is high.

s_1	0.4	00	0.4	00	0.4	00	0.4	00	0.4	1	0.6	0
s_2	0.2	11	0.2	11	0.2	10	0.2	01	0.4	00	0.4	1
s_3	0.1	011	0.1	010	0.2	11	0.2	10	0.2	01		
s_4	0.1	100	0.1	011	0.1	010	0.2	11				
s_5	0.1	101	0.1	100	0.1	011						
s_6	0.05	0100	0.1	101								
s_7	0.05	0101										

(ii) as high as possible

S_1	00
S_2	11
S_3	011
S_4	100
S_5	101
S_6	0100
S_7	0101

$H(S) = 2.421$

$L = 2.5$

Variance $= 0.45$

Probability of 0's $= 0.58$

Probability of 1's $= 0.42$

Efficiency $\eta = 96.84\%$

Redundancy $R_\eta = 3.16\%$

Problem : Consider a source with 8 alphabet A to H with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05, 0.02.

i] Construct a binary compact code and determine the code efficiency.

ii] Construct a ternary compact code and determine efficiency of the code

iii] Construct a quaternary compact code and determine the code efficiency. Compare and comment on the result. Draw code trees for all three cases.

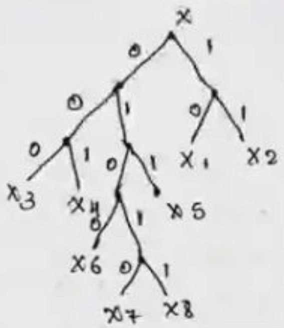
iv] Decode the messages using appropriate code trees

a) 01010010000011010110 01…

b) 12111011020012002 ….

c) 031132020300100231 …..

Solution:



X_6	0.08	0100
	0.07	0101
X_7	0.05	01010
X_8	0.02	01011

$$H(S) = \sum_{i=1}^{9} P_i \log_2 \frac{1}{P_i}$$

$H(S) = 2.75$ bits / symbol

$$\eta_c = \frac{H(S)}{L}$$

$$L = \sum_{i=1}^{9} P_i l_i \qquad R_{\eta_c} = 1.79\%$$

$L = 2.8$

$\eta_c = 98.25\%$

iv a) 0101 00 100 000 11 0101 11 001

$= X_7\ X_6\ X_3\ X_2\ X_8\ X_4$

(ii) $q = 3 + \alpha(2)$

$\alpha = 2.5$

$\alpha = 3 \Rightarrow q = 9$

(iv) b) 121 11 01 10 2 00 120 02

$X_8\ X_6\ X_3\ X_1\ X_2\ X_7\ X_4$

Symbol	Prob							
X_1	0.22 2	0.22 2	0.25	1	0.53	0
X_2	0.20 00	0.20 00	0.22	2	0.25	1
X_3	0.18 01	0.18 01	0.20	00	0.22	2
X_4	0.15 02	0.15 02	0.18	01		
X_5	0.10 10	0.10	0.15	02		
X_6	0.08 11	0.08				
X_7	0.05	0.07				
X_8	0.02					
X_9	0					

(Codes in second stage: 10 0, 11, 12; third stage: 12 0, 12 1, 12 2)

$$H_r(S) = \frac{H(S)}{\log_2 r}$$

$$H_r(S) = \frac{H(S)}{\log_2(3)} = 1.735$$

$$L = \sum_{i=1}^{9} P_i l_i$$

$L = 1.85$

$$\eta_c = \frac{H(S)}{L} = 93.73\%$$

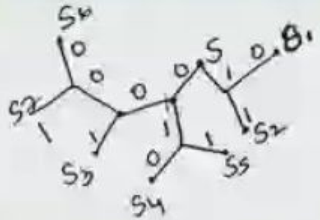
$R_{\eta_c} = 1 - \eta_c = 6.21\%$

$H(S)=2.625$ bits/symbol

$L=2.625$ bits/symbol

$\%\eta=H(s)/L=100\%$

Code tree



1.6 Discrete memoryless Channel

A channel is defined as the medium through which the coded signals are generated by an information source are transmitted. In general, the input to the channel is a symbol belonging to an alphabet 'A' with 'r' symbols, the output of a channel is a symbol belonging to an alphabet 'B' with 's' symbols.

1.6.1 Representation of a channel:

A communication channel may be represented by a set of input alphabets A=$(a_1, a_2, a_3 \dots \dots \dots a_r)$ consisting of 'r' symbols and set of output alphabets B=$(b_1, b_2, b_3 \dots \dots \dots b_s)$ consisting of s symbols and a set of conditional probability $P(b_j/a_i)$ with i=1,2,…. r and j=1,2,…… s

$$A \left.\begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_r \end{matrix}\right\} \rightarrow P(b_j/a_i) \rightarrow \left\{\begin{matrix} b_1 \\ b_2 \\ \vdots \\ b_s \end{matrix}\right. B$$

The conditional probabilities come in to the existence due to the presence of noise in the channel. Because of noise there will be some amount of uncertainty in the reception of any symbols. For this reason there are 's' number of symbols at the receiver from 'r' symbols at transmitter. Tot… form of matrix

$$A \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_r \end{matrix} \Bigg\} \rightarrow P\left(b_j / a_i\right) \rightarrow \Bigg\{ \begin{matrix} b_1 \\ b_2 \\ \vdots \\ b_s \end{matrix} \; B$$

The conditional probabilities come in to the existence due to the presence of noise in the channel. Because of noise there will be some amount of uncertainty in the reception of any symbols. For this reason there are 's' number of symbols at the receiver from 'r' symbols at transmitter. Totally there are r * s conditional probabilities represented in a form of matrix which is called as Channel Matrix or Noise Matrix.

$$P(b_j/a_i) = \begin{array}{c} \\ a_1 \\ \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & & b_s \end{array} \\ \begin{bmatrix} P(b_1/a_1) & P(b_2/a_1) & P(b_3/a_1) & \cdots & P(b_s/a_1) \\ P(b_1/a_2) & P(b_2/a_2) & P(b_3/a_2) & \cdots & P(b_s/a_2) \\ P(b_1/a_3) & P(b_2/a_3) & P(b_3/a_3) & \cdots & P(b_s/a_3) \\ P(b_1/a_4) & P(b_2/a_4) & P(b_3/a_4) & \cdots & P(b_s/a_4) \\ \vdots & & & & \\ P(b_1/a_r) & P(b_2/a_r) & P(b_3/a_1) & \cdots & P(b_s/a_r) \end{bmatrix} \end{array}$$

When a_1 is transmitted, it can be received as any one of the output symbols $(b_1, b_2, b_3 \dots \dots . . b_s)$

Therefore $P_{11} + P_{12} + P_{13} + \cdots \dots P_S = 1$

Therefore $P_{11} + P_{12} + P_{13} + \cdots \ldots P_S$=1

$$\Rightarrow P(^{b_1}/_{a_1}) + P(^{b_2}/_{a_1}) + P(^{b_3}/_{a_1}) + \ldots\ldots\ldots\ldots P(^{b_s}/_{a_1}) = 1$$

In general, $\sum_{j=1}^{s} P(^{b_j}/_{a_i}) = 1$ for i= 1 to r

Thus the sum of all the elements in any row of the channel matrix is equal to UNITY.

1.6.2 Joint Probability:

Joint probability between any input symbol a_i and any output symbol b_j is given by

$$P(a_i \cap b_j) = \boldsymbol{P(a_i, b_j)} = \mathbf{P}(^{b_j}/_{a_i})\boldsymbol{P(a_i)}$$

$$P(a_i, b_j) = P(a_i/b_j)P(b_j)$$

Properties:

Consider the source alphabet A=$(a_1, a_2, a_3 \dots\dots.. a_r)$ and output alphabet B=$(b_1, b_2, b_3 \dots\dots.. b_s)$

- The source entropy is given by $H(A) = \sum_{i=1}^{r} P_{a_i} log_2(\frac{1}{P_{a_i}})$
- The entropy of the receiver or output is given by $H(B) = \sum_{j=1}^{s} P_{b_j} log_2(\frac{1}{P_{b_j}})$
- If all the symbols are equi-probable, then maximum source entropy is $H(A)_{max} = log_2 r$
- Conditional Entropy: The entropy of input symbols $a_1 a_2 a_3 \dots\dots.. a_r$ after the transmission and reception of particular output symbol b_j is defined as conditional entropy, denoted by $H(A/b_j)$

$$H\left(A/b_j\right) = \sum_{i=1}^{r} P\left(a_i/b_j\right) log_2 \frac{1}{P\left(a_i/b_j\right)}.$$

- If the average value of all the conditional probability is taken as j varies from 1 to s denoted