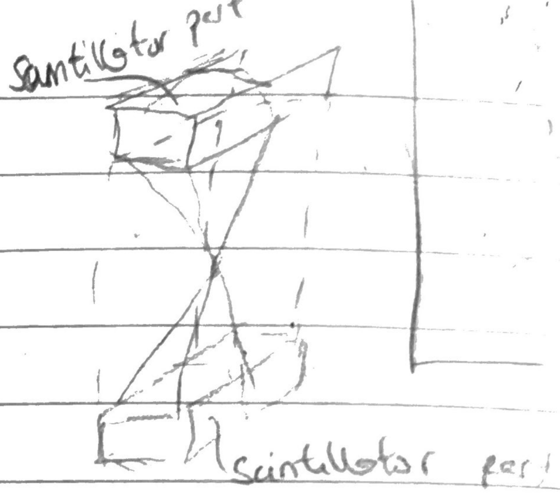
Solid angle



$$d\Omega = \frac{dA_{sphere}}{R^2}$$

The differential solid angle $d\Omega$ subtended by a small patch of area dA at distance R is → THE PROJECTED AREA PERPENDICULAR to the line of sight divided by the distance squared

$$d\Omega = \frac{\hat{R}\, dA_{scintillator}}{R^2} = \qquad \rightarrow \hat{R} = \frac{\vec{R}}{|R|} = \frac{x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}}{\sqrt{x^2+y^2+d^2}}$$

$$dA_{sphere} = \hat{R}\, dA_{scintillator}$$

- If we use as the origin the middle point between both scintillators $(0,0,0)$

$\vec{R} = x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}$ from the origin to the different points on the scintillator

constant $= h/2$ [half of the table height]

The z coordinate is constant. Thus, we only need to integrate in the xy plane in the direction of $\hat{k}$ → $\boxed{dA = dx\,dy\,\hat{k}}$

$\vec{n}$

The normal vector

$$\vec{n} = \hat{k}$$

Therefore $dA_{sphere} = \hat{R}\, dA_{scintillator}$

$$dA_{sphere} = \left(\frac{x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}}{\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}}\right)\cdot(dx\,dy\,\hat{k})$$

If we operate we get: $\left[x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}\right]\cdot\hat{k} = \frac{h}{2}$

$$dA_{sphere} = \left(\frac{\frac{h}{2}\,dx\,dy}{\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}}\right)$$

$$d\Omega = \frac{1}{R^2}\,dA_{sphere} = \frac{1}{R^2}\left(\frac{\frac{h}{2}\,dx\,dy}{\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}}\right)$$

$$d\Omega = \frac{\frac{h}{2}\,dx\,dy}{\left(\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}\right)^2\left(\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}\right)}$$

$$d\Omega = \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\left(\frac{h}{2}\right)^2\right)^{3/2}}$$

$$\boxed{\Omega = \int_{x_i}^{x_F}\int_{y_i}^{y_F} \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\left(\frac{h}{2}\right)^2\right)^{3/2}}}$$

$$\int_{-x_i}\int_{-y_i} \frac{\quad}{\left(x^2+y^2+\frac{h^2}{4}\right)^{3/2}}$$

own work
PLEASE CHECK
if Integrated well

Solving I^2 with respect to x

$$\frac{h}{2}\int_{-y_i}^{y_F}\int_{x_i}^{x_F} \frac{dx\,dy}{\left(x^2+\underbrace{y^2+\frac{h^2}{4}}_{a}\right)^{3/2}}$$

if we call $y^2+\frac{h^2}{4}=a$

$u=\arctan\left(\frac{x}{a}\right)$

we have $\displaystyle\int_{x_i}^{x_F} \frac{dx}{(x^2+a^2)^{3/2}}$

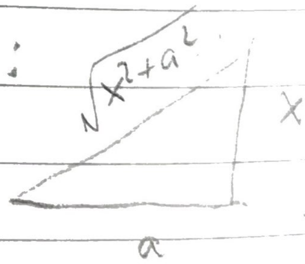
we can do a variable exchange where

$$\boxed{\begin{array}{l} x=a\tan(u) \\ dx=a\sec^2(u)\,du \end{array}}$$

$$\int \frac{a\sec^2(u)\,du}{(a^2\tan^2(u)+a^2)^{3/2}} = \int \frac{a\sec^2(u)\,du}{a^3\,(\sec^2 u)^{3/2}} =$$

$$\frac{1}{a^2}\int \frac{1}{\sec(u)}\,du = \frac{1}{a^2}\int \cos(u)\,du = \frac{1}{a^2}\sin(u)$$

Undoing the first variable exchange:



$$\frac{1}{a^2}\sin\left(\arctan\left(\frac{x}{a}\right)\right)$$

$$\frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}$$ undoing the second variable exchange

$$\left(\frac{1}{y^2+\frac{h^2}{4}}\right)\left(\frac{x}{\sqrt{x^2+y^2+\frac{h^2}{4}}}\right)\Bigg]_{+x_i}^{x_F}$$

$$\Omega = \frac{h}{2}\int_{y_i}^{y_F} \frac{1}{y^2+\frac{h^2}{4}}\left[\frac{x_F}{\sqrt{x_F^2+y^2+\frac{h^2}{4}}} - \frac{x_i}{\sqrt{x_i^2+y^2+\frac{h^2}{4}}}\right]dy$$

$$\Omega = \frac{h}{2}\left[\int_{y_i}^{y_F} \frac{x_F}{\left(y^2+\frac{h^2}{4}\right)} \frac{dy}{\sqrt{x_F^2+y^2+\frac{h^2}{4}}} - \int_{y_i}^{y_F} \frac{x_i}{\left(y^2+\frac{h^2}{4}\right)} \frac{}{\sqrt{x_i^2+y^2+\frac{h^2}{4}}}\right]$$

Because of we choose a symmetric point in the middle $|x_F| = |x_i|$

But $x_i = -x_F$

Hence:

$$\Omega = \frac{h}{2}\int_{y_1}^{y_F} \frac{2x_F\, dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{x_F^2+y^2+\frac{h^2}{4}}}$$

Calculation for the integral in the next page!!

SOLUTION

$$\boxed{\Omega = 4\arctan\left(\frac{x_F\, y_F}{\frac{h}{2}\sqrt{x_F^2+y^2+\frac{h^2}{4}}}\right)}$$

$$\Omega = \frac{h}{2}\int_{y_1}^{y_F} \frac{2x_F\,dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{x_F^2+y^2+\frac{h^2}{4}}} = hx_F\int_{y_1}^{y_F}\frac{dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{x_F^2+y^2+\frac{h^2}{4}}}$$

$$a=\sqrt{\frac{h^2}{4}+x_F^2}$$

$$y = a\tan u \qquad u=\arctan\left(\frac{y}{a}\right)$$

$$dy = a\sec^2 u\,du$$

$$hx_F\int_{y_1}^{y_F}\frac{dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{a^2+y^2}}$$

$$hx_F\int_{y_1}^{y_F}\frac{a\sec^2 u\,du}{\left(a^2\tan^2 u+\frac{h^2}{4}\right)\sqrt{a^2+a^2\tan^2 u}} = hx_F\int_{y_1}^{y_F}\frac{a\sec^2 u\,du}{\left(a^2\tan^2 u+\frac{h^2}{4}\right)a\sec u}$$

$$hx_F\int_{y_1}^{y_F}\frac{\sec u\,du}{\left(a^2\tan^2 u+\frac{h^2}{4}\right)} = hx_F\int_{y_1}^{y_F}\frac{du}{\cos u\left(a^2\frac{\sin^2 u}{\cos^2 u}+\frac{h^2}{4}\right)}$$

$$hx_F\int_{y_1}^{y_F}\frac{du}{\left(a^2\left(\frac{\sin^2 u}{\cos u}\right)+\frac{h^2}{4}\cos u\right)} = hx_F\int_{y_1}^{y_F}\frac{\cos u\,du}{a^2\sin^2 u+\frac{h^2}{4}\cos^2 u}$$

$$t=\sin u$$
$$dt=\cos u\,du$$

$$hx_F\int_{y_1}^{y_F}\frac{dt}{a^2t^2+\frac{h^2}{4}(1-t^2)} = hx_F\int_{y_1}^{y_F}\frac{dt}{a^2t^2+\frac{h^2}{4}-t^2\frac{h^2}{4}}$$

$$\sin^2 u+\cos^2 u=1$$
$$\cos^2 u = 1-\sin^2 u$$
$$\cos^2 u = 1-t^2$$

$$hx_F\int_{y_1}^{y_F}\frac{dt}{t^2\left(a^2-\frac{h^2}{4}\right)+\frac{h^2}{4}} = hx_F\int_{y_1}^{y_F}\frac{\cos u\,du}{\sin^2 u\left(a^2-\frac{h^2}{4}\right)+\frac{h^2}{4}} \quad \triangleright ②$$

$$hx_F\int_{y_1}^{y_F}\frac{\frac{4}{h^2}\cos u\,du}{\frac{4}{h^2}\sin^2 u\left(a^2-\frac{h^2}{4}\right)+1}$$

$$b=\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}\sin u$$

$$db=\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}\cos u\,du$$

$$hx_F\int\frac{\frac{4}{h^2}}{\sqrt{a^2-\frac{h^2}{4}}}\frac{db}{1+b^2} = 4\frac{hx_F}{h^2\sqrt{a^2-\frac{h^2}{4}}}\int\frac{db}{1+b^2} = \frac{4}{h^2}\arctan(b)\left[\frac{hx_F}{\sqrt{a^2-\frac{h^2}{4}}}\right]$$

② $h x_F \int \frac{\frac{4}{h^2}\cos u\, du}{\frac{4}{h^2}\sin^2 u\left(a^2-\frac{h^2}{4}\right)+1}$

$$b^2 = \left(\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}\sin u\right)^2$$

$$db = \sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}\cos u\,du$$

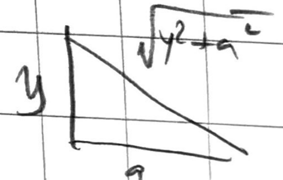
$$\cos u\,du = \frac{db}{\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}}$$

$$h x_F \frac{4}{h^2}\int \frac{db}{\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}}\left(\frac{1}{b^2+1}\right)$$

$$h x_F = \frac{4x_F}{h\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}}\int\frac{db}{b^2+1} =$$

$$\frac{4x_F}{h\,\frac{2}{h}\sqrt{a^2-\frac{h^2}{4}}}\arctan(b)$$

$$\frac{2x_F}{h\sqrt{\frac{h^2}{4}+x_F^2-\frac{h^2}{4}}}\arctan\left(\sqrt{\frac{4}{h^2}\left(a^2-\frac{h^2}{4}\right)}\sin u\right)$$



$$\frac{2\cancel{x_F}}{h\cancel{x_F}}\arctan\left(\sqrt{\frac{4}{h^2}\left(\frac{h^2}{4}+x_F^2-\frac{h^2}{4}\right)}\sin\left(\arctan\left(\frac{y}{a}\right)\right)\right)$$

$$2\arctan\left(\sqrt{\frac{4}{h^2}x_F^2}\left(\frac{y}{\sqrt{y^2+a^2}}\right)\right) = 2\arctan\left(\frac{2}{h}x_F\frac{y}{\sqrt{y^2+\frac{h^2}{4}+x_F^2}}\right)$$

$$2\arctan\left(\frac{2x_F y}{\frac{h}{2}\sqrt{y^2+\frac{h^2}{4}+x_F^2}}\right)\Bigg]_{y_1}^{y_F}$$

$|y_F = -y_1$

$$4\arctan\left(\frac{x_F y}{\frac{h}{2}\sqrt{y^2+\frac{h^2}{4}+x_F^2}}\right)$$