Redundancy $=1-\eta_{ch}$

1.6.6 Symmetry Channel

Symmetry channel is defined as the channel in which the channel matrix has 2nd and subsequent rows, the same elements as the first row, but in different order.

$\therefore H(Y/X) = h$, where →entropy of any single row. The channel capacity with r_s=1 bits/sec is given by,

$$C = Max(R_t)$$

$$= \text{Max}[I(X,Y)]\, r_s$$

$$= \text{Max}[I(X,Y)]$$

$$= \text{Max}\left(H(Y) - H(Y/X)\right)$$

$$= \text{Max}[H(Y)] - \text{Max}[H(Y/X)]$$

$$= \text{Max}[H(Y)] - \text{Max}(h)$$

$$C = \text{Max}[H(Y)] - h$$

- The capacity of a discrete memoryless noisy channel is defined a s maximum possible rate of maximum rate of transmission occurs when the source is matched to the channel.
- $\therefore C = Max(R_t)$
- $= \text{Max}[I(X,Y) \,.\, r_s]$
- $C = \text{Max}\{[H(X) - H(X/Y)]\, r_s\}$

1.6.5 Channel Efficiency

$$\%\eta_{ch} = \frac{R_t}{C} * 100$$

$$= \frac{I(X,Y) \,.\, r_s}{\text{Max}[I(X,Y) \,.\, r_s]} * 100$$

$$\%\eta_{ch} = \frac{H(X) - H(X/Y)}{Max[H(X) - H(X/Y)]} * 100$$

Redundancy =1-η

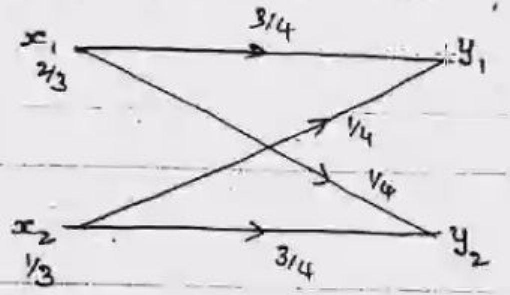
Since it is a binary symmetric channel. $H(Y)_{max} = log_2 s = log_2 2 = 1$

$\therefore C = 1 - h$ bits/sec.

Ex.1: A binary symmetric channel has the following noise matrix with source probabilities of $P(x_1)$=2/3 and $P(x_2)$=1/3. $P\left({}^{Y}/_{X}\right) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$. Determine H(X), H(Y), H(X,Y), I

H(X/Y), I(X,Y), Channel Capacity, Channel efficiency and redundancy.

Solution:



$$H(X) = \sum_{i=1}^{2} P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$= \frac{2}{3} \log_2 \left(\frac{3}{2}\right) + \left(\frac{1}{3}\right) \log_2 (3)$$

$$H(X) = 0.9183 \text{ bits/symbol}$$

$$\text{JPM} \Rightarrow \begin{bmatrix} 2/4 & 1/6 \\ 1/12 & 1/4 \end{bmatrix}$$

$P(y_1) =$

$$H(X) = 0.9183 \text{ bits/symbol}$$

$$JPM \Rightarrow \begin{bmatrix} 2/4 & 1/6 \\ 1/12 & 1/4 \end{bmatrix}$$

$$\therefore P(y_1) = 7/12 \quad ; \quad P(y_2) = 5/12$$

$$H(Y) = \sum_{j=1}^{2} P(y_j) \log_2 \frac{1}{P(y_j)}$$

$$= \frac{7}{12} \log_2 \left(\frac{12}{7}\right) + \frac{5}{12} \log_2 \left(\frac{12}{5}\right)$$

$$H(Y) = 0.9799 \text{ bits/symbol}$$

$$= \tfrac{3}{4}\log(4/3) + 1/4 \log(4/1)$$

$$= 0.8113 \text{ bits/symbols}$$

$$H(X, Y) = H(X) + H(Y/X)$$
$$= 0.9183 + 0.8113$$
$$H(X, Y) = \mathbf{1.7296 \text{ bits/message-symbol}}$$

$$H(Y, X) = H(X, Y) = H(Y) + H(X/Y)$$
$$H(X/Y) = H(X, Y) - H(Y)$$
$$= 1.7296 - 0.9799$$
$$H(X/Y) = \mathbf{0.7497 \text{ bits/message-symbol}}$$

$$I(X, Y) = H(X) - H(X/Y) \text{ [or } H(Y) - H(Y/X)]$$
$$= 0.9183 - 0.7497$$
$$I(X, Y) = \mathbf{0.1686 \text{ bits/message-symbol}}$$

$$C = 1 - h = 1 - H(Y/X)$$
$$= 1 - 0.8113$$
$$C = 0.1887 \text{ bits/message-symbol}$$

$$\text{Channel efficiency} = \frac{I(X,Y)}{C}$$
$$= \frac{0.1686}{0.1887}$$
$$\therefore \quad \eta_{ch} = 89.35\%$$

Channel Redundancy $= \eta_{ch} = 10.65\%$

Problem : Consider a source with 8 alphabet A to H with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05, 0.02.

i] Construct a binary compact code and determine the code efficiency.

ii] Construct a ternary compact code and determine efficiency of the code

iii] Construct a quaternary compact code and determine the code efficiency. Compare and comment on the

The waveform received at the receiver may be accompanied by a waveform which varies with respect to time in an entirely unpredictable manner. This unpredictable voltage waveform is a random process called "noise".

The noise introduced due to thermal motion of electrons is called ***"Johnson Noise"*** or ***"White Noise"*** and the noise resulting due to the flow of electrons across semiconductor junction is called ***"Shot Noise"***. When the noise adds to the signal it is called ***"Additive Noise"*** and if it multiplies, then it is called ***"Fading"***.

In channels, the noise is almost white and it has a distribution which resembles Gaussian (or normal) distribution with zero mean and some variance. Hence, this noise is also called "additive white Gaussian-noise (AWGN)".

Statement of Shannon-Hartley Law :

Shannon-Hartley law also called Shannon's third theorem, states that the capacity of a band-limited Gaussian channel with AWGN is given by

$$C = B \log\left(1+\frac{S}{N}\right) \text{ bits/sec} \quad \ldots (4.90)$$

Where B = Channel bandwidth in Hz

S = Signal power in watts

N = Noise power in watts $= \eta B$

where the two sided power spectral density of noise is $(\eta/2)$ watts/Hz.

1.9 Shannon Hartley Theorem and its Implication

The waveform received at the receiver may be accompanied by a waveform which varies with respect to time in an entirely unpredictable manner. This unpredictable voltage waveform is a random process called "noise".

The noise introduced due to thermal motion of electrons is called ***"Johnson Noise"*** or ***"White Noise"*** and the noise resulting due to the flow of electrons across semiconductor junction is called ***"Shot Noise"***. When the noise adds to the signal it is called ***"Additive Noise"*** and if it multiplies, then it is called ***"Fading"***.

In channels, the noise is almost white and it has a distribution which resembles Gaussian (or normal) distribution with zero mean and some variance. Hence, this noise is also called "additive white Gaussian-noise (AWGN)".

Statement of Shannon-Hartley Law :

Shannon-Hartley law also called Shannon's third [illegible] band-limited Gaussian channel with AWGN is given by

$$C = B \log\left(1+\frac{S}{N}\right) \text{ bits/sec}$$

Where B = Channel bandwidth in Hz

S = Signal power in watts

N = Noise power in watts $= \eta B$

where the two sided power spectral density of noise is $(\eta/2)$ watts/Hz.

Proof :

From equation (4.85), the channel capacity C is given by

$$C = [H(Y) - H(N)]_{max} \qquad \text{...... (4.100)}$$

If the noise is additive, white and Gaussian having a power N in a bandwidth of B Hz, then from equation (4.66), we have

$$H(N)_{max} = B \log 2\pi e N \text{ bits/sec} \qquad \text{...... (4.101)}$$

When the input signal is limited to an average power S, over the same bandwidth of B Hz, and when the signal at the receiver Y = X + N with X and N being independent, then the received signal power is nothing but variance given by

$$\sigma_Y^2 = (S + N) \qquad \text{...... (4.102)}$$

We have seen in the derivation of equation (4.65) that for a given mean square value, the entropy will be maximum if the signal is Gaussian and the maximum entropy is given by

$$H(Y)_{max} = B \log 2\pi e \sigma_Y^2 \text{ bits/sec} \qquad \text{...... (4.103)}$$

Using equation (4.102) in (4.103), we get

$$H(Y)_{max} = B \log 2\pi e(S + N) \text{ bits/sec} \qquad \text{....}$$

Using equations (4.101) and (4.104) in equation (4.100), we get

$$C = B \log 2\pi e(S + N) - B \log 2\pi e N$$

From equation (4.85), the channel capacity C is given by

$$C = [H(Y) - H(N)]_{max} \quad \text{...... (4.100)}$$

If the noise is additive, white and Gaussian having a power N in a bandwidth of B Hz, then from equation (4.66), we have

$$H(N)_{max} = B \log 2\pi eN \text{ bits/sec} \quad \text{...... (4.101)}$$

When the input signal is limited to an average power S, over the same bandwidth of B Hz, and when the signal at the receiver Y = X + N with X and N being independent, then the received signal power is nothing but variance given by

$$\sigma_Y^2 = (S + N) \quad \text{...... (4.102)}$$

We have seen in the derivation of equation (4.65) that for a given mean square value, the entropy will be maximum if the signal is Gaussian and the maximum entropy is given by

$$H(Y)_{max} = B \log 2\pi e\sigma_Y^2 \text{ bits/sec} \quad \text{...... (4.103)}$$

Using equation (4.102) in (4.103), we get

$$H(Y)_{max} = B \log 2\pi e(S + N) \text{ bits/sec} \quad \text{...... (4.104)}$$

Using equations (4.101) and (4.104) in equation (4.100), we get

$$C = B \log 2\pi e(S + N) - B \log 2\pi eN$$

$$= B \log \frac{2\pi e(S+N)}{2\pi eN}$$

$$\text{or} \quad C = B \log \left(1 + \frac{S}{N}\right) \text{ bits/sec} \quad \text{......}$$

or $C = B \log\left(1+\frac{S}{N}\right)$ bits/sec ...... (4.105)

1st Implication :

From Shannon-Hartley law, we have

$$C = B \log_2\left(1+\frac{S}{N}\right) \text{ bits/sec} \quad \text{...... (4.107)}$$

It looks, from the above equation that when B is increased, channel capacity C also increases and since $R_{max} = C$, the maximum rate of information transmission can be enhanced to any large value as we please. However, the channel capacity does not become infinite when the bandwidth is made infinite. This is because, as B increases, the noise power N which is dependent on B, also increases thereby reducing $\left(\frac{S}{N}\right)$. Thus the product of B and $\log_2\left(1+\frac{S}{N}\right)$ will increase only upto a certain value and becomes constant with increasing B. This value is denoted by C_{∞}. Let us calculate that value.

Substituting $N = \eta B$ in equation (4.107), we get

$$C = B \log_2\left(1+\frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\left(\frac{\eta B}{S}\right)\log_2\left(1+\frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\log_2\left(1+\frac{S}{\eta B}\right)^{\left(\eta B/S\right)}$$

Let $x = \frac{S}{\eta B}$

Then $C = \frac{S}{\eta}\log_2 (1+x)^{(1/x)}$

Accordingly when $B \rightarrow \infty$, $x \rightarrow 0$

$$\therefore \lim_{\substack{B\rightarrow\infty \\ x\rightarrow 0}} C = \lim_{\substack{B\rightarrow\infty \\ x\rightarrow 0}} \frac{S}{\eta}\log_2 (1+x)^{(1/x)}$$

$$\therefore C_\infty = \frac{S}{\eta}\log_2 \lim_{x\rightarrow 0}\left[(1+x)^{(1/x)}\right]$$

$$\therefore C_\infty = \frac{S}{\eta}\log_2 e$$

or $C_\infty = B\frac{S}{N}\log_2 e$ bits/sec ...... (4.1