Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

3.3 Peak Overshoot

3.3.1. Find the expression of peak overshoot for a second order control system.

Solution:

The transfer function for a second order control system is:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3.3.1.1}$$

To calculate the unit step response,

$$r(t) = 1 \implies R(s) = \frac{1}{s} \tag{3.3.1.2}$$

On Simplifying,

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d} . \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \tag{3.3.1.3}$$

where,

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \tag{3.3.1.4}$$

In time domain,

$$c(t) = \mathcal{L}^{-1}C(s) \tag{3.3.1.5}$$

$$\implies c(t) = 1 - e^{-}\zeta\omega_n t(cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}.sin\omega_d t) \tag{3.3.1.6}$$

Peak overshoot (Mp) is defined as the deviation of the response at peak time from the final value of response.

$$\implies M_p = c(t_p) - c(\infty) \tag{3.3.1.7}$$

At t_p:

$$\frac{dc(t)}{d(t)} = 0 \tag{3.3.1.8}$$

Applying this condition on (3.3.1.6):

$$\implies t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{3.3.1.9}$$

Substituting t_p in (3.3.1.6):

$$c(t_p) = 1 + e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{3.3.1.10}$$

From (3.3.1.6):

$$\lim_{t\to\infty} c(t) = 1 \tag{3.3.1.11}$$

Substituting the value of $c(t_p)$ and $c(\infty)$ in (3.3.1.7):

$$M_p(PeakOvershoot) = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{3.3.1.12}$$

3.3.2. Find the peak Overshoot for the following second order control system

$$G(S) = \frac{100}{s^2 + 10s + 100} \tag{3.3.2.1}$$

Solution:
For the given Equation :

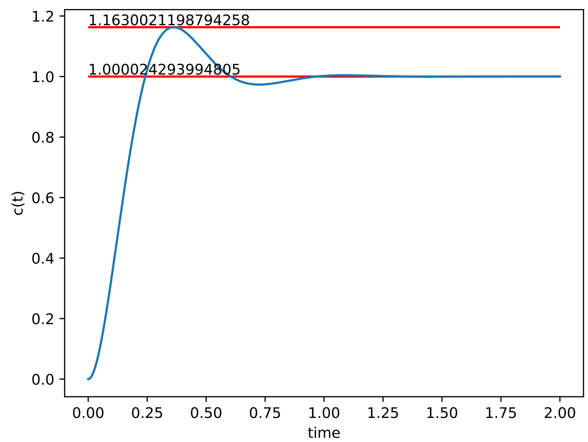
$$\zeta = 0.5 \tag{3.3.2.2}$$

Substitute this value of zeta in (3.3.1.12) to get:

$$M_p = 0.163 \tag{3.3.2.3}$$

3.3.3. Verify using a Python Plot
Solution:

```
codes/ee18btech11045.py
```



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