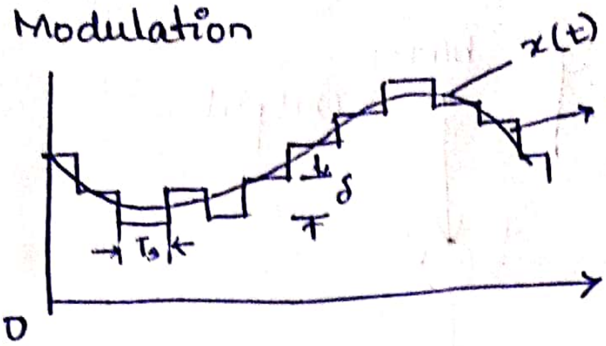
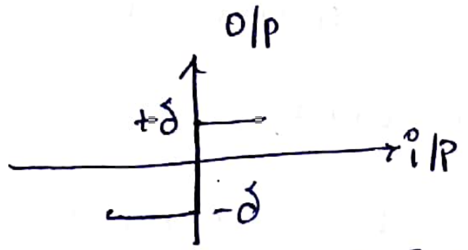
Delta Modulation



Binary seq. 0 0 1 0 1 1 1 1 0 0 0

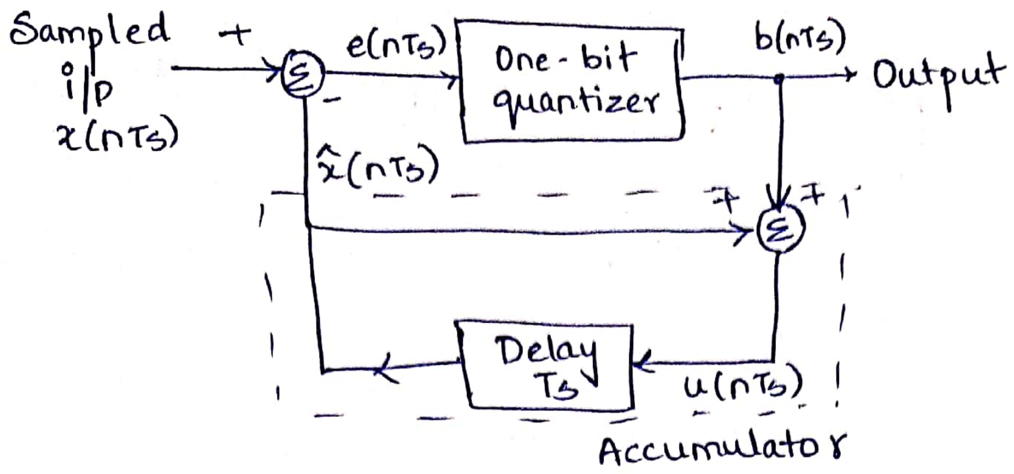
Fig (i) Illustration of delta modulation

1. Delta modulation (DM) is the one-bit (or two-level) version of DPCM.
2. DM provides a staircase approximation to the oversampled version of an i/p baseband signal, as shown in fig (i). The difference betwn the input & the approximation is quantized into only two levels, namely $\pm\delta$, corresponding to +ve & -ve differences resp^ly.
If approximation lies below the signal at any sampling epoch, it is increased by δ. If, on the other hand it lies above the signal, it is decreased by δ.

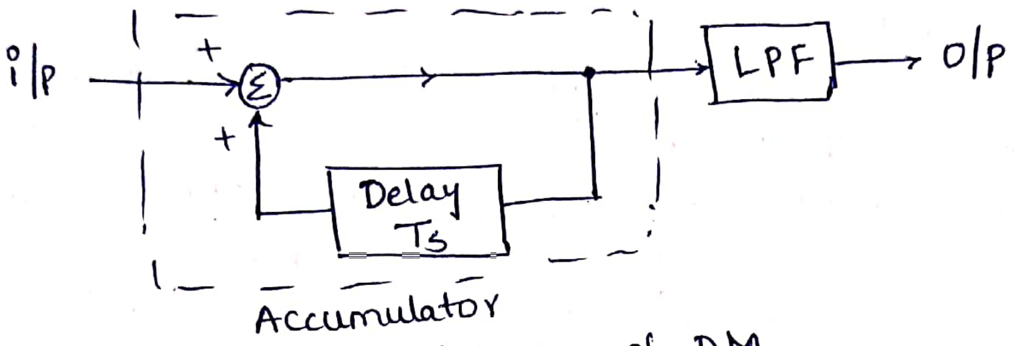


I/p - o/p characteristics of two-level quantizer.

δ denotes the absolute value of the two representa-tion levels of the one-bit quantizer used in DM. These two levels are indicated in the transfer characteristic. The step size Δ of the quantizer is $\therefore$ $\Delta = 2\delta$



(a) Tx of DM



(b) Rx of DM

From fig (a),

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$
$$= x(nT_s) - u(nT_s - T_s) \rightarrow ①$$

$$b(nT_s) = \delta \, \text{sgn}[e(nT_s)]. \rightarrow ②$$

$$u(nT_s) = u(nT_s - T_s) + b(nT_s) \rightarrow ③$$

where T_s is the sampling period

$e(nT_s)$ is prediction error

$b(nT_s)$ is one-bit word transmitted by the DM system.

* DM offers 2 unique features
  i. one-bit code word for the o/p
  ii. simple Tx & Rx design.

However, DM systems are subject to two types of error i) Slope - Overload distortion
ii) Granular Noise.

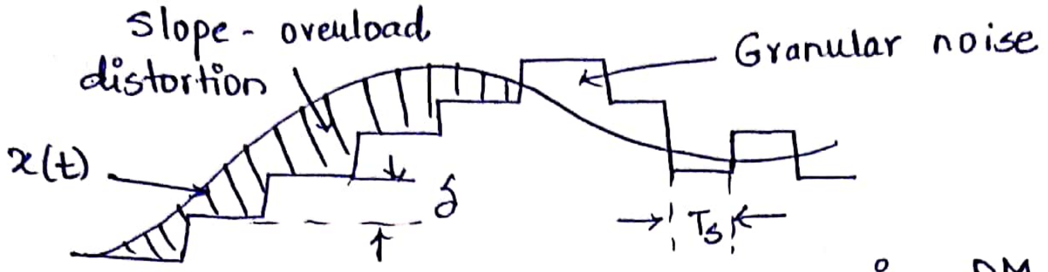


Illustration of quantization error in DM.

Let $q(nT_s)$ denote the quantizing error.

Then, $u(nT_s) = x(nT_s) + q(nT_s)$

$\therefore e(nT_s) = x(nT_s) - x(nT_s - T_s) - q(nT_s - T_s) \rightarrow (4)$

$\therefore$ Eq^n (4) represents that except for the quantization error $q(nT_s - T_s)$, the quantizer i/p is a first backward difference of the i/p signal, which may be viewed as a digital approximation to the derivative of the i/p signal or the inverse of the digital integration process.

The condition to be satisfied to overcome the distortion is.

$$\frac{\delta}{T_s} \geq \max \left| \frac{dx(t)}{dt} \right|$$

When $u(t)$ staircase approx. falls behind $x(t)$ as shown in figure. This condition is called slope-overload and the resulting quantization error is called <u>slope - overload distortion</u>.

<u>Granular noise</u> occurs when step size Δ is too large relative to the local slope characteristics of the input waveform $x(t)$.

$(SNR)_o$

If $x(t) = a_0 \cos(2\pi f_0 t)$

Then maximum slope of the signal $x(t)$ is given by

$$\max\left|\frac{dx(t)}{dt}\right| = 2\pi f_0 a_0$$

$\therefore$ to avoid slope overload, the condition on value of δ :

$$\frac{\delta}{T_s} \geq 2\pi f_0 a_0 .$$

The condition on amplitude ~~mode~~ of the sinusoidal modulation :

$$a_0 \leq \frac{2\pi f_0 T_s}{\delta}$$

$P_{max} = \frac{a_0^2}{2}$, is the max. permissible value of the o/p signal power.

$$\therefore P_{max} = \frac{\delta^2}{8\pi^2 f_0^2 T_s^2}$$

Quantization error is given by its variance σ_Q^2

as $\sigma_Q^2 = E[Q^2]$

$$= \frac{1}{\Delta}\int_{-\delta}^{\delta} q^2 . dq .$$

$$= \frac{1}{2\delta}\left[\frac{q^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta} \cdot \left[\frac{\delta^3 + \delta^3}{3}\right]$$

$$= \frac{\cancel{2}\delta^3}{\cancel{2}.3\delta}$$

$$= \frac{\delta^2}{3}$$

$$\therefore (SNR)_{o,max} = \frac{P_{max}}{\text{Avg. o/p noise power.}}$$

$$= \frac{P_{max}}{WT_s\left(\frac{\delta^2}{3}\right)}$$

$$= \frac{\delta^2}{8\pi^2 f_o^2 T_s^2} \times \frac{3}{WT_s.\delta^2}$$

$$(SNR)_{o,max} = \frac{3}{8\pi^2 W f_o^2 T_s^3}$$

ADAPTIVE DELTA MODULATION

The performance of delta modulator can be improved significantly by making the step size of the modulator assume a time - varying form.

When the segment of i/p signal is steep, the step size is increased & when the i/p signal is varying slowly, the step size is reduced.

$\therefore$ The step size is adapted to the level of the i/p signal. This method is called as adaptive delta modulation (ADM).

Two types of ADM are : (classification based on the type of scheme used for adjusting the step size).

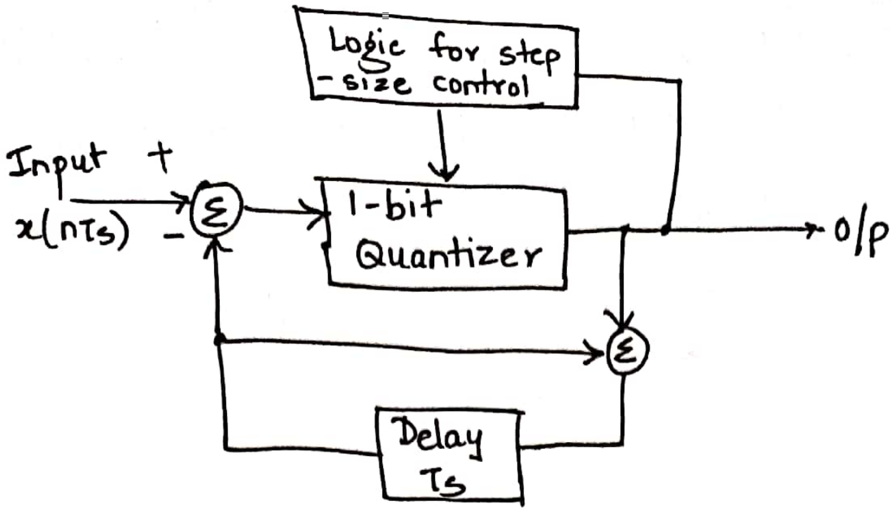
1. Discrete set of values is provided for the step size.
2. Continuous range for step - size variation is provided.

The step size $\Delta(nT_s)$ or $2\delta(nT_s)$ is constrained to lie between min. & max. values as given below

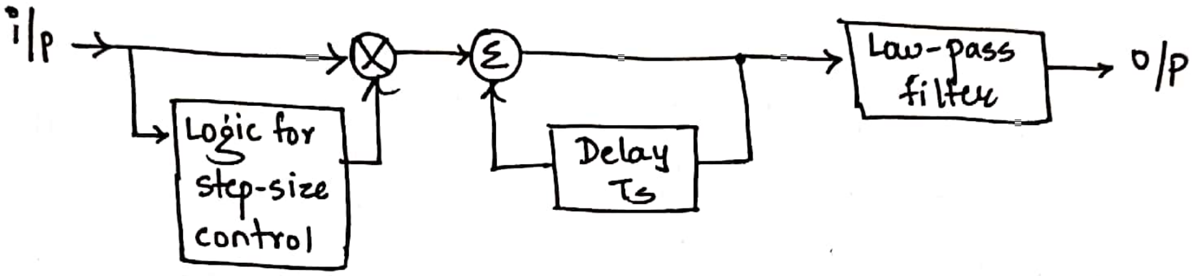
$$\delta_{min} \leq \delta(nT_s) \leq \delta_{max}.$$

δ_{max} : controls the amount of slope - overload distortion.

δ_{min} : controls the amount of idle channel noise.



(a) ADM Tx



(b) ADM Rx.

The adaptation rule for $\delta(nT_s)$ is

$$\delta(nT_s) = g(nT_s) \cdot \delta(nT_s - T_s)$$

where,

$$g(nT_s) = \begin{cases} K & \text{if } b(nT_s) = b(nT_s - T_s) \\ K^{-1} & \text{if } b(nT_s) \neq b(nT_s - T_s). \end{cases}$$

The adaptation algorithm is called a constant factor ADM with one-bit memory.

Digital Multiplexers

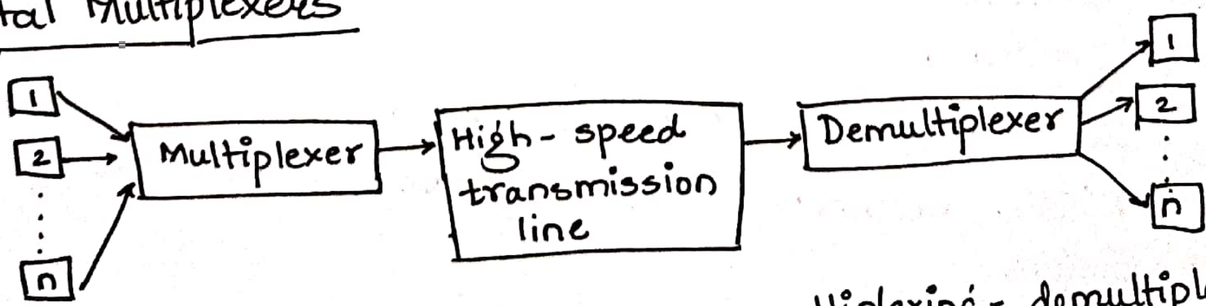


fig: Conceptual diagram of multiplexing - demultiplexing

Here, we consider the multiplexing of digital signals at different bit rates. Digital mux allows us to combine several digital signals, such as computer o/ps, digitized voice signals, digitized facsimile & television signals into a single data stream.

The multiplexing is accompalished by using a bit-by-bit interleaving procedure with a selector switch that sequentially takes a bit from each incoming line & then applies it to the high-speed common line. At the receiving end, the o/p of this common line is separated out into its low-speed individual components & then delivered to their respective destinations.

Basic problems involved in the system

1. Synchronization provision should be made for incoming signals (digital), so that they can be properly interleaved.
2. The signals should include some kind of framing, so that the individual components can be identified.
3. The multiplexer needs to handle small variations in the bit rates of the incoming digital signal.

To overcome these problems, the following can be adopted.

1. Bit shuffling : to accommodate small variations in individual data rates.
2. Use elastic store at mux : to read out the data stream at a rate different from the rate it was read in. Bell system M12 mux. utlized a frame synchronization method.

SIGNAL TO QUANTIZATION NOISE RATIO.

Consider midtread type of quantizer. Then no. of representation levels in the quantizer as

$$L = 1 + \frac{2x_{max}}{\Delta} \rightarrow (1)$$

Since no. of representation levels in the quantizer as mid-tread has odd no. of levels is given by

$$L = 2^n - 1 \rightarrow (2)$$

Comparing (1) & (2),

$$1 + \frac{2x_{max}}{\Delta} = 2^n - 1$$

$$\frac{2x_{max}}{\Delta} = 2^n - 2$$

$$\frac{x_{max}}{2^{n-1} - 1} = \Delta$$

$$\therefore \Delta = \frac{x_{max}}{2^{n-1} - 1} \rightarrow (3)$$

The ratio x_{max}/σ_x is called the loading factor.

To avoid overload distortion $x_{max} = 4\sigma_x$.

$$\Delta = \frac{4\sigma_x}{2^{n-1} - 1}$$

$$\sigma_Q^2 = E[Q^2]$$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 . dq.$$

$$= \frac{1}{\Delta} \left[\frac{q^3}{3} \right]$$

$$= \frac{1}{\Delta 3} \left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right]$$

$$= \frac{2\Delta^3}{\Delta 24} = \frac{\Delta^2}{12}$$

$$\therefore (SNR)_0 = \frac{\sigma_x^2}{\sigma_Q^2}$$

$$= \frac{\sigma_x^2}{\Delta^2/12}$$

$$= \frac{\sigma_x^2}{\left(\frac{4\sigma_x}{2^{n-1}-1}\right)^2} \times 12$$

$$= \frac{\cancel{12}^{3}}{\cancel{16}_{4}} (2^{n-1}-1)^2$$

$$= \frac{3}{4} (2^{n-1}-1)^2$$

$$\left[(2^{n-1})^2 = 2^{2n-2} = \frac{2^{2n}}{2^2} \right.$$

For large n,

$$(SNR)_0 = \frac{3}{4} \frac{2^{2n}}{2^2}$$

Taking log on both sides, to express SNR in dB,

$$10 \log_{10} (SNR)_0 = \left(2n \log 2 + \log \frac{3}{16}\right) \times 10$$

$$= (0.602n - 0.726) \times 10$$

$$10 \log_{10} (SNR)_0 = 6.02\,n - 7.26$$

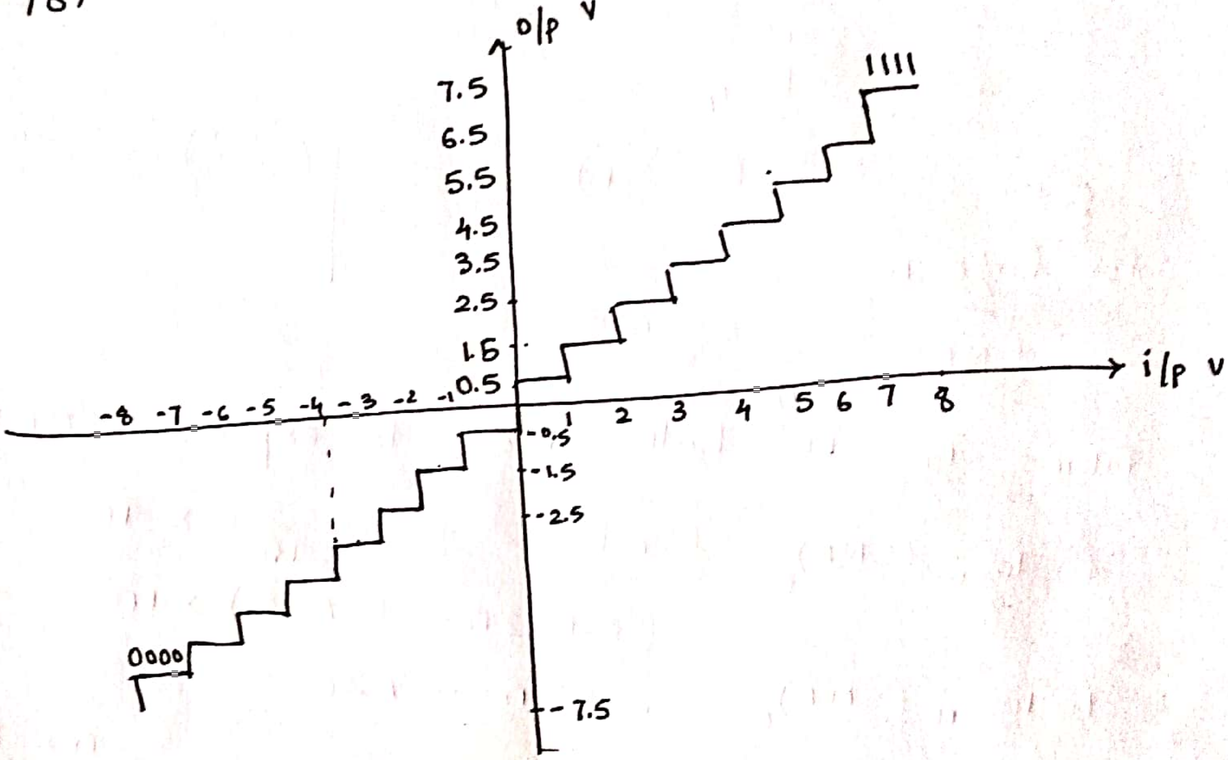
$$n = \frac{B}{W}$$

$$10 \log_{10} (SNR)_0 = 6.02\left(\frac{B}{W}\right) - 7.26$$

This result shows that in PCM system limited by quantizing noise, doubling the channel BW permits twice the no. of bits in a code-word & therefore increases the SNR by 6n dB.

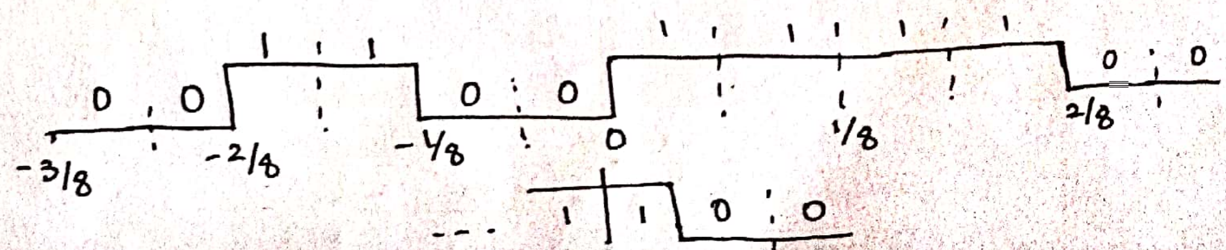
1. A voice signal $g(t) = 6 \sin 2\pi t$ volt is sent using a 4-bit binary PCM system. The quantizer is of midrise type, with a step size of $\Delta = 1V$. Sketch the PCM wave for one complete cycle of the i/p, assuming a sampling rate of 4 samples/sec., with samples taken at $t = \pm 1/8, \pm 3/8, \pm 5/8, \ldots$ sec.

→



Samples of $x(t)$ & corresponding codes.

t	$x(t)$	code
$-3/8$	-4.23	0011
$-1/8$	-4.23	0011
$1/8$	$+4.23$	1100
$3/8$	$+4.23$	1100



ROBUST QUANTIZATION

$\sigma_Q^2 = \frac{\Delta^2}{12}$ → variance of quantization noise

The signal-to-Quantization noise ratio should remain constant for a wide range of i/p power levels. A quantizer that satisfies this requirement is said to be robust.

The provision for such a robust performance necessitates the use of a non-uniform quantizer, characterised by a step size that increases as the spear separation from the origin increases

ROBUST QUANTIZATION (Non-uniform)

If the i/p signal magnitude is too small & if such a signal is quantized using uniform quantizer having fixed step size, then low magnitude signal will not encounter more steps resulting in large quantization noise.

On the other hand, if peak to peak excursion is too large, quantizer may be overload again resulting in more quantization noise.

It is therefore desirable from practical point of view to have constant signal to quantization noise ratio for wide range of i/p levels.

A quantizer that satisfies this requirement is said to be robust quantizer. This also necessiates use of non-uniform quantizer whose step size increases as separation from origin is increased. Due to this weak magnitude signals can be assigned more representation levels compared to loud & high magnitude signals.

This desired form of non-uniform quantization can be achieved by using compressor followed by uniform quantizer. By cascading this combination with an expander, complementary to the compressor, the original signal values can be restored. The model of non-uniform quantizer is as shown in fig (2). The operation of compressor is as shown in fig (1) below.

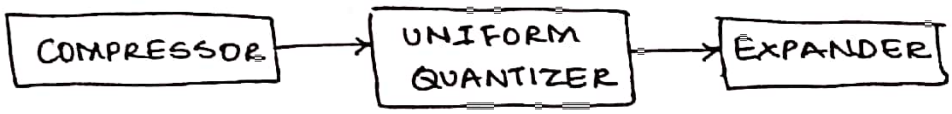


Fig (1). Non-uniform quantizer.

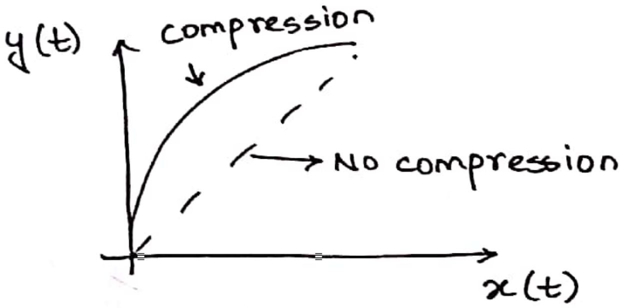


FIG (2). Operation of compression

The combination of compressor & expander together is called as companding.

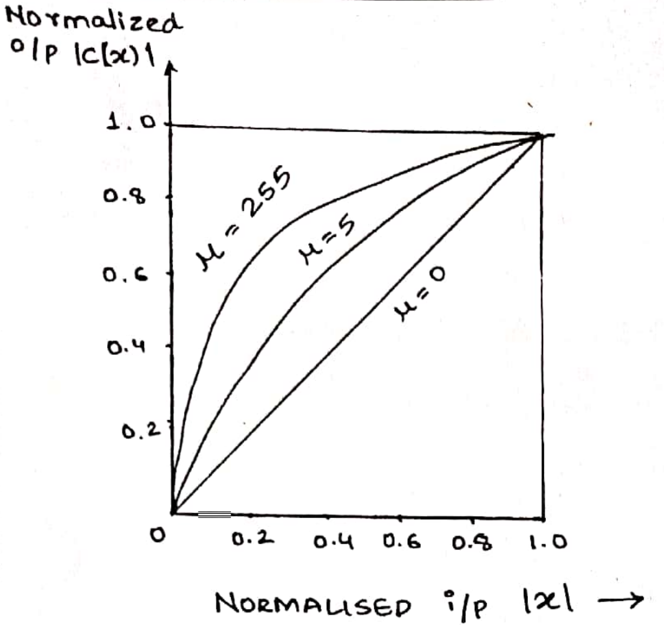
* The normalized avg. signal power

$$P = \frac{A_m^2}{2}$$

The two laws that govern the companding process are

1. μ-law companding
2. A-law companding

1. μ-LAW COMPANDING



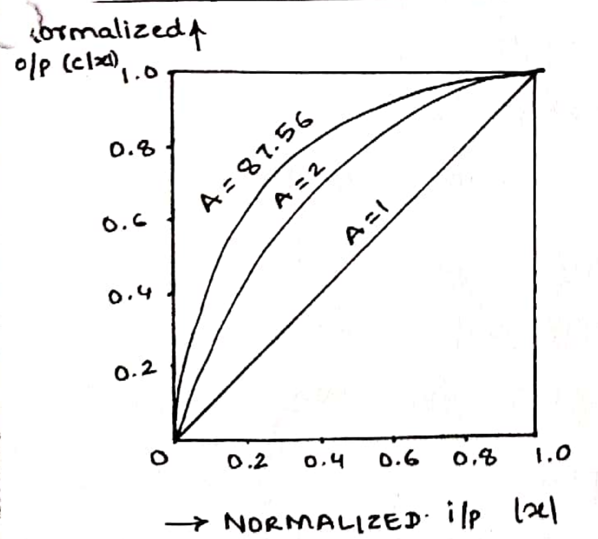
In the μ-law companding, the compressor characteristic c(x) is continuous, approx. a linear dependence on x for low i/p levels & logarithmic one for high i/p levels.

It is described by the eqn.

$$\frac{c|x|}{x_{max}} = \frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)}, \quad 0 \leq \frac{|x|}{x_{max}} \leq 1$$

A practical value for μ = 255. The μ-law is used for PCM systems in United states, Japan, Canada.

2. A-LAW COMPANDING



Here compressor characteristic c(x) is piecewise made up of linear segment for low-level i/p's & logarithmic segment for high i/p levels.

It is described by

$$\frac{c|x|}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{max}} \le 1/A \\ \dfrac{1+\ln(A|x|/x_{max})}{1+\ln A} & 1/A \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

The practical value for A = 87.56. A-law companding is used for PCM systems in European countries, India etc.

POINTS TO REMEMBER

1. For Mid-tread quantizer:
   $10 \log_{10} (SNR)_0 \simeq 6n - 7.2$
2. For Mid-tread quantizer:
   Idle channel noise is zero
3. Robust Quantization is Non-uniform
4. The combination of compressor & expander together is called companding.
5. μ-law companding : $\mu = 255$: US, Japan, Canada.
6. A-law companding: A = 87.56 : European countries, India

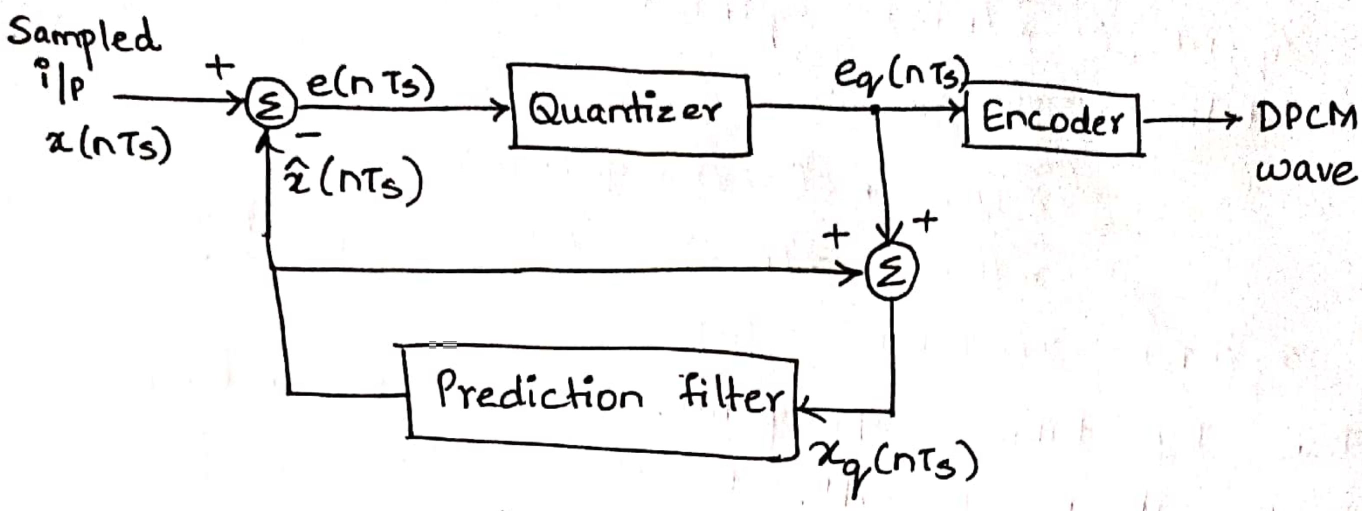
ASSIGNMENT QUESTIONS

1. Derive the equation for signal-to-Quantization noise ratio for mid-tread type of quantizer.
2. Write a short note on: Robust Quantization.
3. Give the comparision between: μ-law companding & A-law companding.

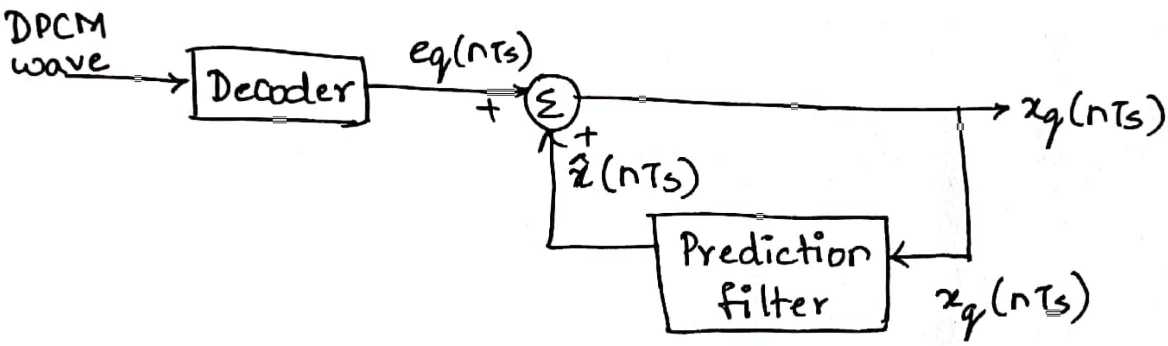
REFERENCE

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008.

Differential Pulse Code Modulation (DPCM)



(a) DPCM Tx.



(b) DPCM Rx.

$x(nT_s)$ is the sampled version of the analog signal $x(t)$ with T_s being the sampling period.

from (a) we have,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

where $\hat{x}(nT_s)$ is the prediction of $x(nT_s)$.

$e(nT_s)$ is then quantized to $e_q(nT_s)$. The quantizer o/p is then encoded to produce a DPCM wave.

In DPCM, the difference in the amplitude of a sample & its prediction is transmitted rather than the actual sample. Since the range of samples & their prediction differences is typically less than conventional PCM.

The quantizer o/p is

$$e_q(nT_s) = e(nT_s) + q_e(nT_s)$$

The i/p to prediction filter is,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

Substituting $e_q(nT_s)$,

$$= \hat{x}(nT_s) + e(nT_s) + q_e(nT_s)$$

$$= \hat{x}(nT_s) + x(nT_s) - \hat{x}(nT_s) + q_e(nT_s)$$

$$x_q(nT_s) = x(nT_s) + q_e(nT_s)$$

Thus, by taking appropriate no. of quantization levels, it is possible to adjust the avg. power of the prediction error.