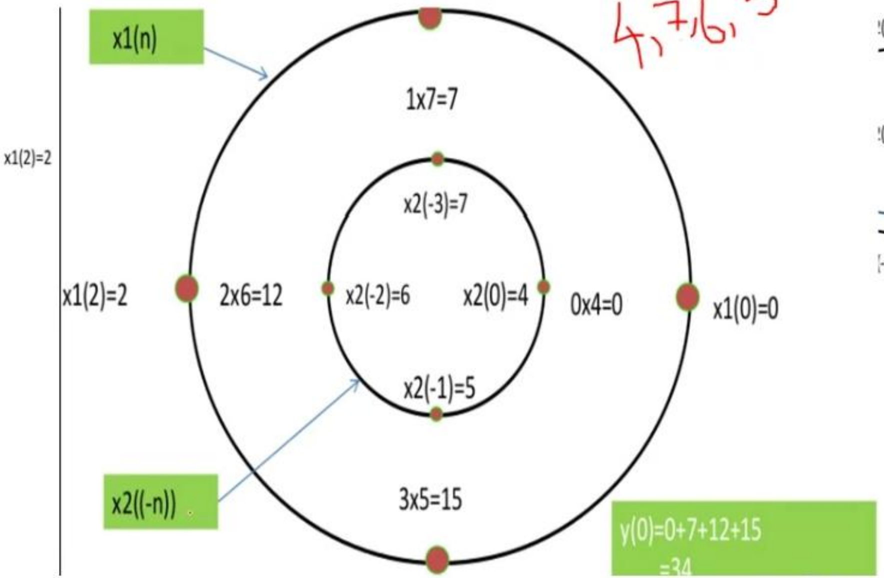
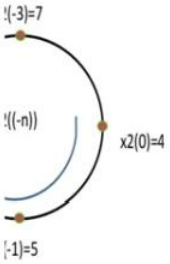
Circular convolution

$x1(n)=\{0, 1, 2, 3\}$ $x2(n)=\{4, 5, 6, 7\}$

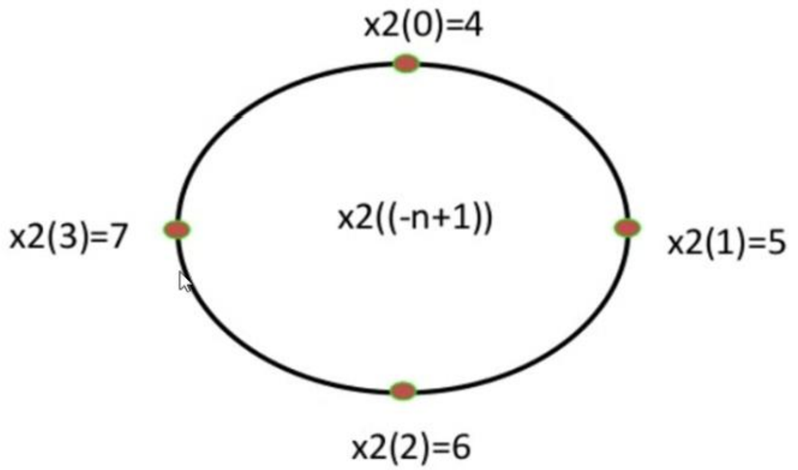
4, 7, 6, 5



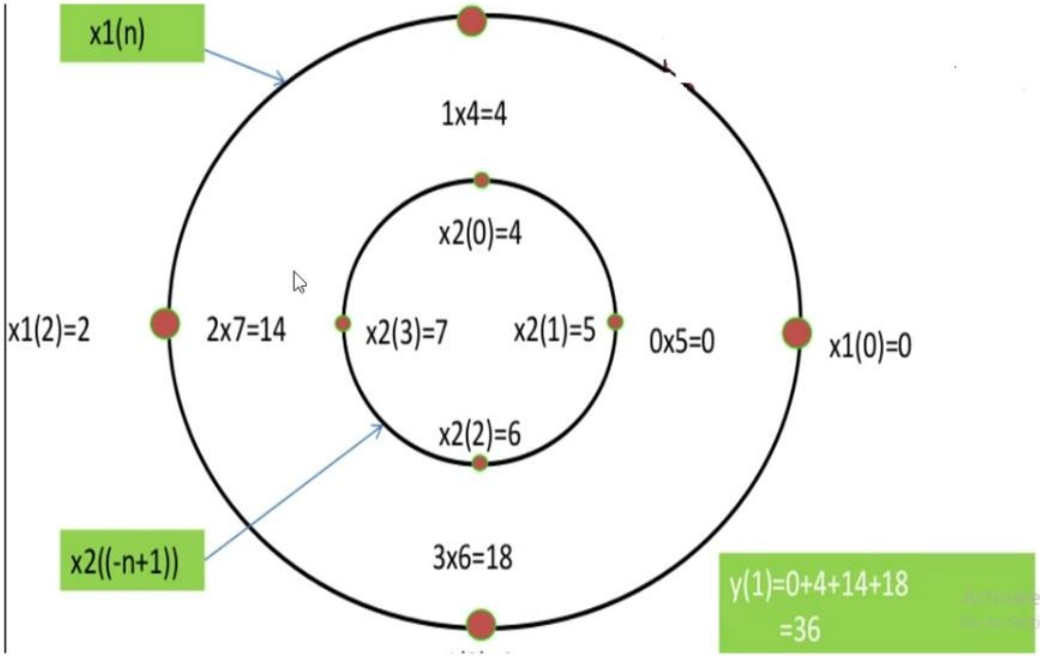


Contd...

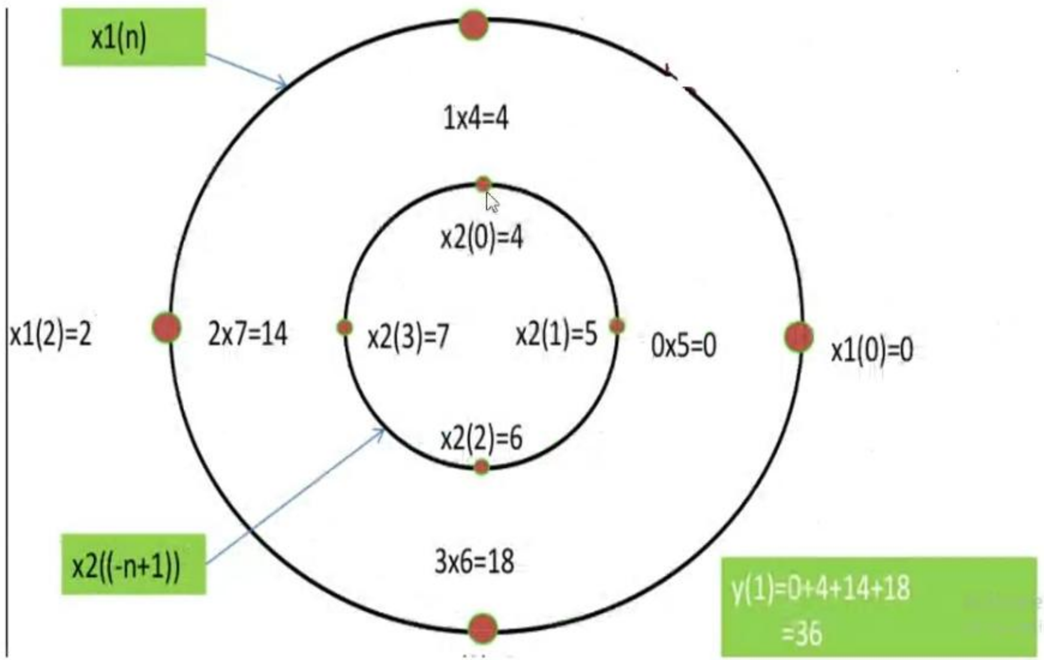
x2(1-n)=x2(-n+1)
i.e. shift x2(-n) by 1 sample anticlockwise



Contd...


x1(n)
1x4=4
x2(0)=4
x1(2)=2
2x7=14
x2(3)=7
x2(1)=5
0x5=0
x1(0)=0
x2(2)=6
x2((-n+1))
3x6=18
y(1)=0+4+14+18
=36

Contd...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x2((-n+1))
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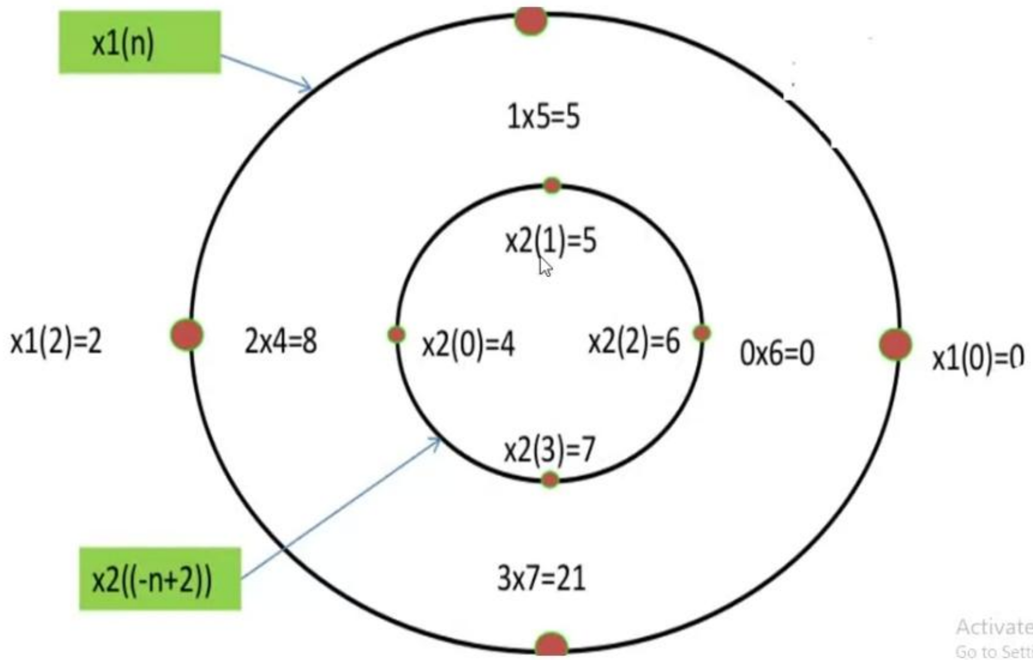
You

Contd...



Circular convolution using Matrix method

N-point DFT :

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}} \qquad \text{For } k=0,1,2.....N-1$$

Twiddle factor: $W_N = e^{-\frac{j2\pi}{N}}$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad \text{For } k=0,1,2.....N-1$$

$$V_N^{kn} = \begin{array}{c c} & \begin{array}{ccccc} n=0 & n=1 & n=2 & \ldots\ldots\ldots & n=N-1 \end{array} \\ \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ \vdots \\ \\ \\ k=N-1 \end{array} & \begin{bmatrix} W_N^{kn} & W_N^{kn} & W_N^{kn} & \ldots\ldots\ldots\ldots & W_N^{kn} \\ W_N^{kn} & W_N^{kn} & W_N^{kn} & \ldots\ldots\ldots\ldots & W_N^{kn} \\ W_N^{kn} & W_N^{kn} & W_N^{kn} & \ldots\ldots\ldots\ldots & W_N^{kn} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ W_N^{kn} & W_N^{kn} & W_N^{kn} & \ldots\ldots\ldots\ldots & W_N^{kn} \end{bmatrix} \end{array}$$

$$W_2^{kn} = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix}$$

$$W_N = e^{-\frac{j2\pi}{N}}$$

$$W_2 = e^{-\frac{j2\pi}{2}} \qquad W_2 = e^{-j\pi}$$

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta) \ldots\ldots\ldots\ldots \text{Euler's Identity}$$

$$W_2^{kn} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

You

Contd...

$$W_4^{kn} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

DFT : $X_N = [W_N] x_N$

- Where

$$X_N = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ . \\ . \\ . \\ X(N-1) \end{bmatrix}$$

$$W_4^{kn} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X_N = [W_N] x_N$$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\{4, 1-j, -2, 1+j\}$$

You

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{array}{c} n \rightarrow \\ k \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \cancel{2} \\ 3 \end{bmatrix}$$

$$(8, -1+j, -2, -1-j)$$

$$(6, -1+j, 0, -1-j)$$

$$Y(k) = X(k) . H(k)$$

$$Y(k) = (48, -2j, 0, 2j)$$

$$y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}$$

Comption of DFT(Computational Complexity)

The DFT pair was given as

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn} \qquad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j(2\pi/N)kn}$$

Baseline for computational complexity:

- Each DFT coefficient requires
  - N complex multiplications;
  - N-1 complex additions
- All N DFT coefficients require
  - N^2 complex multiplications;
  - N(N-1) complex additions

$$X(\overset{K}{0}) = \underline{x(0)W_4^0} + \underline{x(1)W_4^0} + \underline{x(2)W_4^0} + \underline{x(3)W_4^0}$$

$c_1 = 4$ - complex m $\quad c_2$

c_1 $(c+jd)$

$(a+jb)+c+jb \quad 3 \qquad (a+jb)$

$(a+c)+j \quad \cancel{x_4}\,(4) - N^2 \qquad \underset{1}{ac} + \underset{2}{jad} + \underset{3}{jbc} - \underset{4}{bd}$

$4N^2$ - Real

$4(3) - \boxed{N(N-1)} \qquad \underline{2N^2}$ - Real

$N^2 - N$ add

4 real multi

2 real add

$(2N^2 - 2N) + 2$

You

FFT Algorithms