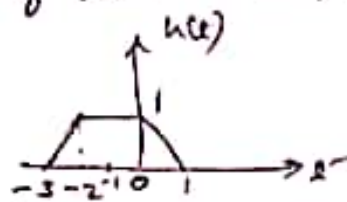
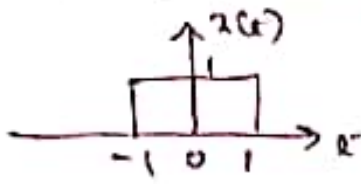


Practice Problems (convolution)

1) Find convolution of $x(t)$ & $h(t)$



$$\text{ans: } y(t) = \begin{cases} 0 & t < -4 \\ \frac{1}{2}t^2 + 4t + 8 & -4 \leq t < -3 \\ 3 + 7/2 & -3 \leq t \leq -2 \\ -\frac{1}{2}t^2 - t + \frac{3}{2} & -2 < t < 0 \\ \frac{3}{2} - t & 0 \leq t < 1 \\ \frac{1}{2}t^2 - 2t + 2 & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

2) Determine $y(t) = e^{-t} u(t) * [\delta(t+1) - \delta(t) + 2\delta(t-2)]$

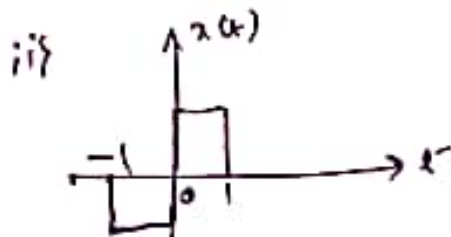
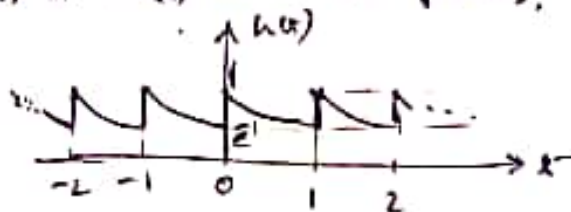
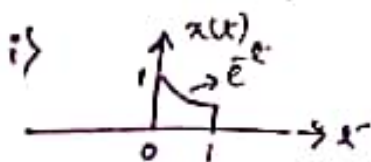
$$\text{ans: } y(t) = e^{-(t+1)} u(t+1) - e^{-t} u(t) + 2e^{-(t-2)} u(t-2)$$

3) Find $y(n)$ if i) $x(n) = 3\delta(n) - 2\delta(n-1)$ & $h(n) = \delta(n+1) + 3\delta(n) + 2\delta(n-1) - \delta(n-2) + \delta(n-3)$

ii) $x(n) = u(n+1) - u(n-3)$ & $h(n)$ as above

ans: i)

4) Find $y(t)$ if $x(t)$ & $h(t)$ are as follows.



$$h(t) = h(t) \text{ of (i)}$$

iii) find $y(t)$ if $x(t) = x(t) \text{ of (i)}$

$$h(t) = x(t) \text{ of (ii)}$$

iv) $y(t) = \cos(2\pi t) [u(t+1) - u(t-1)] * [u(t+1) - u(t-1)]$

5} Find the DT convolutions

i) $y(n) = \left(\frac{1}{4}\right)^n u(n) * u(n+2)$

ii) $\cos\left(\frac{\pi}{2}n\right) u(n) * u(n-1)$

iii) $y(n) = u(n) * \sum_{p=0}^{\infty} \delta(n-4p)$

iv) $x(n) = n \quad -5 \leq n \leq 5$

$h(n) = u(n+3) - 2u(n-1) + u(n-5)$

► **Problem 2.2** Evaluate the following discrete-time convolution sums:

(a) $y[n] = u[n] * u[n - 3]$

(b) $y[n] = (1/2)^n u[n - 2] * u[n]$

(c) $y[n] = \alpha^n \{u[n - 2] - u[n - 13]\} * 2\{u[n + 2] - u[n - 12]\}$

(d) $y[n] = (-u[n] + 2u[n - 3] - u[n - 6]) * (u[n + 1] - u[n - 10])$

(e) $y[n] = u[n - 2] * h[n]$, where

$$h[n] = \begin{cases} \gamma^n, & n < 0, |\gamma| > 1 \\ \eta^n, & n \geq 0, |\eta| < 1 \end{cases}$$

(f) $y[n] = x[n] * h[n]$, where $x[n]$ and $h[n]$ are shown in Fig. 2.8.

Answers:

(a)

$$y[n] = \begin{cases} 0, & n < 3 \\ n - 2, & n \geq 3 \end{cases}$$

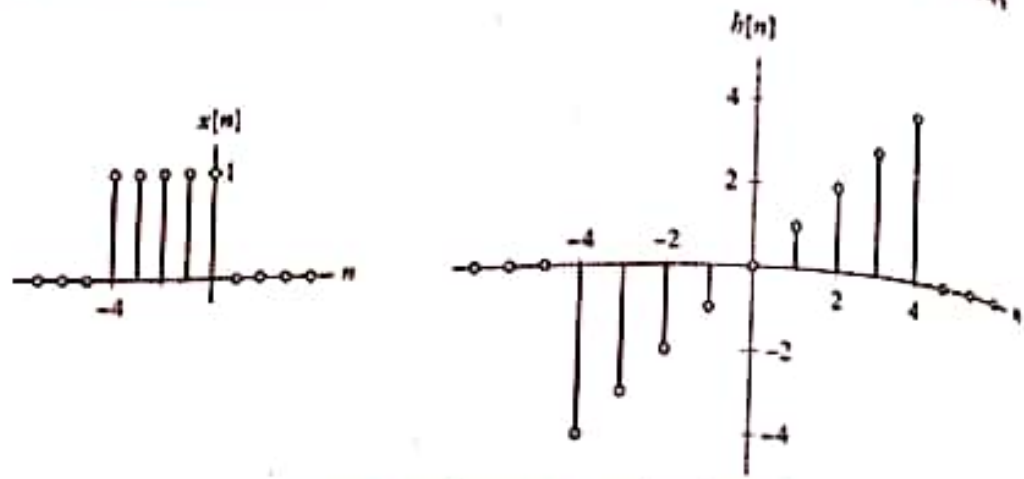


FIGURE 2.8 Signals for Problem 2.2(f).

(b)

$$y[n] = \begin{cases} 0, & n < 2 \\ 1/2 - (1/2)^n, & n \geq 2 \end{cases}$$

(c)

$$y[n] = \begin{cases} 0, & n < 0 \\ 2\alpha^{n+2} \frac{1 - (\alpha)^{-1-n}}{1 - \alpha^{-1}}, & 0 \leq n \leq 10 \\ 2\alpha^{12} \frac{1 - (\alpha)^{-11}}{1 - \alpha^{-1}}, & 11 \leq n \leq 13 \\ 2\alpha^{12} \frac{1 - (\alpha)^{n-24}}{1 - \alpha^{-1}}, & 14 \leq n \leq 23 \\ 0, & n \leq 24 \end{cases}$$

(d)

$$y[n] = \begin{cases} 0, & n < -1 \\ -(n+2), & -1 \leq n \leq 1 \\ n-4, & 2 \leq n \leq 4 \\ 0, & 5 \leq n \leq 9 \\ n-9, & 10 \leq n \leq 11 \\ 15-n, & 12 \leq n \leq 14 \\ 0, & n > 14 \end{cases}$$

(e)

$$y[n] = \begin{cases} \frac{\gamma^{n-1}}{\gamma-1}, & n < 2 \\ \frac{1}{\gamma-1} + \frac{1-\eta^{n-1}}{1-\eta}, & n \geq 2 \end{cases}$$

(f)

$$y[n] = \begin{cases} 0, & n < -8, n > 4 \\ -10 + (n+5)(n+4)/2, & -8 \leq n \leq -5 \\ 5(n+2), & -4 \leq n \leq 0 \\ 10 - n(n-1)/2, & 1 \leq n \leq 4 \end{cases}$$