

\* A linear time-invariant digital filter is specified by the following transfer function.

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{[z - (\frac{1}{2} + j\frac{1}{2})][z - (\frac{1}{2} - j\frac{1}{2})][z - j\frac{1}{4}][z + j\frac{1}{4}]}$$

Realize the system in the following forms.

a) direct form - I and

b) direct form - II

Sol<sup>n</sup>

$$H(z) = \frac{1 - 2z^{-1} - z^{-2} + 2z^{-3}}{1 - z^{-1} + \frac{9}{16}z^{-2} - \frac{1}{16}z^{-3} + \frac{1}{32}z^{-4}}$$

(a) direct form - I realization

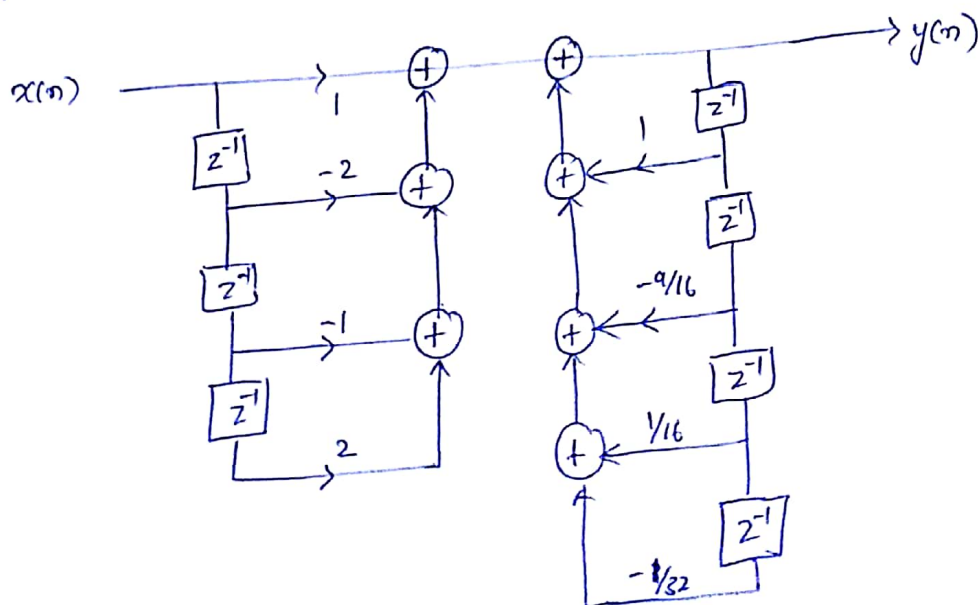
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} - z^{-2} + 2z^{-3}}{1 - z^{-1} + \frac{9}{16}z^{-2} - \frac{1}{16}z^{-3} + \frac{1}{32}z^{-4}}$$

$$Y(z) - z^{-1}Y(z) + \frac{9}{16}z^{-2}Y(z) - \frac{1}{16}z^{-3}Y(z) + \frac{1}{32}z^{-4}Y(z) = X(z) - 2z^{-1}X(z) - z^{-2}X(z) + 2z^{-3}X(z)$$

Inverse Z-transform.

$$y(n) - y(n-1) + \frac{9}{16}y(n-2) - \frac{1}{16}y(n-3) + \frac{1}{32}y(n-4) = x(n) - 2x(n-1) - x(n-2) + 2x(n-3)$$

$$y(n) = y(n-1) - \frac{9}{16}y(n-2) + \frac{1}{16}y(n-3) - \frac{1}{32}y(n-4) + x(n) - 2x(n-1) - x(n-2) + 2x(n-3)$$



(b)

$$V(z) = X(z) H_1(z)$$

$$V(z) = X(z) \left( \frac{1}{1 - z^{-1} + \frac{9}{16} z^{-2} - \frac{1}{16} z^{-3} + \frac{1}{32} z^{-4}} \right)$$

$$V(z) = z^{-1} V(z) + \frac{9}{16} z^{-1} V(z) - \frac{1}{16} z^{-3} V(z) + \frac{1}{32} z^{-4} V(z) + X(z)$$

inverse z-transform.

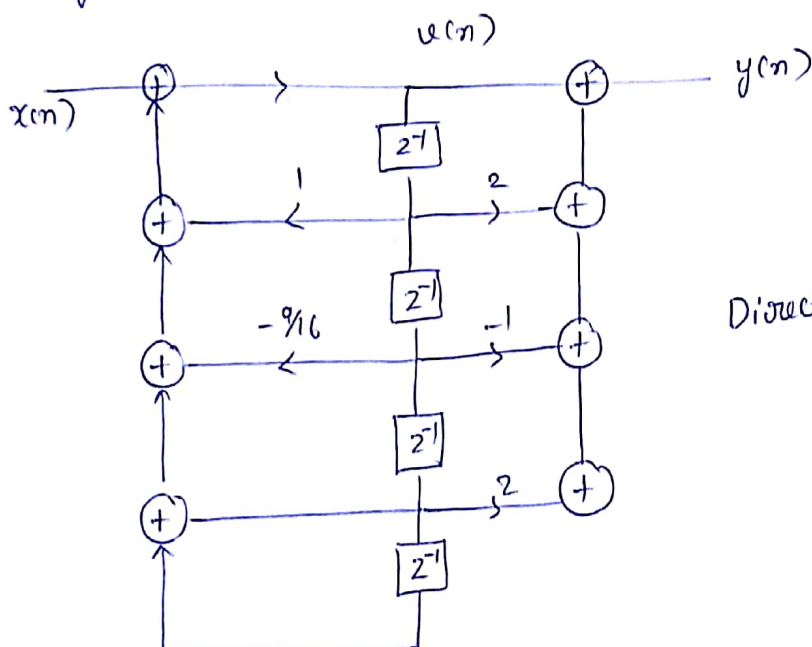
$$V(n) = V(n-1) + \frac{9}{16} V(n-2) - \frac{1}{16} V(n-3) - \frac{1}{32} V(n-4) + x(n)$$

$$v(n) = v(n-1) - \frac{9}{16} v(n-2) + \frac{1}{16} v(n-3) - \frac{1}{32} v(n-4) + x(n)$$

$$Y(z) = V(z) H_2(z)$$

$$= V(z) (1 - 2z^{-1} - z^{-2} + 2z^{-3})$$

$$y(n) = v(n) - 2v(n-1) - v(n-2) + 2v(n-3)$$



Direct form - II realization.

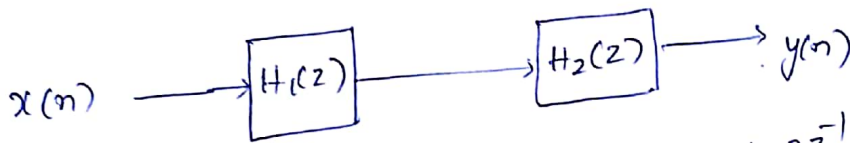
⑧ obtain a cascade realization for a system having the following system function.

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{(z-\frac{1}{2}-j\frac{1}{2})(z-\frac{1}{2}+j\frac{1}{2})(z-j\frac{1}{4})(z+j\frac{1}{4})}$$

Sol<sup>n</sup>

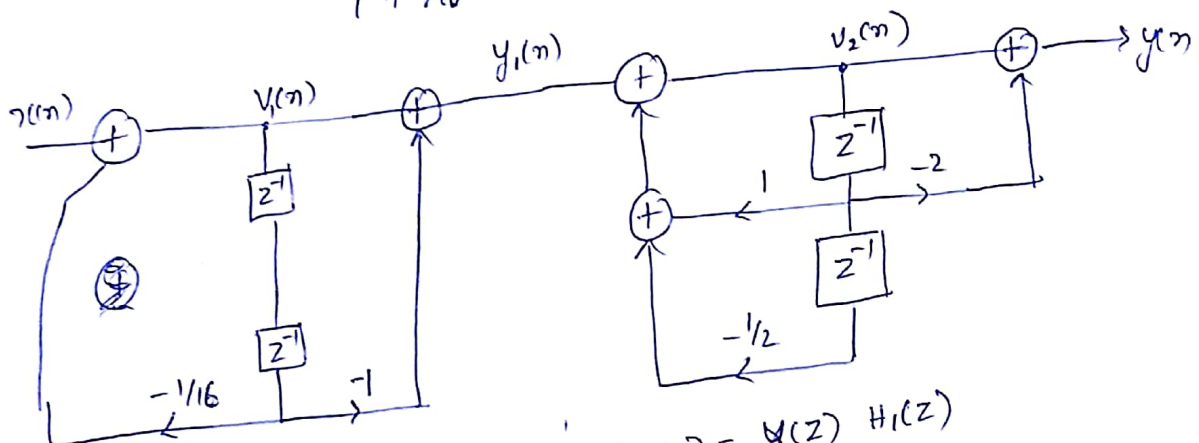
$$H(z) = \frac{(z^2-1)(z^2-2z)}{(z^2+\frac{1}{16})(z^2-z+\frac{1}{2})}$$

$$= \frac{(1-z^{-2})(1-2z^{-1})}{(1+\frac{1}{16}z^{-2})(1-z^{-1}+\frac{1}{2}z^{-2})}$$



$$H_1(z) = \frac{1-z^{-2}}{1+\frac{1}{16}z^{-2}}$$

$$H_2(z) = \frac{1-2z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$



$$V_1(z) = X(z) H_1(z)$$

$$V_1(n) \Rightarrow X(z) \frac{1}{1+\frac{1}{16}z^{-2}}$$

$$V_1(n) = x(n) - \frac{1}{16}x(n-2)$$

$$y_1(n) \Rightarrow V_1(z) (1-z^{-2})$$

$$y_1(n) = v_1(n) - v_1(n-2)$$

$$V_2(z) = Y_1(z) H_2(z)$$

$$= Y_1(z) \frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

$$v_2(n) = y_1(n) + y_1(n-1) - \frac{1}{2}v_2(n-2)$$

$$y(n) = V_2(z) H_2(z)$$

$$= V_2(z) (1-2z^{-1})$$

$$y(n) = v_2(n) - 2v_2(n-1)$$