Information

- The output of a discrete information source is a message that consists of a sequence of symbols.
- Information of an event depends only on its probability of occurrence and is not dependent on its content.
- The randomness of happening of an event and the probability of its prediction as a news is known as information.

- The message associated with the least likelihood event contains the maximum information.
- Ex: A trip to Miami, Florida from Minneapolis in the winter time,
  - **mild and sunny day,**
  - **cold day,**
  - **possible snow flurries.**
- Let information source emits one of ‘q’ possible messages $m_1, m_2 \ldots\ldots m_q$ with $p_1, p_2 \ldots\ldots p_q$ as their probs. of occurrence.

- The information content of the k^{th} message is $I(m_k)\alpha\frac{1}{p_k}$
- Information is a non-negative quantity: $I(p) \geq 0$.
- If an event has probability 1, we get no information from the occurrence of the event $I(1) = 0$.

$$I(m_k) > I(m_j); \quad \text{if } p_k < p_j$$
$$I(m_k) \rightarrow 0\,(m_j); \quad \text{if } p_k \rightarrow 1$$
$$I(m_k) \geq 0; \quad \text{when } 0 < p_k < 1$$

- If two independent events occur (whose joint probability is the product of their individual probabilities), then the information we get from observing the events is the sum of the two information: $I(p_1 * p_2) = I(p_1) + I(p_2)$.

$$I(m_k) = \log\left(\frac{1}{p_k}\right)$$

- Relationship between Bits, Hartleys and Nats
  - **Natural logarithm base : 'nat'**
  - **Base - 10 : Hartley / decit**
  - **Base - 2 : bit**
    - 1 Hartley =2.303 Nats
    - 1 Nats=0.434 Hartley
    - 1 Hartley=3.32 bits
    - 1 bit=0.301 Hartley
    - 1 Nat=1.44 bits
    - 1 bits=0.693 Nats

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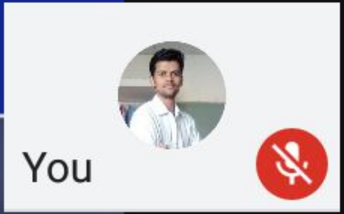

A source puts out one of five possible messages during each message interval. The probs. of these messages are $p_1 = \frac{1}{2}$; $p_2 = \frac{1}{4}$; $p_1 = \frac{1}{4}$: $p_1 = \frac{1}{16}$, p_5 $\frac{1}{16}$

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What is the information content of these messages?

$$I(m_1) = -\log_2\left(\frac{1}{2}\right) = 1 \text{ bit}$$

$$I(m_2) = -\log_2\left(\frac{1}{4}\right) = 2 \text{ bits}$$

$$I(m_3) = -\log\left(\frac{1}{8}\right) = 3 \text{ bits}$$

$$I(m_4) = -\log_2\left(\frac{1}{16}\right) = 4 \text{ bits}$$

$$I(m_5) = -\log_2\left(\frac{1}{16}\right) = 4 \text{ bits}$$

- HomeWork:

The binary symbols 0’s and 1’s are transmitted with probabilities $\frac{1}{4}$ and $\frac{3}{4}$ respectively. Calculate the amount of information.

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Ans: 2 bits and 0.415 bits

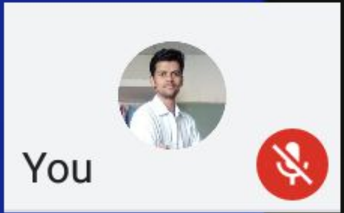
Entropy

Let $p_1, p_2 \ldots\ldots p_q$ be the probabilities of occurrence of the *M* symbols.

In a long message sequence of length *N* symbols,

- **symbol s_1 will occur p_1N times**
- **symbol s_2 will occur p_2N times**
- **symbol s_M will occur p_MN times**

The information content of the ith symbol is



- p_1N number of messages of type s_1 contains $p_1 N \log_2\left(\frac{1}{p_1}\right)$bits of information.
- p_2N number of messages of type s_1 contains $p_2 N \log_2\left(\frac{1}{p_2}\right)$bits bits of information.
- $I_{total} =$
  $p_1 N \log_2\left(\frac{1}{p_1}\right) + p_2 N \log_2\left(\frac{1}{p_2}\right) +$
  $\cdots. + p_M N \log_2\left(\frac{1}{p_M}\right)$

  - $\boldsymbol{I_{total} = N \sum_{i=1}^{M} p_i \, log_2\left(\frac{1}{p_i}\right)}$ **bits**

- The average information *per* symbol is obtained by dividing the total information content of the message by the number of symbols in the message.
- $Entropy = H = \frac{I_{total}}{N} = \sum_{i=1}^{M} p_i \log_2\left(\frac{1}{p_i}\right)$ bits/symbol
- **Average information rate:**
- If the symbols are emitted by source at a fixed time rate r_s, then the average information rate R_s is given by $\boldsymbol{R_s = r_s * H}$ bits/sec

- Consider a discrete memoryless source with a source alphabet A= ($s_{0,}s_{1,}s_2$) with respective probabilities $p_0 = \frac{1}{4}$, $p_1 = \frac{1}{4}$,$p_2 = \frac{1}{2}$. Find the entropy of the source.

Solution: By definition, the entropy of a source is given by

$$H = \sum_{i=1}^{M} p_i \log \frac{1}{p_i} \text{ bits/ symbol}$$

H for this example is

$$H(A) = \sum_{i=0}^{2} p_i \log \frac{1}{p_i}$$

Substituting the values given, we get

$$H(A) = p_o \log \frac{1}{P_o} + P_1 \log \frac{1}{p_1} + p_2 \log \frac{1}{p_2}$$

$$= ¼ \log_2 4 + ¼ \log_2 4 + ½ \log_2 2$$

$$= \left(\frac{3}{2}\right) = 1.5 \text{ bits}$$

if r_s = 1 per sec, then

H′ (A) = r

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2. **An analog signal is band limited to B Hz, sampled at the Nyquist rate, and the samples are quantized into 4-levels. The quantization levels Q_1, Q_2, Q_3, and Q_4 (messages) are assumed independent and occur with probs.**

$$P_1 = P_2 = \frac{1}{8} \text{ and } P_2 = P_3 = \frac{3}{8}.$$ **Find the information rate of the source.**

Solution: By definition, the average information H is given by

$$H = p_1 \log \frac{1}{p_1} + p_2 \log \frac{1}{p_2} + p_3 \log \frac{1}{p_3} + p_4 \log \frac{1}{p_4}$$

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2. **An analog signal is band limited to B Hz, sampled at the Nyquist rate, and the samples are quantized into 4-levels. The quantization levels Q_1, Q_2, Q_3, and Q_4 (messages) are assumed independent and occur with probs.**

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Solution: By definition, the average information H is given by

$$H = p_1 \log \frac{1}{p_1} + p_2 \log \frac{1}{p_2} + p_3 \log \frac{1}{p_3} + p_4 \log \frac{1}{p_4}$$

Substituting the values given, we get

$$H = \frac{1}{8} \log 8 + \frac{3}{8} \log \frac{8}{3} + \frac{3}{8} \log \frac{8}{3} + \frac{1}{8} \log 8$$

$$= 1.8 \text{ bits/ message.}$$

Information rate of the source by definition is

$$R = r_s \, H$$

$$R = 2B, (1.8) = (3.6\ B) \text{ bits/sec}$$

- The collector voltage of a certain circuit is to lie between -5 and -12 volts. The voltage can take on only these values -5, -6, -7, -9, -11, -12 volts with respective probabilities 1/6, 1/3, 1/12,1/12,1/6,1/6. This voltage is recorded with a pen recorder. Determine the average self information associated with the record in terms of bits/level.
- A discrete source emits one of six symbols once every m-sec. The symbol probabilities are ½, ¼, 1/8, 1/16, 1/32 and 1/32 respectively. Find the source entropy and information rate.