

# Control Systems

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## 1 SIGNAL FLOW GRAPH

### 1.1 Mason's Gain Formula

### 1.2 Matrix Formula

## 2 BODE PLOT

### 2.1 Introduction

### 2.2 Example

## 3 SECOND ORDER SYSTEM

### 3.1 Damping

### 3.2 Example

### 3.3 Peak Overshoot

3.3.1. Find the peak overshoot for the second order control system given by:

$$G(S) = \frac{100}{s^2 + 10s + 100} \quad (3.3.1.1)$$

**Solution:** Peak overshoot ( $M_p$ ) is defined as the deviation of the response at peak time from the final value of response.

$$\Rightarrow M_p = c(t_p) - c(\infty) \quad (3.3.1.2)$$

Given,

$$G(S) = \frac{C(s)}{R(s)} = \frac{100}{s^2 + 10s + 100} \quad (3.3.1.3)$$

To calculate the unit step response,

$$r(t) = 1 \Rightarrow R(s) = \frac{1}{s} \quad (3.3.1.4)$$

$$\Rightarrow C(s) = \frac{1}{s} \cdot \frac{100}{s^2 + 10s + 100} \quad (3.3.1.5)$$

$C(s)$  can be expanded as:

$$C(s) = \frac{1}{s} - \frac{s+5}{(s+5)^2 + 75} - \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{75}}{(s+5)^2 + 75} \quad (3.3.1.6)$$

$$c(t) = \mathcal{L}^{-1}C(s) \quad (3.3.1.7)$$

$$\Rightarrow c(t) = 1 - e^{-5t} \cos(\sqrt{75}t) \quad (3.3.1.8)$$

$$- \frac{e^{-5t}}{\sqrt{3}} \cdot \sin(\sqrt{75}t) \quad (3.3.1.9)$$

$c(t)$  can be further simplified to:

$$c(t) = 1 - \frac{2e^{-5t}}{\sqrt{3}} \cdot \sin\left(5\sqrt{3}t + \frac{\pi}{3}\right) \quad (3.3.1.10)$$

From Final Value theorem:

$$\lim_{s \rightarrow 0} s.C(s) = \lim_{t \rightarrow \infty} c(t) \quad (3.3.1.11)$$

$$\Rightarrow c(\infty) = \lim_{s \rightarrow 0} \frac{100}{s^2 + 10s + 100} = 1 \quad (3.3.1.12)$$

At  $t_p$ ,  $c(t)$  is maximum:

$$\begin{aligned} \Rightarrow c'(t)|_{t=t_p} &= \frac{10e^{-5t}}{\sqrt{3}} \cdot \sin\left(5\sqrt{3}t + \frac{\pi}{3}\right) \\ &- \frac{10e^{-5t}}{\sqrt{3}} \cdot \sqrt{3} \cdot \cos\left(5\sqrt{3}t + \frac{\pi}{3}\right) = 0 \end{aligned} \quad (3.3.1.13)$$

$$\Rightarrow \sin\left(5\sqrt{3}t + \frac{\pi}{3}\right) - \sqrt{3} \cdot \cos\left(5\sqrt{3}t + \frac{\pi}{3}\right) = 0 \quad (3.3.1.14)$$

$$\Rightarrow \tan\left(5\sqrt{3}t + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) \quad (3.3.1.15)$$

$$\Rightarrow 5\sqrt{3}t_p = n\pi \quad (3.3.1.16)$$

The maximum overshoot occurs at  $n = 1$ :

$$t_p = \frac{\pi}{\sqrt{75}} = 0.36 \quad (3.3.1.17)$$

Substitute  $t_p$  in (3.3.1.10) to get  $c(t_p)$ :

$$c(t_p) = 1 + e^{\frac{-\pi}{\sqrt{3}}} \Rightarrow c(t_p) = 1.163 \quad (3.3.1.18)$$

Substitute  $c(t_p)$  and  $c(\infty)$  in (3.3.1.2) to get peak overshoot:

$$M_p = 1.16 - 1 = 0.16 \quad (3.3.1.19)$$

3.3.2. Verify using a Python Plot

**Solution:**

codes/ee18btech11045.py

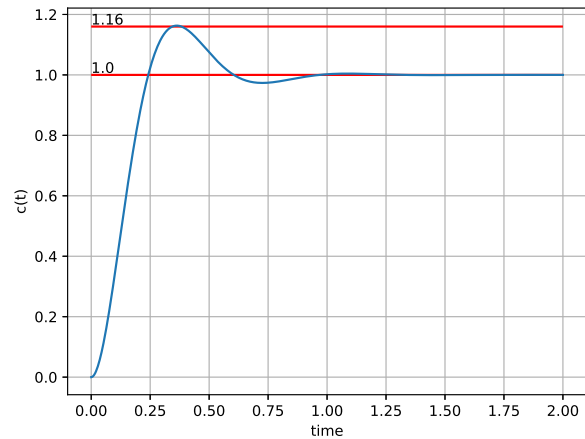


Fig. 3.3.2:  $c(t)$  vs  $t$  plot

## 4 ROUTH HURWITZ CRITERION

### 4.1 *Routh Array*

### 4.2 *Marginal Stability*

### 4.3 *Stability*

### 4.4 *Example*

## 5 STATE-SPACE MODEL

### 5.1 *Controllability and Observability*

### 5.2 *Second Order System*

### 5.3 *Example*

### 5.4 *Example*

### 5.5 *Example*

## 6 NYQUIST PLOT

## 7 COMPENSATORS

### 7.1 *Phase Lead*

### 7.2 *Example*

## 8 GAIN MARGIN

### 8.1 *Introduction*

### 8.2 *Example*

## 9 PHASE MARGIN

## 10 OSCILLATOR

### 10.1 *Introduction*

### 10.2 *Example*