STAT 443: Time Series and Forecasting

Lab 3: Properties of MA and AR Processes

- The aim of this lab is to explore properties of MA and AR processes that were discussed in lectures, including the behaviour of their autocorrelation functions and conditions of invertibility and stationarity.
- You will use the R command `arima.sim()` to simulate realizations from MA and AR time series models. Use the help facility to find out more about this command.
- The lab must be completed in `R Markdown`. Create a **pdf file** or **html file** (via 'knit to pdf' or 'knit to html') and use it as your lab submission.

1. Let $\{Z_t\}_{t\in\mathbb{N}}$ be a white noise process with mean zero and variance 0.2. Use `arima.sim()` function to simulate 500 observations from the process

$$X_t = Z_t - 4.25Z_{t-1} + 5.75Z_{t-2} - 1.80Z_{t-3}.$$

   (a) Create a time series plot for the simulated data. How should the autocorrelation function behave for the process $\{X_t\}_{t\in\mathbb{N}}$?

   (b) Use the command `acf` to plot the sample acf, and comment on its behaviour. Does the sample acf look like you would expect? Repeat the simulation a few times to see what is happening.

   (c) The command `ARMAacf` provides the autocorrelation function for an ARMA model (that is, a model that is an MA, an AR, or a mixture of the two). Learn about this function, and then find the acf for $\{X_t\}_{t\in\mathbb{N}}$.

   (d) Explain the behaviour of the sample acf here.

2. Let $\{Z_t\}_{t\in\mathbb{N}}$ be a white noise process with mean zero and variance 0.9.

   (a) Use `arima.sim()` to simulate 1000 observations from the process

$$X_t = Z_t + 5Z_{t-1}.$$

   Create a plot for the simulated data. Plot the sample acf, and comment on its behaviour.

   (b) Use `arima.sim()` to simulate 1000 observations from the process

$$X_t = Z_t + 0.2Z_{t-1}.$$

   Plot the sample acf, and comment on its behaviour.

(c) Repeat parts (a) and (b) several times. What do you notice? Explain what is happening.

3. Use `arima.sim()` to generate 1000 observations from the model

$$X_t = \alpha X_{t-1} + Z_t \quad \text{for} \quad \alpha = 0.3,$$

where $\{Z_t\}_{t \in \mathbb{N}}$ is a white noise process with $Z_t \sim \mathcal{N}(0, 0.2^2)$ for all t.

(a) Create a time series plot of your simulated data. Plot the sample acf, and comment on its behaviour.

(b) How does the acf of the process change as $\alpha \to 1$? Explore this in `R`, and explain what you observe.

(c) What happens to the sample acf if the parameter α changes sign? Experiment with this, changing the sign of α in your simulations. Explain what is happening.