(IT BRANCHES)

Discrete and Integral Transforms (DIT)

Subject Code: 18MA3GCDIT

Module – 1 (Curve Fitting & Statistics Modelling)

Q.No						(Quest	ions	3					
1.	a) F	ind the linear	· law P	= n	ı W	+ <i>c</i>								
					W	50	70	10	0 1	20				
					P	12	15	21	2	5				
		it the best po or the data	ssible (curv	e of	the fo	rm y	= 0	ax +	<i>b</i> , u	sing	g meth	od of least	square
				2	X	5	10	15	20	25				
				7	У	16	19	23	26	30				
						•	•		•		_			
2.	-	it the best p quare for the		e cu	irve	of th	ie foi	m y	r = a	<i>x</i> +	- b,	using	method o	f least
			X	1	3	4	6	8	9	1	1	14		
			у	1	2	4	4	5	7	8	}	9		
	C	simply sup orresponding given in the stimate Y wh	g to vai	iou: ing	s va tabl	lues o	f X th	e ma	aximu	ım d	efle	ction Y	is measur	ed and
			X	1	00	120	14	0	160	18	0	200]	
			Y	0	.15	0.55	5 0.	6	0.7	0.8	3	0.85	-	
		l								1			J	
3.		it a straight l he year 2006	ine to	the	follo	wing	data.	And	also	find	the	expec	cted produc	tion in
		Year			19	61	197	'1	198	31	1	991	2001	
		Production	in ton	es	8		10		12		1	0	16	
		it the best po or the data	ssible (curv	e of	the fo	rm y	= 0	ax +	<i>b</i> , u	sing	g meth	od of least	square
			X		1		2	3	4		5			
			у		1	4	13	9	5		2			
					l			l		J				

4.	a) Fit the best po	ossib	le cui	ve of	the f	orm	$\overline{y} =$	$\overline{a}x$	<i>b</i> , us	sing m	ethod of least square
	ioi the data			0	1	1 -	<u>, </u>	2	1		\neg
		X		0	1		2	3	4	5	
		У		9	8	4	24	28	26	20	
	b) Fit a parabola	<i>y</i> =	ax^2	+ <i>bx</i> -	+ c t	o the	data				
				х	1		2	3	4	Ī	
				у	1	.7	1.8	2.3	3.2		
				,							
5.	a) Fit a parabola	27 —	ax2	l hx	Lat	o the	data				
3.	aj rita paraboia	у –	ux -						1-	1	
				X	1	2	3	4	5		
				У	10			16	19		
	Fit a parabola <i>y</i> :	= ax	$^{2} + b$	x + c	to tl	he da	ita				
		X	1	.0	1.5	2.0	2.5	3.0	3.5	4.0	0
		у	1	1	1.3	1.6	2.0	2.7	3.4	4.2	1
			ı							l	
6.	a) Fit a parabola	<i>y</i> =	ax^2	+ <i>bx</i>	+ c t	o the	data				
		X		0	1	2		3	4	5	
		у		1	3	7		13	21	31	
	b) Fit a parabola	<i>y</i> =	ax^2	+ <i>bx</i> -	+ c t	o the	data				
		X	-2	2	-1		0	1		2	
		у	-:	3.150	-1.	.390	0.62	20 2	.880	5.378	3
7.	a) The revoluti	on	(r) ·	and	time	(t)	are	rela	ted 1	hv ai	uadratic polynomial
7.	$r = at^2 + bt$	+ c.	Estin	iate t	he nu	ımbe	r of re	evolut	tion fo	r time	e 3.5 units given
		t		1.2	1.6	1.9	2.1	2.4	2.6	3	
		r		5	10	15	20	25	30	35	_
	b) Fit a parabola	<i>y</i> =	ax^2	+ <i>bx</i> -	+ c t	o the	data				
	_	·	X	-3	-2	-1	0	1	2	3	
			у	38	16	4	2	10	28	56	

a)	Calculate		ze of		.and		eviat			101	11		12	7	
			eque		3		Ċ	9 1	13	8	5		4		
b)) Find the	mean	and s	tanda	ırd	deviat	ion f	or the	foll	owir	ıg			_	
		Mid V			.5	20	25	30	35	4		45	5	0	
		Frequ	uency	7 2	2	22	19	14	3	4		6	1		
a)	Compute	e the av	verag	e for	the	follow	ing (data							
	Class interval	l	0- 99	100 199		200- 299		00- 99	400		50 59		600		700- 799
												,	699	,	
	Freque	ncy	10	54		184		64	246		40		1	,	1
b)	Frequent Following the mean Age in yr	ng tablo	e give e grou 0- 2	es the		184 queno	2 2 cy of 0-			fag	40	p of	1		1 hers.
b)	Followin	ng table n of the s 2:	e give e grou 0- 2	es the up.	fre	184 queno - 40 4:	2 2 cy of 0-	the a	ge o:	fag	40 rou	p of	199	teac	1 hers.

11.	a) The index number of prices of two articles A and B for six consecutive weeks are
	given below. Find which has a more variable price?

A:	314	326	336	368	404	412
B:	330	331	320	318	321	330

b) The two observers bring the following two sets of data which represent measurements of the same quantity.

I set	105.1	103.4	104.2	104.7	104.8	105.0	104.9
II set	105.3	105.1	104.8	105.2	106.7	102.9	103.1

Calculate the SD in each case. Which set of data is more reliable?

- a) Define: (i) Correlation (ii) Co-efficient of correlation (iii) Regression (iv) Lines of Regression (v) Regression co-efficient.
 - b) Establish the formula $r = \frac{\sigma_x^2 + \sigma_y^2 \sigma_{x-y}^2}{2 \sigma_x \sigma_y}$

13. a) Find the correlation co-efficient between x and y from the given data:

х	78	89	97	69	59	79	68	57
у	125	137	156	112	107	138	123	108

b) Find the correlation co-efficient between *x* and *y* from the given data:

X	21	23	30	54	57	58	72	78	87	90
у	60	71	72	83	110	84	100	92	113	135

a) Calculate the correlation co-efficient for the following heights in inches of fathers (x) and their sons (y).

X	65	66	67	67	68	69	70	72
У	67	68	65	68	72	72	69	71

b) Find the co-efficient of correlation between industrial production and export using the following data and comment on the result.

Production (in crore tons)	55	56	58	59	60	60	62
Exports(in crore tons)	35	38	38	39	44	43	45

15.	a)			_			-	k an	d x c	n y	an	id he	nce	find	the co	rrelation
		coem	icient i	for the f	onow	ing da	ta:								<u> </u>	
				X	2	4	4	6	Ó		8		10	0		
				y	5	1	7	Ģ)		8		1	1		
	b)	Obtai	n the i	regressi	on lir	es of	v on s	x an	d x d	n v	, an	ıd he	nce	find	_ the co	rrelation
				for the f			•		u 11 (,,,,	412		,,,,,,	111101		110101011
					х	1	2		3	4		5				
					y	2	5		3	8	}	7				
16.	a)			regressi for the f				k an	d x c	n y	an	id he	nce	find	the co	rrelation
		COCIII	iciciit i	ioi tiic i	OHOW	ing da	ta.									
		Ī	x	1	2	3	4	5	1	6	7	1	3	9	10	1
																_
			У	10	12	16	28	2.		36	41		19	40	50	
	b)	Obtai data:	n the l	ines of 1	regres	sion a	nd he	nce	find	the	со-е	effici	ent	of cor	relatio	on for the
		uata:				1		1								
			x	1	3	4	2	5	8	}	9	1	0	13	15	
			у	8	6	10	8	12	1	6	16	1	0	32	32	
17.	a)	The f	ollowii	ng resu	lts we	re ob	 tained	l fro	m re	cor	ds c	of ag	e(x)	and	blood	pressure
		(y) o	f a gro	oup of 1	l0 me	n, giv	en Σ (<i>x</i> –	$\bar{x})(y$	· — j	$\overline{y}) =$	122	0. I	ind t	he ap	propriate
		regre is 45	ssion e	equatior	i ana	use it	to est	ımaı	e tno	e bi	ooa	pres	sur	e or a	man w	hose age
									x		у					
					M	Iean		53		-	142					
					V	arianc	e	13	0		165					
	h)	Cirron	O	O remit								of no	ano	aaian	and h	ongo find
	עט			.o, write obable v			_		O1 (11	.C II	1162	01 16	gre	SSIUII	anu il	ence find
			-						x		y					
						Mea	n	18		10	00					
						S.D		14		20)					
40	-	mi ·					- (1)		-1.7		1			10	10 1	.07
18.	a)															.87 <i>y</i> and rrelation
		coeffi	icient b	etween	x and	<i>y</i> .										
	b)						_				-			-		equations 's and y's
			-	elation	-		_		-		u ui	1111	cuii	varut	.5 UI X	s and y s

19.	 a) In a partially destroyed laboratory data, only the equations giving the two lines of regression of <i>y</i> on <i>x</i> and <i>x</i> on <i>y</i> are7<i>x</i> - 16<i>y</i> + 9 = 0, 5<i>y</i> - 4<i>x</i> - 3 = 0 respectively. Calculate the coefficient of correlation, <i>x̄</i> and <i>ȳ</i>. b) In a partially destroyed laboratory record of correlation data, the following result only are a variable, variance of x is 9, regression equation y on x and x on y are 4<i>x</i> - 5<i>y</i> + 33 = 0, 20<i>x</i> - 9<i>y</i> - 107 = 0 respectively. Calculate the coefficient of correlation, <i>x̄</i>, <i>ȳ</i> and σ_y
20.	a) If θ is the acute angle between the two regression lines relating the variables x and y , show that $\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{1 - r^2}{r}\right)$ b) Find the co-efficient of correlation between x and y given $2\sigma_x = \sigma_y$ and the angle between the lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$

Discrete and Integral Transforms (DIT)

Subject Code: 18MA3GCDIT

Module – 2 (Z - Transform)

Q.No	Questions
1.	a) Find the Z- transform of n^3 and hence find $Z_T(k^n n^3)$
	b) Prove that $Z_T(\cos n\theta) = \frac{Z(Z-\cos\theta)}{Z^2-2Z\cos\theta+1}$ and hence deduce Z-Transform of $(k^n \cos n\theta)$
2.	a) Prove that $Z_T(\sin n\theta) = \frac{Z\sin\theta}{Z^2 - 2Z\cos\theta + 1}$ and hence deduce $Z_T(k^{-n}\sin\theta)$.
	b) Find the Z-Transform of $\cosh n\theta$ and hence find $Z_T(a^n \cosh n\theta)$.
3.	a) Find the Z-Transform of $\sinh n\theta$ and hence find $Z_T(a^n \sinh n\theta)$.
	b) Find the Z-Transforms of $(i)(n-1)^2$ $(ii)\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$
4.	a) Find the Z-Transform of $(i)a^{n+3}$ (ii) $\cosh\left(\frac{n\pi}{2} + \theta\right)$
	b) Find the Z-Transforms of (i) $(n + 1/3)^2$ (ii) $\sin(3n + 5)$.
5.	a) Find the Z-Transforms of (i) $\frac{1}{n!}$ (ii) $n e^{an}$
	b) Find the Z-Transforms of $(i)e^{-an} n (ii) n \cos n\theta$.
6.	a) Find the Z-Transforms of (i) $(\frac{1}{2})^n + (\frac{1}{3})^n$ (ii) $3^n \cos(\frac{\pi n}{4})$.
	b) Find the Z-Transforms of (i) $e^{-an} \cos(n\theta)$ (ii) $e^{-an} n^2$
7.	a) Find the Z-Transforms of $\frac{n}{3^n} + 2^n n^2 + 4 \cos n\theta + 4^n + 8$
	b) Find the Z-Transforms of (i) $(2n-1)^2$ (ii) $3n-4\sin\frac{n\pi}{4}$
8.	a) If $\bar{u}(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$ evaluate u_2 and u_3 .
	b) Given that $Z(u_n) = \frac{2z^2 + 3z + 4}{(z-3)^3}$, $ z > 3$, show that $u_1 = 2$, $u_2 = 21$ and $u_3 = 139$.
9.	a) If $\bar{u}(z) = \frac{5z^2 + 3z + 12}{(z-1)^4}$ Show that $u_2 = 5$ and $u_3 = 23$.
	b) If $\overline{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ evaluate u_2 and u_3 .
10.	a) State Initial Value Theorem in Z-Transforms and given $Z_T(U_n) = \frac{z}{z-1} + \frac{z}{z^2+1}$, Find U_0, U_1, U_2

	b) If $\overline{U}(z) = \frac{2z^2 + 3z + 12}{(z-1)^2}$, find the values of U_0 , U_1 , U_2 , U_3 .
11.	a) Obtain the Inverse Z- transform of $\frac{z^2}{(z-1)(z+3)}$.
	b) Obtain the Inverse Z- transform of $\frac{z}{(2z^2+z-3)}$.
12.	a) Find the Inverse Z- Transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.
	b) Find inverse Z-transform of $\frac{z(z+3)}{(z+1)(z-2)}$
13.	a) Find $Z^{-1}\left(\frac{2z}{(z-1)(z^2+1)}\right)$.
	b) Compute the Inverse Z-Transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$.
14.	a) Find $Z^{-1}\left(\frac{5z}{(2-z)(3z-1)}\right)$.
	b) Find inverse Z-transform of $\frac{10z}{(z-1)(z-2)}$
15.	a) Find the Inverse Z-transform of $\frac{8z-z^3}{(4-z)^3}$
	b) Find the Inverse Z- Transform of $\frac{4z^2-2z}{z^3-5z^2+8z-4}$.
16.	a) Find the Inverse Z- Transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$
	b) Find the Inverse Z- Transform of $\frac{z}{(z+1)^2(z-1)}$
17.	a) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$; $y_0 = 0, y_1 = 0$ using z-transforms.
	b) Using the Z-transform method, solve $U_{n+2} - 2U_{n+1} + U_n = 3n + 5$.
18.	a) Solve the difference equation using Z-transform $y_{n+2} - 4y_{n+1} + 3y_n = 5^n$.
	b) Solve $y_{n+2} - 4y_n = 0$ given that $y_0 = 0, y_1 = 2$ using Z-transform.
19.	a) Solve the difference equation using Z-transform $y_{n+2} - 3y_{n+1} + 2y_n = 0$ given that $y_0 = 0, y_1 = 1$
	b) Solve the difference equation using Z-transform $U_{n+2}-2U_{n+1}+U_n=2^n$; $U_0=2$, $U_1=1$
20.	a) Solve the difference equation $U_{n+2}+2U_{n+1}+U_n=n\;; U_0=U_1=0\;$. Using z-transforms
	b) Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n$, with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for $n = 0,1,2,3,$ by Z-transform method.

Discrete and Integral Transforms (DIT)

Subject Code: 18MA3GCDIT

Module – 3 (Fourier series)

Q.No	Questions
1.	a) Find a Fourier series in $(-\pi,\pi)$ to represent $f(x)=x-x^2$ and hence deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}$
	b) Expand $(x) = \sqrt{1-\cos x}$, $0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots \dots$
2.	a) Find a Fourier series in $(0,2\pi)$ to represent $f(x)=\frac{\pi-x}{2}$ and hence deduce that $\frac{\pi}{4}=1-\frac{1}{2}+\frac{1}{5}-\frac{1}{7}+\cdots\dots\dots\dots\dots\dots$
	b) Find the Fourier series for the function $f(x) = x $ in $-\pi \le x \le \pi$, hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}.$
3.	a) Obtain the Fourier series of the function $f(x)$ defined by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x < \pi \end{cases}$
	and hence prove that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. b) Find the Fourier series of $f(x) = \left\{ \begin{array}{ll} 0 & when & -\pi \leq x \leq 0 \\ x^2 & when & 0 \leq x \leq \pi \end{array} \right\}$
4.	a) Obtain Fourier series of the function $f(x) = \begin{cases} -k & -\pi \le x \le 0 \\ k & 0 \le x \le \pi \end{cases}$ hence deduce that
	$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \dots$ b) Find the Fourier series of $f(x) = x^3$ in $(-\pi, \pi)$
5.	a) Find the Fourier expansion of the function $f(x)$ defined by the $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ x - 2\pi, & \pi \le x \le 2\pi \end{cases}$ and prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
	b) Find the Fourier series expansion of $f(x) = e^{ax}$ in $(0,2\pi)$
6.	a) Find the Fourier series $f(x)=xcosx$ in $(-\pi$, $\pi)$ b) Obtain the Fourier series for $f(x)=e^{-ax}$, $a>0$ in $(0$, $2\pi)$
7.	a) Find the Fourier series of $f(x) = \begin{cases} -\left(\frac{\pi+x}{2}\right) & for \ -\pi \le x < 0 \\ \left(\frac{\pi-x}{2}\right) & for \ 0 \le x < \pi \end{cases}$
	b) Find the Fourier series that represent $f(x) = x^3$, in $(-l, l)$
8.	a) Find the Fourier series expansion of the function $f(x) = 1 - x^2$ in $(-1, 1)$. b) Obtain the Fourier series expansion of $f(x) = \frac{l-x}{2}$ in $0 < x < 2$

9.	a)	Obtain	the Four	ier series	for the fu	unction <i>f</i>	f(x) =	$\frac{\pi x}{\pi(2-}$	$0 \le x$	$x \le 1$ $1 \le x 2$	nd deduc	e that
		$\frac{\pi^2}{8} = \sum_{n=1}^{\infty}$	$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^n}$	1)2								
	b)	Obtain	the Fourier Series for $f(x) = \begin{cases} x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$									
10.			f(x) =									
			the Four			. 0	2 \ \ \	\ 1 ²				
11.			the Four						$\cos \frac{\pi}{b}$	$\frac{x}{t}$ in the in	nterval(—	l, l).
	b)	Obtain	Fourier s	eries $f(z)$	(x) = 4 -	x^2 in ((-2,2)				
12.	a)	Find th	e Fourier	series for	the fund	ction $f(x)$) = {	-1 , —: 2 , 0	2 ≤ < <i>x</i>	$\begin{cases} x < 0 \\ \leq 2 \end{cases}$	defined or	า (-2,2)
			df(x) = 0									
13.	a)	Find th	e Half rar	nge Sine s	eries for	the funct	tion (x	$=\begin{cases} x \\ \pi \end{cases}$, – x,	$0 < 2$ $\frac{\pi}{2}$	$x \le \pi/2$ $\le x \le \pi$	
	b)	Detern	nine the F	lalf range	Fourier	cosine se	ries $f($	(x) = x	² in	$(0,\pi)$		
14.	a)	Expand	df(x) =	$\begin{cases} \frac{1}{4} - x, & 0 \\ x - \frac{3}{4}, & 1 \end{cases}$	$0 < x < \frac{1}{2} < x < $	1/ ₂ in I	Half rai	nge sine	ser	ies.		
	b)	Detern	nine the H	lalf range	Fourier	cosine se	ries f ($f(x) = \left\{ \begin{array}{l} x \\ \end{array} \right.$	$\begin{cases} x \\ l - \end{cases}$	$0 \le x$ $x \frac{l}{2} \le x$	$0 < \frac{l}{2}$ $0 \le l$	
15.	a)	Find th	e half ran	ge cosine	series fo	or the fur	ction j	f(x) =	(x -	- 1) ² in tl	he interva	0 < x < 1
	b)	Obtain	the Four	ier series	of <i>xsinx</i>	as half	range	cosine s	erie	s in $(0,\pi)$	τ)	
16.	a)	Find th	e Half rar	nge Cosine	e series f	or the fui	nction	f(x) =	$\begin{cases} k(t) \end{cases}$	$\frac{1}{kx}$, $0 < \frac{1}{kx}$	$x \le \frac{l}{2}$ $x \le x \le x \le x$!
			e Half rar								_	
17.	a) Find the constant term and the first 2 harmonics for the function $f(x)$ given by the											
		followi	ng table	0	60	120	180	24	10	300	360]
			Х	U		120		22	ŀU	300	300	
		y 0.8 0.6 0.4 0.7 0.9 1.1 0.8										
	h\	Obtoin	the Form	ior corios	noglosti:	og torms	hiahar	+han +h	o ti~	et harme	nicc	
	D)	Obtain	the Four	0	60°		nigner 20°	180°		240°	300°	
			У	7.9	7.2	3	.6	0.5		0.9	6.8	
			,									

a) For the periodic function f(x) of period 6 specified by the following table over the interval (0,6), find the Fourier coefficients a_0,a_1 and b_1

х	0	1	2	3	4	5	6
у	9	18	24	28	26	20	9

b) Express y as a Fourier Series upto the 3rd harmonics given the fallowing values

			•		•		•
Ī	Х	0	1	2	3	4	5
	У	4	8	15	7	6	2

a) The following table gives the variations of periodic current over a period. Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the 1st harmonic.

t(sec)	0	Т/6	T/3	T/2	2T/3	5T/6	Т
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

b) The fallowing values of y and x are given , Find the Fourier series of y up to second harmonics

Х	0	2	4	6	8	10	12
У	9.0	18.2	24.4	27.8	27.5	22.0	9.0

a) Analyze harmonically the data given below & express y as a Fourier series up to 2nd harmonic.

Х	0	$\pi/_3$	$^{2\pi}/_{3}$	π	$4\pi/_{3}$	$5\pi/_3$	2π
у	1.0	1.4	1.9	1.7	1.5	1.2	1.0

b) Determine the constant terms and the first cosine and sine terms of the Fourier series expansion of y from fallowing table

Х	0	45	90	135	180	225	270	315
У	2	3/2	1	1/2	0	1/2	1	3/2

Discrete and Integral Transforms (DIT) Subject Code: 18MA3GCDIT Module – 4 (Integral Transform – I)

Q.No	Question
1.	a) Prove that (i) $L(coshat) = \frac{s}{s^2 - a^2}$ (ii) $L(sinat) = \frac{a}{s^2 + a^2}$
	b) Prove that $L[t^n] = \frac{n!}{s^{n+1}}$, n is a positive integer
2.	Find a) $L(\cos t \cos 2t \cos 3t)$ b) $L(e^{at} + 2t^n - 3\sin 3t + 4\cosh 2t)$
3.	Find a) $L[e^{-2t}(2\cos 5t - \sin 5t]$ b) $L(t\sin t)$
4.	Find a) $L\{e^{3t} \sin 5t \sin 3t\}$ b) $L(e^{-t} \cos^2 4t)$
5.	Find a) $L[t(sin^3t - cos^3t)]$ b) $L(t^5e^{4t}cosh3t)$
6.	Find a) $L(te^{-2t}sin4t)$ b) $L\{e^{-2t}sin3t + e^t t cost\}$
7.	Find a) $L(te^{2t} - \frac{2sin3t}{t})$ b) $L(3^t + \frac{cos2t - cos3t}{t})$
8.	a) If f (t) is a periodic function of period T, then show that
	$L(f(t)) = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$
	b) Prove that L (f (t)) = $\frac{E}{s}$ tanh $\left(\frac{as}{4}\right)$ where f(t+a)=f(t), given $f(t) = \begin{cases} E & 0 \le t \le \frac{a}{2} \\ -E & \frac{a}{2} \le t \le a \end{cases}$
9.	a) Find the Laplace transform of periodic function $f(t) = \begin{cases} t & 0 \le t \le \pi \\ \pi - t & \pi \le t \le 2\pi \end{cases}$
	b) Find the Laplace transform of a periodic function of period $2\pi/\omega$ is
	defined by $f(t) = \begin{cases} Esin\omega t & 0 \le t \le \pi/\omega \\ 0 & \pi/\omega \le t \le 2\pi/\omega \end{cases}$
10.	a) Find $L[2\delta(t-1) + \cosh 3t\delta(t-2)]$
	b) Find $L\left[\frac{2\delta(t-3)+3\delta(t-2)}{t}\right]$
11.	Find the Inverse Laplace transform
	a) $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$ b) $\frac{s^2}{(s+1)^3}$
12.	Find the Inverse Laplace transform
	a) $\frac{4s+5}{(s-1)^2(s+2)}$ b) $\frac{1}{s(s+1)(s+2)(s+3)}$

13.	Find the Inverse Laplace transform
	a) $\frac{s+1}{(s-1)^2(s+2)}$ b) $\log \frac{s^2+1}{s(s+1)}$
14.	Find the Inverse Laplace transform
	a) $\log\left(1 + \frac{a^2}{s^2}\right)$ b) $\log\left(\frac{s+1}{s-1}\right)$
15.	Find the Inverse Laplace transform
	a) $tan^{-1}\left(\frac{a}{s}\right)$ b) $cot^{-1}\left(\frac{s+a}{b}\right)$
16.	a) Find the Inverse Laplace transform $\cot^{-1}\left(\frac{s}{a}\right)$
	b) Using the convolution theorem, obtain Inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$
17.	Using the convolution theorem, obtain Inverse Laplace transform of
	a) $\frac{1}{(s-1)(s^2+1)}$ b) $\frac{s}{(s^2+a^2)^2}$
18.	Using the convolution theorem, obtain Inverse Laplace transform of
	a) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ b) $\frac{1}{s^3(s^2-1)}$
19.	a) Solve the differential equation using the Laplace transform method.
	$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 5e^{2t} \text{ given that y (0)} = 2, \frac{dy(0)}{dt} = 1$
	b) Solve the differential equation by using the Laplace transform method
	y''' + 2y'' - y' - 2y = 0, y = 1, y'' = 2 = y' at t = 0
20.	a) A particle is moving with damping motion according to the law $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 0$. If the initial position of the particle is at $y = 20$ and the initial speed is 10, find the displacement of the particle at any time t using Laplace transforms.
	b) A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L resistance R. Show that the current at any time t is $\frac{E}{R-aL} \left(e^{-at} - e^{-\frac{Rt}{L}} \right)$

Discrete and Integral Transforms (DIT)

Subject Code: 18MA3GCDIT

Module – 5 (Integral Transform – II)

Questions					
a) Find the Fourier transform of $e^{-a x }$. Where a>0 b) Find the Fourier transform of $e^{-a^2x^2}$, $a>0$ ($-\infty < x < \infty$). Hence prove that $e^{-x^2/2}$ is Self- reciprocal					
a) Find Fourier transform of $f(x) = \begin{cases} x, & x \le \alpha \\ 0, & x > \alpha \end{cases}$ where is a α is a positive constant b) Find the Fourier transform of $xe^{-a x }$. Where a>0					
a) Find Fourier transform of $f(x) = \begin{cases} x & for \ 0 < x < 1 \\ 2 - x & for \ 1 < x < 2 \\ 0 & for \ x > 2 \end{cases}$					
b) Find a Fourier transform of $f(x) = \begin{cases} x^2 & for x < a \\ 0 & for x > a \end{cases}$ Where a is a positive constant.					
a) Find the Fourier transform of $e^{- x }$.					
b) Find the Fourier transform of $f(x) = \begin{cases} -e^x & for \ x < 0 \\ e^{-x} & for \ x > 0 \end{cases}$					
a) Find the Fourier transform $f(x) = \begin{cases} 1, & x \le a \\ 0, & x > a \end{cases}$ a>0 evaluate $\int_0^\infty \frac{\sin ax}{x} dx$					
a) Find the Fourier transform $f(x) = \begin{cases} 1, & x \le a \\ 0, & x > a \end{cases}$ a>0 evaluate $\int_0^\infty \frac{\sin ax}{x} dx$ b) Find a Fourier transform of $f(x) = \begin{cases} 1 - x & for \ x \le 1 \\ 0 & for \ x > 1 \end{cases}$ and evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$					
a) Find a Fourier transform of $f(x) = \begin{cases} 1 - x^2 & for \ x \le 1 \\ 0 & for \ x > 1 \end{cases}$ and evaluate $\int_0^\infty \left(\frac{x cosx - sinx}{x^3}\right) dx$					
b) Find a Fourier transform of $f(x) = \begin{cases} 1 - x^2 & for \ x \le 1 \\ 0 & for \ x > 1 \end{cases}$ and evaluate $\int_0^\infty \frac{\sin x - x \cos x}{x^3} \cos\left(\frac{x}{2}\right) dx$					
a) Find the inverse Fourier transform of $e^{-a u }$ where a>0					
b) Find the inverse Fourier transform of e^{-u^2}					
a) Obtain the Fourier Cosine transform of the function $f(x) = \begin{cases} 4x & for \ 0 < x < 1 \\ 4 - x & for \ 1 < x < 4 \\ 0 & for \ x > 4 \end{cases}$					
b) Find the Fourier Cosine transform of e^{-x^2}					
a) Find Fourier Cosine transformation of $f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & elsewhere \end{cases}$ b) Find the Cosine transform of $f(x) = xe^{-ax}$, $a > 0$					

10)	a)	Find the Fourier Cosine transform of e^{-ax}	· (<i>a</i> >	0 hence find	$\int_{-\infty}^{\infty} \frac{\cos x}{\cos x} dx$
,	aj	i ma the i durier cosme transform of e	, ,	<i>u</i> <u>_</u>	o, nence min	$\int_{0}^{1} \int_{0}^{1} a^{2} + x^{2} u x$

b) Solve Integral equation $\int_0^\infty f(x) \cos ux \, dx = \begin{cases} 1-u, & 0 < u < 1 \\ 0, & u \ge 1 \end{cases}$ Hence deduce that $\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$

a) Solve the integral equation
$$\int_0^\infty f(x) \cos ux \ dx = e^{-u}$$
 and hence show that $\int_0^\infty \frac{\cos x \cdot x}{1+x^2} dx = \frac{\pi}{2} e^{-x}$

- b) Obtain the Fourier Sine transform of the function $f(x) = \begin{cases} x & for \ 0 < x < 1 \\ 2 x & for \ 1 < x < 2 \\ 0 & for \ x > 2 \end{cases}$
- a) Find the finite Fourier Sine transform of f(x) = 2x in $0 \le x \le 4$.
 - b) Find the Fourier Sine transform of e^{-ax} , $a \ge 0$.

a) Find the Fourier Sine transform of the Functions
$$f(x) = \begin{cases} sinx, & 0 < x < a \\ 0, & x \ge a \end{cases}$$

- b) Find Fourier Sine transform of $\frac{1}{x}e^{-ax}$, $x \neq 0$,

 a) Find Fourier Sine transformation of $f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & elsewhere \end{cases}$ b) Find the Fourier Sine transform of $f(x) = xe^{-ax}$, a > 0

15) a) Find the Fourier Sine transform of
$$e^{-x}$$
. Hence prove that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$, $m > 0$

b) Find the inverse Fourier Cosine transform of e^{-2u}

a) Solve the integral equation
$$\int_0^\infty f(x) \sin ux \, dx = \begin{cases} 1 & \text{for } 0 \le u < 1 \\ 2 & \text{for } 1 \le u < 2 \\ 0 & \text{for } u \ge 2 \end{cases}$$

b) Find the inverse Fourier Sine transform of $\frac{1}{u}e^{-au}$ where a>0

17) a) Find the inverse Fourier Cosine transform of
$$e^{-au}$$
 where a>0

b) Show that the inverse Fourier Sine transform of
$$F_s(u) = \frac{1}{u} \left(1 + \cos u \pi - 2\cos \frac{u\pi}{2} \right)$$
 is
$$f(x) = \begin{cases} 1, & 0 \le x \le \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x \le \pi \end{cases}$$

18) a) Find the inverse Fourier Sine transform of
$$\frac{u}{1+u^2}$$
.

b) Employ Convolution theorem to find
$$F(f * g)$$
 given $f(x) = g(x) = e^{-x^2}$

a) Employ Convolution theorem to find
$$F(f * g)$$
 given $f(x) = g(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

b) Using Parseval's identities prove that
$$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

20) a) Using Parseval's identities prove that
$$\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$$

b) An infinite string is initially at rest and that the initial displacement is f(x), $(-\infty < x < \infty)$ Determine the displacement y(x, t) of the string.