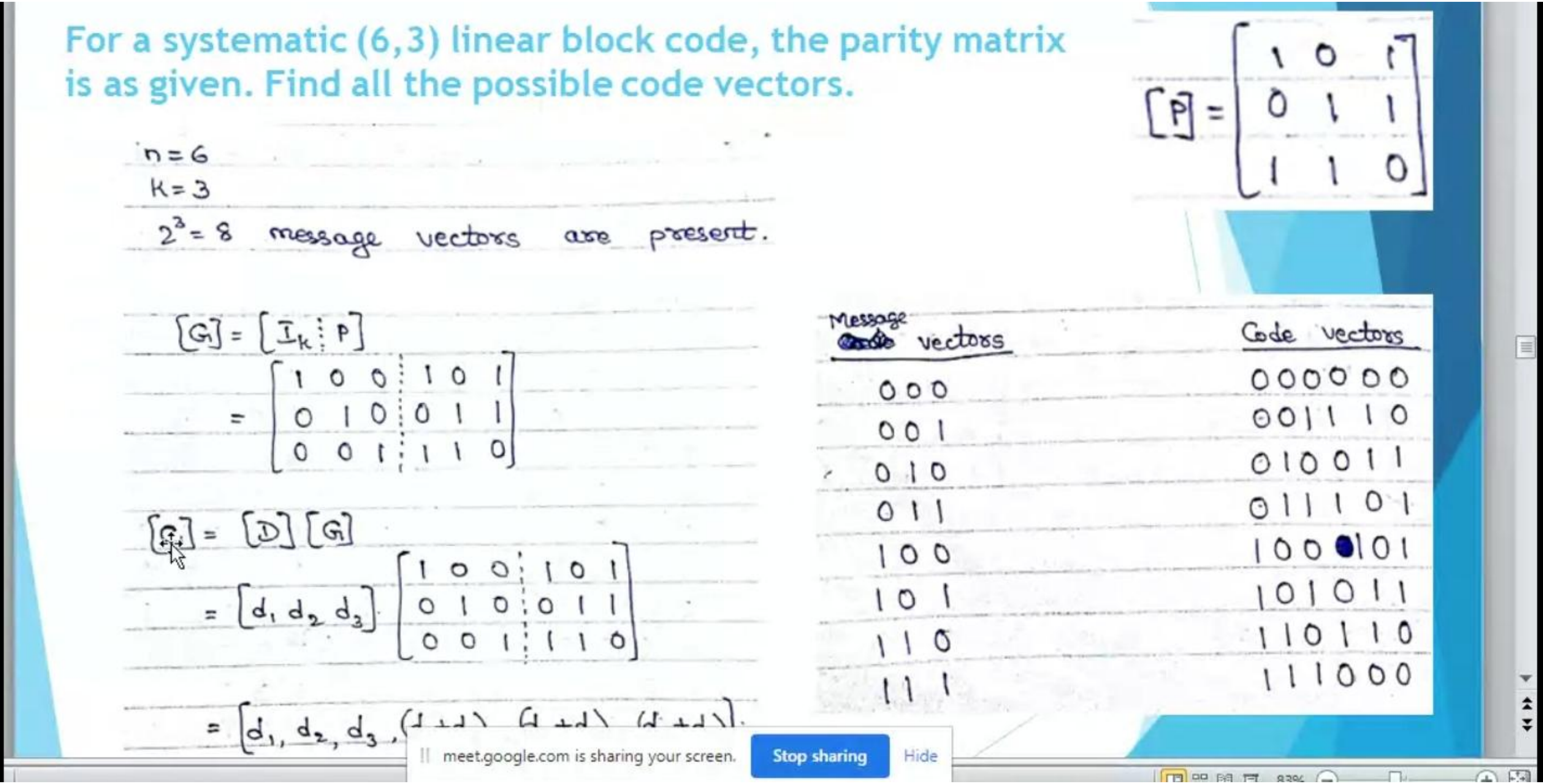


For a systematic (6,3) linear block code, the parity matrix is as given. Find all the possible code vectors.

$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$n = 6$
$K = 3$
$2^3 = 8$ message vectors are present.

$$[G] = [I_k \vdots P]$$

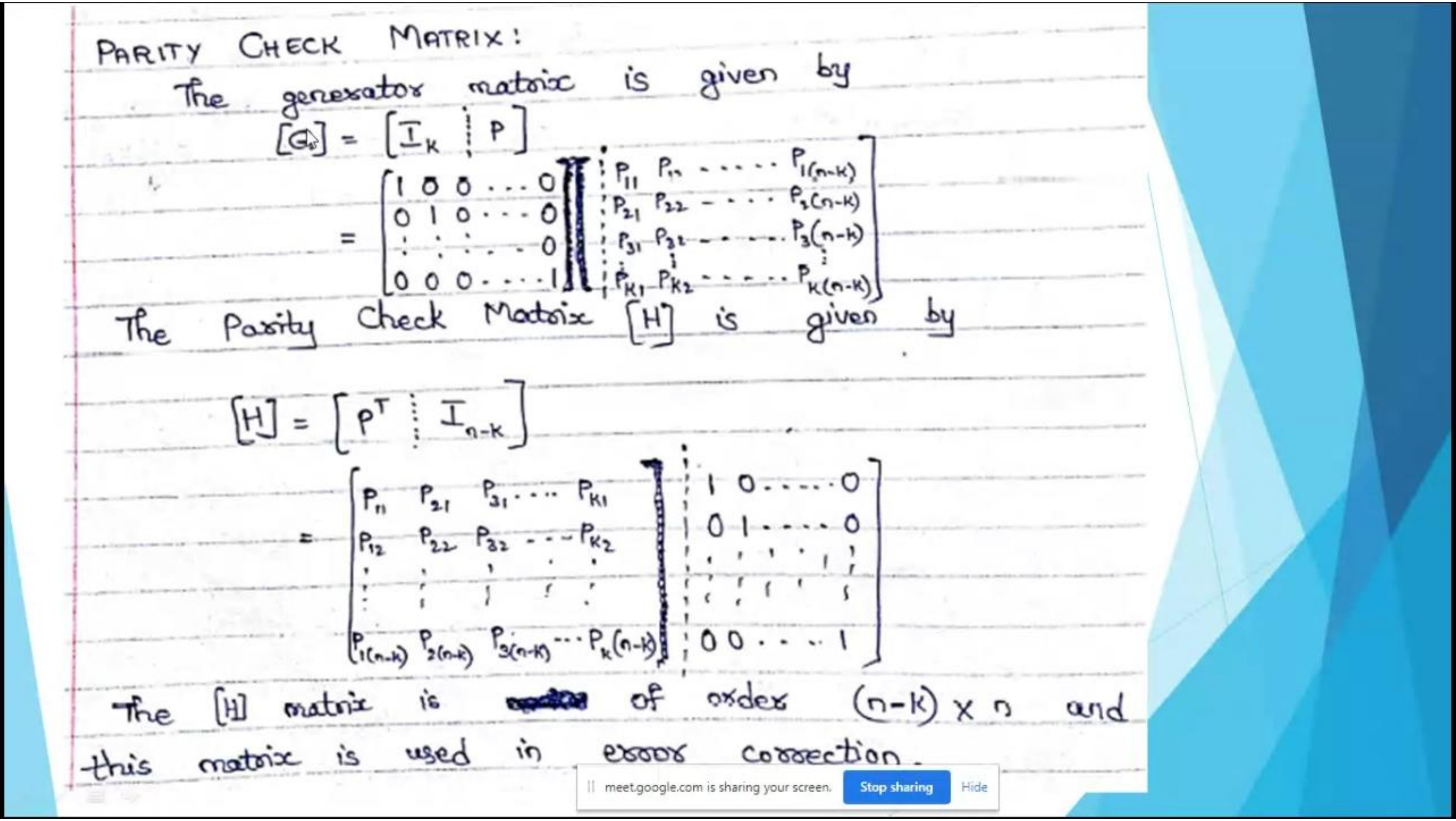
$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$[C] = [D][G]$$

$$= [d_1\ d_2\ d_3] \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= [d_1, d_2, d_3, (d_1 + d_3)\ (d_1 + d_2)\ (d_1 + d_2)]$$

Message vectors	Code vectors
000	000000
001	001110
010	010011
011	011101
100	100101
101	101011
110	110110
111	111000



PARITY CHECK MATRIX:

The generator matrix is given by

$$[G] = [I_K \;\vdots\; P]$$

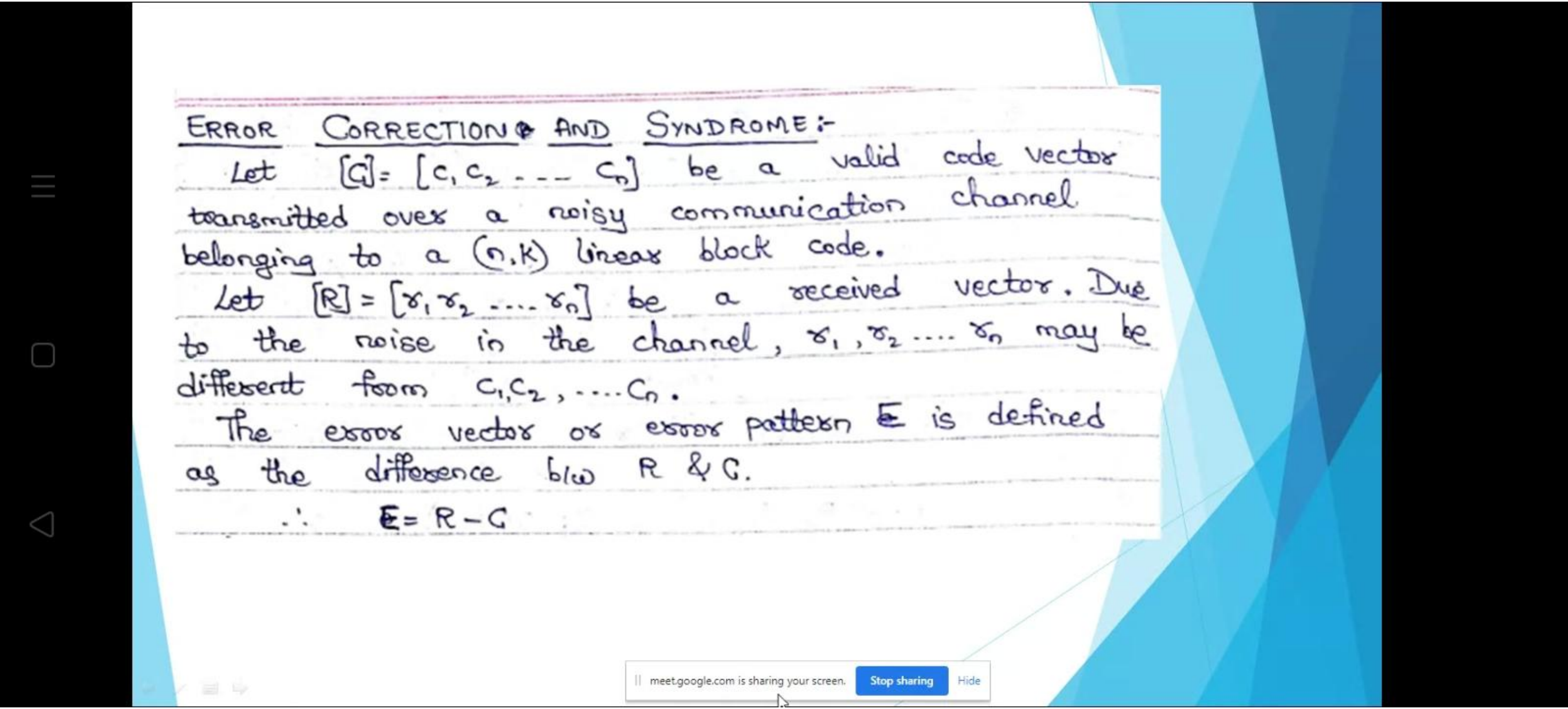
$$= \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & \cdots & 0 & P_{11} & P_{12} & \cdots & P_{1(n-K)} \\ 0 & 1 & 0 & \cdots & 0 & P_{21} & P_{22} & \cdots & P_{2(n-K)} \\ \vdots & \vdots & \vdots & \cdots & 0 & P_{31} & P_{32} & \cdots & P_{3(n-K)} \\ & & & & & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & P_{K1} & P_{K2} & \cdots & P_{K(n-K)} \end{array}\right]$$

The Parity Check Matrix [H] is given by

$$[H] = [P^T \;\vdots\; I_{n-K}]$$

$$= \left[\begin{array}{ccccc|ccc} P_{11} & P_{21} & P_{31} & \cdots & P_{K1} & 1 & 0 \cdots & 0 \\ P_{12} & P_{22} & P_{32} & \cdots & P_{K2} & 0 & 1 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{1(n-K)} & P_{2(n-K)} & P_{3(n-K)} & \cdots & P_{K(n-K)} & 0 & 0 \cdots & 1 \end{array}\right]$$

The [H] matrix is of order $(n-K) \times n$ and this matrix is used in error correction.



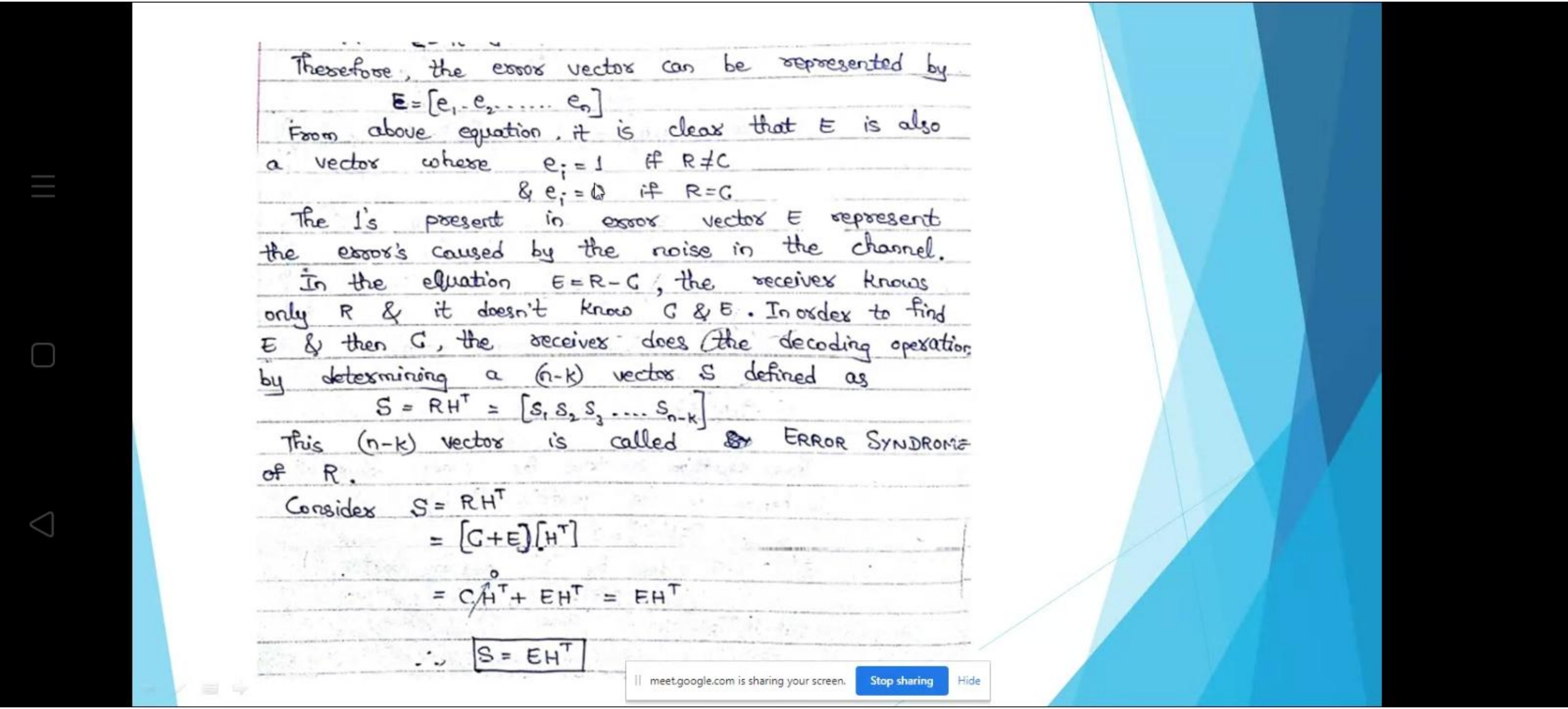
ERROR CORRECTION AND SYNDROME:-

Let $[C] = [c_1, c_2 \dots c_n]$ be a valid code vector transmitted over a noisy communication channel belonging to a (n, K) linear block code.

Let $[R] = [r_1, r_2 \dots r_n]$ be a received vector. Due to the noise in the channel, $r_1, r_2 \dots r_n$ may be different from $c_1, c_2, \dots c_n$.

The error vector or error pattern E is defined as the difference b/w R & C.

$$\therefore \quad E = R - C$$



Therefore, the error vector can be represented by

$$E = [e_1, e_2 \ldots\ldots e_n]$$

From above equation, it is clear that E is also a vector where $e_i = 1$ if $R \neq C$

& $e_i = 0$ if $R = C$

The 1's present in error vector E represent the error's caused by the noise in the channel.

In the equation $E = R - C$, the receiver knows only R & it doesn't know C & E. In order to find E & then C, the receiver does (the decoding operation by determining a (n-k) vector S defined as

$$S = RH^T = [S_1\, S_2\, S_3 \ldots\ldots S_{n-k}]$$

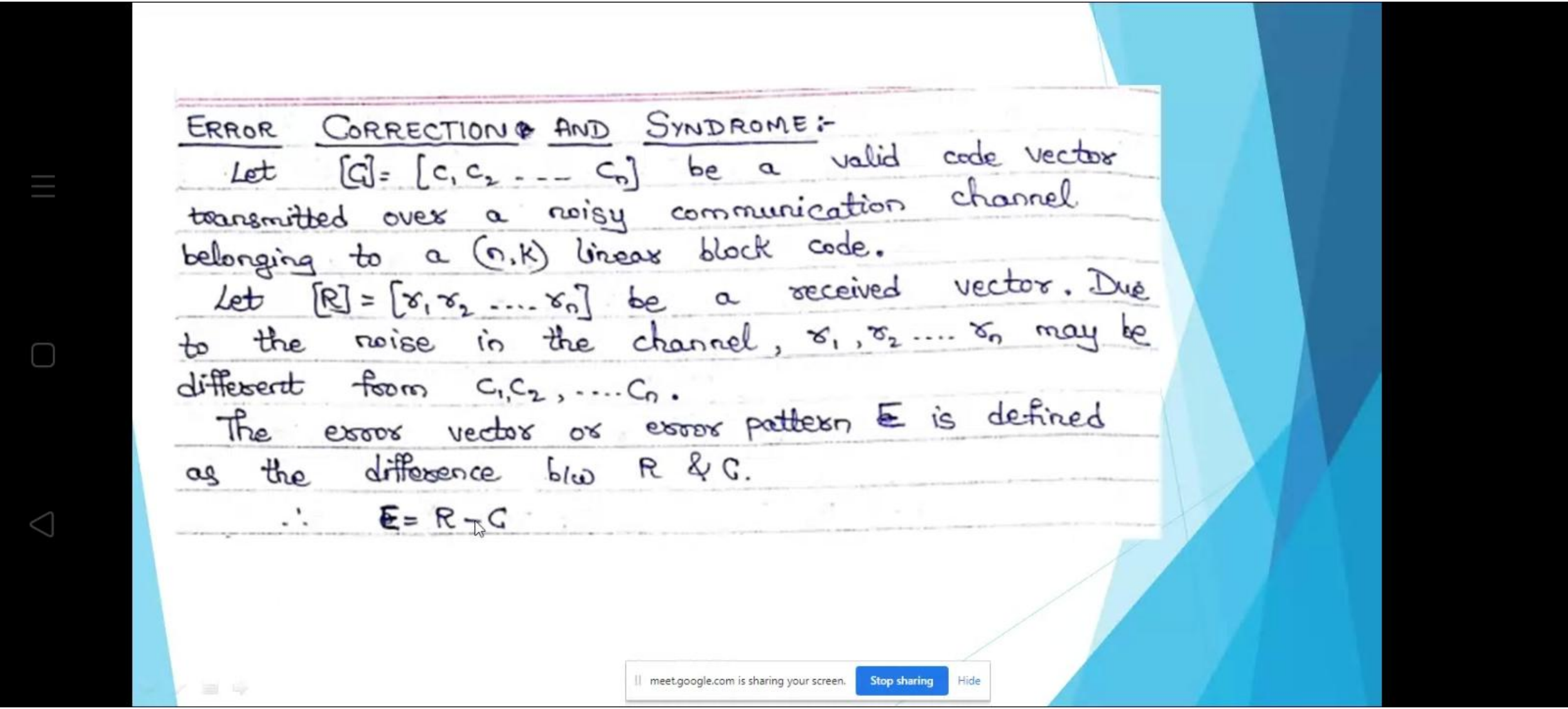
This (n-k) vector is called ERROR SYNDROME of R.

Consider $S = RH^T$

$$= [C+E][H^T]$$

$$= \overset{0}{\cancel{CH^T}} + EH^T = EH^T$$

$$\therefore \boxed{S = EH^T}$$



ERROR CORRECTION AND SYNDROME :-

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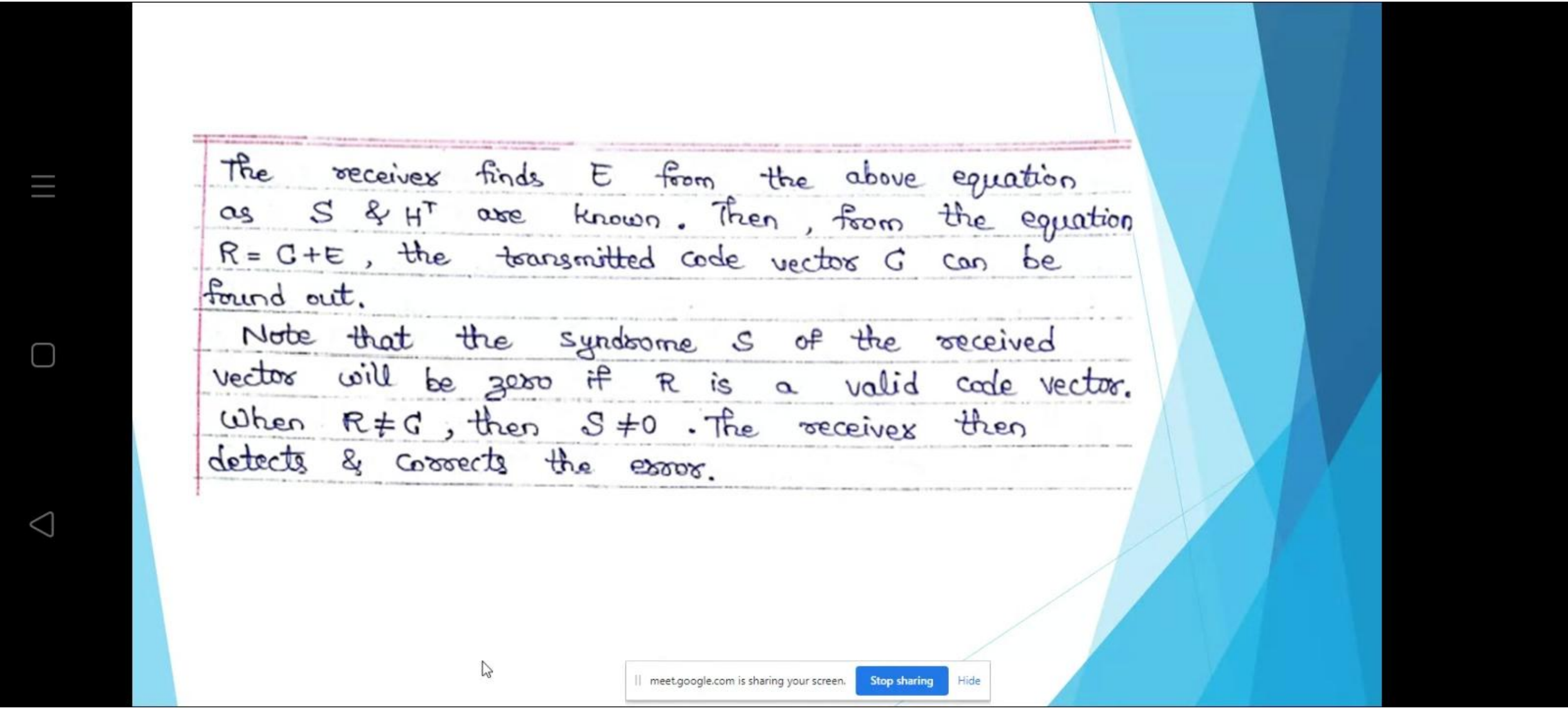
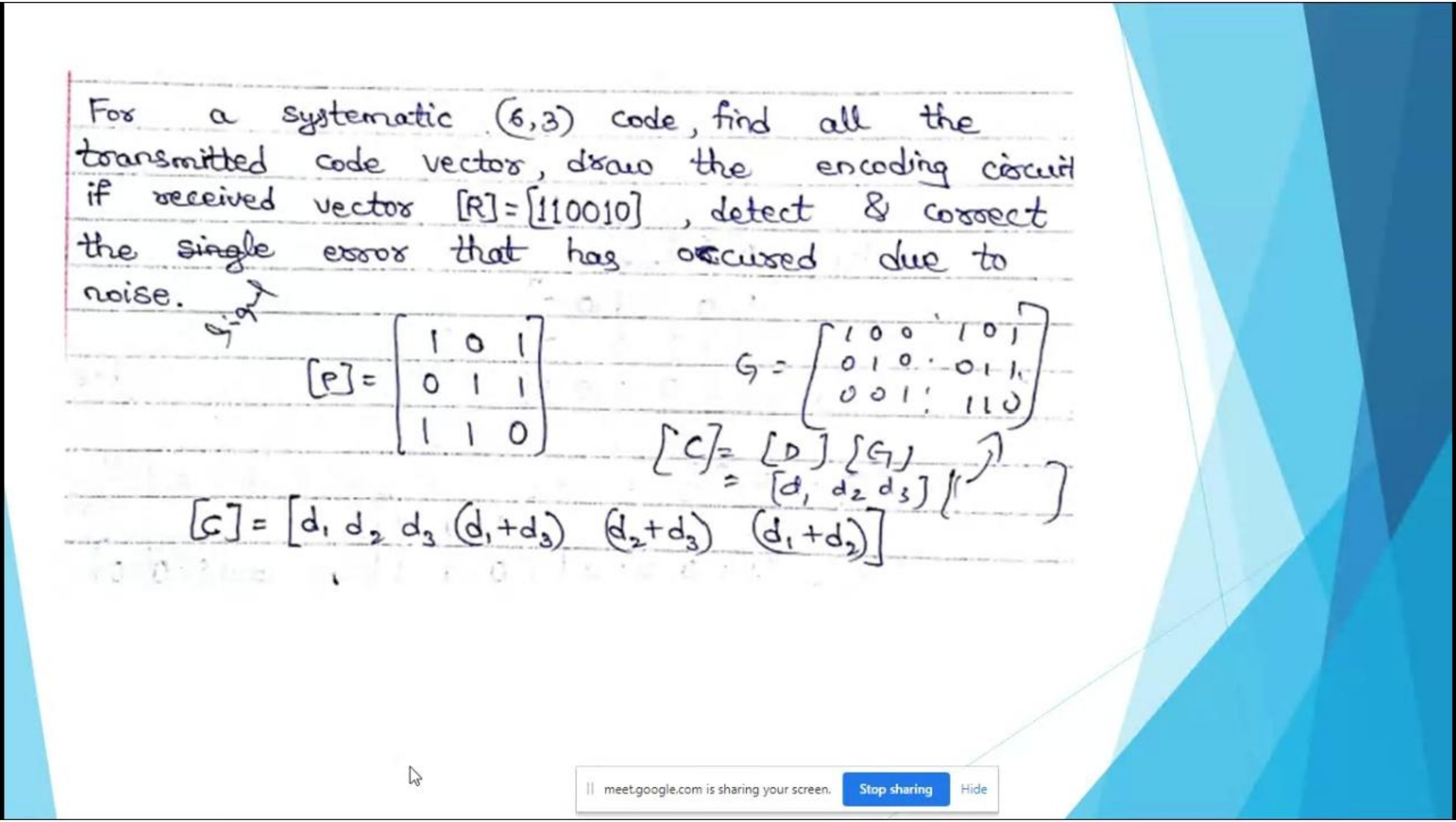
The error vector or error pattern **E** is defined as the difference b/w R & C.

$$\therefore \quad E = R - C$$



The receiver finds E from the above equation as S & H^T are known. Then, from the equation $R = C + E$, the transmitted code vector C can be found out.

Note that the syndrome S of the received vector will be zero if R is a valid code vector. When $R \neq C$, then $S \neq 0$. The receiver then detects & corrects the error.



For a systematic (6,3) code, find all the transmitted code vector, draw the encoding circuit if received vector [R] = [110010], detect & correct the single error that has occurred due to noise.

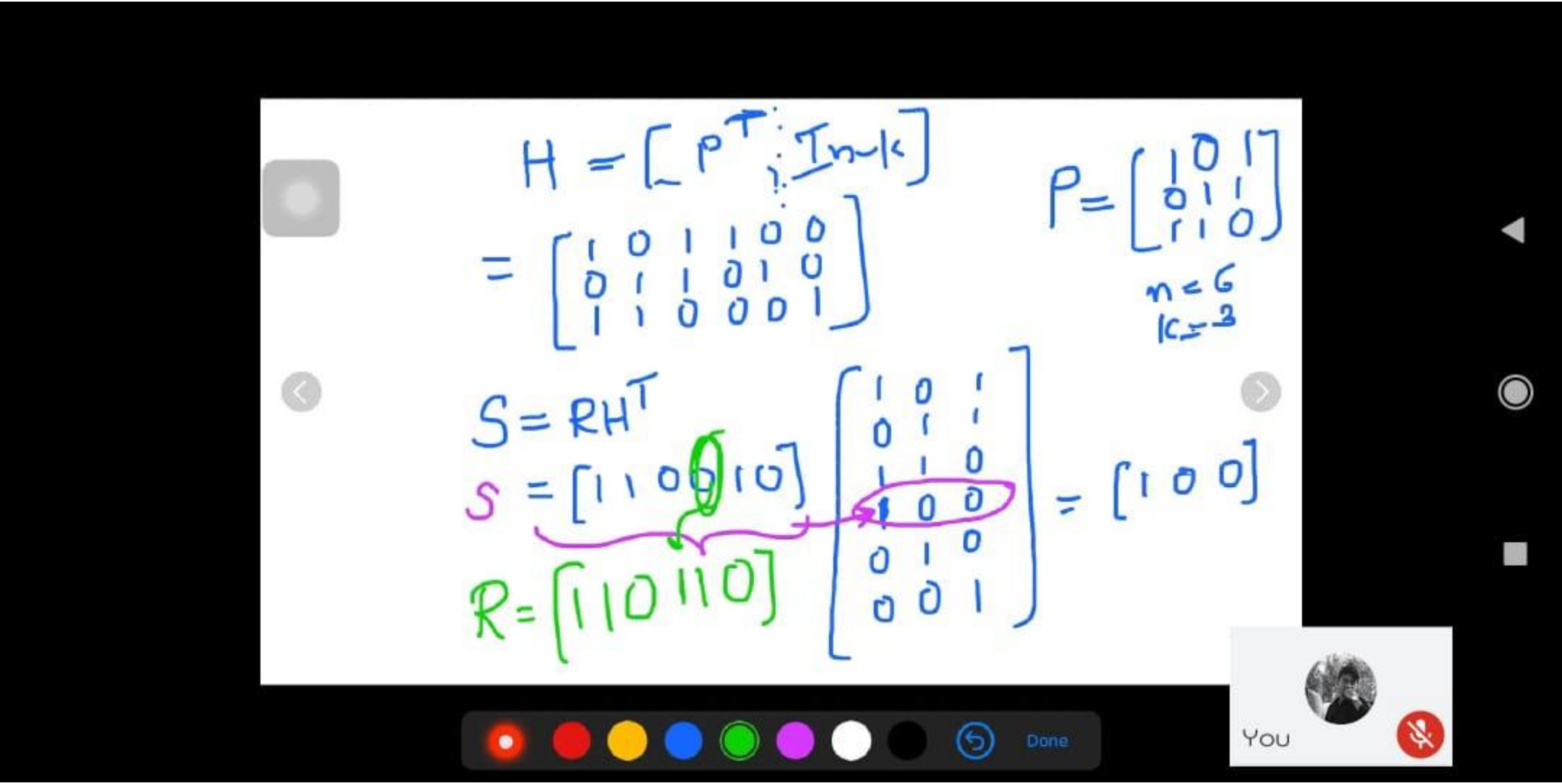
$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$$

$$[C] = [D][G]$$

$$= [d_1,\ d_2\ d_3]\,[\quad]$$

$$[C] = [d_1\ d_2\ d_3\ (d_1+d_3)\ (d_2+d_3)\ (d_1+d_2)]$$



$$H = [P^T \vdots I_{n-k}]$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$n = 6$

$k = 3$

$$S = RH^T$$

$$S = \underbrace{[1\ 1\ 0\ 0\ 1\ 0]} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 0\ 0]$$

$$R = [1\,1\,0\,1\,1\,0]$$

he received vec… correct the error.
ndrome, detect & correct the error.

$$[C] = [D][G]$$

$$[C] = \underset{\substack{1\times k\\ 1\times 3}}{[d_1 \;\; d_2 \;\; d_3]} \underset{\substack{k\times n\\ \cancel{6\times 3}\\ 3\times 6}}{\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]}$$

$$d_1 \quad d_2 \quad d_3 \quad d_1+d_2+d_3 \quad d_1+d_2 \quad d_1+d_3]$$

ctors	Code vectors					
0	0	0	0	0	0	0
1	0	0	1	1	0	1
0	0	1	0	1	1	0
1	0	1	1	0	1	1
0	1	0	0	1	1	1
1	1	0	1	0	1	0
0	1	1	0	0	0	1
1	1	1	1	1	0	0

$$S = RH^T$$

$$= \underset{1\times 6}{[1 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 1]} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5th row →

$$\left.\begin{array}{c:ccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$