UNIVERSITY OF BRITISH COLUMBIA
Department of Statistics
Stat 443: Time Series and Forecasting
Assignment 2: Time Series Models

The assignment is due on **Tuesday, February 15** at **9:00pm**.

- Submit your assignment online in the **pdf format** under module "Assignments".
- This assignment can be hand-written and scanned, or typeset using either LaTeX or R Markdown.
- When answering the questions, writing down the final answers will not be sufficient to receive full marks. Please explain your steps and show all calculations unless otherwise specified.
- Please make sure your submission is clear and neat. It is the student's responsibility that the submitted file is in good order (e.g., not corrupted and is what you intend to submit).
- **Late submission penalty**: 1% per hour or fraction of an hour. (In the event of technical issues with submission, you can email your assignment to the instructor to get a time stamp but submit on canvas as soon as it becomes possible to make it available for grading.)

1. Consider the following AR(3) process:

$$X_t = \frac{2}{5}X_{t-1} + \frac{1}{4}X_{t-2} - \frac{1}{10}X_{t-3} + Z_t, \qquad \{Z_t\}_{t\in\mathbb{N}} \sim WN(0, \sigma^2).$$

   (a) Show that $\{X_t\}_{t\in\mathbb{N}}$ is stationary.

   (b) Using the Yule-Walker equations, derive the autocorrelation function for $\{X_t\}_{t\in\mathbb{N}}$. Show all steps.

   (c) Assuming $Z_t \sim \mathcal{N}(0, 1.96)$, use the `arima.sim()` function to simulate 2000 observations from the process $\{X_t\}_{t\in\mathbb{N}}$ and plot the sample acf for the first 10 lags along with the theoretical acf from part (b).
   Use `set.seed(123)` before simulating the process. Include your R code.

2. Consider the following ARMA(1,1) process:

$$X_t = \frac{7}{10}X_{t-1} + Z_t - \frac{1}{10}Z_{t-1}, \qquad \{Z_t\}_{t\in\mathbb{N}} \sim WN(0, \sigma^2).$$

   (a) Check whether the process is stationary and invertible.

   (b) Write the above ARMA(1, 1) process as a pure MA process.

   (c) Write the above ARMA(1, 1) process as a pure AR process.

   (d) Derive the autocorrelation function for $\{X_t\}_{t\in\mathbb{N}}$. Show all steps.

3. Show that a SARIMA$(2, 1, 0) \times (0, 1, 2)_{12}$ process can be written as an ARMA(p, q) process. Specify the values of p and q.