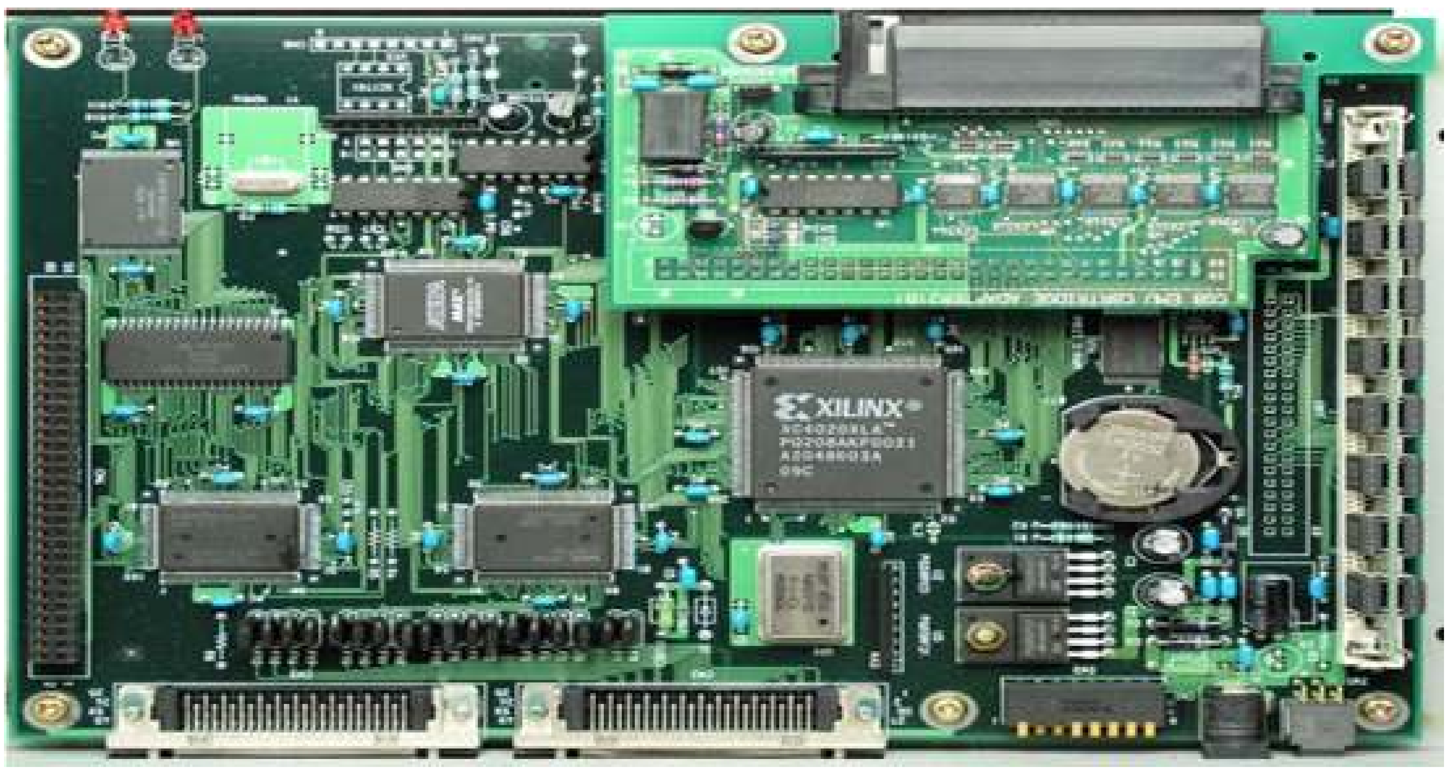
D.C. Circuits

Content

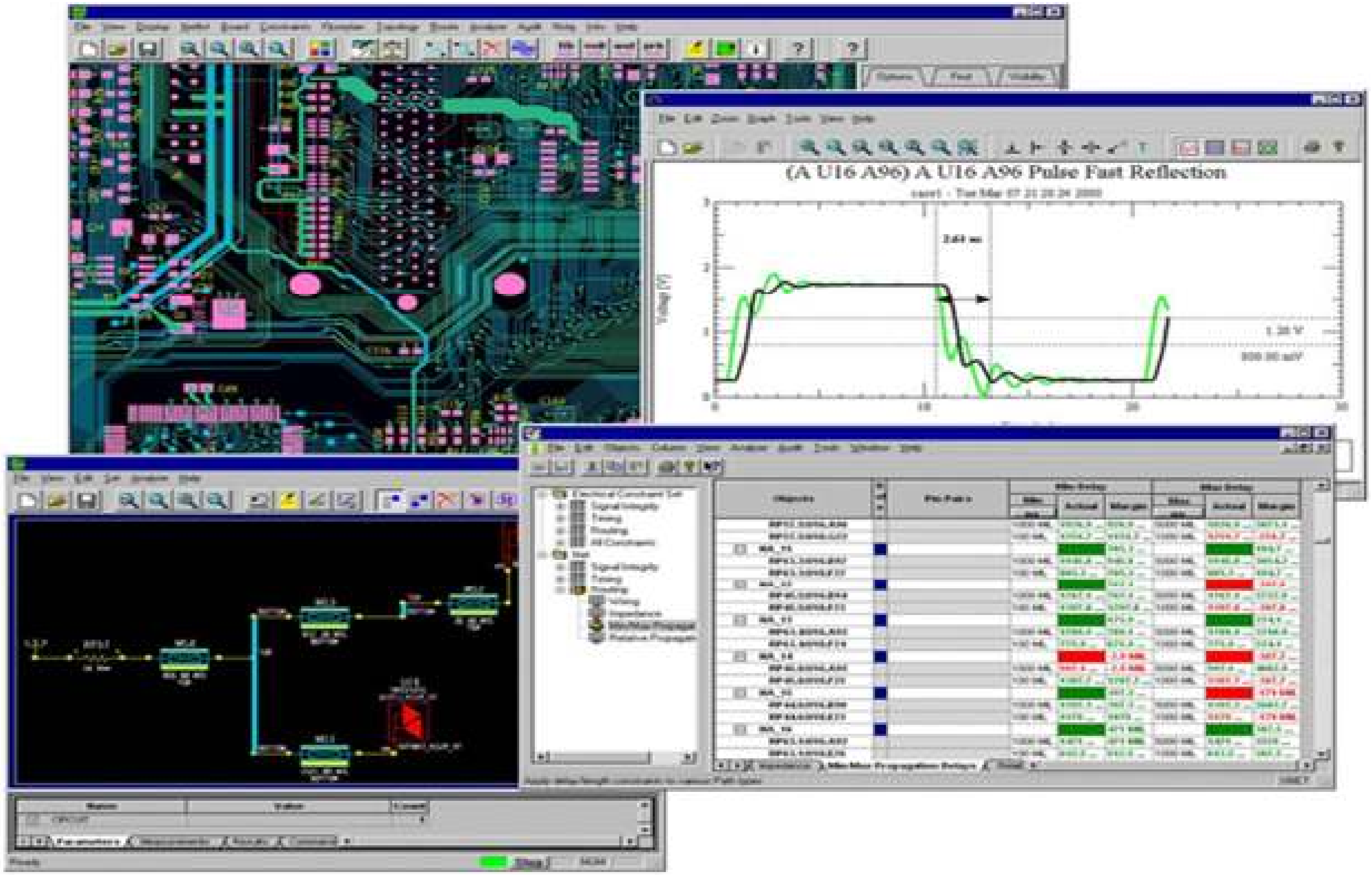
Kirchhoff’s Laws

- Over the years, electrical circuits have become increasing complex, with more and more components combining to achieve very precise results.
- At one time, circuit designers would start with a simple circuit and gradually modify it until the desired result was achieved.
- This is impossible today when circuits include many hundreds or thousands or components!



Kirchhoff’s Laws

- Instead, electronic engineers rely upon computer-based design software which can work out the effect of any combination of components.



Kirchhoff's Laws

- This is only possible because computers can be programmed after knowing the equations which describe how current and voltage behave in a circuit. Some of these basic yet very important equations / laws are the Ohm's Law & the Kirchhoff's two laws.
- There is a powerful set of relations called Kirchhoff's laws which enable one to analyze arbitrary/random circuits .
- There are 2 such laws:

1st law – Current Law

2nd law – Voltage Law

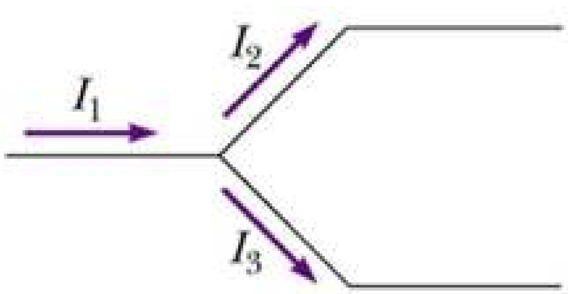
Kirchhoff's 1st law

Current Law

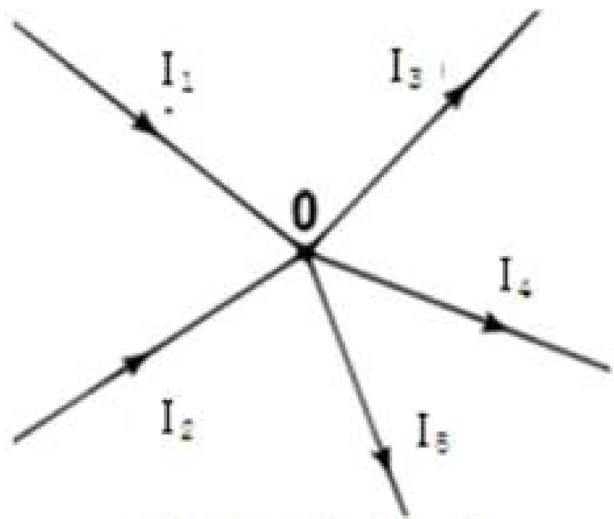
- For a given junction in a circuit, the **SUM** of the currents entering the junction is equal to the sum of currents leaving the junction.

$$\boxed{\sum I_{enter} = \sum I_{leave}}$$

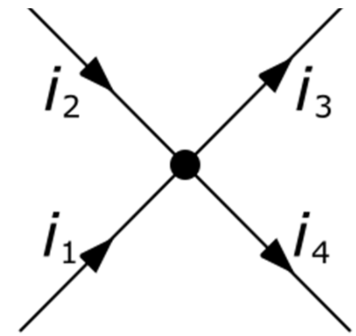
- This law is based on the **law of conservation of charge.**



$I_1 = I_2 + I_3$



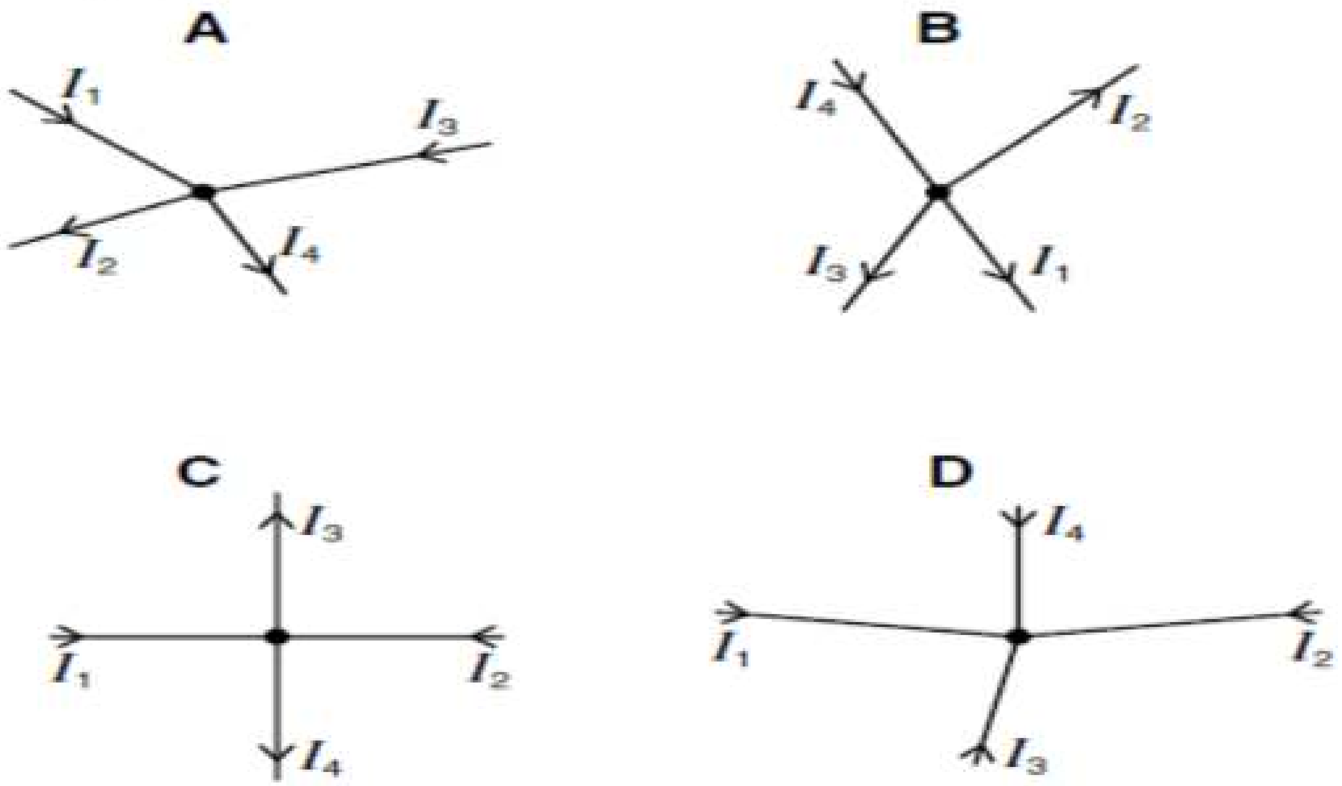
$I_1 + I_2 = I_3 + I_4 + I_5$



$i_1 + i_2 = i_3 + i_4$

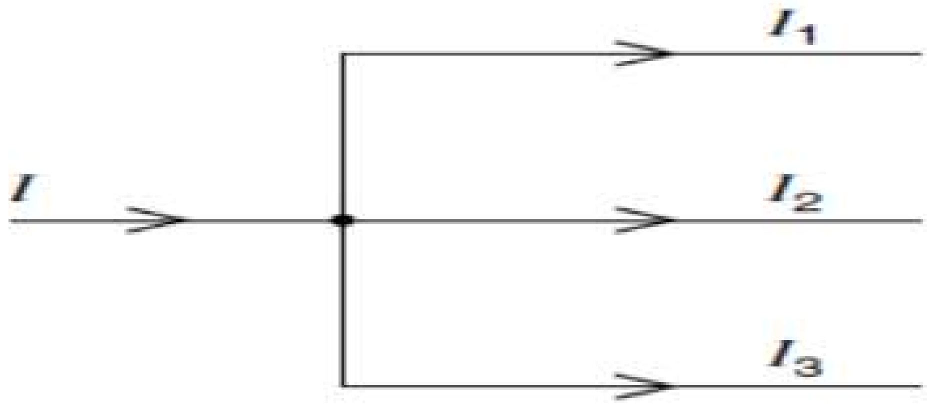
Example 1

- The diagrams show connected wires which carry currents I_1, I_2, I_3 and I_4. The currents are related by the equation $I_1 + I_2 = I_3 + I_4$. To which diagram does this equation apply?



Example 2

- At a circuit junction, a current I divides into currents I_1, I_2 and I_3.



- These currents are related by the equation $I = I_1 + I_2 + I_3$. Which law does this statement illustrate and on what principle is the law based?

  A Kirchhoff’s first law based on conservation of charge

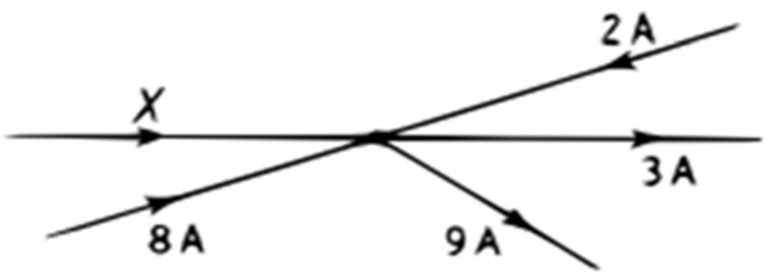
  B Kirchhoff’s first law based on conservation of energy

  C Kirchhoff’s second law based on conservation of charge

  D Kirchhoff’s second law based on conservation of energy

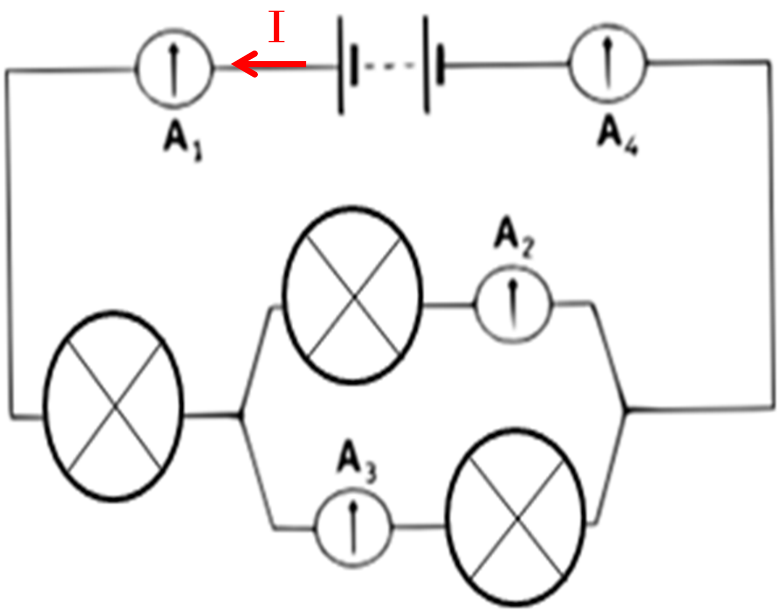
Example 3

- Find the value of X.

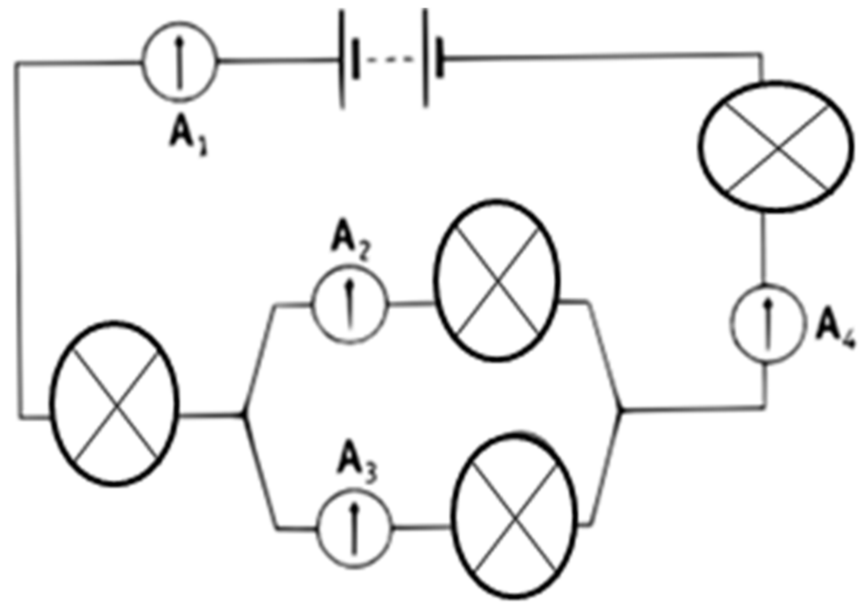


Example 4

- The bulbs are identical. What are the values of A1, A3 & A4 in term of I.



- The bulbs are different. A2 reads 7 A and A4 reads 20 A. Calculate A1 & A3.



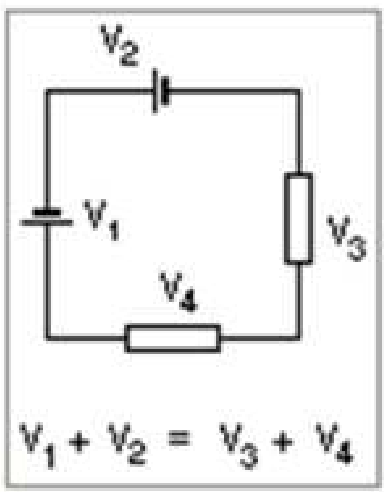
Kirchhoff's 2^{nd} law

Voltage Law

- Around any closed loop in a circuit, the algebraic **SUM** of the e.m.f.s is equal to the algebraic sum of the p.d.s across all elements around the loop.

$$\sum \text{e.m.f.s} = \sum \text{p.d.s}$$

- This law is based on the **law of conservation of energy.**



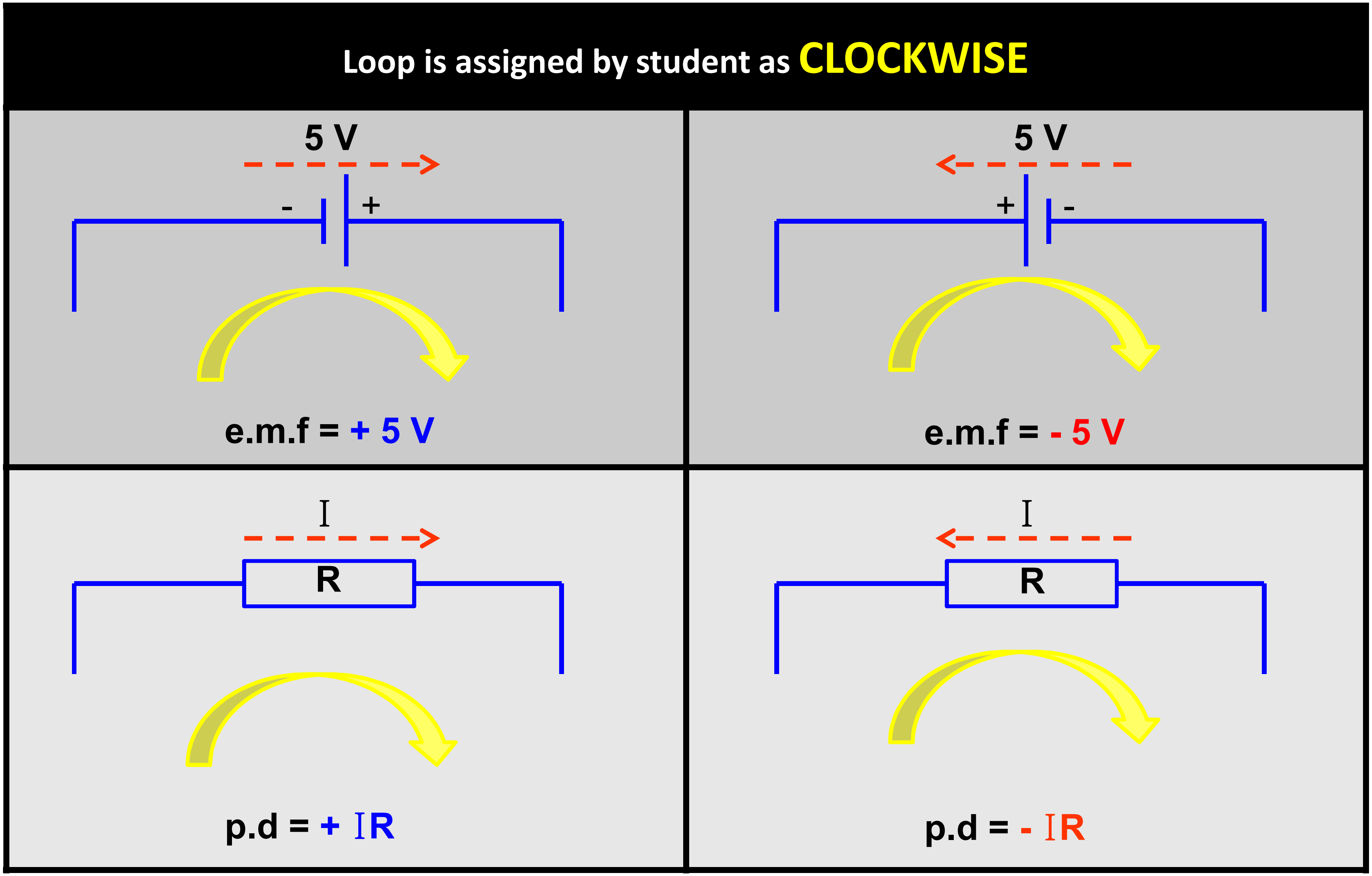
However, it is **VERY** important to correctly determine the sign convention before writing down the value of e.m.f.s / p.d.s. (Should it be positive or negative?)

The sign can be determined by adopting a method through the understanding of the relationship between the:

1.) **direction of the loop assigned** with its **e.m.f.s**

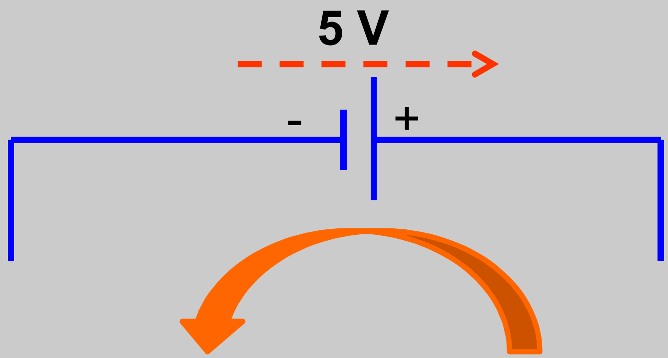
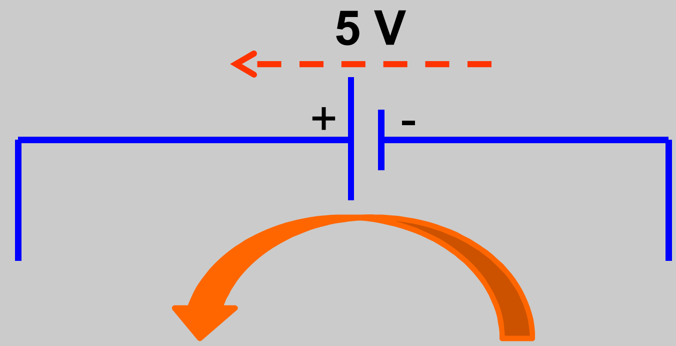
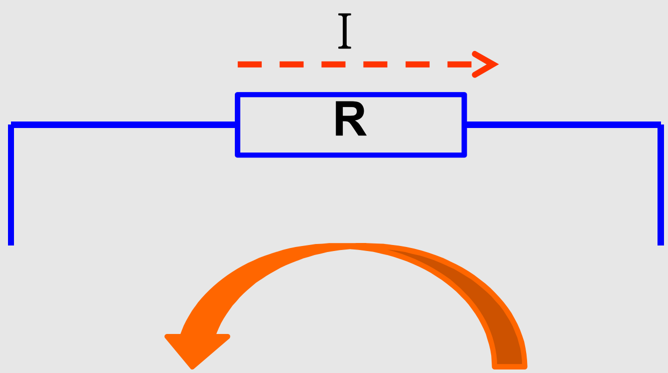
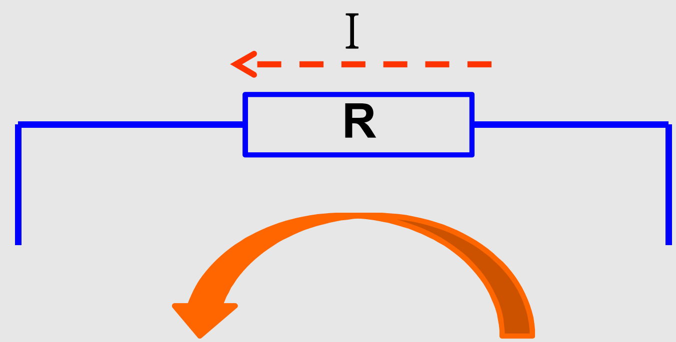
2.) **direction of the loop assigned** with its **current flows.**

Kirchhoff's 2nd law



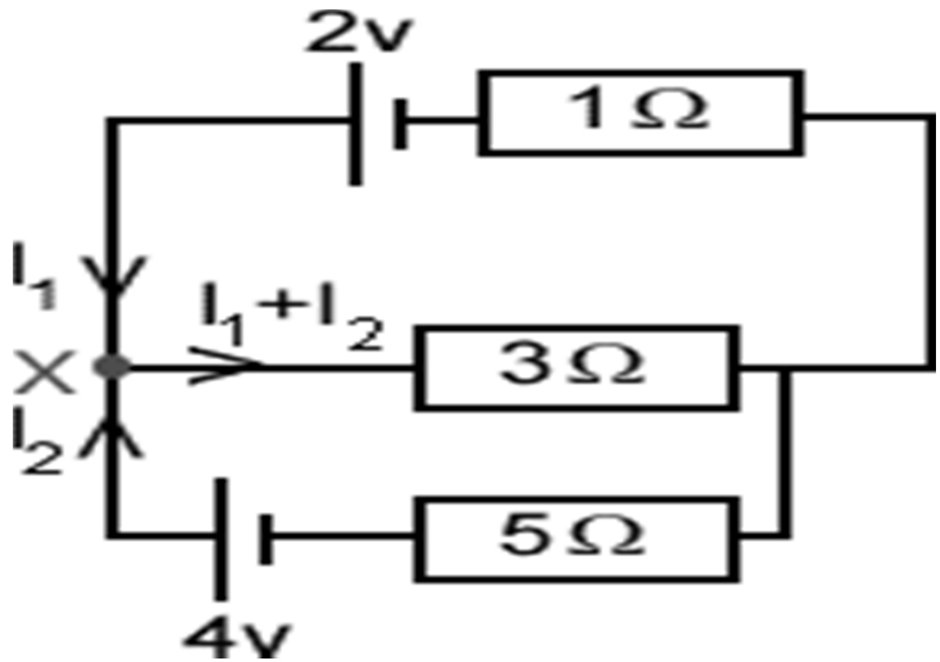
Kirchhoff's 2^{nd} law

Loop is assigned by student as ANTI-CLOCKWISE

 e.m.f = $-\ 5\ V$	 e.m.f = $+\ 5\ V$
 p.d = $-\ IR$	 p.d = $+\ IR$

Example 5

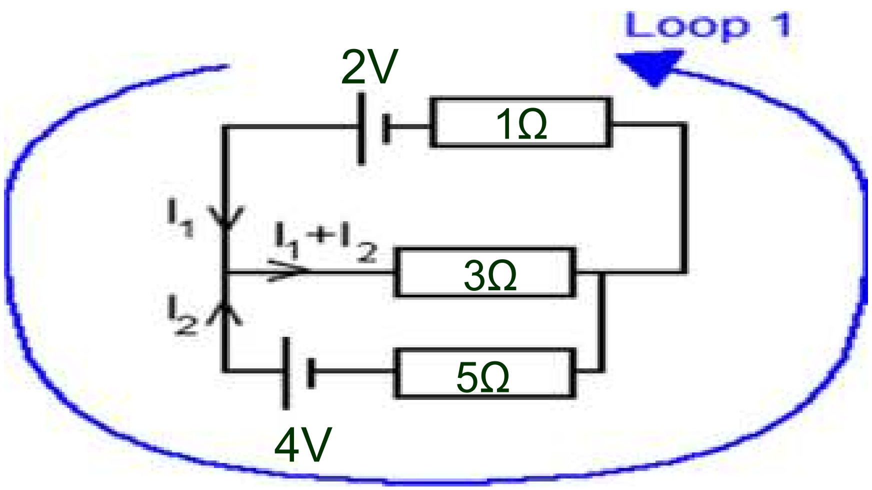
- Calculate the value of I_1 and I_2



Example 5

ASSIGN FIRST LOOP.

- Draw a loop on the diagram showing a possible route around circuit. Label it loop 1.



For loop 1:

Total e.m.f.s = (2 - 4) = -2

Total p.d.s = $-5I_2 + I_1$

Kirchhoff's 2nd law stated that:

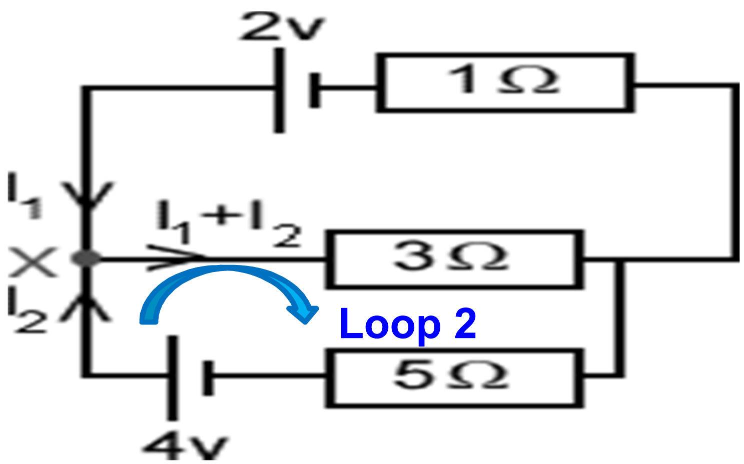
Total e.m.f.s = total p.d.s

$$-2 = -5I_2 + I_1 \quad \text{------} \ (1)$$

Example 5

ASSIGN SECOND LOOP.

- Draw a loop on the diagram showing a possible route around circuit. Label it loop 2.



Solve the 2 simultaneous equations to get

$-2 \quad = \quad -5I_2 + I_1$ ------ (1)

$4 \quad = \quad 3I_1 + 8I_2$ ------ (2)

$I_2 = + 10/23$ A

$I_1 = + 4/23$ A

For loop 1:

Total e.m.f.s = 4

Total p.d.s $= 3(I_1 + I_2) + 5I_2$

$= 3I_1 + 8I_2$

Kirchhoff's 2nd law stated that:

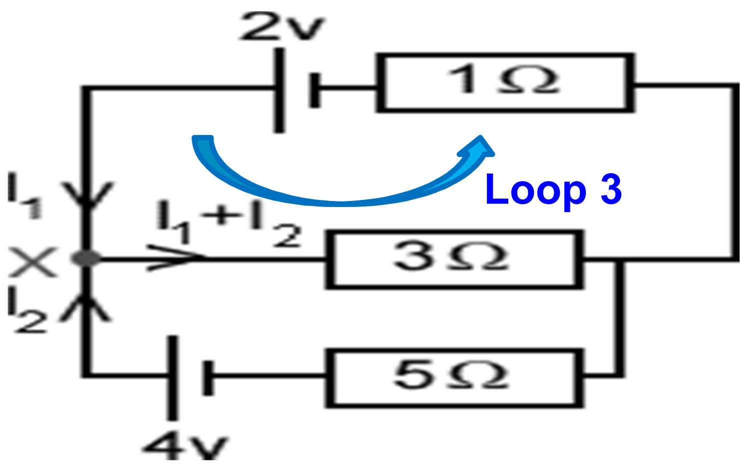
Total e.m.f.s = total p.d.s

$4 \quad = \quad 3I_1 + 8I_2$ ------ (2)

Example 5

TRY USING THE THIRD LOOP.

- One of the simultaneous equation must involve Loop 3.



Solve the 2 simultaneous equations to get

I_1 = ________ A

I_2 = ________ A

For loop 1:

Total e.m.f.s = ___

Total p.d.s = ______________

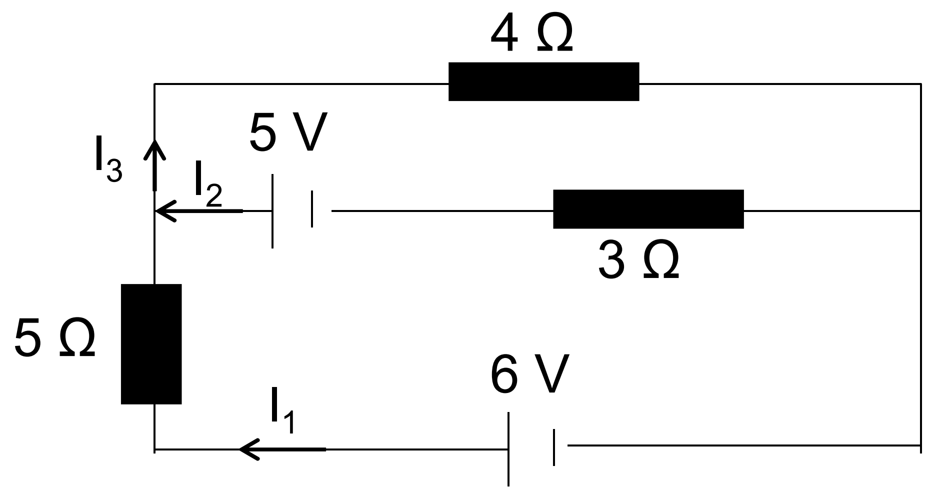
= ______________

Kirchhoff's 2nd law stated that:

Total e.m.f.s = total p.d.s

_____ = _________ ------ (3)

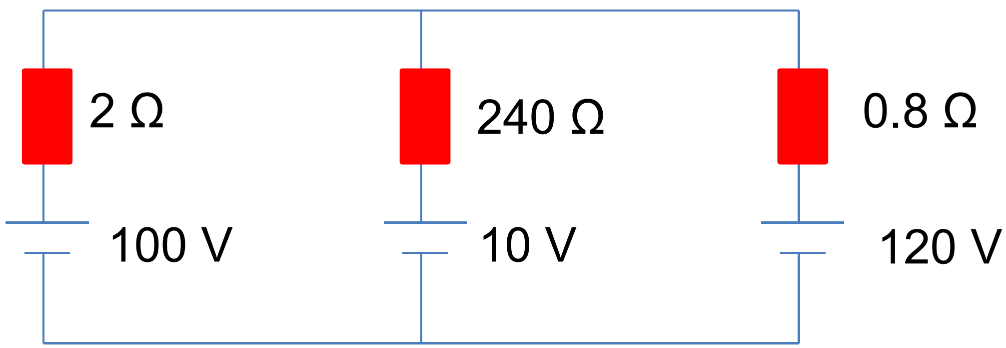
Example 6



- Determine I_1, I_2 and I_3 in the circuit shown.

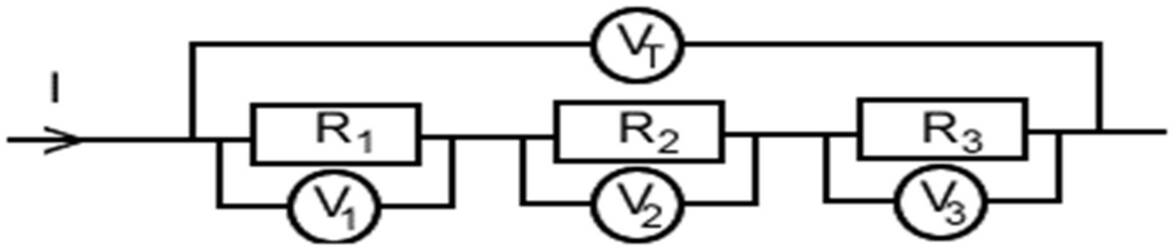
Example 7

- Determine the current flowing through the 240 Ω resistor.



Resistors (in series)

- When resistors are connected in series as shown, the same current flows through all the resistors.



$$\mathbf{I = I_1 = I_2 = I_3}$$

- This is consistent with the principle of conservation of charge.
- Charge flows into R_1 must flow out of it and flow into R_2 and so on.

Resistors (in series)

- Potential difference across R_1, R_2 and R_3:

  $V_1 = IR_1$ $V_2 = IR_2$ $V_3 = IR_3$

- So, total potential difference, $V_T = V_1 + V_2 + V_3$

  $V_T = IR_1 + IR_2 + IR_3$

  $IR_T = I(R_1 + R_2 + R_3)$

- Hence, $R_T = R_1 + R_2 + R_3$

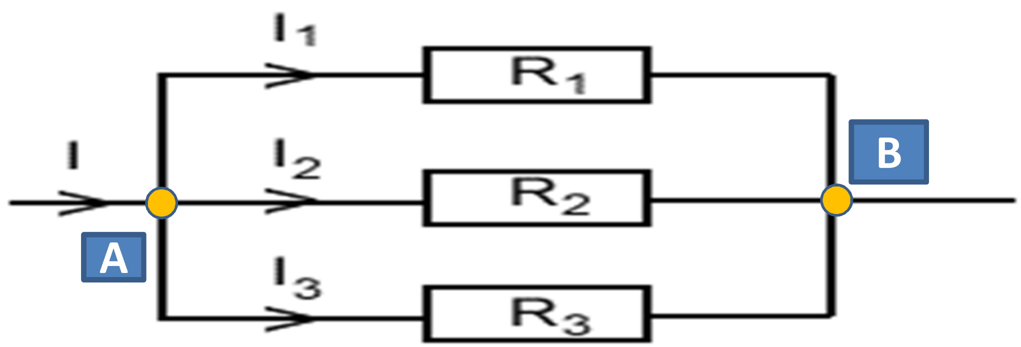
More resistors connected in series will increase the total resistance and decrease the current provided the voltage source is constant.

Example 8

- Calculate the current in a circuit which consists of an emf source 12 V and internal resistance 5 ohm and 3 external resistors of 15 ohm each.

- 3 light bulbs with a rating of 12V, 3W are connected in series. Calculate the total resistance produced by these three bulbs when they are in series and each one is lighted up with normal brightness.

Resistors (in parallel)



- Since 3 resistors are connected across the same points A and B, the potential differences of 3 resistors will be the same.

- The current that flows into A is equal to the sum of current out of B.

  $\mathbf{I = I_1 + I_2 + I_3}$

Resistors (in parallel)

- The current in each of the resistors are:

$I_1 = V/R_1$ $\quad$ $I_2 = V/R_2$ $\quad$ $I_3 = V/R_3$

$$I = I_1 + I_2 + I_3$$
$$V/R_T = V/R_1 + V/R_2 + V/R_3$$
$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$$
$$R_T = [1/R_1 + 1/R_2 + 1/R_3]^{-1}$$

More resistors connected in parallel will decrease the total resistance and increase the total current provided the voltage source is constant.

Example 9

- Find the resistance of the resistor that must be connected in parallel with 50 Ω resistor to obtain a combined resistance of 20 Ω.

Example 10

- Three resistors $R_1 = 3.0\ \Omega$, $R_2 = 5.0\ \Omega$ and $R_3 = 7.0\Omega$ are connected in series to a 15 V battery. Find:

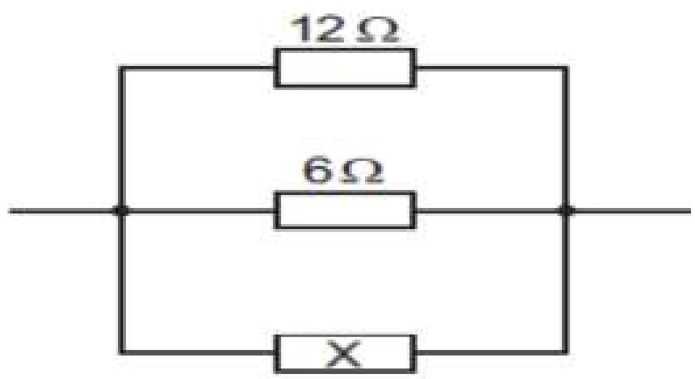
  a.) The equivalent resistance of the arrangement

  b.) The current through each resistor

  c.) The rate at which energy is dissipated in each resistor.

Example 11

- The diagram shows a parallel combination of three resistors. The total resistance of the combination is 3 Ω.



- What is the resistance of resistor X?

Example 12

- The combined resistance R_T of two resistors of resistances R_1 and R_2 connected in parallel is given by the formula

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Which statement is used in the derivation of this formula?

A The currents through the two resistors are equal.

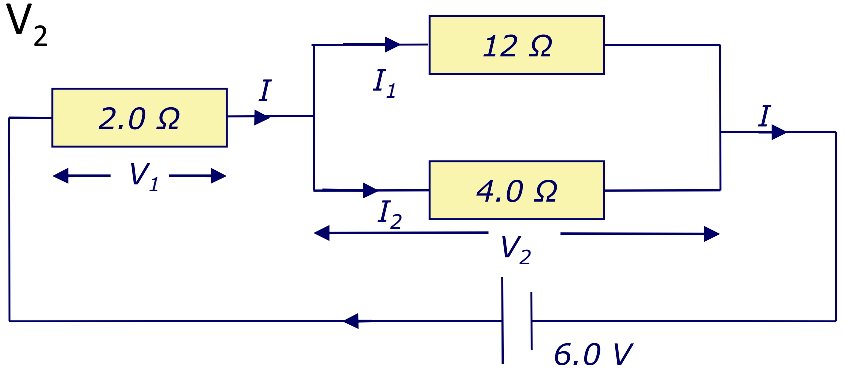
B The potential difference across each resistor is the same.

C The supply current is split between the two resistors in the same ratio as the ratio of their resistances.

D The total power dissipated is the sum of the powers dissipated in the two resistors separately.

Example 13

- Calculate:
  - a.) the combined resistance of the circuit
  - b.) I, I_1, I_2
  - c.) V_1, V_2

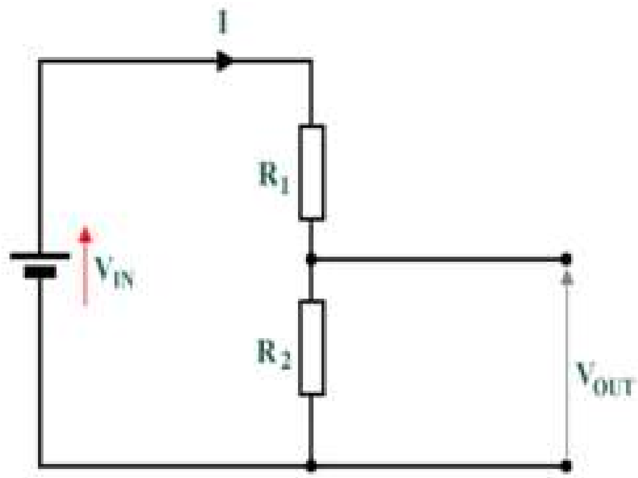


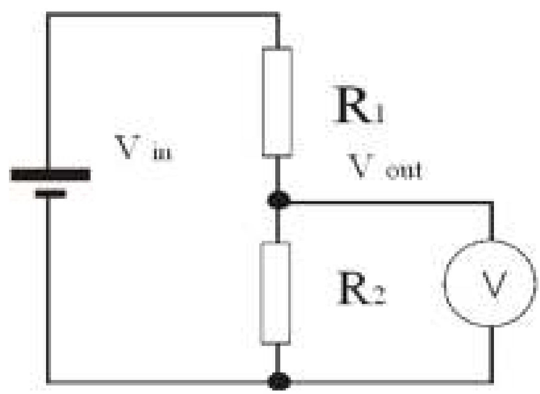
Potential divider circuit

- On certain occasion, we may have devices that only need to use a fraction of the e.m.f of a supply in order to operate.
- Eg: if we have a lamp that only required a voltage of 3.0 V to operate normally and a battery of e.m.f 9.0 V. How can we get an output of 3.0 V from this battery of e.m.f 9.0 V?
- To do this, we construct a circuit consisting of the e.m.f with resistors arranged in series to it, then we connect the desired lamp across a particular resistor in parallel.
- Such circuit or connection is called the potential divider circuit.
- In other words, these resistors in series are able to divide the e.m.f. of the source to a fraction of its total value.

Potential divider circuit

- This is how the potential divider circuit looks like.





- The voltage difference, V_{out} across resistor R_2 will be less than V_{in}
- From the loop rule,

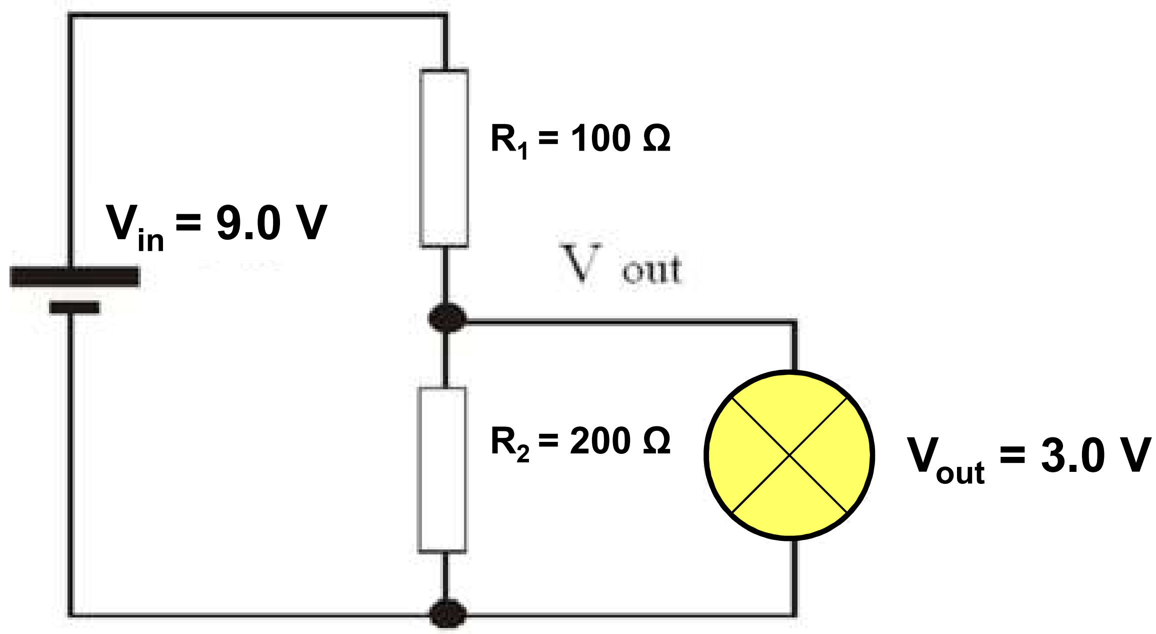
$$V_{in} = IR_1 + IR_2$$
$$V_{in} = I(R_1 + R_2)$$

- So the current is given by $I = V_{in} / (R_1 + R_2)$
- Thus, the voltage difference, V_{out} across resistor R_2 is:

$$\boxed{V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_{in}}$$

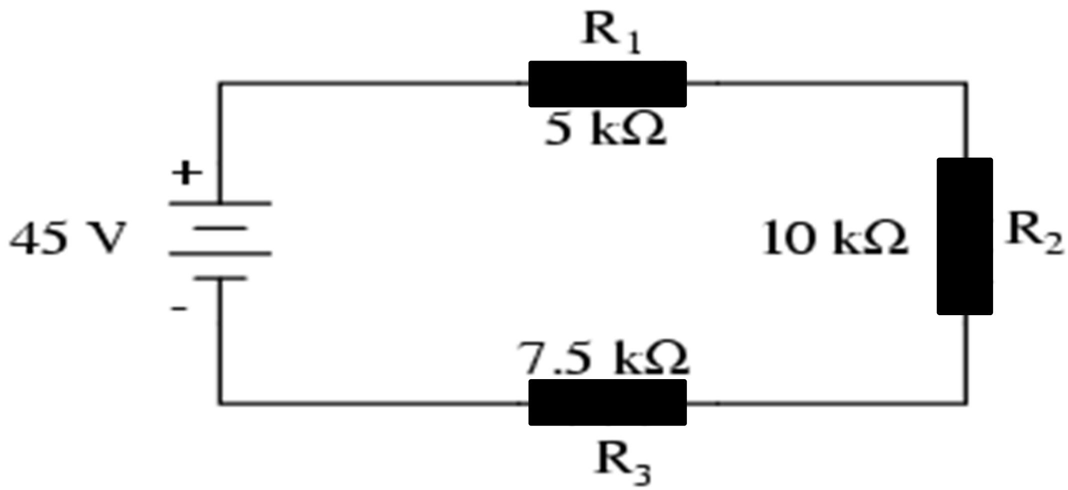
Potential divider circuit

- Back to the question asked earlier.
- If we have a lamp that only required a voltage of 3.0 V to operate normally and a battery of e.m.f 9.0 V. How can we get an output of 3.0 V from this battery of e.m.f 9.0 V?



Example 14

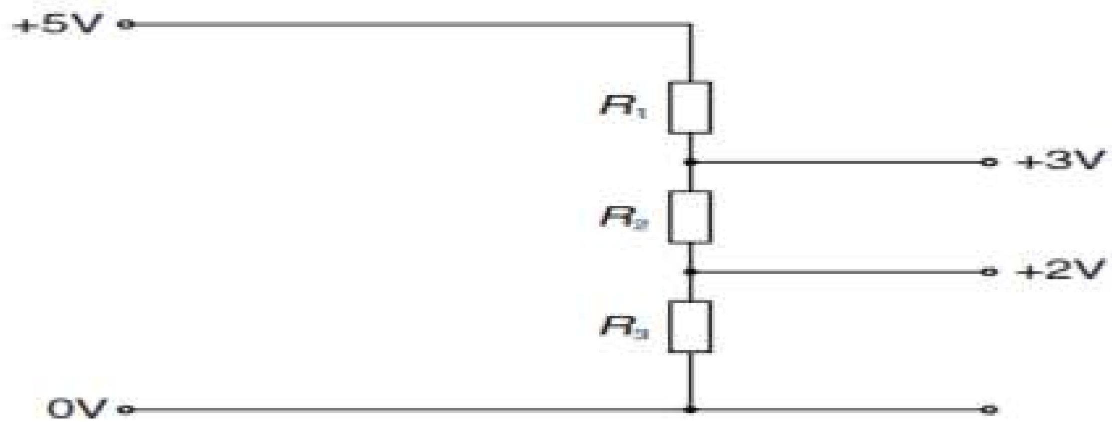
- Let's analyze a simple series circuit, determining the voltage drops across individual resistors



	R_1	R_2	R_3	Total	
E				45	Volts
I					Amps
R	5k	10k	7.5k		Ohms

Example 15

- A potential divider is used to give outputs of 2 V and 3V from a 5 V source, as shown.

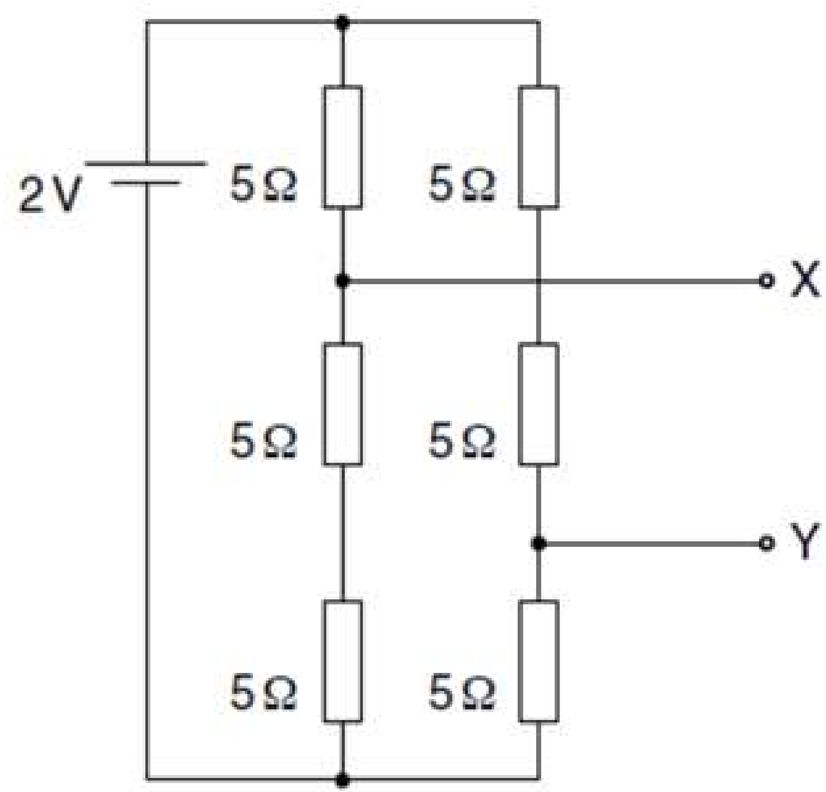


What are possible values for the resistances R_1, R_2 and R_3?

	R_1/kΩ	R_2/kΩ	R_3/kΩ
A	2	1	5
B	3	2	2
C	4	2	4
D	4	6	10

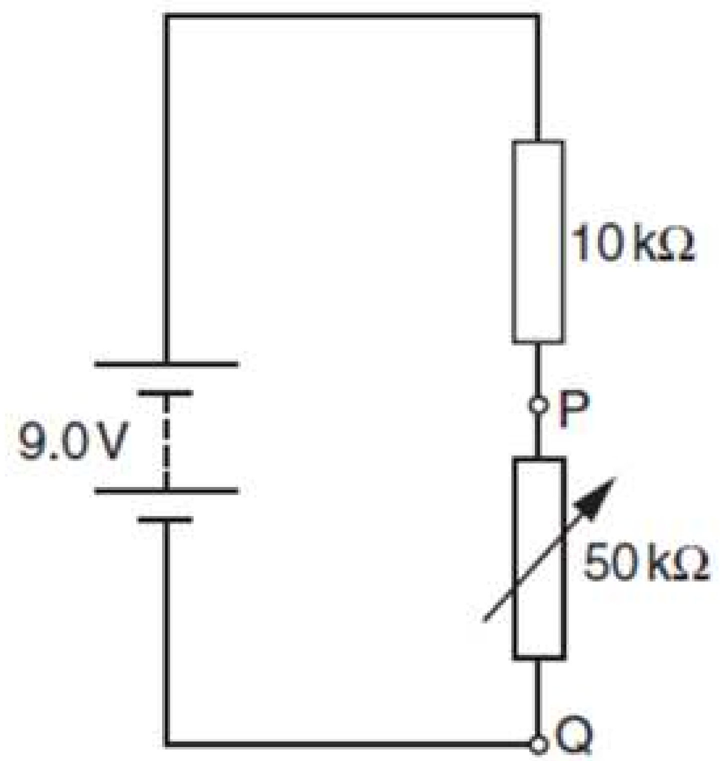
Example 16

- Six resistors, each of resistance 5 Ω, are connected to a 2 V cell of negligible internal resistance. What is the potential difference between terminals X and Y?



Example 17

- The diagram shows a potential divider connected to a 9.0 V supply of negligible internal resistance.



What range of voltages can be obtained between P and Q?

A zero to 1.5V

B zero to 7.5V

C 1.5 V to 7.5V

D 1.5 V to 9.0V

Application of Potential Divider (in LDR case)

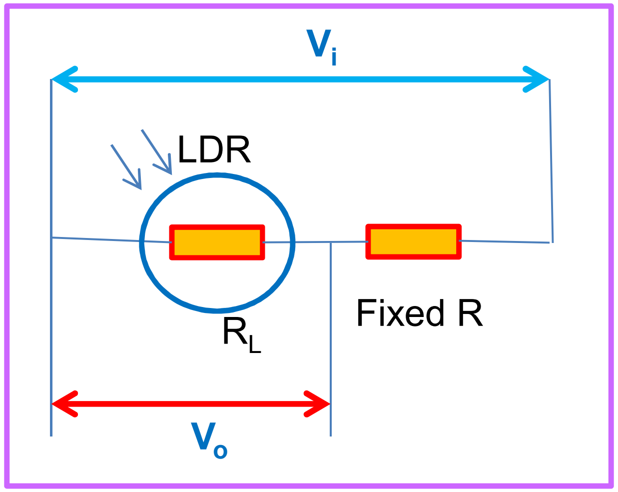


Figure above shows how a light dependent resistor (LDR) can be used in a potential divider to provide an output p.d. (V_o) which varies with light intensity.

The characteristic of a LDR is that light energy brings an effect of reducing the resistance of the device.

- High intensity of light (brightness increase) will decrease the resistance
- Low intensity of light (brightness decrease) will increase the resistance.

$V_o = V_i \times [R_L / (R_L + R)]$; where R_L = LDR resistance.

When brightness increases, R_L will decrease and lower in comparison to R, thus V_o is low.

When brightness decreases, R_L will increase until it reaches its maximum value. V_o will then has its maximum value. (approximately equal to V_i in most cases)

What would happen if R_L & R interchanged. (brightness increases, V_o increases)

Application of Potential Divider (in NTC thermistor case)

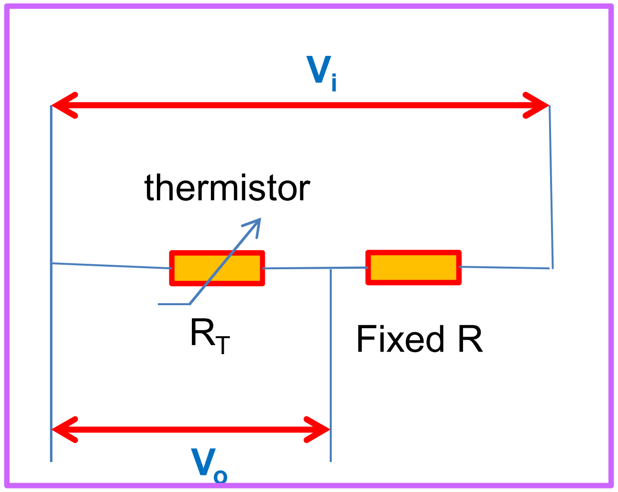


Figure above shows how a thermal resistor (thermistor) can be used in a potential divider to provide an output p.d. (V_o) which varies with temperature.

The characteristic of a thermistor (NTC type) is that

- Increase in temperature will decrease the resistance.

$V_o = V_i \times [\, R_T / (R_T + R) \,]$; where R_T = thermistor resistance.

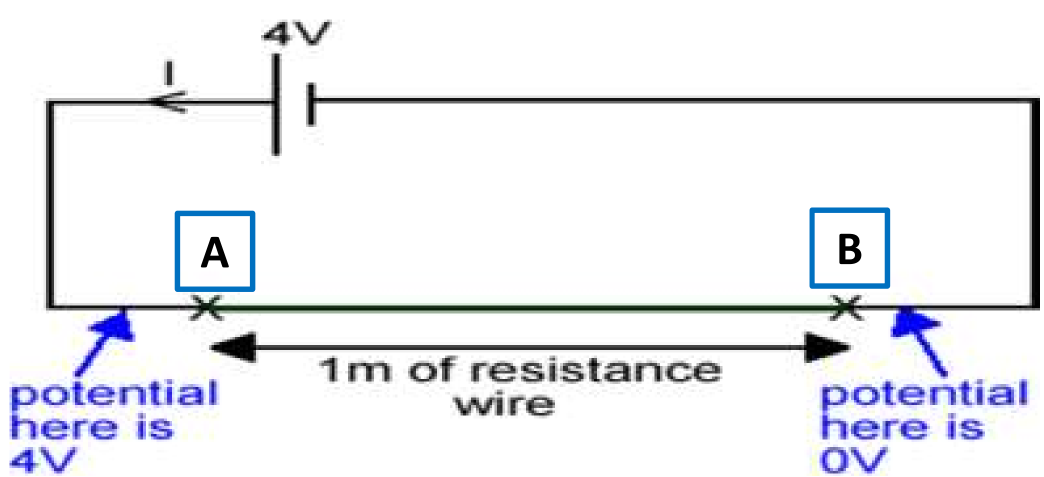
For a NTC thermistor, R_T will decrease as temperature increases.

This means that the output p.d. will drop as temperature rises.

What would happen if R_T & R interchanged. (temperature increases, V_o increases)

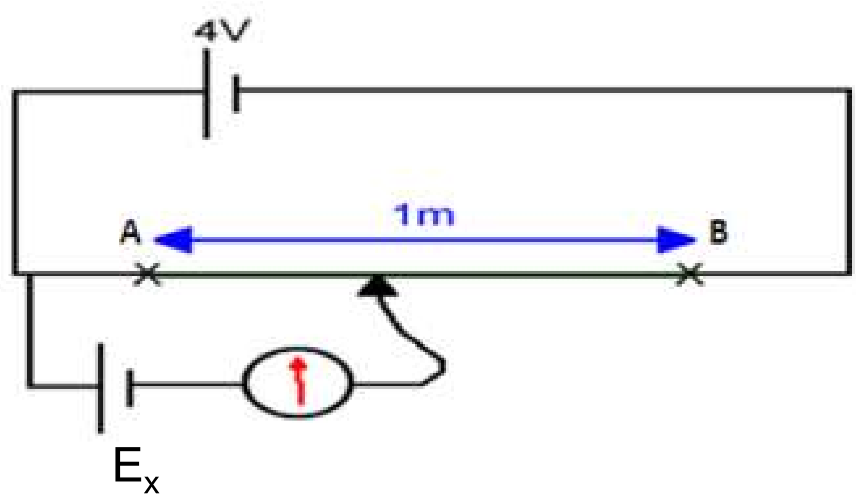
Potentiometer

- A potentiometer is a device used for comparing potential differences.
- For example it can be used to measure the unknown e.m.f of a cell, provided you already have another source whose e.m.f is known.
- Can be thought of as a type of potential divider circuit. A continuously variable potential divider circuit.
- A potentiometer consists of a metre length of resistance wire stretched horizontally between two end points.
- Figure below showed a potentiometer with end points A & B. A driver cell is connected across the length of the wire. Suppose this cell has an e.m.f of 4.0 V.
- We can then say point A has potential of 4.0 V, B is at 0 V, and the midpoint of the wire is 2.0 V. In other words, the voltage decreases uniformly along the length of the wire.



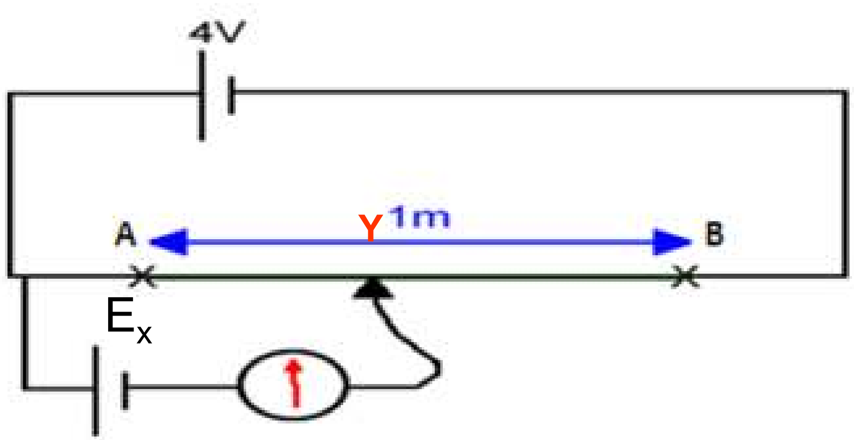
Potentiometer

- Now suppose we wish to measure the e.m.f, E_x of cell x.
- Two important points to take note when we make the connection.
- Firstly, this **unknown e.m.f** **MUST have a value less than** that of **the driver cell** else we cannot find this unknown e.m.f.
- Secondly, the **positive terminal of the cell x** **MUST be connected to the same point** as the **positive terminal of driver cell**, which is to point A.
- The negative terminal of cell x is connected to the galvanometer.
- The other terminal of the galvanometer is connected to the metal jockey.



Potentiometer

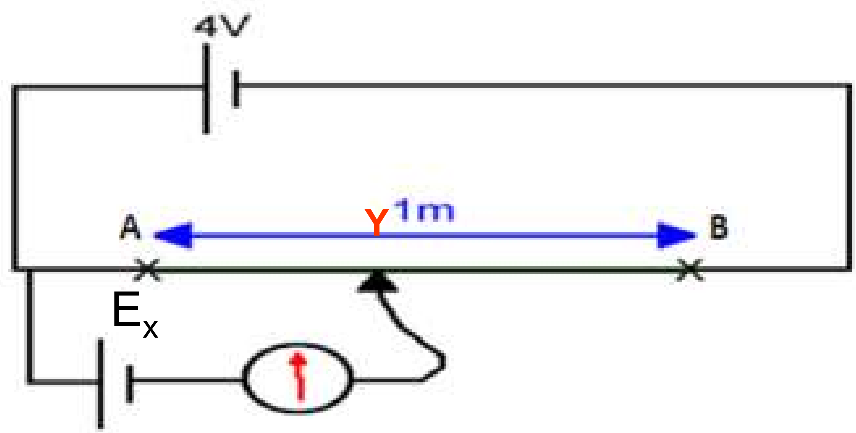
- If the jockey touched onto the metre wire close to point A, the galvanometer needle will deflect in one direction.
- If the jockey touched onto the metre wire close to point B, the galvanometer needle will deflect in the opposite direction.
- Clearly, there must be a point (let say point Y) along the wire which, when touched by the jockey, gives zero deflection.
- When the jockey is positioned at a point where there is no deflection in galvanometer, this indicates that there is NO current flowing through the galvanometer.
- This can only happen if the potential difference across the length of wire AY is equal to the e.m.f of the cell x.
- We can say that the potentiometer is balanced.
- If the balance point was exactly half way along the wire, we would be able to say that the e.m.f of cell x is half that of the driver cell.



Potentiometer

- Since voltage is directly proportional to the length of wire and the resistance of the wire, therefore the ratio of:

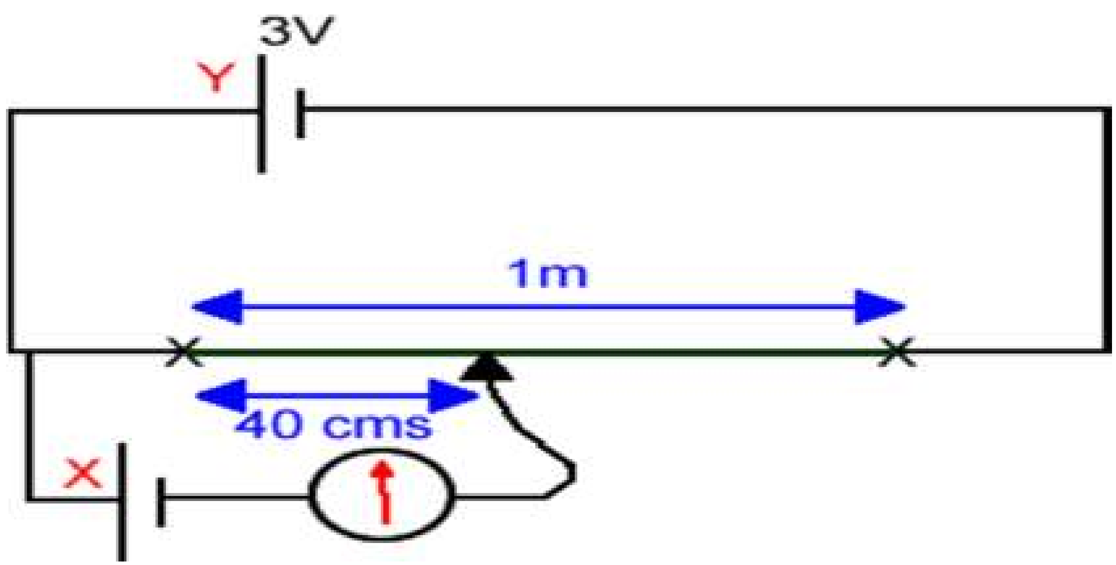
$$\frac{\text{length current has passed through}}{\text{total length}} = \frac{\text{pd}}{\text{total pd}} = \frac{\text{resistance passed}}{\text{total resistance}}$$



Example 18

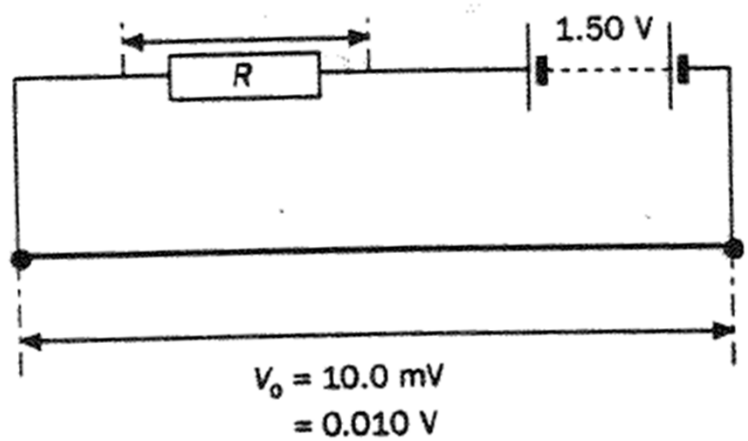
- What's the emf of cell X if cell Y has emf of 3V and the balance point is 40cm (see diagram) along the wire?

  (Total wire length = 1m).



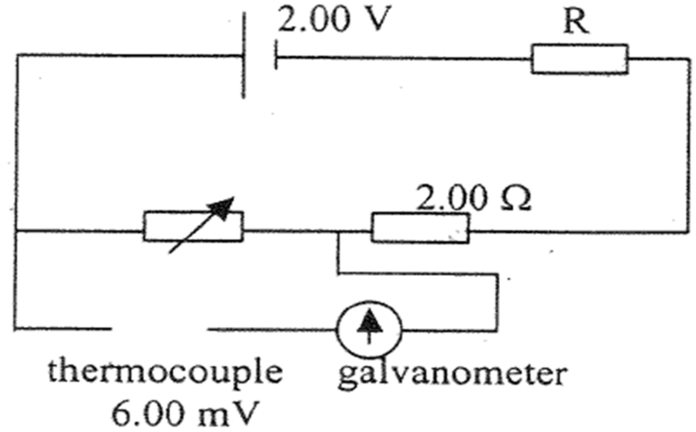
Example 19

- A potentiometer wire has a driver cell of e.m.f. 1.50 V and negligible internal resistance.
- If the 100 cm long slide-wire has a resistance of 2.00 Ω, calculate the value of the resistor which must be connected in series so that the p.d. across the wire is 10.0 mV.



Example 20

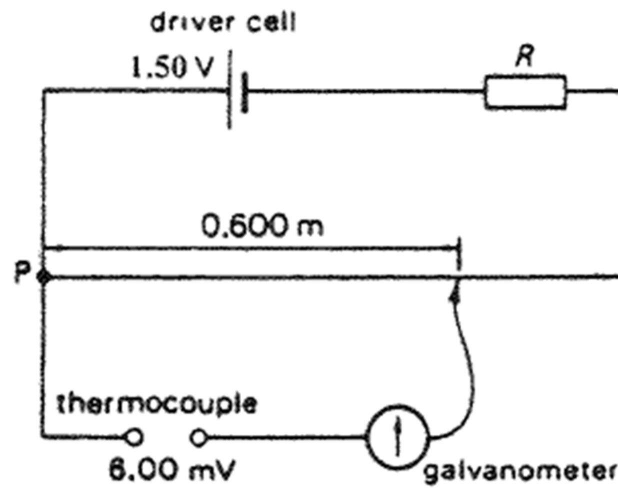
- The diagram shows a circuit for measuring a small e.m.f. produced by a thermocouple.



- There is a zero current in the galvanometer when variable resistor is set at 3.0 Ω. What is the value of resistance, R ?

Example 21

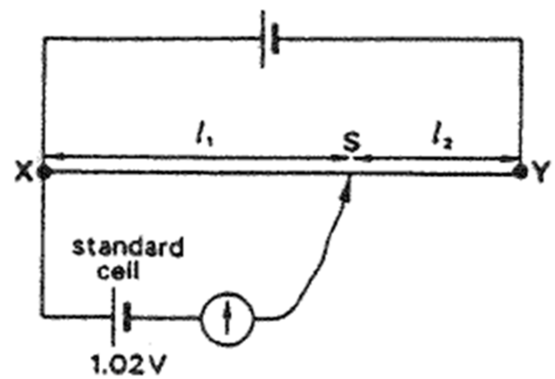
- The diagram shows a simple potentiometer circuit for measuring a small e.m.f. produced by a thermocouple.



- The meter wire PQ has a resistance of 5 Ω and the driver cell has an e.m.f. of 1.50 V. If a balance point is obtained 0.600 m along PQ when measuring an e.m.f. of 6.00 mV, what is the value of the resistance, R ?

Example 22

- A standard cell of e.m.f. 1.02 V is used to find the potential difference across the wire **XY** as shown in the diagram. It is found that there is no current in the galvanometer when the sliding contact is at **S**, l_1 from **X** and l_2 from **Y.**



- What is the potential difference across **XY** ?

A. $1.02\left[\dfrac{l_1+l_2}{l_1}\right]$ V

B. $1.02\left[\dfrac{l_1+l_2}{l_2}\right]$ V

C. $1.02\left[\dfrac{l_2}{l_1}\right]$ V

D. $1.02\left[\dfrac{l_1}{l_2}\right]$ V