Table B.8 Properties of Z-transforms

Property	Response	ROC
	$x_1(n) \xleftrightarrow{ZT} X_1(z)$	R_1
	$x_2(n) \xleftrightarrow{ZT} X_2(z)$	R_2
Linearity	$ax_1(n)+bx_2(n) \xleftrightarrow{ZT} aX_1(z)+bX_2(z)$	$R_1 \cap R_2$
Time shifting	$x(n-m) \xleftrightarrow{ZT} z^{-m}X(z)$	Same as $X(z)$ except $z=0$
	$x(n+m) \xleftrightarrow{ZT} z^{m}X(z)$	Same as $X(z)$ except $z=\infty$
Multiplication by exponential sequence or scaling in z-domain	$a^n u(n) \xleftrightarrow{ZT} X\left(\frac{z}{a}\right)$	$\lvert a\rvert R_1 < \lvert z\rvert < \lvert a\rvert R_2$
Time reversal	$x(-n) \xleftrightarrow{ZT} X(z^{-1})$	$\frac{1}{R_2} < \lvert z\rvert < \frac{1}{R_1}$
Time expansion	$x\left(\frac{n}{k}\right) \xleftrightarrow{ZT} X(z^{k})$	
Differentiation in z-domain	$nx(n) \xleftrightarrow{ZT} -z\frac{d}{dz}X(z)$	$R_1 < \lvert z\rvert < R_2$
Conjugation	$x^*(n) \xleftrightarrow{ZT} X^*(z)^*$	$R_1 < \lvert z\rvert < R_2$
Accumulation	$\sum_{k=-\infty}^{n} x(k) \xleftrightarrow{ZT} \frac{1}{1-z^{-1}}X(z)$	
Convolution	$x_1(n) * x_2(n) \xleftrightarrow{ZT} X_1(z)\,X_2(z)$	At least the intersection of R_1 and R_2
Correlation	$R_{x_1x_2}(n) = x_1(n) \otimes x_2(n) \xleftrightarrow{ZT} X_1(z)\,X_2(z^{-1})$	At least the intersection of the ROC of $X_1(z)$ and $X_2(z^{-1})$
Multiplication	$x_1(n)\,x_2(n) \xleftrightarrow{ZT} \frac{1}{2\pi j}\oint_c X_1(v)\,X_2\left(\frac{z}{v}\right)v^{-1}\,dv$	At least $R_{1l}R_{2l} < \lvert z\rvert < R_{1u}R_{2u}$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x_1(n)\,x_2{}^*(n) = \frac{1}{2\pi j}\oint_c X_1(v)\,X_2{}^*\left(\frac{1}{v^*}\right)v^{-1}\,dv$	
Initial value theorem	$x(0) = \underset{n\to 0}{\mathrm{Lt}}\; x(n) = \underset{z\to\infty}{\mathrm{Lt}}\; X(z)$	
Final value theorem	$x(\infty) = \underset{n\to\infty}{\mathrm{Lt}}\; x(n) = \underset{z\to 1}{\mathrm{Lt}}\; (z-1)\,X(z)$, If $(1-z^{-1})$ has no pole on or outside the unit circle	

Note: The initial value theorem and the final value theorem hold true only for causal signals.

Table B.7 Some Common Z-transform Pairs

Sequence $x(n)$	Z-transform $X(z)$	ROC
1. $\delta(n)$	1	All z
2. $u(n)$	$z/(z-1) = 1/(1-z^{-1})$	$\lvert z\rvert > 1$
3. $u(-n)$	$\dfrac{1}{1-z} = -\dfrac{1}{z-1} = -\dfrac{z^{-1}}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $u(-n-1)$	$z/(z-1) = 1/(1-z^{-1})$	$\lvert z\rvert < 1$
5. $u(-n-2)$	$z^2/(z-1)$	$\lvert z\rvert < 1$
6. $u(-n-k)$	$z^k/(z-1)$	$\lvert z\rvert < 1$
7. $\delta(n-k)$	z^{-k}	All z except at $z = 0$ (if $k > 0$) All z except at $z = \infty$ (if $k < 0$)
8. $a^n u(n)$	$(z/(z-a)) = 1/(1-az^{-1})$	$\lvert z\rvert > \lvert a\rvert$
9. $-a^n u(-n)$	$a/(z-a)$	$\lvert z\rvert < \lvert a\rvert$
10. $-a^n u(-n-1)$	$z/(z-a) = 1/(1-az^{-1})$	$\lvert z\rvert < \lvert a\rvert$
11. $nu(n)$	$-z/(z-1)^2 = -[z^{-1}/(1-z^{-1})^2]$	$\lvert z\rvert > 1$
12. $na^n u(n)$	$az/(z-a)^2 = az^{-1}/(1-az^{-1})^2$	$\lvert z\rvert > \lvert a\rvert$
13. $-na^n u(-n-1)$	$az/(z-a)^2 = az^{-1}/(1-az^{-1})^2$	$\lvert z\rvert < \lvert a\rvert$
14. $e^{-j\omega n} u(n)$	$z/(z-e^{-j\omega}) = 1/(1-z^{-1}e^{-j\omega})$	$\lvert z\rvert > \lvert 1\rvert$
15. $\cos \omega n\, u(n)$	$\dfrac{z(z-\cos\omega)}{z^2-2z\cos\omega+1} = \dfrac{1-z^{-1}\cos\omega}{1-2z^{-1}\cos\omega+z^{-2}}$	$\lvert z\rvert > 1$
16. $\sin \omega n\, u(n)$	$\dfrac{z\sin\omega}{z^2-2z\cos\omega+1} = \dfrac{z^{-1}\sin\omega}{1-2z^{-1}\cos\omega+z^{-2}}$	$\lvert z\rvert > 1$
17. $a^n \cos \omega n\, u(n)$	$\dfrac{z(z-a\cos\omega)}{z^2-2az\cos\omega+a^2} = \dfrac{1-z^{-1}a\cos\omega}{1-2az^{-1}\cos\omega+a^2 z}$	$\lvert z\rvert > \lvert a\rvert$
18. $a^n \sin \omega n\, u(n)$	$\dfrac{az\sin\omega}{z^2-2az\cos\omega+a^2} = \dfrac{az^{-1}\sin\omega}{1-2az^{-1}\cos\omega+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
19. $(n+1)a^n u(n)$	$z^2/(z-a)^2 = 1/(1-az^{-1})^2$	$\lvert z\rvert > \lvert a\rvert$
20. $-nu(-n-1)$	$z/(z-1)^2 = z^{-1}/(1-z^{-1})^2$	$\lvert z\rvert < \lvert 1\rvert$
21. $na^n u(n)$	$z/(z-a)^2$	$\lvert z\rvert > \lvert a\rvert$
22. $[n(n-1)a^{n-2}u(n)]/2!$	$z/(z-a)^3$	$\lvert z\rvert > \lvert a\rvert$
23. $\dfrac{n(n-1)\ldots[n-(k-2)]a^{n-k+1}}{(k-1)!}u(n)$	$z/(z-a)^k$	$\lvert z\rvert > \lvert a\rvert$
24. $1/n,\quad n > 0$	$-\ln(1-z^{-1})$	$\lvert z\rvert > \lvert 1\rvert$
25. $n^k a^n,\ k < 0$	$-\left(-z\dfrac{d}{dz}\right)^n \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
26. $a^{\lvert n\rvert}$ for all n	$(1-a^2)/[(1-az)(1-az^{-1})]$	$\lvert a\rvert < \lvert z\rvert < \lvert 1/a\rvert$