FOURIER REPRENSTATION OF SIGNALS:

1) CTFS :-

$$x(t) = \sum_{K=-\infty}^{\infty} X(K) e^{jK\omega_0 t}$$

$x(t)$ has period $T = \frac{2\pi}{\omega_0}$

$$X(K) = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jK\omega_0 t} dt$$

2) DTFS :-

$$x(n) = \sum_{K=0}^{N-1} X(K) e^{jK\omega_0 n}$$

$N = \frac{2\pi}{\Omega_0}$

$$X(K) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jK\Omega_0 n}$$

3) CTFT :

$$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

4) DTFT :

$$x(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(e^{j\Omega}) e^{j\Omega n} d\Omega$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

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1) For the signal $x(t) = \sin\omega_0 t$, find the Fourier series and draw its spectrum

Ans:- $x(t) = \sin\omega_0 t$

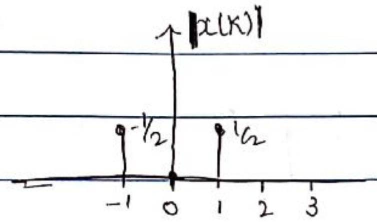
$$x(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

w.k.t
$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t}$$

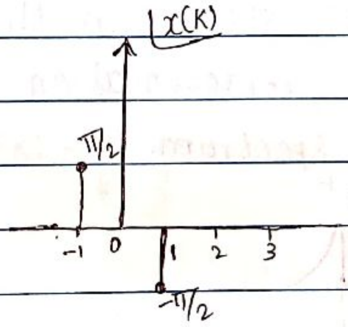
comparing the above two equations we get

$$X[1] = \frac{1}{2j} \qquad X[-1] = \frac{-1}{2j}$$

$$X[k] = 0 \quad \text{for } k \neq \pm 1$$



magnitude spectrum



$z = a+ib$

$\theta = \tan^{-1}(b/a)$

$$X(1) = \frac{-1}{2} j$$

$$X(1) = 0 - \frac{1}{2} j$$

$$\angle X(1) = \tan^{-1}\left(\frac{-1/2}{0}\right) = \tan^{-1}(-\infty) = -\pi/2$$

2) Evaluate Fourier series representation of the signal

$x(t) = \sin 2\pi t + \cancel{\cos\sin}\ \cos 3\pi t$

$\omega_0 = \gcd(2\pi, 3\pi)$

$\omega_0 = \pi$

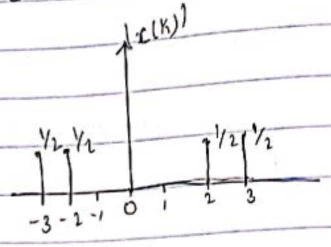
$$x(t) = \frac{1}{2j} e^{j2\pi t} - \frac{1}{2j} e^{-j2\pi t} + \frac{1}{2} e^{j3\pi t} + \frac{1}{2} e^{-j3\pi t}$$

compare with
$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t}$$

$x[2] = \frac{1}{2j}$ $x[3] = \frac{1}{2}$

$x[-2] = \frac{-1}{2j}$ $x[-3] = \frac{+1}{2}$

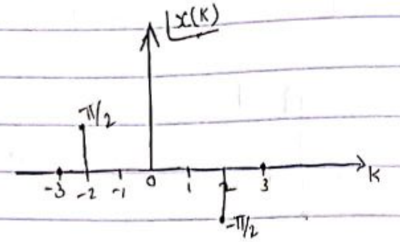
$x[k] = 0$ for $k \neq \pm 2$ & $k \neq \pm 3$



$x[2] = \frac{-1}{2}j$

$x[2] = 0 - \frac{1}{2}j$

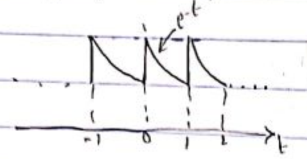
$\tan^{-1}\left(\frac{-1/2}{0}\right) =$



$x[3] = \frac{1}{2} + i0$

$\tan^{-1}\left(\frac{0}{1/2}\right) =$

3) For the signal $x(t)$ shown in the figure, find the Fourier series representation & draw its magnitude & phase spectrum.



Ans:

$x(t) = e^{-t}$ ~~$u(t)$~~ $T = 1$

w.k.t

$\omega_0 = \frac{2\pi}{T} = 2\pi$

$x(k) = \frac{1}{1}\int_0^1 x(k) e^{-jk\omega_0 t} dt$

$= \int_0^1 e^{-t} e^{-jk\omega_0 t} dt \Rightarrow \int_0^1 e^{-(1+jk\omega_0)t} dt \rightarrow (1)$

$\left[\frac{e^{-(1+jk\omega_0)t}}{-(1+jk\omega_0)}\right]_0^1$

$\frac{1}{1+jk\omega_0}\left[1 - e^{-(1+jk\omega_0)(1)}\right]$

$\frac{1}{1+jk\omega_0}\left[1 - e^{-1} \cdot e^{-jk\omega_0}\right]$

$\frac{1}{1+jk\omega_0}\left[1 - \frac{1}{e}\left(\cos k\omega_0 - \sin k\omega_0\right)\right]$

$\frac{1}{1+jk\frac{2\pi}{}}\left[1 - \frac{1}{e}\left(\cos 2\pi k - \sin 2\pi k\right)\right] \Rightarrow \frac{1}{1+jk2\pi}\left(1 - 1/e\right) \Rightarrow \frac{0.632}{1+j2\pi k}$

$\left[-e^{0} + e^{-1}\right]$

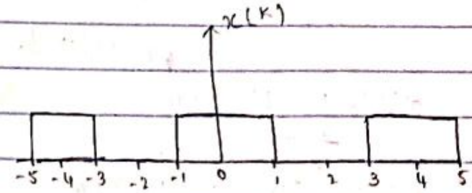
$\left[\frac{1}{e} - 1\right]$

sub. $k = 0$ in (1)

$x(0) = \int_0^1 e^{-t} dt$

$= \left[\frac{e^{-t}}{-1}\right]_0^1 \Rightarrow \left[1 - e^{-1}\right]$

4) Find the Fourier representation of the given signal



$T = 3 - (-1) = 4$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$x(k) = \frac{1}{4}\int_{-1}^{3} x(t) e^{-jk\omega_0 t} dt$

or

$x(k) = \frac{1}{4}\int_{-2}^{2} x(t) e^{-jk\omega_0 t} dt \rightarrow (1)$

$= \frac{1}{4}\int_{-1}^{1} (1) e^{-jk\pi/2 t} dt$

$= \frac{1}{4}\left[\frac{e^{-jk\pi/2 t}}{-jk\pi/2}\right]_{-1}^{1}$

$= \frac{1}{-j4k\pi/2}\left[e^{-jk\pi/2} - e^{+jk\pi/2}\right]$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$= \frac{1}{j2k\pi}\left[e^{jk\pi/2} - e^{-jk\pi/2}\right]$

$x(k) = \frac{1}{j2k\pi} \times 2j \sin(k\pi/2) \Rightarrow \frac{1}{k\pi}\sin(\pi/2\, k)$ for $k \neq$

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w.k.t

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

Sub. $k=0$ in (1)

$$x(0) = \frac{1}{4}\int_{-2}^{2} x(t) e^{0} dt$$

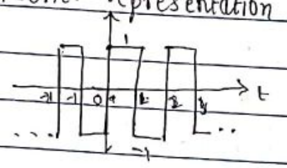
$$= \frac{1}{4}\int_{-1}^{1} (1) dt \Rightarrow \frac{1}{4}[t]_{-1}^{1}$$

$$\frac{1}{4}[1+1] \Rightarrow 1/2$$

$$X(k) = \frac{1}{2} \quad \text{if } k=0$$

$$\therefore X(k) = \begin{cases} \frac{\sin k\pi/2}{k\pi} & \text{if } k \neq 0 \\ \frac{1}{2} & \text{if } k=0 \end{cases}$$

5) Find the Fourier representation



$T = 2$

$$\omega_0 = \frac{2\pi}{T} = \pi$$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(k) e^{-jk\omega_0 t}$$

$$= \frac{1}{\cancel{\pi}2}\int_{0}^{2} x(k) e^{-jk\omega_0 t}$$

$$= \frac{1}{\cancel{\pi}2}\left[\int_{0}^{1} (1) e^{-jk\omega_0 t} dt + \int_{1}^{2} (-1) e^{-jk\omega_0 t} dt\right] \rightarrow (1)$$

$$= \frac{1}{2\cancel{\pi}}\left[\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_0^1 - \left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_1^2\right]$$

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$$\Rightarrow \frac{1}{2\cancel{\pi}}\left[\frac{1}{-jk\omega_0}\left(e^{-jk\omega_0} - 1\right) - \left(\frac{1}{-jk\omega_0}\left(e^{-j2k\omega_0} - e^{-jk\omega_0}\right)\right)\right]$$

$$= \frac{1}{-j2\cancel{\pi}k\omega_0}\left[\left(1 - e^{-jk\omega_0}\right) - \left(e^{-jk\omega_0} - e^{-j2k\omega_0}\right)\right]$$

$$= \frac{1}{j2k\omega_0}\left[1 - 2e^{-jk\omega_0} + \left(e^{-jk\omega_0}\right)^2\right]$$

$$= \frac{1}{j2k\omega_0}\left[\cancel{\left(1 - e^{-jk\omega_0}\right)^2}\right]$$

$\omega_0 = \pi$

$$X(k) = \frac{1}{j2\pi k}\left[1 - 2e^{-jk\pi} + e^{-j2k\pi}\right]$$

$$= \frac{1}{j2\pi k}\left[\cancel{1} - 2\cos k\pi + 2\cancel{\sin k\pi} + \cancel{\cos 2\pi k} - \cancel{\sin 2\pi k}\right]$$

$$X(k) = \frac{1}{j2\pi k}\left[2 - 2\cos k\pi\right] \quad \text{for } k \neq 0$$

Sub. $k=0$ in (1)

$$x(0) = \frac{1}{2}\left[\int_{0}^{1} (1) e^{0} dt + \int_{1}^{2} (-1) dt\right]$$

$$= \frac{1}{2}\left[[t]_0^1 - [t]_1^2\right]$$

$$= \frac{1}{2}\left[1 - 2 + 1\right] = 0$$

$$\therefore X(k) = \begin{cases} \frac{1 - \cos k\pi}{j\pi k} & ; \text{ for } k \neq 0 \\ 0 & ; \text{ for } k = 0 \end{cases}$$

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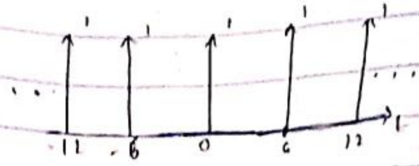
6)



$T = 6$

$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$

$$x(k) = \frac{1}{6}\int_{-3}^{3} x(k)e^{-jk\omega_0 t}\,dt$$

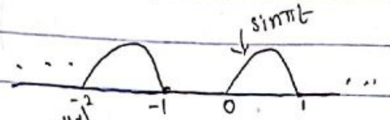
$$= \frac{1}{6}\left[\int_{-3}^{3} \delta(t)e^{-jk\omega_0 t}\,dt\right]$$

$$= \frac{1}{6}\,\delta(t)e^{-jk\omega_0 t}\Big|_{t=0}$$

$$= \frac{1}{6}\,\delta(0)$$

$$\underline{x(k) = \frac{1}{6}} \quad \text{for } \forall k$$

7) Find the fourier representation of the given by



$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$T = 2$

$\omega_0 = \frac{2\pi}{T} = \pi$

$$x(k) = \frac{1}{2}\int_{-1}^{1} x(k)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2}\int_{-1}^{0}(0)\,dt + \frac{1}{2}\int_{0}^{1}\sin\pi t\, e^{-jk\omega_0 t}\,dt$$

$$\frac{1}{2}\int_0^1 e^{-jk\omega_0 t}\sin\pi t\,dt$$

$$\frac{1}{2}\int_0^1 e^{-jk\pi t}\sin\pi t\,dt$$

$e^{-jk\pi} = \cos\pi k - j\sin\pi k$

$e^{-jk\pi} = \cos\pi k - \sin\pi k$

$= (-1)^k$

$$= \frac{1}{2}\int_0^1 e^{-jk\pi t}\left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right)dt$$

$$= \frac{1}{4j}\int_0^1 \left(e^{-j(k\pi-\pi)t} - e^{-j(k\pi+\pi)t}\right)dt$$

$$= \frac{1}{4j}\left[\frac{e^{-j\pi(k-1)t}}{-j\pi(k-1)} - \frac{e^{-j\pi(k+1)t}}{-j\pi(k+1)}\right]_0^1$$

$$= \frac{1}{4j^2\pi}\left[\frac{1}{k-1}\left(e^0 - e^{-j\pi(k-1)}\right) - \frac{1}{k+1}\left(e^0 - e^{-j\pi(k+1)}\right)\right]$$

$$= \frac{1}{-4\pi}\left[\frac{1}{k-1}\left(1 - e^{-j\pi(k-1)}\right) - \frac{1}{k+1}\left(1 - e^{-j\pi(k+1)}\right)\right]$$

$$\Rightarrow \frac{1}{2}\int_0^1 (-1)^k \sin\pi t\,dt$$

$$= \frac{1}{2}(-1)^k\left[-\frac{1}{\pi}\cos\pi t\right]_0^1$$

$$= \frac{-(1)(-1)^k}{2\pi}\left[\cos\pi - \cos(0)\right]$$

$$x(k) = \frac{(-1)(-1)^k(-2)}{2\pi} \Rightarrow \frac{(-1)^k}{\pi}$$

(8) Determine Fourier series representation of the given signal.

$$x(t) = 3\cos\left[\frac{\pi}{2}t + \frac{\pi}{4}\right]$$

Ans:

$$x(t) = \sum_{k=-\infty}^{\infty} x(k)e^{jk\omega_0 t} \rightarrow (1) \qquad \omega_0 = \frac{\pi}{2} \qquad T = \frac{2\pi}{\omega_0} = 4$$

$$x(t) = \frac{3}{2}\left[e^{j(\pi/2\, t + \pi/4)} + e^{-j(\pi/2\, t + \pi/4)}\right]$$

$$= \frac{3}{2}e^{j\pi/4}e^{j\pi/2\, t} + \frac{3}{2}e^{-j\pi/2\, t}e^{-j\pi/4} \rightarrow (2)$$

comparing (1) & (2)

$e^{j\pi/4} = \cos\pi/4 + j\sin\pi/4$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \qquad \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = 1$

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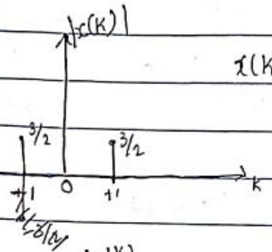
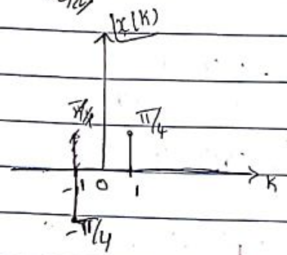
$$x(1) = \frac{3}{2} e^{j\pi/4}$$

$$x(-1) = +\frac{3}{2} e^{-j\pi/4}$$

$$x(k) = 0 \quad , \quad k \neq \pm 1$$



$$x(k) = \begin{cases} \frac{3}{2} e^{j\pi/4} & ; k = 1 \\ \frac{3}{2} e^{-j\pi/4} & ; k = -1 \\ 0 & ; \text{otherwise} \end{cases}$$



$x(1) = \frac{3}{2} e^{j\pi/4}$

$(\cancel{x(t)}\ x(z) = re^{i\theta})$

$\theta = \pi/4$

(9) Evaluate the DTFS representation of the signal

$$x(n) = \cos\frac{\pi}{3}n$$

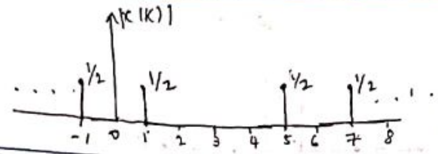
$$= \frac{1}{2} e^{j\pi/3 n} + \frac{1}{2} e^{-j\pi/3 n}$$

w.k.t, $x(n) = \sum_{k=0}^{N-1} x(k) e^{-jk\Omega_0 n}$

$\Omega_0 = \frac{\pi}{3} \qquad N = \frac{2\pi}{\Omega_0} = 6$

$x(1) = \frac{1}{2} \qquad x(-1) = \frac{1}{2}$

$\cancel{x(k) = \frac{1}{2}} \quad \cancel{k \neq \pm 1}$



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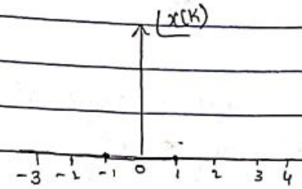


$x(1) = \frac{1}{2} + 0i$

$\tan^{-1}\left(\frac{0}{1/2}\right) = 0$

(10) $x(n) = \sin\left(\frac{4\pi n}{21}\right) + \cos\left(\frac{10\pi n}{21}\right) + 1$

$\Omega_0 = \frac{4\pi}{21} \qquad \Omega_0 = \frac{10\pi}{21}$

$\gcd\left(\frac{4\pi}{21}, \frac{10\pi}{21}\right) \Rightarrow \frac{2\pi}{21}$

$\therefore N = \frac{2\pi}{\Omega_0} = \frac{2\pi}{2\pi/21} = 21$

$$x(n) = \frac{1}{2j}\left[e^{j\frac{4\pi}{21}n} - e^{-j\frac{4\pi}{21}n}\right] + \frac{1}{2}\left[e^{j\frac{10\pi}{21}n} + e^{-j\frac{10\pi}{21}n}\right] + 1$$

$$= \frac{1}{2j} e^{j\frac{4\pi}{21}n} - \frac{1}{2j} e^{-j\frac{4\pi}{21}n} + \frac{1}{2} e^{j\frac{10\pi}{21}n} + \frac{1}{2} e^{-j\frac{10\pi}{21}n} + 1$$

w.k.t,

$x(n) = \sum_{k=0}^{N-1} x(k) e^{-jk\Omega_0 n}$

By comparing, we get

$x(2) = \frac{1}{2j} \qquad x(5) = \frac{1}{2} \qquad x(0) = 1$

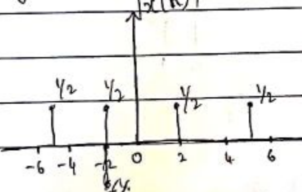
$x(-2) = -\frac{1}{2j} \qquad x(-5) = \frac{1}{2}$

$x(2) = 0 - \frac{1}{2}j$

$\tan^{-1}\left(\frac{-1/2}{0}\right) \Rightarrow -\pi/2$

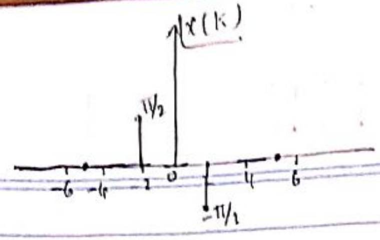
$x(-2) = 0 + \frac{1}{2}j$

$\tan^{-1}\left(\frac{1/2}{0}\right) = \pi/2$



$x(5) = \frac{1}{2} + 0i$

$\tan^{-1}(0) = 0$



11) Find the DTFS representation of the given signal

$$x(n) = \cos\left[\frac{\pi}{8} n + \phi\right]$$

Ans:- w.k.t, $\cos[\Omega_0 n + \phi]$, $\Omega_0 = \pi/8$

w.k.t,

$$x(n) = \sum_{k=0}^{N-1} x(k) e^{+jk\Omega_0 n}$$

$$\Rightarrow x(n) = \frac{1}{2} e^{j(\frac{\pi}{8} n + \phi)} + \frac{1}{2} e^{-j(\frac{\pi}{8} n + \phi)}$$

$\Omega_0 = \frac{2\pi}{N} m$

$N = \frac{2\pi}{\pi/8} \times 1 = 16$

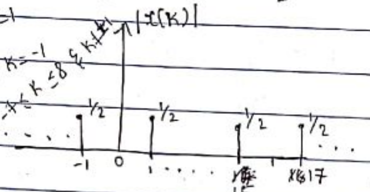
By comparing,

$$x(n) = \frac{1}{2} e^{j\phi} e^{j(\frac{\pi}{8} n)} + \frac{1}{2} e^{-j\phi} e^{-j(\frac{\pi}{8} n)}$$

$$\Rightarrow x(1) = \frac{1}{2} e^{j\phi}, \quad x(-1) = \frac{1}{2} e^{-j\phi}$$

$$|x(1)| = \frac{1}{2} \quad |x(-1)| = \frac{1}{2}$$

$$x(k) = \begin{cases} \frac{1}{2} e^{j\phi} ; & k = 1 \\ \frac{1}{2} e^{-j\phi} ; & k = -1 \\ 0 ; & -1 < k \le 8 \ \& \ k \ne 1 \end{cases}$$

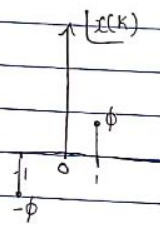


$x(1) = \frac{1}{2} e^{j\phi}$

$(\because x(z) = re^{j\theta})$

$x(1) = \frac{1}{2}[\cos\phi + j\sin\phi]$

$\tan\theta = \{ \sin\phi$



DTFT :-

$$x(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(e^{j\Omega}) e^{j\Omega n} d\Omega$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} = X(\Omega)$$

Relation b/w DTFT & z-transform :-

w.k.t

$$z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = X(z) \rightarrow (1)$$

$$DTFT\{x(n)\} = X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} \rightarrow (2)$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) (e^{+j\Omega})^{-n} \rightarrow (3)$$

comparing (1) & (3)

$z = e^{j\Omega}$

Hence,

$$X(\Omega) = X(z)\big|_{z = e^{j\Omega}}$$

1) Find the DTFT of $x(n) = \delta(n)$

Ans:- $DTFT\{x(n)\} = X(\Omega) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\Omega n}$

$$= \delta(n) e^{-j\Omega n}\big|_{n=0}$$

$$\underline{X(\Omega) = 1}$$

2) $x(n) = a^n u(n)$

$$DTFT\{x(n)\} = X(\Omega) = \sum_{n=\infty}^{\infty} a^n u(n) e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\Omega n}, \quad \left(\because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, |a|<1\right)$$

$$= \sum_{n=0}^{\infty} (a e^{-j\Omega})^n \Rightarrow \frac{1}{1 - a e^{-j\Omega}}, \quad |ae^{-j\Omega}| < 1 \Rightarrow |a| < 1$$

3) $x(n) = (1, 2, \underset{\uparrow}{3}, 2, 1)$

Ans:

$\cancel{x(n) = \delta(n) + 2\delta(n)}$

$x(n) = \delta(n+2) + 2\delta(n+1) + 3\delta(n) + 2\delta(n-1) + \delta(n-2)$

$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$

$= \delta(n+2)e^{-j\Omega n}\Big|_{n=-2} + 2\delta(n+1)e^{-j\Omega n}\Big|_{n=-1} + 3\delta(n)e^{-j\Omega n}$

$+ 2\delta(n-1)e^{-j\Omega n}\Big|_{n=1} + \delta(n-2)e^{-j\Omega n}\Big|_{n=2}$

$X(\Omega) = e^{j2\Omega} + 2e^{j\Omega} + 3 + 2e^{-j\Omega} + e^{-j2\Omega}$

$= \cos 2\Omega + \cancel{j\sin 2\Omega} + 2\cos\Omega + \cancel{j2\sin\Omega} + 3 +$
$2\cos\Omega - \cancel{2j\sin\Omega} + \cos 2\Omega - \cancel{j\sin 2\Omega}$

$X(\Omega) = 2\cos 2\Omega + 4\cos\Omega + 3$

at $\Omega = 0$

$X(0) = 9$

4) $x(n) = -a^n u(-n-1)$

$DTFT\{x(n)\} = X(\Omega) = \sum_{n=-\infty}^{\infty} -a^n u(-n-1) e^{-j\Omega n}$

$u(n): \quad 0 < n < \infty$

$u(n-1): \quad 0 < n-1 < \infty$

$1 < -n < \infty$

$-\infty < n < -1$

$\therefore X(\Omega) = -\sum_{n=-\infty}^{-1} a^n e^{-j\Omega n}$

put $n = -n$

$X(\Omega) = -\sum_{n=\infty}^{1} a^{-n} e^{-j\Omega(-n)} \Rightarrow -\sum_{n=1}^{\infty} (a^{-1} e^{j\Omega})^n$

$= -\sum_{n=1}^{\infty} \cancel{(e^{j\Omega}/a)^n} (a^{-1}e^{j\Omega})^n$

$= -\dfrac{e^{j\Omega}/a}{1 - e^{j\Omega}/a} \qquad \left|\dfrac{e^{j\Omega}}{a}\right| < 1$

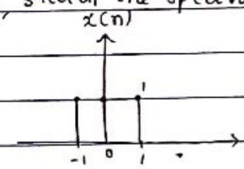
$= \cancel{e^{j\Omega}/a}$

$\Rightarrow \dfrac{\dfrac{e^{j\Omega}}{a}}{\dfrac{e^{j\Omega}}{a} - 1} \qquad |e^{j\Omega}| < |a|$

$1 < |a|$

$X(\Omega) \Rightarrow \dfrac{e^{j\Omega}}{e^{j\Omega} - a} \qquad |a| > 1$

5) $x(n) = u(n+1) - u(n-2)$ $\quad -\pi \le \Omega \le \pi$, sketch the spectrum



$\Rightarrow x(n) = \delta(n+1) + \delta(n) + \delta(n-1)$

$\therefore X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$

$= \sum_{n=-\infty}^{\infty} (\delta(n+1) + \delta(n) + \delta(n-1)) e^{-j\Omega n}$

$= \delta(n+1) e^{-j\Omega n}\Big|_{n=-1} + \delta(n) e^{-j\Omega n}\Big|_{n=0} + \delta(n-1) e^{-j\Omega n}\Big|_{n}$

$= e^{j\Omega} + 1 + e^{-j\Omega}$

$= e^{j\Omega} + e^{-j\Omega} + 1$

$\Rightarrow 2\cos\Omega + 1$

$|X(\Omega)| = |1+2\cos\Omega|$

At $\Omega = 0$

$|X(0)| = |1+2| = 3$

At $\Omega = \pi/2$

$|X(\pi/2)| = |1+0| = 1$

At $\Omega = -\pi/2$

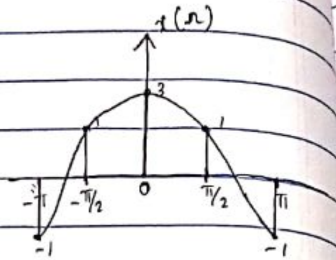
$|X(-\pi/2)| = |1+0| = 1$

At $\Omega = \pi$

$|X(\pi)| = |1+(-2)| = -1$

At $\Omega = -\pi$

$|X(-\pi)| = |1+(-2)| = -1$



6) Find the DTFT of the signal $x(n) = \left(\frac{3}{4}\right)^n u(n-4)$

Ans:- $DTFT\{x(n)\} = X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n}$

$= \sum_{n=-\infty}^{\infty} (3/4)^n u(n-4)\, e^{-j\Omega n}$

$u(n)\ ;\quad 0<n<\infty$

$u(n-4)\ ;\quad 0<n-4<\infty$

$4<n<\infty$

$\therefore\ X(\Omega) = \sum_{n=4}^{\infty} (3/4)^n e^{-j\Omega n}$

$= \sum_{n=4}^{\infty} (3/4\, e^{-j\Omega})^n$

w.k.t $\sum_{k=n}^{\infty} a^k = \frac{a^n}{1-a} \quad |a|<1$

$X(\Omega) = \frac{(3/4\, e^{-j\Omega})^4}{1-3/4\, e^{-j\Omega}} \Rightarrow \frac{(3/4)^4 e^{-j4\Omega}}{1-3/4\, e^{-j\Omega}}, \quad |3/4\, e^{-j\Omega}|<1$

7) Find the DTFT of unit step sequence $x(n) = u(n)$

Ans:- $DTFT\{x(n)\} = X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n}$

$= \sum_{n=-\infty}^{\infty} u(n)e^{-j\Omega n}$

$= \sum_{n=0}^{\infty} e^{-j\Omega n}$

$= \sum_{n=0}^{\infty} (e^{-j\Omega})^n$

w.k.t,

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a|<1$

$X(\Omega) = \frac{1}{1-e^{-j\Omega}} \quad |e^{-j\Omega}|<1$

since $|e^{-j\Omega}| \not< 1$

8)

$\therefore\ x(n) = u(n)$

$= \delta(n) + \delta(n-1) + \delta(n-2) + \dots$

$x(n) = \sum_{k=0}^{\infty} \delta(n-k)$

Let $n-k=m$

At $k=0,\ m=n$

At $k=\infty,\ m=-\infty$

$x(n) = \sum_{m=n}^{-\infty} \delta(m)$

$u(n) = \sum_{m=-\infty}^{n} \delta(m)$

$u(n) = \sum_{k=-\infty}^{n} \delta(k)$

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w.k.t, $\delta(n) \xleftrightarrow{DTFT} 1$

From accumulation property.

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$Y(\Omega) = \pi X(0)\delta(\Omega) + \frac{1}{1-e^{-j\Omega}} X(\Omega)$$

Here, $x(k) = \delta(k)$

$$x(n) = \delta(n) \xleftrightarrow{DTFT} X(\Omega) = 1$$

Hence, $U(\Omega) = \pi(1)\delta(\Omega) + \frac{1}{1-e^{-j\Omega}}(1)$

$$\boxed{U(\Omega) = \pi\delta(\Omega) + \frac{1}{1-e^{-j\Omega}}}$$

<u>Inverse DTFT</u> :-

1) Find Inverse DTFT of $X(\Omega) = \dfrac{3 - \frac{5}{4}e^{-j\Omega}}{\frac{1}{8}e^{-j2\Omega} - \frac{3}{4}e^{-j\Omega} + 1}$

Ans:- Let $e^{-j\Omega} = v$

$$\therefore X(\Omega) = \frac{3 - \frac{5}{4}v}{\frac{1}{8}v^2 - \frac{3}{4}v + 1}$$

$$v = 4, 2$$

$$X(\Omega) = \frac{3 - \frac{5}{4}v}{(v-4)\left(\frac{v}{8} - \frac{1}{4}\right)}$$

$$= \frac{3 - \frac{5}{4}v}{4\left(\frac{v}{4} - 1\right)\frac{1}{4}\left(\frac{v}{2} - 1\right)}$$

$$X(\Omega) = \frac{3 - \frac{5}{4}v}{\left(\frac{v}{4}-1\right)\left(\frac{v}{2}-1\right)}$$

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$$3 - \frac{5}{4}v \Rightarrow A\left(\frac{v}{2} - 1\right) + B\left(\frac{v}{4} - 1\right)$$

Let $v = 2$ — $\frac{1}{2} \Rightarrow -\frac{B}{2}$, $\underline{B = -1}$

Let $v = 4$ — $\underline{-2 = A}$

Hence, $X(\Omega) = \dfrac{-2}{\left(\frac{v}{4}-1\right)} - \dfrac{1}{\left(\frac{v}{2}-1\right)}$

$$X(\Omega) = \frac{2}{1 - \frac{v}{4}} + \frac{1}{1 - \frac{v}{2}}$$

$$= \frac{2}{1 - \frac{e^{-j\Omega}}{4}} + \frac{1}{1 - \frac{e^{-j\Omega}}{2}}$$

w.k.t

$$X(\Omega) = \frac{1}{1 - ae^{-j\Omega}} \xleftrightarrow{INV.DTFT} a^n u(n) = x(n)$$

$$\therefore x(n) = 2\left(\tfrac{1}{4}\right)^n u(n) + \left(\tfrac{1}{2}\right)^n u(n)$$

2. Use convolution property to find inverse DTFT of

$$X(e^{j\Omega}) = \frac{1}{(1 - ae^{-j\Omega})^2} \quad |a| < 1$$

Ans:- w.k.t

$$y(n) = z(n) * h(n) \xleftrightarrow{DTFT} Y(\Omega) = Z(\Omega) \cdot H(\Omega)$$

$$\therefore X(\Omega) = \frac{1}{1 - ae^{-j\Omega}} \cdot \frac{1}{1 - ae^{-j\Omega}}$$

Apply inverse DTFT

$$x(n) = a^n u(n) * a^n u(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} z(k) h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) a^{n-k} u(n-k)$$

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$= \sum_{K=-\infty}^{\infty} \cancel{a^{K}} u(K) a^n \cancel{a^{K}} u(n-K)$

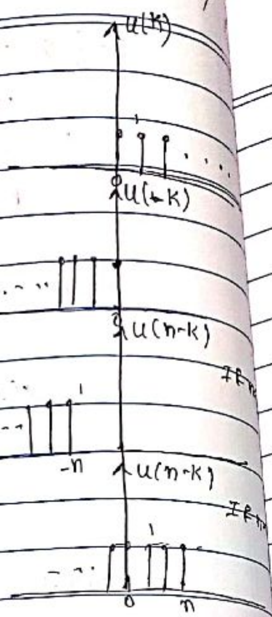
$x(n) = a^n \sum_{K=-\infty}^{\infty} u(K)u(n-K)$

If $n<0$, $x(n)=0$

If $n>0$, $x(n) = a^n \sum_{K=0}^{n} (1)(1)$

$= a^n \sum_{K=0}^{n} (1)$

$= a^n(n+1)$



Hence, $x(n) = \begin{cases} a^n(n+1) ; & n \geq 0 \\ 0 ; & n<0 \end{cases}$

Hence, $\boxed{x(n) = a^n(n+1)u(n)}$

CTFT :-

$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \rightarrow$ Inverse CTFT

$X(j\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$

1) Find CTFT or FT of $x(t) = \delta(t+0.5) - \delta(t-0.5)$

Ans:- $CTFT\{x(t)\} = X(\omega) = \int_{t=-\infty}^{\infty} (\delta(t+0.5) - \delta(t-0.5)) e^{-j\omega t} dt$

$X(\omega) = \int_{t=-\infty}^{\infty} \delta(t+0.5) e^{-j\omega t} dt - \int_{t=-\infty}^{\infty} \delta(t-0.5) e^{-j\omega t} dt$

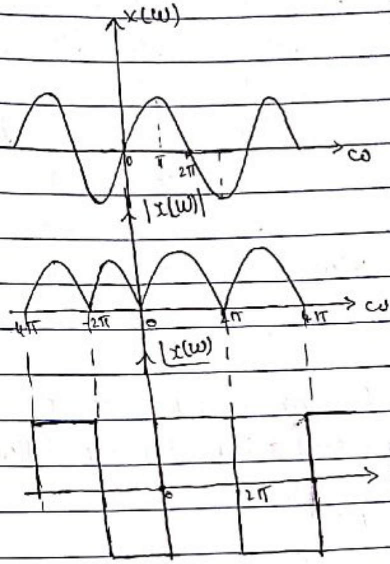
$= \delta(t+0.5) e^{-j\omega t}\Big|_{t=-0.5} - \cancel{\delta(t-0.5)} e^{-j\omega t}\Big|_{t}$

$\frac{3\pi}{4}$ $\sin\theta = \frac{e^{i\theta} - e^{i\theta}}{2i}$ Date: / / /

$= e^{j(\omega/2)} - e^{-j\omega/2}$

$X(\omega) = \cancel{xx0x}\ 2j\sin(\omega/2)$

$\frac{3\pi}{2}$



$X(\omega) = 0 + 2j\sin(\omega/2)$

$\tan^{-1}\left(\frac{2\sin(\omega/2)}{0}\right)$

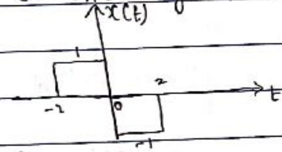
If $2\sin\frac{1}{2}\omega > 0$

$\tan^{-1}(\infty) = \pi/2$

If $2\sin\frac{1}{2}\omega < 0$

$\tan^{-1}(-\infty) = -\pi/2$

2) Determine the FT of the signal shown below



Ans:- $x(t) = \begin{cases} 1 ; & -2 \leq t \leq 0 \\ -1 ; & 0 \leq t \leq 2 \end{cases}$

$X(\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$= \int_{t=-2}^{0} (1) e^{-j\omega t} dt - \int_{t=0}^{2} e^{-j\omega t} dt$

$\Rightarrow \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{t=-2}^{0} - \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{2}$

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$$\Rightarrow \frac{1}{j\omega}\left[e^{j2\omega}-1\right] - \frac{1}{j\omega}\left[1-e^{-j2\omega}\right]$$

$$= \frac{1}{j\omega}e^{j2\omega} - \frac{1}{j\omega} - \frac{1}{j\omega} + \frac{1}{j\omega}e^{-j2\omega}$$

$$\Rightarrow \frac{1}{j\omega}\left(e^{j2\omega}+e^{-j2\omega}\right) - \frac{2}{j\omega}$$

$$= \frac{1 \times 2\cos 2\omega}{j\omega} - \frac{2}{j\omega}$$

$$X(\omega) = \frac{2}{j\omega}\left(\cos 2\omega - 1\right)$$

3) using the defining equation of F.T, evaluate the frequency domain representation for the following signals

a) $x(t) = e^{-2t}u(t-3)$

b) $x(t) = e^{-4|t|}$

Ans:- (a) $x(t) = e^{-2t}u(t-3)$ $\qquad u(t) : 0<t<\infty$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$u(t-3)\ 0<t-3<\infty$

$\Rightarrow 3<t<\infty$

$$= \int_{t=3}^{\infty} e^{-2t}e^{-j\omega t}dt$$

$$= \int_{t=3}^{\infty} e^{-(2+j\omega)t}dt$$

$$\Rightarrow \left[\frac{e^{-(2+j\omega)t}}{-(1+j\omega)}\right]_3^{\infty}$$

$$= \left[0 + \frac{e^{-(2+j\omega)3}}{-2+j\omega}\right]$$

$$= \frac{1}{2+j\omega}e^{-3(2+j\omega)} \Rightarrow \frac{e^{-6}}{2+j\omega}\left[\cos 3\omega - j\sin 3\omega\right]$$

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(b) $x(t) = e^{-4|t|}$

$$x(t) = \begin{cases} e^{-4t} & ; t \geq 0 \\ e^{4t} & ; t<0 \end{cases}$$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$= \int_{t=-\infty}^{0} e^{4t}e^{-j\omega t}dt + \int_{t=0}^{\infty} e^{-4t}e^{-j\omega t}dt$$

$$= \int_{t=-\infty}^{0} e^{+(-j\omega+4)t}dt + \int_{t=0}^{\infty} e^{-(4+j\omega)t}dt$$

$$= \left[\frac{e^{(-j\omega+4)t}}{-j\omega+4}\right]_{-\infty}^{0} + \left[\frac{e^{-(4+j\omega)t}}{-(4+j\omega)}\right]_0^{\infty}$$

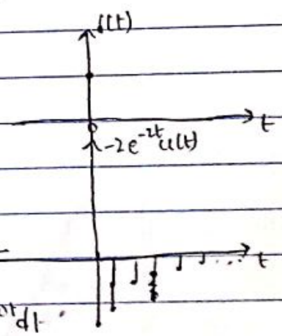
$$= \left[\frac{1}{4-j\omega} + 0\right] + \left[\frac{1}{4+j\omega} - 0\right]$$

$$= \frac{1}{4-j\omega} + \frac{1}{4+j\omega}$$

$$= \frac{4+j\omega+4-j\omega}{16-(j\omega)^2} \Rightarrow \frac{8}{16+\omega^2}$$

4) Sketch the frequency response of the system described by the impulse response using F.T.

Ans:- $h(t) = \delta(t) - 2e^{-2t}u(t)$



$$H(\omega) = \int_{t=-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

$$= \int_{t=-\infty}^{\infty} \left(\delta(t) - 2e^{-2t}u(t)\right)e^{-j\omega t}dt$$

$$= \int_{t=-\infty}^{\infty} \delta(t)e^{-j\omega t}dt - 2\int_{t=-\infty}^{\infty} e^{-2t}u(t)e^{-j\omega t}dt$$

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$$= \int_{t=-\infty}^{\infty} \delta(t)e^{-j\omega t}dt - 2\int_{t=0}^{\infty} 2e^{-2t}e^{-j\omega t}dt$$

$$= \delta(t)e^{-j\omega t}\Big|_{t=0} - 2\left[\frac{e^{-(2+j\omega)t}}{-(2+j\omega)}\right]_0^{\infty}$$

$$= 1 - 2\left[0 + \frac{1}{2+j\omega}\right]$$

$$= 1 - \frac{2}{2+j\omega}$$

$$H(\omega) = \frac{\cancel{2}+j\omega-\cancel{2}}{2+j\omega} = \frac{j\omega}{2+j\omega}$$

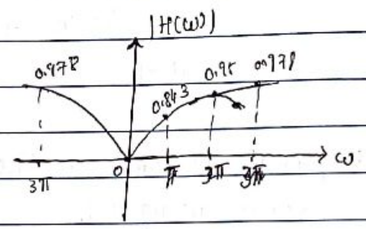
$$|H(\omega)| = \frac{\omega}{\sqrt{4+\omega^2}}$$

$\omega = 0$, $|H(\omega)| = 0.\cancel{843}$

$\omega = \pi$, $|H(\omega)| = 0.843$

$\omega = 2\pi$, $|H(\omega)| = 0.95$

$\omega = 3\pi$, $|H(\omega)| = 0.978$



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MODULE - 5

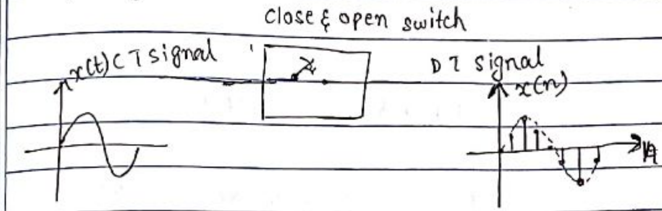
Application of fourier representation of signals

sampling interval:-

The time duration between two samples is known as sampling interval.

Sampling frequency:-

Number of samples per second is known as sampling frequency.

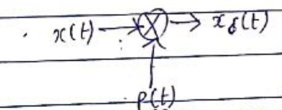


Sampling a continuous time signal:-

consider an arbitary signal $x(t)$ as shown in the figure which need to be sampled with a sampling interval of 'Ts'.

sampled version of $x(t)$ is given as

$$x_\delta(t) = x(t)p(t) \rightarrow (1)$$



where $p(t) = \sum_{K=-\infty}^{\infty} \delta(t - KT_s)$

Taking F.T on both sides of (1), then

$$X_\delta(\omega) = \frac{1}{2\pi}[X(\omega) * P(\omega)] \rightarrow (2)$$

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Recall, $p(t) = \sum_{K=-\infty}^{\infty} \delta(t - K T_s) \xleftrightarrow{F.T} P(\omega)$

$$P(\omega) = \omega_s \sum_{K=-\infty}^{\infty} \delta(\omega - K\omega_s) \longrightarrow (3)$$

where, $\omega_s = \frac{2\pi}{T_s}$

$$X_\delta(\omega) = \frac{1}{2\pi} X(\omega) * \omega_s \sum_{K=-\infty}^{\infty} \delta(\omega - K\omega_s)$$

$$= \frac{1}{2\pi} X(\omega) * \frac{2\pi}{K_s} \sum_{K=-\infty}^{\infty} \delta(\omega - K\omega_s)$$

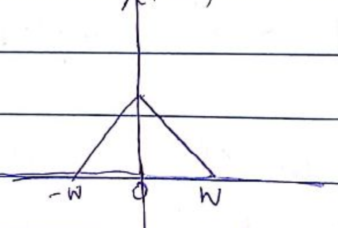
$$= \frac{1}{T_s} \sum_{K=-\infty}^{\infty} X(\omega) * \delta(\omega - K\omega_s)$$

w.k.t,

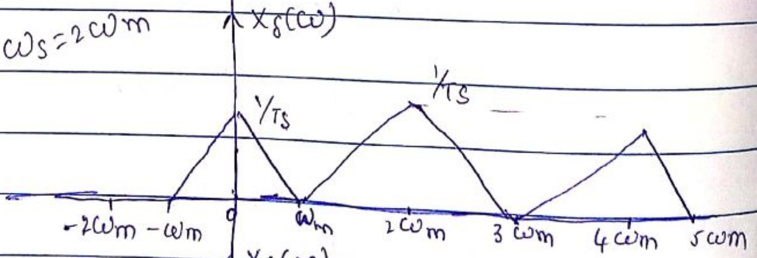
$$x(t) * \delta(t - t_0) = x(t - t_0)$$

$$\therefore \quad X_\delta(\omega) = \frac{1}{T_s} \sum_{K=-\infty}^{\infty} X(\omega - K\omega_s) \rightarrow (4)$$

$$X_\delta(\omega) = \frac{1}{T_s} \left[\ldots + X(\omega + \omega_s) + X(\omega) + X(\omega - \omega_s) + \right.$$



$\omega_s = 2\omega_m$



$\omega_s > 2\omega_m$