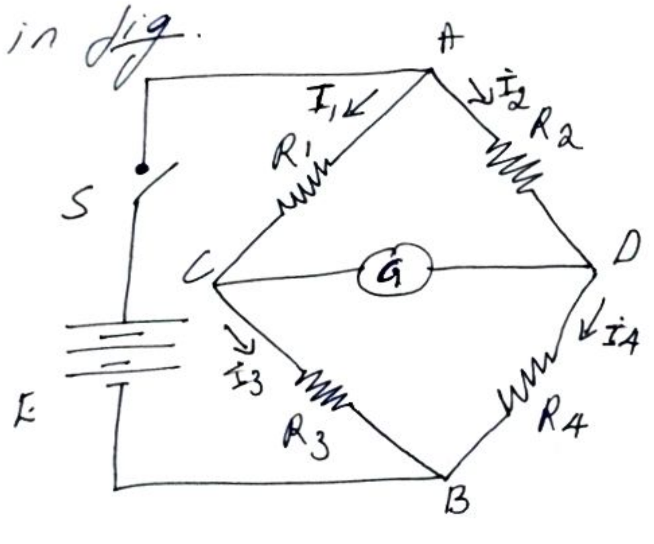
UNIT - 1

Measurement of resistance, Inductance & Capacitance.

Introduction

A bridge ckt in its simplest form consists of a N/w of four resistance arms forming a closed circuit, with a dc source of current applied to two opposite junctions and a current detector connected to the other two junctions as shown in fig.



Bridge circuits are extensively used for measuring component values such as R, L & C. Since the bridge circuit merely compares the value of an unknown component with that of an accurately known component (a standard), its measurement accuracy can be high.

The basic dc bridge is used for accurate measurement of resistance & is called wheatstone's bridge.

<u>wheatstone's bridge</u> (measurement of Resistance).

This is the most accurate method for measuring resistances. The ckt diagram is as in fig above. The source of emf & switch is connected to points A & B, while a sensitive current indicating meter, the galvanometer is connected to points C & D.

The galvanometer is a microammeter, with a zero centre scale. when there is no current <u>thro'</u> the meter, the galvanometer pointer rests at 0, i.e mid scale. current in one direction causes the pointer to deflect on one side & current in the opposite direction to the other side.

When 'S' is closed, current flows & divides into the two arms at point A i.e i_1 & i_2. The bridge is balanced when there is not current <u>thro'</u> the galvanometer or when the potential difference at points C & D is equal

To obtain bridge balance

$$I_1 R_1 = I_2 R_2 \longrightarrow ①$$

for the galvanometer current to be zero. The following conditions should be satisfied

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \longrightarrow ②$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \longrightarrow ③$$

sub ② & ③ in ①

$$\frac{E}{R_1 + R_3} \cdot R_1 = \frac{E}{R_2 + R_4} \cdot R_2$$

$$R_1 (R_2 + R_4) = R_2 (R_1 + R_3)$$

$$R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$R_1 R_4 = R_2 R_3$$

$$R_4 = \frac{R_2 R_3}{R_1}$$

This is the equation for the bridge to be balanced. R_4 can be considered as unknown resistance R_x.

Eg:- fig ⓐ consists of the following parameters $R_1 = 10K$, $R_2 = 15K$, $R_3 = 40K$. Find the unknown resistance R_x.

W.K.T $R_x = \frac{R_2 R_3}{R_1} = \frac{15K \times 40K}{10K} = \underline{60 K\Omega}$

Limitations

1] For low resistance measurement, the resistance of the leads and contacts becomes significant & introduces error (eliminated by Kelvin's Bridge)

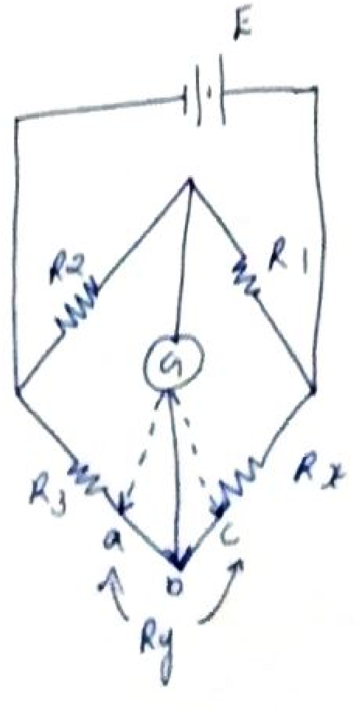
2] For high resistance measurements, the resistance presented by bridge becomes so large that the galvanometer is insensitive to imbalance. Therefore for high resistance measurements wheatstone's bridge cannot be used.

3] When current flows through the resistance due to heating effect there is change in resistance of the bridge arms

4]. Excessive current may cause a permanent change in value.

Kelvin's Bridge

Kelvin's Bridge is a modification of wheatstone's bridge and is used to measure values of resistance below 1Ω



In low resistance measurement, the resistance of the leads connecting the unknown resistance to the terminal of the bridge circuit may affect the measurement.

Let R_y = resistance of the connecting leads from R_3 to R_x

The galvanometer can be connected either to point 'c' or to point 'a'. when it is connected to point a, the resistance R_y of the connecting lead is added to the unknown resistance R_x.

When the connection is made to point 'c'. R_y is added to the bridge arm R_3 & the resulting measurement of R_x is lower than the actual value, because now the actual value of R_3 is higher than its nominal value by the resistance R_y.

If it is connected to point b, in such a way that the ratio of the resistance from c to b & that from a to b equal the ratio of resistances R_1 & R_2 then

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \longrightarrow (1)$$

W.K.T balance Eqn is

$$R_1 R_3 = R_2 R_x$$

applying this to the above bridge.

$$R_1(R_3 + R_{ab}) = R_2(R_x + R_{cb}).$$

$$(R_3 + R_{ab})\frac{R_1}{R_2} = R_x + R_{cb}. \longrightarrow (2).$$

but from (1) W.K.T $\frac{R_1}{R_2} = \frac{R_{cb}}{R_{ab}}.$

$$\& \; R_{ab} + R_{cb} = R_y. \longleftarrow (3)$$

W.K.T $\frac{R_1}{R_2} = \frac{R_{cb}}{R_{ab}}.$

adding 1 to both sides

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}.$$

from (3).

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}.$$

$$\therefore R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \longrightarrow (4).$$

from (3) w.k.T

$$R_{ab} + R_{cb} = R_y .$$

$$R_{cb} = R_y - R_{ab}.$$

Sub (4).

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$\therefore R_{cb} = \frac{R_1 R_y + R_2 R_y - R_2 R_y}{R_1 + R_2}.$$

$$R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \quad \text{— (5)}.$$

Sub (4) & (5) in (2).

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \left(R_3 + \frac{R_2 R_y}{R_1 + R_2} \right) \frac{R_1}{R_2}.$$

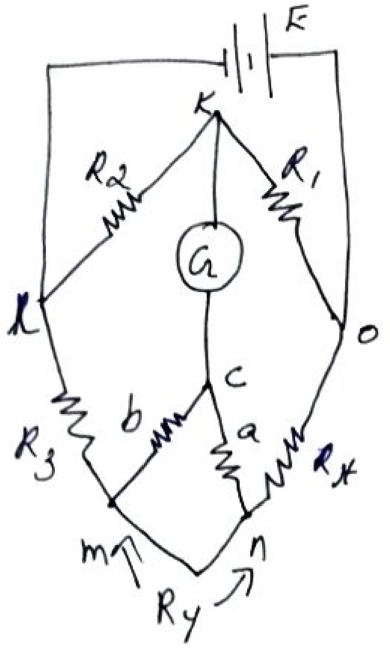
$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}.$$

$$\therefore R_x = \frac{R_1 R_3}{R_2} \quad \text{— (A)}.$$

This is the usual wheatstone's balance eqn & its indicates that the effect of the resistance of the connecting leads from pnt a to pnt c has been eliminated by connecting the galvanometer to an intermediate position b.

Kelvin's Double Bridge.

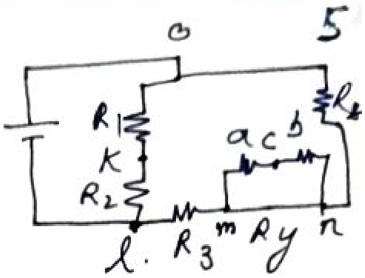
It is called Double bridge because it incorporates a second set of ratio arms.



The second set of arms a & b, connect the galvanometer to point 'c' at the appropriate potential between m & n. connection i.e R_y.

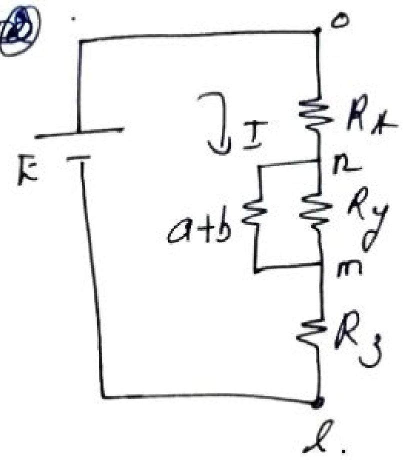
The ratio of the resistances of arms a & b is the same as the ratio of R_1 & R_2. The galvanometer indication is zero when the potentials at K & C are equal.

$$\therefore \; E_{lK} = E_{lmc}$$



But $E_{lK} = \dfrac{E \cdot R_2}{R_1 + R_2}$ —— (1)

& $E = I\left(R_x + R_3 + (a+b) \|^{le} R_y\right)$



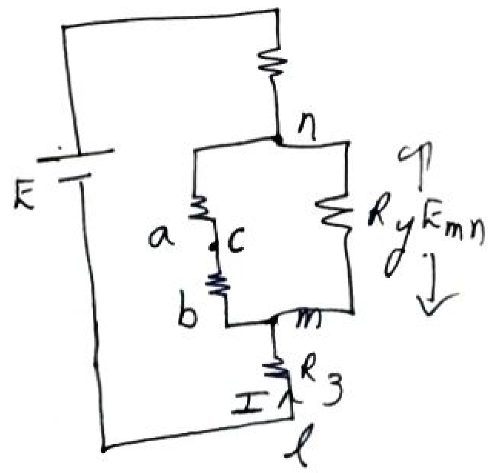
$\therefore E = I\left(R_x + R_3 + \dfrac{(a+b)R_y}{a+b+R_y}\right)$ —— (2)

Sub (2) in (1)

$$E_{lK} = \frac{R_2}{R_1+R_2} \cdot \left[I\left(R_x + R_3 + \frac{(a+b)R_y}{a+b+R_y}\right)\right] \longrightarrow (A)$$

|||ly

$E_{lmc} = \cancel{E_{mc}}\; I.R_3 + E_{mc} \rightarrow (3)$



$E_{mc} = \dfrac{E_{mn} \cdot b}{a+b}$ —— (4)

& $E_{mn} = I.\ R_y \|^{le} (a+b)$

$= I\left[\dfrac{(a+b) \cdot R_y}{a+b+R_y}\right]$ —— (5)

$E_{lmc} = E_{lm} + E_{mc}$
$= I \cdot R_3 + E_{mc}$

Sub (5) in (4)

$$E_{mc} = I\left[\frac{(a+b)R_y}{a+b+R_y}\right] \cdot \frac{b}{a+b} \longrightarrow (6)$$

Sub (6) in (3)

$$E_{lmc} = \cancel{E} I R_3 + I \frac{b}{a+b} \left[\frac{(a+b) R_y}{a+b+R_y} \right]$$

$$E_{lmc} = I \left[R_3 + \frac{b}{a+b} \left[\frac{(a+b) R_y}{a+b+R_y} \right] \right] \longrightarrow (B)$$

But $E_{lK} = E_{lmc}$

using (A) & (B)

$$\frac{I R_2}{R_1 + R_2} \left[R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} \right] = I \left[R_3 + \frac{b}{a+b} \left[\frac{(a+b) R_y}{a+b+R_y} \right] \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2} \left[R_3 + \frac{b R_y}{a+b+R_y} \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = \frac{R_1 R_3}{R_2} + R_3 + \frac{b R_1 R_y}{(a+b+R_y) R_2} + \frac{b R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{(a+b+R_y) R_2} + \frac{b R_y}{a+b+R_y} - \frac{(a+b) R_y}{a+b+R_y}$$

$$= \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 [a+b+R_y]} + \frac{b R_y - a R_y - b R_y}{a+b+R_y}$$

$$= \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 [a+b+R_y]} - \frac{a R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{a+b+R_y}\left[\frac{R_1}{R_2} - \frac{a}{b}\right]$$

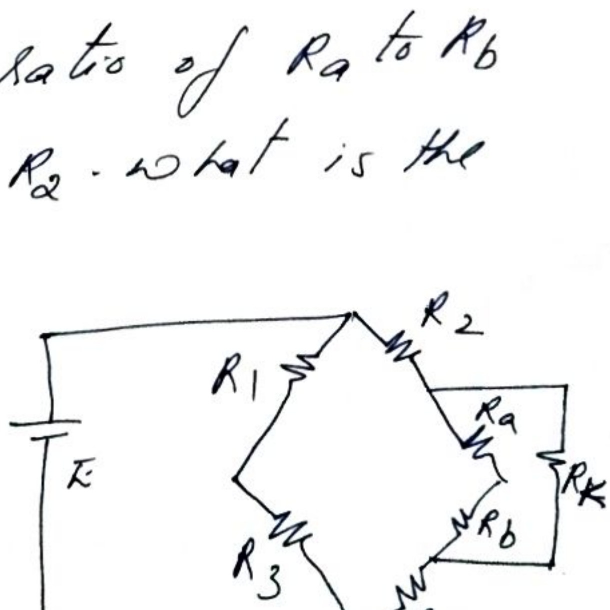
but w.k.T $\frac{R_1}{R_2} = \frac{a}{b}.$

$$\therefore R_x = \frac{R_1 R_3}{R_2}.$$

It indicates that the resistance of the connecting lead R_y, has no effect on the measurement Provided that the ratios of resistance of the two sets of ratio arms are equal

In this bridge the range of resistance covered is $1 - 0.00001\Omega$ ($10\mu\Omega$) with accuracy of $\pm 0.05\%$ to $\pm 0.2\%$.

Eg. If in fig below the ratio of R_a to R_b is 1000Ω, $R_1 = 5\Omega$ & $R_1 = 0.5 R_2$. what is the value of R_x



w.k.T $R_x = \frac{R_2 R_3}{R_1} \rightarrow (1)$.

& $\frac{R_1}{R_3} = \frac{R_a}{R_b} = 1000$

$$\therefore \frac{R_3}{R_1} = \frac{R_b}{R_a} = \frac{1}{1000} \rightarrow (2)$$

$$R_2 = \frac{R_1}{0.5}$$

$$= \frac{5}{0.5} \rightarrow (3)$$

$$R_x = R_2\left(\frac{R_3}{R_1}\right)$$

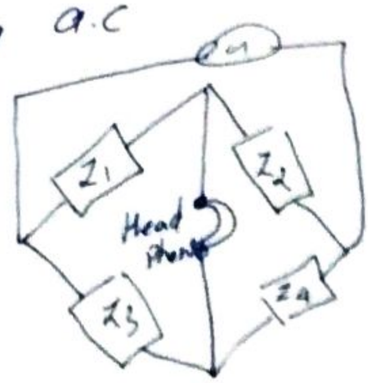
$$= \frac{5}{0.5}\left(\frac{1}{1000}\right)$$

$$= 0.01\,\Omega$$

AC Bridges

Impedance at audio & radio freq is commonly determined by means of a.c bridges.

The a.c bridge in their basic form consists of four arms, an a.c source & a null detector i.e Galvanometer is replaced by detector such as a pair of headphones for detecting a.c



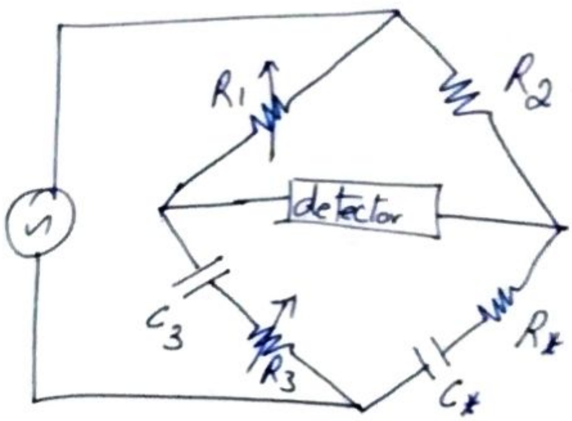
when the bridge is balanced

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

where Z_1, Z_2, Z_3 & Z_4 are the impedances of the arms and are vector complex quantities that possess phase angles. ∴ It is necessary to adjust both the magnitude & phase angles of the impedance arms to achieve balance i.e the bridge must be balanced for both the reactance and the resistive component. The phase angles are +ve for inductive impedances & -ve for capacitive impedance.

$X_L = 2\pi fL$ $(R + jX_L)$ $(R - jX_C)$ $X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$

Capacitance Comparison Bridge



The ratio arms R_1 R_2 are resistive. The known standard capacitor C_3 is in series with R_3.

R_3 may also include an added variable resistance needed to balance the bridge.

C_x is the unknown capacitor & R_x is the small leakage resistance of the capacitor

$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = R_3$ in series with C_3 i.e $R_3 - j/\omega C_3$ $\quad R_3 + \frac{1}{j\omega C_3}$

$Z_x = R_x$ in series with C_x i.e $R_x - j/\omega C_x$

$R - jX_C$

$R - j\frac{1}{\omega C}$

the condition for balance of the bridge is

$$Z_1 Z_x = Z_2 Z_3$$

i.e $R_1\left[R_x - \frac{j}{\omega C_x}\right] = R_2\left\{R_3 - \frac{j}{\omega C_3}\right\}.$

$$R_1 R_x - \frac{jR_1}{\omega C_x} = R_2 R_3 - \frac{jR_2}{\omega C_3}$$

Two complex quantities are equal when both their real & their imaginary terms are equal.

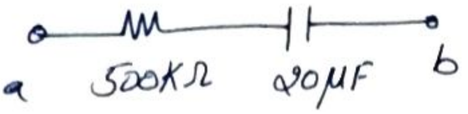
$\therefore \; R_1 R_x = R_2 R_3$ & $\frac{jR_1}{\omega C_x} = \frac{jR_2}{\omega C_3}$

$\therefore \; \underline{\underline{R_x = \frac{R_2 R_3}{R_1}}}.$ $\quad \underline{\underline{C_x = \frac{C_3 R_1}{R_2}}}.$

Eg:- A capacitance comparison bridge is used to measure a capacitive impedance at a freq of 2KHz. The bridge constants at balance are $C_3 = 100\mu F$, $R_1 = 10K\Omega$, $R_2 = 50K\Omega$, $R_3 = 100K\Omega$. Find the equivalent series circuit of the unknown impedance.

w.k.T $R_x = \frac{R_2 R_3}{R_1} = \frac{50\times10^3 \cdot 100\times10^3}{10\times10^3} = \underline{\underline{500K\Omega}}$

Then $C_x = \frac{C_3 R_1}{R_2} = \frac{10\times10^3}{50\times10^3} \times 100\times10^{-6} = \underline{\underline{20\mu F}}$

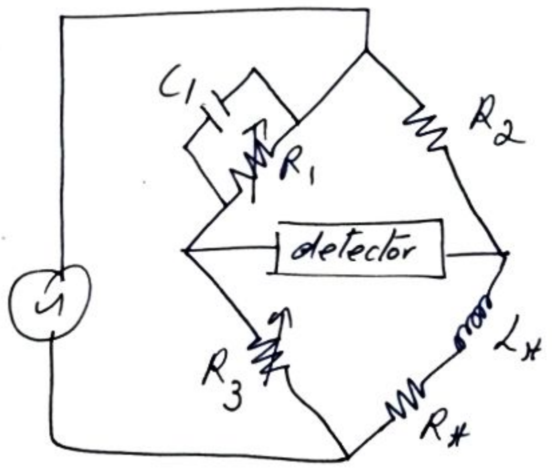


Inductance Comparison bridge — assignment

$R_x = \frac{R_2 R_3}{R_1}$ $L_x = \frac{L_3 R_2}{R_1}$

Maxwell's Bridge

measures an unknown inductance interms of a known capacitor. The capacitor is almost a loss-less component.



one arm has a resistance R_1 in parallel with C_1.

w.k.t $Z_1 Z_x = Z_2 Z_3 \rightarrow (1)$

$Z_1 = R_1$ in parallel with C_1 i.e $Y_1 = 1/Z_1$

$$= \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$
$$Z_3 = R_3$$

$Z_x = R_x$ in series with $L_x \Rightarrow R_x + j\omega L_x$

Eqn (1) becomes

$$Z_x = Z_2 Z_3 Y_1$$

$$R_x + j\omega L_x = R_2 R_3 \left[\frac{1}{R_1} + j\omega C_1\right]$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real terms & imaginary terms

$$R_x = \frac{R_2 R_3}{R_1} \quad \& \quad j\omega L_x = j\omega C_1 R_2 R_3$$

$$\text{i.e } L_x = C_1 R_2 R_3.$$

Also.

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3 R_1}{R_2 R_3} = \omega C_1 R_1.$$

Maxwell bridge is limited to the measurement of low Q values (1—10).

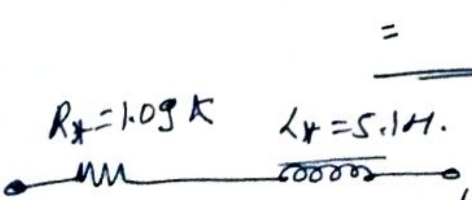
This bridge is suited for inductances measurements i.e from 1—1000H. with ± 2% error.

Eg A maxwell bridge is used to measure an inductive impedance. The bridge constants at balance are $C_1 = 0.01\mu F$, $R_1 = 470 k\Omega$, $R_2 = 5.1 k\Omega$ & $R_3 = 100 k\Omega$. Find the series equivalent of the unknown impedance.

$$R_x = \frac{R_2 R_3}{R_1} = \frac{100\times10^3 \times 5.1\times10^3}{470\times10^3} = 1.09 k\Omega$$

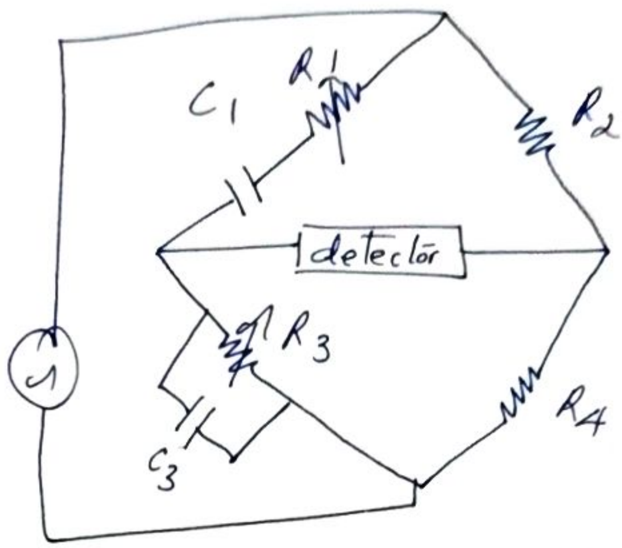
$$L_x = C_1 R_2 R_3 = 0.01\times10^{-6} \times 5.1\times10^3 \times 100\times10^3$$

$$= 5.1H$$



Wien's Bridge

The wien bridge has a series RC combination in one arm and a parallel combination in the adjoining arm.



Wien's bridge is used for the measurement of an unknown capacitance with great accuracy. It is ~~used~~ designed to measure freq.

$$Z_1 = R_1 - j/\omega C_1$$

The admittance of the ||le arm is

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

for bridge balance

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left\{\left(R_1 - j/\omega C_1\right) R_4\right\}\left(\frac{1}{R_3} + j\omega C_3\right) = R_2$$

$$R_2 = \left(R_1 R_4 - \frac{jR_4}{\omega C_1}\right)\left(1/R_3 + j\omega C_3\right)$$

$$= \frac{R_1 R_4}{R_3} - \frac{jR_4}{\omega C_1 R_3} + R_1 R_4 j\omega C_3 + \frac{\omega C_3 R_4}{\omega C_1}$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} - j\left[\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4\right]$$

Equating real & imaginary terms

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \quad \& \quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \& \quad \frac{1}{\omega C_1 R_3} = \omega C_3 R_1$$

$\rightarrow$ A

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3}.$$

$$\omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

w.k.T $\omega = 2\pi f$.

$$f = \frac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}} \quad \text{— B}$$

If we satisfy Eq A & also Excite the bridge with the freq of Eqn B the bridge will be balanced.

In most wien bridge CKT, $R_1 = R_3 = R$

& $C_1 = C_3 = C$.

$\therefore$ Eqn A reduces to

$\frac{R_2}{R_4} = 2$. & Eqn (B) is

$$f = \frac{1}{2\pi RC}.$$

which is the general equation for the freq of the bridge ckt.

Application :-

1) used for measuring freq in the audio range {20 - 200 - 2K - 20K Hz ranges)

2) used for measuring capacitances.

3) used in harmonic distortion analyzer as a Notch filter

4) used in Audio freq & Radio freq oscillators as a freq determining element.

Disadvantage :-

Since this is freq sensitive, it is difficult to balance unless the w/f of the applied V_g is purely sinusoidal.

Eg:- Find the equivalent parallel resistance & capacitance that causes a wien bridge to null with the following component values.

$R_1 = 3.1K\Omega$ $\quad R_2 = 25K\Omega$ $\quad R_4 = 100K\Omega$.

$C_1 = 5.2\mu F$ $\quad f = 2.5KHz$

w.k.t $\omega = 2\pi f$

$= 2 \times \pi \times 2.5 \times 10^3 = 15.71K\ rad/s$

w.k.t $\omega^2 = \dfrac{1}{C_1 R_1 R_3 C_3}$

$$C_3 = \frac{1}{\omega^2 C_1 R_1 R_3}.$$

Sub in $\dfrac{R_2}{R_4} = \dfrac{R_1}{R_3} + \dfrac{C_3}{C_1}$

$$= \frac{R_1}{R_3} + \frac{1}{\omega^2 C_1^2 R_1 R_3}$$

$$\frac{R_2}{R_4} = \frac{1}{R_3}\left[R_1 + \frac{1}{\omega^2 C_1^2 R_1}\right]$$

$$R_3 = \frac{R_4}{R_2}\left[R_1 + \frac{1}{\omega^2 C_1^2 R_1}\right]$$

$R_3 = 12.4 K\Omega$.

$$C_3 = \frac{1}{\omega^2 C_1 R_1 R_3}$$

$$= \frac{1}{(15.7\times10^3)^2 \times 5.2\times10^{-6} \times 3.1\times10^3 * 12.4\times10^3}$$

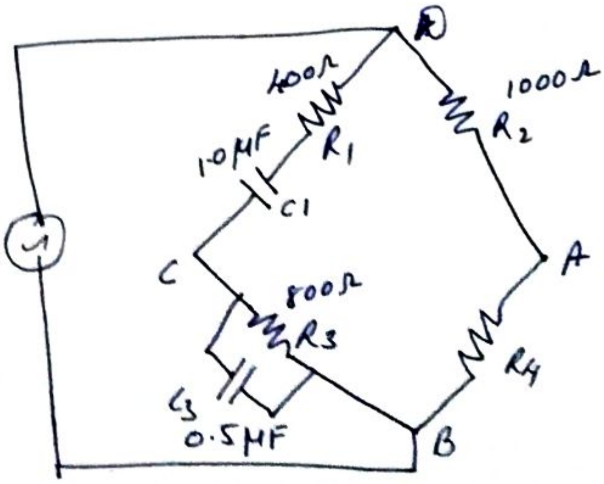
$= 20.4 PF$

Problems

1) An ac bridge with terminals A, B, C, D has in AB a pure resistance, Arm BC has a resistance of 800Ω in parallel with a capacitor of 0.5μF, arm CD has a resistance of 400Ω in series with a capacitor of 1.0μF. Arm DA has a resistance of 1000Ω

i) obtain the value of the frequency for which the bridge can be balanced by first deriving the balance equations connecting the branch impedance.

ii) calculate the value of the resistance in arm AB to produce balance.



i) $f = \dfrac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}}$

$= \dfrac{1}{2\pi\sqrt{1\times10^{-6}\times400\times0.5\times10^{-6}\times800}}$

$= \underline{\underline{397.88\,Hz}}.$

ii) R_4 w.r.t $\dfrac{R_2}{R_4} = \dfrac{R_1}{R_3} + \dfrac{C_3}{C_1}$

$R_4 = \dfrac{R_2}{\left[\dfrac{R_1}{R_3} + \dfrac{C_3}{C_1}\right]} = \dfrac{1000}{\left[\dfrac{400}{800} + \dfrac{0.5}{1}\right]}$

$= \underline{\underline{1000\Omega}}.$

Q2] The wheatstone bridge is shown in figure. The galvanometer has a current sensitivity of 12 mm/μA. The internal resistance of galvanometer is 200Ω. Calculate the deflection of the galvanometer caused due to 5 Ω unbalance in the arm AD.

$V_{th} = I_g \cdot R_{th}$

$= 1\times10^{-9} \cdot 1000$

$= 1\,\mu V$

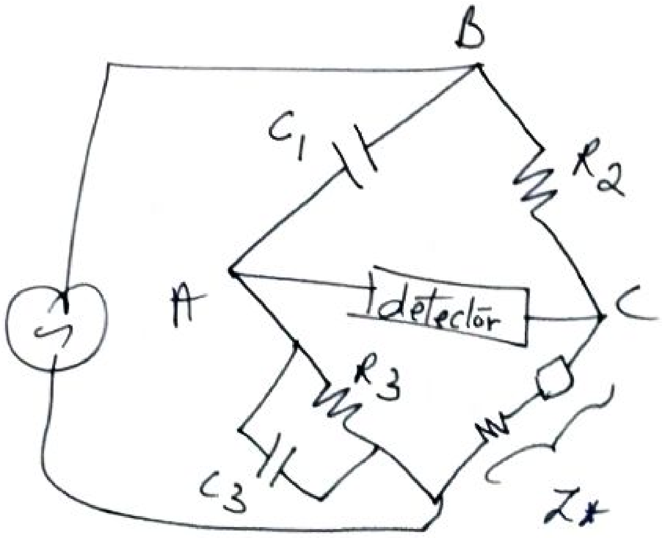
$$\Delta r = \frac{V_{th} \times 4R}{E}$$

$$= \frac{1\times10^{-6} \times 4 \times 1000}{20}$$

$\Delta r = 200\,\mu\Omega$

4] The bridge is balanced at 1000Hz. It has following components Arm AB = 0.2μF pure capacitance Arm BC = 500Ω pure resistance, DA = 300Ω //le with 0.1μF. Find the constants of arm CD, considering it as a series ckt

(Maxwell's ckt)



$Z_1 = 0 - j/\omega C_1 = 0.2\,\mu F$

$Z_2 = R_2 = 500\Omega$

$\frac{1}{Z_3} = Y_3 = \left(\frac{1}{R_3} + j\omega C_3\right)$

$Z_x = R_x + jX_L$

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = \frac{Z_2}{Z_1 Y_3}$$

$$= \frac{R_2}{-j/\omega C_1 \left[\frac{1}{R_3} + j\omega C_3\right]}$$

$$Z_x = \frac{j\omega C_1 R_2}{\frac{1}{R_3} + j\omega C_3}$$

$$= \frac{j 2\pi 1000 \times 0.2\times10^{-6} \times 500}{\frac{1}{300} + j 2\pi \times 1000 \times 0.1\times10^{-6}}$$

$$= \frac{j 0.6283}{3.3333\times10^{-3} + j 6.2831\times10^{-4}}$$

Rec(0.6283, 90) ⇓ ACL-tan

$$= \frac{0 + j0.6283}{3.3333\times10^{-3} + j6.2833\times10^{-4}} = \frac{0.6283 \angle 90^\circ}{3.388\times10^{-3} \angle 10.68^\circ}$$

$$= 185.407 \angle 79.32$$

$$R_x + j\omega L_x = 34.36 + j\,182.19\,\Omega$$

$$\therefore \underline{\underline{R_x = 34.36\,\Omega}}$$

$$\omega L_x = 182.19$$

$$L_x = \frac{182.19}{2\pi \times 1000}$$

$$\underline{\underline{= 28.99\,mH}}$$

5] The four arms of a bridge are.

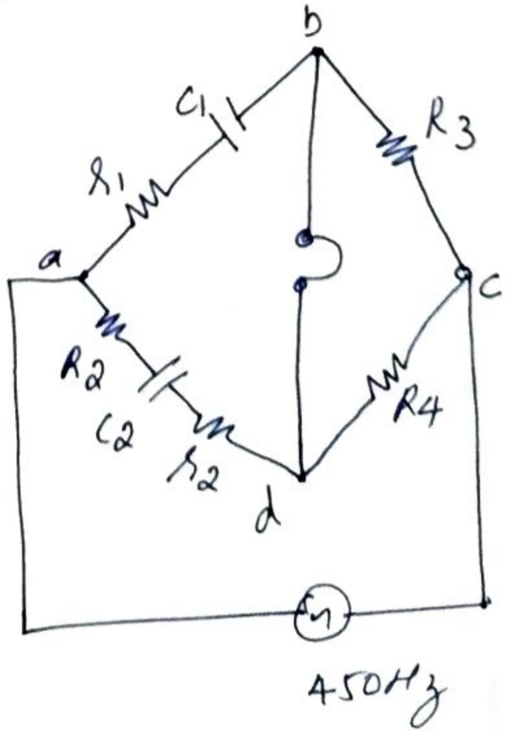
Arm ab: an imperfect capacitor C_1 with an equivalent series resistance r_1

Arm bc: a non-inductive resistance R_3.

Arm cd: " " " R_4

Arm da: an imperfect capacitor C_2 with an equivalent series resistance of r_2 in series with a resistance R_2.

A supply of 450Hz is given betwn terminal a & c & the detector is connected betwn b & d. at balance $R_2 = 4.8\,\Omega$, $R_3 = 200\,\Omega$, $R_4 = 2850\,\Omega$, $C_2 = 0.5\,\mu F$, $r_2 = 0.4\,\Omega$. calculate the value of r_1 & C_1 & also the dissipating factor of this capacitor.



$$Z_1 Z_4 = Z_3 Z_2.$$

$$\left(r_1 - \frac{j}{\omega C_1}\right) R_4 = R_3\left(R_2 + r_2 - \frac{j}{\omega C_2}\right).$$

$$r_1 R_4 - \frac{jR_4}{\omega C_1} = R_2 R_3 + r_2 R_3 - \frac{jR_3}{\omega C_2}.$$

$$r_1 = \frac{R_2 R_3 + r_2 R_3}{R_4} = \frac{4.8(200) + 0.4(200)}{2850}$$

$$= \underline{0.3649\,\Omega}.$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$C_1 = \frac{R_4 C_2}{R_3} = \frac{2850\,(0.5 \times 10^{-6})}{200} = \underline{\underline{7.125\,\mu F}}$$

dissipating factor $= \omega C_1 r_1$

$= 2\pi\,(450) \times 7.125 \times 10^{-6} \times 0.3649$

$= \underline{\underline{7.35 \times 10^{-3}}}$

$= \underline{\underline{0.00735}}$