Assignment 1: CS 754, Advanced Image Processing

Due: 1st Feb before 11:55 pm

Remember the honor code while submitting this (and every other) assignment. All members of the group should work on and understand all parts of the assignment. We will adopt a zero-tolerance policy against any violation.

Submission instructions: You should ideally type out all the answers in Word (with the equation editor) or using Latex. In either case, prepare a pdf file. Create a single zip or rar file containing the report, code and sample outputs and name it as follows: A1-IdNumberOfFirstStudent-IdNumberOfSecondStudent.zip. (If you are doing the assignment alone, the name of the zip file is A1-IdNumber.zip). Upload the file on moodle BEFORE 11:55 pm on 1st Feb. Late assignments will be assessed a penalty of 50% per day late. Note that only one student per group should upload their work on moodle. Please preserve a copy of all your work until the end of the semester. If you have difficulties, please do not hesitate to seek help from me.

1. Your task here is to implement the ISTA algorithm for the following three cases:
   - Consider the barbara image from the homework folder. Add iid Gaussian noise of mean 0 and variance 10 (on a [0,255] scale) to it, using the 'randn' function in MATLAB. Thus $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{\eta}$ where $\boldsymbol{\eta} \sim \mathcal{N}(0, 100)$. You should obtain $\boldsymbol{x}$ from $\boldsymbol{y}$ using the fact that patches from $\boldsymbol{x}$ have a sparse or near-sparse representation in the 2D-DCT basis. [20 points]
   - Divide the barbara image shared in the homework folder into patches of size 8×8. Let $\boldsymbol{x_i}$ be the vectorized version of the i^{th} patch. Consider the measurement $\boldsymbol{y_i} = \boldsymbol{\Phi x_i}$ where $\boldsymbol{\Phi}$ is a 32×64 matrix with entries drawn iid from $\mathcal{N}(0, 1)$. Note that $\boldsymbol{x_i}$ has a near-sparse representation in the 2D-DCT basis $\boldsymbol{U}$ which is computed in MATLAB as 'kron(dctmtx(8)',dctmtx(8)')'. In other words, $\boldsymbol{x_i} = \boldsymbol{U\theta_i}$ where $\boldsymbol{\theta_i}$ is a near-sparse vector. Your job is to reconstruct each $\mathbf{x_i}$ given $\boldsymbol{y_i}$ and $\boldsymbol{\Phi}$ using ISTA. Then you should reconstruct the image by averaging the overlapping patches. You should choose the α parameter in the ISTA algorithm judiciously. Choose $\lambda = 1$ (for a [0,255] image). [20 points]
   - Consider a 100-dimensional sparse signal $\boldsymbol{x}$ containing 10 non-zero elements. Let this signal be convolved with a kernel $\boldsymbol{h} = [1, 2, 3, 4, 3, 2, 1]/16$ followed by addition of Gaussian noise of standard deviation equal to 5% of the magnitude of $\boldsymbol{x}$ to yield signal $\boldsymbol{y}$, i.e. $\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{x} + \boldsymbol{\eta}$. Your job is to reconstruct $\boldsymbol{x}$ from $\boldsymbol{y}$ given $\boldsymbol{h}$. Be careful of how you create the matrix $\boldsymbol{A}$ in the ISTA algorithm. [20 points]

2. We have derived a model showing how the DCT coefficients of an image are Laplacian distributed. This model is derived by assuming a Gaussian distribution for the DCT coefficients of a patch assuming fixed variance, followed by imposing an exponential distribution on the patch variance values. Now suppose that the patch variance values were distributed as Uniform$(0, b)$, as is common in document images. Your job is to derive the resultant distribution of the DCT coefficients. You will not obtain a closed-form expression (unlike the case for natural images which yielded the Laplacian) so you will need to resort to numerical integration and plot the final distribution, as well as its closest Gaussian fit. Refer to the paper 'Analysis of the DCT Coefficient Distributions for Document Coding' by Edmund Lam, IEEE Signal Processing Letters, Feb 2004. You may work with images d5.jpg and d6.png in the homework folder. [20 points]

3. Refer to the paper 'User assisted separation of reflections from a single image using a sparsity prior' by Anat Levin, IEEE Transactions on Pattern Analysis and Machine Intelligence. Answer the following questions:
   - In equation (7), explain how you will obtain matrices $A_{j\rightarrow}$ and b_j.

- In equation (6), which terms are obtained from the prior and which terms are obtained from the likelihood? What is the prior used in the paper? What is the likelihood used in the paper?
- Why does the paper use a likelihood term that is different from the more commonly used Gaussian prior? [20 points]