FT-II UNIT

UNIT-2

ENERGY AND POTENTIALS.

CAPACITANCES

20 pages (1)

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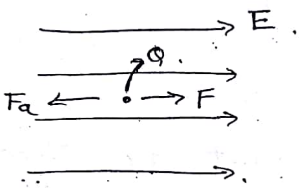
Consider a point charge $+Q$ placed in an electric field. The force exerted on this point charge due to electric field is given by

$$F = Q\vec{E}.$$



The component of force against the electric field is

$$F_a = -Q\vec{E}.$$

The differential amount of work done in moving the charge through distance dl is the product of force & displacement.

$$dw = F_a \cdot dl$$

$$dw = -Q\vec{E} \cdot dl$$

The total workdone from initial to final point is

$$W = -Q \int_{initial}^{final} \vec{E} \cdot dl.$$

~~If~~ W ~~is~~ is positive then work is done against the field.

If W is negative then the field is doing the work.

Potential Difference :

Work done in moving the unit +ve charge from point B to point A.

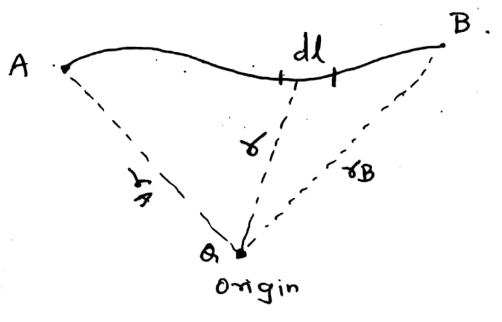
$$V_{AB} = -\int_B^A E \cdot dl.$$

potential :

work done in bringing unit +ve charge from ∞ to the point.

$$V_A = -\int_\infty^A E \cdot dl.$$

Potential of a point charge :



Consider path AB where point charge Q is moved from B to A in presence of Electric field. The amount of work done in moving a charge by distance 'dl'

$$dW = -QE \cdot dl.$$

The total work done from point 'B' to poin

$$W_{AB} = -Q\int_B^A E \cdot dl \quad \text{Joules.} \quad \text{—}$$

Dividing W_{AB} by Q gives potential di
or Energy per unit charge.

$$\frac{W_{AB}}{Q} = V_{AB} = -\int_{B}^{A} E \cdot dl \quad \text{Joules/Columb or volt.} \quad — (2)$$

We know that Electric field is due to point charge Q placed at origin.

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2} \hat{a_r}.$$

Substituting for $\vec{E}$ in Eqn (2)

$$V_{AB} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon r^2} \hat{a_r} \cdot dr\, \hat{a_r}.$$

$$= \frac{-Q}{4\pi\epsilon} \int_{r_B}^{r_A} \frac{1}{r^2}\, dr$$

$$= \frac{-Q}{4\pi\epsilon} \left[\frac{-1}{r}\right]_{r_B}^{r_A}$$

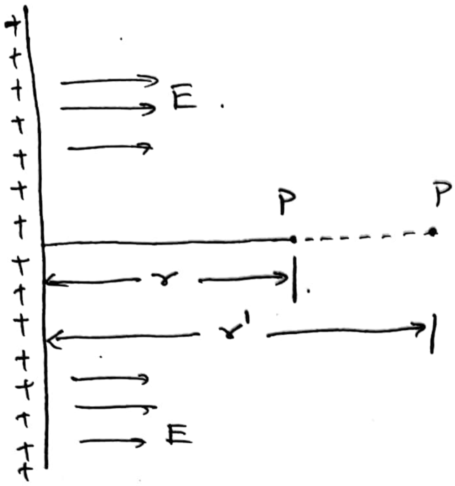
$$V_{AB} = \frac{Q}{4\pi\epsilon} \left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

By Definition of potential, $r_B = \infty$ & $Q = 1$.

$$\boxed{V_A = \frac{Q}{4\pi\epsilon r_A} \quad \text{volt.}}$$

Potential at point 'A':

Potential of a line charge :



Consider a uniform line charge with charge density ρ_l. Let 'p'' be a point at distance 'r'' from the line charge. The differential work against the field is given by

$$dw = -Q\vec{E} \cdot dl \quad \text{--- (1)}$$

If p' is a point at distance r'. The work done in moving a charge from p' to p is

$$W = -Q \int_{r'}^{r} E \cdot dl \quad \text{--- (2)}$$

W.K.T Electric field intensity of line charge $\vec{E} = \frac{\rho_l}{2\pi\epsilon r} \hat{a_r}$.

substituting This in (2)

$$W = -Q \int_{r'}^{r} \frac{\rho_l}{2\pi\epsilon r} \hat{a_r} \cdot \hat{a_r} dr$$

$$= \frac{-Q\rho_l}{2\pi\epsilon} \left[\ln r \right]_{r'}^{r}$$

$$W = \frac{Q\rho_l}{2\pi\epsilon} \ln \frac{r'}{r}$$

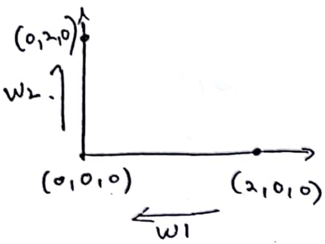
if $Q=1$ Then $W=V$

$$\boxed{V = \frac{\rho_l}{2\pi\epsilon} \ln \left[r'/r \right]}$$

Problems

1. Find The work done in moving a point charge of $+2c$ (i) from $(2,0,0)$ to $(0,0,0)$ & Then from $(0,0,0)$ to $(0,2,0)\,m$. (ii) from $(2,0,0)$ to $(0,2,0)\,m$. Given $\vec{E} = 2x\,\hat{a}_x - 4y\,\hat{a}_y$ V/m.

As



(i) $W = W_1 + W_2$.

$dW_1 = -Q\,\vec{E}\cdot\vec{dL}$

$dW_1 = -2\,(2x\,\hat{a}_x)\cdot(\hat{a}_x\,dx)$

$W_1 = -2\int_2^0 2x\,dx = 8J$.

$dW_2 = -Q\vec{E}\cdot dL$.

$= -2(-4y\,\hat{a}_y)\cdot\hat{a}_y\,dy$.

$W_2 = -2\int_0^2 -4y\,dy = 16J$

$W = W_1 + W_2 = 24J$

(ii) $(2,0,0)$ to $(0,2,0)\,m$.

$dW = -Q\vec{E}\cdot dL$.

$= -2\left(2x\,\hat{a}_x - 4y\,\hat{a}_y\right)\cdot\left(\hat{a}_x\,dx + \hat{a}_y\,dy\right)$

$$W = -2\int_{x=2}^{0}\int_{y=0}^{2}\left(2x\,\hat{a}_x - 4y\,\hat{a}_y\right)\cdot\left(\hat{a}_x\,dx + \hat{a}_y\,dy\right)$$

$$\cancel{W = -2\int_{x=2}^{0}\int_{y=0}^{2}(2x\,dx - 4y\,dy)}$$

$$W = -2\left[\int_{x=2}^{0}2x\,dx - \int_{y=0}^{2}4y\,dy\right] = -2(-12) = \underline{\underline{24J}}.$$

(2) An Electric field $\vec{E} = 20\hat{a_r} - 30\hat{a_\theta} + 60\hat{a_\phi}$ V/m. Find incremental work done in moving a 100 μC to a distance of 0.08 μm in following direction.

(i) $\hat{a_r}$ (ii) a_θ (iii) $\hat{a_\phi}$ (iv) magnitude of $|E|$.

Soln: (i) $dw = -Q\vec{E} \cdot dl$.

$= -100\times10^{-6}(20\hat{a_r}) \cdot \hat{a_r}(0.08\times10^{-6})$

$dw = -160\,pJ$

(ii)

$dw = -Q\vec{E} \cdot dl$.

$= -100\times10^{-6}(-30\hat{a_\theta}) \cdot \hat{a_\theta}(0.08\times10^{-6})$

$dw = 240\,pJ$

(iii) $\cancel{dw = -100\times10^{-6}(60\hat{a_\phi})} \cdot a_\phi(0.08\times10^{-6})$

$= -480\,pJ$

(iv)

$|E| = \sqrt{(20)^2 + (30)^2 + (60)^2}$

$|E| = 70.1$

$dw = -100\times10^{-6}(70)\times0.08\times10^{-6}$

$= -560\,pJ$.

(3) Determine the work done in carrying a −2 μC charge from point P1(2, 1, −1) to point P2(8, 2, −1) m. in Electric field $\vec{E} = \hat{a_x}\,y + \hat{a_y}\,x$ V/m.

(i) along parabola $x = 2y^2$

(ii) along straight line joining points P1 & P2.

Soln:

a) $dw = -Q\vec{E} \cdot dl$

$$W = -Q\int_{P1}^{P2} (\hat{a_x}\, y + \hat{a_y}\, x) \cdot (\hat{a_x}\, dx + \hat{a_y}\, dy + \hat{a_z}\, dz)$$

$$W = -Q\int_{P1}^{P2} (y\,dx + x\,dy) \qquad dz \to 0.$$

(i) along parabola $x = 2y^2$

$dx = 4y\,dy.$

$$W = -Q\left[\int_{P1}^{P2} y(4y\,dy) + \int_{P1}^{P2} (2y^2)\,dy\right]$$

$$= -(-2\times10^{-6})\left[\int_1^2 4y^2 dy + \int_1^2 2y^2 dy\right]$$

$$= 2\times10^{-6}\int_1^2 6y^2\,dy$$

$$W = 28\mu J.$$

(ii) along straight line.

$$\frac{x - x_2}{x_1 - x_2} = \frac{y - y_2}{y_1 - y_2}$$

$$\frac{x-8}{2-8} = \frac{y-2}{1-2}$$

$$x - 8 = 6y - 12$$

$$x = 6y - 4.$$

differentiating $dx = 6\,dy.$

$$W = -Q \int_{P1}^{P2} y\,dx + x\,dy$$

$$= -Q \left[\int_{P1}^{P2} y\,(6\,dy) + (6y - 4)\,dy \right]$$

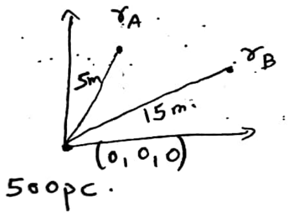
$$= -Q \int_{1}^{2} (12y - 4)\,dy$$

$$= -(-2 \times 10^{-6}) \int_{1}^{2} (12y - 4)\,dy$$

$$W = 28\,\mu J$$

(4) Find The potential at $r_A = 5m$ w.r.t $r_B = 15m$ due to point charge $Q = 500\,pC$ at The origin and zero reference at infinity.

(A)



(i) w.r.t origin.

$$V_{AB} = \frac{Q}{4\pi\epsilon} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

$$= \frac{500 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12}} \left[\frac{1}{5} - \frac{1}{15} \right]$$

$$= 0.6V$$

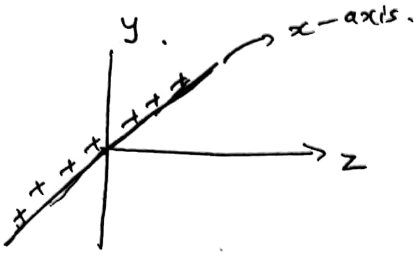
(ii) zero reference at inifinity.

$$V_A = \frac{Q}{4\pi\epsilon(5)} = 0.9V$$

$$V_B = \frac{Q}{4\pi\epsilon(15)} = 0.3V.$$

5 A line charge of $\rho_l = 10 \, nc/m$ is placed along x-axis of rectangular co-ordinate s/m. Given The points A(1, 2, 3) & B(8, 6, 10) m. Find V_{AB} due to line charge. 5

Soln:



we have,

$$V_{AB} = \frac{\rho_l}{2\pi\epsilon} \ln\left(\frac{r_B}{r_A}\right) \quad \text{for line charge.}$$

$r_B \rightarrow \perp^{r}$ distance from line charge

$\because$ line charge is along x-axis, we take only y & z for distance calculation.

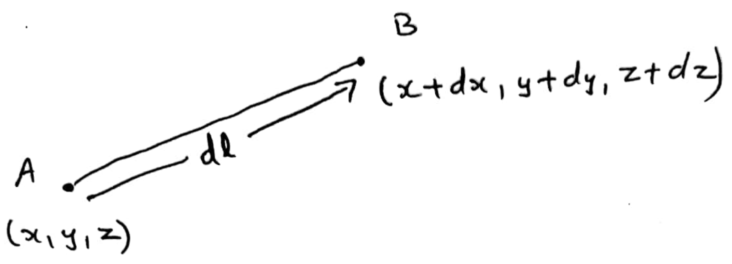
$$r_B = \sqrt{6^2 + 10^2} = \sqrt{136}$$

$$r_A = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$V_{AB} = \frac{10 \times 10^{-9}}{2 \times \pi \times 8.854 \times 10^{-12}} \ln\left(\frac{\sqrt{136}}{\sqrt{13}}\right)$$

V_{AB} = 211 Volts.

Relationship between E & V :



Consider two points A & B such that B is at higher potential compared to A. The distance between two points is given by dl.

ie., $dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$.

The amount of work done in moving a charge from A to B is given by

$$dw = -Q\vec{E}\cdot dl$$

if $Q = 1 \rightarrow$ unit +ve charge

Then $dw = dv$

$$dv = -\vec{E}\cdot(dx\,\hat{a_x} + dy\,dy + dz\,\hat{a_z}) \quad \text{— (1)}$$

We can express dv mathematically w.r.t partial derivatives as

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$dv = \left(\frac{\partial v}{\partial x}\hat{a_x} + \frac{\partial v}{\partial y}\hat{a_y} + \frac{\partial v}{\partial z}\hat{a_z}\right)\cdot\left(dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}\right) \quad \text{(2)}$$

comparing (1) & (2)

$$\vec{E} = -\left(\frac{\partial v}{\partial x}\hat{a_x} + \frac{\partial v}{\partial y}\hat{a_y} + \frac{\partial v}{\partial z}\hat{a_z}\right)$$

$$\boxed{\vec{E} = -\nabla V.} \quad \boxed{\text{or}} \quad \boxed{V = -\int E\cdot dl.}$$

Problems :

1. Find The Electric field strength at The point $(1,2,-1)$ given The potential $V = 3x^2y + 2yz^2 + 3xyz$.

Soln:

Given $V = 3x^2y + 2yz^2 + 3xyz$.

$\vec{E} = -\nabla V$

$= -\left[\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right]$

$\vec{E} = -\left[(6xy+3yz)\hat{a_x} + (3x^2+2z^2+3xz)\hat{a_y} + (4yz+3xy)\hat{a_z}\right]$

at point $(1,2,-1)$ we get

$\vec{E} = -\left[(6\times1\times2 + 3\times2\times(-1))a_x + (3\times1^2 + 2(-1)^2 + 3\times1(-1))\hat{a_y} + [(4\times2\times(-1) + 3\times1\times2)\hat{a_z}]\right]$

$\vec{E} = -6\hat{a_x} - 2\hat{a_y} + 2\hat{a_z}$ V/m.

2. Given $V = 50x^2yz + 20y^2$ V in free space.

(i) find potential at $(1,2,3)$ m.

(ii) Electric field intensity at $P(1,2,3)$ m.

(iii) $\hat{a_r}$ at $P(1,2,3)$ m.

Soln: (i) $V = 50(1)^2(2)(3) + 20(2)^2$

$V = 380$ V

(ii)

$$E = -\left(\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right)$$

$$= -\left(100\,xyz\,\hat{a_x} + (50x^2z + 40y)\hat{a_y} + 50x^2y\,\hat{a_z}\right)$$

at $(1, 2, 3)$

$$E = -\left(600\hat{a_x} + 230\hat{a_y} + 100\hat{a_z}\right)$$

(iii)

$$\hat{a_r} = \frac{E}{|E|} = \frac{-600\hat{a_x} - 230\hat{a_y} - 100\hat{a_z}}{\sqrt{600^2 + 230^2 + 100^2}}$$

$$\hat{a_r} = -0.923\hat{a_x} - 0.354\hat{a_y} - 0.154\hat{a_z}.$$

(3) The Electric potential at an arbitrary point in free space is given as

$V = (x+1)^2 + (y+2)^2 + (z+3)^2$ V, at $P(1,1,1)$. Find (a) V at P (b) E (c) $|E|$ (d) D (e) $|D|$ & (f) ρ_v

Soln:

(i) $$V_P = (x+1)^2 + (y+2)^2 + (z+3)^2 \Big|_{(1,1,1)}$$

$$= (1+1)^2 + (1+2)^2 + (1+3)^2$$

$$= 29V$$

(ii) $$E = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right]$$

$$= -\left[2(x+1)\hat{a_x} + 2(y+2)\hat{a_y} + 2(z+3)\hat{a_z}\right]$$

at $(1,1,1)$

$$= -\left[2(1+1)\hat{a_x} + 2(1+2)\hat{a_y} + 2(1+3)\hat{a_z}\right]$$

$\vec{E} = -4\hat{a_x} - 6\hat{a_y} - 8\hat{a_z}$ V/m.

(iii) $|E_p| = \sqrt{(4)^2+(6)^2+(8)^2} = 10.77$ V/m.

(iv) $D = \epsilon_0 E$

$= 8.854\times10^{-12}\left(-4\hat{a_x} - 6\hat{a_y} - 8\hat{a_z}\right)$

$= -35.42\times10^{-12}\hat{a_x} - 53.12\times10^{-12}\hat{a_y} - 70.83\times10^{-12}\hat{a_z}$ C/m².

(v) $|D| = 95.36\times10^{-12}$ C/m².

(vi) $\rho_v = \nabla\cdot D$

$= \nabla\cdot\epsilon_0 E$

$= \epsilon_0(\nabla\cdot E) = \epsilon_0\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right]$

$= \epsilon_0\frac{\partial}{\partial x}\left[-2(x+1)\right] + \epsilon_0\frac{\partial}{\partial y}\left[-2(y+2)\right] +$
$\epsilon_0\frac{\partial}{\partial z}\left[-2(z+3)\right]$.

$= \epsilon_0(-2-2-2) = -6\epsilon_0$

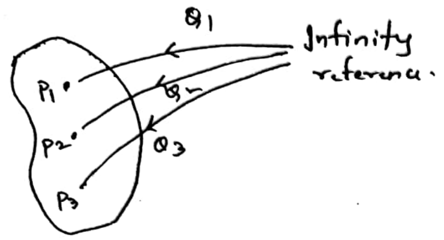
$\rho_v = -6\times8.854\times10^{-12}$

$\rho_v = \underline{\underline{-53.12\times10^{-12}}}$ C/m³

~~ENERGY~~

ENERGY DENSITY :

Consider a region free of Electric fields. Let The charges Q_1 Q_2 & Q_3 be located at infinity.



W.D in bringing Q_1 from ∞ to P1, $W1 = 0$.

W.D in bringing Q_2 from ∞ to P_2, $W_2 = Q_2 V_{21}$

where V_{21} is Loc^n 2 apposed by Loc^n 1.

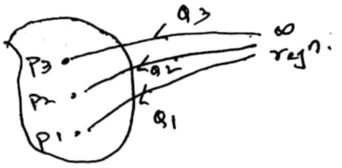
W.D in bringing Q_3 from ∞ to P3,

$W3 = Q_3 V_{31} + Q_3 V_{32}$

total work done, ~~$W = W_1 + W_2 + W3$~~

$= 0 + Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32}$ —— ①

If These charges were positioned in reverse order.



$W_3 = 0$

$W_2 = Q_2 V_{23}$

$W_3 = Q_1 V_{12} + Q_1 V_{13}$.

$W = W_1 + W_2 + W3$

$= 0 + Q_2 V_{23} + Q_1 V_{12} + Q_1 V_{13}$ —— ②.

Adding ① & ②

$2W = Q_1 [V_{12} + V_{13}] + Q_2 [V_{21} + V_{23}] + Q_3 [V_{31} + V_{32}]$

$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$

$W = \frac{1}{2} \sum_{i=1}^{3} Q_i V_i$ —— ③

Eq^n 3 can be generalized for Continous distribution of charges over any volume.

i.e.,

$$W = \frac{1}{2}\int_V \rho_v \, dv \cdot V$$

From Gauss law in point form we have,

$$\rho_v = \nabla \cdot D.$$

$$W = \frac{1}{2}\int_V (\nabla \cdot D) V \, dv$$

By vector calculus

$$(\nabla \cdot D) V = \nabla \cdot (VD) - D \cdot (\nabla V)$$

$$W = \frac{1}{2}\int_V (\nabla \cdot VD) \, dv - \frac{1}{2}\int_V D \cdot (\nabla V) \, dv \quad —— (4)$$

The volume Integral of 1^{st} term on RHS of Eq^n (4) is Converted to surface integral by divergence Theorem.

$$W = \frac{1}{2}\oint_S VD \cdot ds - \frac{1}{2}\int_V D \cdot (\nabla V) \, dv$$

W.K.T $E = -\nabla V$

$$W = \frac{1}{2}\oint_S VD \cdot ds + \frac{1}{2}\int_V D \cdot E \, dv \quad —— (5)$$

if we allow The volume V to include all Space, The surface $S \to \infty$, D decays as $\frac{1}{r^2}$. first term goes to 0.

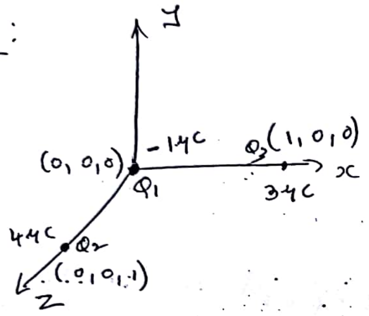
$$\boxed{W = \frac{1}{2}\int_V D \cdot E \, dv} \quad D = \epsilon E \quad \boxed{\text{or}} \quad \boxed{W = \frac{1}{2}\int_V \epsilon E^2 \, dv}$$

problems :

Type (I)

(1) Three charges $-1\mu C$, $4\mu C$ & $3\mu C$ are located in free space at $(0,0,0)$, $(0,0,1)$ & $(1,0,0)$. Find The Energy stored in The system.

Soln :



$$W = \frac{1}{2} \sum_{i=1}^{3} Q_i V_i$$

$$= \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right]$$

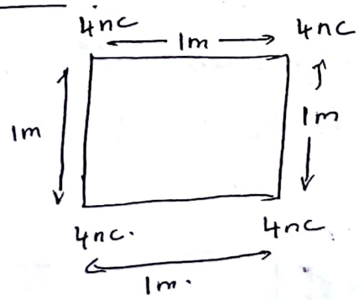
$$W = \frac{Q_1}{2}\left[\frac{Q_2}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(1)}\right] + \frac{Q_2}{2}\left[\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(\sqrt{2})}\right] +$$

$$\frac{Q_3}{2}\left[\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_2}{4\pi\epsilon_0(\sqrt{2})}\right]$$

$W = 13.35 \text{ mJ}$.

(2) Find The Energy stored in a system of four identical charges of 4nc at corners of square of side 1m.

Soln:



$$W = \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3 + Q_4 V_4\right].$$

Here $\because$ $Q_1 = Q_2 = Q_3 = Q_4 = Q$.

$V_1 = V_2 = V_3 = V_4 = V$

$$W = \frac{1}{2}\left[4QV\right].$$

$$V_1 = V = \frac{4\times10^{-9}}{4\pi\times8.854\times10^{-12}\,\cancel{\times 10}}\left[\frac{1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{1}\right]$$

$$V_1 = V = 97.4\,V$$

$$W = \frac{1}{2}\left[4\times 4\times10^{-9}\times 97.4\right]$$

$$\boxed{W = 780\ nJ}$$

Type II

(3) The potential field in free space is Expressed as $V = \frac{20}{xyz}$ volts. Find the energy stored in a cube for which $1 < x, y, z < 2$.

soln:

$$W = \frac{1}{2}\int_V D.E\,dv$$

$$= \frac{1}{2}\int_V \epsilon_0 E.E\,dv = \frac{1}{2}\int_V \epsilon_0 . E^2 dv$$

if field is Constant :

$$W = \frac{\epsilon_0}{2}\int_V |E|^2 dv.$$

$$E = -\nabla V = -\frac{\partial V}{\partial x}\hat{a_x} - \frac{\partial V}{\partial y}\hat{a_y} - \frac{\partial V}{\partial z}\hat{a_z}.$$

$$= -\left[\frac{-20}{x^2yz}\right]\hat{a_x} - \left[\frac{-20}{xy^2z}\right]\hat{a_y} - \left[\frac{-20}{xyz^2}\right]\hat{a_z}.$$

$$E = \frac{20}{x^2yz}\hat{a_x} + \frac{20}{xy^2z}\hat{a_y} + \frac{20}{xyz^2}\hat{a_z}.$$

$$|E|^2 = \frac{400}{x^4y^2z^2} + \frac{400}{x^2y^4z^2} + \frac{400}{x^2y^2z^4}.$$

$$W = \frac{1}{2}\int_{x=1}^{2}\int_{y=1}^{2}\int_{z=1}^{2} \epsilon_0 \left[\frac{400}{x^4y^2z^2} + \frac{400}{x^2y^4z^2} + \frac{400}{x^2y^2z^4}\right] dx\,dy\,dz$$

$$W = \epsilon_0\, 43.75 \text{ Joules.}$$

(4) Determine the Energy stored in freespace of region for given by $0 < \rho < a$, $0 < \phi < \pi$ & $0 < z < 2$

$$V = \frac{V_0 \rho}{a} \text{ volts.}$$

soln:

$$E = -\nabla V$$

$$E = -\frac{\partial V}{\partial r}\hat{a_r} - \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a_\phi} - \frac{\partial V}{\partial z}\hat{a_z}.$$

$\because$ There are no fields in a_ϕ & $\hat{a_z}$.

$$E = -\frac{\partial V}{\partial r}\hat{a_r}$$

$$= -\frac{\partial}{\partial r}\left[\frac{V_0 \rho}{a}\right]\hat{a_r}$$

$$E = \frac{-V_0}{a}\hat{a_r}$$

$$|E|^2 = \frac{V_0^2}{a^2}.$$

$$W = \frac{\epsilon_0}{2}\int_V |E|^2 dv$$

$$= \frac{\epsilon_0}{2}\int_{r=0}^{a}\int_{\phi=0}^{\pi}\int_{z=0}^{2} \frac{V_0^2}{a^2}\, r\,dr\,d\phi\,dz$$

$$dv = r\,dr\,d\phi\,dz.$$

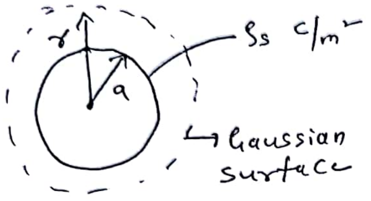
$$W = \frac{\epsilon_0 \pi V_0^2}{2}$$

$$\boxed{W = 1.57\, \epsilon_0 V_0^2 \text{ Joules.}}$$

Type III

(5) A conducting sphere of radius 'a' has surface charge density ρ_s C/m². calculate the Energy stored in the system.

Soln :



We have

$$W = \frac{1}{2}\int_V D \cdot E$$

Assume a Gaussian surface around sphere.

i) $\psi = Q_{enclosed}$

ii) $D = \frac{\psi}{A} = \frac{Q}{4\pi r^2} = \frac{\cancel{4\pi} a^2(\rho_s)}{\cancel{4\pi} r^2}$

iii) $\therefore E = \frac{D}{\epsilon} = \frac{a^2 \rho_s}{\epsilon r^2}$

$$W = \frac{1}{2}\int_V \frac{a^4 \rho_s^2}{\epsilon r^4}\, dv.$$

$$= \frac{1}{2}\int_{r=a}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{a^4\rho_s^2}{\epsilon r^4}\, r^2 \sin\theta \, dr\, d\theta\, d\phi .$$

$$W = \frac{a^4 \rho_s^2}{2\epsilon}\int_{r=a}^{\infty}\frac{r^2}{r^4}dr \int_{\theta=0}^{\pi}\sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$W = \frac{2\pi \rho_s^2 a^3}{\epsilon} \text{ Joules.}$$