Root locus

→ Movement of the poles can be known by Root Locus Method, introduced by W.R. Evans in 1948

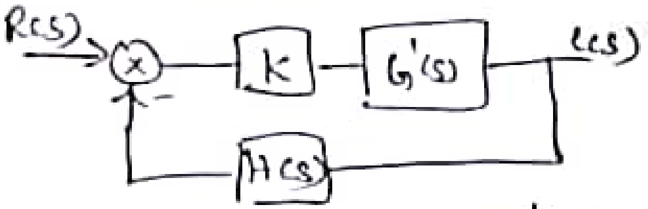
→ Root locus method is a graphical method, in which movement of poles in the s-plane is sketched when a particular parameter of system is varied from zero to infinity

→ The for Root locus method, gain is assumed to be a parameter which is to be varied from zero to infinity.

Basic concept of Root locus.

→ Characteristic Eqn of a closed loop s/m is given as

$1 + G(s)H(s) = 0$



$G(s) = KG'(s)$

K = Gain of the amplifier in forward path or also called system Gain

The characteristic Eqn becomes

$1 + G(s)H(s) = 0$ ie $1 + KG'(s)H(s) = 0$

→ The closed loop poles are now dependent on the values of 'K'

→ When K is varied from 0 to $+\infty$ the plot is called Direct root locus.

→ While K is varied from $-\infty$ to 0 the plot is called Inverse root locus.

Angle and Magnitude condition

for a General closed loop s/m the characteristic eqn is

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

As s-plane is complex we can write as

$$G(s)H(s) = -1 + j0$$

Angle condition

WKT

$$G(s)H(s) = -1 + j0$$

Equating angles of both sides

$$\angle G(s)H(s) = \pm(2q+1)180^\circ \quad q = 0, 1, 2 \ldots$$

$$= \text{odd multiple of } 180^\circ$$

→ $-1+j0$ is a point on negative real axis which can be traced as magnitude 1 at an angle $\pm 180^\circ, \pm 540^\circ, \pm 900^\circ \ldots$ $\pm(2q+1)180^\circ$

→ Any point in s-plane which satisfies the angle condition has to be on the root locus of the corresponding system.

Use of Angle condition

Ex: consider the system with $G(s)H(s) = \dfrac{K}{s(s+2)(s+4)}$

find whether $s = -0.75$ is on the root locus or not using angle condition.

$$\angle G(s)H(s)\Big|_{s=-0.75} = \pm(2q+1)180^\circ \quad q = 0, 1, 2 \ldots$$

$$\angle G(s)H(s)\Big|_{s=-0.75} = \frac{\angle K + j0}{\angle -0.75+j0 \; \angle -0.75+2+j0 \; \angle -0.75+4+j0}$$

$$= \frac{\angle k + j0}{\angle -0.75 + j0 \; \angle 1.25 + j0 \; \angle 3.25 + j0}$$

$$= \frac{0^\circ}{180^\circ \cdot 0^\circ \cdot 0^\circ} = -180^\circ$$

= odd multiple of 180°

Hence the angle condition is satisfied thus the point $s = -0.75$ is on the root locus.

Magnitude condition

$$|G(s)H(s)| = |-1 + j0| = 1$$

So magnitude condition is $|G(s)H(s)|_{\text{at a point in s-plane which is on root locus}} = 1$

→ Magnitude condition can be used only when a point in s-plane is confirmed for its existence on the root locus by use of angle condition.

Use of Magnitude condition

Ex) From previous Example $G(s)H(s) = \frac{k}{s(s+2)(s+4)}$ and $s = -0.75$ is confirmed to be on the root locus. What value of k, $s = -0.75$ is one of the roots of $1 + G(s)H(s) = 0$

$$|G(s)H(s)|_{s=-0.75} = 1.$$

$$\frac{|K|}{|-0.75|\,|-0.75+2|\,|-0.75+4|} = 1$$

$$\frac{|k|}{|0.75||1.25||3.25|} = 1$$

$$k = 3.0468$$

Rules for construction of Root locus

(1) The root locus is always symmetrical about the real axis.

(2) Let G(s) H(s) = open loop T.F of the system

P = No: of open loop poles

Z = No: of open loop zeros

case (i) P > Z

No: of branches N

N = P

Branches start from the poles and terminate at the locations of open loop zeros

case (ii) Z > P

No: of branches N

N = Z

out of Z no: of branches 'P' no: of branches will start from each of the finite open loop pole locations while remaining Z−P no: of branches will originate from infinity and will approach to finite zeros

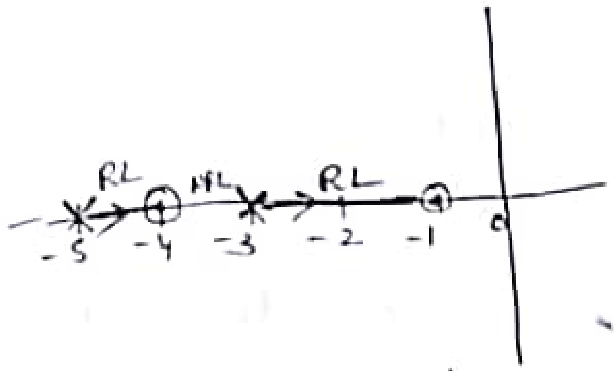
(3) A point on the real axis lies on the root locus if the sum of number of open loop poles and open loop zero's on the real axis, to the right hand side of this point is odd.

Note:- Imaginary poles & zero's are not be considered in the count.

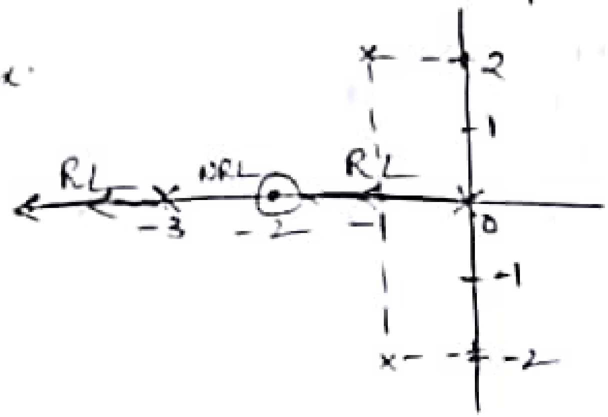
Ex $G(s)H(s) = \dfrac{K(s+1)(s+4)}{(s+3)(s+5)}$

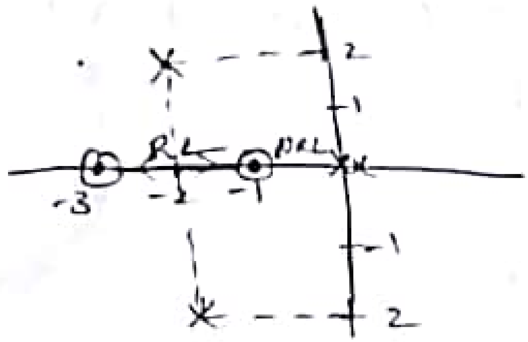
$P = 2 \quad Z = 2, \quad N = 2$

$P = -3, -5$
$Z = -1, -4$



Ex.





(4) The branches which are approaching to infinity, do so along the straight lines called Asymptotes of the roots. Angles of such asymptotes are given by.

$$\theta = \frac{(2q+1)180^\circ}{P-Z} \quad \text{where } q = 0, 1, 2 \ldots (P-Z-1)$$

(5) Location of asymptotes in S-plane is given by this rule

All the asymptotes intersect the real axis at a common point known as Centroid denoted by σ.

The co-ordinates of centroid can be calculated as

$$\sigma = \frac{\Sigma \text{Real parts of poles of } G(s)H(s) - \Sigma \text{Real parts of zeros of } G(s)H(s)}{P-Z}$$

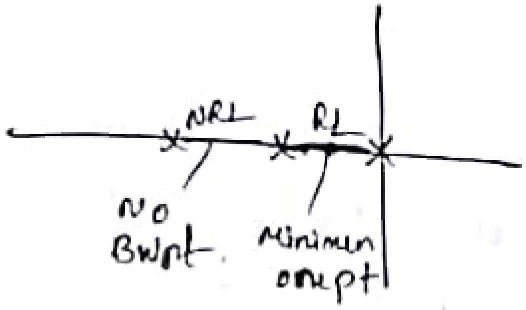
→ Centroid is always real, it may be located on negative or positive real axis. It may or may not be the part of the root locus.

⑥ Breakaway point :-

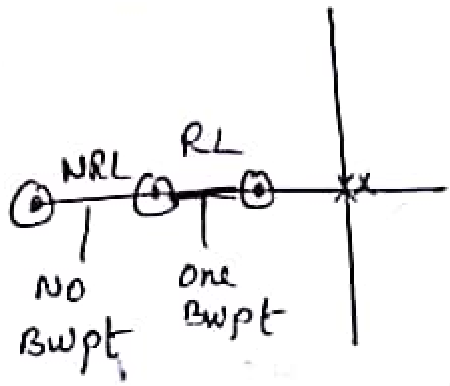
Breakaway point is a point on the root locus where multiple roots of the characteristic eqn. occurs for a particular value of k.

General predictions about existence of breakaway points :

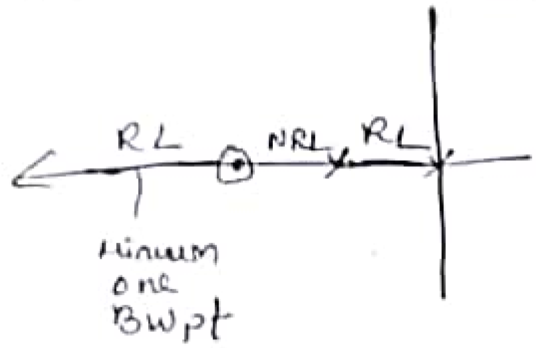
(1) If there are adjacently placed poles on the real axis and the real axis between them is a part of the root locus then there exists minimum one breakaway point in btw adjacently placed poles



(2) If there are two adjacently placed zero's on real axis and section of real axis in between them is a part of the root locus then there exist minimum one breakaway point in between adjacently placed zeros



(3) If there is a zero on the real axis and to the left of that zero there is no pole or zero existing on the real axis and complete real axis to the left of this zero is a part of the root locus then there exists minimum one breakaway point to the left of that zero.



Determination of Breakaway point

Step 1. Construct the characteristic Eqn $1+G(s)H(s)=0$ of the system

Step 2: From this eqn, seperate the terms involving 'k' and terms involving 's'. write the value of 'k' in terms of s.

$$K = F(s)$$

Step 3: Differentiate above Eqn w.r.t 's' equate it to zero

$$\frac{dK}{ds} = 0$$

Step 4: Roots of the Eqn $\frac{dK}{ds}=0$ gives us the breakaway points

→ If the value of 'K' is positive the breakaway point is valid for root locus

(7) Intersection of Root locus with Imaginary axis

→ Step 1 : Consider characteristic eqn $1+G(s)H(s)=0$ as obtained in Rule 6

Step 2 : Construct Routh's array in terms of "K"

Step 3: Determine Kmarginal i.e value of K which creates one of the rows of Routh's array as row of zero's, except the row of s^0

Step 4: Construct auxillary eqn $A(s)=0$

Step 5: Roots of auxiliary eqn $A(s)=0$ for $K=K_{mar}$ are nothing but the intersection points of the root locus with imaginary axis

→ If Kmar is positive, root locus intersects with imaginary axis. But if Kmar is negative root locus does not intersect with imaginary axis and lies totally in left half of s-plane.

(8) Angle of departure at complex pole and Angle of arrival at complex zeros.

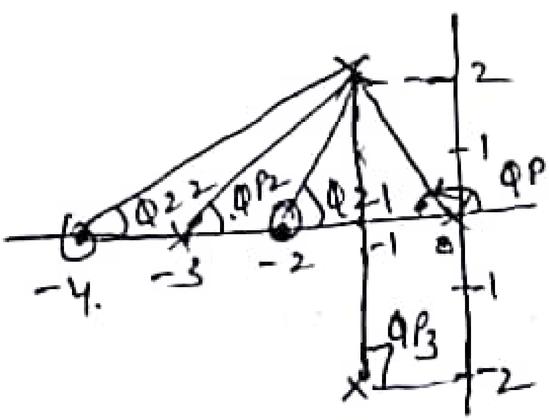
Angle of departure

$$\phi_d = 180^\circ - \phi$$

where $\phi = \Sigma\phi_p - \Sigma\phi_z$

$\Sigma\phi_p$ = contributions by the angles made by remaining open loop poles at the pole at which ϕ_d is to be calculated

$\Sigma\phi_z$ = contributions by the angles made by the open loop zero's at the pole at which ϕ_d is to be calculated



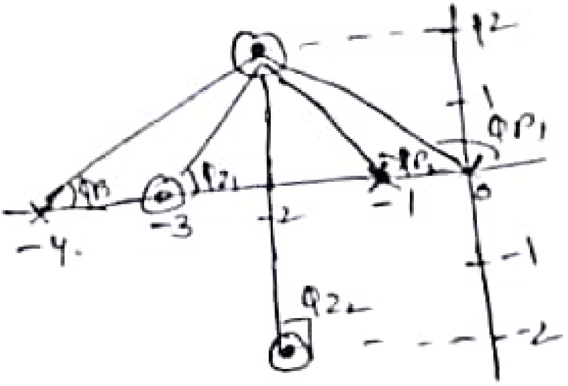
$$\Sigma\phi_p = \phi_{P1} + \phi_{P2} + \phi_{P3}$$

$$\Sigma\phi_z = \phi_{z1} + \phi_{z2}$$

Angle of arrival.

$\phi_a = 180^\circ + \phi$

where $\phi = \Sigma\phi_p - \Sigma\phi_z$



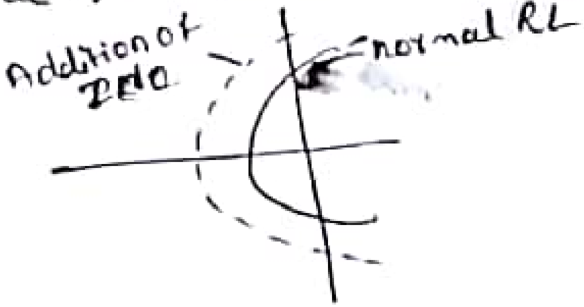
$\Sigma\phi_p = \phi_{p_1} + \phi_{p_2} + \phi_{p_3}$

$\Sigma\phi_z = \phi_{z_1} + \phi_{z_2}$

Effect of Addition of open loop poles an zeros

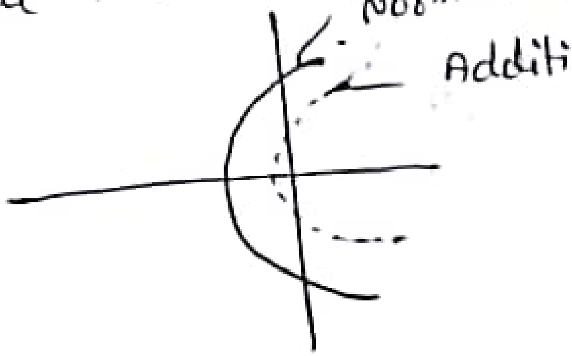
Addition of ~~pole~~ zero

Shifts the root locus to left side of s-plane



Addition of ~~zero~~ pole

Shifts the root locus toward right side of s-plane.



Problems on Root locus.

(1) Draw the approximate root locus diagram for a closed loop system whose loop transfer function is given by

$$G(s) = \frac{K}{S(S+5)(S+10)}$$

Soln.) Step 1: Poles $S = 0, -5, -10$

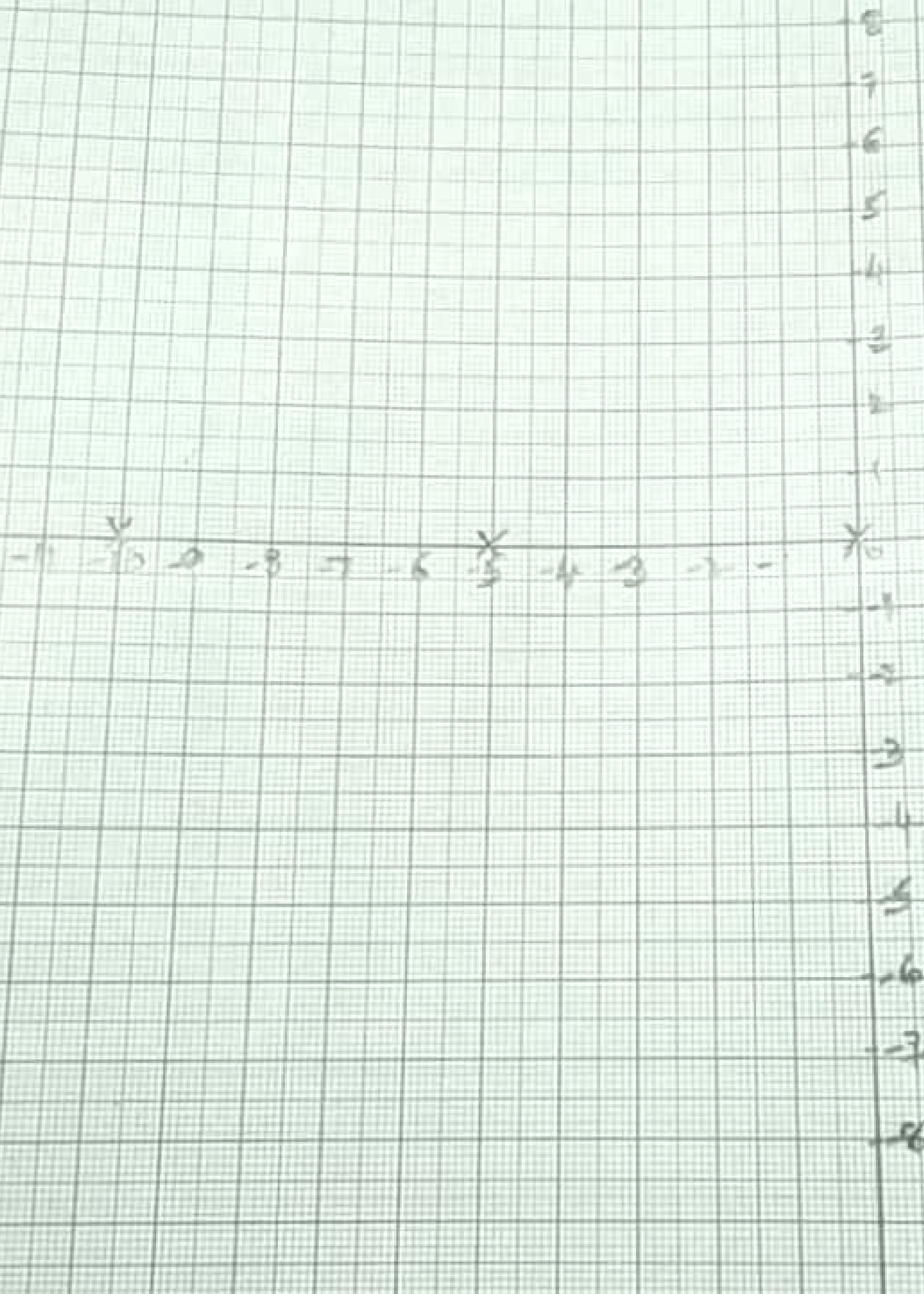
$P = 3$

zeros $= 0$ $Z = 0$

No: of branches approaching to ∞ $= N = P - Z = 3$

Step 2:

Scale
x-axis -
1 unit = 1
y-axis
1 unit = 1



Step-1

Date

① Given $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$

Step 1 $P=3,\ z=0 \quad N=P=3$ branches

Poles $s = 0, -5, -10$

zeros $= 0$

Step 2 :- Sections of Real axis

Step 3 Angles of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-z} \quad q=0,1,2$$

$$\theta_1 = \frac{(2\times0+1)180^\circ}{3} = 60^\circ,\ \theta_2 = \frac{(2\times1+1)180^\circ}{3} = 180^\circ,\ \theta_3 = \frac{(2\times2+1)180^\circ}{3} = 300^\circ$$

Step 4 : centroid.

$$\sigma = \frac{\Sigma RP \text{ of poles} - \Sigma RP \text{ of zeros}}{P-z} = \frac{0-5-10-0}{3} = -5$$

Step 5. Breakaway point

$$1+G(s)H(s) = 1+\frac{K}{s(s+5)(s+10)} = 0$$

$$s^3+15s^2+50s+K=0$$

$$K = -s^3-15s^2-50s$$

$$\frac{dK}{ds} = -3s^2-30s-50=0$$

$$3s^2+30s+50=0$$

$$s = \frac{-10\pm\sqrt{(10)^2-4\times16.67}}{2}$$

$$s = -2.113,\ -7.88$$

$s = -2.113$ is valid according to prediction it lies on par[t] of the Root locus

Step 6: Intersection with Imaginary axis

$$s^3+15s^2+50s+K=0$$

s^3	1	50
s^2	15	K
s^1	$\frac{750-K}{15}$	0
s^0	K	

$750-K=0$

$K_{mar} = 750$

$A(s): 15s^2+K=0$

$15s^2+750=$

$s^2 = \frac{-750}{15} =$

$s = \pm j\sqrt{50}$

$\pm j\,7.0$

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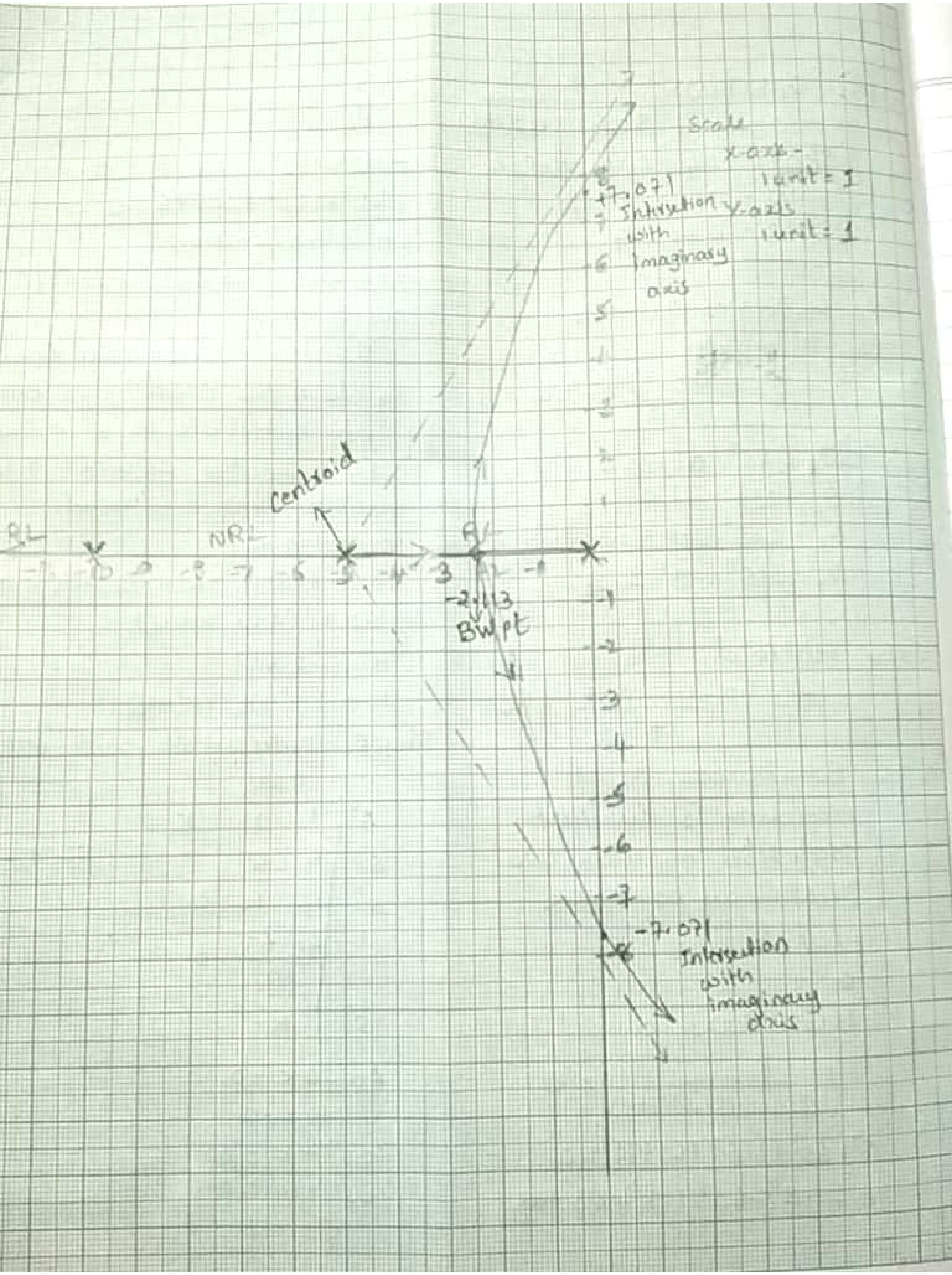
Scale
x-axis - 1 unit = 1
y-axis 1 unit = 1
+7.071 Intersection with imaginary axis
Centroid
RL
NR
-2.113
BW pt
-7.071 Intersection with imaginary axis

Date

(2) $G(s)H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$

Step 1: $P = 4$, $z = 0$, $N = P = 4$

Poles: $0, -1, -2, -3$

$P - z = 4 - 0 = 4$

Step 2: Sections of Real axis

Step 3: Asymptotes

$\theta = \pm \frac{(2q+1)180°}{P-z}$ $q = 0,1,2,3$

$\theta_1 = 45°$, at $q = 0$

$\theta_2 = 135°$ at $q = 1$

$\theta_3 = 225°$ at $q = 2$

$\theta_4 = 315°$ at $q = 3$

Step 4: Centroid

$\sigma = \frac{0-1-2-3-0}{4} = -1.5$

Step 5: Breakaway point

According to the first prediction minimum two Breakaway points are present

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s(s+1)(s+2)(s+3)} = 0$

$s^4 + 6s^3 + 11s^2 + 6s + K = 0$

$K = -s^4 - 6s^3 - 11s^2 - 6s$

$\frac{dK}{ds} = 0$

$-4s^3 - 18s^2 - 22s - 6 = 0$

$4s^3 + 18s^2 + 22s + 6 = 0$

on simplification

$s = -1.5, -0.381, -2.619$

out of these

$s = -1.5$ is not valid Breakaway pt

To check the validity of Breakaway pt substitute the s in Characteristic Eqn. If the obtained k is positive then it is valid.

For $s = -0.381$

$K = -(0.381)^4 - 6(-0.381)^3 - 11(-0.381)^2 - 6(-0.381)$

$K = 1$ (+ve value)

||ly $K = -2.619$

$K = 1$ (+ve value)

Step 6: Intersection with imaginary axis

s^4	1	11	K
s^3	6	6	
s^2	10	K	0
s^1	$\frac{60-6K}{10}$	0	
s^0	K		

$s = \pm j1$

$60 - 6K = 0$

$K = 10$

$10s^2 + K = 0$

$10s^2 + 10 = 0$

$s^2 = -1$

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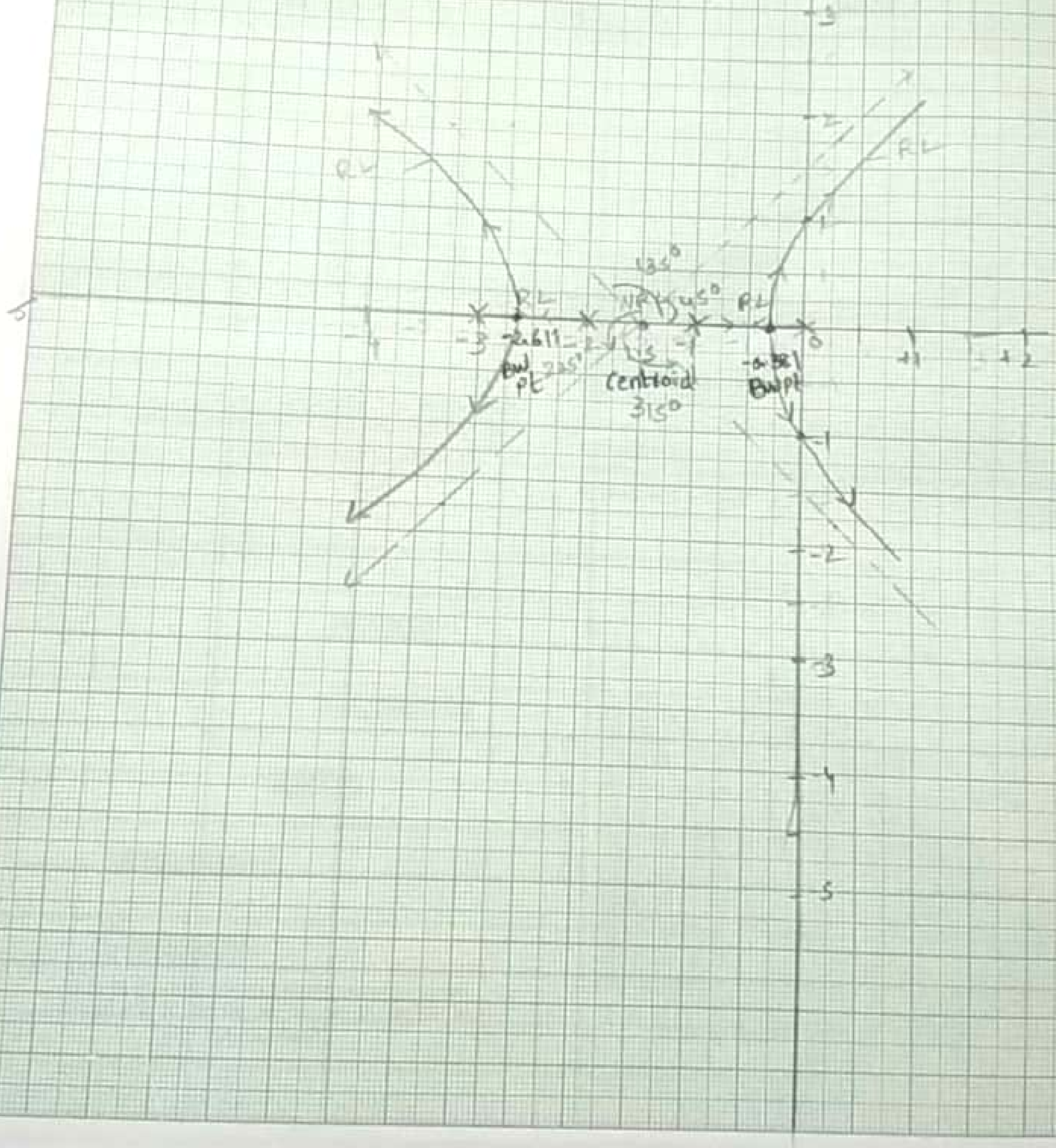
(2)

Scale

X-axis 2unit = 1unit/cm

Y-axis 2unit = 1unit/cm

$$G(s)H(s) = \frac{k}{s(s+1)(s+2)(s+3)}$$



(3) Given $G(s)H(s) = \dfrac{K}{s(s+3)(s^2+3s+4.5)}$

Step 1 $P=4$ $Z=0$ $N=P=4$

$P-Z = 4$

$P = 0, -3, -1.5 \pm j1.5$

(2) section on Real axis

(3) Asymptotes

$\theta = \dfrac{(2q+1)180^\circ}{P-Z}$ $q = 0,1,2,3$

$\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$

$\theta_4 = 315^\circ$

(4) centroid

$\sigma = \dfrac{\Sigma RP\ of\ Poles - \Sigma RP\ of\ zeros}{P-Z}$

$= \dfrac{0 - 3 - 1.5 - 1.5 - 0}{4}$

$\sigma = -1.5$

(5) Breakaway pt

$1 + G(s)H(s) = 0$

$1 + \dfrac{K}{s(s+3)(s^2+3s+4.5)} = 0$

$s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$

$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$

$\dfrac{dK}{ds} = 0$

$-4s^3 - 18s^2 - 27s - 13.5 = 0$

$4s^3 + 18s^2 + 27s + 13.5 = 0$

on solving we get

$s = -1.5, -1.5, -1.5.$

Substituting $s = -1.5$ in characteristic Eqn

$K = -(-1.5)^4 - 6(-1.5)^3 - 13.5(-1.5)^2 - 13.5(-1.5)$

$K = 5.0625$ (+ve Value)

Thus all $s = -1.5, -1.5, -1.5$ are Valid Breakaway point.

(6) Intersection with imaginary axis

s^4	1	13.5	K
s^3	6	13.5	0
s^2	11.25	K	0
s^1	$\dfrac{151.87 - 6K}{11.25}$	0	
s^0	K		

$K_{max} = 25.3125$

$11.25s^2 + K = 0$

$s^2 = 2.25$

$s = \pm j1.5$

(7) Complex pole is available so Angle of departure to be found

$\phi_d = 180^\circ - \phi$

$\phi = \Sigma\phi_p - \Sigma\phi_z$

$\phi_{P_1} = 140^\circ$, $\phi_{P_2} = 90^\circ$

$\phi_{P_3} = 40^\circ$

$\phi = 140^\circ + 90^\circ + 40^\circ = 270^\circ$

$\phi_d = 180^\circ - 270^\circ = -90^\circ$ for $-1.5+$

$\phi_d = +90^\circ$ for $-1.5 - j1.5$.

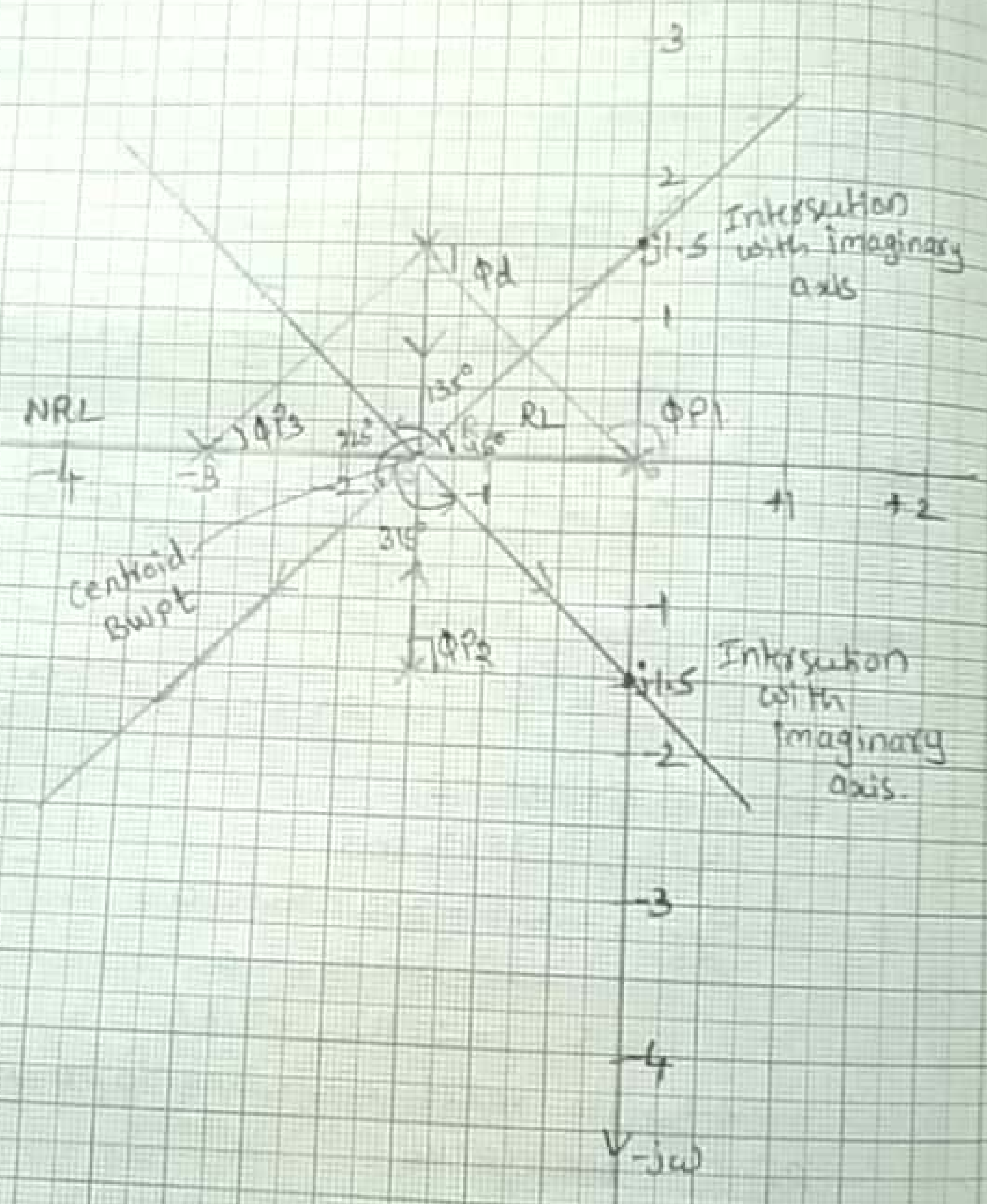
Teachers Signature

Scale

x-axis 2 unit = 1 cm

Y-axis 2 unit = 1 cm.

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+3s+4.5)}$$



(4) $G(s)H(s) = \dfrac{K}{s(s+3)(s^2+3s+11.25)}$

(1) $P = 4,\ Z = 0 \quad N = P = 4$

$P - Z = 4$

$P = 0, -3, -1.5 \pm j3$

(2) Section of real axis

(3) Asymptotes.

$\theta = \dfrac{(2q+1)180^\circ}{P-Z} \quad q = 0,1,2,3$

$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ$

$\theta_4 = 315^\circ$

(4) centroid.

$\sigma = \dfrac{0-3-1.5-1.5-0}{4} = -1.5$

(5) Breakaway pts

$1 + G(s)H(s) = 0$

$1 + \dfrac{K}{s(s+3)(s^2+3s+11.25)} = 0$

$s^4 + 6s^3 + 20.25s^2 + 33.75s + K = 0$

$K = -s^4 - 6s^3 - 20.25s^2 - 33.75s$

$s = -1.5, -1.5, \pm j1.837$

substitute -1.5 in characteristic eqn

$K = +20.25$

BW pt is imaginary so ~~Hey~~ Angle condition is to be verified.

$\angle G(s)H(s) = \pm(2q+1)180^\circ$

$\dfrac{\angle K + j0}{\angle -1.5 + j1.837\ \angle -1.5 + j1.837 + 3\ \angle -1.5 + j1.837 + 1.5 + j3\ \angle -1.5 + j1.837 + 1.5 - j3}$

$= \dfrac{0^\circ}{\angle 1.5 + j1.8371\ \angle 1.5 + j1.837\ \angle j4.837\ \angle -j1.1}$

$= \dfrac{0^\circ}{129.33^\circ\ 50.77^\circ\ 90^\circ\ (90^\circ)} = -180^\circ$

Odd multiple of 180° so $-1.5 \pm$

it is valid.

(6) Intersection with imaginary axis

s^4	1	20.25	K
s^3	6	33.75	0
s^2	14.625	K	0
s^1	$\dfrac{493-6K}{14.625}$	0	
s^0	K		

$K_{mar} = 82.265$

$s = \pm j2.3717$

(7) Angle of departure

$\theta p_1 = 116.56^\circ$

$\phi p_2 = 90^\circ, \ \phi p_3 = 63.43^\circ$

$\Sigma \phi_p = 270^\circ$

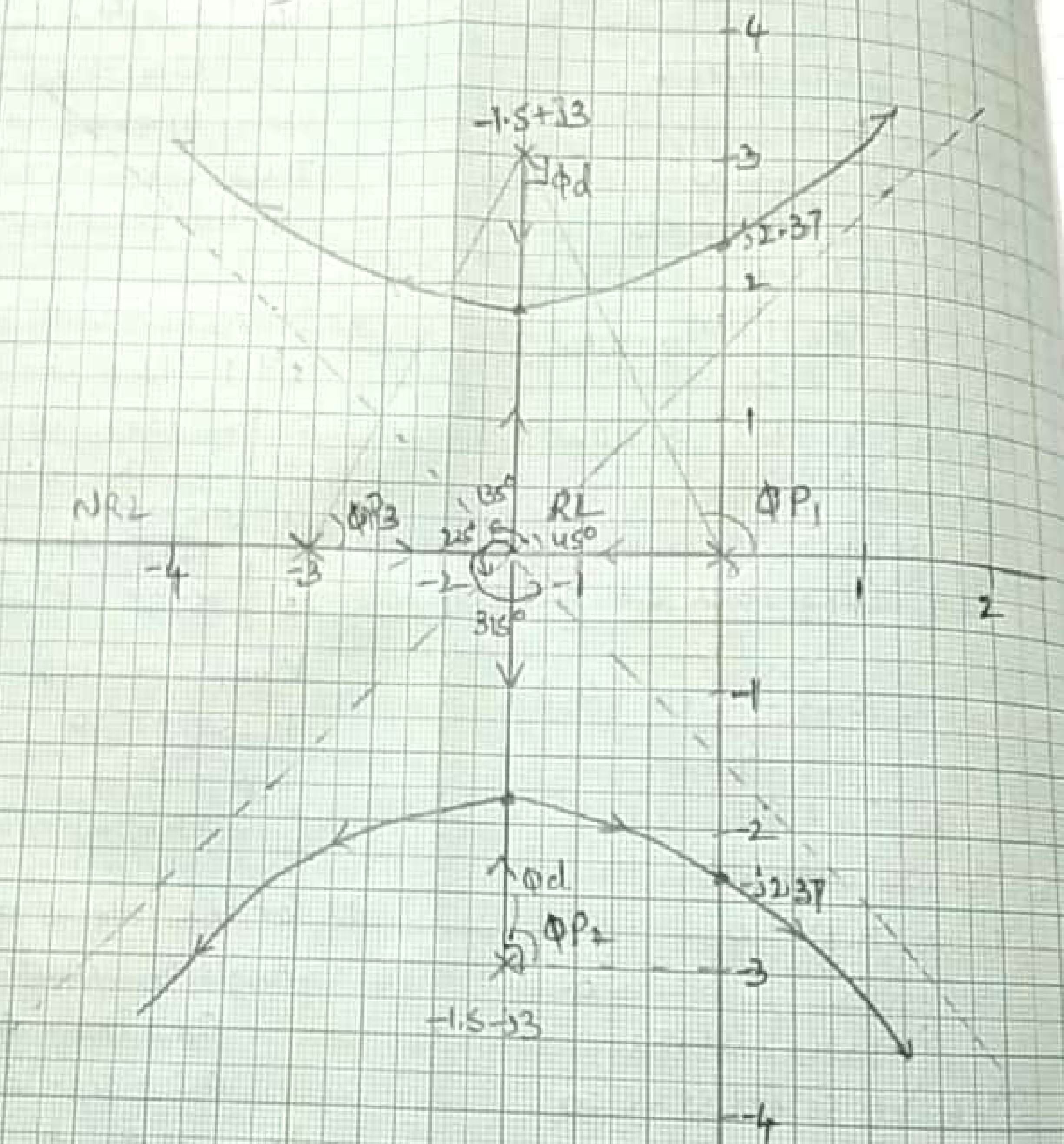
~~$\phi d = 270$~~

$\phi d = 180^\circ - 270^\circ = -90$ at

$\phi d = +90^\circ$ at $-1.5 - j1.8$

Scale
x-axis 1 unit = 1cm
Y-axis 1 unit = 1cm

(4) $G(s)H(s) = \frac{k}{s(s+3)(s^2+3s+11.25)}$



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(5) $G(s)H(s) = \frac{k}{s(s+3)(s^2+3s+3)}$

(1) $P = 4,\ Z = 0,\ N = P = 4$

$P - Z = 4$

Poles $s = 0, -3, -1.5 \pm j0.866$

(2) Sections on Real axis

(3) Asymptotes.

$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ,$

$\theta_4 = 315^\circ$

(4) centroid.

$$\sigma = \frac{0 - 3 - 1.5 - 1.5}{4} = -1.5$$

(5) Breakaway point

$1 + G(s)H(s) = 0$

$$1 + \frac{k}{s(s+3)(s^2+3s+3)} = 0$$

$s^4 + 6s^3 + 12s^2 + 9s + k = 0$

$k = -s^4 - 6s^3 - 12s^2 - 9s$

$$\frac{dk}{ds} = -4s^3 - 18s^2 - 24s - 9 = 0$$

on Solving we get

$s = -1.5, -0.633, -2.366$

Substituting $s = -1.5$ in Characteristic Eqn

$k = -(-1.5)^4 - 6(-1.5)^3 - 12(-1.5)^2 - 9(-1.5)$

$k = 1.6875$

$s = -0.633$

$k = -(-0.633)^4 - 6(-0.633)^3 -$
$12(-0.633)^2 - 9(-0.633)$

$k = 2.25$

IIIly $s = -2.366$

$k = 2.25$

All are positive, Thus all the $s = -1.5, -0.633$ & -2.366 are Valid.

(6) Intersection with imaginary axis

s^4	1	12	k
s^3	6	9	0
s^2	10.5	k	0
s^1	$\frac{94.5 - 6k}{10.5}$	0	
s^0	k		

$94.5 - 6k = 0$

$k_{max} = 15.75$

$A(s) = 10.5s^2 + k = 0$

$s^2 = -1.5$

$s = \pm j1.224$

(7) Angle of departure

$\phi_d = 180^\circ - \phi$

$\phi = \Sigma\phi_p - \Sigma\phi_z$

$\phi_{P1} = 150^\circ \quad \phi_{P2} = 90^\circ, \ \phi_{P3} = 30^\circ$

$\Sigma\phi_p = 150^\circ + 90^\circ + 30^\circ = 270^\circ$

$\phi_d = 180^\circ - 270^\circ = -90^\circ$ for $-1.5 + j0.866$

$\phi_d = +90^\circ$ for $-1.5 - j0.866$.

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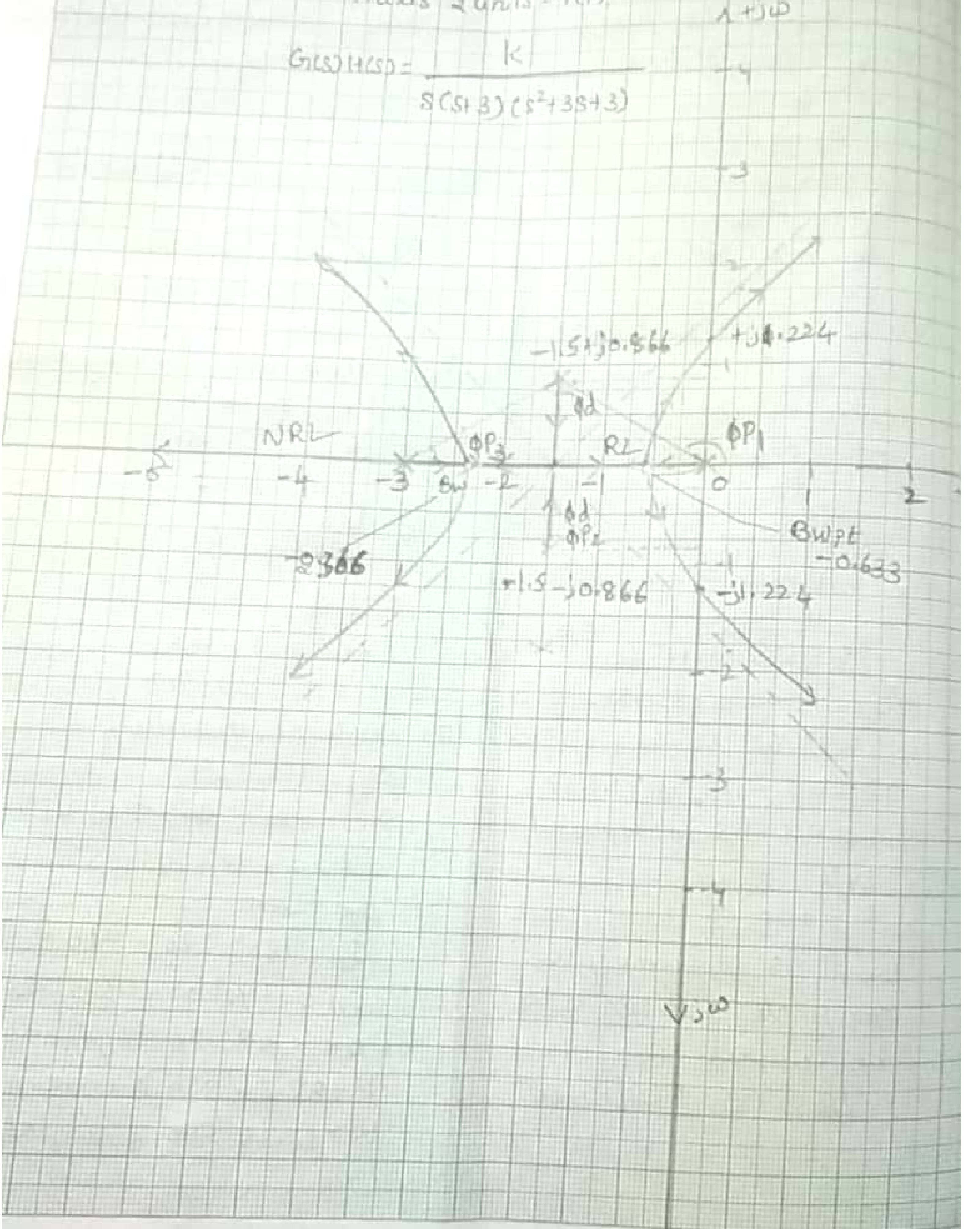
Scale

X-axis 2units = 1cm

Y-axis 2 units = 1cm

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+3s+3)}$$



Date

(6) Sketch the rough nature of the root locus of a certain control system whose characteristic Equation is given as
$s^3+9s^2+ks+k=0$ Comment on the stability

Sol^n) Dividing the characteristic Eqn by K

$s^3+9s^2+k(s+1)=0$

$$1+\frac{K(s+1)}{s^3+9s^2}=0$$

$1+G(s)H(s)=0$

$$G(s)H(s)=\frac{K(s+1)}{s^3+9s^2}$$

$$=\frac{K(s+1)}{s^2(s+9)}$$

$$G(s)H(s)=\frac{K(s+1)}{s^2(s+9)}$$

(1) Poles $s=0,0,-9$.
$P=3,\quad Z=1.\quad N=P=3$
zeros $s=-1$.
$P-Z=3-1=2$

(2) section on Real axis

(3) Asymptotes

$$\theta=\frac{(2q+1)}{P-Z}180^\circ \quad q=0,1$$

$\theta_1=90^\circ \quad \theta_2=180^\circ$

(4) centroid

$$\sigma=\frac{0-0-9+1}{2}=-4$$

(5) As per predictions there is no breakaway points. But need check mathematically

$s^3+9s^2+ks+k=0$

$$K=-\frac{s^3-9s^2}{(s+1)}$$

$$\frac{dk}{ds}=0$$

$$\frac{dk}{ds}=\frac{(s+1)(-3s^2-18s)-(-s^3-9s^2)(1)}{(s+1)^2}=0$$

$(s+1)(-3s^2-18s)-(-s^3-9s^2)=0$

$-3s^3-18s^2-3s^2-18s+s^3+9s^2=0$

$-2s^3-12s^2-18s=0$

$s(2s^2-12s+18)=0$

$s=0 \qquad s^2+6s+9=0$

$$s=\frac{-6\pm\sqrt{(-6)^2-4(1)(9)}}{2}$$

$s=0 \quad s=-3,-3$

$$K=\frac{-(-3)^3-9(-3)^2}{(-3+1)}=\frac{27-81}{-2}=27$$

(6) Intersection with imaginary axis

s^3	1	K
s^2	9	K
s^1	$\frac{8K}{9}$	0
s^0	K	

$9s^2+k=0$

$8K=0$
$K=0$

$s^2=0$

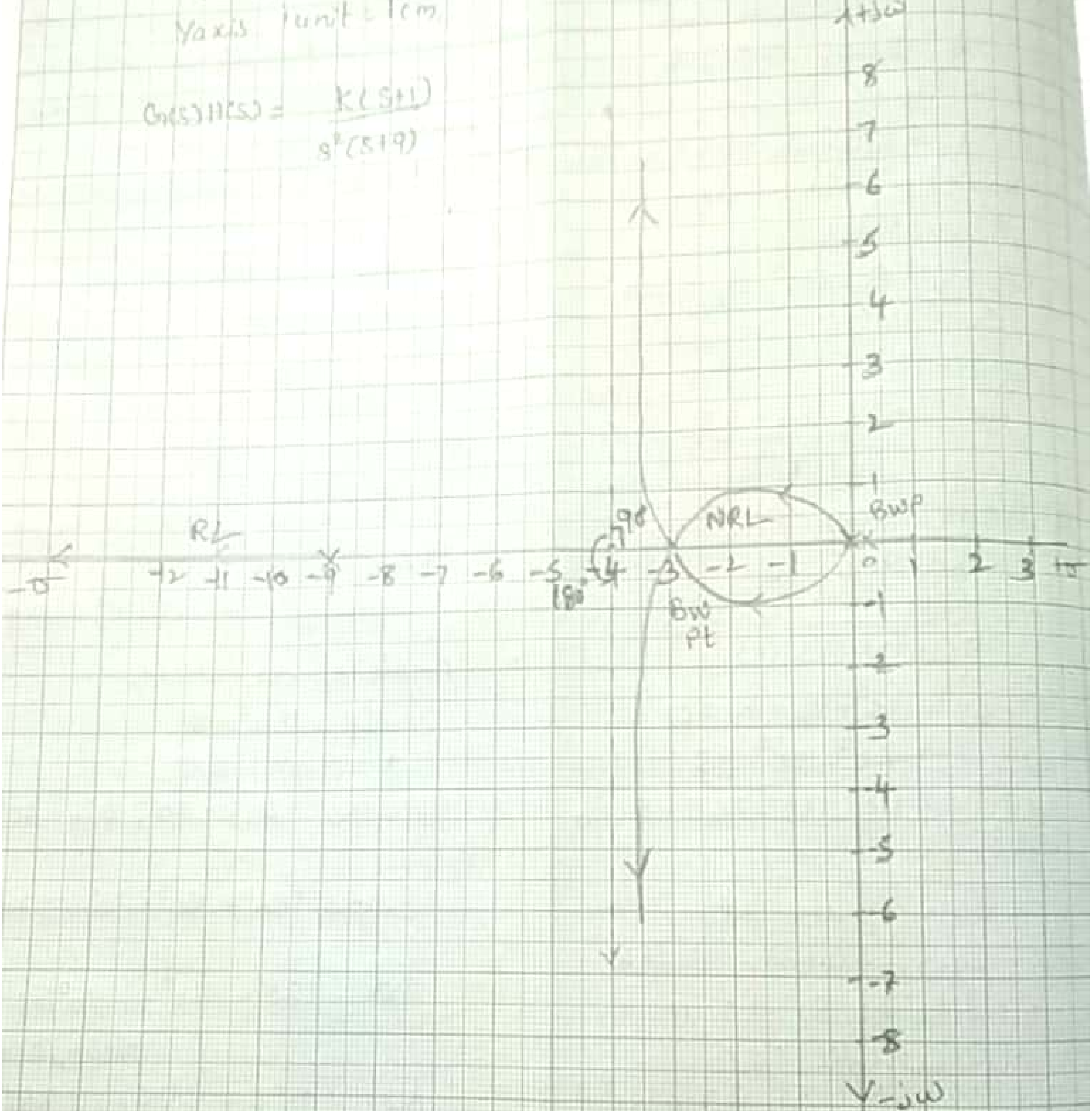
$s=0$

Root locus lies on left side of the s-plane Thus, the system is stable

Teachers Signature

Scale
x axis 1 unit = 1cm
Y axis 1 unit = 1cm

$$G(s)H(s) = \frac{K(s+1)}{s^2(s+9)}$$



Date

(7) $G(s) = \frac{k}{s(s+2)(s^2+6s+25)}$

$H(s) = 1$

1) poles $P=4$ $Z=0$ $N=P=4$

$P-Z = 4$

poles $s = 0, -2, -3 \pm j4$

2) sections on Real axis

3) Asymptotes

$\theta = \frac{(2q+1)180^\circ}{P-Z}$ $q = 0,1,2,3$

$\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

4) Centroid

$\sigma = \frac{0-2-3-3}{4} = -2$

5) Breakaway point

$1 + G(s) = 0$

$1 + \frac{k}{s(s+2)(s^2+6s+25)} = 0$

$s^4 + 8s^3 + 37s^2 + 50s + k = 0$

$k = -s^4 - 8s^3 - 37s^2 - 50s$

$\frac{dk}{ds} = 0$

$-4s^3 - 24s^2 - 74s - 50 = 0$

$s^3 + 6s^2 + 18.5s + 12.5 = 0$

on solving

$s = -0.898$ is valid

$k = +20.2$

(6) Intersection with imaginary axis

s^4	1	37	k
s^3	8	50	0
s^2	30.75	k	
s^1	$\frac{1537.5-8k}{30.75}$	0	
s^0	k		

$1537.5 - 8k = 0$

$k_{mar} = 192.187$

$A(s) = 30.75s^2 + k = 0$

$s^2 = -6.25$

$s = \pm j2.5$

(7) Angle of Departure

$\phi d = 180^\circ - \phi$

$\phi = \Sigma\phi p - \Sigma\phi z$

$\phi p_1 = 126^\circ$

$\phi p_2 = 105^\circ$

$\phi p_3 = 90^\circ$

$\Sigma\phi p = 126 + 105 + 90 = 321$

$\phi d = 180^\circ - 321 = -141^\circ$

$\phi d = -141^\circ$ for $-3+j4$

$\phi d = +141^\circ$ for $-3-j4$

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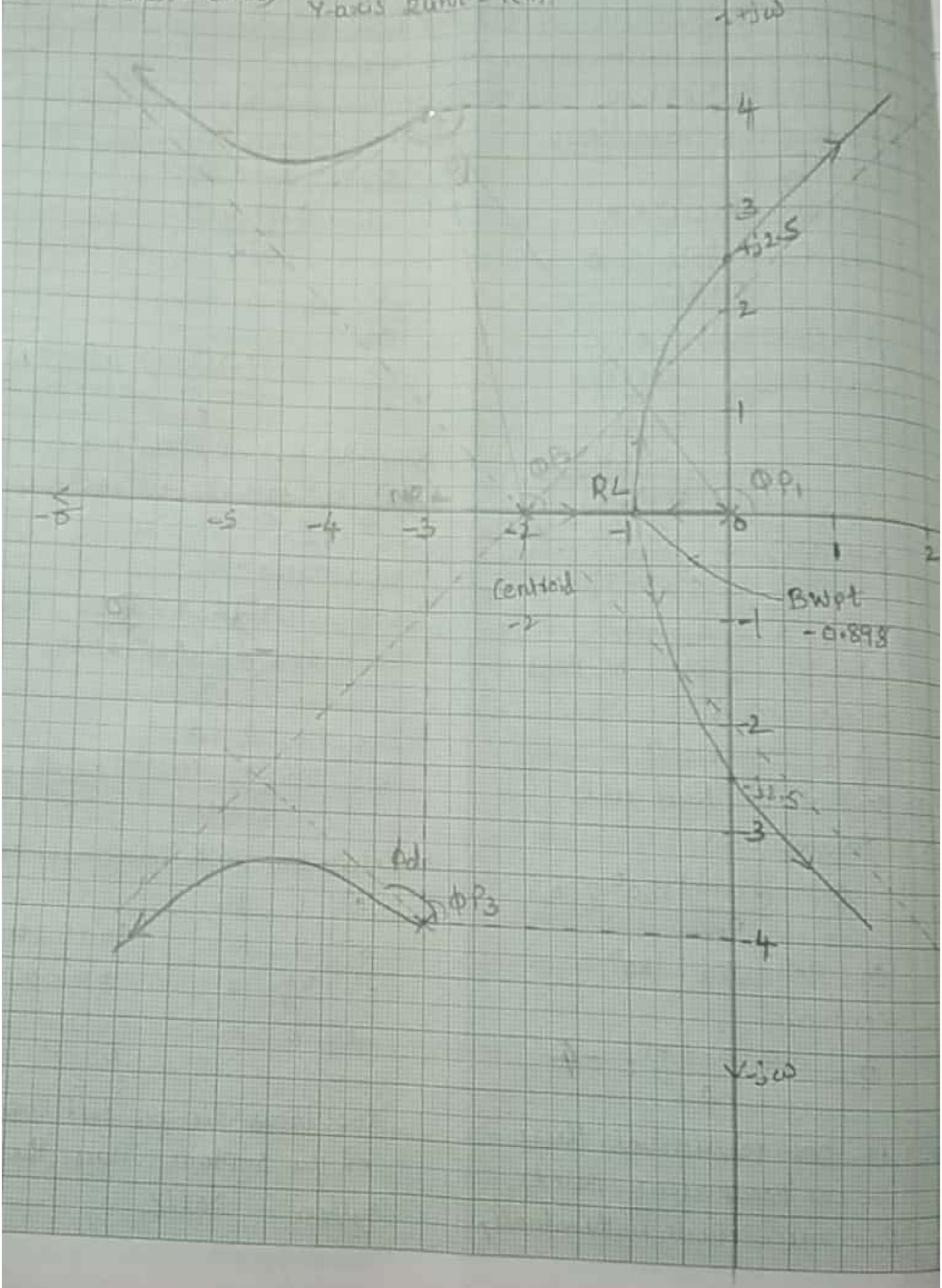
$$G(s) = \frac{K}{s(s+2)(s^2+6s+25)}$$

Scale
x-axis 2unit = 1cm
Y-axis 2unit = 1cm



(8) Sketch the root locus plot of a unity feedback system with an open loop transfer function of $G(s) = \frac{1}{s(s+2)(s+4)}$. Find the range of values of K for which the system has damped oscillatory response. What is the greatest value of K which can be used before continous oscillations occur? Also determine the frequency of continous oscillations. Also determine the value of K so that the dominant pair of complex poles of the system has a damping ratio of 0.5. Corresponding to this value of K determine the closed loop transfer function in factored form.

Sol^{n}) $G(s) = \frac{K}{s(s+2)(s+4)}$

1) Poles $P = 3$, $z = 0$, $N = P = 3$

$P - Z = 3$

Zeros $= 0$

Poles $s = 0, -2, -4$

(2) Sections on Real axis.

(3) Asymptotes

$\theta = \frac{(2q+1)}{P-Z} 180^\circ \quad q = 0, 1, 2$

$\theta_1 = 60^\circ, \theta_2 = 180^\circ, \theta_3 = 300^\circ$

(4) Centroid

$\sigma = \frac{0-2-4-0}{3} = -2$

(5) Breakaway point

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s(s+2)(s+4)} = 0$

$s^3 + 6s^2 + 8s + K = 0$

$K = -s^3 - 6s^2 - 8s$

$\frac{dK}{ds} = 0 \quad -3s^2 - 12s - 8 = 0$

$s = -0.846, -3.153$

$s = -0.846$

$K = -(-0.846)^3 - 6(-0.846)^2 - 8(-0.846)$

$K = 3.079$ (+ve value) valid

(6) Intersection with imaginary axis

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	

$48 - K = 0$

$K_{max} = 48$

$6s^2 + K = 0$

$s^2 = -\frac{48}{6} = -8$

$s = \pm j2.828$

(7) $\xi = 0.5$

$\cos^{-1}\xi = 60^\circ$

(8) $P(-0.75 + j1.25)$ $|G(s)H(s)|$ at $P = 1$

$\left| \frac{K}{(-0.75+j1.25)(-0.75+j1.25+2)(-0.75+j1.25+4)} \right| = 1$

$K = 8.94$

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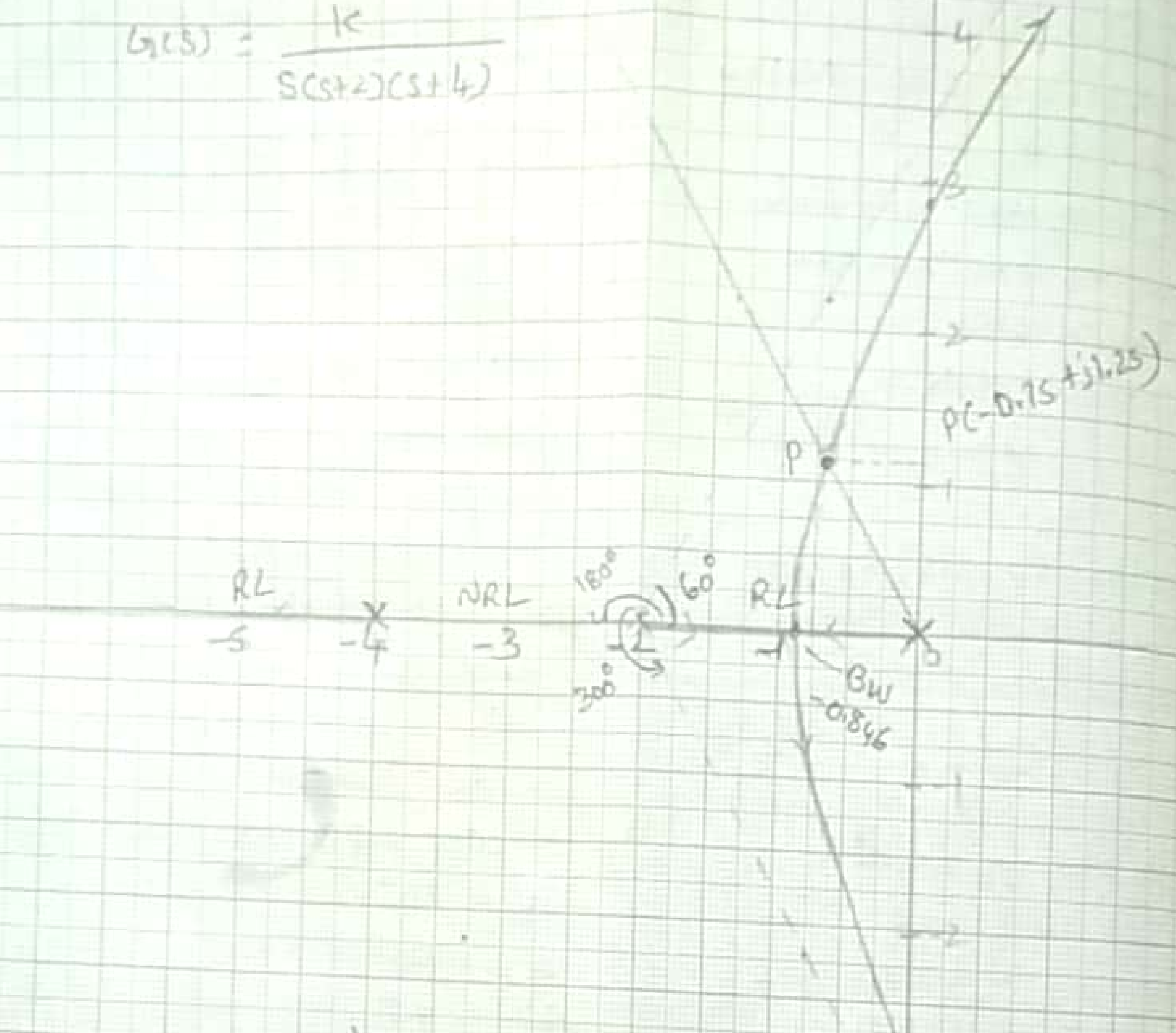
Scale

x axis 2 units = 1 cm

Y-axis 2 units = 1 cm

$$G(s) = \frac{K}{S(S+2)(S+4)}$$



$$R(s) = \frac{1}{S}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(S+0.75+1.25j)(S+0.75+1.25j)}$$

$$\frac{C(s)}{R(s)} = \frac{8.94}{S^2+1.5S+2.125}$$

Date

(1) The characteristic Equation of a single loop unity feedback control system is given by $F(s) = s^3 + 8s^2 + 20s + k = 0$ Sketch the complete root locus diagram and from that find,

i) Two values of k that make the system critically damped

ii) Two Values of k for which the damping ratio is 0.95

iii) write closed loop transfer functions for the values of k found in Part (ii)

Sol^n $F(s) = s^3 + 8s^2 + 20s + k = 0$

$$1 + \frac{k}{s^3 + 8s^2 + 20s} = 0$$

$$1 + \frac{k}{s(s^2 + 8s + 20)} = 0$$

$$G(s)H(s) = \frac{k}{s(s+4+j2)(s+4-j2)}$$

(1) Poles $P = 3$, $Z = 0$, $N = P = 3$

$P - Z = 3 - 0 = 3$

Poles $s = 0, -4 \pm j2$

(2) Sections on Real axis

(3) Asymptotes.

$$\theta = \frac{(2q+1)180^\circ}{P-Z} \quad q = 0,1,2$$

$\theta_1 = 60^\circ$ $\theta_2 = 180^\circ$, $\theta_3 = 300^\circ$

(4) centroid

$$\sigma = \frac{0-4-4}{3} = -2.667$$

(5) Breakaway point

$K = -s^3 - 8s^2 - 20s$

$$\frac{dk}{ds} = 0$$

$-3s^2 - 16s - 20 = 0$

$3s^2 + 16s + 20 = 0$

on solving

$s = -2, -3.33$

At $s = -2$ $k = +16$

At $s = -3.33$ $k = +14.8148$

(6) Intersection with imaginary axis

s^3	1	20
s^2	8	K
s^1	$\frac{160-k}{8}$	0
s^0	K	

$160 - k = 0$

$K_{mar} = 160$

(6) $8s^2 + k = 0$

$s^2 = \frac{-160}{8} = -$

$s = \pm j4.472$

(7) Angle of departure

$\phi_d = 180^\circ - \phi$

$\phi = \Sigma\phi_p - \Sigma\phi_z$

$\phi_{p_1} = 154^\circ$

$\phi_{p_2} = 90^\circ$

$\Sigma\phi_p = 154^\circ + 90^\circ = 244^\circ$

s $\phi_d = 180^\circ - 244 = -64^\circ$

(8) $\xi = 0.95$

$\theta = \cos^{-1}0.95 = 18.19^\circ$

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ndra's

$P(-1.8+j0.55)$ and $Q(-3.6+j1.1)$

$$|G(s)H(s)|_P = \frac{|K|}{|-1.8+j0.55||-1.8+j0.55+4+j2||-1.8+j0.55+4-j2|} = 1$$

$$|K| = 1.8821 \times 3.3678 \times 2.6348 = 16.7011$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2}$$

$$G(s) = \frac{K}{s(s^2+8s+20)} \quad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s(s^2+8s+20)+K}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{16.7011}{s^3+8s^2+20s+16.7011}}$$

$$G(s)H(s)\Big|_{\text{at } Q} = \frac{|K|}{|-3.6+j1.1||-3.6+j1.1+4+j2||-3.6+j1.1+4-j2|} = 1$$

$$|K| = 11.5872$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^3+8s^2+20s+K} = \frac{11.5872}{s^3+8s^2+20s+11.5872}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{11.5872}{s^3+8s^2+20s+11.5872}}$$

Scale

x-axis 2unit = 1cm

Y-axis 2unit = 1cm

$$G(s) = \frac{K}{s(s^2+8s+20)}$$