Assignment

I find Even and odd components of a given signal

① $x(t) = 1 + \tan t + t^2 \tan^2 t$

$x(-t) = 1 - \tan t + t^2 \tan^2 t$ as $\tan(-t) = -\tan t$

Even component $= \frac{x(t)+x(-t)}{2} = \frac{2 + 2\tan^2 t\, t^2}{2}$

$= 1 + t^2 \tan^2 t.$

odd component $= \frac{x(t) - x(-t)}{2} = \frac{2\tan t}{2} = \tan t$

Verification

$x_e(t) + x_o(t) = 1 + t^2 \tan^2 t + \tan t = x(t)$ Hence Proved.

② $x(n) = n^2 \left(\frac{1}{2}\right)^{n-2}$.

$x(-n) = n^2 \left(\frac{1}{2}\right)^{-n-2}$.

Even component $= \frac{x(n)+x(-n)}{2} = \frac{n^2}{2}\left[\frac{1}{2}\right]^{-2}\left[\frac{1}{2^n} + \frac{1}{2^{-n}}\right]$

Odd component $= \frac{x(n) - x(-n)}{2}$

$= n^2/2 \, (1/2)^{-2}\left[1/2^n - 1/2^{-n}\right]$ //.

Verification:-

$x_e(n) + x_o(n) = \frac{n^2}{2}\left(\frac{1}{2}\right)^{-2}\left[\frac{1}{2^n} + \frac{1}{2^{-n}} + \frac{1}{2^n} - \frac{1}{2^{-n}}\right]$

$= \frac{n^2}{2}\left(\frac{1}{2}\right)^{-2}\left[\frac{2}{2^n}\right] = n^2\left[1/2\right]^{n-2}$

$= x(t)$

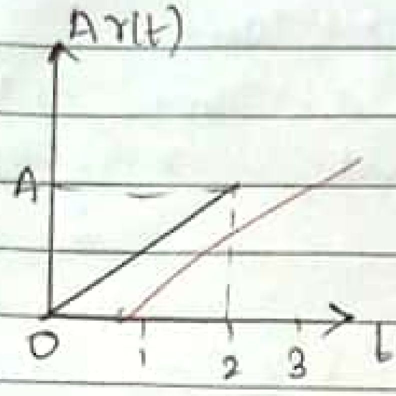
hence proved.

③ $x(t) = A\,r(t) \qquad 0 < t < 2.$

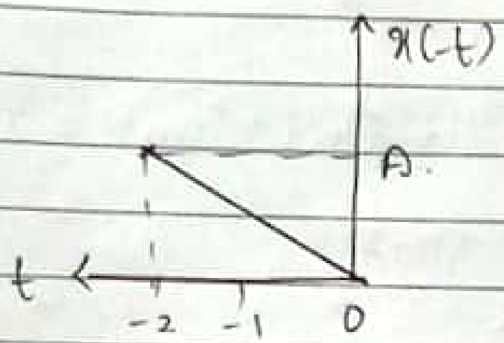
Even component = $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

odd component = $x_o(t) = x(t) - x(-t)/2.$

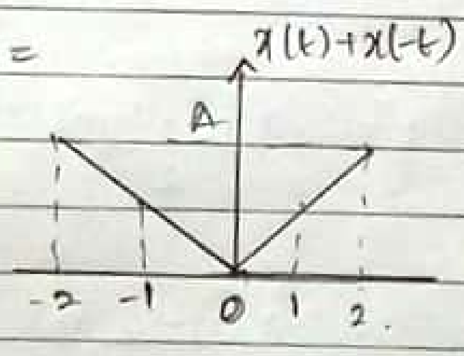
$x(t) =$



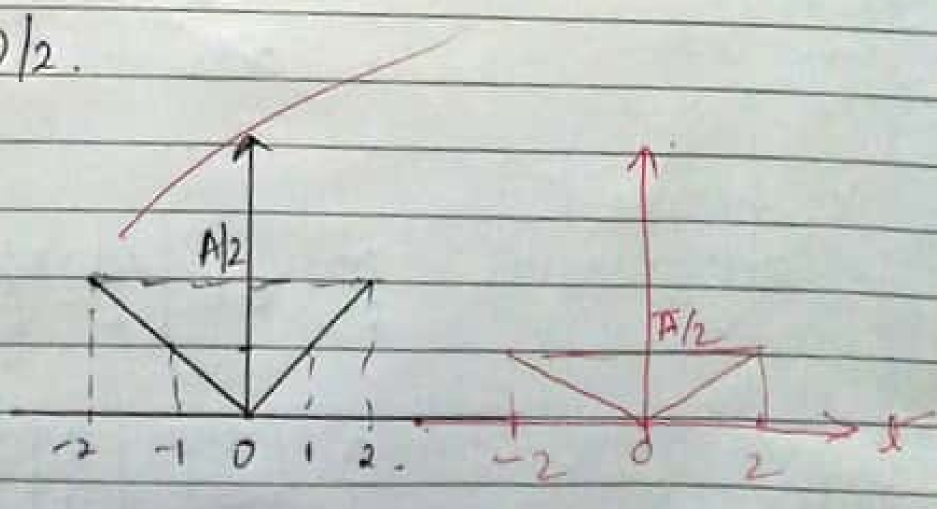
$x(-t) =$



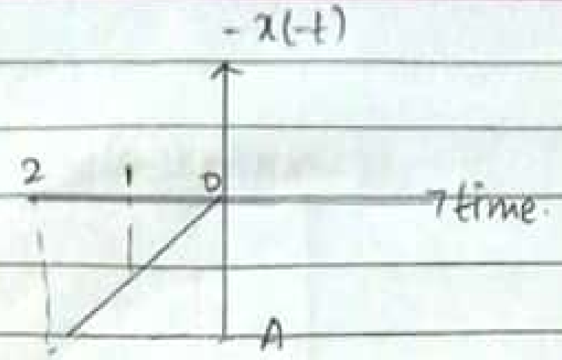
$x(t) + x(-t) =$



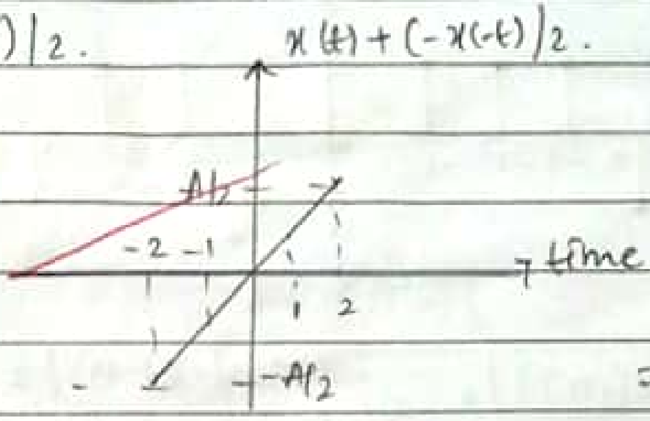
$x(t) + x(-t)/2.$



$-x(-t)$.



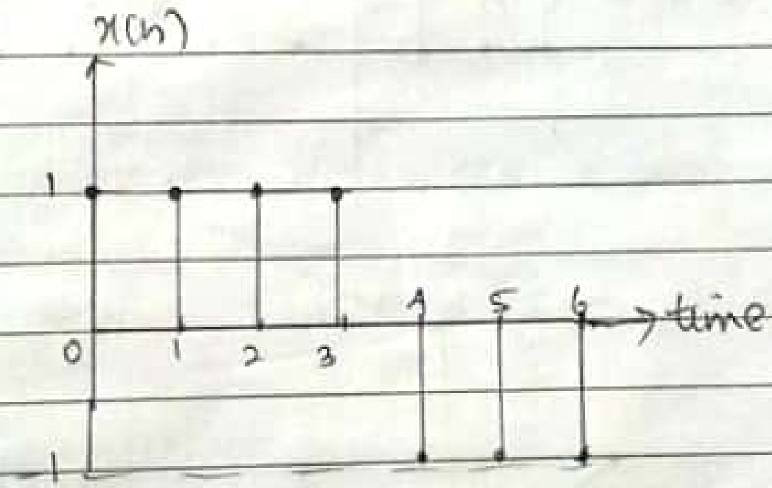
$x(t) + (-x(-t))/2$.



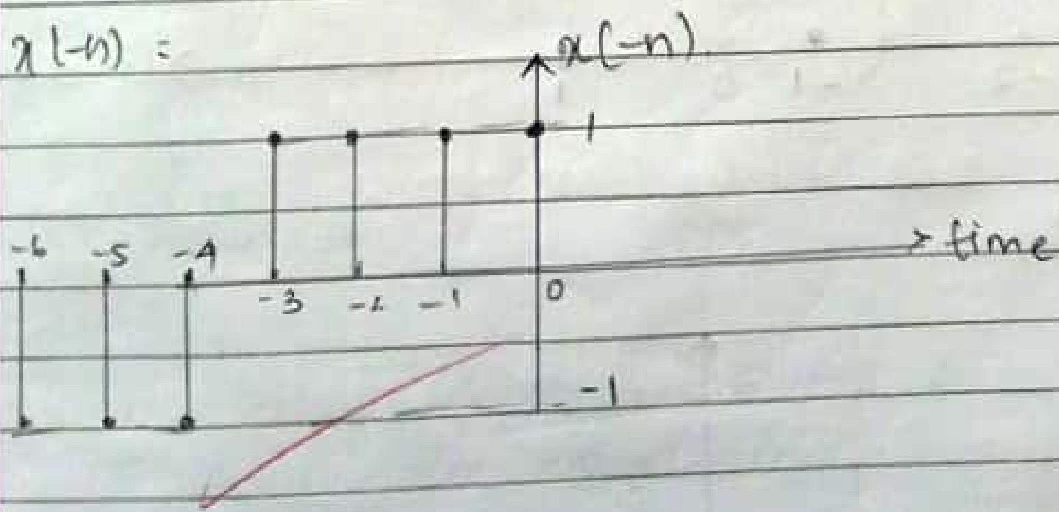
=oddcomponent

(4) $x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ -1 & 4 \le n \le 6 . \end{cases}$

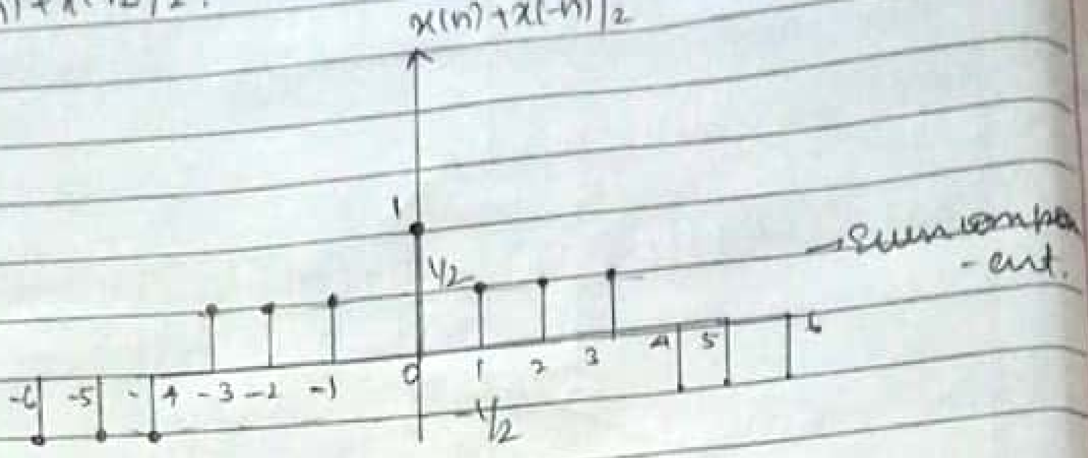
$x(n) =$



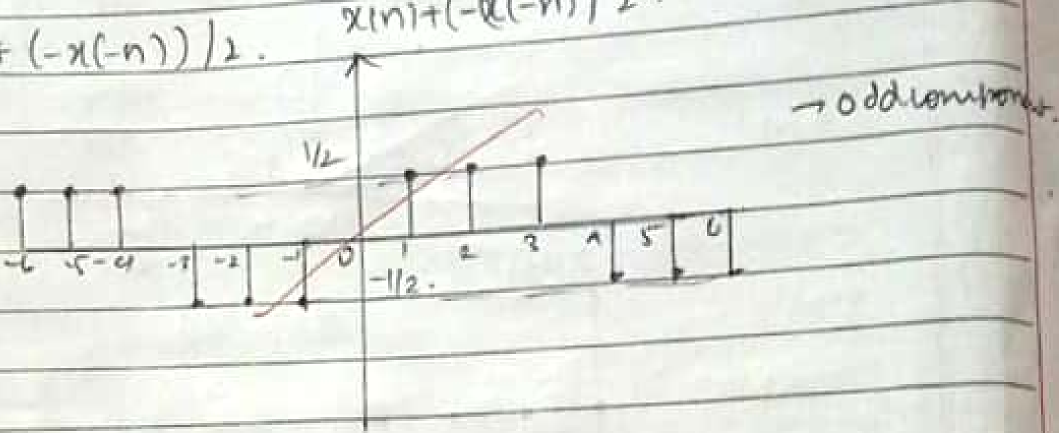
$x(-n) =$



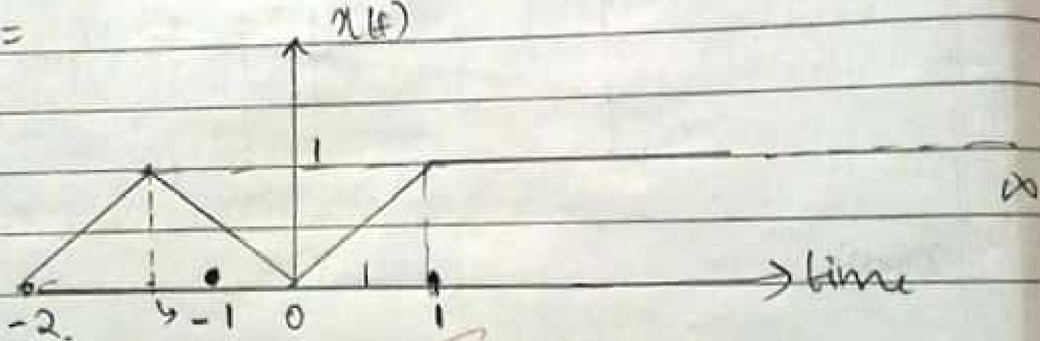
$x(n) + x(-n)/2.$



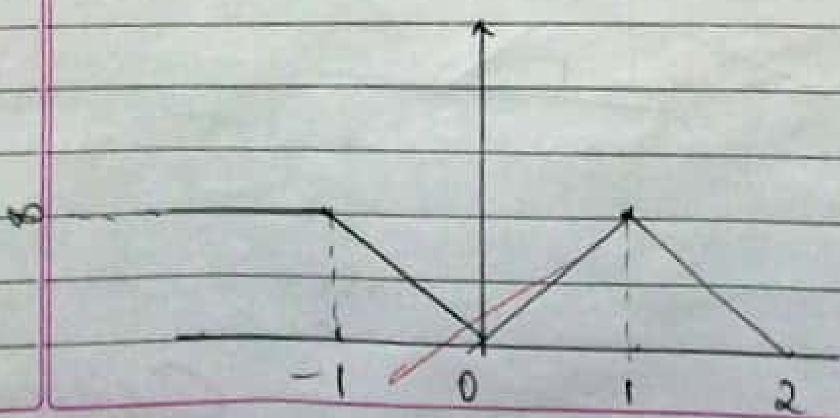
$x(n) + (-x(-n))/2.$



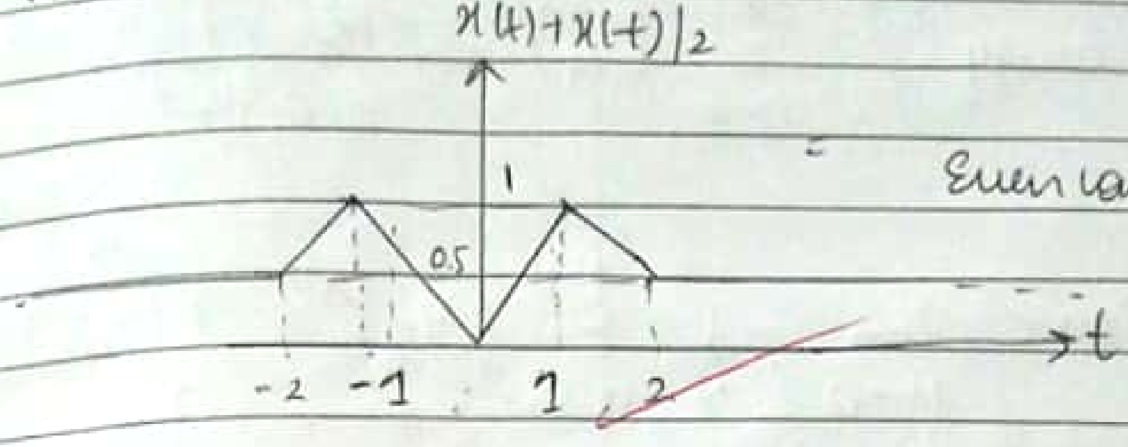
⑤. $x(t) =$



$x(-t)$

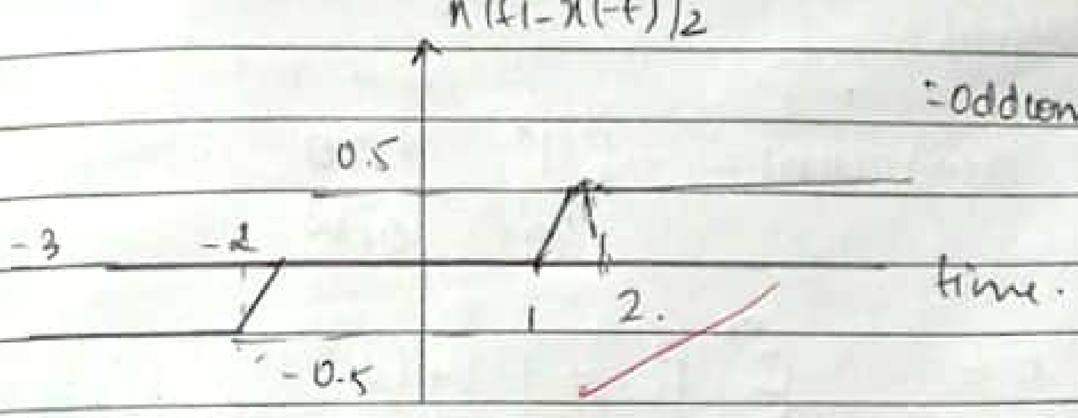


$x(t) + x(-t)/2$.



= Even component

$x(t) - x(-t)/2$



= Odd component

(2) To find Energy or power of a signal.

(1) $x(t) = \begin{cases} A, & -T/2 < t < T/2 \\ 0 & \text{O.W.} \end{cases}$

$$E = \int_{-T/2}^{T/2} A^2\,dt = A^2[t]_{-T/2}^{T/2} = A^2T \text{ Joules}$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_0^T A^2\,dt = \frac{1}{\infty}(A^2T) = 0.$$

as Energy is finite and Average power is equal to Zero the signal is a Energy signal.

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(i) $x(n) = (1/4)^n u(n)$

$$\text{Energy} = \sum_{n=0}^{\infty} (1/4)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{1}{1 - 1/16}$$

$$= \frac{15}{16} \text{ Joules.}$$

$$P_{av} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \cancel{(1/16)^n} = \frac{1}{\infty}(15/16) = 0.$$

As $P_{av} = 0$ and Energy is finite it is a Energy signal.

(2) $x(n) = u(n) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{o.w} \end{cases}$

$$E = \sum_{n=0}^{\infty} 1^2 = 1 + 1 + \cdots \infty = \infty //$$

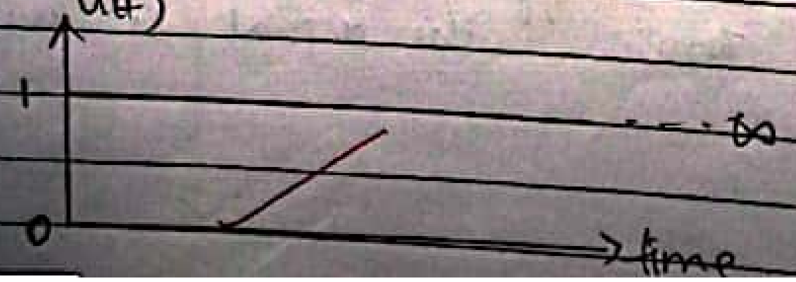
$$P_{av} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} 1 = \lim_{N\to\infty} \frac{1}{N} \times N = 1 \text{ watt}$$

as Pav is finite and Energy is infinite given signal is a power signal.

III find and sketch the following signals and their derivatives.

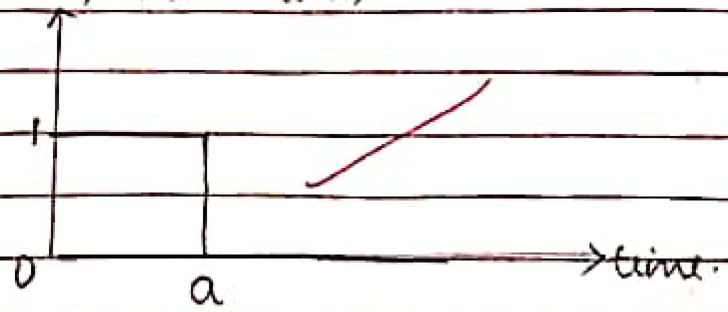
i) $x(t) = u(t) - u(t-a) \quad a > 0$

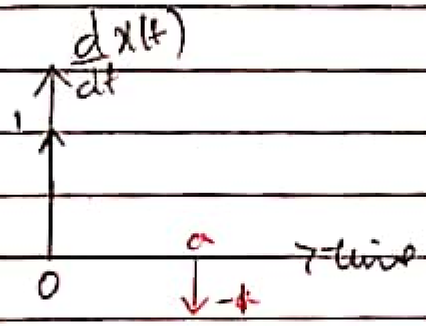
$u(t) =$



$u(t-a)$

$x(t) = u(t) - u(t-a)$

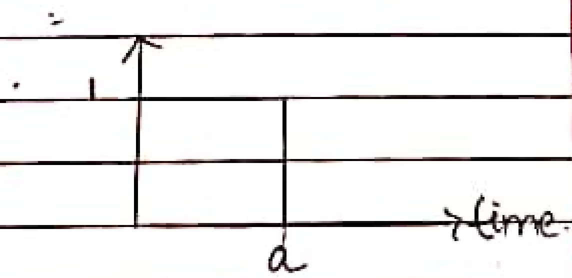




as differentiation of a unit step signal is a impulse signal.

② $x(t) = t(u(t) - u(t-a))$

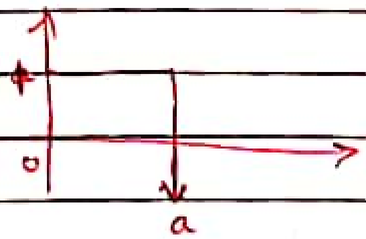
$u(t) - u(t-a)$



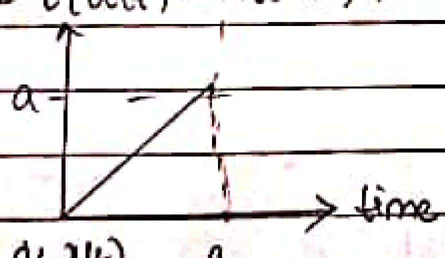
$\frac{d}{dt}\, t\,u(t) = 0 + u(t)$

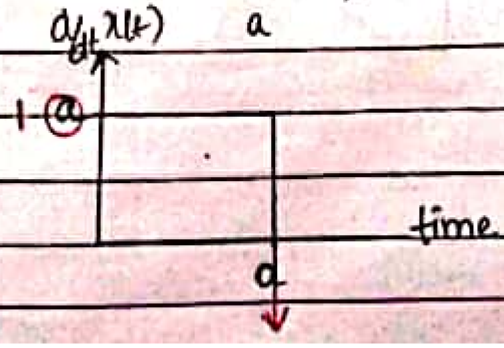
$\frac{d}{dt}\, t\,u(t-a) = a\delta(t-a) + u(t-a)$

$\therefore \frac{dx(t)}{dt} = u(t) - a\delta(t-a) - u(t-a).$



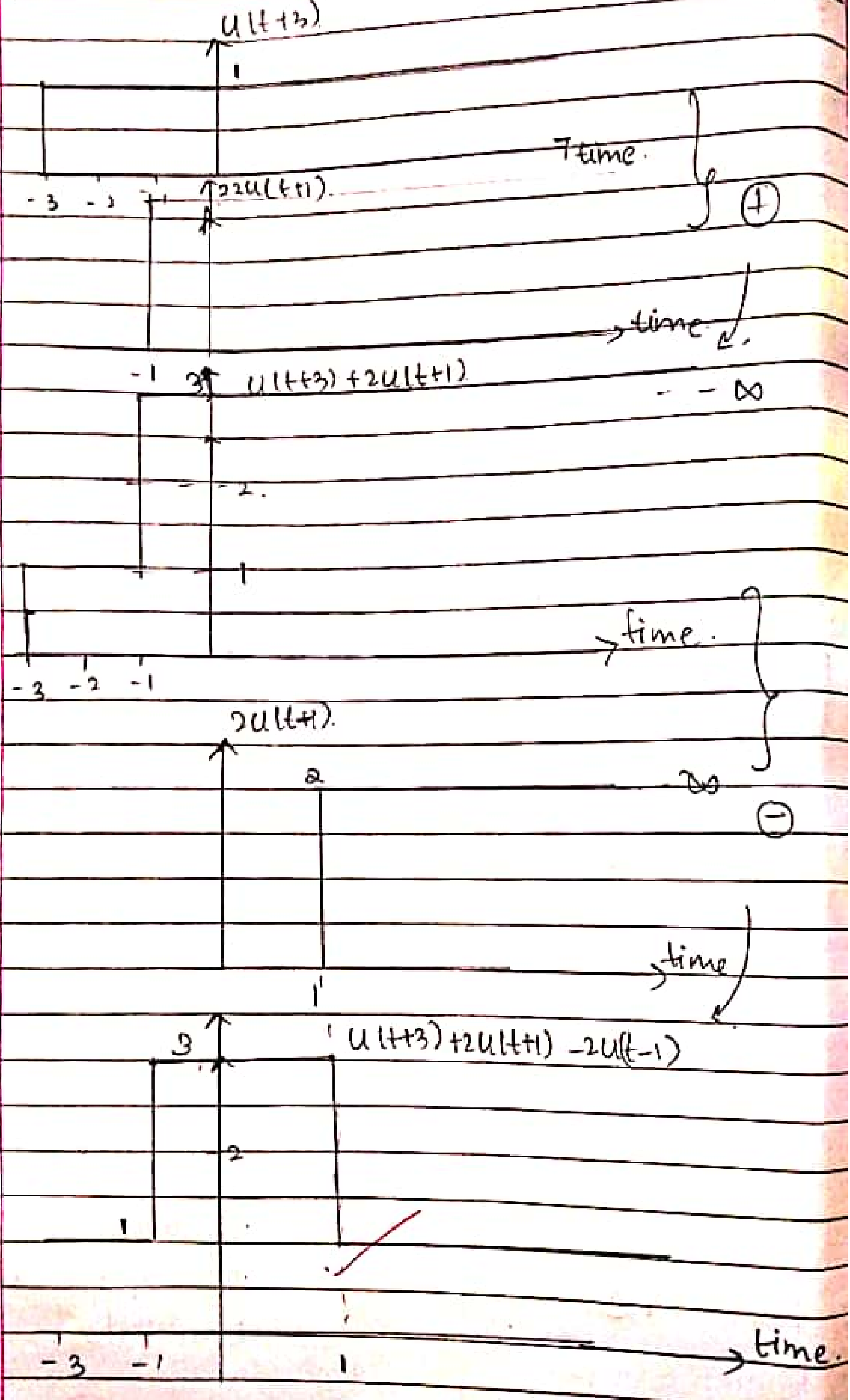
$x(t) = t(u(t) - u(t-a))$

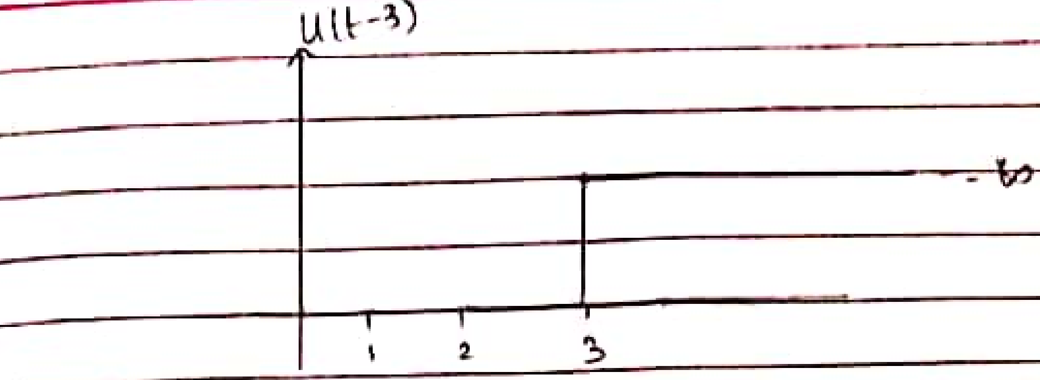




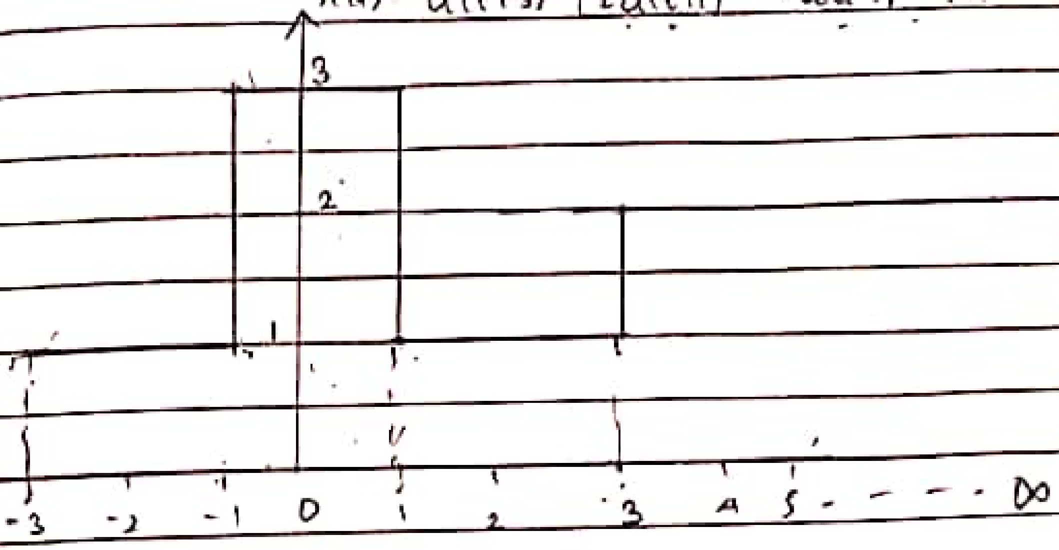
as differentiation of a ramp signal is a step signal.

(iii) $x(t) = u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

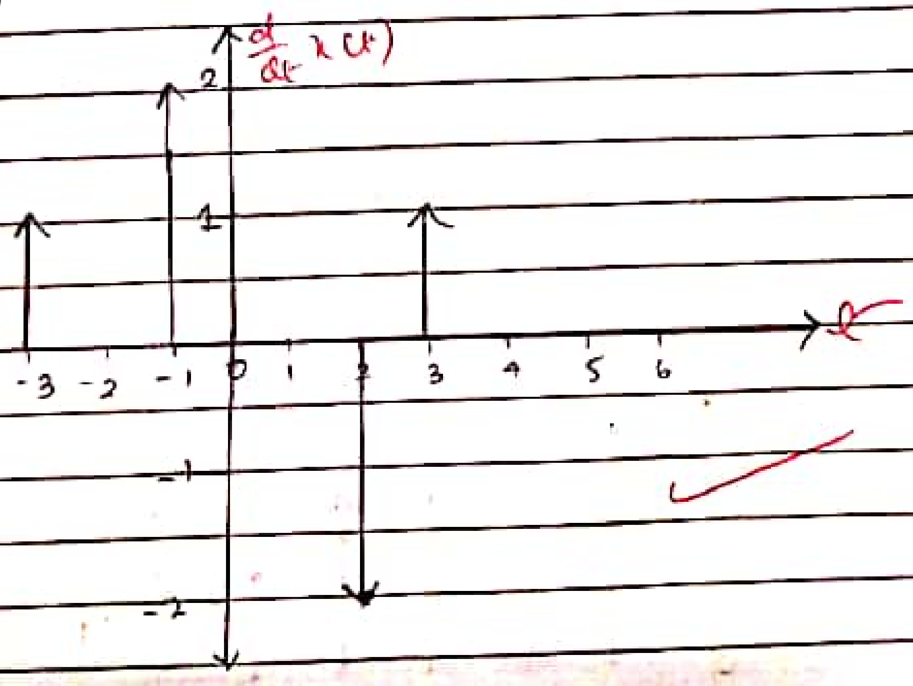




$x(t) = u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$



(4) ~~If x(n) = (8-n) { u(n) - u(n-8) }.~~



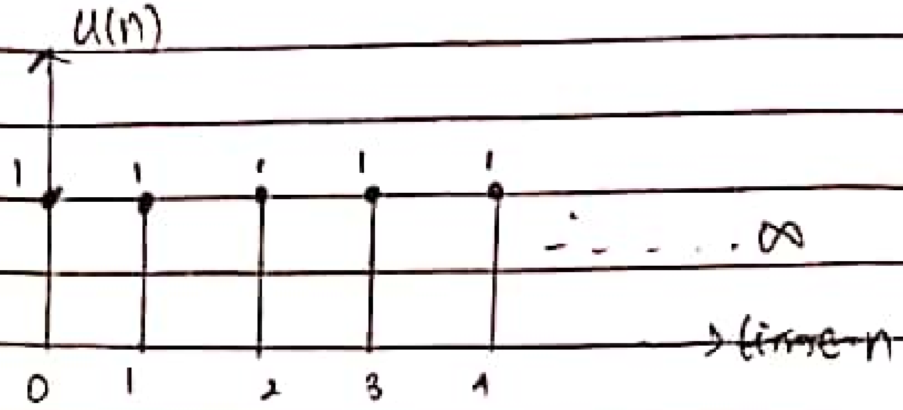
Verification

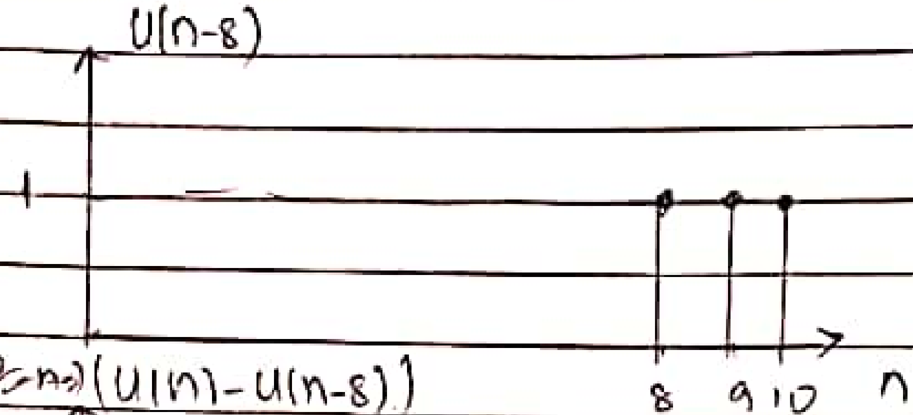
$\frac{d}{dt} u(t) = \delta(t)$ $\qquad$ $\frac{d}{dt} u(t-a) = \delta(t-a)$

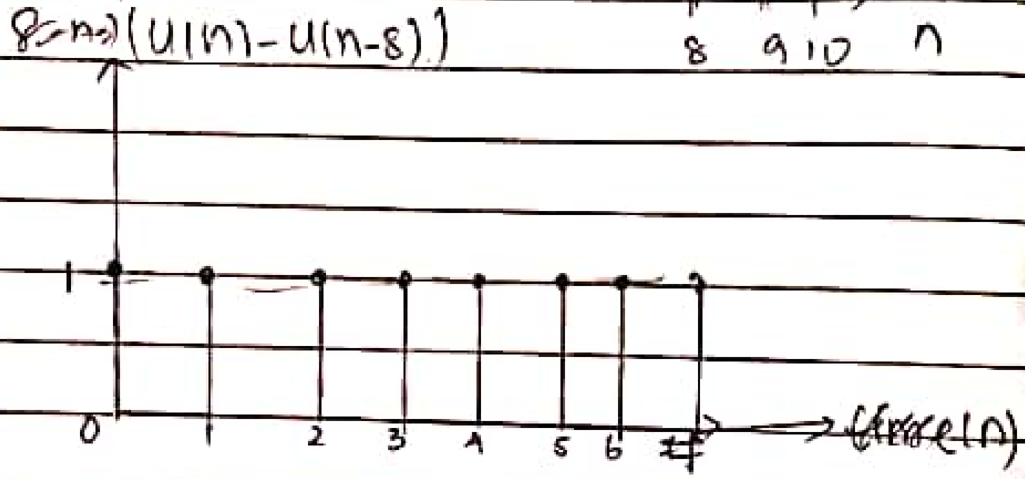
$\therefore \frac{d}{dt} x(t) = \delta(t+3) + 2\delta(t+2) - 2\delta(t-1) + \delta(t-3)$

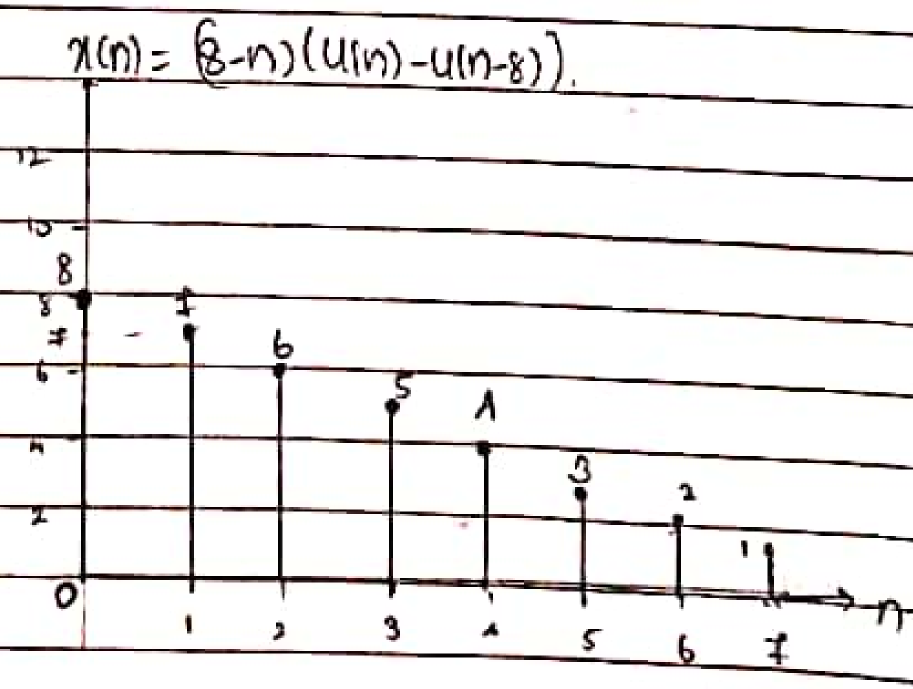
Caliber

(A) $x(n) = (8-n)\{u(n) - u(n-8)\}$


u(n)
1
1
1
1
1
.....∞
time n
0
1
2
3
4

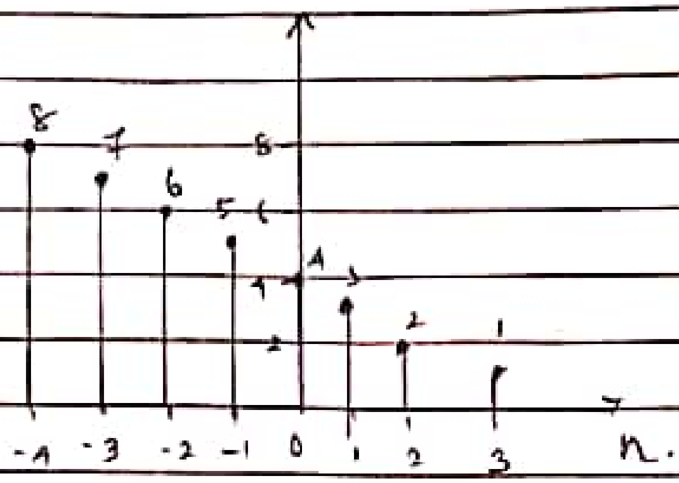

u(n-8)
1
8
9
10
n


(8-n)(u(n)-u(n-8))
1
0
1
2
3
4
5
6
7
time(n)

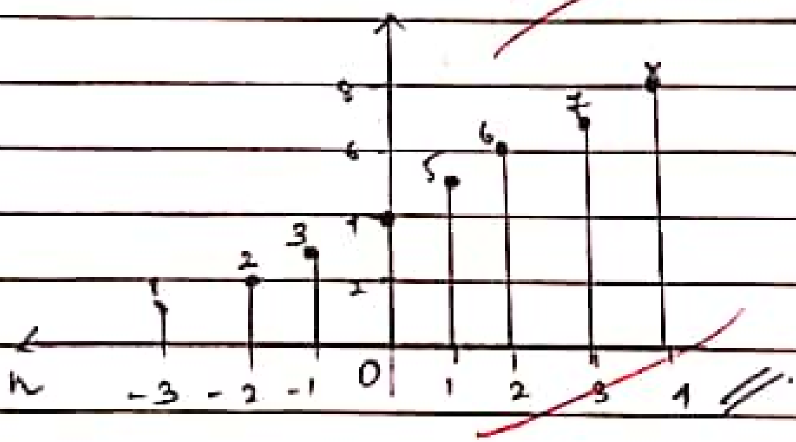

x(n) = (8-n)(u(n)-u(n-8)).
12
10
8
8
7
6
6
5
4
4
3
2
2
1
0
1
2
3
4
5
6
7
n

$y_1(n) = x(-n+4)$.

Consider $v(n) = x(n+4)$

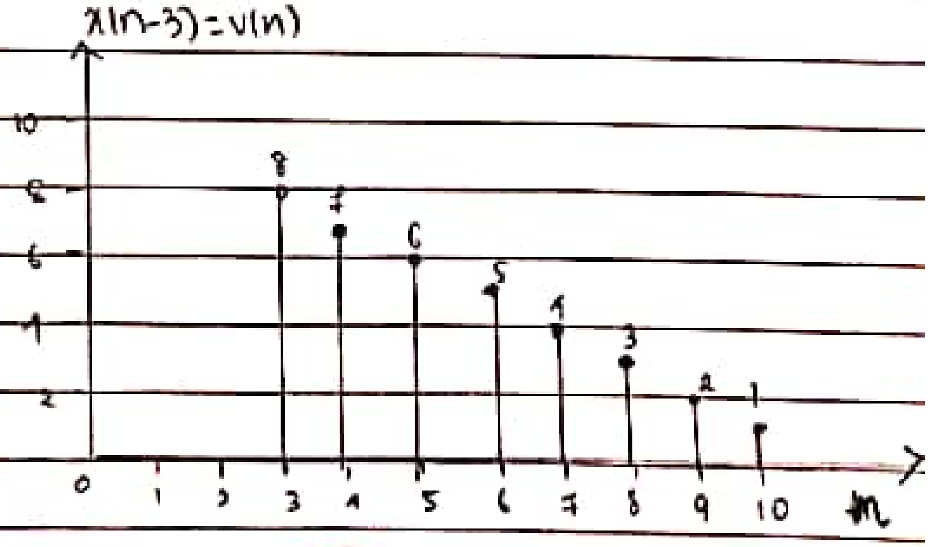


$y_1(n) = v(-n) = x(-n+4)$



$y_2(n) = x(2n-3)$

Consider $v(n) = x(n-3)$



$y_2(n) = u(2n)$ $\quad y_2(-1) = u(-2) = 0$

$y_2(0) = u(0) = 0$

$y_2(1) = u(2) = 0$

$y_2(2) = u(4) = 7$

$y_2(3) = u(6) = 5$

$y_2(4) = u(8) = 3$

$y_2(5) = u(10) = 1$

$y_2(6) = u(12) = 0$

$y_2(n) = x(2n-3)$

⑤ Check whether the signals are periodic or not.

① $x(t) = [\cos(2\pi t)]^2$ ② $x(n) = \cos(20\pi n) + \sin(50\pi n)$

① $x(t) = \cos^2(2\pi t)$

$= \dfrac{1 + \cos 4\pi t}{2}$

$x(t) = \dfrac{1}{2} + \dfrac{\cos 4\pi t}{2}$

The first time represents a DC signal which is periodic with $\boxed{T_1 = 1}$

$\cos 4\pi t \Rightarrow \omega = 4\pi$ $\quad 2\pi f = 4\pi = f = 2$

$T = \frac{1}{2} = \frac{1}{2} = \underline{0.5 sec}$

$\dfrac{T_1}{T_2} = \dfrac{1}{1/2} = \left.\dfrac{2}{1}\right\}$ rational number

fundamental period $= T_2 \times N_2 = 0.5 \times 2 = \underline{1 sec}$

(a) $x(n) = \cos(20\pi n) + \sin(50\pi n)$

$\Omega_1 = 20\pi$

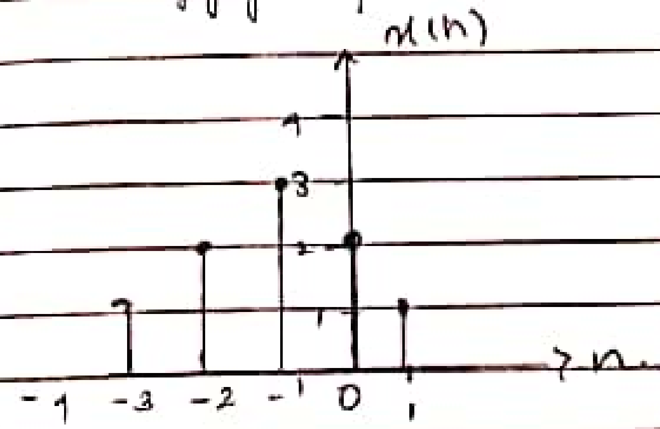
$\Omega = \frac{10 \times 2\pi}{1} \Rightarrow \boxed{N_1 = 1}$

$\Omega_2 = 50\pi = \frac{25 \times 2\pi}{1} \quad N_2 = 1$

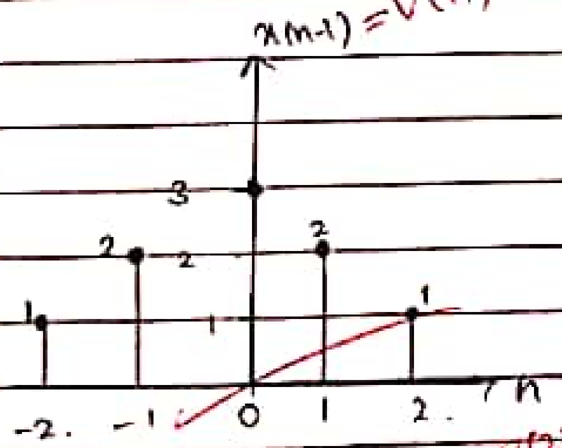
$\frac{N_1}{N_2} = \frac{1}{1}$ } rational number. fundamental period $= N_1 \times n_1 = 1$

periodic with fundamental period $N = 1$ (dc)

(b) find Energy of signal $x(2n-1)$



let $v(n) = x(n-1)$.



$y(n) = v(2n)$

$y(0) = v(0) = 3$

$y(1) = v(2) = 1$

$y(2) = x(4) = 0$

$y(-1) = x(-2) = 1$

$\text{Energy} = \sum_{n=-1}^{1} |y(n)|^2$

$= 1^2 + 3^2 + 1^2$

$= 10$ Joules

(7) Check weather periodic or not

$x(n) = \cos(n\pi/8) \cdot \sin(n\pi/4)$

$x(n) = \frac{1}{2}\left[\sin\left(\frac{n\pi}{8} + \frac{n\pi}{4}\right) - \sin\left(\frac{n\pi}{8} - \frac{n\pi}{4}\right)\right]$

$= \frac{1}{2}\left[\sin(n\,3\pi/8) + \sin(\pi/8\, n)\right]$

$\Omega_1 = 3\pi/8 = \frac{2 \times 3\pi}{16}$

$= 3/16 \times 2\pi \quad \therefore N_1 = 16$

$\Omega_2 = \pi/8 = \frac{2\pi}{16} = \frac{1}{16} \times 2\pi \quad N_2 = 16.$

$\therefore \frac{N_1}{N_2} = \frac{16}{16} = 1/1$ } rational number

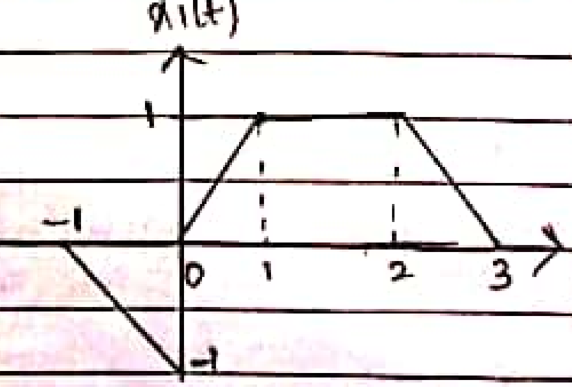
$\therefore$ periodic

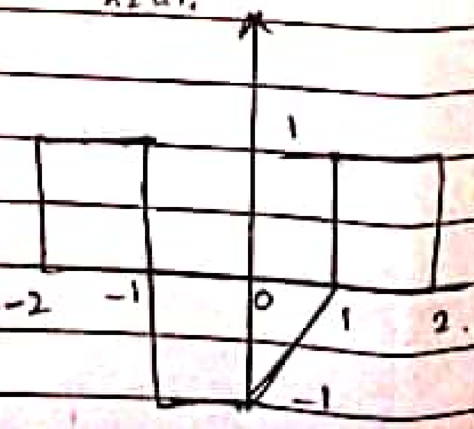
with fundamental period $= m_1 \times N_1 = 1 \times 16 = 16$

(8) $y(t) = x_1(t) \times x_2(t-1)$

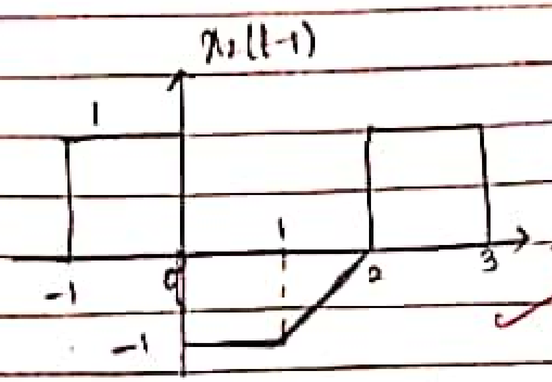
where

$x_1(t) =$ $\qquad$ $x_2(t)$:

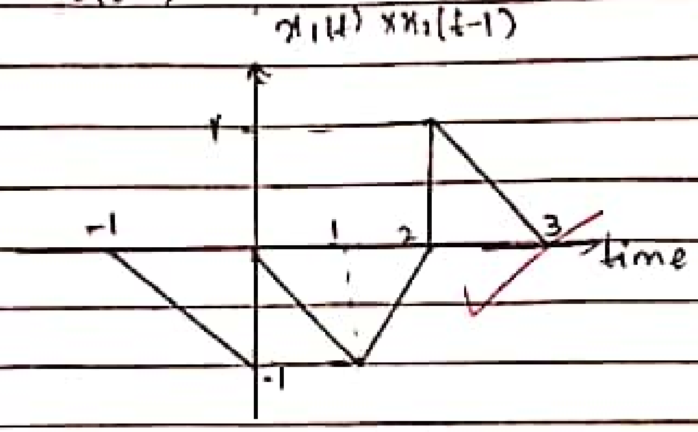




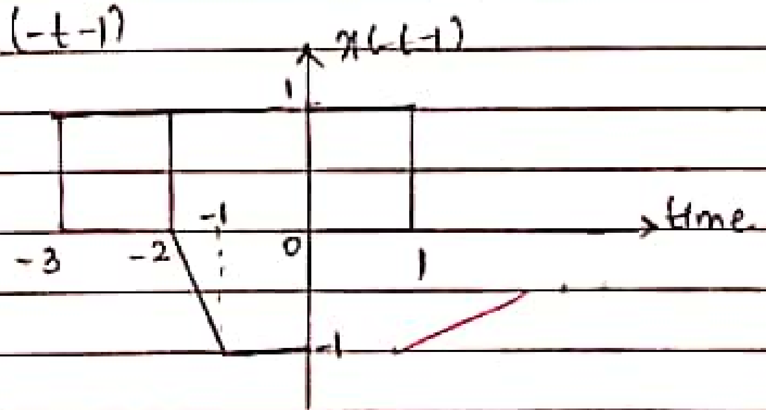
$x_2(t-1)$

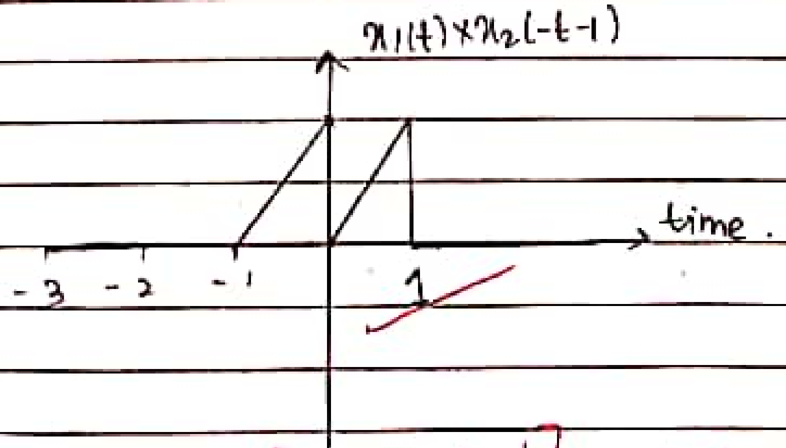


$x_1(t) \times x_2(t-1)$

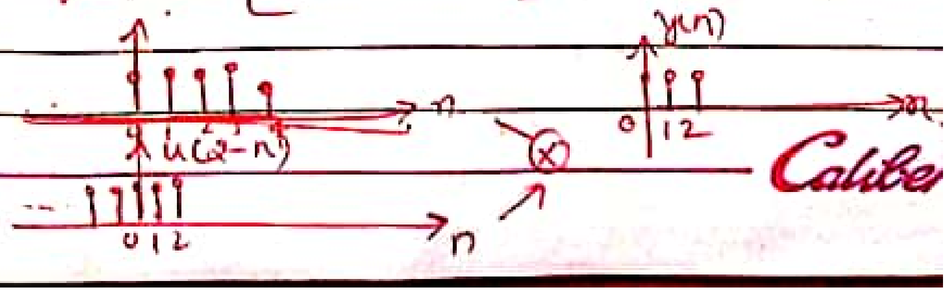


$x_2(-t-1)$



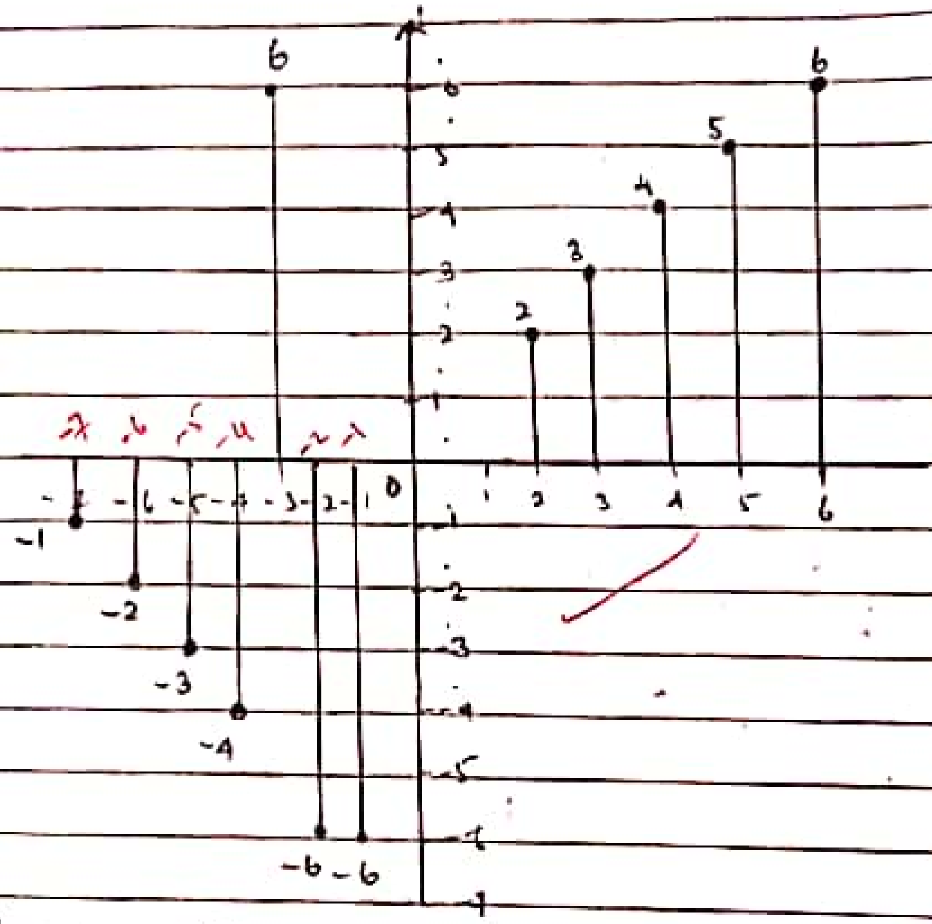


(iii) $y(n) = \left[1 \;\; 1 \;\; 1 \;\; 1 \;\; \tfrac{1}{2}\right] \cdot u(2-n)$

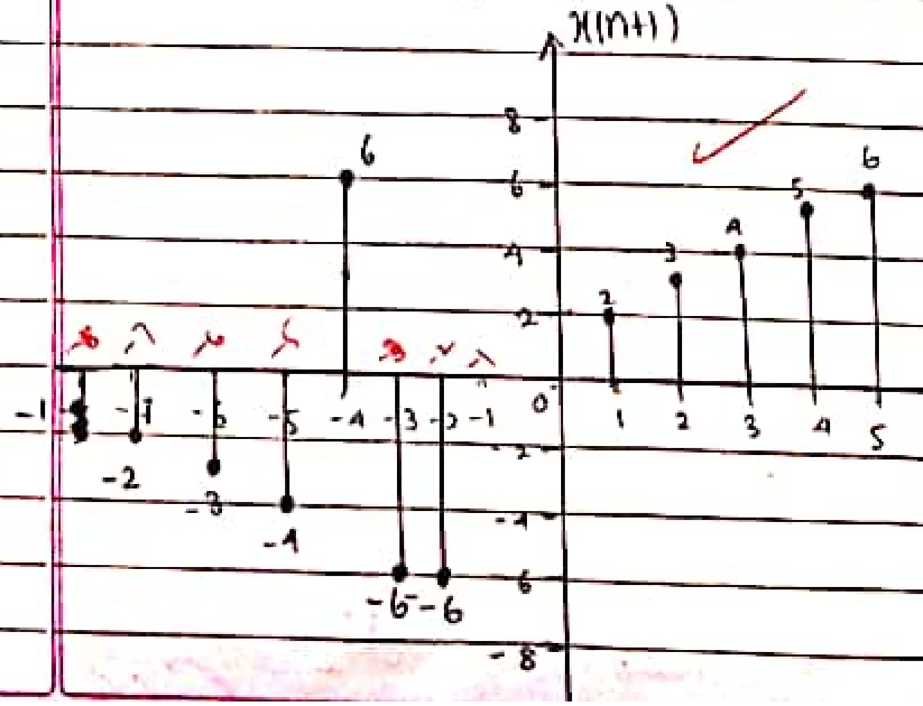


iii) find $y(n)$ if $x(n) = \begin{cases} -(n+8) & -8 < n < -3 \\ 6 & n = -3 \\ -6 & -3 < n < 0 \\ n & 1 < n < 7 \\ 0 & o.w \end{cases}$

$y(n) = 3x\left[\frac{n}{2}+1\right]$



Consider $v(n) = x(n+1)$



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let

$z(n) = v(n/2)$

$z(0) = v(0) = 0$

$z(2) = v(1) = 2$

$z(4) = v(2) = 3$

$z(6) = v(3) = 4$

$z(8) = v(4) = 5$

$z(10) = v(5) = 6$

$z(-4) = v(-2) = -6$

$z(-6) = v(-3) = -6$

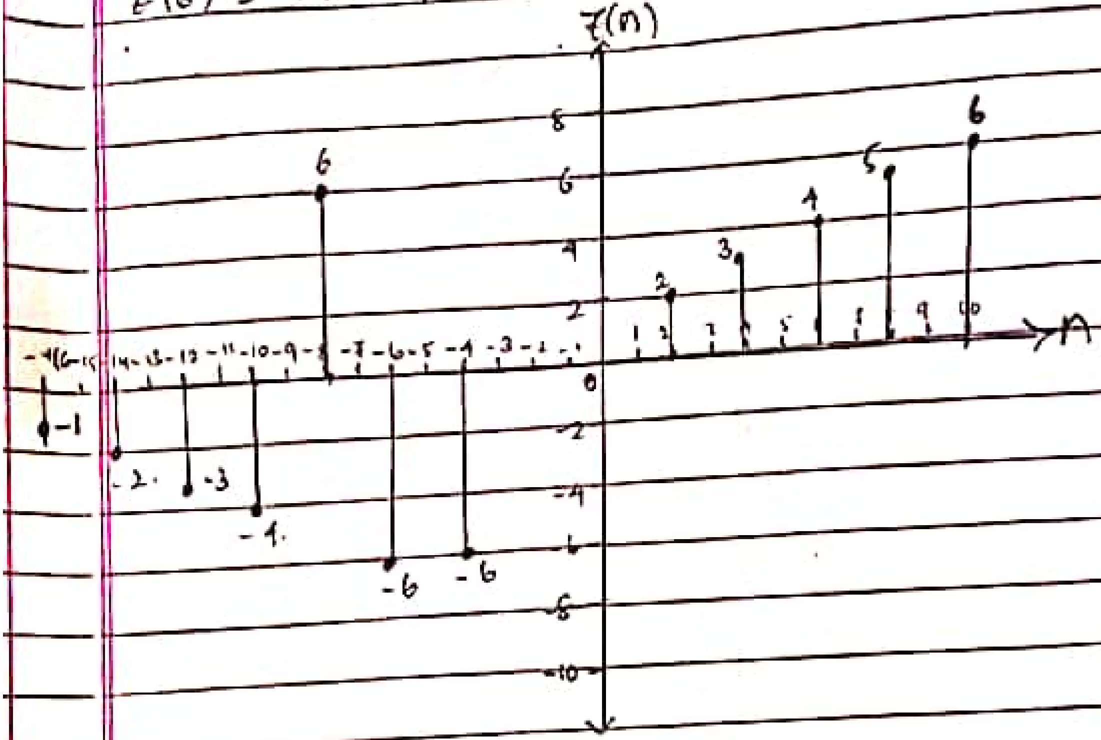
$z(-8) = v(-4) = 6$

$z(-10) = v(-5) = -4$

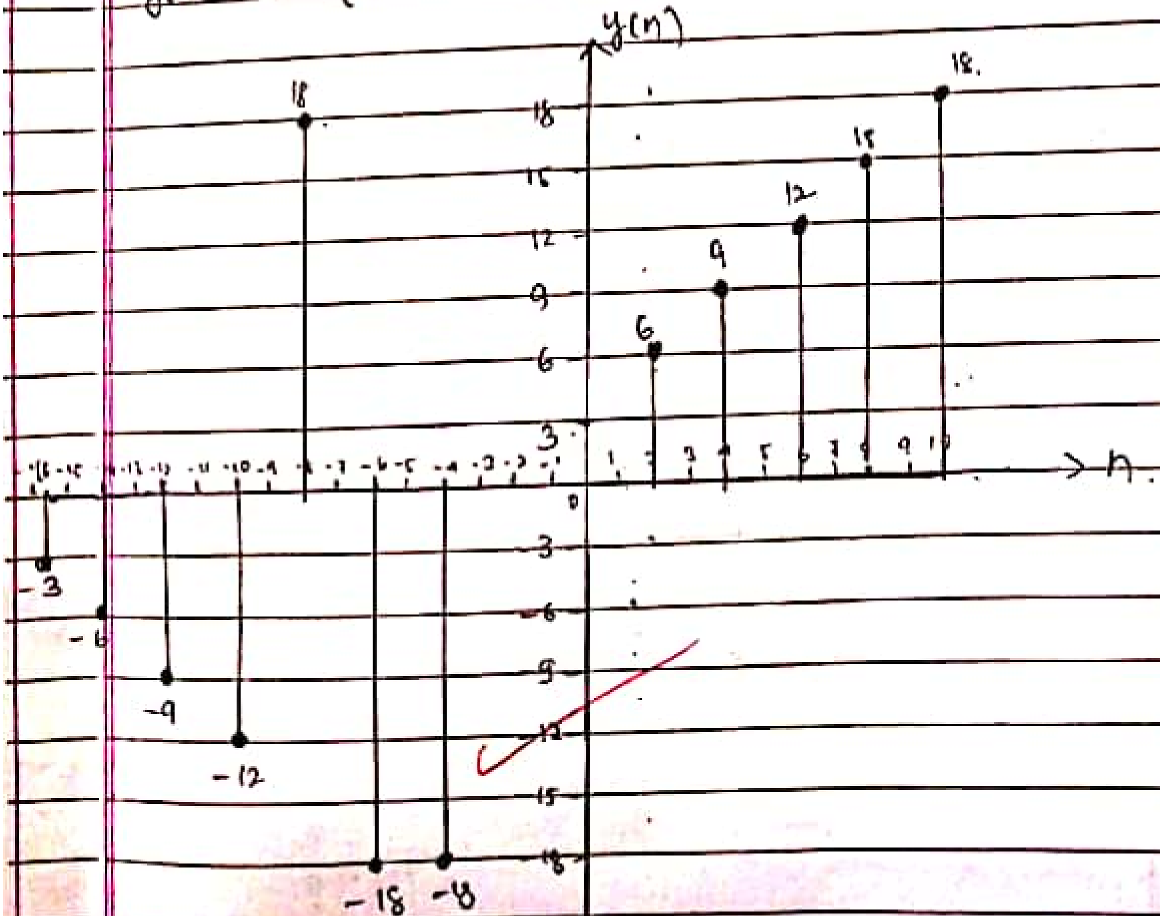
$z(-12) = v(-6) = -3$

$z(-14) = v(-7) = -2$

$z(-16) = v(-8) = -1$

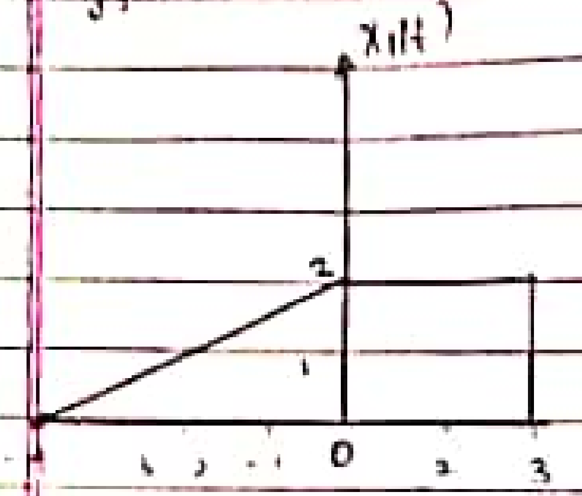


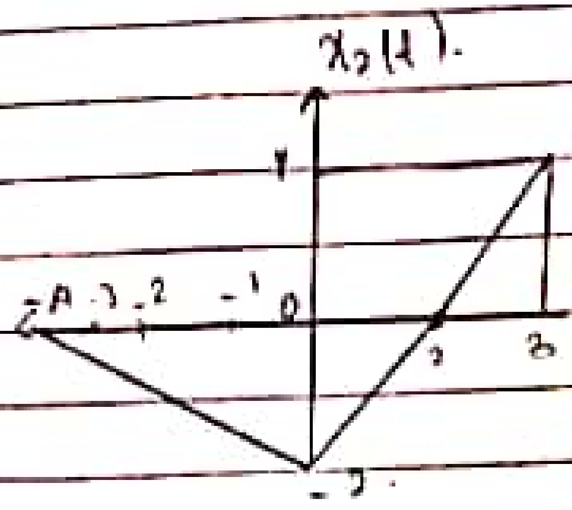
$y(n) = 3x(n/2+1) = 3z(n)$



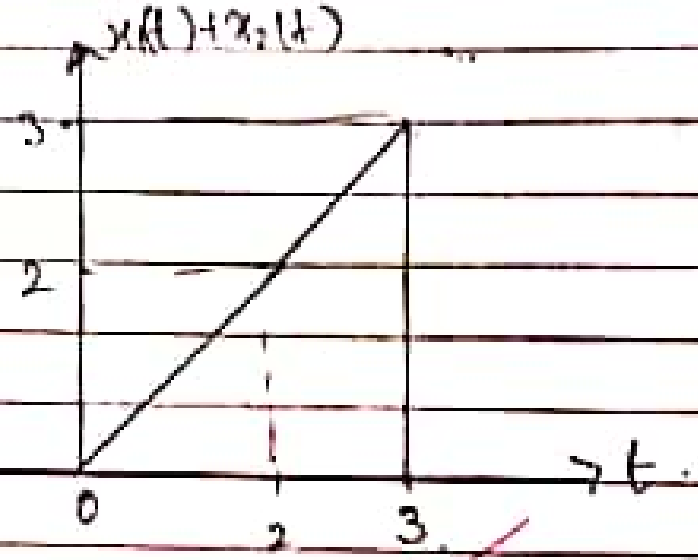
(ii) $y_1(t) = x_1(t) + x_2(t)$

$y_2(t) = x_1(t) * x_2(t)$

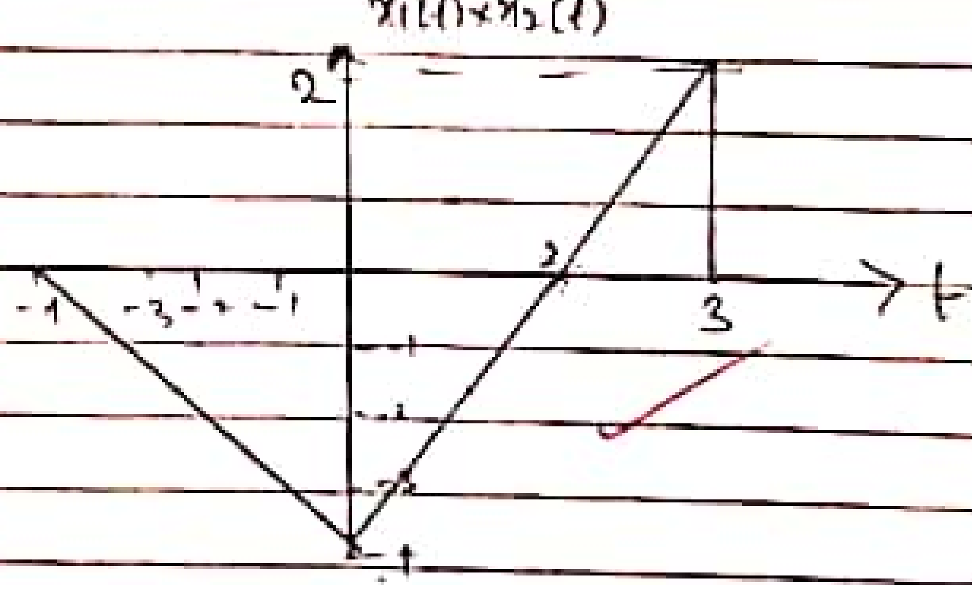




$x_1(t) + x_2(t) = y_1(t)$

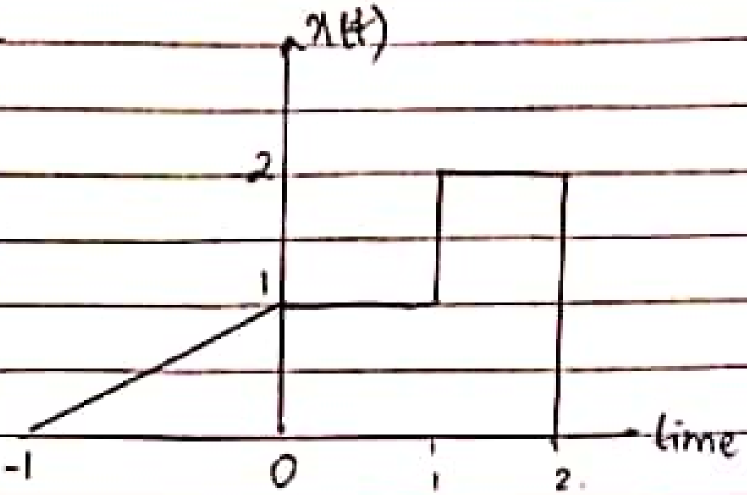


$y_2(t) = x_1(t) \times x_2(t)$.

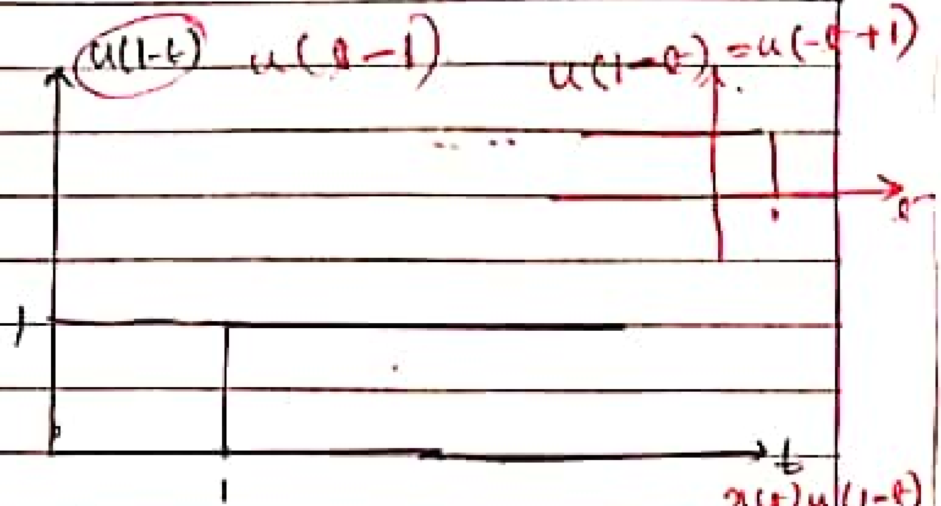


(9) Sketch (i) $x(t)\,u(1-t)$

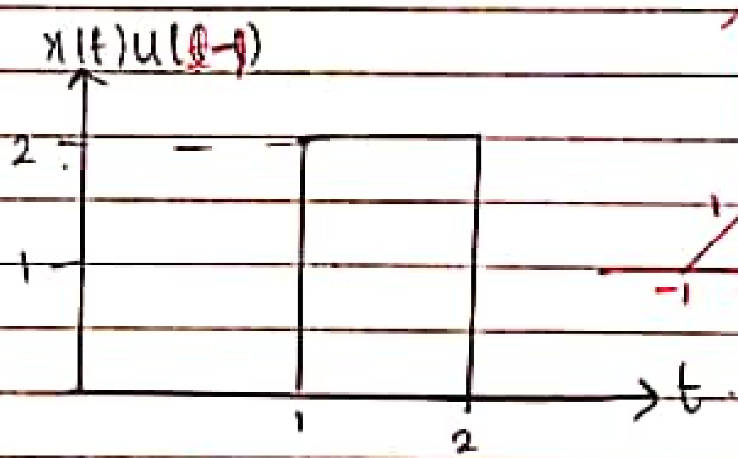
$x(t) =$

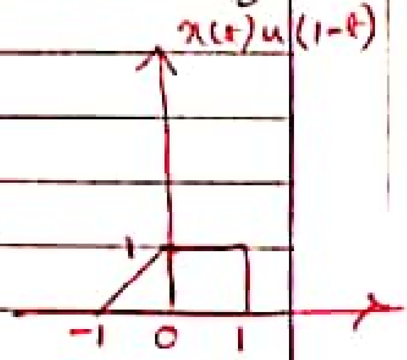


$u(1-t)$:



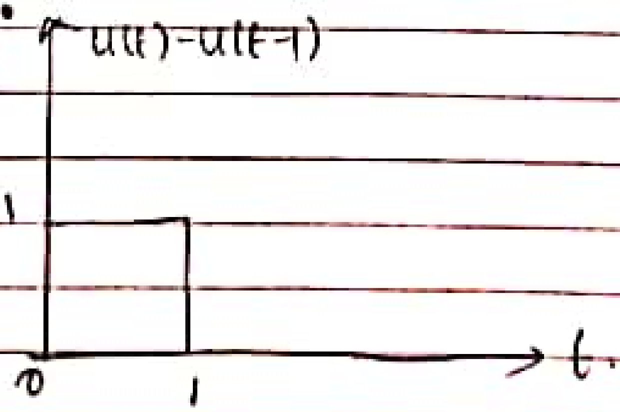
$x(t)\cdot u(1-t)$:



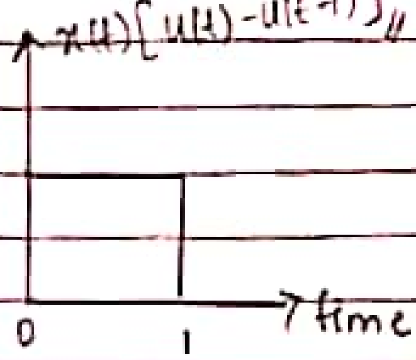


(ii) $x(t)\,[\,u(t) - u(t-1)\,]$

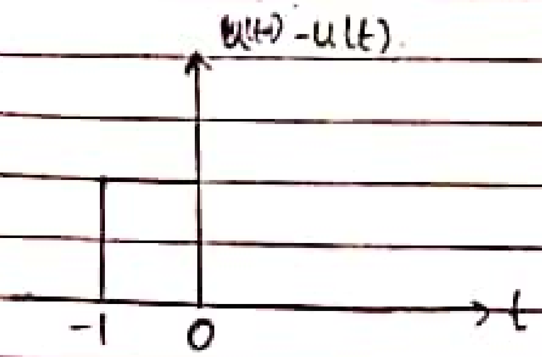
$u(t) - u(t-1)$:



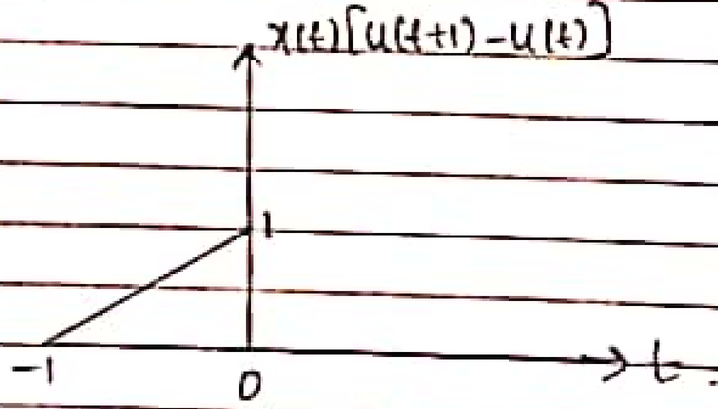
$x(t)\left(u(t) - u(t-1)\right)$



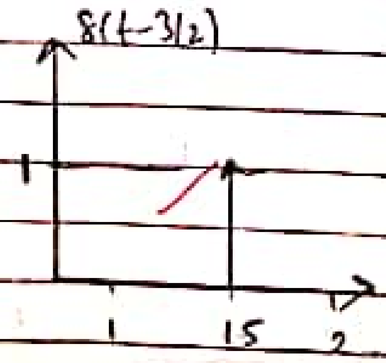
(ii) $x(t)\left[u(t+1) - u(t)\right]$



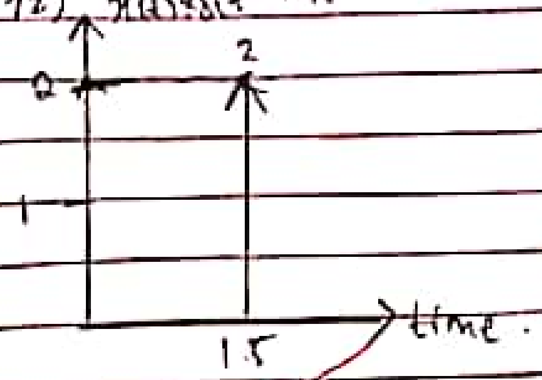
$x(t)\left[u(t+1) - u(t)\right]$



(iv) $x(t) \times \delta(t - 3/2)$



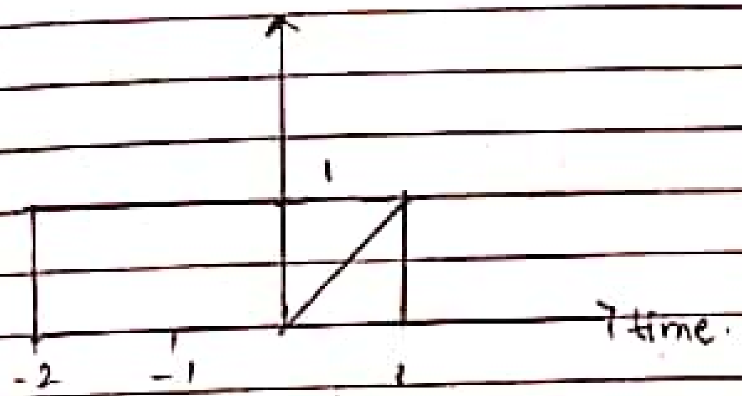
$x(t) \times \delta(t - 3/2)$, $x(t) \times \delta(t - 3/2)$.



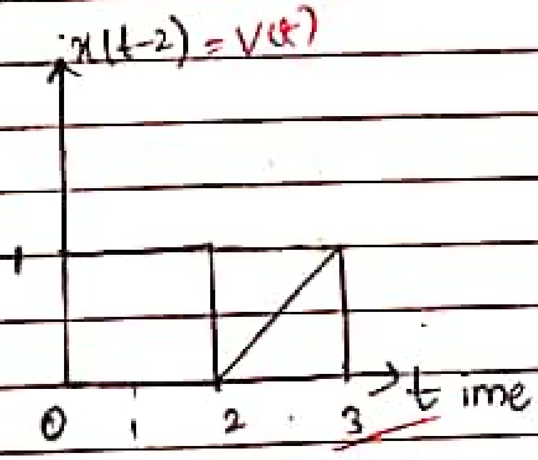
(10) Sketch.

$x(-2t-2)$ where

$x(t) =$



consider $v(t) = x(t-2)$

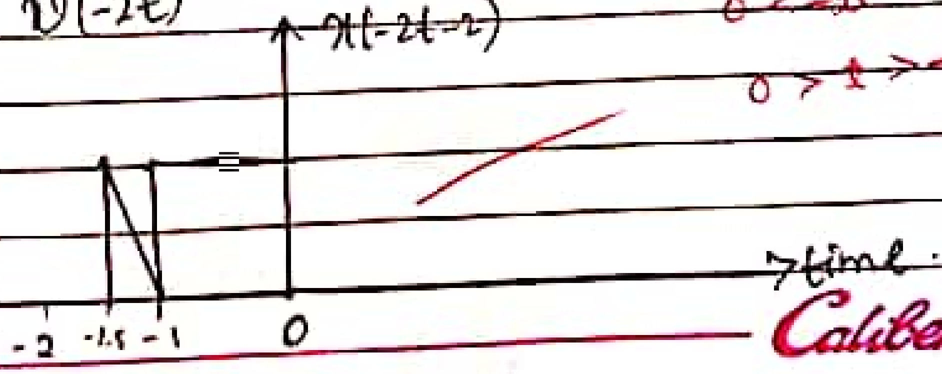


$0 < t < 3$

$0 < -2t < 3$

$0 > t > -1.5$

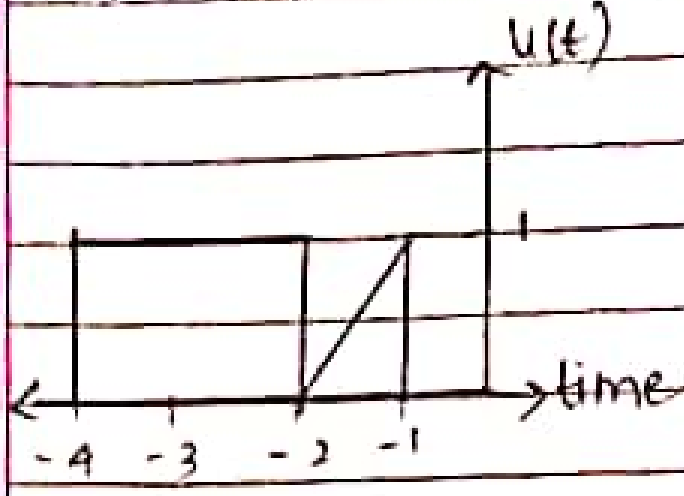
$v_1(t) = v(-2t)$



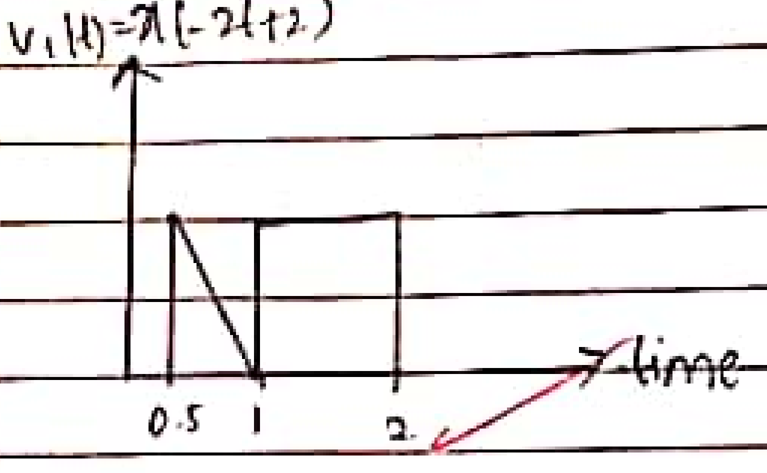
Caliber

(ii) $x(-2t+2)$

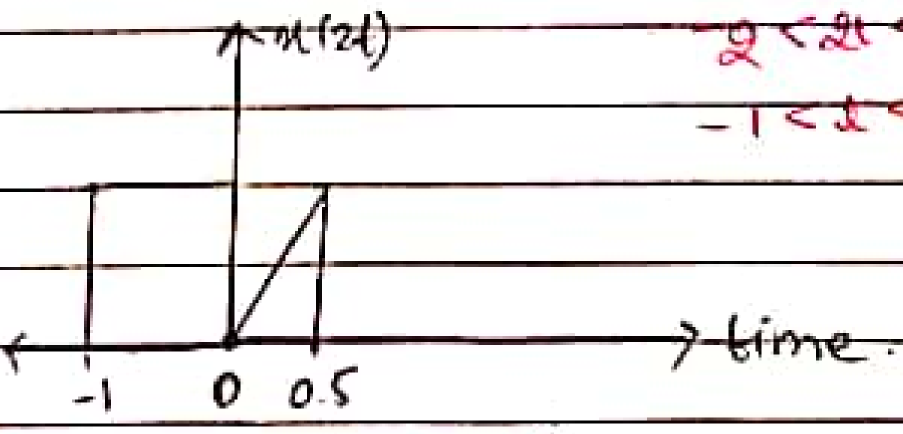
Let $v(t) = x(t+2)$



$v_1(t) = v(-2t)$



(iii) $x(+2t)$.



$-2 < 2t < 1$

$-1 < t < \frac{1}{2}$

(11) find whether each of the signals are periodic or not if periodic find its fundamental period

i) $10\sin(2n)$

here comparing with $10\sin(\Omega n)$

$\Rightarrow \Omega = 2$

which cannot be represented as an integral multiple of 2π hence not periodic

ii) $15\cos(0.2\pi n)$

comparing with $A\cos(\Omega n)$

$\Omega = 0.2\pi$

$= \frac{2}{10}\pi = \frac{1}{10} \times 2\pi = \frac{k}{N} \times 2\pi$

As the signal could be represented as an integral multiple of 2π the signal is periodic with fundamental time period $= \underline{10}$.

(iii) $5\sin(6\pi/35n)$

here $\Omega = 6\pi/35$

$\Omega = \frac{3 \times 2\pi}{35}$

$\therefore N = 35$

$\therefore$ The signal is periodic with $N = 35$.

(iv) $(-1)^n = \cos(n\pi)$

comparing with $\cos(\Omega n)$

$\Omega = \pi$

$\Omega = \frac{2\pi}{2} = \frac{1}{2} \times 2\pi$

$\therefore$ The signal is periodic with $N = 2$.

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v) $x(n) = \cos(\frac{\pi}{8} n^2)$ $= \frac{1}{2}[1 + \cos 2\frac{\pi n}{8}]$

$\Rightarrow N = 8$

$x(0) = 1$

$x(1) = 0.923$

$x(2) = 0$ $\therefore$ The value repeats itself after $N = 8$ values.

$x(3) = -0.923$

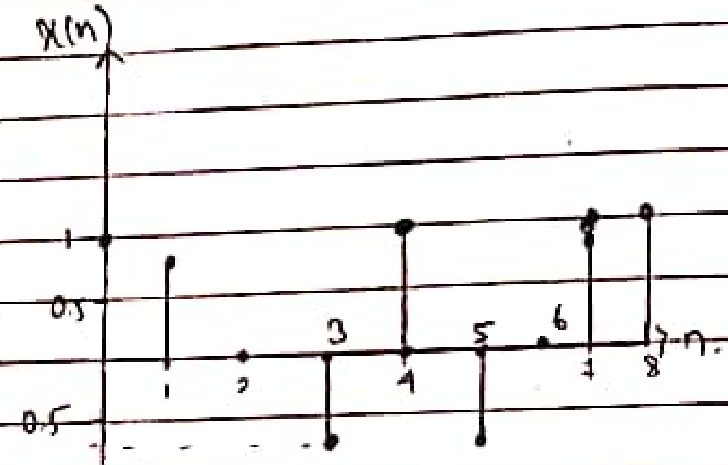
$x(4) = 1$ $\therefore$ The signal is periodic with $N = 8$.

$x(5) = -0.923$

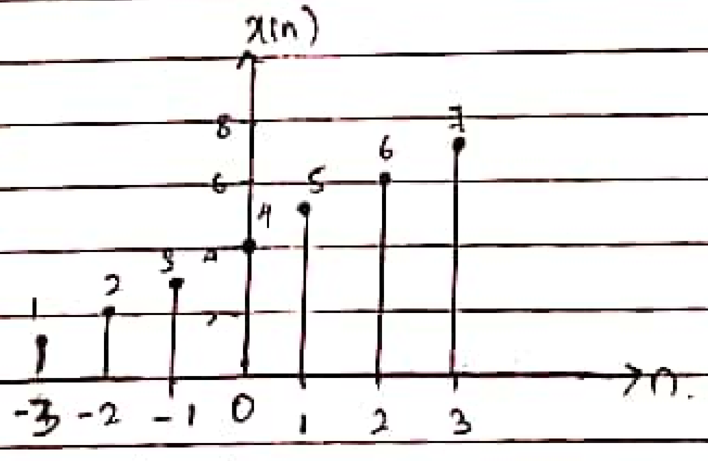
$x(6) = 0$

$x(7) = 0.923$

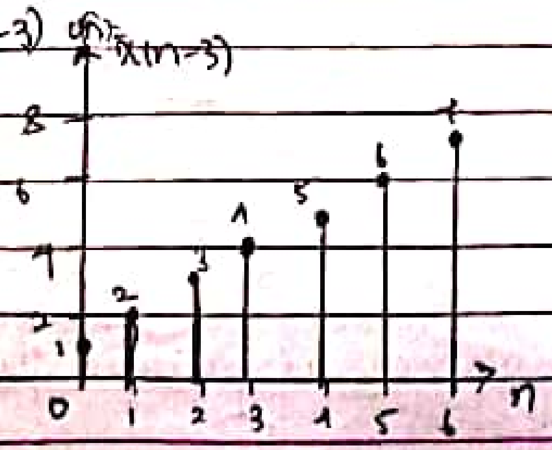
$x(8) = 1$



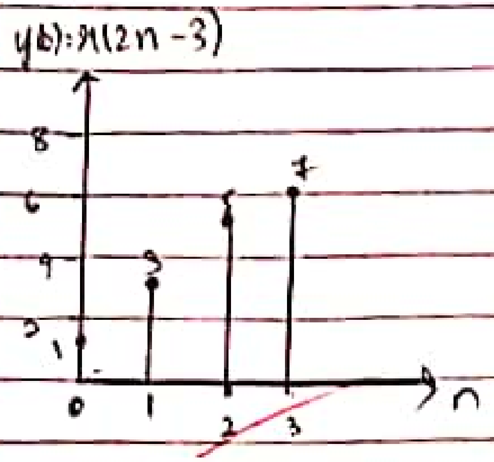
(vi) $x(n) = \{1, 2, 3, \underset{\uparrow}{4}, 5, 6, 7\}$ find $x(2n-3)$ and $x(-2n+1)$



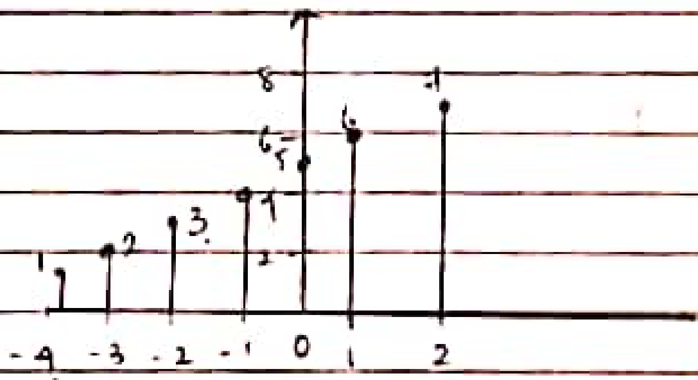
Let $u(n) = x(n-3)$



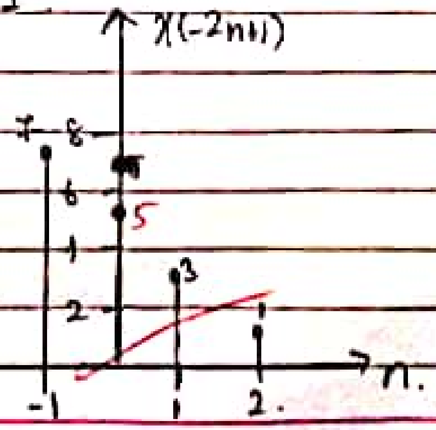
$y(0) = v(0) = 1$
$y(1) = v(2) = 3$
$y(2) = v(4) = 5$
$y(3) = v(6) = 7$



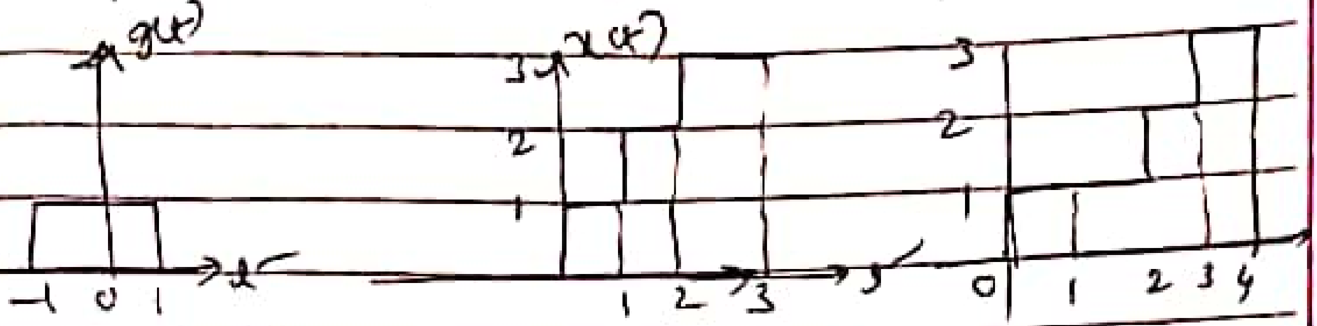
2) $x(-2n+1)$
$v(n) = x(n+1)$



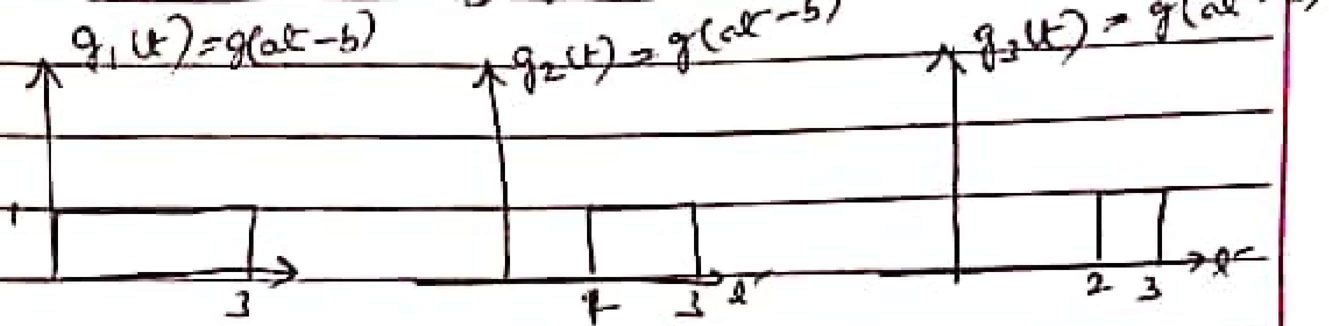
$y(n) = v(2n)$
$y(0) = v(0) = 5$
$y(1) = x(2) = 7$
$y(-1) = x(-2) = 3$
$y(-2) = x(-4) = 1$



Expr^n $x(t)$ & $y(t)$ in terms of $g(t)$



$x(t)$ in terms of $g(t)$

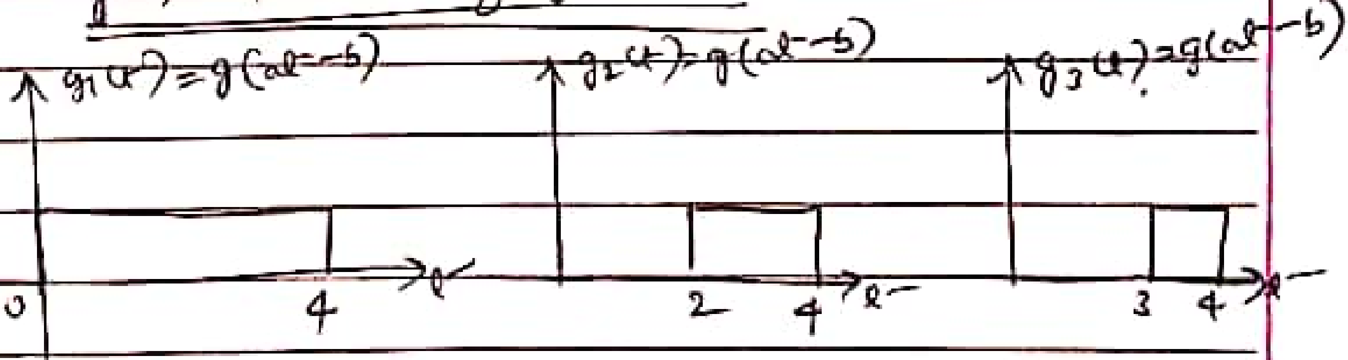


$b = 1,\ a = 2/3$ $\quad$ $b = 2,\ a = 1$ $\quad$ $a = 2,\ b = 5$

$$x(t) = g_1(t) + g_2(t) + g_3(t)$$

$$= g\left(\tfrac{2}{3}t + 1\right) + g(t-2) + g(2t-5)$$

$y(t)$ in term of $g(t)$



$a = \frac{1}{2},\ b = 1$ $\quad$ $a = 1,\ b = 3$ $\quad$ $a = 2.\ b = 7.$

$$y(t) = g_1(t) + g_2(t) + g_3(t)$$

$$= g\left(\tfrac{t}{2} - 1\right) + g(t-3) + g(2t-7)$$