Unit – 1 (Probability Theory)

Q.No	Questions

1.

a) The Random Variable X has the following probability mass function, find (i) k ii) P(X<3) (iii) P(3<X≤ 5) (iv) variance

X	0	1	2	3	4	5
P(X)	K	3K	5K	7K	9K	11K

b) A random variable (X= x) has the following probability distributions. Find (i) k (ii) p(x<6) (iii) p(x>6) (iv) Mean and also find the probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

2.

a) Find 'k' such that the following distribution represents a finite probability distribution. Hence find (i)mean (ii) p(x≤1) (iii) p(x>1) (iv) p(-1<x≤2)

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

b) A random variable (X=x) has the following probability function for various values of x. Find (i)k (ii) p(x<1) (iii) p(x>-1)

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

3.

a) Find the mean and variance of geometric distributions.

b) 3% of the product produced by a machine is found to be defective. Find the probability that first defective occurs in the (i) 5^{th} item inspected (ii) first five inspected (iii) mean (iv) variance.

4.

a) What is the probability that the marketing representative must select (i) more than 6 people (ii) six people, before he finds one who attended the last home? c.d.f of a Geometric R V with 1-p=0.8.

b) If the probability that a target is destroyed on any one shot is 0.5. What is the probability that it would be destroyed on (i) 6^{th} attempt (ii) more than 6 attempt (iii) Mean (iv) variance?

5.

a) Derive mean and variance for the Poisson Distribution

b) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses which contains (i) No defective fuse (ii) 3 or more defective fuses (iii) atleast one defective fuse

6.

a) The probability that an individual's suffers a bad reaction from an injection is 0.001. Find the probability that out of 2,000 individual (i) almost 2 (ii) exactly 2 (iii) more than 2 will get bad reaction

	b) In a certain factory turning out razor blade, there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets
7.	a) The probability density function of continuous random variable is given by $f(x) = ke^{-\|x\|}, -\infty < x < \infty$. Prove that $k = 1/2$ and also find mean and variance. b) A continuous random variable has the following density function P(x)= $\begin{cases} kx^2, -3 \le x \le 3 \\ 0 \quad elsewhere \end{cases}$ Evaluate (i) k, (ii) P(1≤ $x \le 2$), (iii) P($x \le 2$), (iv) P(x>1).
8.	a) A continuous random variable has the density function $f(x) = \frac{k}{1+x^2}, -\infty < x < \infty$. Determine (i) k, (ii) $P(x > 0)$, $(iii) P(0 < x < 1)$. b) The probability density function f(x) of continuous random variable is given by $P(x) = \begin{cases} kx(1-x), 0 \le x \le 1 \\ 0 \quad otherwise \end{cases}$ Determine k, $P(0 < x < 1/3)$.
9.	a) Derive S.D for the Exponential Distribution. b) The sale per day in a shop is exponentially distributed with an average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on a day.
10.	a) In a certain town the duration of a shower is exponentially distributed with mean 5min. what is the probability that the shower will last for (i) 10min or more (ii) less than 10min (iii) between 10 to 12min. b) The life of a compressor manufactured by a company is known to be 200 months on an average following an exponential distribution. Find the probability that the life of a compressor of that company is (i) < 200 months, (ii) between 100 months and 25 years.
11.	a) Derive mean and variance for normal distribution. b) In examination 7% of students score less than 35% marks and 89% of students score less than 60% marks, Find the mean and standard deviation, if the marks are normally distributed. It is given that if p (z) $= \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$ then p (1.2263) = 0.39 p(1.4757) = 0.43.
12.	a) A sample of 100 dry battery cells tested to find the length of life produced by a company and following results are recorded: mean life is 12 hrs, SD is 3 hrs. Assuming data to be normally distributed, find the expected life of a dry cell. (i) have more than 15 hrs (ii) between 10 and 14 hrs.[P(0.667)=0.2486,P(1)=0.3413]. b) The mean weight of 1,000 students during medical examination was found to be 70kg and S.D weight 6kg. Assume that the weight are normally distributed, find the number of students having weight (i) less than 65kg (ii) more than 75kg (iii) between 65kg to 75kg.[P(0.83)=0.2967]
13.	a) Explain the following i) Null hypothesis

	ii) Alternative hypothesis iii) Type I and type II error iv) Level of significance v) Standard error b) The weights of 1,500 ball bearings are normally distributed with a mean of 635gms and S.D of 1.36gms. If 300 random samples of size 36 are drawn from this population, determine the expected mean and S.D of the sampling distribution of means if sampling is done a) with replacement b) without replacement.
14.	a) Certain tubes manufactured by a company have mean life time of 800 hours and S.D of 60hours. Find the probability that a random sample of 16 tubes from the group will have a mean life time a) between 790 hours and 810 hours b) less than 785 hours c) more than 820 hours d) between 770 hours and 830 hours. b) The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find a) 95% b) 99% confidence limits for mean of the maximum loads of all cables by the company.
15.	a) The life of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that $\mu = 800$ hours against the alternate hypothesis $\mu \neq 800$ hours at 5% and 1% level of significance. b) A newly developed pain reliever will more quickly produce perceptible reduction in pain to patients after minor surgeries than a standard pain reliever. The standard pain reliever is known to bring relief in an average of 3.5 minutes with standard deviation 2.1 minutes. To test whether the new pain reliever works more quickly than the standard one, 50 patients with minor surgeries were given the new pain reliever and their times to relief were recorded. The experiment yielded sample mean is 3.1 minutes and sample standard deviation 1.5 minutes. Is there sufficient evidence in the sample to indicate, at the 5% level of significance, that the newly developed pain reliever does deliver perceptible relief more quickly?
16.	a) In a sample of 50 individuals the average income was found to be Rs.21,720 per annum. It is known that the Standard deviation of the distribution of the income is Rs.900. Can we conclude that average annual income is Rs.22,000 at 5% and 1% level of significance? b) A large group of athletes is found to have a average weight of 140lbs and standard deviation 5lbs. One student from a college was found to weight 120lbs. Can it be reasonably concluded that he was not an athlete?
17.	a) An entrance examination was given to a large group of boys and girls who appeared for the selection of professional course in Karnataka who scored on the average 84.5 marks. The same test was given to a sample group of 400 boys and girls. They scored an average of 82.5 marks with a standard deviation 22.5 marks. Examine whether the difference is significant. Can we conclude that the sample is from the Population? b) It is claimed that a random sample of 100 Tyres (with a mean life of 15269km) is

	drawn from a population of Tyres which has mean life of 15,200 km and a standard deviation of 1,248 km. Test the validity of this claim.
18.	a) Daily sales figures of 40 shopkeepers showed that their average sales and standard deviation were Rs.528 and Rs.600 respectively. Is the assertion that daily sale on the average is Rs.400 contradicted at 5% level of significance by the sample? b) A sample of 900 men is found to have a mean height of 64inch. If this sample has been drawn from a normal population with standard deviation 20 inch, find the 99% confidence limits for the mean height of the men in the population
19.	a) A sample of 5000 students in a college was taken and their average weight was found to be 62.5Kg with a standard deviation of 22kg. Find the 95% confidential limits of the average weight of the students in the entire University. b) Systolic blood pressure of 566 males was taken. Mean BP was found to be 128.8mm and SD 13.05mm. Find 95% confidence limits of BP within which the populations mean would lie.
20.	a) Standard deviation of blood sugar level in a population is 6 mg%. If population mean is not known, within what limits is it likely to lie if a random sample of 100 has a mean of 80mg%? b) In a population sample of children with average height 66cm and SD 2.7 cm, can a sample of 100 with an average height of 67cm occur easily?