M-3

Time-domain representations for LTI systems

①

Differential & difference Eq^n Representation of LTI systems, Solving Difference Eq^n.

Z-Transforms : (* Brief review of Z-transforms) properties of ROC, properties of Z-transform, Inversion of the Z-transforms. The transform function, System response, causality & stability of sys.

Differential and Difference Equation Representations of LTI systems

* Linear constant-coefficient difference and differential Eq^ns Provide another representation for the i/p & o/p characteristics of LTI systems.

* Difference Eq^n — DT systems
  Differential Eq^n — CT systems.

$$\boxed{\sum_{K=0}^{N} a_K\, y(n-K) = \sum_{K=0}^{M} b_K\, x(n-K)}$$

Linear Constant-coefficient difference Eq^n

where a_K & b_K are constant co-efficients of system. $x(t)$ = i/p, $y(t)$ = o/p, N, M = order of Syst.

$$\boxed{\sum_{K=0}^{N} a_K \frac{d^K}{dt^K} y(t) = \sum_{K=0}^{M} b_K \frac{d^K}{dt^K} x(t)}$$

Linear constant Co-efficient differential Eq^n.

where a_K & K_K ⇒ constant co-efficients of system.

$x(n)$ = i/p
$y(n)$ = o/p
N, M = order of system.

NOTE: order represents the NO of energy storage devices in sys. often $N \geq M$ ∴ order is described using only N.

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* Difference Equations are easily rearranged to obtain recursive formulas for computing the current o/p of the system from the i/p signal & past o/p's.

Ex: Let $y(n) + y(n-1) + \frac{1}{4} y(n-2) = x(n) + 2x(n-1)$

$$y(n) = x(n) + 2x(n-1) - y(n-1) - \frac{1}{4} y(n-2)$$

$$\boxed{\text{ie } y(n) = \frac{1}{a_0} \sum_{K=0}^{M} b_K x(n-K) - \frac{1}{a_0} \sum_{K=1}^{N} a_K y(n-K)}$$

$y(-1)$ & $y(-2)$ => known as initial conditions.

$n \geq 0 \Rightarrow y(n) = x(n) + 2x(n-1) - y(n-1) - \frac{1}{4} y(n-2)$

$$y(0) = x(0) + 2x(-1) - y(-1) - \frac{1}{4} y(-2)$$
$$y(1) = x(1) + 2x(0) - y(0) - \frac{1}{4} y(-1)$$
$$y(2) = x(2) + 2x(1) - y(1) - \frac{1}{4} y(0)$$
$$\vdots \quad \vdots \quad \vdots$$

NOTE :) on Initial Condition In general the NO of initial conditions required to determine the system.
=) the o/p is Equal to the maximum memory of the system.
→ Differential - related to the initial values of the energy storage devices in the system
→ Difference - summarize all information about the past history of the system that can affect future o/p's.
→ ∴ Initial Conditions also represents the "memory" of continuous-time systems.

(1) Homogeneous Soln
(2) particular soln
(3) complete soln.

① Natural response
② The forced response.

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(*) Find the first 2 o/p values $y(0)$ & $y(1)$ for the system described by $y(n) = x(n) + 2x(n-1) - y(n-1) - \frac{1}{4}y(n-2)$, assuming that the i/p is $x(n) = (1/2)^n u(n)$ and the initial conditions are $y(-1) = 1$ & $y(-2) = -2$.

Sol^n

$$y(n) = x(n) + 2x(n-1) - y(n-1) - \tfrac{1}{4}\,y(n-2)$$

$$n=0 \Rightarrow y(0) = x(0) + 2x(-1) - y(-1) - \tfrac{1}{4}\,y(-2)$$

$$= \cancel{1} + 2\times 0 - \cancel{1} - \tfrac{1}{4}(-2)$$

$$= +\tfrac{1}{2}$$

$$n=1 \Rightarrow y(1) = x(1) + 2x(0) - y(0) - \tfrac{1}{4}\,y(-1)$$

$$= (1/2) + 2\times 1 - 1/2 - 1/4\,(1)$$

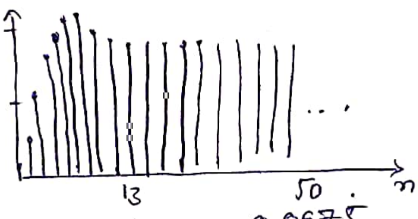
$$= \cancel{1/2} + 2 - \cancel{1/2} - 1/4 = 2 - 1/4 = \frac{8-1}{4} = 7/4$$

(*) A system is described by the difference equation
$y(n) - 1.143\,y(n-1) + 0.4128\,y(n-2) = 0.0675\,x(n) + 0.1349\,x(n-1) + 0.675\,x(n-2)$.

Write a recursive formula that computes the present o/p from the past o/p's and the current inputs. Use a computer to determine (1) the step response of the system, (2) the system o/p when the i/p is zero and the initial conditions are $y(-1) = 1$ & $y(-2) = 2$, and the o/p in response to the sinusoidal i/p's $x_1(n) = \cos(\frac{\pi}{10}n)$, $x_2(n) = \cos(\pi n/5)$ & $x_3(n) = \cos(7\pi n/10)$. Assuming zero initial conditions.

Sol^n

(1) $$y(n) = 0.0675\,x(n) + 0.1349\,x(n-1) + 0.675\,x(n-2) + 1.143\,y(n-1) + 0.4128\,y(n-2)$$



$$x(n) = u(n) \Rightarrow u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$y(0) = 0.0675\,x(0) + 0 + 0 + 0 + 0 = 0.0675\times 1 = 0.0675$$

$$y(1) = 0.0675\,x(1) + 0.1349\,x(0) + 0 + 1.143\,y(0) - 0$$

$$= 0.0675 + 0.1349 + 1.143\times 0.0675 = 0.2795$$

$n \Rightarrow 0$ to $13 \Rightarrow y(n)\uparrow$ & $y(n)$ & later then maintain same dc level.

→ o/p & i/p amplitude are equal ⇒ gain = unity. for constant i/p.

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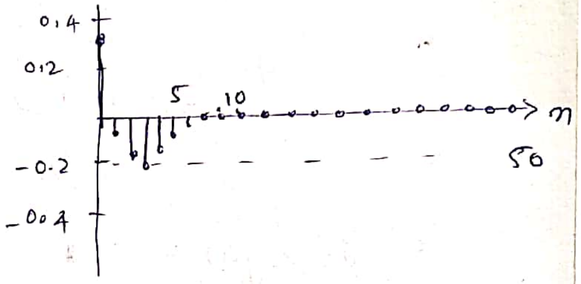
(2) $y(-1) = 1$, $y(-2) = 2$ $\quad$ i/p = 0 = $x(n)$

$$y(n) = 0.0675\,x(n) + 0.1349\,x(n-1) + 0.675\,x(n-2) + 1.143\,y(n-1) - 0.4128\,y(n-2)$$

$$y(0) = 0 + 0 + 0 + 1.143 \times 1 - 0.4128 \times 2$$
$$= 0.3174$$

$$y(1) = 0 + 0 + 0 + 1.143 \times 0.3174 - 0.4128 \times 1 = -0.0500118$$

$$y(2) = 0 + 0 + 0 + 1.143 \times (-0.050) - 0.4128 \times 0.3174 = -0.188172.$$

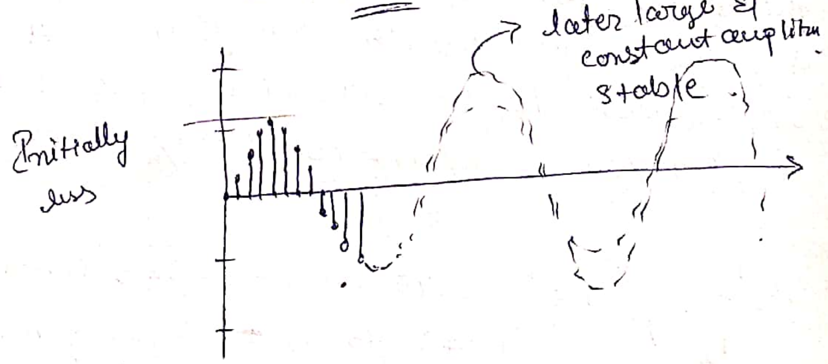


(3) $x_1(n) = \cos\left(\frac{\pi}{10}n\right)$ & zero initial condition.

$$y(n) = 0.0675\,x(n) + 0.1349\,x(n-1) + 0.675\,x(n-2) + 1.143\,y(n-1) - 0.4128\,y(n-2)$$

$$y(0) = 0.0675\cos(0) + 0.1349\cos(-\pi/10) + 0.675\cos(-\pi/5) + 0 + 0$$
$$= 0.0675 + 0.1282 + 0.5460 = 0.7417$$

$$y(1) = 0.0675 \times \cos(-\pi/10) + 0.1349\cos(-\pi/5) + 0.675\cos(-3\pi/10) + 0 + 0$$
$$= 0.06419 + 0.1091 + 0.3967 = 0.5699 + 1.143 \times 0.7417$$
$$= 1.41775$$



(*) Calculate $y(n)$, $n = 0, 1, 2, 3$ for the first-order recursive System $y(n) - (1/2)\, y(n-1) = x(n)$ if the i/p is $x(n) = u(n)$ & the initial condition is $y(-1) = -2$.

Sol^n $\quad y(n) - (1/2)\, y(n-1) = x(n)$ $\qquad x(n) = u(n)$

$y(-1) = -2$

$$y(n) = x(n) + (1/2)\, y(n-1)$$

$n = 0 \Rightarrow y(0) = x(0) + \frac{1}{2} y(-1)$

$= 1 + \frac{1}{2} \times (-2) = 0$

$n = 1 \Rightarrow y(1) = x(1) + 1/2\, y(0) = 1 + 0 = 1$

$n = 2 \Rightarrow y(2) = x(2) + \frac{1}{2} y(1) = 1 + 1/2 = 1.5 = 3/2$

$n = 3 \Rightarrow y(3) = x(3) + \frac{1}{2} y(2) = 1 + \frac{1.5}{2} = 1.75 = 7/4$

Characteristics of Systems Described by Differential & Difference Eqn

* output of a system described by a differential (or) difference Equation as the sum of two components.

  (i) Associated only with the initial conditions (natural response)

  (ii) only to the i/p signal. (forced response)

Total, zero-i/p, zero state response.

y_n^{c} (or) y^n

y_f (or) y^f

$$y = y^{(n)} + y^{(f)}$$

(1) Homogeneous soln
(2) Particular soln
(3) Complete soln.

(i) Natural response :-

→ zero i/p & any stored energy (or) memory of the past represented by non-zero initial conditions.

$$y(t) = \sum_{i=1}^{N} c_i e^{r_i t}$$

$$y(n) = \sum_{i=1}^{N} c_i r_i^{n}$$

c_i → initial condition values to satisfy

$$\sum_{k=0}^{N} a_k r^{N-k} = 0.$$

(ii) The forced response :-

→ o/p due to the i/p signal & zero initial conditions.

→ zero initial condition = system at rest.

→ $t > 0$ (or) $n \geq 0$

→ $t = 0^+$ = $t = 0^-$

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