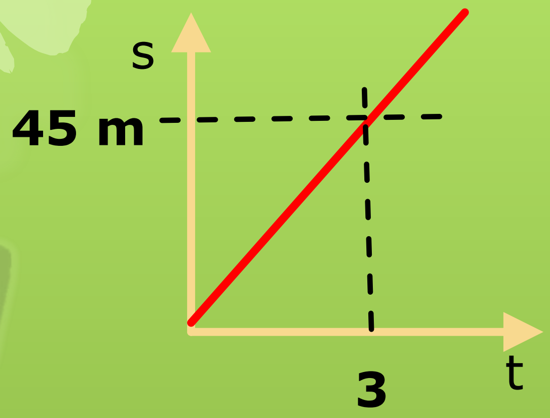
gradient magnitude

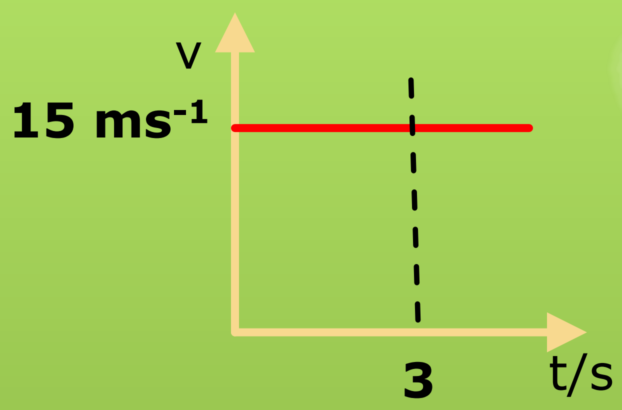


$V = \frac{ds}{dt}$ $a = \frac{dv}{dt}$

gradient magnitude

example





The displacement = 3 x 15 = 45 m from the starting point

Graphical Methods

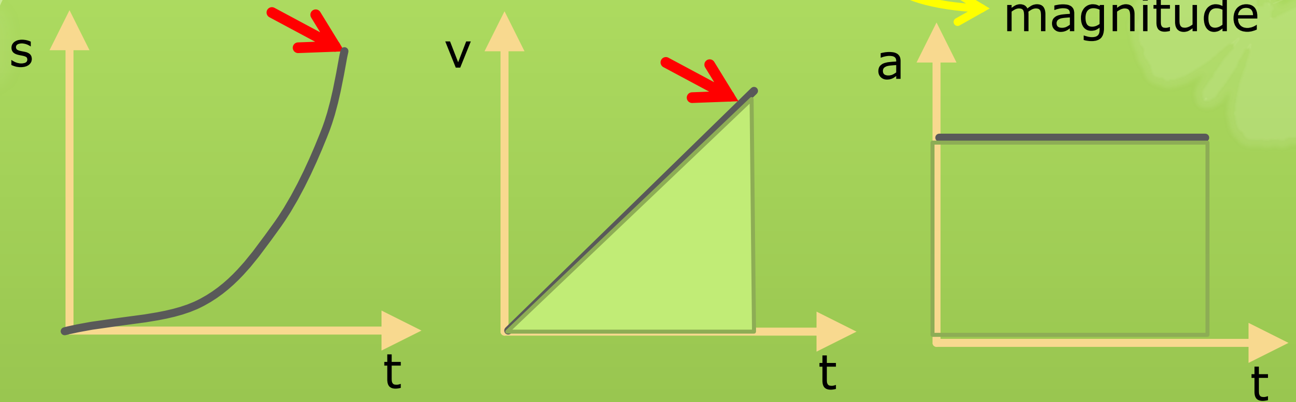
(2) A mass is moving with a uniformly increasing velocity

Change in vel

area

differentiate

magnitude



$V = \frac{ds}{dt}$

$a = \frac{dv}{dt}$

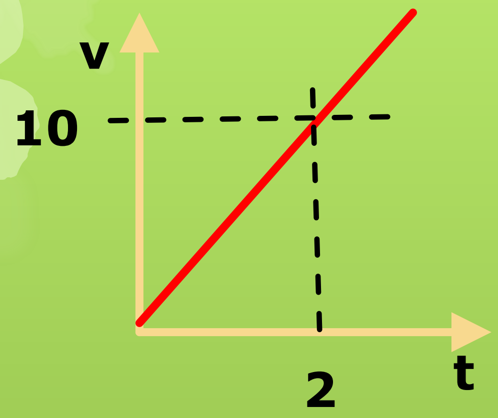
differentiate

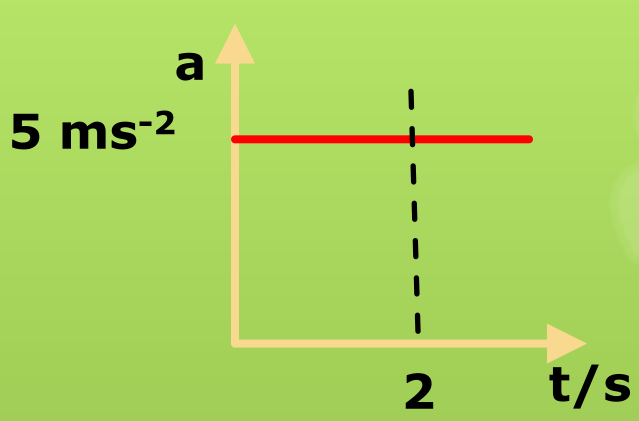
magnitude

Change in disp

area

example

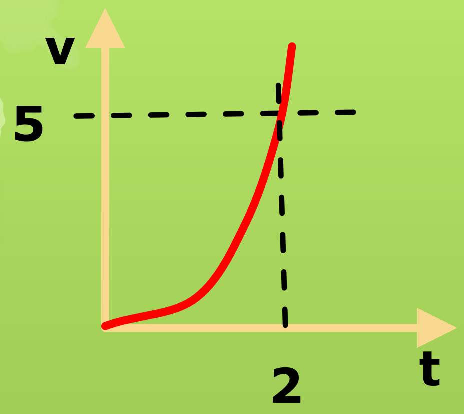


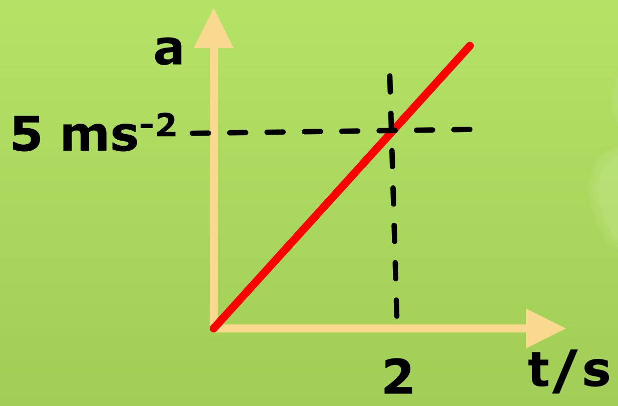


velocity increases uniformly and it is 10 ms^{-1} at t= 2 s

It moves at a constant acceleration of 5 ms^{-2}

example





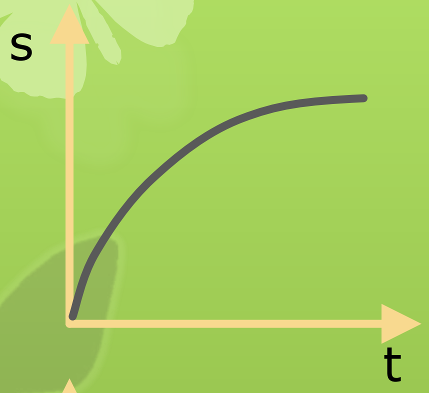
A mass accelerates from rest as shown above.

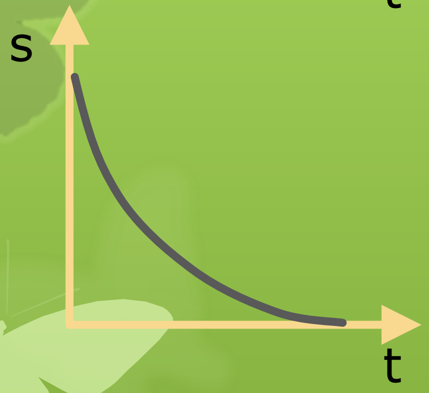
What is its velocity at t= 2 s?

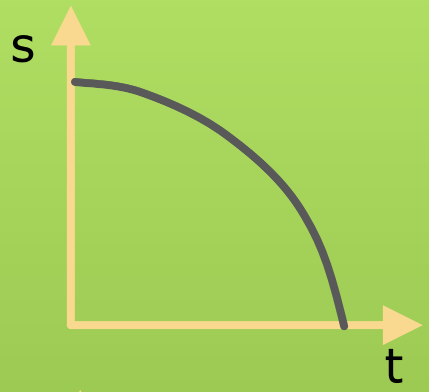
Area under the graph = ½ x2 x 5 = 5 ms^{-1}

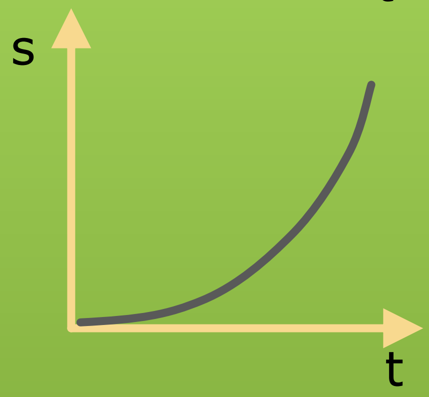
Graphical Methods

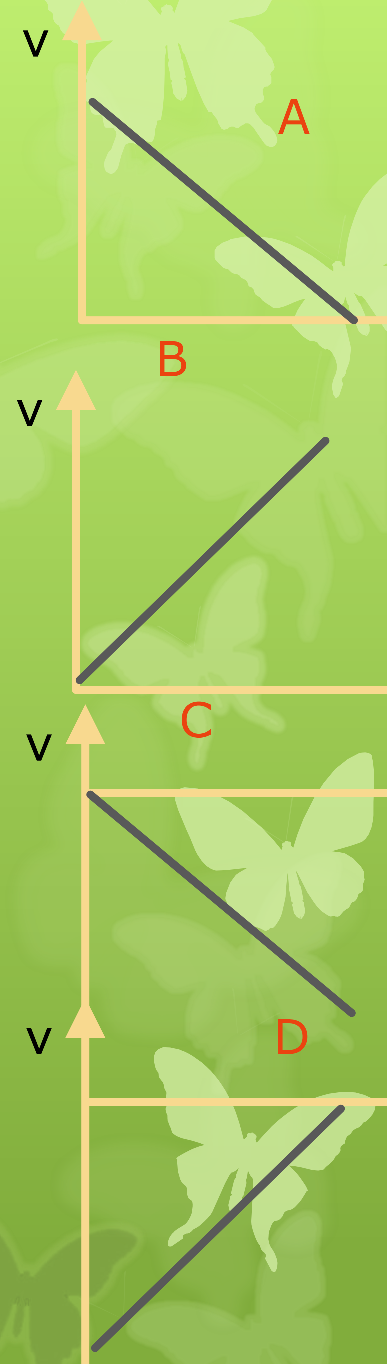
Match these graphs!



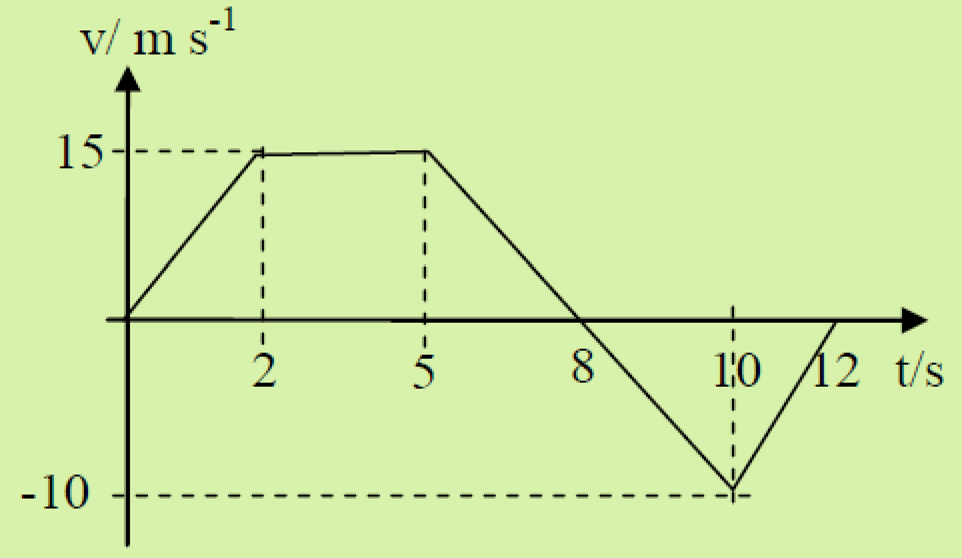








Problem solving (c) to (e)



Calculate:
(a) the acceleration for the motion in the first 2 seconds

(b) the deceleration at t = 7s

(c) total distance travelled for the whole journey

(d) the average speed

(e) Determine the shortest distance between the two towns.

(f) Sketch graphs of s-t & a-t for the whole motion

Linear motion of a car moving from town A to town B is shown in the v-t graph above.

The whole journey takes 12s.

solutions

Calculate:

(a) the acceleration for the motion in the first 2 seconds

$=15/2= 7.5\ ms^{-2}$

(b) the deceleration at t = 7s

$=15/3= 5\ ms^{-2}$

(c) total distance travelled for the whole journey

$=82.5 + 20 = 102.5$ m

(d) the average speed

$=102.5/12 = 8.54\ ms^{-1}$

(e) Determine the shortest distance between the two towns. $=82.5 - 20 = 62.5$ m

Learning outcomes

(f) derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line

(g) solve problems using equations that represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance

(h) recall that the weight of a body is equal to the product of its mass and the acceleration of free fall

(i) describe an experiment to determine the acceleration of free fall using a falling body

(j) describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance

(f) Derivation of kinematics equations

from the definition of acceleration $a = \frac{v-u}{t}$ (1)

from the definition of velocity $\bar{v} = \frac{s}{t}$ and hence $s = \frac{(v+u)t}{2}$ (2)

Show the remaining two kinematics equations:

$s = ut + \frac{1}{2}at^2$ (3)
$v^2 = u^2 + 2as$ (4)

Hints: substitute equation (1) into (2) to eliminate *t*
substitute equation (1) into (2) to eliminate *v*

These 4 equations are used for linear motion with constant acceleration, vertical up/down, horizontal or moving on a ramp.

kinematics equations

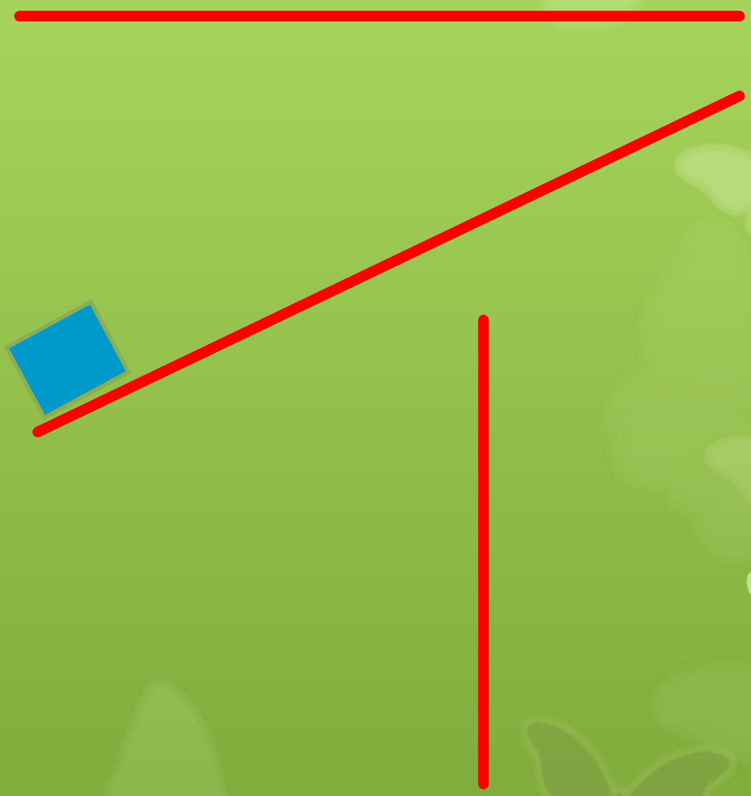
$$a = \frac{v-u}{t} \quad (1)$$

$$s = \frac{(v+u)t}{2} \quad (2)$$

$s = u\,t + \frac{1}{2}at^2$ (3)

$v^2 = u^2 + 2as$ (4)

Linear motion with constant acceleration



(g) Problem solving

Example 1

A car driver travelling in his car at a steady speed of 8 m s^{-1} sees a dog walking across the road 30 m ahead. The driver's reaction time is 0.2 s and the brakes are capable of producing deceleration of 1.2 m s^{-2}.

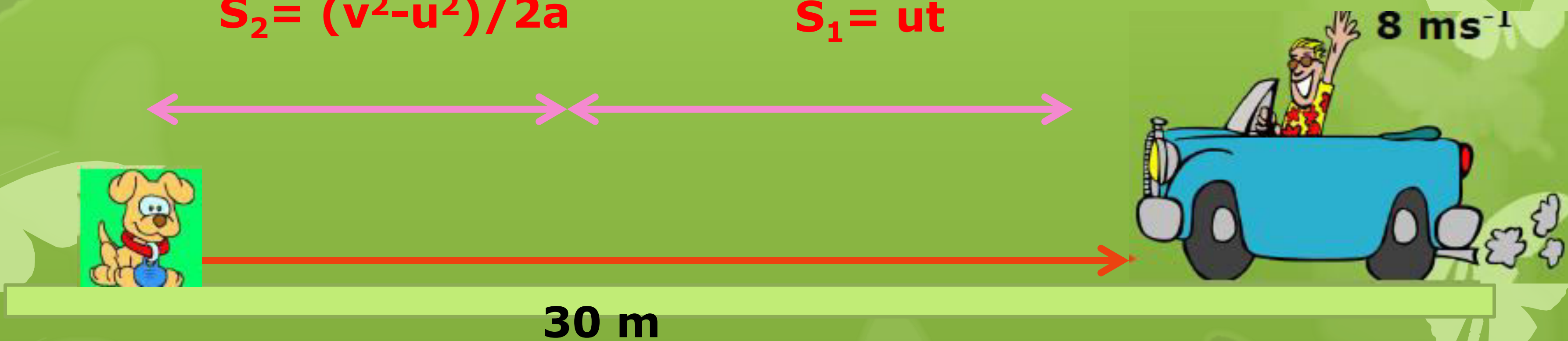
Calculate the distance from where the car stops to where the dog is crossing assuming the driver reacts and brakes as quickly as possible.

During deceleration, distance travelled

$S_2 = (v^2 - u^2)/2a$

During reaction time, the car is driven at a constant speed

$S_1 = ut$



Solution

During reaction time, the car is driven at a constant speed

S_1= ut = 8 x 0.2 = 1.6 m

During deceleration, distance travelled

$S_2 = (v^2-u^2)/2a = (0 – 8^2)/ (2x - 1.2)=26.6667$ m

Total distance travelled = 28.27 m
Thus, distance where the car stops from the dog =1.73 m

Example 2

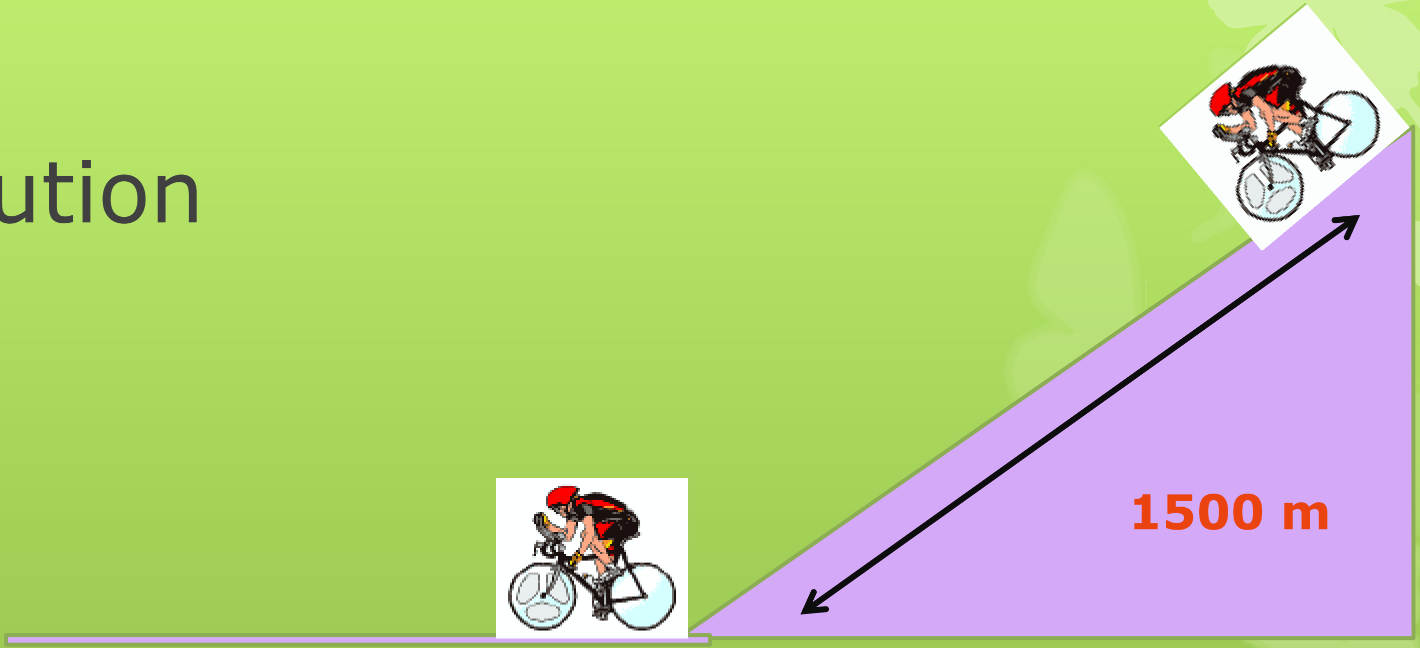
A cyclist accelerates steadily from rest on 1500 m downhill stretch of road, reaching a speed of 22.0 m s^{-1} before free wheeling to a halt 192 seconds after starting.

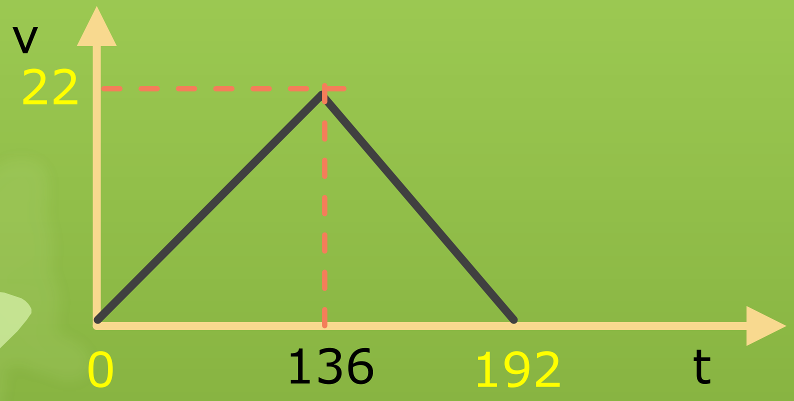
(a) Sketch a speed-time graph to represent the journey

(b) Calculate how long the cyclist takes to reach top speed

(c) Calculate the acceleration in each part of the journey and the average speed for the whole of the journey

Solution





Area of the 1st triangle=1500
Acceleration = gradient

[**ans**: 136, 0.162, 0.393,11]

(g) Acceleration of free fall, $g = 9.81 \text{ m s}^{-2}$

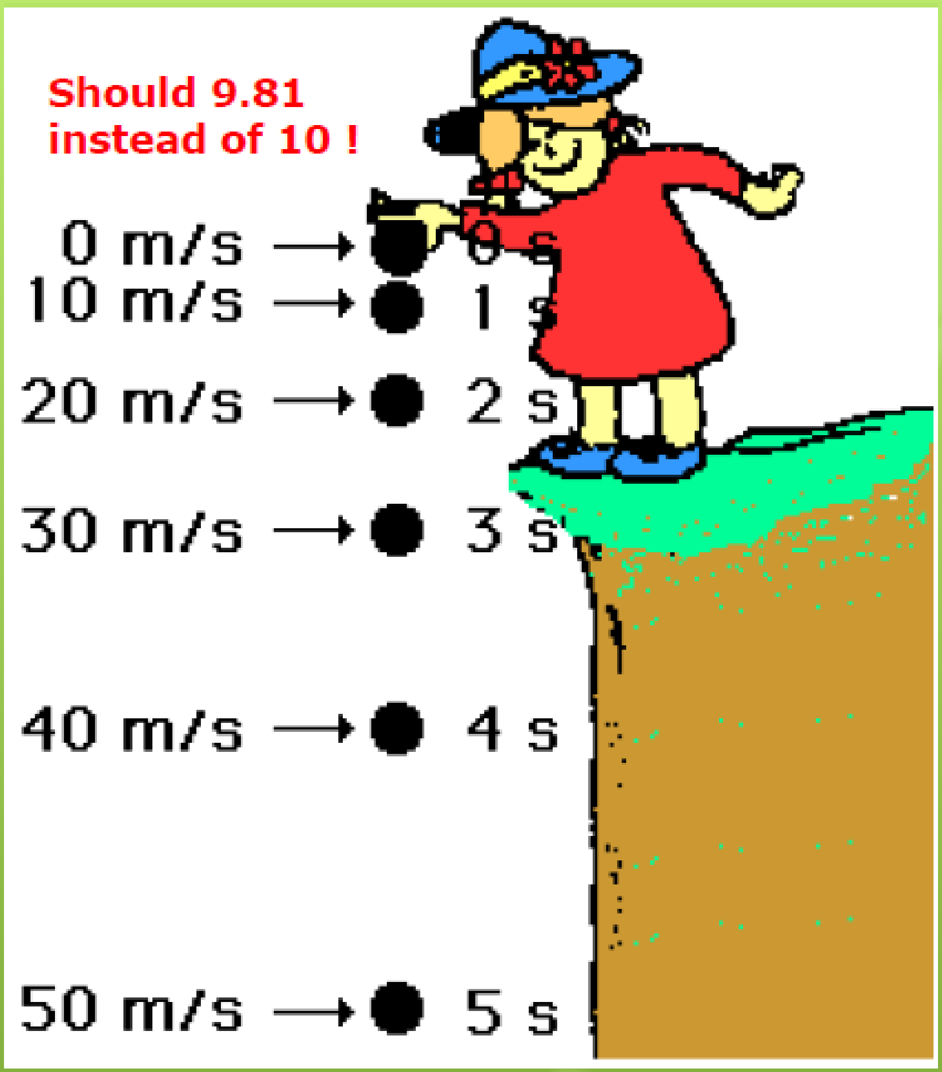
A free-falling object is an object which is falling under the sole influence of gravity i.e. it is moving and being acted upon only by the force of gravity

Two important characteristics:

- Free-falling objects do not encounter air resistance.
- All free-falling objects (on Earth) accelerate downwards at a constant rate of $g = 9.81 \text{ ms}^{-2}$

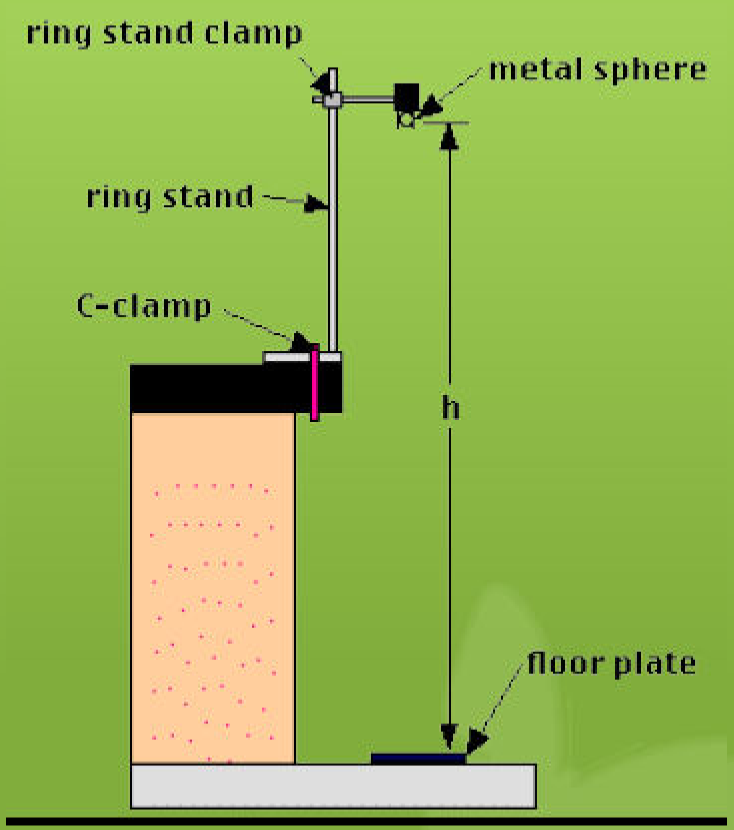
(h) W= mg

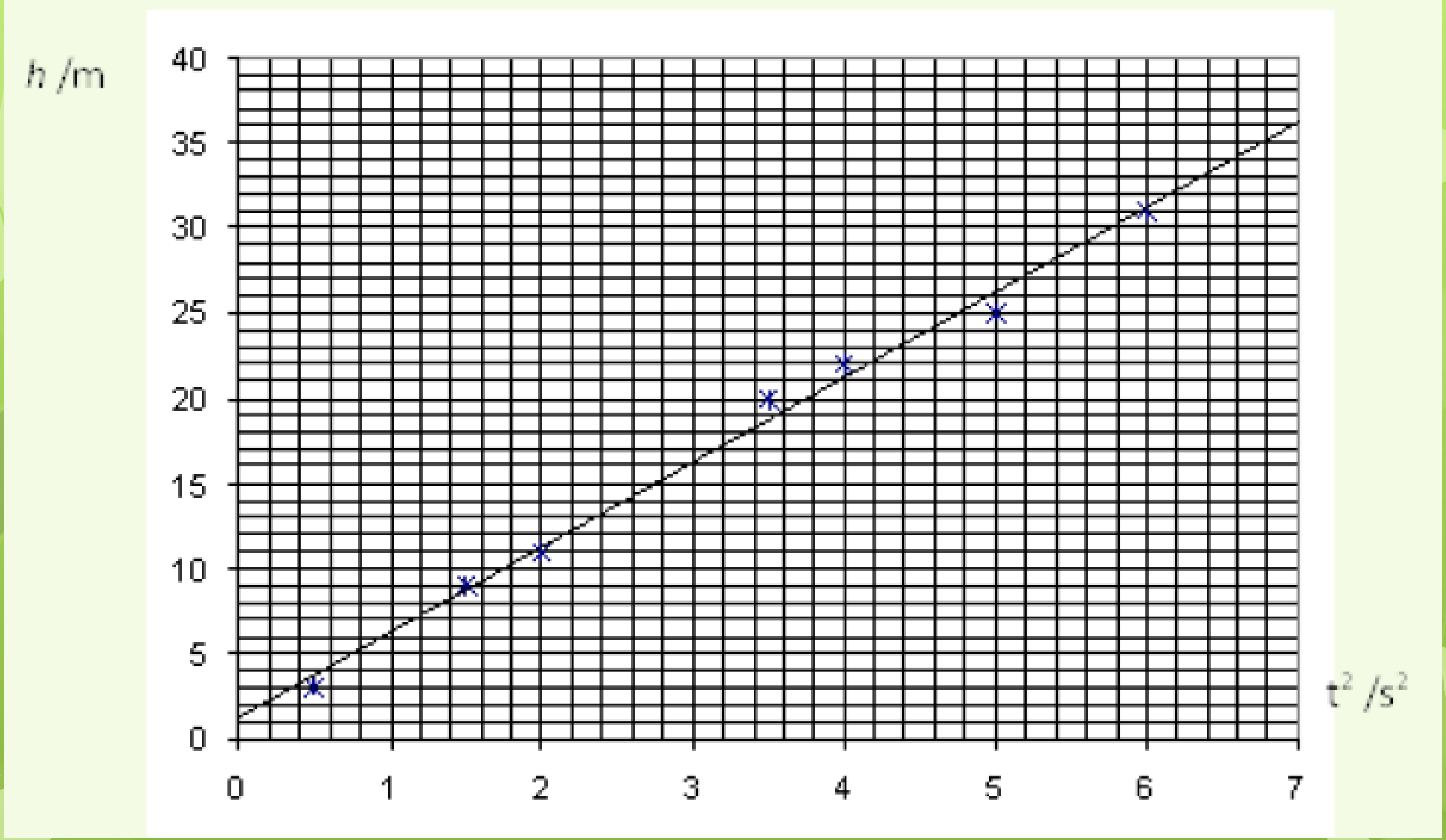
Hence, when a mass *m* is falling, its velocity increases by **9.81 m s^{-1}** in one second. The acceleration is caused by a gravitational force exerted by the Earth on the mass, which is known as its weight. The weight of the body is W= *mg*.


Should 9.81
instead of 10 !
0 m/s
0 s
10 m/s
1 s
20 m/s
2 s
30 m/s
3 s
40 m/s
4 s
50 m/s
5 s

(i) determine the acceleration of free fall, g using a falling body

The time *t* for a ball to fall from rest through a distance *h* is measured and the acceleration, *g* due to gravity is calculated by $h = \frac{1}{2} g t^2$. Measurements of *t* for several different values of *h* are carried out and a graph of *h* vs. t^2 is plotted as shown below.




h /m
40
35
30
25
20
15
10
5
0
0
1
2
3
4
5
6
7
t² /s²

Calculations

Releasing from rest, u=0

$h = \frac{1}{2} g t^2$

Plotting h vs. t^2

Gradient of the graph = ½ *g*

Choose two points from the graphs to calculate the gradient

g **= 2 x gradient = ...**

Problem solving-free falling

Example 1

A mass is thrown up vertically with velocity 6 m s^{-1} from a cliff top.

a) Determine the max height the mass can rise up from where it is thrown. [1.83]

b) How long does it take to reach the max height? [0.612]

c) How long does it take to reach the base of the cliff which is 10 m below the cliff top?

Why is $g = -9.81\ m\ s^{-2}$ throughout the motion?

+ve

Speed decreases when moving up

$0\ m\ s^{-1}$

$4\ m\ s^{-1}$

$5\ m\ s^{-1}$

$6\ m\ s^{-1}$

- ve

Speed increases when moving down

$-4\ m\ s^{-1}$

$-5\ m\ s^{-1}$

$-6\ m\ s^{-1}$

a =dv/dt
= (5-6)/dt
-ve

Accelerating in the positive direction

a =dv/dt
= (-6-(-5))/dt
-ve

decelerating in the negative direction

Taking upwards as positive >> velocity

The sign of acceleration depends on the change in velocity

If a mass decelerates upwards, a –ve

If a mass accelerates downwards, a -ve

Taking upwards as negative >> velocity

The sign of acceleration depends on the change in velocity

If a mass decelerates upwards, a +ve

If a mass accelerates downwards, a +ve

Solution

Taking upwards movement as positive,

u= 6 m s^{-1} ; g = - 9.81 m s^{-2}

a) Determine the max height the mass can rise up from where it is thrown. [1.83]

$v^2 = u^2 + 2as$, v=0
s = $(6)^2 /(2x9.81)$ = ……..

b) How long does it take to reach the max height? [0.612]

V=u+at ; v=0
t = 6/9.81 = ………….

c) How long does it take to reach the base of the cliff which is 10 m below the cliff top?

$s = u\,t + \frac{1}{2} at^2$
$-10 = 6t - \frac{1}{2}\, 9.81\, t^2$

For a falling motion,

A mass is dropped and moving downwards throughout.

Taking downwards as +ve, a= +g as the mass accelerates in the positive direction.

Displacement and velocity are also positive in values.

+ ve

4 m s $^{-1}$

5 m s $^{-1}$

6 m s $^{-1}$

Speed increases when moving down

example

A mass is dropped from a tall building of 10 metres high.

(i) Determine its speed right before it hits the ground.

U=0, a = +g, s = 10

$v^2 = u^2 + 2as$

$v = \sqrt{2 \times 9.81 \times (10)}$

= ……………………

(ii) Determine its distance from the ground when t = 0.5s

U=0, a = +g, t = 2

$s = u\,t + ½\,at^2$

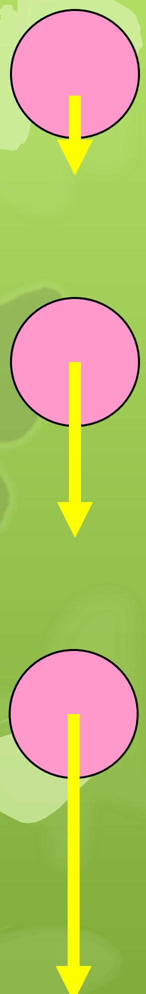
$= ½ \times 9.81 \times 0.5^2 =$……………..

Distance = 10 - …………. = …………………

(j) When a parachutist falls in air…

- Identify forces acting on him

Terminal velocity



Start falling

- Low speed

- Low air resistance F

- Speed increases

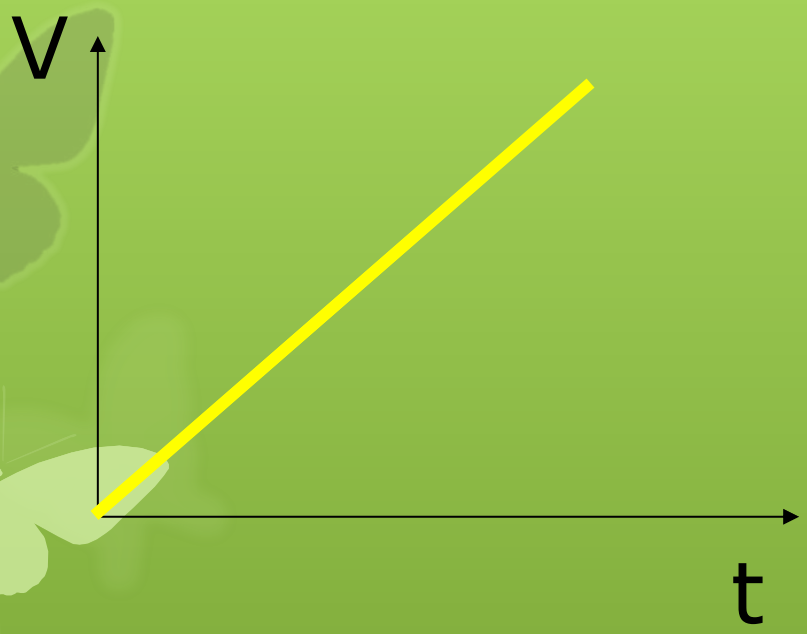
- W >> F
- air resistance increases

- Constant speed

- W = F + U
- a = 0

Falling …

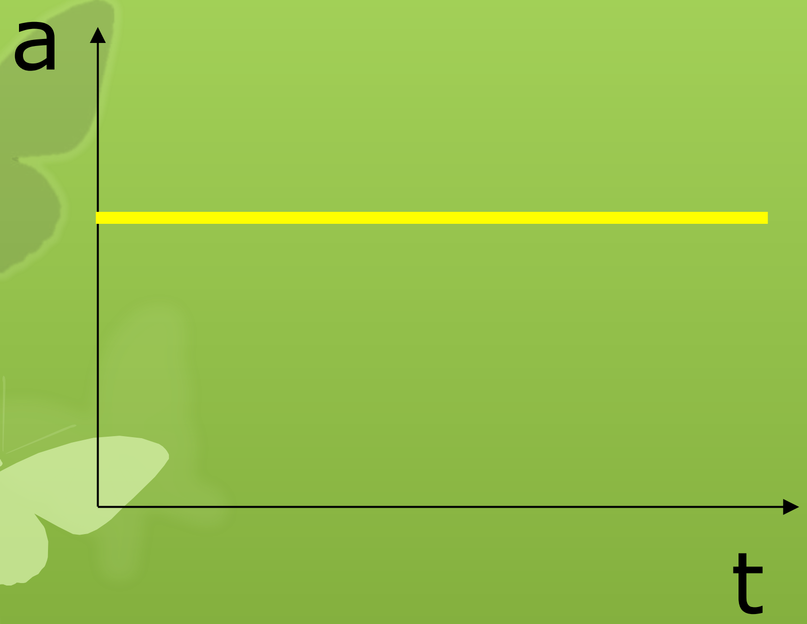
Without air resistance



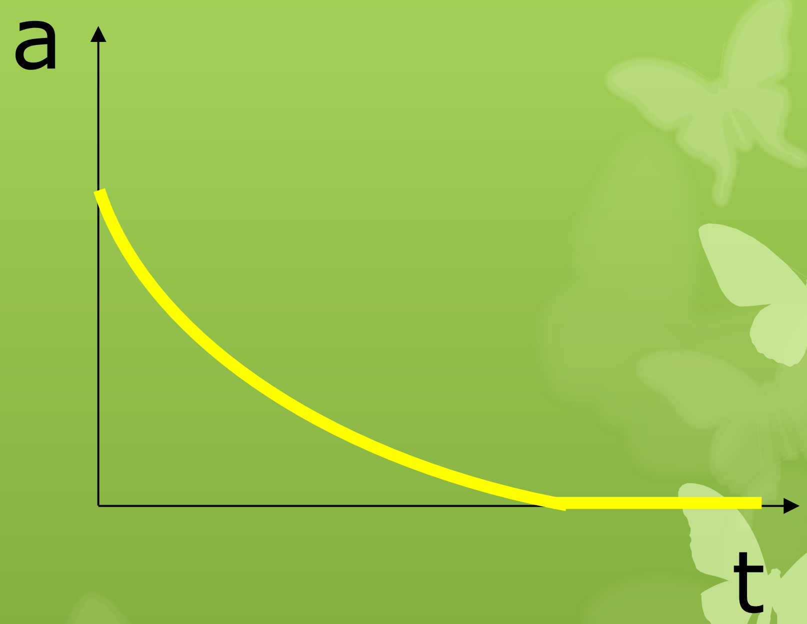
With air resistance

Falling …

Without air resistance

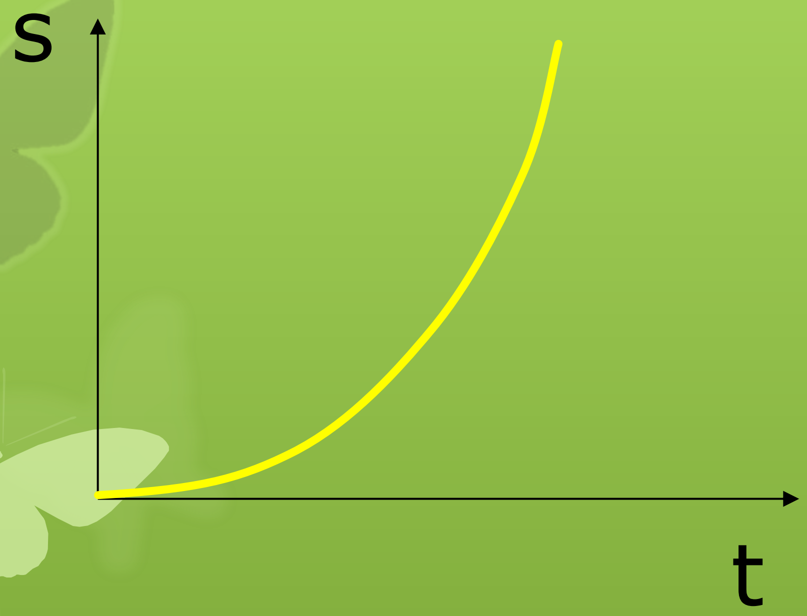


With air resistance

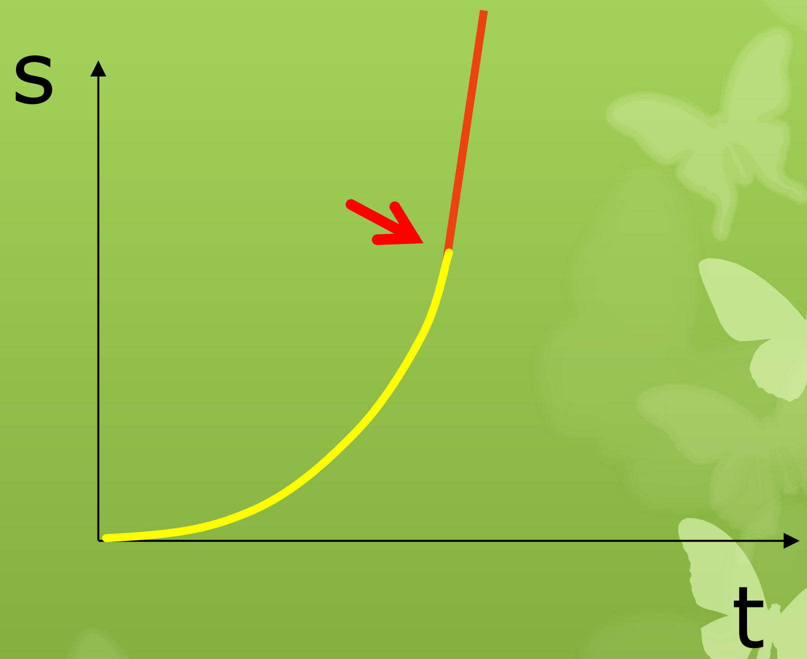


Falling …

Without air resistance



With air resistance

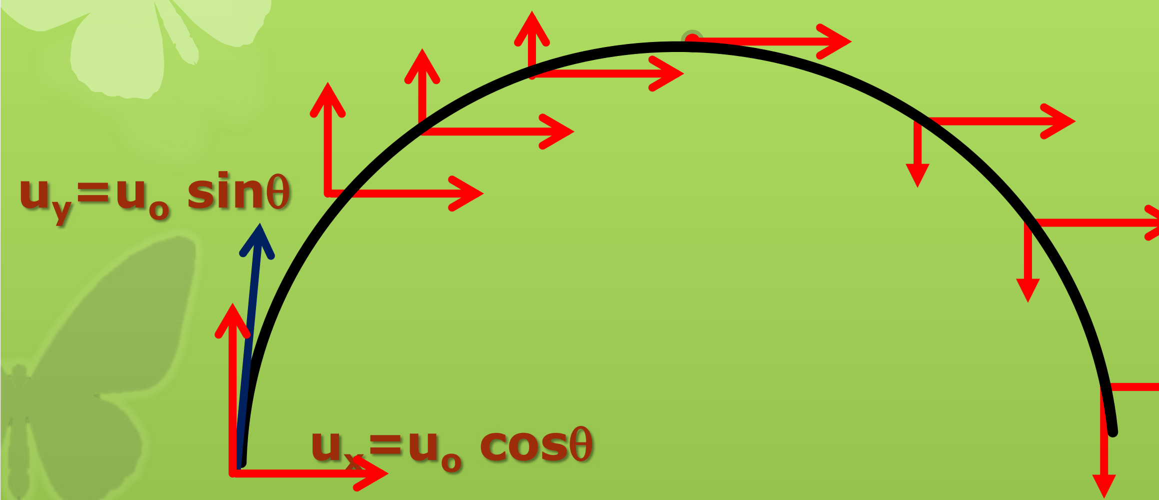


Learning outcomes

(k) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction (projectile motion)

Projectile motion

A mass is thrown up with u_o m s^{-1} at θ to the horizontal.

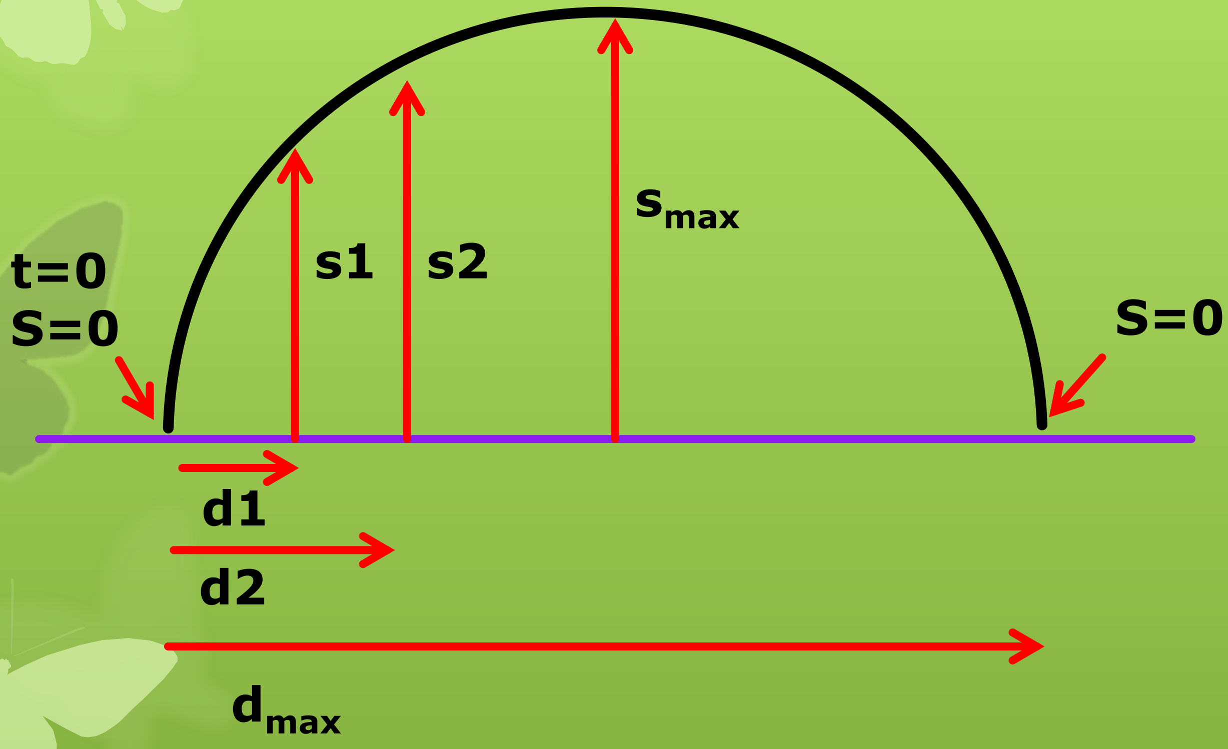


a. **Resolve the initial velocity to get two perpendicular components.**

b. **Vertical speed decreases till zero then increasing when moving down.**

c. **Horizontal velocity remains constant as there is no air resistance and hence no net horizontal force acting on it. The only force i.e. gravitational force acts perpendicularly on it and hence it does no work to change the velocity.**

How are displacements measured?



Vertical movement

$v = u + at$ (1)

$s = t\,(u+v)/2$ (2)

$s = u\,t + \frac{1}{2}at^2$ (3)

$v^2 = u^2 + 2as$ (4)

where $a = -g$ because upward movement is taken as positive and initial vertical velocity, $u = u_o \sin\theta$

How high?

$v^2 = u^2 + 2as$

$s =$ ....

Where $v = 0$, $a = -g$, $u = u_o \sin\theta$

How far?

$u_x = d_x/t$

$d_x =$ ....

Where t = total time, $a = -g$, $u = u_o \cos\theta$

Horizontal movement

$u_x = u_o \cos\theta = d_x/t$

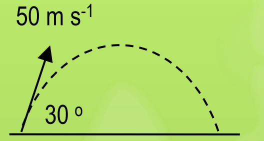
How long it takes to return to the starting level?

$s = u\,t + \frac{1}{2}at^2$

$t =$ ....

Where $s = 0$, $a = -g$, $u = u_o \sin\theta$

Example 1



A projectile is released upwards at an angle of 30^{o} to the horizontal level with an initial velocity of 50 m s^{-1}.

a) Determine the max height that the projectile achieves before falling down [31.86m]

b) How long does it take to reach the ground level? [5.1s]

c) How far can it travel from its starting point? [220.84m]

Solution

a) Determine the max height that the projectile achieves before falling down [31.86m]

$v^2 = u^2 + 2as$
$s = (50\sin 30)^2 / (2 \times 9.81) =$ ……..

b) How long does it take to reach the ground level? [5.1s]

$s = u\,t + \frac{1}{2}\,at^2$
$0 = t\,(u - \frac{1}{2}\,gt)$
$t=0;\ t=2u/g = (2 \times 50\sin 30)/9.81 =$ …………………

c) How far can it travel from its starting point? [220.84m]

$d=ut$
$d = 50\cos 30 \times 5.1 =$ ………………

Example 2

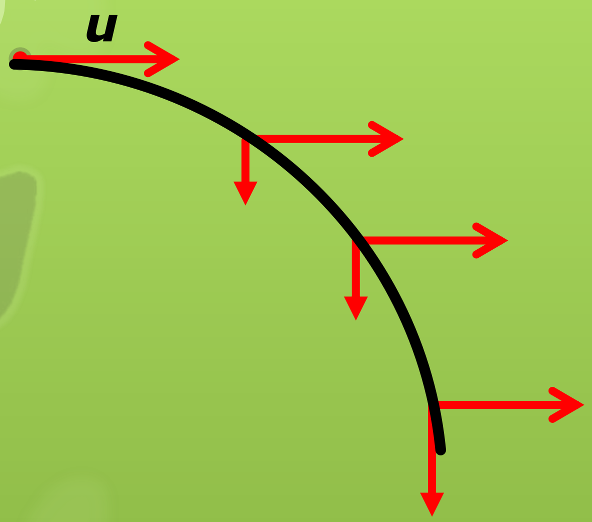
A projectile of mass 1 kg is thrown up in vacuum with 10 m s^{-1} at 30^{o} to the horizontal.

Which of the following statements are correct?

1. The horizontal acceleration is zero.
2. The vertical displacement increases uniformly when rising.
3. The resultant velocity at the maximum height is the horizontal velocity.
4. The KE of the projectile is zero at the max height.
5. The change in momentum is 10 kg m s^{-1}

Projectile motion (II)

A mass is thrown up with a horizontal velocity *u* m s^{-1}.



a. **An initial horizontal velocity has no vertical component, $u_y=0$**

b. **Vertical speed increases**

c. **Horizontal velocity remains constant as there is no air resistance and hence no net horizontal force acting on it. The only force i.e. gravitational force acts perpendicularly on it and hence it does no work to change the velocity.**

d. **Taking downwards as positive, s,v, and a are all positive.**

Relation between vertical displacement & horizontal displacement

Vertical movement

$s = u\,t + \frac{1}{2}\,g\,t^2$; $u=0$

Horizontal movement

$u = d\,/t$

Combining two equations,

$s = \frac{1}{2}\,g\,(d/u)^{2}$

Since g and u are constants,

$$s \propto d^{2}$$

d increases at a constant rate while

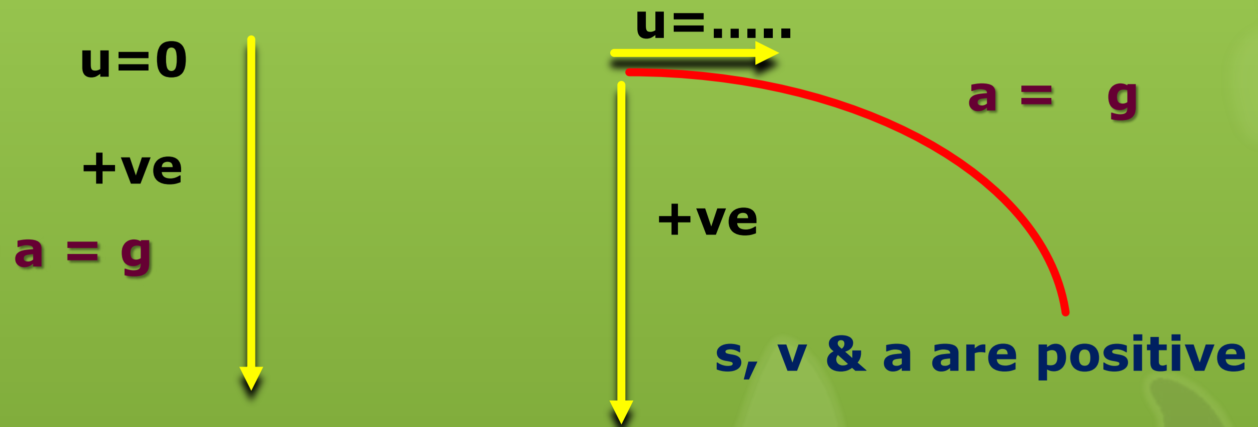
S increases at an increasing rate.

When d is doubled, s is quadruple.

free-falling motions

- **Throw up vertically/ at an angle to the horizontal then moving downwards**

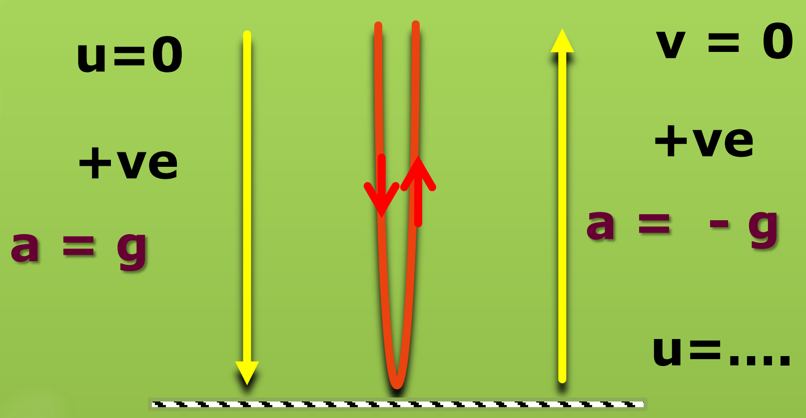
- **Drop down or moves horizontally sideway**



Drop and Bouncing

A ball is dropped and it bounces few times upon hitting the ground.

Separate downwards and upwards movements.



example

A ball is dropped at a height of 2 metres and upon hitting the ground, it bounces with 4 ms^{-1}.

Determine

(i) the speed right before it hits the ground

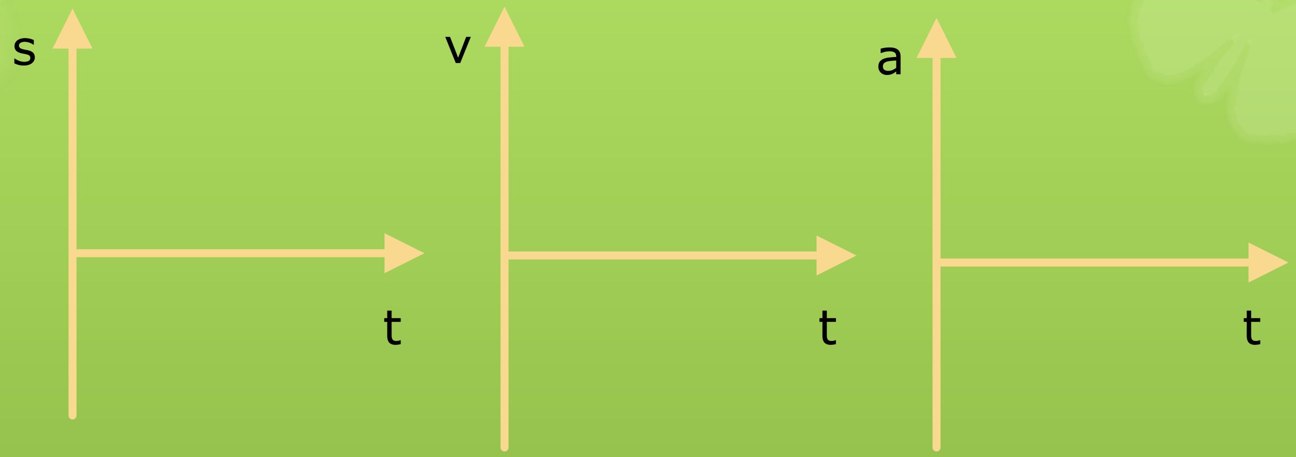
(ii)the max height after the 1st bounce

(iii)the total time taken from where it is dropped to the max height after the first bounce.

Graphical representation

Situation 1:

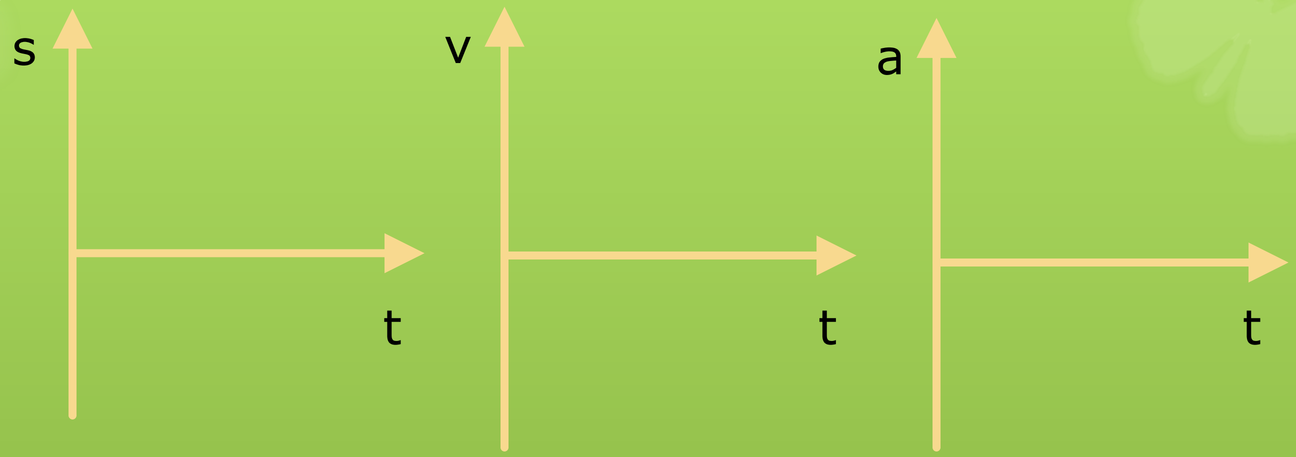
A body is thrown vertically upwards, reaching a max height then falling downwards



Graphical representation

Situation 2:

A body is dropped, upon hitting the ground it then bounces upwards.



Any Questions