Nyquist Criterion for Distortionless Transmission

In baseband transmission of digital data, sampled output y_i or $y(iT_b) = \mu a_i$, which means that ith transmitted bit is decoded correctly in abcence of ISI which is called distortionless transmission of digital data.

In order to minimize the effect of ISI, the designing of the transmitting & receiving filter is based on Nyquist Criterion.

Lets consider the output of overall pulse spectrum,

$$y(t) = \mu \sum_{K=-\infty}^{\infty} a_K \, p(t - KT_b) \rightarrow \text{(Assume) } \cancel{\text{Assume}} \text{ Neglecting noise}$$

Nyquist Criterion is based upon 2 steps:-

Step 1 :- There is process of sampling, that is called <u>extraction</u>

In extraction process, output $y(t)$ is sampled at instant $t = iT_b$, resulting sampled output is called y_i / $y(iT_b)$.

Step 2 :- <u>Decoding</u> is performed in step II, the weighed pulse obtained is free from ISI (which is obtained at $i = k$)

Weighted pulse $\Rightarrow a_K \, p(iT_b - KT_b)$

if $i = K \Rightarrow a_i \, p(0) = a_i$ (since $p(0) = 1$)

normalisation of $p(t)$

After performing extraction & decoding,

$$p(iT_b - KT_b) = \begin{cases} 1 & \text{for } i = k. \\ 0 & \text{for } i \neq k \end{cases} \quad \text{where } p(0) = 1 \text{ due to normalizing}$$

If received pulse $p(t)$ satisfying above condition, then receiver output get reduced to

$y_i = \mu a_i \rightarrow$ distortionless transmission having zero ISI & absence of noise

Let us consider received pulse $p(t)$ sampled at $t = mT_b$ & produces sampled signal $p_\delta(t)$. For sampling process we consider a periodic train of pulse denoted as $S_\delta(t)$

$$S_\delta(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_b).$$

In sampling process $p_\delta(t) = p(t) \cdot S_\delta(t)$. For freq domain we use Fourier transform, $P(f)$ is FT of $p(t)$, $S_\delta(f)$ is FT of $S_\delta(t)$ & $P_\delta(f)$ is FT of $p_\delta(t)$

$p(t)$ ——— switch ——— $p_\delta(t) = p(t) \cdot S_\delta(t)$

$p(t) \xrightarrow{FT} P(f)$

$S_\delta(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_b) \xrightarrow{FT} S_\delta(f)$

$p_\delta(t) \xrightarrow{F.T} P_\delta(f)$

$$p_\delta(t) = p(t) \cdot S_\delta(t) = \sum_{m=-\infty}^{\infty} p(mT_b)\, \delta(t - mT_b)$$

$p(mT_b)$ → sampled func of pulse $p(t)$

F.T of $p_\delta(t)$

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t)\, e^{-j2\pi f t} \cdot dt = \int_{-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} p(mT_b)\, \delta(t - mT_b) \right] e^{-j2\pi f t} \cdot dt$$

Let $m = i - k$ (assume, in order to define Nyquist Criteria

$$P_\delta(f) = \int_{-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} p[(i-k)T_b]\, \delta[t - (i-k)T_b] \right] e^{-j2\pi f t} \cdot dt$$

Consider $p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases}$ $\because p(0) = 1.$

→ Condition for distortionless transmission.

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi f t} \cdot dt, & i = k \\ 0, & i \neq k. \end{cases}$$

For Nyquist Criteria for Distortionless transmission,

we consider case $\Rightarrow i = k$

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi ft} \cdot dt \quad , \text{ for } i = k$$

$$\Rightarrow p(0) \underbrace{\int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft} \cdot dt}_{\to \text{ FT of } \delta(t) = 1}$$

$$\therefore \; P_\delta(f) = p(0)$$

$$= 1 \text{ (after normalisation).}$$

The received pulse is normalised for $i = k$, & in this condition transmitted bit is encoded correctly.

Relation for pulse shaping function $p(t)$ satisfying Nyquist Criteria for Distortionless transmission

Consider $p_\delta(t) = p(t) \cdot S_\delta(t)$. ← Multiplication in time domain

Taking F.T on both sides

$P_\delta(f) = P(f) * S_\delta(f)$ → Convolution in freq domain

$$\boxed{\begin{array}{l} S_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \\ \quad \downarrow \text{FT} \\ S_\delta(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \end{array}}$$

$$\therefore \; P_\delta(f) = P(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Let $f_s = \frac{1}{T_b} = R_b \rightarrow$ bit rate.

$$\therefore \; P_\delta(f) = P(f) * R_b \sum_{n=-\infty}^{\infty} \delta(f - nR_b).$$

To solve above relation, we use convolution property of impulse function (convolution of $P(f)$ & $\delta(f - nR_b)$) $= P(f - nR_b)$.

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b) \rightarrow (A)$$

Using def of Fourier Transform,

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t)\, e^{-j2\pi ft} \cdot dt \Rightarrow \int_{-\infty}^{\infty} p(t)\, S_\delta(t)\, e^{-j2\pi ft} \cdot dt$$

$(\because p(t) \cdot S_\delta(t) = p_\delta(t))$

Sampling at $t = mT_b$

$$P_\delta(f) = \int_{-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} p(mT_b)\, \delta(t - mT_b) \right] e^{-j2\pi ft} \cdot dt \quad \rightarrow (B).$$

Consider received pulse $p(mT_b)$, put $m = i - k$

$$p[(i-k)T_b] = p(mT_b) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \quad \because p(0) = 1, \text{ for } i = k \quad \text{for } i \neq k$$

eq (B) becomes $\Rightarrow$

For Nyquist Criteria we consider $i = k$ / $m = 0$ we get

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi ft} \cdot dt \quad - \quad m = 0 \;/\; i = k.$$

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0. \end{cases}$$

$$P_\delta(f) = p(0)\, e^{-j2\pi ft} \Big|_{t=0.}$$

$$P_\delta(f) = p(0) = 1.$$

In place of $P_\delta(f)$, from eq (A). we can write

$$R_b \sum_{n=-\infty}^{\infty} P(f - nR_b) = 1$$

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = \frac{1}{R_b}$$ $\rightarrow$ Pulse shaping function $p(t)$ with F.T $P(f)$ satisfies the relation & having condition $\rightarrow$ Nyquist criteria.

$$p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

$\rightarrow$ called Nyquist criteria for distortionless baseband transmission with zero ISI