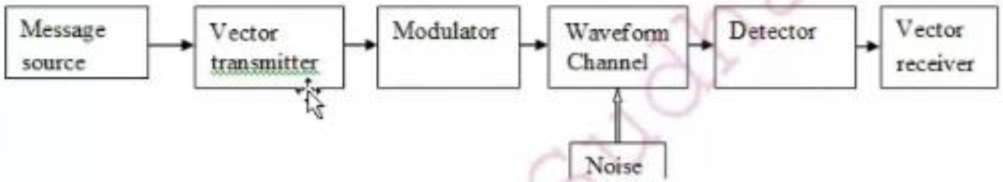
insightful tool for the study of data transmission. Thus, we need to understand signal space concepts as applied to digital communications

5.1 Conceptual model of Digital Communication:



Fig(1) Digital Communication block diagram

A message source emits one symbol every T seconds, with the symbols belonging to an alphabet of M symbols denoted by $m_1, m_2, \ldots, m_M$

- A priori probabilities $p_1, p_2, \ldots, p_M$ specify the message source output probabilities.
- If the M symbols of the alphabet are equally likely, we may express the probability that symbol m_i is emitted by the source as:

$$P_i = P(m_i) = 1/M \text{ for } i = 1,2,3 \ldots M$$

M symbols denoted by $m_1, m_2, \ldots, m_M$

- A priori probabilities $p_1, p_2, \ldots, p_M$ specify the message source output probabilities.
- If the M symbols of the alphabet are equally likely, we may express the probability that symbol m_i is emitted by the source as:

$P_i = P(m_i) = 1/M$ for $i=1,2,3\ldots M$

The transmitter takes the message source output m_i and codes it into a distinct signal $s_i(t)$ suitable for transmission over the channel.

- The signal $s_i(t)$ occupies the full duration T allotted to symbol m.
- Most important, $s_i(t)$ is a real-valued energy signal (i.e., a signal with finite energy), as shown by:

$$E_i = \int_0^T S_i^2(t)\,dt \quad i=1,2,\ldots M$$

The channel is assumed to have two characteristics:

1. The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of signal si(t) with negligible or no distortion.
2. The channel noise, w(t), is the sample function of a zero mean white Gaussian noise process.

We refer to such a channel as an additive white Gaussian noise (AWGN) channel. Accordingly, we may express the received signal x(t) as

1. The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of signal si(t) with negligible or no distortion.

2. The channel noise, w(t), is the sample function of a zero mean white Gaussian noise process. We refer to such a channel as an additive white Gaussian noise (AWGN) channel. Accordingly, we may express the received signal x(t) as

$$X(t)=S_i(t)+w(t) \quad 0 \le t \le T,\ i = 1,,2,3...M$$

The receiver has the task of observing the received signal x(t) for a duration of T seconds and making a best estimate of the transmitted signal $s_i(t)$ or, equivalently, the symbol $\boldsymbol{m_i}$. However, owing to the presence of channel noise, this decision-making process is statistical in nature, with the result that the receiver will make occasional errors.

- The requirement is therefore to design the receiver so as to minimize the average probability of symbol error, defined as:

$$P_e = p(\hat{m} \neq m_i)$$

5.2 Geometric Representation of Signals

$m_1 \; m_2 \; m_3 \ldots\ldots m_M$

$\phi_1(t) \; \phi_2(t) \ldots\ldots \phi_N(t)$

$N \leq M$

1

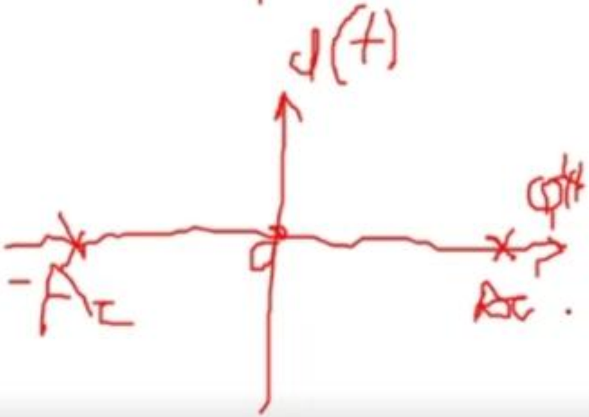
0

* M_{avg} is 1

$x_1(t) = A_c \cos \omega t$

$x_2(t) = -A_c \cos \omega_0 t$ $= 0$

$\phi_1(t) = \cos \omega_0$



The essence of geometric representation of signals is to represent any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \le M$. That is to say, given a set of real-valued energy signals $s_1(t)$, $s_2(t)$, ..., $s_M(t)$, each of duration T seconds, we write

$$s_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t), \qquad \begin{cases} 0\le t\le T \\ i=1,2,\dots,M \end{cases}$$

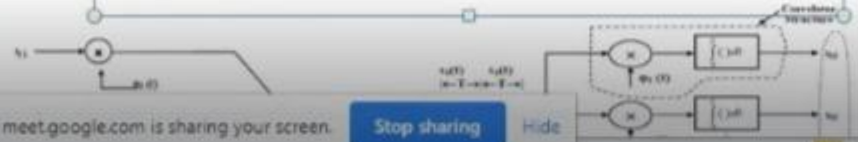
Where the coefficients of the expansion are defined by:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \qquad \begin{cases} i=1,2,\dots,M \\ j=1,2,\dots,N \end{cases}$$

The real-valued basis functions are orthonormal which means

$$s_{ij} = \int_0^T \phi_i(t)\phi_j(t)dt \qquad \begin{cases} 1 & if \quad i=j \\ 0 & if \quad i\ne j \end{cases}$$

The set of coefficients may naturally be viewed as an N- dimensional vector, denoted by S_i. The important point to note here is that the vector S_i bears a one, to, one relationship with the Transmitted signal



$$s_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t), \qquad \begin{cases} 0\le t\le T \\ i=1,2,\dots M \end{cases}$$

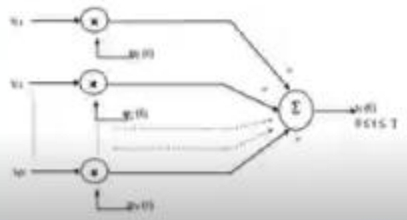
Where the coefficients of the expansion are defined by:

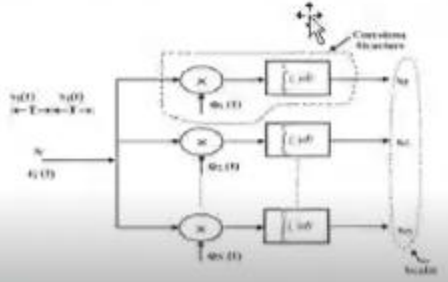
$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt \qquad \begin{cases} i=1,2,\dots M \\ j=1,2,\dots N \end{cases}$$

The real-valued basis functions are orthonormal which means

$$s_{ij} = \int_0^T \phi_i(t)\phi_j(t)\,dt \qquad \begin{cases} 1 & \text{if} \quad i=j \\ 0 & \text{if} \quad i\neq j \end{cases}$$

The set of coefficients may naturally be viewed as an N dimensional vector, denoted by $\boldsymbol{S_i}$. The important point to note here is that the vector $\boldsymbol{S_i}$ bears a one to one relationship with the Transmitted signal





Part – I:

To start with, let us assume that all $s_i(t)$ are not linearly independent. Then, there must exist a set of coefficients $\{a_i\}$, $1 < i \le M$, not all of which are zero, such that, $a_1s_1(t) + a_2s_2(t) + \ldots\ldots + a_Ms_M(t) = 0$, $0 \le t < T$

Verify that even if two coefficients are not zero, e.g. $a_1 \neq 0$ and $a_3 \neq 0$, then $s_1(t)$ and $s_3(t)$ are dependent signals.

Let us arbitrarily set, $a_M \neq 0$. Then,

$$s_M(t) = -\frac{1}{a_M}\left[a_1s_1(t) + a_2s_2(t) + \ldots\ldots + a_{M-1}s_{M-1}(t)\right]$$

$$= -\frac{1}{a_M}\sum_{i=1}^{M-1} a_i s_i(t)$$

The above equations shows that $s_M(t)$ could be expressed as a linear combination of other $s_i(t)$, $i = 1, 2, \ldots, (M-1)$.

We need to check in a similar way any dependent signals are there in the given signal set and eliminate them. Now we are left with signal set which contain only independent signals.

Part – II : We now show that it is possible to construct a set of 'N' orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots\ldots, \phi_N(t)$ from $\{s_i(t)\}$, $i = 1, 2, \ldots\ldots, N$.

Let us choose the first basis function as,

$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

$$= -\frac{1}{a_M}\sum_{i=1}^{M-1} a_i s_i(t)$$

The above equations shows that $s_M(t)$ could be expressed as a linear combination of other $s_i(t)$, $i = 1, 2, \ldots, (M-1)$.

We need to check in a similar way any dependent signals are there in the given signal set and eliminate them. Now we are left with signal set which contain only independent signals.

Part – II : We now show that it is possible to construct a set of 'N' orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ from $\{s_i(t)\}$, $i = 1, 2, \ldots, N$.

Let us choose the first basis function as,

$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

where E_1 denotes the energy of the first signal $s_1(t)$

$$s_{11} = \sqrt{E_1}$$

$$s_{31} = \int_0^T s_3(t)\varphi_1(t)dt \quad \text{and} \quad s_{32} = \int_0^T s_3(t)\varphi_2(t)dt$$

It is now easy to identify that,

$$\varphi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)dt}}$$

Indeed, in general,

$$\varphi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}} = \frac{g_i(t)}{\sqrt{Eg_i}}$$

for i = 1, 2,……, N, where

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\varphi_j(t)$$

$$s_{ij} = \int_0^T s_i(t).\varphi_j(t)dt$$