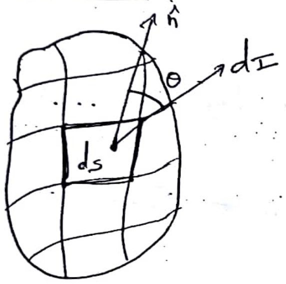
# CONDUCTORS, DIELECTRICS & CAPACITANCE.

**Current:** The amount of charge flowing across a conductor per unit time is called Electric current.

$$I = \frac{dq}{dt}.$$

**Current density:** It is defined as the amount of current flowing through the surface area.



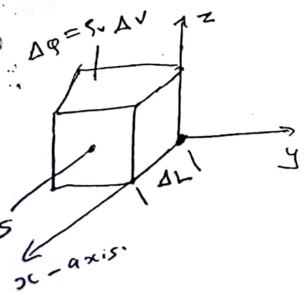

Consider a Elemental surface ds. The amount of current flowing in elemental surface is $dI = J \cdot ds$

where $J$ is current ~~density~~

The total current through the Entire surface is

$$I = \oint J \cdot ds.$$

**Convention current:** This occurs when current flows through an Insulating medium. It does not involve conductors.




$$\Delta I = \frac{\Delta q}{\Delta t}$$

$$\Delta q = \rho_v \Delta V$$
$$= \rho_v \Delta S \Delta L$$
$$= \rho_v \Delta S \Delta x$$

$$\Delta I = \rho_v \Delta S \frac{\Delta x}{\Delta t}$$

$$V_x = \text{velocity} = \frac{\Delta x}{\Delta t}$$

$$\Delta I = \rho_v \Delta S\, V_x$$

$$\boxed{J_x = \frac{\Delta I}{\Delta S} = \rho_v V_x}$$ → Convention current density.

Conduction current : Conduction current flows in a Conductor. A conductor comprises of large quantity of Electrons that provide conduction current due to an Electric field.

The $e^-$ will Expierience a force within field E

$$F = e^- E \quad \text{—— (1)}$$

The current density for any Conductor can be explained in terms of charge density as

$$J = \rho_v V_x \quad \text{—— (2)}$$

$V_x \rightarrow$ velocity.

W.K.T velocity $V_x = -\mu_e E$ —— (3)

substituting Eqn (3) in (2)

~~$J = \mu_e \rho_v V$~~

$$J = \rho_v \mu_e E$$

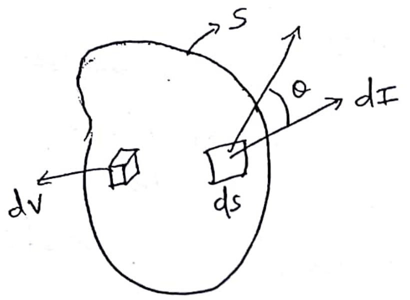
$$\boxed{J = \sigma E}$$ → Ohm's Law

where $\boxed{\sigma = \rho_v \mu_e}$.

Conduction current density.

EQUATION OF CONTINUITY :

To prove $\nabla \cdot J = \dfrac{-\partial \rho_v}{\partial t}$




Consider a small volume dv Enclosed within The surface S. If ds is The small Elemental surface. The amount of current through ds is

$$dI = J \cdot ds$$

The total current over The Entire Surface S,

$$I = \oint_S J \cdot ds \quad \text{——} \ (1)$$

If $\rho_v$ is The charge density over The volume dv, The charge inside The volume can be Expressed as

$$Q = \int_v \rho_v \, dv$$

If There is outward flow of charges through The surface, Then There is decrease in charge inside The volume given by

$$I = -\frac{dQ}{dt}$$

$$= -\frac{d}{dt}\left[\int_v \rho_v \, dv\right]$$

$$I = -\int_v \frac{\partial \rho_v}{\partial t} dv \quad \text{——} \ (2)$$

Equating (1) & (2)

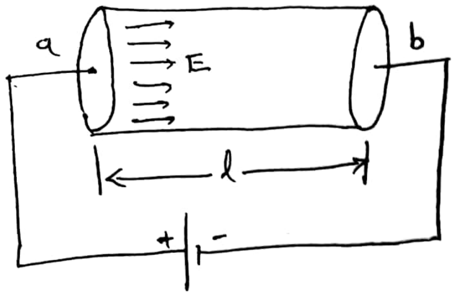
$$\oint_S J \cdot ds = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

⇓ converting into volume Integral

$$\int_v (\nabla \cdot J) dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$ proved.

## Resistance of a Conductor :




Consider a cylindrical surface of length $l$ connected to a battery. The Electric field Exists in The cylinder.

The Resistance is given by, $R = \frac{V}{I}$ — (1)

$$V = -\int_b^a E \cdot dl = -\int_b^a E_x \cdot dl$$

$$\boxed{V = -E_x(l)} \quad \text{— (2)}$$

$$I = \oint J \cdot ds$$

by ohm's Law

$J = \sigma E$.

$$I = \oint_s \sigma E \cdot ds \quad \text{if current is continous}$$

$$= \sigma E_x \int_s ds$$

$$\boxed{I = \sigma E_x S} \quad \text{— (3)}$$

using (2) & (3) in Eqn (1)

$$R = \frac{\cancel{E_x} l}{\sigma \cancel{E_x} S}$$

$$\boxed{R = \frac{l}{\sigma S} \ \Omega.}$$

problems:



1. Find E & J corresponding to a drift velocity of $6 \times 10^{-4}$ m/s in case of silver conductor using the data.

$$\sigma_{silver} = 61.7 \times 10^{6} \ S/m$$

$$\mu_{silver} = 5.6 \times 10^{-3} \ m^3/v\text{-}s.$$

soln:

$$V_x = \mu_e E$$

$$E = \frac{V_x}{\mu} = \frac{6 \times 10^{-4}}{5.6 \times 10^{-3}} = 0.107 \ V/m$$

$$J = \sigma E$$

$$= 61.7 \times 10^{6} \times 0.107$$

$$J = 6.6019 \times 10^{6} \ A/m^2.$$

2. In a certain region of free space, The electric potential is found to be function of x only & is given by $V = 150 x^{4/3}$ volts for $x > 0$. Find

(i) E, D & $\rho_v$ as function of x.

(ii) $J_x$ at $x = 0$ and 1m, if velocity $V_x = 3 \times 10^{6} x^{2/3}$ m/s.

Soln:

$$E = -\nabla V = -\frac{\partial V}{\partial x} \hat{a_x} - \frac{\partial V}{\partial y} \hat{a_y} - \frac{\partial V}{\partial z} \hat{a_z}.$$

$$= -200 x^{1/3} \hat{a_x} \ (V/m)$$

$$D = \epsilon_0 E$$

$$= -1.77 x^{1/3} \hat{a_x} \ nC/m^2$$

$$\rho_v = \nabla \cdot D$$

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= -0.59 x^{-2/3} \ nc/m^3 .$$

(ii)

$$J_x = \rho_v V_x$$

$$= -0.59 x^{-2/3} \times 3 \times 10^6 x^{2/3} \times 10^{-9}$$

$$= -1.77 \ (mA/m^2)$$

(3) Inside the region, $1 < \rho < 5$ (cm), $0 < \phi < 0.3\pi$, $0 < z < 2$ (cm), The current density is given as

$$J = \frac{200 \cos^3\phi}{\rho + 0.01} a_\phi \ A/m^2 .$$

what is the total current in $a_\phi$ direction that crosses the surface: $\phi = 0$, $1 < \rho < 5$ cm & $0 < z < 2$ cm?

Soln:

$$J_\phi = \frac{200}{\rho + 0.01}$$

$$I = \int_{\rho=0.01}^{0.05} \int_{z=0}^{0.02} J_\phi a_\phi \cdot d\rho \, dz \, a_\phi .$$

$$= \int_{0.01}^{0.05} \int_{0}^{0.02} J_\phi \, d\rho \, dz$$

$$= \int_{\rho=0.01}^{0.05} \int_{z=0}^{0.02} \frac{200}{\rho + 0.01} d\rho \, dz$$

$$= 200 \times 0.02 \int_{0.01}^{0.05} \frac{1}{\rho + 0.01} d\rho \quad = \underline{4.39 A}$$

4) The current density due to flow of charges in a very small region in the vicinity of the origin is given by $J = J_0 [x^2 a_x + y^2 a_y + z^2 a_z]$ A/m², where $J_0$ is constant. Find the time rate of increase of charge density at each of following points:

(a) (0.02, 0.01, 0.01) (b) (0.02, −0.01, −0.01) (c) (−0.02, −0.01, 0.01)

Soln:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} = -(\nabla \cdot J)$$

$$= -\left[\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right]$$

$$= -[2xJ_0 + 2yJ_0 + 2zJ_0]$$

(a) $\frac{\partial \rho_v}{\partial t} = -0.08 J_0$ at (0.02, 0.01, 0.01)

(b) $\frac{\partial \rho_v}{\partial t} = 0$ at (0.02, −0.01, −0.01)

(c) $\frac{\partial \rho_v}{\partial t} = 0.04 J_0$ at (−0.02, −0.01, 0.01)

5) (i) Find the total current crossing the surface $z = 3$, $\rho < 6$ in the $a_z$ direction, if the current density in that region is given as

$$J = \left(\frac{100}{\rho^2}\right) a_\rho + \left[\frac{10}{\rho^2 + 1}\right] a_z \ \text{A/m}^2.$$

(ii) Also find $\frac{\partial \rho_v}{\partial t}$

$$J = \left(\frac{100}{\rho^2}\right) a_\rho + \left(\frac{10}{\rho^2+1}\right) \hat{a_z} \quad A/m^2$$

$$I = \iint J \cdot ds = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{6} J_z\, a_z \cdot ds\, a_z$$

$$I = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{6} \frac{10}{\rho^2+1} \underbrace{\rho\, d\rho\, d\phi}_{ds}$$

$$= 2\pi \times 10 \int_0^6 \frac{1}{\rho^2+1} \rho\, d\rho$$

$$= 2\pi \times \frac{1}{2} \times 10 \; \ln(\rho^2+1)\Big|_0^6$$

$$= 113.4 \; A$$

(b)

$$\nabla \cdot J = \frac{-\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot J$$

$$= -\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho J_\rho) + \frac{1}{\rho}\frac{\partial J_\phi}{\partial \phi} + \frac{\partial J_z}{\partial z}\right]$$

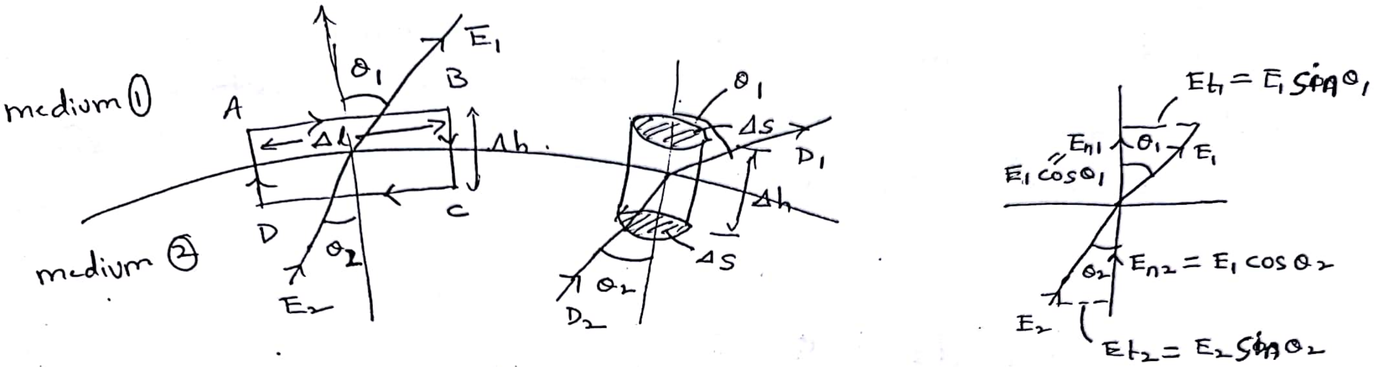
$$= \frac{-1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{100}{\rho}\right) - 10\frac{\partial}{\partial z}\left(\frac{1}{\rho^2+1}\right)$$

$$= -\frac{1}{\rho} \times -\frac{100}{\rho^2}$$

$$\nabla \cdot J = \frac{100}{\rho^3} \quad C/m^3\text{-}s$$

# Boundary Conditions

Case (i) Dielectric – Dielectric Interface.




(a) Tangential components: Consider two dielectric mediums ① & medium ②. Let ABCD be a small rectangular contour at the ~~surface of~~ interface of 2 mediums. The electric field is passed from medium ② to medium ① which abruptly changes the direction of E in medium ①.

The work done in moving in carrying a unit positive charge along countour ABCD is 0

$$\int_{ABCD} E \cdot dl = 0$$

$$\int_A^B E \cdot dl + \int_B^C E \cdot dl + \int_C^D E \cdot dl + \int_D^A E \cdot dl = 0$$

Assuming $\Delta h \rightarrow 0$.

$$\int_A^B E \cdot dl + \int_C^D E \cdot dl = 0.$$

$$E_{t1} \Delta l - E_{t2} \Delta l = 0$$

$$E_{t1} = E_{t2} \quad V/m \quad \text{——} \; (1)$$

This shows tangential components are Equal.

$$\frac{D_{t1}}{\epsilon_1} = \frac{D_{t2}}{\epsilon_2} \quad \text{—— (2)}$$

Normal Components:

Consider a small pill box of gaussian surface extending over small height across the surface. If flux density flows from medium (2) to medium (1).

we have

$$\oint D \cdot ds = Q \qquad \text{Assuming } \Delta h \to 0$$

$$\oint D \cdot ds = \int_{Top} D \cdot ds + \int_{bottom} D \cdot ds = Q$$

$$\oint_S D \cdot ds = \int_T D_{n1}\, \hat{a_N} \cdot ds\, \hat{a_N} + \int_B D_{n2}\, \hat{a_N} \cdot (-ds)\, \hat{a_N} = Q$$

$$D_{n1} \Delta S - D_{n2} \Delta S = \rho_s \Delta S$$

$$D_{n1} - D_{n2} = \rho_s$$

$$\boxed{\rho_s = 0} \rightarrow \because \text{dielectric medium}$$

$$D_{n1} = D_{n2} \quad \text{—— (3)}$$

$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2} \quad \text{—— (4)}$$

By Equation (4) Normal components are not continous across boundary.

Dividing Eqn (1) by Eqn (4)

$$\frac{E_{t1}}{\epsilon_1 E_{n1}} = \frac{E_{t2}}{\epsilon_2 E_{n2}}$$

$$\frac{\cancel{E_1} \sin\theta_1}{\epsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\epsilon_2 \cancel{E_2} \cos\theta_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

$$|D_2| = \sqrt{D_{t2}^2 + D_{n2}^2}$$

$$= \sqrt{\left(\frac{\epsilon_2}{\epsilon_1} D_{t1}\right)^2 + D_{n1}^2}$$

$D_{t1} = D_1 \sin\theta_1$

$D_{n1} = D_1 \cos\theta_1$

$$\boxed{D_2 = D_1 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \sin^2\theta_1 + \cos^2\theta_1}}$$

$\because \quad D = \epsilon E$

$$\epsilon_2 E_2 = \epsilon_1 E_1 \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \sin^2\theta_1 + \cos^2\theta_1}$$

$$\boxed{E_2 = E_1 \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)^2 \cos^2\theta_1 + \sin^2\theta_1}}$$

Case (ii) Conductor – Dielectric medium:

(a) Tangential Components: Same derivation.

$$E_{t1} = E_{t2} \quad \text{—} \ (1)$$

$\because$ There Exists no Electric field inside Conductor

$E_2 = 0$ i.e., $E_{t2} = 0$.

Using This in (1)

$$\boxed{E_{t1} = 0}$$

(b) Normal Components: Same derivation

we have $D_{n1} - D_{n2} = \rho_s$

$\because \quad E_2 = 0, \ D_2 = 0$ & $D_{n2} = 0$

$D_{n1} - 0 = \rho_s$

$$\boxed{D_{n1} = \rho_s}$$

(1) The $z=0$ plane defines the boundary between free space & a dielectric medium with a dielectric constant of 20. The E field next to the interface in free space is $E = 10a_x + 20a_y + 40\hat{a_z}$ V/m. Determine E on the other side of the boundary.

Soln:

Let $z>0$ be medium (1) → dielectric

$z<0$ be medium (2) → free space

Then $E_2 = 10a_x + 20a_y + 40a_z$ V/m.

$z=0$ plane means $x$ & $y$ are tangential

& $z$ → normal comp.

∴ tangential fields are continous.

$E_{x1} = E_{x2} = 10$ V/m

$E_{y1} = E_{y2} = 20$ V/m.

Normal components

$\epsilon_1 E_{z1} = \epsilon_2 E_{z2}$

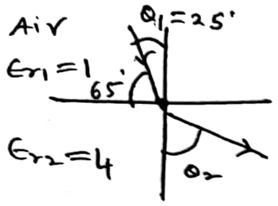
$E_{z1} = \frac{\epsilon_2}{\epsilon_1} E_{z1} = \frac{1}{20} \times 40 = 2$ V/m.

∴ $E_1 = E_{x1}\hat{a_x} + E_{y1}\hat{a_y} + E_{z1}\hat{a_z}$

$$\boxed{E_1 = 10\hat{a_x} + 20\hat{a_y} + 2\hat{a_z} \text{ V/m}}$$

(2) ~~At The~~ An Electric field strength 1.2 V/m is Entering medium of $\epsilon_r = 4$ from air. The orientation of the Electric field in air is 65° w.r.t the boundary. Determine the orientation of Electric field in dielectric medium & its strength in dielectric medium.

soln:




$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$\frac{\tan 25}{\tan\theta_2} = \frac{1}{4}$$

$$\theta_2 = 61.8°.$$

$$E_2 = E_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)^2 \cos^2\theta_1 + \sin^2\theta_1}$$

$$\boxed{E_2 = 0.575\ V/m.}$$

# CAPACITANCE :

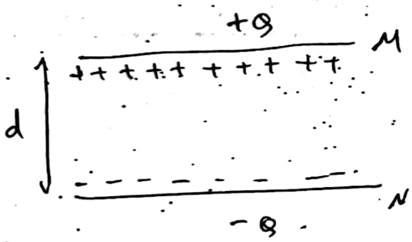
If a battery is connected between two metallic conductors. For a voltage $V_0$, applied between two conductors, resulting in a total free charge $Q$ stored on conductors of capacitor, Then capacitance is given by

$$C = \frac{Q}{V_0} \text{ farads.}$$

$$V = -\int E \cdot dl$$

$$Q = -\int_s \rho_s \, ds.$$

case (i). parallel plate capacitor:




Consider two $\parallel^{le}$ plates ~~M & N~~ each of area $A$ carrying equal & opposite charge seperated by distance $d$. The field $E$ b/w two plates is uniform. If $\rho_s$ is The charge density Then

$$Q = \rho_s A \quad \text{—— (1)}$$

Let $V$ be The p.d b/w two plates, By definition of $V$ → work done in moving +ve charge from $N$ to $M$

$$V = -\int_0^d E \, dr \quad \text{—— (2)}$$

W.K.T $E$ due to sheet charge

$$E = \frac{\rho_s}{2\epsilon}$$

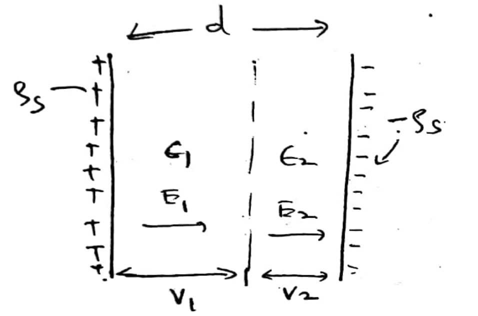
∵ Two plates for 2 sheets, $E = \cancel{2}\left(\frac{\rho_s}{\cancel{2}\epsilon}\right) = \frac{\rho_s}{\epsilon}$

$\therefore$ Using This in Eqn (2)

$$V = -\int_0^d \frac{\rho_s}{\epsilon} dr = -\frac{\rho_s d}{\epsilon}. \quad \text{--- (3)}$$

$$\text{But} \quad C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\epsilon}} = \frac{\epsilon A}{d}.$$

Case (ii) <u>two different dielectric mediums:</u>




Consider the capacitor seperated by two dielectric mediums $\epsilon_1$ & $\epsilon_2$. The flux density D will be same in both dielectrics,

$$D = \epsilon_1 E_1 = \epsilon_2 E_2 .$$

The p.d V between plates is given by.

$$V = V_1 + V_2$$

$$V = E_1 d_1 + E_2 d_2$$

$$= \frac{D}{\epsilon_1} d_1 + \frac{D}{\epsilon_2} d_2$$

$$V = D\left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right]$$

The flux density D is Expressed as $D = \frac{\psi}{A} = \frac{Q}{A}$

$$V = \frac{Q}{A}\left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right]$$

$$\frac{V}{Q} = \frac{1}{A}\left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right]$$

$$C = \frac{Q}{V} = \frac{A}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}}.$$

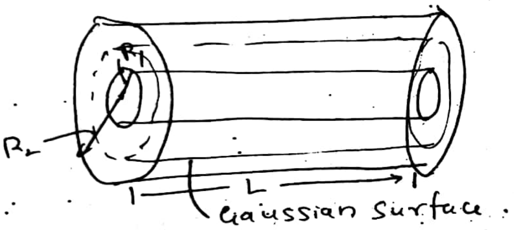
Extending for n dielectrics.

$$c = \frac{A}{\sum_{i=1}^{n} d_i/\epsilon_i}$$

$\boxed{\text{or}}$

$$c = \frac{\epsilon_0 A}{\sum_{i=1}^{n} d_i/\epsilon_{ri}}$$

(iii) <u>Two Concentric cylinders:</u>




consider concentric cylinder with inner radius $R_1$ & outer radius $R_2$. ~~$\rho_L$~~ ~~The charge~~

we have, $c = \frac{Q}{V}$ — ①

If $\rho_L$ is The charge density of inner cylinder.

$Q = \rho_L \times L$ — ②

If V is p.d b/w two cylinders, By definition.

$$V = -\int_{R_2}^{R_1} E\, dr. \quad — ③$$

w.K.T E due to inner ~~solid~~ cylinder at Gaussian surface radius 'r'

$$E = \frac{\rho_L}{2\pi\epsilon r}$$

using This in Eqⁿ ③

$$V = + \int_{R_1}^{R_2} \frac{\rho_L}{2\pi\epsilon r} dr = \frac{\rho_L}{2\pi\epsilon} \int_{R_1}^{R_2} \frac{1}{r} dr$$

$$V = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{R_2}{R_1}\right) \quad \text{--- (4)}$$

subsituting (2) & (4) in (1)

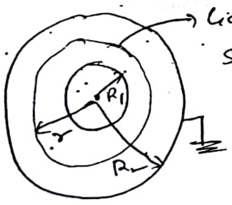
$$c = \frac{Q}{V} = \frac{\cancel{\rho_L} L}{\frac{\cancel{\rho_L}}{2\pi\epsilon} \ln(R_2/R_1)}$$

$$c = \frac{2\pi\epsilon L}{\ln(R_2/R_1)} \quad \text{farads}$$

$$\frac{c}{L} = \frac{2\pi\epsilon}{\ln(R_2/R_1)} \quad \text{farads.}$$

c/unit Length

iv) Concentric spherical shells :




Consider a concentric spherical shell with inner radius $R_1$ & outer Radius $R_2$. We have capacitance

$$c = \frac{Q}{V} \quad \text{--- (1)}$$

V is p.d b/w two shells, By defn, amt of w.D in moving charge from $R_2$ to $R_1$ is

$$V = - \int_{R_2}^{R_1} E \, dr \quad \text{--- (2)}$$

If Q is charge on inner sphere, Then E at a distance r (Gaussian surface) is given by

$$E = \frac{Q}{4\pi\epsilon r^2}$$

using This in Eqn (2)

$$V = -\int_{R_2}^{R_1} \frac{Q}{4\pi\epsilon r^2} dr$$

$$= -\frac{Q}{4\pi\epsilon}\int_{R_2}^{R_1} \frac{1}{r^2} dr$$

$$= \frac{-Q}{4\pi\epsilon}\left[-\frac{1}{r}\right]_{R_2}^{R_1} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad — (3)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon}\left[1/R_1 - 1/R_2\right]}$$

$$\boxed{C = \frac{4\pi\epsilon}{1/R_1 - 1/R_2} \text{ farads}}$$

problems:

~~Energy~~

(1) A $\parallel^{le}$ plate capacitor consists of 3 dielectric layers:

if $\epsilon_1 = 1$, $d_1 = 0.4mm$.

$\epsilon_2 = 2$, $d_2 = 0.6mm$, ~~= 20 cm~~

$\epsilon_3 = 3$, $d_3 = 0.8mm$.

and area of cross-section $= 20\ cm^2$.

find its capacitance $C$.

soln: we have

$$C = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}} + \frac{d_3}{\epsilon_{r3}}}$$

$$= \frac{8.854\times10^{-12} \times 20\times10^{-4}}{\frac{0.4\times10^{-3}}{1} + \frac{0.6\times10^{-3}}{2} + \frac{0.8\times10^{-3}}{3}}$$

$$\boxed{C = 18.32pf}$$

(2) Determine The capacitance of a capacitor consisting of two parallel plates 30 cm x 30 cm, surface area, seperated by 5 mm in air. what is The total Energy stored by The capacitor if capacitor is charged to p.d of 500V? what is The Energy density.

soln:

$$A = 30 \times 30 \times 10^{-4}\ m^2$$

$$d = 5\,mm = 5 \times 10^{-3}\ m$$

$$V = 500\ V$$

$$c = \frac{\epsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 30 \times 30 \times 10^{-4}}{5 \times 10^{-3}}$$

$$c = 159\ pf$$

Energy in capacitor,

$$w = \frac{1}{2} cv^2 = \frac{1}{2} (159 \times 10^{-12}) (500)^2$$

$$w = 1.988 \times 10^{-5}\ J$$

Energy density,

$$w = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left(\frac{v}{d}\right)^2$$

$$= 0.04427\ J/m^3$$

(3) A spherical condensor has capacity of 54 pf. It consists of two concentric spheres differing by radii by 4 cm. & having air as dielectric. Find Their radii.

soln:

$$c = 54 \times 10^{-12}\ F$$

$$R_2 - R_1 = 0.04 m$$

$$\epsilon_r = 1.$$

we have

$$c = \frac{4\pi\epsilon}{1/R_1 - 1/R_2} = \frac{4\pi\epsilon_0\epsilon_r R_1 R_2}{R_2 - R_1}$$

$$R_1 R_2 = \frac{c(R_2 - R_1)}{4\pi\epsilon_0\epsilon_r} = \frac{54\times10^{-12}\times4\times10^{-2}}{4\pi\times8.854\times10^{-12}\times1}$$

$$R_1 R_2 = 0.0194$$

we have $R_2 - R_1 = 0.04$

x'ly by $R_2$

$$R_2^2 - R_1 R_2 = 0.04 R_2$$

$$R_2^2 - 0.04 R_2 - R_1 R_2 = 0$$

$$R_2^2 - 0.04 R_2 - 0.0194 = 0$$

$$R_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\boxed{R_2 = 0.16\,m}$ ~~$R_1 = 0.04+0.16$~~

~~$R_1 = 0.04 - 0$~~

$$R_1 = 0.16 - 0.04 = 0.12\,m.$$

$\boxed{R_1 = 0.12\,m}$