Its $N = 200$ point DFT is illustrated in Fig. 5.16(e). Now the effect of the wider spectral window function is clearly evident. First, the main peak is very wide as a result of the wide spectral window. Second, the sinusoidal envelope variations in the spectrum away from the main peak are due to the large sidelobes of the rectangular window spectrum. Consequently, the DFT is no longer a good approximation of the analog signal spectrum.

5.5 SUMMARY AND REFERENCES

The major focus of this chapter was on the discrete Fourier transform, its properties and its applications. We developed the DFT by sampling the spectrum $X(\omega)$ of the sequence $x(n)$.

Frequency-domain sampling of the spectrum of a discrete-time signal is particularly important in the processing of digital signals. Of particular significance is the DFT, which was shown to uniquely represent a finite-duration sequence in the frequency domain. The existence of computationally efficient algorithms for the DFT, which are described in Chapter 6, make it possible to digitally process signals in the frequency domain much faster than in the time domain. The processing methods in which the DFT is especially suitable include linear filtering as described in this chapter and correlation, and spectrum analysis, which are treated in Chapters 6 and 12. A particularly lucid and concise treatment of the DFT and its application to frequency analysis is given in the book by Brigham (1988).

PROBLEMS

5.1 The first five points of the eight-point DFT of a real-valued sequence are $\{0.25, 0.125 - j0.3018, 0, 0.125 - j0.0518, 0\}$. Determine the remaining three points.

5.2 Compute the eight-point circular convolution for the following sequences.

(a) $x_1(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$$x_2(n) = \sin\frac{3\pi}{8}n \qquad 0 \le n \le 7$$

(b) $x_1(n) = (\frac{1}{4})^n \qquad 0 \le n \le 7$

$$x_2(n) = \cos\frac{3\pi}{8}n \qquad 0 \le n \le 7$$

(c) Compute the DFT of the two circular convolution sequences using the DFTs of $x_1(n)$ and $x_2(n)$.

5.3 Let $X(k)$, $0 \le k \le N-1$, be the N-point DFT of the sequence $x(n)$, $0 \le n \le N-1$. We define

$$\hat{X}(k) = \begin{cases} X(k), & 0 \le k \le k_c,\ N-k_c \le k \le N-1 \\ 0, & k_c < k < N-k_c \end{cases}$$

and we compute the inverse N-point DFT of $\hat{X}(k)$, $0 \le k \le N-1$. What is the effect of this process on the sequence $x(n)$? Explain.

5.4 For the sequences

$$x_1(n) = \cos \frac{2\pi}{N} n \qquad x_2(n) = \sin \frac{2\pi}{N} n \qquad 0 \le n \le N-1$$

determine the N-point:

(a) Circular convolution $x_1(n) \, \text{Ⓝ} \, x_2(n)$

(b) Circular correlation of $x_1(n)$ and $x_2(n)$

(c) Circular autocorrelation of $x_1(n)$

(d) Circular autocorrelation of $x_2(n)$

5.5 Compute the quantity

$$\sum_{n=0}^{N-1} x_1(n) x_2(n)$$

for the following pairs of sequences.

(a) $x_1(n) = x_2(n) = \cos \frac{2\pi}{N} n \qquad 0 \le n \le N-1$

(b) $x_1(n) = \cos \frac{2\pi}{N} n \qquad x_2(n) = \sin \frac{2\pi}{N} n \qquad 0 \le n \le N-1$

(c) $x_1(n) = \delta(n) + \delta(n-8) \qquad x_2(n) = u(n) - u(n-N)$

5.6 Determine the N-point DFT of the Blackman window

$$w(n) = 0.42 - 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1} \qquad 0 \le n \le N-1$$

5.7 If $X(k)$ is the DFT of the sequence $x(n)$, determine the N-point DFTs of the sequences

$$x_c(n) = x(n) \cos \frac{2\pi k n}{N} \qquad 0 \le n \le N-1$$

and

$$x_s(n) = x(n) \sin \frac{2\pi k n}{N} \qquad 0 \le n \le N-1$$

in terms of $X(k)$.

5.8 Determine the circular convolution of the sequences

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$$

using the time-domain formula in (5.2.39).

5.9 Use the four-point DFT and IDFT to determine the sequence

$$x_3(n) = x_1(n) \, \text{Ⓝ} \, x_2(n)$$

where $x_1(n)$ and $x_2(n)$ are the sequence given in Problem 5.8.

5.10 Compute the energy of the N-point sequence

$$x(n) = \cos \frac{2\pi k n}{N} \qquad 0 \le n \le N-1$$

5.11 Given the eight-point DFT of the sequence

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & 4 \le n \le 7 \end{cases}$$

compute the DFT of the sequences:

(a) $x_1(n) = \begin{cases} 1, & n = 0 \\ 0, & 1 \le n \le 4 \\ 1, & 5 \le n \le 7 \end{cases}$

(b) $x_2(n) = \begin{cases} 0, & 0 \le n \le 1 \\ 1, & 2 \le n \le 5 \\ 0, & 6 \le n \le 7 \end{cases}$

5.12 Consider a finite-duration sequence

$$x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4\}$$

(a) Sketch the sequence $s(n)$ with six-point DFT

$$S(k) = W_2^k X(k) \qquad k = 0, 1, \ldots, 6$$

(b) Determine the sequence $y(n)$ with six-point DFT $Y(k) = \mathrm{Re}\,|X(k)|$.

(c) Determine the sequence $v(n)$ with six-point DFT $V(k) = \mathrm{Im}\,|X(k)|$.

5.13 Let $x_p(n)$ be a periodic sequence with fundamental period N. Consider the following DFTs:

$$x_p(n) \underset{N}{\overset{\text{DFT}}{\longleftrightarrow}} X_1(k)$$

$$x_p(n) \underset{3N}{\overset{\text{DFT}}{\longleftrightarrow}} X_3(k)$$

(a) What is the relationship between $X_1(k)$ and $X_3(k)$?

(b) Verify the result in part (a) using the sequence

$$x_p(n) = \{\cdots 1, 2, 1, 2, \underset{\uparrow}{1}, 2, 1, 2 \cdots\}$$

5.14 Consider the sequences

$$x_1(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4\} \qquad x_2(n) = \{\underset{\uparrow}{0}, 1, 0, 0, 0\} \qquad s(n) = \{\underset{\uparrow}{1}, 0, 0, 0, 0\}$$

and their 5-point DFTs.

(a) Determine a sequence $y(n)$ so that $Y(k) = X_1(k)X_2(k)$.

(b) Is there a sequence $x_3(n)$ such that $S(k) = X_1(k)X_3(k)$?

5.15 Consider a causal LTI system with system function

$$H(z) = \frac{1}{1 - 0.5z^{-1}}$$

The output $y(n)$ of the system is known for $0 \le n \le 63$. Assuming that $H(z)$ is available, can you develop a 64-point DFT method to recover the sequence $x(n)$, $0 \le n \le 63$? Can you recover all values of $x(n)$ in this interval?

5.16* The impulse response of an LTI system is given by $h(n) = \delta(n) - \frac{1}{4}\delta(n - k_0)$. To determine the impulse response $g(n)$ of the inverse system, an engineer computes the N-point DFT $H(k)$, $N = 4k_0$, of $h(n)$ and then defines $g(n)$ as the inverse DFT of