MODULE : 2

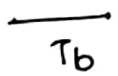
Base Band Transmission

Base - band Shaping for Data Transmission.

* when digital data are transmitted through a band-limited channel, dispersion in the channel causes an overlap in time between successive symbols. This form of distortion, known as Inter-symbol Interference.

Different formats for representation of binary data

1. Unipolar format : (on - off signalling)

'1' →  '0' → 

'1' → symbol 1 is represented by transmitting a pulse

'0' → symbol 0 is represented by switching off the pulse.

When pulse occupies full duration of a symbol, the unipolar format is called NRZ type. When pulse occupies for a fraction of symbol duration, it is called RZ type

NRZ → Non - Return to zero

RZ → Return to zero.

* Simple to implement
* Contains dc component

2. Polar format

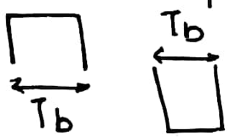
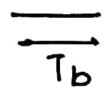
'1' →  by positive pulse

'0' →  by negative pulse

It can be or NRZ or RZ type.

* No dc component if '0's & '1's occur ~~with~~ in equal proportion.

3. <u>Bipolar Format</u> (pseudoternary signaling)

'1' →    '0' → 

Symbol '1' by positive & negative pulses alternately.

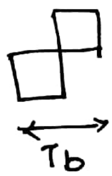
Symbol '0' by no pulse

It can be NRZ or RZ type.

* Here we have 3 levels : +1, 0 & -1.
* No dc component, even though the i/p binary data may contain long strings of 0s & 1s.
* Monitors any isolated error.
* Bipolar format is adopted for use in T_1 carrier system for digital telephony.

4. <u>Manchester format</u> (Biphase baseband signalling)

'1' :  positive pulse for one-half of the symbol duration, followed by a negative pulse for the remainings half of the symbol duration.

'0' : 

* NRZ versions of unipolar, polar and bipolar formats make efficient use of B.W.
* Manchester format alone provides (built-in) synchronization
* Manchester format requires twice that of NRZ unipolar, polar & bipolar formats.

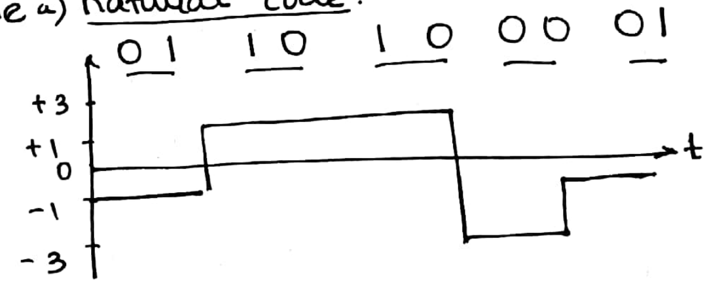
To make efficient utilization of B.W M-ary format representation can be adopted.

Ex:

* <u>Polar Quartenary Format</u> of NRZ type.

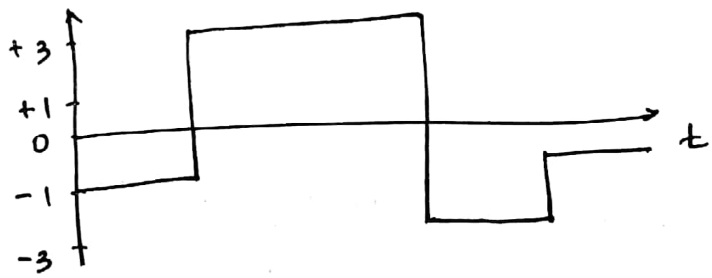
It has 4 distinct symbols, referred as dibits (pairs of bits). Each dibit is assigned a level in accordance with the a) <u>natural code</u>.

Dibit	Level
00	- 3V
01	- 1V
10	+ 1V
11	+ 3V



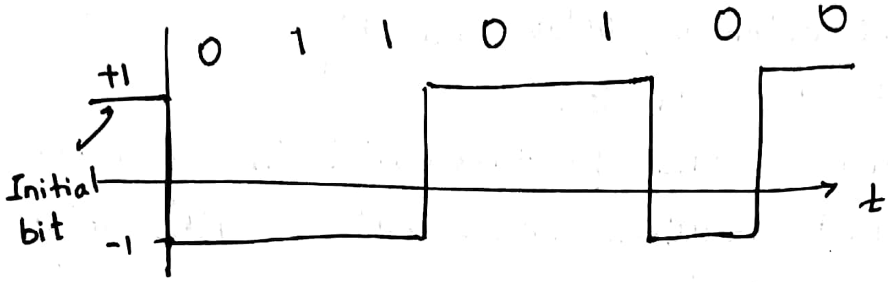
b) <u>Gray code</u>.

Dibit	Level
00	- 3V
01	- 1V
11	+ 1V
10	+ 3V



* <u>Differential Encoding format</u>

This starts with an arbitrary initial bit. For eg., a +ve level is shown for initial bit. The information in the input binary sequence itself is encoded in terms of signal transitions. For eg: a signal transition is used to designate symbol 1.



Power Spectra of Discrete PAM Signals

The different formats are representation of discrete amplitude - modulated pulse train.

Let us represent them by a Random process $x(t)$ given by

$$x(t) = \sum_{K=-\infty}^{\infty} A_K \, v(t - KT)$$

A_K : discrete random variable
$v(t)$: basic pulse shape
T : symbol duration

NRZ format	Co-eff. A_K	Basic pulse $v(t)$
1. Unipolar	$A_K = \begin{cases} a, & \text{symbol '1'} \\ 0, & \text{" '0'} \end{cases}$	$v(t)$ consists of rectangular pulse of unit amplitude & duration T_b.
2. Polar	$A_K = \begin{cases} a; & \text{symbol '1'} \\ -a, & \text{" '0'} \end{cases}$	
3. Bipolar	$A_K = \begin{cases} a, -a, & \text{alternating 1s} \\ 0, & \text{symbol '0'} \end{cases}$	
4. Manchester	$A_K = \begin{cases} a, & \text{symbol 1} \\ -a, & \text{" '0'} \end{cases}$	$v(t)$ consists of doublet pulse of heights ± 1, & total duration T_b.
5. Polar Quaternary (natural)	$A_K = \begin{cases} 3a, & \text{dibit 11} \\ a, & 10 \\ -a, & 01 \\ -3a; & 00 \end{cases}$	$v(t)$ consists of rectangular pulse of unit amplitude & duration $2T_b$

* Data signalling rate is defined as rate, measured in b/s, at which data are transmitted. It is also called as bit rate.

$$R_B = \frac{1}{T_b} \quad , \quad T_b \text{ is duration of the bit.}$$

* Modulation rate is defined as the rate at which the signal level is changed, depending on the nature of the format used to represent digital data. It is measured in bauds or symbols per seconds

$$T = T_b \log_2 M$$

for M-ary format, here M is an integer power of 2.

* One baud equals $\log_2 M$
* Ensemble - averaged autocorrelation function :

$$R_A(n) = E[A_k \cdot A_{k-n}]$$

where E : expectation operation.

* Power spectral density, as per Wiener-Khintchine relation)

$$S_X(f) = \frac{1}{T} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j 2\pi n f T)$$

where V(f) is Fourier transform of basic pulse v(t). Values of V(f) & $R_A(n)$ depend on the type of discrete PAM signal being considered.

* $V(f) = T_b \operatorname{sinc}(f T_b)$

1. NRZ Unipolar Format

Assume that 0's & 1's of a random binary sequence occur with equal probability.
Then, for NRZ unipolar format,

Case 1 $\quad P(A_k = 0) = P(A_k = a) = \frac{1}{2}$

Hence, for $n = 0$,

$$R_A(0) = E[A_k^2] = (0)^2 P(A_k = 0) + (a)^2 P(A_k = a)$$
$$= \frac{a^2}{2}$$

Case 2:

when $n \neq 0$.

The product $A_k . A_{k-n}$ has 4 possible values,

$(0.a) = (a.0) = (0.0) = 0$, $(a.a) = a^2$.

Assuming that successive symbols in the binary sequence are statistically independent, these occur with probability of 1/4 each.

$$E[A_k . A_{k-n}] = 3(0).(1/4) + a^2(1/4)$$

$$= a^2/4 \qquad n \neq 0$$

∴ The autocorrelation function $R_A(n)$ is

$$R_A(n) = \begin{cases} \frac{a^2}{2}, & n = 0 \\ \frac{a^2}{4}, & n \neq 0 \end{cases}$$

for basic pulse $v(t)$, we have rectangular pulse of unit amplitude & duration T_b.

Fourier transform of $v(t)$ equals,

$$V(f) = T_b \operatorname{sinc}(fT_b).$$

∴ Power spectral density of NRZ unipolar format : with $T = T_b$

$$S_X(f) = \frac{1}{T_b} |T_b \operatorname{sinc}(fT_b)|^2 . \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi n f T_b)$$

$$= \frac{a^2}{4} T_b . \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} \exp(-j2\pi n f T_b)$$

By Poisson's formula:

$$\sum_{n=-\infty}^{\infty} \exp(-j2\pi n f T_b) = \frac{1}{T_b} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

where $\delta(f)$ is dirac delta function at $f = 0$.

$$\therefore S_X(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} \delta(f).$$

The presence of the Dirac delta function $\delta(f)$ accounts for one half of the power contained in the unipolar waveform.

2 NRZ polar format.

Assume that symbols '0' & '1' occur with equal probability in the binary data stream.

$$P(A_K=-a) = P(A_K=a) = \frac{1}{2}$$

Case 1 $\quad n = 0,$

$$E[A_K^2] = (-a)^2 \cdot P(A_K=0) + (a)^2 P(A_K=1)$$

$$= +\frac{a^2}{2} + \frac{a^2}{2}$$

$$= a^2$$

Case 2, $\quad n \neq 0,$

4 possibilites : $(0.0) = (-a.-a) = a^2$

$(0.1) = (-a.\, a) = -a^2$

$(1.0) = (a.-a) = -a^2$

$(1.1) = (a.\, a) = a^2$

Each occur with probability of 1/4.

$$E[A_K \cdot A_{K-n}] = (0.0)\, P(A_K=0,\, A_{K-n}=0) + (0.1)\, P(A_K=0 \cdot A_{K-n}=1)$$

$$+ (1.0)\, P(A_K=1 \cdot A_{K-n}=0) + (1.1)\, P(A_K=1,\, A_{K-n}=1)$$

$$= \frac{a^2}{4} - \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4}$$

$$= 0$$

$$\therefore R_A(n) = \begin{cases} a^2, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$V(f) = T_b \operatorname{sinc}(fT_b)$$

$\therefore$ Power Spectral Density for NRZ Polar format,

$$S_X(f) = \frac{1}{T_b} \left| T_b^2 \operatorname{sinc}^2(T_b) \right| . \sum_{n=-\infty}^{\infty} R_A(n) . \exp(-j2\pi n f T_b),$$

$$S_X(f) = a^2 T_b \operatorname{sinc}^2(fT_b)$$

3 NRZ Bipolar Format

It has 3 levels : $a, 0, -a$.

Assume that 0's & 1's in the input binary data occur with equal probability.

$$P(A_K = a) = 1/4$$

$$P(A_K = 0) = 1/2$$

$$P(A_K = -a) = 1/4.$$

Case 1 : with $n = 0$

$$E[A_K^2] = (a)^2 P(A_K = a) + (0)^2 P(A_K = 0) + (-a)^2 P(A_K = -a)$$

$$= a^2 \cdot \frac{1}{4} + 0 + a^2 \cdot \frac{1}{4}$$

$$= \frac{a^2}{2}$$

Case 2 : with $n \neq 0$

for $n = 1$,

(A_{K-1}, A_K), can assume 4 possible forms

$(0, 0)$: $(0 \cdot 0) = 0$.

$(0, 1)$: $(0 \cdot a) = 0$.

$(1, 0)$: $(a \cdot 0) = 0$.

$(1, 1)$: $(a \cdot -a) = -a^2$.

Each occur with a probability of 1/4.

$$\therefore E[A_K \cdot A_{K-1}] = 3(0) \cdot \left(\frac{1}{4}\right) + (-a^2) \cdot \frac{1}{4}$$

$$= -\frac{a^2}{4}.$$

$$\therefore R_A(n) = \begin{cases} a^2/2, & n = 0 \\ -a^2/4, & n = \pm 1 \\ 0, & \text{otherwise} \end{cases}$$

Power spectral density of NRZ bipolar format

$$S_X(f) = \frac{1}{T_b} \cdot |T_b^2 \operatorname{sinc}^2(fT_b)| \left[\frac{a^2}{2} - \frac{a^2}{4} (\exp(j2\pi f T_b) + \exp(-j2\pi f T_b)\right]$$

$$= \frac{a^2 T_b}{2} \operatorname{sinc}^2(fT_b) [1 - \cos(2\pi f T_b)]$$

$$= a^2 T_b \operatorname{sinc}^2(fT_b) \cdot \sin^2(\pi f T_b)$$

4. Manchester format

Refer polar format derivation.

$$R_A(n) = \begin{cases} a^2, & n = 0 \\ 0, & n \neq 0. \end{cases}$$

$$V(f) = j\, T_b \,\text{Sinc}\left(\frac{fT_b}{2}\right) \sin\left(\frac{\pi f T_b}{2}\right)$$

↳ doublet pulse in freq. domain.

$$S_X(f) = \frac{1}{T_b}\left[T_b^2\right]\left[\text{Sinc}^2(fT_b)\right]\left[\text{Sin}^2\left(\frac{\pi f T_b}{2}\right)\right] \cdot a^2$$

$$= a^2\, T_b\, \text{Sinc}^2(fT_b)\, \sin^2\left(\frac{\pi f T_b}{2}\right)$$

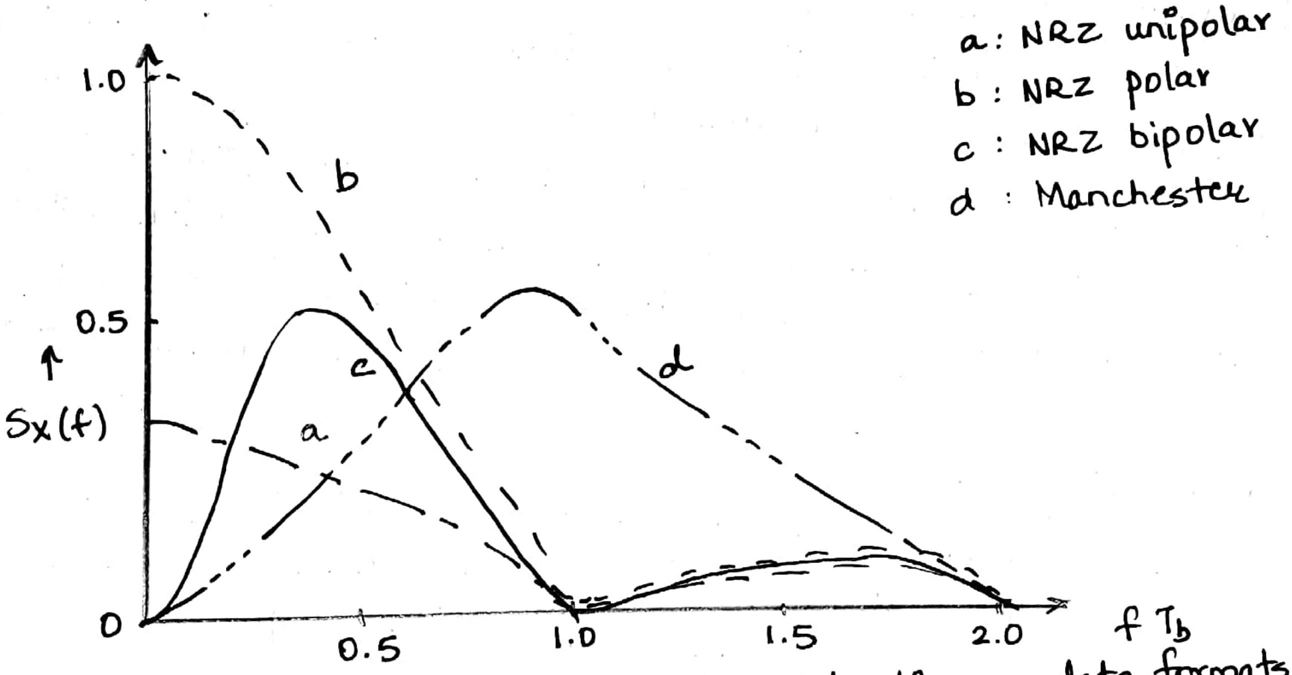


FIG : Power spectra of different binary data formats.

Intersymbol Interference : Baseband binary data transmission system.

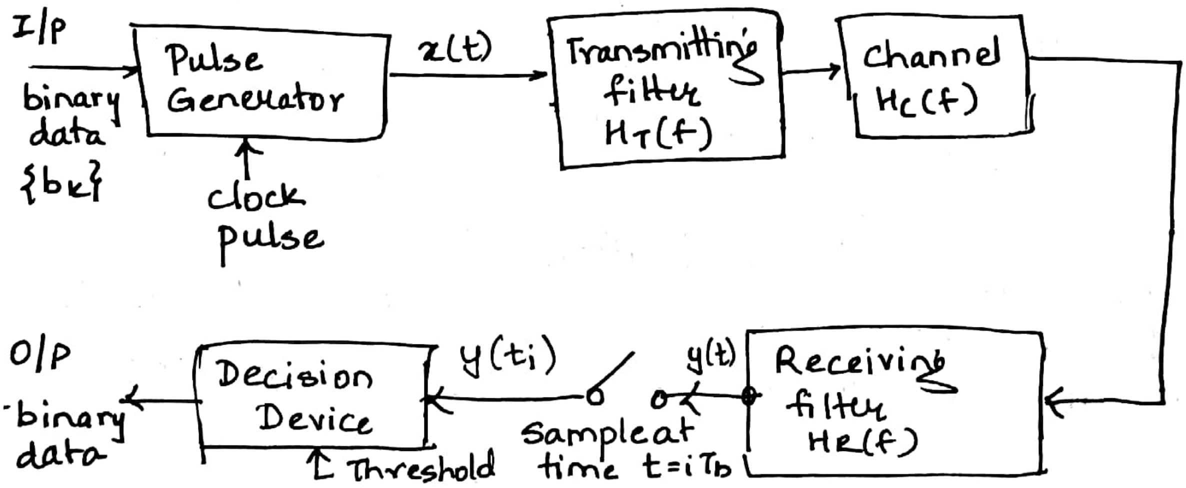


Fig. depicts the basic elements of a baseband binary PAM system.

The i/p signal consists of binary data sequence $\{b_k\}$ with T_b as the bit duration.

Then the discrete PAM signal is given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, v(t - kT_b) \rightarrow (1)$$

$v(t)$: basic pulse shape

a_k : co-eff depends on the i/p data & format used

This $x(t)$ passes thro' a transmitting filter of transfer function $H_T(f)$.

The resulting filter o/p defines the transmitted signal, which is modified as a result of transmission thro' the channel of T.F $H_C(f)$. The channel may be co-axial / optical fiber, where dispersion takes place & hence degradation of signal takes place.

Let us assume that channel is noiseless but dispersive.

The channel o/p is passed thro' a receiving filter of T.F $H_R(f)$. The o/p is next sampled synchronously with the Tx, with appropriate sampling freq.

Finally, the seq. of samples thus obtained is used to reconstruct the original data seq. by means of a decision device. Here, each sample is compared to a threshold. We, assume that symbols '1' & '0' occur with equal probability & set a threshold half way between their representation levels.

If threshold is exceeded, a decision is made in favour of symbol '1', else '0'.

The receiving filter o/p is given by

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b) \rightarrow (2).$$

where μ : scaling factor
$p(t)$: pulse

$\therefore$ The pulse $\mu p(t)$ is the response of cascade connection of transmitting filter, channel & receiving filter.

$\therefore$ In freq. domain,

$$\mu P(f) = V(f).\, H_T(f)\, H_C(f)\, H_R(f) \rightarrow (3)$$

where $V(f) \underset{F.T}{\overset{I.F.T}{\rightleftharpoons}} v(t)$, $P(f) \underset{F.T}{\overset{I.F.T}{\rightleftharpoons}} P(t)$

The receiving filter o/p $y(t)$ is sampled at $t_i = iT_b$.

$$\therefore\ y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k.\, p(iT_b - kT_b)$$

$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k\, p(iT_b - kT_b) \rightarrow (4).$$

where the first term μa_i is produced by the ith transmitted bit & second term represents the residual effect of all other transmitted bits on the decoding of ith bit; this residual effect is called Inter Symbol Interference (ISI).

In absence of ISI,

$$y(t_i) = \mu a_i.$$

The presence of ISI, introduces errors in the decision device at Rx o/p. $\therefore$ The major objective in design of Tx & Rx filters is to minimize effects of ISI & deliver data with smallest error rate.