STAT 443: Time Series and Forecasting

Lab 5: Model Fitting and AR processes

- The lab must be completed in `R Markdown`. Display all the `R` code used to perform your analysis.
- Create a **pdf** or **html** file and use it as your lab submission.
- Please ensure that the file you submit is in good order (e.g., not corrupted and contains the work you intend to submit). No late (re-)submissions will be accepted.

Given a time series, we can fit possible ARIMA models in `R` using the `arima` command. Look at the help page on this function before attempting the following activities.

Suppose $\{Z_t\}_{t\in\mathbb{N}}$ is a white noise with mean zero and variance 0.8. Consider stochastic process $\{X_t\}_{t\in\mathbb{N}}$ with

$$X_t = 0.8\, X_{t-1} - \frac{1}{3} X_{t-2} + \frac{0.6}{\sqrt{3}} X_{t-3} + Z_t. \tag{1}$$

1. Name the process defined in equation (1), specifying its order.
2. Explain how to recognize this process based on an observed time series and how to determine its order.
3. Use the command `set.seed(123456)` to set the random seed for reproducibility and then use function `arima.sim()` to generate 500 observations from the model in (1). Plot the simulated time series.
4. Plot the sample autocorrelation function (acf). Comment on the behaviour of the sample acf, and explain whether it appears as you would expect given the model.
5. Now plot the sample partial autocorrelation function, using command `pacf()`. Comment on the behaviour of the sample pacf, and explain whether it appears as you would expect given the model.
6. Use the `arima` function to fit an ARMA model to the simulated time series. You should specify the order (which determines the class of models to be fitted), decide whether a non-zero mean should be included, and use the default estimation method. Provide your parameter estimates.
7. Now re-fit the model as above, but choose conditional least squares as the estimation method. Provide your parameter estimates, and compare them with those from question 6.