

# Control Systems

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where,

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (3.3.1.4)$$

In time domain,

$$c(t) = \mathcal{L}^{-1}C(s) \quad (3.3.1.5)$$

$$\Rightarrow c(t) = 1 - e^{-\zeta\omega_n t} \left( \cos\omega_d t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin\omega_d t \right) \quad (3.3.1.6)$$

Peak overshoot (Mp) is defined as the deviation of the response at peak time from the final value of response.

$$\Rightarrow M_p = c(t_p) - c(\infty) \quad (3.3.1.7)$$

At  $t_p$ :

$$\frac{dc(t)}{dt} = 0 \quad (3.3.1.8)$$

Applying this condition on (3.3.1.6):

$$\Rightarrow t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad (3.3.1.9)$$

Substituting  $t_p$  in (3.3.1.6):

$$c(t_p) = 1 + e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}} \quad (3.3.1.10)$$

From (3.3.1.6):

$$\lim_{t \rightarrow \infty} c(t) = 1 \quad (3.3.1.11)$$

Substituting the value of  $c(t_p)$  and  $c(\infty)$  in (3.3.1.7):

$$M_p(\text{PeakOvershoot}) = e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}} \quad (3.3.1.12)$$

3.3.2. Find the peak Overshoot for the following second order control system

$$G(S) = \frac{100}{s^2 + 10s + 100} \quad (3.3.2.1)$$

**Solution:**

For the given Equation :

$$\zeta = 0.5 \quad (3.3.2.2)$$

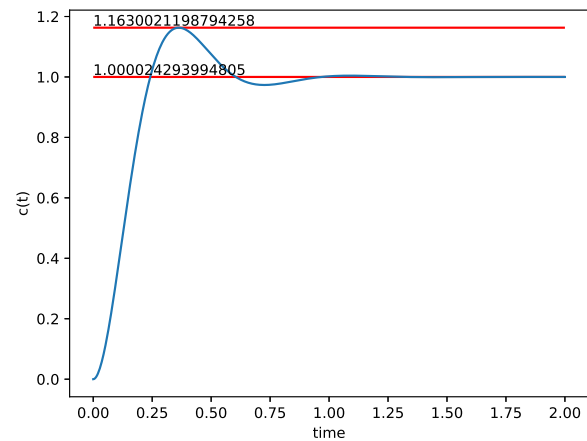
Substitute this value of zeta in (3.3.1.12) to get:

$$M_p = 0.163 \quad (3.3.2.3)$$

3.3.3. Verify using a Python Plot

**Solution:**

codes/ee18btech11045.py



## 4 ROUTH HURWITZ CRITERION

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### 4.2 Marginal Stability

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## 5 STATE-SPACE MODEL

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