A Gray Level Indicator Based Telegraph Diffusion Model: Application to Poisson Noise Reduction

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Abstract

This study describes a gray-level indicator-based telegraph diffusion model application to Poisson noise reduction on digital images. This approach uses the benefit of the combined effect of the diffusion equation and the wave equation. In this model, the diffusion coefficient depends on the image gradient and the gray level of the image, which controls the diffusion process better than only gradient-based techniques. We solve the PDE model using an implicit finite difference method. We also check the noise removal ability of the present model over grayscale and color images corrupted by multiplicative Poisson noise.

Introduction

In digital image processing applications, the detailed analysis of images always relies on the quality of the acquired image. However, due to the natural barriers of acquiring equipment or the presence of random fluctuations in the medium, images produced by a scanner or digital camera are typically corrupted by different types of noises. This contamination affects the purity of the edge and texture information in the images. Hence, image restoration is crucial for high-level image analysis, e.g., image segmentation, object recognition, scene understanding, etc.

Poisson Noise

Poisson noise manifests as an image distortion caused by the Poisson process. Stemming from the stochastic nature of photon emission and detection, digital imaging devices introduce signal-dependent inaccuracies. These inaccuracies result in images that exhibit characteristics consistent with the Poisson distribution. Figure 1 depicts the effect of Poisson noise on the quality of a one-dimensional signal. It indicates that the multiplicative Poisson noise degraded the high gray level regions more than the low gray level regions.

Noisy signal = poissrnd(Original signal/ σ) $\cdot$ σ

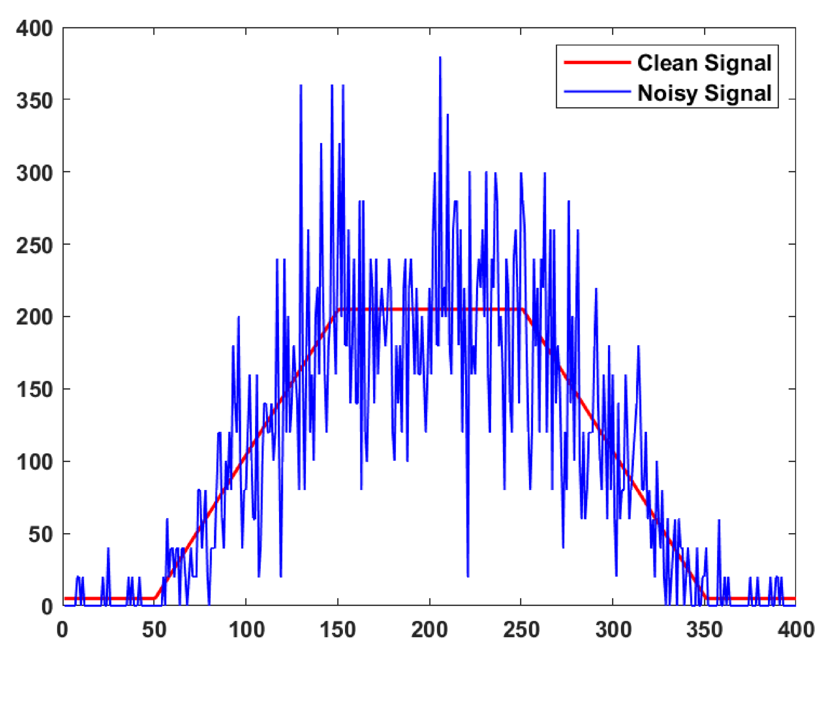


Figure 1: Effect of Poisson noise: σ=20

Proposed Model

It has been known for a long time that partial differential equation (PDE) based models are efficient techniques for noise removal in digital images. We describe the following telegraph diffusion model for Poisson noise reduction.

$$I_{tt} + \gamma I_t - \operatorname{div}\left(g\left(\cdot\right)\nabla I\right) = 0 \quad \text{in } \Omega_T := \Omega\times(0,T), \quad (1)$$

$$I(x,0) = J(x), \quad I_t(x,0) = 0 \quad \text{in } \Omega, \quad (2)$$

$$\partial_n I = 0 \quad \text{on } \partial\Omega_T := \partial\Omega\times(0,T), \quad (3)$$

where the diffusion function g is given by

$$g\left(I_\xi, |\nabla I_\xi|\right) = \left\{\frac{2|I_\xi|^\nu}{\left(M_\xi^I\right)^\nu + |I_\xi|^\nu}\right\}^{1/2} \cdot \frac{1}{1+\left(\frac{|\nabla I_\xi|}{K}\right)^2}.$$

Here, $\nu \geq 1$, $\gamma, K > 0$ are constants, $I_\xi = G_\xi * I$, $M_\xi^I = \max_{x\in\Omega}|I_\xi(x,t)|$. The gray level indicator function

$$b(I_\xi) = \left\{\frac{2|I_\xi|^\nu}{\left(M_\xi^I\right)^\nu + |I_\xi|^\nu}\right\}^{1/2}. \quad (4)$$

From (4), we can see that $b(I_\xi)$ becomes very small at low gray levels, which leads the diffusion coefficient close to zero and preserves low gray level image features. In high gray level regions, $b(I_\xi)$ approaches one and leads to the fact that $1/\left\{1+\left(\frac{|\nabla I_\xi|}{K}\right)^2\right\}$ controls the diffusion process.

Numerical Method

To solve the system (1)–(3) numerically, we construct a weighted-θ finite difference scheme. Using finite difference approximations, we replace the derivative terms in (1)–(3). The discrete form of the equation (1) could be written as

$$(1+0.5\gamma\tau)I_{i,j}^{n+1} - \tau^2\theta_1\left[\nabla(g\nabla I)\right]_{i,j}^{n+1} = 2I_{i,j}^n + \tau^2(1-\theta_1-\theta_2)\left[\nabla(g\nabla I)\right]_{i,j}^n + \tau^2\theta_2\left[\nabla(g\nabla I)\right]_{i,j}^{n-1} + (0.5\gamma\tau-1)I_{i,j}^{n-1} \quad (5)$$

where θ_1 and θ_2 are non negative weights. τ and h denote the time and spatial step sizes, respectively. $I_{i,j}^n = I(t_n, x_i, y_j)$, where $x_i = ih$ $(i = 0,1,2,...,M-1)$, $y_j = jh$ $(j = 0,1,2,...,N-1)$, $t_n = n\tau$ $(n = 0,1,2,\ldots)$, where n is the number of iterations and $M\times N$ is the image dimension. The superscript 'n' denotes the value at the n^{th} time level t_n and

$$\begin{aligned}\left[\nabla(g\nabla I)\right]_{i,j} = \frac{0.5}{\tilde{h}^2}&\Big[(g_{i,j}+g_{i+1,j})I_{i+1,j} + (g_{i,j}+g_{i-1,j})I_{i-1,j} - (g_{i+1,j}+2g_{i,j}+g_{i-1,j})\,I_{i,j}\Big]\\ +\frac{0.5}{\tilde{h}^2}&\Big[(g_{i,j}+g_{i,j+1})I_{i,j+1} + (g_{i,j}+g_{i,j-1})I_{i,j-1} - (g_{i,j+1}+2g_{i,j}+g_{i,j-1})\,I_{i,j}\Big], \quad (6)\end{aligned}$$

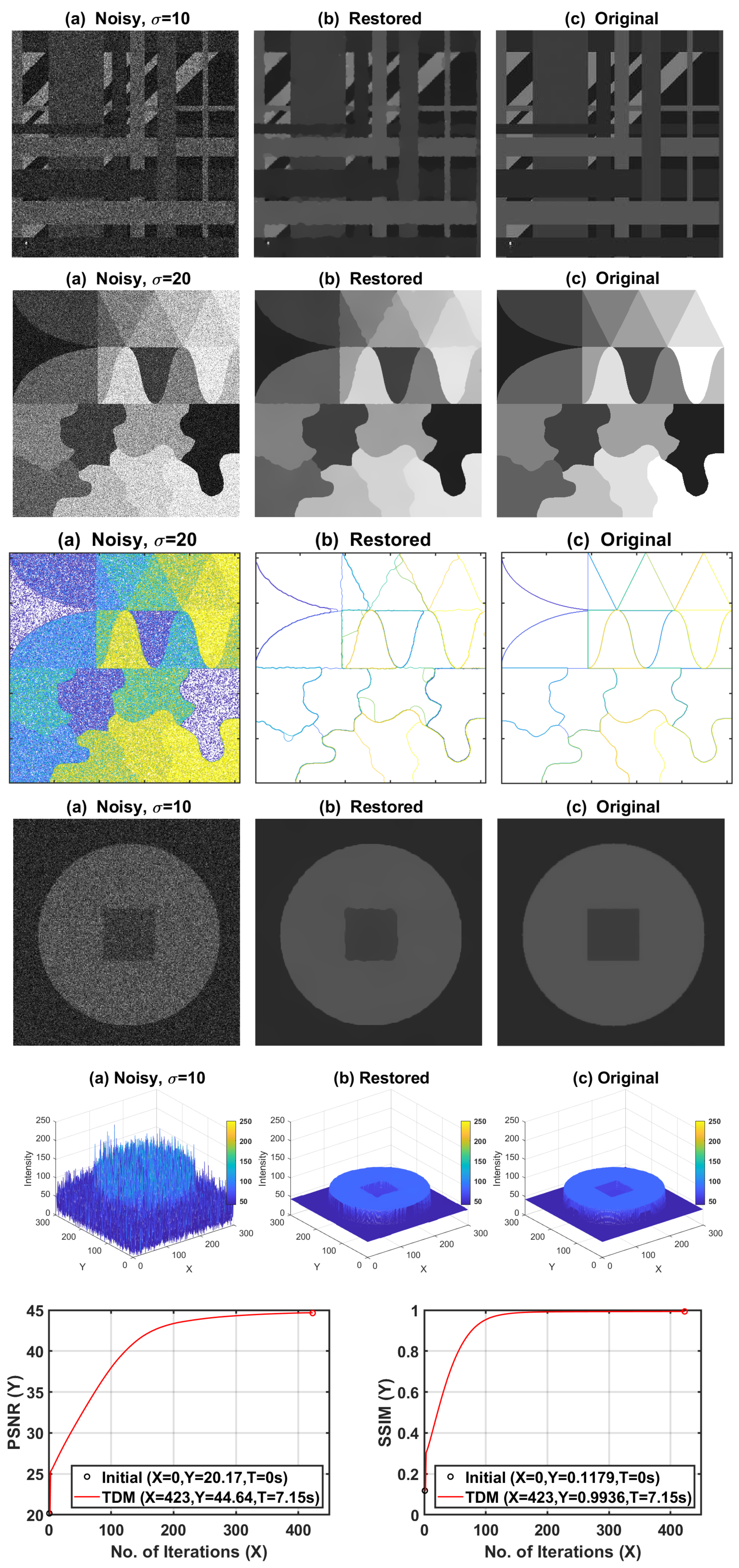
The initial and boundary conditions are given as follows:

$$I_{i,j}^0 = I_0(x_i,y_j)\,, I_{i,j}^1 = I_{i,j}^0,\ 0\leq i\leq M-1\,, 0\leq j\leq N-1\,, \quad I_{0,j}^n = I_{1,j}^n,\ I_{M-1,j}^n = I_{M-2,j}^n,\ I_{i,0}^n = I_{i,1}^n\,,\ I_{i,N-1}^n = I_{i,N-2}^n\,.$$

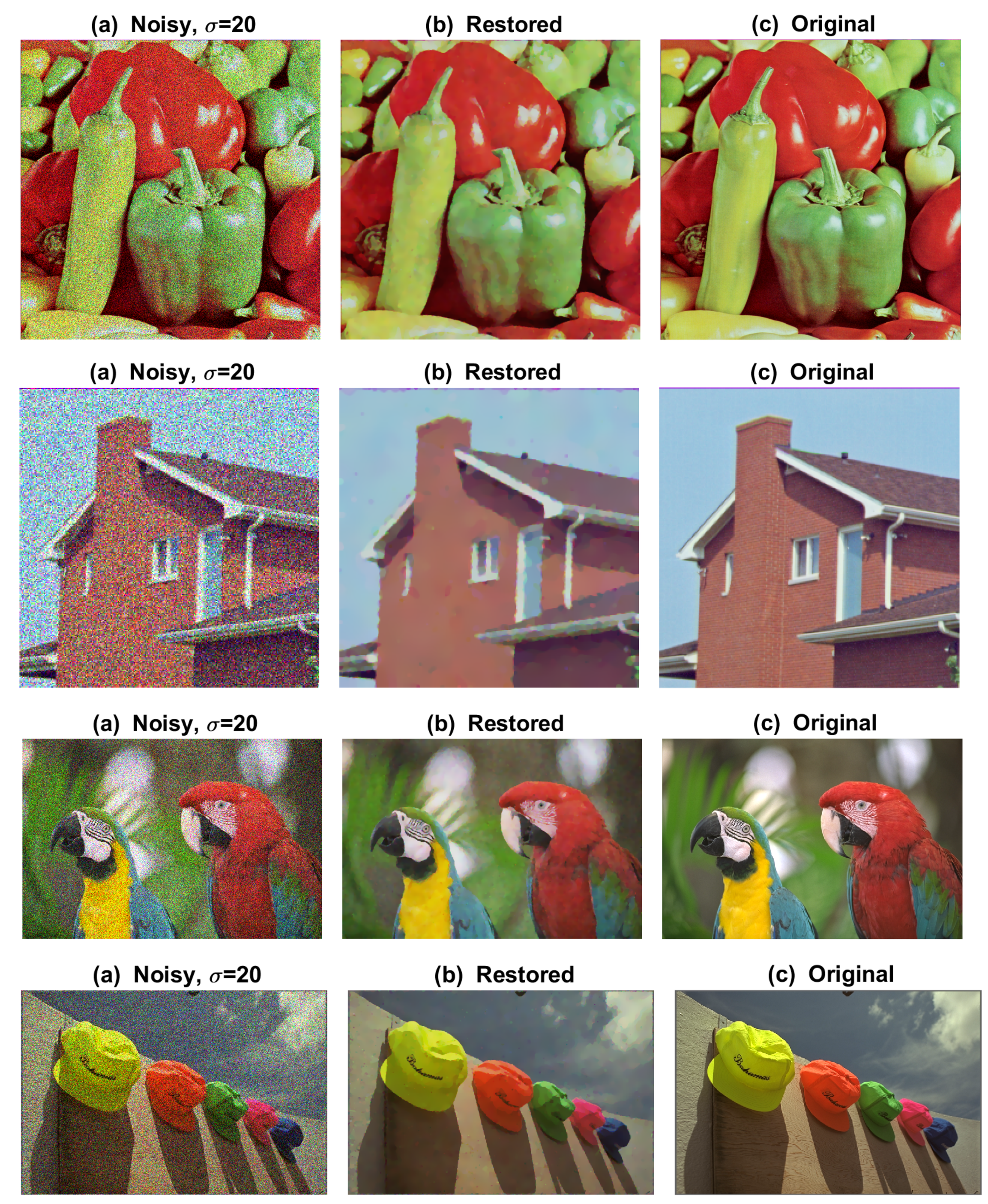
We solve the system (5) using Gauss-Seidel iterative method. We choose a uniform time step $\tau = 0.25$, spatial step $h = 1$, and $\xi = 1$ for all computations. We stop the numerical simulation process after getting the best possible peak signal-to-noise ratio (PSNR) value between the clean image (I) and the restored image (I^k) calculated by the formula

$$\text{PSNR} = 10\log_{10}\left\{\frac{\max(I)^2}{\frac{1}{\text{MN}}\sum_{i=1}^{\text{M}}\sum_{j=1}^{\text{N}}(I(i,j)-I^k(i,j))^2}\right\}. \quad (7)$$

Computational Results: Gray Image



Computational Results: Color Image



Conclusion

This study proposes a telegraph diffusion model for Poisson noise reduction in digital images. Such a method intends to preserve the image edges during the noise-removal process. To overcome the limitations of gradient-based denoising models, we consider a hybrid approach combining a gray-level indicator function with gradient-based diffusion in a telegraph diffusion framework for image denoising. To the best of our knowledge, the gray level indicator-based telegraph diffusion model has not been used before for Poisson noise suppression. Computational results of the present model indicate that the images are suitably recovered without introducing undesired artifacts. A potential direction in the telegraph diffusion model can be extended to handle texture preservation issues in various real-life images degraded by mixed noises.

References

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