Unit-3 Problems on Transient Response Specified

1. For a slm $G(s)H(s) = \dfrac{K}{s^2(s+2)(s+3)}$. Find the value of K to limit steady state error to 10 when i/p to the slm is $1 + 10t + \frac{40}{2}t^2$

Sol^n)

$$K_p = \lim_{s \to 0} G(s)\,H(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s) = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s)\,H(s) = K/6$$

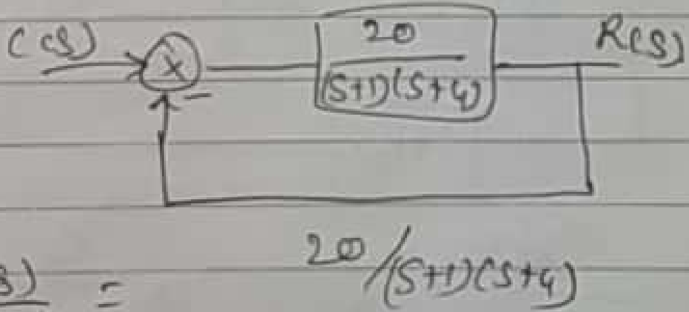
$$e_{ss} = e_{ss_1} + e_{ss_2} + e_{ss_3}$$

$$10 = \frac{1}{1+\infty} + \frac{1}{\infty} + \frac{40}{(K/6)}$$

$$10 = \frac{40 \times 6}{K}$$

$$K = \frac{40 \times 6}{10} = 24$$

2. For the slm shown in the fig. obtain the closed loop T.F. damping ratio, natural freq & expression for the o/p response if subjected to unit step i/p.



Sol^n)

$$\frac{C(s)}{R(s)} = \frac{20/(s+1)(s+4)}{1 + \dfrac{20}{(s+1)(s+4)}} = \frac{20}{(s+1)(s+4) + 20}$$

$$= \frac{20}{s^2 + 5s + 24}.$$

$\omega_n^2 = 24 \quad \omega_n = \sqrt{24} = 4.8989$ rad/sec

$2\xi\omega_n = 5$

$\xi = 0.51031.$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 4.2129$ rad/sec

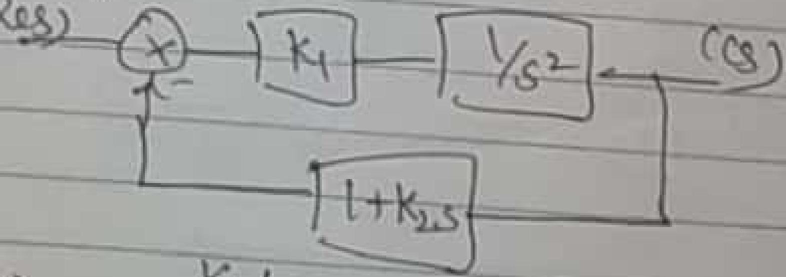
$$\frac{C(s)}{R(s)} = \frac{20}{24} \times \frac{24}{s^2+5s+24}$$

$$= \frac{20}{24}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right]$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 1.03 \text{ radn.}$$

$$= \frac{20}{24}\left[1 - 1.1628\, e^{-2.5t} \sin(4.2129t + 1.03)\right]$$

3. For the CS shown in fig, find the values of K_1 & K_2 so that $M_p = 25\%$, & $T_p = 4$ sec. Assume unit step i/p



Sol)
$$\frac{C(s)}{R(s)} = \frac{K_1/s^2}{1 + (1+K_2 s)\times \frac{K_1}{s^2}} = \frac{K_1}{s^2 + K_1K_2 s + K_1}$$

$\omega_n = \sqrt{K_1}$ $2\xi\omega_n = K_1 K_2$ $\xi = \frac{K_1 K_2}{2\sqrt{K_1}} = \frac{1}{2}\sqrt{K_1}\, K_2$

M_p

$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$

$0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

ln on both sides

$-1.3862 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$ $\xi = 0.4037$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$4 = \frac{\pi}{\omega_n\sqrt{1-(0.403)^2}} =$ $\omega_n = 0.8584$ rad/sec

$\omega_n = \sqrt{K_1}$

$K_1 = \omega_n^2 = (0.8584)^2 = 0.7369$

$\xi = \frac{1}{2}\sqrt{K_1}\, K_2 =$ $K_2 = 0.9405$

4. A s/m is given by a differential eqn $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$, where y = o/p & x = i/p. Determine all time domain specifications for unit step i/p.

Sol^n) $s^2Y(s) + 4sY(s) + 8Y(s) = 8X(s)$

$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$

$\omega_n = \sqrt{8} = 2.83$ rad/sec $\xi = 0.7067$

$\omega_d = 2.002$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.002} = 1.57$ sec

$M_p = 4.33\%$

$T_s = \frac{4}{\xi\omega_n} = 2$ sec

$\theta = \pi/4$.

$c(t) = 1 - 1.41\, e^{-2t} \sin(2t + \pi/4)$.

(5) For a s/m having $G(s) = \frac{15}{(s+1)(s+3)}$ $H(s) = 1$.
Determine.
(i) characteristic eqn (ii) ω_n & ξ
(iii) Time at which 1st undershoot will occur
(iv) Time period of oscillations
(v) no of cycles o/p will perform before settling down

Soln) $$\frac{C(s)}{R(s)} = \frac{15/(s+1)(s+3)}{1 + \frac{15}{(s+1)(s+3)}} = \frac{15}{s^2+4s+18}$$

$\omega_n = \sqrt{18} = 4.2426$

$\xi = 0.4714$

n=1 1st overshoot
n=2 1st undershoot
n=3 2nd overshoot
n=4 2nd undershoot.

$$T_p = \frac{n\pi}{\omega d} = \frac{2\pi}{\omega d} = 1.6792$$

Time period of oscillation

damped freq of oscillation = $\omega d = \frac{2\pi}{T}$

$$T = \frac{2\pi}{\omega d} = 1.6792$$

For 1 cycle o/p will take 1.6792 sec

Scanned by CamScanner

no: of cycles o/p will perform.

$T_s = \frac{4}{\xi\omega_n} = 2sec$

Total no: of cycle = $\frac{2}{1.6792} = 1.191$

$fd = \frac{1}{T} = 0.5955 Hz$

6. A second order s/m is represented by the transfer function

$$\frac{Q(s)}{I(s)} = \frac{1}{Js^2+fs+k}$$

A step i/p of 10Nm is applied to the s/m & the test results are

(a) Max overshoot = 6%

(b) Time at peak overshoot = 1sec

(c) The steady state value of the o/p is 0.5 radians.

Determine the values of J, f & k

Soln)

$$\frac{Q(s)}{I(s)} = \frac{1/J}{s^2+\frac{f}{J}s+\frac{k}{J}}$$

$\omega_n = \sqrt{k/J}$ $\quad 2\xi\omega_n = f/J$ = $\xi = \frac{f/J}{2\sqrt{k/J}} = \frac{1}{2}\cancel{f/J}$

$$\xi = \frac{f}{2\sqrt{KJ}}$$

$M_p = 6\% = 0.06$

$\cdot\ \xi = 0.667$

$T_p = \frac{\pi}{\omega_d} = 1 sec$

$\omega_n = 4.2165 rad/sec$

step i/p 10Nm.

$T(s) = 10/s$

$$Q(s) = \frac{10}{s(Js^2+fs+k)}$$

Steady state o/p $= \lim_{s\to 0} s\,Q(s)$

$$0.5 = \lim_{s\to 0} \frac{\cancel{s}\,10}{\cancel{s}(Js^2+fs+k)}$$

$$0.5 = \frac{10}{K}$$

$$K = \frac{10}{0.5} = 20$$

ω_n $4.2165 = \sqrt{\frac{K}{J}}$ $\quad J = 1.1249$

$0.667 = \frac{f}{2\sqrt{KJ}}$ $\quad f = 6.3274$

7. A s/m has the following transfer function

$$\frac{C(s)}{R(s)} = \frac{20}{s+10}$$

Determine its unit impulse, step & ramp response with zero initial conditions. Sketch the responses

Sol:) $\frac{C(s)}{R(s)} = \frac{20}{s+10}$

ILT

$c(t) = 20e^{-10t}$

Impulse

$$C(s) = \frac{20}{s+10}$$

(b) unit step i/p $R(s) = 1/s$

$$C(s) = \frac{20}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}$$

$A = 2 \quad B = -2$

$$c(t) = \frac{2}{s} - \frac{2}{s+10} = 2 - 2e^{-10t}$$

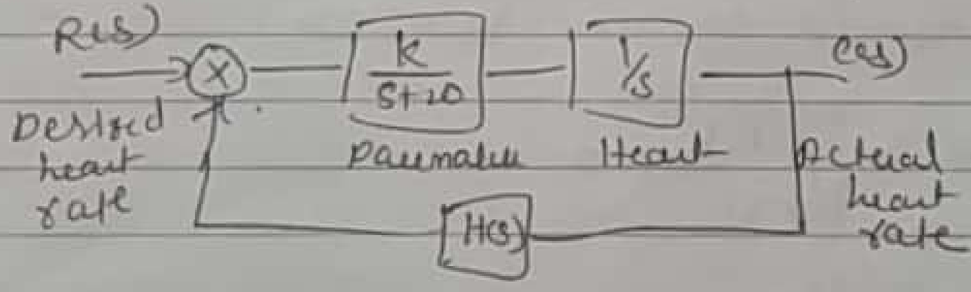
(c) ramp i/p $1/s^2$

$$C(s) = \frac{20}{s^2(s+10)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+10}$$

$$C(s) = \frac{2}{s^2} - \frac{0.2}{s} + \frac{0.2}{s+10}$$

$$c(t) = 2t - 0.2 + 0.2e^{-10t}$$

(8) The block diagram of an electronic pacemaker for controlling the rate of heart beats is shown. Assuming unity feedback & $k=400$.
(1) calculate o/p $c(t)$ for unit step i/p.
(2) steady state error for unit ramp i/p.
(B) Determine K, if error to a ramp i/p should be 0.0



Sol^n 1)
$$\frac{C(s)}{R(s)} = \frac{400}{s^2 + 20s + 400}$$

$\omega_n = 20$ rad/sec

$\xi = 0.5 \qquad \omega_d = \omega_n\sqrt{1-\xi^2} = 17.32$ rad

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 1.047$$

$$c(t) = 1 - 1.154\, e^{-10t} \sin(17.32t + 1.047)$$

(ii) $K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} s \times \frac{k}{s(s+20)} = \frac{k}{20}$

$$K_v = \frac{400}{20} = 20$$

(ii) $e_{ss} = \frac{1}{K_v} = \frac{1}{20} = 0.05$

$$e_{ss} = 0.02$$

$$0.02 = \frac{1}{K_v} = \frac{1}{\left(\frac{k}{20}\right)}$$

$$0.02 = \frac{20}{K}$$

$$K = \frac{20}{0.02} = 1000.$$

9. A s/m has 30% overshoot & settling time of 5 secs for an unit step i/p. Determine;
(i) The transfer function (ii) Peak time (t_p)
(iii) O/P response (Assume e_{ss} as 2%)

Sol^n) $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100.$

$0.3 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$\xi = 0.358$

$T_s = \frac{4}{\xi\omega_n}$. $\omega_n = 2.2346$ rad/sec

$$TF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{5}{s^2 + 1.6s + 5}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 2.0881$$

$$T_P = \frac{\pi}{\omega_d} = \frac{\pi}{2.0881} = 1.5045 \text{ sec}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 1.205 \text{ rad}$$

o/p response

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$c(t) = 1 - 1.0708\, e^{-0.8t} \sin(2.0881t + 1.205)$$

10. The open loop transfer function of a unity FBCS is given by $G(s) = \frac{k}{s(sT+1)}$

i) By what factor the amplifier gain k should be multiplied so that damping ratio is increased from 0.2 to 0.8

ii) By what factor the time constant T should be multiplied so that damping ratio is reduced from 0.6 to 0.3

Sol i)
$$\frac{C(s)}{R(s)} = \frac{k}{Ts^2 + s + k} = \frac{k/T}{s^2 + s/T + k/T}$$

$\omega_n = \sqrt{K/T}$ $\qquad 2\xi\omega_n = 1/T$

$\xi = \dfrac{1}{2\sqrt{KT}}$

case(i) $\xi_1 = 0.2$ $k = k_1$ while $\xi_2 = 0.8$, $k = k_2$

$0.2 = \dfrac{1}{2\sqrt{k_1 T}}$ $\qquad 0.8 = \dfrac{1}{2\sqrt{k_2 T}}$

$\dfrac{0.2}{0.8} = \dfrac{1/2\sqrt{k_1 T}}{1/2\sqrt{k_2 T}}$

$\dfrac{1}{16} = \dfrac{k_2}{k_1}$ $\qquad k_2 = \dfrac{1}{16} k_1$

(ii) $\xi_1 = 0.6$ $T = T_1$ & $\xi_2 = 0.3$ $T = T_2$ k - constant

$0.6 = \dfrac{1}{2\sqrt{KT_1}}$ $\qquad 0.3 = \dfrac{1}{2\sqrt{KT_2}}$

$\dfrac{0.6}{0.3} = \dfrac{\sqrt{T_2}}{\sqrt{T_1}}$ $\qquad 2 = \sqrt{\dfrac{T_2}{T_1}}$

$4 = \dfrac{T_2}{T_1}$ $\qquad \boxed{T_2 = 4T_1}$

Scanned by CamScanner