(3) Determine the response of the Sm described by difference equation

$$y(n) - \frac{1}{4}y(n-1) - 1/8\, y(n-2) = x(n) + x(n-1) \text{ with } y(-1)=0, y(-2)= \ldots \text{ for } x(n) = (1/8)^n u(n)$$

1

Note: The output or response of DT LTI Sm described by a difference eqn can be obtained in 2 different approaches.

Approach I : $y(n) = y_N(n) + y_F(n)$

(a) Natural Response (or zero. input response $y_N(n)$)

The given DE is rewritten as

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = 0 \quad \{\text{since natural response o/p without any i/p} \therefore x(n)=0\}$$

Corresponding chs. eqn is

$$r^2 - 1/4\, r - 1/8 = 0$$

$$\boxed{r_1 = \frac{1}{2}\ ;\ r_2 = -1/4}$$

Corresponding homogeneous solution is $\boxed{y_h(n) = C_1 (1/2)^n + C_2(-1/4)^n}$

To determine natural response, the value of constants need to be determined using the given initial condition

$\therefore$ Sub n=0 in above rewritten DE

$$y(0) - \frac{1}{4}y(-1) - \frac{1}{8}y(-2) = 0 \quad \text{, it's given } y(-1)=0, \,\&\, y(-2)=1 \text{ sub them}$$

$$\boxed{y(0) = 1/8}$$

n=1

$$y(1) - \frac{1}{4}y(0) - \frac{1}{8}y(-1) = 0$$

$$\boxed{y(1) = \frac{1}{32}}$$

Sub n=0 & n=1 in $y_h(n)$ & equate to y(0) & y(1) respectively

$$y_h(0) = C_1 + C_2 = 1/8$$

$$y_h(1) = \frac{C_1}{2} - \frac{C_2}{4} = 1/32$$

Solving for C_1 & C_2

$$\boxed{C_1 = \frac{1}{12}\ ;\ C_2 = \frac{1}{24}}$$

$$\therefore \boxed{y_N(n) = \frac{1}{12}(1/2)^n + \frac{1}{24}(-1/4)^n}$$

(b) Forced response or Zero state response $\boxed{y_F(n) = y_h(n) + y_p(n)}$

For i/p $x(n) = (1/8)^n u(n)$, $y_p(n) = K.(1/8)^n u(n)$ & should satisfy the DE

$$K\left(\frac{1}{8}\right)^n u(n) - \frac{1}{4}K.\left(\frac{1}{8}\right)^{n-1} u(n-1) - \frac{1}{8}K\left(1/8\right)^{n-2} u(n-2) = (1/8)^n u(n) + (1/8)^{n-1} u(n-1)$$

For $n=2$, no term vanishes $\therefore$ consider from $n=2$

at $n=2$

$$\frac{K}{64} - \frac{K}{32} - \frac{K}{8} = \frac{1}{64} + \frac{1}{8}$$

$$\boxed{K = -1 \Rightarrow y_p(n) = (-1)(1/8)^n u(n)}$$

$$y_F(n) = C_1(1/2)^n + C_2(-1/4)^n + (-1)(1/8)^n u(n)$$

Sub $n=0$ in the Given DE & equate $y_F(0)$

$n=0$ DE becomes

$$y(0) - \frac{1}{4}\cancel{y(-1)}^{0} - 1/6\,\cancel{y(-2)}^{0} = x(0) + \cancel{x(-1)}^{0}$$

$$\boxed{y(0) = 1}$$

$n=1$

$$y(1) - \frac{1}{4}y(0) - \frac{1}{6}\cancel{y(-1)}^{0} = x(1) + x(0)$$

$$y(1) = \frac{1}{8} + 1 + \frac{1}{4}$$

$$\boxed{y(1) = \frac{11}{8}}$$

$$y_F(0) = C_1 + C_2 - 1 = 1$$

$$C_1 + C_2 = 2$$

$$y_F(1) = \frac{C_1}{2} - \frac{C_2}{4} - \frac{1}{8} = 11/8$$

$$\frac{C_1}{2} - \frac{C_2}{4} = \frac{12}{8} = \frac{3}{2}$$

$$\boxed{C_1 = 8/3 \;\&\; C_2 = -2/3}$$

$$\boxed{y_F(n) = 8/3(1/2)^n - 2/3(-1/4)^n + (-1)(1/8)^n u(n)}$$

(c) Total response $y_T(n) = y_N(n) + y_F(n)$

$$y_T(n) = \frac{1}{12}(1/2)^n + \frac{1}{24}(-1/4)^n + 8/3(1/2)^n - 2/3(-1/4)^n - (1/8)^n u(n)$$

$$\boxed{y(n) = \frac{33}{12}(1/2)^n - 5/8(-1/4)^n - (1/8)^n u(n)}$$

(3) $y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + x(n-1)$ using

2

Approach II

$\boxed{y_T(n) = y_h(n) + y_p(n)}$ and solving for constants using the given initial conditions.

From approach I w.r.t $\boxed{y_h(n) = C_1(1/2)^n + C_2(-1/4)^n}$

and

$y_p(n) = (-1)(1/8)^n u(n)$

$\Rightarrow \boxed{y_T(n) = C_1(1/2)^n + C_2(-1/4)^n + (-1)(1/8)^n u(n)}$

Sub n=0 in the given DE and using the given initial conditions $y(-1)=0; y(-2)=1$

n=0 in DE

$y(0) - \frac{1}{4}y(\cancel{-1})^{0} - \frac{1}{8}y(\cancel{-2})^{1} = x(0) + x(\cancel{-1})^{0}$

$y(0) = 1 + 1/8 = 9/8$

$\boxed{y(0) = 9/8}$

n=1

$y(1) - \frac{1}{4}y(0) - \frac{1}{8}y(\cancel{-1})^{0} = x(1) + x(0)$

$y(1) = \frac{1}{8} + 1 + \frac{9}{32}$

$y(1) = \frac{4+32+9}{32} = \frac{45}{32}$

$\boxed{y(1) = \frac{45}{32}}$

Equate the above $y(0)$ & $y(1)$ with $y_T(0)$ & $y_T(1)$ respectively

$y_T(0) = C_1 + C_2 - 1 = \frac{9}{8}$

$C_1 + C_2 = \frac{17}{8} \longrightarrow$ (A)

$y_T(1) = \frac{C_1}{2} - \frac{C_2}{4} - \frac{1}{8} = \frac{45}{32}$

$\frac{C_1}{2} - \frac{C_2}{4} = \frac{49}{32} \longrightarrow$ (B)

Solving (A) & (B)

$\boxed{C_1 = \frac{22}{8} \text{ \& } C_2 = -5/8}$

$\boxed{y_T(n) = \frac{22}{8}(1/2)^n - 5/8(-1/4)^n - (1/8)^n u(n)}$

Note: The o/p obtained $y_T(n)$ and $y(n)$ in the previous approach are same. Canbe checked by substituting various value of n in $y(n)$ & $y_T(n)$

Verification of both approaches

Approach I

$$y(n) = \frac{33}{12}\left(\tfrac{1}{2}\right)^n - \tfrac{5}{8}\left(-\tfrac{1}{4}\right)^n - \left(\tfrac{1}{8}\right)^n u(n)$$

n=0

$y(0) = \frac{33}{12} - \frac{5}{8} - 1$

$\boxed{y(0) = 1.125}$

n=1

$y(1) = \frac{33}{24} + \frac{5}{32} - \frac{1}{8}$

$\boxed{y(1) = 1.41325}$

n=2

$y(2) = \frac{33}{48} - \frac{5}{128} - \frac{1}{64}$

$\boxed{y(2) = 0.6328}$

Approach II

$$y_T(n) = \frac{22}{8}\left(\tfrac{1}{2}\right)^n - \tfrac{5}{8}\left(-\tfrac{1}{4}\right)^n - \left(\tfrac{1}{8}\right)^n u(n)$$

n=0

$y_T(0) = \frac{22}{8} - \frac{5}{8} - 1$

$\boxed{y_T(0) = 1.125}$

n=1

$y_T(1) = \frac{22}{16} + \frac{5}{32} - \frac{1}{8}$

$\boxed{y_T(1) = 1.407}$

n=2

$y_T(2) = \frac{22}{32} - \frac{5}{128} - \frac{1}{64}$

$\boxed{y_T(2) = 0.6328}$

Same way for other values of n it can be checked, its observed that both approaches yielded same y(n)

Another way of verification can also be done

Sub n=0 in DE

$y(0) - \frac{1}{4}y(-1) - \frac{1}{6}y(-2) = x(0) + x(-1)$

Sub $y(-1)=0$, $y(-2)=1$, $x(-1)=0$

$y(0) = 1 + 1/6 = 7/6 =$

$\boxed{y(0) = 1.166}$

n=1

$y(1) - \frac{1}{4}y(0) - \frac{1}{6}\overset{0}{y(-1)} = x(1) + x(0)$

$y(1) = 1/8 + 1 + 0.29$

$\boxed{y(1) = 1.415}$

n=2

$y(2) - \frac{1}{4}y(1) - \frac{1}{6}y(0) = x(2) + x(1)$

$y(2) = \frac{1}{64} + \frac{1}{8} + 0.35375 + 0.1943$

$\boxed{y(2) = 0.65}$

① Determine forced response for the s/m described by the difference equation (3)

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n) \quad \text{for } x(n) = 2^n, n \geq 0 \;\&\; = 0 \;\; \text{o/o}$$

Forced Response is also called as zero state response of the s/m due only to the s/m i/p with the assumption the initial conditions are zero.

$y_F(n) = ZSR = y_h(n) + y_p(n)$ where $y_h(n)$ - homogeneous solution, $y_p(n)$ - particular solution

The chs eqn of the given DE is

$$r^2 - \frac{5}{6}r + 1/6 = 0$$

$\boxed{r_1 = \frac{1}{3} \text{ and } r_2 = \frac{1}{2}}$ $\Rightarrow$ $\boxed{y_h(n) = C_1(1/3)^n + C_2(1/2)^n}$

→ For i/p $x(n) = 2^n u(n)$, the $y_p(n)$ is of the form $K 2^n u(n)$

$\boxed{y_p(n) = K.2^n u(n)}$

Sub in the given DE

$$K.2^n u(n) - \frac{5}{6}K.2^{n-1}u(n-1) + \frac{1}{6}K.2^{n-2}.u(n-2) = 2^n u(n)$$

for $n=2$, no term in the above eqn vanishes. ! start from $n=2$

at $\underline{n=2}$

$$4K - \frac{10K}{6} + \frac{K}{6} = 4$$

$$\Rightarrow K = \frac{24}{15} = \frac{8}{5}$$

$\boxed{K = 8/5 \Rightarrow y_p(n) = (8/5)\, 2^n u(n)}$

$$K\, 2^n u(n) - \frac{5}{6}K\, 2^{n-1}u(n-1) + \frac{1}{6}K\, 2^{n-2}u(n-2) = 2^n u(n)$$

$$K - \frac{5}{6}K\, 2^{-1} u(n-1) + \frac{1}{6}K\, 2^{-2}u(n-2) = u(n)$$

$$K - \frac{5}{6}K\, 2u(n-1) + \frac{1}{6}K\, 2u(n-2) = u(n)$$

$$K - \frac{5}{6}K\, 2 \ldots$$

$$K - \cdot 166 K = 4$$

$$4K - 1.66K + 0.166K = 4$$

$$K = \frac{4}{2.5} = 1.596 = 8/5$$

$$\therefore \boxed{y_F(n) = y_h(n) + y_p(n) = C_1(1/3)^n + C_2(1/2)^n + (8/5)\, 2^n u(n)}$$

Sub $n=0$ in the given DE & considering initial conditions

$$y(-1) \;\&\; y(-2) = 0$$

$$y(0) - 5/6\, y(-1) + \frac{1}{6}y(-2) = x(0)$$

$$\boxed{y(0) = 1}$$

$\underline{n=1}$

$$y(1) - 5/6\, y(0) + 1/6\, y(-1) = 2^1$$

$$\Rightarrow \boxed{y(1) = 17/6}$$

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Sub $y(0)$ & $y(1)$ obtained for $y_F(0)$ & $y_F(1)$

$y_F(0) = c_1 + c_2 + 8/5 = 1$

$\Rightarrow \quad c_1 + c_2 = -3/5 \longrightarrow (1)$

$y_F(1) = \frac{c_1}{3} + \frac{c_2}{2} + \frac{16}{5} = \frac{17}{6}$

$\frac{c_1}{3} + \frac{c_2}{2} = \frac{-11}{30} \longrightarrow (2)$

solving (1) & (2) $\boxed{c_1 = 2/5 \; ; \; c_2 = -1}$

$$\boxed{y_F(n) = 2/5\left(1/3\right)^n - 1\left(1/2\right)^n + 8/5\, 2^n u(n) \quad n \geq 0}$$

② $10y(n)+3y(n-1)-y(n-2)=x(n)$, with $y(-1)=0.1$ & $y(-2)=3.3$

$x(n)=6$

Natural response or ZIR (Zero i/p response)

4

$10y(n)+3y(n-1)-y(n-2)=0 \rightarrow ①$

$10r^2+3r-1=0$

$\boxed{r_1=0.2\ ;\ r_2=-0.5}$

$\boxed{y_h(n)=C_1(0.2)^n+C_2(-0.5)^n}$

sub $n=0$ in ①

$10y(0)+3y(\overset{0.1}{-1})-y(\overset{3.3}{-2})=0$

$y(0)=\frac{-0.3+3.3}{10}$

$\boxed{y(0)=0.3}$

$n=1$ in ①

$\Rightarrow$ $10y(1)+3y(\overset{0.3}{0})-y(\overset{0.1}{-1})=0$

$\boxed{y(1)=-0.08}$

Sub $n=0$ in $y_h(n)$ & equate to $y(0)$ obtained

~~$y_h(0)$~~

$y_h(0)=C_1+C_2=0.3 \rightarrow ②$

$n=1$ in $y_h(n)$ & equate to $y(1)$

$y_h(1)=0.2C_1-0.5C_2=-0.08 \rightarrow ③$

solving ② & ③

$\boxed{C_2=0.2\ \&\ C_1=0.1}$

$\therefore$ Zero i/p response or $y_h(n)=0.1(0.2)^n+0.2(-0.5)^n$ for $n\geq -2$ (because of initial condition)

Forced response or Zero state response

$y_F(n)=y_h(n)+y_p(n)$

$y_F(n)=C_1(0.2)^n+C_2(-0.5)^n+0.5$

$y_p(n)$ will take the form of i/p for i/p $x(n)=6$; $y_p(n)=K$.

$10\times K+3(K)-K=6$

$12K=6$

$K=6/12=1/2=0.5$

$\boxed{K=0.5} \Rightarrow \boxed{y_p(n)=0.5}$

sub $n=0$ in the given DE

$10y(0)+3y(\overset{0}{-1})-y(\overset{0}{-2})=6$

$\boxed{y(0)=0.6}$

$n=1$

$10y(1)+3y(\overset{0.6}{0})-y(\overset{0}{-1})=6$

$\boxed{y(1)=+0.42}$

sub $n=0$ in $y_F(n)$ & equate to $y(0)$ obtained

$y_F(0)=C_1+C_2+0.5=0.6$; $y_F(1)=0.2C_1-0.5C_2+0.5=0.42$

$C_1+C_2=0.1 \rightarrow ④$ $\quad 0.2C_1-0.5C_2=-0.08 \rightarrow ⑤$

Solving equations (4) & (5)

$$\boxed{\begin{aligned} C_1 &= -0.0428 \\ C_2 &= 0.1428 \end{aligned}}$$

$$\boxed{y_F(n) = -0.0428(0.2)^n + 0.1428(-0.5)^n + 0.5 \quad n \geq 0}$$

Total Response $y(n) = y_N(n) + y_F(n)$ (or) ZIR + ZSR

$y(n) = (0.2)^n(0.1 - 0.0428) + (-0.5)^n(0.2 + 0.1428) + 0.5$

$\boxed{y(n) = 0.0572(0.2)^n + 0.3428(-0.5)^n + 0.5}$ for $n \geq 2$ (since initial condn effect at $y(-1)$ & $y(-2)$ are added.)

Approach 2

$y_h(n) = C_1(0.2)^n + C_2(-0.5)^n$

$y_p(n) = K = 0.5$

$\boxed{y_T(n) = C_1(0.2)^n + C_2(-0.5)^n + 0.5}$

Sub n=0 in the given DE & use the given initial conditions $y(-1) = 0.1$ & $y(-2) = 3.3$

$10y(0) + 34y(-1) - y(-2) = 6$

$\boxed{y(0) = 0.9}$

n=1

$10y(1) + 34y(0) - y(-1) = 6$

$\boxed{y(1) = 0.34}$

$y_T(0) = C_1 + C_2 + 0.5 = 0.9$

$\Rightarrow C_1 + C_2 = 0.4 \rightarrow$ (6)

$y_T(1) = 0.2C_1 - 0.5C_2 + 0.5 = 0.34$

$\Rightarrow 0.2C_1 - 0.5C_2 = -0.16$

$C_1 - 2.5C_2 = -0.8 \rightarrow$ (7)

Solving (6) & (7)

$\boxed{C_1 = 0.05714 \text{ \& } C_2 = 0.34285}$

$\Rightarrow \boxed{y_T(n) = \text{Total Response} = 0.05714(0.2)^n + 0.34285(-0.5)^n + 0.5}$

Note: Both $y(n)$ are same

verification

$y(0) = 0.05714 + 0.34285 + 0.5 = 0.8994$

$y(1) = 0.011428 - 0.171425 = 0.3399$

verification in the given DE eqn

n=0

$10y(0) + 34y(-1) - y(-2) = 6$

$y(0) = 0.9$

n=1

$10y(1) + 34y(0) - y(-1) = 6$

$y(1) = 0.34$

n=2

$10y(2) + 34y(1) - y(0) = 6$

$y(2) = 0.588$

All values are verified using both approaches.

$y(2) = 0.002285 + 0.08571 + 0.5 = 0.5879$