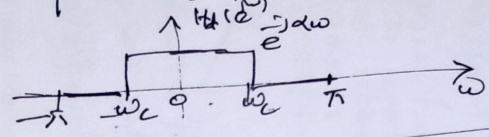
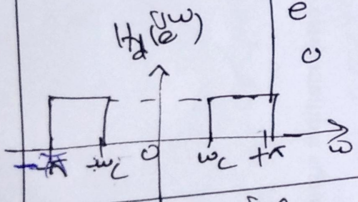
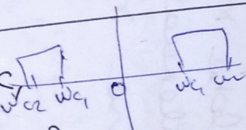
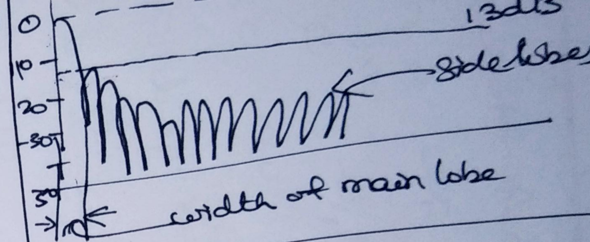
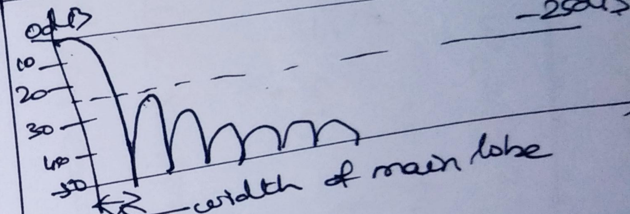
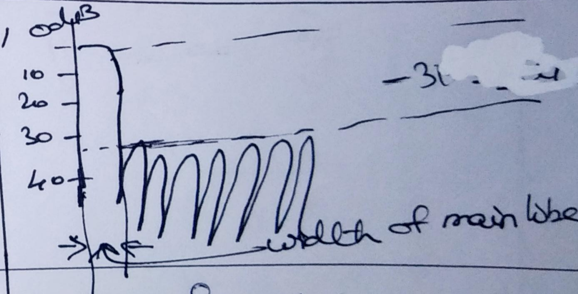
FIR FILTER DESIGN USING WINDOW

Type of filter	Ideal (~~desired~~) Frequency response $H_d(e^{j\omega})$	Ideal (~~desired~~) impulse response $h_d(n)$
		$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{+j\omega n} d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{+\omega_c} e^{-j\alpha\omega} \cdot e^{j\omega n} d\omega$
Low pass filter	$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} ; & -\omega_c \le \omega \le \omega_c \\ 0 ; & -\pi \le \omega \le -\omega_c \\ 0 ; & \omega_c \le \omega \le \pi \end{cases}$ 	$h_d(n) = \begin{cases} \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} ; & \forall\, n \ne \alpha \\ \frac{\omega_c}{\pi} ; & n = \alpha \end{cases}$
High pass filter	$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} ; & -\pi \le \omega \le -\omega_c \\ e^{-j\alpha\omega} ; & \omega_c \le \omega \le \pi \\ 0 ; & -\omega_c < \omega < +\omega_c \end{cases}$ 	$h_d(n) = \begin{cases} \frac{\sin \pi(n-\alpha) - \sin \omega_c(n-\alpha)}{\pi(n-\alpha)} ; & \forall\, n \ne \alpha \\ 1 - \left(\frac{\omega_c}{\pi}\right) ; & n = \alpha \end{cases}$
Band pass filter	 $H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} ; & -\omega_{c_2} \le \omega \le -\omega_{c_1} \\ e^{-j\alpha\omega} ; & +\omega_{c_1} \le \omega \le +\omega_{c_2} \\ 0 ; & -\pi \le \omega \le -\omega_{c_1} \\ 0 ; & -\omega_{c_1} \le \omega < +\omega_{c_1} \\ 0 ; & \omega_{c_2} < \omega < +\pi \end{cases}$	$h_d(n) = \begin{cases} \frac{\sin \omega_{c_2}(n-\alpha) - \sin \omega_{c_1}(n-\alpha)}{\pi(n-\alpha)} ; & \forall\, n \ne \alpha \\ \frac{\omega_{c_2} - \omega_{c_1}}{\pi} ; & n = \alpha \end{cases}$

Different window sequence

Type of window	approximate width of main lobe	~~Minimum SB attenuate~~ min. stop band attenuation	window sequence	Log magnitude Response of filter
Rectangular (Boxcar)	$4\pi/N$	$-21dB$	$w_R(n) = 1$; $0 \le n \le N-1$ $= 0$ ow	
Triangular (Bartlett)	$8\pi/N$	$-25dB$	$w_T(n) = 1 - \frac{2\lvert n - \left(\frac{N-1}{2}\right)\rvert}{N-1}$; $0 \le n \le N-1$ $= 0$; ow	
Hanning	$8\pi/N$	41dB (or) $-44dB$	$w_{han}(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$; $0 \le n \le N-1$ $= 0$; otherwise	
Hamming	$8\pi/N$	or $-58dB$ 53dB	$w_{hamm}(n) = 0.54 \cdot 0.46\cos\left(\frac{2\pi n}{N-1}\right)$ for $n = 0$ to $N-1$ $= 0$; otherwise	mag of 1st side lobe $-41dB$
Blackman	$12\pi/N$	$-74dB$	$w_B(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$ for $0 \le n \le N-1$ $= 0$; ow	$-58dB$

Different window sequence

① Design a linear phase FIR filter using rectangular window by taking 7 samples of window sequence and with a cutoff freq $\omega_c = 0.2\pi$ r/sample

WKT for a LPF

$H_d(e^{j\omega}) = e^{-j\alpha\omega}$; $-\omega_c \le \omega \le \omega_c$

$= 0$ ow

for $\omega_c = 0.2\pi$ r/s & $N = 7$ $\alpha = \frac{N-1}{2} = 3$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; -0.2\pi \le \omega \le 0.2\pi \\ 0 & ow \end{cases}$$

Corresponding impulse response

$$h_d(n) = \begin{cases} \dfrac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} & \forall n \text{ except } n=\alpha \\ \dfrac{\omega_c}{\pi} & ; n=\alpha \end{cases}$$

$$\Rightarrow h_d(n) = \begin{cases} \dfrac{\sin(0.2\pi)(n-3)}{\pi(n-\alpha)} & ; \forall n \ne \alpha \\ \dfrac{0.2\pi}{\pi} & ; n=\alpha \end{cases}$$

$$\Rightarrow \boxed{h_d(n) = \{\underset{h(0)}{0.1009}, 0.1514, 0.1871, \underset{h_d(3)}{0.2}, 0.1871, 0.1514, 0.1009\}}$$

$h_d(n)$ is exhibiting symmetry about point $h(\frac{N-1}{2})$ i.e $h(3)$

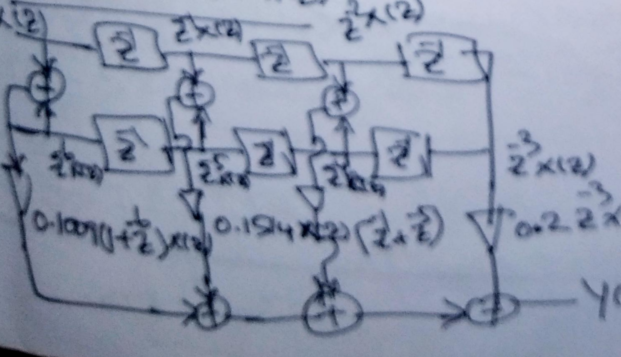
$\boxed{h(n) = h_d(n) \cdot w_R(n) \Rightarrow h_d(n) = h(n)}$

The transfer function of LP FIR filter $H(z) = Z\{h(n)\}$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$\boxed{H(z) = 0.1009(1+z^{-6}) + 0.1514(z^{-1}+z^{-5}) + 0.1871(z^{-2}+z^{-4}) + 0.2z^{-3}}$$

Structure of $H(z)$



Frequency response

When impulse response $h(n)$ is symmetric & N odd with centre of symmetry at $\frac{N-1}{2}$, the magnitude response $|H(e^{j\omega})|$ is given by

$$H(e^{j\omega}) = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos \omega n$$

ie

$$H(e^{j\omega}) = h(3) + \sum_{n=1}^{3} 2h(3-n)\cos \omega n$$

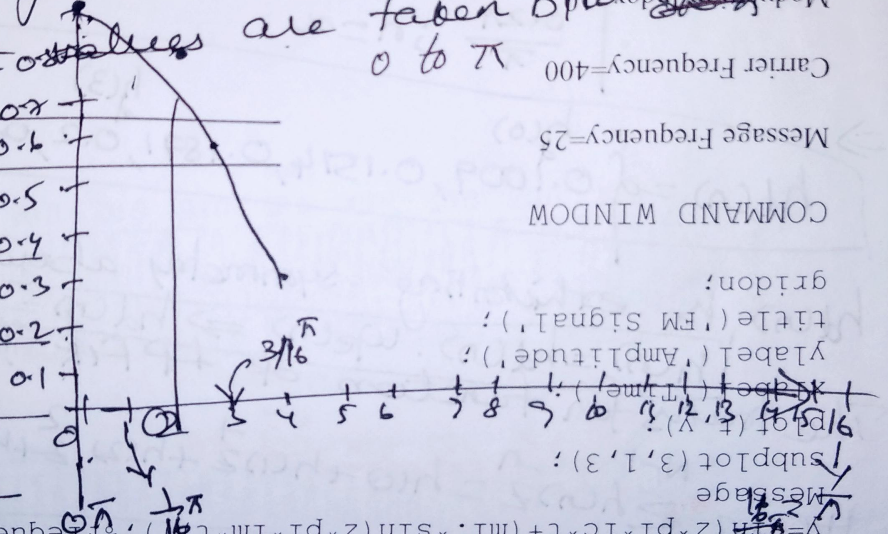
$$\boxed{H(e^{j\omega}) = h(3) + 2h(2)\cos\omega + 2h(1)\cos 2\omega + 2h(0)\cos 3\omega}$$

$$\boxed{H(e^{j\omega}) = 0.2 + 0.3742\cos\omega + 0.3020\cos 2\omega + 0.2018\cos 3\omega}$$

To draw the magnitude response any value of ω can be taken b/w 0 to π

Here 16 different values are taken

ω	$\lvert H(e^{j\omega})\rvert$
$0 \times \frac{\pi}{16}$	1.0788
$1 \times \frac{\pi}{16}$	1.0145
$2 \times \frac{\pi}{16}$	0.8370
$\frac{3\pi}{16}$	0.5876
$4\pi/16$	0.3218
$5\pi/16$	0.0940
$6\pi/16$	0.0573
$7\pi/16$	0.1188
$8\pi/16$	0.1028



at $\omega_c = 0.2\pi \approx \frac{4}{16}\pi$