An Information-Theoretic Meta-Theorem on Edge-Cut bounds and Applications
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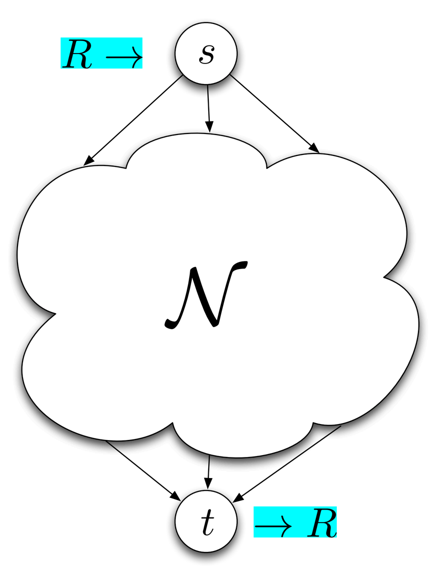
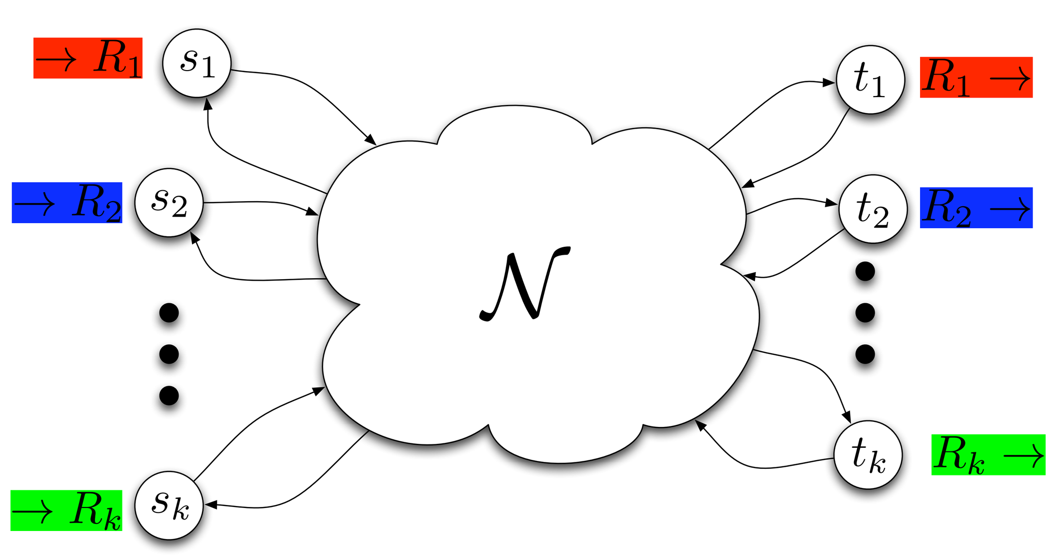
The oldest flow problem ever: Single-unicast



Max-Commodity-Flow = Min-Edge-Cut
[Ford - Fulkerson '56]

Information-Capacity = Min-Edge-Cut
[Ahlswede - Cai - Li - Yeung '00]

Simple generalization: k-unicast



Flow	Edge-cut	Capacity
Easy! ☺	Hard ☹	Notorious ☹
Linear Program	NP-complete [Ambühl - Mastrolilli - Svensson '07]	Linear coding insufficient [Dougherty - Freiling - Zeger '05]
	Hard to approximate [Chuzhoy - Khanna '07]	$\bar{\Gamma}_n^*$ necessary [Chan - Grant '08]

Worst-case relationships

$\text{Flow} \approx \frac{\text{Edge}-\text{cut}}{k}$
[Saks - Samorodnitsky - Zosin '04]

$\text{Edge}-\text{cut} \approx \frac{\text{Capacity}}{k}$
[Harvey - Kleinberg - Lehman '06]

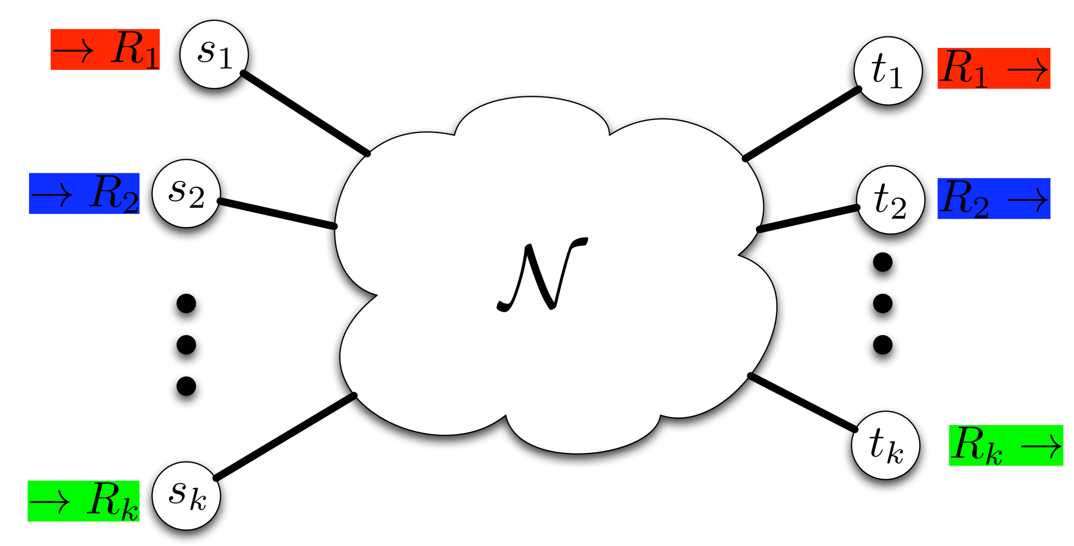
$\text{Flow} \approx \frac{\text{Capacity}}{k}$
[Harvey - Kleinberg - Lehman '06]

Meta-theorems

Suppose the network or traffic pattern has some suitable SYMMETRY.

Algorithmic meta-theorem (CS Theory)	Information-theoretic meta-theorem (this work)	Combined meta-theorem
Flow $\approx$ Edge-cut	Capacity $\lesssim$ Edge-cut	Flow $\approx$ Capacity

Scenario 1: k-unicast in undirected networks



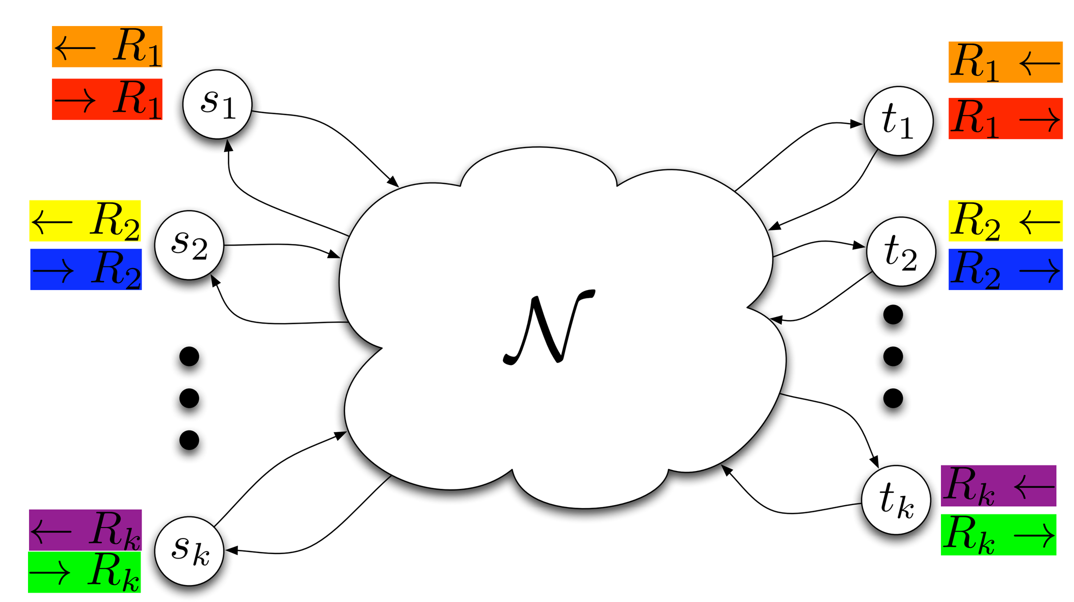
$$\frac{\text{Edge}-\text{cut}}{\Theta(\log k)} \subseteq \text{Flow} \subseteq \text{Edge}-\text{cut}$$

[Leighton - Rao '88]
[Linial - London - Rabinovich '94]

$$\text{Capacity} \subseteq \text{Edge}-\text{cut}$$

(from Cutset bound)

Scenario 2: k-unicast in directed networks with symmetric demands



$$\frac{\text{Edge}-\text{cut}}{\Theta(\log^2 k)} \subseteq \text{Flow} \subseteq \text{Edge}-\text{cut}$$

[Klein - Plotkin - Rao - Tardos '93]

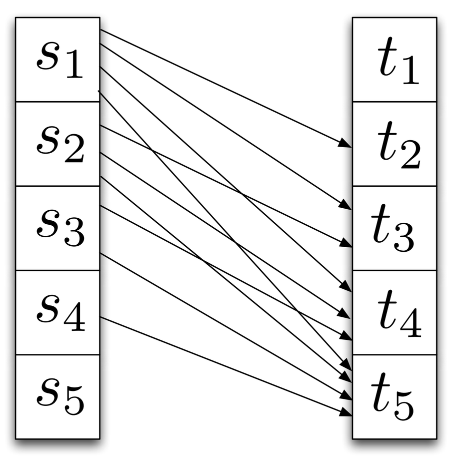
$$\text{Capacity} \subseteq \text{Edge}-\text{cut}$$

(from Generalized Network Sharing bound)

Generalized Network Sharing bound

k-unicast network:

Set of edges E such that $\mathcal{G} \setminus E$ has no paths from s_i to t_j whenever $i \geq j$

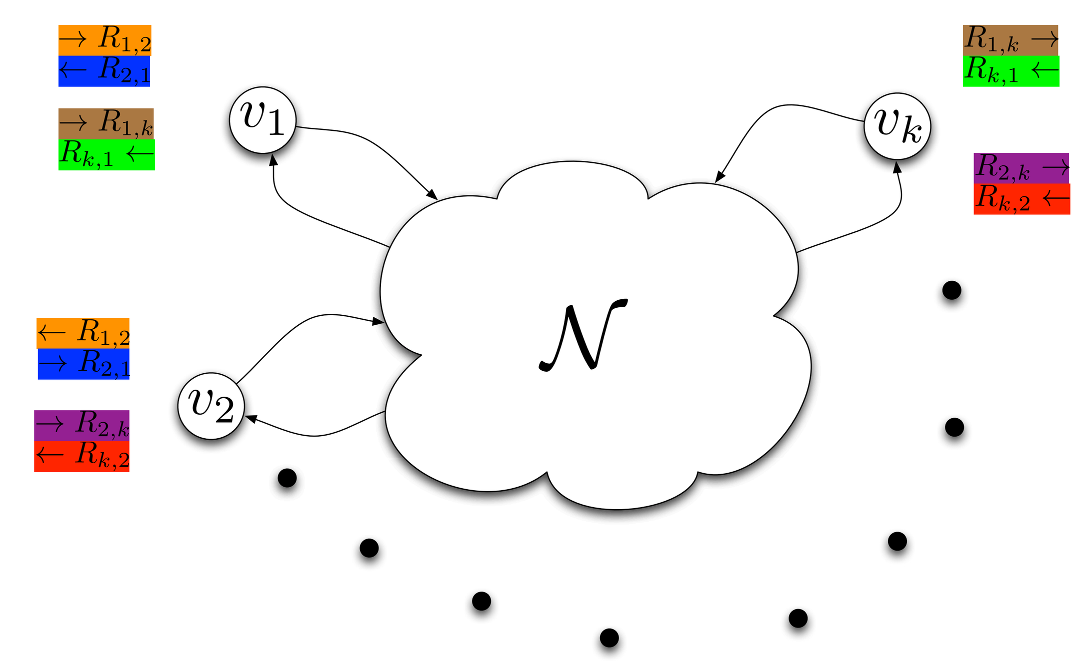


$$R_1 + R_2 + \ldots + R_k \leq \sum_{e \in E} C_e$$

holds for all rate tuples in the capacity region.

[Kamath - Tse - Anantharam '11]

Scenario 3: k-groupcast sum-rate



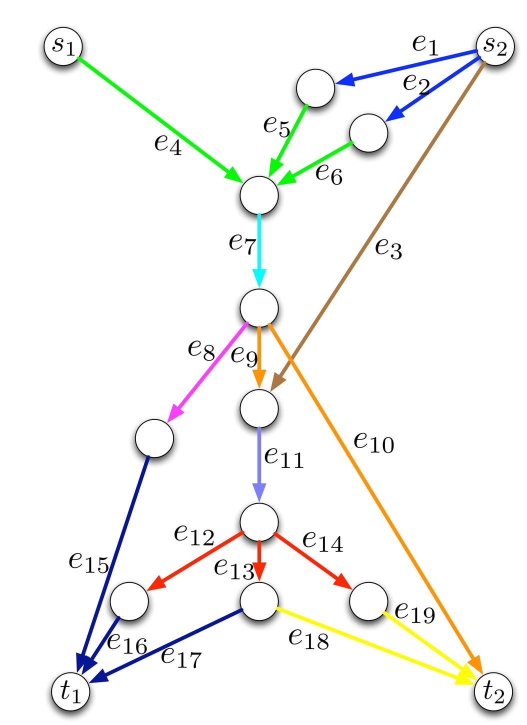
$$\frac{\text{Multicut}}{2} \leq \text{MaxFlowSumRate} \leq \text{Multicut}$$

[Naor - Zosin '01]

$$\text{SumCapacity} \leq 2 \times \text{Multicut}$$

(from Generalized Network Sharing bound)

MAC + BC Gaussian networks



- Simple Scheme:
  - Local code design for MAC's, BC's, point-to-point links
  - Global routing
- Network looks like a polymatroidal network
- Flow-Edge-cut closeness results also hold for polymatroidal networks [Chekuri - Kannan - Raja - Viswanath '11]
- We show a Generalized Network Sharing outer bound for Gaussian networks

Results for Gaussian networks

Bidirected networks: $\frac{\mathcal{R}_{\text{cutset}}(\frac{\bar{P}}{d})}{\Theta(\log k)} \subseteq \mathcal{R}(\bar{P}) \subseteq \mathcal{C}(\bar{P}) \subseteq \mathcal{R}_{\text{cutset}}(\bar{P})$

Symmetric-demand networks: $\frac{\mathcal{R}_{\text{GNS}}(\frac{\bar{P}}{d})}{\Theta(\log^2 k)} \subseteq \mathcal{R}(\bar{P}) \subseteq \mathcal{C}(\bar{P}) \subseteq \mathcal{R}_{\text{GNS}}(\bar{P})$

Groupcast networks: $\frac{R_{\text{GNS}}(\frac{\bar{P}}{d})}{4} \leq R(\bar{P}) \leq C(\bar{P}) \leq R_{\text{GNS}}(\bar{P})$

where $\bar{P}$ = power constraint vector, d = max-degree of MAC or BC

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