

- Pseudo noise sequences-PN Sequence
- Notion of spread spectrum
- Direct sequence spread spectrum
- Frequency Hop Spread Spectrum
- Applications

Spread Spectrum Modulation

- **Bandwidth** and **Power** are the two primary communication resources that they need to be used with care in the design of most communication systems.
- There are situations where it is necessary to sacrifice the efficient utilization of these two resources in order to meet certain other design objectives.
- For example, the system may be required to provide a form of secure communication in a hostile environment such that the transmitted signal is not easily detected or recognized by unwanted listeners. This requirement is catered to by a class of signalling techniques known collectively as spread-spectrum modulation.
- The primary advantage of a spread-spectrum communication system is its ability to reject interference whether it be the unintentional interference of another user simultaneously attempting to transmit through the channel, or the intentional interference of a hostile transmitter attempting to jam the transmission.

Spread Spectrum Modulation

- The definition of spread spectrum may be stated in two parts:
- 1. Spread spectrum is a means of transmission in which the data of interest occupies a bandwidth in excess of the minimum bandwidth necessary to send the data.
- 2. The spectrum spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The same code is used in the receiver (operating in synchronism with the transmitter) to despread the received signal so that the original data may be recovered.

Although standard modulation techniques such as frequency modulation and pulse-code modulation do satisfy Part 1 of this definition, they are not spread spectrum techniques because they do not satisfy Part 2 of the definition.

Cont'd...

- Spread-spectrum modulation was originally developed for **military applications** where resistance to jamming (interference) is of major concern.
- However, there are civilian applications that also benefit from the unique characteristics of spread-spectrum modulation.
 - For example, it can be used to provide multipath rejection in a ground-based mobile radio environment.
 - Yet another application is in multiple-access communication in which a number of independent users are required to share a common channel without an external synchronizing mechanism; here, for example, we may mention a ground-based mobile radio environment involving mobile vehicles that must communicate with a central station.



- A pseudo-noise (PN) sequence is defined as a coded sequence of 1's and 0's with certain autocorrelation properties.
- The class of sequences used in spread-spectrum communications is usually periodic in that a sequence of 1's and 0's repeats itself exactly with a known period.
- The maximum-length sequence, a type of cyclic code, represents a commonly used periodic PN sequence. Such sequences have long periods and require simple instrumentation in the form of a linear feedback shift register. Indeed, they possess the longest possible period for this method of generation.



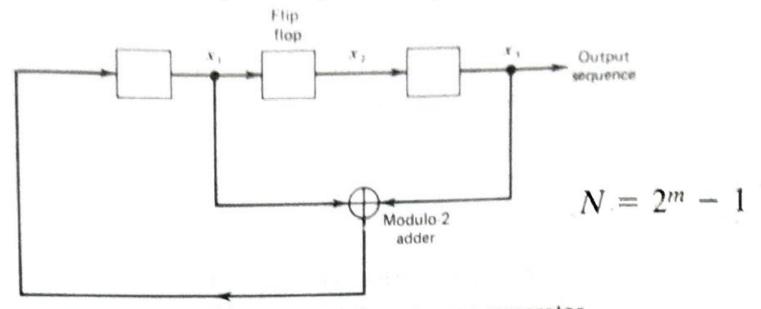
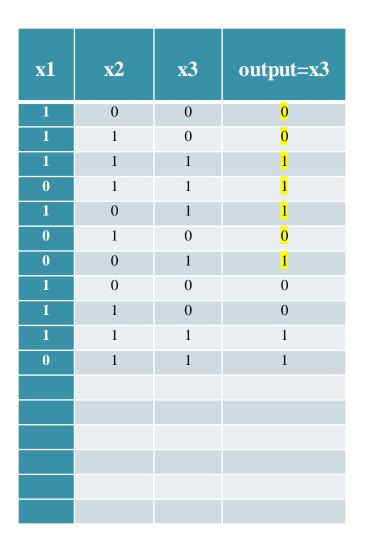


Figure 9.1 Maximum-length sequence generator.

Consider the three-stage feedback shift register shown in Fig. 9.1. It is assumed that the initial state of the shift register is 100 (reading the contents of the three flip-flops from left to right). Then, the succession of states will be as follows:

The output sequence (the last position of each state of the shift register) is therefore 0011101



PN	Sec	aue	nce
		1 W C	

0011101

N=7

Balance Property

Num of 0's= 3

Num of 1's=4

Num of 1's is one(count)

greater than

Num of 0's

Run Property

00 111 **0** 1

Total runs=4

length 1=2 .. (0 1)

50% of Total runs(4/2)

length 2= 1.. (00)

25% of Total runs(4/4)

length 3= 1..(111)

12.5% of Total runs(4/8)

Table 9.1 Range of PN Sequence Lengths

Length of Shift	PN Sequence	
Register, m	Length, N	
7	127	
8	255	
9	511	
10	1023	
11	2047	
12	4095	
13	815,	
17	131071	
19	524287	



Maximum-length sequences* have many of the properties possessed by a truly random binary sequence. A random binary sequence is a sequence in which the presence of a binary symbol 1 or 0 is equally probable. Some properties of maximum-length sequences are listed below:

PROPERTY 1

In each period of a maximum-length sequence, the number of 1s is always one more than the number of 0s. This property is called the balance property.

PROPERTY 2

Among the runs of 1s and of 0s in each period of a maximum-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs. This property is called the run property.

By a "run" we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence. The length of this subsequence is the length of the run. For a maximum-length sequence generated by a feedback shift register of length m, the total number of runs is (m + 1)/2.



The autocorrelation function of a maximum-length sequence is periodic and binary-valued. This property is called the correlation property.

Let binary symbols 0 and 1 be represented by -1 volt and +1 volt, respectively. By definition, the *autocorrelation sequence* of a binary sequence $\{c_n\}$, so represented, equals

$$R_c(k) = \frac{1}{N} \sum_{n=1}^{N} c_n c_{n-k}$$

where N is the *length* or *period* of the sequence and k is the *lag* of the autocorrelation sequence. For a maximum-length sequence of length N, the autocorrelation sequence is periodic with period N and two-valued, as shown by

$$R_c(k) = \begin{cases} 1 & k = lN \\ -\frac{1}{N} & k \neq lN \end{cases}$$

where l is any integer. When the length N is infinitely large, the autocorrelation sequence $R_c(k)$ approaches that of a completely random binary sequence.

EXAMPLE 2

Consider again the maximum-length sequence generated by the feedback shift register of Fig. 9.1. The output sequence (represented in terms of binary symbols 0 and 1) is

$$\{c_n\} = \underbrace{0011101}_{N=7} \dots$$
 (9.4)

In terms of the levels -1 and +1, the output sequence is

$$\{c_n\} = \underbrace{-1, -1, +1, +1, +1, -1, +1}_{N = 7}, \dots$$
(9.5)

We see that there are three 0s (or -1's) and four 1s (or +1's) in one period of the sequence, which satisfies Property 1.

With N = 7, there are a total of four runs in one period of the sequence. Reading them from left to right in Eq. 9.4, the four runs are 00, 111, 0, and 1. Two of the runs (a half of the total) are of length one, and one run (a quarter of the total) is of length two, which satisfies Property 2.

Figure 9.2a shows two full periods of the maximum-length sequence. Figure 9.2b shows the corresponding autocorrelation function $R_c(\tau)$ plotted as a function of the time lag τ . In this figure, the parameter T_c denotes the duration of binary symbol 1 or 0 in the sequence, and N is the length of one period of the sequence. The periodic and two-valued correlation property of the sequence is clearly seen in Fig. 9.2b.

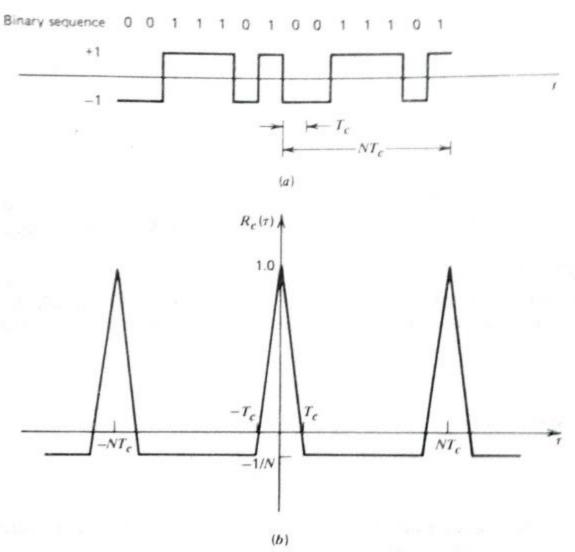
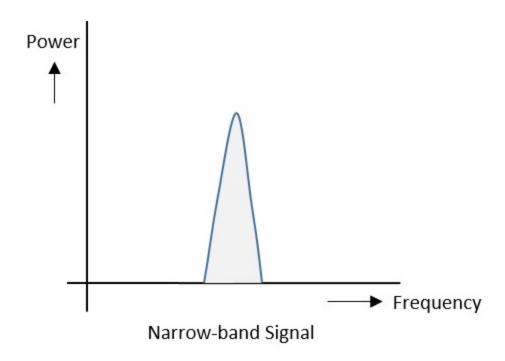


Figure 9.2 (a) Waveform of maximum-length sequence. (b) Autocorrelation of maximum-length sequence. Both parts refer to the output of the feedback shift register of Fig. 9.1.

Narrow-band Signals

The Narrow-band signals have the signal strength concentrated as shown in the following frequency spectrum figure.



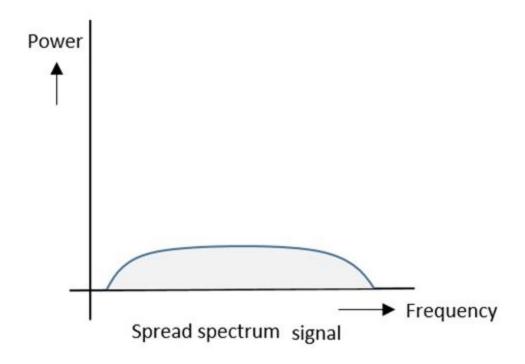
Following are some of its features –

- Band of signals occupy a narrow range of frequencies.
- Power density is high.
- Spread of energy is low and concentrated.

Though the features are good, these signals are prone to interference.

Spread Spectrum Signals

The spread spectrum signals have the signal strength distributed as shown in the following frequency spectrum figure.



Following are some of its features -

- Band of signals occupy a wide range of frequencies.
- Power density is very low.
- Energy is wide spread.

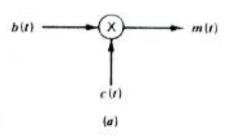
With these features, the spread spectrum signals are highly resistant to interference or jamming. Since multiple users can share the same spread spectrum bandwidth without interfering with one another, these can be called as **multiple access techniques**.

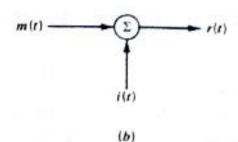
Advantages of Spread Spectrum

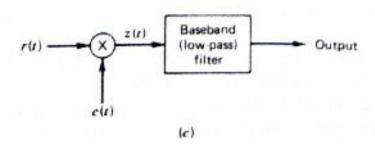
Following are the advantages of spread spectrum -

- Cross-talk elimination
- Better output with data integrity
- Reduced effect of multipath fading
- Better security
- Reduction in noise
- Co-existence with other systems
- Longer operative distances
- Hard to detect
- Not easy to demodulate/decode
- Difficult to jam the signals

A Notion of Spread Spectrum







$$m(t) = c(t)b(t)$$

$$r(t) = m(t) + i(t)$$

$$= c(t)b(t) + i(t)$$

$$z(t) = c(t)r(t)$$

= $c^2(t)b(t) + c(t)i(t)$

$$c^2(t) = 1$$
 for all t

$$z(t) = b(t) + c(t)i(t)$$

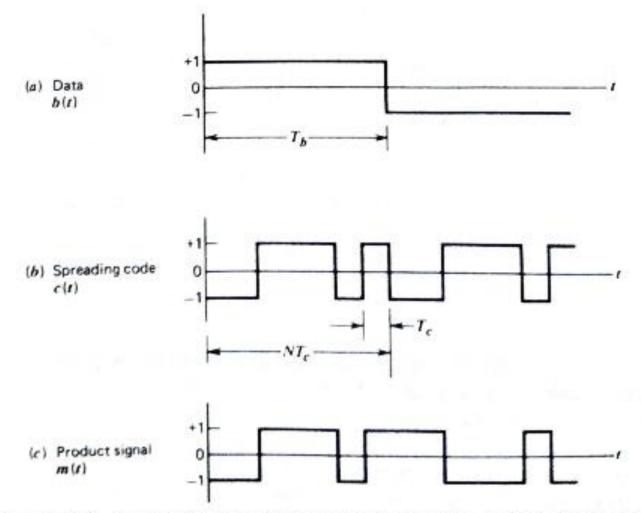
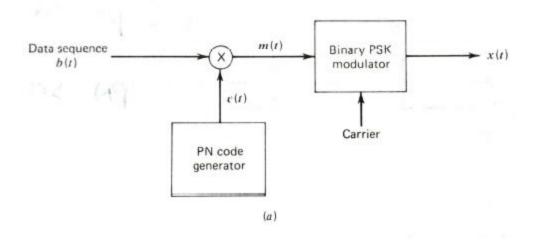
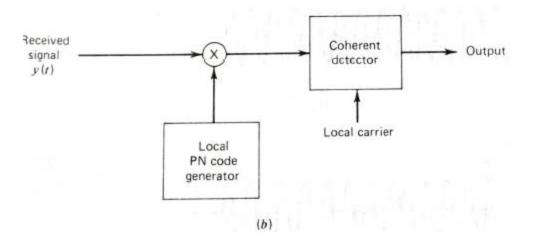
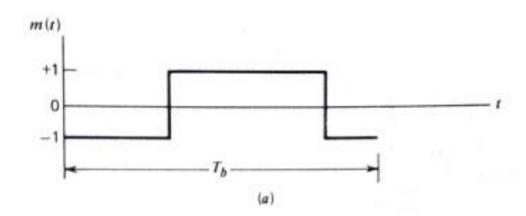


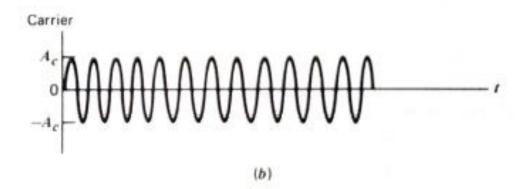
Figure 9.4 Illustrating the waveforms in the transmitter of Fig. 9.3a.

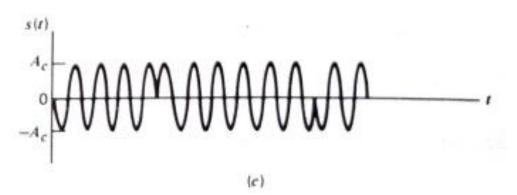
Direct sequence spread spectrum



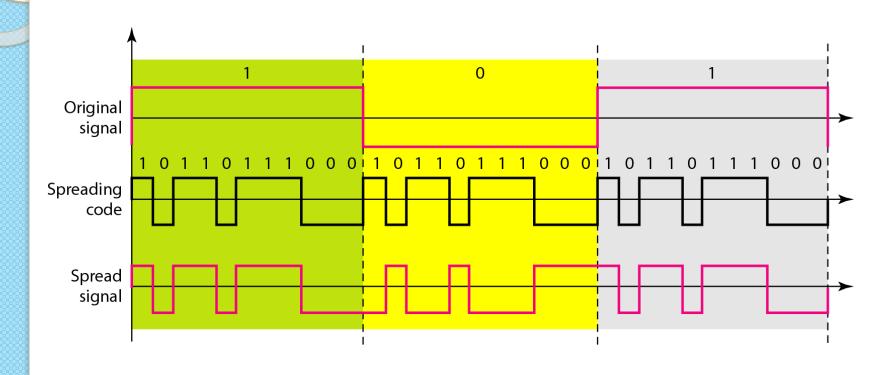


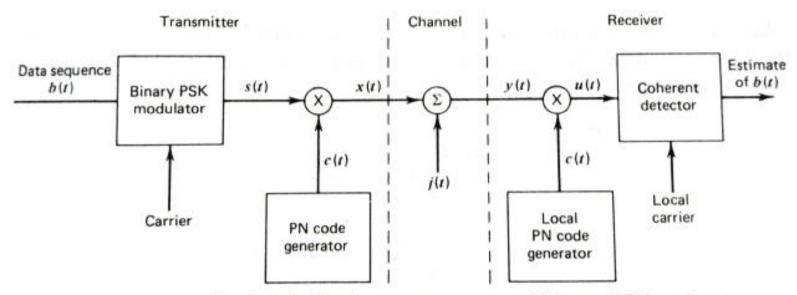






Example





Model of direct-sequence spread binary PSK system.

$$y(t) = x(t) + j(t)$$
 $u(t) = c(t) y(t)$
= $c(t)s(t) + j(t)$ $= c^2(t) s(t) + c(t) j(t)$
= $s(t) + c(t) j(t)$,

$$c^2(t) = 1$$
 for all t

Signal space dimensionality and processing gain

Signal space representation of the transmission and interfering signal

Orthogonal basis functions

$$\phi_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}} \cos(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}} \sin(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$k = 0, 1, \dots, N-1$$

Tc= Chip duration, N=Number of chips per bit

The transmitted signal X(t) is

$$x(t) = c(t)s(t)$$

$$= \pm \sqrt{\frac{2E_b}{T_b}} c(t) \cos(2\pi f_c t)$$

$$= \pm \sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \phi_k(t) \qquad 0 \le t \le T_b$$

$$j(t) = \sum_{k=0}^{N-1} j_k \phi_k(t) + \sum_{k=0}^{N-1} \tilde{j}_k \tilde{\phi}_k(t) \qquad 0 \le t \le T_b$$

$$(SNR)_I = \frac{E_b/T_b}{J} \qquad (SNR)_O = \frac{2T_b}{T_c} (SNR)_I \qquad (9.36)$$

It is customary practice to express signal-to-noise ratios in decibels. We may thus write Eq. 9.36 in the equivalent form

$$10 \log_{10}(SNR)_O = 10 \log_{10}(SNR)_I + 3 + 10 \log_{10}(PG), dB$$
 (9.37)

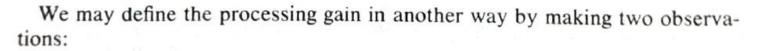
where

$$PG = \frac{T_b}{T_c} \tag{9.38}$$

spread factor $N = T_b/T_c$

Average Interference power of j(t)
$$J = \frac{1}{T_b} \int_0^{T_b} j^2(t) dt$$

The 3-dB term on the right side of Eq. 9.37 accounts for the gain in SNR that is obtained through the use of coherent detection (which presumes exact knowledge of the signal phase by the receiver). This gain in SNR has nothing to do with the use of spread spectrum. Rather, it is the last term, $10 \log_{10}(PG)$, that accounts for the gain in SNR obtained by the use of spread spectrum. The ratio PG, defined in Eq. 9.38, is therefore referred to as the processing gain. Specifically, it represents the gain achieved by processing a spread-spectrum signal over an unspread signal. Note that both the processing gain PG and the spread factor N (i.e., PN sequence length) equal the ratio T_b/T_c . Thus the longer we make the PN sequence (or, correspondingly, the smaller the chip time T_c is), the larger will the processing gain be.



1. The bit rate of the binary data entering the transmitter input is given by

$$R_b = \frac{1}{T_b}$$

2. The bandwidth of the PN sequence c(t), defined in terms of the main lobe of its spectrum, is given by

$$W_c = \frac{1}{T_c}$$

Note that both R_b and W_c are baseband parameters. Hence, we may reformulate the processing gain of Eq. 9.38 as

$$PG = \frac{W_c}{R_b}$$

Probability of Error

BPSK

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_c \simeq \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{JT_c}} \right)$$

Antijam characteristics

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

$$E_b = PT_b$$

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right) \left(\frac{P}{J}\right)$$

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

 $(Jamming margin)_{dB} = (Processing gain)_{dB} - 10 log_{10} \left(\frac{E_b}{N_0}\right)_{min}$

EXAMPLE 3

A spread-spectrum communication system has the following parameters:

Information bit duration, $T_b = 4.095 \text{ ms}$

PN chip duration, $T_c = 1 \mu s$

Hence, using Eq. 9.38, we find that the processing gain is

$$PG = 4095$$

Correspondingly, the required PN sequence is N = 4095, and the feedback shift length is m = 12.

For a satisfactory reception, we may assume that the average probability of error is not to exceed 10^{-5} . From the formula for a coherent binary PSK receiver, we find that $E_b/N_0=10$ yields an average probability of error equal to 0.387×10^{-5} . Hence, using this value for E_b/N_0 , and the value calculated for the processing gain, we find from Eq. 9.49 that the jamming margin is

$$(Jamming margin)_{dB} = 10 log_{10}4095 - 10 log_{10}(10)$$

= 36.1 - 10
= 26.1 dB

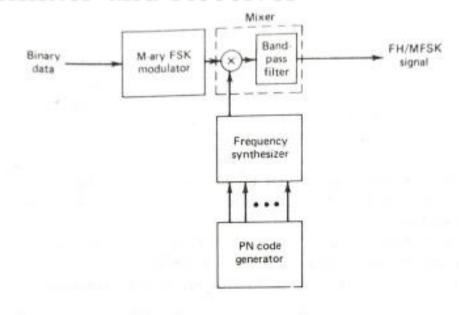


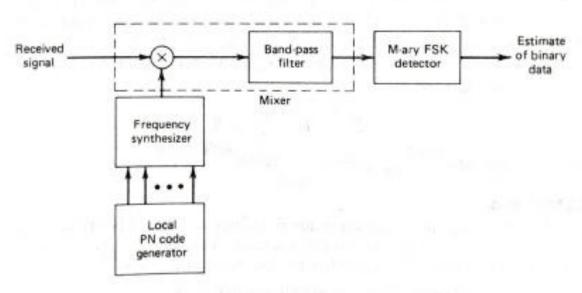
- The type of spread spectrum in which the carrier hops randomly from one frequency to another is called frequency- hop(FH) spread spectrum.
- A common modulation format for FH systems is that of M-ary frequency shift keying(MFSK). The combination is referred as FH/MFSK

Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to consider the rate at which the hops occur. In this context, we may identify two basic (technology-independent) characterizations of frequency hopping:

- 1. Slow-frequency hopping, in which the symbol rate R_s of the MFSK signal is an integer multiple of the hop rate R_h . That is, several symbols are transmitted on each frequency hop.
- 2. Fast-frequency hopping, in which the hop rate R_h is an integer multiple of the MFSK symbol rate R_s . That is, the carrier frequency will change or hop several times during the transmission of one symbol.

Frequency hop spread M-ary frequency shift keying Transmitter and Receiver





Slow Frequency Hopping

EXAMPLE 4

Figure 9.9a illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is $2^4 - 1 = 15$. The FH/MFSK signal has the following parameters:

Number of bits per MFSK symbol	K = 2
Number of MFSK tones	$M = 2^K =$

Length of PN segment per hop k = 3

Total number of frequency hops $2^k = 8$

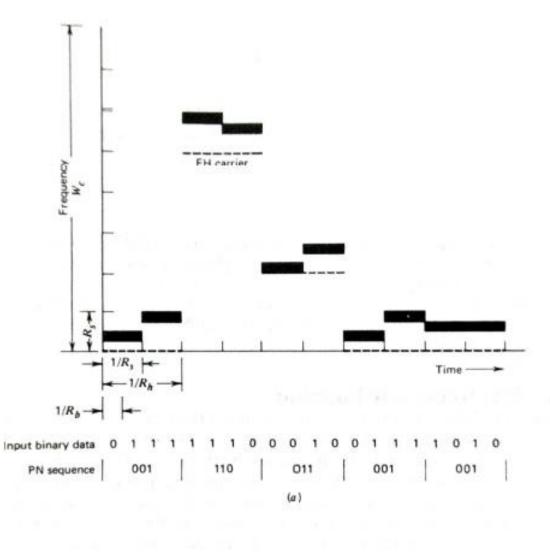
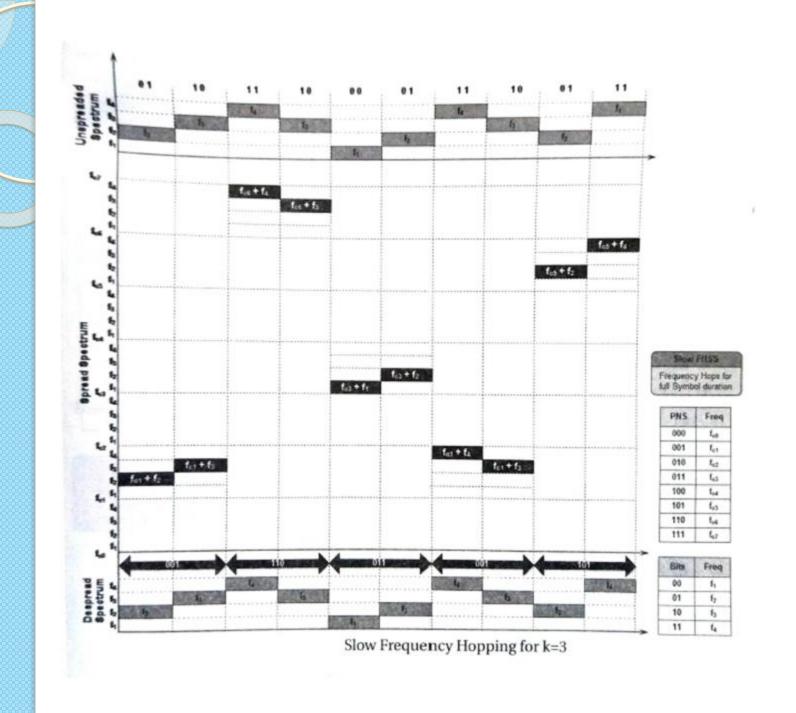




Figure 9.9 Illustrating slow-frequency hopping.





EXAMPLE 5

Figure 9.10a illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time. The signal has the following parameters:

Number of bits per MFSK symbol	K = 2
Number of MFSK tones	$M=2^K=4$
Length of PN segment per hop	k = 3
Total number of frequency hops	$2^k = 8$

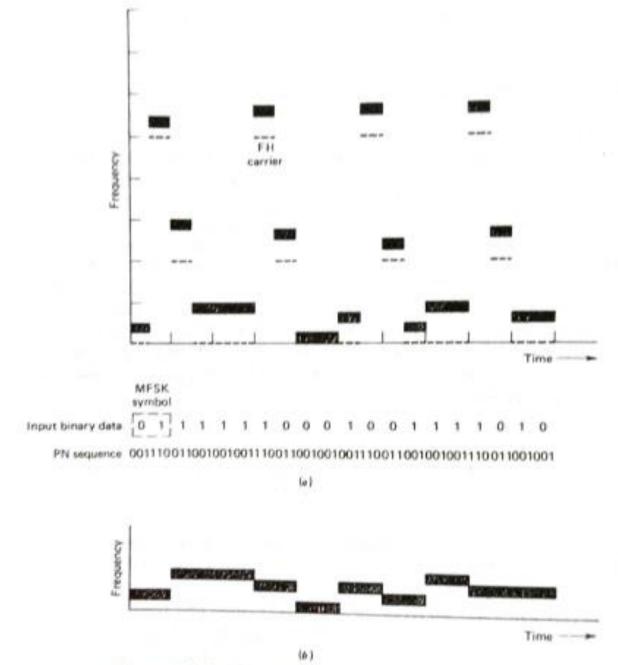
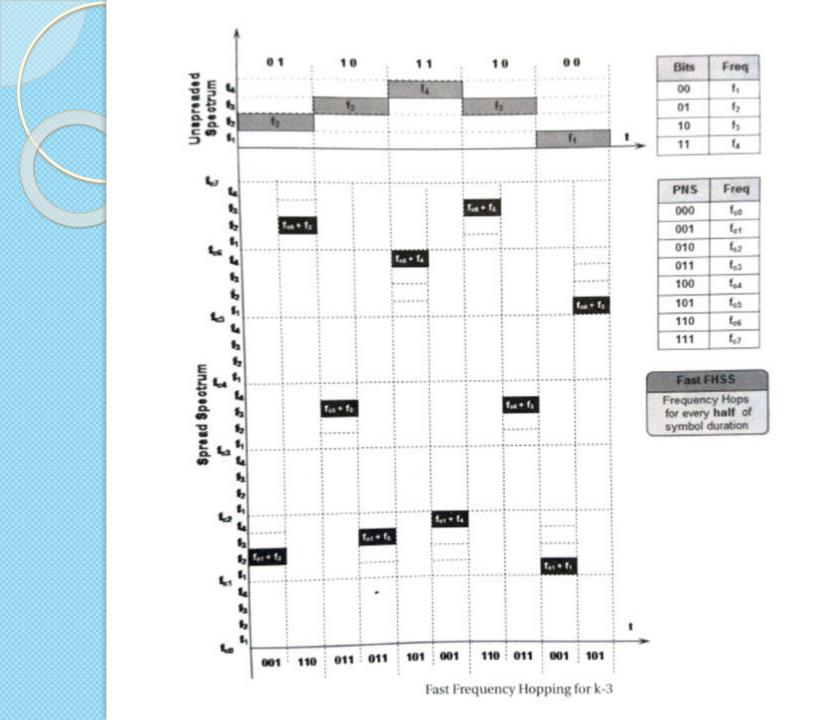
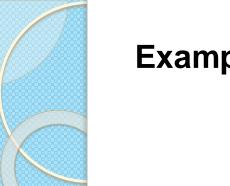


Figure 9.10 Illustrating fast-frequency hopping.





Example

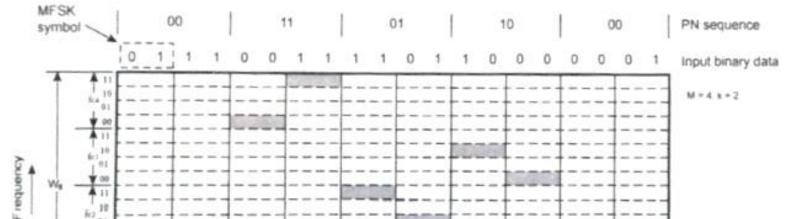
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K = 2Number of bits per MFSK symbol

 $M = 2^K = 4$ Number of MFSK tones

Length of PN segment per hop k = 2

Total number of frequency hops $2^{k} = 4$



Time

Slow Frequency Hop Spread Spectrum for k=2



Number of bits per MFSK symbol

K = 2

Number of MFSK tones

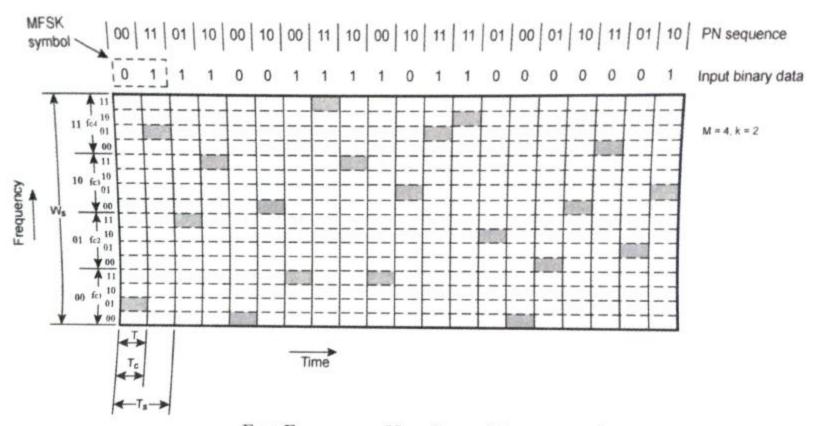
 $M=2^K=4$

Length of PN segment per hop

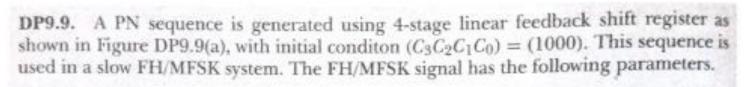
k = 2

Total number of frequency hops

 $2^{k} = 4$



Fast Frequency Hop Spread Spectrum for k=2



Number of bits per MFSK symbol K = 2Number of MFSK tones $M = 2^K = 4$ Length of PN segment per hop k = 3Total number of frequency hops $2^k = 8$

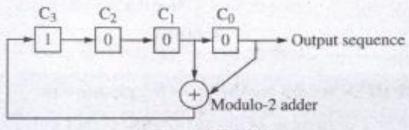


Figure DP9.9(a)

Determine the following:

- a) Period of the PN sequence.
- b) PN sequence for one periodic length.
- c) Illustrate the variation of the frequency of FH/MFSK signal for one complete period of the PN sequence. Assume that the carrier hops to a new frequency after transmitting two MFSK symbols or four information bits. Assume binary data sequence to be 100011010001111111001.
- d) Sketch the variation of dehopped frequency with time.



Solution:

- a) The period of the PN sequence is $2^4 1 = 15$.
- b) For the initial condition shown, the PN sequence is obtained by writing all the successive states of the shift register (SR), for one period. Table DP9.2 gives the successive states, the fed back bit and the output bit.
- ii) The PN sequence of one periodic length is 000100110101111. The carrier is hopped to a new frequency after transmitting two MFSK symbols or four information bits. Number of bits per MFSK symbol K = 2. There are hence four MFSK frequencies corresponding to dibits 00, 01, 10 and 11. Length of PN segment per hop k = 3.

Table DP9.2					
States of SR		SR	Fed back bit	Output bi	
C_3	C_2	C_1	C_0	$C_3 = C_1 \oplus C_0$	C_0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	1	0
1	0	0	1	1	1
1	1	0	0	0	0
0	1	1	0	1	0
1	0	1	1	0	1
0	1	0	1	1	1
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	ô
1	1	1	1	0	1
0	1	1	1	0	î
0	0	1	1	0	1
0	0	0	1	1	i
1	0	0	0	Repeats	Repeats

Hence, there are $2^3=8$ hopping frequencies corresponding to each block of 3 PN sequence bits.

Let the hopping carrier frequencies corresponding to each block of 3 bits be selected as shown in Table DP9.3.

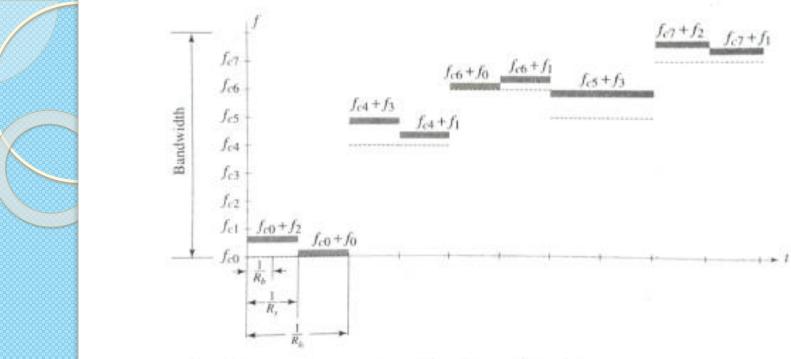
Table DP9.3

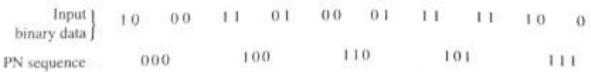
PN sequence segment	Hopping carrier frequency in Hz
000	f _c o
001	f_{c1}
010	fe2
011	fes
100	fc4
101	fes
110	fe6
111	fc7

The 4, MFSK tones be as shown in Table DP9.4.

Table DP9.4

Bits of MFSK symbol	MFSK tone in Hz
00	f_0
01	f_1
10	f_2
11	f_3





Transmitted frequencies versus time for the given binary data and PN sequence.

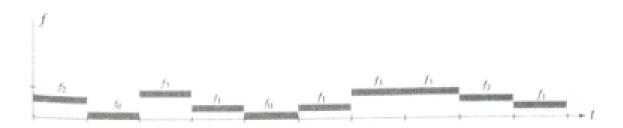
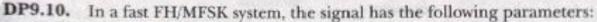


Figure DP9.9(c) Dehopped frequencies.



Number of bits per MFSK symbol: K = 2

Number of MFSK tones: $M = 2^K = 4$

Length of PN segment per hop: k = 3

Total number of frequency hops, $2^3 = 8$

Number of hops per MFSK symbol = 2

Period of PN sequence: L = 15.

i) Determine the relation between bit rate and chip rate.

Sketch the variation of frequency of the transmitted signal with time.
 Assume binary data sequence to be 01101100 and one period of PN sequence is 111100010011010.

iii) Sketch the dehopped MFSK signal.

Solution:

i) In a fast FH/MFSK, there are multiple hops per MFSK symbol. Hence in a fast FH/MFSK system, each hop is a chip. In this example there are 2 bits/MFSK symbol and 2 hops/MFSK symbol. Hence bit rate R_b = hop rate R_h = chip rate R_c.

ii) Let the MFSK tones be denoted by f_0 , f_1 , f_2 and f_3 corresponding to MFSK symbols 00,

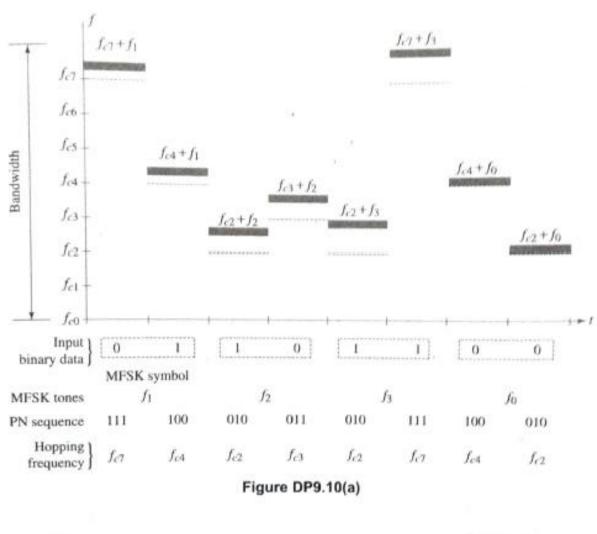
01, 10, 11, respectively.

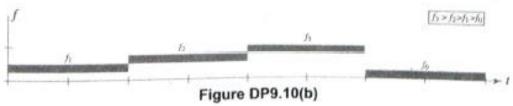
Let the hopping carrier frequencies be denoted by: f_{c0} , f_{c1} , f_{c2} , f_{c3} , f_{c4} , f_{c5} , f_{c6} and f_{c7} which correspond to the PN sequence segments 000, 001, 010, 011, 100, 101, 110, and 111, respectively.

During a hopping interval, if carrier frequency is f_{ij} and MFSK tone is f_i , then the

transmitted frequency is $f_{ij} + f_i$.

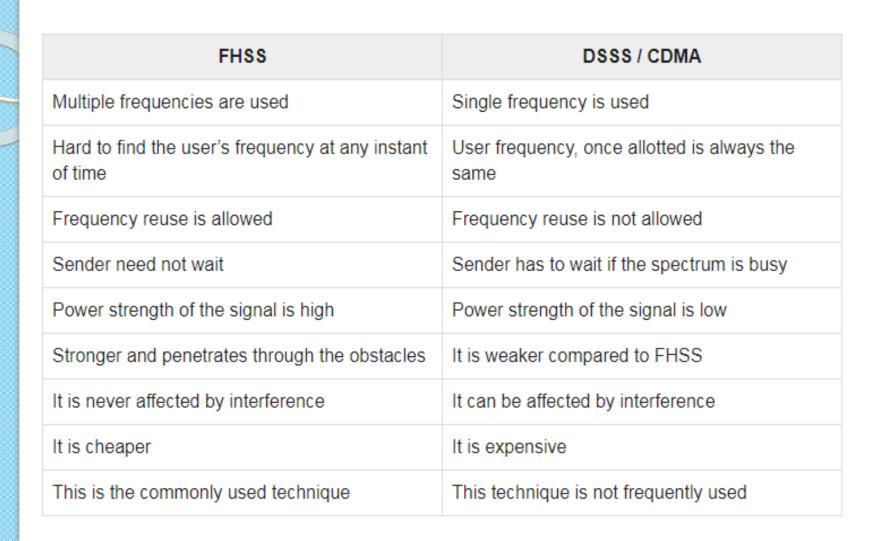
The transmitted frequency and dehopped MFSK signal are shown in Fig-





Comparsion Between Slow Frequency Hopping and Fast Frequency Hopping

Sl. No	Parameter	Slow frequency Hopping	Fast frequency Hopping
1	Definition	Multiple Symbols are transmitted in one hop.	Multiple hops are taken to transmit one symbol
2	Chip Rate	Symbol rate = chip rate	Hop rate = Chip Rate
3	R_h and R_s	$R_h < R_s$	$R_h > R_s$
4	Carrier Frequencies	One or more symbols are transmitted over same carrier frequency	One symbol is transmitter over multiple carriers in different hops.
5	Jammer Interference	This signal can be detected by jammer if carrier frequency in one hop is known	This signal is difficult to detect since one symbol is transmitted on multiple carrier frequencies.



Comprasion Between Direct Sequence Spread Spectrum and Fast Hopping Spread Spectrum

Sl. No	Parameter	DS - SS	FH-SS
1	Definition	PN Sequence of large Bandwidth is multiplied with Narrowband data signal	Data bits are transmitted in different frequency slots which are changed by PN Sequence.
2	Spectrum of Signal	Data Sequence is spread over entire B.W of spread spectrum signal	Data Sequence is spread over small frequency slots of spread spectrum signal
3	Chip rate	$R_c = 1/T_c$	$R_c = \max(R_h, R_s)$
4	modulator	BPSK	M-ary FSK
5	Effect Of Distance	System is distance relative	Less distance effect
6	Acquisition time	Long	Short



- Multipath Suppression
- Code Division Multiple Access

To accomplish CDMA, spread spectrum is always used.* In particular, each user is assigned a code of its own, which performs the direct-sequence or frequency-hop spread-spectrum modulation. The design of the codes has to cater for two provisions:

- Each code is approximately orthogonal (i.e., has low cross-correlation) with all the other codes.
- The CDMA system operates asynchronously, which means that the transition times of a user's data symbols do not have to coincide with those of the other users.

The second requirement complicates the design of good codes for CDMA.†

The use of CDMA offers three attractive features over TDMA:

- CDMA does not require an external synchronization network, which is an essential feature of TDMA.
- CDMA offers a gradual degradation in performance as the number of users is increased. It is therefore relatively easy to add new users to the system.
- CDMA offers an external interference rejection capability (e.g., multipath rejection or resistance to deliberate jamming).

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