Inversion of the z-transform :

→ Recovering a time-domain signal from its z-transform.

$$x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

→ which requires knowledge of complex variable theory ∴

∴ the two alternative methods are there.

(1) Partial fractions
(2) Power series.

(1) Partial fractions : uses knowledge of several basic z-transform pairs & the z-transform properties to invert a large class of z-transforms.

⇒ Relies on an important property of the ROC ie A right-sided time signal has a ROC that lies outside the pole radius

& A left-sided time sided time signal has an ROC that lies ~~out~~ inside the pole radius.

(2) Power series : It expresses $X(z)$ as a power series in z^{-1}

∴ the values of the signal can determined by inspection.

NOTE : (1) Partial fractions :

(i) $M < N$ if not ie $M \geq N$ then we need to use long division to express

$$X(z) = \sum_{k=0}^{M-N} f_k z^{-k} + \underbrace{\frac{\tilde{B}(z)}{A(z)}}_{M<N}$$

rational function.

M → order of Numerator.

N → order of denominator

(ii) If none of the poles are repeated.

then $$X(z) = \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}}$$

where d_k = poles

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(iii) if ROC $|z| > d_K$ (Right sided Signals)

$$A_K (d_K)^n u(n) \xleftrightarrow{Z} \frac{A_K}{1-d_K z^{-1}}$$

(iv) if ROC $|z| < d_K$ (left sided signals

$$-A_K (d_K)^n u(-n-1) \xleftrightarrow{Z} \frac{A_K}{1-d_K z^{-1}}$$

(v) If poles are repeated . ie

$$\frac{A_{i1}}{1-d_i z^{-1}}, \quad \frac{A_{i2}}{(1-d_i z^{-1})^{2}} \quad \cdots \quad \frac{A_{ir}}{(1-d_i z^{-1})^{r}}$$

repeated poles ie roots .

∴ if ROC $|z| > d_i$

$$A \frac{(n+1)\cdots(n+m-1)}{(m-1)!} (d_i)^n u(n) \xleftrightarrow{Z} \frac{A}{(1-d_i z^{-1})^m}$$

Illy ROC $|z| < d_i$

$$-A \frac{(n+1)\cdots(n+m-1)}{(m-1)!} (d_i)^n u(-n-1) \xleftrightarrow{Z} \frac{A}{(1-d_i z^{-1})^m}.$$

(vi) Linearity property : $R_1 \cap R_2$, ∴ need to choose the correct inverse transform .

⇒ location of each pole of $X(z)$ helps in this

⇒ ROC $X(z)$ ⇒ Radius greater than that of the pole associated with a given term . leads to right sided inverse transform

⇒ Radius less than that of the pole , leads to left sided inverse transform

(vii) when poles are complex valued, Expansion co-efficients will be complex valued.

(viii) If the co-efficients in $X(z)$ are real valued , the expansion co-efficients corresponding to complex conjugate poles will be complex conjugates of each other.

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(ix) Causality, stability (or) Existence of DTFT can be used find the inverse transform (other than ROC method)

=> if the signal is causal, right-sided Inverse transforms are chosen

=> If the signal is stable, absolutely summable & has DTFT.

ROC includes the unit circle in the z-plane.

$|z| = 1$

∴ inverse z-transform is determined by comparing the location of the poles. the location of the unit circle, then the

(a) If pole is inside the unit circle Inverse transmission is Right sided

(b) If pole is outside the unit circle, then the Inverse transmission is left sided.

NOTE (2) Power Series Expansion :

(i) Limited to one sided ie Discrete-time signals with ROC $|z| < a$ (or) $|z| > a$.

=> ROC is $|z| > a$, $X(z)$ is expressed as a power series in z^{-1}, can obtain the Right sided signal

=> ROC is $|z| < a$, $X(z)$ is expressed as a power of z, ~~and so~~ can obtain the left sided inverse transform.

(ii) Long division may be used to obtain the power series when $X(z)$ is a ratio of polynomials

=> long division is simple, but long division may not lead to a closed-form expression for $x(n)$

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(iii) power series has a ability to find inverse z-transforms for signals that are not a ratio of polynomials in z

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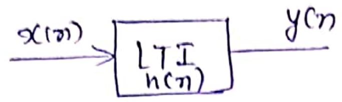
The Transfer function :

→ Relationship b/w the transfer function & i/p - o/p descriptions of LTI discrete - time systems.

→ We know transfer function is Z - transform of impulse response.

→



$$y(n) = x(n) * h(n)$$

$$Z\{y(n)\} = Z\{x(n) * h(n)\}$$ from convolution property of Z - transform.

$$Y(Z) = X(Z)\,H(Z)$$

$$\boxed{H(Z) = \frac{Y(Z)}{X(Z)}}$$ $\forall$ ROC of $X(Z)$ & $Y(Z)$, when $X(Z) \neq 0$

ie "transfer function is the ratio of the Z transform of the output to that of the i/p".

→ impulse response is the inverse Z-transform of the transfer function ie . with known ROC .

$$h(n) = IZT\{H(Z)\}$$
$$= Z^{-1}\{H(Z)\}$$
$$= Z^{-1}\left\{\frac{Y(Z)}{X(Z)}\right\}$$

⇒ If ROC is not known, other system characteristics such as stability (or) causality must be known to determine the unique impulse response.

Relating the Transfer function & the Difference Equation:

→ Transfer function may be obtained directly from the difference - equation description of an LTI System.

WKT $$\sum_{K=0}^{N} a_K\, y(n-K) = \sum_{K=0}^{M} b_K\, x(n-K)$$

n^{th} order difference Eq^n .

→ Transfer function $H(z)$ is eigenvalue with eigenfunction z^n if $x(n) = z^n$ (ie weighted superposition of complex exponentials z^n)

$$x(n)=z^n \longrightarrow \boxed{\begin{matrix}LTI\\ H(z)\end{matrix}} \longrightarrow y(n)$$

$$y(n) = z^n H(z)$$

if $x(n-k) = z^{n-k}$

$$\therefore\ y(n-k) = z^{n-k} H(z)$$

Substitute in difference eqn.

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)$$

$$\sum_{k=0}^{N} a_k\, z^{n-k} H(z) = \sum_{k=0}^{M} b_k\, z^{n-k}$$

$$\boxed{H(z) = \frac{\displaystyle\sum_{k=0}^{M} b_k z^{-k}}{\displaystyle\sum_{k=0}^{N} a_k z^{-k}}}$$

" ratio of polynomials in z^{-1} "

" Rational transfer function ".

where z^{-k} in numerator = coefficient associated with $x(n-k)$ in difference eqn

z^{-k} in denominator = —— " —— $y(n-k)$ —— " ——

Advantage : (1) TF ~~from~~ for given difference eqn

(2) Difference eqn from rational transfer function.

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