Superposition

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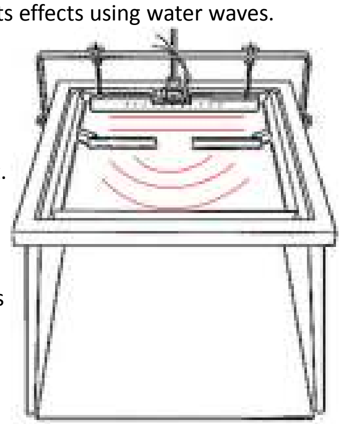
Diffraction

- Diffraction of waves is the ***spreading of waves when it passes through a gap or past the edge of an object.***
- Greatest effect occurs when ***the wavelength is about the same size as the aperture.***
- It is easy to observe this phenomena and investigate its effects using water waves.
- A ripple tank is used to show diffraction.
- Plane waves are generated using a vibrating bar.
- The waves are move towards a gap in a barrier.
- When they strike the barrier, they are reflected back.
- When arrive at gap, they pass through and spread out.
- It is this spreading out of waves as they travel through a gap (or past the edge of a barrier) that is called **diffraction.**
- The extent to which the ripples are diffracted depends on the width of the gap.



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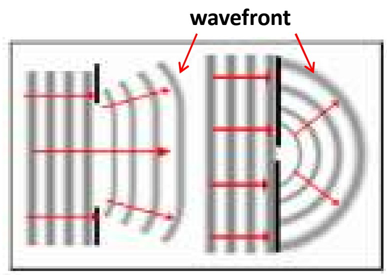
Diffraction

- The lines in the diagram below represents wavefronts. It is as if we are looking down on the ripples from above, and drawing lines to represent the top of the ripples at some instant in time.
- The separation between adjacent wavefronts is equal to the wavelength λ of the ripples.
- There is hardly any diffraction when the gap is very much larger than the wavelength.
- As the gap becomes narrower, the diffraction effect becomes more pronounced.
- It is greatest when the width of the gap is equal to the wavelength of the ripples.
- When the gap is very much smaller than the wavelength, then the wave may not even pass through the gap.
- When diffraction occurs:
  - **Wavelength is unchanged**
  - **Frequency is unchanged**
  - **Speed is unchanged**



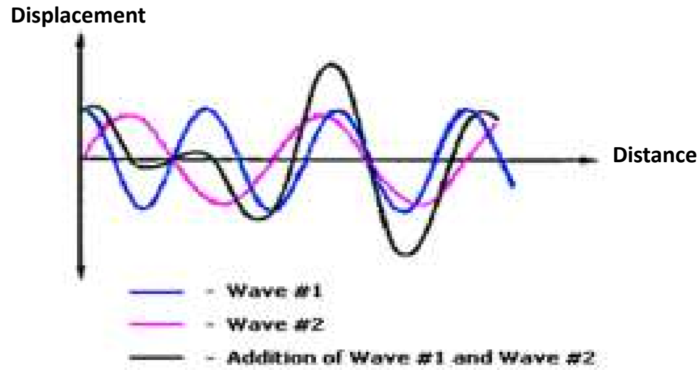
Wave interference

- In previous chapter, we studied the production of waves and the difference between transverse and longitudinal waves.
- Now what happens when we have two or more waves meet at a point in space (same medium) and combine together. This phenomenon is called **wave interference.**
- The interference of waves causes the medium to take on a shape which results from the **net effect of the two individual waves upon the particles of the medium.**
- In short, when two waves meet, they combine, with the displacements of the two waves adding together following the principle of superposition.

Principle of Superposition

- The task of determining the shape of the resultant wave demands that the principle of superposition to be applied.
- The ***principle of superposition*** is generally stated as follows:

  When two or more waves meet at a point, the resultant displacement is the algebraic sum of the displacements of the individual waves.



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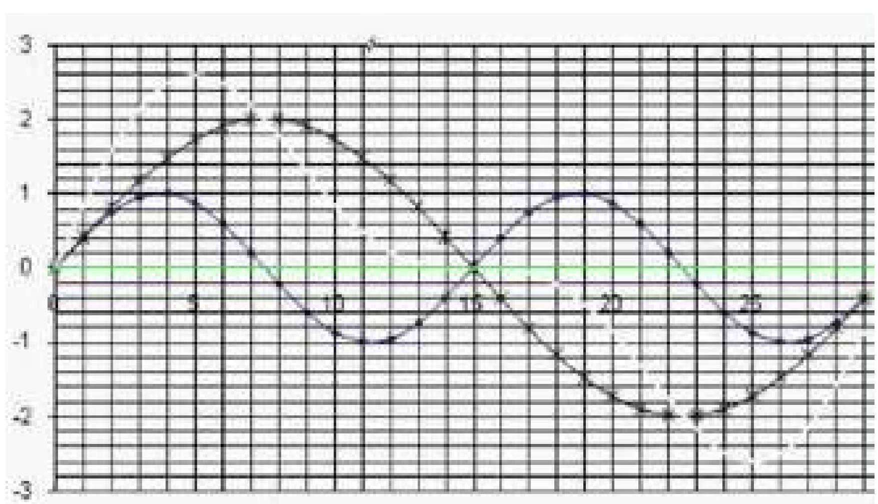
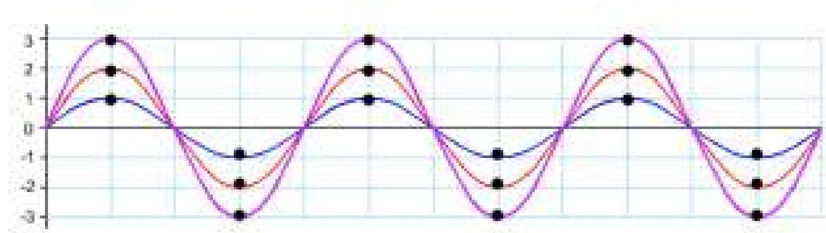
Principle of Superposition (Example)



Constructive Interference

- When there is an interference of 2 waves which are **<u>in phase</u>**, by using the principle of superposition, we would obtain a resultant wave of **maximum amplitude** while maintaining the **same original frequency (as below).**
- **This is called constructive interference.**



- The **purple** wave is the resultant of the **blue** & **red** waves.

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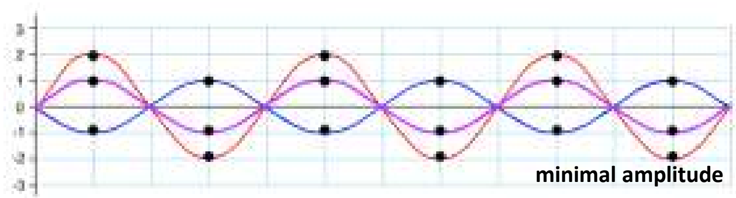
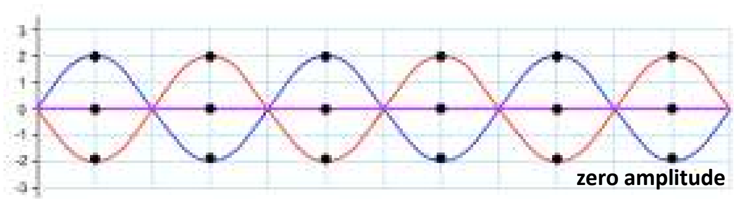
Destructive Interference

- When there is an interference of 2 waves which are **<u>anti-phase</u>**, by using the principle of superposition, we would obtain a resultant wave of **minimum or zero amplitude** while maintaining the **same original frequency (as below).**
- **This is called destructive interference.**





- The **purple** wave is the resultant of the **blue** & **red** waves.

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Path & Phase difference

- Whether the waves combine constructively or destructively at a point depends on the path difference of the waves from the two sources.
- The path difference is defined as the extra distance travelled by one of the waves compared with the other.
- In order to obtain constructive or destructive interference, the path & phase difference must take any of the value as summarized in table below.

Interference	Phase difference	Path difference
Constructive interference (the waves are in phase.)	**0 , 2π , 4π , 6π..** *(nπ, n =even integer)*	**0 , λ , 2λ , 3λ..** *(nλ, n = any integer)*
Destructive interference (the waves are anti phase)	**π , 3π , 5π , 7π..** *(nπ, n =odd integer)*	**λ/2 , 3λ/2 , 5λ/2 , 7λ/2..** *(n+λ/2, n = any integer)*

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Conditions for an interference pattern to be obtained

1.) Amplitude of the 2 waves must be equal or approximately equal.

To obtain a good contrast between maxima & minima intensity.

2.) Wave sources must be coherent.

Coherent sources are sources which have a constant phase difference between them. This means they must have the same frequency.

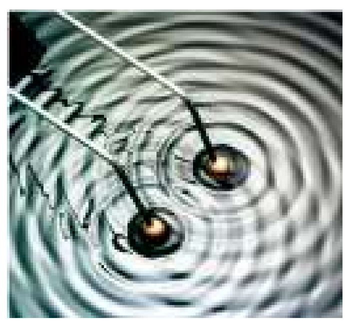
If sources were not coherent, the phase relationship between 2 sets of waves at any point would continuously alter, producing constant changing interference pattern.

Demonstration of interference pattern with:

a) water waves
b) light waves
c.) microwaves

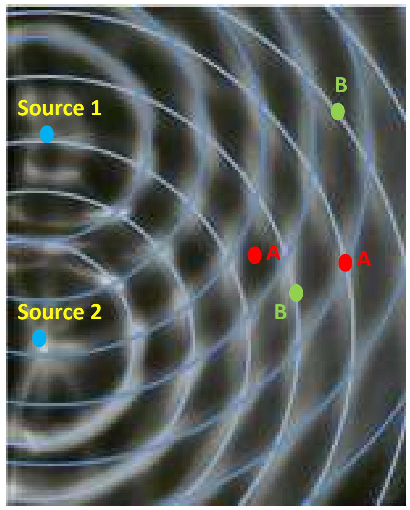
Demonstration of interference pattern (Water wave)

- Two dippers in the ripple tank should be positioned so that the are just touching the surface of the water.
- When the frequency generator is turn on, each dipper acts as a source of circular ripples spreading outwards. Where these sets of ripples overlap, we observe an interference pattern.
- Another way to observe interference in a ripple tank is to use plane waves passing through two gaps in a barrier. The water waves are diffracted at the two gaps and then interfere beyond the gaps.



Demonstration of interference pattern (Water wave)

- At position A, ripples from two sources arrive in phase with one another, and constructive interference occurs. (crest meets crest or trough meets trough)
- At point B, the two sets of ripples arrive anti-phase, and there is a destructive interference. (crest meets trough)
- Although waves are arriving at B, the surface of the water remains approximately flat.



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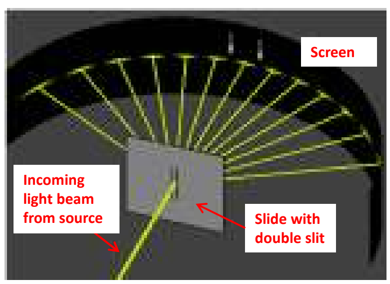
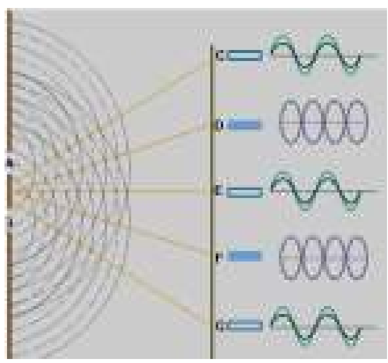
Demonstration of interference pattern (Light wave)

- We can also show interference effects produced by light. A simple arrangement involves directing the light from a laser through two slits.
- The slits are two clear lines on a black slide, separated by a fraction of a milimetre.
- Where the light falls on the screen, a series of equally spaced dots of light are seen.
- These bright dots are referred to as interference fringes and they are regions where light waves from the two slits are arriving in phase with each other. (constructive interference).
- The dark regions in between are the result of destructive interference.

- These bright and dark fringes are the equivalent of the loud and quiet regions that you detected if you investigated the interference pattern of sounds from two loudspeakers.
- Loud sound correspond to constructive interference while soft sound / silence correspond to destructive interference.





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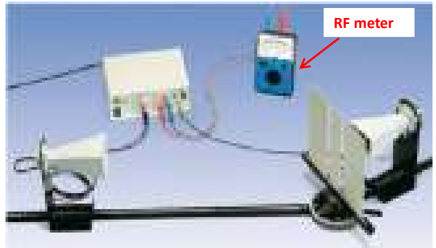
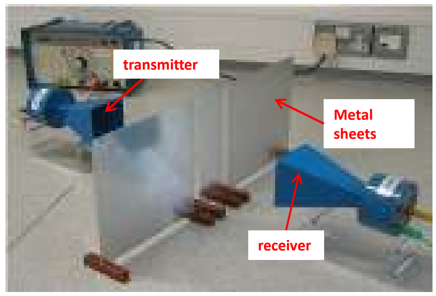
Demonstration of interference pattern (Microwaves)

- Using about 3.0 cm microwave equipment, you can observe an interference pattern.
- The microwave transmitter is directed towards the double gap in a metal barrier. The microwaves are diffracted at the two gaps so that they spread out into the region beyond, where they can be detected using a microwave receiver / detector.
- By moving the detector around, it is possible to detect regions of high intensity (constructive interference) and low intensity (destructive interference)
- The detector may be connected to a RF meter, or an audio amplifier & loudspeaker to give an audible output.





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The Young double-slit experiment

- This is an experiment to show the wave nature of light.
- A beam light is shone on a pair of parallel slits placed at right angles to the beam.
- Light diffracts and spreads outwards from each slit into the space beyond; hence the light from the 2 slits overlaps on a screen.
- An interference pattern of light and dark bands called "fringes" is formed on the screen.
- The distance ***x*** between successive bright fringes on the screen is called the ***fringe width / fringe separation.***
- The fringe width is related to the wavelength λ of the light source by the equation:

$$\boxed{\mathrm{x} = \frac{\lambda D}{a}}$$

where x is the fringe separation,
D is the slits to the screen distance,
a is the slits separation,
λ is the wavelength

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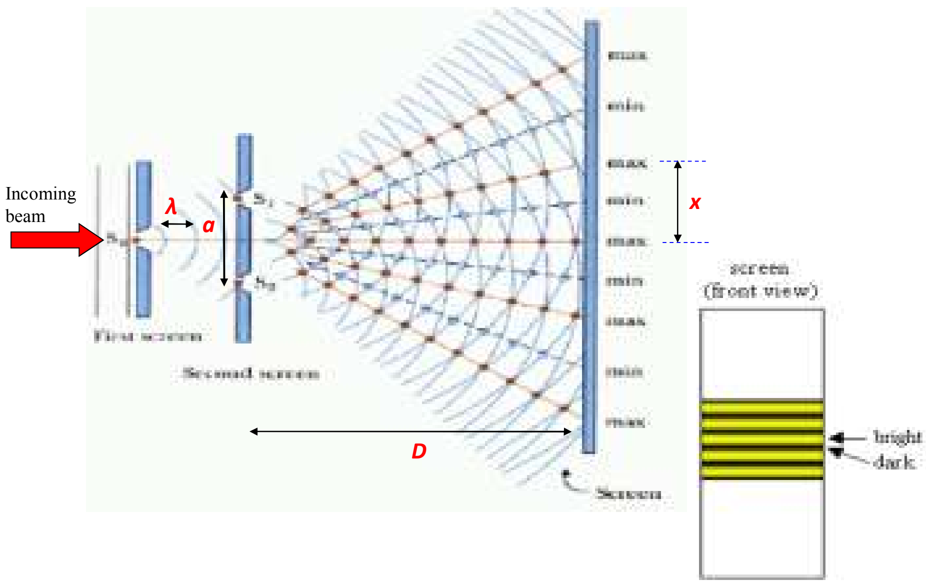
The Young double-slit experiment



Example 1

- Calculate the observed fringe width for a Young's double slit experiment using light of wavelength 600 nm and slits 0.50 mm apart. The distance from the slits to the screen is 0.80 m.

- Two loudspeakers connected to the same signal generator produce sounds of frequency 1000 Hz. The separation between the 2 speakers is 2.0 m. An observer who stands 5.0 m in front hears 3 consecutive loud sounds when he moves through a distance of 1.65 m in the direction parallel to the speakers.

  Determine a) the wavelength of the sound

  b) the speed of the sound waves in air

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Diffraction Grating

- A diffraction grating is similar to the slide used in the double-slit experiment, but with many more slits than just two.
- It consists of a large number of equally spaced lines ruled on a glass or plastic slide. Each line is capable of diffracting the incident light.
- There may be as many as 10 000 lines per centimetre.
- When monochromatic light is incident on this plate, a pattern of narrow bright fringes is produced due to interference and the principle of superposition.
- To obtain constructive interference, the path difference should be an integral / whole number of wavelengths.
- By measuring the angles at which the maxima (bright fringe) occur, we can determine the wavelength λ of the incident light. The wavelength is related to the angle θ by:

$$\boxed{d \sin \theta = n\lambda}$$

where d is the slit separation / grating spacing
n is the order of maximum (values 0,1,2,3…)

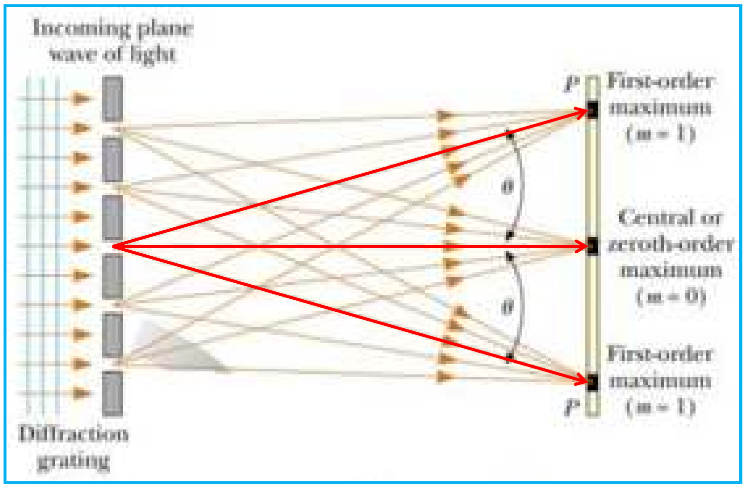
Diffraction Grating

- With double slit, the fringes are equally spaced and the angles are very small.
- With diffraction grating, the angles are much greater and the fringes are not equally spaced. (The fringes are also referred to as **maxima.**)
- The central fringe is called the zeroth-order maximum, the next fringe is called the first-order maximum, and so on.
- The diffracted beams form a symmetrical pattern on either side of the undiffracted central beam, meaning there will be two first-order maxima, two second order maxima, and so on.
- Now, for diffraction grating there are many overlapping beams of light, and these interfere with each other. **It is difficult to achieve constructive interference with many beams, because they all have to be in phase with one another.** Thus, the number of bright fringes seen is also lesser than the double slit.

How the first-order maximum are from?

Ray of light emerge from all of the slits, to form a bright fringe, all the ray MUST be in phase. In the direction of the first-order maximum, ray 1 has travelled the least distance. Ray 2 has travelled an extra distance equal to one wavelength and is therefore in phase with ray 1. The path difference between rays 1 & 2 is equal to one wavelength. Ray 3 would have travelled two extra wavelengths and is in phase with rays 1 & 2. In fact, the rays from all of the slits are in step in this direction, and the bright fringe results!

Diagram of a light passing through a diffraction grating with its zeroth & first order maxima



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Example 2

- Monochromatic light is incident on a diffraction grating with 7.00 x 10^5 lines per meter. A second-order maximum is observed at an angle of diffraction of 40.0°. Calculate the wavelength of the incident light.

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Example 3

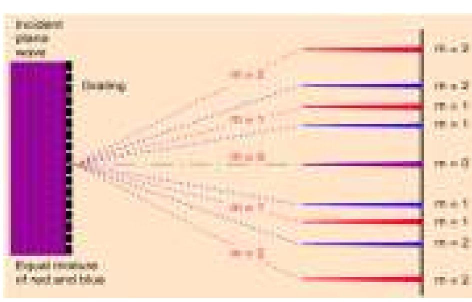
- A diffraction grating is ruled with 3500 lines per cm. parallel light of wavelength 600 nm is viewed through the grating. Find

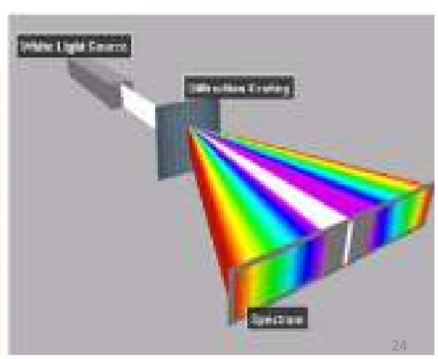
  i) the maximum number of orders of reinforcement that can be seen through the grating

  ii) the number of reinforcement maxima that will be observed

The diffraction grating

- When white light is incident on a diffraction grating, each wavelength making up the white light is diffracted by a different amount as described by ***nλ = d sin ϑ***
- In the visible spectrum(***VIBGYOR***), red light due to it has the longest wavelength is diffracted through the largest angle, whereas violet is the least diffracted.
- Hence a continuous spectrum is produced. This phenomenon is an important use of diffraction grating in a ***spectrometer***, used to investigate spectra.
- By determining the angle of the diffracted image, the wavelength of light producing that image can be determined.

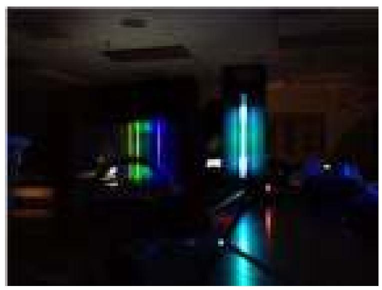




The diffraction grating

White Light

Hydrogen Light



Unknown Gas Light

The diffraction grating

<u>Overlapping of Colors</u>

- Two colors of different orders may overlap if their angles of diffraction θ are equal.
- Since d and θ are the same, the condition for overlapping of spectra of two different colors is

$$\boxed{n_1 \lambda_1 = n_2 \lambda_2}$$

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Example 4

- A parallel beam of white light is incident normally on a diffraction grating. It is noted that the second order and third order spectra partially overlap.

  Which wavelength in the third order spectrum appears at the same angle as the wavelength of 600 nm in the second order spectrum?

Stationary or Standing Waves

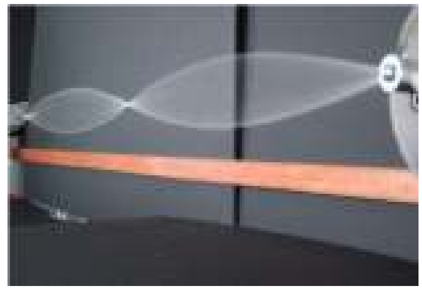
- This is the phenomenon when 2 progressive waves of equal amplitude and frequency travel along the same line with the same speed but in opposite directions.
- One wave is usually called incident wave, the other is called reflected wave.
- The waves interfered to produce a wave profile that does not move, unlike progressive waves, where the wave profile moves.

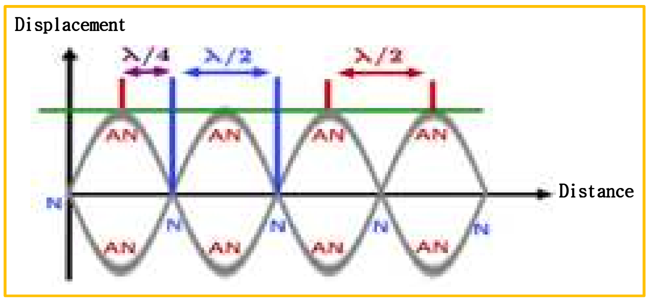
Remember:

- Waves which move energy from place to place are called progressive waves.
- Waves that do not are called stationary waves.

Stationary or Standing Waves

- Stationary waves are the key concept behind strings and tube musical instruments.
- Within a stationary/standing wave, there are regions which are called **antinodes** (where constructive interference occurs) and regions called **nodes** (where destructive interference occurs.)





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Key features distinguishing stationary waves from progressive waves

	Progressive waves	*Stationary waves*
Energy	1. Energy is transferred along the direction of propagation.	1. Energy is confined in a region, i.e. between two nodes. It is not transferred along the direction of wave propagation.
Wave profile	2. Wave profile moves.	2. Wave profile doesn't move.
amplitude	3. Every point has the same amplitude.	3. Points between two nodes have different amplitude.
Displacement	4. Each point is displaced.	4. There are some points which always have zero displacement, i.e. known as nodes.
Phase	5. Neighbouring points are not vibrating in phase.	5. Points between two nodes are vibrating in phase. Points on either side of a node are in antiphase.

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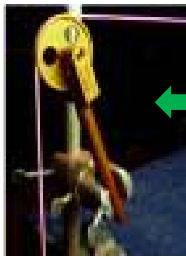
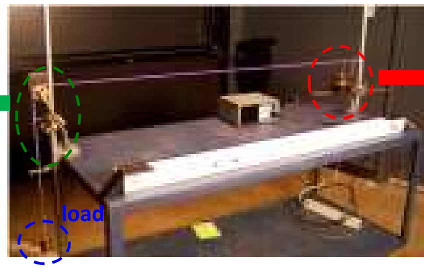
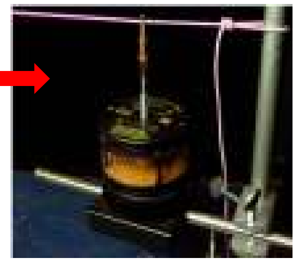
Stationary Waves Experiment: Stretched String

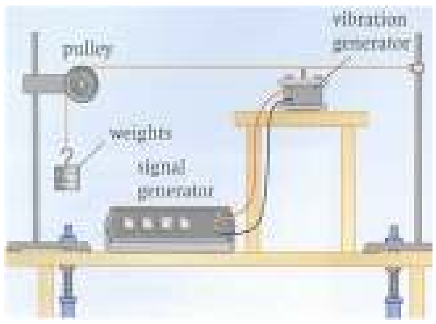






Pulley

Mechanical vibrator



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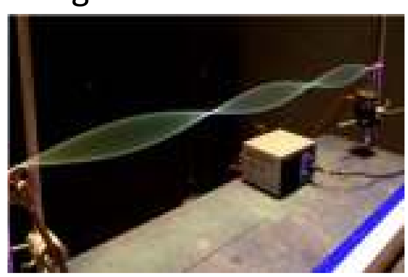
Stationary Waves Experiment: Stretched String



http://www.youtube.com/watch?v=12pjjPIE2IQ&feature=player_embedded

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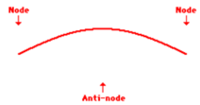
Stationary Waves Experiment: Stretched String



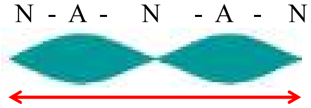
N - A - N

Length of string, L = ½ λ

Fundamental tone frequency, f_o

(1^{st} harmonic)
1 loop = ½ λ

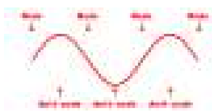
N - A - N - A - N



Length of string, L = λ

1^{st} overtone, $f_1 = 2f_o$

(2^{nd} harmonic)
2 loops = λ



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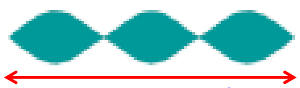

Length of string, L = $^3/_2$ λ

2^{nd} overtone, $f_2 = 3f_o$

(3^{rd} harmonic)
3 loops = 1.5λ

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Stationary waves on strings

- If a stretched string is allowed to vibrate freely, there are certain frequencies at which it will vibrate with a large amplitude.
- This is known as the **resonance frequency.**
- The simplest way a stretched string vibrates, has a wave pattern which has a **single loop** called the **fundamental mode of vibration or the 1^{st} harmonic.**
- We know that v = fλ , where v is the speed of the progressive waves which have interfered to produce the stationary wave.
- λ of fundamental tone is 2L, hence f is $\frac{v}{2L}$
- λ of 1^{st} overtone is L, hence f is $\frac{v}{L}$
- λ of 2^{nd} overtone is $\frac{2}{3}L$, hence f is $\frac{3v}{2L}$
- Since we know the relationship between the wavelength λ & the length of the string L, thus if the speed v is given to us, we can easily calculate the frequency f in order to obtain a stationary waves of desired harmonics.

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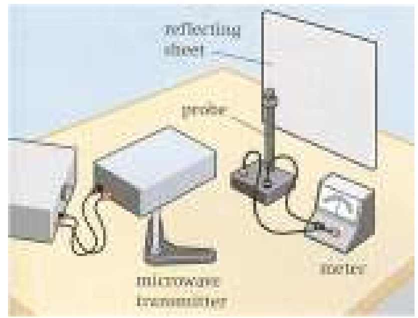
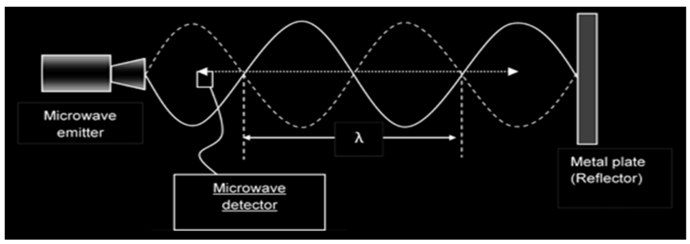
Stationary Waves Experiment: Microwaves

- Start by directing the microwave transmitter at a metal plate, which reflects the waves back towards the source.
- Place the probe close to the metal plate. Adjust the distance of the microwave transmitter until a lowest signal is received at the RF meter. (this will be the position of the node)
- Now move the probe across the space between the transmitter and the metal plate and you will observe position of high intensity signal.





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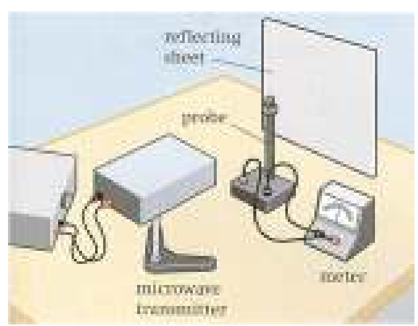
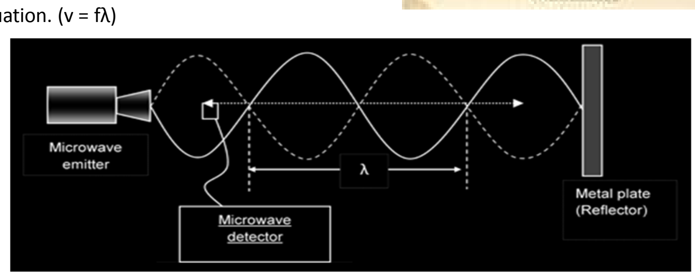
Stationary Waves Experiment: Microwaves

- As you continue moving, you will observe positions of high & low intensity. This is because stationary wave is set up between the transmitter and the metal plate, the positions of high & low intensity are the antinodes a& nodes respectively.
- The wavelength of the microwaves can be determined from the distance between the two nodes. [λ = 2 x (distance between 2 nodes)]
- Knowing the speed of microwaves is 3.0^{8} x 10 ms^{-1}, we can then determine their frequency f using the wave equation. ($v = f\lambda$)





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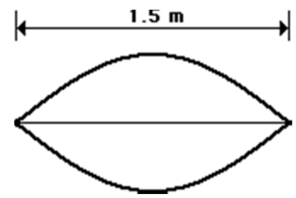
Example 5

- The string at the right is 1.5 meters long and is vibrating as the first harmonic. The string vibrates up and down with 33 complete vibrational cycles in 10 seconds. Determine the frequency, period, wavelength and speed for this wave.



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Example 6

- A stretched wire of length 60.0 cm vibrates transversely. Waves travel along the wire at a speed 210 ms^{-1}. Three antinodes can be found in the stationary waves formed in between the two ends of the wire. Determine
  a.) the wavelength of the waves which move along the wire
  b.) the frequency

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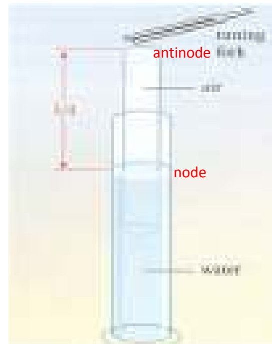
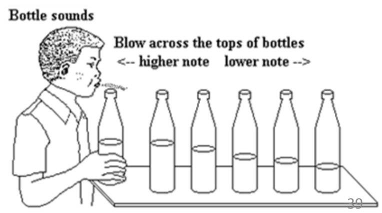
Stationary Waves Experiment: Tubes / Pipes / Air Columns

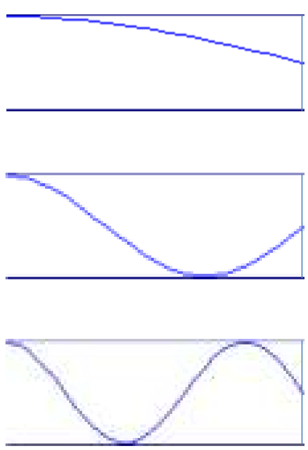
- A glass tube is clamped so that one end dips into a cylinder of water. By adjusting its height in the clamp, you can change the length of the column of air in the tube.
- When you hold a vibrating tuning fork above the end, the air column may be forced to vibrate, and the note of the tuning fork sounds much louder.
- This is another example of resonance.
- For resonance to occur, the length of the air column must be just right. The air at the bottom of the tube is unable to vibrate, so this point must be a node.
- The air at the open end of the tube can vibrate most freely, so this is an antinode.
- Hence the length of the air column must be ¼ of the wavelength, λ.
- By changing the tuning fork to one with another (correct) frequency, we can still hear the loudest sound (resonance again occur), this time the length of the air column would be ¾ of the wavelength, λ.
- This is the case of a **closed-ended tube.**

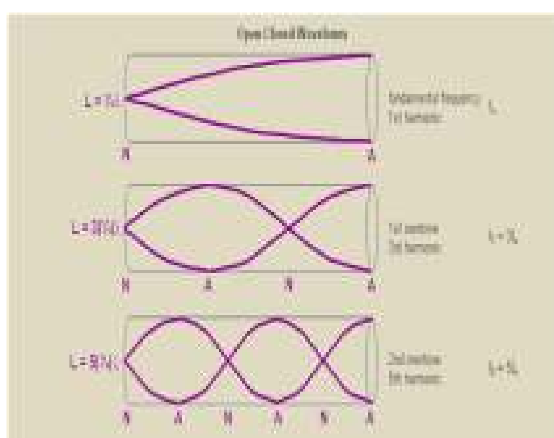




Stationary Waves Experiment: Tubes / Pipes / Air Columns

Standing wave in closed-ended tube.
(one closed ended and one open ended)





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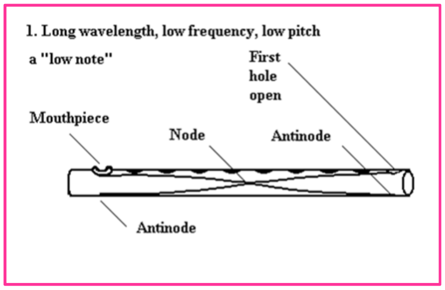
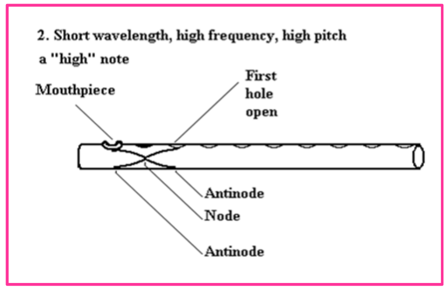
Stationary Waves Experiment: Tubes / Pipes / Air Columns

- The air in an **open-ended tube** which is **open at both ends** will vibrate in similar way to that of the closed end. Take an open-ended tube and blow gently across the top. You should hear a note whose pitch depends on the length of the tube.
- An open-ended tube has antinode at each end.
- There is a node at the midpoint.
- Hence, for the fundamental tone to be heard, the length of the air column must be ½ of the wavelength, λ.
- Example of this principle happens in a flute.





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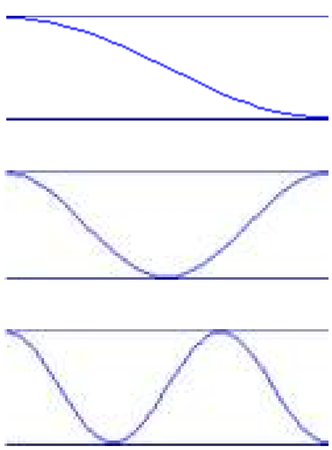
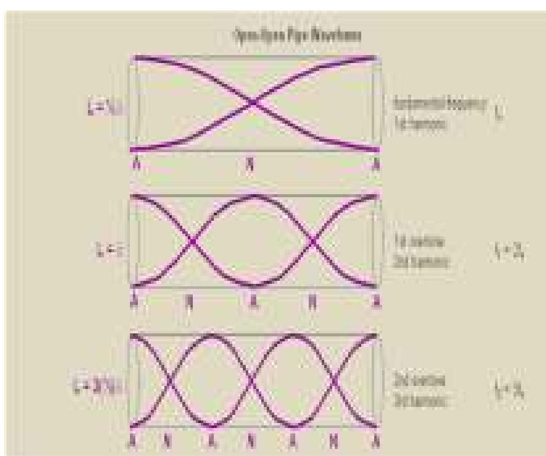
Stationary Waves Experiment: Tubes / Pipes / Air Columns

Standing wave in opened-end tube.
(both sides are open ended)





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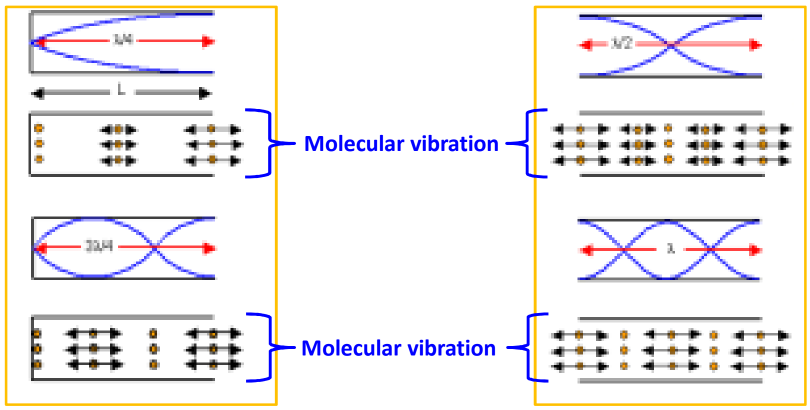
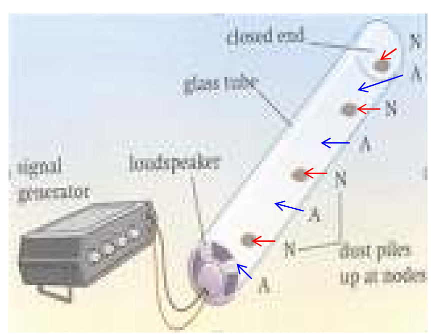
Stationary Waves Experiment: Tubes / Pipes / Air Columns

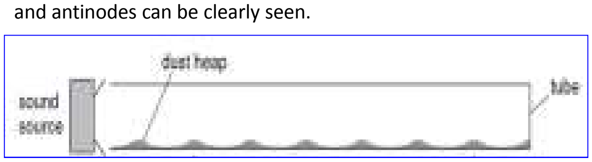
- Please take note.
- The representation of standing waves can be misleading. Remember that a sound wave is a longitudinal wave, but the diagram we draw is more like a transverse wave.
- Just remember that the particles are actually moving front and back!



Determining the wavelength and speed of sound (method 1)

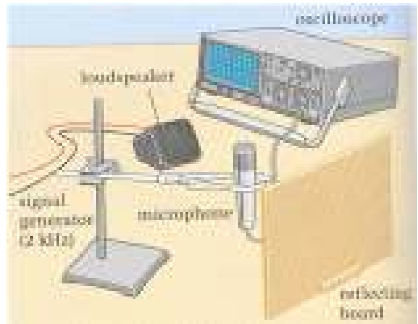
- Since we know that adjacent nodes (or antinodes) of a stationary wave separated by half of a wavelength, we can use this fact to determine the wavelength λ of a progressive wave.
- If we also know the frequency f of the waves, we can then find their speed v using wave equation, $v = f\lambda$
- One method is to use a dust tube. A loudspeaker sends sound waves along the inside of a tube. The sound is reflected at the closed end.
- When stationary wave is established, the dust at the antinodes vibrates violently. It tends to accumulate at the nodes, where the movement of the air is zero. Hence the positions of the nodes and antinodes can be clearly seen.





Determining the wavelength and speed of sound (method 2)

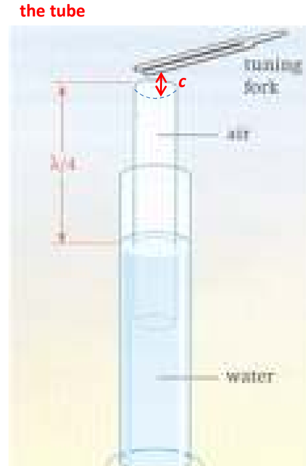
- An alternative method is shown beside. This is the same arrangement used for microwaves.
- The loudspeaker produces sound waves, and these are reflected from the board.
- The microphone detects stationary sound wave in the space between the speaker and the board, and its output is displayed on the oscilloscope.
- By moving the microphone along the line, we can easily detect nodes and antinodes.



Determining the wavelength and speed of sound (method 3)

- The resonance tube is a 3rd alternative to do so. This experiment illustrate an interesting way in which one type of experimental error can be reduced.
- Look at the representation on the right.
- In each case, the antinode at the top of the tube is shown extending slightly beyond the open end of the tube.
- This is because experiment shows that the air is slightly beyond the end of the tube vibrates as part of the stationary wave.

Tuning fork is a distance *c* on top of the tube



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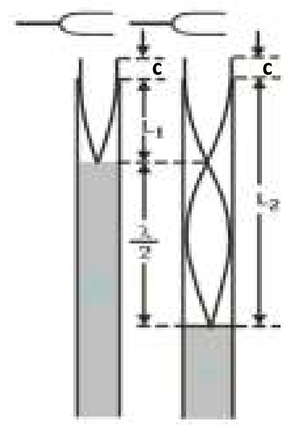
Determining the wavelength and speed of sound

- The antinode is at a distance ***c*** beyond the end of the tube, where ***c*** is called **end-correction.**
- Unfortunately we do not know the value of ***c***. It is difficult to measure directly.
- However, we can eliminate ***c*** following steps as below:

  1st loud sound occurs at : $\lambda/4 = L_1 + c$

  2nd loud sound occurs at: $3\lambda/4 = L_2 + c$
- Subtracting the two equation gives:

  $\lambda/2 = L_2 - L_1$ hence, $\lambda = 2 (L_2 - L_1)$
- Since $v = f\lambda$, $v = f [2 (L_2 - L_1)]$



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