DETECTION AND ESTIMATION.

Detection:

It is the process of making decision as which symbol was transmitted according to some set of rules based on the observation of received signal.

Estimation:

It is the process of extracting the Estimates of physical parameters or waveform of interest.

The results of detection & estimation are always subject to errors.

Model of Digital Communication.

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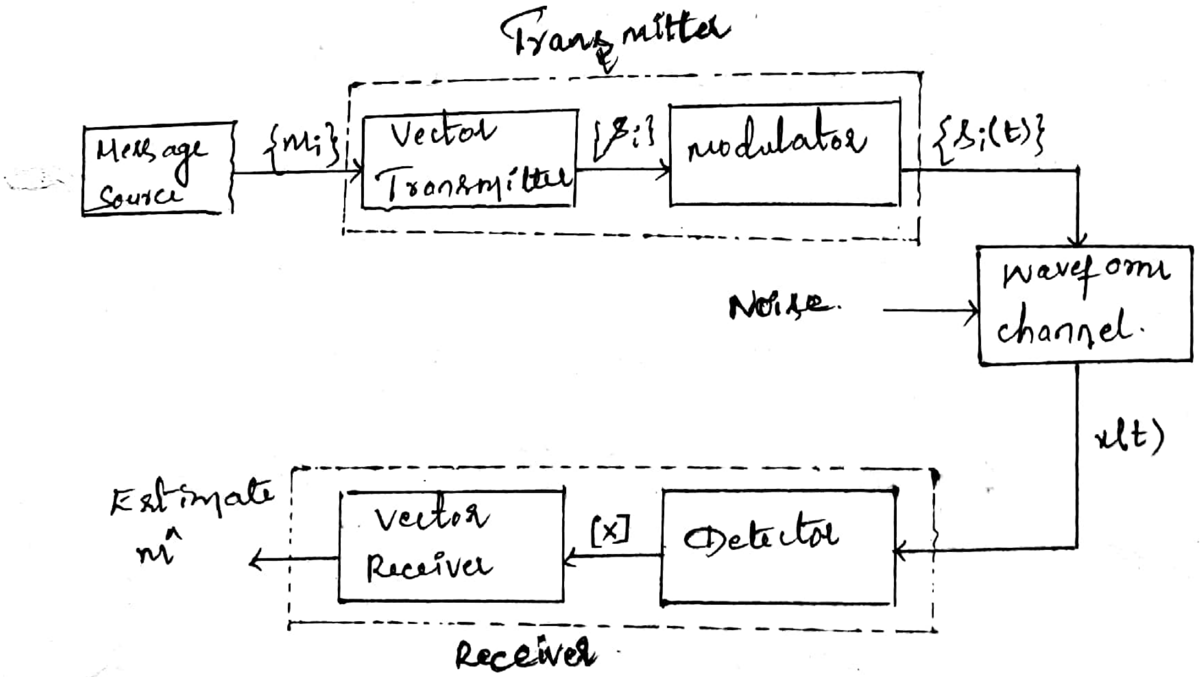


Message Source:

Emits the symbol for every 'T' sec.

Total no of possible symbols is M, namely $m_1, m_2 \ldots m_M$

Assume each symbol occur with equal probability.

Probability of symbol m_i is

$$p_i = \text{Probability of } (m_i \text{ emitted})$$

$$p_i = P(m_i \text{ emitted})$$

$$p_i = \frac{1}{M} \quad \text{for } i = 1, 2 \ldots M.$$

Vector transmitter.

output of message source is given to vector transmitter.

Vector transmitter produces a vector $[s_i]$ for each symbol m_i to be transmitted. as

$$[s_i] = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} \quad i = 1, 2 \ldots M$$

where $N \leq M$.

output of vector transmitter is given to modulator.

Modulator

It produces an energy signal $s_i(t)$ of duration 'T' second for each vector $[s_i]$ it receives.

$$E_i = \int_0^T s_i^2(t)\,dt \qquad i = 1\ 2 \ldots M.$$

$s_i(t)$ is real valued signal.

The signal $s_i(t)$ transmitted for each vector $[s_i]$ depends in some fashion on the incoming message & possibly on the signal transmitted in the preceding slots. It also depends on the physical channel.

Channel :

It is assumed to have two characteristics.

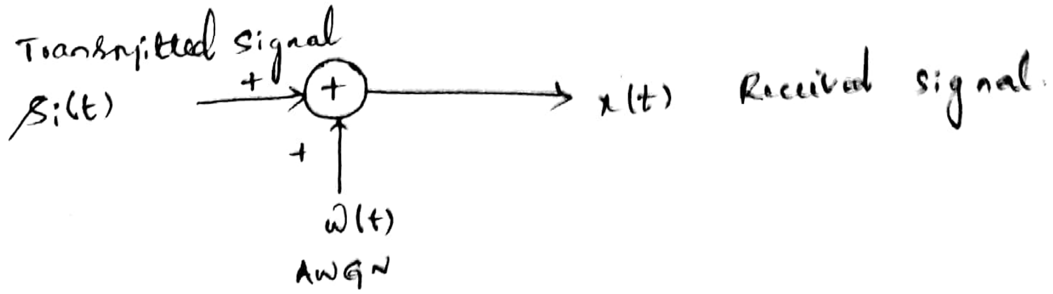
1) The channel is linear with a bandwidth that is large enough to accomodate the transmission of the modulator o/p $s_i(t)$ without distortion.

2) The transmitted signal $s_i(t)$ is ~~white~~ ~~Gaussian~~ ~~noise~~ perturbed by an additive White Gaussian Noise [AWGN] $w(t)$ with zero mean.

Detector :

The received random process $x(t)$ is given as

$$x(t) = s_i(t) + w(t) \qquad 0 < t < T$$
$$i = 1\ 2 \ldots M$$



Receiver:

Receiver = Detector + Vector receiver.

Receiver observes the received signal $x(t)$ for a duration of 'T' seconds & makes a best estimates of the transmitted signal $x(t)$ or equivalently the symbol m_i

This task is accomplished in two stages.

1) The first stage is the detector which operates on the received random process $x(t)$ to produce a vector of random variables.

2) The second stage is the vector receiver By using an observation vector $[X]$ (which is a sample value of X), prior knowledge of the S_i & the prior probability P_i, it produces the estimate $\hat{m}$

Since AWGN is added in channel $\therefore$ the received signal may contain noise & hence errors may occur in the decision making process.

The requirement is to design the vector receiver so as to minimize the average probability of symbol error given as

$$P_e = P(\hat{m} \neq m_i)$$

m_i : transmitted symbol.

$\hat{m}$: estimate produced by receiver.

The resulting receiver is called optimum in the minimum probability of error sense.

Consider there are M symbols

Let $L = M$

Symbol	Notation	vector	transmitted signal
+	m_1	$[s_1] = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$s_1(t)$
−	m_2	$[s_2] = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$s_2(t)$
×	m_3	$[s_3] = \begin{bmatrix} s_{31} \\ s_{32} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$s_3(t)$
÷	m_4	$[s_4] = \begin{bmatrix} s_{41} \\ s_{42} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$s_4(t)$

size of each vector is ie, $N = 2$

In general,

$$[s_i] = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} \quad \begin{matrix} i = 1 \ 2 \ 3 \ 4 \\ i = 1 \ 2 \ .. \ M \end{matrix}$$

Since each symbol represented by two bit $(N = 2)$ vector we need $N = 2$ no of orthonormal basis functions to represent each symbol.

The basis functions are $\phi_1(t)$, $\phi_2(t)$

The transmitted signal $s_i(t)$ is represented by weighted sum of (linear combination of) basis funtions as follows.

Symbol	Transmitted signal $s_i(t)$ $i = 1, 2, 3, 4$
$+$ m_1	$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t)$
$-$ m_2	$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t)$
$\times$ m_3	$s_3(t) = s_{31}\phi_1(t) + s_{32}\phi_2(t)$
$\div$ m_4	$s_4(t) = s_{41}\phi_1(t) + s_{42}\phi_2(t)$

In general the transmitted signal $s_i(t)$ for 'i' th symbol m_i is given as

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iN}\phi_N(t).$$

OR

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \qquad i = 1, 2, \ldots M$$

$$0 \leq t \leq T$$

Orthogonality & orthonormality

Consider two basis functions $\phi_i(t)$ & $\phi_j(t)$, if they satisfy the following condition then they are said to be orthogonal to each other.

i.e.,

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} k & i = j \\ 0 & i \neq j \end{cases}$$

If $k = 1$ then $\phi_i(t)$ & $\phi_j(t)$ are said to be ORTHONORMAL in addition to being orthogonal.

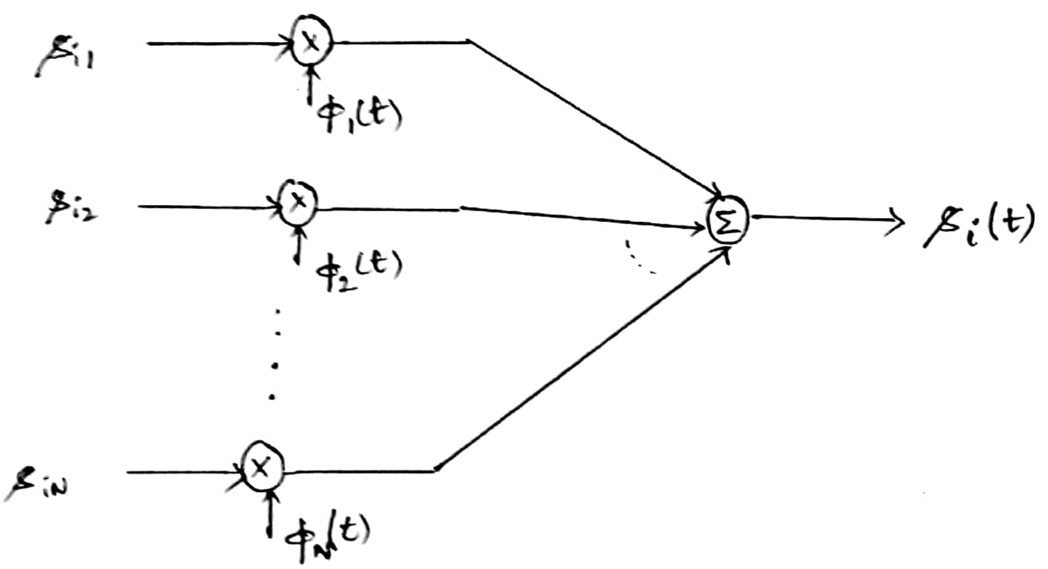
i.e.,

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

Note: All orthonormal signals are orthogonal but converse may not be true.

Gram Schmidt Orthogonalization procedure:

Transmitter.



~~The Gram. Sch~~

The modulated signal $s_i(t)$ transmitted for the symbol m_i is generated as shown in fig above.

Here the symbol vector $[s_i]$ is ~~mad~~ mapped to symbol m_i, where

$$[s_i] = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}$$

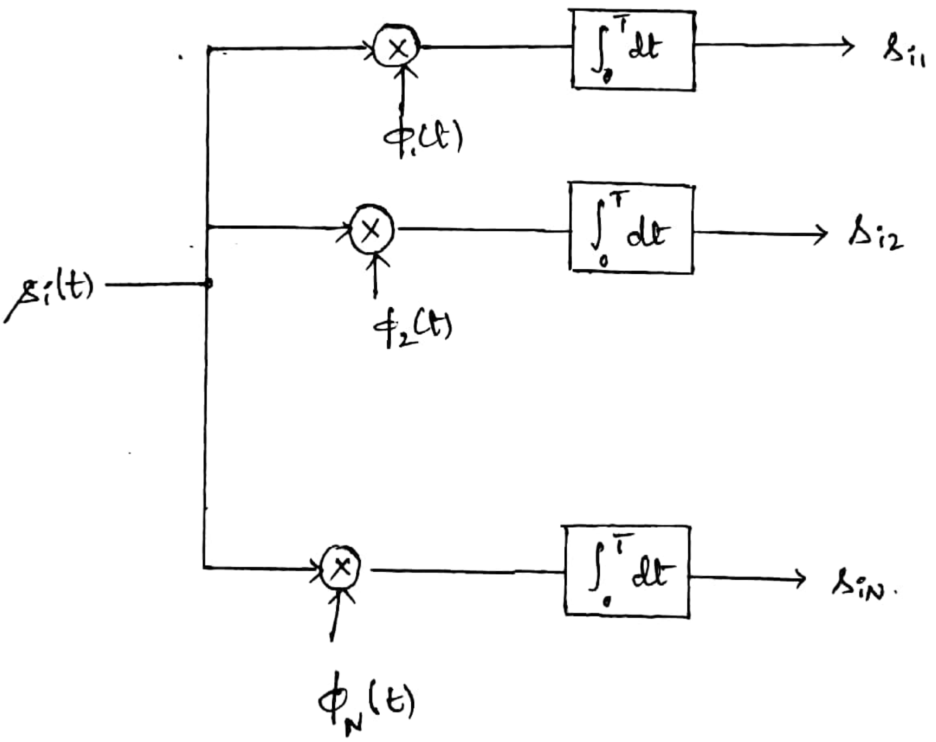
$\phi_1(t)$. $\phi_2(t)$ - - - $\phi_N(t)$ are the orthonormal basis functions.

From figure we can write.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t)$$

Receiver

The symbol m_i is determined at the receiver by determining each element of the vector $[s_i]$ as follows.



For example

$$s_{i1} = \int_0^T s_i(t)\, \phi_1(t)\, dt$$

substituting for $s_i(t)$

$$s_{i1} = \int_0^T \sum_{j=1}^{N} s_{ij}\, \phi_j(t)\, \phi_1(t)\, dt$$

$$= \int_0^T \left[s_{i1}\phi_1(t) + s_{i2}\, \phi_2(t) + \cdots\cdots + s_{iN}\, \phi_N(t) \right] \phi_1(t)\, dt$$

$$= \int_0^T s_{i1}\, \phi_1(t)\, \phi_1(t)\, dt + \int_0^T s_{i2}\, \phi_2(t)\, \phi_1(t)\, dt + \cdots$$

$$\cdots\cdots + \int_0^T s_{iN}\, \phi_N(t)\, \phi_1(t)\, dt$$

Since $\phi_1(t)$ $\phi_2(t)$ ... all are orthogonal, for any combination

$$\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$\therefore$ except for first integral all other integral will turn to zero.

$\therefore$

$$s_{i1} = \int_0^T s_{i1}\, \phi_1(t)\, \phi_1(t)\, dt$$

$$s_{i1} = s_{i1} \int_0^T \underbrace{\phi_1^2(t)\, dt}_{1}$$

$$s_{i1} = s_{i1}$$

Thus any element s_{ij} of any vector $[s_i]$ is determined as

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \qquad \begin{array}{l} i = 1\ 2\ 3 \ldots M \\ j = 1\ 2 \ldots N. \end{array}$$

GEOMETRIC INTERPRETATION of SIGNALS

Consider a set of M no Energy signals as

$\{s_i(t)\}, \quad i = 1, 2, \ldots M$

Consider a set of N no of basis functions as

$\{\phi_j(t)\} \quad j = 1\ 2 \ldots N.$

Any signal $s_i(t)$ can be represented as the linear combination of N no of basis functions as

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1\ 2 \ldots M \end{array}$$

where $$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \qquad \begin{array}{l} i = 1, 2, \ldots M \\ j = 1\ 2 \ldots N. \end{array}$$

i.e., each signal in the set $\{s_i(t)\}$ is completely determined by the vector of its coefficients given as

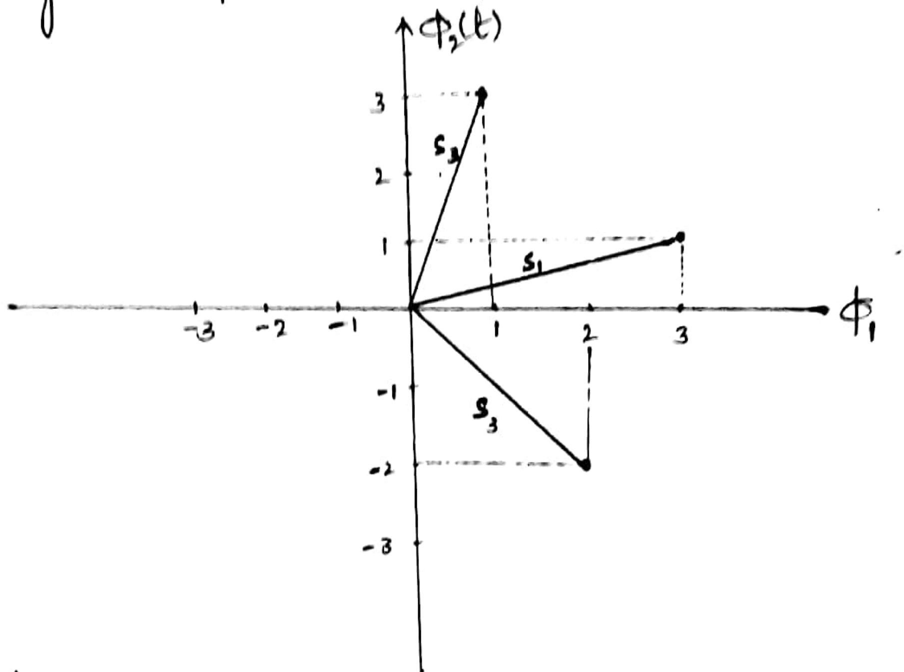
$$[s_i] = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} \qquad i = 1, 2. \ldots M$$

The vector $[s_i]$ is called signal vector.

N dimensional Euclidean space consists of N no of mutually perpendicular axis labelled as

$\phi_1, \phi_2 \ldots \phi_N$.

This N dimensional Euclidean space is also called as signal space.



Consider the case of $N = 2$ $M = 3$

Length of signal vector s_i is denoted as $\|s_i\|$

But inner product or dot product of the signal vector is equal to

$\|s_i\|^2$ computed as

$$\|s_i\|^2 = (s_i, s_i)$$

$$= \sum_{j=1}^{N} s_{ij}^2$$

where s_{ij} are the elements of s_i

$$\left.\begin{array}{l}\text{Cosine of the}\\ \text{angle b/wn two}\\ \text{vector } [s_i] \text{ \& } [s_j]\end{array}\right\} = \frac{([s_i][s_j])}{\|s_i\| \, \|s_j\|}$$

Note: If $([s_i] . [s_j]) = 0$ then the vector $[s_i]$ $[s_j]$ are orthogonal or perpendicular

Now Energy of the signal $s_i(t)$ with duration T is computed as,

$$E_i = \int_0^T s_i^2(t) \, dt$$

$$E_i = \int_0^T s_i(t) \, s_i(t) \, dt$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij} \phi_j(t) \right] \left[\sum_{k=1}^{N} s_{ik} \phi_k(t) \right] dt$$

changing the order of the summation & integration

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} \, s_{ik} \int_0^T \phi_j(t) \, \phi_k(t) \, dt$$

If $j = k$ in the above eqn, it reduces to.

$$E_i = \sum_{j=1}^{N} \sum_{j=1}^{N} s_{ij} \, s_{ij} \int_0^T \underbrace{\phi_j(t) \, \phi_j(t)}_{1} \, dt$$

$$E_i = \sum_{j=1}^{N} s_{ij}^2$$ $\because$ $\phi_j(t)$ & $\phi_k(t)$ are orthogonal signals.

Note: If a pair of signals $s_i(t)$ & $s_k(t)$ represented by the signal vectors $[s_i]$ & $[s_j]$ then we can prove that

$$\|[s_i]-[s_k]\|^2 = \sum_{j=1}^{N}(s_{ij}-s_{kj})^2$$

$$= \int_0^T [s_i(t) - s_k(t)]^2\, dt$$

where $\|[s_i]-[s_k]\|$ is the Euclidean distance b/wn the points seperated by signal vector $[s_i]$ & $[s_k]$

CORRELATION RECEIVER / OPTIMUM RECIEVER.

Assumption: ① The channel is AWGN channel

② The transmitted signals are $s_1(t), s_2(t), \ldots s_M(t)$ are equally likely.

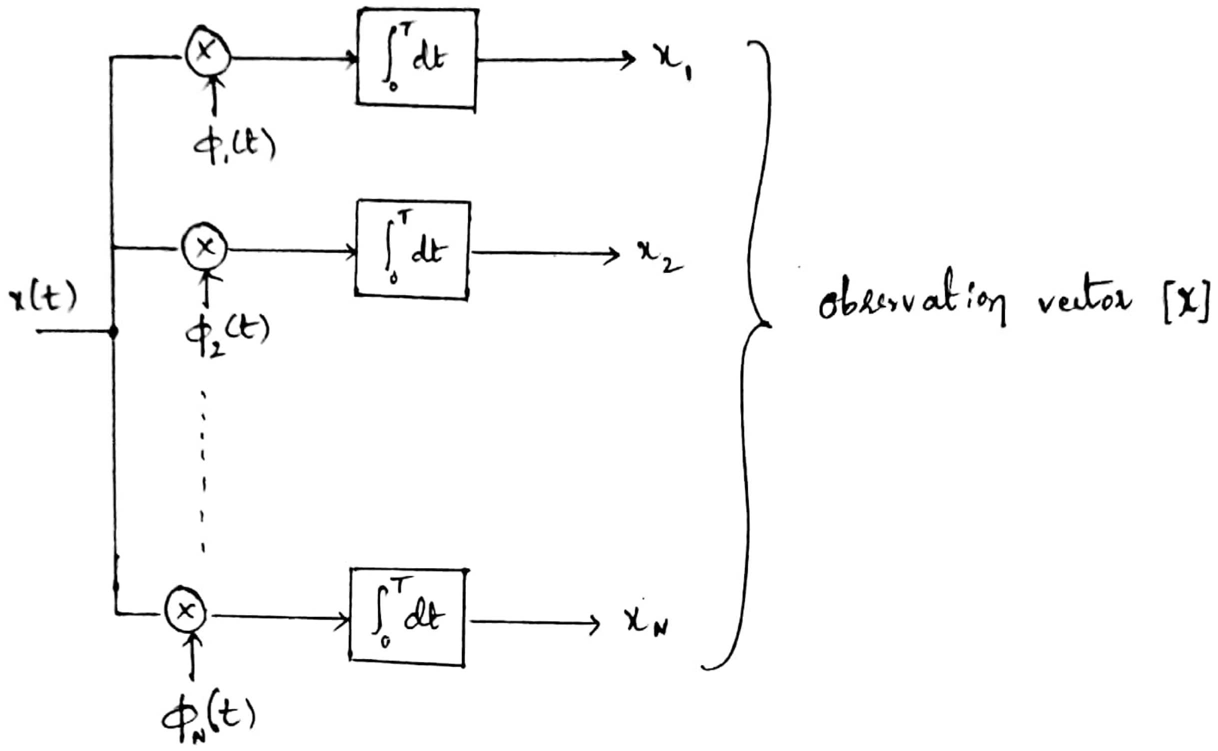
The optimum receiver consists of two parts

1) Detector
2) vector receiver.

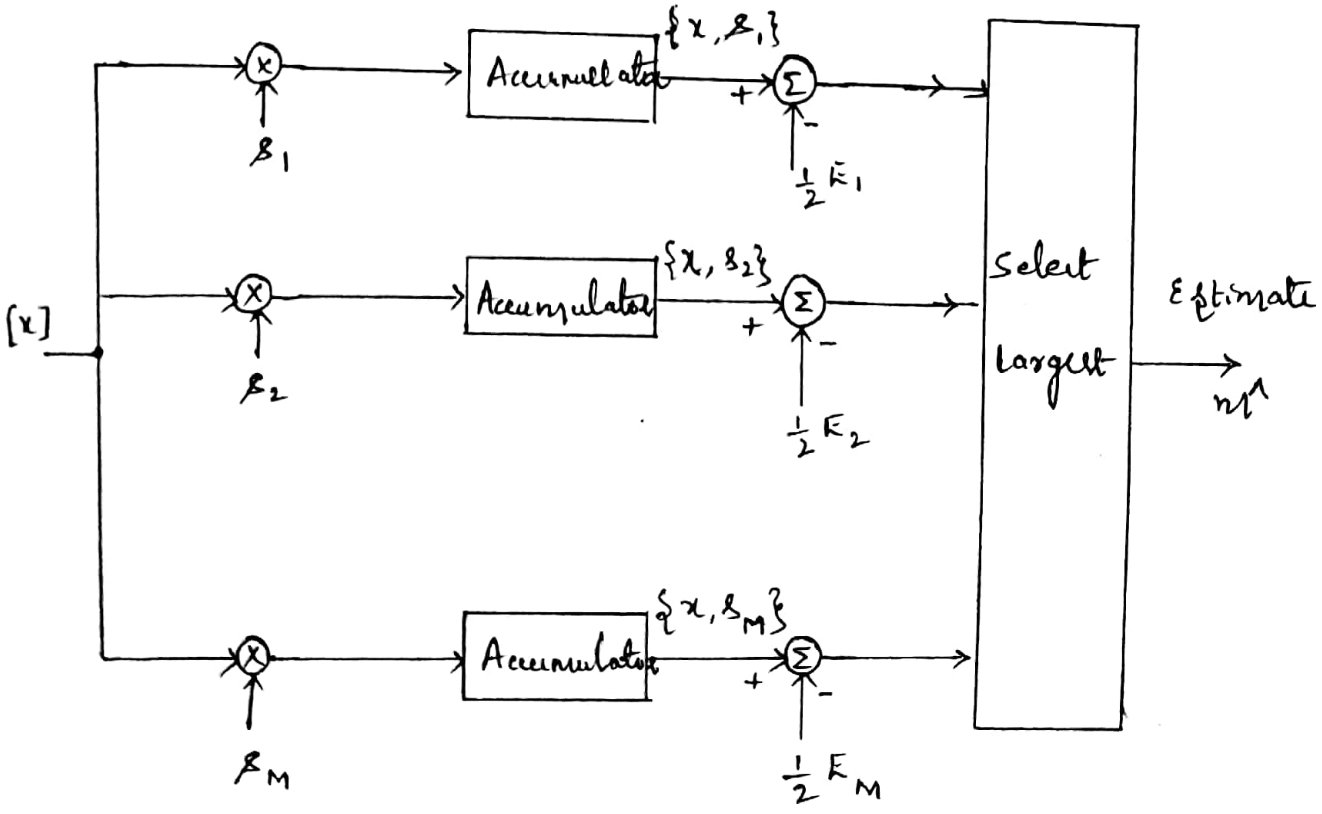
Detector

It consists of bank of M product integrators or correlators supplied with a corresponding set of coherent reference signals or orthonormal basis functions $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ that are generated locally

This bank of correlators operate on the received signal $x(t)$ $0 \le t \le T$, to produce the observation vector $[x]$



Vector Receiver



The vector receiver is implemented in the form of maximum likelyhood detector.

It operates on the observation vector $[x]$ to produce an estimate $\hat{m}$ of the transmitted symbol m_i e $i = 1\ 2\ 3 \ldots M$. in a way that minimizes the average probability of symbol error.

$$\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2} E_k \text{ is max for } k = i \longrightarrow (1).$$

According to Eqn (1) the elements of the observation vector $[x]$ are first multiplied by the corresponding N elements of each of the M signal vectors $[s_1], [s_2] \ldots [s_M]$. and the resulting products are successively summed up in accumulators to form the corresponding set of inner products $\{([x],[s_k])\}$, $k = 1\ 2 \ldots M$.

Next the inner products are corrected for the fact that the transmitted signal energies may be unequal.

Finally, the largest in the resulting set of numbers is selected, and a corresponding decision on the transmitted message is made.

MATCHED FILTER RECEIVER

Since analog multipliers are usually hard to build, we can replace them by matched filters as the orthonormal basis functions $\phi_1(t)$ $\phi_2(t)$ . . . . . $\phi_N(t)$ are zero outside the interval $0 \le t \le T$

$$x(t) \longrightarrow \boxed{h_j(t)} \longrightarrow y_j(t)$$

Consider an LTI filter with impulse response $h_j(t)$. Let $x(t)$ be the received signal which is i/p to the filter.

o/p of the filter is given as

$$y_j(t) = x(t) * h_j(t)$$

$$y_j(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h_j(t-\tau)\, d\tau \longrightarrow (1)$$

Suppose, the impulse response $h_j(t)$ is set as

$$h_j(t) = \phi_j(T-t) \longrightarrow (2)$$

Note: $h_j(t)$ is reflected & time shifted version of $\phi_j(t)$.

put Eqn (2) in (1)

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T-t+\tau)\, d\tau$$

sampling this o/p at time instant T we get

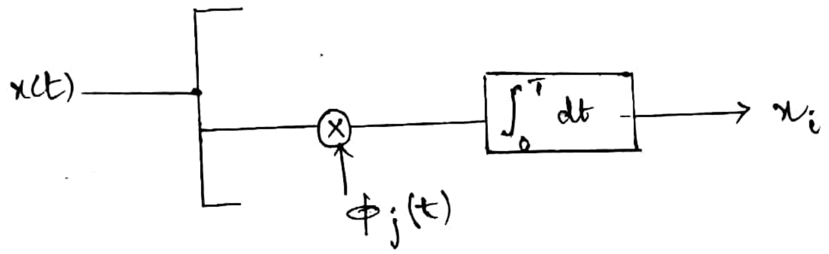
$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T - \cancel{T} + \tau)\, d\tau$$

$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(\tau)\, d\tau$$

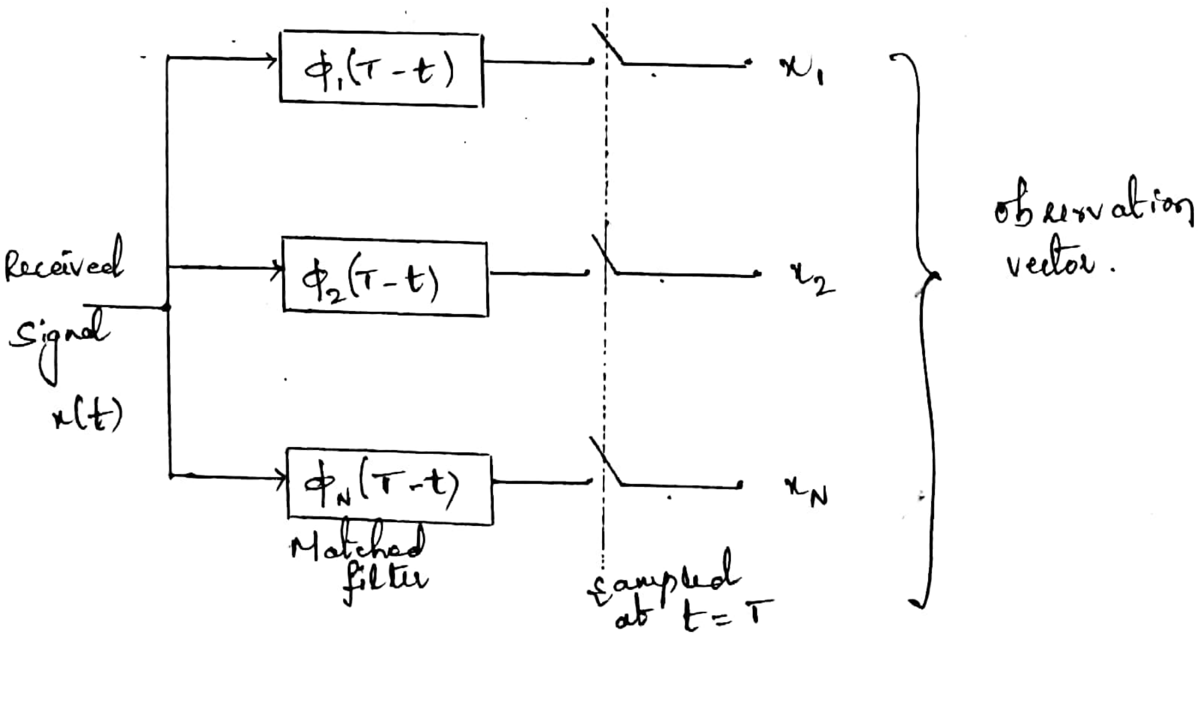
Since $\phi_j(t)$ exists only in the duration $0 \le t \le T$ limits should be changed.

$$y_j(T) = \int_{0}^{T} x(\tau)\, \phi_j(\tau)\, d\tau$$

Here it is observed that $y_j(T)$ is equal to the o/p of the 'j' th correlator, x_i as depicted in figure below.



Therefore detector part of the optimum receiver can also be implemented as shown in figure below.



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A filter whose impulse response is a time reversed & delayed version of some signal $\phi_j(t)$ as in eq ②. is said to be matched to $\phi_j(t)$. Correspondingly, the optimum receiver based on the detector of figure ... is referred to as the matched filter receiver.

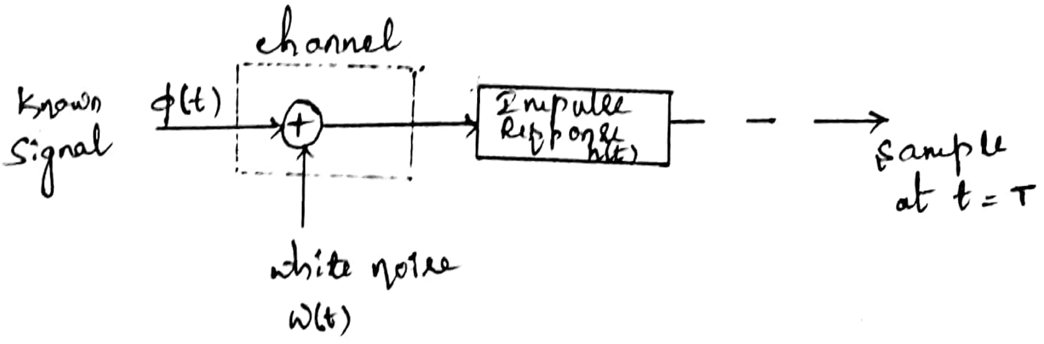
For matched filter to be realized physically, it must be causal. i.e.,

$$h_j(t) = 0 \qquad t < 0.$$

This is possible if $\phi_j(t)$ is zero outside the interval $0 \leq t \leq T$

Maximization of o/p signal to noise Ratio.

Consider a linear filter of impulse response $h(t)$ with an input $x(t)$ that consists of a known signal $\phi(t)$ & an additive noise component $w(t)$ as shown in figure below.



$$x(t) = \phi(t) + w(t) \qquad 0 \leq t \leq T$$

where T is the observation instant.

Let $\phi(t)$ be one of the orthonormal basis functions. The $w(t)$ is the sample function of a stationary white Gaussian noise process of zero mean & power spectral density $\frac{N_0}{2}$

o/p of the filter is $y(t)$ which is modified version of i/p $x(t)$.

Since the filter is linear we can write

$$y(t) = H\{x(t)\}$$

$$= H\{\phi(t) + w(t)\}$$

$$= H\{\phi(t)\} + H\{w(t)\}$$

$$y(t) = \phi_0(t) + n(t)$$

. For the sake of strength of signal $\phi_0(t)$ to be greater than that of the noise $n(t)$, the filter should make sure that the instantaneous power of in the signal $\phi_0(t)$ measured at time $t = T$ is larger than the average power of the noise $n(t)$. at o/p. This is equivalent to maximize the SNR at the o/p of filter.

SNR at the o/p of filter is given as

$$(SNR)_0 = \frac{|\phi_0(T)|^2}{E[n^2(t)]} = \frac{\text{instantaneous power of } \phi_0(t) \text{ at } t = T}{\text{average power of } n(t)}$$

$\longrightarrow$ Eqn (1).

To show that $(SNR)_o$ is maximized if the filter impulse response $h(t)$ is matched to the known signal $\phi(t)$ at the i/p.

$$\phi(t) \xleftrightarrow{F.T} \phi(f)$$

$$h(t) \xleftrightarrow{F.T} H(f)$$

$$\phi_o(t) \xleftrightarrow{F.T} \phi_o(f)$$

Since the filter is linear.

$$x(t) = \phi(t) + w(t) \longrightarrow \boxed{h(t),\ H(f)} \longrightarrow y(t) = \phi_o(t) + n(t)$$

$$\phi_o(f) = H(f)\,\phi(f)$$

Using I.FT

$$\phi_o(t) = \int_{-\infty}^{\infty} \phi_o(f)\, e^{j2\pi f t}\, df$$

$$\phi_o(t) = \int_{-\infty}^{\infty} H(f)\,\phi(f)\, e^{j2\pi f t}\, df$$

Now when filter o/p is sampled at $t = T$, we may write

$$|\phi_o(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)\,\phi(f)\, e^{j2\pi f T}\, df \right|^2 \longrightarrow (2)$$

Power spectral density of the o/p noise $n(t)$ is written as

$$S_N(f) = \begin{pmatrix} \text{Power Spectral density} \\ \text{of i/p noise } w(t) \end{pmatrix} |H(f)|^2$$

$$S_N(f) = \frac{N_0}{2}\, |H(f)|^2$$

10

Average power of o/p noise $n(t)$ is calculated as

$$E\{n^2(t)\} = \int_{-\infty}^{\infty} S_N(f)\, df$$

$$E\{n^2(t)\} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df \longrightarrow (3).$$

$$(SNR)_o = \frac{\left| \int_{-\infty}^{\infty} H(f)\,\phi(f)\, e^{j2\pi f T}\, df \right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} \longrightarrow (4)$$

But according to Schwarz's inequality

$$\left| \int_{-\infty}^{\infty} H(f)\,\phi(f)\, e^{j2\pi f T}\, df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |\phi(f)|^2\, df$$

Using this relation in Eqn (4). we get

$$(SNR)_o \leq \frac{2}{N_0}\int_{-\infty}^{\infty} |\phi(f)|^2\, df \longrightarrow (5).$$

where

(i) $\int_{-\infty}^{\infty} |\phi(f)|^2\, dt = \int_{-\infty}^{\infty} |\phi(t)|^2\, dt$ according to Rayleigh's energy theorem.

(ii) Noise power spectral density $N_0/2$

Maximum value of SNR is written as

$$(SNR)_{o,max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |\phi(f)|^2\, df$$

According to Schwarz's inequality. for $(SNR)_o$ to maximum the transfer function of the filter should be optimum and it is given as.

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$$H_{opt}(f) = \phi^*(f)\exp(-j2\pi f T)$$

To find the impulse response of the optimum filter

$$h_{opt}(t) = \int_{-\infty}^{\infty} H_{opt}(f)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{\infty} \phi^*(f)\exp(-j2\pi f T)\exp(j2\pi f t)\, df$$

Since $\phi(t)$ is real $\phi^*(f) = \phi(-f)$ ∴ above eqn becomes

$$h_{opt}(t) = \underbrace{\int_{-\infty}^{\infty} \phi(-f)\, e^{j2\pi f t}\, df}_{\text{Inverse F.T}} \cdot \underbrace{e^{-j2\pi f t}}_{\text{multiplication by exponential eq} \Longleftrightarrow \text{time shifting.}}$$

$$\boxed{h_{opt}(t) = \phi(T-t)}$$

From above eqn it is clear that the impulse response of optimum filter is a time reversed & delayed version of the i/p signal $\phi(t)$. In otherwords it is matched to the i/p signal.

Note:

The maximization of the $(SNR)_o$ is equivalent to minimization of the average probability of symbol error under two assumptions.

1. The noise input at the receiver is a stationary additive white Gaussian noise.
2. The a priori probability of the transmitted signals are known.

Orthogonality example

Consider two basis functions

$\phi_1(t) = \sin 2\pi f_1 t \quad \phi_2(t) = \sin 2\pi f_2 t$

Let $f_1 = 1\,Hz \quad f_2 = 2\,Hz$

To check if $\phi_1(t)$ & $\phi_2(t)$ are orthogonal to each other

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = ?$$

Mathematical operation shown above is called correlation

If result of correlation is zero b/wn 2 signals then those two signals are said to be ORTHOGONAL.

Let $T = 1\,sec$

$$\int_0^1 \sin 2\pi t \,\sin 2\pi \times 2 \times t \,dt$$

$$= \int_0^1 \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] dt$$

$$= \tfrac{1}{2}\int_0^1 \left[\cos 2\pi t - \cos 6\pi t\right] dt$$

$$= \tfrac{1}{2}\left[\frac{\sin 2\pi t}{2\pi} - \frac{\sin 6\pi t}{6\pi}\right]_0^1$$

$$= \tfrac{1}{2}\left[\frac{\sin 2\pi - \sin 0}{2\pi} - \frac{\sin 6\pi - \sin 0}{6\pi}\right]$$

$$= \tfrac{1}{2}\left[0 - 0\right]$$

$$= 0.$$

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0$$

$\therefore$ $\phi_1(t)$ & $\phi_2(t)$ are orthogonal.

~~Now consider~~

In general

$$\int_0^T \sin(2\pi \times 1 \times t)\,\sin(2\pi \times n \times t)\,dt = 0$$

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where 'n' is any integer ≥ 2

$\therefore$ 1 Hz sinusoid is orthogonal to all other sinusoids with frequency integer multiple of 1

Now consider

$$\int_0^T \phi_1^2(t)\,dt$$

$$= \int_0^1 \sin^2 2\pi t \, dt$$

$$= \int_0^1 \frac{1}{2}(1 - \cos 4\pi t)\,dt$$

$$= \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_0^1 \cos 4\pi t \, dt$$

$$= \frac{1}{2}[t]_0^1 - \frac{1}{2}\left[\frac{\sin 4\pi t}{4\pi}\right]_0^1$$

$$= \frac{1}{2}[1-0] - \frac{1}{2}\left[\frac{\sin 4\pi - \sin 0}{4\pi}\right]$$

$$= \frac{1}{2}$$

In general

$$\int_0^T \sin(2\pi \times n \times t)\sin(2\pi \times m \times t)\,dt = \begin{cases} 0 & n \neq m \\ k & n = m \end{cases}$$

$$\int_0^T \cos(2\pi \times n \times t)\cos(2\pi \times m \times t)\,dt = \begin{cases} 0 & n \neq m \\ k & n = m \end{cases}$$

IIIly

$$\int_0^T \phi_2^2(t)\,dt = \frac{1}{2}$$

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