UNIT 5.

Magnetic forces, materials & Inductance

Steady magnetic field can exert force only on moving charges as we can not expect $\vec{H}$ with stationary charges.

For electrostatics, $\vec{F} = Q\vec{E}$

In magnetostatics; a charged particle moving produces magnetic flux density B & it is found to experience a force which is $\propto^{le}$ to the product of Q, velocity of charge 'v', flux density B & sine of the angle betn $v \times B$. Direction of force is $\perp^{ar}$ to both v & B.

$$\therefore \vec{F} = Q\vec{V} \times \vec{B}$$

The force on a moving particle arising from combined electric & magnetic fields is obtained by superposition

$$\vec{F} = Q\vec{E} + Q\vec{U} \times \vec{B} = Q[\vec{E} + \vec{V} \times \vec{B}]$$

This equation is called Lorentz force equation

The solution to this equ is used in determining electron orbits in the magnetron, proton paths in cyclotron, plasma characteristics in a magnetohydrodynamic (MHD generator or in general charged particle motion in combined electric & magnetic fields.

Prob: The point charge Q = 18 nC has a velocity of 5×10^6 m/s in the direction $a_v = 0.6\vec{a}_x + 0.75\vec{a}_y + 0.3\vec{a}_z$. Calculate the magnitude of force exerted on the charge by the field (a) $B = -3\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$ mTesla

(b) $\vec{E} = -3\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$ KV/m (c) B & E acting together.

Ans

(a) $\vec{F} = Q\vec{v} \times \vec{B}$

$= 18 \times 10^{-9} \times 5 \times 10^{6} [\vec{v} \times \vec{B}]$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0.6 & 0.75 & 0.3 \\ -3\times10^{-3} & 4\times10^{-3} & 6\times10^{-3} \end{vmatrix}$$

$$= (3.3\vec{a}_x - 4.5\vec{a}_y + 4.65\vec{a}_z)10^{-3}$$

$\therefore \vec{F} = 90\times10^{-3} \times 10^{-3} (3.3\vec{a}_x - 4.5\vec{a}_y + 4.65\vec{a}_z]$

$|\vec{F}| = 654 \mu N$.

(b) $\vec{F} = Q\vec{E} = 140 \times 10^{-6} N$

$\therefore$ Total force $= 654 + 140 = 794 \mu N$.

Force on differential current element (Idl)

A differential element of charge is made up of large number of small & discrete charges. The differential force is given by $dF = dQ\,\vec{E}$ N. & $F = Q\vec{E}$ N is the addition of forces on individual charges.

Consider a conductor in which electrons are in motion. Immobile ions also exists in conductor. Electrons in motion forms magnetic field & this field tries to displace centre of gravity of +ve & -ve charges. But this force is opposed by coulomb's forces beth +ve ions & -ve electrons. In conductors, this force is much greater than magnetic force. Thus the seperation of charges is maintained which indicates that small pot diff exists across conductor in a dir $\perp^{lr}$ to the magnetic field intensity as well as velocity of charge. This small voltage across conductor is called Hall effect

* Hall effect is used to measure magnetic flux density
* I is made $\propto^{l}$ to H, Hall effect now can be used to make the device work as electronic wattmeter or squaring device
* In semiconductor physics Hall effect is used to know P & N type materials. (as Hall voltages are diff ~~for~~ both)

we have $d\vec{F} = dQ\, \vec{V} \times \vec{B}$ ~

But $J = \rho_v \vec{V}$, ~~&~~ $dQ = \rho_v dv$

$\therefore d\vec{F} = \rho_v dv\, \vec{v} \times \vec{B}$ But $J = \rho_v \vec{V}$

$= \vec{J} \times \vec{B}\, dv$

W.K.T $\vec{J} dv = \vec{K} ds = I d\vec{l}$

$\therefore d\vec{F} = \vec{K} \times \vec{B}\, ds$

$d\vec{F} = I d\vec{l} \times \vec{B}$ Integrating

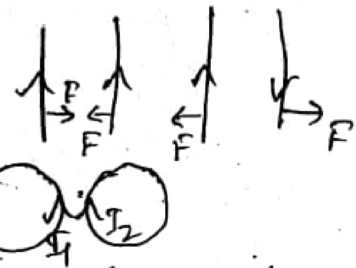
$\vec{F} = \int_v J \times \vec{B}\, dv = \int_s \vec{K} \times \vec{B}\, ds = \oint I d\vec{l} \times \vec{B}$

$\vec{F} = I\vec{L} \times \vec{B}$ $(\because \oint dl = \vec{L})$

or $|\vec{F}| = ILB\sin\theta$

Force between diff current elements

If two current carrying conductors are placed $\parallel^{lel}$ to each other. If currents are flowing in the same direction, the conductors experience force of ~~attracti~~ ~~repulsion~~ attraction & if currents are in opposite direction, then they experience force of repulsion.



Let us consider two current elements $I_1\vec{dL_1}$ & $I_2\vec{dL_2}$ with dir of current same.

Then the force $d(d\vec{F_1})$ exerted on element $I_1\vec{dL_1}$ due to the magnetic field $d\vec{B_2}$ produced by other element $I_2\vec{dL_2}$ is the force of attraction

then $d(d\vec{F_1}) = I_1\vec{dL_1} \times d\vec{B_2}$ [where $d(d\vec{F})$ is differ-ntial amount of diff force]

From Biot savart law

$$d\vec{B_2} = \mu_0 d\vec{H_2} = \mu_0 \left[\frac{I_2\vec{dL_2} \times \vec{a}_{R_{21}}}{4\pi R_{21}^2}\right]$$

substituting $d\vec{B_2}$

$$d(d\vec{F_1}) = \mu_0 \, I_1\vec{dL_1} \times \frac{(I_2\vec{dL_2} \times \vec{a}_{R_{21}})}{4\pi R_{21}^2}$$

By integrating twice

$$\vec{F_1} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1}\oint_{L_2} \frac{\vec{dL_1} \times (\vec{dL_2} \times \vec{a}_{R_{21}})}{R_{21}^2}$$

$|||^{ly}$

$$\vec{F_2} = \frac{\mu_0 I_2 I_1}{4\pi} \oint_{L_2}\oint_{L_1} \frac{\vec{dL_2} \times (dL_1 \times \vec{a}_{R_{12}})}{R_{12}^2}$$

For two current carrying conductors of length l each, the force exerted is given by

$$F = \frac{\mu I_1 I_2 l}{4\pi d}.$$

<u>Prob</u>: Two differential current elements $I_1 \Delta L_1 = 3\times10^{-6}\vec{a}_y$ A.m at $P_1(1,0,0)$ & $I_2\Delta L_2 = 3\times10^{-6}(-0.5\vec{a}_x + 0.4\vec{a}_y + 0.3\vec{a}_z)$ A.m at $P_2(2,2,2)$ are located in free space. Find the vector force exerted on (a) $I_2\Delta L_2$ by $I_1\Delta L_1$ (b) $I_1\Delta L_1$ by $I_2\Delta L_2$

$$d\vec{H}_1 = \frac{I_1 \Delta \vec{L}_1 \times \vec{a}_{R_{12}}}{4\pi(R_{12})^2}$$

$$\vec{R}_{12} = (2-1)\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z$$

$$\therefore |\vec{R}_{12}| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$$

$$\therefore \vec{a}_{R_{12}} = \frac{\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z}{3}$$

$$d\vec{H}_1 = \frac{\cancel{3}\times10^{-6}\vec{a}_y \times (\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z) \times \frac{1}{\cancel{3}}}{4\pi \times 3^2}$$

$$= \frac{10^{-6}}{4\pi\times 9}\left[2\vec{a}_x - \vec{a}_z\right]$$

$$\begin{vmatrix} a_x & a_y & a_z \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} = \vec{a}_x(2) + \vec{a}_z(-1)$$

$$d\vec{B}_1 = \mu_0 d\vec{H}_1 = \frac{\cancel{4}\pi\times10^{-7}\times10^{-6}}{\cancel{4}\pi\times 9}\left[2\vec{a}_x - \vec{a}_z\right]$$

$$= \frac{10^{-13}}{9}\left[2\vec{a}_x - \vec{a}_z\right]$$

$$d(d\vec{F}_2) = I_2\Delta\vec{L}_2 \times d\vec{B}_1$$

$$= \cancel{3}\times10^{-6}(-0.5\vec{a}_x + 0.4\vec{a}_y + 0.3\vec{a}_z)\times\frac{10^{-13}}{\cancel{9}\,3}\left[2\vec{a}_x - \vec{a}_z\right]$$

$$= \frac{10^{-19}}{3}\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -0.5 & 0.4 & 0.3 \\ 2 & 0 & -1 \end{vmatrix} = \frac{10^{-19}}{3}\left[\vec{a}_x[-0.4] - \vec{a}_y[0.5-0.6] + \vec{a}_z[-0.8]\right]$$

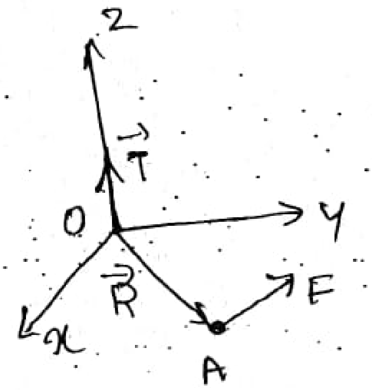
$$= \frac{10^{-19}}{3}\left[-0.4\vec{a}_x + 0.1\vec{a}_y - 0.8\vec{a}_z\right] = \left[-1.33\vec{a}_x + 0.33\vec{a}_y - 2.67\right] \times 10^{-20}$$

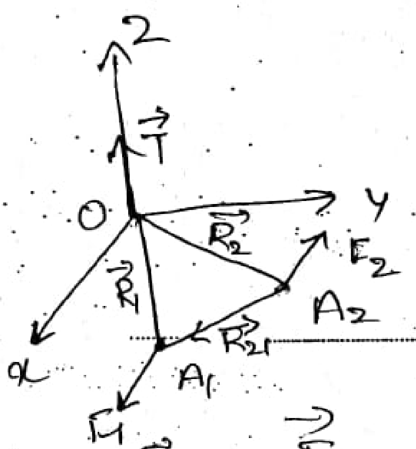
Magnetic Torque (magnetic moment)

Moment of a force or Torque about a specified point is defined as the vector product of the moment arm $\vec{R}$ & force $\vec{F}$. It is measured in N-m.

$$\vec{T} = \vec{R} \times \vec{F} \text{ Nm.}$$

direction of torque is $\perp^{lr}$ to $\vec{R}$ & $\vec{F}$.





Torque is defined w.r.t origin. But when the total force is zero, the torque is independent of the choice of the origin.

If $\vec{F_2} = -\vec{F_1}$ (ie $(F_1 + F_2) = 0$)

$$T = \vec{R_1} \times \vec{F_1} + R_2 \times \vec{F_2}$$

$$= \vec{R_1} \times (\vec{F_1}) - \vec{R_2} \times \vec{F_1}$$

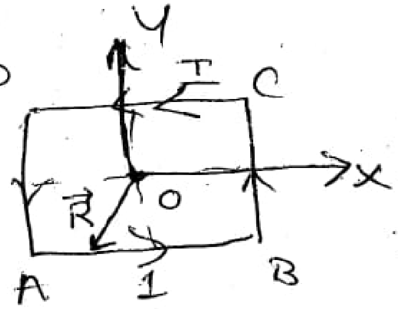
$$= \vec{F_1} \times [\vec{R_1} - \vec{R_2}] = \vec{F_1} \times \vec{R_{21}}$$

$$R_{21} = \vec{R_1} - \vec{R_2}$$

Magnetic moment of a planar coil

Consider a differential current loop of rectangular shape with uniform magnetic field everywhere around it.

Assume that the loop is placed in xy plane. AB ∥ CD ∥ X-axis & AD ∥ CB ∥ Y-axis.



Let Δx & Δy be the lengths of the sides of the rectangular loop.

Let centre of rectangular loop be at origin.

Let B_0 be the magnetic field at O. As differential area is very small, magnetic field can be assumed B_0 everywhere. The total force on the loop is zero & the origin for the torque can be selected as the centre of the loop.

The force exerted on side AB is given by

$$dF_1 = I\,dx\,\vec{a}_x \times \vec{B}_0 \qquad (dF = I\,d\vec{l} \times \vec{B})$$

$$= I\,dx\,[\vec{a}_x \times (B_{0x}\vec{a}_x + B_{0y}\vec{a}_y + B_{0z}\vec{a}_z)]$$

$$= I\,dx\,[B_{0y}\vec{a}_z - B_{0z}\vec{a}_y]$$

For the side AB, the lever arm extends from origin to the midpoint of the side AB.

Lever arm for side AB is given by

$$\vec{R}_1 = \frac{1}{2}dy(-\vec{a}_y) = -\frac{1}{2}dy\,\vec{a}_y$$

∴ Torque on side one is given by

$$d\vec{T}_1 = \vec{R}_1 \times d\vec{F}_1 = -\frac{1}{2}dy\,\vec{a}_y \times I\,dx\,[B_{0y}\vec{a}_z - B_{0z}\vec{a}_y]$$

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$\therefore\ dT_1 = -\frac{1}{2}dx\,dy\, I B_{oy} \vec{a_x}$

Similarly force exerted on side BC is

$dF_2 = I dy\, \vec{a_y} \times B_o$

$= I\,dy \left[\vec{a_y} \times (B_{ox}\vec{a_x} + B_{oy}\vec{a_y} + B_{oz}\vec{a_z})\right]$

$= I\,dy \left[B_{oz}\vec{a_x} - B_{ox}\vec{a_z}\right]$

For BC, lever arm extends from mid point to the side BC then

$\vec{R_2} = \frac{1}{2}dx\,\vec{a_x}$

$dT_2 = \vec{R_2} \times d\vec{F_2} = \frac{1}{2}dx\,\vec{a_x} \times \left[I\,dy(B_{oz}\vec{a_x} - B_{ox}\vec{a_z})\right]$

$= \frac{1}{2}dx\,dy\, I B_{ox}\vec{a_y}$

Torque on CD is same as that of AB & Torque on DA is same as that of BC [though $\vec{R}$ is opp. +ve sign, current also becomes -ve)

$\therefore$ Total torque $= dT_1 + dT_2 + dT_3 + dT_4$

$= -dx\,dy\, I B_{oy}\vec{a_x} + dx\,dy\, I B_{ox}\vec{a_y}$

$= I\,dx\,dy\left[B_{ox}\vec{a_y} - B_{oy}\vec{a_x}\right]$

$= I\,dx\,dy\left[\vec{a_z} \times [B_{ox}\vec{a_x} + B_{oy}\vec{a_y} + B_{oz}\vec{a_z}]\right]$

$= I\,ds(\vec{a_z} \times \vec{B_o})$

$$\boxed{d\vec{T} = I\,d\vec{s} \times \vec{B_o}}$$

Even though the total force exerted on the rectangular loop as a whole is zero, the torque exist along the axis of rotation i.e. in z dir. The exp^n is valid for all loops of any arbitrary shape.

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Magnetic dipole moment

magnetic dipole moment of a current loop is defined as the product of current thro' the loop & the area of the loop, directed normal to the current loop

$$\vec{m} = I\vec{S}\,\hat{a}_n \ Am^2$$

torque along the axis of rotation of a planar coil is

$$dT = I d\vec{s} \times \vec{B}$$

when differential dipole moment is considered

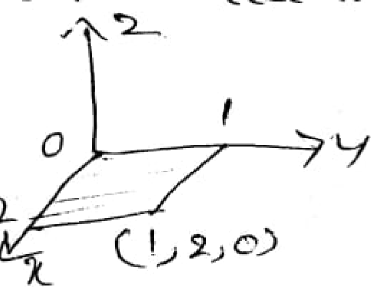
$$d\vec{m} = I d\vec{s}$$

$$\therefore dT = d\vec{m} \times \vec{B}$$

In general $\boxed{T = \vec{m} \times \vec{B} = I\vec{S} \times \vec{B}}$

Prob:

A rectangular coil as shown in fig is in magnetic field given by $B_0 = -0.6a_y + 0.8a_z$ Tesla. Find the torque when the current thro' the loop is 4 mA.



We have $T = I\vec{S} \times \vec{B}$

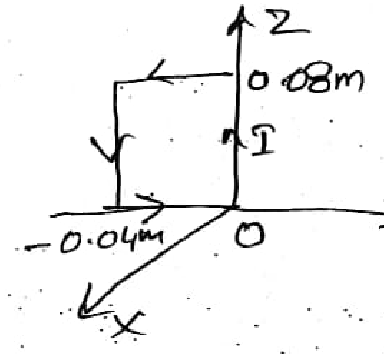
$$= 4\times10^{-3}\, dx\,dy\,\vec{a}_z \times [-0.6\hat{a}_y + 0.8\hat{a}_z]$$

$$= 4\times10^{-3} \times 1 \times 2\,[-0.6(-\hat{a}_x) + 0.8\times 0]$$

$$T = 8\times10^{-3} \times 0.6\,\hat{a}_x = 4.8\times10^{-3}\,\hat{a}_x \ Nm.$$

$$\begin{array}{r} 1375 \\ 687.5 \\ \hline 2062.5 \end{array}$$

A rectangular coil is shown in fig. magnetic field is given by $\vec{B} = \frac{0.05}{\sqrt{2}}(\vec{a_x} + \vec{a_y})$ T. Find torque about z axis when the coil is in the position shown & carries a current of 5A.



$\vec{m} = I S \vec{a_n}$

$S = \text{area} = 0.08 \times 0.04$

$\vec{m} = 5 \times 3.2 \times 10^{-3} m^2 \hat{a}_x$

$\vec{m} = 0.016 \vec{a_x}$ A m^2

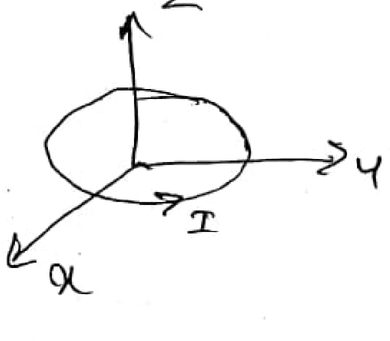
$\vec{T} = \vec{m} \times \vec{B}$

$= 0.016 \vec{a_x} \times 0.05 \left(\frac{\vec{a_x} + \vec{a_y}}{\sqrt{2}} \right)$

$T = 5.6568 \times 10^{-4} \vec{a_z}$ Nm.

A circular loop of radius r & current I lies in z=0 plane. Find the torque which results if the current is in $\vec{a_\phi}$ dir & there is uniform field $\vec{B} = \frac{B_0}{\sqrt{2}}(\vec{a_x} + \vec{a_z})$ Tesla

Let the loop be in z=0 plane.



$\vec{m} = I S \vec{a_n}$

$= I \pi r^2 \vec{a_z}$

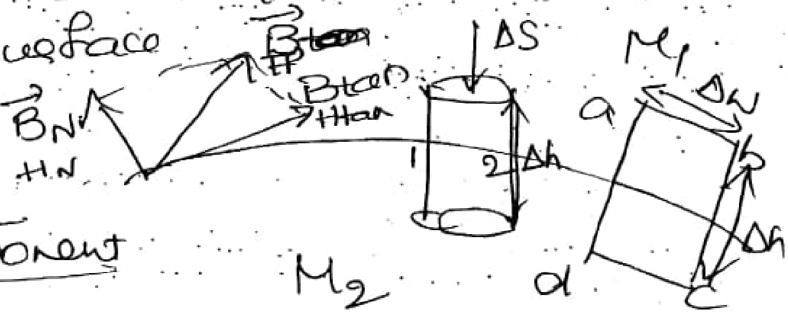
$T = \vec{m} \times \vec{B}$

$= I \pi r^2 \vec{a_z} \times \frac{B_0}{\sqrt{2}}(\vec{a_x} + \vec{a_z})$

$T = \frac{I \pi r^2 B_0}{\sqrt{2}} [\vec{a_y}]$ Nm

Magnetic boundary conditions

When two medias are there & we are considering the magnetic field existing at the boundary, ~~conditions~~ we need to know magnetic boundary conditions. We consider both tangential & normal components of $\vec{H}$ & $\vec{B}$ at the boundary betw two different magnetic materials. Let μ_1 & μ_2 be the permeabilities of two medias. To determine the boundary conditions, let us use the closed path & the Gaussian surface



Boundary conditions for normal component

Consider closed Gaussian surface in the form of a right circular cylinder as shown. $\Delta h \rightarrow$ hight

$\frac{\Delta h}{2} \rightarrow$ placed in medium 1

$\frac{\Delta h}{2} \rightarrow$ —— " ——

Axis of the cylinder is in normal direction to the surface.

From Gauss law $\oint_S \vec{B}\, d\vec{s} = 0$

Surface integral should be evaluated on all 3 surfaces, top, bottom & curved surface

$$\therefore \oint_{top} \vec{B}.\vec{ds} + \oint_{bottom} \vec{B}.\vec{ds} + \oint_{curved} B.ds = 0.$$

As we need boundary values let $\Delta h \to 0$ then we are left with only top & bottom surface. $\left(\oint_{curved} (\vec{B}.\vec{ds}) = 0 \right)$

$$\oint_{top} \vec{B}.\vec{ds} = B_{N_1} \oint_{top} \vec{ds} = B_{N_1} \Delta S$$

$$\oint_{bottom} \vec{B}.\vec{ds} = B_{N_2} \Delta S$$

$$B_{N_1} \Delta S + B_{N_2} \Delta S = 0.$$

As $B_{N_1} = -B_{N_2}$

$$\boxed{B_{N_1} = B_{N_2}}$$

Thus the normal components are continuous at boundary.

$\vec{B} = \mu \vec{H}$ $\therefore \mu_1 H_{N_1} = \mu_2 H_{N_2}$ $\therefore \boxed{\frac{H_{N_1}}{H_{N_2}} = \frac{\mu_2}{\mu_1}}$

$\therefore$ Normal comp. of magnetic field are not continuous at boundary.

<u>Tangential components</u>

According the ampere's circuital law

$$\oint \vec{H}.\vec{dl} = I$$

consider the rectangular closed path indg

$\oint \vec{H}.\vec{dl} = \oint_{AB} ( ) + \int_{BC} ( ) + \int_{cd} ( ) + \int_{da} ( )$ At boundary BC & $\boxed{cd} \to 0$ ad $\Delta h \to 0$.

$\oint_{AB} \vec{H}.\vec{dl} = H_{t_1} \Delta W$ $\oint_{cd} \vec{H}.\vec{dl} = -H_{t_2} \Delta W$

$\vec{\Delta H}$ - $\vec{H_t}$ $H_{t_1} \Delta W - H_{t_2} \Delta W = I = K.\Delta W$ $\therefore H_{t_1} - H_{t_2} = K$ (as $\Delta W \neq \Delta W$)

H_{t_1} AW $H_{t_1} - H_{t_2} = K$

In vector form

$$\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{a}_{N_{12}} \times \vec{K}$$

where $\vec{a}_{N_{12}}$ is a unit vector in the direction normal at the boundary from medium 1 to med 2.

For B, $\frac{B_{tan1}}{\mu_1} = \frac{B_{tan2}}{\mu_2} = K$

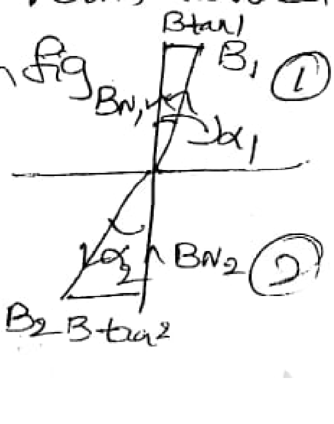
when the boundary is free of current (special case)

$H_{t_1} - H_{t_2} = 0 \quad \therefore H_{t_1} = H_{t_2}$

$|||^{ly}$ $\boxed{\frac{B_{t_1}}{\mu_1} = \frac{B_t}{\mu_2}}$

Hence tangential comp of $\vec{H}$ are continuous while tang comp of $\vec{B}$ are not with the condition that the boundary is current free.

In terms of angles α_1, α_2 with normal to the interface as shown in fig



In med 1 $\tan\alpha_1 = \frac{B_{t1}}{B_{N1}}$

$\tan\alpha_2 = \frac{B_{t2}}{B_{N2}}$.

Dividing $\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{B_{t1}}{B_{N1}} \cdot \frac{B_{N2}}{B_{t2}}$

As $B_{N1} = B_{N2}$

$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{B_{t1}}{B_{t2}} = \frac{\mu_1}{\mu_2}$$

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magnetic dipole moment in the material

1. orbital magnetic dipole moment
2. Electron spin magnetic moment
3. Nuclear spin magnetic moment

The field produced due to movement of bound charges is called magnetization ($\vec{m}$)

$$\vec{m} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_{a=1}^{n\Delta V} \vec{m}_a$$ where m_a = magnetic dipole moment.

Classification of magnetic materials

1) diamagnetic → net magnetic moment = 0 Ex Cu, Si
2) paramagnetic → $m \neq 0$ but random distribution Ex tungsten, potassium
3) ferromagnetic → $m \gg$ - Lineup in $\parallel^{el}$ - Hysteresis Ex iron, nickel
4) Antiferromagnetic → dipoles lineup in anti$\parallel^{el}$, net moment = 0 Ex oxides
5) Ferrimagnetic → antiparallel but net moment ≠ 0 Ex Ferrite
6) Supermagnetic → ferro magnets in which Large magnetic domains exist & they donot overlap due to external field. Ex magnetic tapes.

From boundary relations we have

$$\begin{cases} B_{N_2} = B_{N_1} \\ H_{N_2} = \frac{\mu_1}{\mu_2} H_{N_1} \end{cases}$$ normal comp.

$\therefore \oint H \cdot dl = I$

& $H_{t_1}\Delta L - H_{t_2}\Delta L = I = K \Delta L$

$\therefore H_{t_1} - H_{t_2} = K$

The above eqn gives only magnitude when directions need to be specified, we use cross product

$(\vec{H}_1 - \vec{H}_2) \times \vec{a}_{N12} = \vec{K}$

or $\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{a}_{N12} \times \vec{K}$

& $\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K$

HW

Prob: Let the permittivity be $5 \mu H/m$ in region A where $x<0$ & $20 \mu H/m$ in region B where $x>0$. If there is surface current density $K = 150 \vec{a}_y - 200 \vec{a}_z$ A/m at $x=0$ & if $H_A = 300 \vec{a}_x - 400 \vec{a}_z$ A/m find $|H_{tA}|$, $|H_{NA}|$, $|H_{tB}|$, $|H_{NB}|$

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Let the permittivity be $\mu_1 = 4\,\mu H/m$ in region 1 where $z>0$ while $\mu_2 = 7\,\mu H/m$ in region 2 wherever $z<0$. Let $K = 80\vec{a}_x$ A/m on the surface $z=0$. If the field $B_1 = 2\vec{a}_x - 3\vec{a}_y + \vec{a}_z$ mTesla, in region 1, seek the value of B_2.

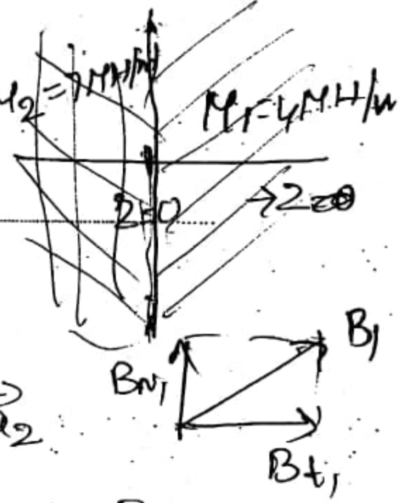
Sol^n: The normal comp of B_1 is

$$B_{N_1} = (B_1 \cdot \vec{a}_{N12})\, a_{N12} = \left[(2\vec{a}_x - 3\vec{a}_y + \vec{a}_z)\cdot(-\vec{a}_z)\right](-\vec{a}_z)$$

$$= \vec{a}_z \text{ mT}$$

At boundary $B_{N_1} = B_{N_2}$

$$B_{N_2} = \vec{a}_z \text{ mT}$$



Tangential comp

$$\vec{B}_{t_1} = \vec{B}_1 - \vec{B}_{N_1}$$

$$= (2\vec{a}_x - 3\vec{a}_y + \vec{a}_z) - \vec{a}_z$$

$$= 2\vec{a}_x - 3\vec{a}_y \text{ mT}$$

$$\vec{H}_{t_1} = \frac{\vec{B}_{t_1}}{\mu_1} = \frac{(2\vec{a}_x - 3\vec{a}_y)\times 10^{-3}}{4\times 10^{-6}}$$

$$= (0.5\vec{a}_x - 0.75\vec{a}_y)\times 10^{3} \text{ A/m}$$

$$\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{a}_{N12}\times\vec{K} = -\vec{a}_z \times 80\vec{a}_x = -80\vec{a}_y$$

$$H_{t_2} = H_{t_1} - \vec{a}_{N12}\times\vec{K} = (0.5\vec{a}_x - 0.75\vec{a}_y)10^{3} + 80\vec{a}_y$$

$$= 500\vec{a}_x - 670\vec{a}_y$$

$$B_{t_2} = \mu_2 H_{t_2} = 7\times 10^{-6}(500\vec{a}_x - 670\vec{a}_y)$$

$$= 3.5\vec{a}_x - 4.69\vec{a}_y \text{ mT}$$

$$\therefore B_2 = B_{N_2} + B_{t_2} = \vec{a}_z + 3.5\vec{a}_x - 4.69\vec{a}_y \text{ mT}$$

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The magnetic circuit

We have already seen that electric circuits are analysed by taking field & pot relations i.e $E = -\nabla V$. In magnetostatic we have similar relations for scalar pot

$$\vec{H} = -\nabla V_m$$

V_m here can also be called as magneto motive force (mmf) $\parallel^{ly}$ to emf for E. potential units of mmf, even though amp, as no of turns of current carrying conductors are used to produce H, V_m ~~unit is taken~~ as ampere turns. mmf is defined only in current free area.

$\parallel^{ly}$ to pot diff $V_{AB} = \int_A^B \vec{E} \cdot \vec{dl}$

mmf can be defined as $V_{mAB} = \int_A^B \vec{H} \cdot \vec{dl} = NI$

$\parallel^{ly}$ to ohms law in point form in magnetostatics i.e $\vec{J} = \sigma \vec{E}$, we have in magnetostatics

$\vec{B} = \mu \vec{H}$ Here flux density is analogous to current density.

I in electrostatics is $I = \int_S \vec{J} \cdot \vec{ds}$ &

ϕ in magnetostatics is $\phi = \int_S \vec{B} \cdot \vec{ds}$

Resistance in Electrostatics is $V = IR$ & in magnetostatics we define "Reluctance" as the ratio of mmf to total flux.

$V_m = \phi R$ where R = Reluctance, At/wb

If σ is conductivity of conductor with surface area S & of length d

$$R = \frac{d}{\sigma S}$$

$|||^{ly}$ for homogeneous magnetic material of length d & uniform cross section S, total reluctance is

$$\mathcal{R} = \frac{d}{\mu S}$$

Permeance is analogous to conductance & $P = \frac{\mu S}{d}$

In electrostatics $\oint E.dl = 0$ gives KCL

In magnetostatics $\oint \vec{H}.\vec{dl} = I_{total}$

Taking N turns coil is there & I is current thro' each coil

$$\oint \vec{H}.\vec{dl} = NI$$

ie in magnetostatics the current carrying coil will surround or link the magnetic ckt. In tracing a magnetic circuit, we shall not be able to identify a pair of terminals at which mmf is applied.

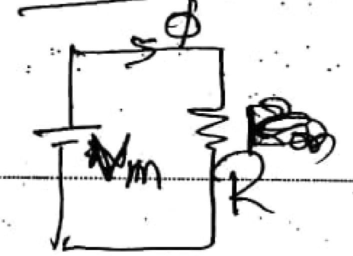
In magnetic circuits, we have Kirchhoff's flux law & mmf law ($|||^{ly}$ to KCL & KVL)

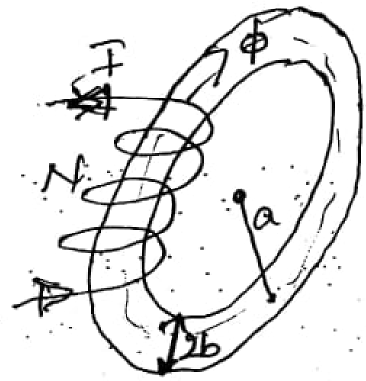
<u>K. Flux law</u> states the total magnetic flux arriving at any junction in a magnetic ckt is equal to the total flux leaving that junction ie $\sum \phi = 0$.

<u>K. mmf</u> law states that the resultant mmf around a closed magnetic ckt is equal to sum of products of flux & reluctance of each part ... $\sum mmf = \sum \phi \mathcal{R} = \sum H.d$

Prob: A toroidal core shown in fig has a mean radius $a = 20cm$ & a circular c/s with $b = 1cm$. If core is made of silicon steel ($\mu_r = 1000$) & has a coil with 200 turns, find the current I that will produce a flux of 0.5 mWb in the core.

Equivalent ckt





Here N_I → shows potential V_m due to N turns of coil, R → Reluctance of the core, ϕ → flux thro' the core

$$V_m = IN = \phi R = \phi \frac{d}{\mu S}$$

$d \rightarrow$ circumfarance $= 2\pi a$

$S \rightarrow$ c/s area $= \pi b^2$

$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1000 = 4\pi \times 10^{-4}$

$$IN = V_m = \frac{0.5 \times 2\pi \times 20 \times 10^{-3} \times 10^{-2}}{4\pi \times 10^{-4} \times \pi \times 1^2 \times (10^{-2})^2}$$

$= 1592\ AT$

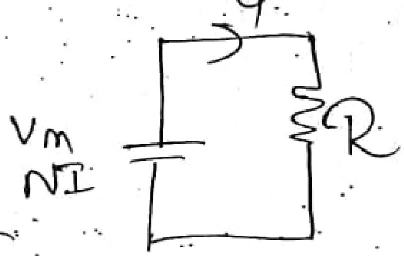
$\therefore I = \frac{1592}{N} = \underline{7.96A}$

If gap of 1cm is there

$V_m = \frac{\phi d}{\mu S} =$ $d = 2\pi a - 1cm$

$V_{mgap} = \frac{\phi \times 1cm}{\mu_0 \times S}$ $\left\{ V_m + V_{mgap} = IN \right.$ $\therefore I = \frac{V_m + H}{N}$

A toroidal ~~core~~ air core is ~~...~~ having 500 turns with c/s area of $6\,cm^2$, a mean radius of 15cm & a coil current of 4A. Find magnetic potential (scalar), Reluctance of core, flux thro' the core, flux density & magnetic field intensity.



$$V_m = NI$$
$$= 500 \times 4 = 2000\ At.$$

$$\mathcal{R} = \frac{l}{\mu S} = \frac{2\pi \times 15 \times 10^{-2}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}}$$
$$= 1.25 \times 10^{9}\ At/wb$$

$$\phi = \frac{V_m}{\mathcal{R}} = \frac{2000}{1.25 \times 10^{9}} = 1.6 \times 10^{-6}\ wb$$

$$B = \frac{\phi}{S} = \frac{1.6 \times 10^{-6}}{6 \times 10^{-4}} = 2.67 \times 10^{-3}\ T$$

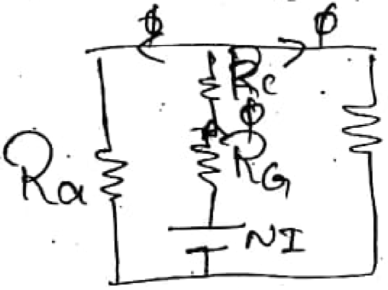
$$H = \frac{B}{\mu_0} = 2120\ At/m.$$

[As a check we can apply ampere's circuital law $\oint H.dl = NI$

ie $H_\phi \cdot 2\pi r = NI$

$$H_\phi = \frac{NI}{2\pi r} = \frac{2000}{2\pi \times 15 \times 10^{-2}} = 2120\ At/m$$

For the magnetic circuit shown below find the current I that will produce a magnetic flux density of 1T in the air gap assuming $\mu_r = 50$ & c/s of core is uniform at 10 cm^2



$R_a \rightarrow$ betⁿ c-b-a

$R_b \rightarrow$ betⁿ c-d-a

$R_c \rightarrow$ c-e + f-a

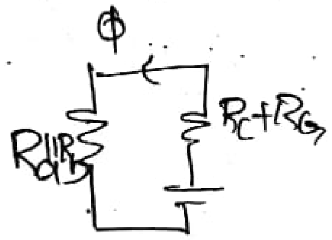
$R_G \rightarrow$ f-e

NI $\rightarrow$ mmf

$$R_a = R_b = \frac{d}{\mu_0 \mu_r S}$$

$$= \frac{0.3}{4\pi \times 10^{-7} \times 50 \times 10 \times 10^{-4}}$$

$$= \frac{3 \times 10^8}{20\pi} \text{ AT/wb}$$



$$R_C = \frac{0.045 \times 2}{\mu_0 \mu_r \times 10 \times 10^{-4}}$$

$$d = \frac{0.1 - 0.01}{2} = \frac{0.09}{2} = 0.045$$

$$R_4 = \frac{0.9 \times 10^8}{20\pi}$$

$$R_G = \frac{0.01}{\mu_0 \times 10 \times 10^{-4}} = \frac{5 \times 10^8}{20\pi}$$

$$\therefore R_T = \frac{R_a R_b}{R_a + R_b} + R_c + R_G = \frac{7.4 \times 10^8}{20\pi}$$

$$\text{NOW } \phi = \frac{NI}{R} = \frac{200 \times I \times 20\pi}{7.4 \times 10^8}$$

$$\phi = BS = 1 \times 10 \times 10^{-4} = \frac{200 \times 20\pi I}{7.4 \times 10^8}$$

$$\therefore \underline{I = 5.889 A}$$

Potential energy & forces on magnetic materials

In electrostatics $W_E = \frac{1}{2}\int_v \vec{D}\cdot\vec{E}\,dv$

$$= \frac{1}{2}\int_v \epsilon E^2\,dv$$

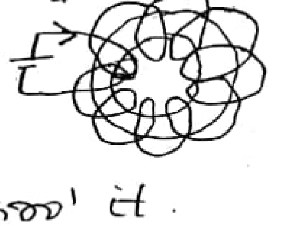
||^{ly} in magnetostatics, assuming magnetic poles, we can write energy eqn

$$W_H = \frac{1}{2}\int_v \vec{B}\cdot\vec{H}\,dv = \frac{1}{2}\int_v \mu \vec{H}^2\,dv = \frac{1}{2}\int_v \frac{\vec{B}^2}{\mu}$$

This result is valid for only linear media. We may also use them to calculate the forces on linear magnetic materials if we focus our attention on the linear media like air.

Inductance & mutual Inductance

When a coil with N turns carrying current I, the flux is produced by it. This flux links with each turn of the coil. Thus total flux linkage of the coil having N turns $N\phi$ Wb-turns



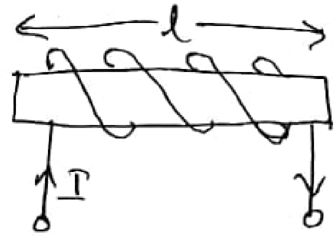
The flux linked with the coil is $\propto$ to the current I flowing thro' it.

∴ The ratio of total flux linkage to the current producing denoted by that flux is called inductance denoted by 'L'. It is measured in Henry (H) $L = \frac{N\phi}{I}$ H

This is also known as self inductance as the flux ... the coil links with ... the current flowing thro' the ...

Inductance of a solenoid

Field intensity inside the solenoid is given by



$$H = \frac{NI}{l}\ A/m.$$

Where $N \to$ No of turns in the coil.

Total flux linkage $= N\phi = NBA$, $A \to$ ~~area~~ C/S area of solenoid

$$= N\mu HA$$

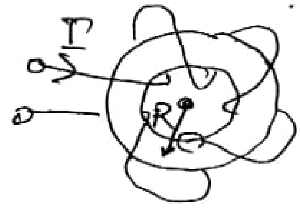
$$= N\mu \frac{NI}{l} A$$

$$= \frac{\mu N^2 IA}{l}$$

Inductance $L = \frac{N\phi}{I} = \frac{\mu N^2 \cancel{I} A}{l \cancel{I}}$

$$\boxed{L = \frac{\mu N^2 A}{l}}$$

Inductance of a toroid



Let No of turns $= N$

Current $= I$

Mean radius of toroid $= R$

The magnetic flux density inside a toroidal ring is given by $B = \frac{\mu NI}{2\pi R}$

$\because B = \mu H$

$HL = NI$

$L = 2\pi R$

Total flux linkage $= N\phi = NBA$

$$= N \times \frac{\mu NI}{2\pi R} \times A$$

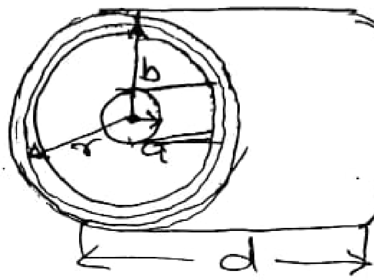
$$L = \frac{N\phi}{I} = \frac{\mu N^2 \cancel{I} A}{2\pi R \cancel{I}} = \frac{\mu N^2 A}{2\pi R}\ H$$

Continue

For a toroid with N turns with hight h, r_1 & r_2 inner & outer radii $L = \frac{\mu N^2 h}{2\pi} \ln \frac{r_2}{r_1}$

Here $A = \pi r^2 \; m^2$ $\therefore L = \frac{\mu N^2 \cancel{\pi} r^2}{2\cancel{\pi} R} = \frac{\mu N^2 r^2}{2R}$

Inductance of a coaxial cable



Inner rad = a
outer rad = b
current thro' the cable = I
the field intensity at any point betn inner & outer conductor is given by

$$H = \frac{I}{2\pi r} \quad a \le r < b$$

$$B = \mu H = \frac{\mu I}{2\pi r}$$

If the axis of the cable is along the z axis, the magnetic flux density will be in radial plane extending from $r = a$ to $r = b$ & $z = 0$ & $z = d$.
ϕ plane

Then $\vec{B} = \frac{\mu I}{2\pi r} \vec{a}_\phi$ Tesla.

Total magnetic flux $\phi = \int_S \vec{B} \cdot d\vec{s}$

$$d\vec{s} = dr\,dz\,\vec{a}_\phi$$

$$\phi = \int_{z=0}^{d} \int_{r=a}^{b} \frac{\mu I}{2\pi r} dr\,dz = \frac{\mu I d}{2\pi} \ln r \Big]_a^b$$

$$= \frac{\mu I d}{2\pi} \ln \frac{b}{a}$$

$$L = \frac{\text{Total flux linkage}}{\text{Total current}} = \frac{\mu \cancel{I} d \ln \frac{b}{a}}{2\pi \times \cancel{I}} = \frac{\mu d \ln \frac{b}{a}}{2\pi}$$

Inductance of coaxial cable may be expressed per unit length $\boxed{\frac{L}{d} = \frac{\mu}{2\pi} \ln \frac{b}{a}}$

But $\vec{A} = \int_{vol} \frac{\mu J}{4\pi R} dv$

$$L = \frac{1}{I^2} \iiint_V \left(\frac{\mu J}{4\pi R} dv\right) \cdot J\, dv$$

If we represent J in terms of I then the volume integral becomes line integral (taking current filament of small cross section for which J dv is replaced by I dl)

ie $L = \frac{1}{I^2} \oint\oint \frac{\mu I}{4\pi R} \vec{dl} \cdot I\vec{dl}$

$= \frac{\mu}{4\pi} \oint\oint \frac{\vec{dl}}{R} \cdot \vec{dl}$

Assuming uniform distribution in a filamentary x section so that J dv becomes I dl

$$L = \frac{1}{I^2} \int A \cdot J\, dv$$

$$= \frac{1}{I^2} \oint A \cdot I\, dl = \frac{1}{I} \oint A \cdot \vec{dl}$$

Applying stokes theorem $\oint A \cdot \vec{dl} = \int_S (\nabla \times A) \cdot \vec{ds}$

$$L = \frac{1}{I} \int_S (\nabla \times \vec{A}) \cdot \vec{ds}$$

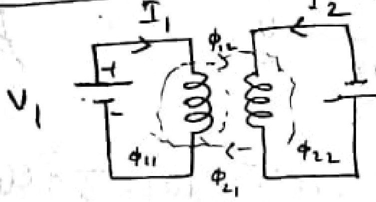
$$= \frac{1}{I} \int_S \vec{B} \cdot \vec{ds} = \frac{\phi}{I}$$

$$L = \frac{\phi}{I}$$

If the filament makes N identical turns about the total flux, an idealization which may be closely realised in some type of inductors

then $L = \frac{N\phi}{I}$

Mutual Inductance



Consider two magnetic ckts with two different coils carrying two different currents as shown.

Let coil 1 be of N_1 turns with inductance L_1 carrying I_1

Magnetic flux produced by I_1 is ϕ_{11}

This ϕ_{11} links with coil 2 to give ϕ_{12}

Likewise I_2 gives ϕ_{22} and links with coil 1 to give ϕ_{21}

Total flux linkage of coil 2 due to I_1 = $N_2 \phi_{12}$

" " " coil 1 " " I_2 = $N_1 \phi_{21}$

Mutual Inductance b/w two coils is ratio of flux linkage of one coil to the current in other coil.

$$M_{12} = \frac{N_2 \phi_{12}}{I_1} \text{ H}$$

$$M_{21} = \frac{N_1 \phi_{21}}{I_2} \text{ H}$$

Force on magnetic materials.

In electromechanical systems, many times, it is required to calculate magnetic force exerted by a magnetic field on a magnetic material.