Control Systems (EE2227) Presentation

Sudeep Veggalam
EE18BTECH11045

12 February 2019

Problem Statement

GATE 2017 (Set II) Question 26

Question

Which of the following systems has maximum peak overshoot due to a unit step input?

- (A) $100/s^2 + 10s + 100$
- (B) $100/s^2 + 15s + 100$
- (C) $100/s^2 + 5s + 100$
- (D) $100/s^2 + 20s + 100$

Theory

The transfer function for the second order control system is given by:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

To calculate the unit step response,

$r(t) = 1$ or $R(s) = \frac{1}{s}$

On Simplifying,

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d} \cdot \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \tag{2}$$

where,

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$

This is the Unit step response in frequency domain

Theory (contd.)

Take inverse laplace transform to get the unit step response in time domain:

$$c(t) = 1 - e^{-}\zeta\omega_n t(cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}.sin\omega_d t) \tag{3}$$

This is the time domain representation of unit step response

Theory (contd.)

What is Peak overshoot?

Maximum overshoot is the difference between the magnitude of the highest peak of time response and magnitude in its steady state. Peak Overshoot is expressed in term of percentage of steady-state value of the response.

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} * 100\% \tag{4}$$

Peak Overshoot

Mathematical expression for Peak Overshoot:

c(t) is max where $\frac{dc(t)}{d(t)}$

On applying this condition on (3), we get

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{5}$$

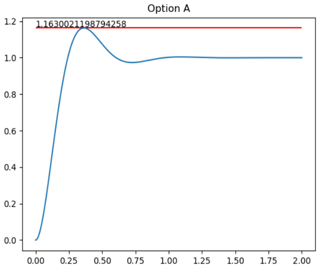
Substituting in the equation for Peak overshoot:

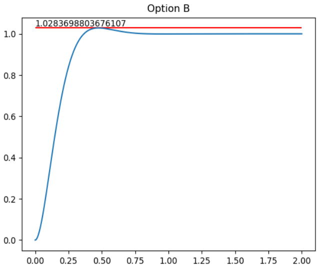
$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{6}$$

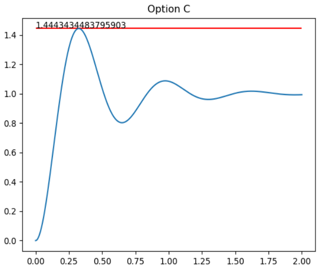
Solution

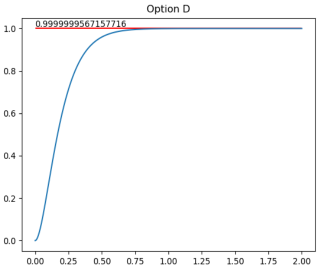
From (6), we can infer that Peak Overshoot is a decreasing function of ζ
i.e The second order equation with the lowest ζ will have the maximum peak overshoot
For option (A), $\zeta = 0.5$
For option (B), $\zeta = 0.75$
For option (C), $\zeta = 0.25$
For option (D), $\zeta = 1$
Therefore, (C) has the maximum peak overshoot
Answer : (C)

Plotting


Option A
1.1630021198794258
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00


Option B
1.0283698803676107
1.0
0.8
0.6
0.4
0.2
0.0
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00


Option C
1.4443434483795903
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00


Option D
0.9999999567157716
1.0
0.8
0.6
0.4
0.2
0.0
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00