Initial value theorem :

$\begin{cases} 0^+ \\ 0^- \end{cases}$

$x(n) \longleftrightarrow X(z)$

$x(n)\big|_{n=0} = x(0) = \lim_{z\to\infty} X(z)$ $\qquad x(n)=0 \quad n<0$

proof wkt $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=0}^{\infty} x(n) z^{-n}$

$= x(0) z^0 + x(1) z^{-1} + \cdots\cdots$

$\lim_{z\to\infty} X(z) = \lim_{z\to\infty} \left\{ x(0)\times 1 + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \cdots \right\}$

$= x(0) + \cancel{\frac{x(1)}{\infty}}^{\,0} + \cancel{\frac{x(2)}{\infty^2}}^{\,0} + \cdots$

$$\boxed{\therefore\ x(0) = \lim_{z\to\infty} X(z)}$$

Final value theorem : $x(n) \longleftrightarrow X(z)$

$\lim_{n\to\infty} x(n) = x(\infty) = \lim_{z\to 1} \left\{ (1-z^{-1}) X(z) \right\}$

Conditions : (i) $x(n) = 0$, $n<0$

if ROC not given then above will be assumed.

(ii) $(1-z^{-1}) X(z)$ should have poles inside unit circle in z-plane.

(*) The z-transform of a signal is given by $C(z) = \dfrac{z^{-1}(1-z^{-4})}{4(1-z^{-1})}$. Its final value is?

Soln

$$c(n) \longleftrightarrow C(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

Condition : (1) ROC Not given ∴ we assume $c(n)=0$, $n<0$

(2) $(1-z^{-1})\,C(z)$

$$= 1-z^{-1} \times \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

$$= \frac{(1-z^{-1})\, z^{-1}\, (1+z^{-2})(1-z^{-2})}{4(1-z^{-1})^2}$$

$$= \frac{\cancel{(1-z^{-1})}\, z^{-1}\, (1+z^{-2})(1+z^{-1})\cancel{(1-z^{-1})}}{4\cancel{(1-z^{-1})^2}}$$

$$= \frac{(1+z^{-2})(1+z^{-1})}{4z}$$

Poles $\Rightarrow z=0$ Satisfied.

$$\therefore\ C(\infty) = \lim_{z\to 1} \underbrace{(1-z^{-1})\,C(z)}$$

$$= \lim_{z\to 1} \frac{(1+z^{-2})(1+z^{-1})}{4z}$$

$$= \frac{4}{4} = 1$$

(*) $X(z) = \dfrac{1}{1-z^{-2}}$ find $x(0)$ & $x(\infty)$?

Soln

$$x(0) = \lim_{z\to\infty} X(z) = \lim_{z\to\infty} \frac{1}{1-z^{-2}}$$

$$= \frac{1}{1-0} = 1$$

$$x(\infty) = \lim_{z\to 1} (1-z^{-1}) \frac{1}{(1-z^{-2})} = \lim_{z\to 1} \frac{\cancel{(1-z^{-1})}}{\cancel{(1-z^{-1})}(1+z^{-1})}$$

$$= \lim_{z\to 1} \frac{1}{1+z^{-2}} = \frac{1}{2}$$

(*) $X(z) = \dfrac{1}{1+2z^{-1}-3z^{-2}}$ $x(0)$ & $x(\infty) = ?$

$$x(0) = \frac{1}{1+0} = 1$$

$$x(\infty) = 1/4$$

Scanned by CamScanner