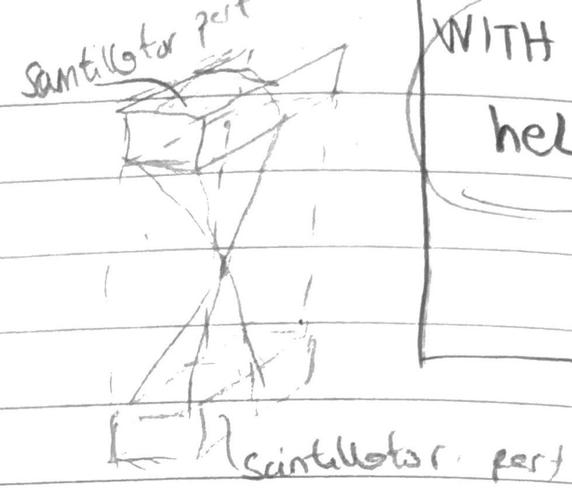


WITH AI help

Solid angle

$$d\Omega = \frac{dA_{sphere}}{R^2}$$

The differential solid angle $d\Omega$ subtended by a small patch of area dA at distance R is $\rightarrow$
THE PROJECTED AREA PERPENDICULAR to the line of sight divided by the distance squared

$$d\Omega = \frac{\hat{R}\, dA_{scintillator}}{R^2} = \qquad \rightarrow \hat{R} = \frac{\hat{r}}{|R|} = \frac{x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}}{\sqrt{x^2+y^2+d^2}}$$

$dA_{sphere} = \hat{R}\, dA_{scintillator}$

- If we use as the origin the middle point between both scintillators $(0,0,0)$

constant $= h/2$ [half of the table height]

$R = x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}$ from the origin to the different points on the scintillator

The z coordinate is constant. Thus, we only need to integrate in the xy plane in the direction of $\hat{k}$. $\rightarrow$ $\boxed{dA = dx\,dy\,\hat{k}}$

Therefore $dA_{sphere} = \hat{R} \, dA_{scintillator}$

$$dA_{sphere} = \left(\frac{x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k}}{\sqrt{x^2 + y^2 + \left(\frac{h}{2}\right)^2}} \right) \cdot \left(dx\,dy\,\hat{k} \right)$$

If we operate we get: $\left[x\hat{i} + y\hat{j} + \frac{h}{2}\hat{k} \right] \cdot \hat{k} = \frac{h}{2}$

$$dA_{sphere} = \left(\frac{\frac{h}{2}\,dx\,dy}{\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}} \right)$$

$$d\Omega = \frac{1}{R^2} dA_{sphere} = \frac{1}{R^2} \left(\frac{\frac{h}{2}\,dx\,dy}{\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}} \right)$$

$$d\Omega = \frac{\frac{h}{2}\,dx\,dy}{\left(\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}\right)^2 \left(\sqrt{x^2+y^2+\left(\frac{h}{2}\right)^2}\right)}$$

$$d\Omega = \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\left(\frac{h}{2}\right)^2\right)^{3/2}}$$

$$\boxed{\Omega = \int_{x_i}^{x_F} \int_{y_i}^{y_F} \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\left(\frac{h}{2}\right)^2\right)^{3/2}}}$$

$$\Omega_1 = \int_{-x_i}^{x_F}\int_{-y_i}^{y_F} \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\frac{h^2}{4}\right)^{3/2}}$$

my own work PLEASE CHECK if Integrated well

Solving with respect to x

$$\frac{h}{2}\int_{-y_i}^{y_F}\int_{x_i}^{x_F} \frac{dx\,dy}{\left(x^2+\underbrace{y^2+\frac{h^2}{4}}_{a}\right)^{3/2}}$$

if we call $y^2+\frac{h^2}{4}=a$

$u = \arctan\left(\frac{x}{a}\right)$

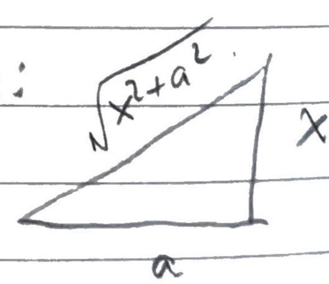
we have $\displaystyle\int_{x_i}^{x_F} \frac{dx}{(x^2+a^2)^{3/2}}$ we can do a variable exchange where

$$\boxed{\begin{aligned} x &= a\tan(u) \\ dx &= a\sec^2(u)\,du \end{aligned}}$$

$$\int \frac{a\sec^2(u)\,du}{(a^2\tan^2(u)+a^2)^{3/2}} = \int \frac{a\sec^2(u)\,du}{a^3\,(\sec^2 u)^{3/2}} =$$

$$\frac{1}{a^2}\int \frac{1}{\sec(u)}\,du = \frac{1}{a^2}\int \cos(u)\,du = \frac{1}{a^2}\sin(u)$$

Undoing the first variable exchange:



$$\frac{1}{a^2}\sin\left(\arctan\left(\frac{x}{a}\right)\right)$$

$$\frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}$$ undoing the second variable exchange

$$\left(\frac{1}{y^2+\frac{h^2}{4}}\right)\left(\frac{x}{\sqrt{x^2+y^2+\frac{h^2}{4}}}\right)\Bigg]_{+x_i}^{x_F}$$

$$\Omega_i = \int_{-x_i}^{x_F}\int_{-y_i}^{y_F} \frac{\frac{h}{2}\,dx\,dy}{\left(x^2+y^2+\frac{h^2}{4}\right)^{3/2}}$$

my own work PLEASE CHECK if Integrated well

Solving with respect to x

$$\frac{h}{2}\int_{-y_i}^{y_F}\int_{x_i}^{x_F} \frac{dx\,dy}{\left(x^2+\underbrace{y^2+\frac{h^2}{4}}_{a}\right)^{3/2}}$$

if we call $y^2+\frac{h^2}{4}=a$

$u=\arctan\left(\frac{x}{a}\right)$

we have $\displaystyle\int_{x_i}^{x_F}\frac{dx}{(x^2+a^2)^{3/2}}$

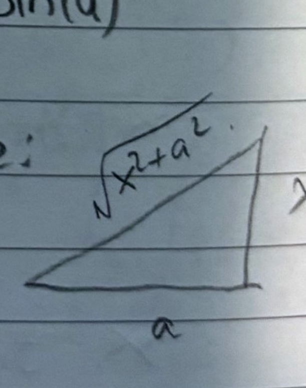
we can do a variable exchange ~~where~~

$$\boxed{\begin{aligned} x&=a\tan(u)\\ dx&=a\sec^2(u)\,du\end{aligned}}$$

$$\int\frac{a\sec^2(u)\,du}{(a^2\tan^2(u)+a^2)^{3/2}} = \int\frac{a\sec^2(u)\,du}{a^3\,(\sec^2 u)^{3/2}} =$$

$$\frac{1}{a^2}\int\frac{1}{\sec(u)}\,du = \frac{1}{a^2}\int\cos(u)\,du = \frac{1}{a^2}\sin(u)$$

Undoing the first variable exchange:



$$\frac{1}{a^2}\sin\left(\arctan\left(\frac{x}{a}\right)\right)$$

$$\frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}$$ undoing the second variable exchange

$$\left(\frac{1}{y^2+\frac{h^2}{4}}\right)\left(\frac{x}{\sqrt{x^2+y^2+\frac{h^2}{4}}}\right)\Bigg]_{+x_i}^{x_F}$$

$$\Omega = \frac{h}{2}\int_{y_i}^{y_F} \frac{1}{y^2+\frac{h^2}{4}} \left[\frac{x_F}{\sqrt{x_F^2+y^2+\frac{h^2}{4}}} - \frac{x_i}{\sqrt{x_i^2+y^2+\frac{h^2}{4}}} \right] dy$$

$$\Omega = \frac{h}{2}\left[\int_{y_i}^{y_F} \frac{x_F}{\left(y^2+\frac{h^2}{4}\right)} \frac{dy}{\sqrt{x_F^2+y^2+\frac{h^2}{4}}} - \int_{y_i}^{y_F} \frac{x_i}{\left(y^2+\frac{h^2}{4}\right)} \frac{}{\sqrt{x_i^2+y^2+\frac{h^2}{4}}}\right]$$

Because of we choose a symmetric point in the middle $|x_F| = |x_i|$

But $x_i = -x_F$

Hence:

$$\Omega = \frac{h}{2}\int_{y_i}^{y_F} \frac{2x_F \, dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{x_F^2+y^2+\frac{h^2}{4}}}$$

Very long calculation → I think I didn't get the official answer

$$h x_F \cdot \int_{-y_i}^{y_F} \frac{dy}{\left(y^2+\frac{h^2}{4}\right)\sqrt{x_F^2+y^2+\frac{h^2}{4}}}$$

& symmetric origin $\boxed{y_F = -y_i}$

$$\left[\arctan\left(\frac{x_F\, y}{\frac{h}{2}\sqrt{x_F^2+y^2+\frac{h^2}{4}}}\right)\right]_{y_i}^{y_F} =$$

$$\boxed{\Omega = 2\arctan\left(\frac{x_F\, y_F}{\frac{h}{2}\sqrt{x_F^2+y^2+\frac{h^2}{4}}}\right)} \equiv 2\arcsin\left(\frac{}{\sqrt{}}\right)$$

$\begin{cases} x_F = \text{half width} \\ y_F = \text{half length} \end{cases}$

but the official solution is

$$\Omega_1 = 4\arctan\left(\frac{LW}{d\sqrt{L^2+W^2+d^2}}\right)$$

wh $d = \frac{h}{2}$ $L =$

$L =$ half length

$W =$ half width