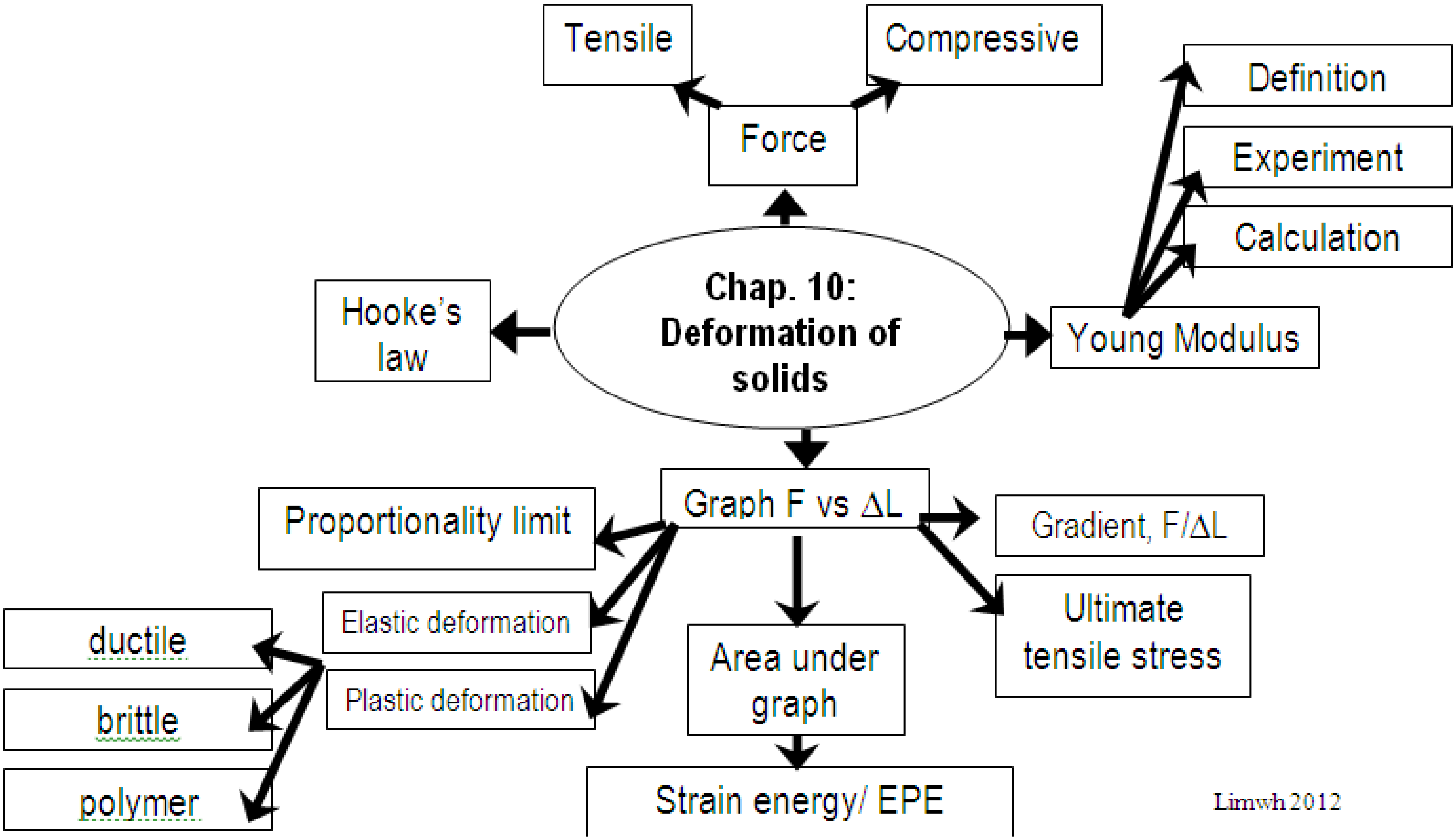
Chap 10. Deformation of Solids

By Lim WH 2012


Tensile
Compressive
Force
Definition
Experiment
Calculation
Hooke's law
Chap. 10: Deformation of solids
Young Modulus
Graph F vs ΔL
Proportionality limit
Gradient, F/ΔL
Elastic deformation
Ultimate tensile stress
ductile
Area under graph
brittle
Plastic deformation
polymer
Strain energy/ EPE
Limwh 2012

Learning Outcomes

(a) appreciate that deformation is caused by a force and that, in one dimension, the deformation can be tensile or compressive

(b) describe the behaviour of springs in terms of load, extension, elastic limit, Hooke's law and the spring constant (i.e. force per unit extension)

(c) define and use the terms stress, strain and the Young modulus and ultimate tensile stress

(e) distinguish between elastic and plastic deformation of a material

Tensile force vs Compressive force

Forces cause deformation of solids

Tensile force causes an increase in length when it applies on a material.

Compressive force causes an decrease in length when it applies on a material.

Example 1

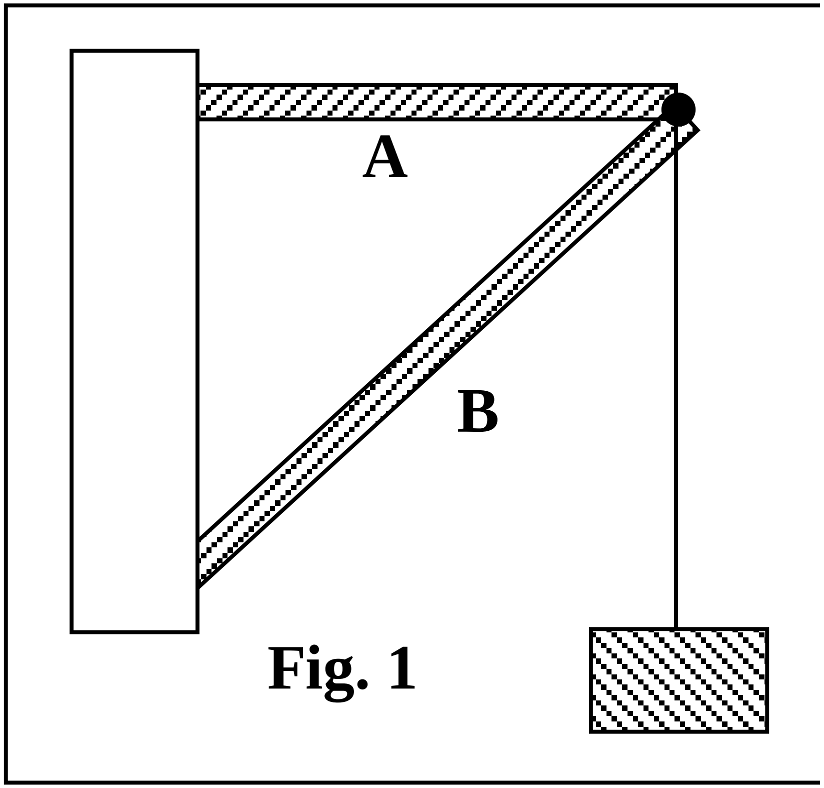


Fig. 1

State the type of force, tensile force or compressive force, acting on poles A and B respectively.

To which direction are the forces acting on Point P?

To which direction are the forces acting on pole A and B?

Example 2

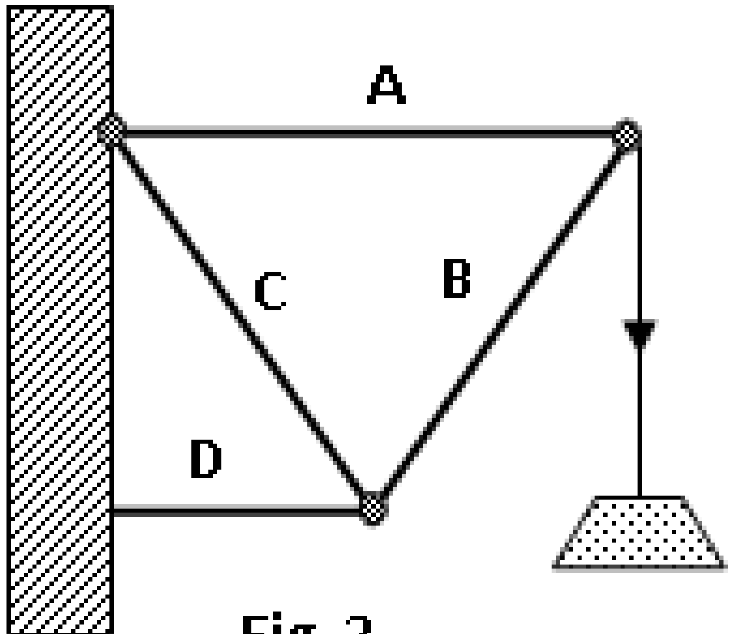


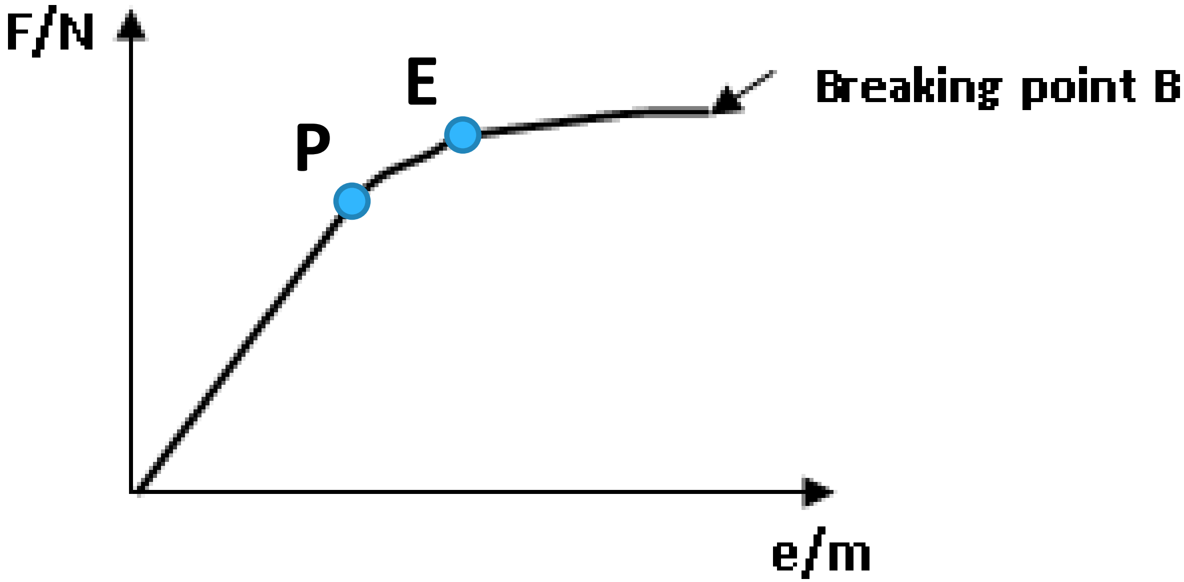
Fig. 2

State the type of force, tensile force or compressive force, acting on poles A, B, C and D respectively.

Hooke's Law

- State Hooke's law.

Force exerted to stretch a spring is directly proportional to the extension [1] provided **proportionality** limit is not exceeded [1]



From the origin to P, obeys Hooke's law & elastic

From P to E, does not obey Hooke's law but elastic

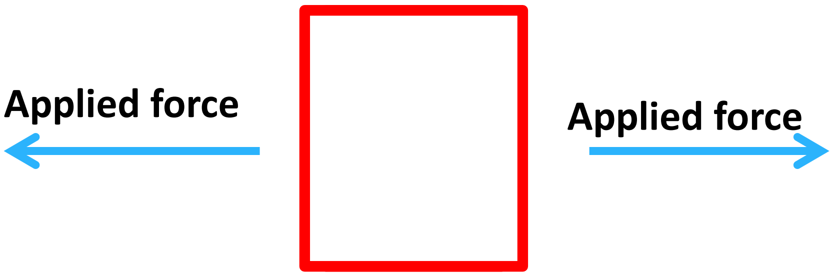
Elastic & Plastic deformations

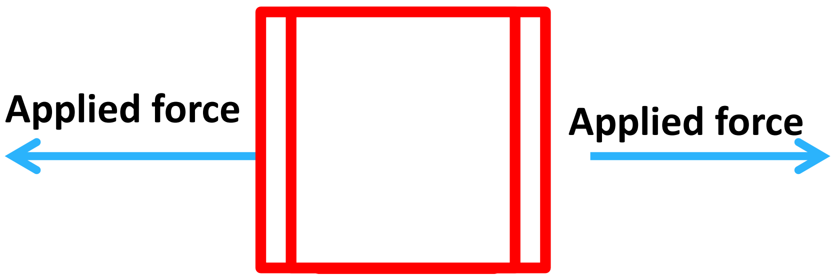
1. Explain what is happened on the shape of material when it is stretched
   (i) before the elastic limit

   the material returns to the original shape when the applied force is removed.

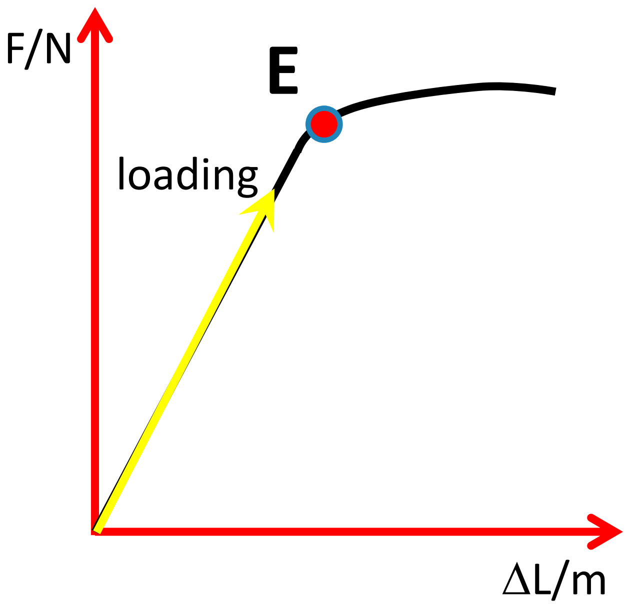
   (i) beyond the elastic limit (i.e. plastic deformation)

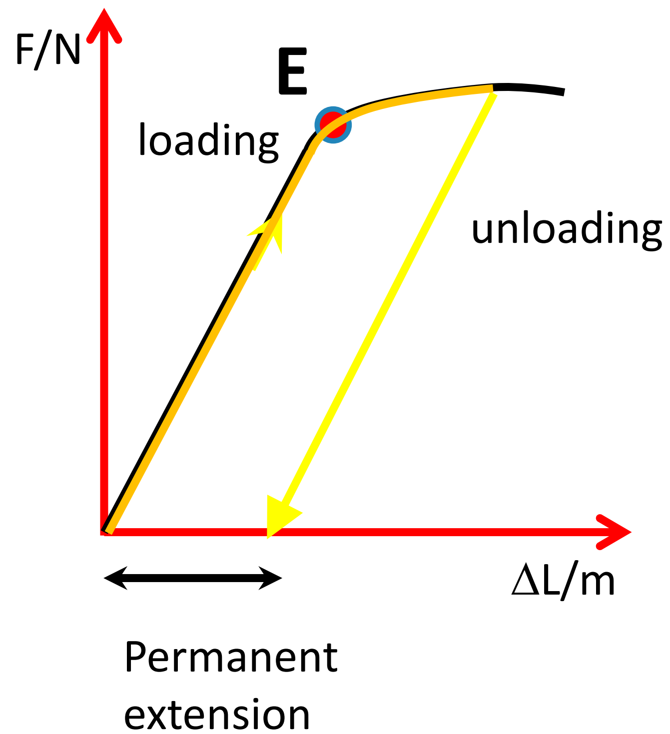
   the material retains some of the permanent extension when the applied force is removed.





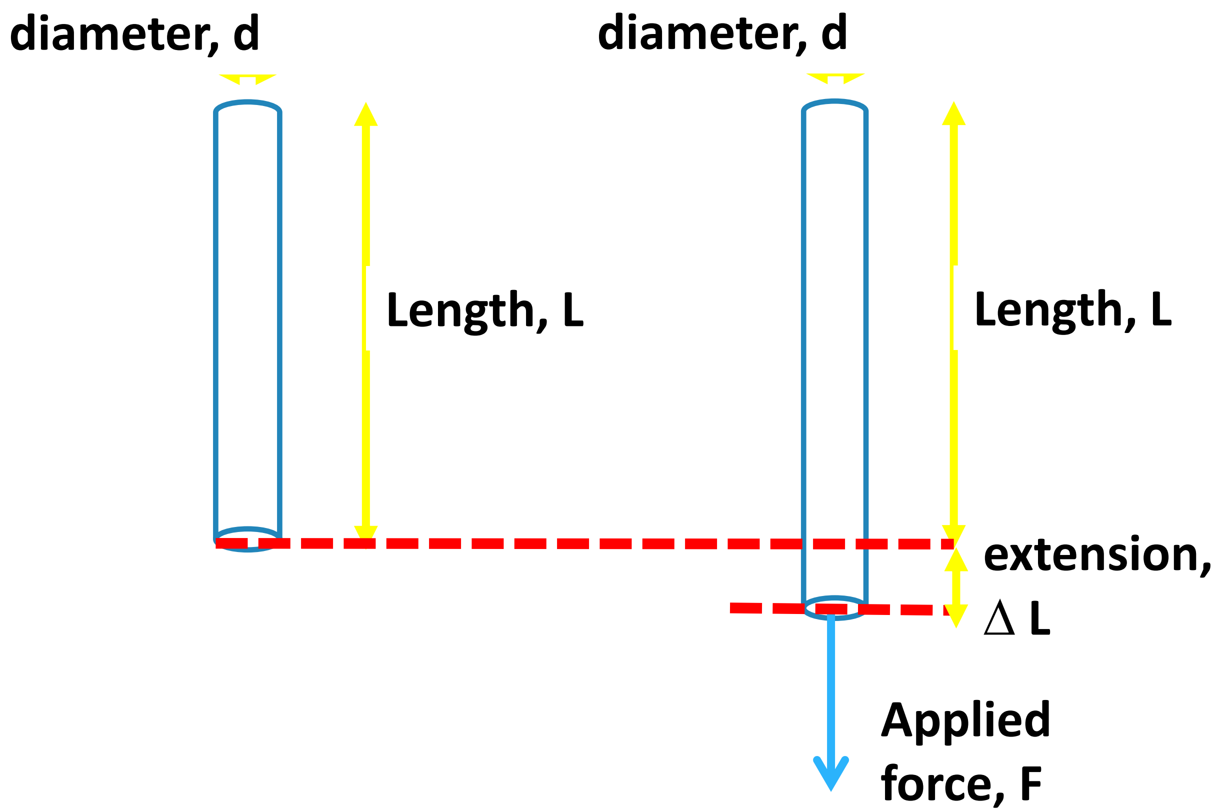
Elastic & Plastic deformations





Stress & strain

1. Define the following terms and state its units:
   (i) Stress – force per unit cross sectional area, units Pa or Nm^{-2}
   (ii) Strain – extension per unit original length



Stress = F/A
where A = $\frac{\pi}{4}d^2$ for a cylindrical wire

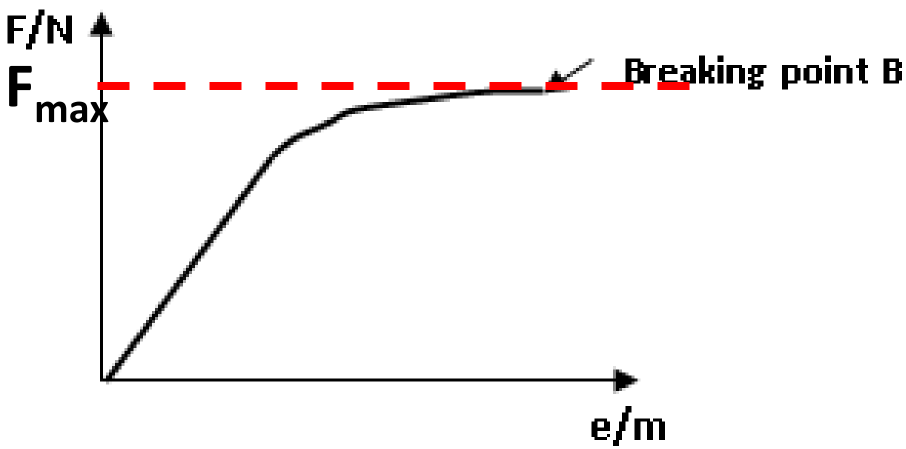
Strain = ΔL/L
where ΔL is the extension

ultimate tensile stress & Young Modulus

1. ultimate tensile stress – maximum force that a material can withstand before breaking
2. Define Young modulus

Stress per unit strain

units Pa or Nm^{-2}



Max Stress = F_{max}/A

where A = $\frac{\pi}{4}d^2$ for a cylindrical wire

Young Modulus, E

$$E = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$E = \frac{F\ L}{\Delta L\ A}$$

Young Modulus

Young modulus, E is a measure of material's resistance to change its length within its proportionality limit.

It is a measure of stiffness/rigidity.

Materials	**E/GPa**
Steel	210
Brass	90-110
Aluminum	70
Concrete	40
Lead	18
Perspex	3
Polystyrene	2.7-4.2
Rubber	0.01
glass	70-80

Example 1:

A steel wire 3.6 m long & 0.6 mm in diameter carries a mass of 7.0 kg. By how much does the wire stretch in excess of its original length? Given $E_{steel} = 1.9 \times 10^{11}\ Nm^{-2}$

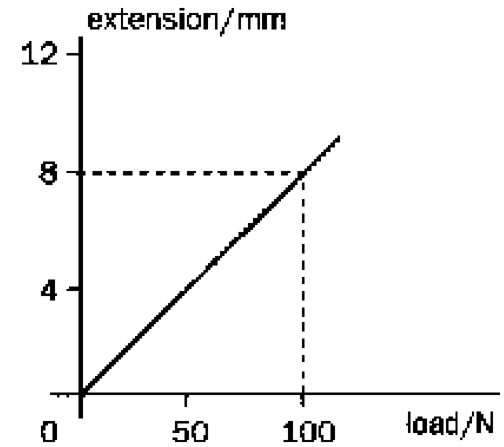
Problem solving – Young Modulus

1. An elastic string of cross-sectional area 3 mm^2 and length 2.5 m stretches by 2.5 cm when a force of 4 N is applied to it. Determine the Young modulus for the material of the string.

2. A rod of cross-sectional area 8.0 x $10^{-6} m^2$ is made of a material for which the Young modulus is 2.1 x 10^{11} Pa and which breaking strain is 1.0 x 10^{-3}. Assuming Hooke's law is obeyed until the rod fractures, calculate the tensile force needed to cause fracture.

Problem solving – Young Modulus

1. The graph shows the variation of extension with applied load for a wire of length 4 m and radius 0.25 mm. Calculate the Young modulus for the wire material.



2. A cylindrical wire of length *L* and diameter *D* is stretched by a load of *W* is hung on it. What happens to the extension when the load is doubled and the diameter is halved.

Learning Outcomes

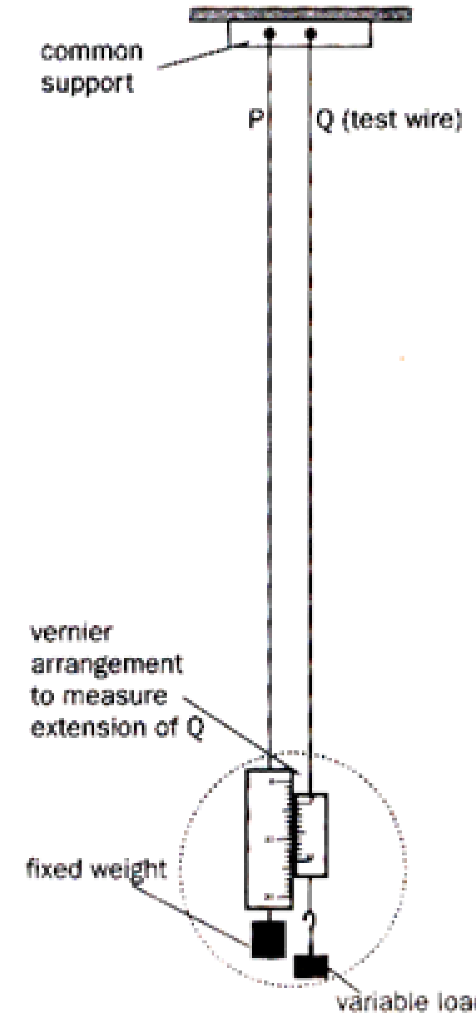
(d) describe an experiment to determine the Young modulus of a metal in the form of a wire

(f) deduce the strain energy (Elastic PE) in a deformed material from the area under the force-extension graph

(g) demonstrate knowledge of the force-extension graphs for typical ductile, brittle and polymeric materials.

Experiment to measure Young Modulus

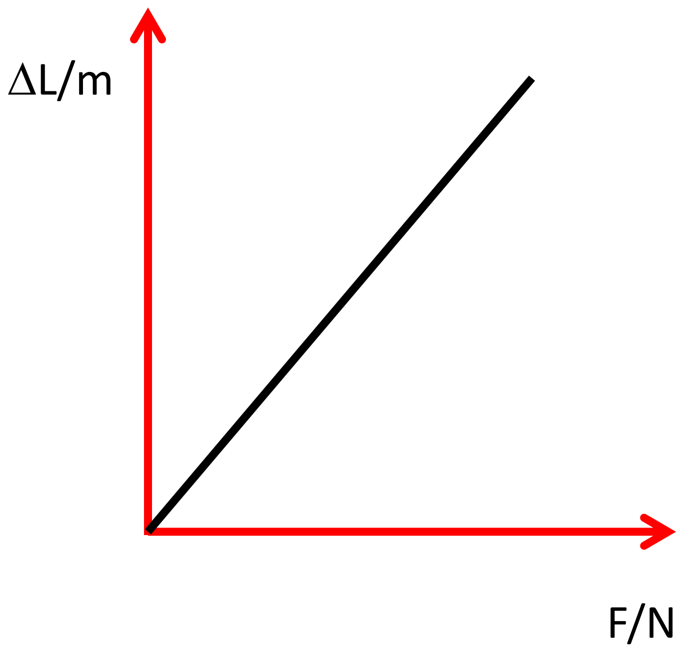
1. Two wires P and Q of the same diameter, material and length are hung from a common support, as shown in the diagram.
2. Wire Q is the wire under test and wire P is the comparison wire. The use of a comparison wire as a reference standard avoids errors due to
   i. Fluctuation in temperature
   ii. Sagging of the support
3. Any small increment in length of wire P is accurately measured by the vernier arrangement between P and Q.
4. The original length of Q (L) is measured using a measuring tape and the cross- sectional area (A) is obtained by measuring the diameter of Q using a micrometer screw gauge at several points and in at least two directions across the wire.
5. These precautions are necessary to minimise errors due to unevenness in the cross-section of the wire. The mean diameter is then calculated and used to calculate the cross-sectional area A = $\frac{\pi}{4}d^2$
6. The test wire is then incrementally loaded and the corresponding extensions are measured and noted. The results are used to plot a graph of extension ΔL against load (F).



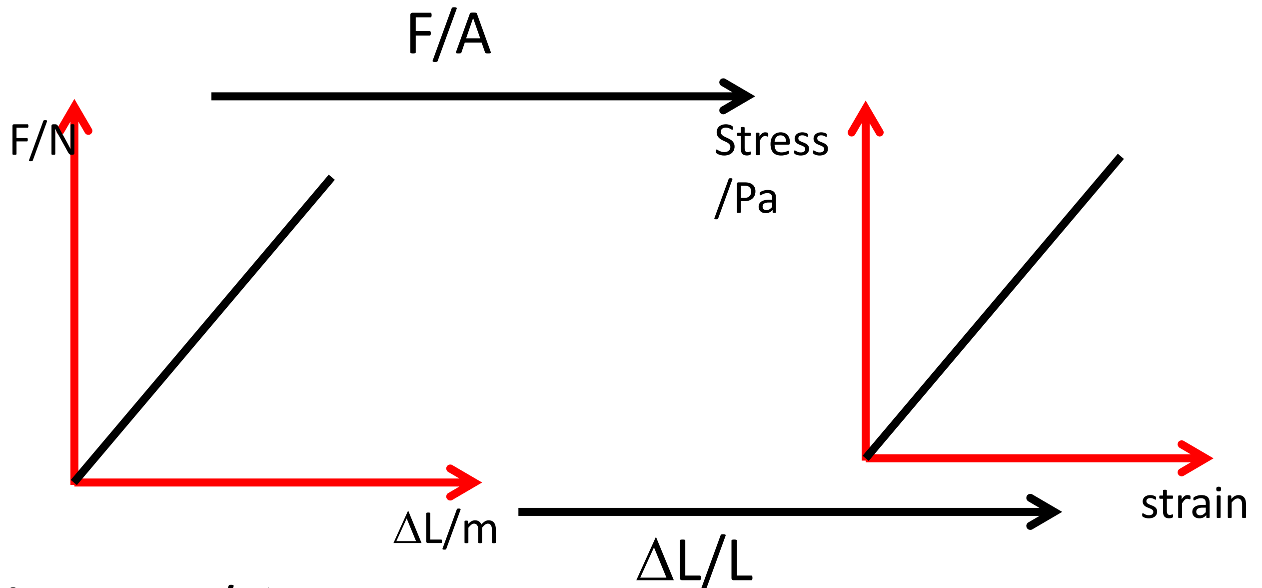
Analysis

1. Determine the gradient of the graph, $\Delta L/F$
2. Substitute the gradient, L and A into the formula

$$E = \frac{F\,L}{\Delta L\,A} = \frac{L}{gradient \times A}$$

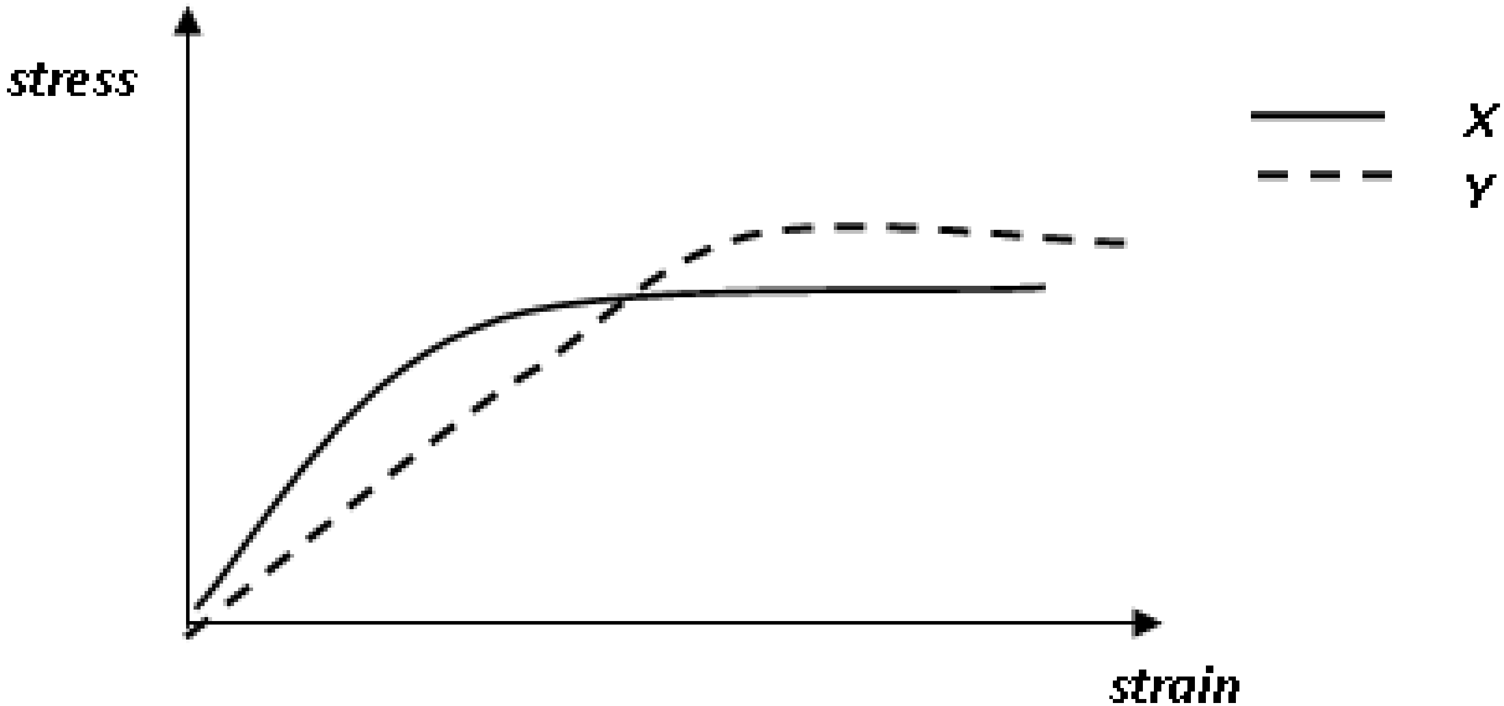


Force vs extension and Stress vs strain



Gradient = F/ ΔL
= force constant k
Area under the graph =
½ F ΔL = EPE or strain
energy
Or ½ kx^2

Gradient = stress/strain
= Young Modulus
Area under the graph = ½
stress x strain
= EPE or strain energy per
unit volume



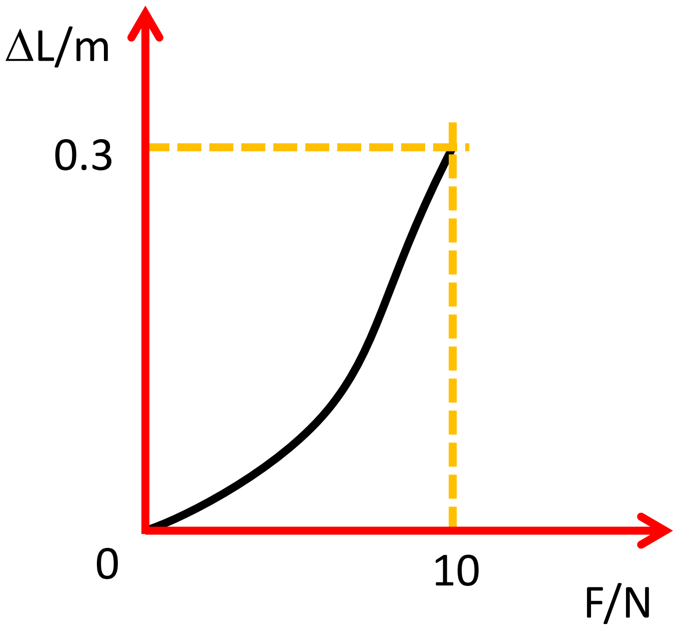
- Both materials have the same dimensions and break at the end of the graph.
- ____ is stronger as the max stress it can withstand breaking is larger.
- ____ is stiffer as it's gradient, i.e. Young modulus= stress/ strain is larger

examples

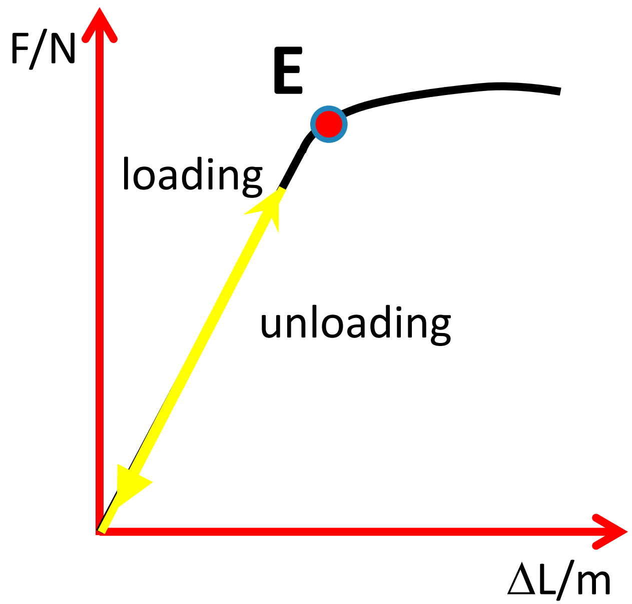
1. A support cable on a bridge has an area of cross-section of 0.0085 m^2 and a length of 35 m. it is made of high tensile steel which Young modulus is 2.8×10^{11}Pa. the tension in the cable is 720 kN. Calculate:
   i. the extension of the cable [10.6mm]
   ii. the strain energy stored in the cable [3.82kJ]

   What is the assumption made in the calculation of part (ii)?

2. Estimate strain energy stored when a wire is stretched to 0.3 m.
   A. 1.2 J
   B. 1.5 J
   C. 1.8 J
   D. 3.0 J



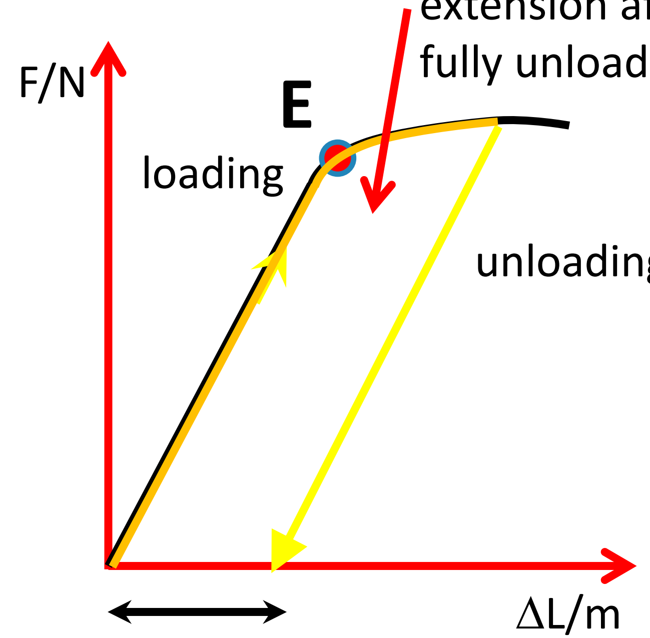
Strain energy in Elastic & Plastic deformations



EPE is stored during loading

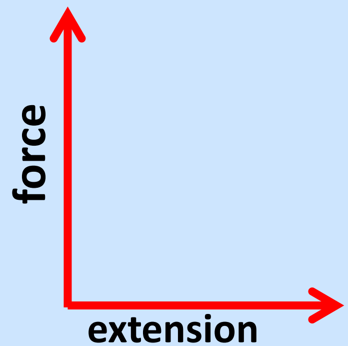
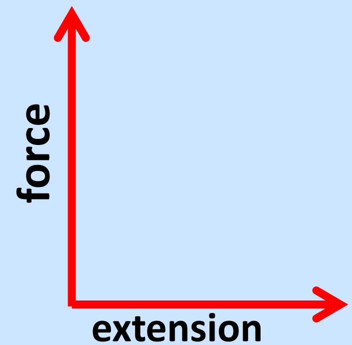
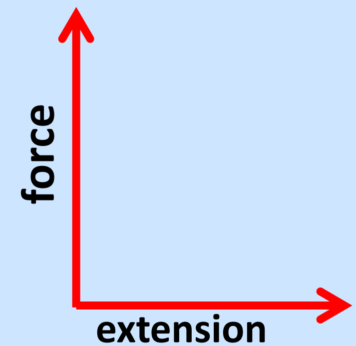
EPE is restored during unloading

Confined area = work done to produce permanent extension after it is fully unloaded.



Permanent extension

Stress-strain graphs
for ductile, brittle & polymeric materials

	Ductile	Brittle	Polymeric
graph of stress vs. strain	 obeys Hooke's law Elastic & Plastic deformations	 obeys Hooke's law It breaks after Elastic limit. Only elastic deformation.	 Does not obey Hooke's law.
Examples	✓ Metallic wires	✓ Ceramics, glass, porcelain	✓ PVC, polyethylene, rubber….

Springs

When a number of springs, with spring constant k1, k2 and so forth, are connected in series, the spring constant k of the combination is given by

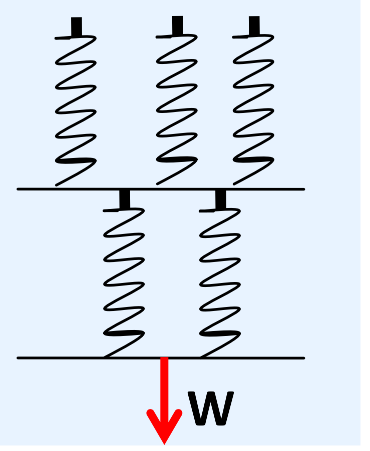
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

When a number of springs, with spring constant k1, k2 and so forth, are connected in parallel, the spring constant k of the combination is given by

$$k = k_1 + k_2 + \ldots$$

Examples

Five identical springs, each has a spring constant k and extends by x when a load of W is hung, are connected as shown below:



Express (i) the spring constant of the combination and (ii) the strain energy stored, in terms of k and x, where appropriate.