Control Systems

G V V Sharma*

Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Bode Plots

1.1 Example

1.1.1. Sketch the bode magnitude and phase plots for the closed loop (negative feedback) system given by:

$$G(s) = \frac{100(s+2)(s+4)}{s^2 - 3s + 10} \tag{1.1.1.1}$$

$$H(s) = \frac{1}{s} \tag{1.1.1.2}$$

Solution: The system can be represented as:

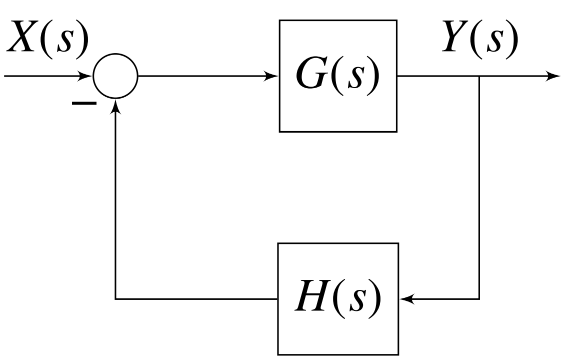

Fig. 1.1.1: Block diagram for the system

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

The closed loop transfer function of the system is given by:

$$G_m(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s).H(s)} \tag{1.1.1.3}$$

$$\implies G_m(s) = \frac{100s(s+2)(s+4)}{s^3 + 97s^2 + 610s + 800} \tag{1.1.1.4}$$

Evaluate at $s = \jmath\omega$:

$$G_m(\jmath\omega) = \frac{100\jmath\omega(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^3 + 97(\jmath\omega)^2 + 610(\jmath\omega) + 800} \tag{1.1.1.5}$$

$$= \frac{-600\omega^2 + \jmath\left(800\omega - 100\omega^3\right)}{800 - 97\omega^2 + \jmath(610\omega - \omega^3)} \tag{1.1.1.6}$$

From (1.1.1.6):

$$\left|G_m(\jmath\omega)\right| = \frac{\sqrt{(600\omega^2)^2 + (800\omega - 100\omega^3)^2}}{\sqrt{(800 - 97\omega^2)^2 + (610\omega - \omega^3)^2}} \tag{1.1.1.7}$$

$$\angle G_m(\jmath\omega) = \tan^{-1}\left(\frac{\omega^2 - 8}{6\omega}\right) - \tan^{-1}\left(\frac{610\omega - \omega^3}{800 - 97\omega^2}\right) \tag{1.1.1.8}$$

The following code plots the bode magnitude and phase plots in Fig. 1.1.1:

```
codes/ee18btech11045/
    ee18btech11045_bode1.py
```

1.1.2. Compute the gain margin of the system.
Solution:

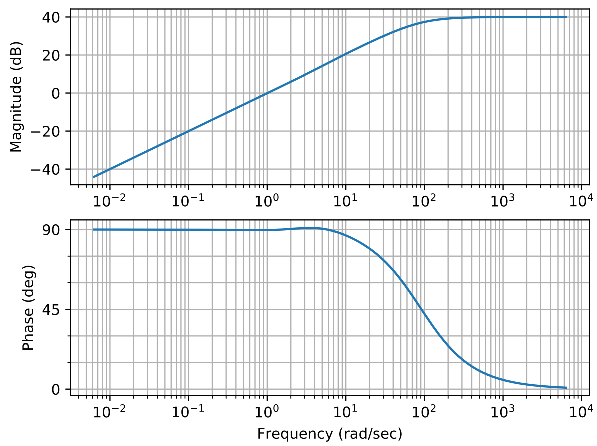

Fig. 1.1.1: Bode plot for $G_m(\jmath\omega)$

$$G(\jmath\omega)H(\jmath\omega) = \left(\frac{100(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^2-3\jmath\omega+10}\right)\left(\frac{1}{\jmath\omega}\right) \tag{1.1.2.1}$$

$$= \frac{100\left(-\omega^2+8+\jmath6\omega\right)}{3\omega^2+\jmath\left(10\omega-\omega^3\right)} \tag{1.1.2.2}$$

Using (1.1.2.2)

$$\left|G(\jmath\omega)H(\jmath\omega)\right| = \frac{100\sqrt{(8-\omega^2)^2+(6\omega)^2}}{\sqrt{(3\omega^2)^2+(10\omega-\omega^3)^2}} \tag{1.1.2.3}$$

$$\angle G(\jmath\omega)H(\jmath\omega) = \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) - \tan^{-1}\left(\frac{10-\omega^2}{3\omega}\right) \tag{1.1.2.4}$$

At the phase crossover frequency ω_{pc}:

$$\left|\angle G(\jmath\omega)H(\jmath\omega)\right| = 180 \tag{1.1.2.5}$$

$$\implies \tan^{-1}\left(\frac{6\omega_{pc}}{8-\omega_{pc}^2}\right) - \tan^{-1}\left(\frac{10-\omega_{pc}^2}{3\omega_{pc}}\right) = 180 \tag{1.1.2.6}$$

Solving the above equation:

$$\frac{6\omega_{pc}}{8-\omega_{pc}^2} = \frac{10-\omega_{pc}^2}{3\omega_{pc}} \tag{1.1.2.7}$$

$$\implies \omega_{pc} = 5.8rad/sec \tag{1.1.2.8}$$

$$\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{pc}} = 28.1dB \tag{1.1.2.9}$$

Gain Margin GM :

$$GM = 0 - \left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{pc}} dB \tag{1.1.2.10}$$

$$= -28.1dB \tag{1.1.2.11}$$

1.1.3. Compute the phase margin of the system.
Solution:
At the gain crossover frequency ω_{gc}:

$$\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{gc}} = 1 \tag{1.1.3.1}$$

From (1.1.2.3),

$$10^4\left(\left(8-\omega^2\right)^2+(6\omega)^2\right) = 9\omega^4+\left(10\omega-\omega^3\right)^2 \tag{1.1.3.2}$$

$$\implies \omega_{gc} = 100.15rad/sec \tag{1.1.3.3}$$

Substitute ω_{gc} in (1.1.2.4):

$$\angle G(\jmath\omega)H(\jmath\omega)_{\omega=\omega_{gc}} = 265^\circ \tag{1.1.3.4}$$

Phase Margin PM:

$$PM = 180^\circ - \angle G(\jmath\omega)H(\jmath\omega)_{\omega=\omega_{gc}} \tag{1.1.3.5}$$

$$= 180^\circ - 265^\circ = -85^\circ \tag{1.1.3.6}$$

1.1.4. Verify the values of Gain Margin and Phase Margin using a python plot.
Solution:
The following code is used to verify the gain and phase margins:

```
codes/ee18btech11045/
    ee18btech11045_bode2.py
```

1.1.5. Comment on the stability of the system
Solution:
As both the Gain Margin (GM) and Phase Margin (PM) are found to be negative, the system is unstable.

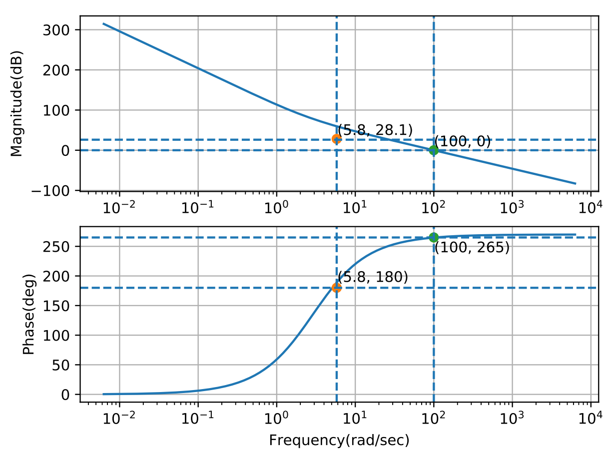

Fig. 1.1.4: Bode plot for $G(\jmath\omega)H(\jmath\omega)$