Wireless Networks with Symmetric Demands

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Abstract—It has been shown recently that a simple layering principle – local physical-layer schemes combined with global routing – can achieve approximately optimal performance in wireless networks. However, this result depends heavily on the assumption of reciprocity of wireless networks, which may be violated due to asymmetric power constraints, directional antennas or frequency-duplexing. In this paper, we show that the approximate optimality continues to hold even for wireless networks modeled as directed graphs as long as there is a symmetric demand constraint: every demand from source s_i to sink t_i at rate R_i has a counterpart demand from source node t_i to sink node s_i at the same rate. This models several practical scenarios including voice calls, video calls, and interactive gaming. We prove this result in the context of several channel models for which good local schemes exist. The key technical contributions are an outer bound based on a Generalized Network Sharing bound for wireless networks and an achievable strategy based on a connection to polymatroidal networks.

I. INTRODUCTION

The capacity region of multiple unicast in general wireless networks is an interesting open problem. Recent work [6], [7], [8] has made progress in this direction by giving an approximate characterization of this capacity region by using the reciprocity in wireless channels. It has been shown that simple layered architectures involving local physical-layer schemes combined with global routing can achieve approximately optimal performance in wireless networks.

In many practical scenarios, the reciprocity may be affected due to asymmetric power constraints, directional antennas or frequency-duplexing. The question we address in this paper is: "do layered architectures continue to be optimal even in this case?" We answer this question in the affirmative under a special traffic model called the *symmetric demands* model: there are k specially-marked source-sink pairs of nodes $(s_i, t_i), i = 1, 2, ..., k$ with s_i wanting to communicate an independent message to t_i at rate R_i *and* t_i wanting to communicate an independent message to s_i at rate R_i. This traffic model is valid in several practical scenarios including voice calls, video calls, and interactive gaming.

The symmetric demands traffic model was originally studied for *wireline networks* by Klein, Plotkin, Rao and Tardos [1], who established that the routing rate region and *edge-cuts* are within a factor $\mathcal{O}(\log^2 k)$ of each other. This result, however, does not establish that routing is approximately optimal since edge-cuts do not, in general, bound the rate of general communication schemes. A companion paper [2] does

so by proving that edge-cuts in fact, form fundamental outer bounds for the communication rates under this traffic model.

In this paper, we show an analogous result for wireless networks under several channel models for which good schemes are known at a local level. Our results for wireless networks with symmetric demands include

1) Capacity approximations for networks comprised of Gaussian MAC and broadcast channels,
2) Degrees-of-freedom approximation for fixed Gaussian networks, and
3) Capacity approximations for fading Gaussian networks.

At the heart of our achievable scheme is a connection to "polymatroidal networks" for which the symmetric demands problem was recently addressed [4]. Our outer bound is based on the Generalized Network Sharing bound [5], which we extend to wireless networks in this work.

A. Prior Work

Single hop wireless networks (called interference and X-networks) have been studied extensively in recent times, starting with the seminal work of [12], which proposed an interference alignment scheme (building on [13]) by which each user in a fast fading interference channel can achieve half the degrees-of-freedom simultaneously. This result has been extended to a variety of other scenarios, including interference channel with fixed channel coefficients [15], [16], approximate capacity characterization of ergodic interference channels [14]. In this paper, we build on these results to obtain results for a network comprised of such channels.

Wireless networks with multiple hops have also been studied under several traffic models, starting from the capacity approximation for unicast and multicast traffic in [17], the multiple-source single-sink case in [18] and broadcast traffic in [19].

Multiple-unicast in general wireless networks has started receiving attention only recently. Certain classes of 2-unicast problems were studied in [21], 2-unicast deterministic networks were studied in [26], and the degrees-of-freedom of 2-unicast was established in [20]. K-unicasts in certain topologies were studied in [22] where K sources communicate to K sinks via L fully-connected layers of K relays each. Compute and forward schemes for layered networks were presented in [23]. Multiple multicasts in networks of 2-user Gaussian MAC and broadcast channels was studied in [24], where a separation scheme was shown to be approximately optimal. While these

existing works attempt to compute the degrees-of-freedom (or approximate capacity) exactly for specific instances of the problem, we adopt a different viewpoint and focus our attention on obtaining general results for arbitrary networks at the expense of obtaining potentially weaker approximation in specific instances.

II. BACKGROUND

In this section, we provide the basic background on which the main results are then built.

A. Directed Graph with Symmetric Demands

We review the directed wireline network problem with symmetric demands. There is a directed wireline graph $G = (V, E)$ with capacity functions $c(e) \ \forall e \in E$ and k source-sink pairs (s_i, t_i), $i = 1, 2, ..., k$. For the symmetric demands problem, a rate tuple $(R_1, ..., R_k)$ is said to be achievable if s_i can communicate to t_i at rate R_i *and* t_i can communicate at s_i at rate R_i. A fundamental question in this context is "How good is routing?".

The optimal performance achievable by routing can be written as a linear program, whose solution can be computed in time that grows only polynomial with the size of the graph (see [1]). Let $\mathcal{R}_f$ denote the set of all k-tuples achievable by routing. An obvious bound on the rate of flow is the edge-cut bound, defined as follows. Given a subset of edges (called an edge-cut) $F \subseteq E$, let $c(F) := \sum_{e \in F} c(e)$ denotes the capacity of the cut. Let $K(F)$ denote the set of $i \in \{1, 2, ..., k\}$ such that either there is no path from s_i to t_i or there is no path between t_i to s_i in $E \setminus F$. Now, the edge-cut bound is given by $\sum_{i \in K(F)} R_i \leq c(F)$. The set of rate tuples that satisfy all the edge-cut bounds is called the edge-cut bound region $\mathcal{R}_{\text{e.c.}}$. Note that if for some index $i \in K(F)$, there is no path from s_i to t_i and no path from t_i to s_i in $E \setminus F$, then we could have a tighter inequality with the co-efficient of R_i being 2 instead of 1. These tighter inequalities are not always fundamental (i.e., they do not upper bound general network coding rates of communication, only those achievable via routing), but in any case, they result in tighter bounds by at most a factor of two. A classical result [1] shows that

$$\frac{1}{\mathcal{O}(\log^2 k)} \mathcal{R}_{\text{e.c.}} \subseteq \mathcal{R}_f \subseteq \mathcal{R}_{\text{e.c.}}, \tag{1}$$

i.e., the rate region achievable by flow is within an $\mathcal{O}(\log^2 k)$ factor of the edge-cut bound.

B. GNS Outer Bound

While the edge-cut bound is an upper bound on the flow, it is not clear if the edge-cut bound is an upper bound on the rates achievable by arbitrary coding schemes. In fact, it is known [25] that edge-cuts do not bound the capacity region for general directed graphs (for general directed graphs, the edge-cut is any subset of edges that disconnects s_i from t_i, for all $i \in K$ for some subset K).

In a recent companion work [2], we show that edge cut bounds give fundamental upper bounds on the capacity region

for the directed graphs with symmetric demands problem. This result along with (1) establishes that routing is within a factor $\mathcal{O}(\log^2 k)$ optimal. The basic engine in the proof is the bound established in [5] called the Generalized Network Sharing (GNS) bound. The GNS bound establishes that certain edge cuts which disconnect s_i from t_j, whenever $i \geq j$ are fundamental. The basic idea in the proof of [2] is a combinatorial argument showing that, in the directed graph with symmetric demands problem, there is a re-labeling of s_i and t_i (by swapping and permutation) such that any given edge cut can be seen to be a so-called *GNS-cut*.

C. Polymatroidal Networks

A standard wireline network model is an edge-capacitated graph; each edge is associated with a capacity that constrains the total amount of information that can be communicated on it. Polymatroidal networks are a strictly more general model and handle additional constraints when edges meet at a node, similar in spirit to the broadcast and interference constraints in wireless. In particular, we study a directed graph $G = (V, E)$ where the constraints on the information flow f_e on edge e are given by

$$\sum_{e: e \in S_v} f(e) \ \leq \ \rho_v^{\text{in}}(S_v) \quad \forall S_v \subseteq \text{In}\{v\} \tag{2}$$

$$\sum_{e: e \in S_v} f(e) \ \leq \ \rho_v^{\text{out}}(S_v) \quad \forall S_v \subseteq \text{Out}\{v\}, \tag{3}$$

where $\rho_v^{\text{in}}(\cdot)$ and $\rho_v^{\text{out}}(\cdot)$ are *polymatroidal* functions on the corresponding input sets.

While prior work establishes a max-flow min-cut result for unicast communication [9], [10] and broadcast traffic [11], recent work has established approximate max-flow min-cut results for the case of both bi-directed polymatroidal networks and for polymatroidal networks with symmetric demands.

We recall the following theorem from [4], which generalizes the results of [1] to the case of polymatroidal capacity networks:

Theorem 1. *[4] For a directed polymatroidal network with k source-destination pairs having symmetric demands,*

$$\frac{1}{\mathcal{O}(\log^2 k)} \mathcal{R}_{e.c.} \subseteq \mathcal{R}_f \subseteq \mathcal{R}_{e.c.}, \tag{4}$$

.

III. GAUSSIAN NETWORKS COMPOSED OF BROADCAST AND MULTIPLE ACCESS CHANNELS

The communication network is represented by a directed graph $G = (V, E)$, and an edge coloring $\psi : E \rightarrow C$, where C is the set of colors. Each node v has a set of colors $C(v) \subseteq C$ on which it operates. Each color can be thought of as an orthogonal resource, and therefore the broadcast and interference constraints for the wireless channel apply only *within* a given color. The channel model can therefore be written as,

$$y_i^c = \sum_{j \in \text{In}_c(i)} h_{ji}^c x_j^c + z_i^c \ \ \forall c \in C(i), \tag{5}$$

where x_i^c, y_i^c, z_i^c are the transmitted vector, received vector and noise vector on color c, h_{ji}^c is the channel coefficient between node i and node j on color c and $\text{In}_c(i)$ represents the set of in-neighbors of node i who are operating on color c and $d_c(i) = |\mathcal{N}_c(i)|$ be the degree of node i in color c. Let $d = \max_{c,v} d_c(v)$ be the maximum degree of any node in a given color; therefore, d is the maximum number of users on any component broadcast or multiple access channel.

A network is composed of broadcast and multiple access channels if and only if no edge is involved simultaneously in a broadcast and interference constraint inside the same color. We will call such a network a "Gaussian MAC+BC network". We will assume that each color represents a distinct MAC or broadcast channel without loss of generality. Each node has a power constraint P to transmit on each *edge*. If there are distinct power constraints for different nodes, they can be absorbed into the channel co-efficient without loss of generality.

1) Edge-cut bound: The edge cut bound for the gaussian MAC+BC network is defined by the following: consider any set $F \subseteq E$, as in the wireline case, let $K(F)$ denote the set of $i \in \{1, 2, ..., k\}$ such that either there is no path from s_i to t_i or there is no path between t_i to s_i in $E \setminus F$. The capacity of the cut $c(F)$ is defined, in the obvious way, as the sum of three terms: the capacities of the orthogonal links in F, the sum-capacity of sub-MAC components of F and the sum-capacity of sub-BC components of F, where sum-capacity of sub-MAC (sub-BC) component is computed assuming complete coordination among source (destination) terminals of the sub-MAC (sub-BC) component. The edge-cut bound region is now given as

$$\mathcal{R}_{\text{e.c.}} = \{(R_1, ..., R_k) : \sum_{i \in K(F)} R_i \le c(F) \ \forall F \subseteq E\}. \quad (6)$$

As in the wireline network case, it is not immediately obvious if $\mathcal{R}_{\text{e.c.}}$ is an outer bound to the capacity region.

A. Multiple Unicast in Gaussian MAC+BC Networks

There are k pairs of nodes $s_i, t_i, \ i = 1, 2, ..., k$, where node s_i has a message to send to t_i and t_i has a message to send to s_i at rate R_i. We would like to characterize the set of all achievable rate tuples, called the capacity region $\mathcal{C}$. We will use $\mathcal{R}_{\text{ach}}$ to denote rates achievable by our proposed simple scheme, $\mathcal{R}_{\text{e.c.}}$ to denote the edge-cut region and $\mathcal{C}$ to denote the capacity region. Our main result is the following.

Theorem 2. *For the k-unicast problem with symmetric demands in a Gaussian MAC+BC network, the edge-cut bound is fundamental and a simple separation strategy can achieve $\mathcal{R}_{ach}(P)$ which satisfies,*

$$\frac{\mathcal{R}_{\text{e.c.}}(\frac{P}{d})}{\mathcal{O}(\log^2 k)} \subseteq \mathcal{R}_{ach}(P) \subseteq \mathcal{C}(P) \subseteq \mathcal{R}_{\text{e.c.}}(P). \quad (7)$$

Thus, the edge-cut bound is a fundamental upper bound on the capacity region and furthermore the edge-cut bound, scaled down in power by a factor d and in rate by a factor $\frac{1}{\mathcal{O}(\log^2 k)}$, can be achieved by the proposed scheme.

B. Outer bound

We first establish that the edge-cut bound is fundamental, i.e., every communication scheme must have rate pairs that lie inside this region: $\mathcal{C} \subseteq \mathcal{R}_{\text{e.c.}}$. A GNS-cut is an edge-cut with stronger disconnection properties. More precisely, the edge-cut that disconnects s_i from t_i for $i = 1, 2, ..., k$ is a GNS-cut if there exists a permutation $\pi : \{1, 2, ..., k\} \to \{1, 2, ..., k\}$ such that the edge-cut disconnects s_i from t_j whenever $\pi(i) \ge \pi(j)$. The key argument is a bound based on GNS cuts for Gaussian MAC+BC networks:

Lemma 1. *For the Gaussian MAC+BC network with symmetric demands, every GNS cut F is fundamental, i.e., $\sum_{i \in K(F)} R_i \le c(F)$ for any communication scheme achieving $(R_1, ..., R_k)$.*

Proof: Without loss of generality, let F be a GNS-cut disconnecting s_i from t_i for $i = 1, 2, ..., k$ with say, the identity permutation π_{id}. Thus, $K(F) = \{1, 2, ..., k\}$. We first provide a proof of the bound for networks with acyclic underlying graph $\mathcal{G}$.

Let $C = \mathcal{M} \cup \mathcal{B}$ where $\mathcal{M}$ consists of the colors of edges involved in MAC components and $\mathcal{B}$ consists of colors of edges involved in broadcast components or orthogonal links. For $\mu \in C$, let A_μ denote the set of edges involved in μ. Now, consider a directed graph $\mathcal{G}'$ with nodes represented by $A_\mu, \mu \in C$ as follows: there exists a directed edge from A_μ to A_ν in $\mathcal{G}'$ if and only if there exists an edge in A_μ that is upstream to some edge in A_ν in the given DAG $\mathcal{G}$. Since the set of all edges with a given color constitute either a MAC or a BC or form a single orthogonal link, we have that $\mathcal{G}'$ is a directed acyclic graph. Thus, we can have a total order on the vertices of $\mathcal{G}'$ consistent with the partial order of ancestry in $\mathcal{G}'$. This gives a total order on $\mathcal{D} := \{\mu \in C : F \cap A_\mu \neq \emptyset\}$, which may be presumed to be, say, $\mu_1 < \mu_2 < ... < \mu_r$, where μ_1 is the most "upstream".

- For $\mu \in \mathcal{M}$, we denote transmissions along edge e in A_μ by X_e and we denote the reception by Y_μ so that $Y_\mu = \sum_{e \in A_\mu} X_e + Z_\mu$ where Z_μ is Gaussian noise. Further, define $U_\mu := \{X_e : e \in F \cap A_\mu\}$, and $V_\mu := \sum_{e \in F \cap A_\mu} X_e + Z_\mu$.
- For $\mu \in \mathcal{B}$, we denote the transmission on the broadcast component or orthogonal link by X_μ and the receptions at heads of $e \in A_\mu$ by Y_e so that $Y_e = X_\mu + Z_e$ where $\{Z_e, e \in A_\mu\}$ are independent Gaussian noise random variables. Further define $U_\mu := X_\mu$, and $V_\mu := \{Y_e : e \in F \cap A_\mu\}$.

Define $\tilde{Y}_{t_i} = \{Y_\mu^n : \text{head}(e) = t_i, e \in A_\mu, \mu \in \mathcal{M}\} \cup \{Y_e^n : \text{head}(e) = t_i, e \in A_\mu, \mu \in \mathcal{B}\}$.

$$n[\sum_{i=1}^{k} R_i - \epsilon_n] \leq \sum_{i=1}^{k} I(W_i; \tilde{Y}_{t_i})$$

$$\leq \sum_{i=1}^{k} I(W_i; \{V_\mu^n : \mu \in \mathcal{D}\}, \{W_j : j < i\})$$

[since $W_i - \{V_\mu^n : \mu \in \mathcal{D}\}, \{W_j : j < i\} - \tilde{Y}_{t_i}$]

$$= \sum_{i=1}^{k} I(W_i; \{V_\mu^n : \mu \in \mathcal{D}\} | \{W_j : j < i\})$$

[since W_i is independent of $\{W_j : j < i\}$]

$$= I(\{W_i : 1 \leq i \leq k\}; \{V_\mu^n : \mu \in \mathcal{D}\})$$
$$= h(\{V_\mu^n : \mu \in \mathcal{D}\})$$
$$\quad - h(\{V_\mu^n : \mu \in \mathcal{D}\} | \{W_i : 1 \leq i \leq k\})$$
$$\leq \sum_{\mu \in \mathcal{D}} h(V_\mu^n) - h(\{V_\mu^n : \mu \in \mathcal{D}\} | \{W_i : 1 \leq i \leq k\})$$
$$=: \sum_{\mu \in \mathcal{D}} h(V_\mu^n) - A.$$

Now, we consider the negative term A above.

$$A = h(\{V_\mu^n : \mu \in \mathcal{D}\} | \{W_i : 1 \leq i \leq k\})$$
$$= h(V_{\mu_1}^n, V_{\mu_2}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\})$$
$$\geq h(V_{\mu_1}^n, V_{\mu_2}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n)$$
$$= h(V_{\mu_1}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n)$$
$$\quad + h(V_{\mu_2}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n)$$
$$= h(V_{\mu_1}^n | U_{\mu_1}^n)$$
$$\quad + h(V_{\mu_2}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n)$$

[since $\{W_i : 1 \leq i \leq k\} - U_{\mu_1}^n - V_{\mu_1}^n$]

$$\geq h(V_{\mu_1}^n | U_{\mu_1}^n)$$
$$\quad + h(V_{\mu_2}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n, U_{\mu_2}^n)$$
$$= h(V_{\mu_1}^n | U_{\mu_1}^n) + h(V_{\mu_2}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n, U_{\mu_2}^n)$$
$$\quad + h(V_{\mu_3}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n, U_{\mu_2}^n, V_{\mu_2}^n)$$
$$= h(V_{\mu_1}^n | U_{\mu_1}^n) + h(V_{\mu_2}^n | U_{\mu_2}^n)$$
$$\quad + h(V_{\mu_3}^n, \ldots, V_{\mu_r}^n | \{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n, U_{\mu_2}^n, V_{\mu_2}^n)$$

[since $\{W_i : 1 \leq i \leq k\}, U_{\mu_1}^n, V_{\mu_1}^n - U_{\mu_2}^n - V_{\mu_2}^n$]

$$\geq \sum_{\mu \in \mathcal{D}} h(V_\mu^n | U_\mu^n),$$

from repeating these steps. Thus, we obtain

$$n[\sum_{i=1}^{k} R_i - \epsilon_n] \leq \sum_{\mu \in \mathcal{D}} I(U_\mu^n; V_\mu^n).$$

The proof can be extended to networks with cyclic underlying graphs by employing a standard time-layering argument (see [17]). ∎

The result from the companion paper [2], states that for the directed graph with symmetric demands problem: any edge-cut is basically a GNS-cut. Using this result in conjunction with Lemma 1, gives us the desired result: $\mathcal{C}(P) \subseteq \mathcal{R}_{\text{e.c.}}(P)$.

C. Coding Scheme

The coding scheme is a separation-based strategy: each component broadcast or multiple access channel is coded for independently creating bit-pipes on which information is routed globally. In order to evaluate the rate region of this scheme, we use polymatroidal networks as an interface for which we can show that routing and edge-cut are close to each other.

Let us first consider the coding for the multiple access channel with channel coefficients $h_1, ..., h_d$ and power constraint P at each of the d nodes. Let the rate region achievable on this multiple access channel be denoted by

$$\mathcal{R}_{\text{ach}}^{\text{MAC}}(P) = \{\overline{R} : \sum_{i \in A} R_i \leq \log\left(1 + \sum_{i \in A} |h_i|^2 P\right) \ \forall A\}.$$

This region is known to be polymatroidal. The outer bound for MAC under arbitrary source cooperation is given by

$$\mathcal{R}_{\text{cut}}^{\text{MAC}}(P) = \{\overline{R} : \sum_{i \in A} R_i \leq \log\left(1 + \left|\sum_{i \in A} |h_i|\right|^2 P\right) \ \forall A\}.$$

Similarly for a broadcast channel with channel $h_1, ..., h_k$ with power constraint P the achievable region includes the polymatroidal region

$$\mathcal{R}_{\text{ach}}^{\text{BC}}(P) = \{\overline{R} : \sum_{i \in A} R_i \leq \log\left(1 + \sum_{i \in A} |h_i|^2 \frac{P}{d}\right) \ \forall A\},$$

and the cutset bound is

$$\mathcal{R}_{\text{cut}}^{\text{BC}}(P) = \{\overline{R} : \sum_{i \in A} R_i \leq \log\left(1 + \sum_{i \in A} |h_i|^2 P\right) \ \forall A\}.$$

Observe that $\mathcal{R}_{\text{cut}}^{\text{MAC}}(P) \subseteq \mathcal{R}_{\text{ach}}^{\text{MAC}}(dP)$ and $\mathcal{R}_{\text{cut}}^{\text{BC}}(P) \subseteq \mathcal{R}_{\text{ach}}^{\text{BC}}(dP)$. Now, each multiple access or broadcast channel can be replaced by a set of d bit-pipes whose rates are jointly constrained by the corresponding polymatroidal constraints. Thus the network induced by using this coding scheme falls under the category of directed polymatroidal networks with symmetric demands. We now invoke Theorem 1 to show that the routing rate region and the edge-cut bounds in the polymatroidal network are within a factor of $\log^2 k$ of each other, i.e.,

$$\frac{\mathcal{R}_{\text{e.c.}}^{\text{poly}}(P)}{\mathcal{O} \log^2 k} \subseteq \mathcal{R}_{\text{flow}}^{\text{poly}}(P) = \mathcal{R}_{\text{ach}}^{\text{g}}(P). \quad (8)$$

Finally, we relate the polymatroidal cuts back to the cuts in the Gaussian network using the relation between MAC (BC) channel achievable regions and the corresponding cuts.

$$\mathcal{R}_{\text{e.c.}}^{\text{g}}(P) \subseteq \mathcal{R}_{\text{e.c.}}^{\text{poly}}(dP).$$

This along with (8) completes the proof of Theorem 2.

IV. General Gaussian Networks

In this section, we consider general Gaussian networks, i.e., networks where broadcast and MAC can occur simultaneously. Our network-level results are under the following two settings: 1) Degrees-of-freedom in fixed Gaussian channels 2) Capacity approximation in ergodic Gaussian channels. Due to lack of space, we only state the main results, referring the reader to [3] for detailed proofs.

A. Fixed Gaussian Channels

We consider a communication network with the edges of the network having fading coefficients on them, each chosen independently from a continuous fading distribution. Our main result is the following:

Theorem 3. *For a directed wireless network with symmetric demands, if the fixed channel coefficients are drawn from a continuous distribution, the DOF region given by $\mathcal{D}_{ach}$ satisfying*

$$\frac{\mathcal{D}_{e.c.}}{\mathcal{O}(\log^2 k)} \subseteq \mathcal{D}_{ach} \subseteq \mathcal{D} \subseteq \mathcal{D}_{e.c.},$$

is achievable.

B. Ergodic Wireless Networks

In an ergodic wireless network, the channel model is similar to the fixed Gaussian network, except that we assume that all the non-zero fading coefficients are varying as a function of time in an i.i.d. manner according to a fading distribution which is assumed to be symmetric and to satisfy a weak tail assumption: $a := e^{-\mathbb{E}(\log|h|^2)} < \infty$. One example of such a fading distribution is the i.i.d. complex gaussian distribution, for which $a \approx 1.78$ [27]. The main result is stated below.

Theorem 4. *For a directed Gaussian network with symmetric demands and with ergodic fading distribution, the rate region given by $\mathcal{R}_{ach}(P)$ satisfying*

$$\frac{\mathcal{R}_{e.c.}(\frac{P}{ad^3})}{\mathcal{O}(\log^2 k)} \subseteq \mathcal{R}_{ach}(P) \subseteq \mathcal{C}(P) \subseteq \mathcal{R}_{e.c.}(P),$$

is achievable.

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