$P(a_i, b_j) = P(a_i/b_j)P(b_j)$

Properties:

Consider the source alphabet A=$(a_1, a_2, a_3 \dots \dots a_r)$ and output alphabet B=$(b_1, b_2, b_3 \dots \dots b_s)$

- The source entropy is given by $H(A) = \sum_{i=1}^{r} P_{a_i} log_2(\frac{1}{P_{a_i}})$
- The entropy of the receiver or output is given by $H(B) = \sum_{j=1}^{s} P_{b_j} log_2(\frac{1}{P_{b_j}})$
- If all the symbols are equi-probable, then maximum source entropy is $H(A)_{max} = log_2 r$
- Conditional Entropy: The entropy of input symbols $a_1 a_2 a_3 \dots \dots a_r$ after the transmission and reception of particular output symbol b_j is defined as conditional entropy, denoted by $H(A/b_j)$

$$H\left(A/b_j\right) = \sum_{i=1}^{r} P\left(a_i/b_j\right) log_2 \frac{1}{P\left(a_i/b_j\right)}$$

- If the average value of all the conditional probability is taken as j varies from 1 to s denoted by $H(A/B) = \sum_{j=1}^{s} P(b_j)\, H(A/b_j)$

- If the average value of all the conditional probability is taken as j varies from 1 to s denoted by $H(^A/_B) = \sum_{j=1}^{s} P(b_j)\ H\left(^A/_{b_j}\right)$

$$=\sum_{j=1}^{s}\sum_{i=1}^{r} P(b_j)\ \mathrm{P}\left(^{a_i}/_{b_j}\right) log_2 \frac{1}{\mathrm{P}\left(^{a_i}/_{b_j}\right)}$$

$\boldsymbol{H(^A/_B) = \sum_{j=1}^{s}\sum_{i=1}^{r} P(a_i, b_j)\ log_2 \frac{1}{P(^{a_i}/_{b_j})}}$ is conditional entropy of transmitter

Similarly $\boldsymbol{H(^B/_A) = \sum_{i=1}^{r}\sum_{j=1}^{s} P(a_i, b_j)\ log_2 \frac{1}{P(^{b_j}/_{a_i})}}$ is conditional entropy of receiver.

- $\boldsymbol{H(A, B) = \sum_{i=1}^{r}\sum_{j=1}^{s} P(a_i, b_j)\ log_2 \frac{1}{P(a_i, b_j)}}$ is joint conditional probability.

1.6.3 Mutual Information:

When an average amount of information H(x) is transmitted over a noisy channel, then an amount of information $H\left({}^{x}/_{y}\right)$ is last in the channel. The balance of the information at the receiver is defined as Mutual Information I(x.y)

$$I(x,y) = H(x) - H\left({}^{x}/_{y}\right)$$

$$= H(y) - H\left({}^{y}/_{x}\right)$$

$$I(x_i) = \log\left(\frac{1}{P(x_i)}\right) \text{ and } I\left({}^{x_i}/_{y_j}\right) = \log\left(\frac{1}{P\left({}^{x_i}/_{y_j}\right)}\right)$$

The difference between the above 2 is the information gained through the channel.

$$I(x_i, y_j) = \log\left(\frac{1}{P(x_i)}\right) - \log\left(\frac{1}{P\left({}^{x_i}/_{y_j}\right)}\right)$$

$$I(x_i, y_j) = \log\frac{P\left({}^{x_i}/_{y_j}\right)}{P(x_i)}$$

$$I(x_i, y_j) = \log\frac{P(x_i, y_j)}{P(x_i)P(y_j)}$$

Properties:

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1.6.4 Channel Capacity

It is known that average information content of the source is $H(X) = \sum_{i=1}^{M} p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)$. Average information per symbol going in to the channel is $R_{in} = r_s * H(X)$. Due to the error, it is not possible to reconstruct the input symbol sequence with certainity on the recovered sequence. Therefore source information is lost due to the errors.

- Therefore average rate of information transmission is given by $R_t = I(X,Y) \,.\, r_s$. Bits/sec.

- The capacity of a discrete memoryless noisy channel is defined a s maximum possible

- The capacity of a discrete memoryless noisy channel is defined a s maximum possible rate of maximum rate of transmission occurs when the source is matched to the channel.
- $\therefore C = Max(R_t)$
- $= \text{Max}[I(X,Y)\ .\ r_s]$
- $C = \text{Max}\{[H(X) - H(X/Y)]\, r_s\}$

1.6.5 Channel Efficiency

$$\%\eta_{ch} = \frac{R_t}{C} * 100$$

$$= \frac{I(X,Y)\ .\ r_s}{\text{Max}[I(X,Y)\ .\ r_s]} * 100$$

$$\%\eta_{ch} = \frac{H(X) - H(X/Y)}{Max[H(X) - H(X/Y)]} * 100$$

Redundancy $= 1 - \eta_{ch}$

1.6.6 Symmetry Channel

Symmetry channel is defined as the channel in which the channel matrix has 2nd and subsequent rows, the same elements as the first row, but in different order.

$$\%\eta_{ch} = \frac{H(X) - H(X/Y)}{Max[H(X) - H(X/Y)]} * 100$$

Redundancy $= 1 - \eta_{ch}$

1.6.6 Symmetry Channel

Symmetry channel is defined as the channel in which the channel matrix has 2nd and subsequent rows, the same elements as the first row, but in different order.

$\therefore H(Y/X) = h$, where $\rightarrow$entropy of any single row. The channel capacity with r_s=1 bits/sec is given by,

$$C = Max(R_t)$$

$$= \text{Max}[I(X, Y)]\, r_s$$

$$= \text{Max}[I(X, Y)]$$

$$= \text{Max}\left(H(Y) - H(Y/X)\right)$$

$$= \text{Max}[H(Y)] - \text{Max}[H(Y/X)]$$

$$= \text{Max}[H(Y)] - \text{Max}(h)$$

$$C = \text{Max}[H(Y)] - h$$

$= \text{Max}[H(Y)] - \text{Max}[H(Y/X)]$

$= \text{Max}[H(Y)] - \text{Max}(h)$

$C = \text{Max}[H(Y)] - h$

H(Y) is the entropy of symbol which becomes maximum if and only if all the receive symbols become equi-probable.

Since there are 's' output symbols

$\text{Max}[H(Y)] = log_2 s$

$\therefore\ C = log_2 s - h$

$\{a_1, a_2, a_3, a_4, a_5\}$ and the receiver

Ex.1: A transmitter has an alphabet containing of 5 letters $\{a_1, a_2, a_3, a_4, a_5\}$ and the receiver has an alphabet of four letters $\{b_1, b_2, b_3, b_4\}$. The joint probabilities of the system are given below. Compute different entropies of this channel.

$$P(A,B) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.10 & 0.30 & 0 & 0 \\ 0 & 0.05 & 0.10 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix} \end{array}$$

Solution:

$P(b_1) = 0.35$, $P(b_2) = 0.35$, $P(b_3) = 0.2$, $P(b_4) = 0.1$

$P(a_1) = 0.25$, $P(a_2) = 0.4$, $P(a_3) = 0.15$, $P(a_4) = 0.15$, $P(a_5) = 0.05$

$$H(A) = \sum P(a_i) \log \frac{1}{P(a_i)}$$

$$= 0.25 \log \frac{1}{0.25} + 0.4 \log \frac{1}{0.4} + 0.15 \log \frac{1}{0.15} + 0.15 \log$$

$$+ 0.05 \log \frac{1}{0.05}$$

H(A) = 2.066 bits/message-symbol

$$
\begin{array}{c} a_4 \\ a_5 \end{array}
\begin{bmatrix} 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix}
$$

Solution:

$P(b_1) = 0.35$, $P(b_2) = 0.35$, $P(b_3) = 0.2$ $P(b_4) = 0.1$

$P(a_1) = 0.25$, $P(a_2) = 0.4$ $P(a_3) = 0.15$, $P(a_4) = 0.15$, $P(a_5) = 0.05$

$$H(A) = \sum_{i=1}^{5} P(a_i) \log \frac{1}{P(a_i)}$$

$$= 0.25 \log \frac{1}{0.25} + 0.4 \log \frac{1}{0.4} + 0.15 \log \frac{1}{0.15} + 0.15 \log \frac{1}{0.}$$

$$+ 0.05 \log \frac{1}{0.05}$$

H(A) = 2.066 bits/message-symbol

$$H(B) = \sum_{j=1}^{4} P(b_j) \log \frac{1}{P(b_j)}$$

$$= 0.35 \log \frac{1}{0.35} + 0.35 \log \frac{1}{0.35} + 0.2 \log \frac{1}{0.2} + 0.1 \log \frac{1}{0.}$$

H(B) = 1.857 bits/message-symbol

$$H(A, B) = \sum_{i=1}^{5}\sum_{j=1}^{4} P(a_i, b_j) \log \frac{1}{P(a_i, b_j)}$$

$$H(B) = \sum_{j=1} P(b_j) \log \frac{1}{P(b_j)}$$

$$= 0.35 \log \frac{1}{0.35} + 0.35 \log \frac{1}{0.35} + 0.2 \log \frac{1}{0.2} + 0.1 \log \frac{1}{0.1}$$

H(B) = 1.857 bits/message-symbol

$$H(A, B) = \sum_{i=1}^{4} \sum_{j=1}^{4} P(a_i, b_j) \log \frac{1}{P(a_i, b_j)}$$

$$H(A, B) = 0.25 \log \frac{1}{0.25} + 0.1 \log \frac{1}{0.1} + 0.3 \log \frac{1}{0.3} + 0.05 \log \frac{1}{0.05}$$

$$+ 0.1 \log \frac{1}{0.1} + 0.05 \log \frac{1}{0.05} + 0.05 \log \frac{1}{0.05} + 0.1 \log \frac{1}{0.1}$$

H(A, B) = 2.666 bits/message-symbol

$$+ 0.1 \log \frac{1}{0.1} + 0.05 \log \frac{1}{0.05} + 0.05 \log \frac{1}{0.05} + 0.1 \log \ldots$$

H(A, B) = 2.666 bits/message-symbol

$$H(B/A) = H(A, B) - H(A)$$
$$= 2.666 - 2.066$$

H(B/A) = 0.6 bits/message-symbol

$$H(A/B) = H(A, B) - H(B)$$
$$= 2.666 - 1.857$$

H(A/B) = 0.809 bits/message-symbol

$$H(B/A) = H(A, B) - H(A)$$
$$= 2.666 - 2.066$$
$$\mathbf{H(B/A) = 0.6 \text{ bits/message-symbol}}$$

$$H(A/B) = H(A, B) - H(B)$$
$$= 2.666 - 1.857$$
$$\mathbf{H(A/B) = 0.809 \text{ bits/message-symbol}}$$

$$I(A, B) = H(A) - H(A/B)$$
$$= 2.066 - 0.809$$
$$\mathbf{I(A, B) = 1.257 \text{ bits/message-symbol}}$$

$$I(A, B) = H(B) - H(B/A)$$
$$= 1.857 - 0.6$$
$$\mathbf{I(A, B) = 1.257 \text{ bits/message-symbol}}$$

Ex 2: A transmitter transmits 5 symbols with probabilities 0.2, 0.3, 0.2, 0.1 and 0.2. Given the