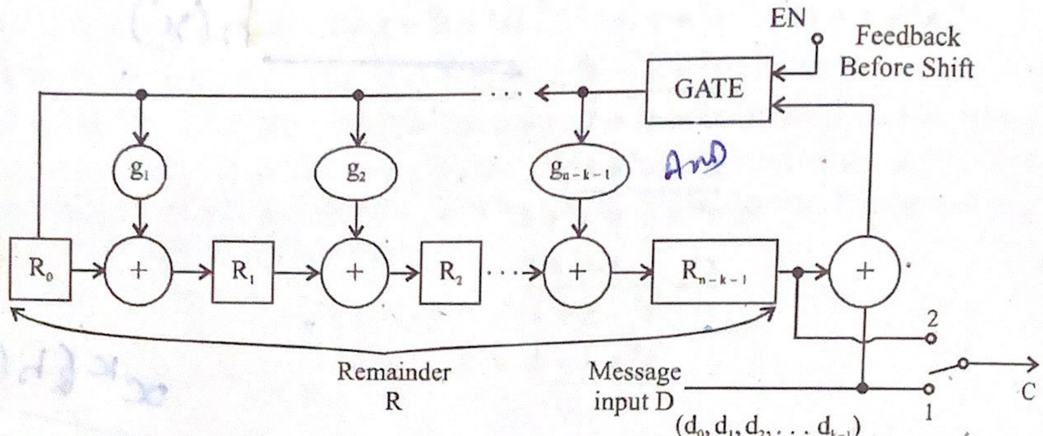
5.14 ENCODING USING (n – k) BIT SHIFT REGISTER

In order to obtain remainder polynomial R(x), we have to perform division of $x^{n-k} D(x)$ by the generator polynomial g(x). This division can be accomplished using the dividing circuit consisting of feedback shift register as shown in figure 5.14.



Symbols :

R → Flipflops that make up a shift register

+ → Modulo-2 adders

g_i → A closed path if $g_i = 1$ and open if $g_i = 0$

Gate → AND gate

Fig. 5.14 : Encoder for an (n, k) cyclic code

OPERATION : It is assumed that at the occurrence of the clock pulse, the register inputs are shifted into the register and appear at the output at the end of the clock pulse.

1. With the gate turned ON and switch in position 1, the information digits (d_0, d_1, ..., d_{k-1}) are shifted into the register (with d_{k-1} first) and simultaneously into the communication channel. As soon as the k information digits have been shifted into the register, the register contains the parity check bits (R_0, R_1, ..., R_{n-k-1}).
2. With the gate turned OFF and the switch in position 2, the contents of the shift register are shifted into the channel. Thus the code-vector (R_0, R_1, ..., R_{n-k-1}, d_0, d_1, ..., d_{k-1}) is generated and sent over the channel.

Example 5.27 : Design an encoder for the (7, 4) binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using the message vectors (1 0 0 1) and (1 0 1 1).

Solution

In general, the generator polynomial is represented as

$$g(x) = g_0 + g_1 x + g_2 x^2 + ... + g_{n-k} x^{n-k} \qquad ...... (5.73)$$

In the problem given, for (7, 4) cyclic code,

$$g(x) = 1 + x + x^3 \qquad ...... (5.74)$$

Comparing the coefficients in equation (5.73) and (5.74), we get

$g_0 = 1, g_1 = 1, g_2 = 0$ and $g_3 = 1$

With these values, the circuit of figure 5.14 reduces to the one shown in figure 5.15 with only 3 flip-flops R_0, R_1 and R_2, two modulo-2 adders and connection from the output of the gate to R_0 and to the first modulo-2 adder.

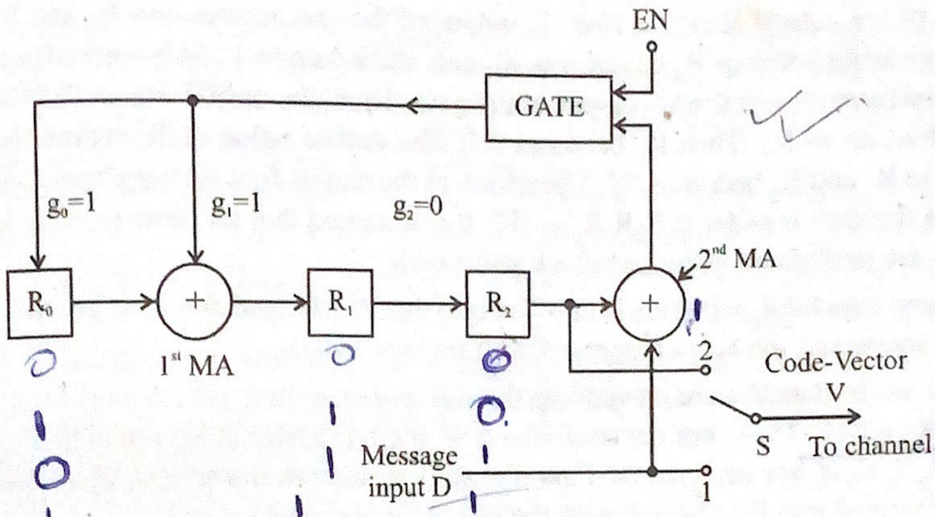


Fig. 5.15 : Encoder for (7, 4) cyclic code of example 5.27

(i) For the message D = [1 0 0 1], the shift register contents are shown in table 5.23.

Number of shifts	Input D	Shift Register Contents R_0 R_1 R_2	Remainder bits → R
Initialisation → switch S is in position-1 and gate is turned ON		0 0 0	–
1	1	1 1 0	–
2	0	0 1 1	–
3	0	1 1 1	–
4	1	0 1 1	–
Switch S moves to position-2 and gate is turned OFF			
5	X	0 0 1	1
6	X	0 0 0	1
7	X	0 0 0	0

Table 5.23 : Contents of shift register in the encoder of figure 5.15 for message sequence D = 1001

Mechanism of Operation : The first process to be carried out is the initialization process. In this process, the shift register contents are cleared to read $R_0R_1R_2 = 000$, the switch S is kept in position-1 and the gate is turned ON [The gate can be a 2-input AND gate with one terminal as enable input EN. With EN = 1, AND gate will be turned ON and the output of the AND gate depends on the other input].

The data input $D = (d_0\ d_1\ d_2\ d_3) = (1\ 0\ 0\ 1)$. The data bit $d_3 = 1$ is moved into 2nd modulo-2 adder (abbreviated as 2nd MA in figure 5.14). Since $R_2 = 0$, the output of 2nd MA is '1' and the output of the gate is also '1'. This '1' output of the gate moves into R_0 and R_0 becomes '1'. But the earlier value of R_0 which was '0' gets shifted on to 1st MA where the addition of this bit is performed with the '1' output of the gate. Then, the output of the 1st MA is '1' and this is shifted on to R_1. Thus R_1 becomes '1'. The earlier value of R_1 which was '0' gets shifted on to R_2 and R_2 becomes '0'. Therefore, at the end of first shifting operation, the new contents of the shift register is $R_0R_1R_2 = 110$. It is assumed that all these shifting and adding operations are performed using one clock pulse only.

The next data bit $d_2 = 0$. This is now fed into the 2nd MA and the same procedure repeats as written above and the new contents of shift register is 011.

When all the data bits are moved into the shift register, the final contents of shift register read $R_0R_1R_2 = 011$. These are the coefficients of the remainder polynomial R(x). When the data bits $d_0\ d_1\ d_2\ d_3$ are being moved into the shift register in the reverse direction, they are also being moved into the channel with the switch in position-1.

Now, the switch S is shifted into position-2 and gate is turned OFF [EN = 0] and the contents of the shift register are shifted into the channel using 3 more shifts. Thus the code-vector 0111001 is generated and sent over the channel. [Note that this code-vector is identical with the code-vector corresponding to the message-vector 1001 in table 5.22].

(ii) For the message $D = [d_0\ d_1\ d_2\ d_3] = [\ 1\ 0\ 1\ 1]$, the shift register contents are shown in table 5.24.

Number of shifts	Input D	Shift Register Contents R_0	R_1	R_2	Remainder bits → R
Initialisation → switch S is in position-1 and gate is turned ON		0	0	0	–
1	1	1	1	0	–
2	1	1	0	1	–
3	0	1	0	0	–
4	1	1	0	0	–
Switch S moves to position-2 and gate is turned OFF					
5	X	0	1	0	0
6	X	0	0	1	0
7	X	0	0	0	1

Table 5.24 : Contents of shift register in the encoder of figure 5.15 for message sequence D = 1011

From table 5.31, we observe that the remainder vector is 0 1 1 1. Hence, the transmitted code-vector through the channel is 0 1 1 1 1 0 0 1 0 1 1 0 1 1 1.

Example 5.32 : A (15, 5) linear cyclic code has a generator polynomial

$$g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$$

(a) Draw the block diagram of an encoder and syndrome calculator for this code.

(b) Find the code polynomial for the message polynomial
$D(x) = 1 + x^2 + x^4$ in systematic form.

(c) Is $V(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ a code polynomial?

[VI Sem, EC/TE, February 2002, Q.7(a)], [VI Sem, EC/TE, July/August 2004, Q.7]
[VI Sem EC/TE, Jan/Feb, 2005, Q.7(b)]

Solution

(a) The generator polynomial is given by

$$g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$$

The coefficients are then given by

$g_0 = 1, g_1 = 1, g_2 = 1, g_3 = 0, g_4 = 1, g_5 = 1, g_6 = 0, g_7 = 0, g_8 = 1, g_9 = 0.$

With these values, the general encoder circuit of figure 5.14 reduces to the one shown in figure 5.22.

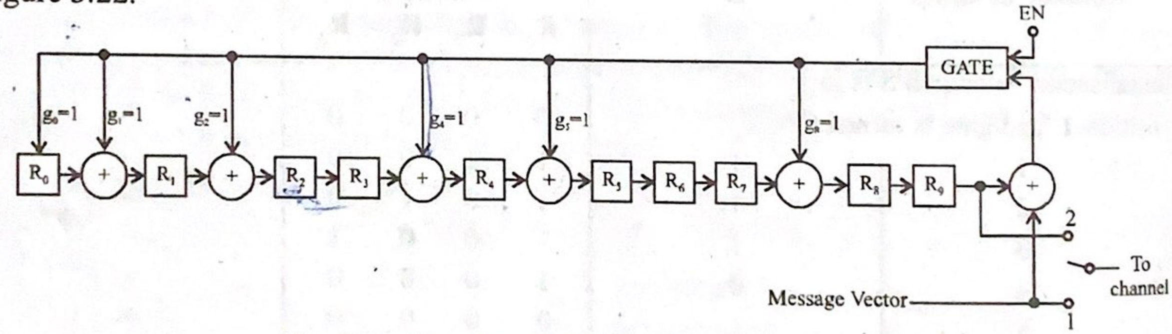


Fig. 5.22 : Encoder for (15, 5) code of example 5.32

(b) For a message input of $D(x) = 1 + x^2 + x^4 \rightarrow 1\ 0\ 1\ 0\ 1$, the contents of shift register with every shift is listed as shown in table 5.32 below.

Number of shifts	Input D	Shift Register Contents R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	Remainder bits-R
Initialisation		0	0	0	0	0	0	0	0	0	0	–
1	1	1	1	1	0	1	1	0	0	1	0	–
2	0	0	1	1	1	0	1	1	0	0	1	–
3	1	0	0	1	1	1	0	1	1	0	0	–
4	0	0	0	0	1	1	1	0	1	1	0	–
5	1	1	1	1	0	0	0	1	0	0	1	–

Table 5.32 : Contents of shift register in the encoder of figure 5.22 for message 1 0 1 0 1

Knowing the coefficients of generator polynomial, the syndrome calculator circuit of figure 5.16 reduces to the one shown in figure 5.23.

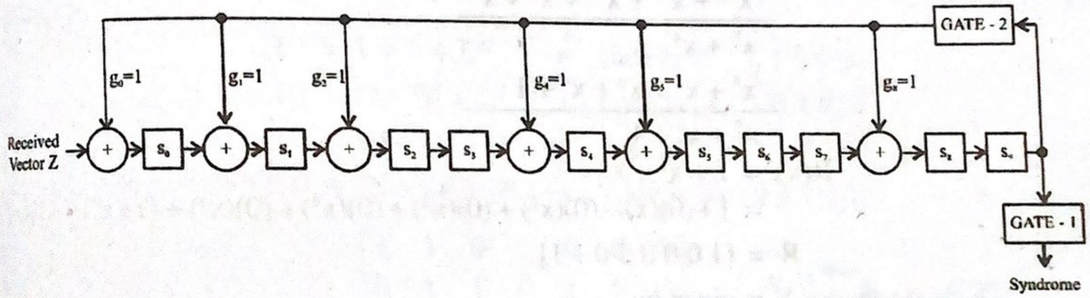


Fig. 5.23 : Syndrome calculator for example 5.32

(c) If V(x) is a code-polynomial, then it should be perfectly divisible by the generator polynomial with remainder zero. If the remainder is not zero, we can conclude that the given polynomial is not a code-polynomial. The division is performed as below :

$$\begin{array}{r} x^4 + x^2 + 1 \\ x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1 \;\big)\; x^{14} + x^8 + x^6 + x^4 + 1 \\ \underline{x^{14} + x^{12} + x^9 + x^8 + x^6 + x^5 + x^4} \\ x^{12} + x^9 + x^5 + 1 \\ \underline{x^{12} + x^{10} + x^7 + x^6 + x^4 + x^3 + x^2} \\ x^{10} + x^9 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 \\ \underline{x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1} \\ R(x) \rightarrow x^9 + x^8 + x^7 + x^6 + x^3 + x + 1 \end{array}$$

V / g.

The remainder polynomial R(x) is a non-zero polynomial. Hence, the given polynomial is not a code polynomial.

Example 5.33 : The generator polynomial for a (15, 7) cyclic code is

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8$$

(i) Find the code-vector is systematic form for the message $D(x) = x^2 + x^3 + x^4$.

(ii) Assume that the first and last bit of the code vector V(x) for $D(x) = x^2 + x^3 + x^4$ suffe transmission errors. Find the syndrome of V(x).

[VI Sem EC/TE, August 2001, Q.7(a)], [VI sem EC/TE, July/August 2002, Q.7(

[VI Sem EC/TE, Jan/Feb. 2003, Q.7(b

Solution

(i) From property (5) of cyclic codes, the systematic code-vector is found by first dividi x^{n-k} D(x) by g(x) to get remainder polynomial R(x).

$$\therefore \quad x^{n-k} D(x) = x^{15-7} D(x) = x^8 (x^2 + x^3 + x^4) = x^{10} + x^{11} + x^{12}$$

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