Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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3.3.1. Find the peak overshoot for the second order control system given by:

$$G(S) = \frac{100}{s^2 + 10s + 100} \tag{3.3.1.1}$$

Solution: Peak overshoot (M_p) is defined as the deviation of the response at peak time from the final value of response.

$$\implies M_p = c(t_p) - c(\infty) \tag{3.3.1.2}$$

Given,

$$G(S) = \frac{C(s)}{R(s)} = \frac{100}{s^2 + 10s + 100} \tag{3.3.1.3}$$

To calculate the unit step response,

$$r(t) = 1 \implies R(s) = \frac{1}{s} \tag{3.3.1.4}$$

$$\implies C(S) = \frac{1}{s}.\frac{100}{s^2 + 10s + 100} \tag{3.3.1.5}$$

C(s) can be expanded as:

$$C(s) = \frac{1}{s} - \frac{s+5}{(s+5)^2 + 75} - \frac{1}{\sqrt{3}}.\frac{\sqrt{75}}{(s+5)^2 + 75} \tag{3.3.1.6}$$

$$c(t) = \mathcal{L}^{-1}C(s) \tag{3.3.1.7}$$

$$\implies c(t) = 1 - e^{-5t}\cos\left(\sqrt{75}t\right) \tag{3.3.1.8}$$

$$-\frac{e^{-5t}}{\sqrt{3}}.\sin\left(\sqrt{75}t\right) \tag{3.3.1.9}$$

c(t) can be further simplified to:

$$c(t) = 1 - \frac{2e^{-5t}}{\sqrt{3}}.\sin\left(5\sqrt{3}t + \frac{\pi}{3}\right) \tag{3.3.1.10}$$

From Final Value theorem:

$$\lim_{s\to 0} s.C(s) = \lim_{t\to\infty} c(t) \tag{3.3.1.11}$$

$$\implies c(\infty) = \lim_{s\to 0}\frac{100}{s^2 + 10s + 100} = 1 \tag{3.3.1.12}$$

At t_p, $c(t)$ is maximum:

$$\implies c'(t)|_{t=t_p} = \frac{10e^{-5t}}{\sqrt{3}}.\sin\left(5\sqrt{3}t + \frac{\pi}{3}\right)$$

$$-\frac{10e^{-5t}}{\sqrt{3}}.\sqrt{3}.\cos\left(5\sqrt{3}t + \frac{\pi}{3}\right) = 0 \tag{3.3.1.13}$$

$$\implies \sin\left(5\sqrt{3}t + \frac{\pi}{3}\right) - \sqrt{3}.\cos\left(5\sqrt{3}t + \frac{\pi}{3}\right) = 0 \tag{3.3.1.14}$$

$$\implies \tan\left(5\sqrt{3}t + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) \tag{3.3.1.15}$$

$$\implies 5\sqrt{3}t_p = n\pi \tag{3.3.1.16}$$

The maximum overshoot occurs at $n = 1$:

$$t_p = \frac{\pi}{\sqrt{75}} = 0.36 \tag{3.3.1.17}$$

Substitute t_p in (3.3.1.10) to get $c(t_p)$:

$$c(t_p) = 1 + e^{\frac{-\pi}{\sqrt{3}}} \implies c(t_p) = 1.163 \tag{3.3.1.18}$$

Substitute $c(t_p)$ and $c(\infty)$ in (3.3.1.2) to get peak overshoot:

$$M_p = 1.16 - 1 = 0.16 \tag{3.3.1.19}$$

3.3.2. Verify using a Python Plot
Solution:

```
codes/ee18btech11045.py
```

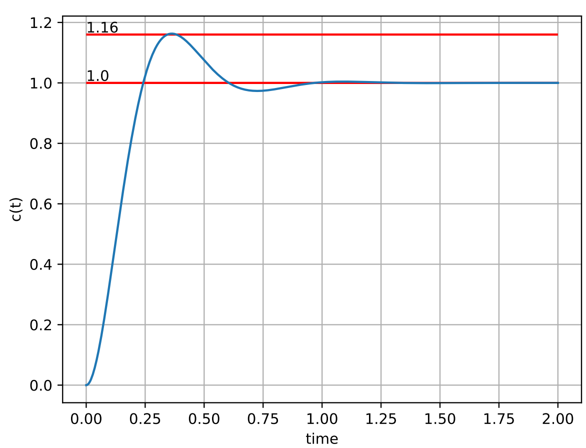

Fig. 3.3.2: c(t) vs t plot