Bode plots

→ Frequency response is to plot magnitude M and angle ϕ against i/p freq ω.

→ ω is varied from 0 to ∞ there is a wide range of variations in M and ϕ & hence it becomes difficult to accomodate all such variations with linear scale

→ Hence Bode p. suggested the method in which logarithmic values of magnitude are to be plotted against logarithmic values of freq

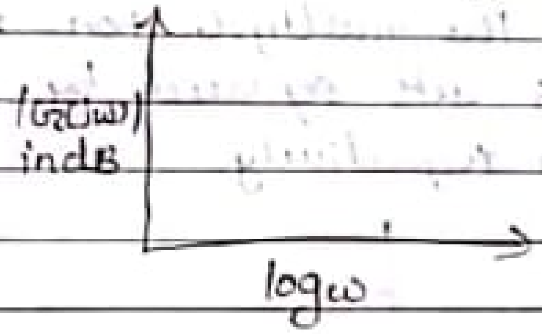
→ Bode plot consist of two plots which are

(1) Magnitude expressed in logarithmic values against logarithmic values of freq called magnitude plot

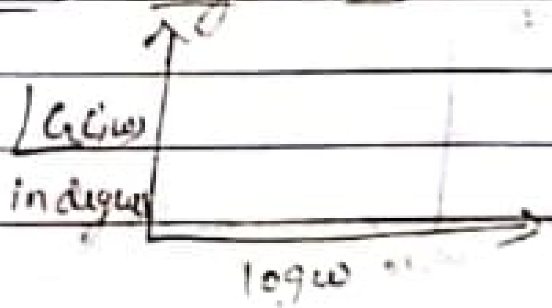
(2) Phase ~~angle~~ angle in degrees against logarithmic values of freq called Phase Angle plot.

Magnitude Plot

$$|G(j\omega)| = 20 \log_{10} |G(j\omega)| \text{ dB}$$



Phase Angle Plot

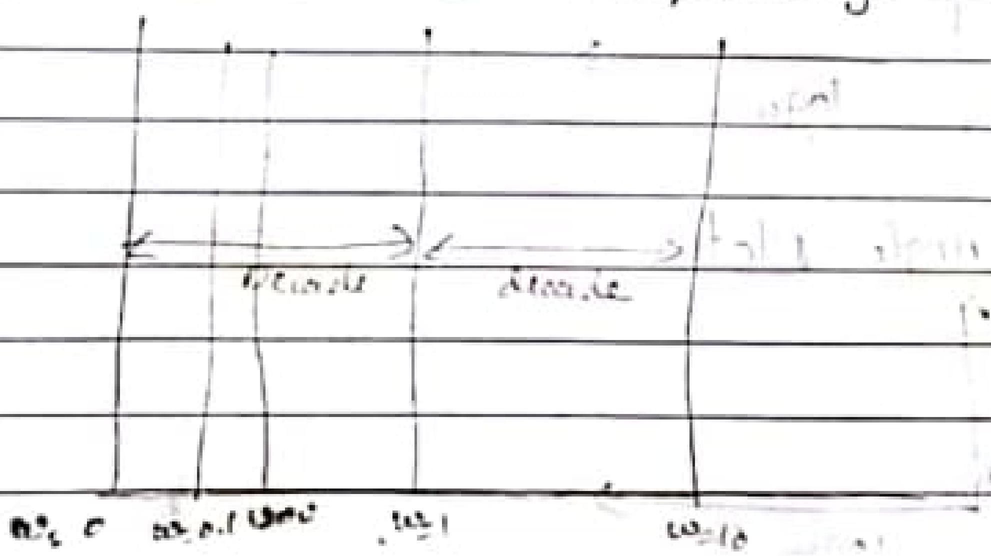


Logarithmic Scales (Semilog paper)

- X-axis is divided into a logarithmic scale which is non linear one while Y-axis is divided into linear scale & hence it is called Semilog paper.
- In x-axis distance b/w 1 & 2 is greater distance b/w 2 & 3 & so on.
- Similarly distance b/w 1 & 10 is same as 10 & 100. This distance is called 1 decade.
- log 1 = 0 & log 10 = 1,
  log 2, log 3 ------
- log 10 = 1   log 100 = 2   log 1000 = 3   distance of 1 decade b/w log 1 & log 10, further divided into parts as log 20, log 30, ...
- It is not necessary to find logarithmic values of ω while plotting on x-axis but logarithmic scale available takes care of logarithmic value of ω.

Standard

The main advantage of using the logarithmic representation is that the multiplication & division of magnitudes gets replaced by the addition & subtraction respectively.



Standard form of open loop TF $G(j\omega)H(j\omega)$

Consider $G(s)H(s) = \dfrac{K' s^z (s+z_1)(s+z_2)}{s^p (s+P_1)(s+P_2)\ldots}$

either s^z or s^p will be present at a time & not both.

Rewrite in the time constant form.

$$G(s)H(s) = \frac{s^z K (1+T_1 s)(1+T_2 s)\ldots}{s^p (1+T_a s)(1+T_b s)\ldots}$$

$$K = \frac{z_1 \times z_2 \times \ldots}{P_1 \times P_2 \times \ldots} \times K'$$

again either s^z or s^p is present & not both.

The standard time constant form can be denoted as

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^p (1+T_a s)(1+T_b s)\ldots}$$

K – resultant sm gain   P – type of the sm

$T_1\ T_2\ T_a\ T_b \ldots$ = Time constants of different poles & zeros

Each of the factor involved in $G(s)H(s)$ above will contribute to magnitude and angle variation of $G(j\omega)H(j\omega)$ in freq domain.

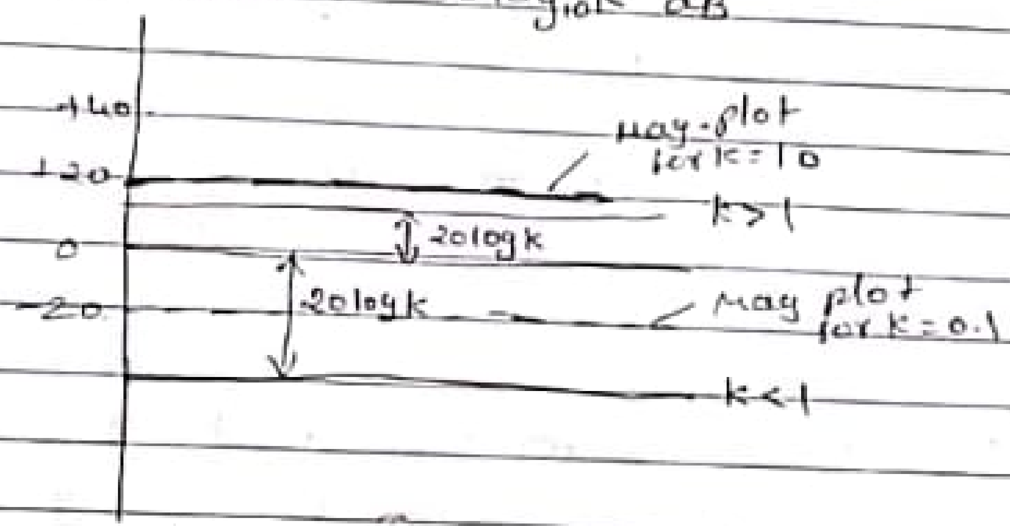
$$G(j\omega)H(j\omega) = \frac{K(1+T_1 j\omega)(1+T_2 j\omega)\ldots}{(j\omega)^p (1+T_a j\omega)(1+T_b j\omega)\ldots}$$

Bode plots of standard factors of $G(j\omega)H(j\omega)$

1) Replace 's' by 'jω' to convert it to freq domain.
2. Find its magnitude as a function of 'ω'.
3. Express the magnitude in dB' by $20\log_{10}|G(j\omega)H(j\omega)|$
4. Find the phase angle by using $\tan^{-1}\left[\frac{\text{imaginary part}}{\text{real part}}\right] = \phi$ in degrees
5. with required approximations, plot magnitude in dB, & phase angle in degrees against $\log\omega$ by varying ω from 0 to ∞

<u>factor 1</u> Steady gain 'k'

$G(s)\,H(s) = K$
$G(j\omega)\,H(j\omega) = K + j0$
$|G(j\omega)\,H(j\omega)| = \sqrt{K^2 + 0} = K$
its 'dB' value $= 20\log_{10}K$ dB

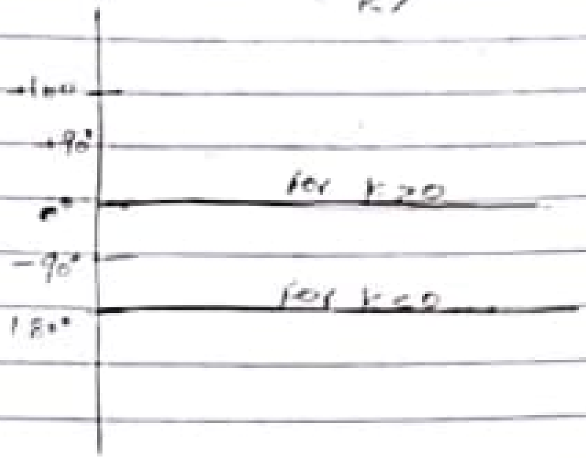


'K' shifts the magnitude plot of $|G(j\omega)H(j\omega)|$ by a distance of $20\log K$ dB upwards if $K>1$ & downwards if $K<1$

Phase angle plot

$G(j\omega)H(j\omega) = K + j0$

$\phi = \tan^{-1}\left(\frac{0}{K}\right) = 0°$ K negative $= -180°$



Factor 2:- poles or zeros at the origin

$G(s)H(s) = \frac{1}{s}$

$G(j\omega)H(j\omega) = \frac{1}{j\omega} = \frac{1}{0 + j\omega}$

Magnitude $= \frac{1}{\sqrt{0^2 + \omega^2}} = \frac{1}{\omega}$

Magnitude in dB $= 20 \log_{10} \frac{1}{\omega}$

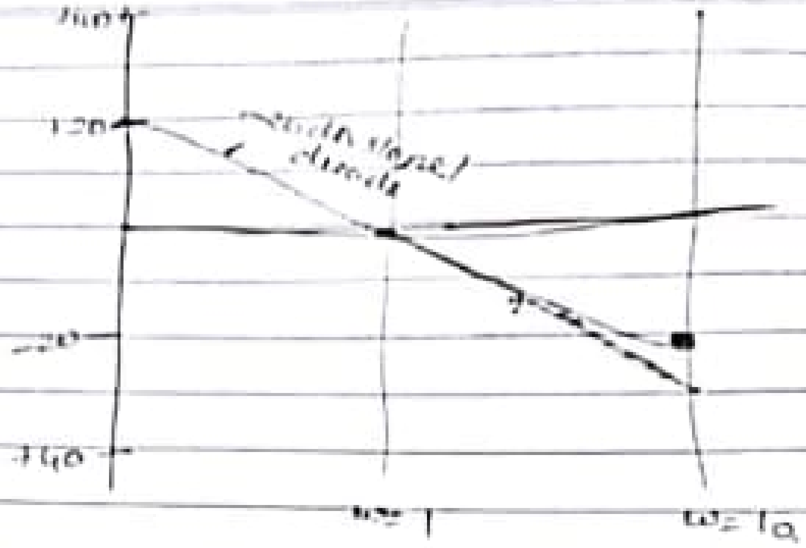
$= -20 \log \omega.$

$\omega = 1$ $|G(j\omega)H(j\omega)| = 0$ dB

$\omega = 10$ $= -20$ dB

$\omega = 100$ $= -40$ dB

$\omega = 0.1$ $= +20$ dB

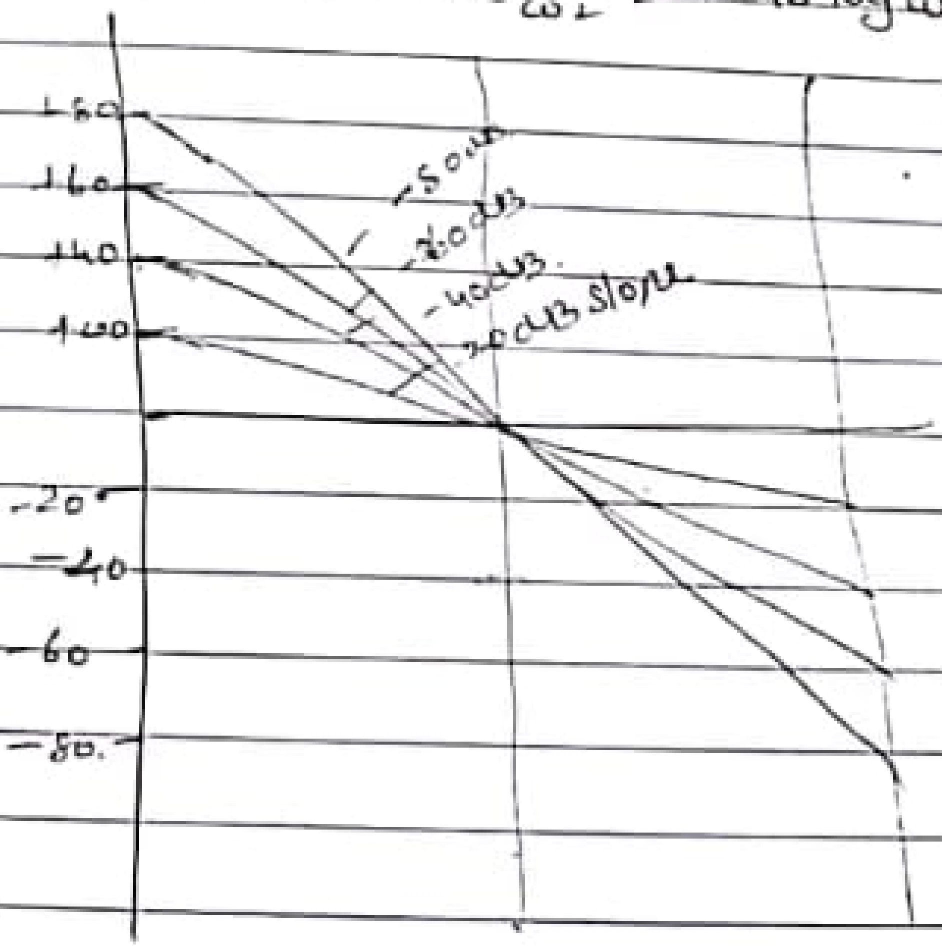


$$G(s)H(s) = \frac{1}{s^2}$$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega}\cdot\frac{1}{j\omega}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\omega}\cdot\frac{1}{\omega} = \frac{1}{\omega^2}$$

$$|G(j\omega)H(j\omega)| = 20\log\frac{1}{\omega^2} = -40\log\omega$$



only zero

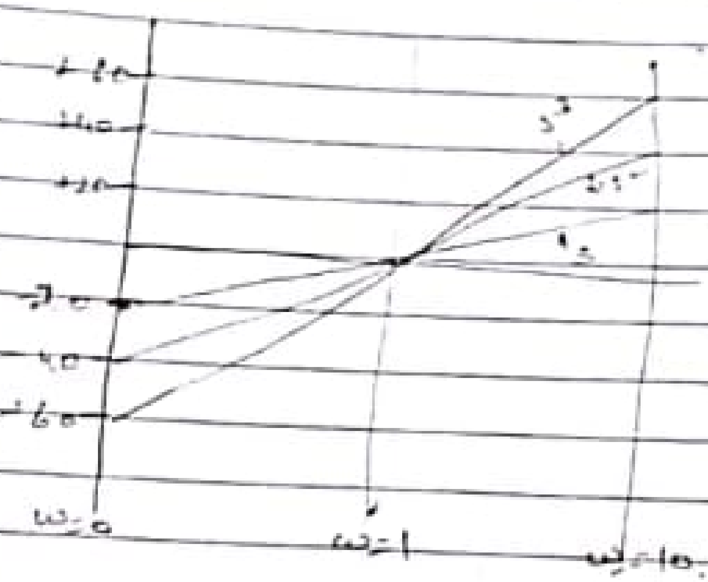
$G(s)H(s) = s$

$G(j\omega)H(j\omega) = j\omega$

$|G(j\omega)H(j\omega)| = \omega$

$= 20 \log \omega$

$\omega = 1$	rad 0 dB
$\omega = 10$	+20 dB
$\omega = 100$	+40 dB
$\omega = 0.1$	−20 dB



Phase plot

$G(s)H(s) = 1/s$

$G(j\omega)H(j\omega) = \frac{1}{j\omega}$

$\angle G(j\omega)H(j\omega) = \angle \frac{1}{j\omega} = \frac{0^\circ}{90^\circ} = -90^\circ$

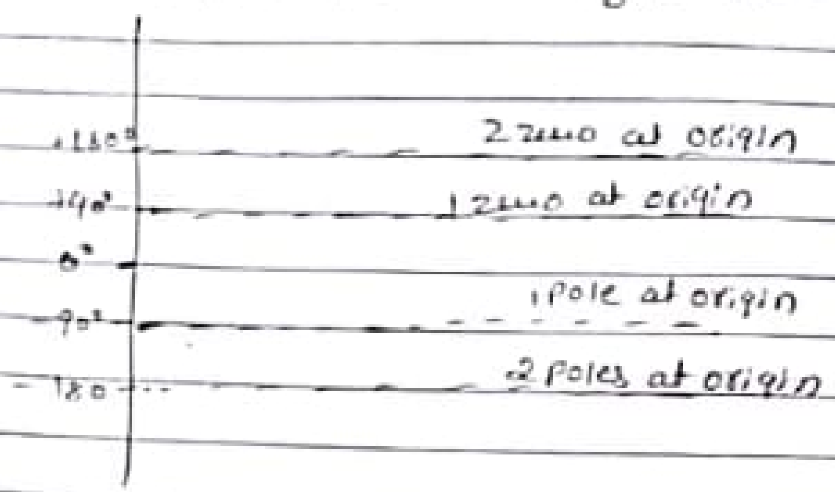
$G(s)H(s) = 1/s^2$

$\angle G(j\omega)H(j\omega) = \frac{1}{j\omega} \times \frac{1}{j\omega} = \frac{0^\circ}{90^\circ\, 90^\circ} = -180^\circ$

$G(s)H(s) = s$

$G(j\omega)H(j\omega) = j\omega$

$\angle G(j\omega)H(j\omega) = \angle j\omega = \tan^{-1}\left(\frac{\omega}{0}\right) = 90^\circ$



The magnitude plots for poles & zeros at the origin are straight lines having slope $-20 \times P$ dB/dec or $+20 \times P$ dB/decade respectively passing through the intersection pt of $\omega = 1$ at 0dB

→ Adding magnitude plot of 'K' to the above means to shift the straight line drawn, upwards or downwards by $20 \log K$ dB depending on whether K is greater or less than 1

→ Net addition of 'K' & pole or zeros at origin will be a line parallel to line representing poles or zeros at the origin at a distance of $20 \log K$ dB upwards or downwards from the 0dB line

Consider $G(s)H(s) = \frac{10}{s}$

$G(j\omega)H(j\omega) = \frac{10}{j\omega}$

(i) constant $K=10$, its contribution to magnitude plot is $20\log_{10}10 = 20dB$

(ii) one pole at the origin whose magnitude plot is straight line of slope $-20dB/decade$ passing through intersection pt of $\omega=1$ & $0dB$ line

Now at $\omega=1$ total magnitude is

$= 20dB$ due to K + $0dB$ due to 1 pole at origin

$= 20dB$

(i) Draw magnitude plot for K.

(ii) Draw straight line representing pole at origin ie slope $-20dB \times P$ dB/decade passing through the intersection pt of $\omega=1$ & $0dB$

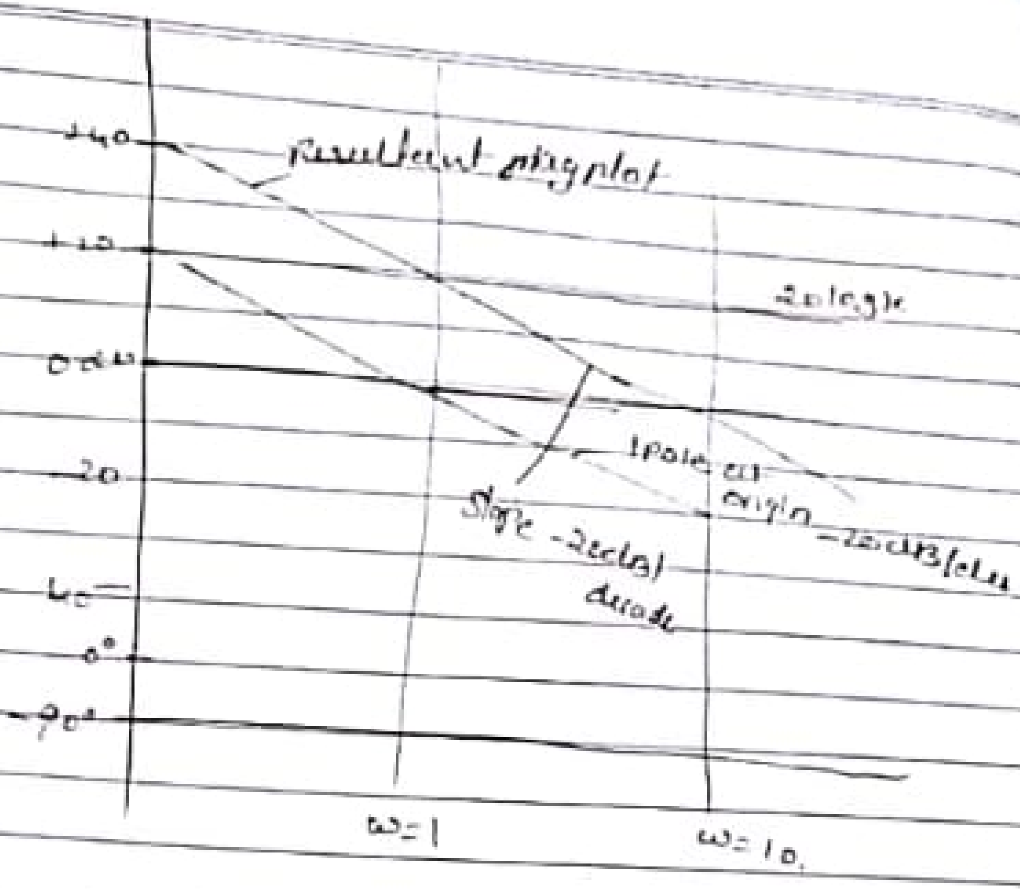
(iii) Shift intersection pt of $\omega=1$ & $0dB$ on the line representing $20\log K$ line

(iv) Draw parallel line to the line representing pole at origin from the point obtained in step (iii)

Phase angle plot:

$\angle G(j\omega)H(j\omega) = \angle\frac{10}{j\omega} = \frac{0^\circ}{\tan^{-1}(\frac{\omega}{0})} = \frac{0^\circ}{90^\circ} = -90^\circ$

ω	contribution by K	by 1 pole at origin	Resultant
0	0°	-90°	-90°
10	0°	-90°	-90°
40	0°	-90°	-90°
1000	0°	-90°	-90°



Simple poles or zeros (first order factor)

$$G(s)\,H(s) = (1+Ts)^{-1} = \frac{1}{1+Ts}$$

$$G(j\omega)\,H(j\omega) = \frac{1}{1+Tj\omega}$$

$$|G(j\omega)\,H(j\omega)| = \frac{1}{\sqrt{1+(T\omega)^2}} = \left[\sqrt{1+(T\omega)^2}\right]^{-1}$$

In dB magnitude $= 20 \log\left[\sqrt{1+(T\omega)^2}\right]^{-1}$

$$= -20 \log (1+\omega^2 T^2)^{1/2}$$

If $\omega \ll \frac{1}{T}$ i.e. $\omega^2 T^2 \ll 1$

$$-20 \log 1 = 0 dB$$

If $\omega \gg 1/T$ $\omega^2 T^2 \gg 1$

$$\text{magnitude} = -20 \log \omega T \ dB$$

For low freq, magnitude plot is 0dB line &
for high freq it is straight line of slope -20dB

Frequency ω, which separates the freq range into two, low & high.
-20dB/decade slope line intersects with 0dB

$$-20 \log \omega T = 0$$
$$-20 \log \omega T = -20 \log 1$$
$$\omega T = 1$$
$$\omega = 1/T$$

This is the corner freq $\omega c = 1/T$

i. Sketch the Bode plot for the sys having
$$G(s)H(s) = \frac{20}{s(1+0.1s)}$$

i) $K = 20$ $\quad 20 \log 20 = +26\,dB$

ii) 1 pole at origin. Its magnitude plot is straight line passing through intersection pt of $\omega = 1$ & $0\,dB$ with slope of $-20\,dB/decade$

iii) simple pole $\rightarrow \frac{1}{1+0.1s}$ compare with $\frac{1}{1+Ts}$

$T = 0.1$

$\omega_c = \frac{1}{T} = \frac{1}{0.1} = 10\,rad/sec$

For magnitude plot

(i) Draw $20 \log K$ line

ii) Draw line for 1 pole at origin.

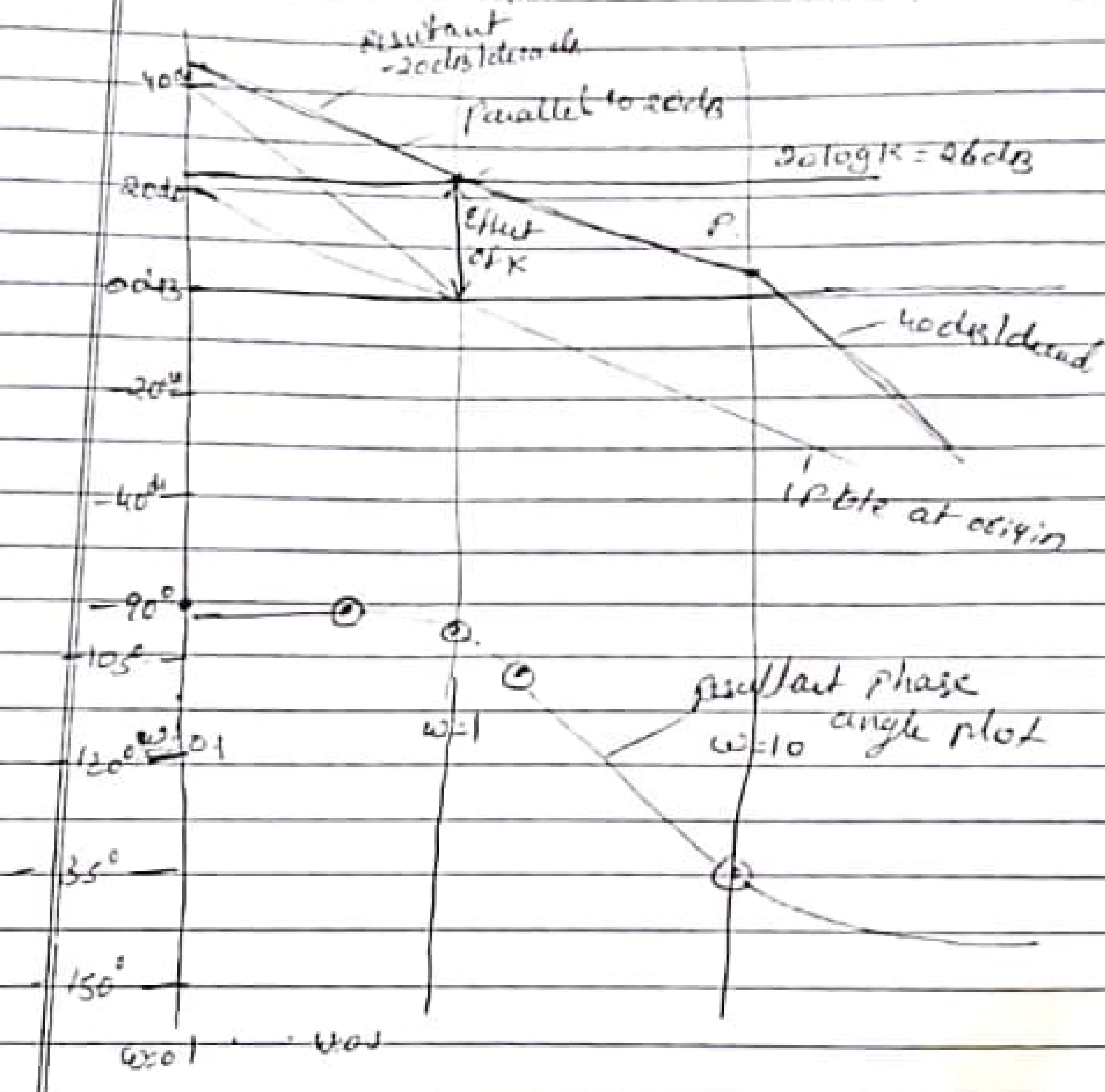
iii) Shift intersection pt of $\omega = 1$ & $0\,dB$ on ~~on~~ $20 \log K$ line & from this pt draw ~~line~~ parallel to ~~a~~ line representing 1 pole at origin. This line will have slope of $-20\,dB/decade$

iv) Thus addition of K & poles at origin will continue till nxt factor becomes dominant ie at $\omega = \omega_c = 10$

(v) from $\omega = 10$ onwards will be ($-20\,dB/decade$ as starting slope) + ($-20\,dB/decade$) due to the simple pole ie resultant is $-40\,dB/decade$

Scanned by CamScanner

ω in rad/sec	φ due to 1 pole at origin	φ due to simple pole $\tan^{-1}(\frac{0.1\omega}{1})$	ϕ_R resultant φ
0.1	-90	-0.57°	-90.57
0.5	-90	-2.86°	-92.86
1	-90	-5.7°	-95.7
2	-90	-11.3°	-101.3°
10	-90	-45°	-135°
50	-90	-78.77°	-168°



Steps to sketch the Bode plot

1. Express given G(s) H(s) into time constant form
2. Draw a line of 20 log K dB
3. Draw a line of appropriate slope representing poles or zeros at the origin, passing through intersection point of $\omega = 1$ & 0 dB
4. Shift this intersection pt on 20 log K line & draw parallel line to the line drawn in step 3. This is addition of constant K & no. of poles or zero's at the origin.
5. Change the slope of this line at various corner freq by appropriate value i.e depending upon which factor is occuring at corner frequency. For a simple pole, slope must be changed by -20 dB/decade, for a simple zero by +20 dB/dec etc. Do not draw these individual lines. Change the slope of line obtained in step 5 by respective value & draw line with resultant slope. Continue this line till it intersects nxt corner freq line change the slope & continue. Apply necessary correction for quadratic factor
6. Prepare the phase angle table & obtain the table of ω & resultant phase angle ϕ_R by actual calculation. Plot these pts & draw the smooth curve obtaining the nessary phase angle plot