$$H(A/B) = 0.809 \text{ bits/message-symbol}$$

$$I(A, B) = H(A) + H(A/B)$$

$$I(A, B) = H(B) - H(B/A)$$

= 1.857 - 0.6
 $I(A, B) = 1.257 \text{ bits/message-symbol}$

Ex.2: A transmitter transmits 5 symbols with probabilities 0.2, 0.3, 0.2, 0.1 and 0.2. Given the channel matrix
$$P(B/A)$$
, Calculate $H(B)$ and $H(A,B)$

nnel matrix
$$P(B/A)$$
, Calculate $H(B)$ and $H(A,B)$

$$P(B/A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I(A, B) = 1.25 / Dits/message-symbol

Ex.2: A transmitter transmits 5 symbols with probabilities 0.2, 0.3, 0.2, 0.1 and 0.2. Given the channel matrix P(B/A), Calculate H(B) and H(A,B)

channel matrix
$$P(B/A)$$
, Calculate $H(B)$ and $H(A,B)$

$$P(B/A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Solution:

We know that,
$$P(a_i, b_i)=P(a_i)P(b_i/a_i)$$

The JPM may now be constructed by multiplying 1st row elements by
$$P(a_1) = 0.2 = \frac{1}{5}$$
, 2^{rod} row by $P(a_2) = 0.3 = \frac{3}{10}$, 3^{rod} row by $P(a_3) = 0.2 = \frac{1}{5}$, 4^{th} row by $P(a_4) = 0.1 = \frac{1}{10}$ and 5^{th} row by $P(a_5) = 0.2 = \frac{1}{5}$.

$$P(A, B) = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \begin{bmatrix} 1/5 & 0 & 0 & 0 \\ 3/40 & 9/40 & 0 & 0 \\ 0 & 1/15 & 2/15 & 0 \\ 0 & 0 & 1/30 & 1/15 \\ 0 & 0 & 1/5 & 0 \end{bmatrix}$$

Adding the element pf each column, we get

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$$P(b_1) = \frac{1}{5} + \frac{3}{40} = \frac{11}{40}$$

$$P(b_2) = \frac{9}{40} + \frac{1}{15} = \frac{7}{24}$$

$$P(b_3) = \frac{2}{15} + \frac{1}{30} + \frac{1}{5} = \frac{11}{30}$$

$$P(b_4) = \frac{1}{15}$$

 $= \frac{1}{5} \log 5 + \frac{3}{40} \log \frac{40}{3} + \frac{9}{40} \log \frac{40}{9} + \frac{1}{15} \log 15 + \frac{2}{15} \log \frac{15}{2}$

$$P(b_4) = \frac{1}{15}$$

$$H(B) = \sum_{i=1}^{4} P(b_i) \log \frac{1}{P(b_i)}$$

$$H(B) = \sum_{j=1}^{4} P(b_j) \log \frac{1}{P(b_j)}$$

$$H(B) = \sum_{j=1}^{4} P(b_j) \log \frac{1}{P(b_j)}$$

$$= \frac{11}{40} \log \frac{40}{11} + \frac{7}{24} \log \frac{24}{7} + \frac{11}{30} \log \frac{30}{11} + \frac{1}{15} \log 15$$

$$\sum_{i=1}^{4} P(h_i) \log_{1} \frac{1}{h_i}$$

H(B) = 1.822 bits/message-symbol

 $H(A, B) = \sum_{i=1}^{3} \sum_{j=1}^{4} P(a_i, b_j) \log \frac{1}{P(a_i, b_j)}$

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Hide $+\frac{1}{30}\log 30 + \frac{1}{5}\log 5 + \frac{1}{15}\log 15$

H(A, B) = 2.7653 bits/message-symbol



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$$P(X/Y) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 15 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15 & 0 & 0 & 25 \end{bmatrix}$$

$$\therefore \qquad \qquad y_1 \qquad y_2$$

$$X_1 \qquad \qquad x_2 \qquad \begin{cases} 1/2 & 0 \\ 0 & 2/3 \\ 0 & 0 \\ 1/2 & 1/3 \end{cases}$$

$$X_{3} = \begin{cases} 0 & 0 & 23 \\ 1/2 & 1/3 & 0 & 2/5 \end{cases}$$

$$H(X/Y) = 0.05 \log 2 + 0.05 \log 2 + 0.1 \log 3/2 + 0.1 \log 3/2$$

$$H(X/Y) = 0.05 \log 2 + 0.05 \log 2 + 0.1 \log 3/2 + 0.05 \log 3 + 0.2 \log 50 + 0.1 \log 5 + 0.2 \log 5/2 + 0.05 \log 5 + 0.1 \log 5/2 + 0.$$

$$+ 0.1 \log 5 + 0.7 \log 5/2 + 0.05 \log 5 + 0.05 \log 5$$

H(X/Y) = 1.379 bits/message symbol

∴
$$H(X/Y) = 0.05 \log 2 + 0.05 \log 2$$

Hide () = $\begin{bmatrix} 0.05 & 0 & 0.20 & 0.05 \\ 0.30 & 0 & 0.30 & 0.30 \\ 0 & 0.10 & 0.10 & 0 \\ 0.20 & 0.20 & 0.20 \\ 0 & 0 & 0.30 & 0.30 \\ 0.05 & 0.05 & 0.20 & 0 & 0.10 \\ 0.20 & 0.20 & 0.20 \end{bmatrix}$

$$P(X/Y) = \frac{x_2}{x_3} \begin{bmatrix} 0 & 2/3 \\ 0 & 0 \\ 1/2 & 1/3 \end{bmatrix}$$

$$\therefore H(X/Y) = 0.05 \log 2 + 0.05 \log 2 + 0.1 \log 5 + 0.7 \log 5/2$$

below:

Stop sharing

reen.

$$P(X/Y) = \frac{x_1}{x_2} \begin{bmatrix} 1/2 & 0 & 2/3 \\ 0 & 2/3 & 1/3 \\ 0 & 0 & 2/3 \\ 1/2 & 1/3 & 0 \end{bmatrix}$$

$$\therefore H(X/Y) = 0.05 \log 2 + 0.05 \log 2 +$$

$$P(X/Y) = \begin{cases} x_1 \\ x_2 \\ x_3 \\ x_4 \end{cases} \begin{bmatrix} y_1 & y_2 \\ 1/2 & 0 \\ 0 & 2/3 \\ 0 & 0 \\ 1/2 & 1/3 \end{bmatrix}$$

$$P(X/Y) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \begin{bmatrix} y_1 & y_2 & y_1 & y_4 \\ 1/2 & 0 & 2/5 & 1/5 \\ 0 & 2/3 & 1/5 & 0 \\ 0 & 0 & 2/5 & 2/5 \\ 1/2 & 1/3 & 0 & 2/5 \end{bmatrix}$$



 $H(Y/X) = \sum_{i=1}^{4} \sum_{j=1}^{4} P(x_i, y_j) \log \frac{1}{P(y_i/x_i)}$ bits/message-symbol.







Using the relationship
$$P(y_j/x_i) = \frac{P(x_i, y_j)}{P(x_i)}$$
, the channel matrix $P(Y/X)$ is constructed.

$$H(Y/X) = \sum_{i=1}^{4} \sum_{j=1}^{4} P(x_i, y_j) \log \frac{1}{P(y_j/x_i)}$$
 bits/message-symbol.

Using the relationship
$$P(y_j/x_j) = \frac{P(x_i, y_j)}{P(x_i)}$$
, the channel matrix $P(Y/X)$ is constructed.

below:

$$P(Y/X) = \begin{bmatrix} 0.05 & 0 & 0.20 & 0.05 \\ 0.30 & 0 & 0.30 & 0.30 \\ 0 & 0.10 & 0.10 & 0 \\ 0 & 0.20 & 0.20 & 0 \\ 0 & 0 & 0.30 & 0.30 \\ 0.05 & 0.05 & 0 & 0.10 \\ 0.20 & 0.20 & 0 & 0.20 \end{bmatrix}$$

$$P(Y/X) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ 1/6 & 0 & 2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix}$$

H(Y/X) =
$$0.05 \log 6 + 0.2 \log 3/2 + 0.05 \log 6 + 0.1 \log 2 + 0.1 \log 2 + 0.2 \log 3/2 + 0.1 \log 3 + 0.05 \log 4 + 0.05 \log 4 + 0.1 \log 2$$

Stop sharing Hide = 1.151 bits/message-symbol

Verification:

1/3 1/2 $H(Y/X) = 0.05 \log 6 + 0.2 \log 3/2 + 0.05 \log 6 + 0.1 \log 2 + 0.1 \log 2$ $+0.2 \log 3/2 + 0.1 \log 3 + 0.05 \log 4 + 0.05 \log 4 + 0.1 \log^2 2$ H(Y/X) = 1.151 bits/message-symbolVerification: = H(X, Y) - H(X)Hide top sharing = 3.122 - 1.971H(Y/X) = 1.151 bits/message-symbol as before

below:

$$Y(X) = \begin{bmatrix} 0 & 0 & 0.20 & 0.10 \\ 0.05 & 0.05 & 0 & 0.30 \\ 0.20 & 0.20 & 0 & 0.10 \\ 0.20 & 0.20 & 0 & 0.20 \end{bmatrix}$$

$$y_1 & y_2 & y_3 & y_4 \\ y_2 & y_3 & y_4 \\ y_3 & 0 & 0.2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix}$$

$$Y(X) = \begin{cases} x_1 \\ x_2 \\ x_3 \\ x_4 \end{cases} \begin{bmatrix} 1/6 & 0 & 2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix}$$

$$Y(X) = 0.05 \log 6 + 0.2 \log 3/2 + 0.05 \log 6 + 0.1 \log 2 + 0.2 \log 3/2 + 0.05 \log 4 + 0.05$$

0.30

0.10

0.10

0.05 0.30

0

0.10

$$H(X/Y) = H(X, Y) - H(Y)$$

= 3.122 - 1.743
 $H(Y/X) = 1.379$ bits/message-symbol as before
 $I(X, Y) = H(X) - H(X/Y)$
= 1.971 - 1.379
 $I(X, Y) = 0.592$ bits/message-symbol
 $I(X, Y) = H(Y) - H(Y/X)$
= 1.743 - 1.151
 $I(X, Y) = 0.592$ bits/message-symbol - verified.

1 7 Rinary Symmetric Channel:

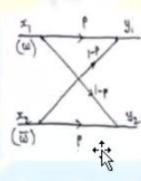
Stop sharing

Hide

ry symmetric channel is one of the most commonly and widely used channel whose

1.7 Binary Symmetric Channel:

The binary symmetric channel is one of the most commonly and widely used channel whose channel diagram is given below



From the above diagram, channel matrix can be written as

$$P(X/Y) = \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix} = \begin{bmatrix} P & \bar{P} \\ \bar{P} & P \end{bmatrix}$$

The matrix is a symmetric matrix. Hence the channel is binary symmetric channel.

Stop sharing Hide inel Capacity

It is known that $C = \text{Max}\{[H(Y) - H(Y/Y)] r_s\}$.

From the above diagram, channel matrix can be written as

$$P(X/Y) = \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix} = \begin{bmatrix} P & \bar{P} \\ \bar{P} & P \end{bmatrix}$$

The matrix is a symmetric matrix. Hence the channel is binary symmetric channel.

1.8 Channel Capacity

It is known that
$$C = \operatorname{Max}\{[H(Y) - H(Y/X)] r_s\}.$$

For symmetry channel,
$$H(Y/X) = h = P \log \frac{1}{p} + \bar{P} \log \frac{1}{\bar{p}}$$

Since it is a binary symmetric channel, $H(Y)_{max} = log_2 s = log_2 2 = 1$

$$\therefore C = 1 - h \text{ bits/sec.}$$

For symmetry channel,
$$H(Y/X) = h = P \log \frac{1}{p} + \overline{P} \log \frac{1}{\overline{p}}$$

Since it is a binary symmetric channel, $H(Y)_{max} = log_2 s = log_2 2 = 1$

$$\therefore C = 1 - h \, \underline{\text{bits/sec}}.$$

Solution:

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DCS

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Ex.1: A binary symmetric channel has the following noise matrix with source probabilities of
$$P(x_1)=2/3$$
 and $P(x_2)=1/3$. $P(Y/X)=\begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$. Determine $H(X)$, $H(Y)$, $H(Y)$, $H(Y/X)$, $H(X/Y)$, $I(X,Y)$, Channel Capacity, Channel efficiency and redundancy.