

# Control Systems

G V V Sharma\*

## CONTENTS

<b>1</b>	<b>Bode Plots</b>	<b>1</b>
1.1	Example . . . . .	1

**Abstract**—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

## 1 BODE PLOTS

### 1.1 Example

1.1.1. Sketch the bode magnitude and phase plots for the closed loop (negative feedback) system given by:

$$G(s) = \frac{100(s+2)(s+4)}{s^2 - 3s + 10} \quad (1.1.1.1)$$

$$H(s) = \frac{1}{s} \quad (1.1.1.2)$$

**Solution:** The system can be represented as:

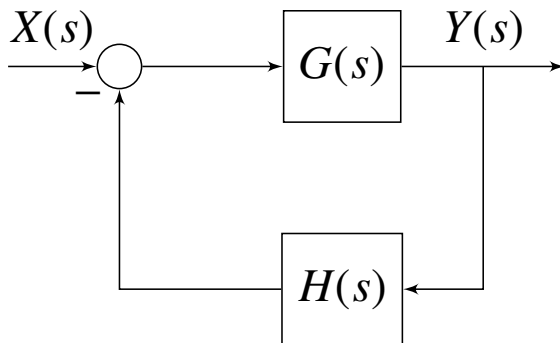


Fig. 1.1.1: Block diagram for the system

The closed loop transfer function of the system is given by:

$$G_m(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} \quad (1.1.1.3)$$

$$\Rightarrow G_m(s) = \frac{100s(s+2)(s+4)}{s^3 + 97s^2 + 610s + 800} \quad (1.1.1.4)$$

Evaluate at  $s = j\omega$ :

$$G_m(j\omega) = \frac{100j\omega(j\omega+2)(j\omega+4)}{(j\omega)^3 + 97(j\omega)^2 + 610(j\omega) + 800} \quad (1.1.1.5)$$

$$= \frac{-600\omega^2 + j(800\omega - 100\omega^3)}{800 - 97\omega^2 + j(610\omega - \omega^3)} \quad (1.1.1.6)$$

From (1.1.1.6):

$$|G_m(j\omega)| = \frac{\sqrt{(600\omega^2)^2 + (800\omega - 100\omega^3)^2}}{\sqrt{(800 - 97\omega^2)^2 + (610\omega - \omega^3)^2}} \quad (1.1.1.7)$$

$$\angle G_m(j\omega) = \tan^{-1}\left(\frac{\omega^2 - 8}{6\omega}\right) - \tan^{-1}\left(\frac{610\omega - \omega^3}{800 - 97\omega^2}\right) \quad (1.1.1.8)$$

The following code plots the bode magnitude and phase plots in Fig. 1.1.1:

```
codes/ee18btech11045/
ee18btech11045_bode1.py
```

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

1.1.2. Compute the gain margin of the system.  
**Solution:**

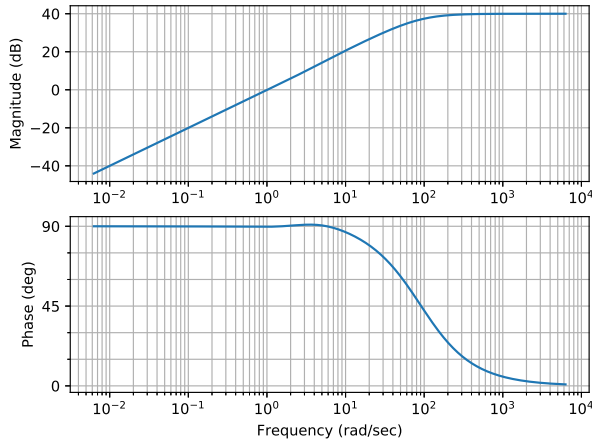


Fig. 1.1.1: Bode plot for  $G_m(j\omega)$

$$G(j\omega)H(j\omega) = \left( \frac{100(j\omega + 2)(j\omega + 4)}{(j\omega)^2 - 3j\omega + 10} \right) \left( \frac{1}{j\omega} \right) \quad (1.1.2.1)$$

$$= \frac{100(-\omega^2 + 8 + j6\omega)}{3\omega^2 + j(10\omega - \omega^3)} \quad (1.1.2.2)$$

Using (1.1.2.2)

$$|G(j\omega)H(j\omega)| = \frac{100\sqrt{(8 - \omega^2)^2 + (6\omega)^2}}{\sqrt{(3\omega^2)^2 + (10\omega - \omega^3)^2}} \quad (1.1.2.3)$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{6\omega}{8 - \omega^2}\right) - \tan^{-1}\left(\frac{10 - \omega^2}{3\omega}\right) \quad (1.1.2.4)$$

At the phase crossover frequency  $\omega_{pc}$ :

$$|\angle G(j\omega)H(j\omega)| = 180 \quad (1.1.2.5)$$

$$\Rightarrow \tan^{-1}\left(\frac{6\omega_{pc}}{8 - \omega_{pc}^2}\right) - \tan^{-1}\left(\frac{10 - \omega_{pc}^2}{3\omega_{pc}}\right) = 180 \quad (1.1.2.6)$$

Solving the above equation:

$$\frac{6\omega_{pc}}{8 - \omega_{pc}^2} = \frac{10 - \omega_{pc}^2}{3\omega_{pc}} \quad (1.1.2.7)$$

$$\Rightarrow \omega_{pc} = 5.8 \text{ rad/sec} \quad (1.1.2.8)$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = 28.1 \text{ dB} \quad (1.1.2.9)$$

Gain Margin  $GM$  :

$$GM = 0 - |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} \text{ dB} \quad (1.1.2.10)$$

$$= -28.1 \text{ dB} \quad (1.1.2.11)$$

1.1.3. Compute the phase margin of the system.

**Solution:**

At the gain crossover frequency  $\omega_{gc}$ :

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \quad (1.1.3.1)$$

From (1.1.2.3),

$$10^4 \left( (8 - \omega^2)^2 + (6\omega)^2 \right) = 9\omega^4 + (10\omega - \omega^3)^2 \quad (1.1.3.2)$$

$$\Rightarrow \omega_{gc} = 100.15 \text{ rad/sec} \quad (1.1.3.3)$$

Substitute  $\omega_{gc}$  in (1.1.2.4):

$$\angle G(j\omega)H(j\omega)_{\omega=\omega_{gc}} = 265^\circ \quad (1.1.3.4)$$

Phase Margin  $PM$ :

$$PM = 180^\circ - \angle G(j\omega)H(j\omega)_{\omega=\omega_{gc}} \quad (1.1.3.5)$$

$$= 180^\circ - 265^\circ = -85^\circ \quad (1.1.3.6)$$

1.1.4. Verify the values of Gain Margin and Phase Margin using a python plot.

**Solution:**

The following code is used to verify the gain and phase margins:

```
codes/ee18btech11045/
ee18btech11045_bode2.py
```

1.1.5. Comment on the stability of the system

**Solution:**

As both the Gain Margin (GM) and Phase Margin (PM) are found to be negative, the system is unstable.

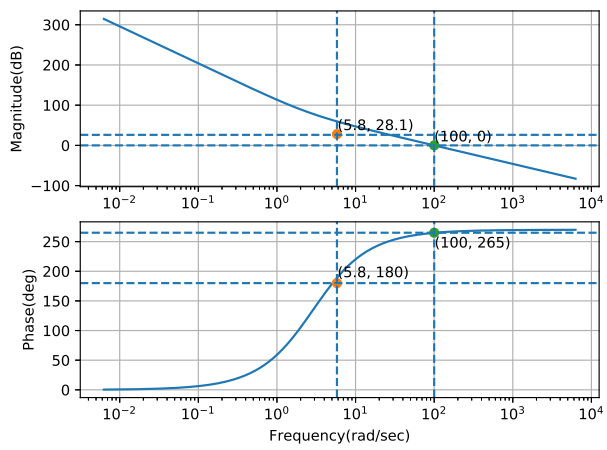


Fig. 1.1.4: Bode plot for  $G(j\omega)H(j\omega)$