equal to $2/T_b$ (i.e., twice that of unipolar, polar, and bipolar formats of the NRZ type).

6.3 INTERSYMBOL INTERFERENCE

Consider Fig. 6.5, which depicts the basic elements of a *baseband binary PAM system*. The input signal consists of a binary data sequence $\{b_k\}$ with a bit duration of T_b seconds. This sequence is applied to a pulse generator, producing the discrete PAM signal

$A_k \rightarrow$ random variable

$a_k \rightarrow$ sample value

$$x(t) = \sum_{k=-\infty}^{\infty} a_k v(t - kT_b) \tag{6.18}$$

where $v(t)$ denotes the basic pulse, normalized such that $v(0) = 1$, as in Eq. 6.2. The coefficient a_k depends on the input data and the type of format used. The waveform $x(t)$ represents one realization of the random process $X(t)$ considered in Section 6.2. Likewise, a_k is a sample value of the random variable A_k.

The PAM signal $x(t)$ passes through a transmitting filter of transfer function $H_T(f)$. The resulting filter output defines the transmitted signal, which is modified as a result of transmission through the channel of transfer function $H_C(f)$. The channel may represent a coaxial cable or optical fiber, where the major source of system degradation is *dispersion* in the channel. In any event, for the present we assume that the channel is *noiseless* but dispersive. The channel output is passed through a receiving filter of transfer function $H_R(f)$. This filter output is sampled synchronously with the transmitter, with the sampling instants being determined by a clock or timing signal that is usually extracted from the receiving filter output. Finally, the sequence of samples thus obtained is used to reconstruct the original data sequence by means of a decision device. Each sample is compared to a threshold. We assume that symbols 1 and 0 are equally likely, and the threshold is set half way between their representation levels. If the threshold is exceeded, a decision is made in favor of symbol 1. If, on the other hand, the threshold is not exceeded, a decision is made in favor of symbol 0. If the sample value equals the threshold exactly, the flip of a fair coin will determine which symbol was transmitted. — 1a

The receiving filter output may be written as*

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \tag{6.19}$$

where μ is a scaling factor and the pulse $p(t)$ is *normalized* such that

$$p(0) = 1 \tag{6.20}$$

* To be precise, an arbitrary time delay t_0 should be included in the argument of the pulse $p(t - kT_b)$ in Eq. 6.19 to represent the effect of transmission delay through the system. For convenience, we have put this delay equal to zero in Eq. 6.19.

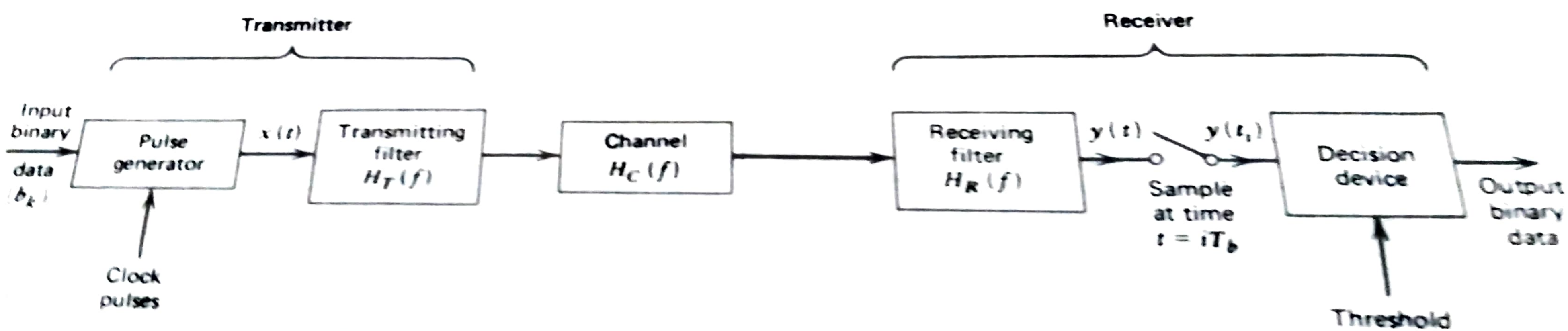


Figure 6.5 Baseband binary data transmission system.

The output $y(t)$ is produced in response to the binary data waveform applied to the input of the transmitting filter. Specifically, the pulse $\mu p(t)$ is the response of the cascade connection of the transmitting filter, the channel, and the receiving filter, which is produced by the pulse $v(t)$ applied to the input of this cascade connection. Therefore, we may relate $p(t)$ to $v(t)$ in the frequency domain by writing

$$\mu P(f) = V(f)H_T(f)H_C(f)H_R(f) \tag{6.21}$$

where $P(f)$ and $V(f)$ are the Fourier transforms of $p(t)$ and $v(t)$, respectively. Note that the normalization of $p(t)$ as in Eq. 6.20 means that the total area under the curve of $P(f)$ equals unity.

The receiving filter output $y(t)$ is sampled at time $t_i = iT_b$ (with i taking on integer values), yielding

$$\begin{aligned} y(t_i) &= \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b) \\ &= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b) \end{aligned} \tag{6.22}$$

In Eq. 6.22, the first term μa_i is produced by the ith transmitted bit. The second term represents the residual effect of all other transmitted bits on the decoding of the ith bit; this residual effect is called *intersymbol interference* (ISI).

In physical terms, ISI arises because of imperfections in the overall frequency response of the system. When a short pulse of duration T_b seconds is transmitted through a band-limited system, the frequency components constituting the input pulse are differentially attenuated and, more significantly, differentially delayed by the system. Consequently, the pulse appearing at the output of the system is *dispersed* over an interval longer than T_b seconds. Thus, when a sequence of short pulses (representing binary 1s and 0s) are transmitted through the system, one pulse every T_b seconds, the dispersed responses originating from different symbol intervals will interfere with each other, thereby resulting in intersymbol interference.

In the absence of ISI, we observe from Eq. 6.22 that

$$y(t_i) = \mu a_i$$

which shows that, under these conditions, the ith transmitted bit can be decoded correctly. The presence of ISI in the system, however, introduces errors in the decision device at the receiver output. Therefore, in the design of the transmitting and receiving filters, the objective is to minimize the effects of ISI, and thereby deliver the digital data to its destination with the smallest error rate possible.

6.4 NYQUIST'S CRITERION FOR DISTORTIONLESS BASEBAND BINARY TRANSMISSION

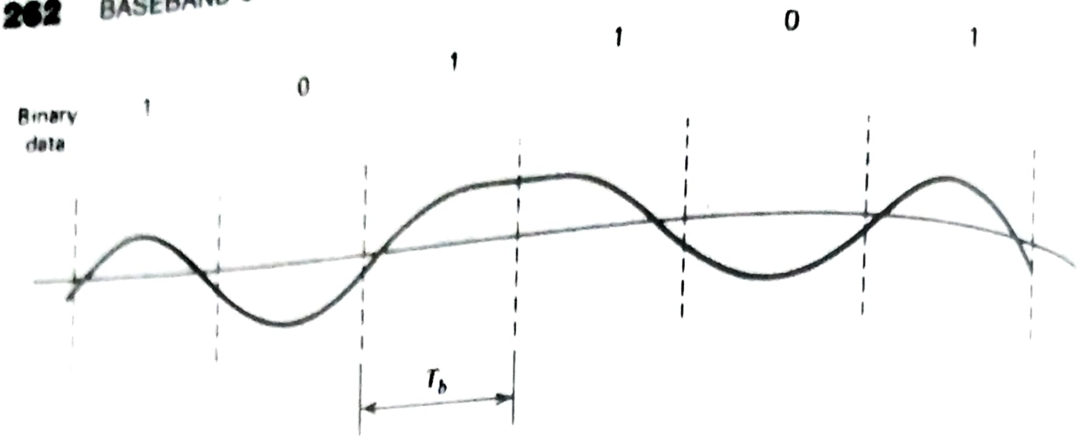
Typically, the transfer function of the channel and the transmitted pulse shape are specified, and the problem is to determine the transfer functions of the transmitting and receiving filters so as to reconstruct the transmitted data se-

6.6 EYE PATTERN

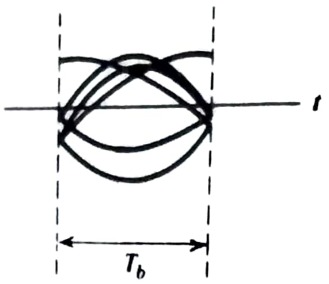
One way to study intersymbol interference in a PCM or data transmission system experimentally is to apply the received wave to the vertical deflection plates of an oscilloscope and to apply a sawtooth wave at the transmitted symbol rate $R = 1/T$ to the horizontal deflection plates. The waveforms in successive symbol intervals are thereby translated into one interval on the oscilloscope display, as illustrated in Fig. 6.18 for the case of a binary wave for which $T = T_b$. The resulting display is called an *eye pattern* because of its resemblance to the human eye for binary waves. The interior region of the eye pattern is called the *eye opening*.

An eye pattern provides a great deal of information about the performance of the pertinent system, as described next (see Fig. 6.19):

1. The width of the eye opening defines the time interval over which the received wave can be sampled without error from intersymbol interfer-



(a)



(b)

Figure 6.18 (a) Distorted binary wave. (b) Eye pattern.

ence. It is apparent that the preferred time for sampling is the instant of time at which the eye is open widest.

2. The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.
3. The height of the eye opening, at a specified sampling time, defines the margin over noise.

When the effect of intersymbol interference is severe, traces from the upper portion of the eye pattern cross traces from the lower portion, with the result that the eye is completely closed. In such a situation, it is impossible to avoid errors due to the combined presence of intersymbol interference and noise in the system, and a solution has to be found to correct for them.

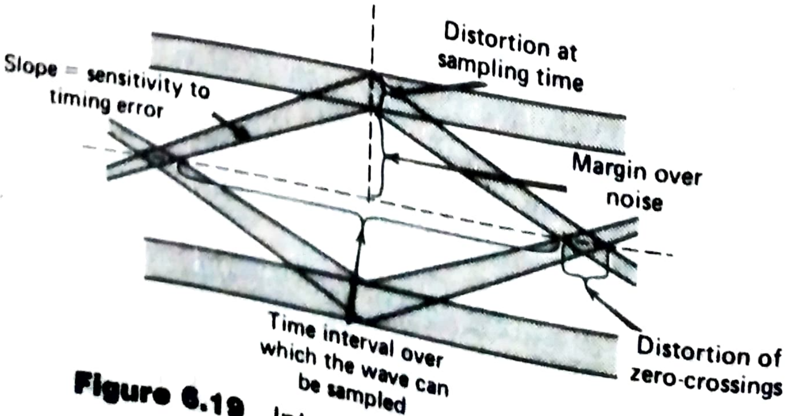


Figure 6.19 Interpretation of eye pattern.

6.8 ADAPTIVE EQUALIZATION FOR DATA TRANSMISSION

An efficient approach to *high-speed transmission* of digital data (e.g., computer data) over a voice-grade telephone channel (which is characterized by a limited bandwidth and high signal-to-noise ratio) involves the use of two basic signal processing operations:

1. Discrete PAM by encoding the amplitudes of successive pulses in a periodic pulse train with a discrete set of possible amplitude levels.
2. A linear modulation scheme that offers bandwidth conservation to transmit the encoded pulse train over the telephone channel. (Spectrally efficient modulation schemes are considered in the next chapter).

At the receiving end of the system, the received wave is demodulated, and then synchronously sampled and quantized. As a result of dispersion of the pulse shape by the channel, we find that the number of detectable amplitude levels is often limited by intersymbol interference rather than by additive noise. In principle, if the channel is known precisely, it is virtually always possible to make the intersymbol interference (at the sampling instants) arbitrarily small by using a suitable pair of transmitting and receiving filters, so as to control the overall pulse shape in the manner described previously. The transmitting filter is placed directly before the modulator, whereas the receiving filter is placed directly after the demodulator. Thus, insofar as intersymbol interference is concerned, we may consider the data transmission as being essentially baseband.

However, in a *switched telephone network*,* we find that two factors contribute to pulse distortion on different link connections: (1) differences in the transmission characteristics of the individual links that may be switched together, and (2) differences in the number of links in a connection. The result is that the telephone channel is random in the sense of being one of an ensemble of possible channels. Consequently, the use of a fixed pair of transmitting and receiving filters designed on the basis of average channel characteristics may not adequately reduce intersymbol interference. To realize the full transmission capability of a telephone channel, there is need for *adaptive equalization*.† By equalization we mean the process of correcting channel-induced distortion. This process is said to be adaptive when it adjusts itself continuously during data transmission by operating on the input signal.

Among the philosophies for adaptive equalization of data transmission systems, we have *prechannel equalization* at the transmitter, and *postchannel equalization* at the receiver. Because the first approach requires a feedback channel, we consider only adaptive equalization at the receiving end of the system. This equalization can be achieved, prior to data transmission, by training the filter with the guidance of a suitable *training sequence* transmitted through the channel so as to adjust the filter parameters to optimum values. The typical telephone channel changes little during an average data call, so that precall equalization with a training sequence is sufficient in most cases encountered in practice. The equalizer is positioned after the receiving filter in the receiver.

The adaptive equalizer consists of a tapped-delay-line filter (with as many as 100 taps or more), whose coefficients are updated (starting from zero initial values) in accordance with the *least-mean square* (LMS) algorithm; the LMS algorithm was described in Section 3.15. The adjustments to the filter coefficients are made in a step-by-step fashion synchronously with the incoming data.‡

There are two modes of operating the adaptive equalizer, as shown in Fig. 6.20. During the *training period*, a known sequence is transmitted and a syn-

* For a description of switched-communication networks, see Section 10.2.

† For tutorial papers on adaptive equalization, see Qureshi (1982, 1985).

‡ In a *synchronous equalizer*, the delay-line taps are spaced at the reciprocal of the symbol rate. In the equalization of telephone channels, the equalizer is sometimes designed with the taps spaced closer than the reciprocal of the symbol rate. Such equalizers are called *fractionally spaced equalizers*. For the same total time span, a fractionally spaced equalizer can effectively compensate for more severe delay distortion than the conventional synchronous equalizer. For details of fractionally spaced equalizers, see Gitlin and Weinstein (1981).

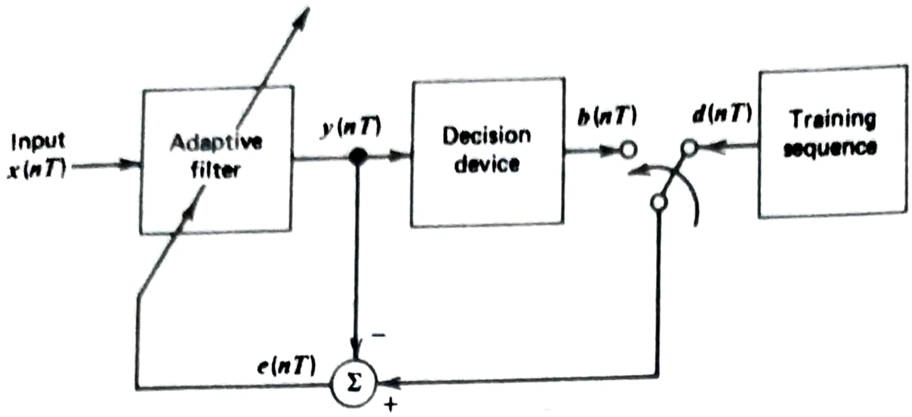


Figure 6.20 Illustrating the modes of operation of an adaptive equalizer.

chronized version of this signal is generated in the receiver where it is applied to the adaptive equalizer as the desired response. The training sequence may, for example, consist of a linear maximal-length or pseudo-noise (PN) sequence.* The length of this training sequence must be equal to or greater than that of the adaptive equalizer.

When the training period is completed, the adaptive equalizer is switched to its second mode of operation, the *decision-directed mode*. In this mode of operation, the error signal equals

$$e(nT) = b(nT) - y(nT)$$

where $y(nT)$ is the equalizer output and $b(nT)$ is the final (not necessarily) correct estimate of the transmitted symbol $b(nT)$. Now, in normal operation the decisions made by the receiver are correct with high probability. This means that the error estimates are correct most of the time, thereby permitting the adaptive equalizer to operate satisfactorily. Furthermore, an adaptive equalizer operating in a decision-directed mode is able to *track* relatively slow variations in channel characteristics.

The methods of implementing adaptive equalizers may be divided into three broad categories: *analog*, *hardwired digital*, and *programmable digital*, as described here:

1. The analog approach is primarily based on the use of *charge-coupled device* (CCD) technology. The basic circuit realization of the CCD is a row of *field-effect transistors* (FET) with drains and sources connected in series, and the drains capacitively coupled to the gates. The set of adjustable tap weights are stored in digital memory locations, and the multiplications of the analog sample values by the digitized tap weights take place in analog fashion. This approach has significant potential in applications where the symbol rate is too high for digital implementation.
2. In hardwired digital implementation of an adaptive equalizer, the equalizer input is first sampled and then quantized into a form suitable for storage in shift registers. The set of adjustable tap weights are also stored in shift registers. Logic circuits are used to perform the required digital arithmetic

* Pseudo-noise (PN) sequences are discussed in Chapters 8 and 9.

(e.g., multiply and accumulate). In this approach the circuitry is hard-wired for the sole purpose of performing equalization. Nonetheless, it is the most widely used method of implementing adaptive equalizers.

3. The use of a programmable digital processor in the form of a *microprocessor*, for example, offers flexibility in that the adaptive equalization is performed as a series of steps or instructions in the microprocessor. An important advantage of this approach is that the same hardware may be time-shared to perform a multiplicity of signal-processing functions such as filtering, modulation, and demodulation in a modem (*mo*dulator-*dem*odulator) used to transmit digital data over a telephone channel.