Impulse Invariance Transformation(IIV)

$$h_a(t) = LT^{-1}\{H_a(s)\}$$

$\Downarrow$ sampling

$$h(n) = h_a(nT), \ t = 0,1,2,\cdots$$

Pole $S_i \Rightarrow e^{S_i T}$

Single Pole

$-S_i \Rightarrow e^{-S_i T}$

$$\boxed{H_a(s) = \sum_{i=1}^{N} \frac{A_i}{s - s_i}}$$

Applying L^{-1}

$$h_a(t) = \sum_{i=1}^{N} A_i e^{s_i t} u(t)$$

$$h(n) = h_a(t) = \sum_{i=1}^{N} A_i e^{s_i nT} u(nT)$$

Z.T

$$H(z) = \sum_{i=1}^{N} \frac{A_i}{1 - e^{s_i T} z^{-1}}$$

$Z = re^{jw}$

$S = \sigma + j\Omega$

$$z = e^{sT}$$

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IIV Contd...

Relation between s-plane and z-plane

Let $s=\sigma+j\Omega$, $z=re^{j\omega}$, we obtain

Z_i – digital Pole

S_i Analog Pole

$$r = e^{\sigma T}$$

$$\omega = \Omega T$$

$Z_i = e^{S_i T}$ $(0-1)e^{-S_i T}$

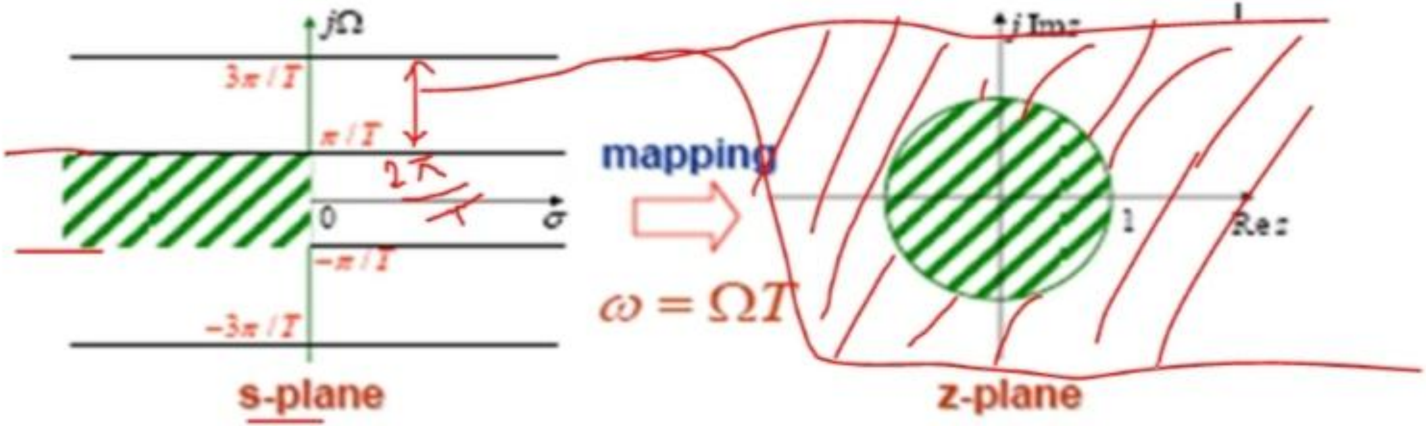
$$\begin{cases} \sigma = 0, r = 1 \\ \sigma < 0, r < 1 \\ \sigma > 0, r > 1 \end{cases}$$

A point on the **frequency axis in the s-plane** is mapped to a point on the **unit circle in the z-plane**

A point on the left-half s-plane with $\sigma_0<0$ is mapped to z-plane with $|z|<1$, i.e., the **left-half s-plane** is mapped **inside the unit circle**

Similarly, A point on the right-half s-plane with $\sigma_0>0$ is mapped to z-plane with $|z|>1$, i.e., the **right-half s-plane** is mapped **outside the unit circle**

IIV Contd...



- Thus, the impulse invariance mapping has the desired properties:
  1) Frequency axis $j\Omega$ corresponds to unit circle
  2) Stability is preserved

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IIV - Inferences

Due to sampling the mapping is *many-to-one*

The strips of length 2π/T are all mapped onto the unit circle

Only if $h_a(t)$ is a band-limited signal, no alias will occur

Hence, this method is not suitable for highpass and bandstop filters design

Assume that $H_a(s)$ has the form of

$$H_a(s) = \frac{A}{s+\alpha}$$

The corresponding signal in time-domain is

$$h_a(t) = \mathrm{ST}^{-1}\{H_a(s)\} = Ae^{-\alpha t}u(t)$$

By sampling $h_a(t)$

$$h(n) = h_a(nT) = Ae^{-\alpha nT}u(nT)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} = A\sum_{n=0}^{\infty} e^{-\alpha nT}z^{-n} = \frac{A}{1-e^{-\alpha T}z^{-1}}$$

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The Standard form

$$\frac{1}{s+P_1} \Longrightarrow \frac{1}{1-e^{-P_1T}z^{-1}} \quad \text{or} \quad \frac{z}{z-e^{-P_1T}}$$

$$\frac{s+a}{(s+a)^2+b^2} \Longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \Longrightarrow \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$H_a(s) = \frac{2}{s^2+3s+2}$$ ① $T = 1\,sec$ ② $T = 0.1\,sec$

s^2+3s+2

$s=-1=P_1$

$s=-2=P_2$

$s^2+2as+a^2$

$$H_a(s) = \frac{2}{(s+P_1)(s+P_2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A=2$

$B=-2$

$$H(z) = \frac{(0.465z^{-1})z^2}{(1-0.5032z^{-1}+0.0498z^{-2})z^2}$$

$$H_a(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

$$H(z) = \frac{2}{1-e^{-T}z^{-1}} + \frac{-2}{1-e^{-2T}z^{-1}}$$

$$H(z) = \frac{2(1-e^{-2T}z^{-1}) - 2(1-e^{-T}z^{-1})}{(1-e^{-T}z^{-1})(1-e^{-2T}z^{-1})} = \frac{\cancel{2} - 2e^{-2T}z^{-1} - \cancel{2} + 2e^{-T}z^{-1}}{1-e^{-2T}z^{-1}-e^{-T}z^{-1}+e^{-3T}z^{-2}}$$

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$0.707 \le |H(e^{j\omega})| \le 1.0 \; ; \; 0 \le |\omega| \le 0.3\pi$

$|H(e^{j\omega})| \le 0.2 \; ; \; 0.75\pi \le |\omega| \le \pi$

① Analog prototype LPF — $A_p, A_s, \Omega_p, \Omega_s$

$$A_p = 20\log(0.707) = -3\,dB$$

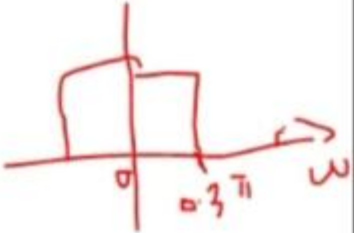
$$A_s = 20\log(0.2) = -13.97\,dB$$

$$\Omega_p = \frac{\omega_p}{T} = \frac{0.3\pi}{1} = 0.3\pi\ r/s$$

$$\Omega_s = \frac{0.75\pi}{T} = 0.75\pi\ r/s$$

IIV

$$\boxed{\omega = \Omega T}$$



$\Omega_s' =$

$\Omega_p' = 1\ r/s$

$A_p = -3.01\,dB$ al

$A_s = -13.97\,dB$

$\Omega'_p = 1\ r/s$

$\Omega'_s = 2.5\ r/s$

N

Ω_c

S_k

$H_n(S)$

$H_a(s) \xrightarrow{\text{IIV}} H(z$