Practice Problems on Properties of ZT

1) $x(n) = \alpha^{n-2} u(n-2)$

ans: $X(z) = \frac{1}{z(z-\alpha)}$ ROC $|z| > |\alpha|$

2) $x(n) = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{o.w} \end{cases}$

ans: $X(z) = \frac{z}{z-1}\left[1 - z^{-N}\right]$

ROC: $|z| > 1$

3) $x(n) = n^2 \left(\frac{1}{3}\right)^n u(n-2)$

$X(z) = \frac{1}{9}\left[\frac{4z^2 - z + (1/9)}{z(z-1/3)^3}\right]$

ROC: $|z| > 1/3$

4) $x(n) = n 2^n \sin\left(\frac{\pi}{2} n\right) u(n)$

ans: $X(z) = \frac{2z(z^2-4)}{(z^2+4)^2}$

5) Find ZT of $x_1(n) = x(n+2)$ if $x(n)$ is given by

$x(n) = \begin{cases} (1/2)^n & n \ge 0 \\ (1/4)^{-n} & n < 0 \end{cases}$ Sketch ROC

$X(z) = \frac{(7/2) z^3}{(z-4)(z-1/2)}$, ROC: $1/2 < z < 4$.

6) $x(n) = n^2 u(n)$

$X(z) = \frac{z(z+1)}{(z-1)^3}$

7) $x(n) = n\left(\frac{1}{2}\right)^n u(n) * \delta(n) - \frac{\delta(n-1)}{2}$

$X(z) = \frac{1/2}{*(z-1/2)}$ ROC $|z| > 1/2$.

8) $x(n) = n\left[\left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n)\right]$

ans: $X(z) = \frac{(7/6) z^2 [z - (2/3)]}{[z-1/2]^2 [z-1/3]^2}$

9) Find ZT of $y(n) = x(n) * h(n)$ if $x(n) = \{2 \;\; 1 \;\; 0 \;\; -1 \;\; 3\}$

$h(n) = \{1 \;\; -3 \;\; 2\}$

10) i) $x(n) = u(2-n) - u(-2-n)$

ii) $x(n) = u(-n) - u(-n-3)$

i) $X(z) = z + 1 + z^{-1} + z^{-2}$

ii) $X(z) = 1 + z + z^2$.