* **Prob 5:-** 05/09/2020

- The O/P of an inf. source consist of 150 symbols, 32 of which occur with a probability of $\frac{1}{64}$ and the remaining 118 occur with a probability of $\frac{1}{236}$. The source emits 2000 symbols/sec. Assuming that the symbols are choosen independently, find the average information rate of this source.

$$H\ \cancel{I_{total}} = \sum_{i=1}^{32} \frac{1}{64}\log_2(64) + \sum_{i=1}^{118} \frac{1}{236}\log_2(236)$$

$$H = 3 + 3.94 = 6.94$$

$$R_s = r_s \times H = 2000 \times 6.94$$

$$R_s = 7882.64 \text{ bits/sec}$$

Shannon's Encoding Algorithm:

O/P of an inf. source is converted into an r-array sequence.

$X = \{q_1, q_2, \ldots q_r\}$

- In binary : $\Rightarrow X = (0,1)$
- In ternary: $\Rightarrow X = (0,1,2)$

* Steps to solve problems.

1. Arrange the source symbol in the decreasing probability.
2. Compute the sequence.

   * $\boxed{\alpha_1 = 0}$ If $\alpha_0 = 0$, continue upto α_q

   * $\alpha_2 = P_1 = P_1 + \alpha_1$

   * $\alpha_3 = P_2 + P_1 = P_2 + \alpha_2$

   * $\alpha_4 = P_3 + P_2 + P_1 = P_3 + \alpha_3$ (* - -)

   * $\boxed{\alpha_{q+1} = P_q + \alpha_q = 1}$

3. Determine the smallest integer for l_i (length of the code word) using the inequality

   * $\boxed{2^{l_i} \geq \frac{1}{P_i}}$ for all $i = 1$ to q

4. Expand the decimal numbers α_i in binary form upto l_i places neglecting the expansion beyond l_i places.

5. Remove the decimal point to get the desired code.

* Code efficiency: The average length 'L' of any code is given by $L = \sum_{i=1}^{q} P_i l_i$ where $l_i = l_1 l_2 \ldots$

* Code efficiency, $\boxed{\eta_c = \frac{H(S)}{L} * 100}$

(10m) Prob.6:-

Construct the Shannon's binary code for the following message symbols $S = (S_1, S_2, S_3, S_4)$ with probabilities $P = (0.4, 0.3, 0.2, 0.1)$.

Solⁿ:-

Step 1:- Arranging probability in decreasing order.

$0.4 > 0.3 > 0.2 > 0.1$

Step 2:-

$\alpha_0 = 0$

$\alpha_1 = P_1 + \alpha_0 = 0.4$

$\alpha_2 = P_2 + \alpha_1 = 0.4 + 0.3 = 0.7$

$\alpha_3 = 0.4 + 0.3 + 0.2 = 0.7 + 0.2 = 0.9$

$\alpha_4 = 0.9 + 0.1 = 1.0$ Here, $\boxed{q = 4}$

Step 3:-

$-2^{l_i} \geq \frac{1}{P_i}$

$\boxed{2^{-l_i} \leq P_i}$

$2^{-l_1} \leq 0.4$

$-2^{l_1} \geq \frac{1}{0.4}$

$\boxed{l_1 = 2}$

$-2^{l_2} \geq \frac{1}{0.3}$

$\boxed{l_2 = 2}$

$-2^{l_3} \geq \frac{1}{0.2} \rightarrow \boxed{l_3 = 3}$

$-2^{l_4} \geq \frac{1}{0.1} \rightarrow \boxed{l_4 = 4}$

- **Ex: 1**. Construct the Shannon's binary code for the following message symbols S=(s_1, s_2, s_3, s_4) with probabilities P=(0.4, 0.3,0.2,0.1).
- Solution:
- ***0.4 > 0.3 > 0.2 > 0.1***

$\alpha_0 = 0,$
$\alpha_1 = 0.4$
$\alpha_2 = 0.4+0.3=0.7$
$\alpha_3 = 0.7+0.2=0.9$
$\alpha_4 = 0.9 + 0.1 = 1.0$

$2^{-l_1} \le 0.4 \rightarrow l_1 = 2$
$2^{-l_2} \le 0.3 \rightarrow l_2 = 2$
$2^{-l_3} \le 0.2 \rightarrow l_3 = 3$
$2^{-l_4} \le 0.1 \rightarrow l_4 = 4$

$\alpha_0 = 0 = 0.00 \,|\, \underline{0}$
$\alpha_1 = 0.4 = 0.01 \,|\, \underline{10}$
$\alpha_2 = 0.7 = 0.101 \,|\, \underline{10}$
$\alpha_3 = 0.9 = 0.1110 \,|\, \underline{01}$

0.4×2
$0.8 \times 2 \rightarrow 0$
$1.6 \rightarrow 1$
0.6×2
$1.2 \rightarrow 1$
0.2×2
$0.4 \rightarrow 0$
$\Rightarrow 0.0110\cdots$

0.7×2
$1.4 \rightarrow 1$
0.4×2
$0.8 \times 2 \rightarrow 0$
$1.6 \rightarrow 1$
0.6×2
$1.2 \rightarrow 1$
0.2×2
$0.4 \times 2 \rightarrow 0$
$0.8 \rightarrow 0$
$\Rightarrow$ [illegible]

0.9×2
$1.8 \rightarrow 1$
0.8×2
$1.6 \rightarrow 1$
0.6×2
$1.2 \rightarrow 1$
$0.2 \times 2 \rightarrow 0$
0.4×2
$0.8 \times 2 \rightarrow 0$
$\Rightarrow 0.11100\cdots$