Supplementary Problems

Discrete Fourier Series

6.40 Find the DFS coefficients for the sequence

$$\tilde{x}(n) = \cos\left(\frac{2\pi n}{10}\right) + \sin\left(\frac{2\pi n}{10}\right)$$

6.41 Find the DFS coefficients for the sequence of period $N = 8$ whose first four values are equal to 1 and the last four are equal to 0.

6.42 If $\tilde{x}(n)$ is a periodic sequence with a period N,

$$x(n) = x(n + N)$$

$\tilde{x}(n)$ is also periodic with period $3N$. Let $\tilde{X}(k)$ denote the DFS coefficient of $\tilde{x}(n)$ when considered to be periodic with a period N, and let $\tilde{X}_3(k)$ be the DFS coefficients of $\tilde{x}(n)$ when considered to be periodic with a period $3N$. Express the DFS coefficients $\tilde{X}_3(k)$ in terms of $\tilde{X}(k)$.

6.43 If the DFS coefficients of a periodic sequence $\tilde{x}(n)$ are real, $\tilde{X}(k) = \tilde{X}^*(k)$, what does this imply about $\tilde{x}(n)$?

The Discrete Fourier Transform

6.44 Find the 10-point DFT of each of the following sequences:

(*a*) $x(n) = \delta(n) + \delta(n - 5)$

(*b*) $x(n) = u(n) - u(n - 6)$

6.45 Find the 10-point DFT of the sequence

$$x(n) = \cos\left(\frac{3\pi}{5}n\right) \cdot \sin\left(\frac{4\pi}{5}n\right)$$

6.46 Find the 10-point inverse DFT of

$$X(k) = \begin{cases} 3 & k = 0 \\ 2 & k = 3, 7 \\ 1 & \text{else} \end{cases}$$

6.47 Find the N-point DFT of the sequence

$$x(n) = (-1)^n \qquad 0 \leq n \leq N - 1$$

where N is an even number.

6.48 Find the 16-point inverse DFT of

$$X(k) = \cos\left(\frac{2\pi}{16}3k\right) + 3j\sin\left(\frac{2\pi}{16}5k\right)$$

DFT Properties

6.49 If $x(n)$ is a finite-length sequence of length four with a four-point DFT $X(k)$, find the four-point DFT of each of the following sequences in terms of $X(k)$:

(*a*) $x(n) + \delta(n)$

(*b*) $x((3-n))_4$

(*c*) $\frac{1}{2}[x(n)+x^*((-n))_4]$

6.50 If $X(k)$ is the 10-point DFT of the sequence

$$x(n)=\delta(n-1)+2\delta(n-4)-\delta(n-7)$$

what sequence, $y(n)$, has a 10-point DFT

$$Y(k)=2X(k)\cos\left(\frac{6\pi k}{N}\right)$$

6.51 If the 10-point DFTs of $x(n)=\delta(n)-\delta(n-1)$ and $h(n)=u(n)-u(n-10)$ are $X(k)$ and $H(k)$, respectively, find the sequence $w(n)$ that corresponds to the 10-point inverse DFT of the product $H(k)X(k)$.

6.52 Let $x(n)$ be a sequence that is zero outside the interval $[0, N-1]$ with a z-transform $X(z)$. If

$$y(n)=x(n)+x(N-n)$$

find the $2N$-point DFT of $y(n)$, and express it in terms of $X(z)$.

6.53 If $x(n)$ is real and $x(n)=x(N-n)$, what can you say about the N-point DFT of $x(n)$?

6.54 If $x(n)=\delta(n)+2\delta(n-2)-\delta(n-5)$ has a 10-point DFT $X(k)$, find the inverse DFT of (*a*) $\mathrm{Re}[X(k)]$ and (*b*) $\mathrm{Im}[X(k)]$.

6.55 If $x(n)$ has an N-point DFT $X(k)$, find the N-point DFT of $y(n)=\cos(2\pi n/N)x(n)$.

6.56 Find the inverse DFT of $Y(k)=|X(k)|^2$ where $X(k)$ is the 10-point DFT of the sequence $x(n)=u(n)-u(n-6)$.

6.57 If $X(k)$ is the N-point DFT of $x(n)$, what is the N-point DFT of the sequence $y(n)=X(n)$?

6.58 Evaluate the sum

$$S=\sum_{n=0}^{15}x_1(n)x_2^*(n)$$

when

$$x_1(n)=\cos\left(\frac{3\pi n}{8}\right)$$

and

$$X_2(k)=3 \qquad 0\le k\le 15$$

Sampling the DTFT

6.59 The z-transform of the sequence

$$x(n)=u(n)-u(n-7)$$

is sampled at five points around the unit circle,

$$X(k)=X(z)\Big|_{z=e^{j2\pi k/5}} \qquad k=0,1,2,3,4$$

Find the inverse DFT of $X(k)$.

Linear Convolution Using the DFT

6.60 How many DFTs and inverse DFTs of length $N=128$ are necessary to linearly convolve a sequence $x(n)$ of length 1000 with a sequence $h(n)$ of length 64 using the overlap-add method? Repeat for the overlap-save method.

6.61 A sequence $x(n)$ of length $N_1=100$ is circularly convolved with a sequence $h(n)$ of length $N_2=64$ using DFTs of length $N=128$. For what values of n will the circular convolution be equal to the linear convolution?