# Digital Filters

Infinite impulse response (IIR) filters

$$H(z) = \frac{\sum_{r=0}^{M} b_r z^{-r}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \qquad H(z) = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + \cdots + p_M z^{-M}}{d_0 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_N z^{-N}}$$

Finite impulse response (FIR) filters

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

# DIGITAL IIR FILTERS

A digital filter, $H(e^{jw})$, with infinite impulse response (IIR), can be designed by first transforming it into a prototype analog filter $H_c(j\Omega)$ and then design this analog filter using a standard procedure.

Once the analog filter is properly designed, it is then mapped back to the discrete-time domain to obtain a digital filter that meets the specifications.

# Contd..

**The commonly used analog filters are**

1. **Butterworth filters:** no ripples at all,
2. **Chebyshev filters:** ripples in the passband OR in the stopband, and
3. **Elliptical filters:** ripples in BOTH the pass and stop bands.

A disadvantage of IIR filters is that they usually have nonlinear phase. Some minor signal distortion is a result.

**main techniques used to design IIR filters:**

1. The Impulse Invariant method,
2. Matched z-transform method, and
3. The Bilinear transformation method

# Impulse Invariance Transformation(IIV)

$$h_a(t) = LT^{-1}\{H_a(s)\}$$

$\Downarrow$ sampling

$$h(n) = h_a(nT), \ t = 0,1,2,\cdots$$

$$H_a(s) = \sum_{i=1}^{N} \frac{A_i}{s - s_i}$$

$$h_a(t) = \sum_{i=1}^{N} A_i e^{s_i t} u(t)$$

$$h(n) = h_a(t) = \sum_{i=1}^{N} A_i e^{s_i nT} u(nT)$$

$$H(z) = \sum_{i=1}^{N} \frac{A_i}{1 - e^{s_i T} z^{-1}}$$

$$z = e^{sT}$$

$\Omega \Rightarrow \omega$

$$z = e^{sT}$$

$(\sigma + j\Omega)T$

one-to-one

IIV

Ω- Analog

$$re^{j\omega} = e$$

ω- digital

$\sigma < 0 \;, \quad 0 \le r \le 1$

many to one

$\sigma > 0 \;, \quad r > 1$

$\sigma = 0 \;, \quad r = 1$

$\Omega_1, \Omega_2, \Omega_3 \to \omega$