Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Frequency Response Analysis

1.1 Polar Plot

1.2 Direct and Inverse Polar Plot

1.3 Bode Plot

1.3.1. Sketch the bode magnitude and phase plots for the closed loop (negative feedback) system given by:

$$G(s) = \frac{100(s+2)(s+4)}{s^2 - 3s + 10} \tag{1.3.0.1}$$

$$H(s) = \frac{1}{s} \tag{1.3.0.2}$$

Solution: The closed loop transfer function of the system is given by:

$$G_m(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s).H(s)} \tag{1.3.0.3}$$

$$\implies G_m(s) = \frac{100s(s+2)(s+4)}{s^3 + 97s^2 + 610s + 800} \tag{1.3.0.4}$$

Evaluate at $s = \jmath\omega$:

$$G_m(\jmath\omega) = \frac{100\jmath\omega(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^3 + 97(\jmath\omega)^2 + 610(\jmath\omega) + 800} \tag{1.3.0.5}$$

$$= \frac{-600\omega^2 + \jmath\left(800\omega - 100\omega^3\right)}{800 - 97\omega^2 + \jmath\left(610\omega - \omega^3\right)} \tag{1.3.0.6}$$

From (1.3.0.6):

$$\left|G_m(\jmath\omega)\right| = \frac{\sqrt{\left(600\omega^2\right)^2 + \left(800\omega - 100\omega^3\right)^2}}{\sqrt{\left(800 - 97\omega^2\right)^2 + \left(610\omega - \omega^3\right)^2}} \tag{1.3.0.7}$$

$$\underline{/G_m}(\jmath\omega) = \tan^{-1}\left(\frac{\omega^2 - 8}{6\omega}\right) - \tan^{-1}\left(\frac{610\omega - \omega^3}{800 - 97\omega^2}\right) \tag{1.3.0.8}$$

The following code plots the bode magnitude and phase plots in Fig. 1.3.1:

```
codes/ee18btech11045/
    ee18btech11045_bode1.py
```

1.3.2. Compute the gain margin of the system.
Solution:

$$G(\jmath\omega)H(\jmath\omega) = \left(\frac{100(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^2 - 3\jmath\omega + 10}\right)\left(\frac{1}{\jmath\omega}\right) \tag{1.3.0.9}$$

$$= \frac{100\left(-\omega^2 + 8 + \jmath 6\omega\right)}{3\omega^2 + \jmath\left(10\omega - \omega^3\right)} \tag{1.3.0.10}$$

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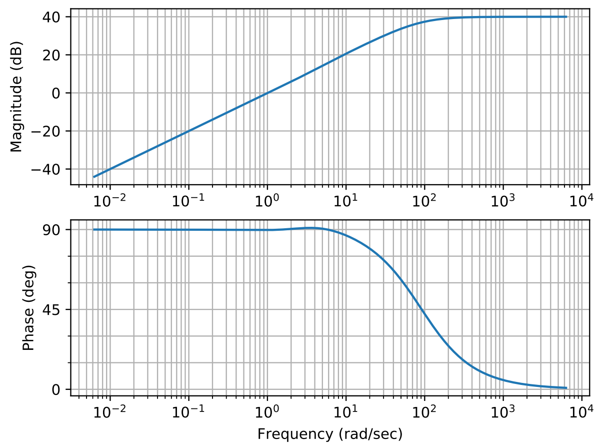


Fig. 1.3.1

Using (1.3.0.10)

$$\left|G(\jmath\omega)H(\jmath\omega)\right| = \frac{100\sqrt{(8-\omega^2)^2+(6\omega)^2}}{\sqrt{(3\omega^2)^2+(10\omega-\omega^3)^2}} \tag{1.3.0.11}$$

$$\begin{aligned}\angle G(\jmath\omega)H(\jmath\omega) &= \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right)\\ &-\tan^{-1}\left(\frac{10-\omega^2}{3\omega}\right)\end{aligned} \tag{1.3.0.12}$$

At the phase crossover frequency ω_{pc}:

$$\left|\angle G(\jmath\omega)H(\jmath\omega)\right| = 180 \tag{1.3.0.13}$$

$$\implies \tan^{-1}\left(\frac{6\omega_{pc}}{8-\omega_{pc}^2}\right)-\tan^{-1}\left(\frac{10-\omega_{pc}^2}{3\omega_{pc}}\right) = 180 \tag{1.3.0.14}$$

Solving the above equation:

$$\frac{6\omega_{pc}}{8-\omega_{pc}^2} = \frac{10-\omega_{pc}^2}{3\omega_{pc}} \tag{1.3.0.15}$$

$$\implies \omega_{pc} = 5.8rad/sec \tag{1.3.0.16}$$

$$\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{pc}} = 26.9dB \tag{1.3.0.17}$$

Gain Margin (GM :

$$GM = 0-\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{pc}} dB \tag{1.3.0.18}$$

$$= -26.9dB \tag{1.3.0.19}$$

1.3.3. Compute the phase margin of the system.
Solution:
At the gain crossover frequency ω_{gc}:

$$\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{gc}} = 1 \tag{1.3.0.20}$$

From (1.3.0.11), At ω_{gc}:

$$10^4\left(\left(8-\omega^2\right)^2+(6\omega)^2\right) = \left(3\omega^2\right)^2+\left(10\omega-\omega^3\right)^2 \tag{1.3.0.21}$$

$$\implies \omega_{gc} = 100.15rad/sec \tag{1.3.0.22}$$

Substitute ω_{gc} in (1.3.0.12):

$$\angle G(\jmath\omega)H(\jmath\omega)_{\omega=\omega_{gc}} = 265^\circ \tag{1.3.0.23}$$

Phase Margin PM:

$$PM = 180^\circ - \angle G(\jmath\omega)H(\jmath\omega)_{\omega=\omega_{gc}} \tag{1.3.0.24}$$

$$= 180^\circ - 265^\circ = -85^\circ \tag{1.3.0.25}$$

1.3.4. Verify the values of Gain Margin and Phase Margin using a python plot.
Solution:
The following code is used to verify the gain and phase margins:

```
codes/ee18btech11045/
    ee18btech11045_bode2.py
```

1.3.5. Comment on the stability of the system
Solution:
As both the Gain Margin (GM) and Phase Margin (PM) are found to be negative, the system is unstable.

2 Stability in Frequency Domain

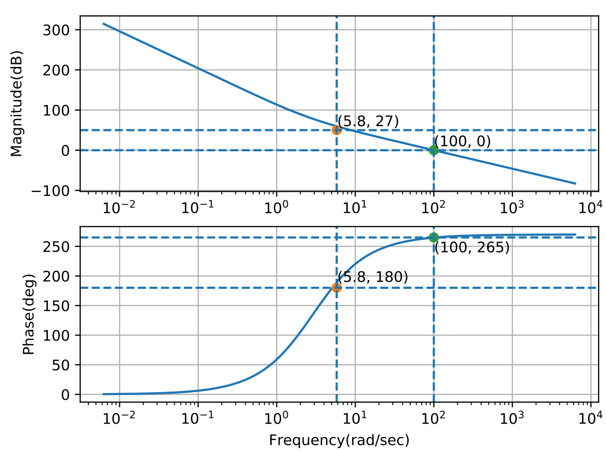
2.1 Nyquist Criterion

2.2

3 Design in Frequency Domain

4 PID Controller Design

4.1 Introduction



Magnitude(dB)
300
200
100
0
−100
(5.8, 27)
(100, 0)
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}
Phase(deg)
250
200
150
100
50
0
(100, 265)
(5.8, 180)
Frequency(rad/sec)

Fig. 1.3.2