# Assignment 1

**1.develop a function poly (double x, int n, double [] a) to evaluate the n-degree polynomial of x (array a store the coefficients of polynomial).**

**poly (x, n, a) =** $a[0] + a[1] * x + a[2] * x^2 + \dots + a[n] * x\hat{}n$

**then discuss the time and storage complexity of your algorithm.**

**Solution**: Polynomial function code:

```
 double void polyEvaluate (double coeff[], int order, double x)
   {
        double res = 0.0;    //initialize result
                 //Evaluating using Horner's method
        for (int i = 0; i <= order; i++)
       {
          res += coeff[i] * pow (x, i);
       }
     print(res);
   }
```

Time Complexity: Here, the for loop is executed if there are n values then the loop executes n times.

Hence, the time complexity is n

Storage Complexity: It will be the size of an array.

**2. Prove by induction that:** $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ .

**Solution:**

Basic Step: when n=1

| Left Hand Side (L.H.S) | Right Hand Side (R.H.S) |
|---|---|
| L.H. S=$n^2$ <br> $= 1^2$ <br> $= 1$ | R.H.S = $\frac{n(n+1)(2n+1)}{6}$ <br> $= \frac{1(1+1)(2(1)+1)}{6}$ <br> $= \frac{1(2)(3)}{6}$ <br> $= \frac{6}{6}$ <br> $= 1$ |

Hence L.H.S = R.H.S when n=1.

Inductive Step:

Inductive Hypothesis: Assuming that n=k holds true then

$$1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Let's prove that it holds true when n=k+1

| Left Hand Side (L.H.S) | Right Hand Side (R.H.S) |
|---|---|
| L.H. S=$1^2 + 2^2 + 3^2 + \ldots + k^2 + (k + 1)^2$ <br> $= \frac{k(k+1)(2k+1)}{6} + (k + 1)^2$ <br> $= \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$ <br> $= 1/6[k(k + 1)(2k + 1) + 6(k + 1)^2]$ <br> $= \frac{(k+1)[k(2k+1)+6(k+1)]}{6}$ <br> $= \frac{(k+1)[(2k^2+k+6k+1)]}{6}$ <br> $= \frac{(k+1)(2k^2+7k+1)}{6}$ <br> $= \frac{(k+1)(2k+3)(k+2)}{6}$ | R.H.S = $\frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ <br> $= \frac{(k+1)(k+2)(2k+2+1)}{6}$ <br> $= \frac{(k+1)(k+2)(2k+3)}{6}$ <br> $= \frac{(k+1)(2k+3)(k+2)}{6}$ |

Therefore, L.H.S = R.H.S when n=k+1 is true by Induction.

**3. Prove by induction that** $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{(-1)^n}{n}$ **is always positive**

Or $\sum_{i=1}^{n} - \frac{(-1)^n}{n} > 0$(0 is neutral)

**Solution:** Let's assume that summation as $x_n$

Basic Step: when n=1

$$x_n = \frac{-(-1)^n}{n}$$

$$= -(-1)^1/1$$

$= 1 > 0$

Hence basic step holds true.

Inductive Step:

Let's assume that $x_k > 0$ *for an integer k when is even.*

Then, $\sum_{i=1}^{k} -\frac{(-1)^k}{k} > 0$

Then note that k+1 will be odd (as we have assumed k as even) then

$$\sum_{i=1}^{k+1} -\frac{(-1)^{k+1}}{k+1} > 0$$

When k+1 is odd then it becomes: $-\left(\frac{-1^{k+1}}{k+1}\right) = -\frac{-1}{k+1}$

So, we can write this as: $x_{k+1} = x_k + \left(-\left(-\frac{1^{k+1}}{k+1}\right)\right) = x_k + \left(-\left(-\frac{1}{k+1}\right)\right)$

Or $x_{k+1} = x_k + \frac{1}{k+1}$

Hence, both the steps hold true that is >0(by basic step 1>0 and by inductive step) because when you sum even number with any number then it will be greater that zero therefore the case is true when k is even and k+1 is odd.

**4. Prove by induction that: 1 + 3 + 5 + ... + (2n-1) = $n^2$**

**Solution:**

Basic Step: when n=1

| Left Hand Side (L.H.S) | Right Hand Side (R.H.S) |
|---|---|
| L.H. S= 2n-1<br>=2(1)-1<br>=1 | R.H. S=$n^2$<br>$= 1^2$<br>=1 |

L.H. S=R.H. S holds true.

Inductive Step:

Inductive Hypothesis: Assuming that n=k holds true

Then: 1 + 3 + 5 + ... + (2k-1) =$k^2$

Let’s prove that it holds true when n=k+1

| Left Hand Side (L.H.S) | Right Hand Side (R.H.S) |
|---|---|
| =1 + 3 + 5 + ... + (2k-1) + (2(k+1)-1)<br>=$k^2 + 2(k + 1) - 1$<br>=$k^2 + 2k + 2 - 1$<br>=$k^2 + 2k + 1$<br>=$(k + 1)^2$ | =$(k + 1)^2$ |

L.H.S = R.H.S when n=k+1 is true by induction.

**5. For all positive integers n, ($n^2$+n+1) is odd.**

**Solution:**

Basic step: Suppose when n=1

$$n^2 + n + 1 = 1^2 + 1 + 1$$

=3

This is odd Hence the basic step holds True

Inductive Step:

For some integers of k:$k^2 + k + 1$ is odd.

From the definition of odd, $\exists p \in Z,\ such\ that\ k^2 + k + 1 = 2p + 1$

Subtract 1 from both sides, $k^2 + k = 2p$

Now let us take $k + 1$:

$(k + 1)^2 + (k + 1) + 1 = (k + 1)(k + 1) + (k + 1) + 1$

$= k^2 + 2k + 1 + k + 1 + 1$

$= \left(k^2 + k\right) + 2k + 2 + 1$

$= 2p + 2k + 2 + 1$

$= 2(p + k + 1) + 1 \qquad by\ inductive\ hypothesis\ where \in Z$

Hence that $(k + 1)^2 + (k + 1) + 1, \exists p \in Z,\ such\ that\ k^2 + k + 1 = 2p + 1$

Hence by definition that $(k + 1)^2 + (k + 1) + 1$ is odd.

Therefore, it is true by Induction.