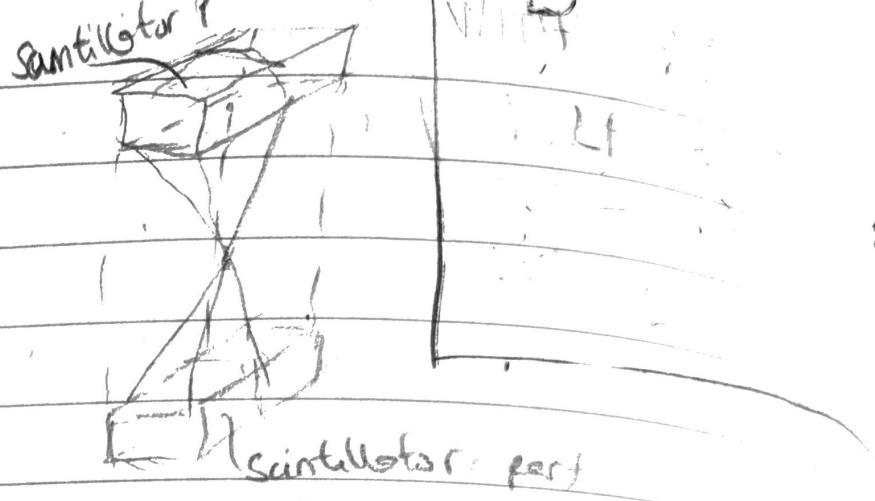


Solid angle

$$d\Omega = \frac{dA_{\text{sphere}}}{R^2}$$



For...

The differential solid angle $d\Omega$ subtended by a small patch of area dA at distance R is →

THE PROJECTED AREA PERPENDICULAR
to the
line of sight divided by the distance squared

$$d\Omega = \frac{\hat{R} dA_{\text{scintillator}}}{R^2} =$$

$$\hat{R} = \frac{\hat{r}}{|R|} = \frac{\sqrt{x^2 + y^2 + \frac{h^2}{4}}}{\sqrt{x^2 + y^2 + z^2}}$$

$$dA_{\text{sphere}} = \hat{R} dA_{\text{scintillator}}$$

° If we use as the origin the middle point between both scintillators $(0,0,0)$

constant = $\frac{h}{2}$ [half of the table height]
 $R = x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}$ from the origin to the different points on the scintillator

The z coordinate is constant. Thus, we only need to integrate in the xy plane in the direction of \hat{k} . → $dA = dx dy \hat{k}$

Therefore $dA_{\text{Sphere}} = \hat{R} dA_{\text{cylinder}}$

$$dA_{\text{Sphere}} = \left(\frac{\hat{x}\hat{i} + \hat{y}\hat{j} + \frac{h}{2}\hat{k}}{\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2}} \right) (dx dy \hat{k})$$

If we operate we get: $[\hat{x}\hat{i} + \hat{y}\hat{j} + \frac{h}{2}\hat{k}] \cdot \hat{k} = \frac{h}{2}$

$$dA_{\text{Sphere}} = \left(\frac{\frac{h}{2} dx dy}{\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2}} \right)$$

$$d\Omega_1 = \frac{1}{R^2} dA_{\text{Sphere}} = \frac{1}{R^2} \left(\frac{\frac{h}{2} dx dy}{\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2}} \right)$$

$$d\Omega_1 = \frac{\frac{h}{2} dx dy}{\left(\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2} \right)^2 \left(\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2} \right)}$$

$$d\Omega_1 = \frac{\frac{h}{2} dx dy}{\left(x^2 + y^2 + \left(\frac{h}{2}\right)^2 \right)^{3/2}}$$

$$\Omega_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\frac{h}{2} dx dy}{\left(x^2 + y^2 + \left(\frac{h}{2}\right)^2 \right)^{3/2}}$$

$$\int_{-x_i}^{-y_i} \left(x^2 + y^2 + \frac{h^2}{4} \right)^{3/2} dy$$

own work
PLEASE CHECK

Solving \int^2 with respect to x

If Integrated well

Diagram showing a right-angled triangle with base a , height $\frac{h}{2}$, and hypotenuse $\sqrt{x^2 + y^2 + \frac{h^2}{4}}$. The area is labeled $\int_{-y_i}^{y_F} \int_{-x_i}^{x_F} dx dy$.

if we call $y^2 + \frac{h^2}{4} = a$

$$U = \arctan\left(\frac{y}{a}\right)$$

we have

$$\int_{-x_i}^{x_F} \frac{dx}{\sqrt{x^2 + a^2}} =$$

we can do a
variable exchange

where

$$x = a \tan(u)$$

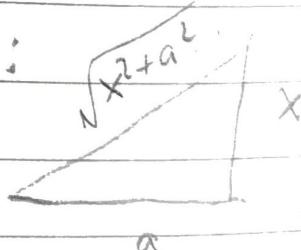
$$dx = a \sec^2(u) du$$

$$\int \frac{a \sec^2(u) du}{(a^2 \tan^2(u) + a^2)^{3/2}} = \int \frac{a \sec^2(u) du}{a^3 (\sec^2 u)^{3/2}} =$$

$$\frac{1}{a^2} \int \frac{1}{\sec(u)} du = \frac{1}{a^2} \int \cos(u) du = \frac{1}{a^2} \sin(u)$$

Undoing the first variable exchange:

$$\frac{1}{a^2} \sin\left(\arctan\left(\frac{x}{a}\right)\right)$$



$$\frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

undoing the second variable

exchange $\left(\frac{1}{y^2 + \frac{h^2}{4}} \right) \left(\frac{x}{\sqrt{x^2 + y^2 + \frac{h^2}{4}}} \right) \right]_{-x_i}^{x_F}$

$$\Omega_1 = \frac{1}{2} \int_{y_i}^{y_F} \frac{1}{\sqrt{y^2 + \frac{h^2}{4}}} \left[\frac{x_F}{\sqrt{x_F^2 + y^2 + \frac{h^2}{4}}} - \frac{x_i}{\sqrt{x_i^2 + y^2 + \frac{h^2}{4}}} \right] dy$$

$$\Omega_1 = \frac{1}{2} \int_{y_i}^{y_F} \frac{x_F}{\left(y^2 + \frac{h^2}{4}\right)} \frac{dy}{\sqrt{x_F^2 + y^2 + \frac{h^2}{4}}} - \int_{y_i}^{y_F} \frac{x_i}{\left(y^2 + \frac{h^2}{4}\right)} \frac{dy}{\sqrt{x_i^2 + y^2 + \frac{h^2}{4}}}$$

Because of we choose a symmetric point
in the middle $|x_F| = |x_i|$

$$\text{But } x_i = -x_F$$

Hence:

$$\Omega_1 = \frac{1}{2} \int_{y_i}^{y_F} \frac{2x_F}{\left(y^2 + \frac{h^2}{4}\right)} \frac{dy}{\sqrt{x_F^2 + y^2 + \frac{h^2}{4}}}$$

Calculation for the integral

in the next page!!

$$y_i$$

$$or$$

SOLUTION

$$\Omega_1 = \frac{1}{2} \operatorname{Harctan} \left(\frac{x_F y_F}{\frac{3}{2} \sqrt{x_F^2 + y^2 + \frac{h^2}{4}}} \right)$$

$$= \frac{1}{2} \operatorname{Harctan} \left(\frac{y_F}{\sqrt{x_F^2 + y^2 + \frac{h^2}{4}}} \right)$$

$$dy = \frac{h}{2} \int_{y_1}^{y_F} \frac{2xF dy}{(y^2 + \frac{h^2}{4}) \sqrt{x_F^2 + y^2 + \frac{h^2}{4}}} = h x_F \int_{y_1}^{y_F} \frac{dy}{(y^2 + \frac{h^2}{4}) \sqrt{x_F^2 + y^2 + \frac{h^2}{4}}}$$

$$y = \sqrt{\frac{h^2}{4} + x_F^2} \quad dy = \sqrt{\frac{h^2}{4} + x_F^2} dx$$

$$h x_F \int_{y_1}^{y_F} \frac{a \sec^2 u du}{\left(a^2 \tan^2 u + \frac{h^2}{4}\right) \sqrt{a^2 + a^2 \tan^2 u}}$$

$$h x_F \int_{y_1}^{y_F} \frac{\sec u du}{\left(a^2 \tan^2 u + \frac{h^2}{4}\right)} = h x_F \int_{y_1}^{y_F} \frac{du}{\cos u \left(a^2 \frac{\sin^2 u}{\cos^2 u} + \frac{h^2}{4}\right)}$$

$$h x_F \int_{y_1}^{y_F} \frac{du}{\left(a^2 \frac{\sin^2 u}{\cos^2 u} + \frac{h^2}{4} \cos u\right)} = h x_F \int_{y_1}^{y_F} \frac{\cos u du}{a^2 \sin^2 u + \frac{h^2}{4} \cos^2 u}$$

$$\begin{aligned} t &= \sin u \\ dt &= \cos u du \end{aligned} \quad h x_F \int_{y_1}^{y_F} \frac{dt}{a^2 t^2 + \frac{h^2}{4} (1-t^2)} = h x_F \int_{y_1}^{y_F} \frac{dt}{a^2 t^2 + \frac{h^2}{4} - t^2 \frac{h^2}{4}}$$

$$\sin^2 u + \cos^2 u = 1$$

$$\cos^2 u = 1 - \sin^2 u$$

$$\cos^2 u = 1 - t^2$$

$$h x_F \int_{y_1}^{y_F} \frac{dt}{t^2 \left(a^2 - \frac{h^2}{4}\right) + \frac{h^2}{4}} = h x_F \int_{y_1}^{y_F} \frac{\cos u du}{\sin^2 u \left(a^2 - \frac{h^2}{4}\right) + \frac{h^2}{4}}$$

$$h x_F \int_{y_1}^{y_F} \frac{\frac{4}{h^2} \cos u du}{\frac{4}{h^2} \sin^2 u \left(a^2 - \frac{h^2}{4}\right) + 1}$$

$$b = \sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4}\right)} \sin u$$

$$db = \sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4}\right)} \cos u du$$

$$h x_F \int_{y_1}^{y_F} \frac{\frac{4}{h^2} db}{1 + b^2} = 4 h x_F \int_{\frac{h^2}{4} \sqrt{a^2 - \frac{h^2}{4}}}^{\frac{4}{h^2} b} \frac{db}{1 + b^2} = \frac{4}{h^2} \arctan(b) \left[\frac{h x_F}{\sqrt{a^2 - \frac{h^2}{4}}} \right]$$

②

$$h x_F = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{4}{h^2} \cos u \, du$$

$$\frac{4}{h^2} \sin u \left(a^2 - \frac{h^2}{4} \right) + C$$

$$b^2 = \left(\sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)} \sin u \right)^2$$

$$db = \sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)} \cos u \, du$$

$$\cos u \, du = \frac{db}{\sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)}}$$

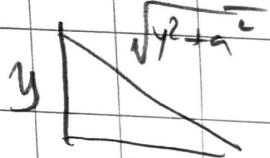
$$h x_F = \frac{4}{h^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)}} \left(\frac{1}{b^2 + 1} \right) db$$

$$h x_F = \frac{4 x_F}{h \sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)}} \frac{db}{b^2 + 1}$$

$$\frac{4 x_F}{h^2} \arctan(b)$$

$$\frac{4 x_F}{h \sqrt{a^2 - \frac{h^2}{4}}}$$

$$h x_F = \arctan \left(\sqrt{\frac{4}{h^2} \left(a^2 - \frac{h^2}{4} \right)} \sin \alpha \right)$$



$$\frac{2 x_F}{h x_F} \arctan \left(\sqrt{\frac{4}{h^2} \left(\frac{h^2}{4} + x_F^2 - \frac{h^2}{4} \right)} \sin \left(\arctan \left(\frac{4}{a} \right) \right) \right)$$

$$2 \arctan \left(\sqrt{\frac{4}{h^2} x_F^2} \left(\frac{y}{\sqrt{y^2 + a^2}} \right) \right) = 2 \arctan \left(\frac{2}{h} x_F \frac{y}{\sqrt{y^2 + \frac{h^2}{4} + x_F^2}} \right)$$

$$2 \arctan \left(\frac{2 x_F y}{\frac{h}{2} \sqrt{y^2 + \frac{h^2}{4} + x_F^2}} \right) \begin{matrix} y_F \\ y_1 \end{matrix}$$

$$y_F = -y_1$$

$$4 \arctan \left(\frac{x_F y}{\frac{h}{2} \sqrt{y^2 + \frac{h^2}{4} + x_F^2}} \right)$$