1. Digital modulation techniques

Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave. In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it and the carrier is sinusoidal wave. Different Shift keying methods that are used in digital modulation techniques are

- **Amplitude shift keying [ASK]**
- **Frequency shift keying [FSK]**
- **Phase shift keying [PSK]**

Fig shows different modulations

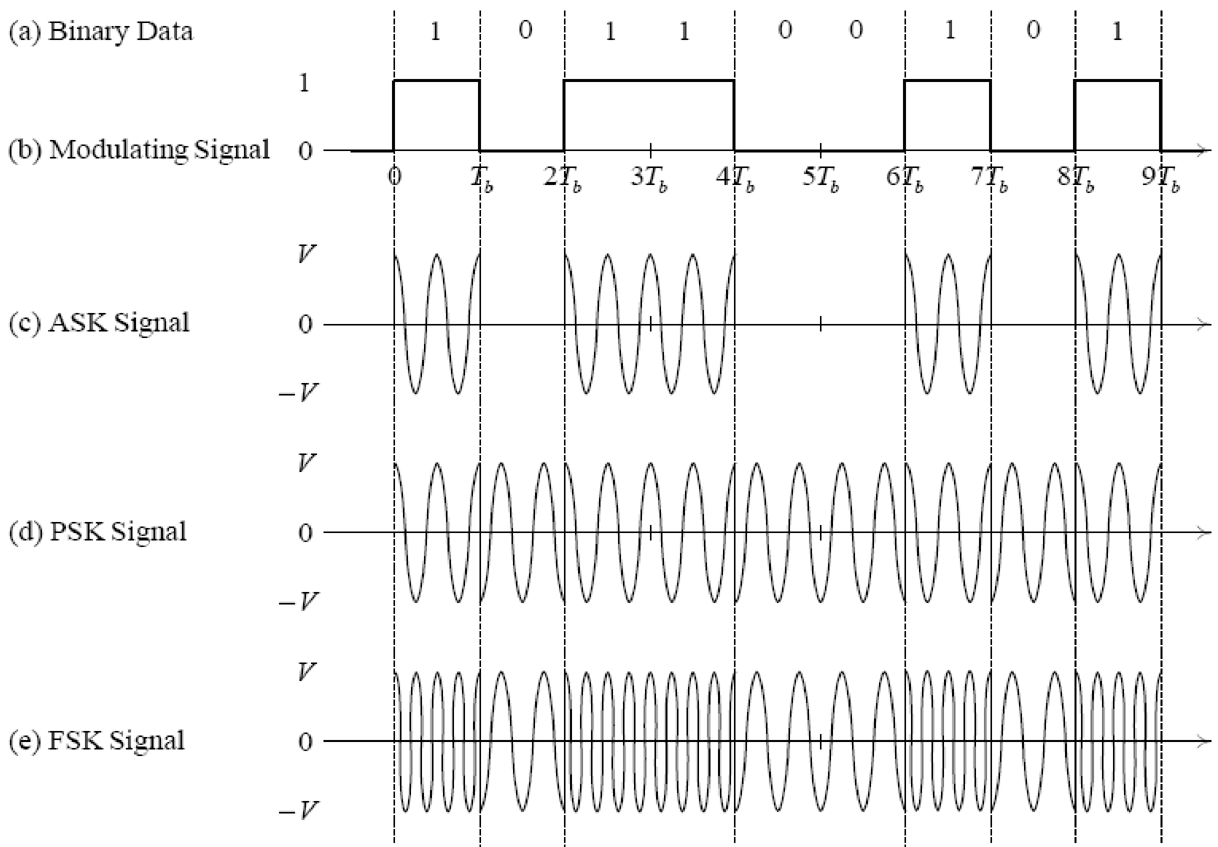


Figure 5.1 Waveforms of ASK,PSK and FSK

Primary resources of digital communication:

- Transmitted Power.
- Channel Bandwidth.

Design goals:

To achieve

- Maximum data rate.
- Minimum probability of symbol error.
- Minimum transmitted power.
- Minimum channel bandwidth.
- Maximum resistance to interfering signals.
- Minimum circuit complexity.

5.1 BINARY PSK

Binary data are represented by two signals with different phases in BPSK. Typically these two phases are 0 and π, the signals are

$s_1(t) = A\cos 2\pi f_c t,\ 0 \le t \le T$, for 1

$s_2(t) = -A\cos 2\pi f_c t,\ 0 \le t \le T$, for 0

These signals are called ***antipodal.*** The waveform of a BPSK signal generated by the modulator is shown in Figure 5.1. The waveform has a constant envelope. Its frequency is constant too. In general the phase is not continuous at bit boundaries. If the $f_c = mR_b = m/T$, where m is an integer and R_b is the data bit rate, and the bit timing is synchronous with the carrier, then the initial phase at a bit boundary is either 0 or π corresponding to data bit 1 or 0.

Signal energy (E_S) = Bit Energy (E_b), given by:

$$E_S = E_b = \frac{A^2T}{2}$$

Hence

$$A = \sqrt{\frac{2E_b}{T_b}}$$

Let's also rewrite the signal amplitudes as a function of their energy

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

Gram-Schmidt Orthogonalization: This allows representation of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where N<=M.

Basis functions of signals are both ortogornal and normalized to have unit energy.

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Let's consider the one dimensional basis function (N=1) where:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad 0 \leq t \leq T$$

Therefore, we can write the signals $s_1(t)$ and $s_2(t)$ in terms of $\phi 1(t)$:

$$s_1(t) = \sqrt{E_b}\phi_1(t) \qquad 0 \leq t \leq T_b$$

$$s_2(t) = -\sqrt{E_b}\phi_1(t) \qquad 0 \leq t \leq T_b$$

PSK signals can be graphically represented by a *signal constellation* in a single -dimensional coordinate system with basis function $\Phi_1(t)$ as its horizontal axis. Here $s_1(t)$ and $s_2(t)$ are represented by two points on the horizontal axis.

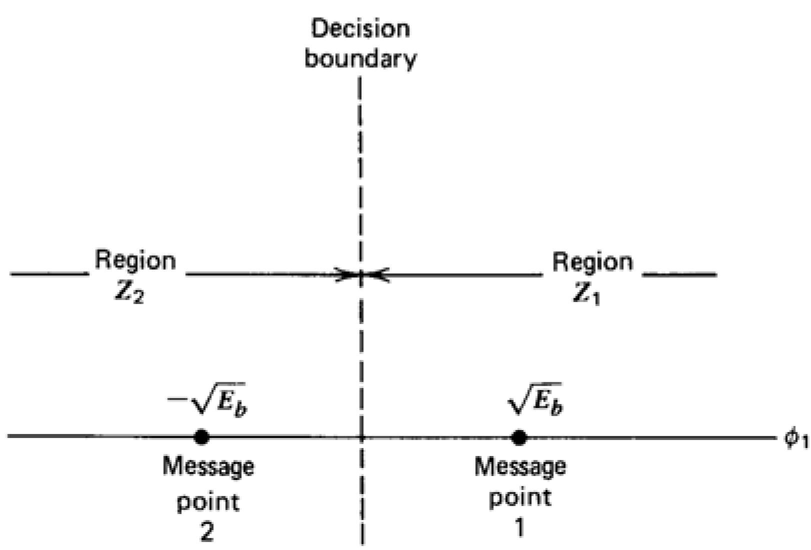


Figure 5.2 Signal space diagram for PSK

The modulator which generates the BPSK signal is quite simple. First a bipolar data stream a(t) is formed from the binary data stream

$$a(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

where $a_k \in \{+1,-1\}$, p(t) is the rectangular pulse with unit amplitude defined on [0, T]. Then a(t) is multiplied with a sinusoidal carrier Acos 2πfct. The result is the BPSK signal

$s(t) = Aa(t) \cos 2\pi f_c t$, $-\infty < t < \infty$

The coherent detector could be in the form of a correlator or matched filter. The correlator's reference signal is the basis function $\Phi_1(t)$. The basis function must be synchronous to the received signal in frequency and phase.

Channel Model

The transmitted waveform gets corrupted by noise n, typically referred to as **Additive White Gaussian Noise** (AWGN).

Additive : As the noise gets 'added' (and not multiplied) to the received signal

White : The spectrum of the noise if flat for all frequencies.

Gaussian : The values of the noise n follows the Gaussian probability distribution function,
$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ with $\mu = 0$ and $\sigma^2 = \frac{N_0}{2}$.

Computing the probability of error

The received signal,

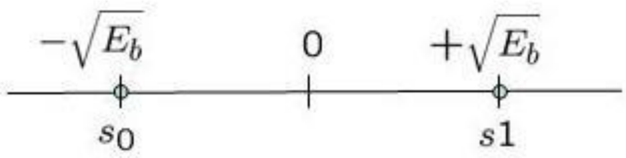
$y = s_1 + n$ when bit 1 is transmitted and

$y = s_0 + n$ when bit 0 is transmitted.

The conditional probability distribution function (PDF) or likely hood function of y for the two cases are:

$$p(y|s_0) = \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(y+\sqrt{E_b})^2}{N_0}}$$

$$p(y|s_1) = \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(y-\sqrt{E_b})^2}{N_0}}.$$



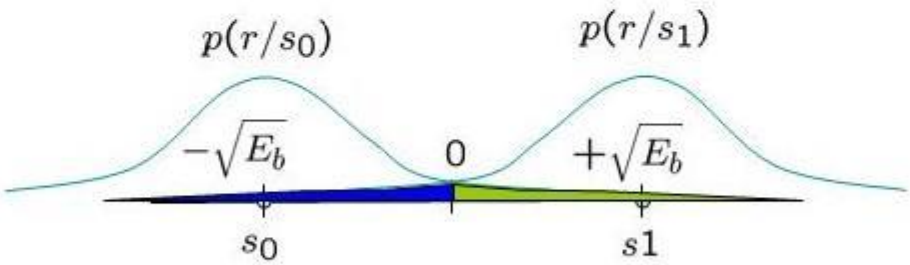


Figure5.3 : Conditional probability density function with BPSK modulation

Assuming that s_1 and s_0 are equally probable i.e. $p(s_1) = p(s_0) = 1/2$, the **threshold 0** forms the optimal decision boundary.

- if the received signal y is greater than 0, then the receiver assumes s_1 was transmitted.
- if the received signal y is less than or equal to 0, then the receiver assumes s_0 was transmitted.

i.e. $y > 0 \Rightarrow s_1$ and $y \leq 0 \Rightarrow s_0$.

Probability of error given s_0 is transmitted

With this threshold, the probability of error given s_0 is transmitted is (the area in green region):

$$P(e/s_0) = \int_0^{\infty} p(y/s_0)dy$$

$$= \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(y+\sqrt{E_b})^2}{N_0}} dy$$

Let

$$\frac{y+\sqrt{E_b}}{\sqrt{N_0}} = z,$$

Then $$P(e/s_0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz$$

Compare the above equation with the complementary error function, given by

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-x^2} dx$$.

We can write $P(e/s_0) = \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}})$

Probability of error given *s_1* is transmitted

Similarly the probability of error given s_1 is transmitted is (the area in blue region):

. $P(e/s_1) = \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}})$

Total probability of bit error

$$P_b = p(s_1)p(e|s_1) + p(s_0)p(e|s_0).$$

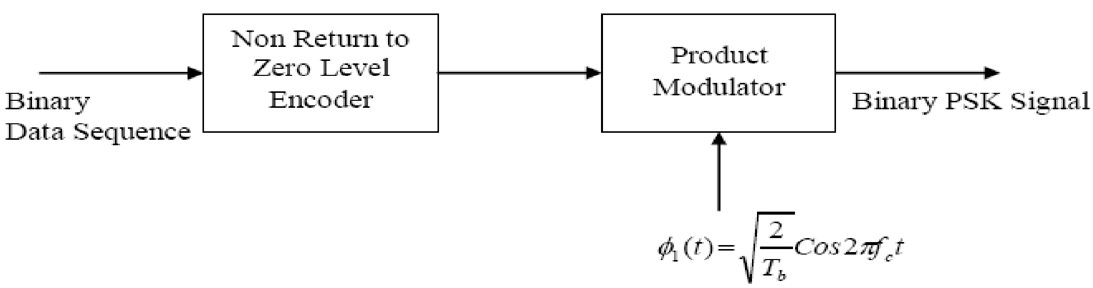
We assumed that s_1 and s_0 are equally probable i.e. $p(s_1) = p(s_0) = 1/2$, the **bit error probability** is,

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right).$$

PSK generation and Detection

In a Coherent binary PSK system the pair of signals $S_1(t)$ and $S_2(t)$ are used to represent binary symbol "1" and "0" respectively. To generate a binary PSK signal we have to represent the input binary sequence in polar form with symbol 1 and 0 represented by constant amplitude levels. Multiply message signal with basis function as shown in fig a.

To detect the original binary sequence of 1's and 0's we apply the noisy PSK signal x(t) to a Correlator, which is also supplied with a locally generated coherent reference signal. The correlator output x_1 is compared with a threshold of zero volt. If $x_1 > 0$, the receiver decides in favour of symbol 1. If $x_1 < 0$, the receiver decides in favour of symbol 0.



Fig(a) Block diagram of BPSK transmitter

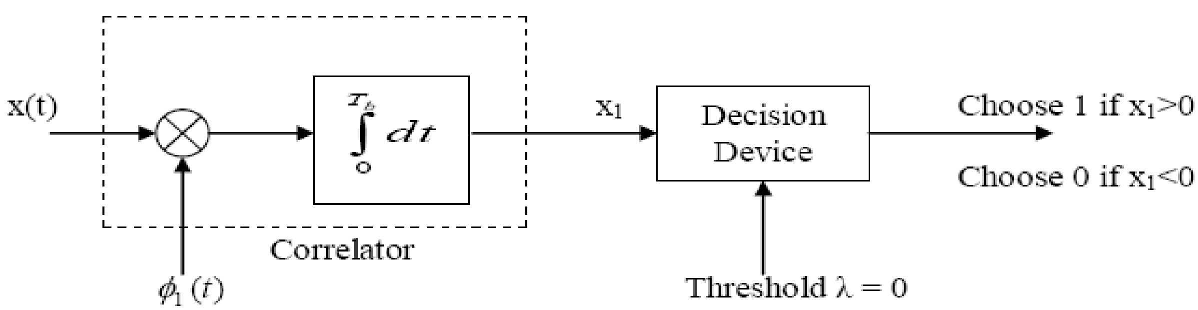


Figure 5.4 PSK Transmitter and receiver

5.2 Frequency Shift Keying

Binary data are represented by two signals with different frequencies in BFSK. Typically these two frequencies are f1 and f2, the signals are

$s_1(t) = A \cos 2\pi f_1 t,\ 0 \le t \le T$, for 1

$s_2(t) = A \cos 2\pi f_2 t,\ 0 \le t \le T$, for 0

In terms of bit energy

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$$
$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$$

We have two basis functions for this case

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \qquad 0 \le t \le T$$

We know $s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt$

Thus,
$$s_{11} = \sqrt{E_b}$$
$$s_{12} = 0$$
$$s_{21} = 0$$
$$s_{22} = \sqrt{E_b}$$

With this, We can write signal space diagram with signal points $s_1(t)$ and $s_2(t)$ as shown below.

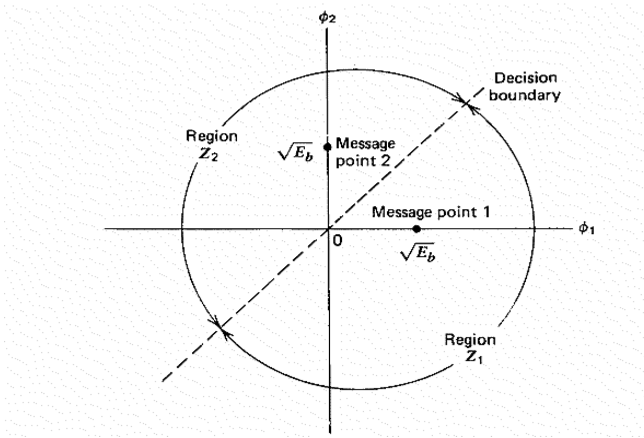


Figure 5.5 Signal space diagram for FSK

Computing the probability of error

The received signal,

$x_1(t)=s_1(t)+N$ when bit 1 is transmitted and

$x_2(t)=s_2(t)+N$ when bit 0 is transmitted.

Let us define a new random variable $l=x_1(t)-x_2(t)$, so that calculating probability of error become simpler as the problem get reduced to single dimension.

The mean value of random variable ***l*** knowing 1 is transmitted has mean value given by

$E[L/1]=E[X_1/1]-E[X_2/1]= +\sqrt{E_b}$

Similarly

$E[L/0]=E[X_1/0]-E[X_2/0]=-\sqrt{E_b}$

Variance of ***l*** is N_0

Then

$$P_e(0)=\int_0^\infty p_l(l/0)dl$$

$$=\int_0^\infty \frac{1}{\sqrt{2\pi N_0}} e^{\frac{-(l+\sqrt{E_b})^2}{2N_0}} dl$$

Let

$$\frac{l + \sqrt{E_b}}{\sqrt{2N_0}} = z,$$

Then

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} dz$$

Compare the above equation with the complementary error function, given by

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-x^2} dx$$

.

We can write $P_e(0) = \frac{1}{2} erfc(\sqrt{\frac{E_b}{2N_0}})$

Similarly we can show $P_e(1)$=0

Total probability of bit error

$$P_b = p(s_1)p(e|s_1) + p(s_0)p(e|s_0).$$

We assumed that s_1 and s_0 are equally probable i.e. $p(s_1) = p(s_0) = 1/2$, the **bit error probability** is,

$$P_e = \frac{1}{2} erfc(\sqrt{\frac{E_b}{2N_0}})$$

With this it is clear that probability of bit error for FSK is more compared to PSK. This supports the result that if signal points are widely separated, P_e will be less.

5.21Modulator and demodulator for FSK

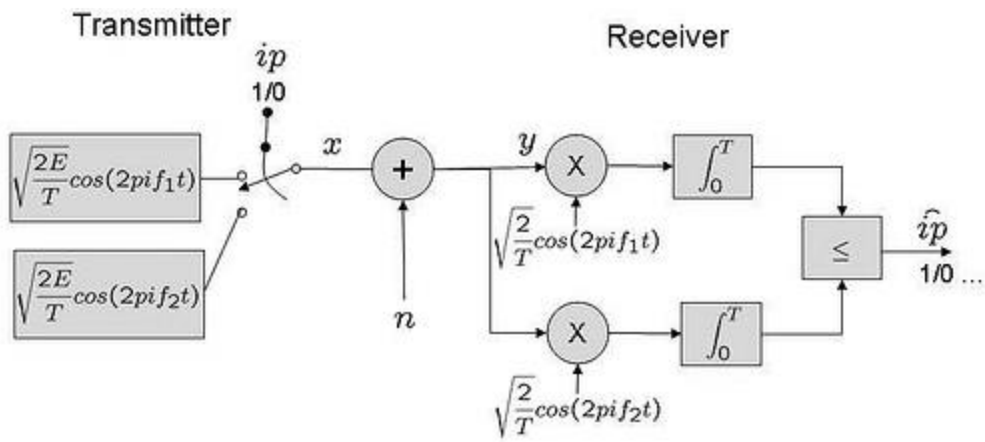


Figure 5.6 Generation and Coherent detection of FSK

In transmitter, depending on the input bit 1 or 0, the waveform is switched between s1(t) and s2(t). At the receiving end, FSK signal is multiplied by basis functions separately in two channels and give it to integrate and dump receiver. The difference of two channels is used to identify the transmitted bit.

5.22 Non coherent Detection of FSK

Non coherent or envelope detection can be performed for FSK signals. In this case the receiver takes the following form shown in fig 5.7

The bit error probability can be shown to be

$$p_\epsilon = \frac{1}{2}e^{-E/2\eta}$$

which under normal operating conditions corresponds to less than a 1dB penalty over coherent detection. In practice almost all FSK receivers are of this form.

In order for envelope detection to be successful, the peaks in the frequency domain at f1 and f2 must be widely separated with respect to the bandwidth of the baseband signal.

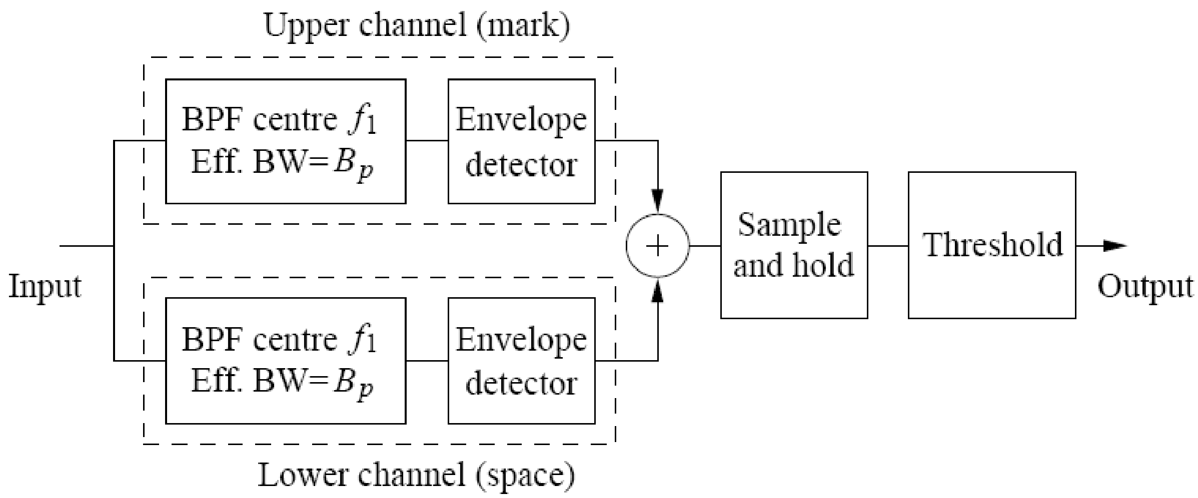


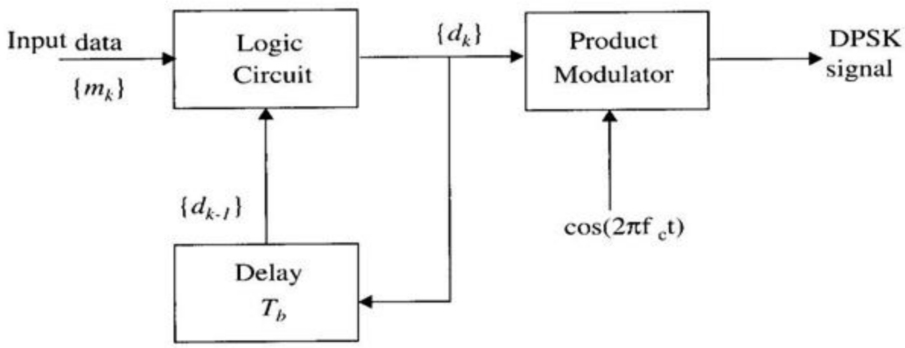
Figure 5.7 Noncoherent detection of FSK

5.3 DIFFERENTIAL PSK

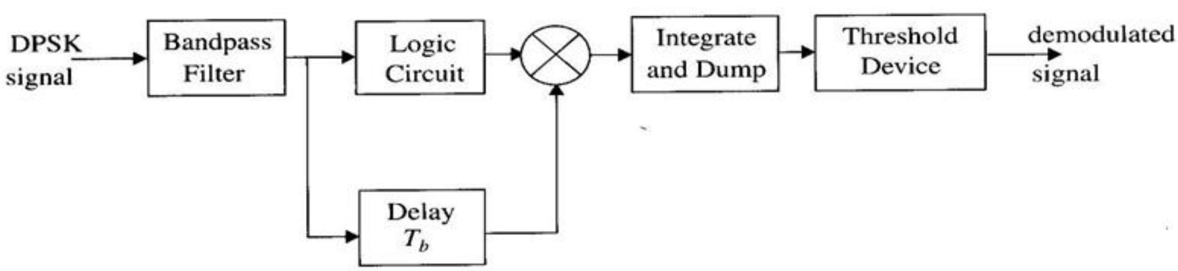
- **DPSK → Differential Phase Shift Keying**
  - Non-coherent Rx can be used
    - easy & cheap to build
    - no need for coherent reference signal from Tx
  - Bit information determined by transition between two phase states
    - incoming bit = 1 → signal phase stays the same as previous bit
    - incoming bit = 0 → phase switches state
  - Same BW properties as BPSK, uses same amount of spectrum
  - Non-coherent detection → all that is needed is to compare phases between successive bits, not in reference to a Tx phase.
  - Power efficiency is 3 dB worse than coherent BPSK (higher power in E_b / N_o is required for the same BER)

DPSK is noncoherent PSK in which detection is done without the help of carrier. While modulating, the binary data is represented with differential encoded format. This is done with the help of logic circuit which is a XNOR gate. The output of differential encoder is given to level shifter which gives polar formatted waveform. This is used to get PSK waveform In the demodulator, the received signal is compared with the 1 bit delayed signal. If the phase of both signals is same, output is 1 otherwise it is 0.

. **Logic circuit**



Block diagram of a DPSK transmitter.



Block diagram of DPSK receiver.

Figure 5.8 DPSK transmitter and receiver

Generation of D PSK signal									
b(k)		1	0	0	1	0	0	1	1
d(k-1)		1	1	0	1	1	0	1	1
d(k) = b(k) XNOR d(k-	1	1	0	1	1	0	1	1	1
transmitted phase	0	0	π	0	0	π	0	0	0

Detection of D PSK signal									
received phase (rad)	0	0	π	0	0	π	0	0	0
b(k)		1	0	0	1	0	0	1	1

5.4 Quadrature phase shift keying (QPSK)

In QPSK, the data bits to be modulated are grouped into symbols, each containing two bits, and each symbol can take on one of four possible values: 00, 01, 10, or 11.During each symbol interval, the modulator shifts the carrier to one of four possible phases corresponding to the four possible values of the input symbol.

$$S_{QPSK} = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right) \quad \text{for } i = 1,2,3,4$$

In a sense, QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a dibit. So, four dibits are possible. Each symbol carries same energy. Let, E: Energy per Symbol and T: Symbol Duration = 2.* Tb, where Tb: duration of 1 bit.

Expanding the above equation, we get

$$s_i(t) = \sqrt{\frac{2E}{T}}.\cos\left[(2i-1)\pi/4\right]\cos w_c t - \sqrt{\frac{2E}{T}}.\sin\left[(2i-1)\frac{\pi}{4}\right]\sin w_c t,\ 1 \le \text{i} \le 4$$

Thus we have two basis functions with which we can represent all 4 signals of QPSK

They are $\Phi_1(t) = \sqrt{\frac{E}{2}}\cos(\omega_c t)$ and $\Phi_2(t) = \sqrt{\frac{E}{2}}\sin(\omega_c t)$

Thus signal space diagram is 2 dimensional.

Coefficients S_{ij} can be calculated using these basis functions as $S_{ij} = \int_{-\infty}^{\infty} S_i(t).\Phi_j(t)dt$

The table below shows the coefficient values for different dibits. Based on the coefficient values, the signal points are identified in signal space diagram as shown below.

Input	Dibit		Phase of QPSK	Coordinates of signal points		
	(b_0)	(b_e)		s_{i1}	s_{i2}	i
$\overline{s_1}$	1	0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	1
$\overline{s_2}$	0	0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	2
$\overline{s_3}$	0	1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	3
$\overline{s_4}$	1	1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	4

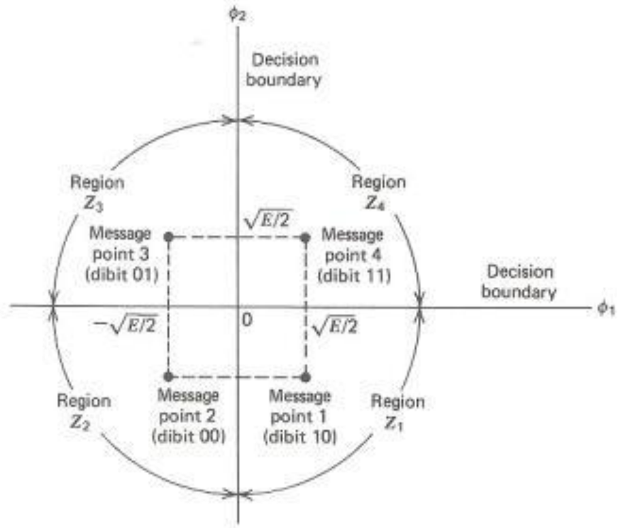


Figure 5.9 (a) truth table (b) constellation diagram

The corresponding signal at the input of a QPSK receiver is $r(t) = s_i(t) + w(t)$, $0 \leq t \leq T$, where 'w(t)' is the noise sample function and 'T' is the duration of one symbol.

As we have two basis functions, received signal have two components given by

$$r_1 = \int_0^T r(t)\varphi_1(t)\,dt = \sqrt{E}.\cos\left[(2i-1)\frac{\pi}{4}\right] + w_1$$

$$r_2 = \int_0^T r(t)\varphi_2(t)\,dt = -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] + w_2$$

Note that if $r_1>0$, it implies that the received vector is either in decision zone Z_1 or in decision zone Z_4. Similarly, if $r_2 > 0$, it implies that the received vector is either in decision zone Z_3 or in decision zone Z_4.

The noise elements w_1 and w_2 are independent, identically distributed (iid) Gaussian random variables with zero mean and variance $N_0/2$. Further, r_1 and r_2 are also sample values of independent Gaussian random variables with means $\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]$ and $-\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]$ respectively and with same variance $N_0/2$

Let us now assume that $S_4(t)$ is transmitted and that we have received r. For a change, we will first compute the probability of correct decision when a symbol is transmitted.

Let Pc = Probability of correct decision when $S_4(t)$ is transmitted. Then mean of r_1 and r_2 are $\sqrt{\frac{E}{2}}$

Taking Gaussian distribution for r_1 and r_2, Joint probability of the event that, $r_1 > 0$ and $r_2 > 0$ is

$$Pc = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} . \exp\left[-\frac{\left(r_1 - \sqrt{\frac{E}{2}} \right)^2}{N_0} \right] dr_1 . \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} . \exp\left[-\frac{\left(r_2 - \sqrt{\frac{E}{2}} \right)^2}{N_0} \right] dr_2$$

As r_1 and r_2 are statistically independent, putting

$$\frac{r_j - \sqrt{\frac{E}{2}}}{\sqrt{N_0}} = Z, \text{ j} = 1, 2,$$

$$Pc = \left[\frac{1}{\sqrt{\pi}} . \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \exp\left(-Z^2 \right) dz \right]^2$$

But

$$\frac{1}{\sqrt{\pi}} \int_{-a}^{\infty} e^{-x^2} dx = 1 - \frac{1}{2} erfc(a)$$

$$\therefore Pc = \left[1 - \frac{1}{2} erfc\left(\sqrt{\frac{E}{2N_0}} \right) \right]^2$$

$$= 1 - erfc\left(-\sqrt{\frac{E}{2N_0}} \right) + \frac{1}{4} . erfc^2\left(\sqrt{\frac{E}{2N_0}} \right)$$

Probability of error= 1- probability of correct reception

$$Pe = 1 - Pc = erfc\left(\sqrt{\frac{E}{2N_0}} \right) - \frac{1}{4} erfc^2\left(\sqrt{\frac{E}{2N_0}} \right)$$

The value of erfc(x) decreases fast with increase in its argument. This implies that, for moderate or large value of E_b/N_0, the second term on the above equation may be neglected to obtain a shorter expression for the average probability of symbol error, P_e:

$$P_e \cong erfc\left(\sqrt{\frac{E}{2N_o}}\right) = erfc\left(\sqrt{\frac{2.E_b}{2.N_o}}\right) = erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

The above equation is for receiving signal $S_4(t)$. Other 3 signals will have same probability of error because of symmetry of signal space diagram. Assuming equal probability of occurrence of all 4 symbols, total probability of error for QPSK is

$$P(e) = erfc\sqrt{\frac{E_b}{N_0}}$$

5.41 QPSK Transmitter and receiver

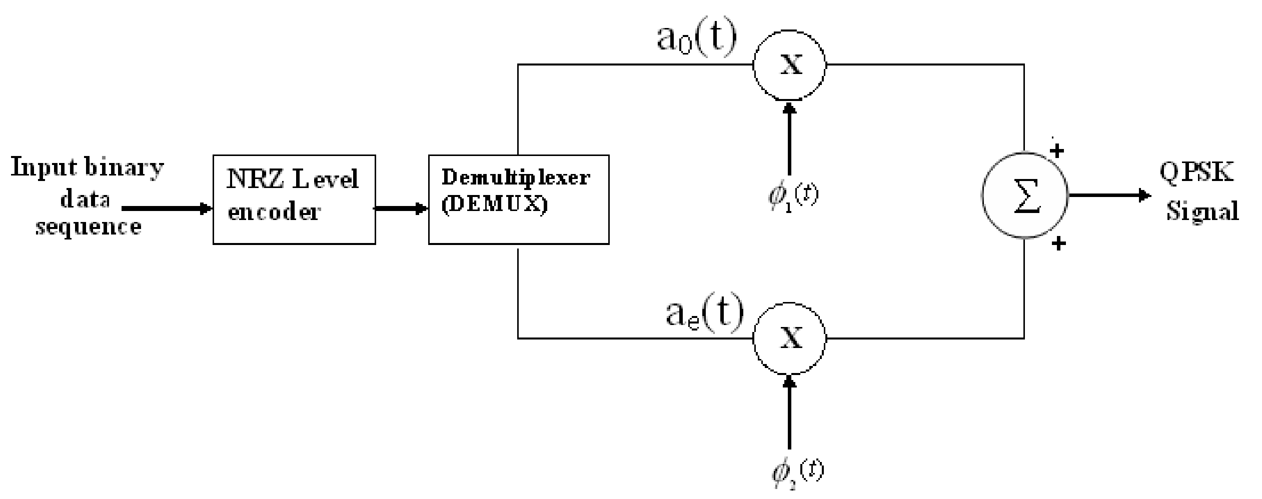


Figure 5.10 QPSK transmitter

A block diagram of a QPSK modulator is shown in Figure5.10 Two bits (a dibit) are clocked into the bit splitter or de multiplexer. The I bit (odd bit) modulates a carrier that is in phase with the reference oscillator (hence the name "I" for "in phase" channel), and the Q bit(even bit) modulates a carrier that is 90 degree out of phase with the reference oscillator.(basis functions $\Phi_1(t)$ and $\Phi_2(t)$

QPSK Receiver:- The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\Phi_1(t)$ & $\Phi_2(t)$ as shown in fig(b).The correlator outputs x_1 and x_2 produced in response to the received signal x(t) are each compared with a threshold value of zero.

The in-phase channel output : If $x_1 > 0$ a decision is made in favour of symbol 1 $x_1 < 0$ a decision is made in favour of symbol 0

Similarly quadrature channel output: If $x_2 > 0$ a decision is made in favour of symbol 1 and $x_2 < 0$ a decision is made in favour of symbol 0 Finally these two binary sequences at the in phase and quadrature channel outputs are combined in a multiplexer (Parallel to Serial) to reproduce the original binary sequence.

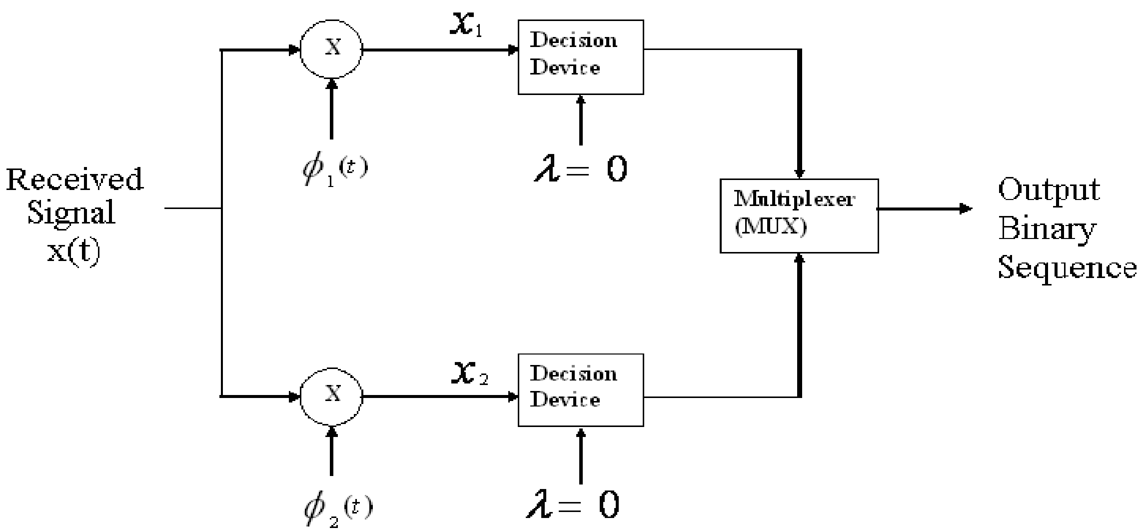
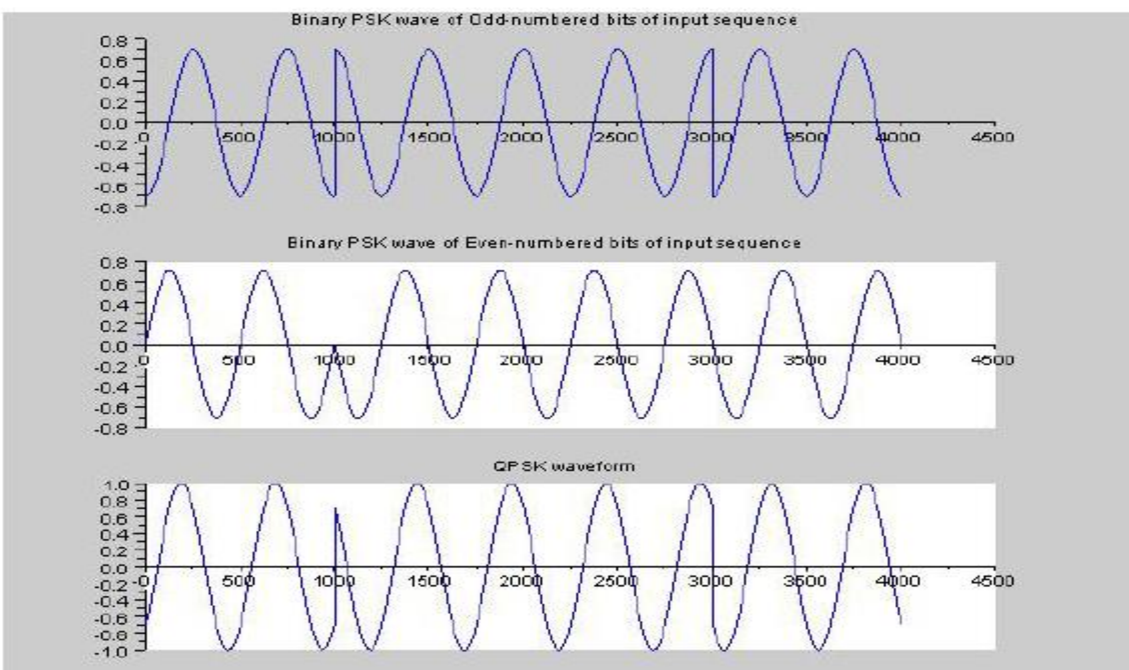


Figure 5.10 QPSK receiver

QPSK for data input 01101000



5.5 Power spectra of digital modulation schemes:

5.51 Power Spectrum for BPSK Modulated Signal

Continuing with our simplifying assumption of zero initial phase of the carrier and with no pulse shaping filtering, we can express a BPSK modulated signal as:

$$s(t) = \sqrt{\frac{E_b.2}{T_b}}.d(t)\cos w_c t \text{ , where } \quad d(t) = \pm 1$$

The baseband equivalent of s(t) is,

$$\tilde{u}(t) = u_I(t) = \sqrt{\frac{2E_b}{T_b}}.d(t) = \pm g(t),$$

$$\text{where } g(t) = \sqrt{\frac{2E_b}{T_b}} \text{ and } u_Q(t) = 0.$$

Now, $u_I(t)$ is a random sequence of

$$+\sqrt{\frac{2E_b}{T_b}} \text{ and } -\sqrt{\frac{2E_b}{T_b}}$$

which are equi-probable. So, the power spectrum of the base band signal is:

$$U_B(f) = \frac{2E_b.\sin^2(\pi T_b f)}{(\pi T_b f)^2} \qquad = 2.E_b.\sin c^2(T_b f)$$

Now, the power spectrum S(f) of the modulated signal can be expressed in terms of UB(f) as:

$$S(f) = \frac{1}{4}\left[U_B(f - f_c) + U_B(f + f_c)\right]$$

Fig shows the normalized base band power spectrum of BPSK modulated signal. The spectrum remains the same for arbitrary non-zero initial phase of carrier oscillator

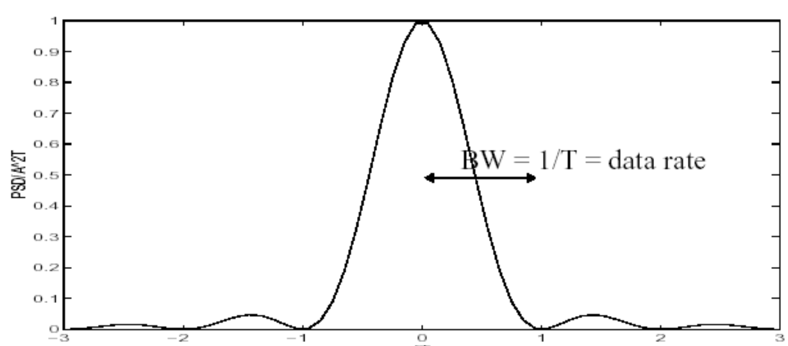


Fig5.51 Normalized base band power spectrum of BPSK modulated signal

5.52 Power Spectrum of BFSK

Power spectrum of an FSK modulated signal depends on the choice of f1 and f2, i.e. on 'm' and 'n'. When (n-m) is large, we may visualize BFSK as the sum of two ASK signals with carriers f1 and f2. However, such choice of (n-m) does not a result in bandwidth efficiency. In the following, we consider

n = (m+1), i.e. $f_1 = m.R_b = m/T_b$, $f_2 = f_1 + \frac{1}{T_b} =$ $n.R_b = (m+1).R_b = (m+1)/T_b$ and $f_c = \frac{f_2 + f_1}{2} = f_1 + \frac{1}{2T_b} = \varpi_c/2\pi$ $= f_{free}$.

BFSK modulated signal can be expressed as:

$$s(t) = \sqrt{\frac{2E_b}{T_b}}.\cos\left[w_c t \pm \frac{\pi t}{T_b}\right] = \underbrace{\sqrt{\frac{2E_b}{T_b}}.\cos\left(\frac{\pi t}{T_b}\right)}_{u_I(t)}.\cos w_c t \mp \sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right)\sin w_c t$$

The '±' sign in the above expression contains information about the information sequence, d(t). It is interesting to note that $u_I(t)$, the real part of the lowpass complex equivalent of the modulated signal s(t) is independent of the information sequence d(t) This portion of s(t) gives rise to a set of two delta functions, each of strength $E_b/2T_b$ and located at

$$f = +\frac{1}{2T_b} = \frac{f_b}{2} \text{ and } f = -\frac{1}{2T_b} = -\frac{f_b}{2}, \text{ where } f_b = R_b.$$

The imaginary part of the low pass complex equivalent of the modulated signal s(t) can be expressed in terms of a shaping function $g_Q(t)$ as,

$$u_Q(t) = \mp\sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right) = \mp g_Q(t),$$

$$\text{where } g_Q(t) = \sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right), \quad 0 \le t \le T_b$$

Now energy spectral density of the shaping function $g_Q(t)$ is

$$\Psi g_Q(f) = \frac{8\, E_b T_b \cos^2(\pi T_b f)}{\pi^2 \left(4T_b^2 f^2 - 1\right)^2}$$

From the above expression, we define the psd of $u_Q(t)$ as:

$$\frac{esd\ of\ g(t)}{T_b} = \frac{8\, E_b \cos^2(\pi T_b f)}{\pi^2 \left(4T_b^2 f^2 - 1\right)^2}$$

As $u_I(t)$ and $u_Q(t)$ are statistically independent of each other, we can now construct the baseband spectrum $U_B(f)$ of the BFSK modulated signal s(t) as:

$$U_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b \cos^2(\pi T_b f)}{\pi^2\left(4T_b^2 f^2 - 1\right)^2}$$

Fig. below shows a sketch (approximate) of the power spectrum of binary FSK signal.

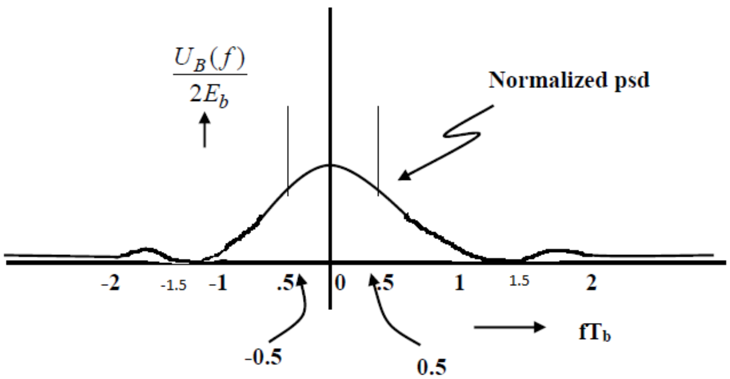
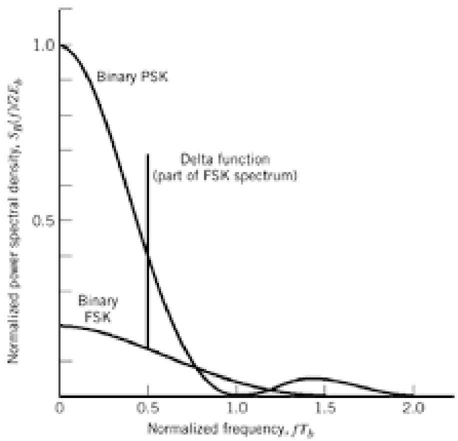


Fig 5.52 Sketch of the power spectrum of binary FSK signal vs. 'fT_b' when $(f_2-f_1) = 1/T_b$.

Comparison of PSK and FSK spectrum



5.53 Power spectrum of QPSK

In QPSK, each symbol is made out of 2 bits and depending on dibit sent during the signaling interval $-T_b<t<T_b$, the in phase component equals $+g(t)$ or $-g(t)$ and similarly for quadrature component. Here again g(t) form the symbol shaping function defined by

$$g(t) = \begin{cases} \sqrt{\frac{E}{T}} & 0 \le t \le T \\ 0 & otherwise \end{cases}$$

Hence inphase and quadrature components have common PSD, namely ***$E\,sinc^2(Tf)$***

The baseband PSD of QPSK signal equals the sum of the individual PSD of inphase and quadrature components, and so we can write

$S_B(f)=2E\,sinc^2(Tf)$

$=4\,E_b\,sinc^2(Tf)$

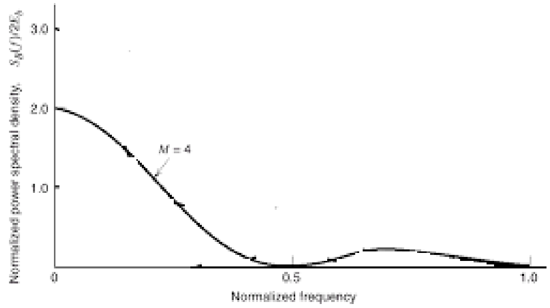


Fig5.31 PSD of QPSK

From the fig above, it is clear that QPSK uses half the BW as that of BPSK and is spectrally efficient.

Problem 1: Suppose, 1 million information bits are modulated by BPSK scheme & the available E_b/N_0 is 5.0 dB in the receiver. Determine approximately how many bits will be erroneous at the output of the demodulator. Find the same if QPSK modulator is used instead of BPSK.

Probability of bit error expression for BPSK is

$$P(e)=\frac{1}{2}erfc\sqrt{\frac{E_b}{N_0}}$$

Here Eb/N0=5 dB, Absolute value Eb/N0= $10^{0.5}$=3.16227766

$$P(e)=\frac{1}{2}erfc\sqrt{3.1622}$$

i.e

$$P(e)=\frac{1}{2}erfc(1.778)$$

From complementary error function table, we find erfc(1.788)=0.0118

Therefore P(e)=0.0059 i.e out of 10000 bits transmitted, 59 bits will be erroneous.

For QPSK,

$$P(e)=erfc\sqrt{\frac{E_b}{N_0}}$$

P(e)= 0.0118 i.e out of 10000 bits transmitted, 118 bits will be erroneous.

Problem 2: A binary FSK system transmits binary data at a rate of 2 MBPS. Assuming channel AWGN with zero mean and power spectral density of N0/2 = 1×10^{-20} W/Hz. The amplitude of the received signal in the absence of noise is 1 microvolt. Determine the average probability of error for coherent detection of FSK.

$$A = \sqrt{\frac{2E_b}{T_b}} \qquad \Rightarrow E_b = \frac{1}{2}A^2 \times T_b$$

Signal energy per bit

$$E_b = \frac{1}{2}A^2 \times T_b = \frac{1}{2}(1 \times 10^{-6})^2 \times 0.5 \times 10^{-6} = 0.25 \times 10^{-18} \; Joulels$$

Given $N_0/2 = 1 \times 10^{-20}$ W/Hz $\qquad \Rightarrow N_0 = 2 \times 10^{-20}$ W/Hz

The average probability of error for coherent PSK is

$$P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = 0.5 \times erfc\left(\sqrt{\frac{0.25 \times 10^{-18}}{4 \times 10^{-20}}}\right) = 0.5 \times erfc(\sqrt{6.25}) = 0.5 \times erfc(2.5)$$

From error function table

$$erfc(2.5) = 0.000407$$

$$P_e = 0.5 \times 0.000407 = 0.0002035$$

Problem 3: A binary data are transmitted at a rate of 106 bits per second over the microwave link. Assuming channel AWGN with zero mean and power spectral density of 1 × 10^{-10} W/Hz. Determine the average carrier power required to maintain an average probability of error probability of error Pe ≤ 10^{-4} for coherent binary FSK. Determine the minimum channel bandwidth required.

The average probability of error for binary FSK is

$$P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = 10^{-4} \qquad \therefore \quad erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = 2 \times 10^{-4} = 0.0002$$

From complementary error function table erfc(2.63) = 0.0002

$$\sqrt{\frac{E_b}{2N_0}} = 2 \times 10^{-4} \simeq 2.63 \qquad \therefore \quad \frac{E_b}{2N_0} = 6.9169$$

Given $N_0/2 = 1 \times 10^{-10}$ W/Hz $\qquad \Rightarrow N_0 = 2 \times 10^{-10}$ W/Hz

$$E_b = 6.9169 \times 2 \times 2 \times 10^{-10} = 27.6676$$

The average carrier power required is

$$\frac{E_b}{T_b} = \frac{27.667610^{-10}}{10^{-6}} = 27.667610^{-4} Watts$$

The minimum channel bandwidth required approximately is

$$\frac{1}{T_b} = \frac{1}{10^{-6}} = 1MHz$$

		Complementary Error Function Table											
x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644
0.2	0.777297	0.7	0.322199	1.2	0.089686	1.7	0.016210	2.2	0.001863	2.7	0.000134	3.2	0.00000603
0.21	0.766478	0.71	0.315335	1.21	0.087045	1.71	0.015593	2.21	0.001776	2.71	0.000127	3.21	0.00000564
0.22	0.755704	0.72	0.308567	1.22	0.084466	1.72	0.014997	2.22	0.001692	2.72	0.000120	3.22	0.00000527
0.23	0.744977	0.73	0.301896	1.23	0.081950	1.73	0.014422	2.23	0.001612	2.73	0.000113	3.23	0.00000493
0.24	0.734300	0.74	0.295322	1.24	0.079495	1.74	0.013865	2.24	0.001536	2.74	0.000107	3.24	0.00000460
0.25	0.723674	0.75	0.288845	1.25	0.077100	1.75	0.013328	2.25	0.001463	2.75	0.000101	3.25	0.00000430
0.26	0.713100	0.76	0.282463	1.26	0.074764	1.76	0.012810	2.26	0.001393	2.76	0.000095	3.26	0.00000402
0.27	0.702582	0.77	0.276179	1.27	0.072486	1.77	0.012309	2.27	0.001326	2.77	0.000090	3.27	0.00000376
0.28	0.692120	0.78	0.269990	1.28	0.070266	1.78	0.011826	2.28	0.001262	2.78	0.000084	3.28	0.00000351
0.29	0.681717	0.79	0.263897	1.29	0.068101	1.79	0.011359	2.29	0.001201	2.79	0.000080	3.29	0.00000328
0.3	0.671373	0.8	0.257899	1.3	0.065992	1.8	0.010909	2.3	0.001143	2.8	0.000075	3.3	0.00000306
0.31	0.661092	0.81	0.251997	1.31	0.063937	1.81	0.010475	2.31	0.001088	2.81	0.000071	3.31	0.00000285
0.32	0.650874	0.82	0.246189	1.32	0.061935	1.82	0.010057	2.32	0.001034	2.82	0.000067	3.32	0.00000266
0.33	0.640721	0.83	0.240476	1.33	0.059985	1.83	0.009653	2.33	0.000984	2.83	0.000063	3.33	0.00000249
0.34	0.630635	0.84	0.234857	1.34	0.058086	1.84	0.009264	2.34	0.000935	2.84	0.000059	3.34	0.00000232
0.35	0.620618	0.85	0.229332	1.35	0.056238	1.85	0.008889	2.35	0.000889	2.85	0.000056	3.35	0.00000216
0.36	0.610670	0.86	0.223900	1.36	0.054439	1.86	0.008528	2.36	0.000845	2.86	0.000052	3.36	0.00000202
0.37	0.600794	0.87	0.218560	1.37	0.052688	1.87	0.008179	2.37	0.000803	2.87	0.000049	3.37	0.00000188
0.38	0.590991	0.88	0.213313	1.38	0.050984	1.88	0.007844	2.38	0.000763	2.88	0.000046	3.38	0.00000175
0.39	0.581261	0.89	0.208157	1.39	0.049327	1.89	0.007521	2.39	0.000725	2.89	0.000044	3.39	0.00000163
0.4	0.571608	0.9	0.203092	1.4	0.047715	1.9	0.007210	2.4	0.000689	2.9	0.000041	3.4	0.00000152
0.41	0.562031	0.91	0.198117	1.41	0.046148	1.91	0.006910	2.41	0.000654	2.91	0.000039	3.41	0.00000142
0.42	0.552532	0.92	0.193232	1.42	0.044624	1.92	0.006622	2.42	0.000621	2.92	0.000036	3.42	0.00000132
0.43	0.543113	0.93	0.188437	1.43	0.043143	1.93	0.006344	2.43	0.000589	2.93	0.000034	3.43	0.00000123
0.44	0.533775	0.94	0.183729	1.44	0.041703	1.94	0.006077	2.44	0.000559	2.94	0.000032	3.44	0.00000115
0.45	0.524518	0.95	0.179109	1.45	0.040305	1.95	0.005821	2.45	0.000531	2.95	0.000030	3.45	0.00000107
0.46	0.515345	0.96	0.174576	1.46	0.038946	1.96	0.005574	2.46	0.000503	2.96	0.000028	3.46	0.00000099
0.47	0.506255	0.97	0.170130	1.47	0.037627	1.97	0.005336	2.47	0.000477	2.97	0.000027	3.47	0.00000092
0.48	0.497250	0.98	0.165769	1.48	0.036346	1.98	0.005108	2.48	0.000453	2.98	0.000025	3.48	0.00000086
0.49	0.488332	0.99	0.161492	1.49	0.035102	1.99	0.004889	2.49	0.000429	2.99	0.000024	3.49	0.00000080

One mark questions

1. ______________ is defined as a process by which some characteristic of carrier is varied in accordance with a modulating wave.
2. In digital modulation techniques, ____ and ______ signals have constant envelop.
3. In M-ary modulation techniques, the symbol length is___________
4. Two types of detection techniques are________ and ________
5. Non coherent detection, complexity of circuit is_________ compared to coherent techniques.
6. Non coherent detection, probability of error is_________ compared to coherent techniques.
7. Among BPSK and QPSK, if you need to use minimum channel bandwidth, which modulation technique you prefer?__________
8. Among BPSK and BFSK, if you need have maximum resistance to interfering signals, which modulation technique you prefer?__________
9. Two signals of BPSK are called as antipodal signals because they have relative phase shift of _______
10. If Euclidian distance between the two signals is large, Probability of error is_______
11. Euclidian distance between the two signals in BFSK is________
12. QPSK has _________basis functions
13. QPSK wave requires __________the transmission BW of the corresponding binary PSK wave
14. Non coherent BFSK detector uses _______ instead of correlators.
15. ____________ is non coherent version of BPSK.
16. Among ASK, PSK and FSK, which modulation technique gives minimum probability of error?______

Question bank

1. What are the design goals of selecting digital modulation techniques?
2. Compare 3 main digital modulation techniques considering any 5 parameters.
3. Identify basis functions for BASK, draw signal space diagram and mark signal points.
4. Derive an expression for the probability of bit error for coherent binary ASK system.
5. Draw the block diagram for BASK transmitter and receiver and explain the working.
6. Draw the block diagram for BPSK transmitter and receiver. From the basic principles prove that BER for BPSK is $P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$
7. Explain in detail along with block diagram a coherent FSK transmitter and receiver.
8. Derive an expression for the average probability of symbol error for coherent binary FSK system.

9. Identify basis functions for QPSK, draw signal space diagram and mark signal points
10. For the binary sequence 0110100, draw waveform and explain the signal space constellation diagram for coherent QPSK system.
11. With a neat block diagram, explain DPSK transmitter and receiver. Illustrate the generation of differentially encoded sequence for the binary input sequence 00100110011110.
12. Obtain the differential encoded sequence and the transmitted phase for the binary input data 10010011.
13. Compare the probability of error based on the distance between the message points in signal space diagram of different modulation techniques and justify probability of error values.
14. Explain with block diagram, the non coherent detection of binary frequency shift keying.
15. A binary data are transmitted at a rate of 106 bits per second over the microwave link. Assuming channel AWGN with zero mean and power spectral density of 1×10^{-10} W/Hz. Determine the average carrier power required to maintain an average probability of error probability of error $P_e \leq 10^{-4}$ for coherent binary FSK. Determine the minimum channel bandwidth required.
16. A binary data is transmitted over an AWGN channel using binary PSK at a rate of 1MBPS. It is desired to have average probability of error $P_e \leq 10^{-4}$. Noise power spectral density is $N0/2 = 1 \times 10^{-12}$ W/Hz. Determine the average carrier power required at the receiver input, if the detector is of coherent type.
17. Suppose that binary PSK is used for transmitting information over an AWGN with a power spectral density of $N0/2 = 10^{-10}$ W/Hz. The transmitted signal energy is $Eb = 1/2A^2T$ where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6} when the data rate is a) 10 kbits/s, b) 100 kbits/s, and c)1 Mbits/s
18. Find the expected number of bit errors made in one day by the following continuously operating coherent BPSK receiver. The data rate is 5000 bps. The input digital waveforms are $s_1(t) = A\cos w_0 t$ and $s2(t) = -A\cos w_0 t$ where A= 1 mV and the single-sided noise power spectral density is $N_0 = 10^{-11}$W/Hz. Assume that signal power and energy per bit are normalized to a 1Ω resistive load.

Spread Spectrum Communications

Spread-spectrum communications technology was first described in a paper by an actress and a musician! In 1941 Hollywood actress Hedy Lamarr and pianist George Antheil described a secure radio link to control torpedoes. The term "spread spectrum" refers to the expansion of signal bandwidth, by several orders of magnitude in some cases, which occurs when a key called pseudonoise sequence is attached to the communication channel.

SS is an RF communications system in which the baseband signal bandwidth is intentionally spread over a larger bandwidth by injecting a higher frequency signal.

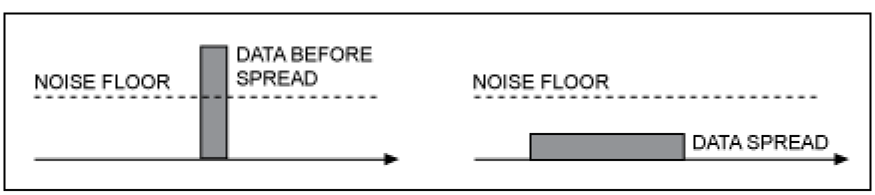
Benefits of Spread Spectrum

1. Resistance to Interference and Antijamming Effects

There are many benefits to spread-spectrum technology. Resistance to interference is the most important advantage. Intentional or unintentional interference and jamming signals are rejected because they do not contain the spread-spectrum key. Only the desired signal, which has the key, will be seen at the receiver when the despreading operation is exercised.

2. Resistance to Interception (Low probability of Intercept-LPI)

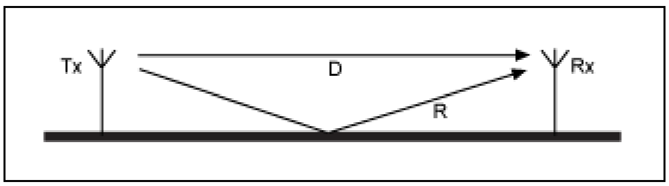
Resistance to interception is the second advantage provided by spread-spectrum techniques. Because non authorized listeners do not have the key used to spread the original signal, those listeners cannot decode it. Without the right key, the spread-spectrum signal appears as noise or as an interferer. (Scanning methods can break the code, however, if the key is short.) Even better, signal levels can be below the noise floor, because the spreading operation reduces the spectral density. Other receivers cannot "see" the transmission; they only register a slight increase in the overall noise level!



Fig(1) Spread-spectrum signal is buried under the noise level. The receiver cannot "see" the transmission without the right spread-spectrum keys.

3. Resistance to Fading (Multipath Effects)

Wireless channels often include multiple-path propagation in which the signal has more than one path from the transmitter to the receiver Such multipaths can be caused by atmospheric reflection or refraction, and by reflection from the ground or from objects such as buildings. The reflected path (R) can interfere with the direct path (D) in a phenomenon called fading. Because the despreading process synchronizes to signal D, signal R is rejected even though it contains the same key. Methods are available to use the reflected-path signals by despreading them and adding the extracted results to the main one.



Fig(2) Illustration of how the signal can reach the receiver over multiple paths.

4. Spread Spectrum Allows Multiple Access

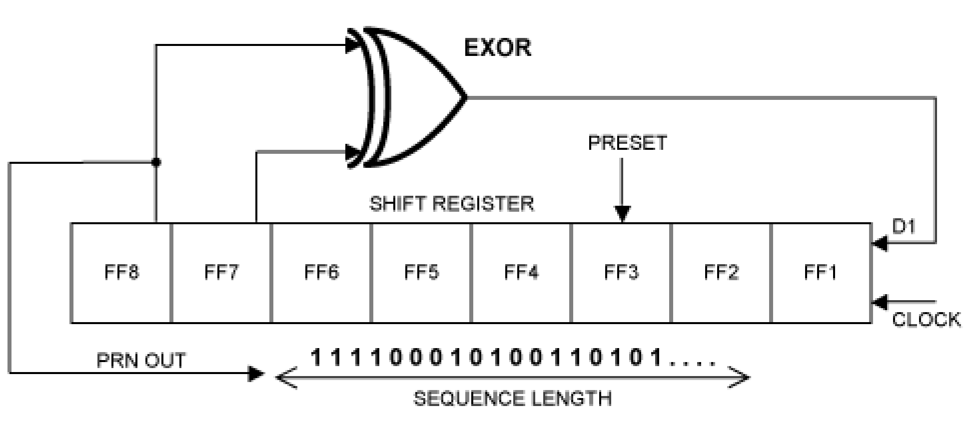
Spread spectrum can also be used as a method for implementing multiple access (i.e., the real or apparent coexistence of multiple and simultaneous communication links on the same physical media). called CDMA (Code Division Multiple Access)

CDMA access to the air is determined by a key or PN code. In that sense, spread spectrum is a CDMA access. The key must be defined and known in advance at the transmitter and receiver ends. Growing examples are IS-95 (DS), IS-98, Bluetooth, and WLAN.

PN sequence generation:

In SS communications the codes used as keys are digital sequences that must be as long and as random as possible to appear as "noise-like" as possible. But in any case, the codes must remain reproducible, or the receiver cannot extract the message that has been sent. Thus, the sequence is "nearly random." Such a code is called a pseudo-random number (PRN) or sequence. The method most frequently used to generate pseudo-random codes is based on a feedback shift register and is called maximum length sequence.One example of a PRN is shown in **Figure**. The shift register contains eight data flip-flops (FF). At the rising edge of the clock, the contents of the shift register are shifted one bit to the left. The data clocked in by FF1 depends on the contents fed back from

FF8 and FF7. The PRN is read out from FF8. The contents of the FFs are reset at the beginning of each sequence length.

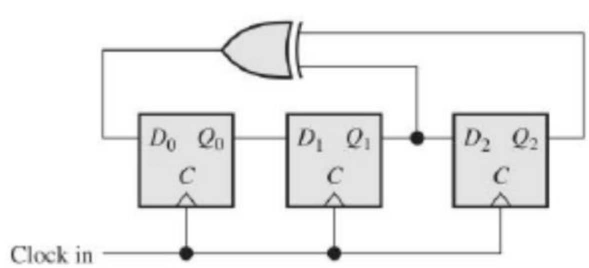


Fig(3) Block diagram of a sample PRN generator.

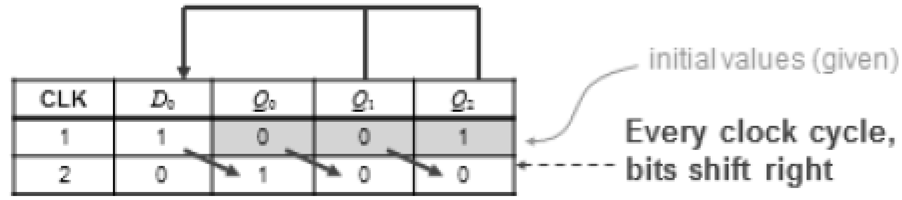
When the length of such a shift-register is n, the period N of the above mentioned PN sequence is $N=2^n-1$

Evaluating a PN sequence Given initial shift register values (seed value), we can determine successive states. Although not explicitly shown on the picture, let's say the PN generator output is taken as Q_2

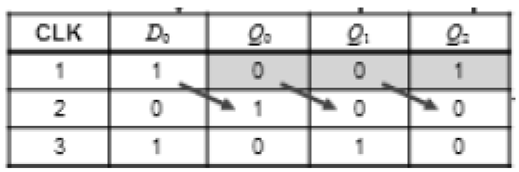
CLK	D_0	Q_0	Q_1	Q_2
1		0	0	1
2				
3				
4				
5				
6				
7				
1				
2				
3				



Fig(4) Generation of PN sequence



CLK	D_0	Q_0	Q_1	Q_2
1	1	0	0	1
2	0	1	0	0



CLK	D_0	Q_0	Q_1	Q_2
1	1	0	0	1
2	0	1	0	0
3	1	0	1	0

First step: Fix the initial value of D_0. Then, determine successive states. While determining successive states, fill in the table above.

In reality, PN sequence generators used in practice contain many shift registers to achieve very long sequences. Connection tapping for different length shift registers for generating PN sequence is given in the table below as generator polynomial.

Length of shift reg	**Generator Polynomial**	Length of shift reg	**Generator Polynomial**
2	[2 1 0]	21	[21 19 0]
3	[3 1 0]	22	[22 21 0]
4	[4 3 0]	23	[23 18 0]
5	[5 3 0]	24	[24 23 22 17 0]
6	[6 5 0]	25	[25 22 0]
7	[7 6 0]	26	[26 25 24 20 0]
8	[8 6 5 4 0]	27	[27 26 25 22 0]
9	[9 5 0]	28	[28 25 0]
10	[10 7 0]	29	[29 27 0]
11	[11 9 0]	30	[30 29 28 7 0]
12	[12 11 8 6 0]	31	[31 28 0]
13	[13 12 10 9 0]	32	[32 31 30 10 0]
14	[14 13 8 4 0]	33	[33 20 0]
15	[15 14 0]	34	[34 15 14 1 0]
16	[16 15 13 4 0]	35	[35 2 0]
17	[17 14 0]	36	[36 11 0]

18	[18 11 0]	37	[37 12 10 2 0]
19	[19 18 17 14 0]	38	[38 6 5 1 0]
20	[20 17 0]	39	[39 8 0]
40	[40 5 4 3 0]	47	[47 14 0]
41	[41 3 0]	48	[48 28 27 1 0]
42	[42 23 22 1 0]	49	[49 9 0]
43	[43 6 4 3 0]	50	[50 4 3 2 0]
44	[44 6 5 2 0]	51	[51 6 3 1 0]
45	[45 4 3 1 0]	52	[52 3 0]
46	[46 21 10 1 0]	53	[53 6 2 1 0]

To guarantee efficient spread-spectrum communications, the PRN sequences must respect certain rules, such as run length, autocorrelation, cross-correlation and bits balancing.

a. **Balance property**: M-sequences are balanced i.e: the number of one's exceeds the number of zeros with only 1.

   Example: in 7 bit PN sequence 0011101, there are four 1's and three 0's

b. **Run length property**: Among the runs of 1^s and 0^s in each period of a ML sequence, ½ of the total runs are of kind one, ¼ are of kind two, 1/8 are of kind three and so on as long as these functions represent meaningful number of runs.

   Run mean a subsequence of identical symbols (1's or 0's)

   Example: in 7 bit PN sequence 0011101,

There are two runs of length 1,

One run of length 2 and

One run of length 3.

Thus total no of runs=4.

Out of this, ½ of total run =4/2=2 runs are of length 1,

¼ of total run=4/4=1 runs are of length 2

c. **Autocorrelation property**: The auto-correlation function is two-valued for PN sequence i.e :

$$R(\tau) = \begin{cases} 1 & \tau = 0, N, 2N, \ldots \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$

ACF of two sequences B_k and $B_{k-\tau}$ is given by

$$R(\tau) = \frac{1}{N}\sum_{k=1}^{N} B_k B_{k-\tau}.$$

ACF is comparing two set of shifted sequences.

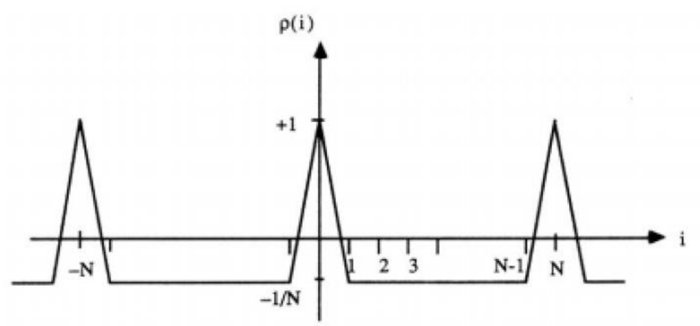
For example If PN sequence considered is 0011101, sequence shifted by 2 bits is 1110100

Represent both by polar format. Then {0011101}={-1,-1,1,1,1,-1,1}

{1110100}= {1,1,1,-1,1,-1,-1}

$$R(\tau) = \frac{1}{N}\sum_{k=1}^{N} B_k B_{k-\tau}.$$

$R(\tau)$ =1/7{-1-1+1-1+1+1-1}=1/7{-1}= -1/7



Different Spreading Techniques:

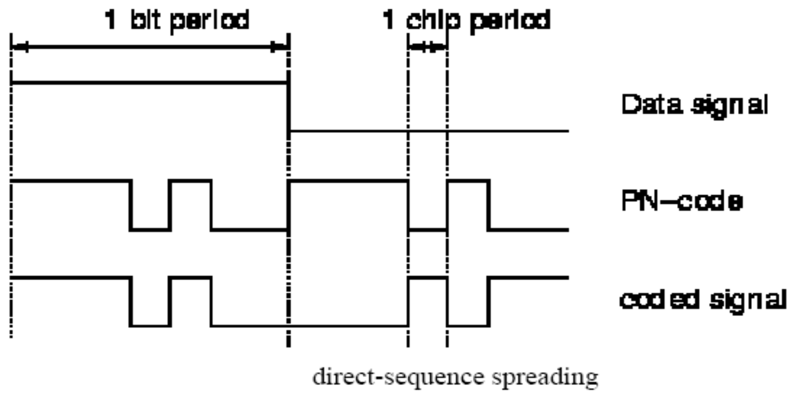
Two different spread-spectrum techniques are commonly used

Direct-Sequence Spread Spectrum (DSSS)

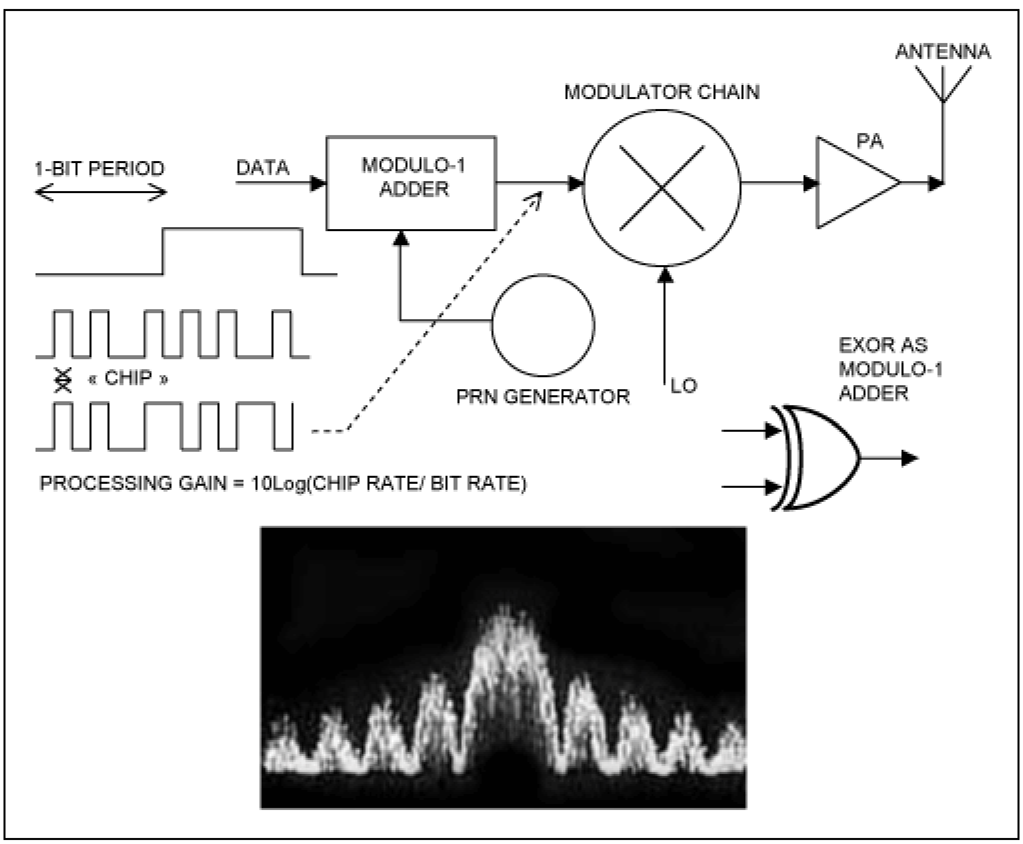
Direct Sequence is the best known Spread Spectrum Technique. The data signal is multiplied by a Pseudo Random Noise Code (PN code). Here PN code is a sequence of chips valued -1 and 1 (polar) which has noise-like properties. This results in low cross-correlation values among the codes and the difficulty to jam or detect a data message. In direct-sequence systems the length of the code is the same as the spreading-factor with the consequence that: $G_{pDS}=N$

This can also be seen from figure below, where we show how the PN code is combined with the data-signal, in this example N=7. The bandwidth of the data signal is now multiplied by a factor

N. The power contents however stays the same, with the result that the power spectral density lowers.



Fig(5) Direct sequence Spreading



Fig(6) Generation of DSSS, and its spectrum

In the receiver, the received signal is multiplied again by the same (synchronized) PN code. Since the code existed of +1s and -1s, this operation completely removes the code from the signal and the original data-signal is left. Another observation is that the de-spread operation is the same as the spread operation. The consequence is that a possible jamming-signal in the radio channel will be spread before data-detection is performed. So jamming effects are reduced.

The main problem with Direct Sequence spreading is the so-called **Near-Far effect.** This effect is present when an interfering transmitter is much closer to the receiver than the intended transmitter. Although the cross-correlation between codes A and B is low, the correlation between the received signal from the interfering transmitter and code A can be higher than the correlation between the received signal from the intended transmitter and code A. The result is that proper data detection is not possible.

Processing gain:

The main parameter in spread spectrum systems is the **processing gain**:

Processing gain is the ratio of transmission and information bandwidth and is given by

$G_p=W_{ss}/W_m=T_b/T_c$=bit duration/chip duration

which is basically the "spreading factor''.

The processing gain determines the number of users that can be allowed in a system, the amount of multi-path effect reduction, the difficulty to jam or detect a signal etc. For spread spectrum systems it is advantageous to have a processing gain as high as possible.

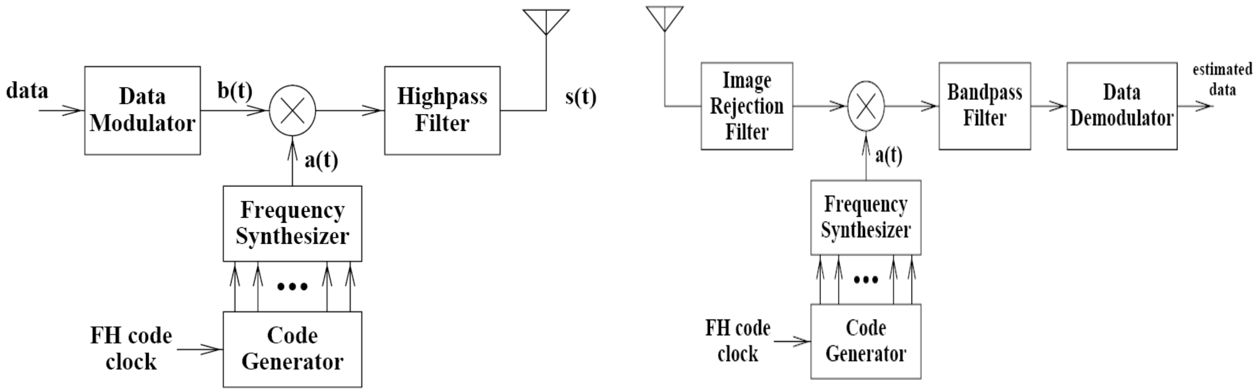
Frequency-Hopping Spread Spectrum (FHSS)

Another common method to spread the transmission spectrum of a data signal is to (pseudo) randomly hop the data signal over different carrier frequencies. This spreading method is called frequency hop spread spectrum (FH-SS). Usually, the available band is divided into non-overlapping frequency bins. The data signal occupies one and only one bin for a duration Tc and hops to another bin afterward. When the hopping rate is faster than the symbol rate (i.e.,$T>T_c$), the FH scheme is referred to as **fast hopping**. Otherwise, it is referred to as **slow hopping.**

.

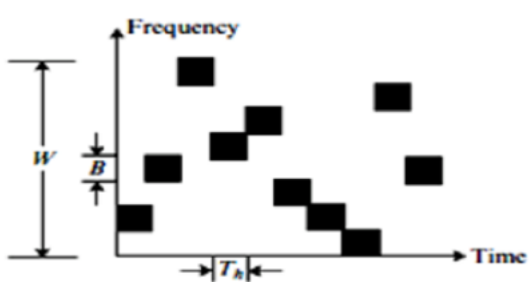
A typical FH-SS transmitter and the corresponding receiver are shown in Figures below.

The transmitted spectrum of a frequency-hopping signal is quite different from that of a direct-sequence system. The frequency hopper's output is flat over the band of frequencies used. The bandwidth of a frequency-hopping signal is simply N times the number of frequency slots available, where N is the bandwidth of each hop channel.



Fig(7) Transmitter and receiver block diagram of a FHSS signal.

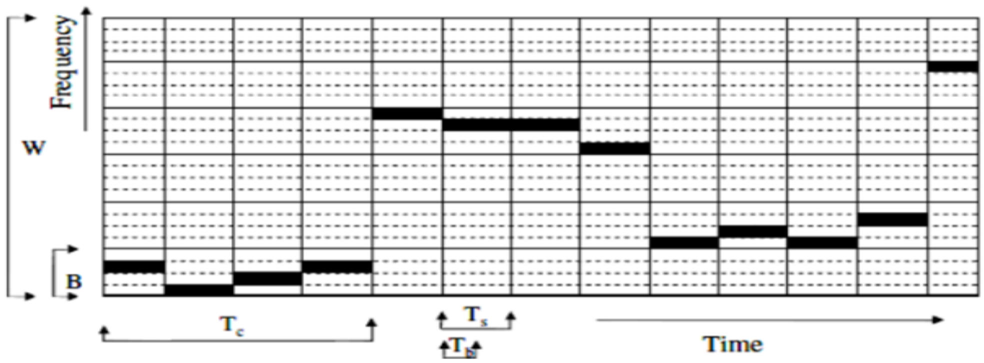
Frequency hopping was first used for military electronic counter measures, because the transmitted signal that uses frequency hopping is difficult to detect and monitor. In an FHSS system the signal frequency is constant for specified time duration, referred to as a hop period Th . The hop period is the time spent in transmitting a signal in a particular frequency slot of bandwidth $B \ll W$, where W and B are spread and symbol bandwidth, respectively. The sequence of carrier frequencies is called the frequency hopping pattern. The set of L possible carrier frequencies $\{f1, f2, \ldots, fL\}$ is called the hopset. The rate at which the carrier frequency changes, is called hop rate, R_h. The carrier frequencies are changed periodically. Hopping occurs over a frequency band called the hopping band that includes L frequency channels. This hopping is typically done in a pseudo-random manner. Each frequency channel is defined as a spectral region that includes a single carrier frequency of the hopset as its center frequency and has a bandwidth B large enough to include most of the power in a signal pulse with a specific carrier frequency. Fig. Illustrates the frequency channels associated with a particular frequency hopping pattern.



Fig(8) Description of Slow Frequency Hopping

Slow frequency hopping:

First, let us consider the case where $Tc > Ts$, which is called slow frequency hopping. A slow FH/MFSK signal is characterized by having multiple symbols transmitted per hop. Hence, each symbol of a slow FH/MFSK signal is a chip. Constraint is that $Tc = NT_s$ for slow frequency hopping.



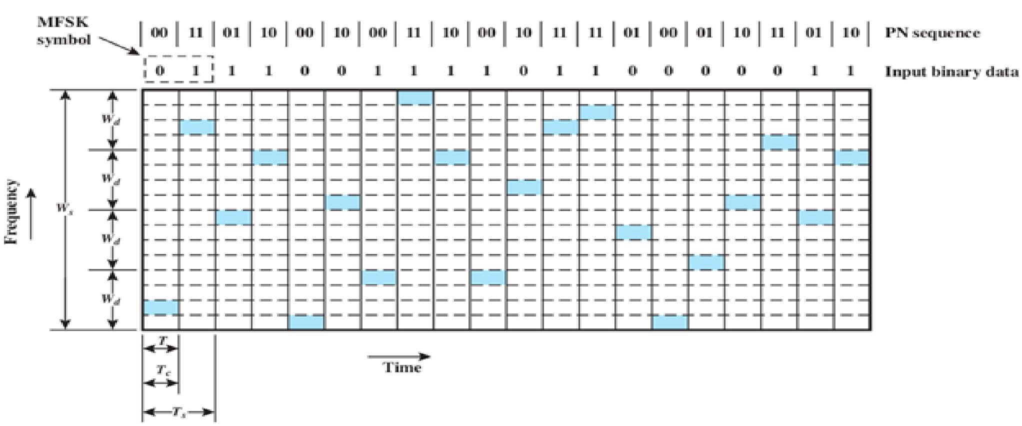
Fig(9) Time-frequency plot for slow frequency hopping (Tc = hop period, Ts = symbol period, Tb = bit period, W = spread bandwidth, B = symbol bandwidth

Fast Frequency-Hopping Spread Spectrum (FHSS): In FFHSS each symbol is transmitted on any of several frequency bands. Pseudo-random (PN) sequence is used to spread the frequencies of MFSK over multiple "blocks"; each MFSK instance uses one block.

T_c: time for each unit in the PN sequence

T_s: time to send one block of L bits

The PN sequence chooses the appropriate frequency; the four possible values of a data symbol choose one of the four sub-bands within a broad band of 4 frequencies.



Fig(10) Fast frequency hopping

For FH/MFSK system chip rate is defined by

R_c=max (R_h,R_s)

Chip rate is maximum rate out of hop rate and symbol rate.

Applications

1. **Code division multiple access**

Code division multiple access (CDMA). Here we can allow different users to use the channel simultaneously by assigning different spreading code sequences to them. Thus there is no physical separation in time or in frequency between signals from different users. The physical channel is divided into many logical channels by the spreading codes. Different from TDMA and FDMA, spread signals from different users do interfere with each other unless the transmissions from all users are perfectly synchronized and orthogonal codes are used.

The interference from other users is known as mulitple access intereference (MAI). In general, synchronization across users is hard to achieve in the uplinks of most practical wireless systems. CDMA system with processing gain N can accommodate about N/5 users without using any error-correcting code. With a powerful error-correcting code, this number can be increased.

2. **Multipath combining**

One advantage of spreading the spectrum is frequency diversity, which is a desirable property when the channel is fading. Fading is caused by destructive interference between time-delayed replica of the transmitted signal arise from different transmission paths (multipaths). The wider the transmitted spectrum, the finer are we able to resolve multipaths at the receiver. Loosely speaking, we can resolve multipaths with path-delay differences larger than 1/W seconds when the transmission bandwidth is W Hz. Therefore, spreading the spectrum helps to resolve multipaths and, hence, combats fading.

The best way to explain multipath fading is to go through the following simple example. Assume that there are two transmission paths leading from the transmitter to the receiver. The first path is the direct line-of-sight path which arrives at a delay of 0 seconds. The second path is a reflected path which arrives at a delay of 2Tc seconds where Tc is the chip duration of the DS-SS system. Receiver can receive signal of path 1 only as synchronization for PN sequence happens for first path alone. Other path signals are considered as noise by the receiver.

One mark questions

1. Spread spectrum communication uses ___________BW compared to ordinary communication system.
2. In SS communication, spectrum spreading is done with the help of __________
3. SS communication is conceived for ________ application
4. Two main types of SS communication are_______ and ______
5. The length of maximum length PN sequence generated by 6 bit shift register is_____
6. If the length of ML PN sequence is 63, then number of flip flops required in PN sequence generating shift register is ________
7. Total number of runs in the ML sequence 0011101 is__________
8. Processing gain in DSSS is dependent on _____________
9. __________effect is a problem with DSSS.
10. Two types of FHSS are __________&__________
11. FHSS in which more than one symbol is transmitted with one hopping frequency is called as _____________
12. LPI expansion is___________
13. Say true or false: "LPI is good in fast FHSS compared to Slow FHSS"
14. Complexity of circuit in FHSS is ________ compared to DSSS
15. Expansion for CDMA is_________

Question bank

1. Define spread spectrum communication. List out the advantages and disadvantages of SSS
2. What is PN sequence? Give general block diagram to generate ML PN sequence.
3. Generate PN sequence using 4 bit shift register with connection $x_3 \oplus x_4 = x_0$. State all properties of PN sequence and verify these properties for the generated PN sequence.
4. With neat block diagram and equations, explain the generation of DSSS.
5. With neat block diagram explain the generation of FHSS.
6. A DS spread binary PSK uses feedback register of length 19 for the generation of PN sequence. Calculate PN sequence length, Chip duration, PN sequence period and processing gain.

7. Draw time-frequency diagram for slow frequency hopping spread spectrum system considering ML sequence 100110101111000, and message sequence 100011101010. Consider BFSK modulation and 2 symbols per hop. Represent each frequency with 4 bits of PN sequence.
8. Discuss about any two civilian applications of Spread spectrum.
9. Compare CDMA with TDMA and FDMA.
10. Distinguish between slow frequency and fast frequency hopping SS.