$$\text{or} \quad C_\infty = B \frac{S}{N} \log_2 e \text{ bits/sec} \qquad \text{...... (4.108)}$$

SHANNON'S LIMIT :

We define an ***"ideal system"*** as one that transmits data at a bit rate R_i equal to the channel capacity C. We may then express the average transmitted power as

$$S = E_b C \qquad \text{...... (4.109)}$$

Where, E_b = transmitted energy per bit in joules.

Using $N = \eta B$ and $S = E_b C$ in equation (4.107), we get for an ideal system

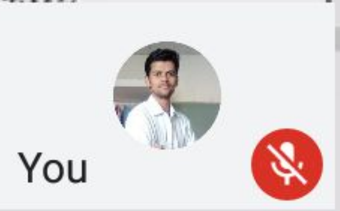
$$C = B \log_2 \left(1 + \frac{E_b}{\eta} \frac{C}{B}\right)$$

$$\text{or} \quad \frac{C}{B} = \log_2 \left(1 + \frac{E_b}{\eta} \frac{C}{B}\right) \qquad \text{...... (4.110)}$$

The quantity $\left(\frac{C}{B}\right)$ is called ***"Bandwidth-efficiency"*** and the quantity (E_b/η) is given by

$$\frac{E_b}{\eta} = \frac{2^{C/B} - 1}{(C/B)} \qquad \text{...... (4.111)}$$

When (R_i/B) is plotted as a function of (E_b/η), we get the bandwidth-efficiency di which is shown in figure 4.3. The resulting curve represents the capacity boundary for



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$R = C$ corresponding to equation (4.111). Based on this diagram, the following observations are made:

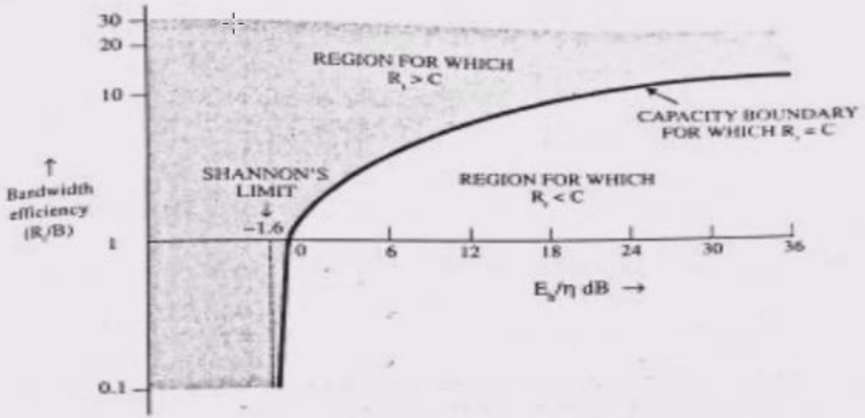

Fig. 4.3 : Illustrating Bandwidth-efficiency diagram

1. For infinite bandwidth, th ... e limiting value.

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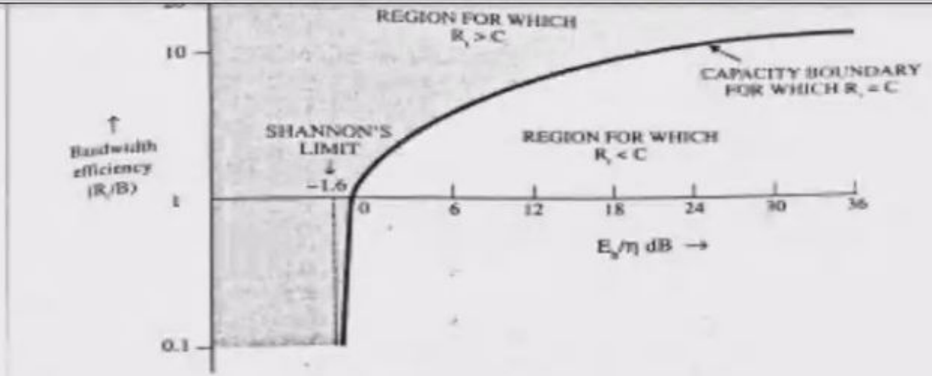

Fig. 4.3 : Illustrating Bandwidth-efficiency diagram

1. For infinite bandwidth, the signal energy-to-noise ratio E_b/η approaches the limiting value.

$$\left(\frac{E_b}{\eta}\right)_{\infty} = \lim_{B\to\infty}\left(\frac{E_b}{\eta}\right) = \lim_{B\to\infty}\left[\frac{2^{C/B}-1}{(C/B)}\right]$$

Let $\frac{C}{B} = x$. As $B \to \infty$, $x \to 0$

$$\therefore \left(\frac{E_b}{\eta}\right)_{\infty} = \lim_{x\to 0}\left[\frac{2^x-1}{x}\right] \quad \text{...... (4.112)}$$

Using L'Hospital Rule, the above limit can be evaluated as below:

Let $y = 2^x$

Taking ln on both sides

$$ln\, y = x\, ln2$$

Differentiating, $\frac{1}{y}\, dy = (ln2)\, dx$ ...... (4.113)

$$\therefore \frac{dy}{dx} = y\,(ln2) = 2^x\,(ln2)$$

Differentiating both nume... of the RHS of equation (4.112) with respect to 'x', we get

$$\left(\frac{E_b}{\eta}\right)_{\infty} = \lim_{x\to 0}\left[\frac{\frac{d}{dx}(2^x-1)}{\frac{d}{dx}(x)}\right]$$

$$= \lim_{x\to 0}\left[\frac{2^x(ln2)}{1}\right] \text{ by using equation (4.113)}$$

$$= 2^0\, ln2$$

$$\therefore \left(\frac{E_b}{\eta}\right)_{\infty} = ln2 = 0.693$$

$$\text{or} \quad \left(\frac{E_b}{\eta}\right)_{\infty} \text{in dB} = 10\log_{10}(0.693)$$

$$\therefore \left(\frac{E_b}{\eta}\right) \text{in dB} \cong -1.6 \text{ dB}$$

This value of −1.6 dB is called the "Shannon's Limit". The corresponding value of

$$\therefore \left(\frac{E_b}{\eta}\right)_{\infty} = \ln 2 = 0.693$$

or $$\left(\frac{E_b}{\eta}\right)_{\infty} \text{ in dB} = 10 \log_{10}(0.693)$$

$$\therefore \left(\frac{E_b}{\eta}\right)_{\infty} \text{ in dB} \cong -1.6 \text{ dB} \qquad \text{...... (4.114)}$$

This value of – 1.6 dB is called the **"Shannon's Limit"**. The corresponding value of channel capacity is given by equation (4.108) as

$$C_{\infty} = B \frac{S}{N} \log_2 e \text{ bits/sec}$$

2. The capacity boundary, defined by the curve for critical bit rate $R_t = C$, separates combinations of system parameters that have the potential for supporting error free transmission ($R_t < C$) from those for which error-free transmission is not possible ($R_t > C$). The latter region is shown using dots in figure 4.3.
3. The diagram of figure 4.3 highlights trade-offs between (E_b/η) and (R_t/B). This is discussed in a difficult aspect in the 2nd implication of Shannon-Hartley law.

2nd Implication :

Bandwidth - (S/N) Trade Off :

An important im[...] xchange of bandwi[...] signal to noise power [...] versa as given below :

$N \log_2 e$ bits/sec

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2nd Implication :

Bandwidth - (S/N) Trade Off :

An important implication of Shannon-Hartley law is the exchange of bandwidth with signal to noise power ratio and vice-versa as given below :

$$\text{Suppose } \left(\frac{S_1}{N_1}\right) = 7 \text{ and } B_1 = 4 \text{ KHz.}$$

$$\therefore \text{ Channel capacity } C_1 = B_1 \log\left(1 + \frac{S_1}{N_1}\right)$$

$$= 4 \times 10^3 \log(1 + 7)$$

$$= 12 \times 10^3 \text{ bits/sec.}$$

Keeping the channel capacity C_2 same as C_1 and if signal-to-noise ratio is increas[e]

then

$$C_2 = C_1 = 12 \times 10^3 = B_2 \log\left(1 + \frac{S_2}{N_2}\right)$$
$$= B_2 \log(1 + 15)$$
$$\therefore \quad B_2 = 3 \text{ KHz}$$

Since the noise power $N = \eta B$, as the bandwidth gets reduced from 4 to 3 KHz, the noise also decreases indicating an increase in signal power as shown below.

We have $N_1 = \eta B_1 = (\eta)(4 \text{ KHz})$

and $N_2 = \eta B_2 = (\eta)(3 \text{ KHz})$

Consider $\dfrac{(S_2/N_2)}{(S_1/N_1)} = \dfrac{15}{7}$

$$\therefore \quad \frac{S_2}{S_1} = \frac{15 N_2}{7 N_1} = \frac{(15)(\eta)(3\text{KHz})}{(7)(\eta)(4\text{KHz})} = \frac{45}{28} \approx 1.6$$

Thus a 25% reduction in bandwidth from 4 KHz to 3 KHz requires a 60% approximate increase in signal power for maintaining the same channel capacity. Let us look into the exact significance by drawing the ***"trade-off curve"***.

From Shannon-Hartley law

$$\frac{B}{C} = \frac{1}{\log_2\left(1 + \frac{S}{N}\right)} \qquad \text{...... (4.115)}$$

The values of (B/C) for different values of (S/N) are listed in table 4.1 below :

$\frac{S}{N}$	0.5	1	2	5	10	15	20	30
$\frac{B}{C}$	1.7[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

and $N_2 = \eta B_2 = (\eta)\,(3\text{ KHz})$

$$\text{Consider } \frac{(S_2/N_2)}{(S_1/N_1)} = \frac{15}{7}$$

$$\therefore \quad \frac{S_2}{S_1} = \frac{15N_2}{7N_1} = \frac{(15)(\eta)(3\,\text{KHz})}{(7)(\eta)(4\,\text{KHz})} = \frac{45}{28} = 1.6$$

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The values of (B/C) for different values of (S/N) are listed in table 4.1 below :

$\frac{S}{N}$	0.5	1	2	5	10	15	20	30
$\frac{B}{C}$	1.71	1	0.63	0.37	0.289	0.25	0.23	0.2

Table 4.1 : Table of values of (B/C) for different values of (S/N)

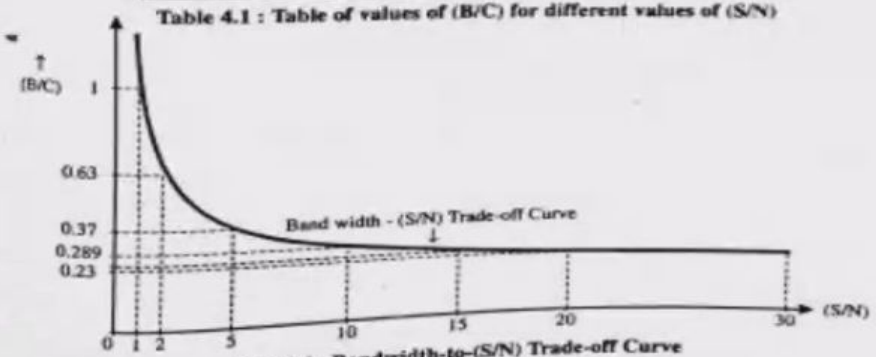

Fig. 4.4 : Bandwidth-to-(S/N) Trade-off Curve

Figure 4.4 above ... S/N). Using this trade-off

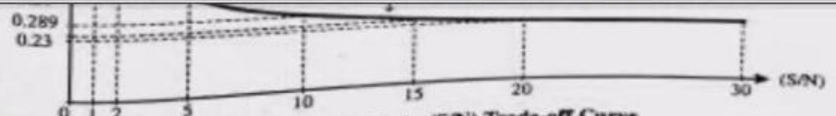

Fig. 4.4 : Bandwidth-to-(S/N) Trade-off Curve

Figure 4.4 above shows a plot of (B/C) as a function of (S/N). Using this trade-off curve the same channel capacity may be obtained by increasing bandwidth if (S/N) is small. Furthermore, the curve also indicates that there exists a threshold point at around (S/N) = 10 up to which the exchange rate of bandwidth with (S/N) is advantageous. Beyond (S/N) = 10, the reduction in B with increasing (S/N) is very poor. FM, PM and PCM systems including DM and ADM systems require larger bandwidths with reasonably good (S/N) ratio.

Ex.1:

Alphanumeric data are entered into a computer from a remote terminal through a voice-grade telephone channel. The channel has a bandwidth of 3.4 KHz, and output signal-to-noise ratio of 20 dB. The terminal has a total of 128 symbols. Assume that the symbols are equiprobable and the successive transmissions are statistically independent.

(a) Calculate channel capacity.

(b) Find the average information content per character.

(c) Calculate the maximum symbol rate for which error-free transmission over the
is possible.