# Structures of FIR Filter

$H(e^{jw}) = |H(e^{jw})| \angle H(e^{jw})$

In general, an FIR system is described by the difference equation

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

h(n) — coeff of x(n) in the

$g_i$

or, equivalently, by the system function

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k} \rightarrow \text{zero's}$$

$H(e^{jw}) = A = C \cdot e^{-j\alpha w}$

Furthermore, the unit sample response of the FIR system is identical to the coefficients $\{b_k\}$, that is,

$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

$|H(e^{jw})| = C$

$\angle H(e^{jw}) = -\alpha w$

## Direct-Form Structure

The direct form realization follows immediately from the non recursive difference equation given below

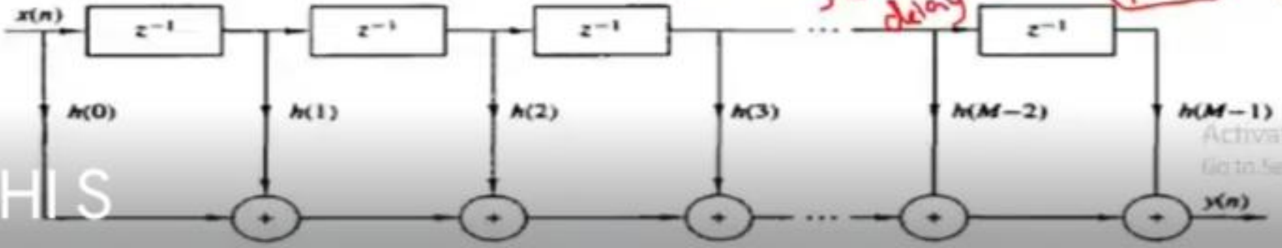
$$y(n) = \sum_{k=0}^{M-1} h(k) x(n-k)$$

$T_g = -\frac{d(\theta(w))}{dw} = \alpha$

phase delay $T_p = -\frac{\theta(w)}{w} = -\frac{(-\alpha w)}{w}$

group delay $T_g$

$T_p = \alpha = \frac{N-1}{2}$



# Cascade Structure

The cascaded realization follows naturally system function given by equation. It is simple matter to factor H(z) into second order FIR system so that

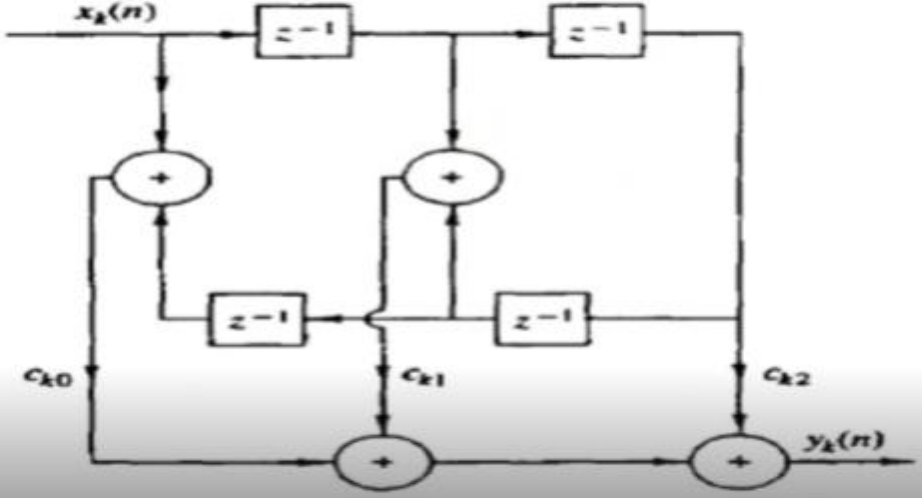
$$H(z) = \prod_{k=1}^{K} H_k(z)$$

where

$$H_k(z) = b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2} \qquad k = 1, 2, \ldots, K$$

$$H_k(z) = c_{k0}(1 - z_k z^{-1})(1 - z_k^* z^{-1})(1 - z^{-1}/z_k)(1 - z^{-1}/z_k^*)$$

$$= c_{k0} + c_{k1}z^{-1} + c_{k2}z^{-2} + c_{k1}z^{-3} + z^{-4}$$

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# Lattice Structure

$$H_m(z) = A_m(z) \qquad m = 0, 1, 2, \ldots, M-1 \tag{7.2.17}$$

where, by definition, $A_m(z)$ is the polynomial

$$A_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k} \qquad m \geq 1 \tag{7.2.18}$$

and $A_0(z) = 1$. The unit sample response of the mth filter is $h_m(0) = 1$ and $h_m(k) = \alpha_m(k)$, $k = 1, 2, \ldots, m$. The subscript $m$ on the polynomial $A_m(z)$ denotes the degree of the polynomial. For mathematical convenience, we define $\alpha_m(0) = 1$.

If $\{x(n)\}$ is the input sequence to the filter $A_m(z)$ and $\{y(n)\}$ is the output sequence, we have

$$y(n) = x(n) + \sum_{k=1}^{m} \alpha_m(k) x(n-k) \tag{7.2.19}$$

Next, let us consider an FIR filter for which $m = 2$. In this case the output from a direct-form structure is

$$y(n) = x(n) + \alpha_2(1)x(n-1) + \alpha_2(2)x(n-2) \tag{7.2.22}$$

By cascading two lattice stages as shown in Fig. 7.10, it is possible to obtain the same output as (7.2.22). Indeed, the output from the first stage is

$$\begin{aligned} f_1(n) &= x(n) + K_1 x(n-1) \\ g_1(n) &= K_1 x(n) + x(n-1) \end{aligned} \tag{7.2.23}$$

The output from the second stage is

$$\begin{aligned} f_2(n) &= f_1(n) + K_2 g_1(n-1) \\ g_2(n) &= K_2 f_1(n) + g_1(n-1) \end{aligned} \tag{7.2.24}$$



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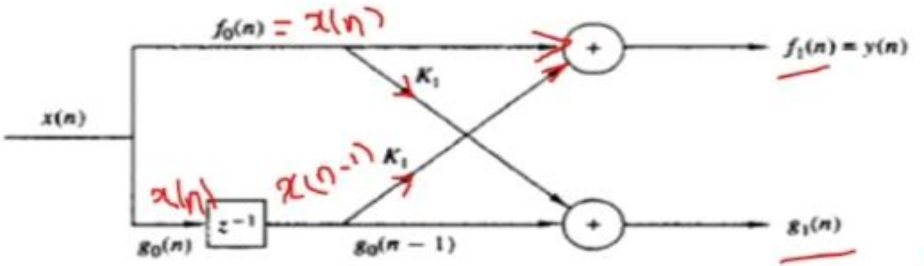
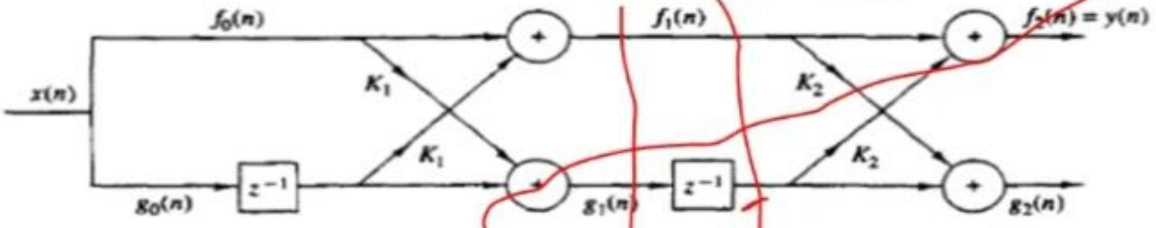
# Contd...




$$f_0(n) = g_0(n) = x(n)$$
$$f_1(n) = f_0(n) + K_1 g_0(n-1) = x(n) + K_1 x(n-1)$$
$$g_1(n) = K_1 f_0(n) + g_0(n-1) = K_1 x(n) + x(n-1)$$

$f_2(n) = K_2 [f_1(n)] + K_2 [g_1(n$




$$f_2(n) = x(n) + K_1 x(n-1) + K_2[K_1 x(n-1) + x(n-2)]$$
$$= x(n) + K_1(1 + K_2)x(n-1) + K_2 x(n-2)$$

The general form of lattice structure for m stage is given by'

$$f_0(n) = g_0(n) = x(n)$$
$$f_m(n) = f_{m-1}(n) + K_m g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$
$$g_m(n) = K_m f_{m-1}(n) + g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$

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# Contd...

**Conversion of lattice coefficients to direct-form filter coefficients.** The direct-form FIR filter coefficients $\{\alpha_m(k)\}$ can be obtained from the lattice coefficients $\{K_i\}$ by using the following relations:

$$A_0(z) = B_0(z) = 1 \qquad (7.2.47)$$

$$A_m(z) = A_{m-1}(z) + K_m z^{-1} B_{m-1}(z) \qquad m = 1, 2, \ldots, M-1 \qquad (7.2.48)$$

$$B_m(z) = z^{-m} A_m(z^{-1}) \qquad m = 1, 2, \ldots, M-1 \qquad (7.2.49)$$

**Conversion of direct-form FIR filter coefficients to lattice coefficients.** Suppose that we are given the FIR coefficients for the direct-form realization or, equivalently, the polynomial $A_m(z)$, and we wish to determine the corresponding lattice filter parameters $\{K_i\}$. For the $m$-stage lattice we immediately obtain the parameter $K_m = \alpha_m(m)$. To obtain $K_{m-1}$ we need the polynomials $A_{m-1}(z)$ since, in general, $K_m$ is obtained from the polynomial $A_m(z)$ for $m = M-1, M-2, \ldots, 1$. Consequently, we need to compute the polynomials $A_m(z)$ starting from $m = M-1$ and "stepping down" successively to $m = 1$.

$$K_m = \alpha_m(m) \qquad \alpha_{m-1}(0) = 1$$

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - K_m \beta_m(k)}{1 - K_m^2}$$

$$= \frac{\alpha_m(k) - \alpha_m(m)\alpha_m(m-k)}{1 - \alpha_m^2(m)} \qquad 1 \le k \le m-1$$



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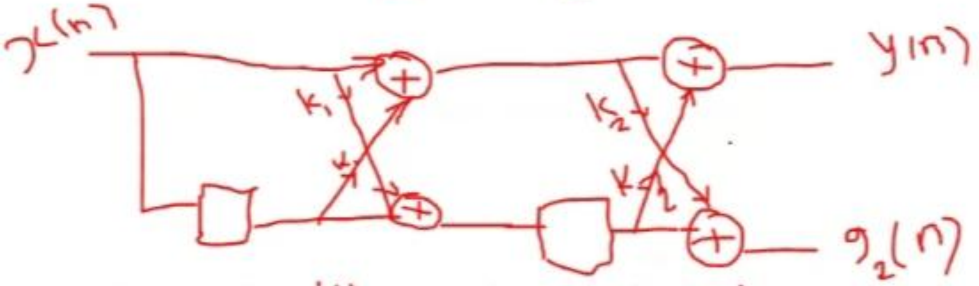
$$H(z) = \underset{a_2(0)}{1} + \underset{a_2(1)}{2} z^{-1} + \underset{a_2(2)}{\tfrac{1}{3}} z^{-2}$$

$$K_m = a_m(m)$$

$$K_2 = a_2(2) = \frac{1}{3}$$

$$K_1$$




$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - a_m^2(m)}$$

$m = 2$

$i = 1$

$$a_{2-1}(1) = \frac{a_2(1) - a_2(2) \cdot a_2(2-1)}{1 - a_2^2(2)} = \frac{2 - \frac{1}{3}(2)}{1 - \left(\frac{1}{3}\right)^2}$$

$$\boxed{a_1(1) = \frac{3}{2}} = K_1$$

$K_1 = \frac{1}{2}, K_2 = \frac{1}{3}, K_3 = \frac{1}{4}$

$K_1 = a_1(1)$

$K_2 = a_2(2)$

$1 + \frac{3}{4}x(n-1) + \frac{1}{2}x(n-2) + \frac{1}{4}x(n-3)$

$y(n) = \underset{a_3(0)}{?} + \underset{a_3(1)}{?}\, x(n-1) + \underset{a_3(2)}{?}\, x(n-2) + \underset{a_3(3)}{?}\, x(n-3)$

$a_3(0) = 1$

$\boxed{K_3 = \frac{1}{4}}$

$a_m(k) = a_{m-1}(k) + K_m a_{m-1}(m-k)$

$m = 2$, $k = 1$

$a_2(1) = a_1(1) + K_2 a_1(1)$

$\boxed{a_2(1) = \frac{1}{2} + \frac{1}{3}\left(\frac{1}{2}\right) = 0.66}$

$a_3(2) = a_2(2) + K_3 a_2(1)$

$= \frac{1}{3} + \frac{1}{4}\left(\frac{2}{3}\right) = \frac{1}{2}$

$m = 3, k = 1$

$a_3(1) = a_2(1) + K_3 \cdot a_2(2)$

$\boxed{a_3(1) = \frac{3}{4}}$

~~$m = 2, k = 2$~~

$a_2(2) = a_1(2)$