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Fractional Order Partial Differential Equations in Image Processing

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Abstract

This poster introduces advantages of non-linear fractional order partial differential equation based methods for different image processing applications such as image filtering, de-blurring and segmentation. Fractional order non-linear partial differential equations are generalizations of classical partial differential equations, in which the fractional derivatives for spatial as well as time can be used at the same time. Here we also describe numerical approximations for the fractional derivative of order 0 < α ≤ 2 along the applications. In sum, it is well proved that as a fundamental mathematic tool, fractional-order derivative shows great success in image processing.

A Brief Review

- Classical Non-linear diffusion:

$$u_t = \nabla(c(|\nabla u|)\nabla u) \qquad in\ \Omega\times(0,T)$$
$$\frac{\partial u}{\partial n} = 0 \qquad in\ \partial\Omega\times(0,T)$$
$$u(x,y,0) = noisy\,image \qquad in\ \Omega$$

- Over the last decade, it has been demonstrated that many systems in science and engineering can be modeled more accurately by the following fractional-order than integer-order derivatives,
  - Riemann-Liouville Derivatives

$${}_aD_t^\alpha = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau \qquad (n-1<\alpha<n)$$

  - Grunwald-Letnikov Derivatives

$${}_aD_t^\alpha = \lim_{h\to 0} h^{-\alpha}\sum_{j=0}^{[\frac{t-a}{h}]}(-1)^j\binom{\alpha}{j}f(t-jh)$$

  - Caputo Derivatives

$${}_aD_t^\alpha = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau \qquad (n-1<\alpha<n)$$

- Two dimensional space fractional partial differential equation is,

$$\frac{\partial u}{\partial t} = p(x,y)\frac{\partial^\alpha u}{\partial x^\alpha} + q(x,y)\frac{\partial^\beta u}{\partial y^\beta}, \quad u(x,y,t)\in\Omega\times(0,T]$$

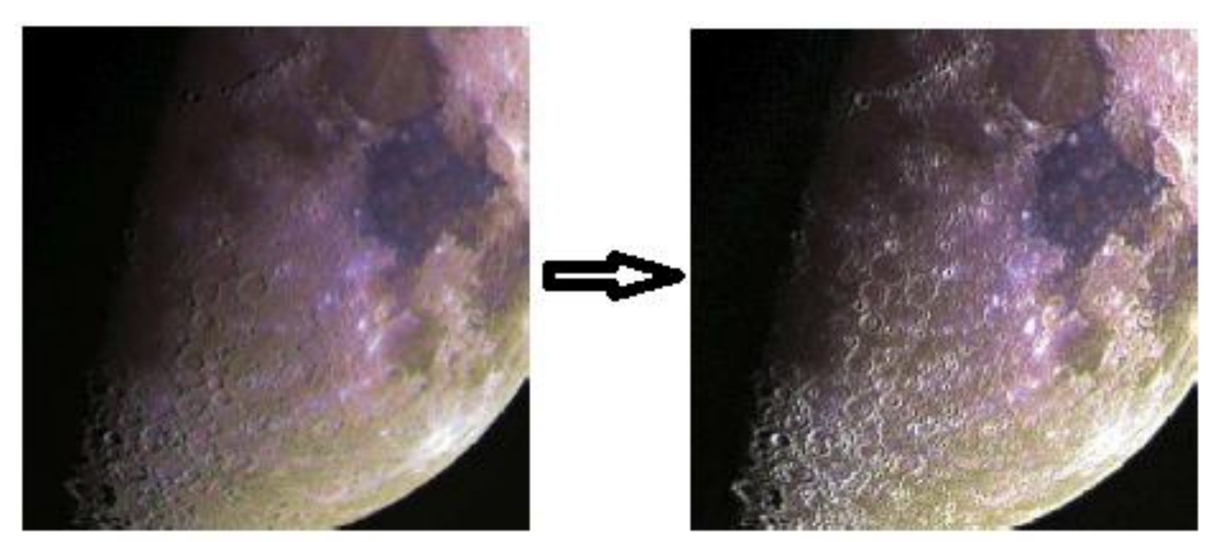
- Finite difference methods are developed to provide the numerical approximation of fractional systems as follows :

$$\frac{u_{i,j}^{n+1}-u_{i,j}^n}{\Delta t} = \frac{p_{i,j}}{(\Delta x)^\alpha}\sum_{k=0}^{i+1} g_{\alpha,k}u_{i-k+1,j}^{n+1} + \frac{q_{i,j}}{(\Delta y)^\beta}\sum_{k=0}^{j+1} g_{\beta,k}u_{i,j-k+1}^{n+1}$$

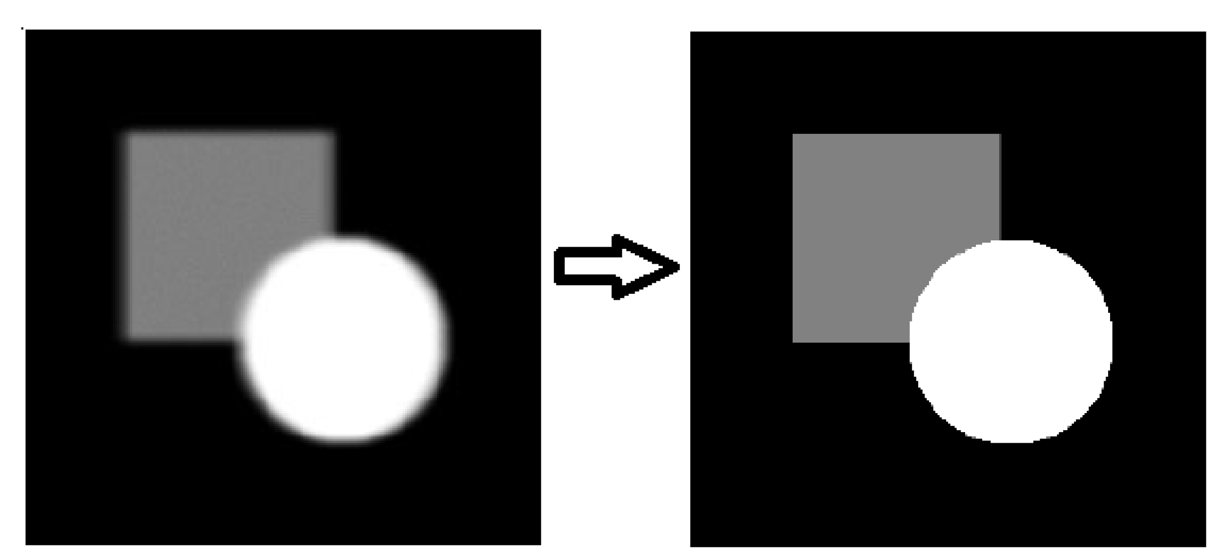
Applications

The filling of missing information, removal of noise and classify the region of interest in an image are very important topics in digital image processing applications. We are trying to develop some fraction derivatives based mathematical models as well as efficient numerical algorithms to address this type of problems.

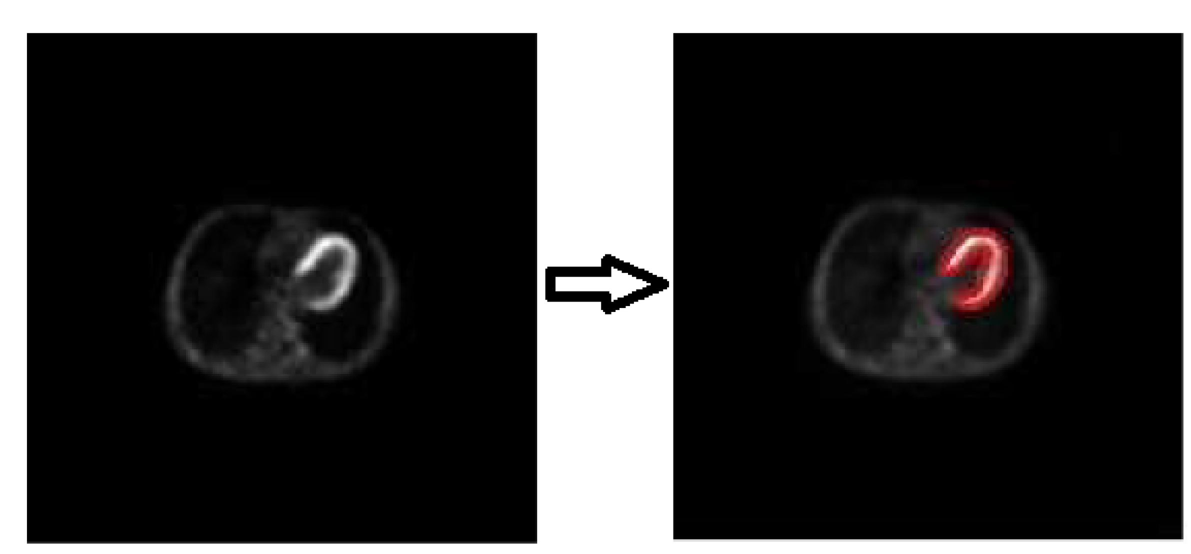
- Image Enhancement



- Image Deblurring



- Image Segmentation



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