Control Systems

G V V Sharma*

Contents

1 Signal Flow Graph 1
- 1.1 Mason's Gain Formula . . . 1
- 1.2 Matrix Formula 1

2 Bode Plot 1
- 2.1 Introduction 1
- 2.2 Example 1

3 Second order System 1
- 3.1 Damping 1
- 3.2 Example 1
- 3.3 Peak Overshoot 1

4 Routh Hurwitz Criterion 2
- 4.1 Routh Array 2
- 4.2 Marginal Stability 2
- 4.3 Stability 2
- 4.4 Example 2

5 State-Space Model 2
- 5.1 Controllability and Observability 2
- 5.2 Second Order System 2
- 5.3 Example 2
- 5.4 Example 2
- 5.5 Example 2

6 Nyquist Plot 2

7 Compensators 2
- 7.1 Phase Lead 2
- 7.2 Example 2

8 Gain Margin 2
- 8.1 Introduction 2
- 8.2 Example 2

9 Phase Margin 2

10 Oscillator 2
- 10.1 Introduction 2
- 10.2 Example 2

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

3.3 Peak Overshoot

3.3.1. Find the peak overshoot for the second order control system given by:

$$G(S) = \frac{100}{s^2 + 10s + 100} \tag{3.3.1.1}$$

Solution: Peak overshoot (M_p) is defined as the deviation of the response at peak time from the final value of response.

$$\implies M_p = c(t_p) - c(\infty) \tag{3.3.1.2}$$

Given,

$$G(S) = \frac{C(s)}{R(s)} = \frac{100}{s^2 + 10s + 100} \tag{3.3.1.3}$$

To calculate the unit step response,

$$r(t) = 1 \implies R(s) = \frac{1}{s} \tag{3.3.1.4}$$

$$\implies C(S) = \frac{100}{(s)(s^2 + 10s + 100)} \quad (3.3.1.5)$$

C(s) can be expanded as:

$$C(s) = \frac{1}{s} - \frac{s+5}{(s+5)^2 + 75} - \frac{1}{\sqrt{3}}.\frac{\sqrt{75}}{(s+5)^2 + 75} \quad (3.3.1.6)$$

In time domain,

$$c(t) = \mathcal{L}^{-1} C(s) \quad (3.3.1.7)$$

$$\implies c(t) = 1 - e^{-5t} cos(\sqrt{75}t) - \frac{e^{-5t}}{\sqrt{3}}.sin(\sqrt{75}t) \quad (3.3.1.8)$$

From (3.3.1.8):

$$\lim_{t \to \infty} c(t) = 1 \quad (3.3.1.9)$$

At t_p, $c(t)$ is maximum:

$$\implies \frac{dc(t)}{d(t)} = 0 \quad (3.3.1.10)$$

Applying this condition on (3.3.1.8), we get:

$$t_p = \frac{\pi}{\sqrt{75}} \quad (3.3.1.11)$$

Substitute t_p in (3.3.1.8) to get $c(t_p)$:

$$c(t_p) = 1 + e^{\frac{-\pi}{\sqrt{3}}} \implies c(t_p) = 1.163 \quad (3.3.1.12)$$

Substitute $c(t_p)$ and $c(\infty)$ in (3.3.1.2) to get peak overshoot:

$$M_p = 1.163 - 1 = 0.163 \quad (3.3.1.13)$$

3.3.2. Verify using a Python Plot
Solution:

codes/ee18btech11045.py

/ee18btech11045-eps-converted-to.pdf

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 State-Space Model

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

6 Nyquist Plot

7 Compensators

7.1 Phase Lead

7.2 Example

8 Gain Margin

8.1 Introduction

8.2 Example

9 Phase Margin

10 Oscillator

10.1 Introduction

10.2 Example