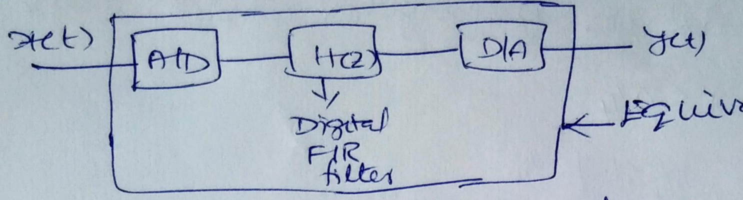
Design an FIR LPF to meet the given specifications $N=30$, $F_s = 8000\,Hz$, $f_c = 1000\,Hz$ or f_p, Bartlett window to be used in the A/D – H(z) – D/A structure



→ Here we are required to design a linear phase FIR filter H(z) to be used in the A/D – H(z) – D/A structure, so that the cascade combination behaves like an equivalent analog filter having the following specification

① $\Omega_p = 2\pi f_c = 2\pi(1000) = 6280$ r/s (or $2\pi f_p$)

$\omega_p = \Omega_p T = 6280\left(\frac{1}{8000}\right) = 0.25\pi$ r/s (or 0.79 r/s)

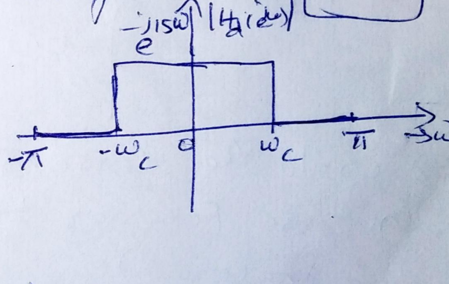
② Bartlett window offers a transition width of $\Delta\omega = \frac{8\pi}{N}$ & minimum stop band attenuation of 25dB

& window function $w(n) = 1 - \frac{2\left|n - \frac{N-1}{2}\right|}{N-1}$

For $N = 30$

$\alpha = \frac{N-1}{2} = 14.5$ to have α an integer select $\boxed{N = 31}$

$\therefore H_d(e^{j\omega}) = \begin{cases} e^{-j15\omega} & ; |\omega| \le \omega_c \\ 0 & ; \omega_c \le |\omega| \le \pi \end{cases}$



$\Rightarrow h_d(n) = \begin{cases} \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} & \text{for all } n, \text{except } n=\alpha \\ \frac{\omega_c}{\pi} & \text{at } n=\alpha \end{cases}$

③ ω_c need to be calculated using the relation

$\omega_c = \omega_p + \frac{\Delta\omega}{2}$

$= 0.25\pi + \frac{8\pi}{\cancel{2}} = \boxed{0.51\pi \text{ r/s} = \omega_c}$

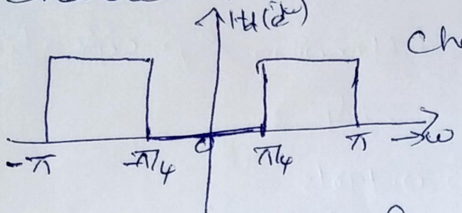
④ $h_d(n) = \begin{cases} \frac{\sin(0.5\pi(n-15))}{\pi(n-15)} & ; \text{for } n \ne 15 \\ \frac{0.51\pi}{\pi} & n = 15 \end{cases}$

$h_d(n) = \{-0.0227, -0.01, 0.02, 0.01, \ldots$ (complete the problem)

(2) Design a HP FIR Filter with the following desired frequency response. Use Rectangular window of size 5.

$$H_d(e^{j\omega}) = \begin{cases} 0 ; & |\omega| \leq \pi/4 \\ e^{-j\omega\left(\frac{N-1}{2}\right)} ; & \pi/4 \leq |\omega| \leq \pi \end{cases}$$

(1) From the specification given, 1st draw magnitude response characteristics as below



Characteristics of a HP FIR filter

(2) Determine α from given N. N is mention in the size of window ie $N = 5$

$$\Rightarrow \alpha = \frac{N-1}{2} = 2$$

(3) Rewrite the given desired frequency response $H_d(e^{j\omega})$

$$\Rightarrow H_d(e^{j\omega}) = \begin{cases} 0 ; & |\omega| \leq \pi/4 \\ e^{-j2\omega} ; & \pi/4 \leq |\omega| \leq \pi \end{cases}$$

(3) write the corresponding impulse response $h_d(n)$ by applying inverse Fourier transform. (Directly the below formula can be used)

$$h_d(n) = \begin{cases} \dfrac{\sin[\pi(n-\alpha)] - \sin(\omega_c(n-\alpha))}{\pi(n-\alpha)} & \forall n \neq \alpha \\ 1 - \left(\dfrac{\omega_c}{\pi}\right) ; & n = \alpha \end{cases}$$

(4) ω_c is given / shown as $\pi/4$

$$\boxed{\omega_c = \pi/4}$$

(5) Determine $h_d(n) = \{-0.16, -0.22, 0.75, -0.22, -0.16\}$

(b) Frequency response for N=5 odd

$$H_d(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n) \cos\left(\omega\left(n-\left(\frac{N-1}{2}\right)\right)\right) \right\}$$

$$= e^{-j2\omega} \left\{ h(2) + \sum_{n=0}^{1} 2h(n) \cos(\omega(n-2)) \right\}$$

$$= e^{-j2\omega} \left\{ 0.75 + 2h(0)\cos(2\omega) + 2h(1)\cos(-\omega) \right\}$$

$$= e^{-j2\omega} \left\{ 0.75 - 2(0.159)\cos(2\omega) + 2(-0.225)\cos(\omega) \right\}$$

Complete this