**Solution**

$$\text{Given} \quad B = 3.4 \text{ KHz} = 3400 \text{ Hz}$$

$$10 \log_{10} \frac{S}{N} = 20 \text{ dB} \quad \therefore \frac{S}{N} = 100$$

$$\text{Number of characters} = q = 128 \text{ equiprobable characters.}$$

(a) Channel capacity from equation (4.107)

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 3400 \log_2(1 + 100)$$

$$\therefore \quad \mathbf{C = 22638 \ bits/sec}$$

(b) Average information content per character [it is maximum since all the characters are equiprobable] is

$$H = H_{max} = \log_2 q = \log_2 128$$

$$\therefore \quad \mathbf{H = 7 \ bits/character}$$

(c) Average information rate $= R_s = r_s H$

For error-free transmission we must have $R_s < C$

$$\therefore \quad r_s H < C$$

$$\therefore \quad r_s < \frac{C}{H}$$

$$\therefore \quad r_s < \frac{22638}{7}$$

$$\therefore \quad \mathbf{H = 7\ bits/character}$$

(c) Average information rate $= R_s = r_s H$

For error-free transmission we must have $R_s < C$

$$\therefore \quad r_s H < C$$

$$\therefore \quad r_s < \frac{C}{H}$$

$$\therefore \quad r_s < \frac{22638}{7}$$

$$\therefore \quad \mathbf{r_s < 3234\ symbols/sec}$$

$\therefore$ **The maximum symbol rate for which error-free transmission over the channel is possible = 3234 symbols/sec.**

Ex.2: A CRT terminal is used to enter alphanumeric data into a chamber. The CRT is connected through a voice-grade telephone line having usable band width of 3KHz and an output (S/N) of 10 dB. Assume that the terminal has 128 characters and data is sent in an independent manner with equal peobability.

(i) Find the average information per characters.

(ii) Find capacity of the channel.

(iii) Find maximum rate at which data can be sent from terminal to the computer error.

is

$$H = H_{max} = \log_2 q = \log_2 128$$

or $$\mathbf{H = 7 \text{ bits/character}}$$

(ii) Given, $$B = 3 \text{ KHz} = 3000 \text{ Hz}$$

$$10 \log_{10} \frac{S}{N} = 10 \quad \therefore \quad \frac{S}{N} = 10$$

From equation (4.107), channel capacity is given by

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

$$= 3000 \log_2 (1 + 10)$$

$\therefore$ $$\mathbf{C = 10378.295 \text{ bits/sec}}$$

(iii) Average information rate $= R_s = r_s H$

For error-free transmission we must have

$$R_s = C$$

i.e., $$r_s H < C$$

$\therefore$ $$r_s < \frac{C}{H}$$

$\therefore$ $$r_s < \frac{10378.295}{7}$$

For error-free transmission we must have

$$R_s = C$$

i.e., $$r_s H < C$$

$$\therefore \quad r_s < \frac{C}{H}$$

$$\therefore \quad r_s < \frac{10378.295}{7}$$

$$\therefore \quad \mathbf{r_s < 1482.614 \text{ symbols/sec}}$$

$\therefore$ The maximum rate at which data can be sent from terminal to the computer without error = 1482.614 symbols/sec.

Ex.3: A voice-grade channel of the telephone network has a bandwidth of 3.4 KHz.

(a) Calculate channel capacity of the telephone channel foe a signal-to-noise ratio of 30dB.

(b) Calculate the minimum signal-to-noise ratio required to support information transmission through the telephone channel at the rate of 4800 bits/sec.

**Solution**

$$\text{Given} \quad B = 3.4 \text{ KHz} = 3400 \text{ Hz}$$

$$10 \log_{10}\frac{S}{N} = 30 \text{ dB} \quad \therefore \quad \frac{S}{N} = 1000$$

(a) Channel capacity from equation (4.107)

$$C = B \log_2\left(1+\frac{S}{N}\right)$$

$$= 3400 \log_2(1 + 1000)$$

$$\therefore \quad \mathbf{C = 33889 \text{ bits/sec}}$$

(b) $$\text{Given} \quad C = 4800 \text{ bits/sec}, \quad \frac{S}{N} = ?$$

$$\text{We have} \quad C = B \log_2\left(1+\frac{S}{N}\right)$$

$$\therefore \quad 4800 = 3400 \log_2\left(1+\frac{S}{N}\right)$$

$$\therefore \quad \frac{S}{N} = 2^{48/34} - 1$$

$$\therefore \quad \frac{S}{N} = 1.66$$

$$\text{or} \quad \frac{S}{N} \text{ in dB} = 10 \log_{10} 1.66 = 2.2 \text{ dB}$$

$$C = B \log_2\left(1+\frac{S}{N}\right)$$

$$= 3400 \log_2(1 + 1000)$$

$$\therefore \quad \mathbf{C = 33889 \text{ bits/sec}}$$

(b) Given $C = 4800$ bits/sec, $\frac{S}{N} = ?$

We have $C = B \log_2\left(1+\frac{S}{N}\right)$

$$\therefore \quad 4800 = 3400 \log_2\left(1+\frac{S}{N}\right)$$

$$\therefore \quad \frac{S}{N} = 2^{48/34} - 1$$

$$\therefore \quad \frac{S}{N} = \mathbf{1.66}$$

or $\frac{S}{N}$ in dB $= 10 \log_{10} 1.66 = 2.2$ dB

$$\therefore \quad \frac{S}{N} = \mathbf{2.2 \text{ dB}}$$

Ex.4:

and $R = r_s H = 2B(2) = (4B)$ bits/sec

## PROPERTIES OF ENTROPY:

1. Entropy is average self information. It is given by

$$H(S) = \sum_{i=1}^{q} P_i \log(1/P_i)$$

$$H(S) = \sum_{i=1}^{q} P_i I_i \quad \text{bits/symbol}$$

where $q$ represents number of symbols and also $\sum_{i=1}^{q} P_i = 1$.

2. For null event and sure event, the entropy vanishes.

3. The entropy is a symmetrical function of its arguments. The value of $H(S)$ remains the same irrespective of location of probabilities.

4. <u>Extremal property</u>: When all the source symbols become equiprobable, then the entropy attains the maximum value.

$$H(S)_{max} = \log_2 q.$$

5. When source symbols are not equiprobable, then entropy is

$$H(S) = \sum_{i=1}^{q} P_i \log (1/P_i)$$

$$H(S) = \sum_{i=1}^{q} P_i I_i \quad \text{bits/symbol}$$

where $q$ represents number of symbols and also $\sum_{i=1}^{q} P_i = 1$.

2. For null event and sure event, the entropy vanishes.
3. The entropy is a symmetrical function of its arguments. The value of $H(S)$ remains the same irrespective of location of probabilities.
4. <u>Extremal property</u>: When all the source symbols become equiprobable, then the entropy attains the maximum value.

$$H(S)_{max} = \log_2 q.$$

5. When source symbols are not equiprobable, then entropy is less than maximum value.
6. The source efficiency, $\eta_s$ is given by

$$\eta_s = \frac{H(S)}{}$$

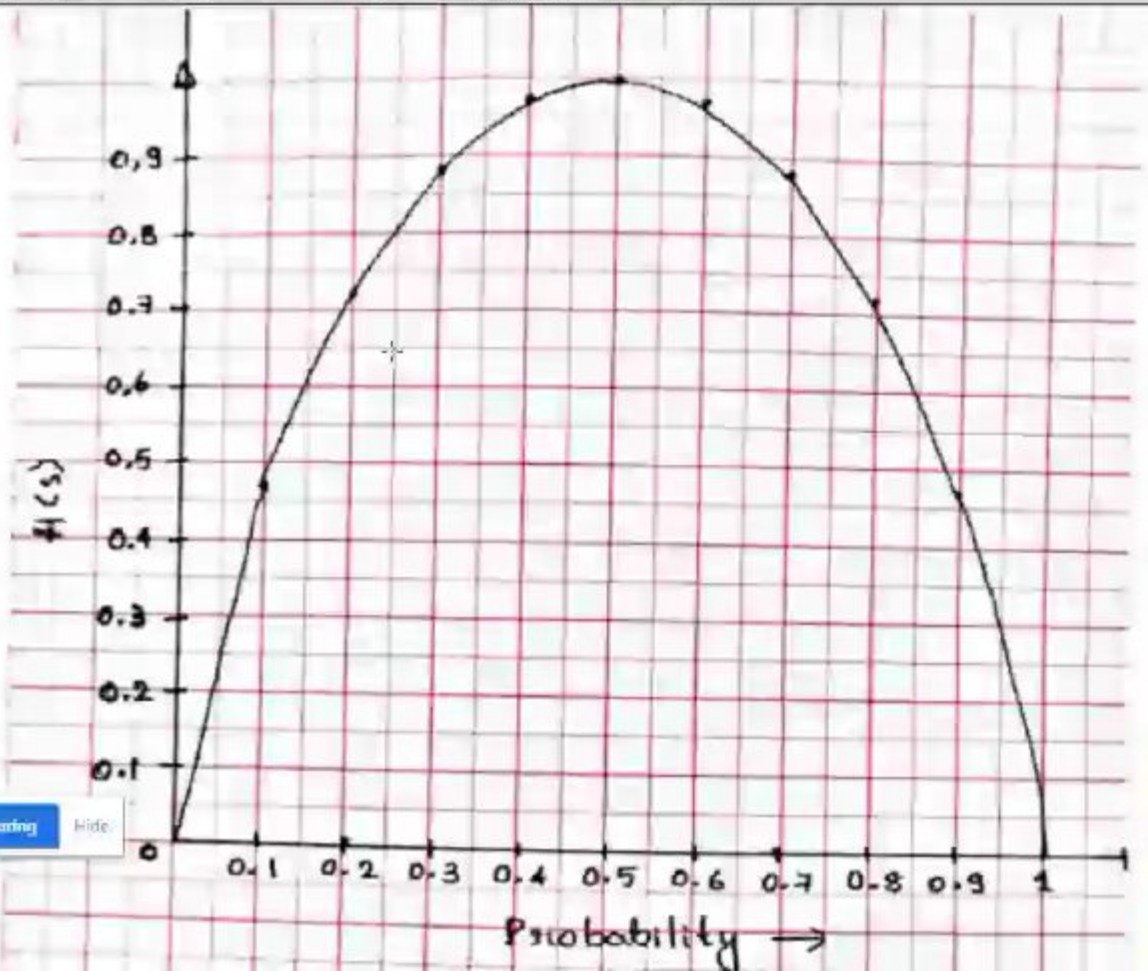
Q> A binary source is emitting an independent sequence of 0's and 1's with probabilities p and (1-p) respectively. Plot the entropy of the source versus p.

Sol$^n$: The entropy of the binary source is given by

$$H(S) = P \log_2\left(\frac{1}{P}\right) + (1-P)\log_2\left(\frac{1}{1-P}\right) \rightarrow (1)$$

Type eq$^n$ (1) in calculator and 'calculate' the values of H(S) for assumed values of P. P should vary from 0.1 to 1

| P | H(S) |
|---|---|
| 0.1 | 0.469 |
| 0.2 | 0.722 |
| 0.3 | 0.881 |
| 0.4 | 0.971 |
| 0.5 | 1 |
| 0.6 | 0.971 |
| 0.7 | 0.881 |
| 0.8 | 0.722 |
| 0.9 | 0.469 |
| 1.0 | — |



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability →