Trans-Resistance Amplifier

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Contents

For the feedback transresistance amplifier in Fig. , use small-signal analysis to find the open-loop gain G, the feedback H, and the closed loop gain G_m. Neglect r_o of each of the transistors and assume $R_C << \beta_2 R_E$ and $R_E << R_F$, and that the feedback causes the signal voltage at the input node to be nearly zero. Evaluate $/ frac V_o I_s$ for the following component values: $\beta_1 = \beta_2 = 100$, $R_C = R_E = 10k\Omega$ and $R_F = 100k\Omega$.

1. Draw the small-signal equivalent of the circuit in Fig.1.

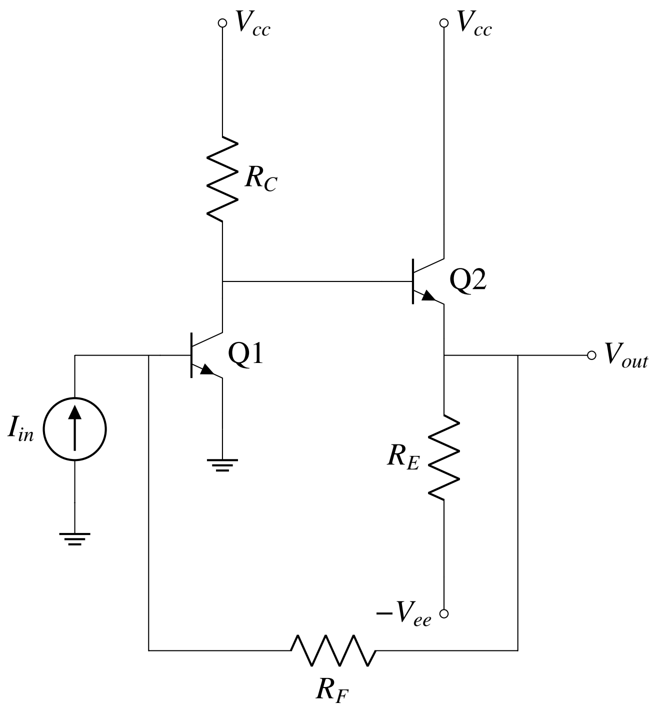

Fig. 1

Solution:
The equivalent circuit is Fig.1

2. Find the expression for the open loop Gain(G) of the system.

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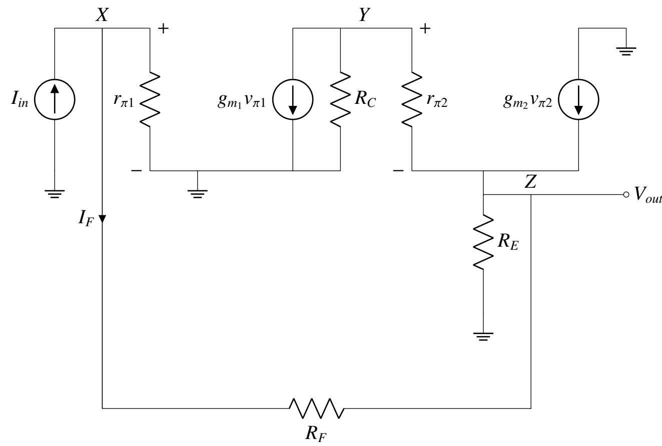

Fig. 1

Solution:
The given system is a cascaded system of Q_1 and Q_2. The signal flow graph is illustrated in Fig. 2

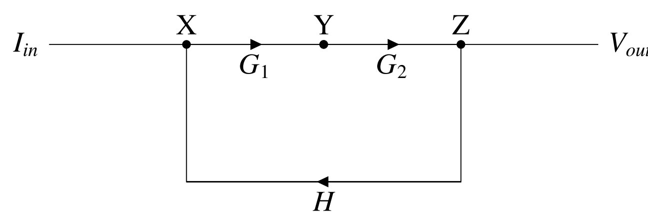

Fig. 2

So, if the gain of Q_1 and Q_2 are G_1 and G_2 respectively, the open-loop gain (G) is given by:

$$G = G_1 G_2 \tag{2.1}$$

Q_1 is in CE(Common-emitter) stage. The input signal is I_{in}. From fig. 1,

$$I_X = I_{in} \tag{2.2}$$

$$\beta = \frac{I_c}{I_b} \tag{2.3}$$

Applying Kirchoff's Law in the loop connecting Y to ground,

$$\implies V_Y = \beta I_{in} R_C \tag{2.4}$$

$$G_1 = \frac{V_{out}}{I_{in}} = \frac{V_Y}{I_X} \tag{2.5}$$
$$= \beta R_c \tag{2.6}$$

Q_2 is in emitter follower topology.

$$V_{\pi 2} = V_Y - V_Z \tag{2.7}$$

Applying Kirchoff's Law,

$$\frac{V_Y - V_Z}{r_\pi} + g_{m2}(V_Y - V_Z) = \frac{V_Z}{R_E} \tag{2.8}$$

$$\implies \frac{V_Z}{V_Y} = \frac{R_E}{\frac{1}{g_{m2}} + R_E} \tag{2.9}$$

$$\implies G_2 = \frac{R_E}{\frac{1}{g_{m2}} + R_E} \tag{2.10}$$

From (2.1), the open loop gain (G):

$$G = (\beta_1 R_c)\left(\frac{R_E}{\frac{1}{g_{m2}} + R_E}\right) \tag{2.11}$$

3. Find the feedback factor(H) of the given circuit.
Solution:
From Fig.1, the feedback circuit consists of only a resistor R_F:

$$\therefore H = \frac{I_F}{V_{out}} = \frac{1}{R_F} \tag{3.1}$$

4. Find the closed loop gain of the system.
Solution:
The closed loop gain of a system is given by:

$$G_L = \frac{G}{1 + GH} \tag{4.1}$$

From (2.11) and (3.1). The closed loop gain of the circuit is given by:

$$G_L = \frac{(\beta_1 R_c)\left(\frac{R_E}{\frac{1}{g_{m2}} + R_E}\right)}{1 + \frac{(\beta_1 R_c)\left(\frac{R_E}{\frac{1}{g_{m2}} + R_E}\right)}{R_F}} \tag{4.2}$$

$$= \frac{R_F R_C R_E \beta}{\beta R_C R_E + R_F\left(\frac{1}{g_{m2}} + R_E\right)} \tag{4.3}$$

5. Find G,H and G_L for the given problem. Parameters are summarised in table 5.

Parameters	Value
V_{cc}	$5V$
β_1	100
β_2	100
R_C	$10K\Omega$
R_E	$10K\Omega$
R_F	$100K\Omega$

TABLE 5

Solution:
To calculate the bias values of Q1, Q2. Remove the input and output, the resultant circuit is shown in fig.5
Applying KVL to the circuit, we get:

$$0.7 + I_{b1}R_F + (I_{b1} - (\beta + 1)I_{b2})R_E = -V_{ee} \tag{5.1}$$

$$0.7 + I_{b1}R_F + 0.7 + (\beta I_{b1} + I_{b2})R_C = V_{cc} \tag{5.2}$$

Solving the above equations, we get:

$$I_{b1} = \frac{\frac{V_{cc}-1.4}{R_C} - \frac{V_{ee}+0.7}{R_E(\beta+1)}}{\frac{R_F+\beta R_C}{R_C} + \frac{R_E+R_F}{R_E(\beta+1)}} \tag{5.3}$$
$$= 3.22 * 10^{-6} \tag{5.4}$$

$$I_{b2} = \frac{\frac{V_{cc}-1.4}{\beta R_C+R_F} + \frac{V_{ee}+0.7}{R_E+R_F}}{\frac{R_C}{R_F+\beta R_C} + \frac{R_E(\beta+1)}{R_E+R_F}} \tag{5.5}$$

we know,

$$g_m = \frac{I_c}{V_T} \tag{5.6}$$

where, V_T = 26mV, and

$$r_\pi = \frac{\beta}{g_m} \tag{5.7}$$

$$\therefore g_{m1} = \frac{\beta I_{b1}}{V_T} \tag{5.8}$$
$$= 0.0123 \tag{5.9}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} \tag{5.10}$$
$$= 8130\Omega \tag{5.11}$$

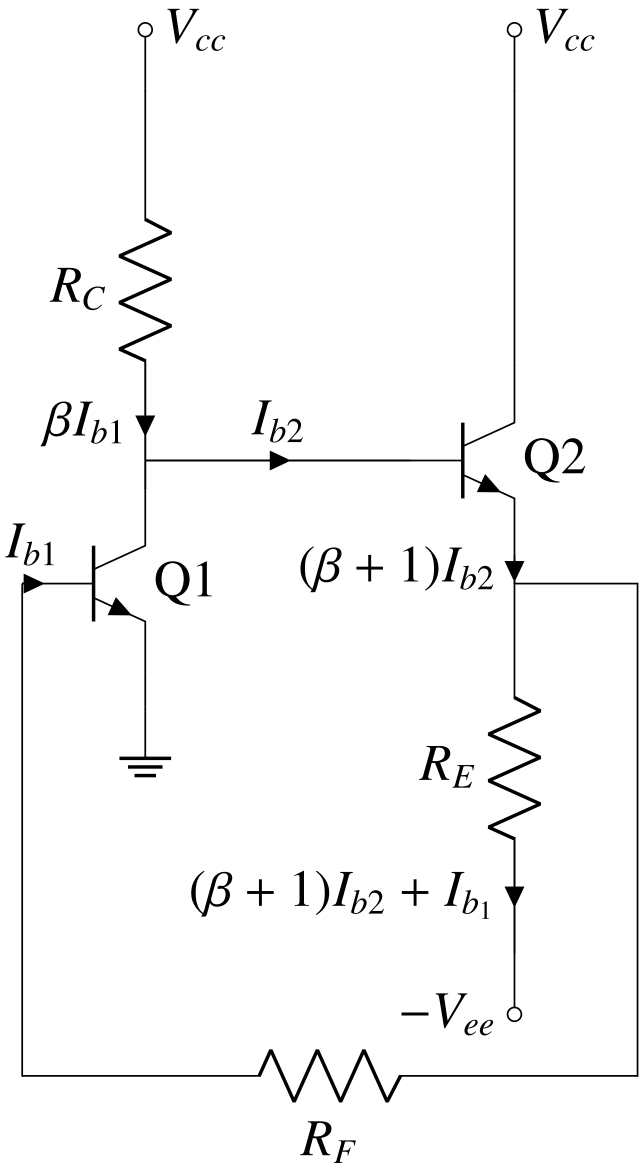

Fig. 5

$$\therefore g_{m2} = \frac{\beta I_{b2}}{V_T} \tag{5.12}$$
$$= 0.023 \tag{5.13}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} \tag{5.14}$$
$$= 4347.8\Omega \tag{5.15}$$

From (2.11), the open loop gain (G):

$$G = (100 * (10K)) \left(\frac{10^4}{10^4 + \frac{1}{0.023}} \right) \Omega \tag{5.16}$$
$$= \left(10^6\right)(0.995) \tag{5.17}$$
$$= 995670\Omega \tag{5.18}$$

From (3.1), the feedback (H):

$$H = \frac{1}{100K}\Omega^{-1} \implies H = 10^{-5}\Omega^{-1} \tag{5.19}$$

From (4.3), the closed loop gain (G_L):

$$G_L = \frac{995670}{1 + (995670)(10^{-5})}\Omega \tag{5.20}$$
$$= 99006.52\Omega \tag{5.21}$$

6. Verify the result using spice simulation.
Solution:
The following netlist simulates the closed loop gain for a sinusoidal signal of amplitude 10^{-6}

```
codes/ee18btech11045/spice/
    ee18btech11045_clresult.net
```

The output is plotted using the folowing code.

```
codes/ee18btech11045/spice/
    ee18btech11045_clresult.py
```

The output is plotted in fig. 6. The output amplitude is shown to be 0.1 .

$$\therefore \frac{V_{out}}{I_{in}} \approx 10^5 \tag{6.1}$$

This proves the value calculated in (5.21).

Fig. 6

7. Represent the circuit using a Feedback Block diagram.
Solution:
8. Calculate the input resistance of the open loop and closed loop system and compare.
Solution:
For the cascaded system of $Q1$ and $Q2$, the input resistance of the system R_{in},

$$R_{in} = R_{in_{Q1}} \tag{8.1}$$

To calculate the input resistance of the system, shot the independent sources and find the ratio

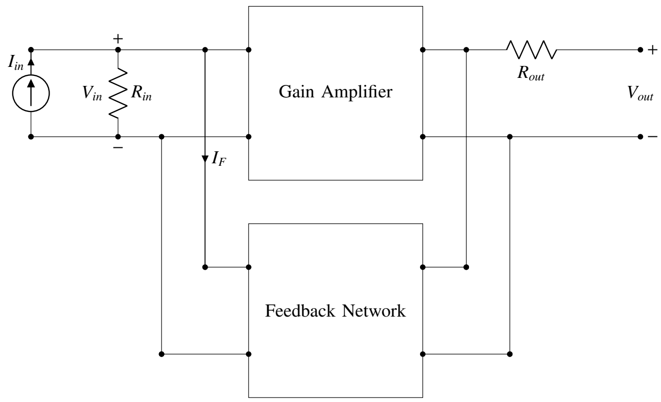

Fig. 7: Shunt-Shunt Feedback

$\frac{V_in}{I_in}$. As, $Q1$ is in common-emitter stage, from the fig. 1,

$$R_in = r_{\pi 1} \quad (8.2)$$
$$= 8130\Omega \quad (8.3)$$

After feedback is applied, to calculate input resistance, consider an input source V_{in} is connected to the input,

$$V_{out} = I_{in}G \qquad = \frac{V_{in}}{R_{in}}G \quad (8.4)$$

But as feedback is applied,

$$V_{in} = V_{out}H \qquad = \frac{V_{in}}{R_{in}}GH \quad (8.5)$$

Applying KVL for input loop in fig. 7,

$$\left(I_{in} - \frac{V_{in}}{R_{in}}GK\right)R_{in} = V_{in} \quad (8.6)$$

$$\implies R_{in_{cl}} = \frac{V_{in}}{I_{in}} = \frac{R_in}{1+GH} \quad (8.7)$$

where, R_in is the input resistance of open loop.Therefore, R_{in} after feedback:

$$R_{in} = \frac{r_{pi1}}{1+GH} \quad (8.8)$$
$$= \frac{8130}{10.95} = 742\Omega \quad (8.9)$$

The input should act as an ideal current source, so as the input resistance is decreased, the feedback gives more favourable value of R_{in}.

9. Calculate the output resistance of the open loop and closed loop system and compare.
Solution:
Similar to the input resistance, the output resistance of the cascaded system is the output resistance of $Q2$. As $Q2$ is in emitter follower configuration, from the fig. 1,

$$R_{out} = \frac{1}{g_{m2}} \quad (9.1)$$
$$= \frac{1}{0.023} = 43.37\Omega \quad (9.2)$$

To calculate output resistance after feedback is applied, consider a voltage source V_X applied at V_{out} with output current I_X :

$$V_{in} = HV_X \quad (9.3)$$

$$\implies V_{out} = GHV_X \quad (9.4)$$

Applying KVL at ouput loop,

$$\frac{GHV_X + V_X}{R_{out}} = I_X \quad (9.5)$$

The closed loop output impedance,

$$R_{out_{cl}} = \frac{V_X}{I_X} \quad (9.6)$$
$$= \frac{R_{out}}{1+GH} \quad (9.7)$$

The output resistance after feedback:

$$R_{out} = \frac{43.37}{10.95} = 3.96\Omega \quad (9.8)$$

The ouput should act as an ideal voltage source, i.e the output resistance should be as low as possible. As feedback reduces the values of R_{out}, it causes the output resistances to be more favourable.