STAT 443: Lab 4

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07/02/2022

Question 1

A time series is considered stationary when observing it at different periods of time wouldn't change its properties.

For a process to be considered stationary, (1) the mean of the process has to be constant:

$$E(X_t) = \mu \qquad for\ all\ t$$

This means that there is no trend in the data.

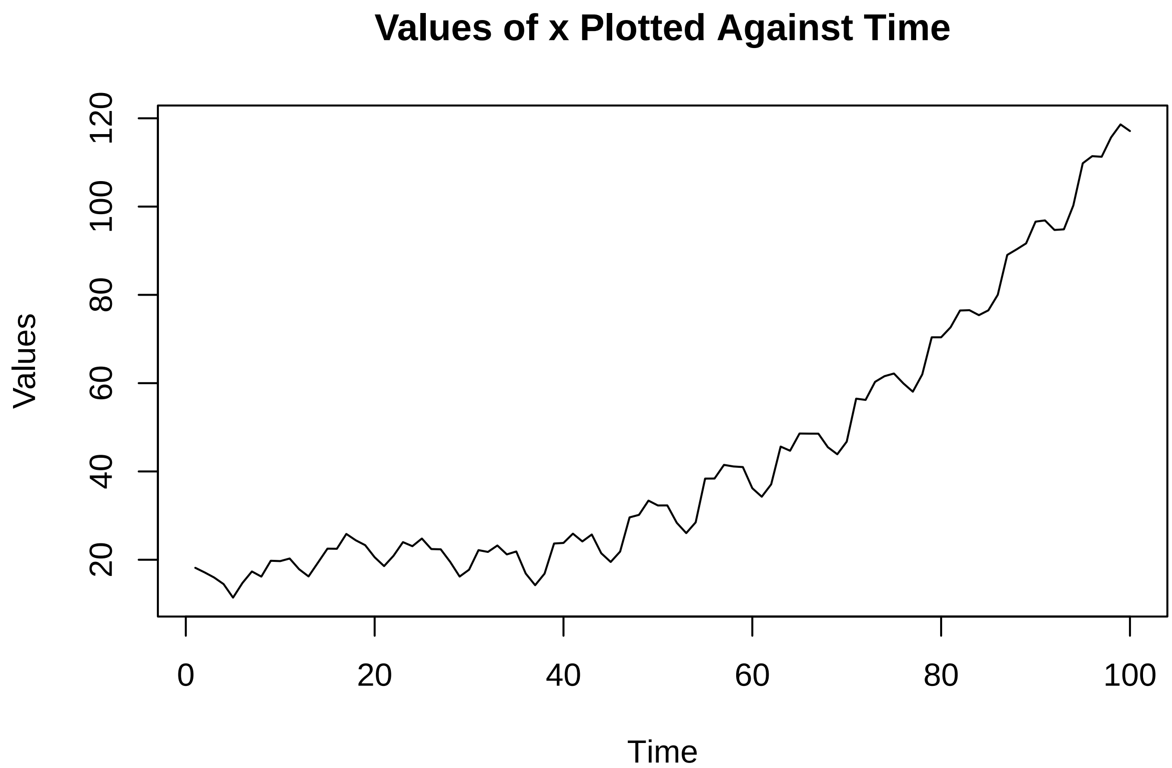
(2) The variance of the process also has to be finite.

And (3) the autocovariance function only depends on the lag, and not at the time at which it is observed.

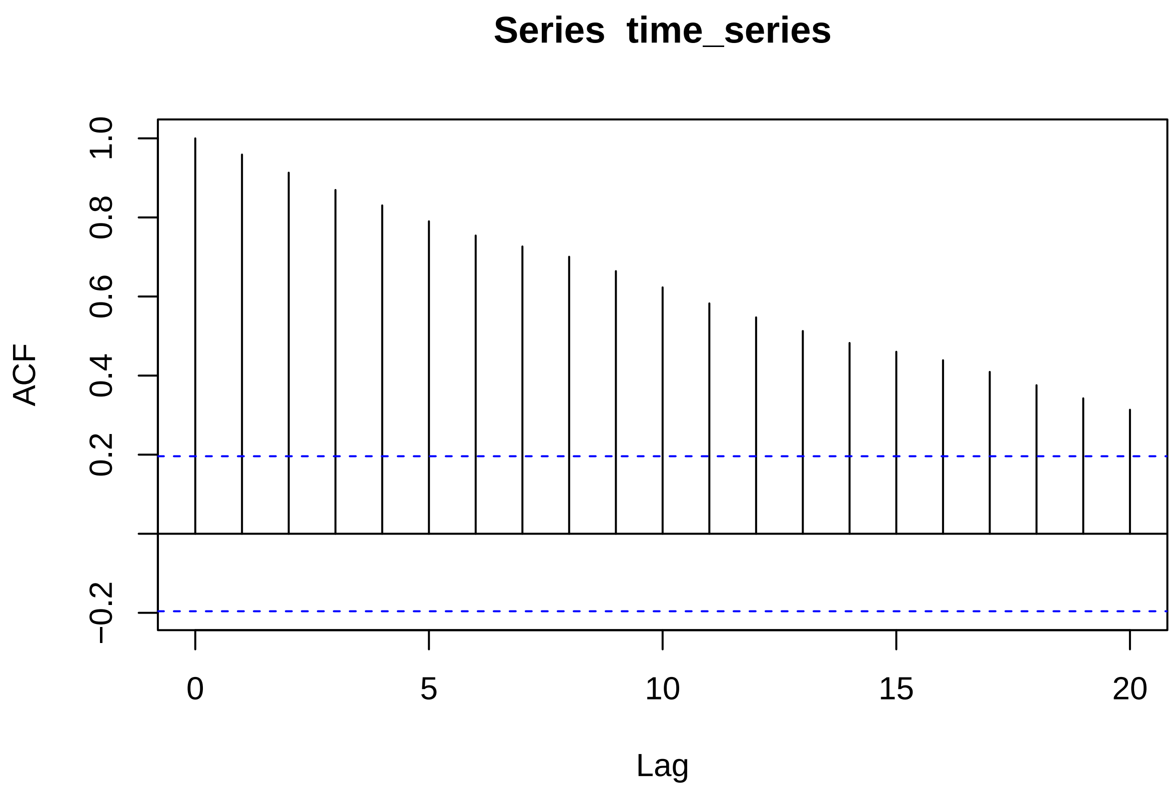
Question 2

```
# Reading in data and converting to time series
dat <- read.csv("lab4data.csv")
time_series <- ts(dat$x)

# Plotting
plot(time_series, xlab = "Time", ylab = "Values", main = "Values of x Plotted Against Time")
```




```
acf(time_series)
```



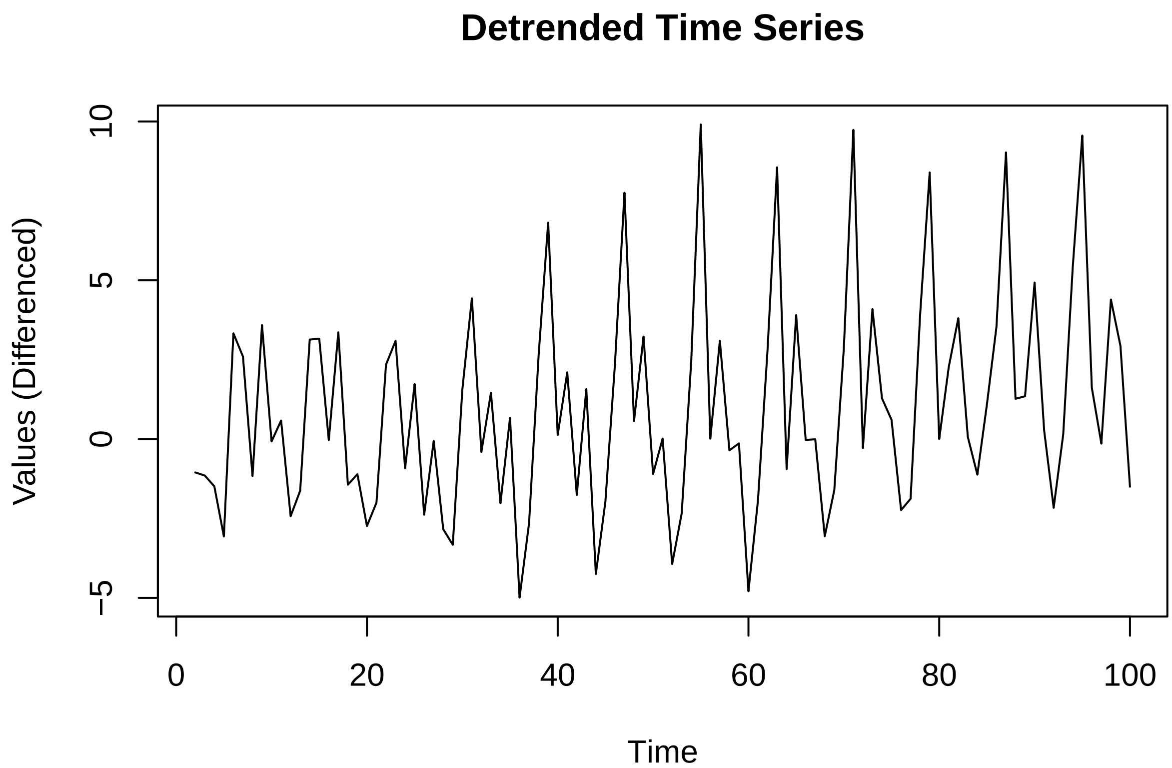
The time series seems to have an increasing trend thus it cannot be stationary. It also looks to have some seasonal or cyclical effects.

The ACF also supports the idea that the time series is not stationary.

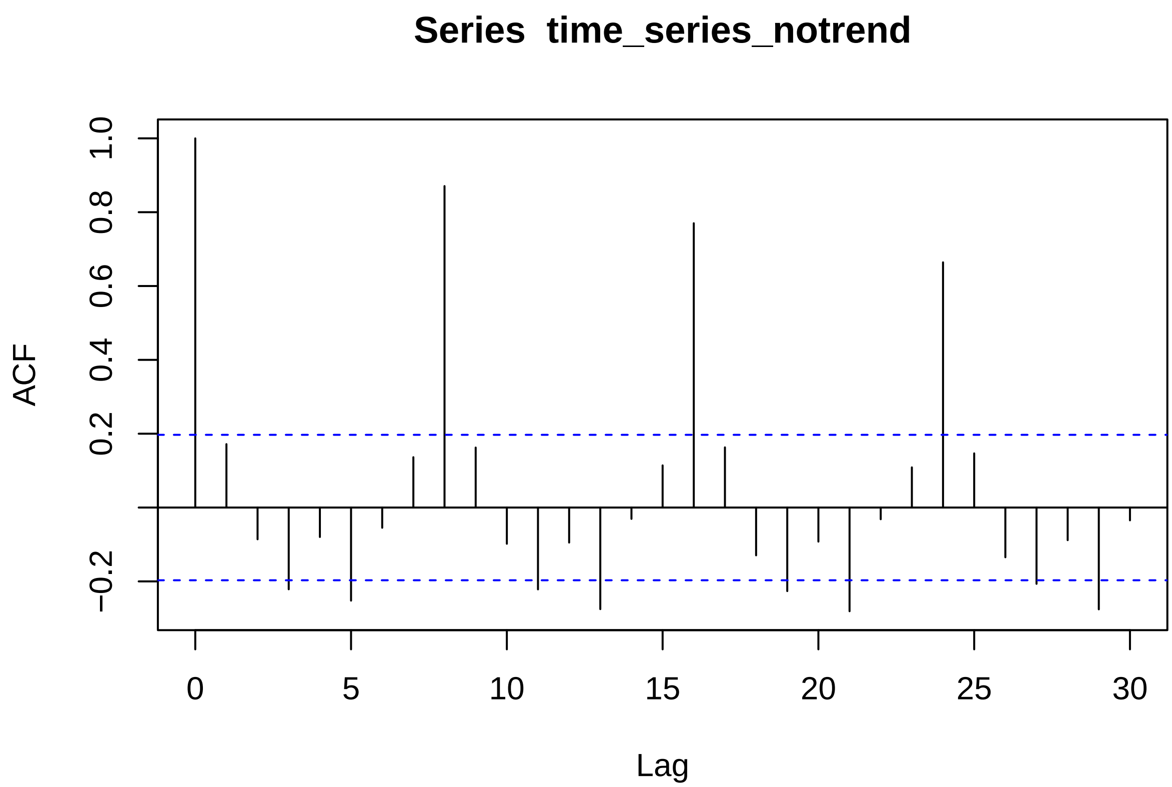
Question 3

```
# Removing trend
time_series_notrend <- diff(time_series, lag = 1, differences = 1)

# Plotting
plot(time_series_notrend, xlab = "Time", ylab = "Values (Differenced)", main = "Detrended Time Series")
```




```
acf(time_series_notrend, lag.max = 30)
```



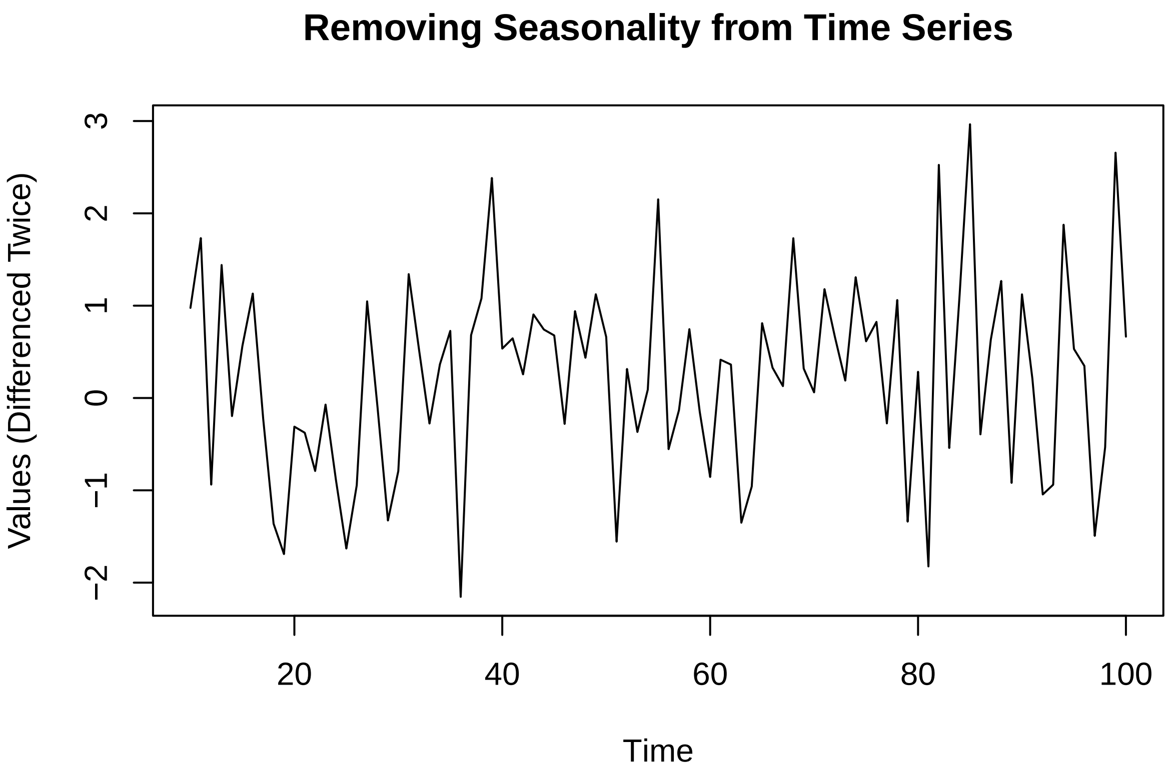
The time series as described by y_t is de-trended but we cannot say for sure its stationary, as it still has seasonality.

The ACF also shows the period of the seasonality, which is 8. The ACF has spikes at lags of 8, 16 and so on.

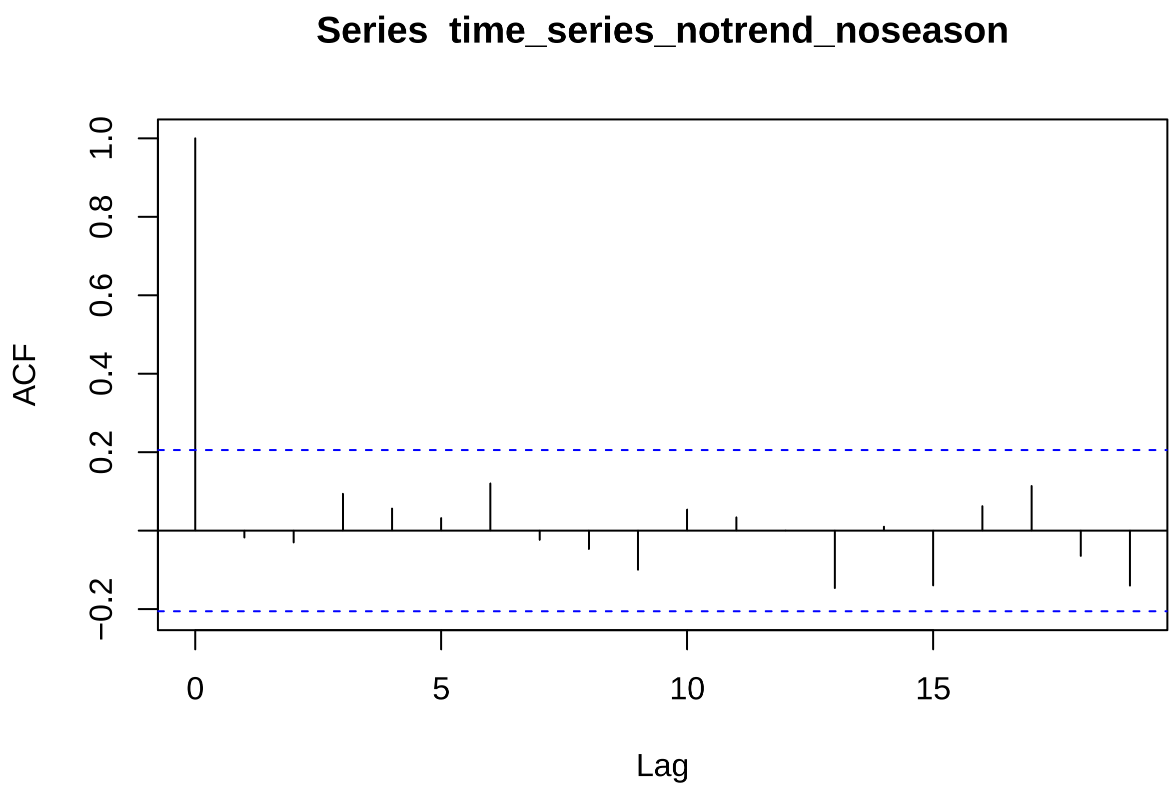
Question 4

```
# Differencing again
time_series_notrend_noseason <- diff(time_series_notrend, lag = 8, differences = 1)

# Plotting
plot(time_series_notrend_noseason, xlab = "Time", ylab = "Values (Differenced Twice)", main = "Removing
```




```
acf(time_series_notrend_noseason)
```



The ACF plots supports that the time series after differencing again for seasonality, is stationary, and the plots also appears to be stationary.

The plot does resemble that of white noise, and so does the ACF

Question 5

The original data can be described by,

$$(p, d, q) \times (P, D, Q)_s = (0, 1, 0) \times (0, 1, 0)_8$$

Question 6

(a)

$$W_t = Y_t - Y_{t-s}$$
$$W_t = (X_t - X_{t-1}) - (X_{t-s} - X_{(t-s)-1}))$$
$$W_t = X_t - X_{t-1} - X_{t-s} + X_{t-(s+1)}$$

(b)

$$We\ know\ that\ BX_t = X_{t-1}$$
$$Y_t = X_t - X_{t-1}$$

$$Y_t = X_t - BX_t = (1 - B)X_t$$
$$W_t = Y_t - Y_{t-s}$$
$$W_t = (1 - B)X_t - X_{t-s} + X_{t-(s+1)}$$

(c)

$$We\ know\ that\ B^s Y_t = Y_{t-s}$$
$$W_t = (1 - B)X_t - B^s X_t + B \cdot B^s X_t$$
$$W_t = (1 - B)X_t - (1 - B)B^s X_t$$
$$W_t = (1 - B)[X_t - B^s X_t]$$