Module 4: Complex Variables II

Q.No	Question
1.	a) Evaluate $I = \int_C \|z\|^2 \, dz$, where C is square with the following vertices (0, 0), (1, 0), (1, 1) and (0, 1). b) Evaluate $I = \int_{z=0}^{2+i} (\bar{z})^2 \, dz$, along (i) the straight line $y = \frac{x}{2}$, (ii) the real axis from 0 to 2 and then vertically to $2 + i$.
2.	a) Evaluate $I = \int_C z^2 \, dz$ along the curve made up of two line segment, one from z = 0 to z = 3 and another from $z = 3 \; to \; z = 3 + i$. b) Evaluate $I = \int_{z=0}^{z=1+i} (x^2 - iy) \, dz$ along (i) the straight line $y = x$ and (ii) the parabola $y = x^2$.
3.	a) Evaluate $I = \int_C \|z\| \, dz$, where (i) C is the straight line from the point $z = -i$ to the point $z = i$ (ii) C is the left half of the circle $\|z\| = 1$ from $z = -i$ to $z = i$. b) Evaluate $\int_{z=1-i}^{z=2+3i} (z^2 + z) \, dz$ along the straight line joining the points $(1, -1), \& \; (2, 3)$.
4.	a) Evaluate $\int_{z=3i}^{z=2+4i} (2y + x^2) \, dx + (3x - y) dy$ along the straight from (0, 3) to (2, 4) b) State and prove Cauchy's theorem.
5.	a) Verify Cauchy's theorem for the analytic function $f(z) = z^2$ where C is the square having vertices (0, 0), (1, 0), (1, 1) and (0, 1) b) Verify Cauchy's theorem for the analytic function $f(z) = 3z^2 + iz - 4$ where C is the square having vertices $1 \pm i \, , -1 \pm i$
6.	a) Verify Cauchy's theorem for the analytic function $f(z) = ze^{-z}$ over the unit circle with origin as the Centre. b) Verify Cauchy's theorem for the analytic function $f(z) = \frac{1}{z}$ taken over the triangle formed by the points (1, 2), (3, 2) and (1, 4).
7.	Verify Cauchy's theorem for the analytic function $f(z) = z^3$ taken over the boundary of the (i) rectangle with vertices $-1, 1, 1 + i, -1 + i,$ (ii) triangle with (1, 2) , (1, 4), (3, 2) .
8.	a) Verify Cauchy's theorem for the analytic function of $f(z) = e^{iz}$ along the boundary of the triangle with the vertices $1 + i \, , -1 + i, -1 - i$. b) Expand $f(z) = \frac{1}{z^2}$ in Taylor's Series about the point $z = -1$.
9.	a) Expand $f(z) = sinz$ in Taylor's Series about the point $z = \frac{\pi}{4}$. b) Find the Taylor's Series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$.

10.	Find the Taylor's Series expansion of $f(z) = \frac{z-1}{z+1}$ about the points (i) $z = 0$ (ii) $z = 1$.
11.	a) Find the Taylor's Series expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = i$ up to second order. b) Find the Taylor's Series expansion of $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the point z=2 upto to first four terms.
12.	Expand $f(z) = \frac{7z^2+9z-18}{z^3-9z}$ in Laurent's series that is valid for (i) $\|z\| > 3$ (ii) 0 <$\|z-3\| < 3$.
13.	Expand $f(z) = \frac{3z^2-6z+2}{z^3-3z^2+2z}$ as Laurent's series in the region (i) $1 < \|z\| < 2,$ (ii) $\|z\| > 2.$
14.	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ as Laurent's series in the region (i) $0 < \|z+1\| < 2,$ (ii) $\|z\| > 3.$
15.	Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent's series in the region (i) $1 < \|z\| < 2,$ (ii) $\|z\| > 2.$
16.	a) Define the following for an analytic function f(z) (i) Singularity, (ii) Isolated singularity, (iii) Removable singularity, (iv) Essential singularity b) Determine the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its simple poles.
17.	a) Find the residue at z = 0 of $z\cos\frac{1}{z}$. b) State Cauchy's Residue Theorem and hence evaluate $\int_C \frac{e^{2z}}{(z+2)(z+4)(z+7)} dz$ where C is $\|z\| = 3$
18.	a) Evaluate $\int_C \frac{zdz}{(z-1)^2(z-3)} dz$ where C is $\|z\| = 2$ using Cauchy's Residue Theorem. b) Evaluate $\int_C \frac{z\cos z}{(z-\frac{\pi}{2})^3} dz$, where C is $\|z-1\| = 1$ by using Cauchy's Residue Theorem.
19.	a) Evaluate $\int_C \frac{e^{2z}}{(z+1)^3} dz$ where C is $\|z\| = 3$, by using Cauchy's Residue Theorem. b) Using Cauchy's Residue Theorem ,evaluate $\int_C \frac{2z^2+1}{(z+1)^2(z-2)} dz$ where C is $\|z\| = 3.$
20.	a) Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$ where C is the circle $\|z+1-i\| = 2$ by using Cauchy's Residue Theorem. b) Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ where C is the circle $\|z\| = 10$ by using Cauchy's Residue Theorem.