SENIOR DIVISION

Questions 1 - 10, 3 marks each

1. The value of $\frac{2004+6}{100}$ is

(A) 30 (B) 2.1 (C) 201 (D) 20.1 (E) 2.01

2. The value of $\frac{4}{5}$ is closest to

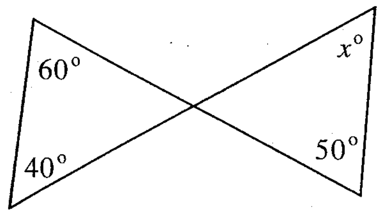
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. If $y = 3x$ and $z = 2 - 3y$, then z equals

(A) $9x$ (B) $2-3x$ (C) $8x-2$ (D) $2+3x$ (E) $2-9x$

4. The value of x in the diagram is

(A) 50 (B) 100 (C) 80
(D) 40 (E) 70



5. If $2x + 3 > 9$ then

(A) $x > 3$ (B) $x \le 6$ (C) $x > 6$ (D) $x < 3$ (E) $x \le 3$

6. If $2^{15} = 4 \times 2^n$, what is the value of n?

(A) 11 (B) 13 (C) 14 (D) 16 (E) 17

7. A rectangle has its length 25 times its width. What is the ratio of its perimeter to the perimeter of the square of the same area?

(A) 13 : 5 (B) 13 : 10 (C) 5 : 1 (D) 51 : 20 (E) 51 : 10

8. If a person's wage rose 20% to \$360 per week, the wage before the rise was

(A) \$288 (B) \$300 (C) \$310 (D) \$280 (E) \$320

9. I have a patio which is 3 m wide. The roof of this consists of sheets of plastic which are 900 mm wide. I have used as few sheets as possible and made all overlaps the same width. What is the width, in millimetres, of this overlap?

(A) 100 (B) 150 (C) 200 (D) 250 (E) 300

10. Consider the five expressions $\sqrt{x}$, x^2, $\frac{1}{\sqrt{x}}$, x^3 and $2x$. If $x > 2$ and these five expressions are arranged in ascending order of magnitude, the middle one will be

(A) $\sqrt{x}$ (B) x^2 (C) $\frac{1}{\sqrt{x}}$ (D) x^3 (E) $2x$

Questions 11 - 20, 4 marks each

11. As one way of saving water during the drought, Holly changed from using the washing machine three times a week to using it once every three days. If the machine uses 120 L of water each time, over a long period the average number of litres saved each week was

(A) 60 (B) 72 (C) 80 (D) 90 (E) 96

12. Which of the following is the sum of four consecutive integers?

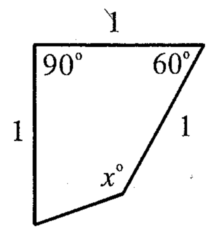
(A) 2000 (B) 2001 (C) 2002 (D) 2003 (E) 2004

13. The yearly changes in population of a mining town for four consecutive years were, respectively, 20% increase, 20% increase, 20% decrease and 20% decrease. The net change over the four years, to the nearest percent, was

(A) -8 (B) -4 (C) 0 (D) 4 (E) 8

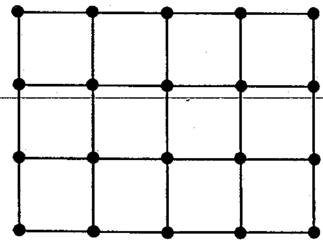
14. In the diagram, the value of x is

(A) 90 (B) 120
(C) 135 (D) 137.5
(E) 140



15. Two ordinary dice are tossed and the difference between the numbers appearing uppermost on the dice is recorded. What difference is most likely to occur?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. In the country Roadonia there are exactly 20 cities and 31 roads connecting neighbouring cities as shown in the diagram. Unfortunately, all the roads are in bad condition and need repair. What is the maximum number of roads which can be closed for repair at the same time so that it is still possible to travel from each city to any other along roads?



(A) 10 (B) 12 (C) 13 (D) 14 (E) 16

17. Natural fruit juice contains 80% water. In concentrating the juice, 75% of the water is removed. What is the percentage of water in the concentrated juice?
(A) 25 (B) 40 (C) 50 (D) 60 (E) 75

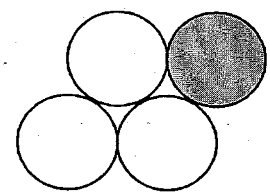
18. John tells the truth on Monday, Tuesday, Wednesday and Thursday. He lies on all other days. Dieter tells the truth on Monday, Friday, Saturday and Sunday. He lies on all other days. One day they both said, 'Yesterday I lied'. The day they said that was
(A) Monday (B) Wednesday (C) Thursday
(D) Friday (E) Saturday

19. If a, b and c are positive integers such that

$$a+\cfrac{1}{b+\cfrac{1}{c}}=\frac{37}{16},$$

then $a+b+c$ is equal to
(A) 10 (B) 16 (C) 21 (D) 14 (E) 11

20. Four 10 c coins lie on a table as shown. Keeping in contact with the other three coins, the shaded coin is rolled around the other three coins until it returns to its starting place. Through what angle does the shaded coin turn, on its axis, in rolling once around the other three coins?



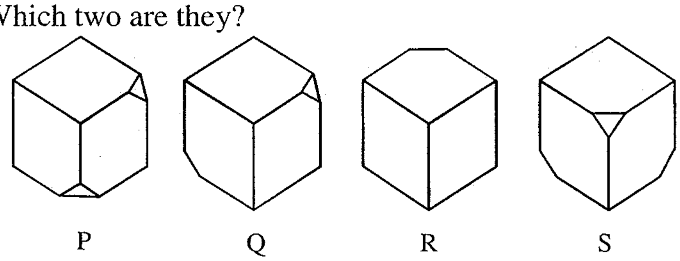
(A) 360° (B) 540° (C) 720°
(D) 900° (E) 1080°

Questions 21-30: MINIMUM 30 MARKS FOR THIS SECTION GUARANTEED
8 marks each correct response, 0 marks each incorrect response, 3 marks each no response. If your total for this section is less than 30, you will score 30 marks.

21. Let $PQRSTU$ be a convex hexagon (every angle is less than $180°$). The lines defined by the sides PQ, RS and TU intersect at the vertices of an equilateral triangle and so do the lines formed from the sides QR, ST and UP. At most, how many different angle sizes does the hexagon have?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

22. Ann, Ben and Cathy have their birthday today. The sum of their ages is 23. The product of their ages is 113 more than the product of their ages on their birthday last year. What is the sum of the squares of their ages?
(A) 209 (B) 185 (C) 189 (D) 241 (E) 259

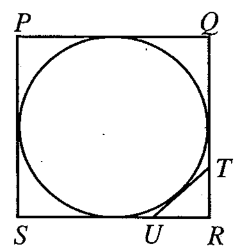
23. Some corners are cut off four cubes. Afterwards, only two of the solids formed are the same shape. Which two are they?



(A) P and Q (B) P and R (C) Q and R (D) P and S (E) Q and S

24. The integers 1, 2, 3, . . . , 100 are written on the board. What is the smallest number of these integers that can be wiped off so that the product of the remaining integers ends in 2?
(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

25. In the diagram, the square circumscribes the circle, UT is tangent to the circle and RU is one quarter of RS. What fraction of RQ is RT?



(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{3}{8}$
(D) $\frac{3}{10}$ (E) $\frac{2}{9}$

26. Seven numbers, each 1 or -1, are listed in a row in such a way that adding the numbers successively from left to right never gives a negative answer. For example, $1\ -1\ 1\ 1\ -1\ -1\ 1$ has successive sums 1, 0, 1, 2, 1, 0, 1 and is valid, while $1\ 1\ -1\ -1\ -1\ 1\ 1$ has successive sums 1, 2, 1, 0, -1, 0, 1, and is not valid. How many valid lists are there?

(A) 35 (B) 34 (C) 33 (D) 32 (E) 31

27. What is the largest possible size of an angle of a triangle formed by joining the midpoints of three edges of a cube?

(A) 60° (B) 90° (C) 120° (D) 135° (E) 150°

28. There are exactly 3 integers x satisfying the inequality

$$x^2 + bx + 2 \leq 0.$$

How many integer values of b are possible?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

29. The smallest possible value that

$$\sqrt{x^2+y^2}+\sqrt{(x-1)^2+y^2}+\sqrt{x^2+(y-1)^2}+\sqrt{(x-3)^2+(y-4)^2}$$

can have is

(A) 5 (B) $4+\sqrt{3}$ (C) 6 (D) $5+\sqrt{2}$ (E) 7

30. Lois and Ben are playing a game with red, yellow, green and blue counters. They are making as long a line as possible while obeying the following two rules:-

(1) No two adjacent counters can be the same colour.
(2) If, in the sequence, any colour occurs twice, no colour between them can occur elsewhere.

Thus *rygbgg* would be banned by only the first rule, and *rbgygbrg* would be banned by only the second rule.

Lois has started her line with *ryr* and Ben has started his with *ryg*.
Which of the following statements is true?

(A) It is possible for Lois to make a longer sequence than Ben can make.
(B) It is possible for Ben to make a longer sequence than Lois can make.
(C) It is possible for both to make sequences of length 6, but no longer.
(D) It is possible for both to make sequences of length 7, but no longer.
(E) There is no limit on the length of the sequence either can make.

SENIOR DIVISION

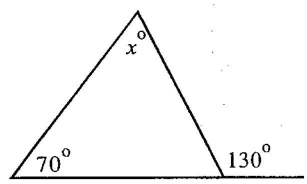
Questions 1 - 10, 3 marks each

1. The value of $(4 \times 5) \div (2 \times 10)$ is

(A) 1 (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{2}$ (E) 1

2. In the diagram, the value of x is

(A) 20 (B) 90
(C) 30 (D) 80
(E) 60



3. $1+\cfrac{1}{3+\cfrac{1}{2}}$ equals

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{7}$

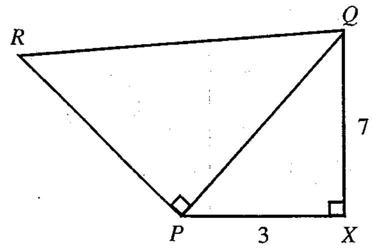
4. The straight line $y = x + g$ passes throught the point (2,3). The value of g is

(A) 0 (B) 1 (C) 2 (D) 3 (E) -1

5. A two-digit number has tens digit t and its units digit u. If the digit 8 is placed between these digits, the value of the three-digit number is

(A) $t + u + 8$ (B) $10t + 80 + u$ (C) $10t + u + 8$
(D) $100t + 10u + 8$ (E) $100t + 80 + u$

6. ΔPXQ is a right angled triangle with sides of length 3 and 7 as shown. At P, PR is drawn so that $\angle RPQ = 90°$ and $PR = PQ$. The area of ΔPRQ is



(A) $\frac{21}{2}$ (B) 29 (C) $\sqrt{58}$
(D) 58 (E) 100

7. In our school the average mark in Year 11 for a test was 70 and in Year 12 it was 80 for the same test. There were 20 students in Year 11 and 30 students in Year 12 who sat the test. The average mark for the two groups was

(A) 72 (B) 75 (C) 76 (D) 78 (E) 74

8. Different tyres were fitted to a car, increasing the circumference of the wheels from 200 cm to 225 cm. On a journey of 1800 km, the number of revolutions of each wheel was reduced by

(A) 50 000 (B) 1000 (C) 2000 (D) 100 000 (E) 7 200 000

9. The sum of all but one of the internal angles of a pentagon is $400°$. The number of degrees in the remaining angle is

(A) 40 (B) 120 (C) 140 (D) 160 (E) 400

10. The value of $\sqrt[4]{2}\times\sqrt{32\sqrt{2}}$ is

(A) 8 (B) 4 (C) $4\sqrt{2}$ (D) $4\sqrt[4]{2}$ (E) $16\sqrt[4]{2}$

Questions 11 to 20, 4 marks each

11. The difference between a positive fraction and its reciprocal is $\frac{9}{20}$. The sum of the fraction and its reciprocal is

(A) $\frac{20}{9}$ (B) $\frac{41}{20}$ (C) $\frac{25}{16}$
(D) 5 (E) not uniquely determined

12. At time $t = 0$ a split forms in a balloon and the quantity Q of gas left in the balloon at time t is given by

$$Q=\frac{100}{(1+2t)^2}$$

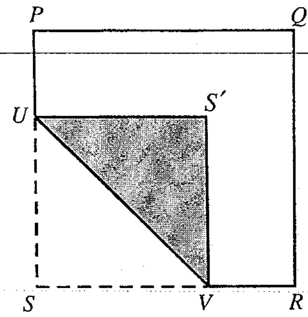
The time taken for half the gas to escape is

(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1+\sqrt{2}}{2}$ (D) $\sqrt{2}$ (E) $\frac{10\sqrt{2}-1}{10}$

13. Two dice are thrown at random. The probability that the two numbers obtained are the two digits of a perfect square is

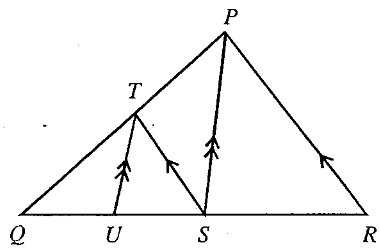
(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{7}{36}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

14. A square piece of paper has area 12 cm^2. It is coloured white on one side and shaded on the other. One corner of the paper has been folded over so that the sides of the triangle formed are parallel to the sides of the square as shown. The total visible area of the paper is half shaded and half white. What is the length, in centimetres, of the fold line UV?



(A) 4 (B) $\sqrt{12}$ (C) 3
(D) 6 (E) $\sqrt{8}$

15. In the triangle PQR shown, S and U are points on QR and T is a point on PQ such that $TS \parallel PR$ and $UT \parallel SP$. If QS = 4cm and SR = 2.4 cm, then the length of QU, in centimetres, is

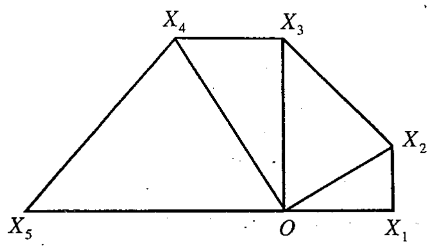


(A) 2.4 (B) 2.5 (C) 3
(D) 3.2 (E) 4

16. A train leaves Canberra for Sydney at 12 noon, and another train leaves Sydney for Canberra forty minutes later. Both trains follow the same route and travel at the same uniform speed, taking $3\frac{1}{2}$ hours to complete the journey. At what time will they pass?

(A) 1:45 pm (B) 2:00pm (C) 2:05 pm (D) 2:15pm (E) 2:25 pm

17. A spiral is formed by starting with an isosceles right-angled triangle OX_1X_2, where OX_1 is of length 1, then using the hypotenuse OX_2 as a shorter side of another isosceles right-angled triangle, and so on. The first few steps are shown in the diagram. Eventually we will reach for the first time a situation where a side OX_k of a triangle overlaps OX_1. What is the length of X_1X_k?

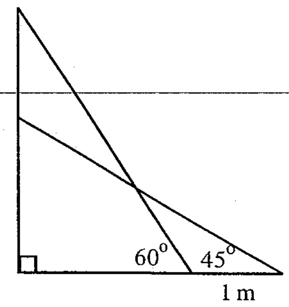


(A) 8 (B) $8\sqrt{2}-1$ (C) $8\sqrt{2}$ (D) 15 (E) 14

18. The number of 5-digit numbers in which every two neighbouring digits differ by 3 is

(A) 40 (B) 41 (C) 43 (D) 45 (E) 50

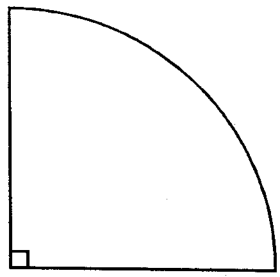
19. A ladder resting against a wall makes an angle of $60°$ with the ground. When the base of the ladder is moved 1m further from the wall it makes an angle of $45°$ with the ground. The length of the ladder, in metres, is

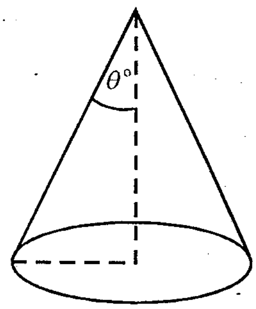


(A) 2 (B) $2(\sqrt{2}+1)$ (C) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

(D) $\sqrt{5}$ (E) $\frac{2}{\sqrt{2}+1}$

20. A quarter circle is folded to form a cone.





If $\theta°$ is the angle between the axis of symmetry and the slant height of the cone, then $\sin\ \theta°$ equals

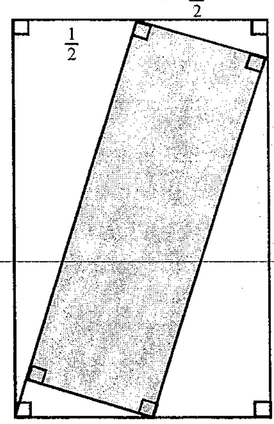
(A) $\frac{1}{4}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1}{\sqrt{3}}$

Questions 21 to 30, 5 marks each

21. The number of real solutions of $x+\sqrt{x^2+\sqrt{x^3+1}}=1$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. The area of the shaded rectangle is



(A) between $\frac{1}{4}$ and $\frac{5}{16}$

(B) between $\frac{5}{16}$ and $\frac{3}{8}$

(C) between $\frac{3}{8}$ and $\frac{7}{16}$

(D) between $\frac{7}{16}$ and $\frac{1}{2}$

(E) more than $\frac{1}{2}$

23. When $(1-2x)^3(1+kx)^2$ is expanded, two values k_1 and k_2 of k give the coefficient of x^2 as 40. The value of k_1+k_2 is

(A) -1 (B) 8 (C) 10 (D) 12 (E) 14

24. What is the area, in square units, enclosed by the figure whose boundary points satisfy

$$|x| + |y| = 4?$$

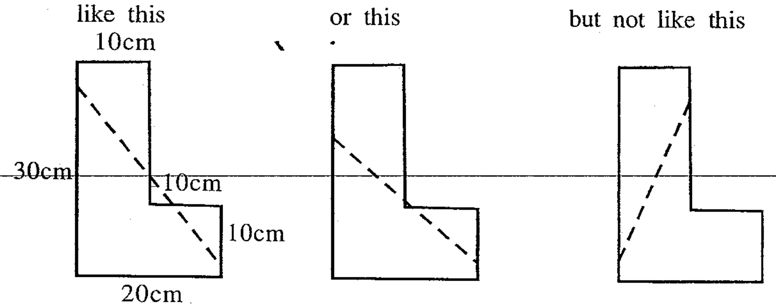
(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

25. The number of digits in the decimal expansion of 2^{2005} is closest to

(A) 400 (B) 500 (C) 600 (D) 700 (E) 800

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

26. My name is Louis and my father has cooked me an L-shaped cake for my birthday. He says that I must cut it into three pieces with a single cut, so that my brother and sister can have a piece too. So, I have to cut it



He says that I have to be polite and let them have the first choice of the pieces, but I just know they'll be greedy and leave the smallest possible piece for me. So I want to cut the cake so that my little piece will be as big as possible. If I do this, how big, in square centimetres, will my piece be?

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27. The function $y = f(x)$ is a function such that $f(f(x)) = 6x - 2005$ for every real number x. An integer t satisfies the equation $f(t) = 6t - 2005$. What is this value of t?

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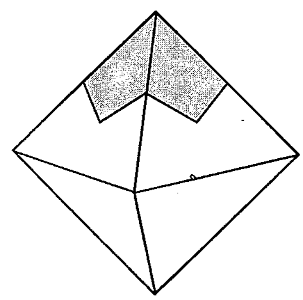
28. A regular octahedron has eight triangular faces and all sides the same length. A portion of a regular octahedron of volume 120 cm^3 consists of that part of it which is closer to the top vertex than to any other one. In the diagram, the outside part of this volume is shown shaded, and it extends down to the centre of the octahedron. What is the volume, in cubic centimetres, of this unusually shaped portion?



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29. If x, y and z satisfy the system of equations

$$x + y + z = 5$$
$$x^2 + y^2 + z^2 = 15$$
$$xy = z^2,$$

determine the value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$.

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30. A positive integer is equal to the sum of the squares of its four smallest positive divisors. What is the largest prime that divides this positive integer?

SENIOR DIVISION

Questions 1 - 10, 3 marks each

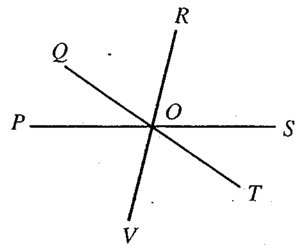
1. The value of $\dfrac{6\times 25}{3\times 5\times 2}$ is

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

2. If $a = 2b - 5$, then b equals

(A) $\dfrac{a}{2}$ (B) $\dfrac{a}{2}+5$ (C) $\dfrac{a-5}{2}$ (D) $\dfrac{a+5}{2}$ (E) $2a+5$

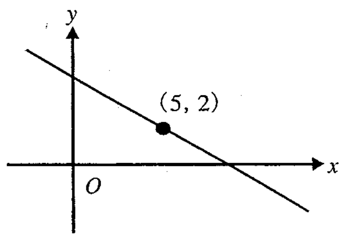
3. In the diagram, $\angle POR = 120°$ and $\angle QOS = 145°$.
The size of $\angle TOV$ is
(A) 45° (B) 60°
(C) 85° (D) 90°
(E) 95°



4. Which of the following is equal to $\dfrac{7}{x^2}$?

(A) $(7x)^{-2}$ (B) $\dfrac{1}{7x}$ (C) $\dfrac{1}{7x^2}$ (D) $\dfrac{x^2}{7}$ (E) $7x^{-2}$

5. In the figure, if the line has gradient -1, what is the y-intercept?
(A) 4 (B) 2
(C) 6 (D) 7
(E) 5



6. The pages of a book are consecutive whole numbers. If you begin reading at the top of page x and stop reading at the bottom of page y, the number of pages you have read is
(A) $x-y$ (B) $y-x$ (C) $x+y$ (D) $y-x+1$ (E) $y-x-1$

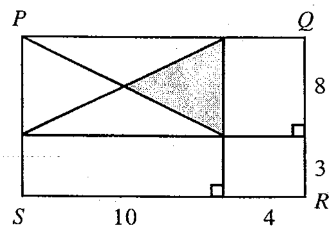
7. A rectangular box has faces with areas of 35, 60 and 84 square centimetres. The volume of the box, in cubic centimetres, is

(A) 420 (B) 480 (C) 512 (D) 563 (E) 635

8. If $x = 3^n + 3^n + 3^n$, which of the following is equal to x^2?

(A) 9^{3n} (B) 3^{2n+2} (C) 27^{2n} (D) 3^{2n} (E) 3^{n^2+6n+9}

9. What fraction of the rectangle $PQRS$ in the diagram is shaded?



(A) $\frac{1}{16}$ (B) $\frac{3}{5}$ (C) $\frac{1}{8}$

(D) $\frac{1}{10}$ (E) $\frac{10}{77}$

10. A train travelling at constant speed takes a quarter of a minute to pass a signpost and takes three-quarters of a minute to pass completely through a tunnel which is 600m in length. The speed of the train, in kilometres per hour, is

(A) 50 (B) 56 (C) 64 (D) 72 (E) 80

Questions 11 to 20, 4 marks each

11. In a container are 8 red, 3 white and 9 blue balls. If 3 balls are selected at random, the probability of getting 2 red balls and 1 white ball is

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $\frac{7}{285}$ (D) $\frac{2}{3}$ (E) $\frac{7}{95}$

12. The number of digits in the answer to the product $16^8 \times 5^{25}$ is

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

13. If $x < y < 0 < z$, which of the following must be true?

(A) $x + y + z > 0$ (B) $(x + y)^2 - z > 0$ (C) $x + y + z^2 > 0$

(D) $x + y - z > 0$ (E) $x + y - z < 0$

14. In a triangle PQR, $\sin\angle P = \frac{1}{3}$ and $\sin\angle Q = \frac{1}{4}$. How many different values can the size of $\angle R$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. How many different pairs of 2-digit numbers multiply to give a 3-digit number with all digits the same?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

16. I have 450 grams of salt and flour mix. How many grams of flour should I add to reduce the percentage of salt in the mixture to 90% of what it was?

(A) 50 (B) 10 (C) 30 (D) 45 (E) 60

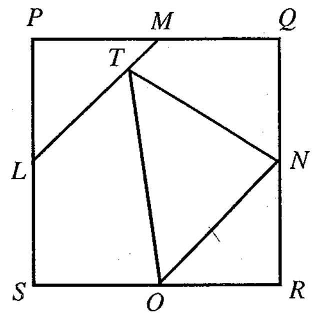
17. Five bales of hay are weighed two at a time in all possible combinations. The weights, in kilograms, are:-

110, 112, 113, 114, 115, 116, 117, 118, 120 and 121.

What is the weight, in kilograms, of the heaviest bale?

(A) 58 (B) 59 (C) 60 (D) 61 (E) 62

18. In the diagram, $PQRS$ is a square of side 2 units. M, N, O and L are the midpoints of PQ, QR, RS and SP respectively, and T is a point on LM.



The area, in square units, of ΔTNO is

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$

19. If $7^{x+1} - 7^{x-1} = 336\sqrt{7}$, then the value of x is

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{-3}{2}$ (D) $\frac{7}{2}$ (E) $\frac{1}{2}$

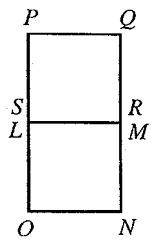
20. The nine squares of a 3×3 grid painted on a wall are to be coloured red, white and blue so that no row or column contains squares of the same colour. One such pattern is shown in the diagram. How many different patterns can be made?

R	W	B
B	R	W
W	B	R

(A) 15 (B) 6 (C) 9
(D) 12 (E) 24

Questions 21 to 30, 5 marks each

21. The squares $PQRS$ and $LMNO$ have equal sides of 1m and are initially placed so that the side SR touches LM as shown. The square $PQRS$ is rotated about R until Q coincides with N. The square is then rotated about Q until P coincides with O.
It is then rotated about P until S coincides with L and then finally rotated about S until R coincides with M and the square is now back to its original position.



The length, in metres, of the path traced out by the point P in these rotations is

(A) $\pi(2+\sqrt{2})$ (B) 4π (C) $2\pi(2+\sqrt{2})$ (D) 2π (E) $\pi(3+\sqrt{2})$

22. The vertices of a cube are each labelled with one of the integers 1, 2, 3, . . ., 8. A *face-sum* is the sum of the labels of the four vertices on a face of the cube. What is the maximum number of equal face-sums in any of these labellings?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

23. In a tetrahedron $PQRS$, $\angle PSR = 30°$ and $\angle QSR = 40°$. If the size of $\angle PSQ$ is an integral number of degrees, how many possible values can it have?

(A) 9 (B) 59 (C) 69 (D) 90 (E) 180

24. For how many positive integer values of a does the equation

$$\sqrt{a+x} + \sqrt{a-x} = a$$

have a real solution for x?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

25. Eight points lie on the circumference of a circle. One of them is labelled P. Chords join some or all of the pairs of these points so that the seven points other than P lie on different numbers of chords. What is the minimum number of chords on which P lies?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

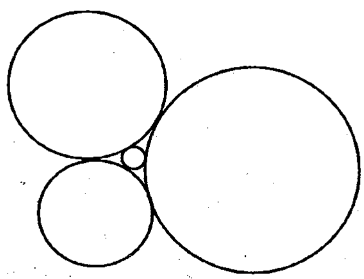
26. Each of the students in a class writes a different 2-digit number on the whiteboard. The teacher claims that no matter what the students write, there will be at least three numbers on the whiteboard whose digits have the same sum. What is the smallest number of students in the class for the teacher to be correct?

27. The sum of three numbers is 4, the sum of their squares is 10 and the sum of their cubes is 22. What is the sum of their fourth powers?

28. In a regular polygon there are two diagonals such that the angle between them is 50°. What is the smallest number of sides of the polygon for which this is possible?

29. The sum of n positive integers is 19. What is the maximum possible product of these n numbers?

30. Three circles of radius 1, 2 and 3 centimetres just touch each other as shown. A smaller circle lies in the space between them, just touching each one.



The radius of the smallest circle is, in centimetres, $\frac{p}{q}$, where p and q are integers with no common factors. What is the value of $p + q$?

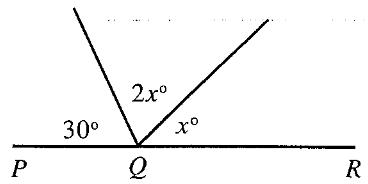
SENIOR DIVISION

Questions 1 - 10, 3 marks each

1. $2(5.61 - 4.5)$ equals
(A) 3.1 (B) 10.48 (C) 2 (D) 2.22 (E) 6.72

2. If $2^n + 2^n = 2^m$, then
(A) $n + n = m$ (B) $n + 1 = m$ (C) $4n = m$ (D) $m + 1 = n$ (E) $n^2 = m$

3. PQR is a straight line. The value of x is
(A) 30 (B) 45
(C) 50 (D) 60
(E) 150

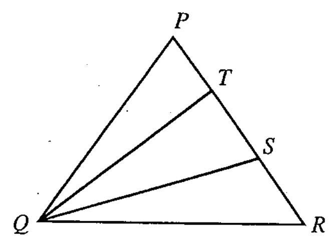


4. Of the following, which is the largest fraction?
(A) $\frac{7}{15}$ (B) $\frac{3}{7}$ (C) $\frac{6}{11}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

5. Nicky started a mobile phone call at 10:57am. The charge for the call was 89 cents per minute and the total cost for the call was \$6.23. Nicky's call ended at
(A) 11:27am (B) 11:14am (C) 11:04am (D) 11:46am (E) 11:05am

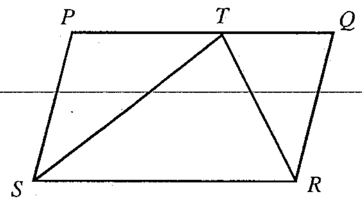
6. The straight lines with equations $2x + y = q$ and $y = x - p$ meet at the point $(2, k)$. The value of $p + q$ is
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. PQR is an equilateral triangle, QS and QT divide $\angle PQR$ into three equal parts. The size of $\angle QTS$, in degrees, is
(A) 40 (B) 70 (C) 80
(D) 90 (E) 100



8. Jane's age is a prime number. Andy's age has 8 factors and he is one year older than Jane. Of the following numbers, which could be the sum of their ages?
(A) 27 (B) 39 (C) 75 (D) 87 (E) 107

9. $PQRS$ is a parallelogram and T lies on PQ such that $PT : TQ = 3 : 2$. The ratio of the area of $PTRS$ to the area of $PQRS$ is



(A) 1 : 2 (B) 2 : 3 (C) 3 : 4
(D) 4 : 5 (E) 5 : 6

10. Five positive integers have a mean of 5, a median of 5 and just one mode of 8. What is the difference between the largest and the smallest integers in the set?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Questions 11 to 20, 4 marks each

11. Dad filled his sprayer with 8 litres of water. He then added 16 drops of insecticide instead of the recommended dosage of 32 drops. After using 2 litres of the spray, he realised his mistake, refilled the sprayer with another 2 litres of water and added a sufficient number of drops of insecticide to reach the recommended concentration. The number of extra drops that dad needed to add was
(A) 20 (B) 12 (C) 8 (D) 16 (E) 24

12. The game of *Four Tofu* is played on a 4×4 grid. When completed, each of the numbers 1, 2, 3 and 4 occurs in each row and column of the 4×4 grid and also in each 2×2 corner of the grid.
When the grid shown is completed, the sum of the four numbers in the corners of the 4×4 grid is

	2		
			1
	1	3	
4			

(A) 13 (B) 11 (C) 15 (D) 12 (E) 10

13. Holly writes down all the two-digit numbers which can be formed using the digits 1, 3, 7 and 9 (including 11, 33, 77 and 99). Warren selects one of these numbers at random. What is the probability that it is prime?
(A) $\frac{5}{8}$ (B) $\frac{1}{2}$ (C) $\frac{9}{16}$ (D) $\frac{11}{16}$ (E) $\frac{3}{4}$

14. Two rectangular garden beds have a combined area of 40m^2. The larger bed has twice the perimeter of the smaller and the larger side of the smaller bed is equal to the smaller side of the larger bed. If the two beds are not similar, and if all edges are a whole number of metres, what is the length, in metres, of the longer side of the larger bed?

(A) 7 (B) 8 (C) 10 (D) 14 (E) 27

15. I take a two-digit positive number and add to it the number obtained by reversing the digits. For how many two-digit numbers is the result of this process a perfect square?

(A) 1 (B) 3 (C) 5 (D) 8 (E) 10

16. Ann, Bill and Carol sit on a row of 6 seats. If no two of them sit next to each other, in how many different ways can they be seated?

(A) 12 (B) 24 (C) 18 (D) 36 (E) 48

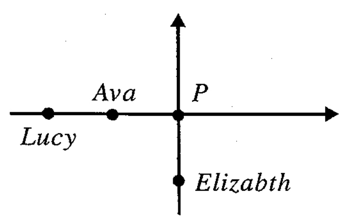
17. The number of integer solutions of the equation

$$\left(x^2-3x+1\right)^{x+1}=1$$

is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. Ava and Lucy both jog at 8 km/h along a straight path with Lucy staying 12m behind Ava. Elizabeth jogs at 6 km/h along a straight path which meets the first path at right-angles at P. When Elizabeth is at P she is the same distance from Ava as from Lucy. When Ava was first 50m from P, how far, in metres, was Elizabeth from P?

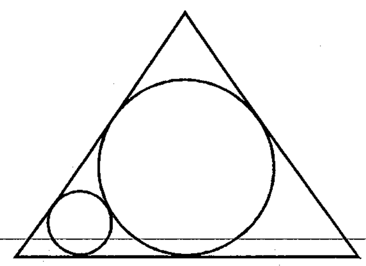


(A) 40 (B) 42 (C) 44 (D) 46 (E) 48

19. On a 3×5 chessboard, a counter can move one square at a time along a row or a column, but not along any diagonal. Starting from some squares, it can visit each of the other 14 squares exactly once, without returning to its starting square. Of the 15 squares, how many could be the counter's starting square?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

20. The inscribed circle of an equilateral triangle has radius 1 unit. A smaller circle is tangent to this circle and to two sides of the triangle as shown. The radius of this smaller circle is



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{6}$

(D) $\frac{\sqrt{3}-1}{2}$ (E) $\frac{1}{5}$

Questions 21 to 25, 5 marks each

21. There are four lifts in a building. Each makes three stops, which do not have to be on consecutive floors or include the ground floor. For any two floors, there is at least one lift which stops on both of them. What is the maximum number of floors that this building can have?

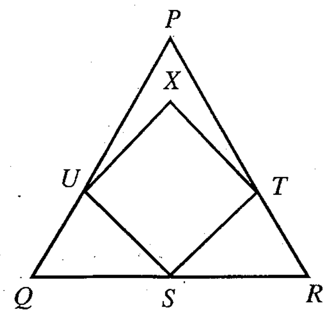
(A) 4 (B) 5 (C) 6 (D) 7 (E) 12

22. A bee can fly or walk only in a straight line between any two corners on the inside of a cubic box of edge length 1. The bee managed to move so that it visited every corner of the box without passing through the same point twice in the air or on the wall of the box. The largest possible length of such a path is

(A) $2+5\sqrt{2}$ (B) $1+6\sqrt{2}$ (C) $7\sqrt{2}$

(D) $\sqrt{3}+6\sqrt{2}$ (E) $4\sqrt{3}+3\sqrt{2}$

23. *PQR* is an equilateral triangle with side length 2. *S* is the midpoint of *QR* and *T* and *U* are points on *PR* and *PQ* respectively such that *STXU* is a square. The area of this square is



(A) $6-3\sqrt{3}$ (B) $\frac{5-2\sqrt{3}}{2}$ (C) $\frac{3}{4}$

(D) $\frac{2\sqrt{2}}{3}$ (E) $\frac{1+\sqrt{2}}{2}$

24. How many functions $f(x) = ax^2 + bx + c$ are there with the property that, for all x, $f(x) \times f(-x) = f(x^2)$?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

25. Let $\left(\sqrt{2}+1\right)^{2007} = a + b\sqrt{2}$, where a and b are integers. The highest common factor of b and 81 is

(A) 1 (B) 3 (C) 9 (D) 27 (E) 81

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A rectangular area measuring 3 units by 6 units on a wall is to be covered with 9 tiles each measuring 1 unit by 2 units. In how many ways can this be done?

27. There are 42 points $P_1, P_2, P_3, \ldots, P_{42}$, placed in order on a straight line so that each distance from P_i to P_{i+1} is $\frac{1}{i}$ where $1 \le i \le 41$. What is the sum of the distances between every pair of these points?

28. A *lucky number* is a positive integer which is 19 times the sum of its digits. How many different lucky numbers are there?

29. On my calculator screen the number 2659 can be read upside down as 6592. The digits that can be read upside down are 0, 1, 2, 5, 6, 8, 9 and are read as 0, 1, 2, 5, 9, 8, 6 respectively. Starting with 1, the fifth number that can be read upside down is 8 and the fifteenth is 21. What are the last three digits of the 2007th number that can be read upside down?

30. Consider the solutions (x, y, z, u) of the system of equations

$$x + y = 3(z + u)$$
$$x + z = 4(y + u)$$
$$x + u = 5(y + z)$$

where x, y, z and u are positive integers. What is the smallest value that x can have?

SENIOR DIVISION

Questions 1 - 10, 3 marks each

1. The value of $8002 - 2008$ is

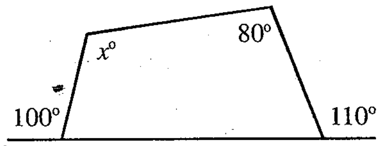
(A) 200 (B) 8 (C) 6006 (D) 1060 (E) 5994

2. The difference between $\frac{1}{20}$ and $\frac{2}{10}$ is

(A) 0 (B) $\frac{1}{10}$ (C) $\frac{3}{5}$ (D) $\frac{3}{10}$ (E) $\frac{3}{20}$

3. In the diagram, x equals

(A) 100 (B) 110 (C) 120
(D) 130 (E) 140



4. The value of $\dfrac{200 \times 8}{200 \div 8}$ is

(A) 1 (B) 8 (C) 16 (D) 64 (E) 200

5. The smallest value that $x^2 - 4x + 3$ can have is

(A) -1 (B) -3 (C) 1 (D) 3 (E) 2

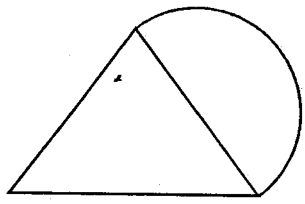
6. $3 is shared between two people. One gets 50 cents more than the other. The ratio of the larger share to the smaller share is

(A) 6 : 1 (B) 7 : 5 (C) 4 : 3 (D) 5 : 3 (E) 7 : 4

7. When 1000^{2008} is written as a numeral, the number of digits written is

(A) 2009 (B) 6024 (C) 6025 (D) 8032 (E) 2012

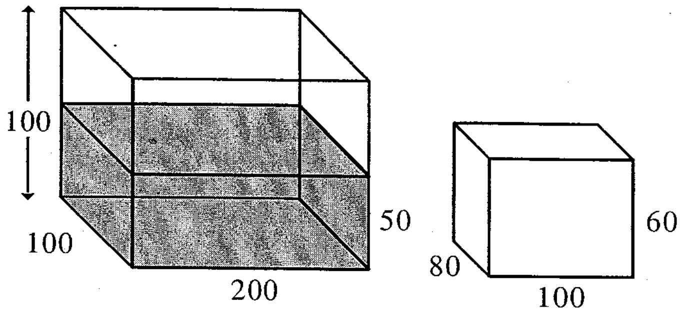
8. A semicircle is drawn on one side of an equilateral triangle. The ratio of the area of the semicircle to the area of the triangle is



(A) $1 : 1$ (B) $\pi : 2\sqrt{3}$ (C) $\pi : \sqrt{3}$
(D) $\sqrt{3} : \pi$ (E) $3 : \pi$

9. Given that $\cos x = 0.5$ and $0° < x < 90°$, which of the following has the greatest value?
(A) $\cos^2 x$ (B) $\cos x$ (C) 0.75 (D) $\sin x$ (E) $\tan x$

10. A fishtank with base 100 cm by 200 cm and depth 100 cm contains water to a depth of 50 cm. A solid metal rectangular prism with dimensions 80 cm by 100 cm by 60 cm is then submerged in the tank with an 80 cm by 100 cm face on the bottom.



The depth of water, in centimetres, above the prism is then
(A) 12 (B) 14 (C) 16 (D) 18 (E) 20

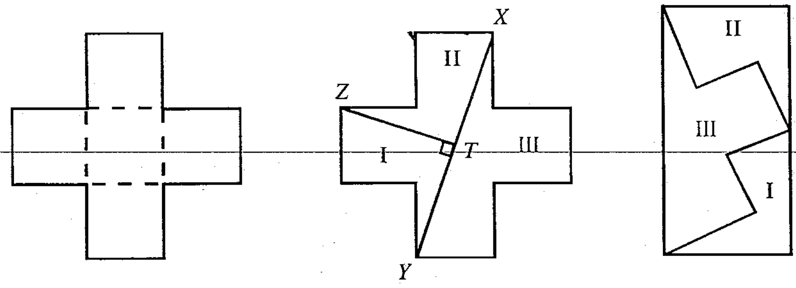
Questions 11 to 20, 4 marks each

11. Which of the following numbers is the largest?
(A) 2^{500} (B) 3^{400} (C) 4^{300} (D) 5^{200} (E) 6^{100}

12. A normal die is thrown 100 times. The sum of the numbers obtained will most likely be
(A) 200 (B) 250 (C) 300 (D) 350 (E) 400

13. What is the smallest whole number which gives a square number when multiplied by 2008?
(A) 2 (B) 4 (C) 251 (D) 502 (E) 2008

14. A cross is made up of five squares, each with side length 1 unit. Two cuts are made, the first from X to Y and the second from Z to T, so that ZTX is a right angle. The three pieces are then arranged to form a rectangle.



What is the ratio of the length to the width of the rectangle?

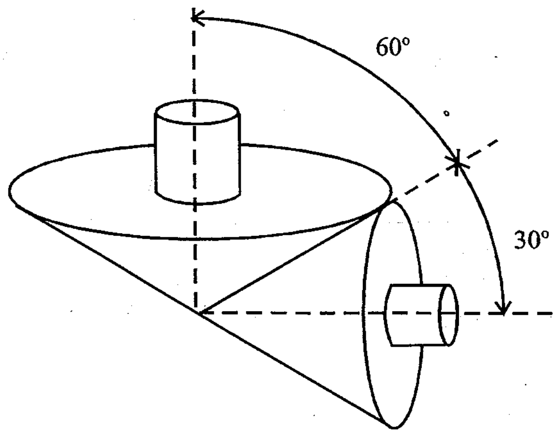
(A) 3 : 1 (B) $\sqrt{10} : 1$ (C) 2 : 1 (D) $2\sqrt{3} : 1$ (E) 5 : 2

15. A function is said to be a toggle function on (p, q, r) if $f(p) = q, f(q) = r$ and $f(r) = p$. The function $f(x) = ax^2 + bx + c$ is a toggle function on (1, 2, 3).

What is the value of c?

(A) -2 (B) 0 (C) 3 (D) 9 (E) 14

16. Two conical rollers with perpendicular axes touch on a line that is 30° to the axis of the smaller roller and 60° to the axis of the larger roller. If the larger roller makes 1 revolution per second and there is no slipping, how many revolutions per second does the smaller roller make?



(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$

(D) $\sqrt{3}$ (E)2

17. Consider the set $X = \{1, 2, 3, 4, 5, 6\}$.

How many subsets of X, with at least one element, do not contain two consecutive integers?

(A) 16 (B) 18 (C) 20 (D) 21 (E) 24

18. Farmer Taylor of Burra has two tanks. Water from the roof of his farmhouse is collected in a 100 kL tank and water from the roof of his barn is collected in a 25 kL tank. The collecting area of his farmhouse roof is 200 square metres while that of his barn is 80 square metres. Currently, there are 35 kL in the farmhouse tank and 13 kL in the barn tank.
Rain is forecast and he wants to collect as much water as possible. He should:
(A) empty the barn tank into the farmhouse tank
(B) fill the barn tank from the farmhouse tank
(C) pump 10 kL from the farmhouse tank into the barn tank
(D) pump 10 kL from the barn tank into the farmhouse tank
(E) do nothing

19. A sequence $\{u_1, u_2, \ldots, u_n\}$ of real numbers is defined by

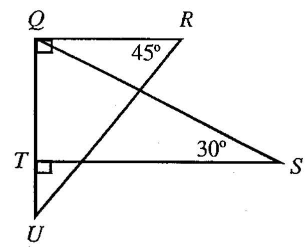
$$u_1 = \sqrt{2},\ u_2 = \pi,$$

$$u_n = u_{n-1} - u_{n-2} \quad \text{for} \quad n \geq 3.$$

What is u_{2008}?

(A) $-\sqrt{2}$ (B) $2008\left(\sqrt{2} - 2008\pi\right)$ (C) $1003\sqrt{2} - 1004\pi$
(D) π (E) $\sqrt{2}$

20. In the diagram, RU is equal in length to ST. What is the ratio of the area of ΔQRU to the area of ΔQST ?



(A) $\sqrt{3} : 1$ (B) $2 : 1$ (C) $\sqrt{6} : 1$
(D) $\sqrt{3} : 2$ (E) $\sqrt{6} : 2$

Questions 21 to 25, 5 marks each

21. P, Q, R, S and T are consecutive vertices of a regular polygon. When extended, the lines PQ and TS meet at U with $\angle QUS = 160°$. How many sides has the polygon?
(A) 36 (B) 42 (C) 48 (D) 52 (E) 54

22. How many numbers from 1, 2, 3, 4, . . ., 2008 have a cubic number other than 1 as a factor?
(A) 346 (B) 336 (C) 347 (D) 251 (E) 393

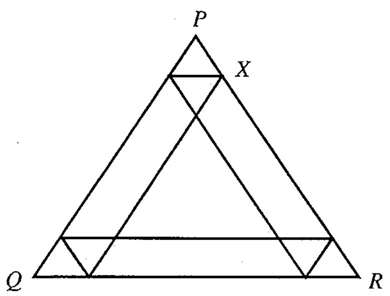
23. The numbers 828 and 313 are 3-digit palindromes where $828 - 313 = 515$, which is also a palindrome. How many pairs (a, b) of 3-digit palindromes are there with $a > b$ and with $a - b$ also a 3-digit palindrome?

(A) 1972 (B) 1980 (C) 1988 (D) 1996 (E) 2008

24. The centres of all faces of a cube are joined to form an octahedron. The centers of all faces of this octahedron are now joined to form a smaller cube. What is the ratio of an edge of the smaller cube to an edge of the original cube?

(A) $1:\sqrt{2}$ (B) $1:\sqrt{3}$ (C) $1:2$ (D) $1:3$ (E) $1:4$

25. In the figure, all line segments are parallel to one of the sides of the equilateral triangle PQR which has side length 1 unit. How long should PX be to maximise the smallest of the ten areas defined?



(A) $\frac{1}{3}$ (B) $\frac{4-\sqrt{2}}{14}$ (C) $\frac{1}{4}$

(D) $\frac{1}{5}$ (E) $\frac{1}{\sqrt{10}}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.
Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. All possible straight lines joining the vertices of a cube with mid-points of its edges are drawn. At how many points inside the cube do two or more of these lines meet?

27. Let us call a sum of integers *cool* if the first and last terms are 1 and each term differs from its neighbours by at most 1. For example, the sum $1+2+3+4+3+2+3+3+3+2+3+3+2+1$ is cool.

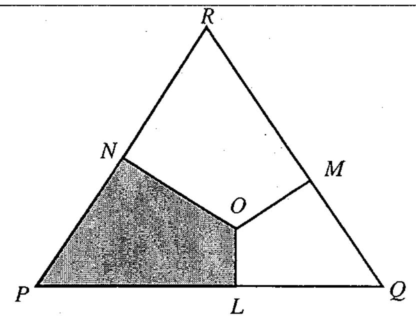
How many terms does it take to write 2008 as a cool sum if we use no more terms than necessary?

28. The positive integers x and y satisfy

$$3x^2 - 8y^2 + 3x^2y^2 = 2008.$$

What is the value of xy?

29. A point O is inside an equilateral triangle PQR and the perpendiculars OL, OM and ON are drawn to the sides PQ, QR and RP respectively.
The ratios of lengths of the perpendiculars $OL : OM : ON$ is $1 : 2 : 3$.
If $\dfrac{\text{area of } LONP}{\text{area of } \Delta PQR} = \dfrac{a}{b}$, where a and b are integers with no common factors, what is the value of $a + b$?



30. What is the smallest value that

$$\sqrt{49 + a^2 - 7\sqrt{2}a} + \sqrt{a^2 + b^2 - \sqrt{2}ab} + \sqrt{50 + b^2 - 10b}$$

can have for positive real numbers a and b?

ANSWERS

SENIOR DIVISION — 2004-2008

QUESTION	2004	2005	2006	2007	2008
1	D	E	D	D	E
2	B	E	D	B	E
3	E	E	C	C	D
4	A	B	E	C	D
5	A	E	D	C	A
6	B	B	D	E	B
7	A	C	A	C	C
8	B	D	B	E	B
9	C	C	E	D	E
10	E	A	D	D	B
11	C	B	E	A	B
12	C	A	E	E	D
13	A	B	E	A	D
14	C	A	C	A	C
15	B	B	C	D	A
16	B	C	A	B	D
17	C	D	E	D	C
18	D	D	B	B	D
19	A	B	A	D	A
20	E	A	D	A	D
21	B	B	A	B	E
22	E	D	E	D	B
23	D	D	B	A	B
24	B	E	D	C	D
25	A	C	C	A	C
26	A	80	35	41	14
27	C	401	50	861	89
28	C	20	18	11	28
29	D	5	972	898	47
30	D	13	29	83	13