Discrete and Integral Transforms (IT)
Subject Code : 18MA3GCDIT
Module – 4 (Integral Transform – I)

Q.No	Question
1.	a) Prove that (i) $L(coshat) = \frac{s}{s^2-a^2}$ (ii) $L(sinat) = \frac{a}{s^2+a^2}$ b) Prove that $L[t^n] = \frac{n!}{s^{n+1}}$, n is a positive integer
2.	Find a) $L(cost\ cos2t\ cos3t)$ b) $L(e^{at} + 2t^n - 3sin3t + 4cosh2t)$
3.	If $L[f(t)] = \bar{f}(s)$, then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[\bar{f}(s)]$ where n is a positive integer.
4.	Find a) $L\{e^{3t}\ sin5t\ sin3t\}$ b) $L(e^{-t}cos^2 4t)$
5.	Find a) $L[t(sin^3 t - cos^3 t)]$ b) $L(t^5 e^{4t} cosh3t)$
6.	Find a) $L(te^{-2t}sin4t)$ b) $L\{e^{-2t}\sin 3t + e^t\ t\ cost\}$
7.	Find a) $L(te^{2t} - \frac{2sin3t}{t})$ b) $L(3^t + \frac{cos2t - cos3t}{t} + tsint)$
8.	a) If f (t) is a periodic function of period T, then show that $L\big(f(t)\big) = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$ b) Prove that L (f (t)) = $\frac{E}{s}\tanh\left(\frac{as}{4}\right)$ where f(t+a)=f(t),given $f(t) = \begin{cases} E & 0 \le t \le \frac{a}{2} \\ -E & \frac{a}{2} \le t \le a \end{cases}$
9.	a) Find the Laplace transform of periodic function $f(t) = \begin{cases} t & 0 \le t \le \pi \\ \pi - t & \pi \le t \le 2\pi \end{cases}$ b) Find the Laplace transform of a periodic function of period $2\pi/\omega$ is defined by $f(t) = \begin{cases} Esin\omega t & 0 \le t \le \pi/\omega \\ 0 & \pi/\omega \le t \le 2\pi/\omega \end{cases}$
10.	a) Prove that $L[\delta(t-a)] = e^{-as}$ b) Find (i) $L\left[\frac{2\delta(t-3)+3\delta(t-2)}{t}\right]$ (ii) $L[2\delta(t-1) + cosh3t\delta(t-2)]$
11.	Find the Inverse Laplace transform a) $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$ b) $\frac{s^2}{(s+1)^3}$
12.	Find the Inverse Laplace transform a) $\frac{4s+5}{(s-1)^2(s+2)}$ b) $\frac{1}{s(s+1)(s+2)(s+3)}$

13.	Find the Inverse Laplace transform a) $\frac{s+1}{(s-1)^2(s+2)}$ b) $\log\frac{s^2+1}{s(s+1)}$
14.	Find the Inverse Laplace transform a) $\log\left(1+\frac{a^2}{s^2}\right)$ b) $\log\left(\frac{s+1}{s-1}\right)$
15.	Find the Inverse Laplace transform a) $\tan^{-1}\left(\frac{a}{s}\right)$ b) $\cot^{-1}\left(\frac{s+a}{b}\right)$
16.	a) Find the Inverse Laplace transform $\cot^{-1}\left(\frac{s}{a}\right)$ b) Using the convolution theorem, obtain Inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$
17.	Using the convolution theorem, obtain Inverse Laplace transform of a) $\frac{1}{(s-1)(s^2+1)}$ b) $\frac{s}{(s^2+a^2)^2}$
18.	Using the convolution theorem, obtain Inverse Laplace transform of a) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ b) $\frac{1}{s^3(s^2-1)}$
19.	a) Solve the differential equation using the Laplace transform method. $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=5e^{2t}$ given that y (0) =2, $\frac{dy(0)}{dt}=1$ b) Solve the differential equation by using the Laplace transform method $y'''+2y''-y'-2y=0, y=1, y''=2=y'\ at\ t=0$
20.	a) A particle is moving with damping motion according to the law $\frac{d^2y}{dt^2}+6\frac{dy}{dt}+25y=0$. If the initial position of the particle is at $y=20$ and the initial speed is 10, find the displacement of the particle at any time t using Laplace transforms. b) A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L resistance R. Show that the current at any time t is $\frac{E}{R-aL}\left(e^{-at}-e^{-\frac{Rt}{L}}\right)$