Deformation of Solids

Content

What is deformation of a solid?

- When an external force is applied, solids change shape or size or both. The change may be very small but nevertheless the spacing between the atoms is altered to a tiny extent causing its external dimensions to change.This change of shape is known as **deformation.**

- **Deformation is distortion or change in the configuration of the particles of a solid due to forces that change the length or thickness or shape of the solid.**

- **Stretching causes tensile deformation / tension.**

  - Causes material to become longer and thinner if its volume is constant.

- **Squashing causes compressive deformation / compression.**

  - Causes material to become shorter and thicker if its volume is constant.

- Material may experience both tension and compression at the same time, example if it is bent.

Characteristics of deformation

- When a metal wire or a spring is loaded at one end with weights and the other end is fixed, the increase in length of the wire or spring called **extension**, is **directly proportional** to the stretching force of the weights called **load**. The same is true for compression.

- For small loads, when the load is removed, the spring or wire **returns to its original shape, size and length**. The spring is said to have undergone an elastic change or **elastic deformation.**

- If the load is increased greatly, the spring or wire will be permanently changed in shape and it **does not return to its original shape** and the change is said to have undergone a plastic change or **plastic deformation.**

- This change from elastic deformation to plastic deformation causes a threshold to be exceeded known as the **elastic limit.**

Hooke's Law

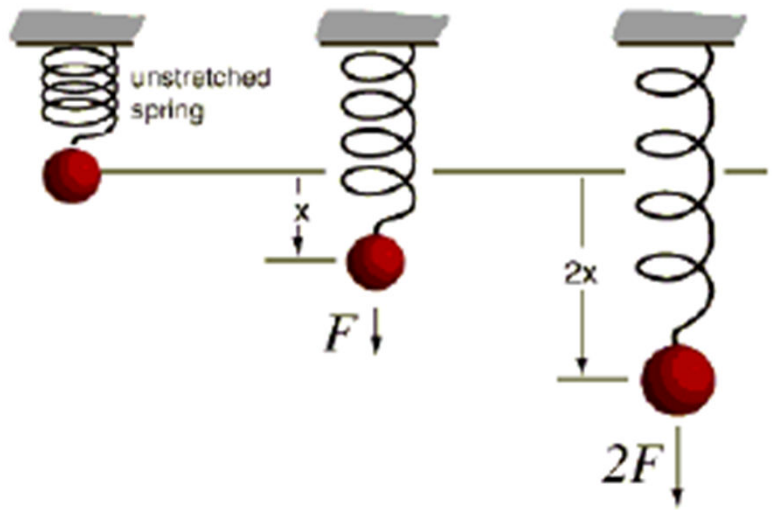
- **Hooke's Law states that, the extension of a spring is directly proportional to the load applied, (F) if the limit of proportionality is not exceeded.**
- Equation for Hooke's Law is

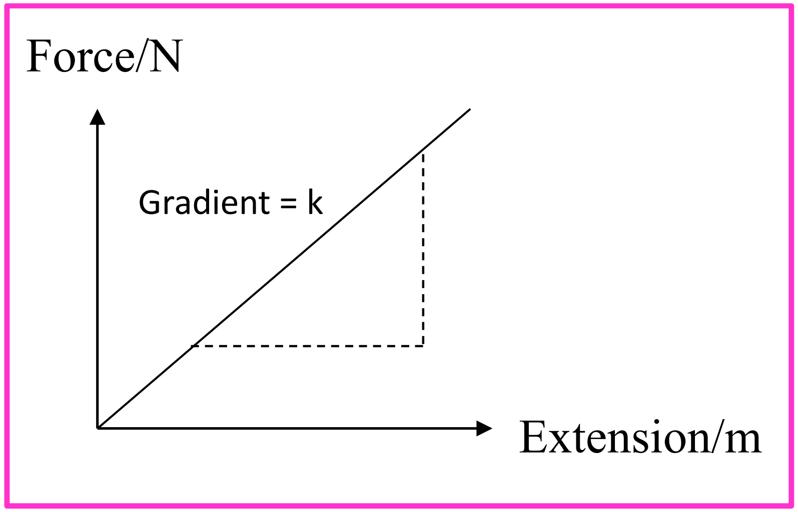
$F = kx$

where F is the force,

x is the extension

k is the **spring constant / force constant** (unit of k is Nm^{-1})

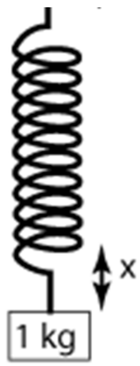




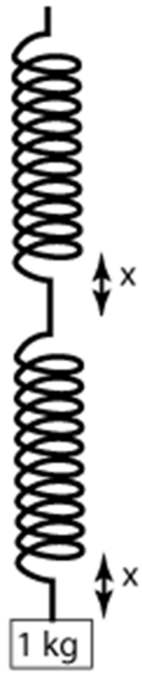
Hooke's Law

- Now, what will happen to the **spring constant, k** if there are few springs arranged together in series, parallel or both?

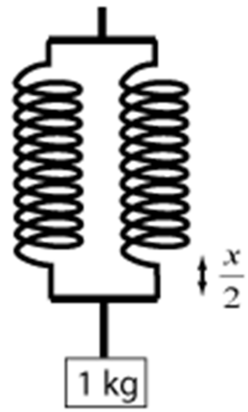
1 spring



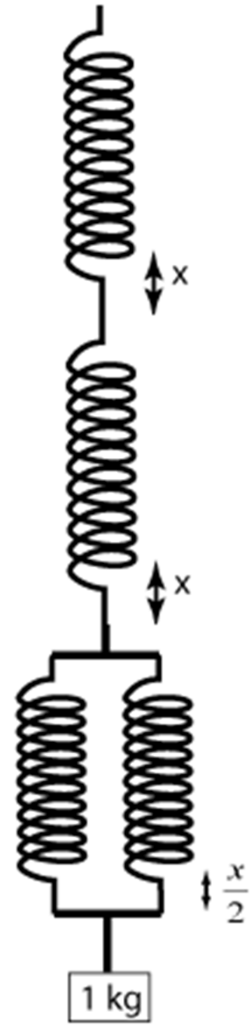
2 springs in series



2 springs in parallel



2 springs in series and 2 springs in parallel



Example 1

- A spiral spring is hung vertically from a fixed point. Its length when unloaded is 240 mm. When a mass of 0.15 kg is hung from its lower end, its length becomes 300 mm with the mass at rest. Calculate the spring constant of the spring.

- Two light springs P and Q are each 1.00 m long and have force constants of 98.0 N m^{-1} and 196 N m^{-1} respectively. They are joined end-to-end and P is attached to the ceiling. A mass of 2.00 kg is hung from the bottom of Q. Find:

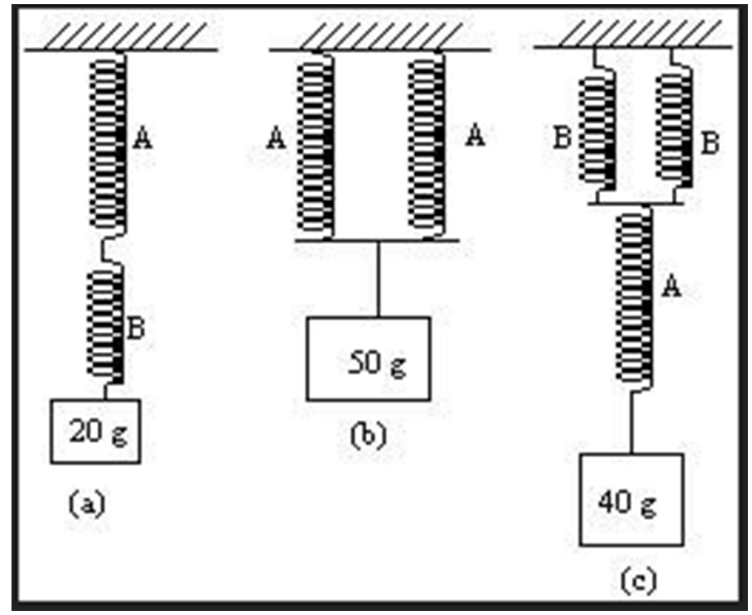
  a) the extension in P;

  b) the extension in Q;

  c) the force-constant of the combination.

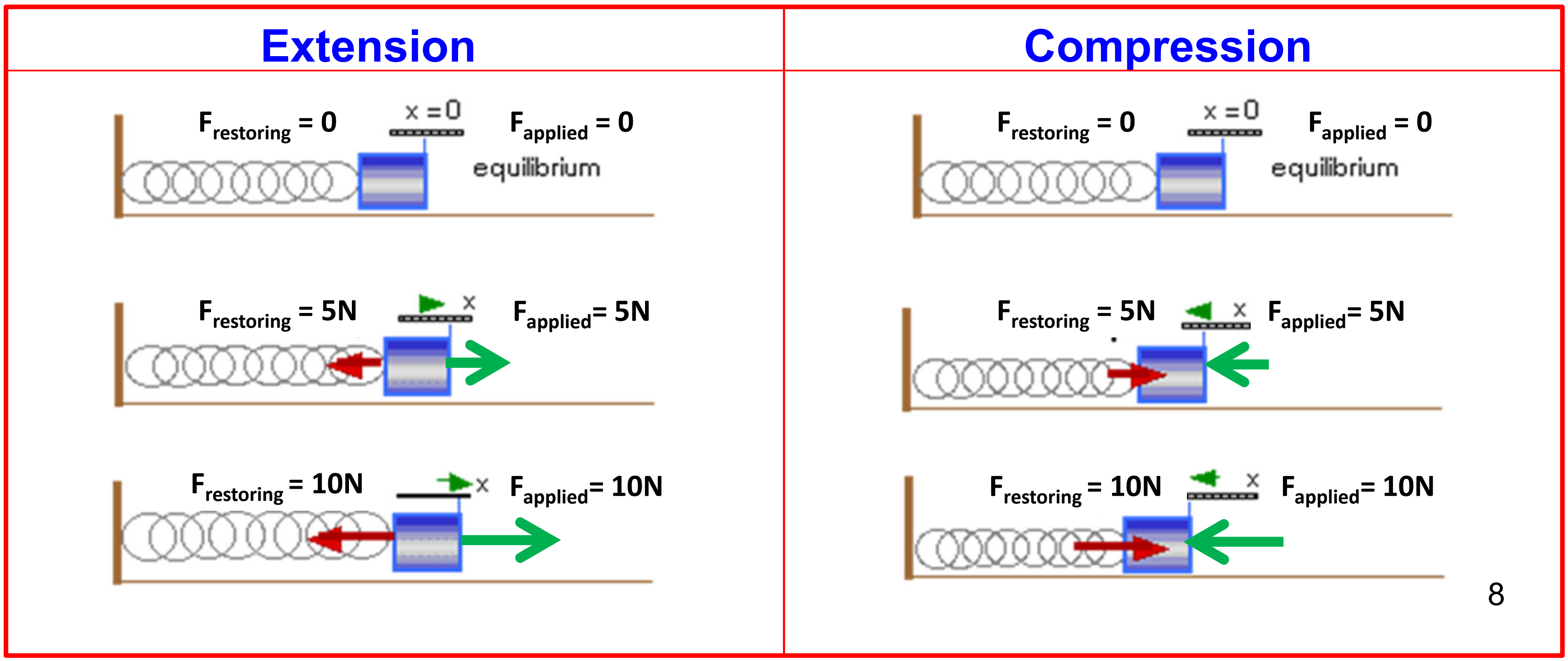
Example 2

- Spring A extends by 2 cm when it hung with a 10 g weight. Spring B extends by 4 cm when it hung with a 10g weight. Find the total stretch in each of the spring systems shown in the following figure on the right.



Restoring Force

- Restoring force is a variable force that gives rise to an equilibrium in a physical system.
- If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back toward equilibrium. The restoring force is always directed back toward the equilibrium position of the system.
- An example is the action of a spring. An idealized spring exerts a force that is proportional to the amount of deformation of the spring from its equilibrium length, exerted in a direction to oppose the deformation. Pulling the spring to a greater length or compressing the spring to a shorter length causes it to exert a force that brings the spring back toward its equilibrium length.



Stress and Strain

- **Stress** is defined as the force acting normally per unit cross sectional area.

$$Stress = \frac{Force\ (N)}{Area\ (m^2)}$$

$$(sigma)\sigma = \frac{F}{A}\ (Nm^{-2})$$

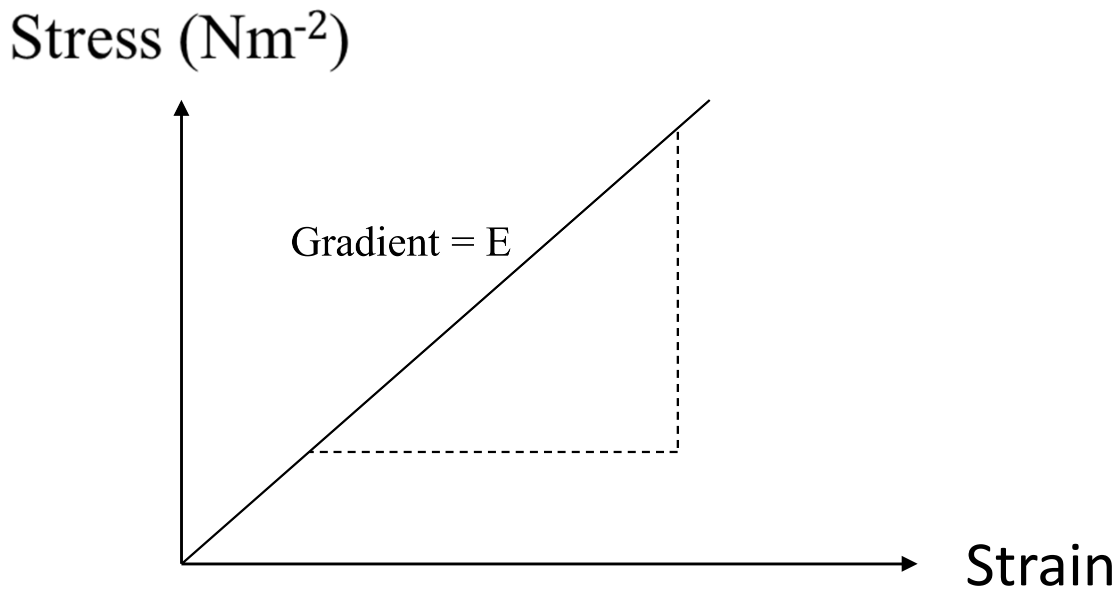
- **Strain** is the ratio of change in length (extension) to the original length.

$$Strain = \frac{extension\ (m)}{length\ (m)}$$

$$(epsilon)\varepsilon = \frac{e}{l}(unitless)$$

Stress and Strain Graph

- The graph of *stress vs strain* for a metal wire below proportional limit is shown below. It looks very much the same as that of *F vs x.*



- The gradient of the straight line equals to the Young Modulus, E.

The Young Modulus

- The **Young Modulus, E** is defined as **the ratio of stress to strain.**
- So we can write:

$$\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$$

- By substituting the equation for stress and strain into this above equation, we would obtain the equation for Young Modulus as:

$$\boxed{E = \frac{F \times l}{A \times e} = \frac{k \times l}{A}}$$

The Young Modulus

- Rearranging the equation in to find the extension would be:
- Extension, $e = \dfrac{F \times l}{A \times E}$
- Hence, extension in a wire will be greater,
  a.) for bigger applied external force, F
  b.) for thinner wires (smaller cross-sectional area, A)
  c.) for longer wires, l
  d.) for wires with smaller values of E (Young Modulus)
- Some typical values of Young's modulus, E for different materials

Aluminium	0.70×10^{11} Pa
Steel	1.1×10^{11} Pa
Copper	2.1×10^{11} Pa
Glass	0.41×10^{11} Pa
Carbon	4.1×10^{11} Pa
Rubber	0.00007×10^{11} Pa

Example 3

- A wire made of a particular material is loaded with a load of 500 N. The diameter of the wire is 1.0 mm. The length of the wire is 2.5 m, and it stretches 8 mm when under load. What is the Young Modulus of this material?

Example 4

- A wire A has length 150 cm and diameter 1.0 mm. A wire B has length 200 cm and diamter 2.0 mm. Forces of same magnitude pull both wires and the extensions of A and B produced are 0.10 mm and 0.08 mm respectively. Determine the ratio of the Young's Modulus for wire A to that for wire B.

Example 5

- An object of mass 3.0×10^3 kg is suspended from a steel cable of length 40 m and cross sectional area 1.0×10^{-4} m^2. Determine the extension of the cable when this mass-cable system

  a.) is at rest,

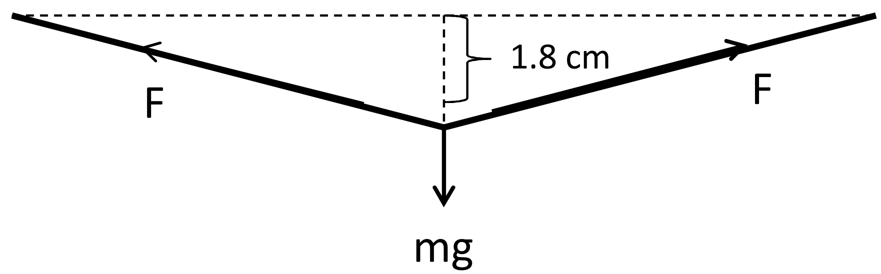
  b.) moves upwards at constant speed

  c.) moves upwards at constant acceleration 3.0 ms^{-2}

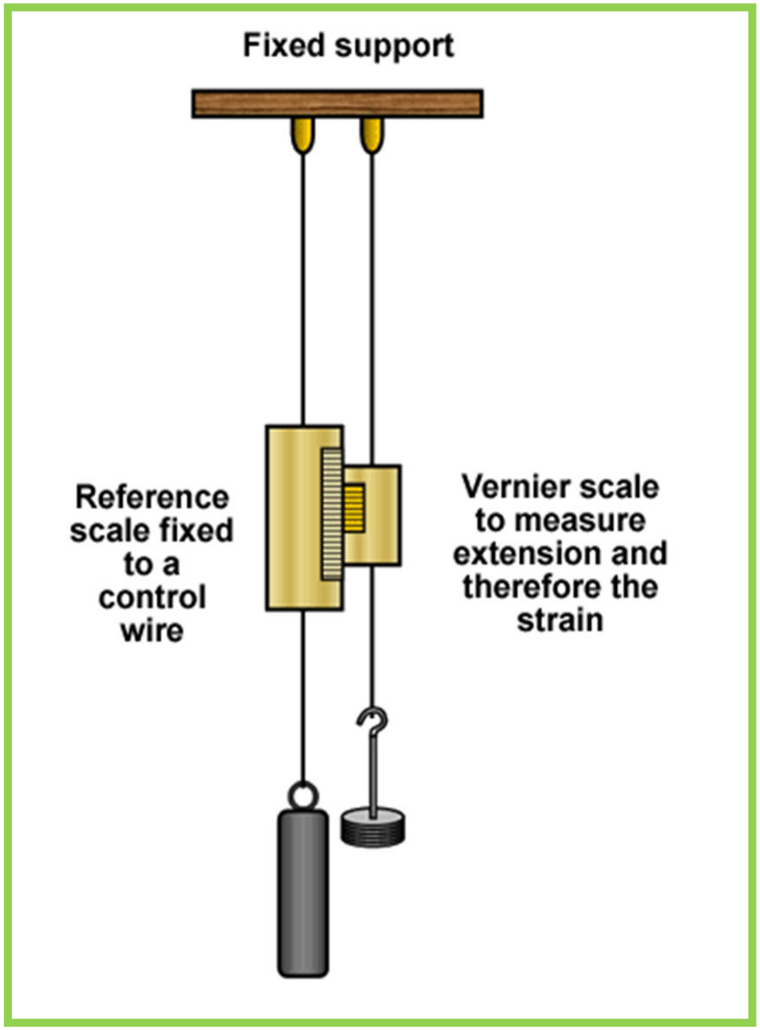
  Assuming the cross sectional area is constant. (Young's Modulus for steel is 2.0×10^{11} Pa)

Example 6

- Each end of a wire of diameter 2.0 mm Is held by a holder. The holders are 2.0 m apart and pass through a common horizontal line. The wire is straight and the tension in it is zero. A mass of 0.10 kg is then suspended in the middle of the wire. The mass is observed to have moved vertically downwards by 1.8 cm as a result of the elongation of the wire. Determine the Young's Modulus for the wire.



Young's Modulus Experiment



Young Modulus experiment

Measure the Young modulus of **steel** in the form of a wire, using the apparatus shown.

1.) Two long, identical steel wires are hung from a beam. The **test** wire and **reference** wire are linked by a vernier scale.

2.) A long wire is used to produce a big extension. A vernier scale is used to measure the extension.

3.) A reference wire is used to eliminate any inaccuracy due to changes in temperature and sagging of the support.

Young's Modulus Experiment

4.) Before we start adding weights to the test wire, we need to measure its dimension.

- Measure the length, l using ruler.
- Measure the diameter, d of the wire using m.s. gauge.
- The cross sectional area can be calculated, $A = \pi(d/2)^2$

5.) Load (F) is then placed gently on the wire and the extension of the wire is recorded. (reading from vernier scale). Repeat the test by increasing the load successively.

6.) To check that the proportionality limit is not exceeded, unloading is carried out to make sure that the wire returns to its original length.

7.) A graph of F against e is plotted which yields a straight line with gradient, m.

8.) From the equation of $E = \frac{(F \times l)}{(A \times e)}$, we can rearranged the equation to suit the *F against e* graph which will give us the equation, $F = \frac{(E \times A)}{l} \times e$

9.) Thus, gradient, $m = \frac{(E \times A)}{l}$; Young Modulus, **E** = $\boldsymbol{\frac{(m \times l)}{A}}$

WHAT HAPPEN WHEN MATTER UNDERGO DEFORMATION?

- Matter undergoes deformation when an amount of force is applied.
- Different type of matter experienced different scale of deformation. Most deformation occur in molecular scale, scale too small to be seen with human's naked eye.

ELASTIC & PLASTIC DEFORMATION

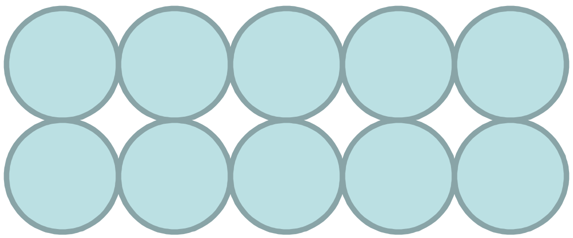
- Solid undergoes two types of deformation(distortion) when an amount of force is applied, which is ***elastic deformation*** and ***plastic deformation***.

- When force applied,

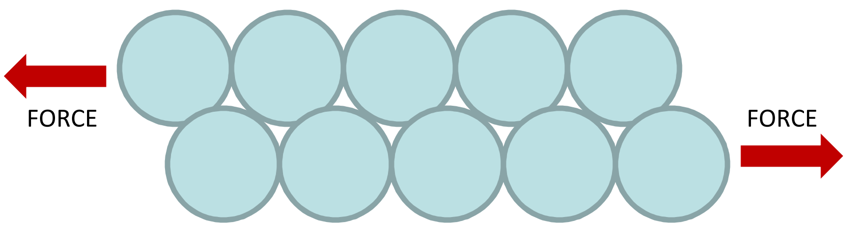
a.) Elastic deformation – The force applied did not exceed the elastic limit. Particle arrangement of the solid may change but it goes back to the original position when the force is removed.

b.) Plastic deformation – The force applied exceed the elastic limit. Particle arrangement of the solid change and maintain the shape after force applied even though force is removed after that.

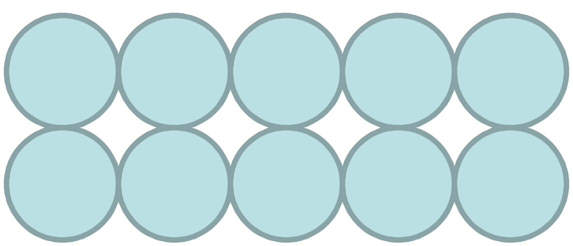
Elastic Deformation



When no force is applied, the particle arrange themselves orderly and in equilibrium position.

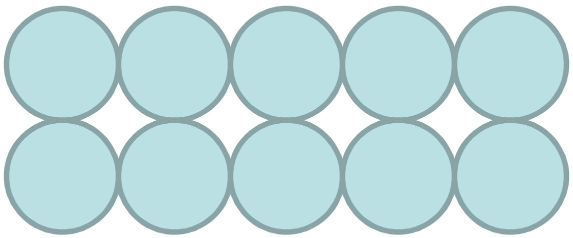


When an amount of force is applied, with condition that the force exerted did not exceed the elastic strain, the particle moved from their equilibrium position.

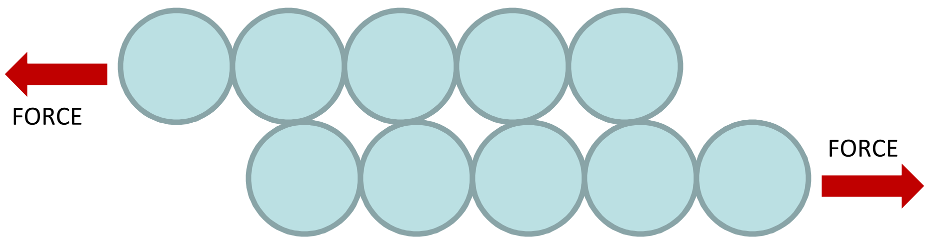


Since the force has been removed, the particle moved back to it's original position or equilibrium position.

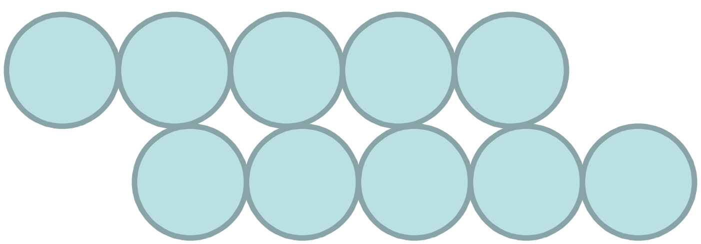
➢ Plastic Deformation



When no force is applied, the particle arrange themselve orderly and in equilibrium position.



When an amount of force is applied, with condition that the force exerted exceed the elastic limit / strain, the particle moved from their equilibrium position.



When force is removed, the particle did not change back to it's original position because the arrangement has changed in such a large scale that the attraction force between the particle unable to force the particle to restore it's original position.

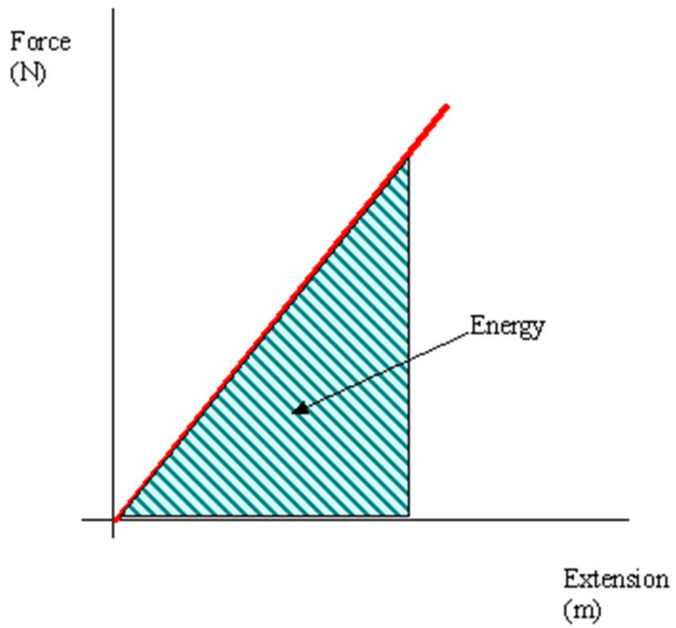
Elastic potential energy

- When an object has its shape altered by forces acting on it, the object is said to be ***strained.***
- When a force moves an object, work is done by the force. Hence, when a force stretches a spring or wire, work is done by the force.
- This work is stored as ***elastic potential energy*** in the spring a.k.a ***strain energy***.
- ***Work done*** = average force x extension

  = ½ Fx = ½ kx^2 since F = kx

  = ***Strain energy = Energy stored***

- **The area under the force extension graph represents the work done <u>even when the graph is not linear.</u>**

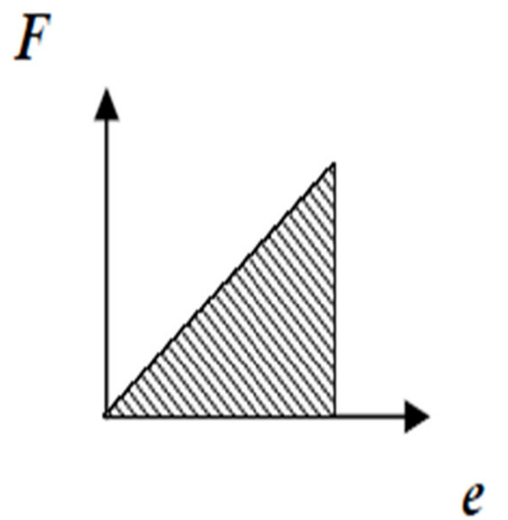


Example 7

What is the elastic strain energy contained in a copper wire of diameter 0.8 mm that has stretched by 4 mm under a load of 400 N?

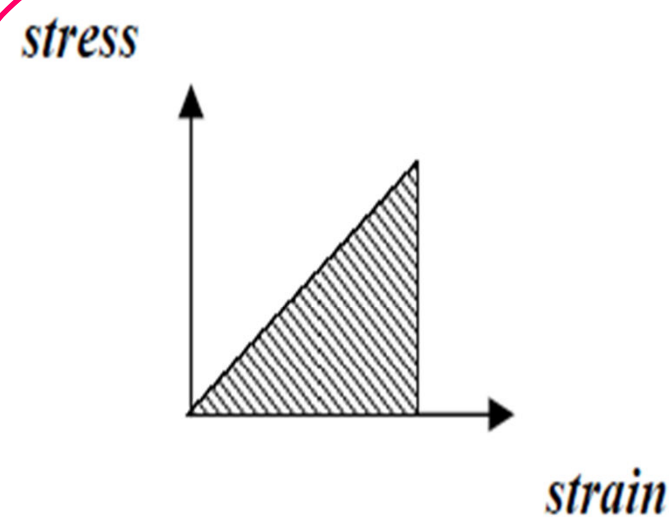
A spring has an elastic constant 65 N m^{-1} and is extended elastically by 1.2 cm. Calculate the strain energy in the spring.

Difference between force-extension & stress-strain graph



Gradient = F/e = spring constant k: force per unit extension

Area = strain energy/ elastic PE



Gradient = stress/ strain = young modulus : a measure of stiffness
The larger the gradient, the stiffer the material is.

Area = strain energy per unit volume

Elastic & Plastic Behaviour

- ***Strength*** - a material is said to be strong if it can withstand a force without breaking. A solid is ***strong*** if it has a ***high tensile stress***. The opposite will be ***weak.***

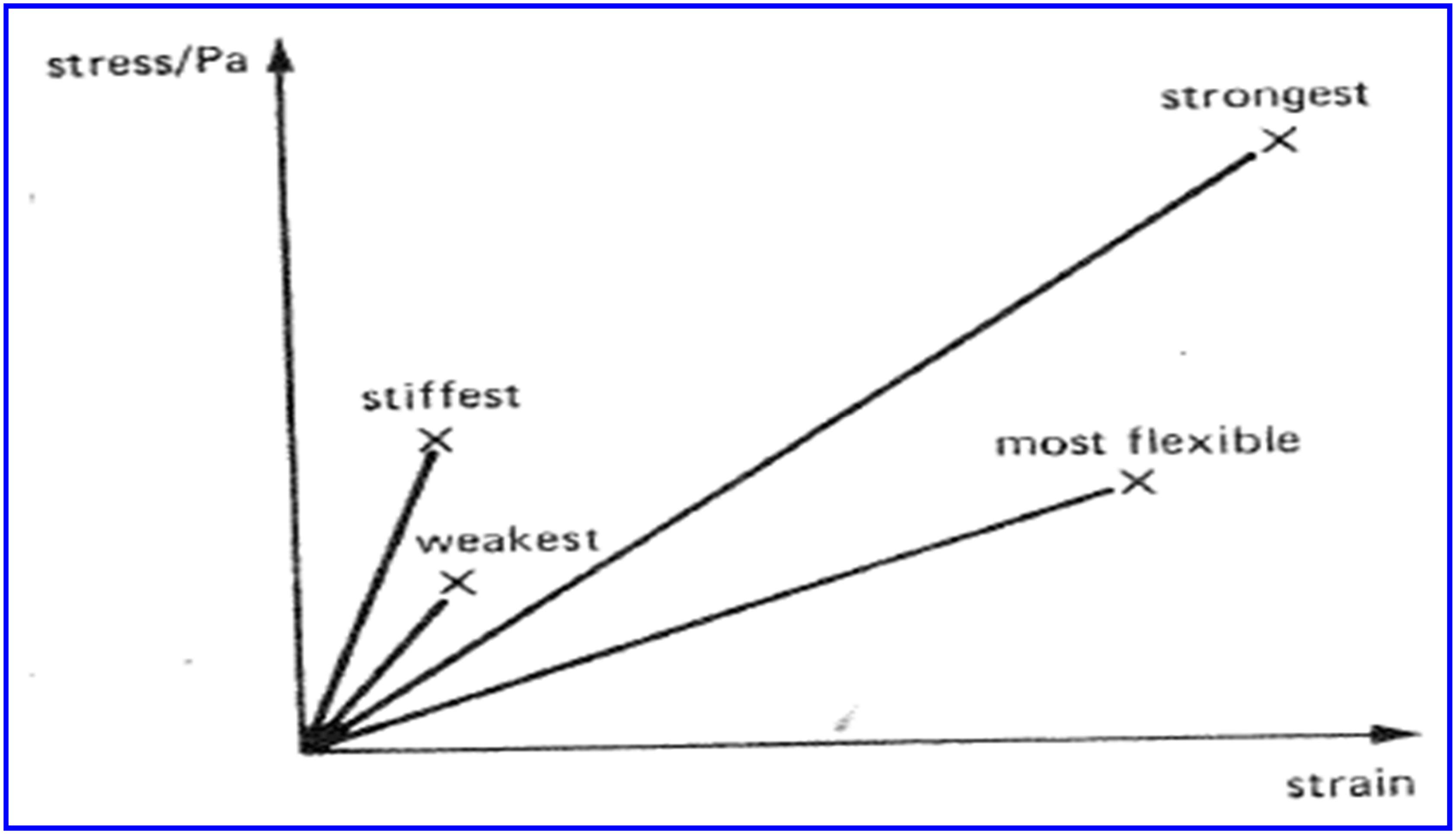
- ***Stiffness*** - tells us about a material's opposition to changes in shape and size. Flexibility is the opposite description. A solid is ***stiff*** if it has a ***high Young Modulus***. The opposite will be ***flexible / elastic.***

- ***Ductility*** - Able to suffer permanent deformation before fracture.

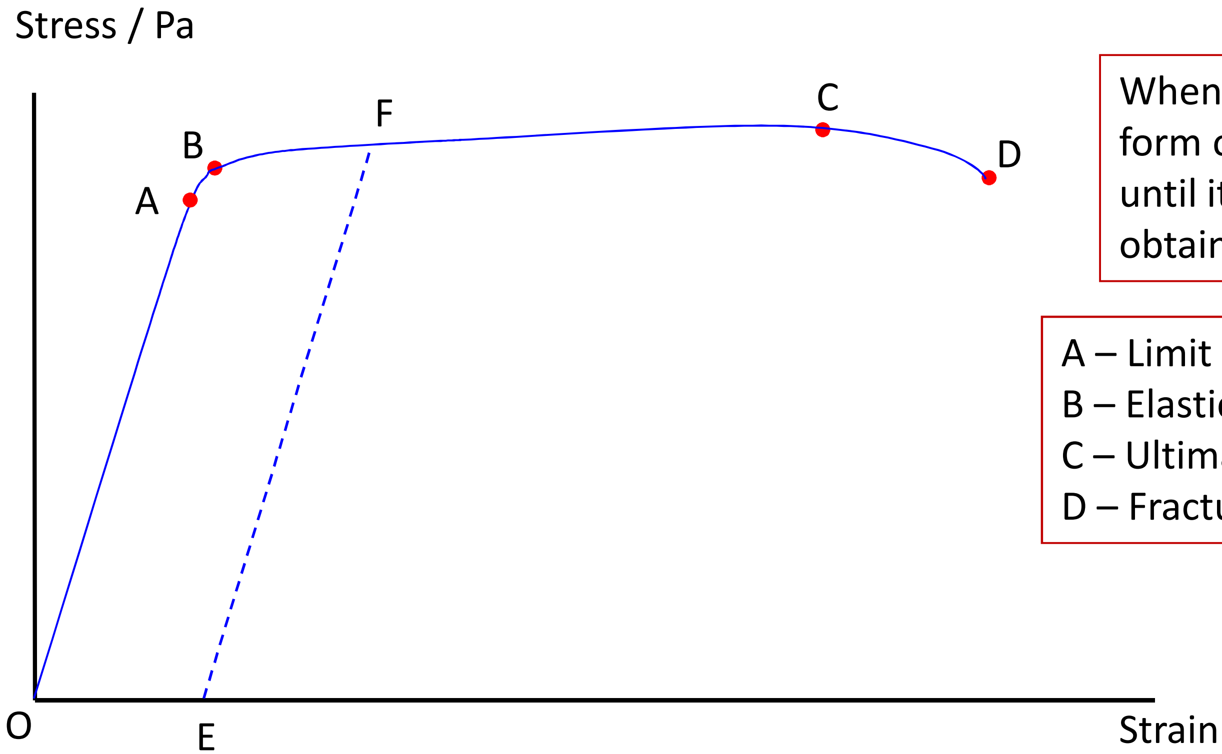
- ***Brittleness*** - Suffer little or no plastic deformation before fracture.

- ***Ultimate tensile stress*** - maximum force divided by the original cross sectional area of a wire ; i.e the maximum stress a material can withstand before breaking.

Elastic & Plastic Behaviour



Ductile Material (Copper)



When a ductile material (copper) in the form of a wire is progressively loaded until it breaks, the stress-strain graph obtained is as follows:

A – Limit of proportionality
B – Elastic limit
C – Ultimate tensile stress/ breaking point
D – Fracture point

- **Section *OA*** is called **proportionality region**. Strain of the wire is directly proportional to the applied stress and wire suffers elastic deformation. (wire returns to original length when the deforming stress is removed). ***A*** is the **limit of proportionality.**

Ductile Material (Copper)

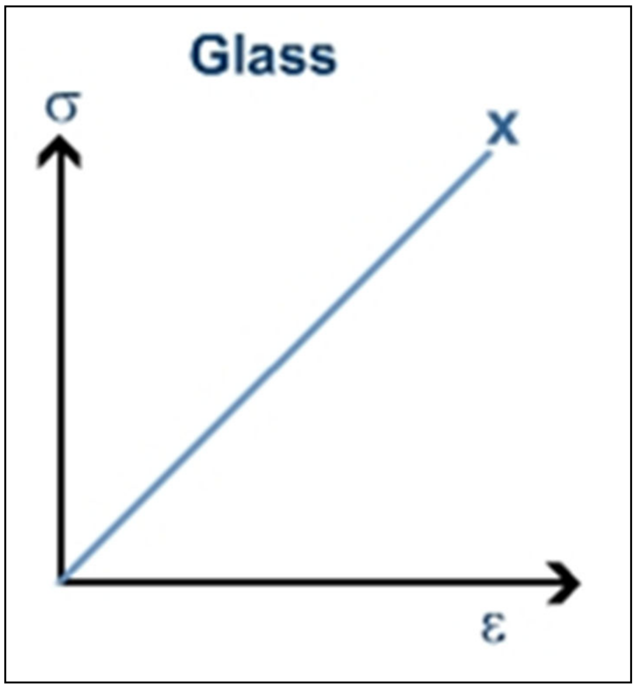
- **Section *AB*,** the strain is no longer proportional to the stress but the material still behaves elastically.
- **Beyond point *B*,** plastic deformation occurs where the wires retains permanently some of its extension after the stress is removed. For example, if the stress corresponding to point F is applied to the wire, a permanent **strain *EF*** will remain after the stress is being removed.
- ***Point C*** is the maximum tensile stress that the wire can withstand before breaking. ***Section CD*** is the region whereby the wire develops a **"waist"** , or often refer to as "**necking"** – before fracture occurs at ***point D.***

Proportionality limit vs. elastic limit

Proportionality limit is lower in position than elastic limit in the graph of force-extension or stress-strain for a stretched material. Up to proportionality limit, the material still obeys Hooke' law, i.e. the extension is proportional to the force exerted. And it is elastic. After proportional limit, it is still elastic but no longer obeys Hooke's law. After elastic limit is exceeded, for ductile material it performs plastic deformation.

Brittle Material (Glass)

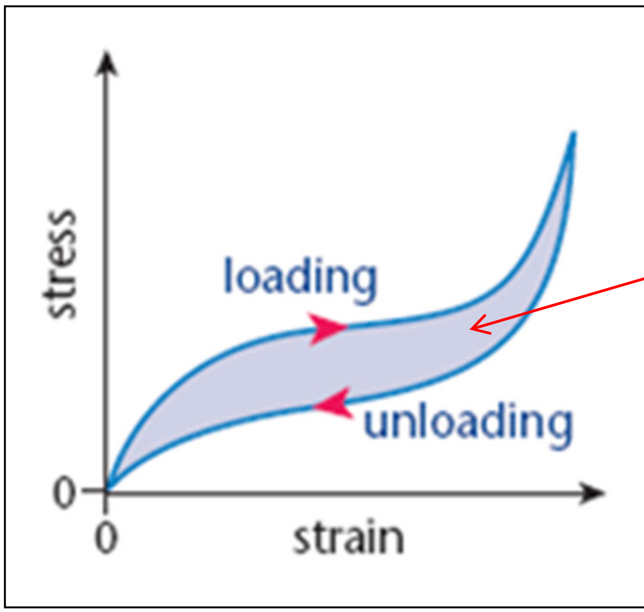
- A brittle material does not possess yield point and thus does not undergo significant plastic deformation.
- The material simply breaks when the stress is too great for it to withstand.



x – fracture point

Polymeric Material (Rubber)

- Different polymers behave differently, depending on their molecular structure and their temperature.
- However, it is extremely elastic. It breaks after a large extension has been produced.



This area is the energy lost as heat