In general for a m-stage filter

$$\boxed{\begin{aligned} \alpha_{m-1}(0) &= 1 \\ k_m &= \alpha_m(m). \\ \alpha_{m-1}(k) &= \frac{\alpha_m(k) - \alpha_m(m)\alpha_m(m-k)}{1-\alpha_m^2(m)}, \quad 1 \le k \le m-1 \end{aligned}} \tag{4.41}$$

Example 4.6

An FIR filter is given by the difference equation

$$y(n) = x(n) + \frac{4}{3}x(n-1) + \frac{1}{2}x(n-2) + \frac{2}{3}x(n-3)$$

Determine its lattice form.

Solution Given

$$y(n) = x(n) + \frac{4}{3}x(n-1) + \frac{1}{2}x(n-2) + \frac{2}{3}x(n-3)$$

$$\alpha_3(0) = 1, \quad \alpha_3(1) = \frac{4}{3}, \quad \alpha_3(2) = \frac{1}{2}, \quad \alpha_3(3) = \frac{2}{3}$$

From equation (4.41) we get

$$\alpha_2(0) = 1$$

$$k_3 = \alpha_3(3) = \frac{2}{3}$$

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - \alpha_m(m)\alpha_m(m-k)}{1 - \alpha_m^2(m)}, \quad 1 \le k \le 2$$

For $m = 3$ and $k = 1$

$$\alpha_2(1) = \frac{\alpha_3(1) - \alpha_3(3)\alpha_3(2)}{1 - \alpha_3^2(3)}$$

$$= \frac{\frac{4}{3} - \frac{2}{3} \cdot \frac{1}{2}}{1 - (\frac{2}{3})^2}$$

$$= \frac{\frac{4}{3} - \frac{2}{6}}{1 - \frac{4}{9}}$$

$$= 1.8$$

For $m = 3$ and $k = 2$

$$k_2 = \alpha_2(2) = \frac{\alpha_3(2) - \alpha_3(3)\alpha_3(1)}{1 - \alpha_3^2(3)}$$

$$= \frac{\frac{1}{2} - \frac{2}{3} \cdot \frac{4}{3}}{1 - (\frac{2}{3})^2}$$

$$= \frac{\frac{1}{2} - \frac{8}{9}}{1 - \frac{4}{9}}$$

$$= -0.7$$

For $m = 2$ and $k = 1$

$$k_1 = \alpha_1(1) = \frac{\alpha_2(1) - \alpha_2(2)\alpha_2(1)}{1 - \alpha_2^2(2)}$$

$$= \frac{1.8 - (-0.7)(1.8)}{1 - (-0.7)^2}$$

$$= \frac{3.06}{0.51}$$

$$= 6$$

$$\boxed{k_1 = 6, \quad k_2 = -0.7, \quad k_3 = 0.6667}$$

The lattice structure is shown in Figure 4.15.

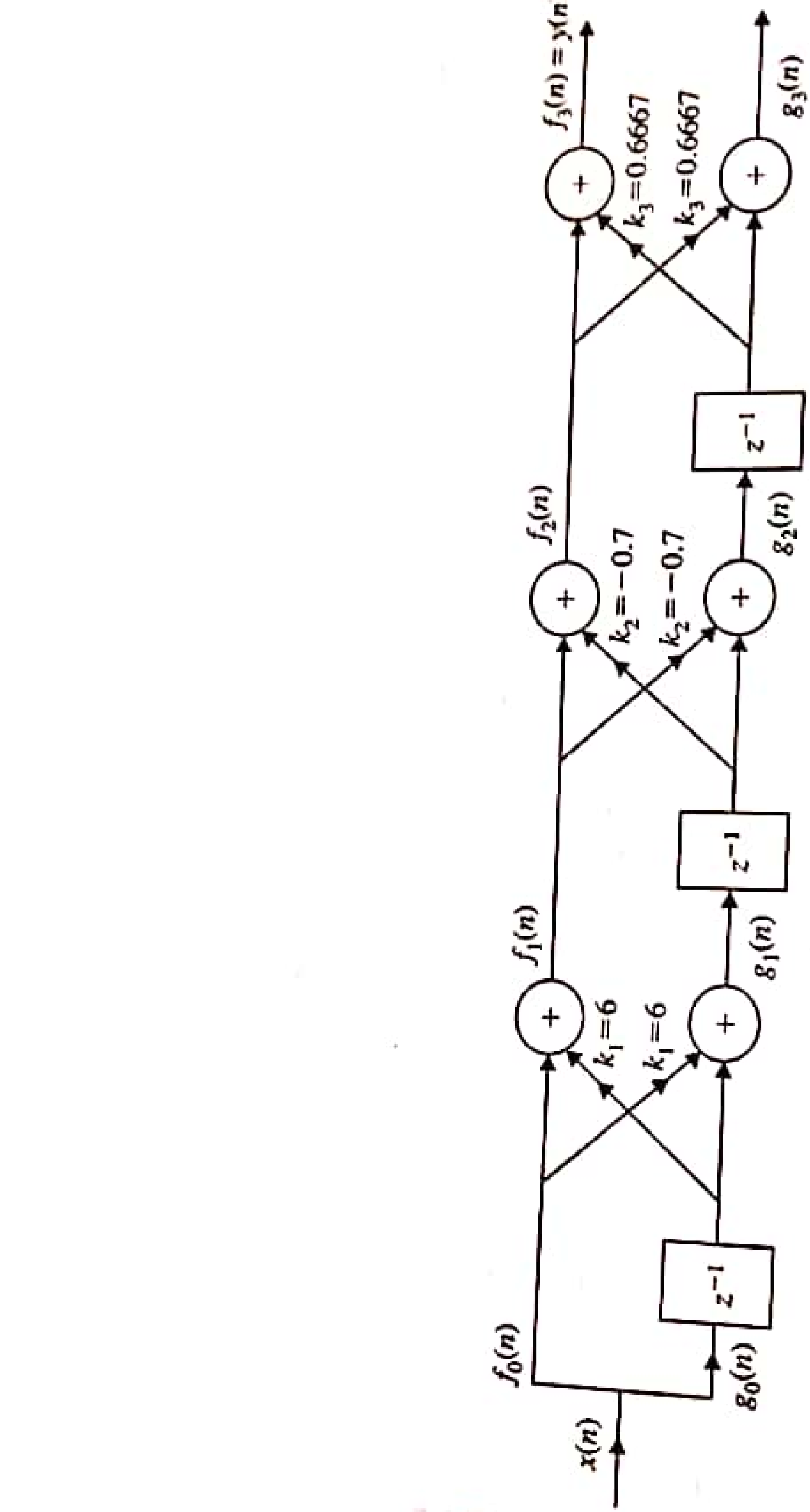

Figure 4.15 Lattice structure for the Example 4.6.

Example 4.7(a)

An FIR filter is given by the difference equation

$$y(n) = 2x(n) + \frac{4}{5}x(n-1) + \frac{3}{2}x(n-2) + \frac{2}{3}x(n-3)$$

Determine the lattice form.

(Anna University, May 2007)

Solution Given

$$\begin{aligned} y(n) &= 2x(n) + \frac{4}{5}x(n-1) + \frac{3}{2}x(n-2) + \frac{2}{3}x(n-3) \\ &= 2\left[x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)\right] \\ &= k_0\left[1 + \sum_{k=1}^{3}\alpha_m(k)x(n-k)\right] \end{aligned}$$

where $k_0 = 2$

$$\alpha_3(0) = 1,\ \alpha_3(1) = \frac{2}{5},\ \alpha_3(2) = \frac{3}{4},\ \alpha_3(3) = \frac{1}{3}$$

From Equation (4.24), we get

$$\alpha_2(0) = 1$$

$$k_3 = \alpha_3(3) = \frac{1}{3}$$

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - \alpha_m(m)\alpha_m(m-k)}{1-\alpha_m^2(m)}, \quad 1 \le k \le 2$$

For $m = 3$ and $k = 1$

$$\begin{aligned} \alpha_2(1) &= \frac{\alpha_3(1) - \alpha_3(3)\alpha_3(2)}{1-\alpha_3^2(3)} \\ &= \frac{\frac{2}{5} - \frac{1}{3}\cdot\frac{3}{4}}{1-(\frac{1}{3})^2} \\ \alpha_2(1) &= 0.16875 \end{aligned}$$

For $m = 3$ and $k = 2$

$$k_2 = \alpha_2(2) = \frac{\alpha_3(2) - \alpha_3(3)\alpha_3(1)}{1 - \alpha_3^2(3)}$$

$$= \frac{\frac{3}{4} - \frac{1}{3}\cdot\frac{2}{3}}{1 - (\frac{1}{3})^2}$$

$$= \frac{111}{160}$$

$$\boxed{k_2 = 0.69375}$$

For $m = 2$ and $k = 1$

$$k_1 = \alpha_1(1) = \frac{\alpha_2(1) - \alpha_2(2)\alpha_2(1)}{1 - \alpha_2^2(2)}$$

$$= \frac{0.16875 - (0.69375)(0.16875)}{1 - (0.69375)^2}$$

$$\boxed{k_1 = 0.0996}$$

Lattice form coefficients are,

$$k_1 = 0.0996, \quad k_2 = 0.69375, \quad k_3 = 0.3333.$$

Example 4.7(b)

Given a three stage lattice filter with coefficients $k_1 = \frac{1}{4}$ and $k_2 = \frac{1}{4}$, $k_3 = \frac{1}{3}$. Determine the FIR filter coefficients for the direct form structure?

(Anna University, May, 2007)

Solution From the given data and from Equation (4.38), we can find that

$$\alpha_3(0) = 1, \quad \alpha_3(3) = k_3 = \frac{1}{3}$$

$$\alpha_1(1) = k_1 = \frac{1}{4}, \quad \alpha_2(2) = k_2 = \frac{1}{4}$$

We know that

$$\alpha_m(k) = \alpha_{m-1}(k) + k_m\alpha_{m-1}(m-k)$$

For $m = 2$ and $k = 1$

$$\alpha_2(1) = \alpha_1(1) + k_2\alpha_1(1)$$

$$\alpha_2(1) = \frac{1}{4} + \frac{1}{4}.\frac{1}{4} = \frac{1}{4} + \frac{1}{16}$$

$$= \frac{4+1}{16} = \frac{5}{16}$$

For $m = 3$ and $k = 1$

$$\alpha_3(1) = \alpha_2(1) + \alpha_3(3)\alpha_2(2)$$

$$= \frac{5}{16} + \frac{1}{3}.\frac{1}{4} = \frac{5}{16} + \frac{1}{12}$$

$$= \frac{60+16}{192} = \frac{76}{192}$$

$$= 0.3958$$

$$= \frac{19}{48}$$

For $m = 3$ and $k = 2$

$$\alpha_3(2) = \alpha_2(2) + k_3\alpha_2(1)$$

$$= \frac{1}{4} + \frac{1}{3}.\frac{5}{16}$$

$$\alpha_3(2) = \frac{1}{4} + \frac{5}{48} = \frac{12+5}{48} = \frac{17}{48}$$

Direct form coefficients are,

$$\boxed{\alpha_3(0) = 1, \quad \alpha_3(1) = \frac{19}{48}, \quad \alpha_3(2) = \frac{17}{48}, \quad \alpha_3(3) = \frac{1}{3}}$$