Digital Communication

VI Sem EC/TC

ARUNKUMAR G M.Tech

Lecturer in E&CE Dept.,

Sri Taralabalu Jagadguru Institute of Technology (S.T.J.I.T), Ranebennur.

For suggestions and feedback:

Cell: 9731311770, 8553655825, 8095382275

e-mail: arunkumar.stjit@gmail.com

ARUNKUMAR G M.Tech
Lecturer in E&CE Department
S.T.J.I.T., Ranebennur

INTRODUCTION

1) Bring out merits and demerits of digital Communication over analog Communication Jan-08, 4M

2) Mention the advantages of digital communication System Jan-09, 4M

3) Explain merits of digital Communication System over analog Communication System. Jan-10, 4M

Advantages of digital Communication Systems are :-

1) Immunity to transmission noise & Interference.

2) Digital Circuits are less Subjected to distortion & Interference than analog Circuits.

3) In digital Communication Systems error detection & Correction is possible, easy to remove noise, low error rates & high Fidelity.

4) Multiplexing of Signals is easier with digital Signals rather than analog Signals.

5) Regeneration of the Coded Signals along the transmission path is possible.

6) Digital Communication Systems provides Security & privacy to the data transmitted (Encryption).

7) Transmission rate can be changed easily.

8) Digital storage is cheaper & flexible.

9) A common format for encoding different kinds of message signals for the purpose of transmission.

10) Signal jamming can be avoided by using spread spectrum technique.

11) Flexible in configuring digital communication system.

Disadvantages :-

(DCS)

1) Digital communication systems requires larger transmission bandwidth.

2) Digital communication systems requires synchronization of transmitter & receiver.

3) System complexity is increased.

Channel :-

Channel is a medium through which electrical signal is sent from one place to another.

Characteristics or parameters of channels :-

The different channels are used/selected in digital communication is based on the following parameters :-

1) Bandwidth

2) Power

3) Amplitude & phase requirements at the o/p.
4) Linear & Non-linear characteristics requirement.
5) Effect of external Interference on the channel.

---

1) Specify the types of digital communication channels. Compare co-axial cable & optical channels June-09, 6M
2) Explain the salient features of different types of channels used for digital communication. Jan-10, 6M

July-06, 6M	Jan-08, 8M

* The channels used in digital communication are classified into
  1) Telephone channels
  2) Co-axial cables
  3) optical fibers
  4) Microwave radio
  5) Satellite channels

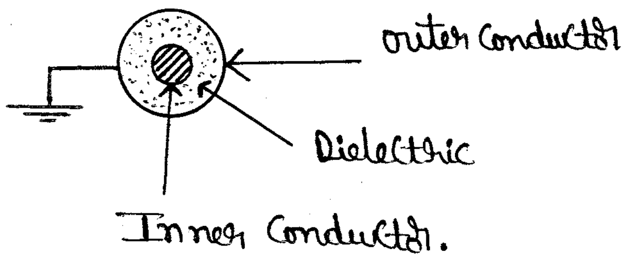
1) Telephone channels :-

* Telephone channel is designed to provide voice grade communication.
* Telephone channels are used for long distance communication.
* The transmission media used are open wire cables, optical fibers, microwave radio & satellite.
* Telephone channels has band-pass characteristics & a linear response.

* The Characteristics of telephone Channels are:

Sl No	Parameter	value
1)	Frequency range	300 Hz - 3,400 Hz
2)	Signal to Noise ratio (SNR)	30 dB
3)	Maximum Txission rate	16.8 Kbps.

3) Co-axial Cable :-



* A dielectric material Separates the Inner & the outer Conductor.

* The Co-axial Cable has 2 advantages:
  i) Wide bandwidth
  ii) Free from external Interference.

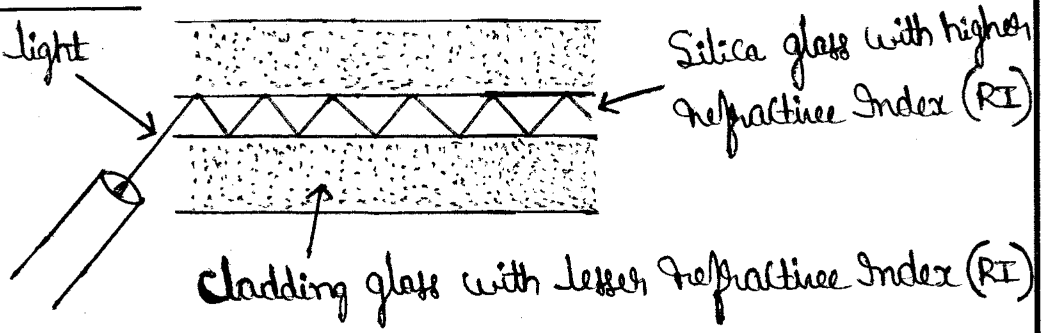
* The disadvantage is that it requires Closely Spaced repeaters.

* Characteristics of Co-axial Cable are:-

Sl No	Parameter	value
1)	Frequency range	upto 400 Hz
2)	Maximum Txission rate	274 Kbps
3)	Maximum repeater Spacing	1 Km

P.T.O

Optical Fiber Channel :-



* It consists of a very fine inner core of Silica glass, surrounded by a concentric layer called cladding, that is also made up of glass.

* The glass in the core has a higher refractive Index (RI) than that of the glass in the cladding.

* OFC work on the principle of Total Internal reflection. They are free from external electrical Interference.

* OFC has wide bandwidth & longer repeater separations.

* characteristics of OFC are :-

Sl No.	parameter	value
1)	Frequency range	10^{14} to 10^{15} Hz
2)	Maximum repeater spacing	2 Km
3)	Effect of external Noise	Minimum

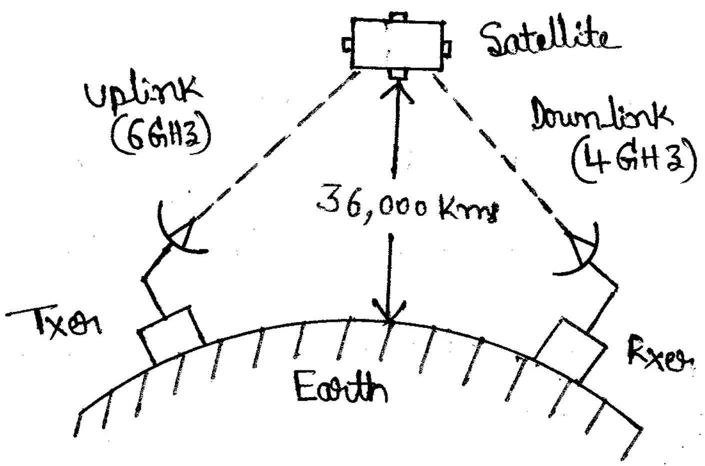
4) Microwave radio channel :-

* It operates on line of sight link consists of transmitter & receiver equiped with antenna of their own.
* These antennas are placed on towers of sufficient height to cover maximum distance.
* The major problem with this channel is "Multipath" reception i.e. propagation takes place in several paths. Due to phase shift among these path the received signal strength is increased & decreased called "FADING".

* Characteristics of microwave channel :-

SL No	parameter	value
1)	Frequency range	1 to 30 GHz
2)	Maximum distance covered	50 Kms
3)	Maximum Transmission rate	7500 Mbps

5) Satellite Channel :-



* It consists of a satellite in a geosynchronous orbit, an uplink from a ground station & a downlink to another ground station.
* uplink & downlink operates at microwave frequencies.
* The satellite channel acts like a repeater in the sky

Downloaded from www.citystudentsgroup.blogspot.com

permitting communication over long distance at high - bandwidth & relatively low cost.

* Satellite offers the following unique system capability

i) Broad area coverage
ii) Reliable transmission links
iii) Wide transmission bandwidth.

* Characteristics of Satellite channel :-

Sl No	parameter	value
1)	Frequency range	0.25 to 64 GHz
2)	Maximum distance covered	1/3 part of earth
3)	Maximum transmission rate	2500 Kbps.

* Comparision of Analog & Digital Communication System :-

Sl No	parameter	Analog Comm System	Digital Comm System
1)	Bandwidth	Less	More
2)	Error detection & correction	Not possible	possible
3)	Immune to Noise	Less	More
4)	Complexity	More	Less
5)	Cost	More	Less
6)	Quality of reconstruction	Good	very good
7)	Synchronization	Not required	Required.
8)	Security to data	Not possible	Possible
9)	Flexibility & reliability	Less	More

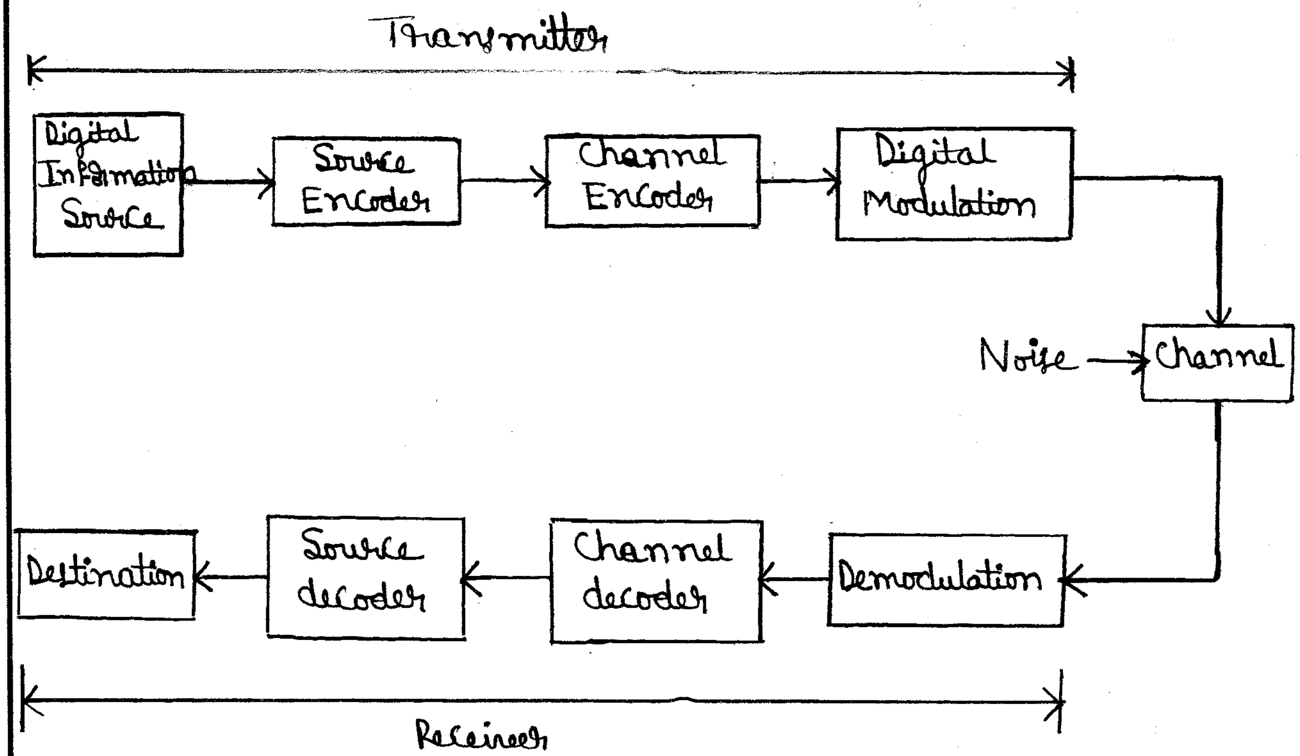
10)	power required	More	Less
11)	Multiplexing	FDM (Complex process)	TDM (Best Suited)
12)	Amplitude & time	Amplitude & time in message vary continuously w.r.to time	Both amplitude & time take discrete values.
13)	Implementation	Difficult to Implement analog ckts	Easier to Implement digital ckts.
14)	programmable	Not possible	possible

1) With neat block diagram explain the operation of digital Communication System. Explain the functioning of each block. Jan-06, 6M

2) With a neat block diagram, describe the basic Signal-processing operations involved in a digital Communication System

Jan-07, 7M	July-06, 6M

P.T.O



* The block diagram shown above consists of 3 main blocks:
  i) Transmitter
  ii) Communication Channel
  iii) Receiver

Digital Information Source :-

The Source of information is assumed to be digital i.e. Symbols, letters etc.

* If I/p is analog Signal, then it is Converted into digital form by using Sampler & Quantizer.
* The Sources of information are human voice, Television picture, Teletype data, atmospheric temperature & pressure etc.

Source Encoder & Decoder :-

Digital information coming out of

Source consists of lots of redundancy which when transmitted as it is results in " Improper utilization of bandwidth". Hence results in poor efficiency. The objective of Source encoder is to eliminate or reduce redundancy.

Source decoder :-

Source decoder at the receiver behaves exactly in a reverse way to Source encoder.

* Decoder converts the codes back to symbols i.e. converts digital information to discrete symbols.

Channel Encoder/Decoder :-

* Channel encoder & decoder are used to reduce the channel Noise effect.
* Channel Coding is the process of adding controlled redundancy to the data to be transmitted, to detect and/or correct the errors caused by the channel noise at the receiver.
* Addition of redundancy increases bit rate & hence increases bandwidth.
* Decoder detects the error in the received data & correct the error.

eg:- Error correcting codes like Linear block codes, cyclic codes & convolution codes.

Modulator & Demodulator :-

* Modulator converts the bitstream into a waveform suitable for transmission over the communication channels.

eg:- ASK, FSK, PSK, QPSK etc.

* Demodulator converts the waveform into digital data.
{optimum detectors are used to minimise the probability of error.}

Communication Channel :-

It is the media through which signal can be transmitted.

ex:- Free Space, Twisted wire, co-axial cable, waveguide, OFC etc.

Sampling :-

Sampling is a process where an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

i.e. process of converting continuous time signal into discrete time signal.

There are two types of Sampling :

1) Ideal Sampling or Impulse Sampling or Instantaneous Sampling
2) practical Sampling.
   i) Natural Sampling or Chopper Sampling
   ii) Flat Top Sampling or Sample & Hold Sampling.

1) Ideal Sampling or Impulse Sampling or Instantaneous Sampling:-

1) State & prove Sampling theorem as applied to low pass Signal.

2) State & prove Sampling & reconstruction of low pass Signals using Nyquist Criterion

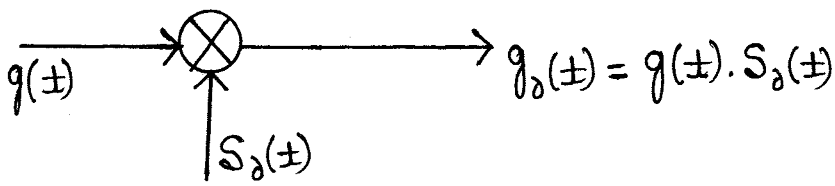
3) State Sampling theorem. Write the equations for the Spectrum of finite energy Signal $g(t)$ Sampled at $\frac{1}{2W}$ Seconds & $G(f)$, if 'W' is the highest frequency content of $g(t)$. Draw the diagrams of $G(f)$ & Sampled Signal $G_\delta(f)$.

July-06, 6M	July-07, 7M	July-09, 8M	Jan-07, 7M

Statement :

1) " A band limited Signal having frequency 'W' Hz can be completely described by its Samples if the Sampling frequency 'f_s' is greater than or equal to twice the highest frequency of message Signal."

2) A band limited Signal having highest frequency 'W' Hz can be completely recovered from its Samples if the Samples are taken at the rate 'f_s' greater than or equal to twice the highest frequency of message Signal i.e. $f_s \geq 2W$

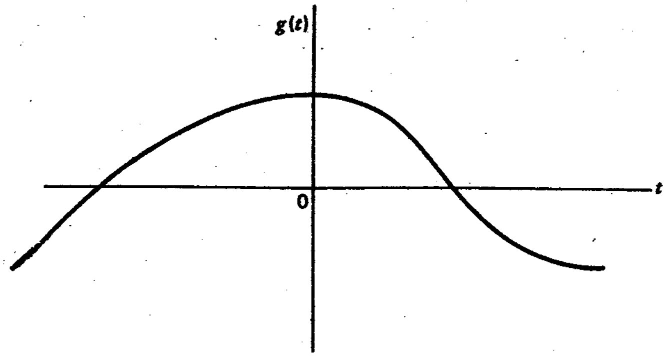


* The Sampled Signal $g_\delta(t)$ is obtained by multiplying the analog Signal $g(t)$ by a Sequence of Impulse $g_\delta(t)$ which is

periodic with period 'T_s' Seconds.

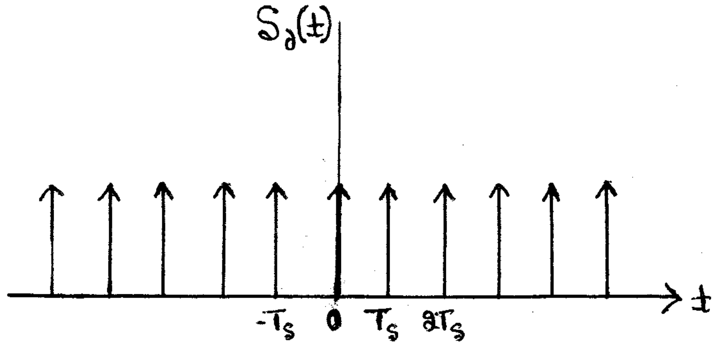
Time-domain analysis :-

ⓐ : Analog Signal

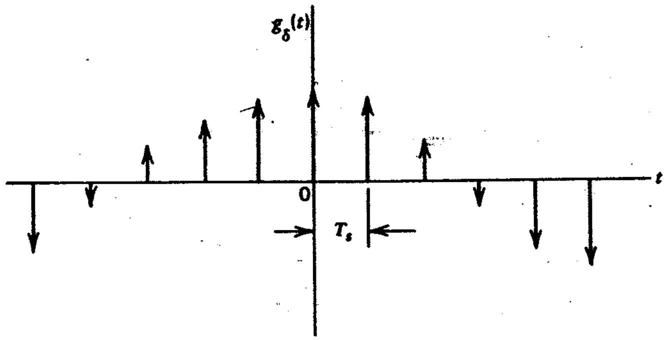


(a)

ⓑ : periodic Impulse



ⓒ : Discrete-time Signal



* Sampled o/p $g_\delta(t)$ is given by

$$\boxed{g_\delta(t) = g(t) \cdot S_\delta(t)} \longrightarrow (1)$$

* periodic Impulse train $S_\delta(t)$ can be represented as :

$$\boxed{S_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Using Shifting property of Impulse function

W.K.T $\quad g(t) \cdot \delta(t - nT_s) = g(nT_s)\, \delta(t - nT_s)$

$$\therefore \quad \boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s)} \longrightarrow (3)$$

Frequency domain analysis (Spectrum analysis):

Taking F.T of eq (1), we get

{ consider eq (1)
$g_\delta(t) = g(t) \cdot S_\delta(t) \rightarrow (1)$ }

$$G_\delta(f) = G(f) * S_\delta(f) \longrightarrow (4)$$

Where $\boxed{S_\delta(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)} \longrightarrow (5)$

Substituting eq (5) in eq (4), we get

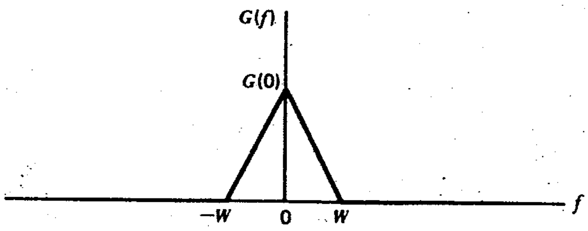
$$G_\delta(f) = G(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

From the convolution property of Impulse function

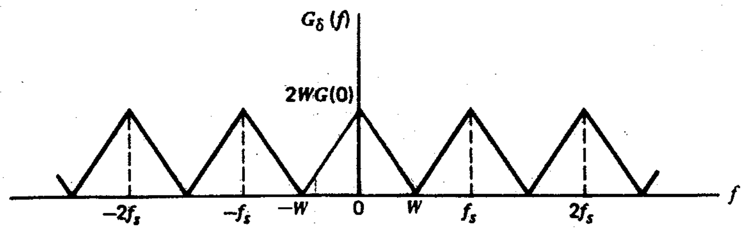
W.K.T $\quad G(f) * \delta(f - nf_s) = G(f - nf_s)$

$$\boxed{G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)} \longrightarrow (6)$$

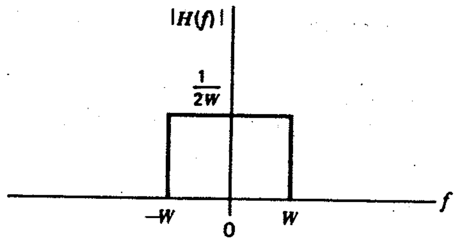
P.T.O



(a) Spectrum of signal $g(t)$.



(b) Spectrum of sampled signal $g_\delta(t)$ for a sampling rate $f_s = 2W$.



(c)
Ideal amplitude response of reconstruction filter.

* Equation 6 can be written as

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} G(f - nf_s) \longrightarrow (7)$$

When the Spectrum of $G_\delta(f)$ i.e. eq 7 is passed through a LPF, then the 2nd terms in R.H.S of eq 7 is eliminated resulting in $\boxed{G_\delta(f) = f_s G(f)}$

$$\therefore \; G(f) = \frac{1}{f_s} G_\delta(f) \longrightarrow (8)$$

Let $f_s = 2W$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Another useful expression for the FT of $G_\delta(f)$ may be obtained by taking the FT on both Side of eq ③

$$FT\left\{ g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \underline{\delta(t - nT_s)} \right\}$$

W.K.T $\quad \delta(t-nT_s) \xrightarrow{FT} e^{-j2\pi n f T_s}$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s} \longrightarrow ⑨$$

Substituting eq ⑨ in eq ⑧ & putting $f_s = 2W$

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{\frac{-j\cancel{2}\pi n f}{\cancel{2}W}}$$

$$\boxed{G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{\frac{-j\pi n f}{W}}} \longrightarrow ⑩$$

* If Sample values of $g\left(\frac{n}{2W}\right)$ are known for all values of n, then $G(f)$ of the Signal is uniquely determined by eq ⑩.

Reconstruction of the Signal from its Samples :-

Taking IFT of eq ⑩, we get

$$g(t) = \int_{-\infty}^{\infty} \underline{G(f)}\, e^{j2\pi f t}\, . df \longrightarrow ⑪$$

Substituting eq ⑩ in eq ⑪, we get

$$g(t) = \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{\frac{-j\pi n f}{W}} . e^{j2\pi f t} . df$$

Interchanging the order of Integration & Summation & Combining the exponential, we get

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \int_{-W}^{W} e^{j2\pi f\left(t - \frac{n}{2W}\right)} . df$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \left[\frac{e^{j2\pi f(t - n/2W)}}{j2\pi(t - n/2W)} \right]_{-W}^{W} \qquad \because \int e^{af} \cdot df = \frac{e^{af}}{a}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \left[\frac{e^{j2\pi W(t - \frac{n}{2W})} - e^{-j2\pi W(t - \frac{n}{2W})}}{j2\pi(t - \frac{n}{2W})} \right]$$

W.K.T $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$ Where $\boxed{\theta = 2\pi W\left(t - \frac{n}{2W}\right)}$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \frac{\sin 2\pi W\left(t - \frac{n}{2W}\right)}{\pi\left(t - \frac{n}{2W}\right)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin \cancel{2}\pi \cancel{W}\left(\frac{2Wt - n}{\cancel{2W}}\right)}{\cancel{2W}\pi\left(\frac{2Wt - n}{\cancel{2W}}\right)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin\pi(2Wt - n)}{\pi(2Wt - n)}$$

WKT

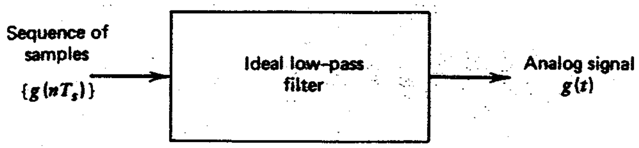
$$\boxed{\frac{\sin\pi\theta}{\pi\theta} = \operatorname{Sinc}\theta}$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{Sinc}(2Wt - n)} \rightarrow (12)$$

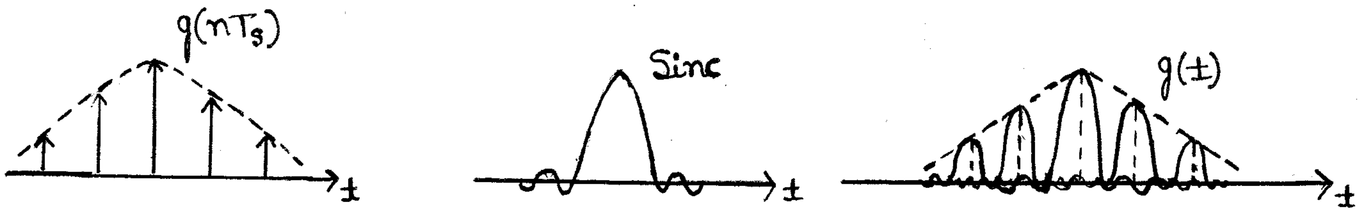
* Eq (12) is Known as Interpolation formula for reconstruction of $g(t)$ from Sequence of Samples $g\left(\frac{n}{2W}\right)$ each Sample is multiplied by a delayed Sinc function.
* All delayed Sinc function are added to obtain $g(t)$.

NOTE:-

* Explaination in time-domain :-

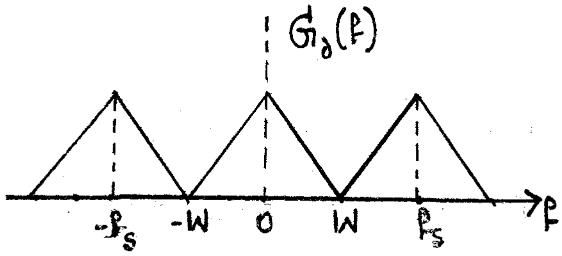


Reconstruction filter.

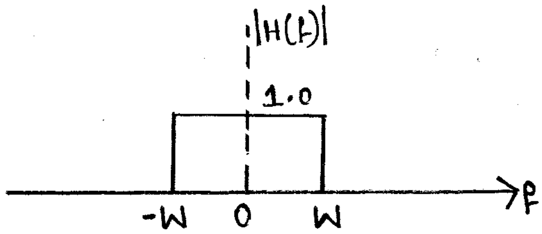


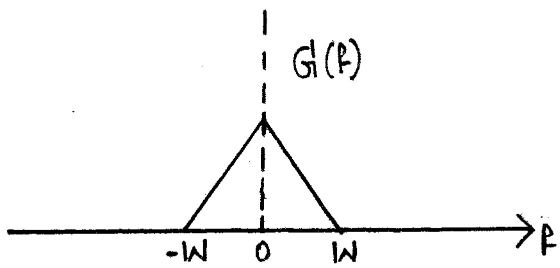
* If we pass Samples '$g(nT_s)$' through a filter whose Impulse-response is Sinc function then o/p is the convolution of I/p & Sinc function.
* The result is periodic repetition of Sinc pulses weighted by Sample magnitude.

* Frequency domain :-



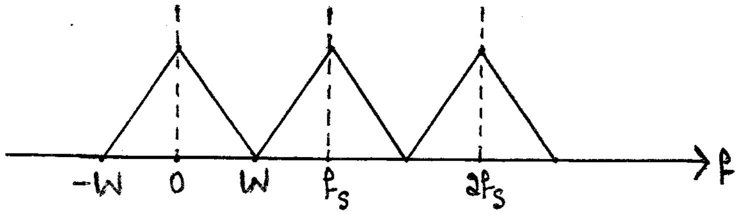
* Sampled Signal in frequency domain is the periodic repitition of the Spectrum of original message Signal at the Instants 'nf_s'. When this Signal is passed through filter whose response is rectangle ranges b/w -W & W centered at '0' HZ known as LPF. The o/p of LPF is Spectrum Component ranging b/W -W & W Centered at '0' HZ. Hence it is the original message Spectrum.





Case i : When $f_s = 2W$

* When $f_s = 2W$, Information Signal can be received from its Samples Ideally.

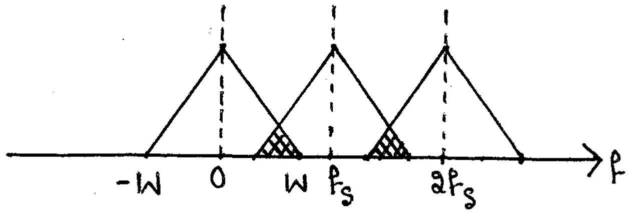


* Practically no filter has Sharp roll-off. Therefore practically it is NOT possible to recover the Signal $g(t)$.
* $f_s = 2W$ is Known as "Nyquist rate" &

$$T_s = \frac{1}{f_s} = \frac{1}{2W}$$ is Known as "Nyquist Interval."

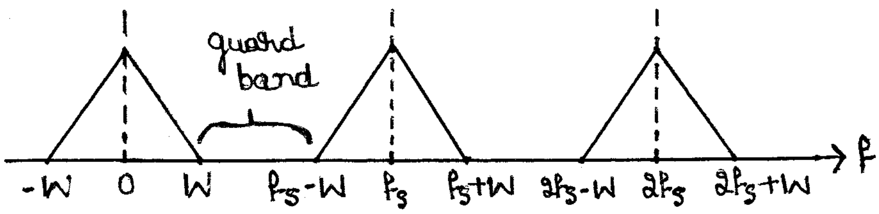
Case ii : When $f_s < 2W$

* When $f_s < 2W$ there will be overlapping of Spectrum Component. Hence o/p of LPF will have distortion due to unwanted frequency Component.



* This is Known as "Aliasing". This type of Sampling is Known as "Under Sampling."
* Aliasing can be overcome by using LPF as antialiasing filter before sampling which will Strictly band limit the Signal to be Sampled.
* Second technique of eliminating aliasing is increasing the Sampling frequency.

Case iii : When $f_s > 2W$



* When $f_s > 2W$, there will be "guard band" b/w components of the spectrum components. Hence original signal can be reconstructed from the samples.

* $f_s = 2.2W$ is the commercially used sampling rate & is known as Engineer's sampling rate.

Disadvantages of Ideal Sampling :-

1) The disadvantage of ideal sampling is that due to very narrow pulses, the transmitted power is very small & signal to noise ratio is very low. Thus ideally sampled pulses may get lost in the back ground noise.

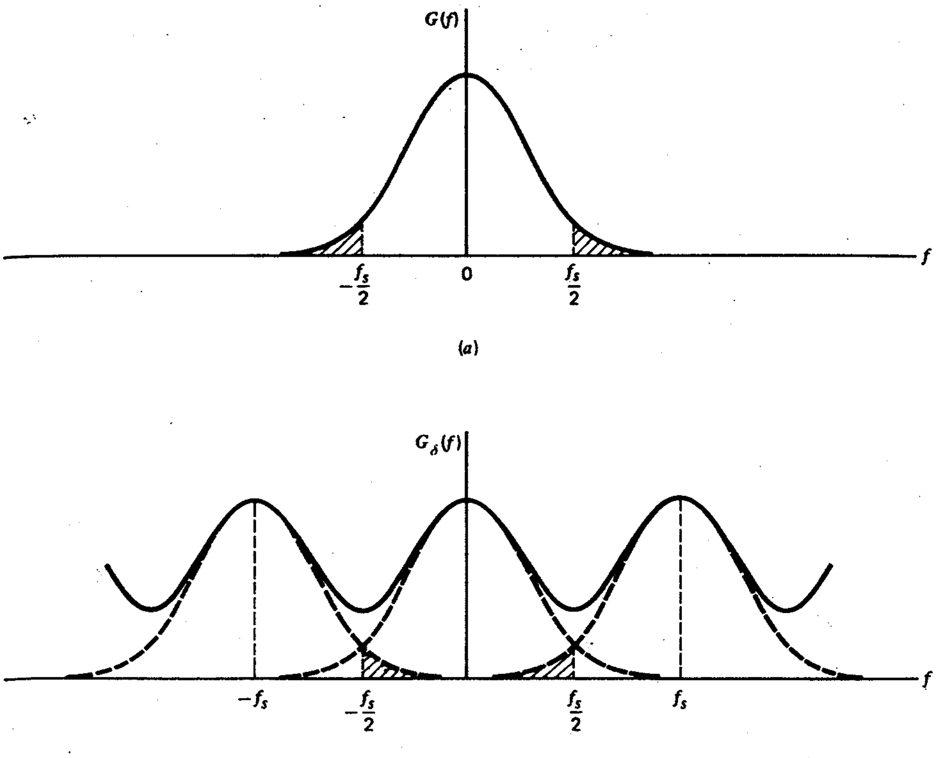
2) Ideal sampling is not possible to achieve practically, because practically it is impossible to have pulses of width approaching zero.

Signal distortion in Sampling :-

* The Sampling theorem was derived by assuming the Signal $g(t)$ is "Strictly bandlimited" i.e. its Spectrum is of finite bandwidth.

* If the Signal $g(t)$ is not Strictly bandlimited & if the Sampling frequency f_s is less than $2W$, then an error called "Aliasing" or foldover error occurs.

* If $f_s < 2W$, the adjacent Spectrum overlaps as shown in fig (b).



(*a*) Spectrum of a finite-energy signal $g(t)$ whose spectrum decreases with increasing frequency without limit. (*b*) Illustrating the composition of the spectrum $G_\delta(f)$ of the discrete-time signal $g_\delta(t)$; the dashed curves represent replicas of $G(f)$.

* Since the Signal is not Strictly bandlimited. Hence the Spectrum of the Signal $g(t)$ is as shown in fig (a).

* The Spectrum $G_\delta(f)$ of the discrete time Signal $g_\delta(t)$, which is the Sum of $G(f)$ & infinite number of frequency Shifted replica of it. The replicas of $G(f)$ are Shifted in frequency by multiples of the Sampling rate 'f_s'.

* Two replicas of $G(f)$ are Shown in fig (b), one at f_s & other at $-f_s$. We find that portions of the frequency Shifted - replicas are "Folded over" inside the desired Spectrum as Shown in fig (b).

* Due to this fold over, high frequency in $G(f)$ are reflected into low frequencies in $G_\delta(f)$.

* The phenomenon of a high frequency in the Spectrum of the original Signal $g(t)$ Seemingly taking on the Identity of a lower frequency in the Spectrum of the Sampled Signal $g(t)$ called aliasing or Foldover.

Bound on Aliasing Error or Analysis of Aliasing error :-

{ * WKT because of aliasing Some of the Information Contained in the original Signal may be lost & also the effect on aliasing on the o/p of reconstruction filter depends upon the original Spectrum $G(f)$.

* Hence the exact analysis of the aliasing become difficult So the worst Case analysis is Considered to obtain the effect of aliasing. } (Dont Write this in exam)

* Consider an arbitrary Signal $g(t)$ whose Samples are obtained by replacing 't' by nT_s.
* Let $g_i(t)$ be the Signal reconstructed from $g(t)$ by the Interpolation formula (derived earlier).

i.e.,

$$\boxed{g_i(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \mathrm{Sinc}\left[f_s t - n\right]} \longrightarrow (1)$$

replace T_s by $1/f_s$

$$g_i(t) = \sum_{n=-\infty}^{\infty} g\left(n/f_s\right) \mathrm{Sinc}\left[f_s t - n\right] \longrightarrow (2)$$

* The aliasing error is given by

$$\boxed{\varepsilon = \left|g(t) - g_i(t)\right|} \longrightarrow (3)$$

* Now we have to find the upper bound on aliasing error.
* Using the definition of IFT, we can write

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t} \cdot df \longrightarrow (4)$$

* Eq (4) can be written by its equivalent equation

$$g(t) = \sum_{m=-\infty}^{\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi f t} \cdot df \longrightarrow (5)$$

* Now by poisson's formula

$$\underbrace{\sum_{m=-\infty}^{\infty} G(f - mf_s)}_{\text{Spectrum of } G(f) \text{ repeats itself}} = \underbrace{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{f_s}\right) e^{-j2\pi (nf/f_s)}}_{\text{Complex Fourier Series}}$$

* Multiplying above equation by $e^{j2\pi f t}$ & Integrating w.r.to f from $-f_s/2$ to $+f_s/2$, we get

$$\int_{-f_s/2}^{f_s/2} \sum_{m=-\infty}^{\infty} G(f-mf_s)\, e^{j2\pi f t} \cdot df = \frac{1}{f_s} \int_{-f_s/2}^{f_s/2} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{f_s}\right) e^{-j2\pi(nf/f_s)}\, e^{j2\pi f t} \cdot dt$$

* Interchanging the order of Integration & Summation on both Sides of the above equation, we get

$$\sum_{m=-\infty}^{\infty} \int_{-f_s/2}^{f_s/2} G(f-mf_s)\, e^{j2\pi f t} \cdot df = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{f_s}\right) \int_{-f_s/2}^{f_s/2} e^{f[j2\pi t - j2\pi(n/f_s)]} \cdot df$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{f_s}\right) \underbrace{\frac{1}{f_s} \left[\frac{e^{f[j2\pi t - j2\pi(n/f_s)]}}{j2\pi t - j2\pi(n/f_s)} \right]_{f=-f_s/2}^{f_s/2}}_{\text{Sinc}(f_s t - n)}$$

$$\sum_{m=-\infty}^{\infty} \int_{-f_s/2}^{f_s/2} G(f-mf_s)\, e^{j2\pi f t} \cdot df = \underbrace{\sum_{n=-\infty}^{\infty} g\left(\frac{n}{f_s}\right) \text{Sinc}(f_s t - n)}_{g_i(t)} \longrightarrow (6)$$

From eq (1), we can write eq (6) as

$$\sum_{m=-\infty}^{\infty} \int_{-f_s/2}^{f_s/2} G\underbrace{(f-mf_s)}_{f}\, e^{j2\pi f t} \cdot df = g_i(t) \longrightarrow (7)$$

* Now change variable of Integration from f to $(f-mf_s)$

$+f_s/2 = f - mf_s$	$-f_s/2 = f - mf_s$
$f = mf_s + f_s/2$	$f = mf_s - f_s/2$
$f = f_s[m + 1/2]$	$f = f_s[m - 1/2]$

$\therefore$ Equation ⑦ becomes

$$g_i(t) = \sum_{m=-\infty}^{\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi(f - mf_s)t} . df$$

$$g_i(t) = \sum_{m=-\infty}^{\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi ft} . e^{-j2\pi mf_s t} . df$$

$$g_i(t) = \sum_{m=-\infty}^{\infty} e^{-j2\pi mf_s t} . \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi ft} . df \longrightarrow ⑧$$

Substituting eq ⑤ & eq ⑧ in eq ③, we get

$$\varepsilon = \left| \underbrace{\sum_{m=-\infty}^{\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi ft} . df}_{\text{Taking Common}} - \sum_{m=-\infty}^{\infty} e^{-j2\pi mf_s t} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f) . e^{j2\pi ft} . df \right|$$

$$\varepsilon = \left| \sum_{m=-\infty}^{\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi ft} . df \left[1 - e^{-j2\pi mf_s t} \right] \right| \longrightarrow ⑨$$

* It is noted that, the term $\left[1 - e^{-j2\pi f_s m t}\right] = 0$, for $m=0$

* The absolute value of $\left[1 - e^{-j2\pi f_s m t}\right] \leq 2$.

* The absolute value of the Integral in the eq ⑨ is bounded as

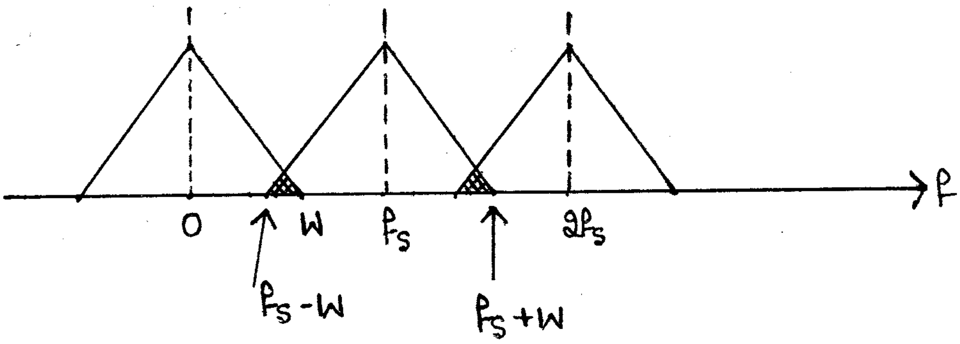
$$\left| \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f)\, e^{j2\pi ft} . df \right| \leq \int_{(m-1/2)f_s}^{(m+1/2)f_s} |G(f)|\, df$$

* Apply all these observation to eq ⑨, the error obtained as

$$\boxed{\varepsilon \leq 2 \int_{|f| > f_s/2} G(f) . df}$$

* Explain the effect of Aliasing & undersampling? Explain how it can be avoided.

Sol :-



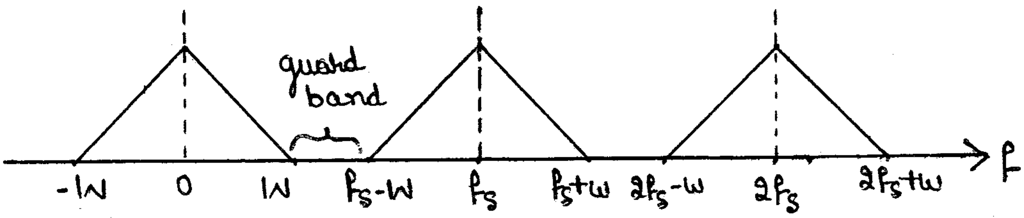
* When $f_s < 2W$ there will be overlapping of spectrum components i.e. high frequency interferes with low frequency & appears as low frequency, this phenomenon is called aliasing.

* Due to aliasing the o/p of LPF will have distortion.

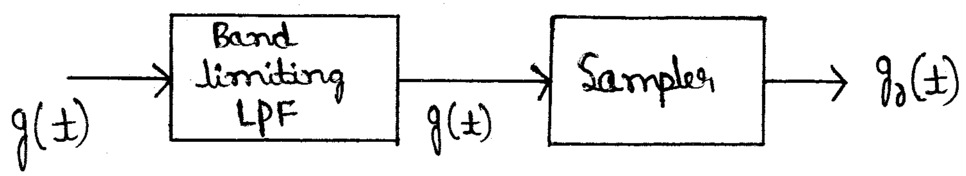
* Aliasing can be avoided by two methods.
  i) Sampling rate $f_s \geq 2W$.
  ii) Strictly bandlimit the signal to 'W' Hz.

i) Sampling rate $f_s \geq 2W$:-

When the sampling rate is made higher than 2W, then the spectrums will not overlap & there will be sufficient gap between the individual spectrums called guard band as shown in fig.



ii) Bandlimiting the Signal :-



* The filter shown above has a cut-off frequency at $f_c = W$. Hence $g(t)$ will be strictly bandlimited by this cut-off frequency. Such filters are called as Anti-aliasing filter or pre-alias filter.

Quadrature Sampling of bandpass Signals

OR

Sampling theorem for bandpass Signals :-

1) Explain the term quadrature sampling of bandpass signal with help of spectrum & block diagrams. July-05, 10M

2) Define the sampling theorem for bandpass signals with necessary diagrams. Explain the generation & reconstruction of bandpass signals. July-07, 7M | July-06, 6M | Jan-08, 8M

Sampling theorem for bandpass signal states that, "The bandpass signal $g(t)$ whose maximum bandwidth is $2W$ can be completely represented into & recovered from its samples if it is sampled at the minimum rate of twice the bandwidth".

* In Quadrature Sampling of bandpass Signal, bandpass Signals are Converted to Lowpass Signals by multiplying bandpass Signals with two Sinusoidal Signals which are phase Quadrature (90°) with each other. The Signals are Sampled using Low pass Sampling theorem.

* At the receiving end low pass Signals are reconstructed from Low pass Samples & then it is Converted to bandpass Signals by multiplying the low pass Signals by 2 Sinusoidal which are phase Quadrature (90°) with each other.

* Since two Sinusoidal Signals which are 90° out of phase are used, we call this techniques as "Quadrature band pass Sampling".

* Consider a bandpass Signal $g(t)$ whose Spectrum is limited to a bandwidth $2W$, centered around the frequency 'f_c' where $f_c > W$. Write Fig 1 (a) & Fig 1 (b) → page No → 30

* Any bandpass Signal can be represented interms of its INPHASE & Quadrature components as:

$$\boxed{g(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t}$$

Where

$g_I(t) \longrightarrow$ Inphase Component

$g_Q(t) \longrightarrow$ Quadrature Component

Downloaded from www.citystudentsgroup.blogspot.com

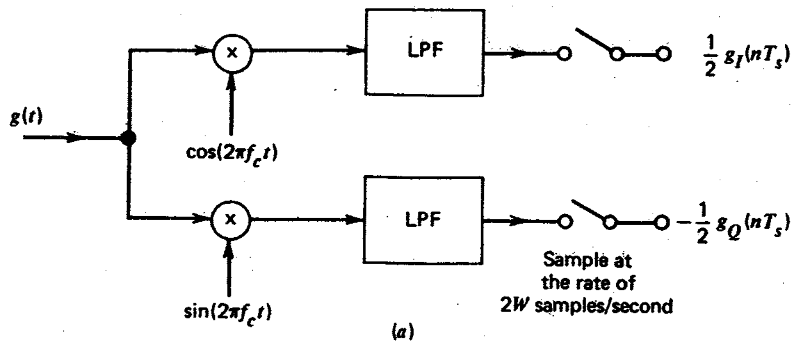


Fig 2: (a) Generation of in-phase and quadrative samples from band-pass signal $g(t)$.

* $g_I(t)$ & $g_Q(t)$ may obtained by multiplying the bandpass Signal $g(t)$ by $\cos 2\pi f_c t$ & $\sin 2\pi f_c t$ & passing the product through the LPF.

* The o/p of the Top LPF is $\frac{g_I(t)}{2}$ & is called Inphase Component

* Similarly the of the bottom LPF is $\frac{g_Q(t)}{2}$ & is called - Quadrature Component.

* Thus the o/p of low pass Signals $g_I(t)$ & $g_Q(t)$ are limited to 'W' Hz & then Sampled at a minimum rate of 2W Samples/Sec. The o/p of Sampler will be $\frac{1}{2} g_I(nT_s)$ & $\frac{1}{2} g_Q(nT_s)$.

Reconstruction:

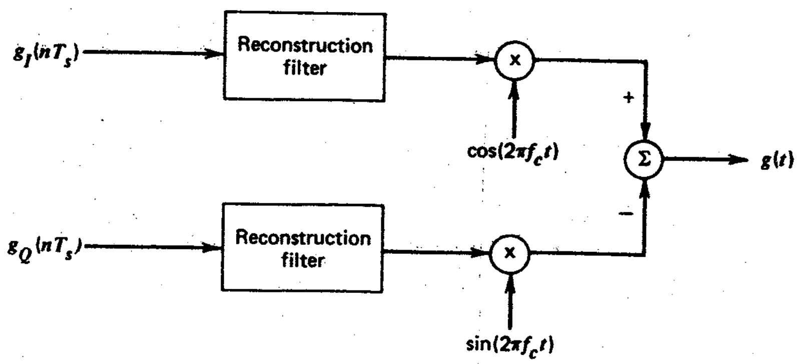


Fig 2: (b) Reconstruction of band-pass signal $g(t)$.

* To reconstruct the original band pass Signal from its -

Quadrature Sampled version we reconstruct $g_I(t)$ & $g_Q(t)$ from their Samples & multiply them by $\cos 2\pi f_c t$ & $\sin 2\pi f_c t$ & then add the result.

* The o/p is the required bandpass Signal $g(t)$ given by:

$$\boxed{g(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t}$$

Continued

Fig ①

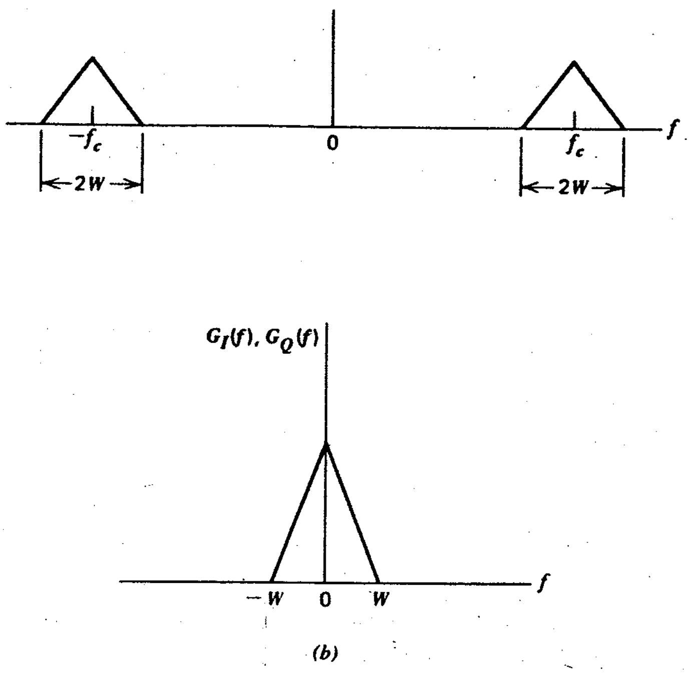


Fig ①: (*a*) Spectrum of bond-pass signal $g(t)$. (*b*) Spectrum of low-pass in-phase component $g_I(t)$ and quadrature component $g_Q(t)$.

Sampling procedure :-

The procedure involve the use of two corrective measure.

1) Prior to Sampling, a low-pass pre-alias filter of high order is used to attenuate the high frequency components of the Signal that do not contribute Significantly to the Information content of the Signal.

2) The filtered Signal (after LPF) is Sampled at a rate Slightly higher than the Nyquist rate 2W, where 'W' is the cut-off frequency of the pre-alias filter.

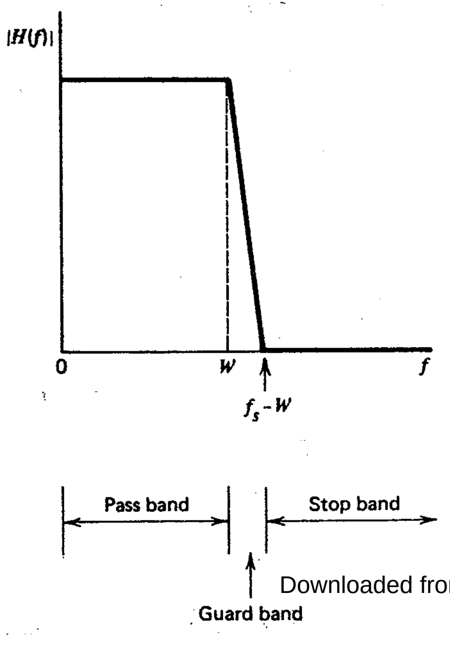
---

* Explain the characteristics of reconstruction filter ?

The characteristics of reconstruction filter are :

1) The pass band of the reconstruction filter extends from 'Zero' to 'W' hertz.

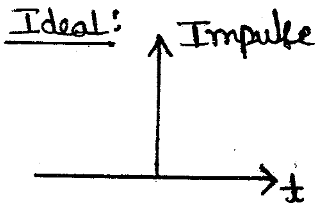
2) The amplitude response of the reconstruction filter rolls off gradually from W to $(f_s - W)$ Hz i.e. guard band as shown in fig.

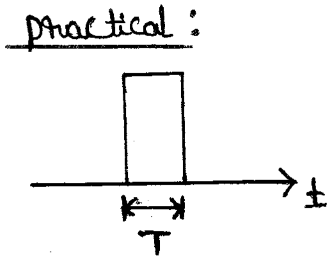


Illustrating the amplitude response of reconstruction filter for positive frequencies.

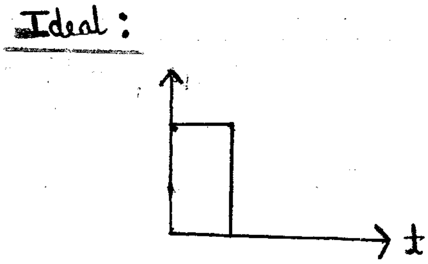
Practical aspects of Sampling :-

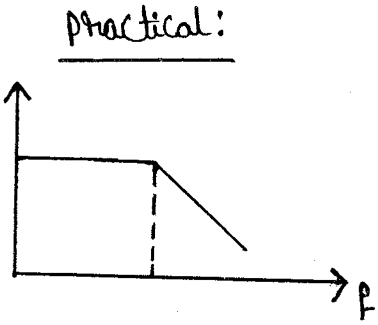
1) The Sampled pulses will have finite duration rather than Impulses. Amplitude of the pulses is also finite.





2) The practical reconstruction filters are not Ideal filters. These filters need a guard band or gap between the Spectrum component of Sampled Signal.





3) The Signals to be Sampled are not Strictly band limited.
∴ There is always problem in Selection of 'f_s'.

There are two types of practical Sampling:

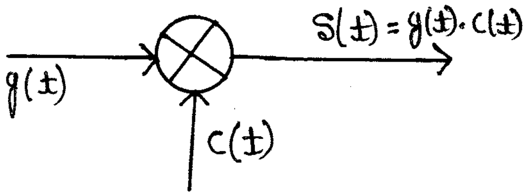
1) Natural Sampling
2) Flat Top Sampling

Natural Sampling or Chopper Sampling :-

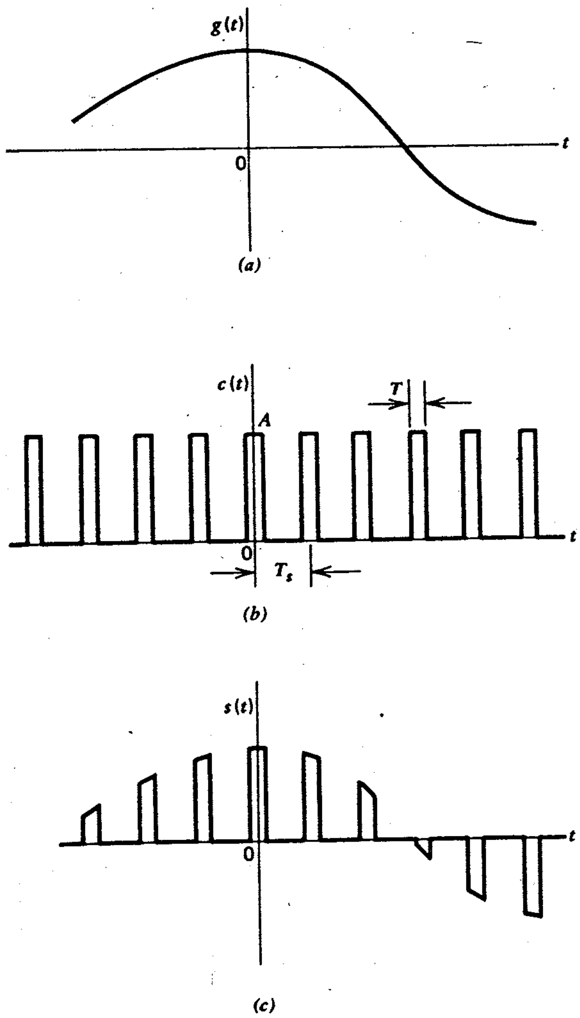
{
* Ideal Sampling results in the Samples where width 'T' approaches to 'Zero'. practically it is not possible to have such a pulse.

∴ practically we use pulse with width 'T' known as "Sample period"
}

* A Sampled Signal is a product of Sampling function $c(t)$ & message Signal $g(t)$ whose pulse width is 'T' Second repeated at the rate of 'f_s' Hz i.e.



$$\boxed{S(t) = g(t) \cdot c(t)} \longrightarrow (1)$$



(a) Analog signal. (b) Sampling function. (c) Sampled signal.

* Such a Sampling is termed as "Natural Sampling".

* Since the top of each pulse retains the shape of the analog Segment during that pulse interval.

P.T.O

* The pulse train $c(t)$ can be represented by using complex Fourier analysis as:

$$\boxed{c(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_s n t}} \longrightarrow (2)$$

Where c_n is the complex Fourier co-efficient given by

$$\boxed{c_n = ATf_s \,\text{Sinc}(nf_sT)} \longrightarrow (3)$$

Substituting eq (3) in eq (2), we get

$$c(t) = \sum_{n=-\infty}^{\infty} ATf_s \,\text{Sinc}(nf_sT)\, e^{j2\pi f_s n t} \longrightarrow (4)$$

Now, Substituting eq (4) in eq (1), we get

$$s(t) = g(t) \sum_{n=-\infty}^{\infty} ATf_s \,\text{Sinc}(nf_sT) \cdot e^{j2\pi f_s n t} \longrightarrow (5)$$

$$s(t) = ATf_s \sum_{n=-\infty}^{\infty} \text{Sinc}(nf_sT) \cdot g(t)\, e^{j2\pi f_s n t} \longrightarrow (6)$$

Taking FT of eq (6), we get

$$\boxed{g(t) \cdot e^{j2\pi f_s n t} \longleftrightarrow G(f - nf_s)}$$

$$\boxed{S(f) = ATf_s \sum_{n=-\infty}^{\infty} \text{Sinc}(nf_sT) \cdot G(f - nf_s)} \longrightarrow (6)$$

Eq (6) shows that the spectrum of $S(f)$ is periodic signal weighted by a sinc function.

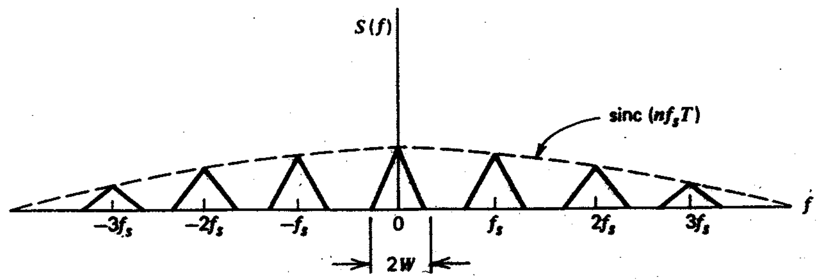


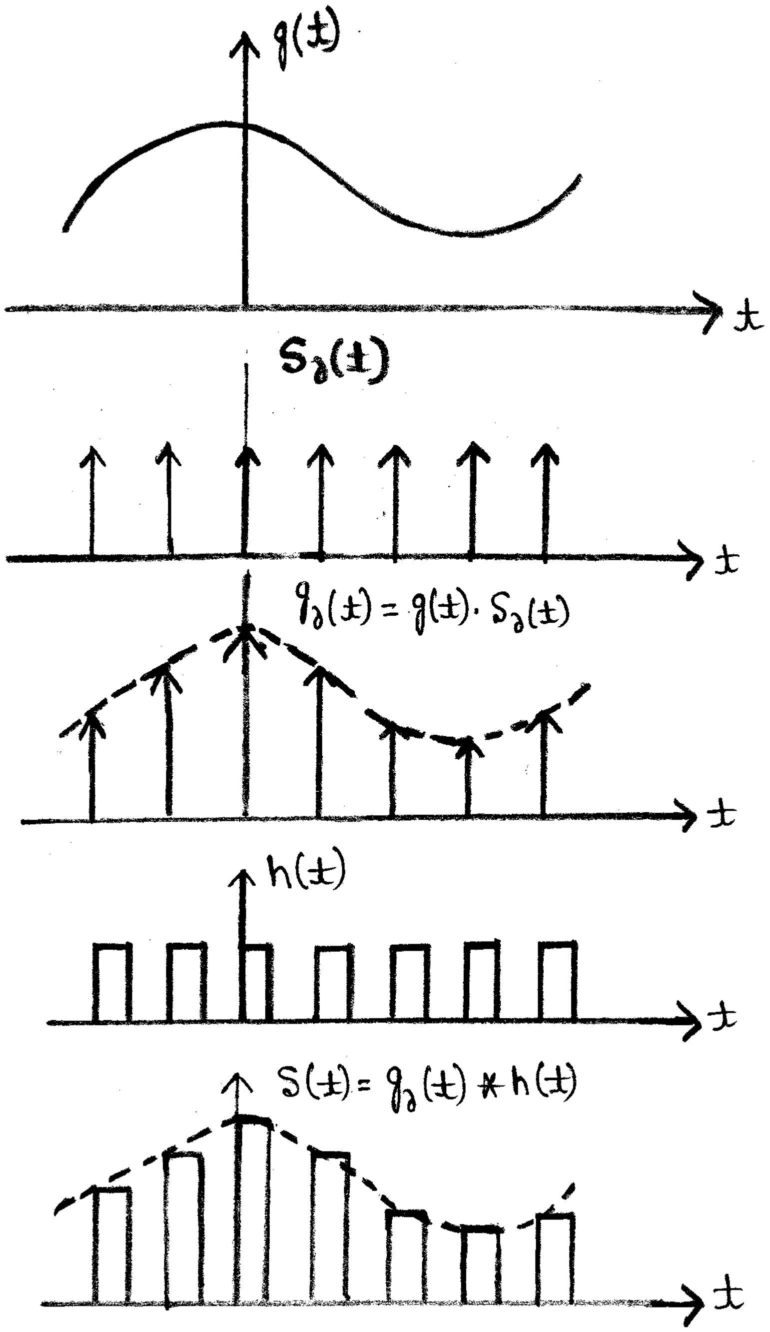
Fig: Spectrum of Naturally Sampled Signal

July-06, 10M	Jan-10, 8M	Jan-08, 7M

Flat - Top Sampling (OR) Rectangular pulse Sampling :-

* As the name itself indicates after Sampling, the pulses will have "Flat Top". It is very easy to obtain flat of Samples Compared to Natural Samples.

* The Top of the Samples remains constant & equal to Instantaneous value of the base band Signal $g(t)$ at the Start of Sampling.



* It is observed from the figure that only starting edge of the pulse represents the Instantaneous value of base band signal $g(t)$.

* The Sampled Signal $g_\delta(t)$ is given by

$$g_\delta(t) = g(t) \cdot S_\delta(t)$$

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s)} \longrightarrow (1)$$

* Convolving $g_\delta(t)$ with the pulse $h(t)$, we get

$$S(t) = g_\delta(t) * h(t) \longrightarrow (2)$$

$$S(t) = \int_{-\infty}^{\infty} \underline{g_\delta(\tau)} \cdot h(t-\tau)\, d\tau \longrightarrow (3)$$

Substituting eq (1) in eq (2), we get

$$S(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

Interchanging the order of Integration & Summation, we get

$$S(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \int_{-\infty}^{\infty} \underbrace{\delta(\tau - nT_s) \cdot h(t-\tau)\, d\tau} \longrightarrow (4)$$

{ From Shifting property

$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0)\, dt = x(t_0)$$

$$\therefore \int_{-\infty}^{\infty} h(t-\tau)\, \delta(\tau - nT_s)\, d\tau = h(t - nT_s)$$

}

Applying Shifting property in eq (4), we get

$$\boxed{S(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot h(t - nT_s)} \longrightarrow (5)$$

* Eq (5) represents the value of $s(t)$ interms of Sampled value $g(nT_s)$ & function $h(t-nT_s)$ for Flat Top Sampled Signal.

W.K.T $\quad s(t) = g_\delta(t) * h(t)$

Taking F.T on both Sides of above equation, we get

$$S(f) = \underline{G_\delta(f)} \cdot H(f)$$

Substituting $G_\delta(f)$ in above equation, we get

$$\boxed{S(f) = f_s \sum_{n=-\infty}^{\infty} G(f-nf_s) \cdot H(f)} \longrightarrow (6)$$

Eq (6) represents the Spectrum of Flat Top Sampled Signal.

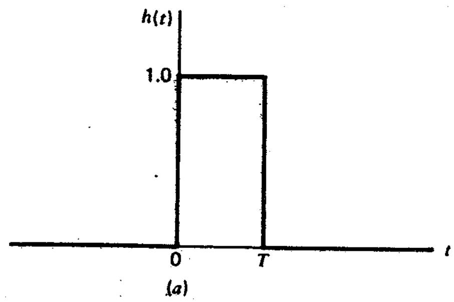
Aperture effect :-

June-09, (4M)

* **What is aperture effect? How is it eliminated?**

W.K.T

$$\boxed{H(f) = T\,\text{Sinc}(fT)\, e^{-j\pi f T}}$$



* By using Flat Top Samples an "amplitude distortion" introduced in reconstructed Signal (i.e. $g(t)$ from $s(t)$)

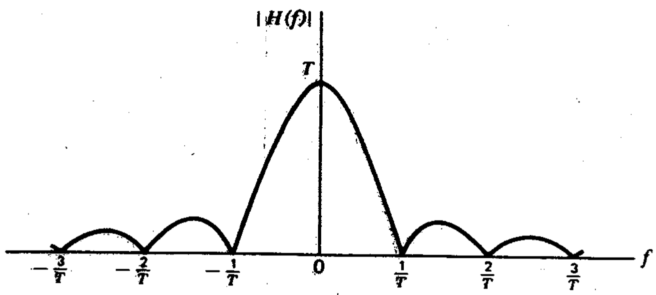
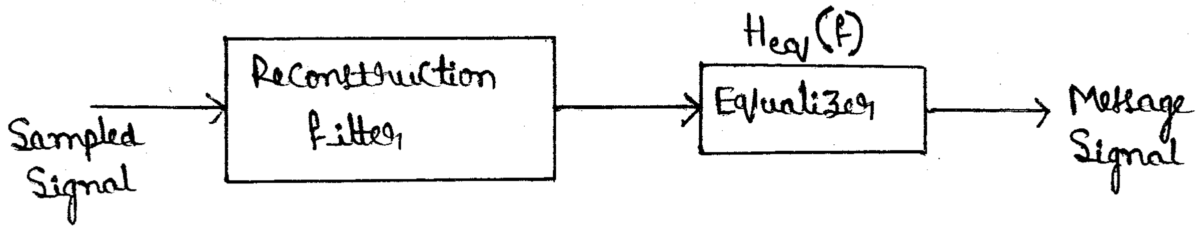


Fig: Spectrum of H(f)

* The high frequency roll-off of H(f) acts as LPF & attenuates upper portion of the message spectrum. This effect is known as "aperture effect."

Compensation & Remedy :-

1) Aperture effected can be eliminated by using "Equalizer" in cascaded with the reconstruction filter at the receiver as shown in fig.



* The transfer function of equalizer is given by

$$H_{eq}(f) = \frac{K e^{-j2\pi f t_d}}{H(f)}$$

Where t_d is the delay introduced by LPF which is equal to $T/2$

$$H_{eq}(f) = \frac{K e^{-j\cancel{2}\pi f T/\cancel{2}}}{T\,\mathrm{Sinc}(fT)\, e^{-j\pi f T}} = \frac{K \cancel{e^{-j\pi f T}}}{T\,\mathrm{Sinc}(fT)\, \cancel{e^{-j\pi f T}}}$$

$$\boxed{H_{eq}(f) = \frac{K}{T\,\mathrm{Sinc}\, fT}}$$

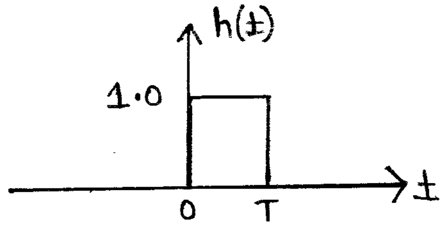
This is the transfer function of the equalizer.

2) Aperture effect can be compensated by selecting the pulse width 'T' of h(t) as very small.

* obtain the expression for fourier transform of Sampling function h(t) used for flat top Sampling. Hence explain aperture effect with the help of Spectral diagrams. Bring out the differences between aperture effect & aliasing effect.

$\boxed{\text{Jan-10, 8M}}$

Sol :-



$$H(f) = \int_0^T h(t)\, e^{-j2\pi f t} \cdot dt \longrightarrow (1)$$

$$H(f) = \int_0^T 1 \cdot e^{-j2\pi f t} \cdot dt$$

$$= \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right]_0^T$$

$$= \frac{1}{-j2\pi f}\left[e^{-j2\pi f T} - e^{0}\right]$$

$$= \frac{1}{-j2\pi f}\left[e^{-j2\pi f T} - 1\right]$$

$$= \frac{1}{+j2\pi f}\left[1 - e^{-j2\pi f T}\right]$$

$$= \frac{e^{-\frac{j\cancel{2}\pi f T}{\cancel{2}}}\left[e^{\frac{j\cancel{2}\pi f T}{\cancel{2}}} - e^{-\frac{j\cancel{2}\pi f T}{\cancel{2}}}\right]}{j2\pi f}$$

$$= \frac{e^{-j\pi f T}\left[e^{j\pi f T} - e^{-j\pi f T}\right]}{2j\pi f}$$

$$H(f) = e^{-j\pi f T}\,\frac{\sin \pi f T}{\pi f} \longrightarrow (2)$$

W.K.T

$$\boxed{\begin{aligned} \frac{1-e^{-\theta}}{\theta} &= \frac{e^{-\theta/2}\left[e^{\theta/2} - e^{-\theta/2}\right]}{\theta} \\ &= \frac{e^{-\theta/2+\theta/2} - e^{-\theta/2-\theta/2}}{\theta} \\ \frac{1-e^{-\theta}}{\theta} &= \frac{e^{0} - e^{-\theta}}{\theta} \end{aligned}}$$

$$\because \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

X^{ly} & ÷ eq (2) by 'T', we get

$$\operatorname{sinc}\theta = \frac{\sin\pi\theta}{\pi\theta}$$

$$H(f) = T\, e^{-j\pi f T} \cdot \frac{\sin \pi f T}{\pi f \cdot T}$$

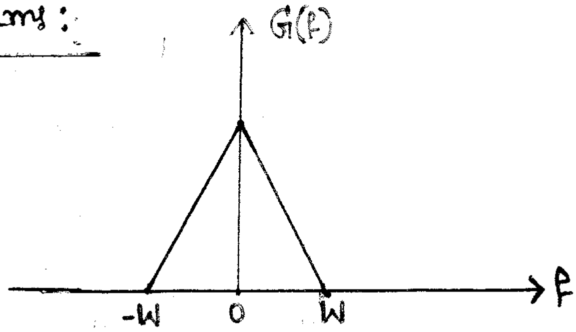
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

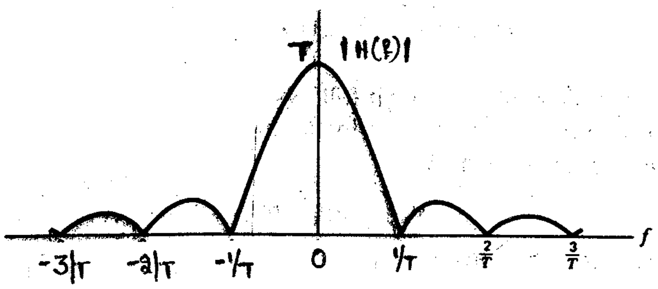
$$H(f) = T e^{-j\pi f T} \operatorname{Sinc} fT$$

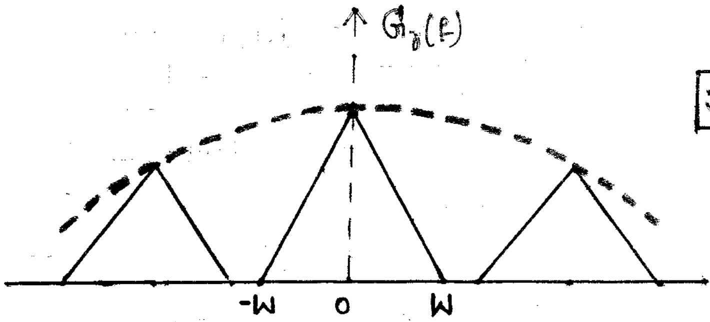
$$\boxed{H(f) = T \operatorname{Sinc}(fT)\, e^{-j\pi f T}} \longrightarrow \boxed{\text{3 Marks}}$$

Aperture effects with Spectral diagrams:

* The high frequency roll-off of H(f) acts as LPF & attenuates upper portion of the message spectrum. This effect is known as "aperture effect".



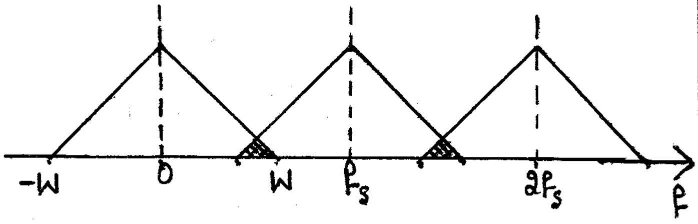




3 Marks

* Aliasing effect is due to wrong choice of sampling frequency 'f_s'.

When $f_s < 2W$, there will be overlapping of spectrum component. Hence o/p of LPF will have distortion due to unwanted frequency component. This is known as "Aliasing" effect.

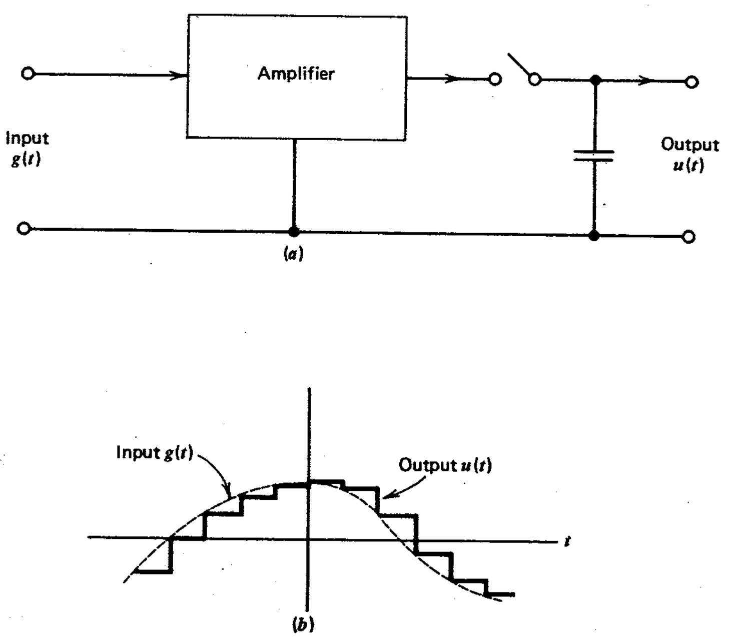


2-Marks

Downloaded from www.citystudentsgroup.blogspot.com

Sample & Hold circuit for Signal recovery :-

* In Natural Sampling & Flat-Top Sampling, the Signal power at the o/p of the Low pass reconstruction filter in the receiver is Small. To overcome this Sample & hold circuit is used.



(a) Sample-and-hold circuit. (b) Idealized output waveform of the circuit.

* Fig (a) Shows the Sample & hold circuit. It Consists of an amplifier of unity gain & low o/p Impedance, a Switch & a capacitor.
* The Switch is timed to Close only for the Small duration 'T' of each Sampling pulse, during which the capacitor charges upto a voltage level equal to that of the I/p Sample.
* When Switch is open, the capacitor retains its voltage level until the next closure of the Switch.
* Thus Sample & Hold circuit produces an o/p Waveform the - represents a Staircase Interpolation of the original analog Signal as

Shown in fig (b).

* The o/p of the Sample & hold Circuit is defined by

$$\boxed{u(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t - nT_s)}$$ (from flat-top Sampling)

Where $h(t)$ is an Impulse response

$$h(t) = \begin{cases} 1 & 0 < t < T_s \\ 0 & t < 0 \ \& \ t > T_s \end{cases}$$

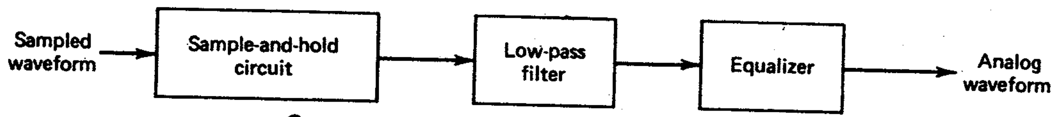
* The Spectrum of the o/p of the Sample & hold is given by

$$\boxed{U(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)\, H(f)}$$

Where $H(f)$ is the transfer function of Sample & Hold Circuit given by

$$\boxed{H(f) = T_s \, \text{Sinc}(fT_s)\, e^{-j\pi f T_s}}$$

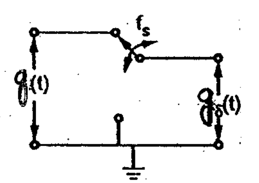
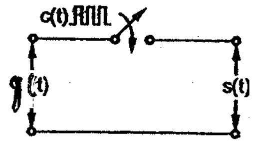
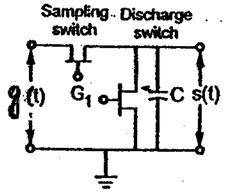
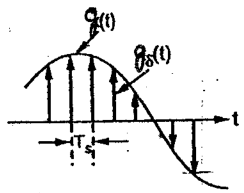
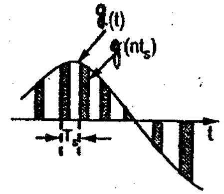
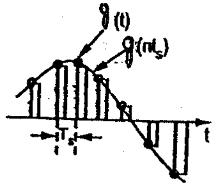
Reconstruction :-



Components of a scheme for signal reconstruction.

* In order to recover the original Signal $g(t)$ without distortion, the o/p of the Sample & hold Circuit is passed through a LPF designed to remove the Spectrum Components of $U(f)$ at multiplies of the Sampling rate f_s & an equalizer whose amplitude response equal $\frac{1}{|H(f)|}$

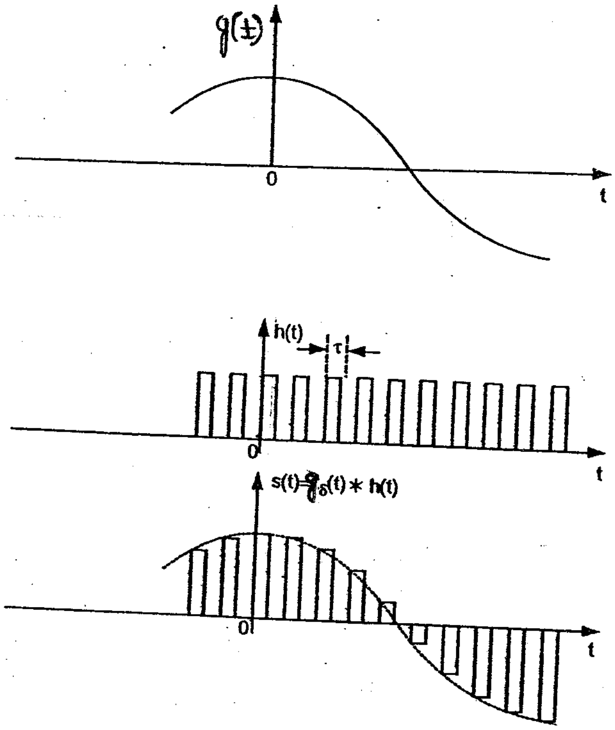
Comparision of various Sampling Techniques:-

Sl No	Parameter	Ideal & Instantaneous Sampling	Natural Sampling	Flat Top Sampling
1)	Principle of Sampling	It uses multiplication by an Impulse function	It uses Chopping function	It uses Sample & hold Circuit.
2)	Circuit of Sampler			
3)	Waveforms			
4)	Realizability	This is NOT practically possible method	This method is used practically	This method is used practically.
5)	Sampling rate	Sampling rate tends to Infinity	Sampling rate Satisfies Nyquist Criteria	Sampling rate Satisfies Nyquist Criteria.
6)	Noise Interference	Maximum	Minimum	Maximum
7)	Time domain representation	$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)$	$s(t) = g(t) \sum_{n=-\infty}^{\infty} AT f_s \,\text{Sinc}(nT f_s)\, e^{j2\pi n f_s t}$	$s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t - nT_s)$
8)	Frequency domain representation	$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$	$S(f) = AT f_s \sum_{n=-\infty}^{\infty} \text{Sinc}(nT f_s)\, G(f - nf_s)$	$S(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)\, H(f)$

Pulse - Amplitude Modulation (PAM) :-

* In PAM, the amplitude of a carrier (consisting of a periodic train of rectangular) pulses is varied in proportion to sample values of a message signal by keeping constant pulse duration.

* PAM is same as Flat - Top Sampling



* The PAM wave $S(t)$ is defined as,

$$\boxed{S(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\ h(t - nT_s)}$$

Where $g(nT_s)$ are sample values of the message signal $g(t)$ & T_s is the sampling period.

* The transmission bandwidth B_T of PAM signal is very large compared to highest frequency of the signal $g(t)$

i.e. $\boxed{B_T \gg W}$

Disadvantages of PAM :-

1) Bandwidth needed for transmission of PAM Signal is very - large compared to its maximum Signal frequency.

2) The amplitude of PAM pulses varies according to modulating Signal. Therefore Interference of noise is maximum for the PAM Signal & this noise cannot be removed very easily.

3) Since amplitude of PAM Signal varies, this also varies the peak power required by the transmitter with modulating Signal.

FORMULAE

1) Nyquist rate $f_S = 2W$

2) Sampling period $T_S = \frac{1}{f_S}$

3) $W = 2\pi f_m$, $f_m = \frac{W}{2\pi}$

4) $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

5) $\operatorname{Sinc}\theta = \frac{\sin \pi\theta}{\pi\theta}$

6) $\operatorname{Sinc}^2\theta = \frac{\sin^2 \pi\theta}{(\pi\theta)^2}$

7) $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

8) $A_c \cos 2\pi f_c t \xrightarrow{FT} \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$

9) $A_c \sin 2\pi f_c t \xrightarrow{FT} \frac{A_c}{2j}\left[\delta(f-f_c) + \delta(f+f_c)\right]$

10) $H(f) = T \operatorname{Sinc}(fT)\, e^{-j\pi f T}$

11) Inphase & Quadrature Component

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$

12) $A \operatorname{Sinc} 2Wt \xrightarrow{FT} \frac{A}{2W} \operatorname{rect}\left[\frac{f}{2W}\right]$

13) $E = \int_{-\infty}^{\infty} |g(t)|^2\, dt = \int_{-\infty}^{\infty} g(t) \cdot g^*(t) \cdot dt$

14) $G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$

15) For eg: Consider 3 frequencies $f_1 = 100Hz$, $f_2 = 110Hz$, $f_3 = 200Hz$

$$f_m = \max(f_1, f_2, f_3)$$

16)

Problems

1) An analog Signal is expressed by the equation

$$x(t) = 3\cos 50\pi t + 10\sin 300\pi t + \cos 100\pi t.$$

Calculate the Nyquist rate & Nyquist Interval for this Signal.

Sol:-

$$x(t) = 3\cos 50\pi t + 10\sin 300\pi t + \cos 100\pi t \longrightarrow (1)$$

equation (1) is in the form

$$x(t) = 3\cos \omega_1 t + 10\sin \omega_2 t + \cos \omega_3 t \longrightarrow (2)$$

comparing eq (1) & (2), we get

$\omega_1 = 50\pi$	$\omega_2 = 300\pi$	$\omega_3 = 100\pi$
$\cancel{2\pi} f_1 = \cancel{50\pi}\ 25$	$\cancel{2\pi} f_2 = \cancel{300\pi}\ 150$	$\cancel{2\pi} f_3 = \cancel{100\pi}\ 50$
$f_1 = 25\,Hz$	$f_2 = 150\,Hz$	$f_3 = 50\,Hz$

* $f_m = \max(f_1, f_2, f_3)$

$f_m = 150\,Hz$

* Nyquist rate 'f_s' $= 2f_m = 2 \times 150\,Hz$

$$\boxed{f_s = 300\,Hz}$$

* Nyquist Interval 'T_s' $= \frac{1}{f_s} = \frac{1}{300\,Hz}$

$$\boxed{T_s = 3.3\,ms\text{ec}}$$

2) Find the Nyquist rate & Nyquist Interval for the Signal

$$x(t) = \frac{1}{2\pi} \cos(4000\pi t)\cos(1000\pi t)$$

Sol:- Given

$$x(t) = \frac{1}{2\pi} \cos(4000\pi t)\cos(1000\pi t)$$

W.K.T $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$x(t) = \frac{1}{4\pi}\left[\cos(4000\pi - 1000\pi)t + \cos(4000\pi + 1000\pi)t\right]$$

$$\boxed{x(t) = \frac{1}{4\pi}\left[\cos(3000\pi t) + \cos(5000\pi t)\right]} \longrightarrow (2)$$

eq (2) is in the form of

$$x(t) = \frac{1}{4\pi}\left[\cos\omega_1 t + \cos\omega_2 t\right]$$

$\omega_1 = 3000\pi$	$\omega_2 = 5000\pi$
$\cancel{2\pi} f_1 = \cancel{3000\pi}\ 1500$	$\cancel{2\pi} f_2 = \cancel{5000\pi}\ 2500$
$\boxed{f_1 = 1500\text{Hz}}$	$\boxed{f_2 = 2500\text{Hz}}$

* $f_m = \max(f_1, f_2) = f_2 = 2500\text{Hz}$

* Nyquist rate 'f_s' $= 2 \times f_m = 2 \times 2500\text{Hz}$

$$\boxed{f_s = 5000\text{Hz}}$$

* Nyquist Interval $T_s = 1/f_s = 1/5000\text{Hz}$

$$\boxed{T_s = 0.2\text{ msec}}$$

3) A Signal $g(t) = 2\cos 400\pi t + 6\cos 640\pi t$ is ideally Sampled at $f_s = 500Hz$. If the Sampled Signal is passed through an Ideal low pass filter with Cut-off frequency $f_c = 400Hz$. Find

i) $G(f)$ & Sketch its Spectrum

ii) Sampled Signal $G_\delta(f)$ & Sketch its Spectrum.

iii) The components that will appear at the filter o/p

$\boxed{\text{Jan-08, 8M}}$

OR

3) A Signal $g(t) = 2\cos(400\pi t) + 6\cos(640\pi t)$ is ideally Sampled at 500Hz. If the Sampled Signal is passed through an Ideal LPF with a Cut-off frequency at 400Hz, what Components will appear in the filter o/p? $\boxed{\text{June-09, 6M}}$

July-08, 5M	July-09, 6M

Sol:-

i) $g(t) = 2\cos(400\pi t) + 6\cos(640\pi t) \longrightarrow (1)$

From eq (1)

$\omega_1 = 400\pi$	$\omega_2 = 640\pi$
$2\pi f_1 = 400\pi$	$2\pi f_2 = 640\pi$
$\boxed{f_1 = 200Hz}$	$\boxed{f_2 = 320Hz}$

$$g(t) = 2\cos 2\pi f_1 t + 6\cos 2\pi f_2 t$$

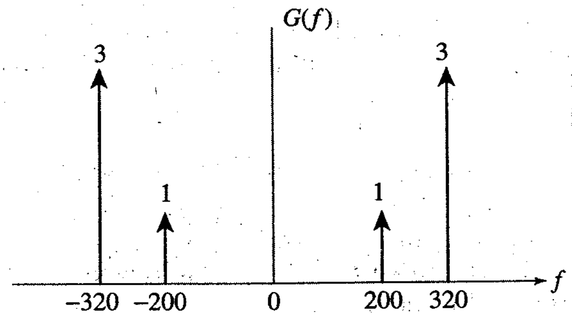
$$g(t) = 2\cos 2\pi(200)t + 6\cos 2\pi(320)t \longrightarrow (2)$$

Taking FT on both Sides of eq (2), we get

$$G(f) = \frac{\cancel{2}}{\cancel{2}}\left[\delta(f-200) + \delta(f+200)\right] + \frac{\cancel{6}3}{\cancel{2}}\left[\delta(f-320) + \delta(f+320)\right]$$

$$G(f) = \left[\delta(f-200) + \delta(f+200)\right] + 3\left[\delta(f-320) + \delta(f+320)\right] \longrightarrow (3)$$

Using eq ③, the Spectrum of the Signal $g(t)$ is drawn & it appears as shown in fig ①.

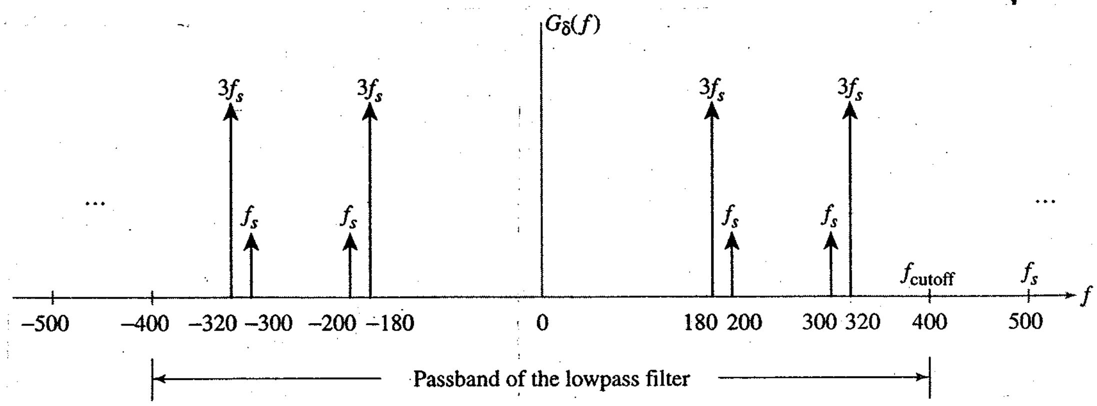


ii) W.K.T $G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$ Given $f_s = 500\,Hz$

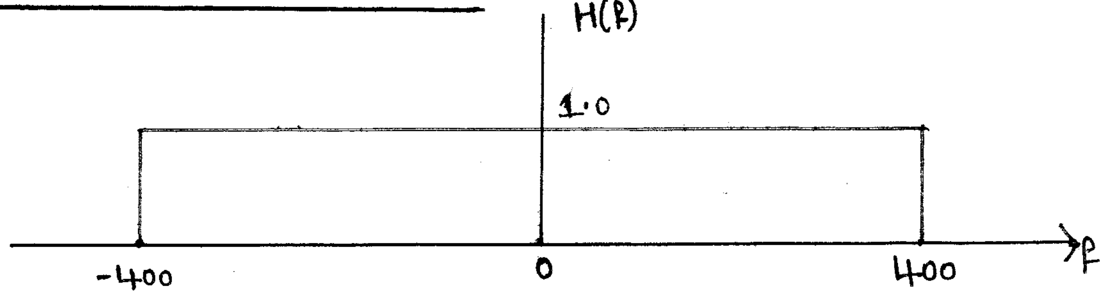
$$= f_s \sum_{n=-\infty}^{\infty} G(f - 500n)$$

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} \left[\delta(f - 500n - 200) + \delta(f - 500n + 200)\right]$$
$$+ 3f_s \sum_{n=-\infty}^{\infty} \left[\delta(f - 500n - 320) + \delta(f - 500n + 320)\right] \longrightarrow ④$$

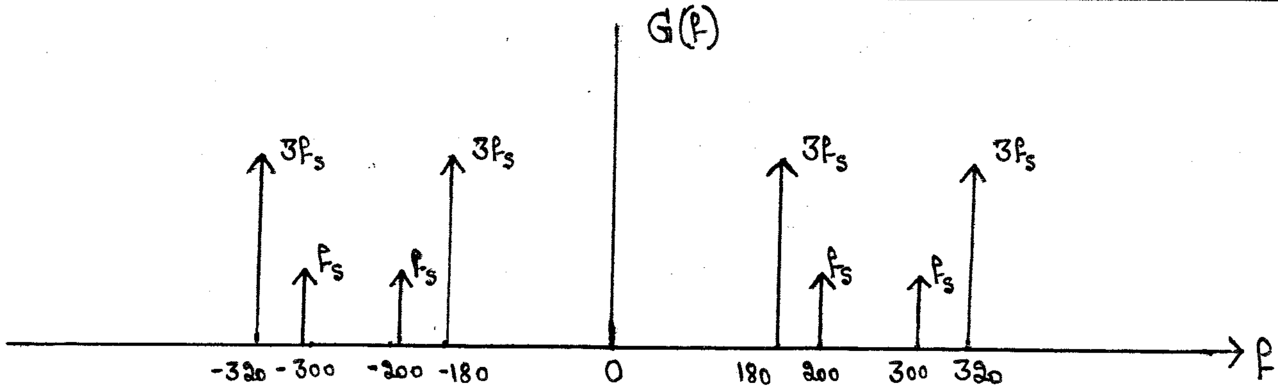
* The Spectrum of the Sampled Signal $g_\delta(t)$ is drawn using eq ④



iii) Spectrum of Ideal LPF :-



ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.



* The Components that are appeared at the filter o/p are $\underline{180Hz, 200Hz, 300Hz, 320 Hz}$.

4) The Signal $g(t) = 10 \cos 20\pi t \cdot \cos 200\pi t$ is Sampled at rate of 250 Samples/Sec.

i) Determine the Spectrum of resulting Sampled Signal.

ii) Specify the Cut-off frequency of the Ideal reconstruction filter So as to recover $g(t)$ from its Sampled version.

iii) What is the Nyquist rate for $g(t)$?

July-05, 10M	July-07, 6M	Jan-07, 6M

Sol:-

$$g(t) = 10 \cos 20\pi t \cos 200\pi t \longrightarrow (1) \qquad \boxed{A = 200,\ B = 20}$$

W.K.T $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$g(t) = \frac{\cancel{10}^{5}}{\cancel{2}}\left[\cos(200-20)\pi t + \cos(200+20)\pi t\right]$$

$$g(t) = 5\left[\cos 180\pi t + \cos 220\pi t\right]$$

$$\boxed{g(t) = 5 \cos 180\pi t + 5 \cos 220\pi t} \longrightarrow (2)$$

From eq (2)

$\omega_1 = 180\pi$	$\omega_2 = 220\pi$
$\cancel{2\pi} f_1 = \cancel{180\pi}\ 90$	$\cancel{2\pi} f_2 = \cancel{220\pi}\ 110$
$\boxed{f_1 = 90\,Hz}$	$\boxed{f_2 = 110\,Hz}$

* $f_m = \max(f_1, f_2) = f_2 = 110\,Hz$ $\boxed{f_m = 110\,Hz}$

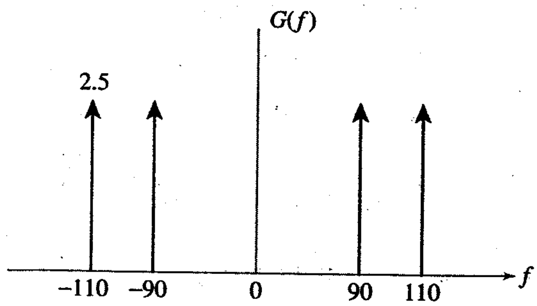
$$\boxed{g(t) = 5\cos 2\pi(90)t + 5\cos 2\pi(110)t} \longrightarrow (3)$$

* Taking FT on both sides of eq (3), we get

$$G(f) = \frac{5}{2}\left[\delta(f-90) + \delta(f+90)\right] + \frac{5}{2}\left[\delta(f-110) + \delta(f+110)\right]$$

$$\boxed{G(f) = 2.5\left[\delta(f-90) + \delta(f+90)\right] + 2.5\left[\delta(f-110) + \delta(f+110)\right]} \longrightarrow (4)$$

* The spectrum of the signal $g(t)$ is drawn using eq (4) as shown in figure below:



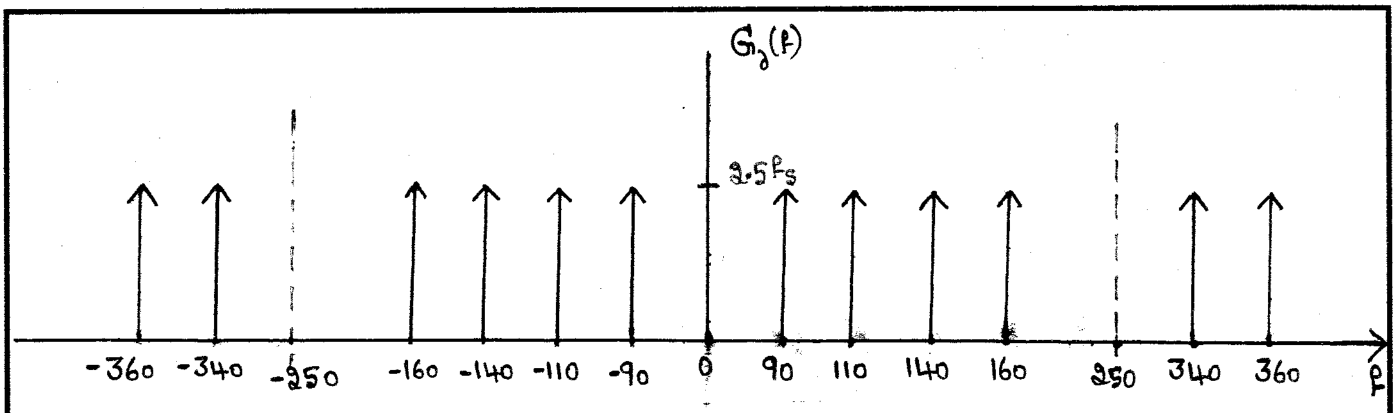
i) WKT

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

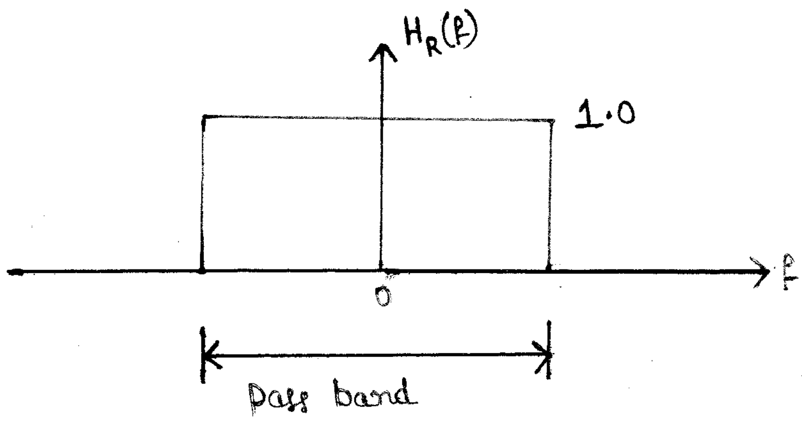
$\boxed{\text{Given: } f_s = 250}$

$$G_\delta(f) = 2.5 f_s \sum_{n=-\infty}^{\infty} \left[\delta(f - 250n - 90) + \delta(f - 250n + 90)\right]$$

$$+ 2.5 f_s \sum_{n=-\infty}^{\infty} \left[\delta(f - 250n - 110) + \delta(f - 250n + 110)\right]$$



ii)



* The Cut-off frequencies of the Ideal LPF Should be more than 110Hz & less than 140Hz for recovering $g(t)$ from $g_\delta(t)$.

iii) Nyquist rate for $g(t)$:

$$f_s = 2f_m = 2 \times 110 \text{Hz}$$

$$\boxed{f_s = 220 \text{Hz}}$$

---

5) The Signal $g(t) = 4\cos(4\pi t)\cos(400\pi t)$ is Sampled at the rate of 500 Samples/Sec.

i) Determine the Spectrum of the resulting Sampled Signal.

ii) What is the Nyquist rate for $g(t)$?

iii) What is the Cut-off frequencies of Ideal reconstruction filter?

July-06, 8M

Sol :- Given : $g(t) = 4\cos(4\pi t)\cos(400\pi t)$ $\boxed{A = 400,\ B = 4}$

W.K.T $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$g(t) = \frac{4}{2}\left[\cos(400-4)\pi t + \cos(400+4)\pi t\right]$$

$$\boxed{g(t) = 2\left[\cos(396\pi t) + \cos(404\pi t)\right]} \longrightarrow (1)$$

From eq (1)

$\omega_1 = 396\pi$	$\omega_2 = 404\pi$
$\cancel{2\pi} f_1 = \cancel{396\pi}\ 198$	$\cancel{2\pi} f_2 = \cancel{404\pi}\ 202$
$\boxed{f_1 = 198\ Hz}$	$\boxed{f_2 = 202\ Hz}$

$f_m = max(f_1, f_2) = f_2$

$\boxed{f_m = 202\ Hz}$

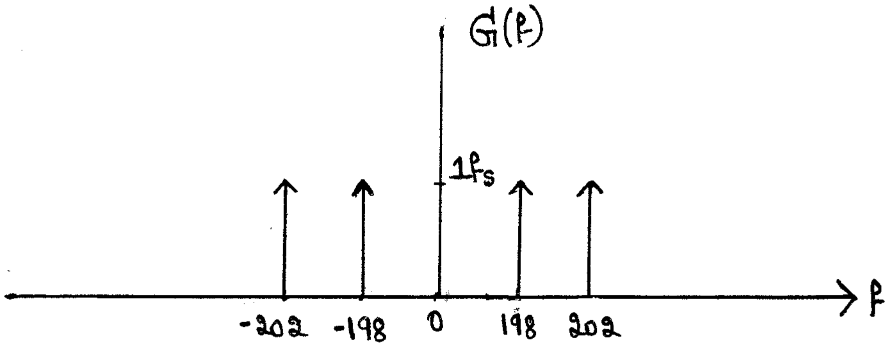
Eq (1) can be written as :

$$\boxed{g(t) = 2\cos 2\pi(198)t + 2\cos 2\pi(202)t} \longrightarrow (2)$$

Taking F.T on both sides of eq (2), we get

$$G(f) = \frac{2}{2}\left[\delta(f-198) + \delta(f+198)\right] + \frac{2}{2}\left[\delta(f-202) + \delta(f+202)\right]$$

* Spectrum of $G(f)$ is as shown in below figure :

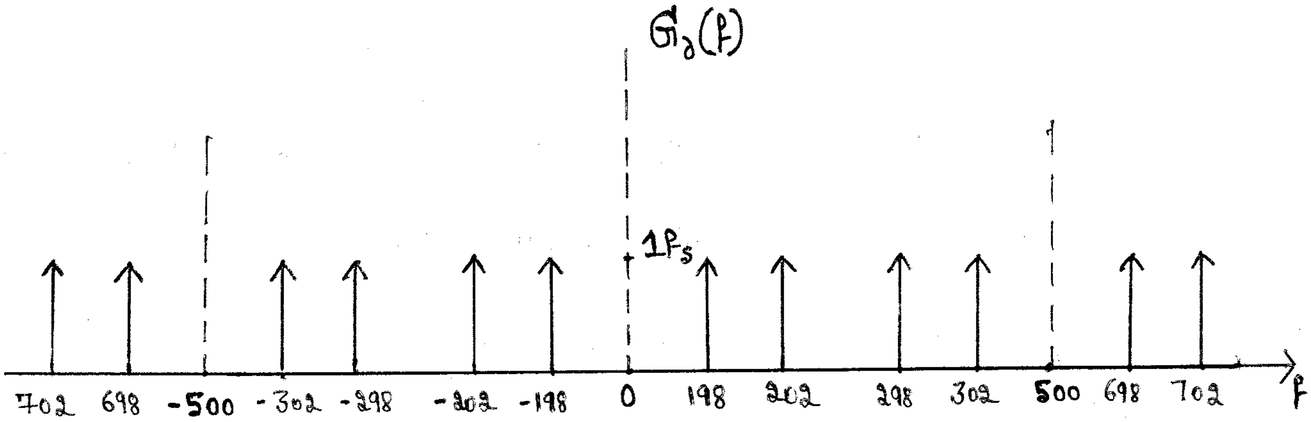


i) $G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$

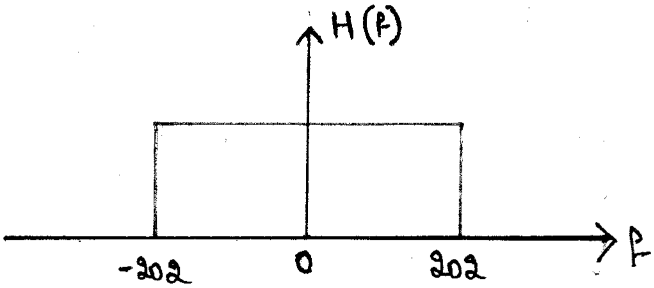
$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - 500n)$$

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} \left[G(f - 500n - 198) + G(f - 500n + 198) \right]$$

$$+ f_s \sum_{n=-\infty}^{\infty} \left[G(f - 500n - 202) + G(f - 500n + 202) \right]$$



iii)



* Cut-off frequency of Ideal reconstruction filter is 202 Hz.

ii) Nyquist rate for g(t) is

$$f_s = 2f_m$$

$$f_s = 2 \times 202 Hz$$

$$\boxed{f_s = 404 Hz}$$

4.1.1 (a) ← Simon Haykins

6) Consider the signal $g(t) = A\cos(2\pi f_o t)$. Plot the spectrum of the discrete time signal $g_\delta(t)$ derived by sampling $g(t)$ at the time $t_n = n/f_s$. Where $n = 0, \pm 1, \pm 2, \ldots$ and

i) $f_s = f_o$ ii) $f_s = 2f_o$ iii) $f_s = 3f_o$

Jan-10, 6M

Sol:- Given: $g(t) = A\cos(2\pi f_o t) \longrightarrow (1)$

Taking FT on both sides of eq (1), we get

$$\boxed{G(f) = \frac{A}{2}\left[\delta(f-f_o) + \delta(f+f_o)\right]} \longrightarrow (2)$$

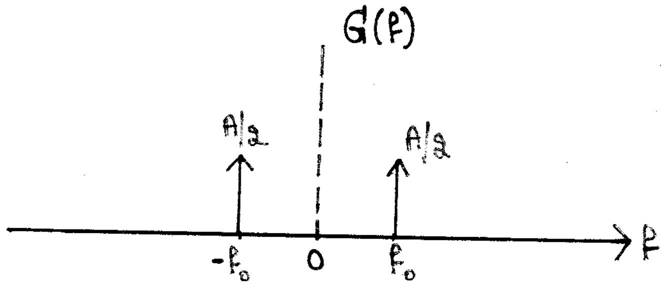


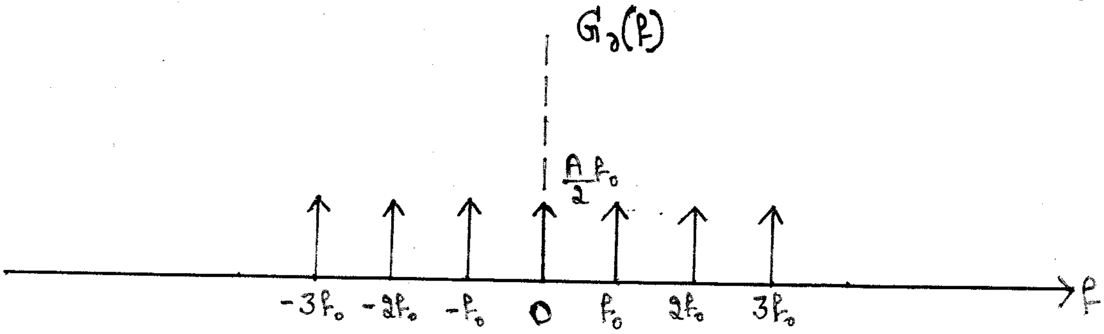
Fig (1) Spectrum of message signal $g(t)$.

W.K.T

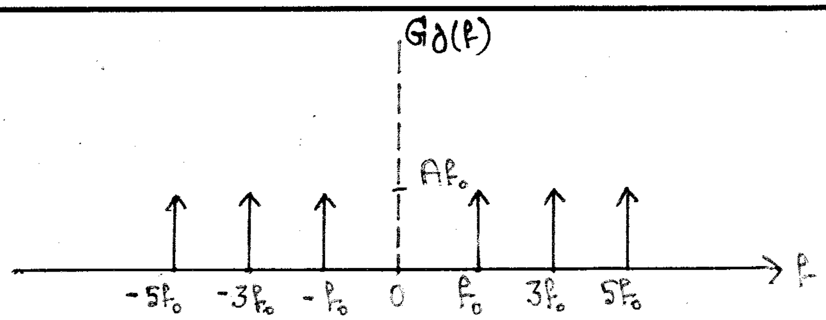
$$\boxed{G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)} \longrightarrow (3)$$

i) $\boxed{f_s = f_o}$

$$G_\delta(f) = \frac{Af_o}{2} \sum_{n=-\infty}^{\infty} \left[\delta(f - f_o - nf_o) + \delta(f + f_o + nf_o)\right]$$



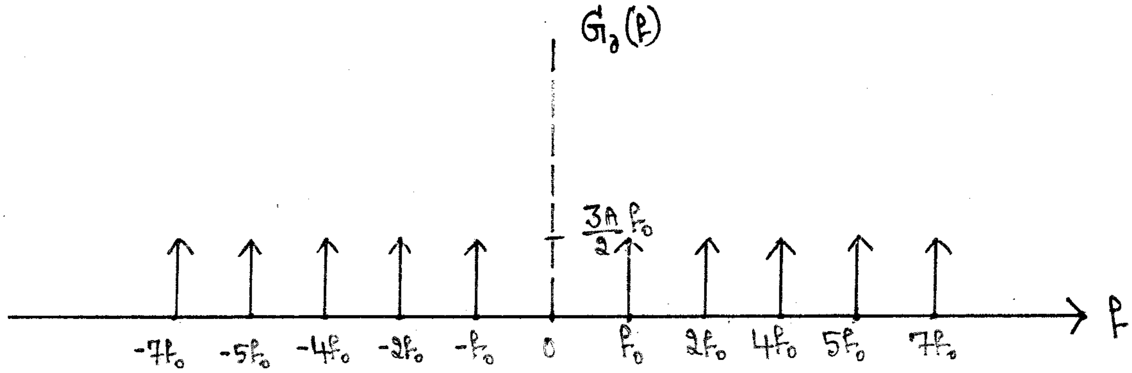
ii) $\boxed{f_s = 2f_0}$



$$\boxed{G_\delta(f) = \frac{A}{\cancel{2}} \cancel{2}f_0 \sum_{n=-\infty}^{\infty} \left[\delta(f - f_0 - n2f_0) + \delta(f + f_0 - n2f_0) \right]}$$

iii) $\boxed{f_s = 3f_0}$

$$\boxed{G_\delta(f) = \frac{3Af_0}{2} \sum_{n=-\infty}^{\infty} \left[\delta(f - f_0 - n3f_0) + \delta(f + f_0 - n3f_0) \right]}$$



7) Consider the Signal $g(t) = A\sin(2\pi f_0 t)$. plot the Spectrum of the discrete time Signal $g_\delta(t)$ derived by Sampling $g(t)$ at the time $t_n = n/f_s$. Where $n = 0, \pm 1, \pm 2, \ldots$ and

i) $f_s = f_0$ ii) $f_s = 2f_0$ iii) $f_s = 3f_0$

Simon Haykin Exercise problem 4.1.1 (b)

Sol :- Given $g(t) = A\sin(2\pi f_0 t) \longrightarrow (1)$

Taking FT on both Sides of eq (1), we get

$$\boxed{G(f) = \frac{A}{2j} \left[\delta(f - f_0) + \delta(f + f_0) \right]} \longrightarrow (2)$$

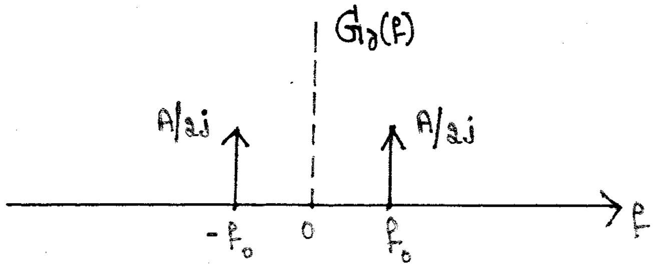


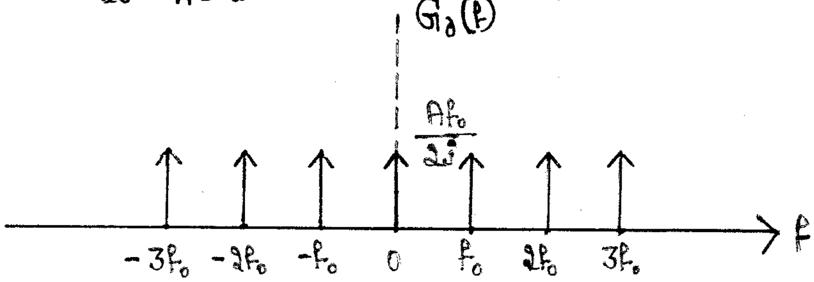
Fig ①: Spectrum of message Signal $g(t)$.

W.K.T

$$\boxed{G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)} \longrightarrow ③$$

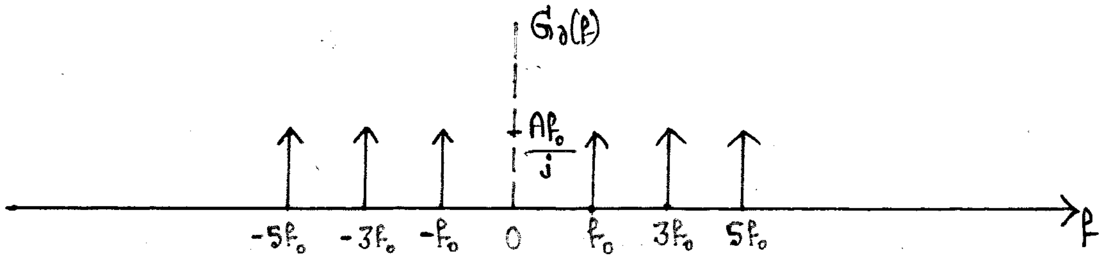
i) $\boxed{f_s = f_0}$

$$G_\delta(f) = \frac{Af_0}{2j} \sum_{n=-\infty}^{\infty} \left[\delta(f - f_0 - nf_0) + \delta(f + f_0 - nf_0)\right]$$



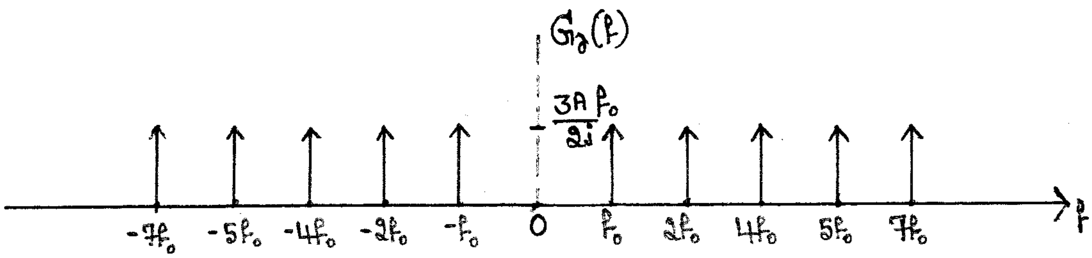
ii) $\boxed{f_s = 2f_0}$

$$\boxed{G_\delta(f) = \frac{A}{\cancel{2}j}\cancel{2}f_0 \sum_{n=-\infty}^{\infty} \left[\delta(f - f_0 - n2f_0) + \delta(f + f_0 - n2f_0)\right]}$$



iii) $\boxed{f_s = 3f_0}$

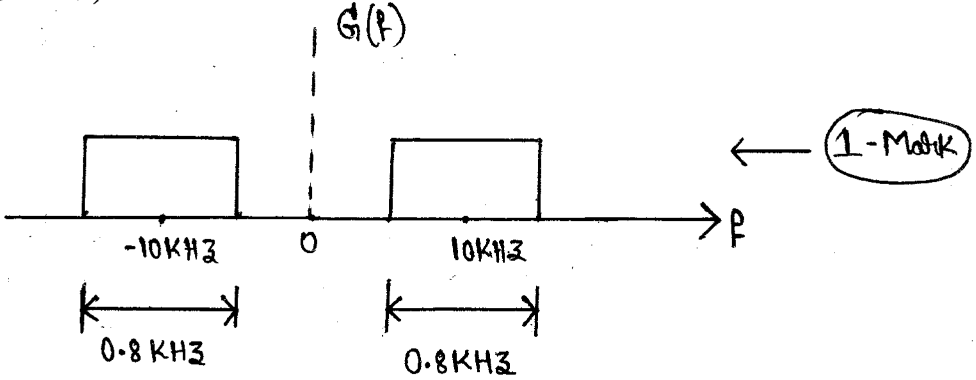
$$\boxed{G_\delta(f) = \frac{3Af_0}{2j} \sum_{n=-\infty}^{\infty} \left[\delta(f - f_0 - n3f_0) + \delta(f + f_0 - n3f_0)\right]}$$



8) The Spectrum of band pass Signal $g(t)$ has a bandwidth of 0.8KHZ Centered around ± 10KHZ. Write the equation for $g(t)$ interms of quadrature Components. Find the Nyquist rate & Nyquist Interval. July-09, 4M

Sol:- Given: $f_c = \pm 10$KHZ, BW = 0.8KHZ.

* Spectrum of $G(f)$



* $g(t)$ can be expressed interms of Inphase & Quadrature phase Components:

$$\therefore \quad g(t) = g_I(t) \cos(2\pi f_c t) - g_Q(t) \sin(2\pi f_c t)$$

$$\boxed{g(t) = g_I(t) \cos(2\pi \times 10 \times 10^3 t) - g_Q(t) \sin(2\pi \times 10 \times 10^3 t)}$$ ← 1-Mark

* $$W = \frac{BW}{2} = \frac{0.8\text{KHZ}}{2} = 0.4\text{KHZ}$$

$\therefore$ Nyquist rate $f_s = 2W = 2 \times 0.4$KHZ

$$\boxed{f_s = 0.8\text{KHZ}}$$ ← 2-Marks

9) The Spectrum of BP Signal $g(t)$ has a BW of 0.6KHz Centered around $\pm$12KHz. Find the Nyquist rate for quadrature Sampling of inphase & Quadrature Components of the Signal $g(t)$.

Sol:- Given : $f_c = \pm 12KHz$, $BW = 0.6KHz$

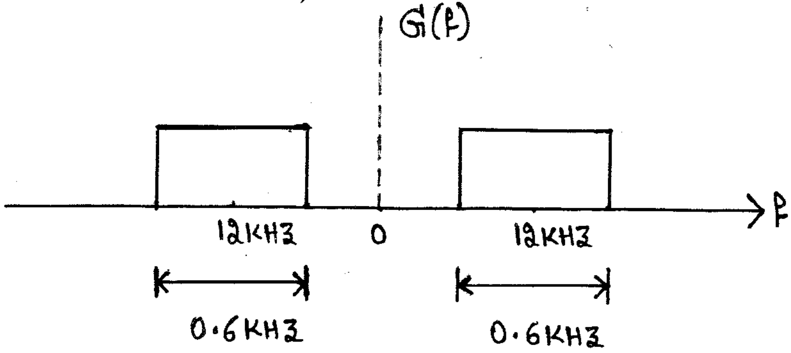


Fig ①: Spectrum of $G(f)$.

* $g(t)$ Can be expressed interms of Inphase & Quadrature phase Components.

$$\therefore \quad g(t) = g_I(t) \cos(2\pi f_c t) - g_Q(t) \sin(2\pi f_c t)$$

$$g(t) = g_I(t) \cos(2\pi \times 12 \times 10^3 t) - g_Q(t) \sin(2\pi \times 12 \times 10^3 t)$$

* $W = \dfrac{BW}{2} = \dfrac{0.6KHz}{2} = 0.3\ KHz$

* $\therefore$ Nyquist rate $f_s = 2W = 2 \times 0.3\ KHz$

$$\boxed{f_s = 0.6KHz}$$

10) The Spectrum of a bandpass Signal occupies a band of width 0.5 KHz Centered around $\pm$10KHz. Find the Nyquist rate for Quadrature Sampling the in-phase & Quadrature Components of the Signal

Sol:- Given : $f_c = \pm 10KHz$

$BW = 0.5KHz$

Simon - Haykins 4.2.1

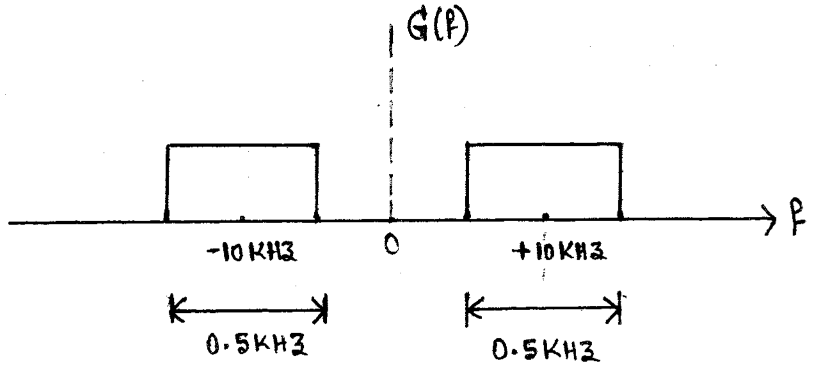


Fig ①: Spectrum of G(f)

* g(t) can be expressed interms of Inphase & Quadrature phase components

$$\therefore \; g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$

$$g(t) = g_I(t)\cos(2\pi \times 10\times10^3 t) - g_Q(t)\sin(2\pi\times10\times10^3 t)$$

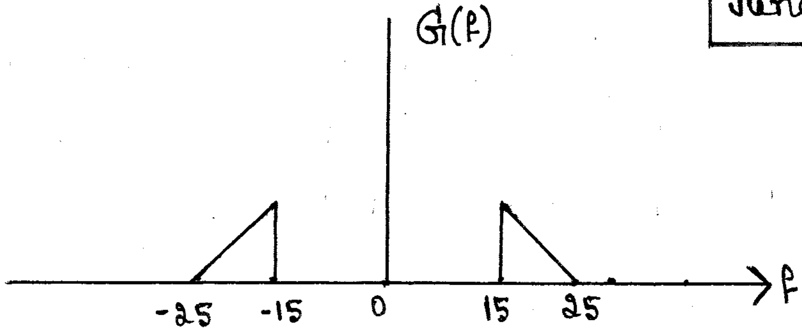
* $W = \frac{BW}{2} = \frac{0.5\,KHz}{2} = 0.25\,KHz$

$\therefore$ Nyquist rate $f_s = 2W = 2\times 0.25\,KHz$

$$\boxed{f_s = 0.5\,KHz}$$

11) A band pass Signal g(t) with a Spectrum Shown below is ideally Sampled. Sketch the Spectrum of Sampled Signals at $f_s = 25, 45$ & $50\,Hz$. Indicate if & how the Signal can be recovered.

June-09, 8M



Sol :-

Given: $f_1 = 15\,Hz$, $f_2 = 25\,Hz$,

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* $BW = f_2 - f_1 = 25Hz - 15Hz = 10Hz$

* $f_c = 20Hz$

* The Spectrum of the Sampled Signal is Sketched using the expression

$$\boxed{G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)}$$

* $f_s = 25Hz$:

$$\boxed{G_\delta(f) = 25 \sum_{n=-\infty}^{\infty} G(f - 25n)}$$ ← 1-Mark

2-Marks

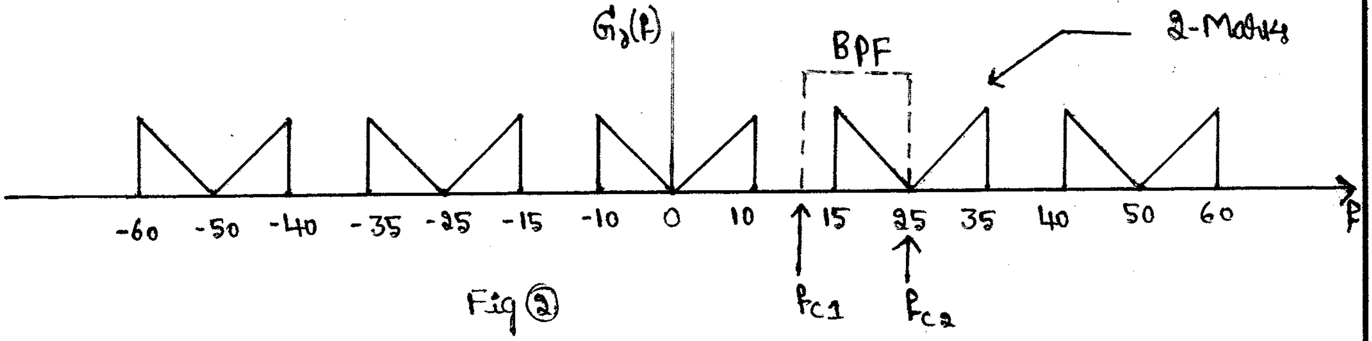


Fig ②

* From Fig ①, we See that $g(t)$ can be recovered from $g_\delta(t)$ by using an Ideal BPF with lower Cut-off frequency f_{c1} greater than 10 Hz & less than 15 Hz, & upper Cut-off frequency f_{c2} equal to 25 Hz.

1- Mark

i.e. Ideal BPF : $BW = 10Hz$
$f_c = 20Hz$

* $f_s = 45Hz$:

$$\boxed{G_\delta(f) = 45 \sum_{n=-\infty}^{\infty} G(f - 45n)}$$ ← 1-Mark

← 2-Marks

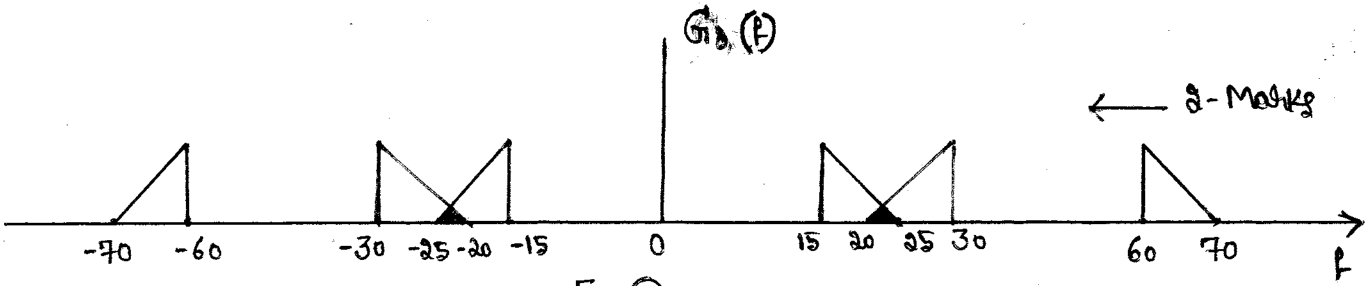


Fig ②

* It is not possible to recover $g(t)$ because of alias error. ← 1-Mark

* $f_s = 50\,Hz$

$$\boxed{G_\delta(f) = 50 \sum_{n=-\infty}^{\infty} G(f - 50n)}$$

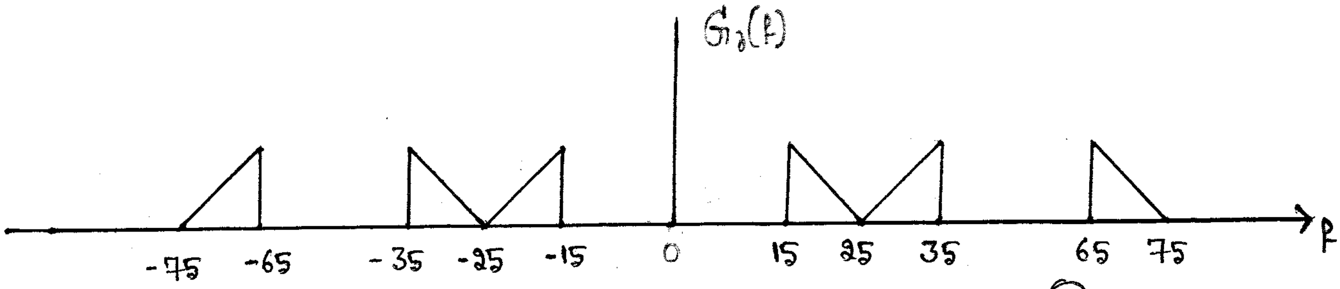
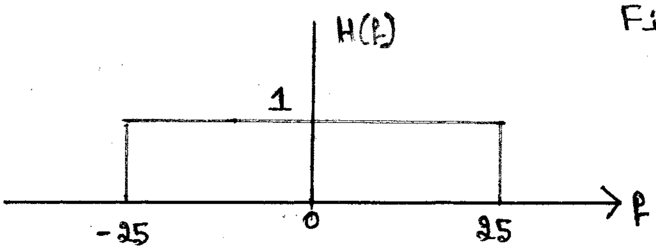
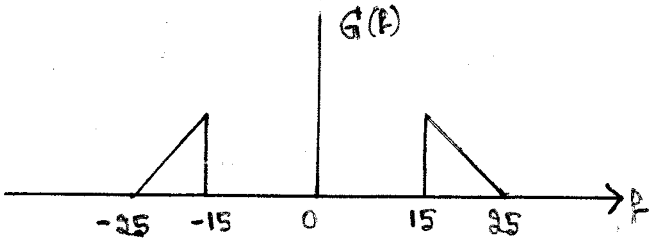


Fig ③

Ideal LPF:





* From fig ③, $g(t)$ can be recovered by using an Ideal LPF with cut-off frequency $f_c = 25\,Hz$.

---

12) A low pass signal $x(t)$ has a spectrum $X(f)$ given by

$$X(f) = \begin{cases} 1 - \dfrac{|f|}{200} & ; |f| < 200 \\ 0 & ; \text{elsewhere} \end{cases}$$

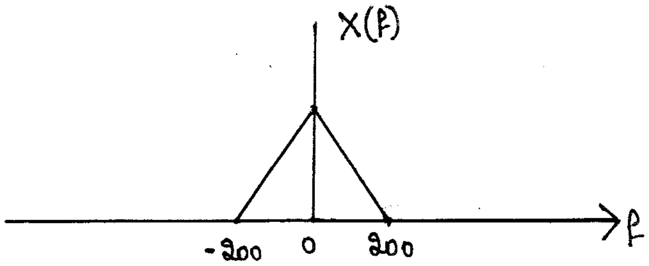
i) Sketch the spectrum $X_\delta(t)$ for $|f| < 200\,Hz$ if $x(t)$ is ideally sampled at $f_s = 300\,Hz$

ii) Repeat part (i) for $f_s = 400\,Hz$

Jan-06, 6M

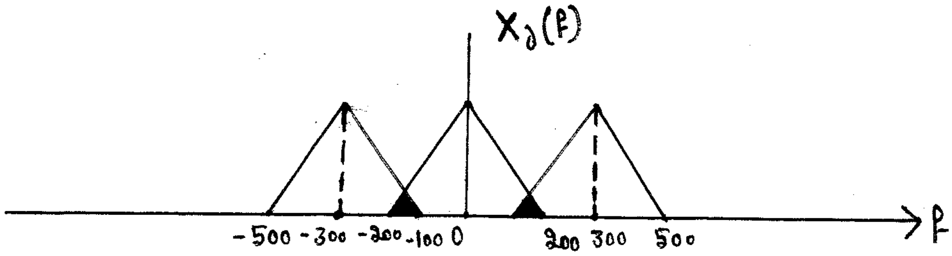
Sol :-

f in Hz	0	10	20	50	75	100	150	180	190	200
$x(f) = 1 - \frac{\lvert f \rvert}{200}$	1	0.95	0.9	0.75	0.625	0.5	0.25	0.10	0.05	0

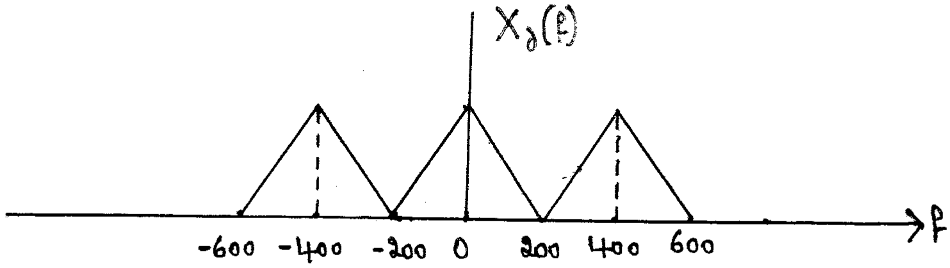


W.K.T $\boxed{X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)}$

i) $f_s = 300\,Hz$ $\boxed{X_\delta(f) = 300 \sum_{n=-\infty}^{\infty} X(f - 300n)}$



ii) $f_s = 400\,Hz$ $\boxed{X_\delta(f) = 400 \sum_{n=-\infty}^{\infty} X(f - 400n)}$



13) A Low pass Signal has the Spectrum given by:

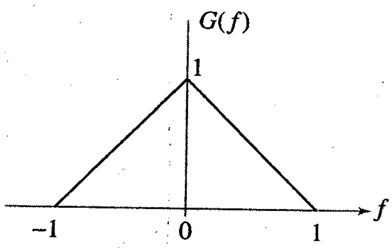
$$G(f) = \begin{cases} 1-|f|, & |f|<1 \\ 0, & |f|>1 \end{cases}$$

Assume that $g(t)$ is Sampled at 1.5 Hz & then applied to a Low pass reconstruction filter with Cut-off frequency at 1 Hz. plot the Spectrum of the resulting Signal.

Sol:-

f in Hz	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$G(f)=1-\|f\|$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

* The Spectrum of $G(f)$ is as shown below:

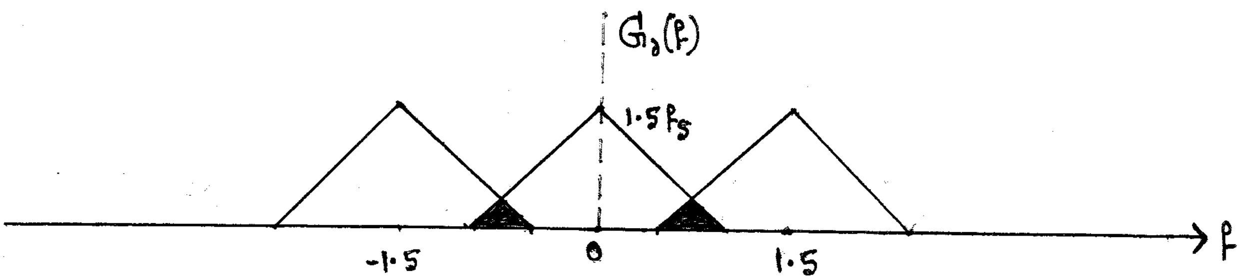


* The Spectrum of the Sampled Signal is given by

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

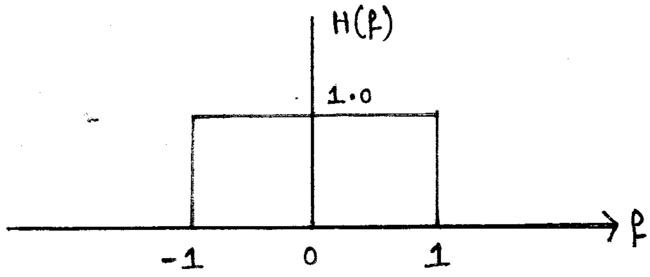
* $f_s = 1.5$ Hz,

$$\boxed{G_\delta(f) = 1.5 \sum_{n=-\infty}^{\infty} G(f - 1.5n)}$$



Downloaded from www.citystudentsgroup.blogspot.com

* Ideal LPF :



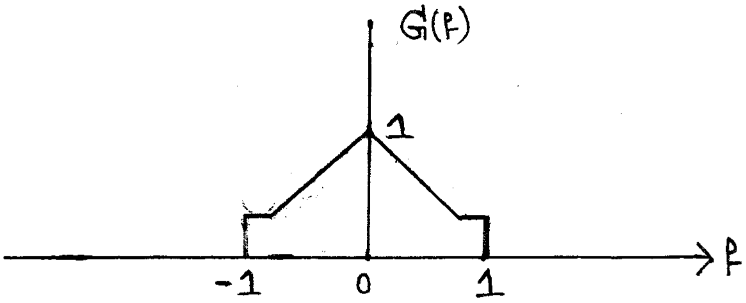


Fig: Spectrum of filter o/p.

Simon-Haykin → 4.4.2

14) Figure ① shows the spectrum of a LP signal $g(t)$. The signal is sampled at the rate of 1.5 Hz, & then applied to a low-pass reconstruction filter with cut-off frequency at 1 Hz. plot the spectrum of the resulting signal.

OR

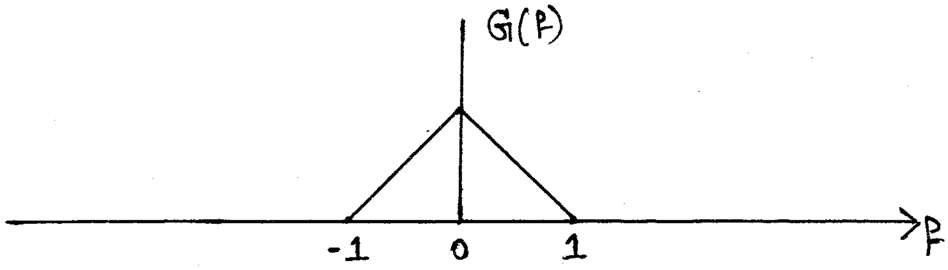
14) Fig ① depicts the spectrum of a message signal $g(t)$. The signal is undersampled at a rate of 1.5 Hz.

ⓐ Sketch the spectrum of the sampled version of this signal

ⓑ $G_\delta(t)$ is passed through Ideal LPF of BW = 1 Hz. Sketch the spectrum of the resulting filter o/p.

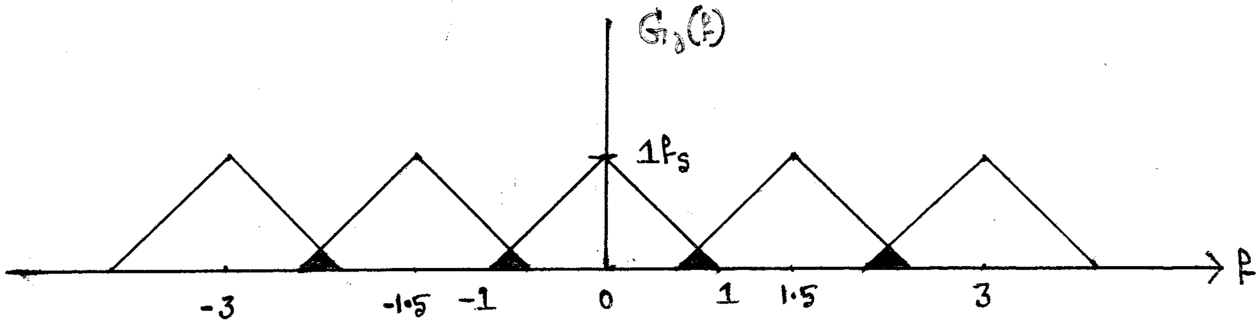
ⓒ Comment on your results.

P.T.O

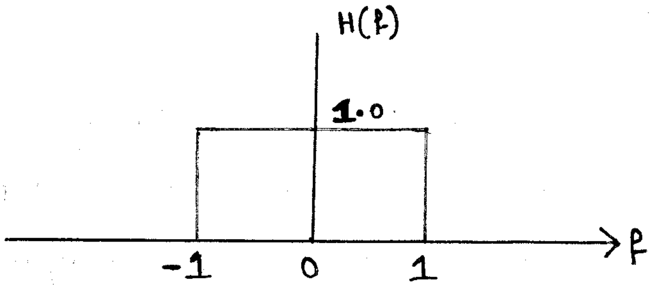


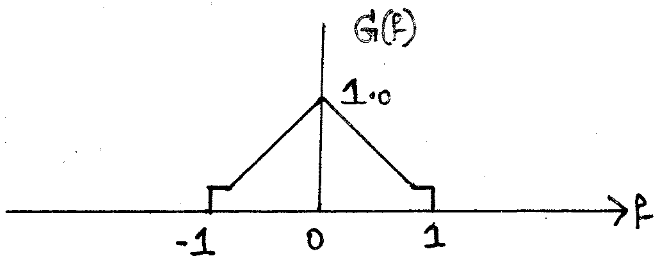
(a) $f_s = 1.5\,Hz$

$$\boxed{G_\delta(f) = 1.5 \sum_{n=-\infty}^{\infty} G(f - 1.5n)}$$



(b)





(c) The spectrum of the reconstruction filter o/p is significantly different from that of the original signal, indicating - distortion due to aliasing.

15) Specify the Nyquist rate & the Nyquist Interval for each of the following signals.

i) $x(t) = \text{Sinc}(200t)$

W.K.T $\boxed{\text{Sinc}\,\theta = \dfrac{\sin \pi\theta}{\pi\theta}}$

$$x(t) = \frac{\sin \pi(200t)}{\pi(200t)}$$

W.K.T

$$\omega = 200\pi$$

$$\cancel{2}\pi f = \cancel{200}\pi \; 100$$

$$\therefore \boxed{f_m = 100Hz}$$

* Nyquist rate 'f_s' $= 2 \times f_m = 2 \times 100Hz$

$$\boxed{f_s = 200Hz}$$

* Nyquist Interval $T_s = 1/f_s = 1/200$

$$\boxed{T_s = 5\,msec}$$

ii) $x(t) = \text{Sinc}^2(200t)$

$$= \frac{\sin^2(200\pi t)}{(200\pi t)^2}$$

$$= \frac{1 - \cos(400\pi t)}{(200\pi t)^2}$$

$$x(t) = \underbrace{\frac{1}{(200\pi t)^2}}_{\text{DC - term}} - \frac{\cos(400\pi t)}{(200\pi t)^2}$$

$$\text{Sinc}^2\theta = \frac{\sin^2\pi\theta}{(\pi\theta)^2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\text{Sinc}^2\theta = \frac{1-\cos 2(200\pi t)}{(200\pi t)^2}$$

$$\omega = 400\pi$$

$$\cancel{2}\pi f = 4\cancel{00}\pi \; 200$$

$$\boxed{f = 200Hz}$$

* Nyquist rate 'f_s' $= 2f_m = 2 \times 200 =$ 400HZ
* Nyquist Interval 'T_s' $= 1/f_s = 1/400 =$ 2.5msec

iii) $x(t) = \text{Sinc}\, 200t + \text{Sinc}^2 200t$

$$= \frac{\sin \pi(200t)}{\pi(200t)} + \frac{\sin^2(200\pi t)}{(200\pi t)^2}$$

$$= \frac{\sin \pi(200t)}{\pi(200t)} + \frac{1-\cos(400\pi t)}{(200\pi t)^2}$$

$$x(t) = \frac{\sin \pi(200t)}{\pi(200t)} + \underbrace{\frac{1}{(200\pi t)^2}}_{\boxed{\text{DC term}}} - \frac{\cos(400\pi t)}{(200\pi t)^2} \longrightarrow (1)$$

From eq (1)

$\omega_1 = 200\pi$	$\omega_2 = 400\pi$
$\cancel{2\pi} f_1 = \cancel{200\pi}\ 100$	$\cancel{2\pi} f_2 = \cancel{400}\pi\ 200$
$f_1 = 100\text{HZ}$	$f_2 = 200\text{HZ}$

* $f_m = \max(f_1, f_2) = f_2$

$\boxed{f_m = 200\text{HZ}}$

* Nyquist rate $f_s = 2f_m = 2 \times 200 =$ 400HZ
* Nyquist Interval $T_s = 1/f_s = 1/400\text{HZ} =$ 2.5msec

June-2004, 8M

16) The Signal $g_1(t) = 10\cos 100\pi t$ & $g_2(t) = 10\cos 50\pi t$ are both Sampled at times nT_s with a Sampling rate $f_s = 1/T_s = 75$ Samples/sec. Show that two Sequences of Samples obtained are Identical in both time & frequency domain.

Sol: Given : $g_1(t) = 10\cos 100\pi t$, $g_2(t) = 10\cos 50\pi t$ & $f_s = 75\,Hz$

Time domain :

$$g_1(t) = 10\cos 100\pi t$$

$$g_1(t)\Big|_{t=nT_s} = g_1(nT_s) = 10\cos 100\pi\, nT_s$$

$$= 10\cos \overset{4}{\cancel{100}}\pi \times n \times \frac{1}{\underset{3}{\cancel{75}}}$$

$$g_1(nT_s) = 10\cos \frac{4\pi n}{3}$$

$\because$ $\boxed{\cos(2\pi - \theta) = \cos\theta}$

$$g_1(nT_s) = 10\cos\left[2\pi - \frac{2\pi}{3}\right]n$$

$$\boxed{g_1(nT_s) = 10\cos\left(\frac{2\pi n}{3}\right)} \longrightarrow (1)$$

$||^{ly}$

$$g_2(t) = 10\cos 50\pi t$$

$$g_2(t)\Big|_{t=nT_s} = g_2(nT_s) = 10\cos 50\pi(nT_s)$$

$$= 10\cos \overset{2}{\cancel{50}}\pi\, n \cdot \frac{1}{\underset{3}{\cancel{75}}}$$

$$\boxed{g_2(nT_s) = 10\cos \frac{2\pi n}{3}} \longrightarrow (2)$$

* Eq (1) & (2) are Identical & hence the Samples in time domain are Same.

Frequency domain :-

$$g_1(t) = 10 \cos 100\pi t$$

$$g_1(t) = 10 \cos 2\pi (50) t \longrightarrow (3)$$

$W_1 = 100\pi$

$\cancel{2\pi} f_1 = \cancel{100\pi}\ 50$

$\boxed{f_1 = 50Hz}$

Taking F.T on both Sides of eq (3), we get

$$G_1(f) = \frac{10}{2}\left[\delta(f-50) + \delta(f+50)\right]$$

$$\boxed{G_1(f) = 5\left[\delta(f-50) + \delta(f+50)\right]} \longrightarrow (4)$$

Consider,

$$g_2(t) = 10 \cos 50\pi t$$

$$g_2(t) = 10 \cos 2\pi (25) t \longrightarrow (5)$$

$\omega_2 = 50\pi$

$\cancel{2\pi} f_2 = \cancel{50\pi}\ 25$

$\boxed{f_2 = 25 Hz}$

Taking FT on both Sides of eq (5), we get

$$G_2(f) = \frac{10}{2}\left[\delta(f-25) + \delta(f+25)\right]$$

$$\boxed{G_2(f) = 5\ \left[\delta(f-25) + \delta(f+25)\right]} \longrightarrow (6)$$

* The Spectrum of the Sampled version of $g_1(t)$ is

$$G_{1\delta}(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

$\boxed{f_s = 75Hz}$

$$\boxed{G_{1\delta}(f) = \underset{(375)}{5 \times 75} \sum_{n=-\infty}^{\infty} \left[\delta(f-50-75n) + \delta(f+50-75n)\right]} \longrightarrow (7)$$

* The Spectrum of the Sampled version of $g_2(t)$ is

P.T.O

$$G_{\delta\delta}(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

$\boxed{f_s = 75\,Hz}$

$$\boxed{G_{\delta\delta}(f) = 5 \times 75 \sum_{n=-\infty}^{\infty} \left[\delta(f - 25 - 75n) + \delta(f + 25 - 75n)\right]} \longrightarrow (8)$$

* Letting $\underline{n = l - 1}$ in the 1st term of eq (8) & $\underline{n = K + 1}$ for the 2nd term, we get

$$G_{\delta\delta}(f) = 375 \sum_{l=-\infty}^{\infty} \delta\left[f - 25 - 75(l-1)\right] + 375 \sum_{K=-\infty}^{\infty} \delta\left[f + 25 - 75(K+1)\right]$$

$$= 375 \sum_{l=-\infty}^{\infty} \delta\left[f - 25 - 75l + 75\right] + 375 \sum_{K=-\infty}^{\infty} \delta\left[f + 25 - 75K - 75\right]$$

$$G_{\delta\delta}(f) = 375 \sum_{l=-\infty}^{\infty} \delta\left[f + 50 - 75l\right] + 375 \sum_{K=-\infty}^{\infty} \delta\left[f - 50 - 75K\right] \longrightarrow (9)$$

Since l & K are dummy variables, they can be replaced by n. Consequently eq (9) becomes

$$\boxed{G_{\delta\delta}(f) = 375 \sum_{n=-\infty}^{\infty} \left[\delta(f - 50 - 75n) + \delta(f + 50 - 75n)\right]} \longrightarrow (10)$$

* Eq (7) & (10) are Identical & hence the Samples in frequency domain are Same.

---

17) A Signal $g(t)$ Consists of two frequency Components $f_1 = 3.9\,KHz$ & $f_2 = 4.1\,KHz$ in Such a relationship that they just Cancel each other $g(t)$ is Sampled at the instants $t = 0, T, 2T, \cdots$ where $T = 125\,\mu sec$. The Signal $g(t)$ is defined by

$\boxed{\text{Aug-2001, 8M}}$

$$g(t) = \cos(2\pi f_1 t + \pi/2) + A\cos(2\pi f_2 t + \phi)$$

Find the values of amplitude A & ϕ of the Second frequency component.

Sol :- Given : $f_1 = 3900\,Hz$, $f_2 = 4100\,Hz$, $T_s = 125\,\mu sec$ & $g(t)$.

$$\boxed{g(t) = \cos(2\pi f_1 n T_s + \pi/2) + A\cos(2\pi f_2 n T_s + \phi)} \longrightarrow (1)$$

Substituting f_1, f_2 & T_s values in eq (1), we get

$$g(t) = \cos(2\pi \times 3900 \times n \times 125 \times 10^{-6} + \pi/2) + A\cos(2\pi \times 4100 \times n \times 125 \times 10^{-6} + \phi)$$

$$g(t) = \cos(0.975\pi n + \pi/2) + A\cos(1.025\pi n + \phi) \longrightarrow (2)$$

i) Let $n = 0$; then eq (2) becomes

$$g(t) = \cos(0 + \pi/2) + A\cos(0 + \phi)$$

$$g(t) = \underline{\cos(\pi/2)} + A\cos(\phi) \qquad \cos(\pi/2) = 0$$

$$g(t) = 0 + A\cos(\phi)$$

* $g(t)$ must be Zero i.e.

$$A\cos(\phi) = 0 \longrightarrow (3)$$

Eq (3) is Satisfied if $A = 0$ or $\phi = \pm\pi/2$. But A (Constant) cannot be Zero. Hence ϕ must be $\pm\pi/2$ i.e. $\boxed{\phi = \pm\pi/2}$

ii) Let $n = 1$, eq (2) becomes,

$$g(t) = \cos(0.975\pi + \pi/2) + A\cos(1.025\pi + \phi)$$

To make $g(t)$ Zero,

$$\cos(0.975\pi + \pi/2) + A\cos(1.025\pi + \phi) = 0$$

$$\boxed{\text{Let } \phi = \pm\pi/2}$$

$$\cos(0.975\pi + \pi/2) + A\cos(1.025\pi \pm \pi/2) = 0$$

$$-\sin(0.975\pi) \mp A\sin(1.025\pi) = 0$$

$$-\sin(\pi - 0.025\pi) \mp A\sin(\pi + 0.025\pi) = 0$$

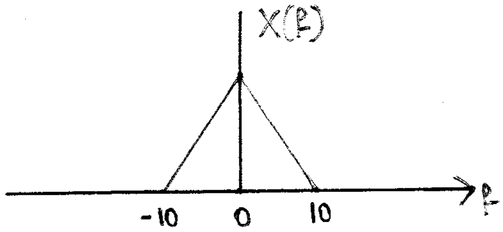
$$-\sin(0.025\pi) \pm A\sin(0.025\pi) = 0$$

$$-1.37\times10^{-3} \pm A \cdot 1.37\times10^{-3} = 0 \longrightarrow (4)$$

* The eq (4) is satisfied only when $A = 1$ & +ve sign for the second term is taken. The +ve sign can appear in eq (4), only when $\phi = +\pi/2$ in eq (4).

Thus $\boxed{A = 1 \;\&\; \phi = \pi/2}$

18) The spectrum of signal $x(t)$ is shown below. This signal is sampled at the Nyquist rate with a periodic train of rectangular pulses of 50/3 milliseconds. Find the spectrum of the sampled signal for frequencies upto 50Hz giving relevant expression.



Given: $f_m = 10Hz$, $T = \frac{50}{3}\times10^{-3}$

Sol :- Nyquist rate $f_s = 2f_m = 2\times10Hz = 20Hz$.

* The spectrum of the flat-top sampled signal is given as

$$\boxed{S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)\, H(f)} \longrightarrow (1)$$

W.K.T

$$H(f) = T\,\text{Sinc}(fT)\, e^{-j\pi fT} \longrightarrow (2)$$

* Given value of rectangular sampling pulse is $\frac{50}{3}$ msec

i.e. $T = \frac{50}{3}\times10^{-3} = \frac{0.05}{3}$ Sec

Putting the value of T in eq (2), we get

$$H(f) = \frac{0.05}{3} \operatorname{Sinc}\left(f \frac{0.05}{3}\right) e^{-j\pi f\left(\frac{0.05}{3}\right)} \longrightarrow (3)$$

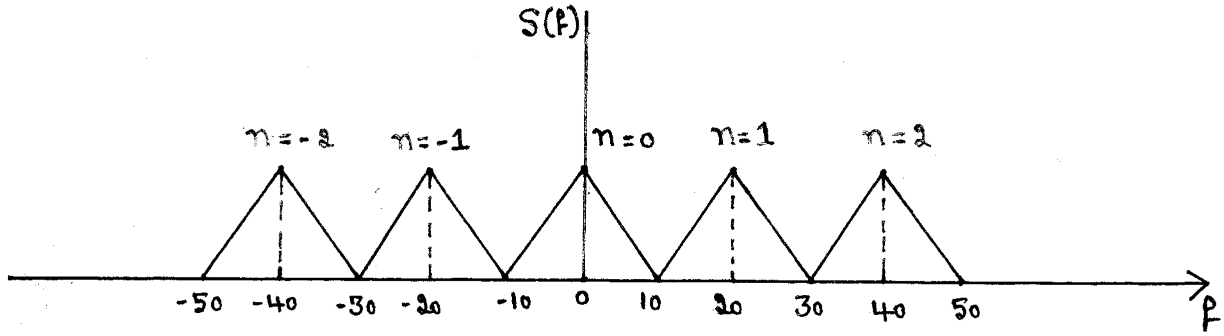
Substituting eq (3) in eq (1), we get

$$S(f) = 20 \sum_{n=-\infty}^{\infty} X(f - 20n) \cdot \frac{0.05}{3} \operatorname{Sinc}\left(f \cdot \frac{0.05}{3}\right) e^{-j\pi f\left(\frac{0.05}{3}\right)}$$

$$\boxed{S(f) = \frac{1}{3} \sum_{n=-3}^{3} X(f - 20n) \cdot \operatorname{Sinc}\left(f\frac{0.05}{3}\right) e^{-j\pi f\left(\frac{0.05}{3}\right)}}$$

$$\boxed{20 \times \frac{0.05}{3} = \frac{1}{3}}$$

* The above equation gives the Spectrum upto 50Hz



Arunkumar G M.Tech
Lecturer in E&CE Dept.
S.T.J.I.T., Ranebennur.

TIME DIVISION MULTIPLEXING (TDM) :

July-05, 5M	Jan-07, 5M	Jan-08, 5M	July-09, 5M

* An important feature of TDM is conservation of time i.e. different time intervals (periods) are allocated for different message signals, so that a common channel is utilized for transmission of these signals without any interference.

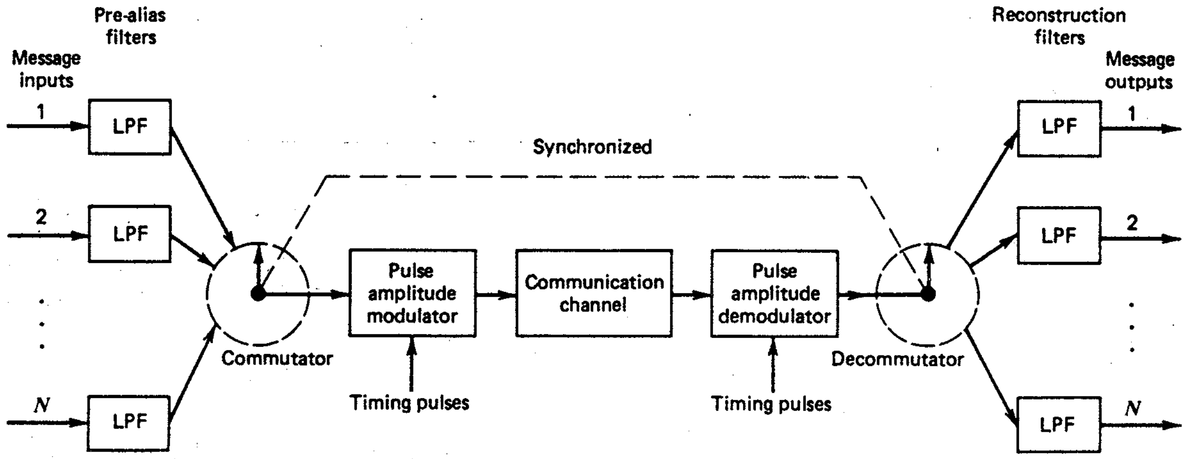


Fig ①: Block diagram of TDM system.

The concept of TDM is illustrated in the block diagram.

* The low pass pre-alias filters are used to remove high-frequency components which may be present in the message signal.
* The o/p of the pre-alias filters are then fed to a commutator, which is usually implemented using electronic switching circuitry.
* The function of commutator is 2 fold:
  1) To take a narrow sample of each of the 'N' I/p signals at a rate $f_s \geq 2W$, where 'W' is the cut-off frequency of pre-alias

filter.

2) To Sequentially interleave these 'N' Samples inside a Sampling interval $T_S = 1/f_S$.
This Interleaving is nothing but multiplexing.

* The multiplexed Signal is applied to a pulse amplitude modulator whose purpose is to transform the multiplexed Signal into a form Suitable for transmission over a Common Channel.

* At the receiving end, the pulse amplitude demodulator performs the reverse operation of PAM & the decommutator distributes the Signals to the appropriate low pass reconstruction filters.

* The decommutator operates in Synchronization with the commutator.

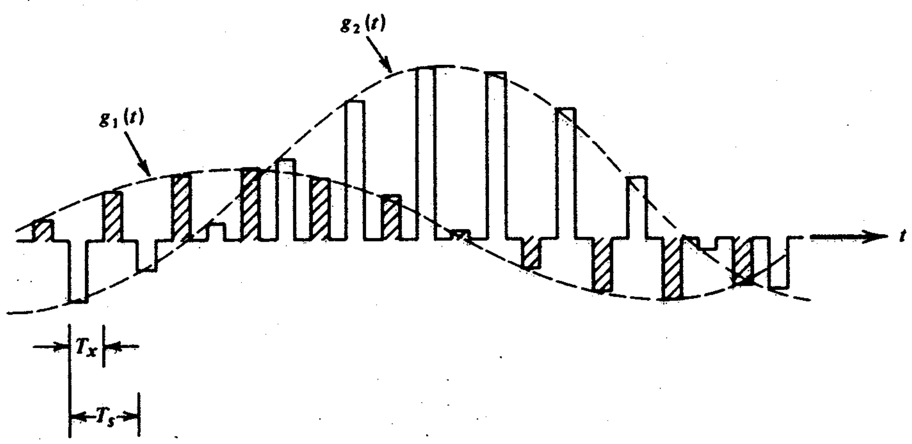


Fig ③: Waveforms illustrating TDM for two message signals.

* Suppose that the 'N' message Signals to be multiplexed (Txed) have the Same Spectral properties (BW). Then the Sampling rate rate for each message Signal is determined in accordance with the Sampling theorem.

* Let 'T_S' denotes the Sampling period.
Let 'T_x' denote the time Spacing between adjacent Samples in the

TDM Signal.

i.e. $\boxed{T_x = \frac{T_s}{N}}$ as shown in fig ②.

NOTE :-

* Spacing b/w two samples 'T_x' $= \frac{T_s}{N}$
* No of pulses per second $= \frac{1}{T_x} = \frac{1}{T_s/N} = \frac{N}{T_s} = Nf_s$
* No of pulses per second is also called as Signalling rate 'r'

i.e. $\boxed{r = Nf_s}$

Since $f_s \geq 2f_m$

$\boxed{r \geq N2f_m}$

* Transmission Bandwidth $= \frac{\text{Signalling rate}}{2}$

(TDM) FORMULAE

1) Speed of the Commutator in revolution per second (rps) = '2W'
Where 'W' is the minimum BW of message signal.
2) Speed of the Commutator in Samples/Sec
= Total number of Segments x Speed of Commutator in 'rps'
3) Minimum transmission BW $= \frac{1}{2}$[Sum of Nyquist rate]
4) Minimum bit rate $= \frac{1}{2}$[Sum of Nyquist rate] bits/Sec.
5) Angle of Separation of Corresponding Segments $= \frac{360^\circ}{n}$
Where 'n' is the number of Segments.
6) Angle of Separation b/w each Segment (pole) $= \frac{360^\circ}{\text{Total Number of Segments}}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

 June-09, 6M

1) A Signal $m_1(t)$ is band-limited to 3 KHz & 3 other Signals $m_2(t)$, $m_3(t)$ & $m_4(t)$ are bandlimited to 1.5 KHz each. These are transmitted by means of TDM.

i) Set up a Commutator Scheme to realize the multiplexing with each Signal Sampled at Nyquist rate.

ii) Find the Speed of the Commutator in Samples/Sec & the minimum band-width of the Channel

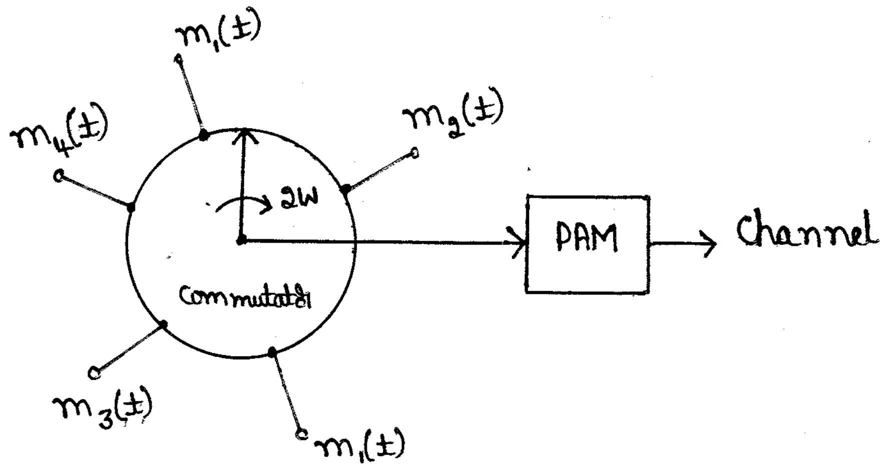
Sol :-

i)

Message Signal	BW	Nyquist rate	No of Segments 'N'	Angle of Separation of Corresponding Segments $= \frac{360°}{N}$
$m_1(t)$	3KHz	6KHz	2	180°
$m_2(t)$	1.5KHz	3KHz	1	360°
$m_3(t)$	1.5KHz	3KHz	1	360°
$m_4(t)$	1.5KHz	3KHz	1	360°

* Angle of Separation b/w each Segment (pole) $= \frac{360°}{\text{Total No of Segments}}$

$$= \frac{360°}{5} = \underline{72°}$$



(4M)

* If the Commutator is rotated at 3000 rev/Sec, then in each revolution we obtain one Sample each for $m_2(t)$, $m_3(t)$ & $m_4(t)$ & 2 Samples from $m_1(t)$.

* Commutator Speed in rps = 2W = 2×1.5KHz = 3000 rps

ii) * Speed of the Commutator in Samples/Sec

= [Total No of Segments × Speed of Commutator in rps]

= 5 × 3000 rps

= 15,000 Samples/Sec. ⟵ 1M

* Minimum BW of Channel $= \frac{1}{2}$ [Sum of Nyquist rate]

$= \frac{1}{2}[6KHz + 3KHz + 3KHz + 3KHz]$

$= 15KHz/2$

$= 7.5$ KHz. ⟵ 1M

---

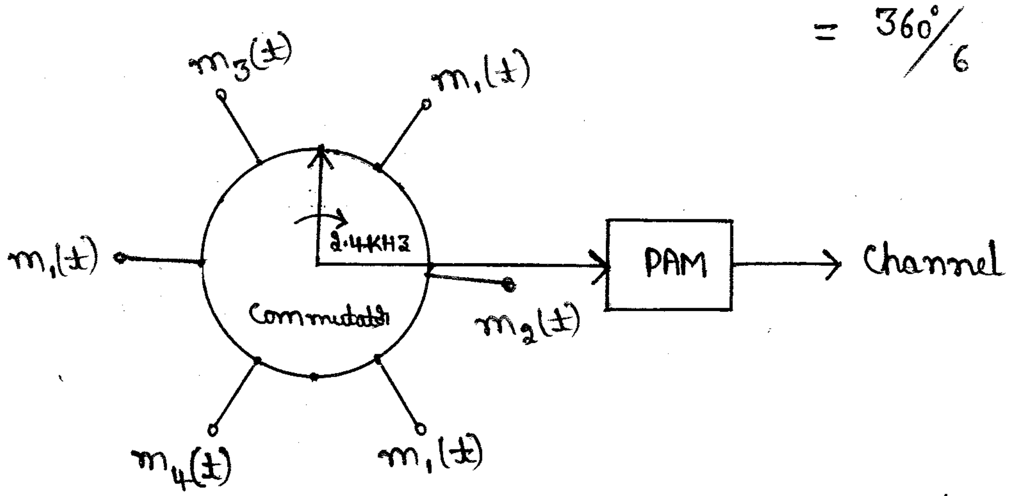
2) A Signal $m_1(t)$ is band limited to 3.6KHz & three other Signals $m_2(t)$, $m_3(t)$ & $m_4(t)$ are band limited to 1.2KHz each. These Signals are to be transmitted by means of TDM. Sketch set up a Scheme for realizing this multiplexing requirements with each Signal Sampled at its Nyquist rate. Determine the Speed of Commutator in Samples per Second. Jan-08, 8M

Sol:-

Message Signal	BW	Nyquist rate $f_s = 2W$	No of Segments 'N'	Angle of Separation of corresponding Segments $= \frac{360°}{N}$
$m_1(t)$	3.6KHz	7.2KHz	3	120°
$m_2(t)$	1.2KHz	2.4KHz	1	360°
$m_3(t)$	1.2KHz	2.4KHz	1	360°
$m_4(t)$	1.2KHz	2.4KHz	1	360°

* Angle of Separation b/w each Segment $= \dfrac{360^\circ}{\text{Total No of Segments}}$

$= 360^\circ/6 = \underline{60^\circ}$



* If the Commutator is rotated at 2400 rev/Sec then in each revolution, we obtain one Sample each for $m_2(t)$, $m_3(t)$ & $m_4(t)$ and 3 Samples from $m_1(t)$.

* Commutator Speed in rps $= 2W = 2 \times 1.2\,KHz = \underline{2400\text{ rps}}$

* Speed of Commutator in Samples/Sec

$=$ Total No of Segments $\times$ Speed of Commutator in rps.

$= 6 \times 2400$

$= \underline{14{,}400\text{ Samples/Sec.}}$

* Minimum BW of the Channel $= \dfrac{1}{2}\left[7.2\,KHz + (3 \times 2.4\,KHz)\right]$

$= \underline{7.2\,KHz.}$

3) Three Independent message Sources of bandwidths 1KHz, 1KHz, 2KHz respectively are to be transmitted using TDM Scheme. Determine:

i) the Commutator Segment arrangement.

ii) the Speed of Commutator if each Signal is Sampled at its Nyquist rate.

iii) Minimum transmission bandwidth. July-06, 6M

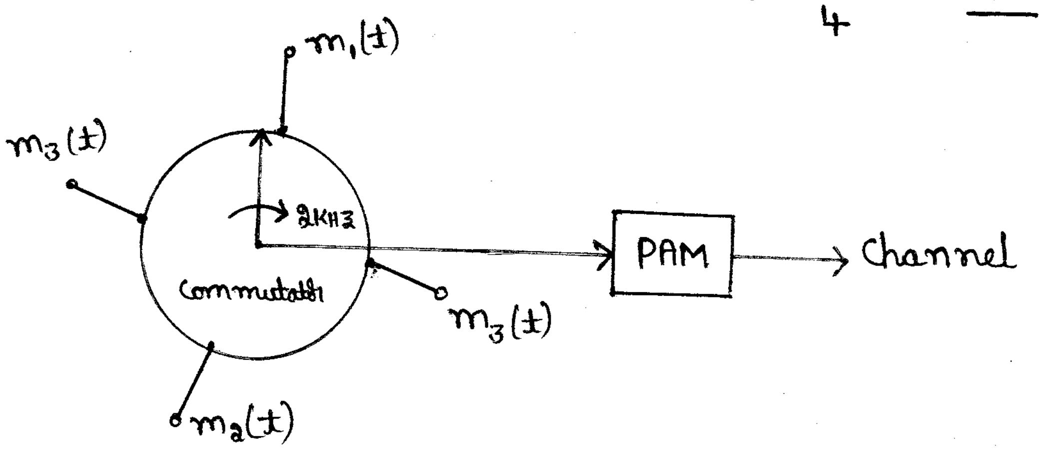
Sol :-

i)

Message Signal	BW	Nyquist rate $f_S = 2W$	No of Segments 'N'	Angle of Separation of corresponding Segments $= \frac{360^\circ}{N}$
$m_1(t)$	1 KHz	2KHz	1	360°
$m_2(t)$	1 KHz	2KHz	1	360°
$m_3(t)$	2KHz	4KHz	2	180°

* Angle of Separation b/w each Segment $= \dfrac{360^\circ}{\text{Total No of Segments}}$

$$= \frac{360^\circ}{4} = \underline{90^\circ}$$



* Commutator Speed in rps = 2W = 2 × 1KHz = <u>2000 rps</u>

* If the Commutator is rotated at <u>2000</u> rev/Sec then in each revolution we obtain <u>one Sample</u> each from $m_1(t)$, $m_2(t)$ & <u>2 Samples</u> from $m_3(t)$.

ii) Speed of the Commutator in Samples/Sec = Total No of Segments × Speed of Commutator in rps

= 4 × 2000 rps = <u>8000 Samp/Sec</u>

iii) Minimum $BW = \frac{1}{2}\left[2KHz + 2KHz + 4KHz\right] = \underline{4KHz}$

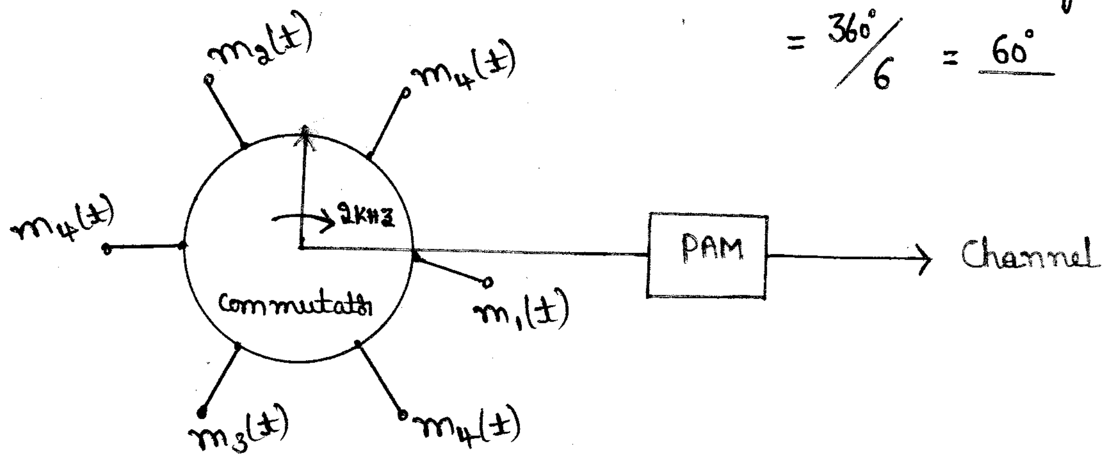
4) Four messages bandlimited to W, W, W & 3W are to be TDM, with W being 2000Hz. Set up a TDM Scheme for the Same & find Speed of the Commutator in Samples per Second. $\boxed{\text{Jan-10, 8M}}$

Sol:- Given: $W = 2KHz$.

Message Signal	BW	Nyquist rate $f_s = 2W$	No of Segments (N)	Angle of Separation of Corresponding Segments $= \frac{360°}{N}$
$m_1(t)$	W = 2KHz	4KHz	1	360°
$m_2(t)$	W = 2KHz	4KHz	1	360°
$m_3(t)$	W = 2KHz	4KHz	1	360°
$m_4(t)$	3W = 6KHz	12KHz	3	120°

* Angle of Separation b/w each Segment $= \frac{360°}{\text{Total No of Segments}}$

$= 360°/6 = \underline{60°}$



* Commutator Speed in rps = 2W = 2 x 2KHz = $\underline{4000 \text{ rps}}$

* If the Commutator is rotated at $\underline{4000 \text{ rev/Sec}}$ then in each revolution, we obtain $\underline{\text{one Sample}}$ each for $m_1(t)$, $m_2(t)$ & $m_3(t)$ and $\underline{\text{3 Samples}}$ from $m_4(t)$.

* Speed of Commutator in Samples per Sec

= Total No of Segments X Speed of Commutator in rps

= 6 X 4000 rps

= 24,000 Samples/Sec.

* Minimum transmission BW $= \frac{1}{2}\left[4KHz + 4KHz + 4KHz + 12KHz\right]$

$= \frac{24KHz}{2}$

$= 12KHz$

5) 6 Independent message Sources of BW's W, W, 2W, 2W, 3W & 3W Hz are to be transmitted on a TDM basis using a Common Communication Channel.

i) Set up a Scheme for accomplishing this multiplexing requirements with each message Signal Sampled at its Nyquist rate.

ii) Determine the minimum transmission BW of the Channel.

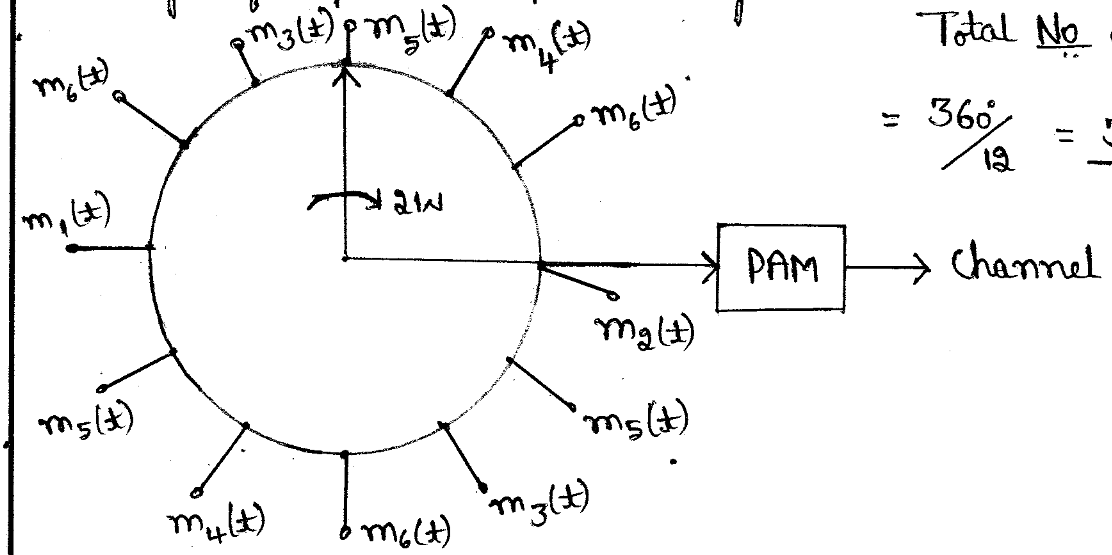
July-08, 5M

Sol :-

Message Signal	BW	Nyquist rate $f_s = 2W$	No of Segments (N)	Angle of Separation of Corresponding Segments $= \frac{360°}{N}$
$m_1(t)$	W	2W	1	360°
$m_2(t)$	W	2W	1	360°
$m_3(t)$	2W	4W	2	180°
$m_4(t)$	2W	4W	2	180°
$m_5(t)$	3W	6W	3	120°
$m_6(t)$	3W	6W	3	120°

* Angle of Separation b/w each Segment $= \dfrac{360^\circ}{\text{Total No of Segments}}$

$= \dfrac{360^\circ}{12} = \underline{30^\circ}$



* Speed of Commutator in rps = $\underline{2W}$.

* If the Commutator is rotated at $\underline{2W}$ rev/Sec, then in one rotation of the Commutator we get $\underline{\text{one Sample}}$ each of $m_1(t)$ & $m_2(t)$ and $\underline{2 \text{ Samples}}$ each of $m_3(t)$ & $m_4(t)$ and $\underline{3 \text{ Samples}}$ each of $m_5(t)$ & $m_6(t)$.

* Speed of the Commutator in Samples per Second

= Total No of Segments × Speed of Commutator in rps.

$= 12 \times 2W$

$= \underline{24W \text{ Samples/Sec}}$.

* Minimum transmission BW $= \frac{1}{2}$ [Sum of Nyquist rate]

$= \frac{1}{2}\left[2W + 2W + 4W + 4W + 6W + 6W\right]$

$= \underline{12W}$

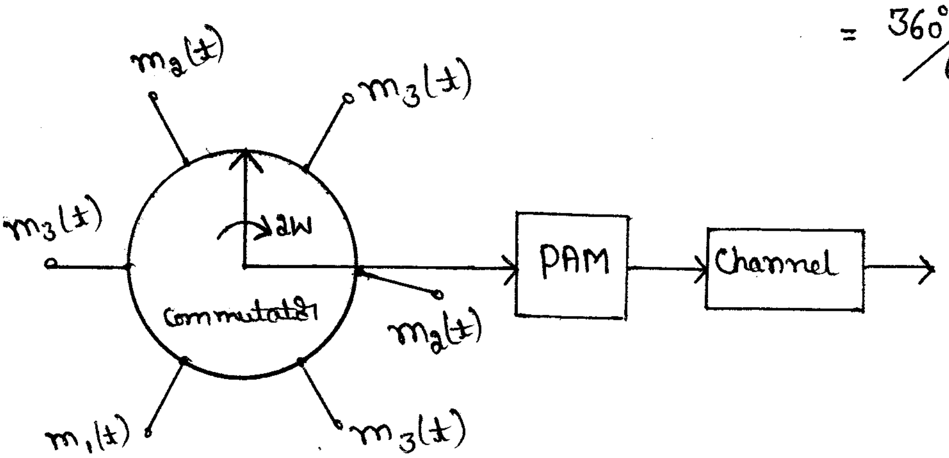
6) 3 Signals m_1, m_2, m_3 have BW 1KHz, 2KHz & 3KHz - respectively. Design a Commutator Switching System to multiplex these Signals Sampled at the Nyquist rate.

Sol :-

Message Signal	BW	Nyquist rate	No of Segments (N)	Angle of Separation of corresponding Segments $= \frac{360°}{N}$
$m_1(t)$	1KHz	2KHz	1	360°
$m_2(t)$	2KHz	4KHz	2	180°
$m_3(t)$	3KHz	6KHz	3	120°

* Angle of Separation b/w each Segment $= \frac{360°}{\text{Total No of Segments}}$

$$= \frac{360°}{6} = \underline{60°}$$



* Speed of the Commutator in rps = 2W = 2x1KHz = 2000 rps.

* If the Commutator is rotated at 2W rev/Sec, then in one rotation of the Commutator, we get one Sample from $m_1(t)$, 2 Samples from $m_2(t)$ & 3 Samples from $m_3(t)$.

* Speed of the Commutator in Samples per Second
= Total No of Segments x Speed of Commutator in rps

$= 6 \times 2KHz$

$= \underline{12KHz}$ Samples/Sec

* Minimum bit rate = Minimum transmission bandwidth.

$= \frac{1}{2}$ [Sum of Nyquist rate]

$= \frac{1}{2}\left[2KHz + 4KHz + 6KHz\right]$

$=$ <u>6 Kilo bits/Sec.</u>

Quantization

The process of transforming **sampled amplitude values** of a message signal into a **discrete amplitude value** (levels) is referred to as **quantization.**

Quantization **approximates** each of input sample value to **nearest prefixed level**.

Or

The conversion of an **analog sample** of the signal into a **digital form** is called the **quantizing process**.

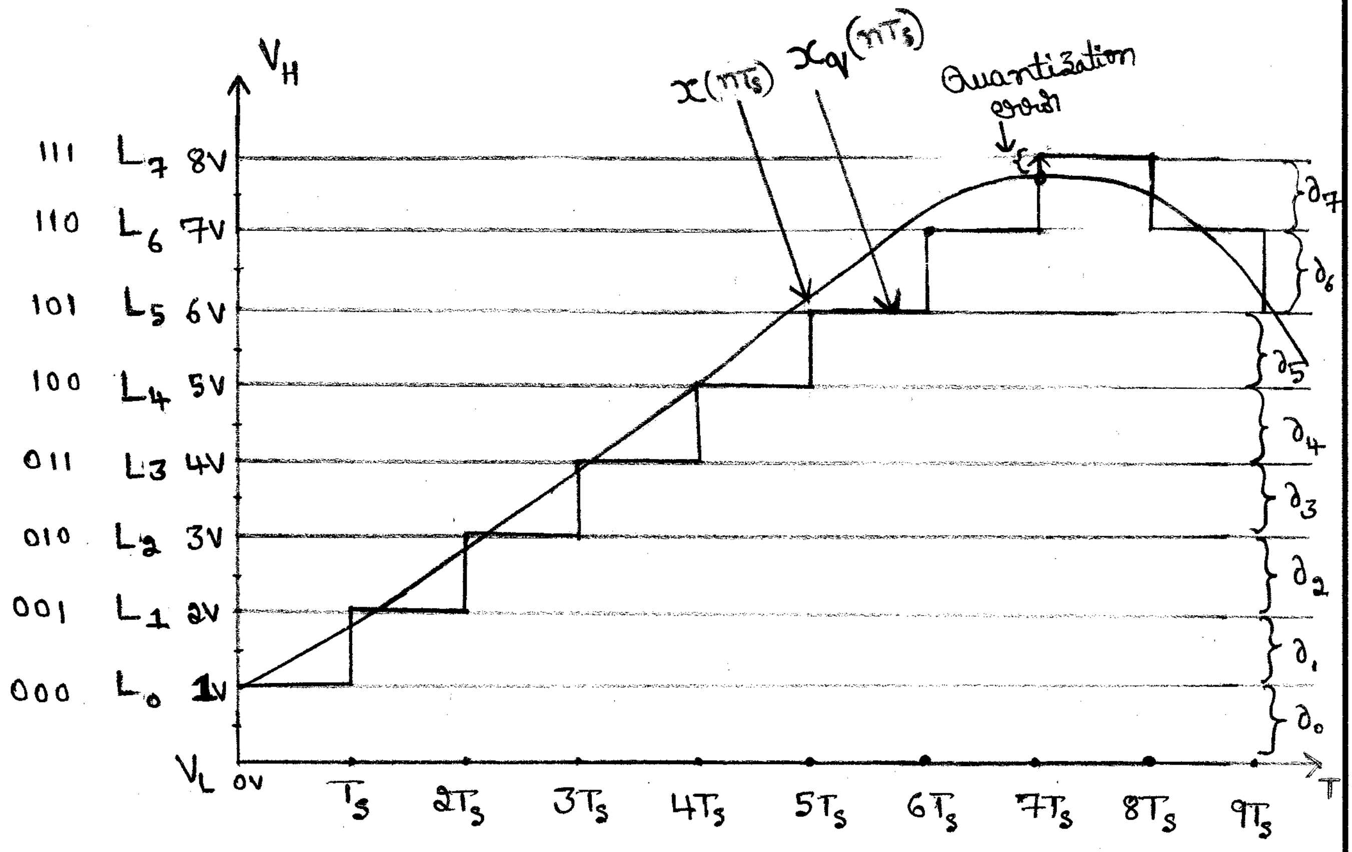


Fig ①: Quantizing operation.

* The Signal $x(t)$ whose excursion is confined to the range from V_L to V_H being divided into 8 - equal levels.

$$x(nT_s) \longrightarrow \boxed{\text{Quantizer}} \longrightarrow x_q(nT_s)$$

* Step Size is denoted by '∂' or 'Δ' & is given by

$$\boxed{\partial = \frac{V_H - V_L}{L}}$$

Where $L = 2^N$

$N \rightarrow$ No of bits

$$\therefore \quad \boxed{\delta = \frac{V_H - V_L}{2^N}}$$

* If the Step Size 'δ' is maintained Same through the process of Quantization, then it is called "Uniform Quantization."

Quantization error :-

* The difference between the Continuous amplitude Sample level & Quantized Signal level is known as Quantization error.

$$\boxed{e(t) = x_q(nT_s) - x(nT_s)}$$

* Quantization error varies from $-\delta/2$ to $+\delta/2$.

* The random errors due to Quantization process produces a noise at the o/p of the Quantizer & this noise is referred to as Quantization Noise.

NOTE :- 1) Step Size :-

* From fig ①: $V_H = 8V$ & $V_L = 0V$.

$L = 2^N$, Where $N = 3$, $L = 2^3 = 8$ levels

$$\delta = \frac{V_H - V_L}{L} = \frac{8V - 0V}{8} = 1V$$

$$\boxed{\delta = 1V}$$

2) Quantization error :-

* Actual value of the Signal is 7.7V.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Quantized value of the Signal is 8V.

∴ Quantization error

$$e(t) = x_q(nT_s) - x(nT_s)$$

$$= 8V - 7.7V$$

$$\boxed{e(t) = 0.3V}$$

Sampling Quantizing & Coding of an analog Signal :-

Consider Fig ①

Sampled values of an analog Signal	1.7V	2.7V	3.9V	5V	6.2V	7.2V	7.7V	7.4V
Nearest quantizer level	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_6
Quantizer level voltage	2V	3V	4V	5V	6V	7V	8V	7V
Binary Code.	001	010	011	100	101	110	111	110

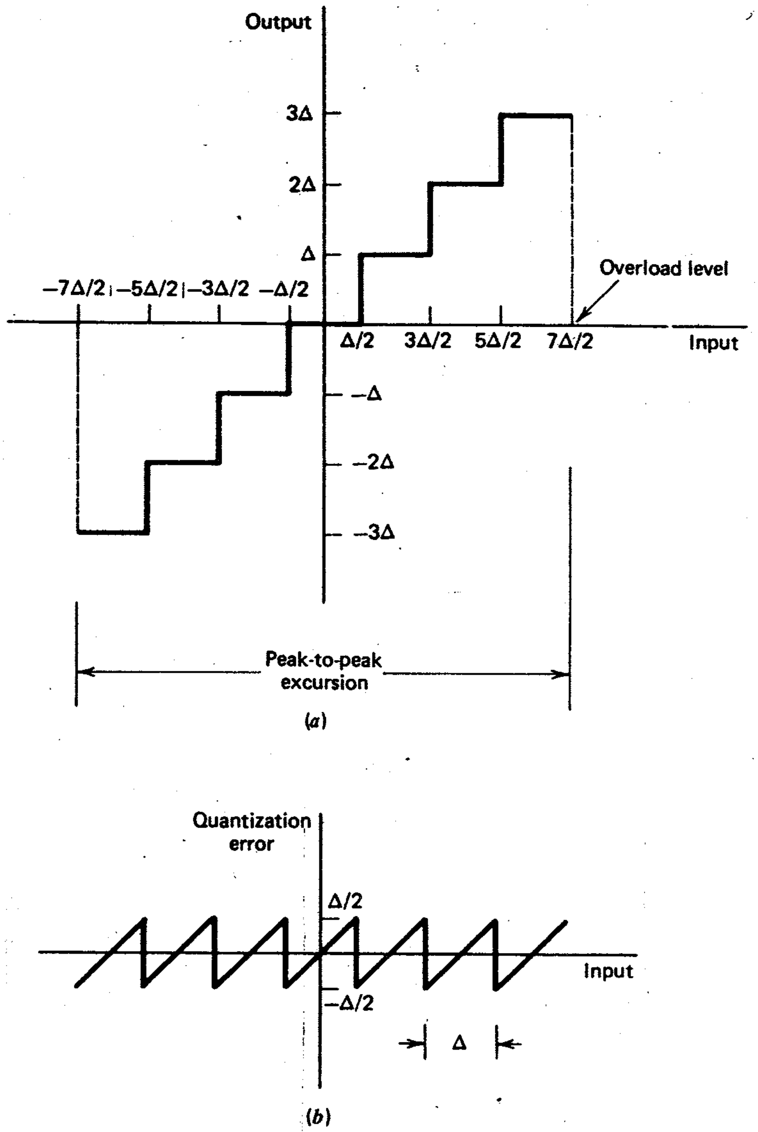
Types of uniform quantizer :-

1) Mid-tread type quantizer

2) Mid-riser type quantizer

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) Mid-Tread type Quantizer :-

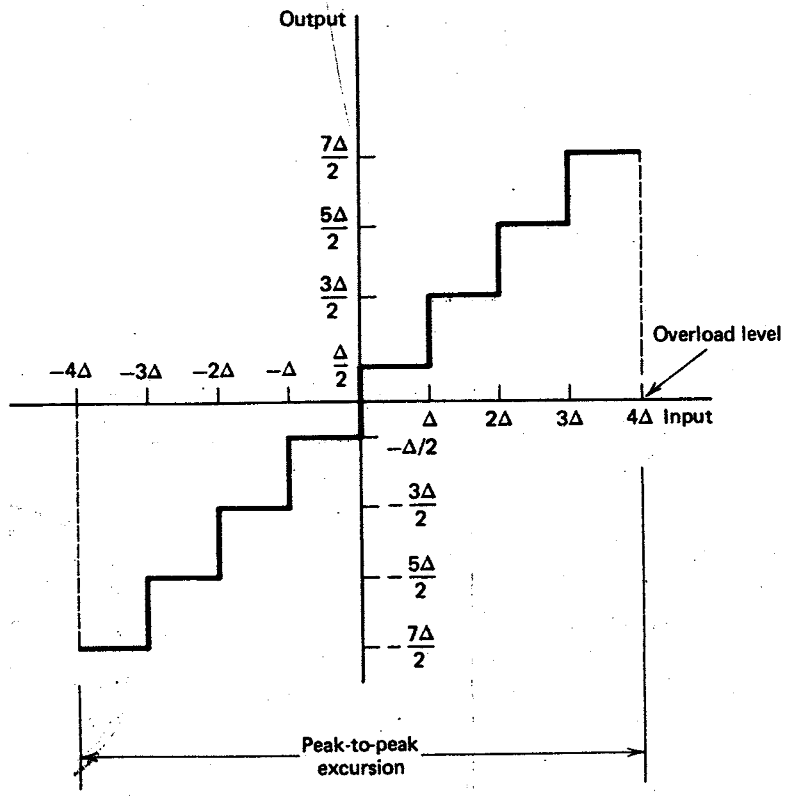


(*a*) Transfer characteristic of quantizer of midtread type. (*b*) Variation of the quantization error with input.

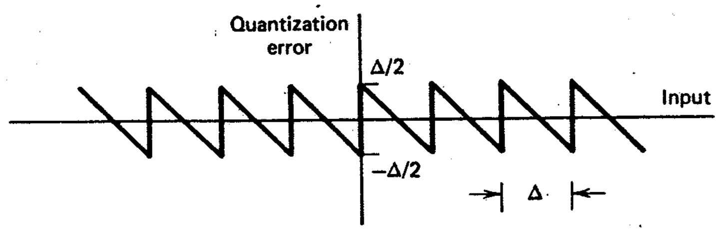
* In midtread Quantizer, the decision threshold of the quantizers are located at $\pm \delta/2, \pm 3\delta/2, \pm 5\delta/2, \dots$, and the representation levels are located at $0, \pm\delta, \pm 2\delta, \dots$, Where δ is the Step Size.
* A uniform Quantizer characterized in this way is referred to as a Symmetric Quantizer of the midtread type, because the origin lies in the middle of a tread of a Staircase.
* Quantization levels are odd Number.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Midriser type Quantizer :-



(a)



(b)

(a) Transfer characteristic of quantizer of midriser type. (b) Variation of the quantization error with input.

* In midriser quantizer, the decision thresholds of the quantizers are located at $0, \pm\delta, \pm 2\delta, \cdots,$ & the representation levels are located at $\pm\delta/2, \pm 3\delta/2, \pm 5\delta/2, \cdots,$ Where δ is the Step Size.
* A uniform quantizer characterized in this way is referred to as a Symmetric quantizer of the midriser type, because the origin lies in the middle of a riser of the Staircase.
* Quantization levels are even number.

* With a neat block diagram explain the Concept of PCM.

Pulse Code Modulation (PCM) :-

Jan-06, 6M	Jan-10, 7M

* PCM is an analog to digital Converter where the Information Contained in the Instantaneous Samples of an analog Signal are represented by digital Codes in a Serial bitstream manner.

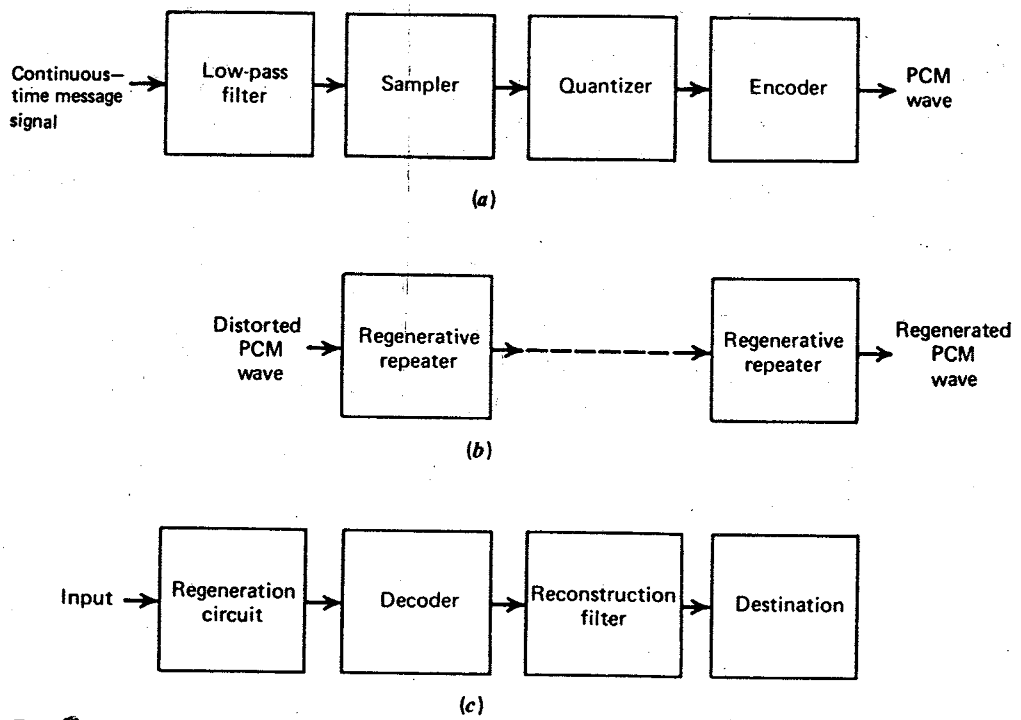


Fig ① : Basic elements of a PCM system. (*a*) Transmitter. (*b*) Transmission path. (*c*) Receiver.

The block diagram of a PCM System is Shown in Fig ①.

It Consists of
1) Transmitter
2) Regenerative repeater &
3) Receiver.

PCM Txer:

In practice the low pass filter (pre-alias filter) is used before Sampler in order to limit the frequency greater than 'W' Hz. Hence message Signal is bandlimited to 'W' Hz.

Sampler :- The incoming message Signal is Sampled with a train

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

of Narrow rectangular pulses. The Sampling rate 'f_s' is Selected above Nyquist rate to avoid aliasing i.e. $\boxed{f_s \geq 2W}$.

Quantization :-

The Sampled Signal is fed to the quantizer. The quantizer approximates each I/p Signal level to the nearest prefixed level.

The o/p of the quantizer is discrete time discrete valued Signal known as "Quantized Signal".

Encoding :-

The quantized Samples are then encoded in the encoder. The process of encoding involves allocating some digital code to each level. These coded levels are transmitted as a bitstream of data i.e. 0's & 1's.

The encoder o/p consists of pulses depending on code combination.

Regenerative repeater :-

The PCM Signal is reconstructed by means of a regenerative repeater located at Sufficiently closed Spacing along the transmission path.

The regenerative networks are used at intermediate points between transmitter & receiver in order to Boost up the pulse amplitude.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

PCM receiver :

Decoder:- The 1st operation in the receiver is to generate the received pulses.

The decoder converts binary coded signal to a approximated pulses of discrete magnitude.

Reconstruction filter:-

The final operation in the receiver is to recover the original analog signal. This is done by passing the decoder o/p through a LPF.

The o/p of LPF is an analog signal.

Advantages:-

1) Relatively inexpensive digital circuitry is involved in PCM.
2) PCM signals can be multiplexed & transmitted over a common high speed communication link.
3) In long distance transmission, clean waveforms can be regenerated using repeaters.
4) The noise performance of digital system is superior to that of an analog system.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Pulse degradation :-

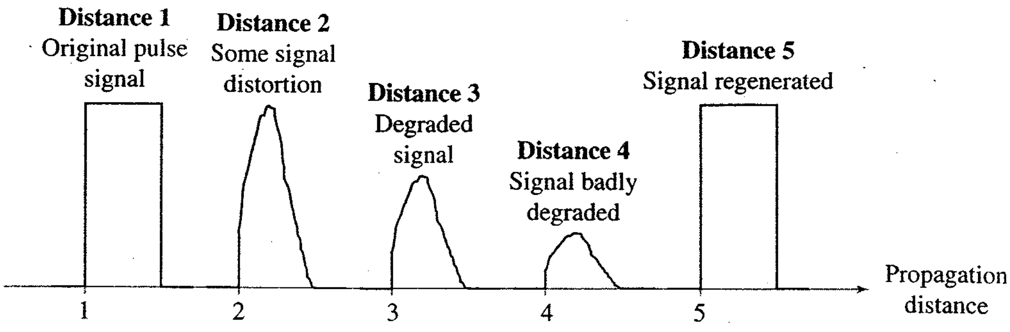


Figure Pulse degradation and regeneration.

The Shape of the pulse is affected mainly by two mechanisms:

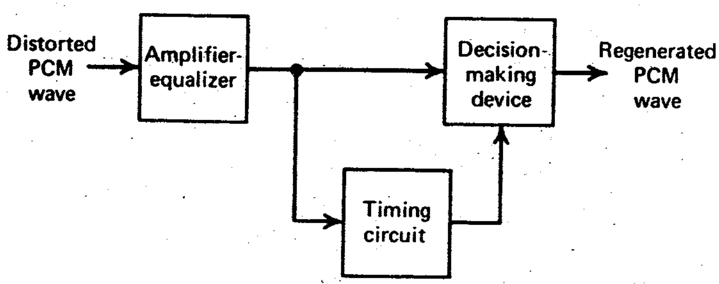
1) Unwanted electrical noise & other disturbance.

2) Nonideal transfer function of the transmission medium.

Both the mechanisms cause the pulse Shape to get degraded as a function of the distance traveled as Shown in fig ①. This degraded pulse can be regenerated by using circuit called regenerative repeaters.

Regenerative repeater :-

The regenerative repeater is used to remove noise from the incoming PCM data. Regenerative repeater are installed (placed) at regular interval along the Channel through which PCM Signal is transmitted.



Block diagram of a regenerative repeater.

The regenerative repeater performs three basic functions:

1) Equalization 2) Timing & 3) Decision making.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Equalizer :-

The equalizer shapes the received pulses so as to compensate for the effects of amplitude & phase distortion produced by the transmission channel.

Timing circuitry :-

The timing circuit provides a periodic pulse train derived from the received pulses.

The o/p of amplifier equalizer is sampled at different sampling instants where the signal to noise ratio is maximum.

Decision making device :-

The sample so extracted is compared with a predetermined threshold (λ) in the decision making device.

* During each bit interval, a decision is then made whether the received symbol is a '1' or '0'.
* If the threshold is exceeded, a clean new pulse representing symbol '1' is transmitted, otherwise a clean base line representing '0' is transmitted.

Limitations of regenerative repeater :-

1) The presence of channel noise & interference causes the regenerative repeater to make wrong decision occassionally. This introduces bit errors into the regenerated signal.

2) Timing jitter is introduced into the regenerated pulses due to decision device. This causes distortion.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Quantization Noise & Signal to Noise ratio :-

1) Derive the expression for Signal to quantization noise ratio for PCM system that employs linear quantization technique?

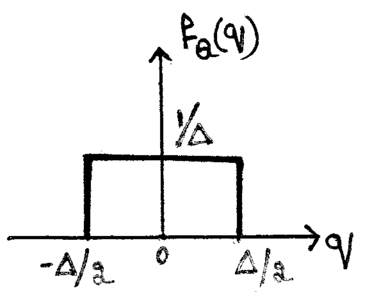
2) A PCM System uses a uniform quantizer followed by a 'ν' bit encoder. S.T rms Signal to quantization noise ratio is approximately given by $(1.8 + 6\nu)$ dB. **Jan-07, 10M**

3) Derive an expression for maximum Signal to quantization noise ratio for PCM System that employs linear quantization techniques. What will be the $(SNR)_{dB}$ if the destination power & signal amplitude are Normalized $\rightarrow \{(SNR)_{dB} = 4.8 + 6N\}$ **Jan-06, 8M**

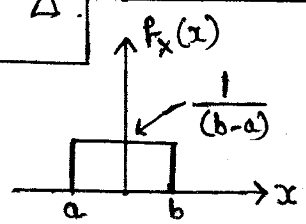
4) obtain an expression for the Signal to quantization noise power ratio in the case of PCM. Assume that the amplitude of Signal is uniformly distributed. **July-06, 6M**

Sol :-

* Let the random variable 'Q' denotes the quantization error & 'q' its Sample value.
* Let us assume that the quantization error 'Q' is uniformly distributed over a Single Quantizer Interval 'Δ'.



WKT PDF,



$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

∴ Probability density function (PDF) of Quantization error 'Q' is then

$$f_Q(q) = \begin{cases} 1/\Delta & \text{for } -\Delta/2 \leq q \leq \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore f_Q(q) = \frac{1}{\Delta/2 - (-\Delta/2)} = \frac{1}{\cancel{2}\,\Delta/\cancel{2}}$$

$$\boxed{f_Q(q) = 1/\Delta}$$

W.K.T mean

$$\mu = \frac{a+b}{2} = \frac{-\Delta/2 + \Delta/2}{2}$$

$$\boxed{\mu = 0}$$

* The mean quantization error $\mu = 0$

* The variance of Quantization error is

WKT

$$\boxed{\begin{array}{c} f_Q(q) = 1/\Delta \\ \mu = 0 \end{array}}$$

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} (q-\mu)^2 \cdot f_Q(q) \cdot dq$$

$$= \int_{-\Delta/2}^{\Delta/2} (q-0)^2 \cdot \frac{1}{\Delta}\, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \cdot \frac{1}{\Delta}\, dq = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2 \cdot dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta}\left[q^3\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} - \frac{-\Delta^3}{8}\right]$$

$$= \frac{1}{3\Delta}\left[\Delta^3/8 + \Delta^3/8\right] = \frac{1}{3\cancel{\Delta}}\,\cancel{2}\left(\frac{\cancel{\Delta^3}\,\Delta^2}{\cancel{8}\,4}\right) = \frac{1}{3}\cdot\frac{\Delta^2}{4}$$

$$\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}} \longrightarrow (1)$$

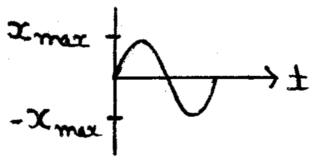
* Eq (1) is known as "Mean Squared Quantization error" or Normalized Noise power or Quantization error interms of power.

* Let us consider 'N'-bits to represent 'L' Quantized level, then

$$\boxed{L = 2^N}$$

$\therefore$ Step Size $\Delta = \dfrac{2x_{max}}{L}$

$$\boxed{\Delta = \frac{2x_{max}}{2^N}} \longrightarrow (2)$$



$$\Delta = \frac{x_{max} - (-x_{max})}{L}$$

$$\boxed{\Delta = \frac{2x_{max}}{2^N}}$$

Substituting eq (2) in eq (1), we get

$$\sigma_Q^2 = \frac{(2x_{max}/2^N)^2}{12}$$

$$\sigma_Q^2 = \frac{4x_{max}^2}{2^{2N}} \Big/ 12 = \frac{\cancel{4}x_{max}^2}{2^{2N}} \times \frac{1}{\cancel{12}\,3}$$

$$\boxed{\sigma_Q^2 = \frac{1}{3} x_{max}^2 \cdot 2^{-2N}} \longrightarrow (3)$$

* Let 'P' denotes the average power of the message signal $x(t)$, then the o/p SNR of a uniform quantizer is

$$\frac{S}{N} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{P}{\sigma_Q^2}$$

$$= \frac{P}{\frac{1}{3} x_{max}^2 \, 2^{-2N}} = \frac{3P}{x_{max}^2} \, 2^{2N}$$

$$\boxed{(SNR)_o = \frac{3P}{x_{max}^2} \cdot 2^{2N}} \longrightarrow (4)$$

* For Normalized I/p voltage $x_{max} = 1$ & Power $P \leq 1$.

$$\therefore \quad SNR = \frac{3(1)}{(1)^2} \cdot 2^{2N}$$

$$(SNR) = 3 \cdot 2^{2N}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$(SNR)_{dB} = 10\log_{10}(3\cdot 2^{2N})$$

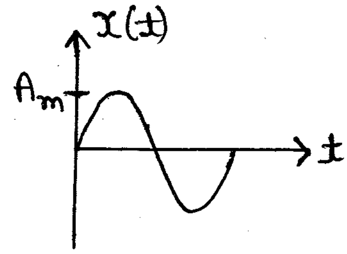
$$= 10\log_{10}(3) + 10\log_{10}(2^{2N})$$

$$= 4\cdot 8 + 20N\log_{10}(2)$$

$$\boxed{(SNR)_{dB} = 4\cdot 8 + 6N} \longrightarrow (5)$$

Eq (5) is the Normalized Signal to quantization noise ratio in dB for any message Signal.

For Sinusoidal message Signal



* Let $x(t) = A_m \cos 2\pi f_m t$

$$\boxed{x_{max} = A_m}$$

∴ The power of this Signal is

$$P = \frac{V^2}{R}$$

V = rms value

$$V = \frac{A_m}{\sqrt{2}}$$

When R=1, the power 'P' is normalized

$$P = \frac{(A_m/\sqrt{2})^2}{R} = \frac{A_m^2}{2\times 1}$$

$$\boxed{P = \frac{A_m^2}{2}}$$

W.K.T $\quad (SNR)_o = \frac{3P}{x_{max}^2}\cdot 2^{2N}$ From eq (4)

$$= \frac{3(\cancel{A_m^2}/2)}{\cancel{A_m^2}}\cdot 2^{2N}$$

$$(SNR)_o = \frac{3}{2}\cdot 2^{2N}$$

$$(SNR)_{dB} = 10 \log_{10} \left(\frac{3}{2} \cdot 2^{2N} \right)$$

$$= 10 \log_{10} (3/2) + 10 \log_{10} (2^{2N})$$

$$= 1.76 + 20 N \log_{10} (2)$$

$$\boxed{(SNR)_{dB} = 1.76 + 6.02N} \longrightarrow 6$$

* Eq 6 is known as "6 dB rule" for uniform quantization. This is because each additional bit of quantization level increases the Signal to Noise ratio by 6 dB.

---

* Derive the equation for Signal to quantization noise ratio if probability of overload is less than 10^{-4}, in the case of a uniform quantizer. Further, if a binary code of n-bits ($n > 6$), write the equation for (SNR)₀.

Jan-09, 9M

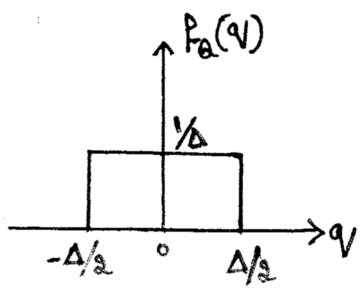
Sol :-

* Let the random variable 'Q' denotes the quantization error & 'q' its sample value.

* Let us assume that the quantization error 'Q' is uniformly distributed over a single quantizer interval 'Δ'.

∴ PDF of quantization error 'Q' is then

$$f_Q(q) = \begin{cases} 1/\Delta & \text{for } -\Delta/2 \le q \le \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$



ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

* The mean quantization error $\mu = 0$.

* The variance of quantization error is

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} (q-\mu)^2 f_Q(q)\, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} (q-0)^2 \frac{1}{\Delta}\, dq$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{3\cancel{\Delta}}\left[\frac{\cancel{2}\Delta^{\cancel{3}\,2}}{\cancel{8}\,4}\right] = \frac{1}{3}\cdot\frac{\Delta^2}{4}$$

$$\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}} \longrightarrow (1)$$

* WKT o/p signal to quantization noise ratio $(SNR)_o$ is

$$(SNR)_o = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\sigma_X^2}{\Delta^2/12}$$

$$\boxed{(SNR)_o = \frac{12\sigma_X^2}{\Delta^2}} \longrightarrow (2)$$

* Let x_{max} denote the absolute value of the overload level of the quantizer & Δ denotes its step-size. We may express the number of levels in the quantizer as

$$L = 1 + \frac{2x_{max}}{\Delta} \longrightarrow (3)$$

* Since the number of levels is odd for midtread quantizer, we write

$$\boxed{L = 2^n - 1} \longrightarrow (4)$$

Substituting eq (4) in eq (3), we get

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$2^n - 1 = 1 + \frac{2x_{max}}{\Delta}$$

$$2^n = 2 + \frac{2x_{max}}{\Delta}$$

$$2^n = 2\left[1 + \frac{x_{max}}{\Delta}\right]$$

$$\frac{2^n}{2} = 1 + \frac{x_{max}}{\Delta}$$

$$2^n \cdot 2^{-1} = 1 + \frac{x_{max}}{\Delta}$$

$$2^{n-1} - 1 = \frac{x_{max}}{\Delta}$$

$$\boxed{\Delta = \frac{x_{max}}{2^{n-1} - 1}} \longrightarrow (5)$$

If probability of overload is $< 10^{-4}$ & $x_{max} = 4\sigma_x$

$$\boxed{\Delta = \frac{4\sigma_x}{2^{n-1} - 1}} \longrightarrow (6)$$

WKT

$$(SNR)_o = \frac{12\sigma_x^2}{\Delta^2} \quad \text{from eq (2)}$$

Substituting Δ value in above equation, we get

$$(SNR)_o = \frac{12\sigma_x^2}{\left(\frac{4\sigma_x}{2^{n-1}-1}\right)^2} = \frac{\cancel{12}^{3}\cancel{\sigma_x^2}}{\frac{\cancel{16}^{4}\cancel{\sigma_x^2}}{(2^{n-1}-1)^2}}$$

$$\boxed{(SNR)_o = \frac{3}{4} \cdot (2^{n-1} - 1)^2} \longrightarrow (7)$$

$$(SNR)_o = \frac{3}{4}\left(2^n \cdot 2^{-1} - 1\right)^2 = \frac{3}{4}\left(2^n - 1\right)^2 \cdot 2^{-2}$$

$$= \frac{3}{4 \cdot 2^2} (2^n - 1)^2 = \frac{3}{16}(2^n - 1)^2$$

for larger n i.e. $n > 6$, we can neglect 1 i.e. $2^n - 1 = 2^n$

$$(SNR)_o = \frac{3}{16} \cdot 2^{2n}$$

$$(SNR)_{dB} = 10 \log_{10}\left(\frac{3}{16}\right) + 10 \log_{10}(2^{2n})$$

$$= -7.2 + 20n \log_{10}(2)$$

$$= -7.2 + 6.02n$$

$$\boxed{(SNR)_{dB} = 6.02n - 7.2}$$

---

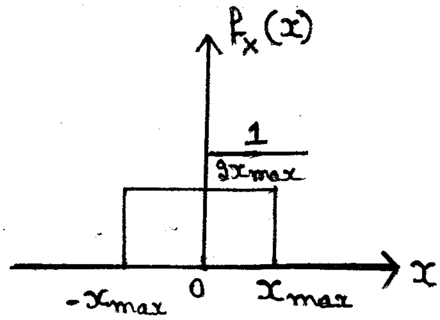
* A Signal $x(t)$ is uniformly distributed in the range $\pm x_{max}$. Calculate Signal to Noise ratio for pulse code modulation of this Signal.

Jan-10, 8M

Sol:- WKT $\boxed{SNR = \frac{\sigma_x^2}{\sigma_Q^2}} \rightarrow (1)$

* The PDF of $x(t)$ is

$$f_X(x) = \begin{cases} \frac{1}{2x_{max}} & \text{for } -x_{max} \leq x \leq x_{max} \\ 0 & \text{otherwise} \end{cases}$$



* The mean Square value of $x(t)$ is

WKT $\boxed{\mu = 0}$

$$\sigma_X^2 = \int_{-x_{max}}^{x_{max}} (x - \mu)^2 f_X(x)\, dx$$

$$= \int_{-x_{max}}^{x_{max}} (x - 0)^2 \frac{1}{2x_{max}} dx$$

$$= \frac{1}{2x_{max}} \int_{-x_{max}}^{x_{max}} x^2\, dx$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

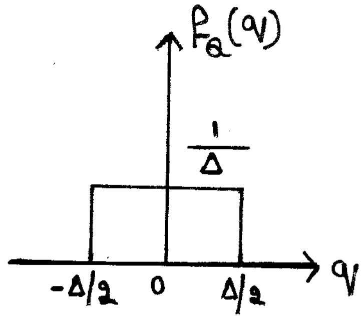
$$= \frac{1}{2x_{max}}\left[\frac{x^3}{3}\right]_{-x_{max}}^{x_{max}} = \frac{1}{6x_{max}}\left[x_{max}^3 - (-x_{max})^3\right]$$

$$= \frac{1}{3\cancel{6}\cancel{x_{max}}}\left[\cancel{2}x_{max}^{\cancel{3}\,2}\right]$$

$$\boxed{\sigma_X^2 = \frac{x_{max}^2}{3}} \longrightarrow (2) \longrightarrow \boxed{\text{3-Marks}}$$

* The PDF of Quantization error 'Q' is

$$f_Q(q) = \begin{cases} \frac{1}{\Delta} & \text{for } -\Delta/2 \leqslant q \leqslant \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$



* The variance of quantization error is

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} (q-\mu)^2 f_Q(q)\cdot dq$$

WKT

$$\boxed{\mu = 0}$$

$$= \int_{-\Delta/2}^{\Delta/2} (q-0)^2 \cdot \frac{1}{\Delta}\, dq$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2 \cdot dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{3\cancel{\Delta}}\,\cancel{2}\left(\frac{\Delta^{\cancel{3}\,2}}{\cancel{8}\,4}\right)$$

$$\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}} \longrightarrow (3) \longrightarrow \boxed{\text{3-Marks}}$$

Substituting eq (2) & (3) in eq (1), we get

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\frac{x_{max}^2}{3}}{\frac{\Delta^2}{12}} = \frac{x_{max}^2}{\cancel{3}} \times \frac{\cancel{12}\,4}{\Delta^2} = \frac{4x_{max}^2}{\Delta^2} \quad (4)$$

WKT $\boxed{\Delta = \frac{2x_{max}}{M}} \longrightarrow (5)$

Substituting eq (5) in eq (4), we get

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$(SNR) = \frac{4x_{max}^2}{\left(\frac{2x_{max}}{M}\right)^2} = \frac{\cancel{4x_{max}^2}}{\frac{\cancel{4x_{max}^2}}{M^2}} = M^2$$

WKT $\boxed{M = 2^N}$

$$(SNR) = (2^N)^2$$

$$\boxed{(SNR) = 2^{2N}} \longrightarrow \boxed{\text{1 - Marks}}$$

$$\therefore (SNR)_{dB} = 10 \log_{10}\left[2^{2N}\right]$$

$$= 2N \cdot 10 \log_{10}(2)$$

$$= 20 \cdot N\,(0.30)$$

$$= N \cdot 20 \times 0.3$$

$$\boxed{(SNR)_{dB} = 6N} \longrightarrow \boxed{\text{1 - Marks}}$$

Bandwidth of PCM Signal :-

* Let the quantizer uses 'N' number of binary digits to represent each level.

$\therefore$ Number of levels that can be represented by 'N' digit will be 'L' i.e $L = 2^N$

ex :- If $N = 3$, then total number of levels will be

$L = 2^3 = 8$ levels.

* The number of bits per second is called Signalling rate of PCM & is denoted by 'r' i.e. $r = Nf_s$ bits/sec.

* Bandwidth needed for PCM transmission will be given by half of the Signalling rate.

∴ Transmission bandwidth of PCM is

$$B_T \geqslant \frac{1}{2} r$$

WKT

$$\boxed{f_s \geqslant 2W}$$

$$B_T \geqslant \frac{1}{2} N f_s$$

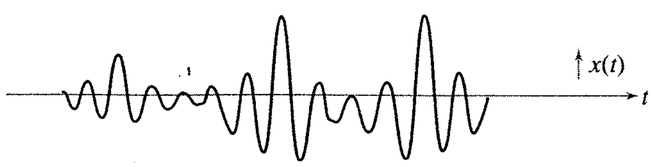
$$B_T \geqslant \frac{1}{\cancel{2}} N \cdot \cancel{2}W$$

$$\boxed{B_T \geqslant NW}$$

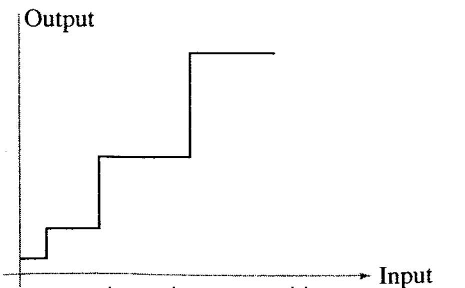
Non uniform quantization or Robust quantization :-

* Speech Communication is very important in digital Communication Systems. If uniform quantization is used, the Step Size will be Constants.

* The System that uses equally Spaced quantization levels, the quantization noise is Same for all Signal amplitudes. Hence Small amplitude Samples are more affected than the bigger Sample values.



Continuous time speech signal $x(t)$.



Transfer characteristics of a nonuniform quantizer.

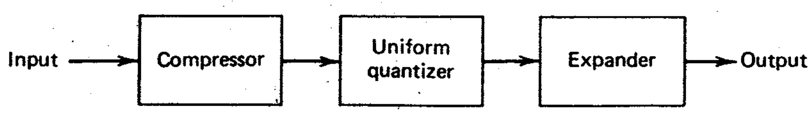
∴ To Keep Signal to quantization noise ratio high, we must use a Signal which is large in Comparision with StepSize. This requirement is not Satisfied when Signal is Small i.e. we need Smaller Step Size for low magnitude Signal Samples & higher Step-Size for higher magnitude Signals.

* Changing StepSize according to Signal magnitude is not preferable one. Instead, Change the Characteristics of the Signal Such that lower amplitudes are amplified without changing the maximum value of the Signal.

* Maintaining of Constant SNR throughout the Signal range is called "Robust quantization"

* To achieve this, the Signal is passed through a Combination of Compressor - Expander Circuit respectively at the transmitter & receiver.

This technique is Known as COMPANDING.



Model of nonuniform quantizer.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

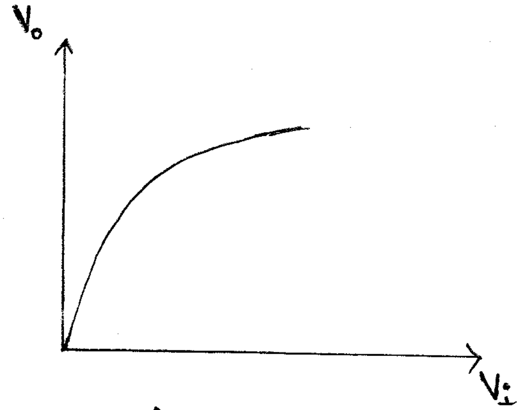


fig ⓐ

Compressor Transfer Characteristics

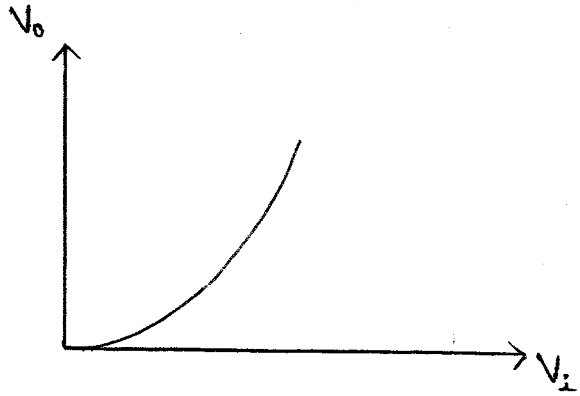


fig ⓑ

Expander Transfer characteristics

* To achieve Robust quantization, the Signal is passed through a a network which has an I/p - o/p Characteristics as Shown in fig ⓐ & ⓑ.

* The Signal is Changed Such that Small amplitude Signals are boosted up without altering the maximum amplitude of the Signal, Small amplitude Signals range through more quantization levels.

* Any Signal when passed through Such a network gets Compressed, leading to Signal distortion.

* To remove this distortion, the Signal is passed through an Inverse Network at the receiver Called as Expander.

* The Complete process of Compressing & expanding the Signal is referred to as <u>COMPANDING.</u>

There are two types of Companding

1) μ-law Companding
2) A-law Companding.

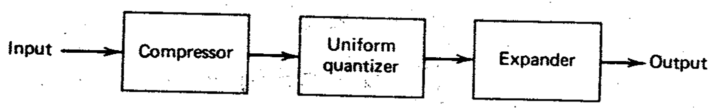
* Explain the Need for Non-uniform quantization. Also explain μ-law & A-law Companding. June-09, 8M

* In uniform quantization, the Step Size will be Constant. Hence Small amplitude Samples are more affected than the bigger Sample values. Thus Non-uniform quantization is used.

* In Non-uniform quantization for Smaller amplitude Samples Step Size is Small & for larger amplitude Samples Step Size is large. Hence Step Size varies.

Thus in Non-uniform quantization SNR remains Constant for a wide range of I/p power levels.

* Non uniform quantization is equivalent to passing the baseband Signal through a Compressor & then applying the Compressed Signal to a uniform quantizer. The resultant Signal is then transmitted.



Model of nonuniform quantizer.

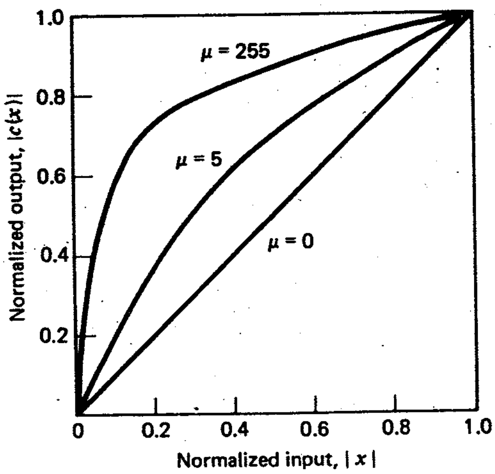
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* At the receiver, a Inverse network called <u>Expander</u> is used to restore the signal samples to their correct relative level.

* The complete process of compressing & expanding the signal is referred to as <u>COMPANDING.</u>

There are two types of Companding

1) μ-law Companding
2) A-law Companding.



(a)

1) μ-law Companding :-

* μ-law is defined by following equation

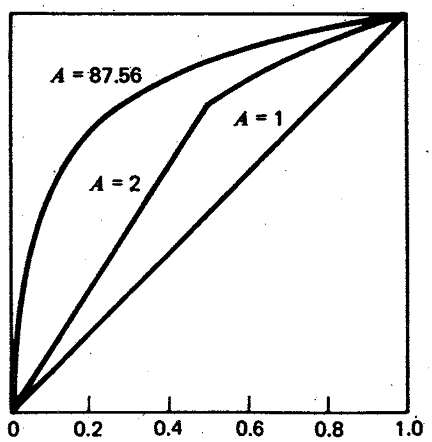
$$\frac{c|x|}{x_{max}} = \frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \qquad 0 \leq \frac{|x|}{x_{max}} \leq 1$$

* μ-law companding is a type of companding used in United States, Canada & Japan.
* When μ=0, compression corresponds to uniform quantization.
* In μ-law companding, the compression characteristic is continuous.

2) A-law Companding :-

* A-law is defined by following equation

$$\frac{c(|x|)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \leqslant \dfrac{|x|}{x_{max}} \leqslant \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|x|/x_{max})}{1+\ln A} & \dfrac{1}{A} \leqslant \dfrac{|x|}{x_{max}} \leqslant 1 \end{cases}$$



(b)

* A-law is used in India & Europe.
* For digital telephony $A = 87.6$ is the Standard value used in europe.
* The Compressor Characteristic is piece wise.

Advantages of Non-uniform quantizer:-

1) Reduced quantization Noise
2) High average SNR

---

* ST for large values of $\mu = A$, the μ-law & A-law have the Same companding gain G_c

$\boxed{\text{Aug-2004, 8M}}$

Sol:-

* The Companding gain 'G_c' is given by

$$G_c = \left.\frac{d\,c(x)}{dx}\right|_{x\to 0} \longrightarrow ①$$

Where $c(x)$ is the compressor Characteristics.

WKT $c(x)$ for μ-law is $c(x) = \dfrac{\ln(1+\mu x)}{\ln(1+\mu)} \longrightarrow ②$

Substituting eq ② in eq ①, we get

$$G_c = \frac{d}{dx}\left.\left(\frac{\ln(1+\mu x)}{\ln(1+\mu)}\right)\right|_{x\to 0}$$

$$= \frac{1}{\ln(1+\mu)}\frac{d}{dx}\left(\ln(1+\mu x)\right)_{x\to 0}$$

$$= \frac{1}{\ln(1+\mu)}\left(\frac{\mu}{1+\mu x}\right)_{x\to 0}$$

WKT

$$\frac{d}{dx}\ln(x) = \frac{1}{x}\frac{d}{dx}(x) = \frac{1}{x}$$

$\|^{ly}$

$$\frac{d}{dx}\ln(1+\mu x) = \frac{1}{1+\mu x}\frac{d}{dx}(1+\mu x)$$

$$= \frac{1}{1+\mu x}(\mu)$$

$$= \frac{\mu}{1+\mu x}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$= \frac{1}{\ln(1+\mu)} \cdot \frac{\mu}{(1+\mu(0))}$$

$$\boxed{G_c = \frac{\mu}{\ln(1+\mu)}} \longrightarrow (3)$$

* Companding gain for A-law

$$G_c = \frac{d}{dx} c(x) \longrightarrow (4)$$

WKT $c(x)$ for A-law is $\boxed{c(x) = \frac{A|x|}{1+\ln A}} \longrightarrow (5)$

Substituting eq (5) in eq (4), we get

$$G_c = \frac{d}{dx}\left[\frac{A|x|}{1+\ln A}\right]$$

$$= \frac{1}{1+\ln A} \frac{d}{dx} (A|x|)$$

$$= \frac{1}{1+\ln A} A(1)$$

$$\boxed{G_c = \frac{A}{1+\ln A}} \longrightarrow (6)$$

For $\mu = A$, eq (3) becomes

$$G_{c(\mu\text{-law})} = \frac{A}{\ln(1+A)} \longrightarrow (7) \ \&$$

$$G_{c(A\text{-law})} = \frac{A}{1+\ln A} \longrightarrow (8)$$

Comparing eq (7) & (8) for large values of A & μ

$$G_{c(\mu\text{-law})} = G_{c(A\text{-law})}$$

Downloaded from www.citystudentsgroup.blogspot.com
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

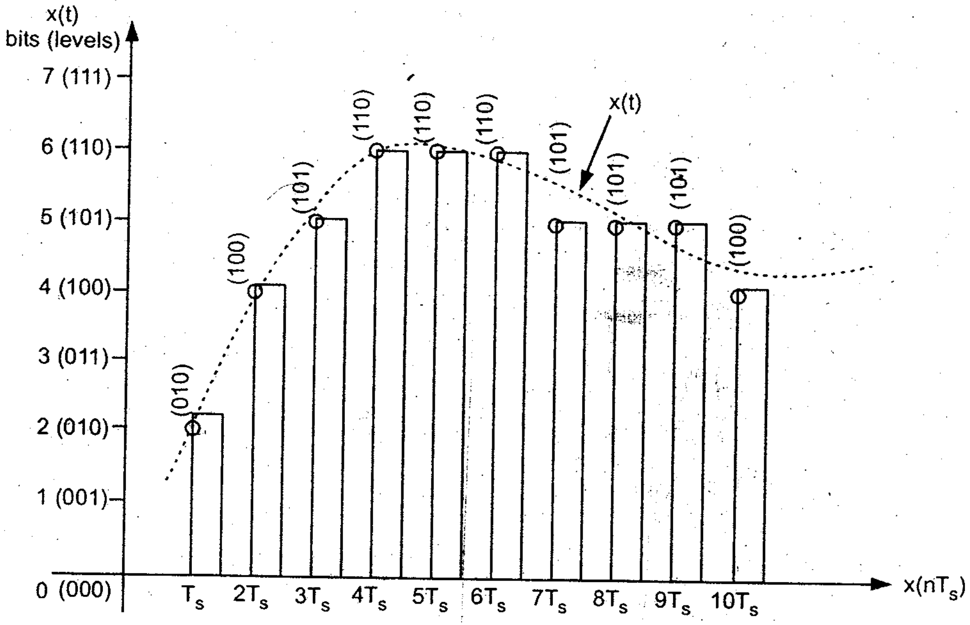
$$\frac{\cancel{A}}{\ln(1+A)} = \frac{\cancel{A}}{1+\ln A}$$

$$\boxed{1+\ln A \simeq \ln(1+A)}$$

Hence proved.

Differential pulse Code Modulation :-

(Basic) → For understanding DPCM.



Redundant information in PCM

* In PCM System, each Sample of waveform is encoded independ-tly of other Samples.
* The Samples of Signals are highly Correlated this is because
  i) Any Signal does not Change fast.
  ii) We are taking the Samples above Nyquist rate i.e. $f_s > 2W$.

Thus the Signal doesnot change rapidly from one Sample to the next Sample.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* When these highly correlated samples are encoded, the resulting encoded signal contains Redundant information.

* The redundancy can be eliminated by using DPCM.

Fig① shows the continuous time signal $x(t)$ sampled using flat-top sampling. This signal is sampled at the instants $T_s, 2T_s, 3T_s, \ldots, nT_s$. The sampling frequency is selected higher than the Nyquist rate & encoded using 3-bit encoder. The samples quantized to nearest level is shown in fig①.

* We can see that samples taken at $4T_s, 5T_s, 6T_s$ are encoded to same value "110". This information can be carried by one sample. But three samples are carrying the same information means it is Redundant.

* If this redundancy is reduced, then overall bitrate will decrease & number of bits required to transmit one sample is also reduced. This type of digital pulse modulation scheme is known as DPCM.

DPCM :- $\boxed{\text{Jan-08, 6M}}$

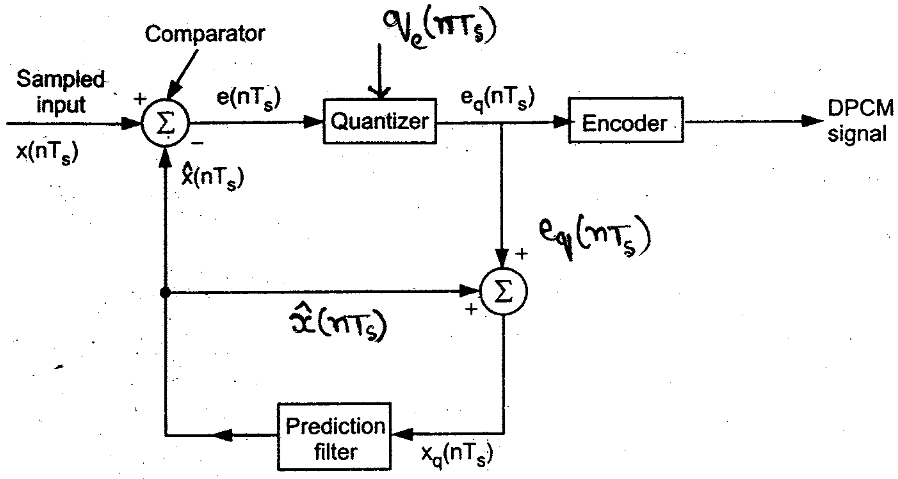
* When an analog signal is sampled at a rate slightly higher than the Nyquist rate, resulting high correlated samples i.e. the signal does not change rapidly from one sample to next.
* If these highly correlated samples are encoded, the resulting encoded signal contains redundant information.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* If we remove this redundancy before encoding, efficiency of the coded signal can be increased.

* DPCM works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value.

DPCM Transmitter :-



Differential pulse code modulation transmitter

From fig ①, $x(nT_s)$ represents the sampled version of the analog signal $x(t)$.

* The o/p of the comparator is the difference between the unquantized sampled I/p & prediction of its $\hat{x}(nT_s)$

i.e. $\boxed{e(nT_s) = x(nT_s) - \hat{x}(nT_s)} \longrightarrow (1)$

Where $\hat{x}(nT_s)$ is the prediction of $x(nT_s)$.

* The prediction error $e(nT_s)$ is the quantized to produce $e_q(nT_s)$. In the quantizer the noise $q_e(nT_s)$ gets added.

∴ The o/p of quantizer can be written as :

$$\boxed{e_q(nT_s) = e(nT_s) + q_e(nT_s)} \longrightarrow ②$$

* From fig ①, the I/p to the prediction filter may be written as :

$$x_q(nT_s) = \hat{x}(nT_s) + \underline{e_q(nT_s)} \longrightarrow ③$$

Substituting eq ② in eq ③, we get

$$x_q(nT_s) = \hat{x}(nT_s) + \underline{e(nT_s)} + q_e(nT_s) \longrightarrow ④$$

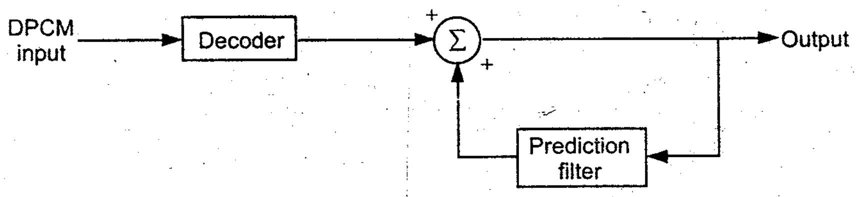
Substituting eq ① in eq ④, we get

$$x_q(nT_s) = \hat{x}(\cancel{n}T_s) + x(nT_s) - \hat{x}(\cancel{n}T_s) + q_e(nT_s)$$

$$\boxed{x_q(nT_s) = x(nT_s) + q_e(nT_s)} \longrightarrow ⑤$$

Where $x_q(nT_s)$ is the quantized version of $x(nT_s)$.

DPCM Receiver :-



DPCM receiver

* The decoder 1st reconstruct the quantized error signal from incoming binary signal.

* The prediction filter o/p & quantized error signals are summed up to give the quantized version of the original signal.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Thus the Signal at the receiver differs from actual Signal by quantization error $q_e(nT_s)$, which is introduced permanently in the reconstructed Signal.

Prediction gain :-

* The o/p Signal to noise ratio of a DPCM System is

$$\boxed{(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2}} \longrightarrow (1)$$

Where, σ_x^2 is the variance of the original I/p $x(nT_s)$, assumed to be of Zero mean, & σ_Q^2 is the variance of the quantization error $q(nT_s)$.

Eq (1) can be rewritten as

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2} \cdot \frac{\sigma_E^2}{\sigma_E^2}$$

$$= \frac{\sigma_x^2}{\sigma_E^2} \cdot \frac{\sigma_E^2}{\sigma_Q^2}$$

$$\boxed{(SNR)_o = G_p \cdot (SNR)_p}$$

Where $(SNR)_p$ is the prediction error to quantization noise ratio defined by $\boxed{(SNR)_p = \frac{\sigma_E^2}{\sigma_Q^2}}$ &

G_p is the prediction gain defined by

$$\boxed{G_p = \frac{\sigma_x^2}{\sigma_E^2}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Delta Modulation :-

Jan-06, 8M	Jan-07, 10M

Delta modulation transmits only one bit per sample i.e. the present sample value is compared with the previous sample value & the indication, whether the amplitude is increased or decreased is sent.

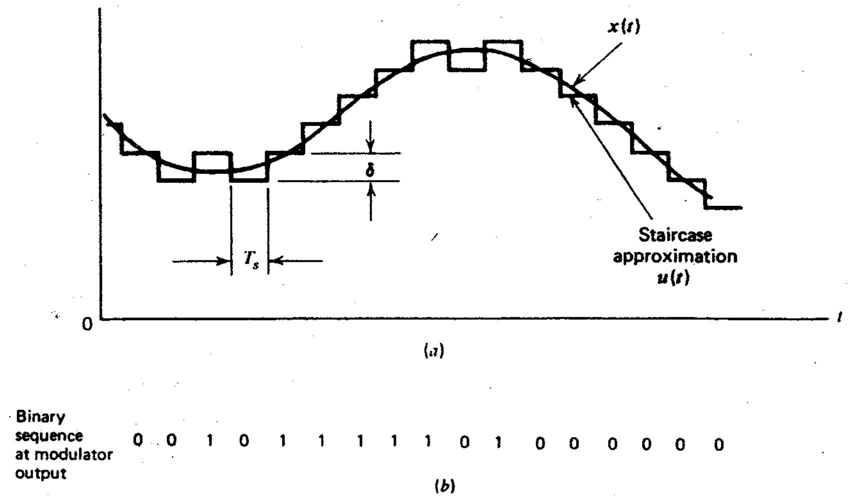


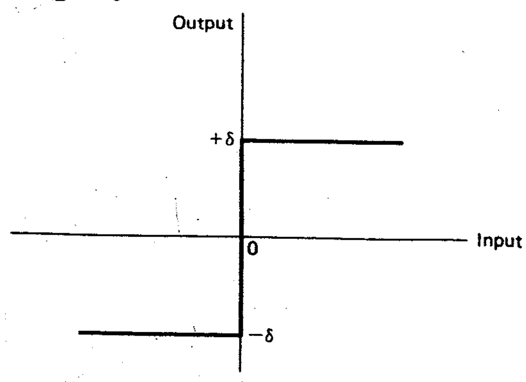
Illustration of delta modulation.

* The I/p signal $x(t)$ is approximated to step signal by the delta modulator. The difference between I/p signal $x(t)$ & staircase approximated signal is quantized into only two levels i.e. $+\delta$ or $-\delta$.

* If the difference is +ve, then approximated signal is increased by one step i.e. $+\delta$ & bit 1 is transmitted.

* If the difference is -ve, then approximated signal is reduced by one step i.e. $-\delta$ & bit 0 is transmitted.

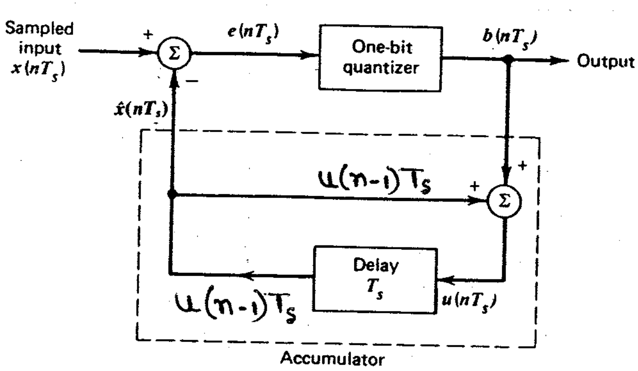
* Thus for each sample only one-bit is transmitted.



Input–output characteristic of two-level quantizer.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

DM Transmitter :-



* The error between the Sampled value $x(nT_s)$ & last approximated Sample is given by

$$\boxed{e(nT_s) = x(nT_s) - \hat{x}(nT_s)} \longrightarrow (1)$$

* Let $u(nT_s)$ be the present Sample approximation of Staircase o/p.

From Fig (3) : $\hat{x}(nT_s) = u(n-1)T_s$

$$\boxed{\hat{x}(nT_s) = u(nT_s - T_s)} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$\boxed{e(nT_s) = x(nT_s) - u(nT_s - T_s)} \longrightarrow (3)$$

* The binary quantity $b(nT_s)$ is the algebraic Sign of the error $e(nT_s)$, except for the Scaling factor δ.

i.e. $$\boxed{b(nT_s) = \delta \, \mathrm{Sgn}[e(nT_s)]} \longrightarrow (4)$$

$b(nT_s)$ depends on the Sign of error $e(nT_s)$, the Sign of Step-Size 'δ'

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

will be decided

i.e. $b(nT_s) = +\delta$, if $x(nT_s) \geq \hat{x}(nT_s)$

$b(nT_s) = -\delta$, if $x(nT_s) \leq \hat{x}(nT_s)$

* If $b(nT_s) = +\delta$, then binary '1' is transmitted

If $b(nT_s) = -\delta$, then binary '0' is transmitted.

$$\therefore \quad \boxed{u(nT_s) = u[nT_s - T_s] + b(nT_s)}$$

* The previous Sample approximation $u[nT_s - T_s]$ is restored by delaying one Sample period 'T_s'.

DM Receiver:-

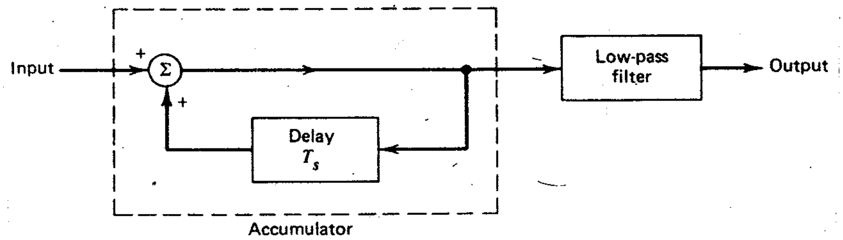


Fig Shows the block diagram of DM receiver.

* The accumulator generates the Staircase approximated Signal o/p & is delayed by one Sampling period 'T_s'. It is then added to the I/p Signal.
* If I/p is binary '1' then it adds $+\delta$ Step to the previous o/p
* If I/p is binary '0' then one Step 'δ' is Subtracted from the delayed Signal.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* The LPF is used to remove step variation & to get smooth reconstructed message signal $x(t)$.

Advantages of DM :-

The DM has following advantages over PCM

1) DM transmits only one bit for one sample. Thus the signalling rate & transmission channel bandwidth is quite small for DM.

2) Simplicity of design for both the transmitter & the receiver.

3) A one-bit code word for the o/p, which eliminates the need for word framing.

Drawbacks of DM :-

DM systems are subjected to two types of quantization error:

1) Slope overload distortion

2) Granular Noise.

* Explain disadvantages of DM

July-04, 6M	July-05, 5M

DM systems are subjected to two types of quantization error

1) Slope-overload distortion

2) Granular Noise

P.T.O

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

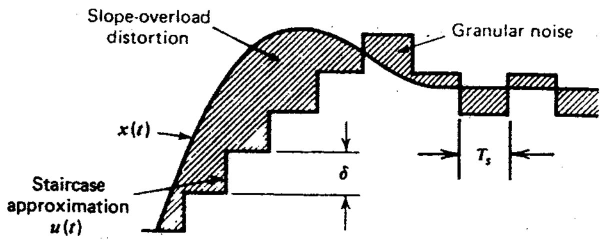


Illustration of quantization error in delta modulation.

1) Slope overload distortion :-

* Slope overload distortion arises because of the large dynamic range of the I/p Signal.

* In fig ① it can be seen, the rate of rise of I/p Signal $x(t)$ is so high that the Staircase Signal cannot approximate it, the Step Size 'δ' becomes too Small for Staircase Signal $x(t)$ to follow the Steep Segment of $x(t)$. Thus large error between the Staircase approximated Signal & the original I/p Signal $x(t)$. This error is called Slope overload distortion.

* To reduce this error, the Step-Size Should be increased when Slope of the Signal $x(t)$ is high.

i.e. Slope of the Staircase $u(t) \geq$ Slope of the message Signal.

$$\boxed{\frac{\delta}{T_s} \geq \max \left| \frac{d}{dt}[x(t)] \right|}$$

Granular Noise :-

* This noise occurs when the Step Size is too large Compared to Small variations in the I/p Signal i.e. for very Small variations in the I/p Signal, the Staircase Signal is Changed by large

amount because of large Step Size 'δ'.

* In fig ①, I/p Signal is almost flat, the Staircase Signal $u(t)$ Keeps on oscillating by $\pm\delta$ around the Signal.

* The error between the I/p & approximated Signal is called Granular Noise. The Solution of this problem is to make Step Size Small.

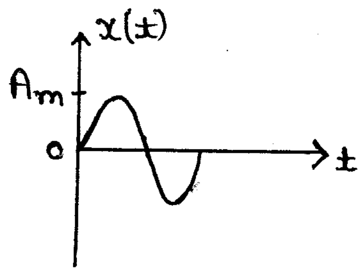
* Consider a Sinewave of frequency f_m & amplitude A_m applied to a delta modulator of Step Size δ. S.T the Slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

Where 'T_s' is the Sampling period.

Dec-04, 4M

Sol:-



* Let the message Signal be represented as $x(t) = A_m \sin(2\pi f_m t)$

* Consider the uniform DM with Step Size δ & Sampling interval T_s.

$\therefore$ The Slope of DM $= \dfrac{\text{Step Size}}{\text{Sampling period}}$

$$= \frac{\delta}{T_s}$$

* Slope overload distortion will take place if Slope of Sine wave is

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

greater than Slope of delta modulator i.e.

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\partial}{T_s}$$

$$\max\left|\frac{d}{dt} A_m \sin(2\pi f_m t)\right| > \frac{\partial}{T_s}$$

$$\max\left| A_m \cos(2\pi f_m t)\, 2\pi f_m \right| > \frac{\partial}{T_s}$$

Slope will be maximum When

$\cos 2\pi f_m t = 1$ i.e. at $t=0$

$$A_m (1)\, 2\pi f_m > \frac{\partial}{T_s}$$

$$\therefore \boxed{A_m > \frac{\partial}{2\pi f_m T_s}}$$

---

* Derive an expression for Signal to Quantization noise power ratio for delta modulation. Assume that no Slope overload distortion exists

July-02, 8M	July-08, 6M

OR

* Prove that the maximum Signal to Noise ratio of a DM System is given by

$$(SNR)_o = \frac{3}{8\pi^2 f_o W T_s^3}$$

Where W is the Cut-off frequency of LPF in the DM receiver.

OR

* For the Sinusoidal modulating Signal $x(t) = a_0 \cos 2\pi f_0 t$, ST the o/p Signal to Quantization noise ratio in a DM System under the assumption of no Slope overload is given by

$$(SNR)_o = \frac{3f_s^3}{8\pi^2 f_o^2 f_M}$$

Where $f_s = 1/T_s$ Sampling frequency &

f_M = Cut-off frequency of the LPF in the receiver.

Sol :- Given:

Signal power $x(t) = a_0 \cos(2\pi f_0 t)$

* The maximum Slope of the Signal $x(t)$ is given by

$$\left|\frac{d\,x(t)}{dt}\right|_{max} = \left|\frac{d}{dt}\, a_0 \cos(2\pi f_0 t)\right|_{max}$$

$$= \left|-a_0\, 2\pi f_0 \sin 2\pi f_0 t\right|_{max}$$

$$\boxed{\left|\frac{d\,x(t)}{dt}\right|_{max} = a_0 2\pi f_0} \longrightarrow ①$$

$\because \sin 2\pi f_0 t = 1$ at $t = 90°$

* The condition for no Slope overload is

$$\frac{\partial}{T_s} \geqslant \left|\frac{d[x(t)]}{dt}\right|_{max} \longrightarrow ②$$

Substituting eq ① in eq ②, we get

$$\frac{\partial}{T_s} \geqslant a_0 2\pi f_0$$

$$a_0 2\pi f_0 \leqslant \frac{\partial}{T_s}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$a_0 = \frac{\partial}{2\pi f_0 T_s}$$

$$\boxed{a_0 = \frac{\partial f_s}{2\pi f_0}} \longrightarrow (3)$$

* The maximum average power of the Signal $x(t)$ is given by

$$P_{max} = \frac{V^2}{R}$$

Here 'V' is the r.m.s value of the Signal. $\therefore\ V = \frac{a_0}{\sqrt{2}}$

* Normalized Signal power is obtained by taking R=1

$$P_{max} = \left(\frac{a_0}{\sqrt{2}}\right)^2 \Big/ 1$$

$$P_{max} = \frac{a_0^2}{2} \longrightarrow (4)$$

Substituting eq (3) in eq (4), we get

$$P_{max} = \frac{\partial^2 f_s^2}{4\pi^2 f_0^2} \times \frac{1}{2}$$

$$\boxed{P_{max} = \frac{\partial^2 f_s^2}{8\pi^2 f_0^2}} \longrightarrow (5)$$

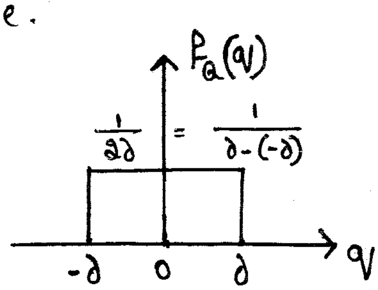
Eq (5) is the Signal power in delta modulation.

Noise power:

When there is no Slope overload, the maximum quantization error is $\pm \partial$. Assuming that the quantization error is uniformly distributed.

* Consider the PDF of the quantization error i.e.

$$f_Q(q) = \begin{cases} \frac{1}{2\delta} & \text{for } -\delta \le q \le \delta \\ 0 & \text{otherwise} \end{cases}$$



* The variance of Quantization error is

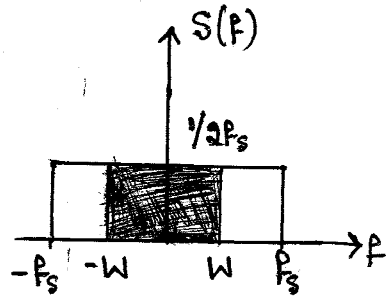
$$\sigma_Q^2 = \int_{-\delta}^{\delta} q^2 \cdot f_Q(q) \cdot dq$$

$$= \int_{-\delta}^{\delta} q^2 \cdot \frac{1}{2\delta} \, dq = \frac{1}{2\delta} \int_{-\delta}^{\delta} q^2 \, dq$$

$$= \frac{1}{2\delta} \left[\frac{q^3}{3} \right]_{-\delta}^{\delta} = \frac{1}{2\delta} \left[\frac{\delta^3}{3} - \frac{(-\delta)^3}{3} \right]$$

$$= \frac{1}{2\delta} \left[\frac{\delta^3}{3} + \frac{\delta^3}{3} \right] = \frac{1}{\cancel{2\delta}} \cancel{2} \left(\frac{\delta^{\cancel{3}\,2}}{3} \right)$$

$$\boxed{\sigma_Q^2 = \frac{\delta^2}{3}} \longrightarrow (7)$$



* The DM receiver contains a LPF whose cut-off frequency $f_c = W$ Hz.

* Assuming that the average power of the quantization error is uniformly distributed over a frequency interval extending from $-1/T_s$ to $+1/T_s$, we get

Average o/p Noise power 'N_o' $= \left(\frac{f_c}{f_s}\right) \frac{\delta^2}{3} = WT_s \left(\frac{\delta^2}{3}\right) \longrightarrow (8)$

* Signal to Noise power ratio at the o/p of DM receiver is

$$(SNR)_o = \frac{P_{max}}{N_o}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$= \frac{\delta^2 f_s^2 / 8\pi^2 f_0^2}{W\delta^2 T_s / 3}$$

$$= \frac{\cancel{\delta^2} f_s^2}{8\pi^2 f_0^2} \times \frac{3}{W \cancel{\delta^2} T_s}$$

$$= \frac{3}{8\pi^2 f_0^2 T_s^2 \cdot W T_s}$$

$$\boxed{(SNR)_0 = \frac{3}{8\pi^2 f_0^2 W T_s^3}} \longrightarrow (9)$$

Eq (9) Shows that, under the assumption of no-Slope overload distortion, the maximum o/p Signal to Noise ratio of a delta-modulator is proportional to the Sampling rate <u>Cubed</u>.

---

* Consider the test Signal $m(t)$ defined by a hyperbolic tangent function $m(t) = A\tanh(\beta t)$ where A & β are Constants. Determine the minimum Step Size 'δ' for delta modulation of this Signal, which is required to avoid Slope overload.

July-01, 7M

<u>Sol:-</u> The given Signal is

$$m(t) = A\tanh(\beta t)$$

To avoid Slope overload

$$\frac{\delta}{T_s} \geq \max\left|\frac{d}{dt} m(t)\right|$$

$$\geq \max\left|\frac{d}{dt} A\tanh(\beta t)\right|$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\geq \max \left| A \cdot \frac{\beta}{\cosh^2(\beta t)} \right|$$

$$\frac{\delta}{T_s} \geq A\beta \max \left| \frac{1}{\cosh^2(\beta t)} \right|$$

Maximum value of $\frac{1}{\cosh^2(\beta t)}$ is 1. Hence above equation becomes,

$$\frac{\delta}{T_s} \geq A\beta$$

$$\delta \geq A\beta T_s$$

Thus the minimum value of Step Size is

$$\boxed{\delta_{min} = A\beta T_s}$$

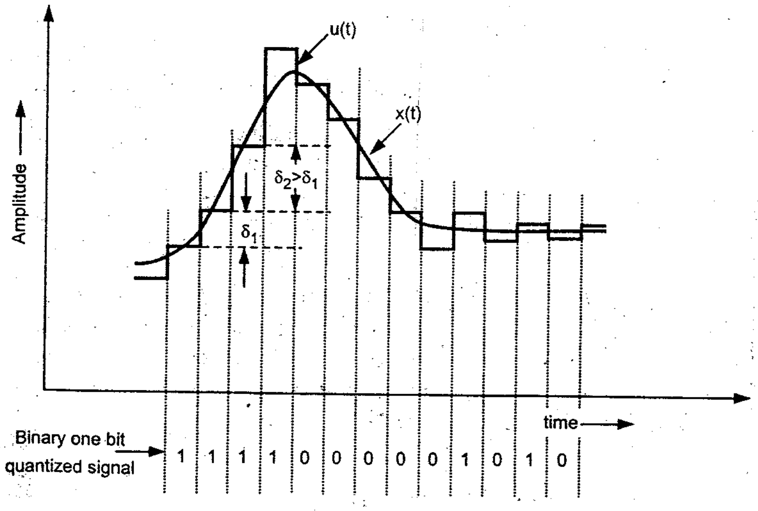
To avoid Slope overload.

Adaptive Delta Modulation :-

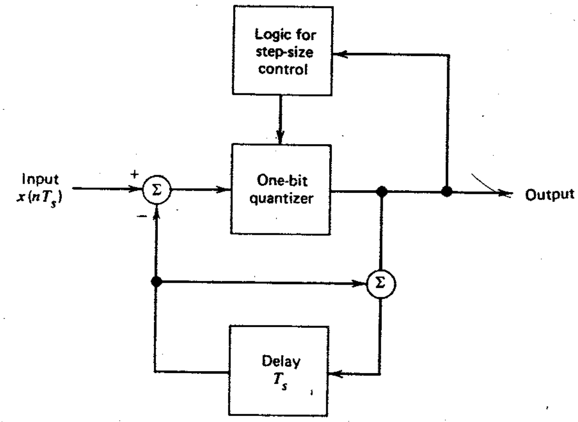
* To overcome Slope overload distortion & Granular distortion, Step Size 'δ' is made adaptive to variations in the I/p Signal $x(t)$.
* The Step Size is increased for the Steep Segment of $x(t)$ & Step Size is decreased for Slowly varying Segment.

This method is called Adaptive delta modulation.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.



Waveforms of adaptive delta modulation



* In practical adaptive DM the Step size $\delta(nT_s)$ is constrained to lie between maximum & minimum values.

i.e.

$$\boxed{\delta_{min} \leq \delta(nT_s) \leq \delta_{max}}$$

* The upper limit δ_{max}, controls the amount of slope overload distortion

* The lower limit δ_{min}, controls the amount of Granular noise.

* Inside these limits the adaptive algorithm rule for $\delta(nT_s)$ expressed in general form

$$\boxed{\delta(nT_s) = g(nT_s)\,\delta(nT_s - T_s)}$$

Where the time varying function $g(nT_s)$ depends on the present & previous binary o/p of the delta modulator.

* The adaptive algorithm is initiated with a starting step

$$\boxed{\delta_{start} = \delta_{min}}$$

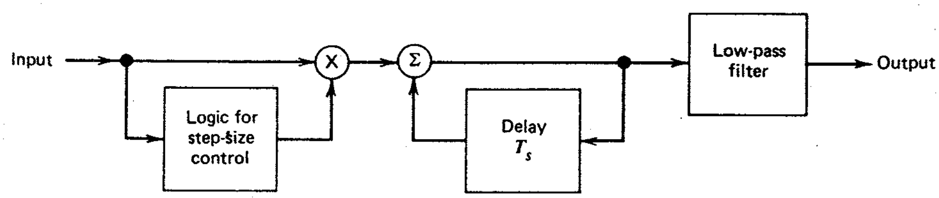
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* The function $g(nT_s)$ is given by

$$g(nT_s) = \begin{cases} K & \text{if } b(nT_s) = b(nT_s - T_s) \\ K^{-1} & \text{if } b(nT_s) \neq b(nT_s - T_s) \end{cases}$$

* This adaptive algorithm is called a Constant factor ADM with one-bit memory.
* The ADM is also known as Continuous variable Slope DM.

ADM receiver :



* In the receiver the 1st part generates Step Size from each incoming bit which is variable in Size. The previous I/p & present I/p decides the Step Size.
* The LPF then Smoothens out the Staircase waveform to reconstruct the Smooth Signal.

Advantages of ADM :-

1) The Signal to Noise ratio is better than ordinary delta modulation because of the reduction in the Slope overload distortion & granular noise.
2) Utilization of bandwidth is better than Delta modulation.

Comparison of Digital pulse Modulation Methods :-

Sr. No.	Parameter	PCM	Differential Pulse Code Modulation (DPCM)	Delta modulation (DM)	Adaptive Delta Modulation (ADM)
1.	Number of Bits	It can use 4, 8 or 16 bits per sample.	Bits can be more than one but are less than PCM.	It uses only one bit for one sample.	Only one bit is used to encode one sample.
2	Levels, step size	The number of levels depend on number of bits. Level size is fixed.	Fixed number of levels are used.	Step size is fixed and cannot be varied.	According to the signal variation, step size varies (Adapted).
3	Quantization error and Distortion	Quantization error depends on number of levels used.	Slope overload distortion and quantization noise is present.	Slope overload distortion and granular noise is present.	Quantization error is present but other errors are absent.
4	Bandwidth of trans-mission channel	Highest bandwidth is required since number of bits are high.	Bandwidth required is lower than PCM.	Lowest bandwidth is required.	Lowest bandwidth is required.
5	Feedback.	There is no feedback in transmitter or receiver.	Feedback exists.	Feedback exists in transmitter.	Feedback exists.
6	System Complexity	System is complex.	Simple.	Simple.	Simple.
7	SNR	Good	Fair	Poor	Good
8	Applications	Audio & video telephony	Speech & video	Speech & Images	Speech & Images.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Requires the use of MODEM's in order to convert the digital format into an analog format suitable for transmission over telephone channels.

2) Designed to operate at much higher bit rates, for example T_1 carrier system.

1) Digital Hierarchy (T_1 to T_4 carrier system) :-

or

July-08, 5M	Jan-10, 5M

2) TDM PCM Telephony

or

3) Digital Multiplex T_1.

4) T_1 carrier system or T_1 system

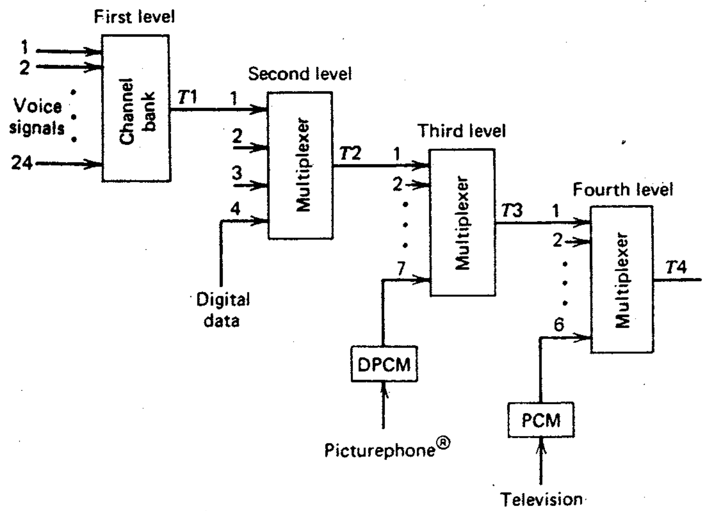


Fig ① Digital hierarchy, Bell system.

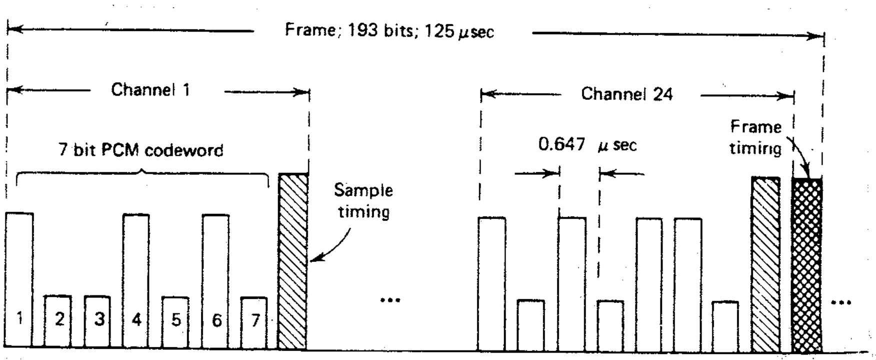
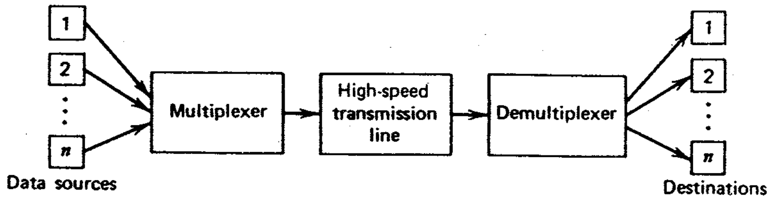


Fig ②: T1 - frame

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Digital Multiplexers :-



Conceptual diagram of multiplexing–demultiplexing.

* Multiplexing of digital Signals is also possible when the data Sources are operating at different bit rate.
* Multiplexing is done to Combine Several digital Signals Such as Computer o/p, digitized telephone Signals, digitized TV Signals etc into a Single data Streams.
* The multiplexing of digital Signals may be accomplished by using a bit by bit interleaving procedure with a Selector Switch that Sequentially takes a bit from each incoming line & then applies to the high-Speed Common line.
* At the receiving end of the System the o/p of this common line is Separated out into its low-Speed individual Components & then deliver to their respective destinations.

Two major groups of digital multiplexers are used in practice:

1)* one group of multiplexers is designed to Combine relatively low Speed digital Signals, upto a maximum rate of 4800 bps into a higher Speed multiplexed Signal with a rate of upto 9600 bps

* Used to transmit data over voice-grade Channels of a telephone Network.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* The T_1 - Carrier System is designed to accomodate 24 voice Channels, primarily for Short - distance, heavy usage in metropolitan areas.

* This System was developed by the Bell System in the united States in the early 1960's. The T_1 System has been adopted for use throughout the united States, Canada & Japan.

* voice Signal is bandlimited to 300Hz - 3400Hz even through the Nyquist rate is 6.8KHz, the Standard Sampling rate used in digital telephony is 8000 Samples/Sec.

* So 24 voice Signals are Sampled at a rate of 8KHz & the resulting Samples are quantized & converted into 7-bit PCM Codewords.

* At the end of each 7-bit Codeword, an additional binary bit is added for Synchronizing purpose. At the end of every group of twenty four 8-bit Codewords, another additional bit is inserted to give frame Synchronization.

* The overall frame Size in the T_1 - Carrier is

$$T_1 = [(24 \text{ voice Signals} \times 8\text{-bit}) + 1\text{-bit}] \text{ bits/frame}$$

$$T_1 = (192 + 1) \text{ bits/frame}$$

$$\boxed{T_1 = 193 \text{ bits/frame}}$$

These frames are transmitted at 8KHz rate

$\therefore$ Bit rate = 193 bits/frame × 8000 frames/Sec

$$\boxed{\text{Bit rate} = 1.544 \text{ Mbps}}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

* Sampling period $T_s = 1/f_s = 1/8000 = \underline{125\ \mu sec}$

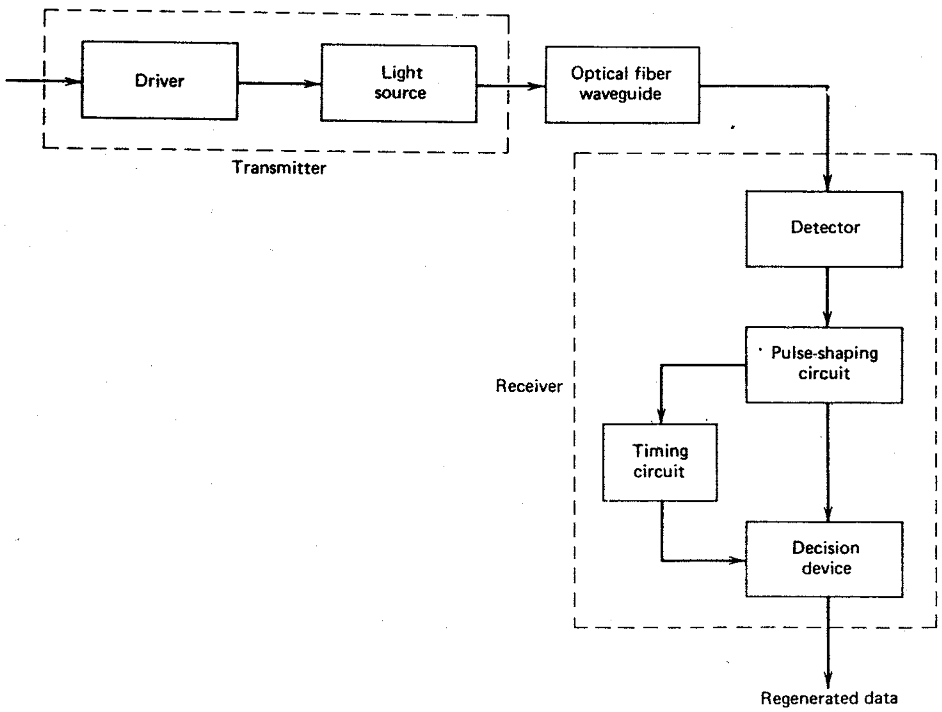
* For 193 bits, time period is $\underline{125\ \mu sec}$

$\therefore$ Duration of each bit $= \frac{125\ \mu sec}{193} = \underline{0.647\ \mu sec}$

* The maximum length of the T_1 System is now limited to <u>50 to 100 miles</u> with a repeater Spacing of <u>1-Mile</u>.

* The overall T-carrier System is designed for accomodating voice Channels, picture phone Service, TV Signals & digital data.

Lightwave Transmission :-



Optical fiber link.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* optical fiber waveguides has low transmission losses & high bandwidth, which are important for long distance, high-Speed communication.

* The other advantages are Small Size, light weight & immunity to electromagnetic interference.

* The optical fiber link consisting of a transmitter, an optical fiber waveguide, & a receiver.

* The binary data is fed into the transmitter, where it emits pulses of optical power, with each pulse being ON or OFF in accordance with the I/p data.

* Light Source used may be :
  1) Laser diode or
  2) LED

* The driver for the light Source, Consists of a high current - low voltage device.

* The light Source is thus turned ON & OFF by Switching the driver current ON & OFF in a corresponding manner.

* The ON-OFF light pulses produced by the transmitter are launched into the optical fiber waveguide.

* The Source to fiber Coupling loss (occurs) takes place while Coupling light Signal into optical fiber that varies over a wide range, depending on the particular Combination of light Source & optical fiber Selected.

* While propagating along the fiber, a light pulse Suffers an additional loss or attenuation that increases exponentially with

distance called fiber loss.

* Another important phenomenon that occurs during propagation is dispersion, which causes light originally concentrated into a short pulse to spread out into a broader pulse as it propagates along the optical fiber waveguide.

At the receiver, the original I/p data are regenerated by performing three basic operations:

1) Detection:- The light pulses impinging on the receiver I/p are converted back into pulses of electrical current.

2) Pulse Shaping & timing circuit:- This involves amplification, filtering & equalization of the electrical pulses as well as the extraction of timing information

3) Decision Making device:- This device decides the received pulse as ON or OFF with reference to threshold voltage.

The detector used may be photodiode or PIN diode.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) Prove that the Shannon's Channel Capacity theorem can be easily achieved by increasing the average power in PCM System by $K^2/12$

July-07, 6M

2) Explain a M-ary PCM System interms of transmitted power (P), Noise variance $(\sigma_N)^2$ & bit rate (R_b). Specify the assumptions when R_b is compared with channel capacity 'C' for an Ideal System.

June-09, 6M

Sol :- M-ary PCM System :-

* M-levels in powers of 2
* All M-levels are equiprobable
* The average transmitted power will be least if the amplitude range is Symmetrical about Zero.
* Discrete amplitude levels, normalized w.r.to Separation

$$K\sigma_N = \pm 1/2,\ \pm 3/2,\ \pm \cdots \pm (M-1)/2.$$

* The average transmitted power is given by

$$\boxed{P = K^2\sigma_N^2\left(\frac{M^2-1}{12}\right)} \longrightarrow (1)$$

re-arrange the eq (1)

$$\frac{P}{K^2\sigma_N^2} = \frac{M^2-1}{12}$$

$$M^2 = 1 + \frac{12P}{(K\sigma_N)^2}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$M = \left[1 + \frac{12p}{(K\sigma_N)^2}\right]^{1/2} \longrightarrow (2)$$

* The rms Noise voltage is equal to σ_N. Hence Normalized noise power

$$N = \frac{\sigma_N^2}{R} = \frac{\sigma_N^2}{1} \qquad R = 1$$

$$\boxed{N = \sigma_N^2} \longrightarrow (3)$$

Substituting eq (3) in eq (2), we get

$$M = \left[1 + \frac{12p}{K^2 N}\right]^{1/2}$$

Now Substitute $N = N_0 B$

$$M = \left[1 + \frac{12p}{K^2 N_0 B}\right]^{1/2} \longrightarrow (4)$$

* The maximum rate of Information transmission of the PCM System is given by

$$R_b = 2W \log_2 L \quad \text{bits/Sec}$$

WKT $\boxed{L = M^n}$

$$R_b = 2W \log_2 M^n \quad \text{bits/Sec}$$

$$R_b = 2Wn \log_2 M \quad \text{bits/Sec} \longrightarrow (5)$$

Substituting eq (4) in eq (5), we get

$$R_b = 2Wn \log_2 \left[1 + \frac{12p}{K^2 N_0 B}\right]^{1/2}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$R_b = \cancel{2}Wn \frac{1}{\cancel{2}} \log_2\left[1 + \frac{12p}{K^2 N_o B}\right]$$

$$R_b = Wn \log_2\left[1 + \frac{12p}{K^2 N_o B}\right]$$

$$\boxed{R_b = B \log_2\left[1 + \frac{12p}{K^2 N_o B}\right]} \longrightarrow (6) \qquad \because \boxed{B = Wn}$$

$\therefore$ The ideal System described for Shannon's Channel Capacity theorem is given by

$$\boxed{R_b = C = B\log_2\left[1 + \frac{P}{N_o B}\right]} \longrightarrow (7)$$

Where 'C' is the Channel Capacity.

* Comparing eq (6) & (7), we See that they are identical if the average transmitted power in PCM System is increased by the factor $K^2/12$, Compared with the ideal System.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

PCM - FORMULAE

NOTE:

L or M or q → levels
N or v bits

1) Number of levels $L = 2^N$

2) Number of bits $N = \log_2(L)$

$$N = \frac{\log_{10}(L)}{\log_{10}(2)}$$

3) Sampling rate 'f_s' $\geq 2W$
Where 'W' is the highest frequency of message signal

4) Signalling rate or Bit transmission rate

$$r_b = Nf_s \quad \text{or} \quad r_b = N \cdot 2W$$

5) Transmission bandwidth 'B_T'

$$\boxed{B_T = \frac{1}{2} N f_s}$$

$$B_T = \frac{1}{\cancel{2}} N \cdot \cancel{2} W$$

$$\boxed{B_T = NW}$$

$\because r_b = Nf_s$

$$\boxed{B_T = \frac{1}{2} r_b}$$

6) Bit rate 'R_b' $= Nf_s$

7) Bit duration $T_b = 1/R_b$

8) Sampling frequency 'f_s' $= R_b/N$

9) Message bandwidth 'W' $= f_s/2$

10) Maximum signal to quantization ratio

$$\boxed{(SNR)_o = \frac{3P}{x^2_{max}} \cdot 2^{2N}}$$

NOTE:

x_{max} or V_{max} or g_{max}

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Where x_{max} = Maximum amplitude of message Signal.

P = Signal power

11) Signal power

$$\boxed{P = \frac{A_m^2}{2}}$$

12) Quantization Noise power or Quantization error or Normalized Noise power

$$\boxed{\sigma_Q^2 = \frac{\delta^2}{12}}$$

NOTE

$\boxed{\delta \text{ or } \Delta}$

13) Step Size

$$\delta = \frac{x_{max} - (-x_{max})}{L}$$

$$\boxed{\delta = \frac{2x_{max}}{2^N}}$$

NOTE

$\boxed{V_m^2 \text{ or } A_m^2}$

$$P = \frac{V^2}{R}$$

here V is rms value

$$V = \frac{A_m}{\sqrt{2}}$$

$$V^2 = \frac{A_m}{2}$$

$\boxed{R = 1\Omega}$ ← for Normalized power

14) Normalized Signal to Quantization Noise ratio

$$\boxed{(SNR)_{dB} = (4\cdot 8 + 6N)}$$

15) Signal to Quantization Noise ratio for <u>Sinusoidal Signal</u>

$$\boxed{(SNR)_{dB} = 1\cdot 76 + 6N}$$

16) $$(SNR)_o = \frac{P}{\sigma_Q^2} = \frac{A_m^2/2}{\Delta^2/12}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

17) Bit duration 'T_b' $= \frac{T_S}{N}$

$$\boxed{T_S = NT_b}$$

18) $(SNR)_o$ When Compression parameter 'μ' is given

$$(SNR)_o = \frac{3L^2}{[\ln(1+\mu)]^2}$$

19) Maximum quantization error for an uniform quantizer is given by $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$

$$\boxed{\delta = 2\varepsilon_{max}}$$

20) r.m.s Quantization error $\sigma_Q = \sqrt{\frac{\Delta^2}{12}}$

$$\sigma_Q = \frac{\Delta}{\sqrt{12}}$$

21) Noise power or Mean Square value of quantization error

$$\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}}$$

22) $\boxed{N = \frac{r_b}{f_s}}$ $\because \boxed{r_b = Nf_s}$

23) Nyquist rate = 2W

24) $(SNR)_{dB}$ for Non-Sinusoidal Signal (ex:- Telephone Signal)

$$\boxed{(SNR)_{dB} = 4.8 + 6N}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) A Signal of BW 3.5 KHz is Sampled & Coded by a PCM System. The Coded Signal is then transmitted over a Channel at a rate of 50 Kbps. Calculate the maximum SNR that can be obtained by this System. The I/p Signal has a peak value of 4V.

Jan-10, 6M

Sol :- Given : $W = 3.5\,KHz$

$A_m = x_{max} = 4V$, $r_b = 50\,Kbps$, $A_m = 4V$. $(SNR)_o = ?$, $N = ?$

WKT

* $f_s = 2W = 2 \times 3.5\,KHz = \underline{7\,KHz}$

* $r_b = N f_s$

$$N = \frac{r_b}{f_s} = \frac{50\,Kbps}{7\,KHz} = \underline{7.142}$$

Choose $\boxed{N = 8}$ ⟶ 2 Marks

* $$(SNR)_o = \frac{3P\,2^{2N}}{x_{max}^2}$$ ⟶ 1-Mark

$$P = \frac{A_m^2}{2} = \frac{4^2}{2} = \underline{8W}$$ ⟶ 1-Mark

$$(SNR)_o = \frac{3 \times 8 \times 2^{2 \times 8}}{(4)^2} = \underline{98304}$$

$$(SNR)_{dB} = 10 \log_{10}(98304)$$

$$\boxed{(SNR)_{dB} = 49.9\,dB}$$ ⟶ 2-Marks

NOTE:- $(SNR)_{dB} = 1.8 + 6N = 1.8 + 6 \times 8$

$$\boxed{(SNR)_{dB} = 49.8\,dB}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

2) A 10KHz Sinusoid With amplitude 1V peak is quantized to have SNR of about 45 dB. Find the number of bits required per Sample, bit rate & bandwidth of the System if Sampling frequency is twice the Nyquist rate.

Jan-10, 6M

Sol:- Given: $W = 10\,KHz$, $A_m = 1V$, $(SNR)_{dB} = 45\,dB$

$N = ?$, $r_b = ?$, $B_T = ?$

WKT $(SNR)_o = 1.8 + 6N$

$45 = 1.8 + 6N$

$6N = 45 - 1.8 = 43.2$

$N = \frac{43.2}{6} = 7.2$

Choose $\boxed{N = 8}$ → 2-Marks

* Nyquist rate $= 2W = 2 \times 10\,KHz = 20\,KHz$.

* Sampling frequency 'f_s' $= 2 \times$ Nyquist rate

$= 2 \times 20\,KHz$

$\boxed{f_s = 40\,KHz}$

* Bit rate

$r_b = Nf_s$

$= 8 \times 40\,KHz$

$\boxed{r_b = 320\,Kbps}$ → 2-Marks

* Bandwidth $B_T = \frac{1}{2} r_b = \frac{320\,Kbps}{2} = \underline{160\,KHz}$ → 2-Marks

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$B_T = \frac{1}{2} N f_s = \frac{1}{2} 8 \times 40\,KHz = \underline{160\,KHz}$$

---

3) A PCM System uses a uniform quantizer followed by a 7-bit encoder. The bit rate of the System is equal to 50×10^6 bits/Sec

i) What is the maximum message bandwidth for which the System operates Satisfactorily?

ii) Determine the o/p Signal to quantization noise ratio when a full load Sinusoidal modulating wave of frequency 1 MHz is applied to the I/p

July-07, 6M

Sol:- Given: $N = 7$-bit, $r_b = 50\times10^6$ bits/Sec. $W = ?$ $(SNR)_{dB} = ?$

i) WKT $r_b = N f_s$

$$r_b = N\,2W$$

$$W = \frac{r_b}{2N} = \frac{50\times10^6}{2\times7}$$

$$\boxed{W = 3.57\,MHz} \longrightarrow \text{3 - Marks}$$

ii) $$(SNR)_{dB} = 1.8 + 6N$$

$$= 1.8 + 6\times7$$

$$\boxed{(SNR)_{dB} = 43.8\,dB} \longrightarrow \text{3 - Marks}$$

4) A telephone Signal with bandwidth 4-KHz is digitized into an 8-bit PCM, Sampled at Nyquist rate. Calculate PCM transmission bandwidth & Signal to quantization noise ratio (SNR).

Jan-08, 4M

Sol :- Given: $W = 4\,KHz$, $N = 8$-bit

$B_T = ?$, $(SNR)_{dB} = ?$

* PCM transmission bandwidth

$$B_T = \frac{1}{2} N f_s$$

$$f_s = 2W = 2 \times 4KHz = 8KHz$$

$$B_T = \frac{1}{2} 8 \times 8 KHz$$

$$\boxed{B_T = 32KHz}$$

OR

$$B_T = NW$$

$$B_T = 8 \times 4KHz$$

$$\boxed{B_T = 32KHz}$$

* Telephone Signal is Non-Sinusoidal Signal

$$\therefore \quad (SNR)_{dB} = 4.8 + 6N$$

$$= 4.8 + 6 \times 8$$

$$\boxed{(SNR)_{dB} = 52.8\ dB}$$

5) A PCM System which employs uniform quantization & produces a binary o/p, given an I/p Signal whose amplitude varies from +4Volt to -4 volt & having average power of 40 mW. Calculate the number of bits/Sample if the required Signal to Noise ratio is 20 dB

Jan-06, 6M

Sol:- Given : $x_{max} = 4V$, $P = 40mW$, $(SNR)_{dB} = 20\,dB$, $N = ?$

* WKT $(SNR)_{dB} = 10\log_{10}(SNR)$

$$20\,dB = 10\log_{10}(SNR)$$

$$\log_{10}(SNR) = \frac{20}{10}$$

$$(SNR) = \log_{10}^{-1}(2)$$

$$\boxed{SNR = 100}$$

* WKT

$$(SNR) = \frac{3P\,2^{2N}}{x_{max}^2}$$

$$100 = \frac{3 \times 40 \times 10^{-3} \times 2^{2N}}{(4)^2}$$

$$2^{2N} = \frac{100 \times 16}{3 \times 40 \times 10^{-3}}$$

$$2^{2N} = 13333.33$$

$$2N = \log_2(13333.33)$$

$$2N = \frac{\log_{10}(13333.33)}{\log_{10}(2)}$$

$$2N = 13.70274$$

$$N = \frac{13.70274}{2}$$

$$N = 6.85$$

Choose

$$\boxed{N = 7}$$

WKT

$$2^N = L$$

$$N = \log_2(L)$$

Where

$$\log_2(L) = \frac{\log_{10}(L)}{\log_{10}(2)}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$(SNR)_{dB} = 10\log_{10}(36922.84)$$

$$\boxed{(SNR)_{dB} = 45.67 \text{ dB}}$$

7) The Signal $g(t) = 2\cos(2000\pi t) - 4\sin(4000\pi t)$ is quantized by rounding off, using a 12-bit quantizer. What is the rms quantization error & the quantization SNR? **June-09, 6M**

Sol :- Given : $g(t) = 2\cos(2000\pi t) - 4\sin(4000\pi t)$

$N = 12$ bits

$A_{m1} = 2V$

$A_{m2} = 4V$

WKT $p = \dfrac{A_m^2}{2}$

$$p = \frac{A_{m1}^2}{2} + \frac{A_{m2}^2}{2} = \frac{2^2}{2} + \frac{4^2}{2} = \underline{10W}$$

WKT Step-Size Δ is given by

$$\Delta = \frac{2g_{max}}{L}$$

* The peak value of $g(t)$ to be less than $\underline{6V}$, then

$$\Delta = \frac{2\times 6}{2^N} = \frac{12}{2^{12}}$$

$$\boxed{\Delta = 0.0029}$$

* Quantization error

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{(0.0029)^2}{12} = \underline{0.7008\times 10^{-6}}$$

* $$(SNR)_o = \frac{P}{\sigma_Q^2} = \frac{10}{0.7008\times 10^{-6}} = \underline{14.268\times 10^{6}}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

6) For a binary PCM Signal, determine L, if the Compression parameter μ is 100 & the minimum $(SNR)_{dB}$ is 45 dB. Determine the $(SNR)_o$ dB with this value of L, where L is the number of levels

June-09, 5M

Sol:- Given : $\mu = 100$, $(SNR)_{dB} = 45\,dB$, $L = ?$

$$(SNR)_{dB} = 45\,dB$$

$$10 \log_{10} (SNR) = 45\,dB$$

$$(SNR) = \log_{10}^{-1} \left(\frac{45}{10}\right)$$

$$\boxed{(SNR)_o = 31622.7766}$$

WKT
$$(SNR)_o = \frac{3L^2}{[\ln(1+\mu)]^2} = \frac{3L^2}{[\ln(1+100)]^2} = \frac{3L^2}{[\ln(101)]^2}$$

$$31622.77 = \frac{3L^2}{(4.615)^2}$$

$$3L^2 = 673544.0473$$

$$L^2 = 224514.68$$

$$\boxed{L = 473.829}$$

Since, $L = 2^N$, we Select

$$\boxed{L = 512}$$

* $$(SNR)_o = \frac{3L^2}{[\ln(1+\mu)]^2} = \frac{3(512)^2}{[\ln(101)]^2}$$

$$(SNR)_o = 36922.84$$

* $(SNR)_{o\,dB} = 10\log_{10}(14.268\times10^{6})$

$$(SNR)_{o\,dB} = \underline{71.54\text{ dB}}$$

* rms Quantization Error

$$\sigma_Q = \frac{\Delta}{\sqrt{12}} = \frac{0.0029}{\sqrt{12}} = \underline{8.45\times10^{-4}}$$

8) A PCM System uses a uniform quantizer followed by a 7-bit encoder. The bit rate of the System is 50 Mbits/Second.

ⓐ What is the message bandwidth for which the System operates Satisfactorily?

ⓑ Determine the o/p Signal to quantizing noise ratio when a Sinusoidal modulating wave of frequency 1 MHz is applied to the I/p.

Sol:- Given: $R_b = \frac{1}{T_b} = 50$ Mbits/Sec , $N = 7$

$W = ?$, $(SNR)_{dB} = ?$

WKT

$T_b = \frac{T_s}{N}$.

$T_s = NT_b$

$f_s = 2W$

$W = f_s/2$

* WKT bit rate $R_b = Nf_s$

$$f_s = R_b/N = \frac{50 \text{ Mbits/sec}}{7}$$

$$\boxed{f_s = 7.14 \text{ MHz}}$$

$$W = f_s/2 = 7.14 \text{ MHz}/2$$

$$\boxed{W = 3.57 \text{ MHz}}$$

* $(SNR)_{dB} = 1.76 + 6N$

$= 1.76 + 6 \times 7$

$$\boxed{(SNR)_{dB} = 43.76 \text{ dB}}$$

9) If the data rate is 32 Kbps, which is the same bit rate obtained by sampling at 8KHz with 4 bits/sample in a PCM system. Find the average o/p signal to noise ratio of a 4-bit PCM quantizer for the sampling of a full scale sinusoid with $f_s = 8KHz$.

Sol:- Given : $N = 4$, $R_b = 32 Kbps$, $f_s = 8KHz$, $(SNR)_{dB} = ?$

$$(SNR)_{dB} = 1.76 + 6N$$

$$= 1.76 + 6 \times 4$$

$$\boxed{(SNR)_{dB} = 25.76 \text{ dB}}$$

---

10) A 10-KHz sinusoid with amplitude levels of ±1 volt is to be sampled & quantized by rounding off. How many bits are required to ensure a quantization SNR of 45dB? What is the bit rate of the digitized signal if the sampling rate is chosen as twice the Nyquist rate?

Sol:- Given : $A_m = \pm 1V$, $(SNR)_{dB} = 45 dB$, $W = 10KHz$.

WKT $(SNR)_{dB} = 1.76 + 6N$

$$45 = 1.76 + 6N$$

$$6N = 43.24$$

$$N = 7.206$$

Choose

$$\boxed{N = 8}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Nyquist rate $= 2W = 2 \times 10 KHz = 20 KHz$
* Sampling rate 'f_s' $= 2 \times$ Nyquist rate $= 2 \times 20 KHz$

$$\boxed{f_s = 40 KHz}$$

* Bit rate $R_b = N f_s = 8 \times 40 KHz$

$$\boxed{R_b = 320 \text{ Kbs}}$$

11) Consider an audio Signal with Spectral Components limited to a frequency band of 300 - 3000 Hz. A PCM Signal is generated with the Sampling rate of 8000 Samples/Sec. The required o/p SNR must be atleast 30 dB.

(a) What is the minimum number of quantization levels needed & What is the minimum number of bits/Sample needed.

(b) Repeat part (a) when μ-law Companding is used with $\mu = 255$?

Sol :- Given :- $(SNR)_{dB} = 30 \text{ dB}$.

(a) WKT $(SNR)_{dB} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{P}{\sigma_Q^2} \longrightarrow (1)$

WKT $\sigma_Q^2 = \frac{\Delta^2}{12}$

Where $\Delta = \frac{2x_{max}}{L}$

$$\sigma_Q^2 = \left(\frac{2x_{max}}{L}\right)^2 \cdot \frac{1}{12} = \frac{\cancel{4}x_{max}^2}{3\,\cancel{12}L^2}$$

$$\boxed{\sigma_Q^2 = \frac{x_{max}^2}{3L^2}} \longrightarrow (2)$$

* WKT $\boxed{P = \frac{x_{max}^2}{2}} \rightarrow (3)$

Substituting eq (2) & (3) in eq (1), we get

$$(SNR)_o = \frac{P}{\sigma_Q^2} = \frac{\cancel{x_{max}^2}/2}{\cancel{x_{max}^2}/3L^2} = \frac{1/2}{1/3L^2}$$

$$\boxed{(SNR)_o = \frac{3L^2}{2}}$$

$$(SNR)_{dB} = 10\log_{10}\left(\frac{3}{2}\cdot L^2\right)$$

$$= 10\log_{10}\left(\frac{3}{2}\right) + 10\log_{10}\left(L^2\right)$$

$$\boxed{(SNR)_{dB} = 1.76 + 20\log_{10}(L)}$$

* It is given in the problem that $\underline{(SNR)_{dB} \geq 30\ dB}$

$$1.76 + 20\log_{10}(L) \geq 30\,dB$$

$$20\log_{10}(L) \geq 30 - 1.76$$

$$\log_{10}(L) \geq \frac{28.24}{20}$$

$$(L) \geq \log_{10}^{-1}(1.41)$$

$$\boxed{L \geq 25.8}$$

* WKT

$$L = 2^N$$

$$25.8 = 2^N$$

$\boxed{N = 5}$ bits/Sample

b) When μ-law Companding is used. Given: $\mu = 255$

$$(SNR)_o = \frac{3L^2}{[\ln(1+\mu)]^2} = \frac{3L^2}{[\ln(1+255)]^2}$$

$$(SNR)_o = 0.098\, L^2$$

∵ ⇒ $\log_{10}(a \cdot b)$
→ $\log_{10}(a) + \log_{10}(b)$

$$(SNR)_{dB} = 10\log_{10}(0.098\, L^2)$$

$$30\,dB = 10\log_{10}(0.098) + 10\log_{10}(L^2)$$

$$30\,dB = -10.08 + 20\log_{10}(L)$$

$$20\log_{10}(L) = 40.08$$

$$\log_{10}(L) = \frac{40.08}{20}$$

$$L = \log^{-1}(2.004)$$

$$\boxed{L = 100.9}$$

WKT

$$L = 2^N$$

$$100.9 = 2^N$$

$\boxed{N = 7}$ bits/Sample

---

12) A TV Signal with a BW of 4.2MHz is transmitted using PCM with 512 Quantizer levels.

Calculate: i) Binary code word length

ii) Transmitted bit rate.

Sol:- Given: $L = 512$, $W = 4.2MHz$, $N = ?$, $R_b = ?$

i) Binary code word length

$$L = 2^N$$

$$512 = 2^N$$

$$N = 9 \text{ bits}$$

ii) Bit rate

$$R_b = Nf_s$$

$$= 9 \times 2W$$

$$= 9 \times 2 \times 4.2\text{MHz}$$

$$\boxed{R_b = 75.6\text{Mbps}}$$

13) A telephone signal bandlimited to 4KHz is to be transmitted using PCM. The signal to quantization noise is to be atleast 40dB. Find the number of levels into which the signal has to be encoded. Find the BW of transmission & bit rate.

Sol:- Given : $W = 4\text{KHz}$, $SNR = 40\text{dB}$.

WKT Telephone signal is a Non-sinusoidal signal.

$$\therefore (SNR)_{dB} = 4.8 + 6N$$

$$40\text{ dB} = 4.8 + 6N$$

$$6N = 35.2$$

$$N = 5.86$$

Choose $N = 6$ bits

$\therefore$ Number of levels $L = 2^N$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$L = 2^7$

$\boxed{L = 128 \text{ Levels}}$

* Txission BW $B_T = \frac{1}{2} N f_s$

$= \frac{1}{\cancel{2}} N.\cancel{2}W$

$= NW$

$= 7 \times 4KHz$

$\boxed{B_T = 28KHz}$

* Bit rate $R_b = N f_s$

$= N \times 2W$

$= 7 \times 2 \times 4KHz$

$\boxed{R_b = 56 \text{ Kbps}}$

---

14) An analog Signal is Sampled at the Nyquist rate $f_s = 20KHz$ & quantized into $L = 1024$ levels. Find bit rate & the time-duration T_b of one bit of the binary encoded Signal.

Sol:- Given: $f_s = 20KHz$, $L = 1024$.

WKT. $L = 2^N$

$512 = 2^N$

$\boxed{N = 10}$

* Bit rate $R_b = N f_s = 10 \times 20KHz = \underline{200KHz}$

* Bit duration $T_b = 1/R_b = 1/200KHz = \underline{5 \mu sec}$

---

15) A PCM System uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the System is 56 Mega bits/Sec. Find the o/p Signal to quantization Noise ratio when a Sinusoidal

Wave of 1 MHz frequency is applied to the I/p.

Sol :- Given: $N=7$, $R_b = 56$ Mbps, $W = 1$ MHz.

* WKT $R_b = Nf_s$

$$f_s = R_b/N = 56\text{ Mbps}/7 = \underline{8\text{ MHz}}.$$

* Message bandwidth $W = f_s/2 = 8\text{MHz}/2 = \underline{4\text{MHz}}$. $\because \boxed{f_s = 2W}$

---

15) A telephone Signal with cut-off frequency of 4KHz is digitized into 8-bit PCM, Sampled at Nyquist rate. Calculate baseband transmission bandwidth & quantization $\frac{S}{N}$ ratio.

Sol :- Given: $W = 4$ KHz, $N=8$ Jan-08, 4M

* $B_T = NW = 8 \times 4\text{KHz} = \underline{32\text{ KHz}}$

* Telephone Signal is a <u>Non-Sinusoidal Signal</u>.

$$\therefore (SNR)_{dB} = 4.8 + 6N = 4.8 + 6 \times 8$$

$$\boxed{(SNR)_{dB} = 52.8\text{ dB}}$$

---

16) A telephone Signal bandlimited to 4KHz is to be transmitted by PCM. The Signal to quantization Noise is to be atleast <u>40dB</u>. Find the number of levels into which the Signal has to be encoded. Also find the bandwidth of Txission. Jan-04, 4M

Sol :- Given: $W = 4$ KHz, $(SNR)_{dB} = 40$ dB

WKT $(SNR)_{dB} = 4.8 + 6N$

$$40 = 4.8 + 6N$$

$N = 5.866$

Choose $\boxed{N = 6 \text{ bits}}$

* WKT $L = 2^N = 2^6$

$\boxed{L = 64 \text{ levels}}$

* $B_T = NW = 6 \times 4KHz = \underline{24KHz}$

---

17) Consider an audio Signal Comprised of the Sinusoidal term $S(t) = 3\cos(500\pi t)$

i) Find the Signal to Quantization noise ratio when this is Quantized using 10-bit PCM.

ii) How many bits of Quantization are needed to achieve a Signal to Quantization Noise ratio of atleast 40dB?

Sol :- Given : $S(t) = 3\cos(500\pi t)$, $A_m = 3V$.

$S(t)$ is a Sinusoidal Signal.

i) $\therefore$ $(SNR)_{dB} = 1.76 + 6N$ Given : $N = 10$

$= 1.76 + 6 \times 10$

$\boxed{(SNR)_{dB} = 61.76 \text{ dB}}$

ii) Given : $(SNR)_{dB} = 40$ dB.

$(SNR)_{dB} = 1.76 + 6N$

$40 - 1.76 = 6N$

$N = 6.37$

Choose $\boxed{N = 7 \text{ bits}}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

18) A PCM System uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the System is equal to 50 Mbps.

(a) What is the maximum message bandwidth for which the System operates Satisfactorily?

(b) Determine the o/p Signal to quantization noise ratio when a full load Sinusoidal modulating wave of frequency 1 MHz is applied to the I/p.

July-07, 6M	July-03, 6M

Sol:- Given: $N = 7$, $R_b = 50\,Mbps$.

(a) WKT

$$R_b = N f_s$$

$$R_b = N \cdot 2W$$

$$W = R_b/2N = \frac{50\times10^6}{2\times7}$$

$$\boxed{W = 3.57\ MHz}$$

(b) For Sinusoidal Wave

$$(SNR)_{dB} = 1.8 + 6N$$

$$= 1.8 + 6\times7$$

$$\boxed{(SNR)_{dB} = 43.8\ dB}$$

19) The Information in an analog waveform with maximum frequency $f_m = 3KHz$ is to be transmitted over an M-level PCM System where the number of pulse levels is $M=16$. The quantization distortion is Specified not to exceed 1% of peak to peak analog Signal.

i) What is the maximum number of bits per Sample that Should

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

be used in this PCM System?

ii) What is the minimum Sampling rate & What is the resulting bit transmission rate?

Sol:- Given: $f_m = 3KHz$, $L = M = V = 16$.

i) WKT $L = 2^N$

$16 = 2^N$

$\boxed{N = 4}$

ii) $f_s \geq 2W$

∴ Minimum Sampling rate

$f_s = 2W = 2 \times 3KHz = \underline{6KHz}$

* Bit Transmission rate

$R_b = Nf_s = 4 \times 6 \times 10^3$

$\boxed{R_b = 24 \times 10^3 \text{ bps}}$

---

20) The bandwidth of TV video plus audio Signal is 4.5MHz. If the Signal is Converted to PCM bit Stream with 1024 Quantization levels, determine the number of bits/Sec generated by the PCM System. Assume that the Signal is Sampled at the rate of 20% above Nyquist rate.

Sol:- Given: $W = 4.5 MHz$, $L = 1024$, $N = ?$

* Nyquist rate $= 2W = 2 \times 4.5 \times 10^6 = \underline{9MHz.}$

* Sampling rate 'f_s' $= 1.2 \times$ Nyquist rate $= 1.2 \times 9MHz = \underline{10.8MHz}$

* WKT $L = 2^N$

$1024 = 2^N$

$\boxed{N = 10 \text{ bits}}$

* Signalling rate $R_b = N \times f_s = 10 \times 10.8 \times 10^6$

$\boxed{R_b = 108 \times 10^6 \text{ bits/Sec}}$

---

21) A Compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15KHz.

i) What is Nyquist rate?

ii) If the Nyquist samples are quantized into L = 65,536 levels & then binary coded, determine the number of binary digits required to encode a sample.

iii) Determine the number of binary digits per second required to encode the audio signal.

iv) For practical reasons, the signals are sampled at a rate above Nyquist rate at 44100 samples per second. If L = 65,536 determine number of bits per second required to encode the signal & transmission bandwidth of encoded signal.

Sol:- Given : W = 15KHz, L = 65,536.

i) Nyquist rate $f_s = 2W = 2 \times 15\text{KHz} = \underline{30\text{KHz}}$

ii) WKT $L = 2^N$

$65,536 = 2^N$

$\boxed{N = 16 \text{ bits}}$

iii) Signalling rate $R_b = Nf_s = 16 \times 30\text{KHz} = \underline{480 \text{ Kbps}}$

iv) Given : $f_s = 44.1\text{KHz}$, $L = 65,536 \rightarrow$ then $\underline{N = 16}$ bits

$B_T = \frac{1}{2} Nf_s = \frac{1}{2} \times 16 \times 44.1\text{KHz} = \underline{352.8 \text{ KHz}}$.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Delta Modulator - Formulae :

1) Slope overload distortion will occur if

$$\boxed{A_m \geq \frac{\delta}{2\pi f_m T_s}}$$

2) Slope overload distortion will not occur if

$$\boxed{A_m \leq \frac{\delta}{2\pi f_m T_s}}$$

3) Nyquist rate = $\underline{2W}$

Where 'W' $\longrightarrow$ Maximum frequency of the Signal.

4) Sampling Interval $\boxed{T_s = 1/f_s}$

5) Post filtered $(SNR)_o$

$\boxed{f_o \text{ or } f_m}$

$$\boxed{(SNR)_o = \frac{3}{8\pi^2 W f_m^2 T_s^3}}$$

Where 'W' is the cut-off frequency of the LPF i.e. BW of the LPF.

$f_m \rightarrow$ Signal frequency

$T_s = 1/f_s$

6) Prefiltered $(SNR)_o$ $\boxed{(SNR)_o = \frac{3}{8\pi^2 f_m^2 T_s^2}}$

7) Granular Noise power $\boxed{'N_o' = \frac{W T_s \delta^2}{3}}$

8) $(SNR)_o = \dfrac{P}{\sigma_Q^2} = \dfrac{A_m^2/2}{\delta^2/3}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

ex:-

1) Bit rate = 20k bits/Sec Sampling rate 'f_s' = 20K Samples/Sec	2) Bit rate = 60 K bits/Sec Sampling rate 'f_s' = 60 K Samples/Sec

$\therefore$ $\boxed{R_b \text{ in bits/Sec} = f_s \text{ in Samples/Sec.}}$

DM problems

1) Consider a Speech Signal with a maximum frequency of 3.4KHz & maximum amplitude of 1V. The Speech Signal is applied to a DM with its bit rate at 20 K bits/Sec. Discuss the choice of an appropriate Step Size for the delta modulator.

June-09, 5M
July-06, 6M

Sol:- Given: $W = f_0 = 3.4KHz$, $A_m = 1V$

$r_b = 20Kbps = f_s = 20K$ Samples/Sec.

WKT the Slope overload distortion will not occur if

$$A_m \leqslant \frac{\delta}{2\pi f_0 T_s}$$

$$\delta \geqslant A_m 2\pi f_0 T_s$$

$$\delta \geqslant A_m 2\pi f_0 \frac{1}{f_s}$$

$$\delta \geqslant 1 \times 2\pi \times 3.4 \times 10^3 \cdot \frac{1}{20\text{K Samples/Sec}}$$

$$\boxed{\delta \geqslant 1.068\,V}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

2) Consider a Speech Signal with maximum frequency of 3.4 KHz & maximum amplitude of 1 Volt. This Speech Signal is applied to a delta modulator whose bit rate is Set at 60 Kbit/Sec. Explain the choice of an appropriate Step Size for the modulator.

Sol:- Given: $W = f_0 = 3.4\,KHz$, $A_m = 1V$,

$R_b = 60\,Kbps = f_s = 60K$ Samples/Sec.

WKT the Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_0 T_s}$$

$$\delta \geq A_m 2\pi f_0 T_s$$

$$\delta \geq 1 \times 2\pi \times 3.4 \times 10^3 \times \frac{1}{60 \times 10^3}$$

$$\boxed{\delta \geq 0.356\ \text{Volts}}$$

---

3) Assume a Speech Signal with a minimum frequency of 3.4 KHz & a maximum amplitude of 1V. The Speech Signal is applied to a delta modulator with its bit rate at 25 Kbps. Discuss the choice of an appropriate Step Size for the delta modulator.

Sol:- Given: $f_0 = 3.4\,KHz$, $A_m = 1V$, $R_b = 25\,Kbps = f_s = 25K$ Samples/Sec

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_0 T_s}$$

$$\delta \geq A_m 2\pi f_0 T_s$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$\delta \geq A_m 2\pi f_0 \cdot \frac{1}{f_s}$$

$$\delta \geq 1 \times 2\pi \times 3.4\times10^3 \times \frac{1}{25\times10^3}$$

$$\boxed{\delta \geq 0.855 \text{ volts}}$$

---

4) A delta modulator system is designed to operate at five times the Nyquist rate for a signal with 3 KHz bandwidth. Determine the maximum amplitude of a 2KHz I/p sinusoid for which the delta modulator does not have slope overload. Quantizing step size is 250 mV. Derive the formula that you use.

Sol:- Given: $W = 3\text{KHz}$, $f_m = 2\text{KHz}$, $\delta = 250\text{mV}$.

$f_s = 5\times$ Nyquist rate.

For derivation refer page No. 36 & 37

* Nyquist rate $= 2W = 2\times 3\text{KHz} = 6\text{KHz}$
* Sampling rate 'f_s' $= 5 \times$ Nyquist rate $= 5\times 6\text{KHz} = 30\text{KHz}$
* WKT slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$A_m \leq \frac{\delta f_s}{2\pi f_m}$$

$$\leq \frac{250\times10^{-3}\times 30\times10^3}{2\pi\times 2\times10^3}$$

$$\boxed{A_m \leq 0.59 \text{ volts}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

5) Find the Signal amplitude for minimum quantization error in a delta modulation System if Step Size is 1 volts having repetition period 1 msec. The Information Signal operates at 100 Hz.

Sol:- Given : $\delta = 1V$, $f_m = 100\,Hz$, Sampling duration $T_s = 1\,msec$

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$A_m \leq \frac{1V}{2\pi \times 100 \times 1 \times 10^{-3}}$$

$$\boxed{A_m \leq 1.6 \text{ volts}}$$

6) In a Single integration DM Scheme the voice Signal is Sampled at a rate of 64 KHz. The maximum Signal amplitude is 2 volts. voice Signal bandwidth is 3.5 KHz. Determine the minimum value of Step Size to avoid Slope overload & Calculate granular noise power.

Sol:- Given : $f_s = 64 \times 10^3\,Hz$, $A_m = 2V$, $W = 3.5\,KHz$, $\delta = ?$

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$\boxed{T_s = \frac{1}{f_s} = 15.62 \times 10^{-6}}$$

$$\delta \geq A_m 2\pi f_m T_s$$

$$\delta \geq A_m 2\pi f_m \frac{1}{f_s}$$

$$\geq 2 \times 2\pi \times 3.5 \times 10^3 \times \frac{1}{64 \times 10^3}$$

$$\boxed{\delta \geq 0.6872V}$$

* Granular Noise power is given by

$$N_o = \frac{W T_s \delta^2}{3} = \frac{3500 \times \frac{1}{64\times10^3} \times (0.6872)^2}{3}$$

$$\boxed{N_o = 8.6\,mW}$$

---

7) A DM Systems is designed to operate at 3 times Nyquist rate for a Signal with 3KHz bandwidth. The quantizing Step-Size is 250 mv.

a) Determine the maximum amplitude of a 1-KHz I/p Sinusoid for which DM System does not Show Slope overload.

b) Determine the post filtered o/p Signal to Noise ratio for the Signal of part ⓐ.

July-01, 8M
Jan-06, 8M

Sol:- Given : $\delta = 250\,mv$, $W = 3KHz$,

$f_s = 3 \times$ Nyquist rate

$= 3 \times 2W$

$= 3 \times 2 \times 3KHz$

$$\boxed{f_s = 18\,KHz}$$

ⓐ Given : $f_m = 1KHz$.

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

∴ Maximum Amplitude $A_{max} = \frac{\delta}{2\pi f_m T_s}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$A_{max} = \frac{\delta f_s}{2\pi f_m} = \frac{250mv \times 18KHz}{2\pi \times 1\times10^3}$$

$$\boxed{A_{max} = 716.197\ mv}$$

b) Post filtered o/p SNR is f_m & f_o

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 W T_s^3} = \frac{3 f_s^3}{8\pi^2 f_m^2 W}$$

$$= \frac{3\times 18\times10^3}{8\times 9.869\times(1\times10^3)^2\times 3\times10^3} = \frac{1.7496\times10^{13}}{2.368\times10^{11}}$$

$$\boxed{(SNR)_o = 73.86}$$

$$(SNR)_{dB} = 10\log_{10}(73.86)$$

$$\boxed{(SNR)_{dB} = 18.684\ dB}$$

---

8) Consider a DM System designed to operate at four times the Nyquist rate for a Signal with a 4KHz BW. The Step Size of the quantizer is 400mv.

ⓐ Find the maximum amplitude of a 1KHz I/p Sinusoid for which the DM does not Show Slope overload.

ⓑ Find post-filtered o/p SNR.

Sol :-

$W = 4KHz, \quad \delta = 400mv$

Nyquist rate $= 2W = 2\times 4KHz = \underline{8KHz}$

* Sampling rate 'f_s' = 4 × Nyquist rate = 4 × 8 KHz = 32k Samples/Sec

(a) Given: $f_m = 1\,KHz$

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

∴ Maximum Amplitude

$$A_{max} = \frac{\delta}{2\pi f_m T_s} = \frac{400\,mv \times 32 \times 10^3}{2\pi \times 1 \times 10^3} = \frac{12800}{6283.18}$$

$$\boxed{A_{max} = 2.037\,V}$$

(b) Post-filtered $(SNR)_o$ Given: $\boxed{W = 4\,KHz}$

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 W T_s^3} = \frac{3 f_s^3}{8\pi^2 f_m^2 W}$$

$$= \frac{3 \times (32 \times 10^3)^3}{8 \times 9.869 \times (1 \times 10^3)^2 \times 4 \times 10^3} = \frac{9.8304 \times 10^{13}}{3.15808 \times 10^{11}}$$

$$\boxed{(SNR)_o = 311.27}$$

$$(SNR)_{dB} = 10 \log_{10}(311.27)$$

$$\boxed{(SNR)_{dB} = 24.93\,dB}$$

---

9) A DM System is tested with a 10 KHz Sinusoidal Signal with 1V peak to peak at the I/p. It is Sampled at 10 times the Nyquist rate.

i) What is the Step-Size required to prevent Slope overload?

ii) What is the Corresponding SNR?

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Sol:- Given : $f_m = 10KHz$, $A_{m(p-p)} = 1V$ $\therefore A_{m(p)} = 0.5V$

* Nyquist rate $= 2W = 2 \times 10KHz = 20 KHz$.

* Sampling rate $f_s = 10 \times$ Nyquist rate $= 10 \times 20 KHz = 200 KHz$.

i) WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s} \qquad T_s = \frac{1}{f_s} = 5 \times 10^{-6}$$

$$\therefore \delta \geq A_m 2\pi f_m T_s$$

$$\geq (0.5) \times 2\pi \times 10 \times 10^3 \times 5 \times 10^{-6}$$

$$\boxed{\delta \geq 0.157V}$$

ii) SNR of DM System is given by

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 T_s^2} = \frac{3 f_s^2}{8\pi^2 f_m^2} = \frac{3 \times (200 \times 10^3)^2}{8 \times \pi^2 \times (10 \times 10^3)^2}$$

$$\boxed{(SNR)_o = 15.198}$$

$$(SNR)_{dB} = 10 \log_{10} (15.198)$$

$$\boxed{(SNR)_{dB} = 11.817 \text{ dB}}$$

---

10) Consider a lowpass Signal with a bandwidth of 3.4KHz. A linear modulation System with Step-Size $\delta = 0.1$ volts, is used to process this Signal at a Sampling rate 5 times the Nyquist rate

ⓐ Evaluate the maximum amplitude of a test Sinusoidal Signal of Frequency 1KHz, which can be processed by the System without Slope-overload distortion.

(b) For the Specifications given in part (a), find the o/p SNR under i) prefiltered, and ii) postfiltered Conditions.

Sol:- Given : $W = 3.4\,KHz$, $\delta = 0.1V$.

* Nyquist rate $= 2W = 2 \times 3.4\,KHz = \underline{6.8\,KHz}$

* Sampling rate 'f_s' $= 5 \times$ Nyquist rate $= 5 \times 6.8\,KHz = \underline{34\,KHz}$

* $T_s = 1/f_s = \underline{29.4 \times 10^{-6}}$

(a) $f_m = 1\,KHz$, $A_m = ?$

WKT, Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$\leq \frac{0.1}{2\pi \times 1\times10^{3} \times 29.4\times10^{-6}}$$

$$\boxed{A_m \leq 0.541 \text{ volts.}}$$

(b) i) prefiltered (SNR)

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 T_s^2} = \frac{3}{8 \times 9.869 \times (1\times10^3)^2 \times (29.4\times10^{-6})^2}$$

$$\boxed{(SNR)_o = 43.960}$$

$$(SNR)_{dB} = 10 \log_{10}(SNR) = 10 \log_{10}(43.960)$$

$$\boxed{(SNR)_{dB} = 16.430 \text{ dB}}$$

ii) post filtered (SNR)

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 W \times T_s^3} = \frac{3}{8 \times 9.869 \times (1\times10^3)^2 \times 3.4\times10^3 \times (29.4\times10^{-6})^3}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{(SNR)_o = 439.8}$$

$$(SNR)_{dB} = 10\log_{10}(439.8)$$

$$\boxed{(SNR)_{dB} = 26.43\text{ dB}}$$

11) A DM System is designed to operate at 3 times Nyquist rate for a Signal with 3 KHz bandwidth. The quantizing Step-Size is 250mV.

(a) Determine the maximum amplitude of a 1 KHz I/P Sinusoid for which DM System doesnot show slope overload.

(b) Determine the post filtered o/p Signal to Noise ratio for the Signal of part (a)

July-01, 8M
Jan-06, 6M

Sol:- Given: $\delta = 250\text{mV}$, $W = 3\text{KHz}$.

* Nyquist rate $= 2W = 2 \times 3\text{KHz} = 6\text{ KHz}$

* Sampling rate 'f_s' $= 3\times$ Nyquist rate $= 3\times 6\text{KHz} = 18\text{KHz}$.

$T_s = 1/f_s = 55.5\times10^{-6}$

(a) Given: $f_m = 1\text{KHz}$.

WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$\therefore$ Maximum amplitude

$$A_{max} = \frac{\delta}{2\pi f_m T_s} = \frac{250\text{mV}}{2\pi\times1\times10^{3}\times55.5\times10^{-6}}$$

$$\boxed{A_{max} = 716.197\text{mV}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

(b) Post filtered (SNR)

$$(SNR)_o = \frac{3}{8\pi^2 f_m^2 \times W \times T_s^3} = \frac{3}{8\times 9.869 \times (1\times10^3)^2 \times 55.5\times10^{-6}}$$

$$\boxed{(SNR)_o = 73.86}$$

$$(SNR)_{dB} = 10\log_{10}(73.86)$$

$$\boxed{(SNR)_{dB} = 18.684 \text{ dB}}$$

12) In a Single integration DM Scheme, the voice Signal is Sampled at a rate of 64 KHz. The maximum Signal amplitude is 1 volt, voice Signal bandwidth is 3.5 KHz.

i) Determine the minimum value of Step Size to avoid Slope overload.

ii) Determine granular noise N_o

iii) Assuming Signal to be Sinusoidal, Calculate Signal power & Signal to Noise ratio.

iv) Assuming that noise Signal amplitude is uniformly distributed in the range (-1, 1) determine the Signal power & Signal to Noise ratio.

Sol :- Given ; $f_s = 64$ KHz, $A_m = 1V$, $W = 3.5$ KHz.

i) WKT Slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$\delta \geq A_m 2\pi f_m T_s$$

$$\delta \geq 1 \times 2\pi \times 3.5\times10^3 \times \frac{1}{64\times10^3}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{\delta \geq 0.3436\,V}$$

ii) Granular Noise 'N_o'

$$N_o = \frac{W T_s \delta^2}{3} = \frac{W \delta^2}{3 f_s} = \frac{3.5\times10^3 \times (0.343)^2}{3\times 64\times10^3}$$

$$\boxed{N_o = 2.144\,mW}$$

iii) $$P = \frac{A_m^2}{2} = \frac{1^2}{2}$$ Given : $A_m = 1V$

$$\boxed{P = 0.5V}$$

$$(SNR)_o = \frac{\text{Signal power}}{\text{Noise power}} = \frac{0.5}{2.144\,mW}$$

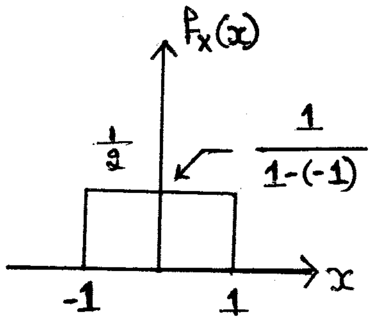
$$\boxed{(SNR)_o = 233.13}$$

$$(SNR)_{dB} = 10\log_{10}(233.13)$$

$$\boxed{(SNR)_{dB} = 23.676\ dB}$$

iv) PDF of the Signal is

$$f_X(x) = \begin{cases} \frac{1}{2} & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



* The mean Square value of the Signal is given by

$$\sigma_X^2 = \int_{-1}^{1} (x-\mu)^2 f_X(x)\,dx$$ WKT $\mu = 0$

$$= \int_{-1}^{1} x^2 \cdot \frac{1}{2}\,dx = \frac{1}{2}\int_{-1}^{1} x^2\,dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{1}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$= \frac{1}{6}\left[(1)^3 - (-1)^3\right] = \frac{1}{\cancel{6}\,3} \times \cancel{2}$$

$$\boxed{\sigma_x^2 = \frac{1}{3}} \longleftarrow \text{Signal power}$$

* Signal to Noise ratio is given by

$$(SNR)_o = \frac{\text{Signal power}}{\text{Noise power}} = \frac{1/3}{2.15 \times 10^{-3}}$$

$$\boxed{(SNR)_o = 155}$$

$$(SNR)_{dB} = 10 \log_{10}(155)$$

$$\boxed{(SNR)_{dB} = 21.9 \text{ dB}}$$

13) Find the o/p Signal to Noise ratio in a delta modulated System for a 1KHz Sinusoid, which is Sampled at 32KHz without Slope overload. The bandwidth of the reconstruction filter used is 4KHz.

Given: $f_s = 32\text{KHz}$, $f_m = 1\text{KHz}$, $W = 4\text{KHz}$

$$(SNR)_o = \frac{3 f_s^3}{8\pi^2 W f_m^2} = \frac{3 \times (32\times10^3)^3}{8\pi^2 \times 4\times10^3 \times (1\times10^3)^2} = \frac{9.8304\times10^{13}}{315.82\times10^3 \times (1\times10^3)^2}$$

$$\boxed{(SNR)_o = 311.25}$$

$$(SNR)_{dB} = 10 \log_{10}(311.25)$$

$$\boxed{(SNR)_{dB} = 24.93 \text{ dB}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Discrete PAM Signals (Lines Codes)

Line coding:

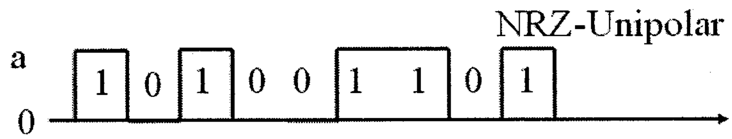
Digital data i.e. **binary digits** are **represented** by **different waveforms** for the purpose of **transmission** over the **channel**. This process is called **line coding** or transmission coding

Unipolar Format or ON-OFF Signaling:

In the **UNIPOLAR** format, symbol **1** is represented by **transmitting** a **pulse**, whereas symbol **0** is represented by **switching off** the **pulse**.

Uni-Polar NRZ- signaling:

When the pulse occupies the **full duration** of a symbol, the unipolar format is said to be of **UNIPOLAR *nonreturn-to-zero*** (**NRZ**) type.



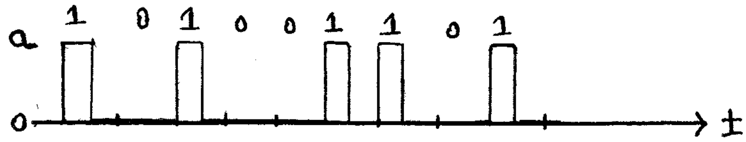
In this scheme the signals are represented as

$$S_1(t) = +a \quad for \quad 0 \le t \le T_b \quad for \quad Symbol\ 1$$

$$S_2(t) = 0 \quad for \quad 0 \le t \le T_b \quad for \quad Symbol\ 0$$

Uni-Polar RZ- signaling:

When the pulse occupies the **one-half** of the symbol duration, it is said to be of ***return-to-zero*** (**RZ**) type.



$$S_1(t) = +a \quad for \quad 0 \le t \le T_b/2 \quad for \quad Symbol\ 1$$

$$= 0 \quad for \quad T_b/2 \le t \le T_b$$

$$S_2(t) = 0 \quad for \quad 0 \le t \le T_b \quad for \quad Symbol\ 0$$

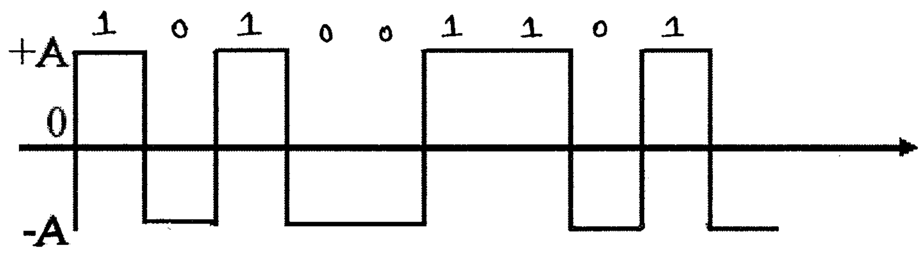
Polar Format:

In polar format, a **positive pulse** is transmitted for symbol **1**, and a **negative pulse** for symbol **0**.

A polar waveform has no DC component.

Polar NRZ- signaling:

When the pulse occupies the **full duration** of a symbol, the unipolar format is said to be of **POLAR *nonreturn-to-zero*** (**NRZ**) type.



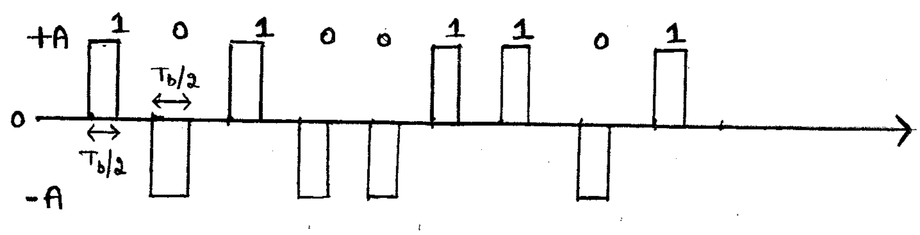
In this scheme the signals are represented as

$$S_1(t) = +a \qquad 0 \le t \le T_b \qquad for \quad Symbol\ 1$$

$$S_2(t) = -a \qquad 0 \le t \le T_b \qquad for \quad Symbol\ 0$$

Polar RZ- signaling:

When the pulse occupies the **one-half** of the symbol duration, it is said to be of **POLAR *return-to-zero*** (**RZ**) type.



$$S_1(t) = +a \quad for\ 0 \le t \le T_b/2 \qquad for \quad Symbol\ 1$$

$$= 0 \quad for \quad T_b/2 \le t \le T_b$$

$$S_2(t) = -a \quad for \quad 0 \le t \le T_b/2 \qquad for \quad Symbol\ 0$$

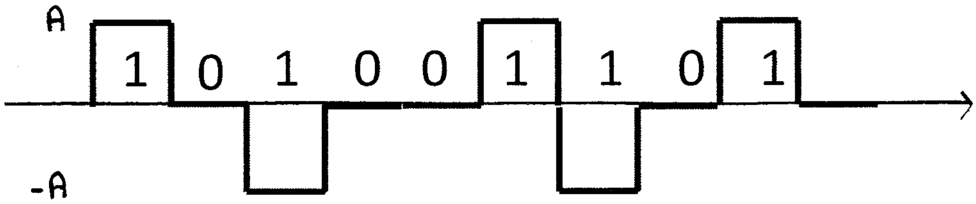
$$= 0 \quad for \quad T_b/2 \le t \le T_b$$

Bipolar Format or Pseudoternary signaling:

In bipolar format, positive and negative pulses are used alternately for transmission of 1s and no pulses for the transmission of 0s.

BIPOLAR NRZ Signalling:

When the pulse occupies the **full duration** of a symbol, the unipolar format is said to be of **BIPOLAR *nonreturn-to-zero*** (**NRZ**) type.



$$S_1(t) = \pm a \quad for \; 0 \le t \le T_b \quad for \quad Symbol\ 1$$

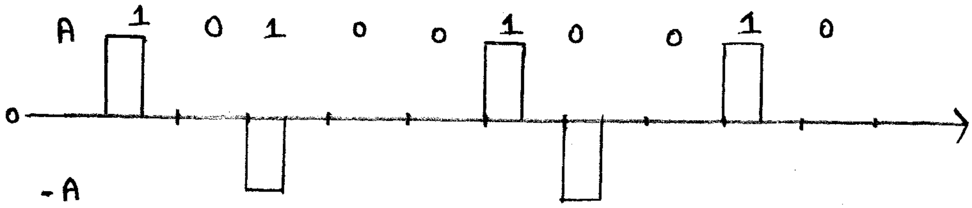
odd no. 1's +a

even no. of 1's -a

$$S_2(t) = 0 \quad for \; 0 \le t \le T_b \quad for \quad Symbol\ 0$$

BIPOLAR RZ Signalling:

When the pulse occupies the **one-half** of the symbol duration, it is said to be of **BIPOLAR *return-to-zero*** (**RZ**) type.



odd no. of 1's +a

even no. of 1's -a

$$S_1(t) = \pm a \quad for \; 0 \le t \le T_b/2 \quad for \quad Symbol\ 1$$

$$= 0 \quad for \; T_b/2 \le t \le T_b$$

$$S_2(t) = 0 \quad for \; 0 \le t \le T_b \quad for \quad Symbol\ 0$$

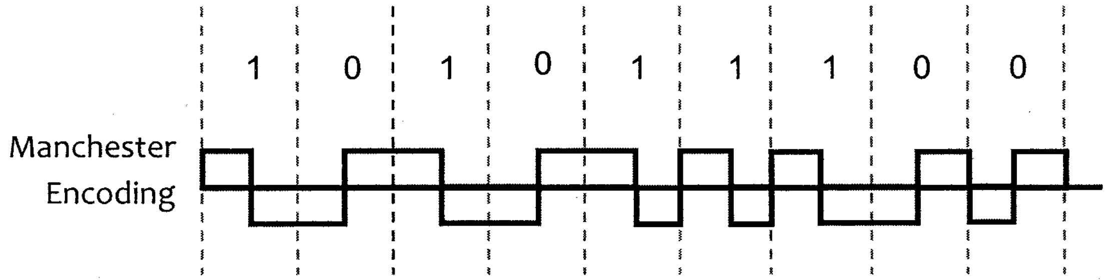
ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Manchester Format or Biphase baseband signaling:

Symbol 1 is represented by transmitting a **positive pulse** for **one-half** of the **symbol duration**, followed by a **negative pulse** for the **remaining half** of the **symbol duration.**

Symbol **0** is represented by transmitting a **negative pulse** for **one-half** of the **symbol duration**, followed by a **positive pulse** for the **remaining half** of the **symbol duration.**

A Manchester waveform has no DC component.



$$S_1(t) = +a \quad for \;\; 0 \le t \le T_b/2 \qquad for \;\; Symbol\; 1$$

$$= -a \quad for \;\; T_b/2 \le t \le T_b$$

$$S_1(t) = -a \quad for \;\; 0 \le t \le T_b/2 \qquad for \;\; Symbol\; 0$$

$$= +a \quad for \;\; T_b/2 \le t \le T_b$$

Polar Quaternary NRZ :-

* It has 4-distinct symbols of dibits (pair of bits) i.e. four possible combinations 00, 01, 10 & 11.

To these 4 combinations, four different amplitude levels are assigned as shown in table.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Message Combination	Signal amplitude
00	$-3A/2$
01	$-A/2$
10	$A/2$
11	$3A/2$

* This System is designed to reduce the Signalling rate & hence the bandwidth. Thus for two message bits only one pulse is transmitted with duration $2T_b$.

* Signalling rate is given by $r_{b/2} = \frac{1}{2T_b}$

Gray Coding :-

Natural Message	Gray Code	Signal Amplitude
00	00	$-3A/2$
01	01	$-A/2$
10	11	$A/2$
11	10	$3A/2$

* The messages are gray coded & polar quarternary NRZ encoding is done.

M-ary NRZ Format :-

* In polar Quarternary NRZ type of Coding we Combine two Successive bits
* In M-ary Coding, we Combine 'N' Successive message bits i.e. $M=2^N$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

for $N=3$, $M=2^3=8$ distinct levels known as (8-ary) octary coding. So the duration of each symbol will be equal to $3T_b$ i.e. 3-bits. Thus the signalling rate is reduced.

* In general signalling rate for M-ary coding is given as

$$\boxed{r = \frac{r_b}{N}} \longrightarrow (1)$$

WKT $M = 2^N$

Taking $\log_2$ on both sides

$$\log_2 M = \log_2 2^N$$

$$\log_2 M = N \log_2 2 \qquad \text{WKT } \log_2(2) = 1$$

$$\log_2 M = N(1)$$

$$\boxed{N = \log_2 M} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$\boxed{r = \frac{r_b}{\log_2 M}}$$

Properties of Line Codes :- **Jan-07, 7M**

* Discuss the various properties of line codes (digital formats).

1) Timing Content :-

The transmitted digital waveform should have adequate timing content information, required for the purpose of synchronizing the receiver to the transmitter.

2) Ruggedness :-

The waveform Should be immune to Channel noise & interference.

3) Error detection capability :-

The transmitted digital waveform Should allow error detection & Correction.

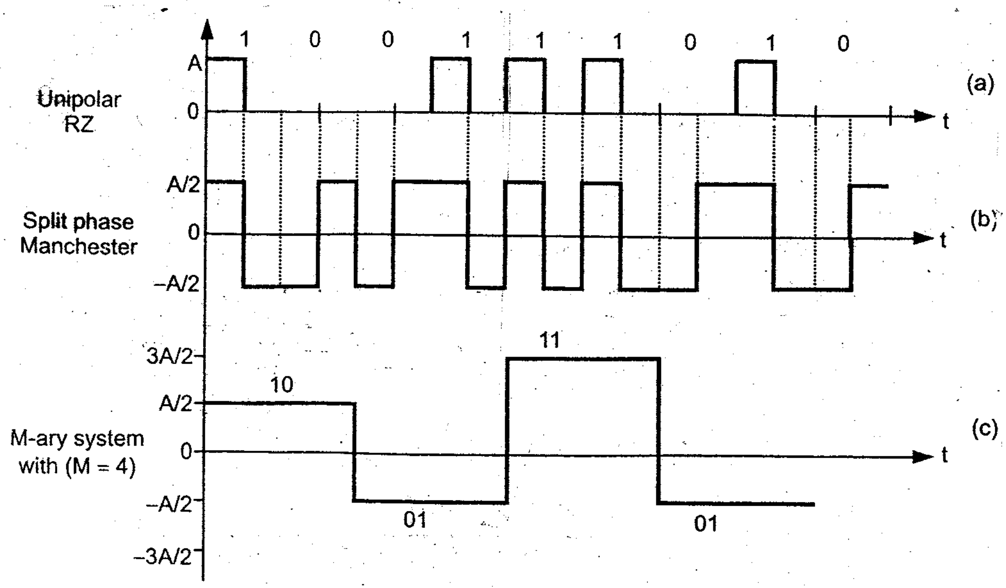
4) Transparency :-

The transmission of digital data over a Channel Should be transparent to the pattern of 1's & 0's in the data.

5) Matched power Spectrum :-

The power Spectrum of the transmitted digital waveform Should match to the Channel to reduce Signal distortion.

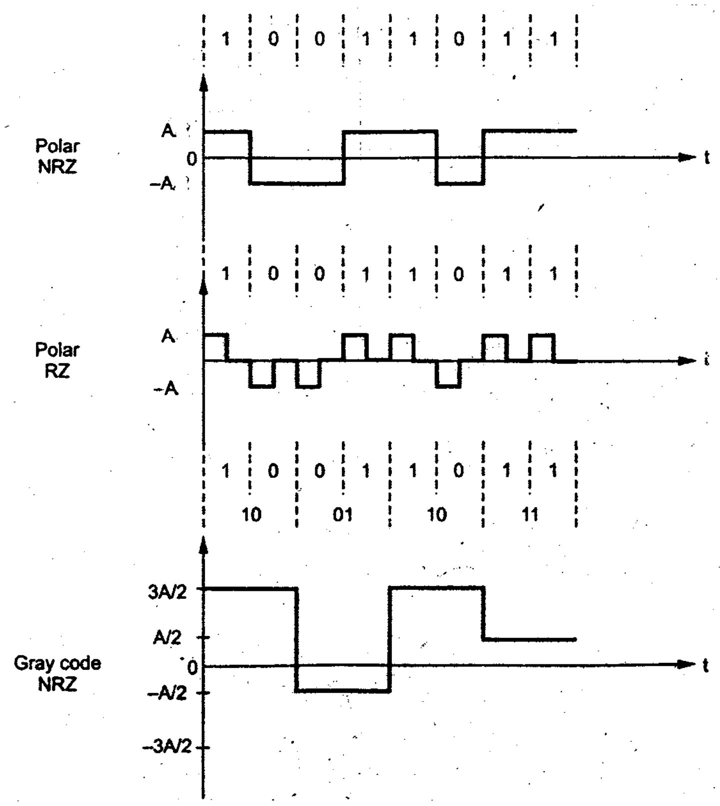
Represent the data 100111010 using following digital data formats.
i) Unipolar RZ
ii) Split phase manchester
iii) M-ary system where m=4.



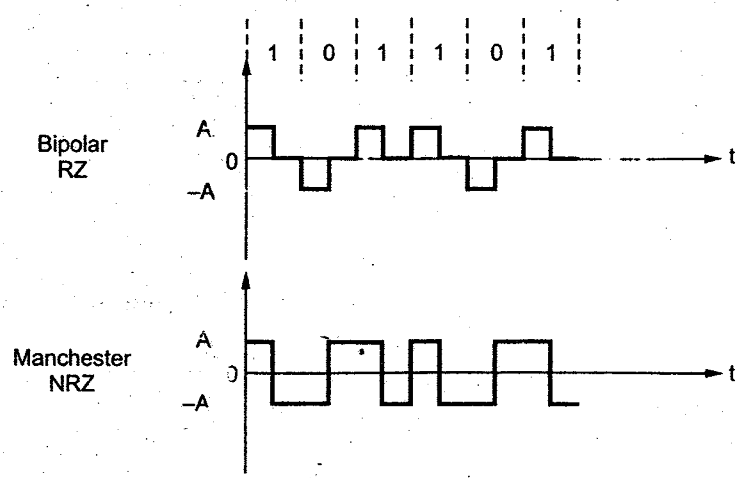
Digital data formats of sequence given
(a) Unipolar RZ format
(b) Split phase manchester format
(c) M-ary system with M = 4

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

For the binary bit stream 10011011 draw the waveforms for the following cases.
i) Polar NRZ
ii) Manchester RZ
iii) Gray code NRZ **[Bangalore University April-98]**



Write the waveforms for a binary sequence 101101 for the following cases.
i) Bipolar RZ
ii) Manchester NRZ **[Bangalore University April-98]**



For a given binary sequence 111000110101, draw the digital format waveforms corresponding to

(i) Polar manchester coding waveform

(ii) Bipolar NRZ waveform

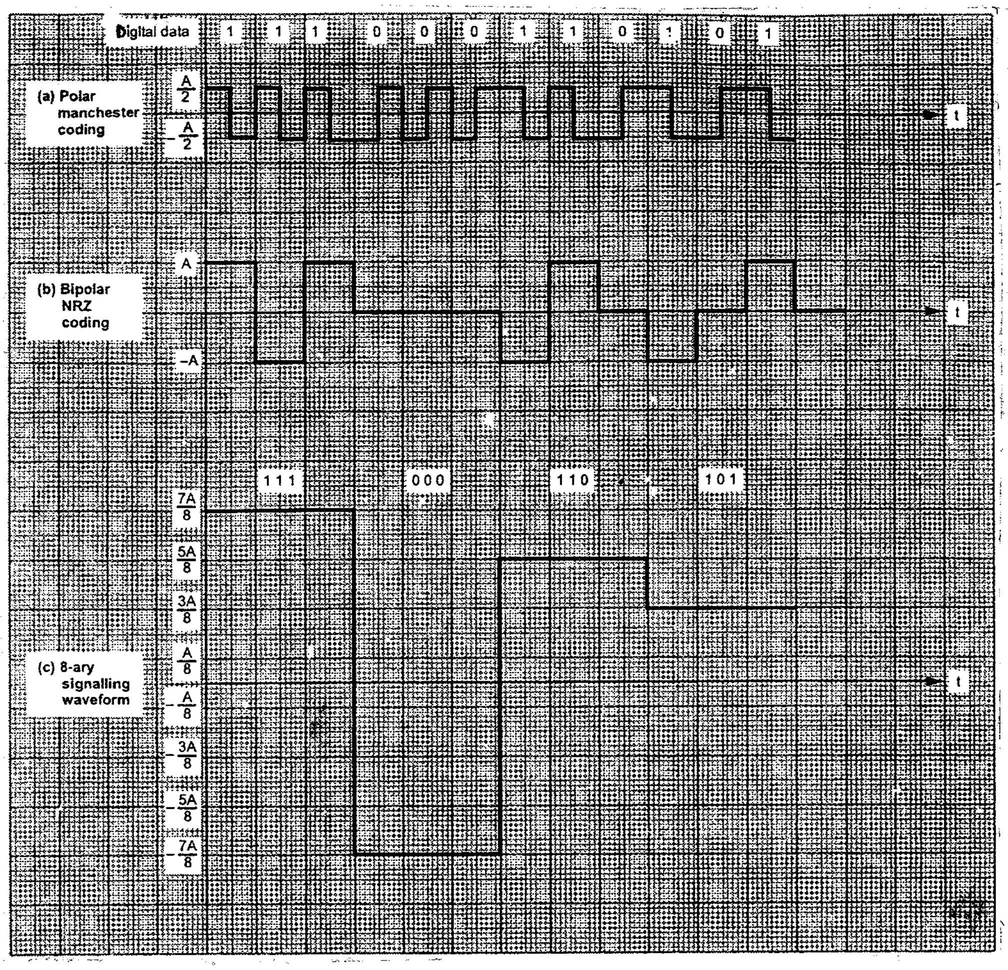
(iii) 8-ary signalling waveform

(VTU, Aug.-2002, 7-Marks)

Amplitude levels for M-ary system (M = 8)

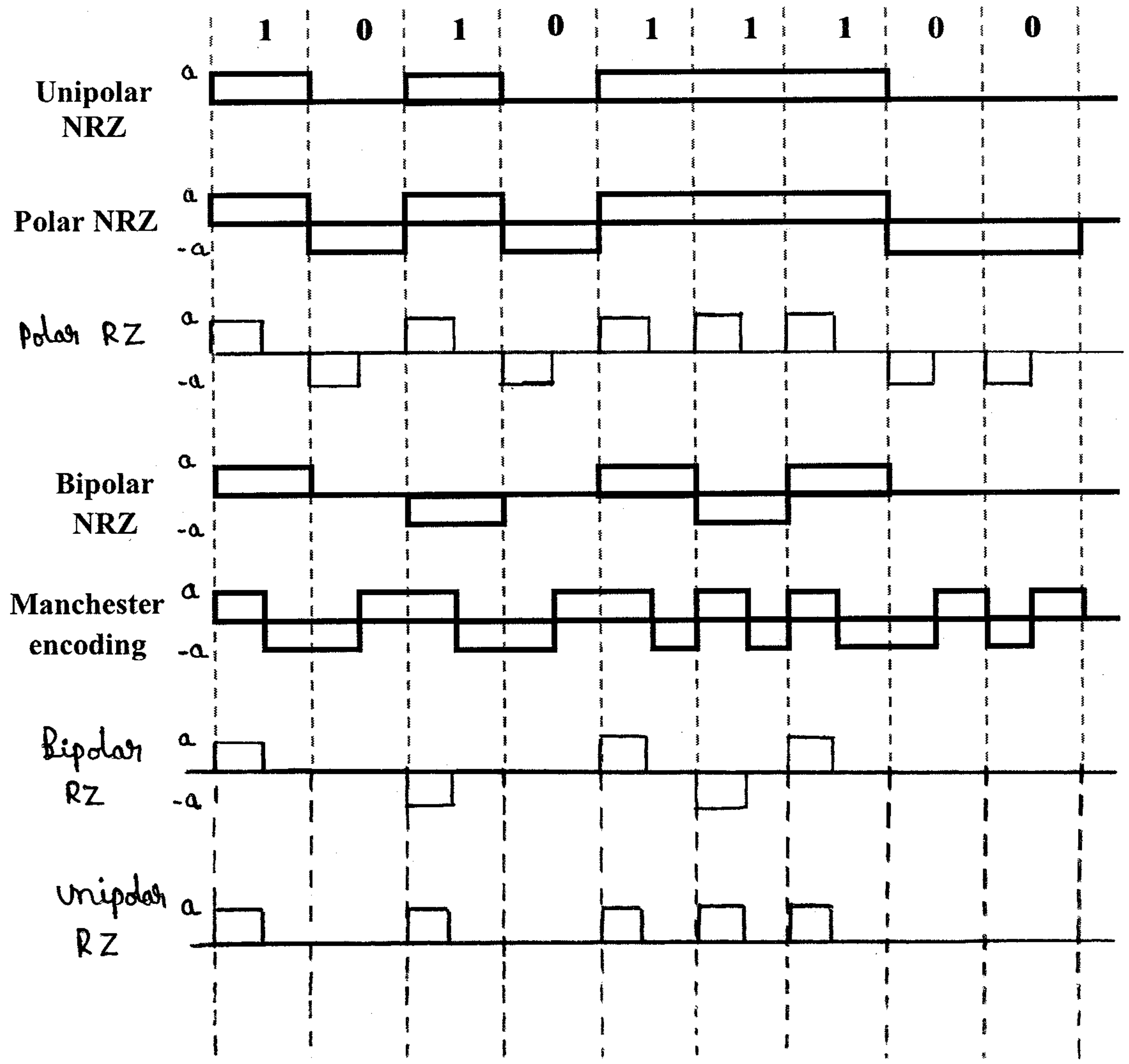
Serial No.	Message bits	Amplitude level
1.	0 0 0	$-\frac{7A}{8}$
2.	0 0 1	$-\frac{5A}{8}$
3.	0 1 0	$-\frac{3A}{8}$
4.	0 1 1	$-\frac{A}{8}$
5.	1 0 0	$\frac{A}{8}$
6.	1 0 1	$\frac{3A}{8}$
7.	1 1 0	$\frac{5A}{8}$
8.	1 1 1	$\frac{7A}{8}$

shows the 8-ary signalling waveform based on above table.


Digital data
1 1 1 0 0 0 1 1 0 1 0 1
(a) Polar manchester coding
A/2
-A/2
t
A
(b) Bipolar NRZ coding
-A
t
111
000
110
101
7A/8
5A/8
3A/8
A/8
-A/8
-3A/8
-5A/8
-7A/8
(c) 8-ary signalling waveform
t

1. **For the binary sequence 101011100 draw the digital format waveforms corresponding to**

i) **Unipolar NRZ** ii) **Polar NRZ** iii) **Bipolar NRZ**
iv) **Manchester encoding** V) Polar RZ VI) Bipolar RZ VII) unipolar RZ



ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Power Spectral Density of Line Codes

Formulae :

1) The PSD of the discrete PAM Signal $x(t)$ defined by

$$\boxed{S_x(f) = \frac{1}{T_b}\left|V(f)\right|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi n f T_b}}$$

2) **Autocorrelation function :-**

$$R_A(n) = E\left[A_K \cdot A_{K-n}\right]$$

$$\boxed{R_A(n) = \sum_{i=1}^{I}\left[A_K A_{K-n}\right] P_i}$$

3) Unipolar, polar & Bipolar :-

$$\boxed{V(f) = T_b \,\text{Sinc}(fT_b)}$$

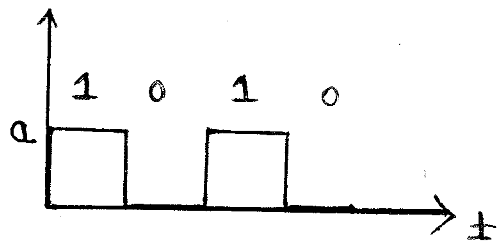
4) Manchester :-

$$\boxed{|V(f)|^2 = T_b^2 \sin^2\left(\frac{\pi f T_b}{2}\right) \text{Sinc}^2\left(f\frac{T_b}{2}\right)}$$

NRZ unipolar format :-

* In NRZ unipolar format

$$A_K = \begin{cases} +a & \text{for Symbol } 1 \\ 0 & \text{for Symbol } 0 \end{cases}$$



* Let us assume that Symbol 0 & 1 occurs with equal probabilities

i.e. $p(A_K = a) = 1/2$ &

$p(A_K = 0) = 0$

* WKT auto correlation function

$$R_A(n) = E\left[A_K A_{K-n}\right]$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Case i : for $n=0$

$$R_A(0) = E\left[A_K A_{K-0}\right] = E\left[A_K^2\right]$$

$$\therefore R_A(0) = \sum_{i=1}^{2} (A_K A_K)_i P_i$$

$$= a^2\left(\tfrac{1}{2}\right) + 0^2\left(\tfrac{1}{2}\right)$$

$$\boxed{R_A(0) = a^2/2}$$

Case ii : for $n \neq 0$

$A_K A_{K-n}$ will have four possibilities with probabilities $1/4$ each

A_K	A_{K-n}	$A_K A_{K-n}$	Equally Probable
0	0	0	$1/4$
0	a	0	$1/4$
a	0	0	$1/4$
a	a	a^2	$1/4$

$$R_A(n) = \sum_{i=1}^{4} (A_K A_{K-n})_i P_i$$

$$= 0(1/4) + 0\times(1/4) + 0\times(1/4) + a^2(1/4)$$

$$\boxed{R_A(n) = \frac{a^2}{4}}$$

* Thus, we may express autocorrelation function $R_A(n)$ for unipolar NRZ format as:

$$R_A(n) = \begin{cases} \dfrac{a^2}{2} & \text{for } n=0 \\ \dfrac{a^2}{4} & \text{for } n\neq 0 \end{cases}$$

* WKT $\boxed{V(f) = T_b \operatorname{Sinc}(fT_b)}$

The PSD of the unipolar NRZ format is

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{S_X(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi n f T_b}} \longrightarrow ①$$

Substituting the value of $V(f)$ & $R_A(n)$ in eq ①, we get

$$S_X(f) = \frac{1}{T_b}\left[T_b \mathrm{Sinc}(fT_b)\right]^2 \left[\sum_{n=0} \frac{a^2}{2} e^{-j2\pi \underline{n} fT_b} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi n f T_b}\right]$$

$$S_X(f) = \frac{1}{\cancel{T_b}} T_b^{\cancel{2}} \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{2} e^0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi n f T_b}\right]$$

$$S_X(f) = T_b \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi n f T_b}\right]$$

$$= T_b \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left\{\frac{a^2}{4} e^{-j2\pi n f T_b} + \frac{a^2}{4} - \frac{a^2}{4}\right\}\right]$$

$$= T_b \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{2} + \sum_{n=-\infty}^{\infty}\left\{\frac{a^2}{4} e^{-j2\pi n f T_b} - \frac{a^2}{4}\right\}\right]$$

$$= T_b \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{2} - \frac{a^2}{4} + \sum_{n=-\infty}^{\infty} \frac{a^2}{4} e^{-j2\pi n f T_b}\right]$$

$$= T_b \mathrm{Sinc}^2(fT_b)\left[\frac{a^2}{4} + \sum_{n=-\infty}^{\infty} \frac{a^2}{4} e^{-j2\pi n f T_b}\right]$$

$$S_X(f) = \frac{a^2}{4} T_b \mathrm{Sinc}^2(fT_b) + \frac{a^2}{4} T_b \mathrm{Sinc}^2(fT_b) \underbrace{\sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b}} \longrightarrow ②$$

From poisson Sum formula

$$\boxed{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)} \longrightarrow ③$$

Substituting eq ③ in eq ②, we get

$$S_X(f) = \frac{a^2}{4} T_b \mathrm{Sinc}^2(fT_b) + \frac{a^2}{4} \underline{T_b \mathrm{Sinc}^2(fT_b)} \cdot \frac{1}{T_b} \underline{\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)} \rightarrow ④$$

From fig ①,

$$\mathrm{Sinc}^2(fT_b) \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) = \delta(f) \longrightarrow ⑤$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Dont Write fig (1)

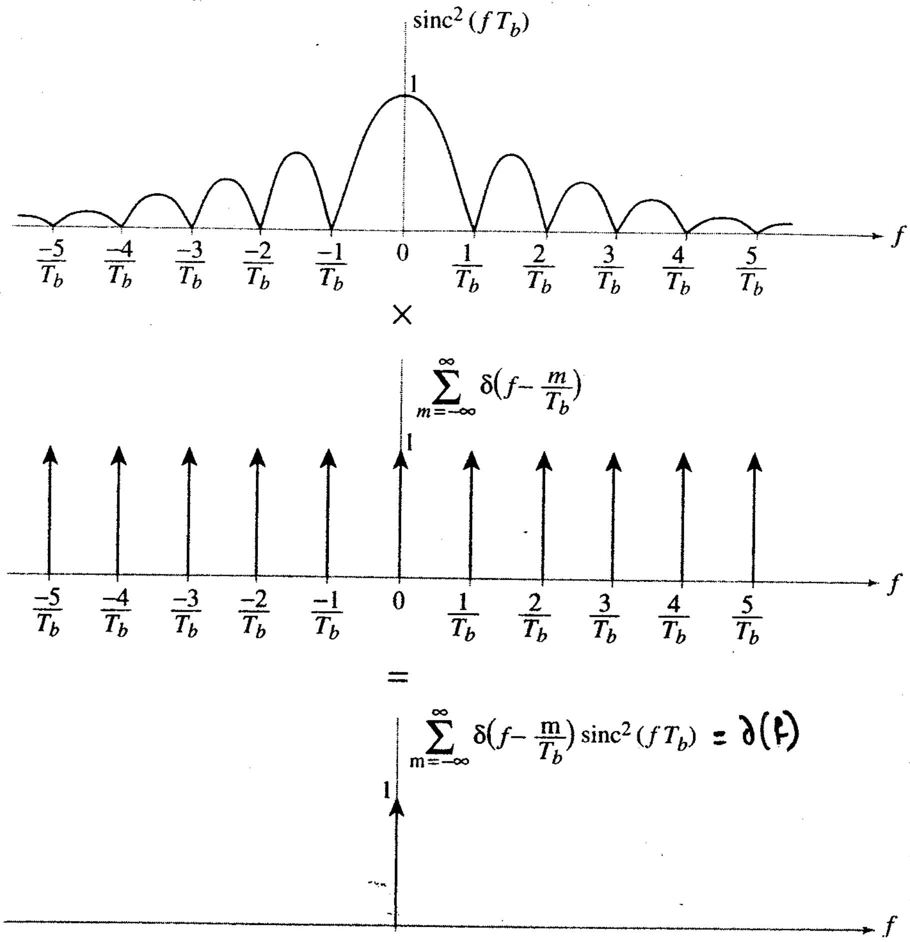


Figure (1) Multiplication of $\text{sinc}^2(f\,T_b)$ and $\sum_{m=-\infty}^{\infty} \delta(f - m/T_b)$.

Substituting eq (5) in eq (4), we get

$$\boxed{S_X(f) = \frac{a^2}{4} T_b \,\text{Sinc}^2(f T_b) + \frac{a^2}{4}\,\delta(f)}$$

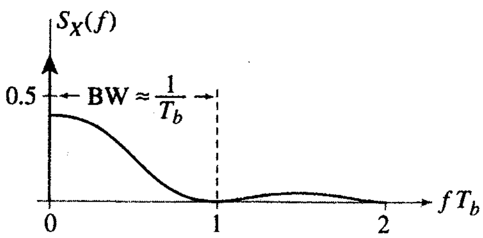


Figure (2) Normalized PSD of NRZ unipolar line code.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Polar NRZ format :-

* In polar NRZ format

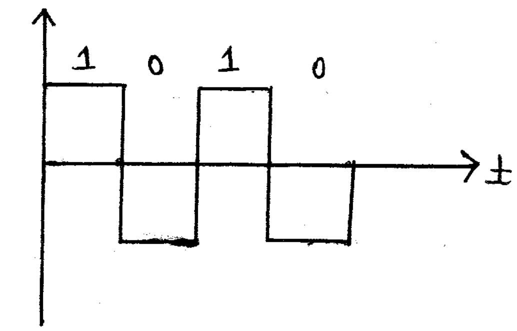
$$A_K = \begin{cases} +a & \text{for Symbol } 1 \\ -a & \text{for Symbol } 0 \end{cases}$$

* Let us assume that Symbol 0 & 1 occurs with equal probabilities

i.e. $P(A_K = +a) = 1/2$

$P(A_K = -a) = 1/2$

Polar NRZ-format



A_K	A_K^2	Equally probable
$+a$	a^2	$1/2$
$-a$	a^2	$1/2$

* WKT autocorrelation function

$$R_A(n) = E\left[A_K \cdot A_{K-n}\right]$$

Case i: for $n=0$

$$R_A(0) = E\left[A_K^2\right]$$

$$= a^2(1/2) + a^2(1/2)$$

$$= a^2/2 + a^2/2$$

$$\boxed{R_A(0) = a^2}$$

NOTE :-

$0 \rightarrow -a$
$1 \rightarrow +a$

Case ii : for $n \neq 0$

$A_K \cdot A_{K-n}$ will have four possibilities with probabilities $1/4$ each

A_K	A_{K-n}	$A_K \cdot A_{K-n}$	Equally probable
$-a$	$-a$	a^2	$1/4$
$-a$	a	$-a^2$	$1/4$
a	$-a$	$-a^2$	$1/4$
a	a	a	$1/4$

$$R_A(n) = E\left[A_K A_{K-n}\right]$$

$$= \cancel{a^2(1/4)} + \cancel{(-a^2)}(1/4) + \cancel{(-a^2)}(1/4) + \cancel{a^2(1/4)}$$

$$\boxed{R_A(n) = 0}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Thus, we may express auto correlation function $R_A(n)$ for polar NRZ format

$$R_A(n) = \begin{cases} a^2 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

* The PSD of the polar NRZ format is

$$\boxed{S_X(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi n f T_b}} \longrightarrow 1$$

WKT $V(f) = T_b \,\text{Sinc}(fT_b)$

Substituting the value of $V(f)$ & $R_A(n)$ in eq 1, we get

$$S_X(f) = \frac{1}{\cancel{T_b}} \left[T_b^{\cancel{2}}\, \text{Sinc}^2(fT_b) \right] \left[\sum_{n=0} a^2 e^{-j2\pi n f T_b} + 0 \right]$$

$$= T_b \,\text{Sinc}^2(fT_b) \left[a^2 e^0 \right]$$

$$\boxed{S_X(f) = a^2 T_b \,\text{Sinc}^2(fT_b)}$$

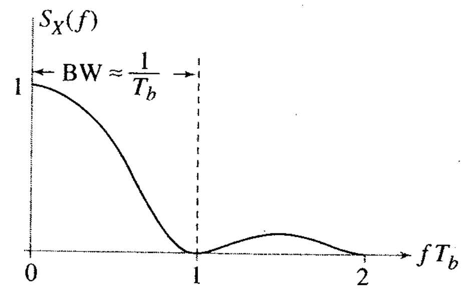
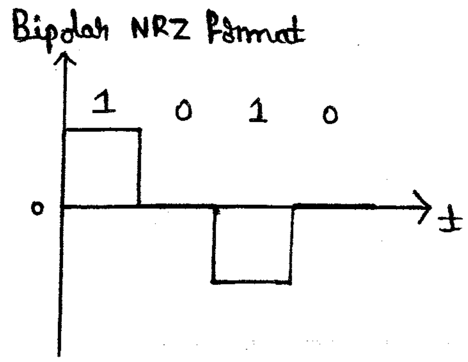


Figure Normalized PSD of NRZ polar format.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Bipolar NRZ Format :-



* In Bipolar NRZ Format

$$A_K = \begin{cases} \pm a & \text{for Symbol 1} \\ 0 & \text{for Symbol 0} \end{cases}$$

* Let us assume that Symbol '0' & '1' occur with equal probabilities

i.e. $P(A_K = 0) = 1/2$

$P(A_K = +a) = 1/4$ &

$P(A_K = -a) = 1/4$

* WKT auto correlation function

$$R_A(n) = E[A_K \cdot A_{K-n}]$$

Case i : for $n=0$, we get

$$R_A(0) = E[A_K^2]$$

$$= 0(1/2) + a^2(1/4) + (-a)^2(1/4)$$

$$\boxed{R_A(0) = \frac{a^2}{2}}$$

Case ii : for $n=1$, we get

$$R_A(1) = 0(1/4) + 0(1/4) + 0(1/4) + (-a^2)(1/4)$$

$$\boxed{R_A(1) = \frac{-a^2}{4}}$$

A_K	A_{K-1}	$A_K A_{K-1}$	Equally probable
0	0	0	1/4
0	$\pm a$	0	1/4
$\pm a$	0	0	1/4
$\pm a$	$\mp a$	$-a^2$	1/4

NOTE:

Since $\boxed{R_A(1) = R_A(-1) = \frac{-a^2}{4}}$

Case iii :- for $n > 1$

Dont write this

A_K	A_{K-1}	A_{K-2}	$A_K \cdot A_{K-2}$	Equally Probable.
0	0	0	0	1/8
0	$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	0	0	1/8
0	0	$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	0	1/8
0	$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	$1\begin{Bmatrix}-a\\+a\end{Bmatrix}$	0	1/8
$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	0	0	0	1/8
$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	$1\begin{Bmatrix}-a\\+a\end{Bmatrix}$	0	0	1/8
$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	0	$1\begin{Bmatrix}-a\\+a\end{Bmatrix}$	$1\begin{Bmatrix}-a^2\\-a^2\end{Bmatrix} = -a^2$	1/8
$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	$1\begin{Bmatrix}-a\\+a\end{Bmatrix}$	$1\begin{Bmatrix}+a\\-a\end{Bmatrix}$	$1\begin{Bmatrix}+a^2\\+a^2\end{Bmatrix} = +a^2$	1/8

When $n = 2$

$$R_A(2) = E[A_K \cdot A_{K-2}]$$

$$= [0(1/8)] \times 6 + \cancel{(-a^2)(1/8)} + \cancel{(+a^2)}(1/8)$$

$$\boxed{R_A(2) = 0}$$

* Thus, we may express auto correlation function $R_A(n)$ for Bipolar NRZ format

$$R_A(n) = \begin{cases} \dfrac{a^2}{2} & \text{for } n = 0 \\ -\dfrac{a^2}{2} & \text{for } n = \pm 1 \\ 0 & \text{for } |n| > 1 \end{cases}$$

* The PSD of the bipolar NRZ format is

$$\boxed{S_X(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}} \longrightarrow (1)$$

Substituting the values of V(f) & $R_A(n)$ in eq ①, we get

$$S_X(f) = \frac{1}{\cancel{T_b}}\left[T_b^{\cancel{2}} Sinc^2(fT_b)\right]\left[\sum_{n=-1} R_A(-1)\, e^{j2\pi f n T_b} + \sum_{n=0} R_A(0)\, e^{j2\pi f n T_b} + \sum_{n=1} R_A(1)\, e^{j2\pi f n T_b}\right]$$

$$= T_b\, Sinc^2(fT_b)\left[R_A(-1)\, e^{-j2\pi f n T_b} + R_A(0) e^{0} + R_A(1)\, e^{j2\pi f n T_b}\right]$$

W.K.T $\quad R_A(-1) = R_A(1)$

$$= T_b\, Sinc^2(fT_b)\left[R_A(1)\, e^{-j2\pi f n T_b} + R_A(0) + R_A(1)\, e^{j2\pi f n T_b}\right]$$

$$S_X(f) = T_b\, Sinc^2(fT_b)\left[R_A(1)\left(e^{j2\pi f n T_b} + e^{-j2\pi f n T_b}\right) + R_A(0)\right]$$

W.K.T. $\quad Cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$$\boxed{2Cos\theta = e^{j\theta} + e^{-j\theta}}$$

$$S_X(f) = T_b\, Sinc^2(fT_b)\left[R_A(1)\, 2Cos(2\pi f T_b) + R_A(0)\right]$$

$$= T_b\, Sinc^2(fT_b)\left[\frac{-a^2}{\cancel{4}2}\, \cancel{2}\, Cos(2\pi f\ T_b) + \frac{a^2}{2}\right]$$

$$= T_b\, Sinc^2(fT_b)\left[\frac{-a^2}{2} Cos(2\pi f T_b) + \frac{a^2}{2}\right]$$

$$= \frac{a^2}{2} T_b\, Sinc^2(fT_b) - \frac{a^2}{2} T_b\, Sinc^2(fT_b)\, Cos(2\pi f T_b)$$

$$= \frac{a^2}{2} T_b\, Sinc^2(fT_b)\left[1 - Cos(2\pi f T_b)\right]$$

W.K.T

$$Sin^2\theta = \frac{1 - Cos2\theta}{2}$$

$$\boxed{2Sin^2\theta = 1 - Cos2\theta}$$

$$= \frac{a^2}{\cancel{2}} T_b\, Sinc^2(fT_b)\left[\cancel{2} Sin^2(\pi f T_b)\right]$$

$$\boxed{S_X(f) = a^2 T_b\, Sinc^2(fT_b)\, Sin^2(\pi f T_b)}$$

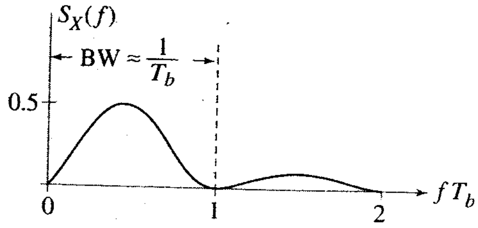
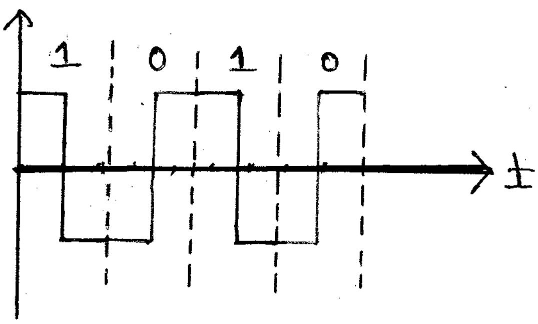


Figure PSD of bipolar NRZ format.

Manchester format :-



* Let us assume that Symbol '0' & '1' occur with equal probabilities

i.e. $P(A_K = 0) = 1/2$

$P(A_K = 1) = 1/2$

* WKT auto correlation function

$$R_A(n) = E[A_K \cdot A_{K-n}]$$

Case i :- for $n = 0$

A_K	A_K	A_K^2	Equally Probable
$0\binom{-a}{a}$	$0\binom{-a}{a}$	a^2	$1/2$
$1\binom{a}{-a}$	$1\binom{a}{-a}$	a^2	$1/2$

$$R_A(0) = E[A_K^2]$$

$$R_A(0) = a^2(1/2) + a^2(1/2)$$

$$\boxed{R_A(0) = a^2}$$

Case ii :- for $n \neq 0$

A_K	A_{K-1}	$A_K A_{K-1}$	Equally Probable
$0\binom{-a}{a}$	$0\binom{-a}{a}$	$-a^2$	$1/4$
$0\binom{-a}{a}$	$1\binom{a}{-a}$	$-a^2$	$1/4$
$1\binom{a}{-a}$	$0\binom{-a}{a}$	$-a^2$	$1/4$
$1\binom{a}{-a}$	$1\binom{a}{-a}$	a^2	$1/4$

$$R_A(n) = a^2(\cancel{1/4}) - a^2(\cancel{1/4}) - a^2(\cancel{1/4}) + a^2(\cancel{1/4})$$

$$\boxed{R_A(n) = 0}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

* Thus, we may express auto correlation function $R_A(n)$ for Manchester format

$$R_A(n) = \begin{cases} a^2 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

* The PSD of the manchester format is

$$\boxed{S_X(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}} \longrightarrow (1)$$

W.K.T. for manchester format $V(f)$ is

$$\boxed{|V(f)|^2 = T_b^2 \sin^2\left(\frac{\pi f T_b}{2}\right) \text{Sinc}^2\left(f \frac{T_b}{2}\right)}$$

Substituting the value of $V(f)$ & $R_A(n)$ in eq (1), we get

$$S_X(f) = \frac{1}{\cancel{T_b}} T_b^{\cancel{2}} \sin^2\left(\frac{\pi f T_b}{2}\right) \text{Sinc}^2\left(f \frac{T_b}{2}\right) \left[\sum_{n=0} R_A(0) e^{-j2\pi f n T_b} + 0 \right]$$

$$= T_b \sin^2\left(\frac{\pi f T_b}{2}\right) \text{Sinc}^2\left(f \frac{T_b}{2}\right) \cdot a^2 e^0$$

$$\boxed{S_X(f) = a^2 T_b \,\text{Sinc}^2\left(f\, T_b/2\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

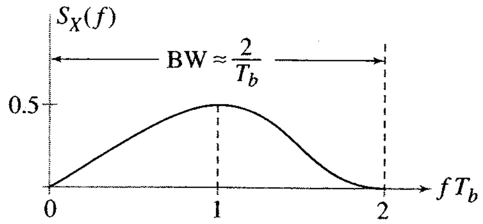
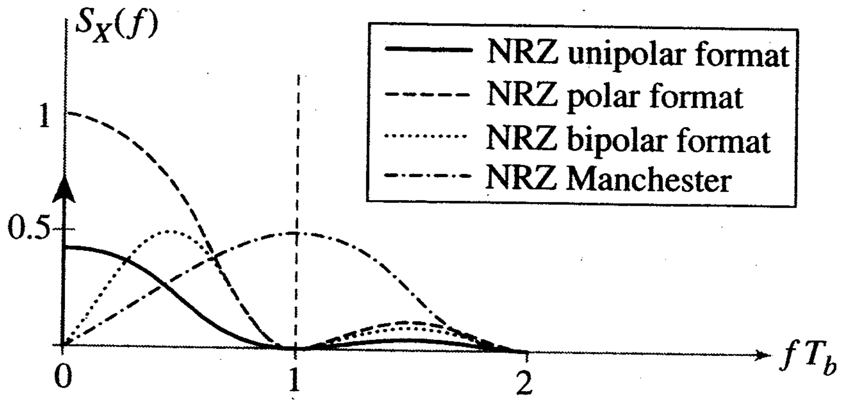


Figure Normalized power spectrum of Manchester format.

Comparision of PSD for binary line Codes :-



Comparison of PSD for binary line codes.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

ARUNKUMAR G M.Tech
Lecturer in E&CE Department
S.T.J.I.T.. Ranebennur

SPREAD SPECTRUM MODULATION

Introduction:

* The digital modulation techniques are more concerned about the efficient utilization of bandwidth & transmitted power.

* There are some other applications where it is necessary for the system to resist external interference & to make it difficult for unauthorized receiver to receive the message being transmitted i.e. Secure communication.
  Such communication is called Spread Spectrum Modulation.

* The advantage of a Spread-Spectrum communication system is its ability to reject interference whether it be the unintentional interference of another user simultaneously attempting to transmit through the channel or the intentional interference of a hostile transmitter attempting to jam the transmission.

---

* What is Spread Spectrum Communication? What is its primary advantage? What are the commonly used SS-technique

  June-09, 7M

* What is Spread Spectrum technique? How are they classified?

  Jan-06, 6M

July-05, 6M	July-08, 4M

The definition of SS may be Stated in two parts :

1) It is a means of transmission in which the transmitted data Sequence occupies a larger bandwidth than the minimum bandwidth necessary to Send the data.

2) Spreading of data is done before transmission through the Channel using a Code which is independent of data Sequence. The Same Code is used at the receiving end to despread the received Signal So that original data may be recovered. ⟶ 2-Marks

* The primary advantage of a SS - Communication System is its ability to reject interference whether it be the - unintentional Interference of another user Simultaneously attempting to transmit through the Channel or the intentional Interference of a hostile transmitter attempting to jam the transmission. ⟶ 3-Marks

Classification of Spread Spectrum Systems :-

1) Direct Sequence - SS
2) Frequency-hopping - SS
3) Time - hopping - SS
4) Chirp - SS
5) Hybrid SS
6) pulsed FM - SS

⟶ 2-Marks

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

PN Sequence :-

July-07, 6M

Discuss the Pseudo-Noise (PN) Sequence with a neat diagram - Showing the maximum length (ML) Sequence generator.

* A Pseudo-noise (PN) Sequence is defined as a Coded Sequence of 0's & 1's with certain autocorrelation properties.

* The PN Sequence used in SS- Communication are periodic.

* The Length of the PN-Sequence is given by

$$\boxed{N = 2^m - 1}$$

Where, m → Number of flip-flops

ex:-
With 3-FF's
$N = 2^3 - 1$
$N = 7$ - bits

* Using m-Stage Shift registers (m-FF's), it is possible to generate a periodic Sequence i.e. $2^m - 1$ bits. Such Sequences are also called Maximum length (ML) Sequence.

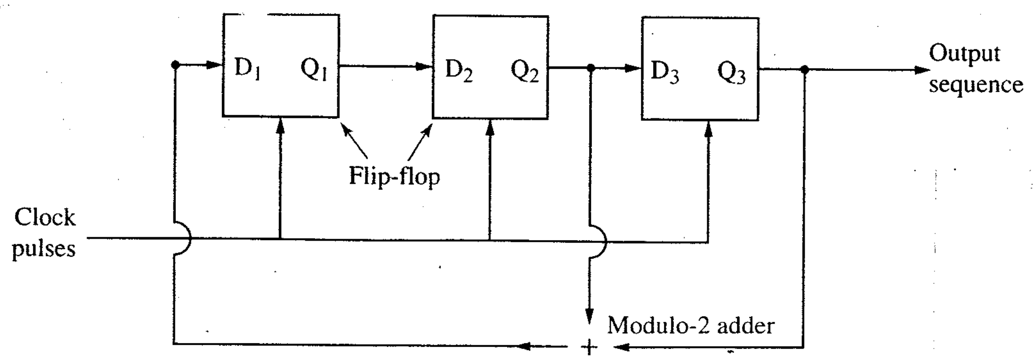


Fig ①: 3 stage maximum length sequence generator.

Clock pulse	Present state (Q_1, Q_2, Q_3)	Next state $(Q_2 \oplus Q_3, Q_1,$
(Initial)	(1,1,0)	(1,1,1)
1	(1,1,1)	(0,1,1)
2	(0,1,1)	(0,0,1)
3	(0,0,1)	(1,0,0)
4	(1,0,0)	(0,1,0)
5	(0,1,0)	(1,0,1)
6	(1,0,1)	(1,1,0)
7	(1,1,0)	(1,1,1)

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Fig ① Shows a ML Sequence generator having 3-Stages.

* The Shift register operation is controlled by a Sequence of Clock pulses. For every clock pulse, the contents of each Stage of the register is Shifted by one position to the right.
* For every clock pulse the contents of 2^{nd} & 3^{rd} Stages are modulo-2 added & the result is fed back to the 1^{st} Stage.
* The o/p of the Shift register is taken at the last Stage of the flip-flop i.e. Q_3.
* The length of PN-Sequence is given by:

$$N = 2^m - 1 = 2^3 - 1$$

$$\boxed{N = 7\text{-bits}}$$

* For an initial State 1,1,0 the o/p Sequence is

$$\underbrace{0,1,1,1,0,0,1,}_{N=7}\ \underbrace{0,1,1,1,0,0,1}_{N=7}\ldots$$

* The PN Sequence is periodic with period equal to 7-bits.

NOTE :-

* Suppose initially all the contents of the Shift registers are Zero i.e. $Q_1 = Q_2 = Q_3 = 0$.
* Modulo-2 addition of Q_2 & Q_3 is '0', which is fed back to Stage 1.
* Hence, when the clock pulses are applied, only 0's are Shifted at I/p D_1, Shift register State remain at State 000 & the o/p Sequence i.e. 'Q_3' is a Sequence of all 0's.

Properties of PN Sequence :-

July-09, 4M

1) Specify the properties of ML-Sequences possessed by a random binary Sequence.

2) Explain properties of PN Sequence July-06, 8M

There are 3 properties of PN Sequence :

1) Balanced property
2) Run property
3) Auto correlation property.

1) Balanced property :-

In each period of a ML-Sequence, the number of 1's is always one more than the number of 0's. (i.e. no of 1's exceeds the no of 0's by one).

ex :- For 3-Stage Shift register,

$$N = 2^3 - 1$$

$\boxed{N = 7}$ i.e. 0010111

No of 1's $\rightarrow$ 4

No of 0's $\rightarrow$ 3

2) Run property :-

A run is defined as a Subsequence of identical Symbols within the ML-Sequence. The length of the Subsequence is known as the run-length.

* The total number of runs = $\frac{(N+1)}{2}$.

{ In ML-Sequence :

i) one-half the runs are of length one

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

ii) one-fourth the runs are of length two.
iii) one-eigth the runs are of length three.
}

ex:- 0010111

* Total No of runs $= \frac{N+1}{2} = \frac{7+1}{2} = $ 4-runs.

00, 1, 0, 111 = 4 runs.

{
i) 1,0 → Two runs are of length one
ii) 00 → one run are of length two
iii) 111 → one run are of length three.
}

3) Auto correlation property :-

The auto correlation function of a ML-Sequence is periodic and binary valued.

$$\boxed{R_c(K) = \frac{1}{N} \sum_{n=1}^{N} c_n c_{n-K}}$$

Where,

N is the length or period of the PN Sequence &

K is the lag of the autocorrelation Sequence &

$$R_c(K) = \begin{cases} 1 \; ; & \text{for } K = lN \\ -1/N \; ; & \text{for } K \neq lN \end{cases}$$ Where 'l' is any Integer.

Notion of Spread Spectrum :- $\boxed{\text{Jan-10, 6M}}$ $\boxed{\text{Jan-09, 6M}}$

* Explain the working of direct Sequence Spread Spectrum transmitter & receiver. (DS-SS) $\boxed{\text{Jan-06, 8M}}$
* Explain the principle of DS-SS Communication System $\boxed{\text{July-06, 8M}}$
* Define SS. Explain the principle of DS-SS System $\boxed{\text{Jan-08, 9M}}$

4) Define the spread spectrum develop a model for the base band spread spectrum system.

July-05, 6M

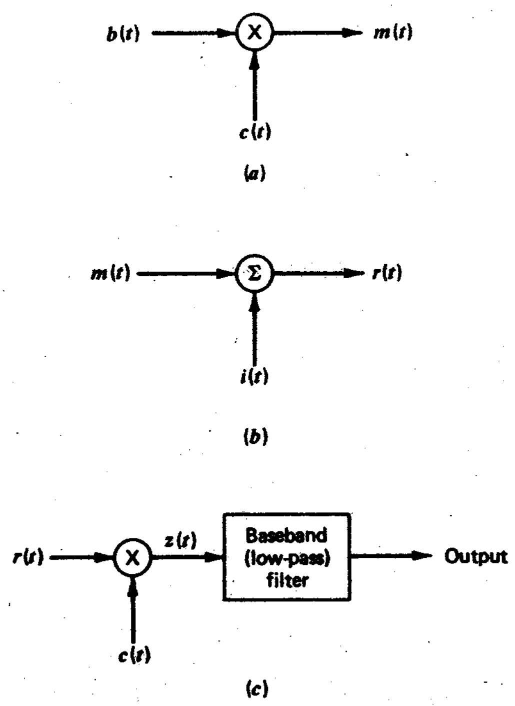


Fig ①: Idealized model of baseband spread-spectrum system. (a) Transmitter. (b) Channel. (c) Receiver.

* The baseband DS-SS does not use any digital modulation techniques.
* The digital signals bandwidth is widen by using only spreading codes (PN sequence).
* The baseband signal $b(t)$, which is of low bandwidth (Narrowband) is multiplied with the Wideband signal $c(t)$ to obtain spread spectrum signal $m(t)$ as shown in fig ⓐ.

i.e. $$\boxed{m(t) = b(t) \cdot c(t)} \longrightarrow ①$$

* The $m(t)$ is transmitted through the channel where additive noise $i(t)$ is added (fig b).
* The received signal consists of transmitted signal $m(t)$ plus an additive interference denoted by $i(t)$

i.e. $$\boxed{r(t) = m(t) + i(t)} \longrightarrow ②$$

Substituting eq ① in eq ②, we get

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{r(t) = c(t)\, b(t) + i(t)} \longrightarrow (3)$$

* To recover the original data sequence $b(t)$, the received signal $r(t)$ is applied to a demodulator that consists of a multiplier followed by a LPF.

* The multiplier is supplied with a locally generated PN-sequence i.e. an exact replica of that used in the transmitter.

$$\boxed{Z(t) = c(t) \cdot r(t)} \longrightarrow (4)$$

Substituting eq (3) in eq (4), we get

$$Z(t) = c(t)\left[c(t)\, b(t) + i(t)\right]$$

$$Z(t) = c^2(t) \cdot b(t) + c(t) \cdot i(t)$$

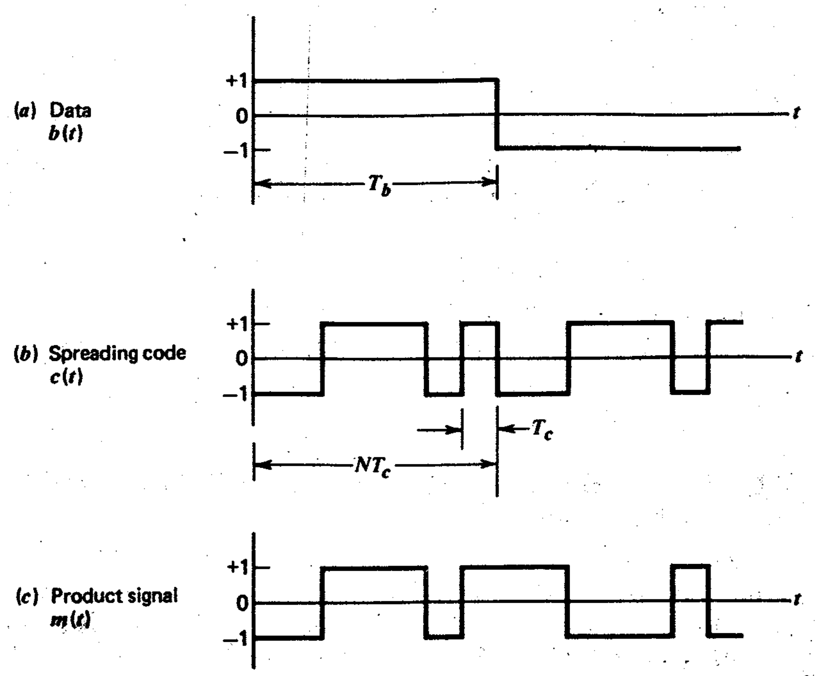
Where $c^2(t) = 1$ for all 't'

$$\boxed{Z(t) = b(t) + c(t) \cdot i(t)} \longrightarrow (5)$$

* From eq (5), $b(t)$ can be recovered by passing $Z(t)$ through a LPF, which removes the effect of the interference represented by $c(t)\, i(t)$ & reproduces the original data.

P.T.O

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*



Illustrating the waveforms in the transmitter of Fig. 1

Direct Sequence Spread Coherent Binary phase Shift Keying (DS-BPSK):-

1) Explain with block diagram the model of DS-SS BPSK System. July-08, 10M

2) Explain with a neat block diagram, the direct Sequence Spread Coherent PSK transmitter & receiver. Jan-10, 8M

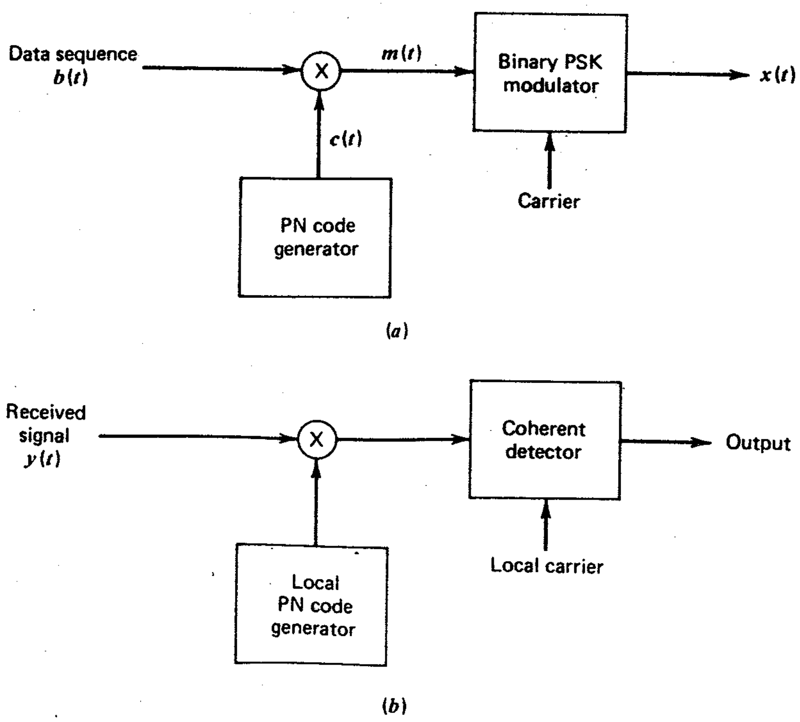


Figure 9 Direct-sequence spread coherent phase-shift keying. (a) Transmitter. (b) Receiver.

The transmitter involves two stages of modulation

* In 1st stage, the data sequence $b(t)$ is modulated with the code sequence $c(t)$. So the spread signal is

$$\boxed{m(t) = b(t) \cdot c(t)}$$

* In 2nd stage, the spread signal $m(t)$ is modulated with the binary PSK modulation.

Table Truth Table for Phase Modulation $\theta(t)$, Radians

		Polarity of Data Sequence $b(t)$ at Time t	
		+	−
Polarity of PN sequence	+	0	π
$c(t)$ at time t	−	π	0

* When the polarity of $b(t)$ & $c(t)$ are same, the product $b(t) \cdot c(t) = 1$, hence the phase of the BPSK signal is $(2\pi f_c t)$ radians.

* IIly when $b(t)$ & $c(t)$ are of different polarities the product $b(t) \cdot c(t) = -1$, hence the phase of the BPSK signal is $(2\pi f_c t + \pi)$ radians.

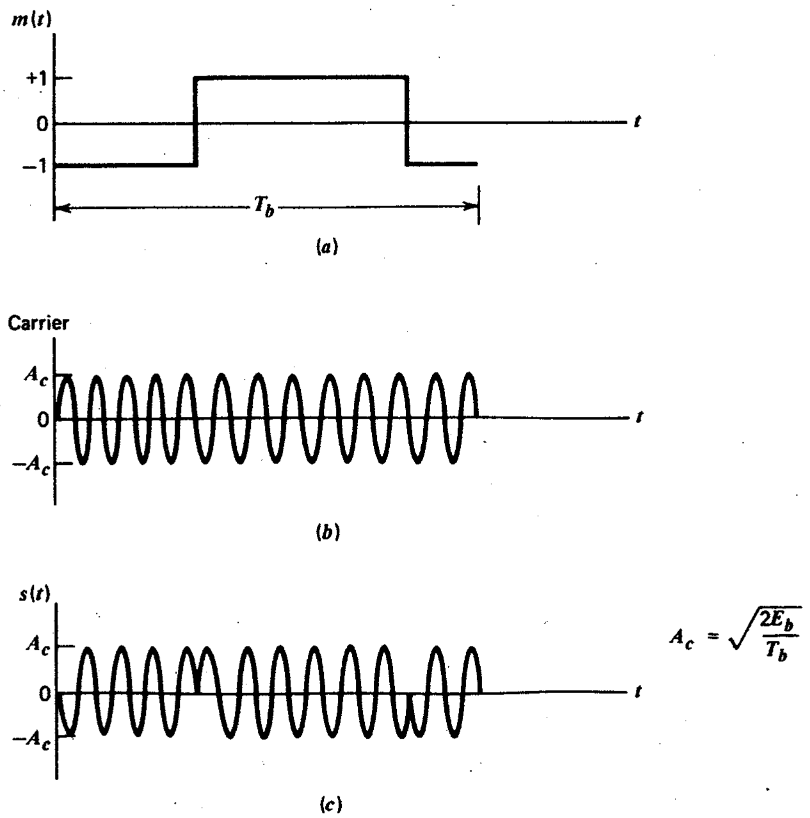


Figure (a) Product signal $m(t) = c(t)b(t)$. (b) Sinusoidal carrier. (c) DS/BPSK

The receiver consist of two stages of demodulation.

* In 1st stage demodulation, the received signal $y(t)$ & a locally generated replica of the PN sequence are applied to a multiplier.

* In 2nd stage, $m(t)$ is despread by multiplying it by $c(t)$ i.e. it consists of a coherent detector, the o/p which provides an estimate of the original data sequence.

Model for analysis :-

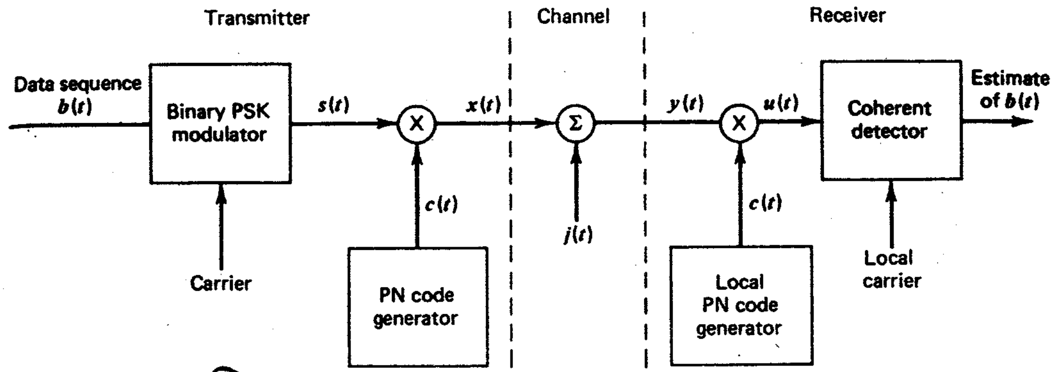


Figure (3) Model of direct-sequence spread binary PSK system.

* The spread spectrum i.e. $m(t) = b(t) \cdot c(t)$ can also be performed performed prior to phase modulation.

* For analysis purpose, the spectrum spreading & the BPSK are interchanged because both are linear operation.

* In this model, it is assumed that the interference $j(t)$ limits performance, so that the effect of channel noisy may be ignored.

From fig (3), channel o/p is given by:

$$y(t) = x(t) + j(t)$$

$$\boxed{y(t) = c(t)\, s(t) + j(t)} \longrightarrow (1)$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Where,

$S(t)$ is the binary PSK Signal &

$c(t)$ is the PN Sequence.

* In the receiver, the received Signal $y(t)$ is 1st multiplied by the PN Sequence $c(t)$.

Thus $\boxed{u(t) = y(t) \cdot c(t)} \longrightarrow$ ②

Substituting eq ① in eq ②, we get

$$u(t) = \left[c(t) \cdot S(t) + j(t)\right] c(t)$$

$$u(t) = \underline{c^2(t)}\, S(t) + j(t) \cdot c(t) \qquad \because c^2(t) = 1$$

$$\boxed{u(t) = S(t) + c(t)\, j(t)}$$

* This $u(t)$ is fed to Coherent detector to detect the information Signal $b(t)$.

---

performance of DS-SS System :-

The performance of DS-SS System can be evaluated on the basis of processing gain & probability of error.

processing gain (PG) :- PG is defined as the ratio of the bandwidth of Spreaded message Signal to the bandwidth of the unspreaded data Signal i.e.

$$\boxed{PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of unspreaded Signal}}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Where BW of data Signal $R_b = \frac{1}{T_b}$

BW of Spreaded message Signal $W_c = \frac{1}{T_c}$

$$PG = \frac{1/T_c}{1/T_b}$$

$$\boxed{PG = \frac{T_b}{T_c}} \longrightarrow (1)$$

WKT one bit period T_b of data Signal is equal to 'N' bits periods of PN Code Signal

i.e. $\boxed{T_b = NT_c} \longrightarrow (2)$

Substituting eq (2) in eq (1), we get

$$PG = \frac{N\cancel{T_c}}{\cancel{T_c}}$$

$$\boxed{PG = N}$$

Probability of error for DS-BPSK System :-

$$\boxed{P_e \approx \frac{1}{2}\, \text{erfc}\left(\sqrt{E_b/JT_c}\right)}$$

Where

J is the average Interference power.

P.T.O

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Jamming Margin & Antijam Characteristics:

WKT for Coherent PSK

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

* In DS-BPSK, the Interference may be treated as Wideband noise of power Spectral density $N_0/2$ defined by:

$$\frac{N_0}{\cancel{2}} = \frac{J T_c}{\cancel{2}}$$

$$\boxed{N_0 = J T_c}$$

Since the bit energy $E_b = P T_b$.

Where, P is the average Signal power &

T_b is the bit duration

* We may express the bit energy to Noise density ratio as:

$$\frac{E_b}{N_0} = \frac{P T_b}{J T_c}$$

$$\frac{E_b}{N_0} = \frac{T_b}{T_c} \quad \frac{P}{J}$$

$$\frac{E_b}{N_0} = PG\,\frac{P}{J}$$

$$\boxed{\frac{J}{P} = \frac{PG}{E_b/N_0}}$$

The ratio of J/P is termed as Jamming margin.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\begin{pmatrix}\text{Jamming}\\ \text{margin}\end{pmatrix}_{dB} = 10\log_{10}\left(\frac{PG}{E_b/N_0}\right)$$

$$= 10\log_{10}(PG) - 10\log_{10}(E_b/N_0)_{min}$$

$$\boxed{\begin{pmatrix}\text{Jamming}\\ \text{margin}\end{pmatrix}_{dB} = (PG)_{dB} - 10\log_{10}(E_b/N_0)_{min}}$$

---

* Explain how a PN Sequence performs the operation of a Spreading Code with related equations [June-09, 4M]

* What is Spread Spectrum? What is the role of PN Code in Spread Spectrum? [July-08, 4M]

If data Sequence $b(t)$ is narrow band & PN Sequence $c(t)$ is wideband;

Transmitted Signal $m(t) = c(t)\cdot b(t)$

Received Signal $r(t) = \underline{m(t)} + i(t)$

$$r(t) = b(t)\,c(t) + i(t)$$

Where $i(t)$ is interference Signal.

* The received Signal is demodulated by using a locally generated PN Sequence

$\therefore$ Demodulated Signal

$$z(t) = c(t)\cdot r(t)$$

$$= c(t).\left[\,b(t)\,c(t) + i(t)\right]$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$Z(t) = b(t).c^2(t) + c(t)\,i(t)$$

* As $c(t)$ alternates between -1 & $+1$, it is destroyed when squared as $c^2(t) = 1$ for all 't'.

$$\therefore \quad Z(t) = \underbrace{b(t)}_{\text{Narrow band Signal}} + \underbrace{c(t)\,i(t)}_{\text{Spreading code affects only } i(t)}$$

* Then $Z(t)$ is passed through LPF to extract only $b(t)$.

NOTE :- Refer Fig ①, for above Question.

NOTE :-

* In DS-SS technique, the ability to overcome jamming is determined by the processing gain of system.

WKT $\boxed{PG = N}$

i.e. PG depends on the length of PN sequence.

* As the number of bits 'N' of PN sequence are increased per data bits, the bandwidth of o/p signal becomes more. Hence PG is also more.

* But there is limitation of the physical devices which generates PN sequence. Hence very large bandwidth are not possible with direct sequence modulation.

To overcome this problem, frequency hop spread spectrum is used.

Frequency Hop - Spread Spectrum :-

Jan-10, 2M

* What is meant by frequency-hop spread spectrum?

The SS in which the carrier hops randomly from one frequency to another is called "Frequency Hop" spread spectrum technique.

The modulation used is MFSK (M-ary FSK).

∴ The combination of FH & MFSK is referred to as FH/MFSK.

P.T.O

Classification of FH-SS :-

FH-SS are of two-types:

1) Slow FH-SS

2) Fast FH-SS

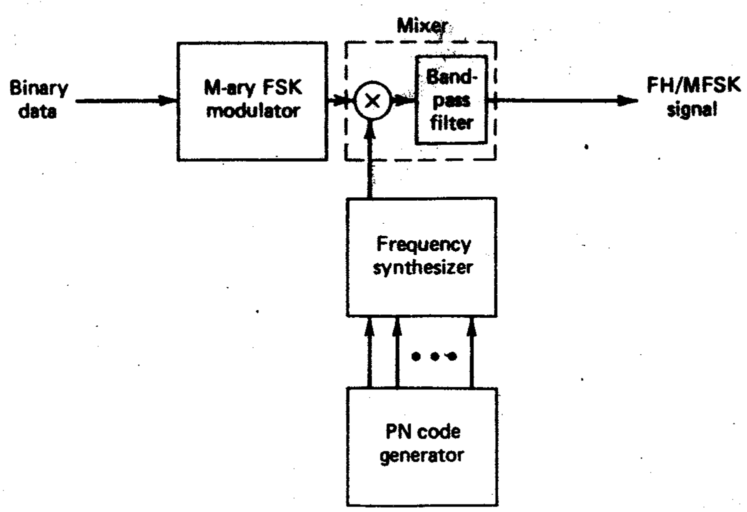
1) Slow FH-SS, in which the Symbol rate 'R_s' of the MFSK Signal is an integer multiple of the hop rate 'R_h'. i.e. Several Symbols are transmitted on each frequency hop.

2) Fast FH-SS, in which hop rate 'R_h' is an integer multiple of the MFSK Symbol rate 'R_s' i.e. the carrier frequency will change or hop Several times during the transmission of one Symbol.

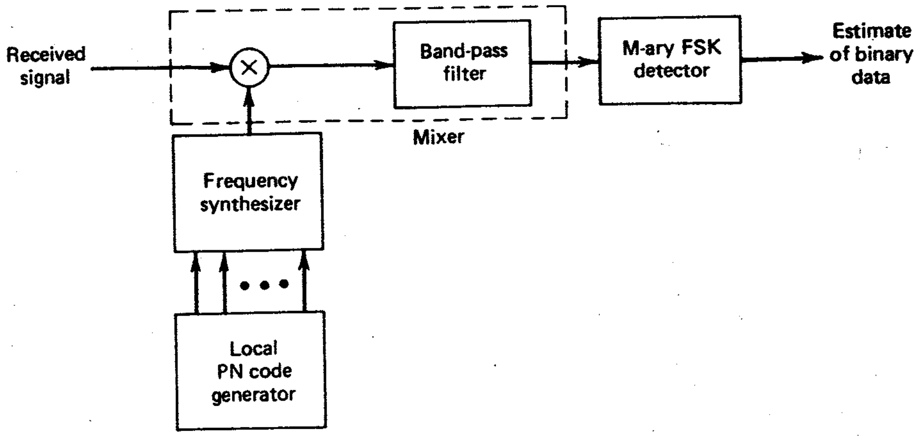
Slow-frequency hopping :-

July-09, 6M	Jan-10, 8M	Jan-07, 10M

In a Slow FH/MFSK System one or more message Symbols are transmitted per hop.



(a)



(b)

Figure Frequency hop spread M-ary frequency-shift keying. (a) Transmitter. (b) Receiver.

* Fig (a) Shows the block diagram of an FH/MFSK transmitter, which involves frequency modulation followed by mixer.

* 1^{st} the incoming data is applied to M-ary FSK System. The resulting modulated wave & the o/p from a digital frequency synthesizer are then applied to a mixer that consists of a multiplier followed by a filter.

* The BPF is designed to select the sum frequency component resulting from the multiplication process.

* PN sequence will drive the frequency synthesizer, which enables carrier frequency over 2^K distinct values.

* on single hop, bandwidth of the transmitted signal is same as that resulting from the use of conventional MFSK format.

* for complete range of 2^K - frequency hops, FH/MFSK signal occupies a much larger bandwidth.

* The Non-Coherent MFSK detector is used

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Fig (b) Shows block diagram of FH/MFSK receiver.

* The frequency hopping is 1^{st} removed by mixing the received Signal with the o/p of a local frequency Synthesizer that is Synchronously Controlled in the Same manner as that in the transmitter.
* The o/p of mixer is then passed through a BPF which Selects the difference frequency Component from the mixer.
* The o/p of a BPF is MFSK Signal which is demodulated using non-Coherent MFSK detector to get original data.
* In FH System an FH tone of Shortest duration is referred as "Chip".
* The Chip rate R_c for an FH-System is defined by

$$\boxed{R_c = \max(R_h, R_s)}$$

Where
R_h is the hop rate
R_s is the Symbol rate.

$$\therefore \boxed{R_c = R_s}$$

$$\therefore \boxed{R_c = R_s = \frac{R_b}{K} \geq R_h}$$

Where $K = \log_2 M$

NOTE:- In Slow FH-SS

$$\boxed{\text{Symbol rate } `R_s' > \text{Hop rate } `R_h'}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

processing gain :-

$$PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of unspreaded Signal}} = \frac{\text{BW of FH- Signal}}{\text{BW of baseband Signal}}$$

* Let 'f_s' be the Symbol frequency.
* WKT there are 2^K frequency hops generated because of K-bits of PN Sequence.

$\therefore$ BW of the FH-Signal is $2^K f_s$ &

BW of the unspreaded Signal is f_s.

$$PG = \frac{2^K \cancel{f_s}}{\cancel{f_s}}$$

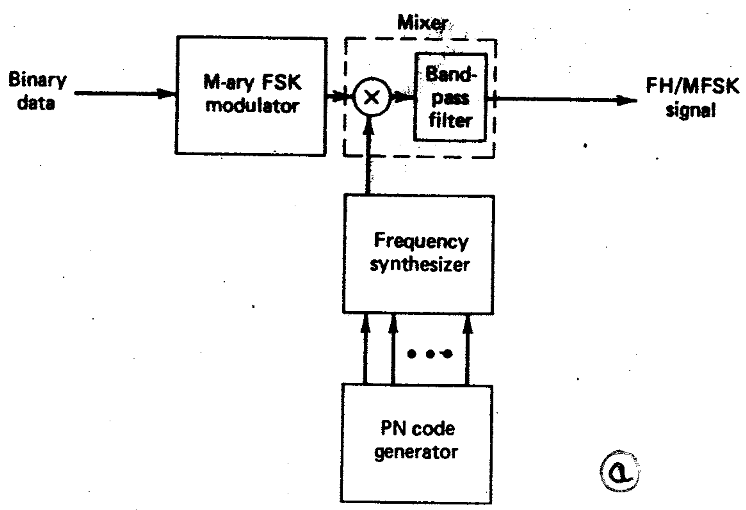
$$\boxed{PG = 2^K}$$

Fast - frequency hopping - SS :-

Jan-10, 10M	July-07, 8M

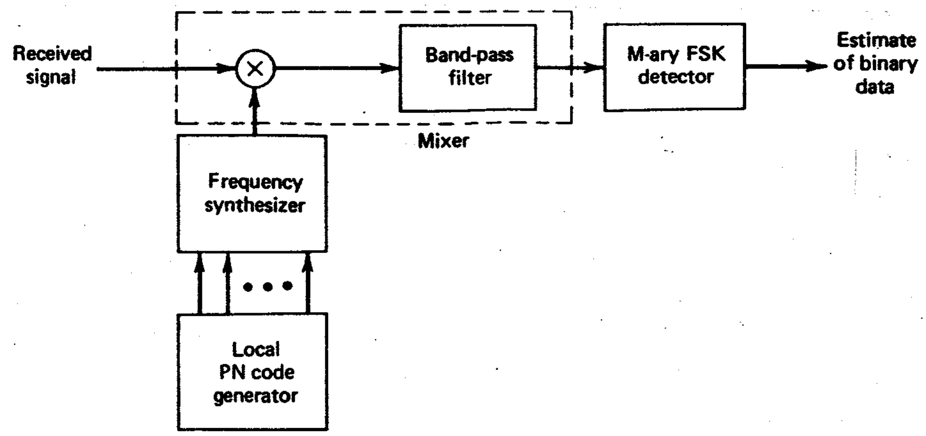
* In fast FH-SS, multiple hops are used to transmit one Symbol

$$\boxed{\text{Hop rate } 'R_H' > \text{Symbol rate } 'R_s'}$$



(a)

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.



(b)

Figure Frequency hop spread M-ary frequency-shift keying. (a) Transmitter. (b) Receiver.

* Fig (a) Shows the block diagram of an FH/MFSK transmitter, which involves frequency modulation followed by mixer.

* 1st the incoming data is applied to M-ary FSK System. The resulting modulated wave & the O/P from a digital frequency Synthesizer are then applied to a mixer that Consists of a multiplier followed by a filter.

* The BPF is designed to Select the Sum frequency Component resulting from the multiplication process.

* PN Sequence will drive the frequency Synthesizer, which enables carrier frequency over 2^K distinct values.

* on Single hop, bandwidth of the transmitted Signal is Same as that resulting from the use of Conventional MFSK format.

* for Complete range of 2^K- frequency hops, FH/MFSK Signal occupies a much larger bandwidth.

* The Non- Coherent MFSK detector is used.

Fig (b) Shows block diagram of FH/MFSK receiver.

* The frequency hopping is 1st removed by mixing the received

Signal with the o/p of a local frequency synthesizer that is synchronously controlled in the same manner as that in the transmitter.

* The o/p of mixer is then passed through a BPF which selects the difference frequency component from the mixer.

* The o/p of a BPF is MFSK signal which is demodulated using non-coherent MFSK detector to get original data.

* In FH system an FH tone of shortest duration is referred as "chip".

* The chip rate R_c for an FH-system is defined by

$$\boxed{R_c = \max(R_h, R_s)}$$

∴ Chip rate equal to hop rate

i.e. $\boxed{R_c = R_h}$

* The advantage of fast FH-SS is that before the jammer tries to complete reception of one symbol, the carrier frequency is changed.

Advantages of Fast FH-SS :-

1) Greatest amount of spreading i.e. system bandwidth are very large.

2) More secured than DS-SS

3) Fast FH-SS have relatively short acquisition time.

4) The distance effect is less.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Disadvantages :-

1) Frequency Synthesizer is Complex
2) Coherent demodulation is not possible.
3) Fast FH-SS need error correction.

Comparison b/w Slow & Fast FH-SS :-

Jan-07, 5M

Advantages of direct Sequence-SS (DS-SS) :-

1) Best Antijam performance.
2) Best Noise performance.
3) Coherent demodulation of the SS-Signal is possible.
4) The generation of the Coded Signal is easy. It Can be done by a Simple multiplication.
5) DS-SS has best discrimination against multipath Signals.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Disadvantages of DS-SS :-

1) It has long acquisition time.
2) Susceptible to the Near-Far problem.
3) The PN generator should generate sequence at high rates.
4) It requires wideband channel with small phase distortion.

Advantages of Spread Spectrum System :-

June-05, 6M

1) Best antijam performance
2) Best Noise performance
3) Secured Communication
4) Multipath protection
5) Improved Spectral efficiency
6) Multiple access Capability
7) Interference rejection.

Application of Spread Spectrum modulation :-

July-09; 8M

1) i) Mention the applications of SS-Communication 4M
   ii) Multipath Suppression & Range determination 4M
2) Write the applications of SS-techniques July-09, 3M
3) Explain the application of SS-technique to CDMA Jan-07, 5M
4) Briefly discuss the applications of SS-techniques July-07, 8M

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) Most important application of SS- technique is in the field of Secure Communication where it has to provide immunity against jamming & low probability of Interception.

2) Code Division Multiple Access

3) Multipath Suppression

4) Range determination

CDMA :-

5-Marks

The most common multiple access techniques for Satellite Communications are FDMA & TDMA

* In FDMA all users access the Satellite Channel by transmitting - Simultaneously but using disjoint frequency bands (i.e different carrier frequencies are used for each Channel).

* In TDMA all users occupy the Same RF bandwidth of the Satellite Channel, but they transmit Sequentially in time.

* If all the users wants to transmit Simultaneously & also occupy the Same RF bandwidth of the channel, then CDMA can be used.

* In CDMA, each user is assigned a Code of its own, which performs the direct Sequence or frequency hop Spread Spectrum modulation.

* In CDMA:
  1) Each Code is approximately orthogonal
  2) The CDMA System operates asynchronously.

* The CDMA has 3 advantages over TDMA :

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) CDMA does not require an external Synchronization network.

2) CDMA offers a gradual degradation in performance as the number of users is increased.
∴ It is easy add new users to the System.

3) CDMA offers an external interference rejection capability.

Multipath Suppression :-

* In many radio channels, the transmitted Signal reaches the receiver I/p via more than one path.
* In mobile Communication, the transmitted Signal reflects - from trees, buildings, moving vehicles etc. Thus the transmitted Signal reach the receiver by Several other indirect path (Instead direct path).
* The received Signals from Indirect path may have different attenuations & different delays. The interference caused by these indirect path is called multipath Interference or Simply multipath, results in Fading.

(The variation in received Signal amplitude due to multipath interference is called fading.)

Range determination using DS-SS :-

* In this application, only the PN Sequence is transmitted. The reflected Signal is received & Correlated with delayed version of the Same Sequence.
* The delay 'D' for which the Correlator o/p is maximum corresponds

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

to two-way path delay of the Signal.

∴ Rang 'd' of the target is

$$\boxed{d = \frac{1}{2} CD}$$

Where 'c' is velocity of light

* The precision of the measurement depends on the chip-duration 'T_c' & hence range 'd' is given by

$$\boxed{d = \frac{c}{2}\left[D \pm T_c\right]}$$

Comparison b/w Slow & Fast FH-SS :-

Jan-07, 5M

Sl No	Slow FH-SS	Fast FH-SS
1)	Multiple Symbols are transmitted in one frequency hop.	Multiple hops are taken to transmit one Symbol
2)	Symbol rate = Chip rate ($R_s = R_c$)	Hop rate = Chip rate ($R_h = R_c$)
3)	Hop rate is lower than Symbol rate ($R_h < R_s$)	Hop rate is higher than Symbol rate ($R_h > R_s$)
4)	one or more Symbols are transmitted over the Same Carrier frequency	one Symbol is transmitted over multiple Carriers in different hops.
5)	Less Secure than fast FH-SS	More Secure than Slow FH-SS.
6)	$PG = 2^K$	$PG = 2^K$

* With related equations define the following terms with respect to interference & Jammer Signal $j(t)$;

i) variance $var[V_{cj}/j]$

ii) $(SNR)_o$

iii) processing gain (PG)

June-09, 6M

i) variance

$$\boxed{V_{cj} = var[V_{cj}/j] = \frac{1}{N} \sum_{K=0}^{N-1} j_K^2}$$

Where Spread factor $N = \frac{T_b}{T_c}$

$$\therefore \boxed{var[V_{cj}/j] = \frac{JT_c}{2}}$$

Where J = Jammer power

T_c = Chip duration

T_b = bit duration

ii) $$(SNR)_o = \frac{\text{Signal power}}{\text{Noise power}} = \frac{P}{N_o} = \frac{E_b}{\frac{JT_c}{2}}$$

$$\boxed{(SNR)_o = \frac{2E_b}{JT_c}} \longrightarrow (1)$$

* The average Signal power at the receiver I/p equals E_b/T_b

$\therefore$ I/p Signal-to-Noise ratio as

$$(SNR)_I = \frac{E_b/T_b}{J} = \frac{E_b}{JT_b} \longrightarrow (2)$$

Sub eq (2) in eq (1), we get

$$(SNR)_o = \frac{2E_b}{JT_c} = \frac{2}{T_c} \cdot \frac{E_b}{J} \longrightarrow (3)$$

x^{ly} & ÷ eq (3) by T_b

$$(SNR)_o = \frac{2}{T_c} T_b \left(\frac{E_b}{T_b J}\right)$$

$$\boxed{(SNR)_o = \frac{2T_b}{T_c} (SNR)_I}$$

iii) processing gain (PG) :-

$$PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of unspreaded Signal}}$$

$$\boxed{PG = \frac{T_b}{T_c}} \longrightarrow (1)$$

WKT $T_b = NT_c$

$$PG = \frac{N\cancel{T_c}}{\cancel{T_c}}$$

$$\boxed{PG = N}$$

FORMULAE :

1) Maximum Length (ML) Sequence or PN Sequence

$$\boxed{N = 2^m - 1}$$

Where 'm' is the number of FF's.

2) No. of runs $= \dfrac{N+1}{2}$

3) DS-SS 'PG' :

$$\boxed{PG = \frac{T_b}{T_c}}$$

$$\boxed{T_b = NT_c}$$

$$\boxed{PG = N}$$

4) FH-SS 'PG' $\boxed{PG = 2^K}$

Where 'K' is the number of bits in PN-Sequence

5) $$\boxed{(\text{Jamming margin})_{dB} = 10\log_{10} PG - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{min}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

1) In a DS-SS modulation Scheme, a 14-Stage linear F/B Shift register is used to generate the PN Code Sequence. Find

(a) The period of Code Sequence

(b) processing gain

July-07, 4M

Sol :- Given : $m = 14$

(a) $N = 2^m - 1 = 2^{14} - 1 = \underline{16,383}$

(b) WKT $PG = N = \underline{16,383}$

$\therefore (PG)_{dB} = 10\log_{10}(16383) = 42.143$ dB

---

2) In a DS-SS modulation, it is required to have a jamming margin greater than 26 dB. The ratio E_b/N_0 is set at 10. Determine the minimum processing gain & the minimum number of stages required to generate the maximum length sequence.

June-09, 6M

Sol :- Given: $\frac{E_b}{N_0} = 10$, $\begin{pmatrix}\text{Jamming}\\ \text{margin}\end{pmatrix}_{dB} = 26\,dB$, $PG = ?$

WKT $\begin{pmatrix}\text{Jamming}\\ \text{margin}\end{pmatrix}_{dB} = 10\log_{10} PG - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{min}$

$26 = 10\log_{10} PG - 10\log_{10}(10)$

$26 = 10\log_{10} PG - 10$

$36 = 10\log_{10} PG$

$PG = \log_{10}^{-1}\left(\frac{36}{10}\right)$

$\boxed{PG = 4000}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

WKT $PG = N$ & $N = 2^m - 1$

With $m = 11$, we have $N = 2047$ &

With $m = 12$, we have $\underline{N = 4095}$

* To have PG of 4000

$\underline{m = 12}$ is the minimum number of Stages.

---

3) A PN Sequence is generated using linear feedback Shift register with number of Stages equal to 10. The Chip rate is 10^7 per Sec. Find the following:

(a) PN Sequence length

(b) Chip duration of the PN Sequence

(c) period of the PN Sequence.

Sol:- Given: $m = 10$, Chip rate $= \underline{10^7 \text{ per Sec.}}$

(a) PN Sequence $N = 2^m - 1 = 2^{10} - 1 = \underline{1023}$

(b) Chip duration $T_c = \dfrac{1}{10^7} = \underline{0.1\ \mu\text{sec.}}$

(c) period of the PN Sequence

$$T = N \times T_c$$

$$= 1023 \times 0.1\ \mu\text{sec}$$

$$\boxed{T = 102.3\ \mu\text{sec}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

4) A DS-SS uses a linear feedback shift register of 20 stages for the generation of PN sequence. Calculate the processing gain of the sequence, in dB.

July-06, 6M

Sol:- Given: $m = 20$

PN Sequence length $N = 2^m - 1 = 2^{20} - 1 \approx \underline{2^{20}}$

* WKT $\boxed{PG = N = 2^{20}}$

$$(PG)_{dB} = 10 \log_{10}(N)$$

$$= 10 \log_{10}(2^{20})$$

$$\boxed{PG = 60 dB}$$

5) A Slow FH/MFSK has the following parameters:

Number of bits/MFSK Symbol = 4

Number of MFSK Symbols/hop = 5

Calculate the processing gain of the System.

Jan-08, 3M
July-05, 6M
Jan-06, 4M

Sol:- Given:

BW of Spread Signal $5f_s$

BW of unspreaded Signal $\frac{f_s}{4}$

$$PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of unspreaded Signal}} = \frac{5\cancel{f_s}}{\frac{\cancel{f_s}}{4}}$$

$PG = 5 \times 4 = \underline{20}$

$(PG)_{dB} = 10 \log_{10}(20) = \underline{13 dB}$

6) A Slow FH/MFSK System has the following parameters:

The number of bits / MFSK Symbol = 4

The number of MFSK Symbols per hop = 6

Calculate the processing gain of the System.

Sol: Given:

BW of Spreaded Signal $6f_s$

BW of unspreaded Signal $f_s/4$

$$PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of unspreaded Signal}} = \frac{6\cancel{f_s}}{\frac{\cancel{f_s}}{4}} = \underline{24}$$

$$(PG)_{dB} = 10\log_{10}(24)$$

$$\boxed{(PG)_{dB} = 13.80\ dB}$$

---

7) Explain the properties of ML Sequence for a Sequence generated from 3-Stage Shift register with linear feedback. verify these properties & determine the period of the given PN Sequence 01011100101110

July-08, 6M

Sol:- Given: $m = 3$.

$$\therefore\ N = 2^m - 1 = 2^3 - 1 = \underline{7}$$

1) Balanced property:

0101110 | 0101110

7-bits | 7-bits

No of 1's → 4

No of 0's → 3

* No of 1's is always more than number of 0's by one in each period of ML-Sequence.

Run property:-

Total No of runs = $\frac{N+1}{2} = \frac{7+1}{2} = 4$ runs

i.e. 0, 1, 0, 111, 0

* If we take the Sequence 0101110, we get 5 runs, which is not in agreement.

Hence, we give a Circular Shift to right & get a new Sequence 0010111 i.e. 00, 1, 0, 111

Run 1 → 00

Run 2 → 1

Run 3 → 0

Run 4 → 111

Autocorrelation property:-

Autocorrelation function of ML-Sequence is periodic & it is binary valued.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

DP9.5. Figure DP9.5 shows a 4-stage linear feedback shift register. If the initial state is 1111, find the output sequence of the shift register.

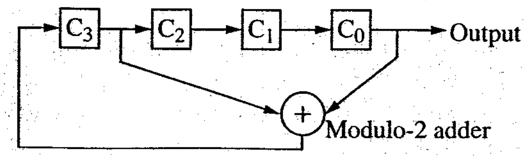


Figure DP9.5

Solution: The successive states of the linear feedback shift register is tabulated in Table DP9.1.

Table DP9.1

Shift	State of shift register C_3	C_2	C_1	C_0	Fed back bit $C_3 \oplus C_0$	Output bit
0	1	1	1	1	0	1
1	0	1	1	1	1	1
2	1	0	1	1	0	1
3	0	1	0	1	1	1
4	1	0	1	0	1	0
5	1	1	0	1	0	1
6	0	1	1	0	0	0
7	0	0	1	1	1	1
8	1	0	0	1	0	1
9	0	1	0	0	0	0
10	0	0	1	0	0	0
11	0	0	0	1	1	1
12	1	0	0	0	1	0
13	1	1	0	0	1	0
14	1	1	1	0	1	0
(The states repeat)						
15	1	1	1	1	0	1
⋮						

The output sequence is taken from last stage and is shown in the last column.

$$\underbrace{111101011001000}_{\text{one period}}1111$$

DP9.9. A PN sequence is generated using 4-stage linear feedback shift register as shown in Figure DP9.9(a), with initial conditon $(C_3C_2C_1C_0) = (1000)$. This sequence is used in a slow FH/MFSK system. The FH/MFSK signal has the following parameters.

Number of bits per MFSK symbol $K = 2$

Number of MFSK tones $M = 2^K = 4$

Length of PN segment per hop $k = 3$

Total number of frequency hops $2^k = 8$

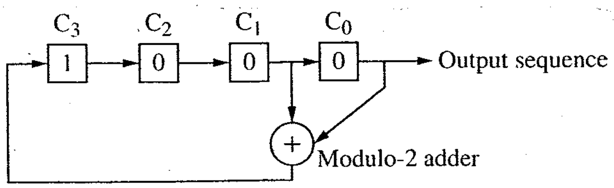


Figure DP9.9(a)

Determine the following:

a) Period of the PN sequence.
b) PN sequence for one periodic length.
c) Illustrate the variation of the frequency of FH/MFSK signal for one complete period of the PN sequence. Assume that the carrier hops to a new frequency after transmitting two MFSK symbols or four information bits. Assume binary data sequence to be 100011010001111111001.
d) Sketch the variation of dehopped frequency with time.

Solution:

a) The period of the PN sequence is $2^4 - 1 = 15$.
b) For the initial condition shown, the PN sequence is obtained by writing all the successive states of the shift register (SR), for one period. Table DP9.2 gives the successive states, the fed back bit and the output bit.
ii) The PN sequence of one periodic length is 000100110101111. The carrier is hopped to a new frequency after transmitting two MFSK symbols or four information bits. Number of bits per MFSK symbol $K = 2$. There are hence four MFSK frequencies corresponding to dibits 00, 01, 10 and 11. Length of PN segment per hop $k = 3$.

Table DP9.2

States of SR				Fed back bit	Output bit
C_3	C_2	C_1	C_0	$C_3 = C_1 \oplus C_0$	C_0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	1	0
1	0	0	1	1	1
1	1	0	0	0	0
0	1	1	0	1	0
1	0	1	1	0	1
0	1	0	1	1	1
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	0
1	1	1	1	0	1
0	1	1	1	0	1
0	0	1	1	0	1
0	0	0	1	1	1
1	0	0	0	Repeats	Repeats

Hence, there are $2^3 = 8$ hopping frequencies corresponding to each block of 3 PN sequence bits.

Let the hopping carrier frequencies corresponding to each block of 3 bits be selected as shown in Table DP9.3.

PN sequence segment	Hopping carrier frequency in Hz
000	f_{c0}
001	f_{c1}
010	f_{c2}
011	f_{c3}
100	f_{c4}
101	f_{c5}
110	f_{c6}
111	f_{c7}

Table DP9.4

Bits of MFSK symbol	MFSK tone in Hz
00	f_0
01	f_1
10	f_2
11	f_3

Figure DP9.9(b) gives the transmitted frequency, in any MFSK symbol interval, which is equal to $f_{ci}+f_j$ where f_{ci} is the carrier frequency which depends on the 3 bits of PN sequence and f_j is the MFSK tone which depends on the 2 bits of MFSK symbol. The transmitted frequency as a function of time is shown corresponding to the given binary data sequence and the PN sequence.

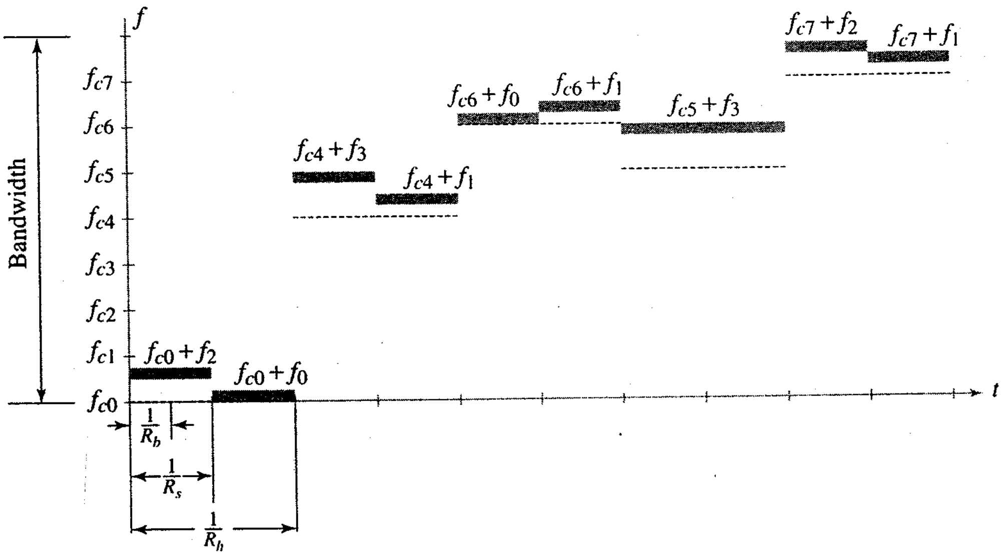


Figure DP9.9(b) Transmitted frequencies versus time for the given binary data and PN sequence.

DP9.10. In a fast FH/MFSK system, the signal has the following parameters:

Number of bits per MFSK symbol: $K = 2$
Number of MFSK tones: $M = 2^K = 4$
Length of PN segment per hop: $k = 3$
Total number of frequency hops, $2^3 = 8$
Number of hops per MFSK symbol = 2
Period of PN sequence: $L = 15$.

i) Determine the relation between bit rate and chip rate.
ii) Sketch the variation of frequency of the transmitted signal with time. Assume binary data sequence to be 01101100 and one period of PN sequence is 111100010011010.
iii) Sketch the dehopped MFSK signal.

ARUNKUMAR G. M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

Solution:

i) In a fast FH/MFSK, there are multiple hops per MFSK symbol. Hence in a fast FH/MFSK system, each hop is a chip. In this example there are 2 bits/MFSK symbol and 2 hops/MFSK symbol. Hence bit rate R_b = hop rate R_h = chip rate R_c.

ii) Let the MFSK tones be denoted by f_0, f_1, f_2 and f_3 corresponding to MFSK symbols 00, 01, 10, 11, respectively.

Let the hopping carrier frequencies be denoted by: $f_{c0}, f_{c1}, f_{c2}, f_{c3}, f_{c4}, f_{c5}, f_{c6}$ and f_{c7} which correspond to the PN sequence segments 000, 001, 010, 011, 100, 101, 110, and 111, respectively.

During a hopping interval, if carrier frequency is f_{cj} and MFSK tone is f_i, then the transmitted frequency is $f_{cj} + f_i$.

The transmitted frequency and dehopped MFSK signal are shown in Figure DP9.10(a) and (b) respectively.

Note:

i) For the sake of convenience spacing is shown between segments of PN sequence.

ii) After one period of 15 bits, the sequence repeats (Figure DP9.10(a)).

The dehopped MFSK signal is as shown below.

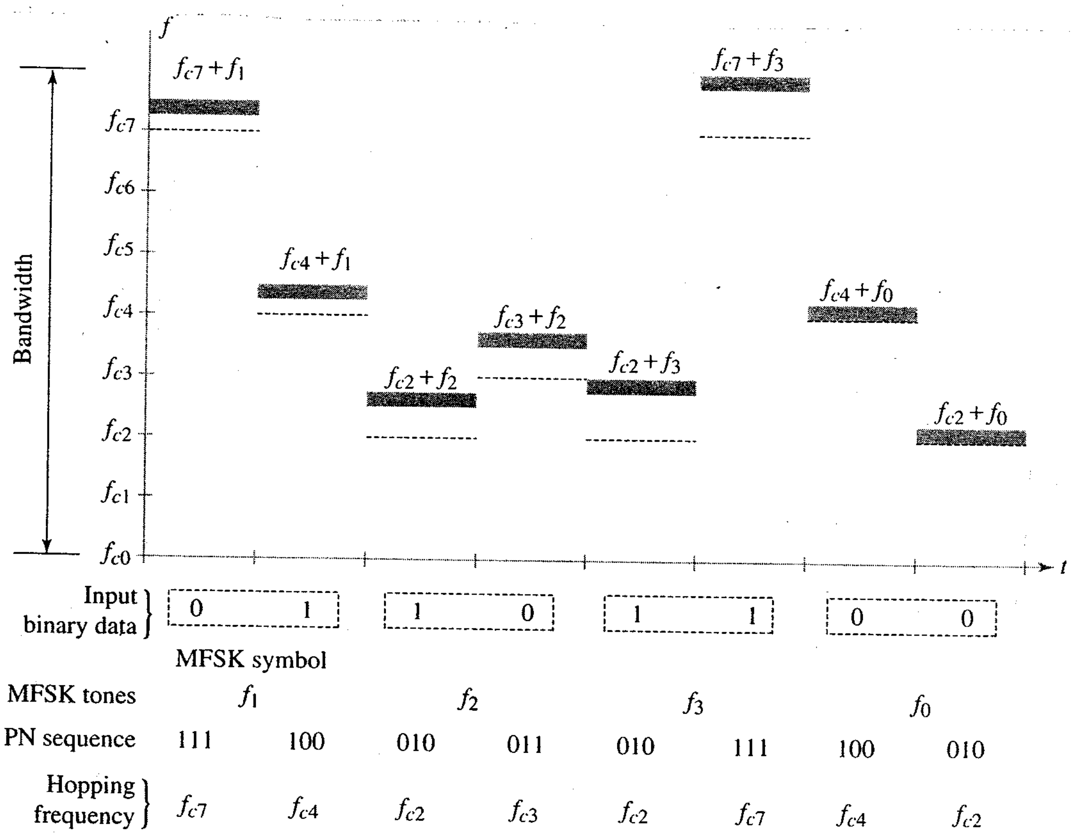


Figure DP9.10(a)

FORMULAE

1) $S_{11} = \int_0^{T_b} S_1(t)\, \phi_1(t).\, dt$ Where $S_1(t) = \sqrt{\frac{E_{max}}{T_b}}$

$\boxed{S_{11} = \sqrt{E_{max}}}$ $\phi_1(t) = \sqrt{\frac{1}{T_b}}$

2) $S_{21} = \int_0^{T_b} S_2(t)\, \phi_1(t)\, dt$ $S_2(t) = \sqrt{\frac{E_{max}}{T_b}}$

$\boxed{S_{21} = 0}$ $\phi_1(t) = \sqrt{\frac{1}{T_b}}$

3) PDF of a random variable X, having gaussian distribution

$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$$

Where σ^2 is the variance &

μ is the mean

4) ex: if $S_1(t) = \sqrt{\frac{E_{max}}{T_b}}$

mean	$\mu = \sqrt{E_{max}}$
variance	$\sigma^2 = \frac{N_0}{2}$

ex:- if $S_2(t) = 0$

$\mu = 0$
$\sigma = \frac{N_0}{2}$

5) $P_e(0)$ denotes the Conditional probability of deciding in Favour of Symbol '1' When '0' is transmitted

$$\boxed{P_e(0) = \int_0^{\infty} f_{X_1}(x_1|0)\, dx_1}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

6) $P_e(1)$ denotes the Conditional probability of deciding in favour of Symbol '0' When '1' is transmitted.

$$\boxed{P_e(1) = \int_0^{-\infty} f_{X_1}(x_1/0)\, dx_1}$$

7) Complementary error function

i) $$\boxed{\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-z^2}\, dz}$$

ii) $$\boxed{\operatorname{erfc}(u) = 1 - \operatorname{erf}(u)}$$

8) Energy

$$E = P T_b$$

$$E_{max} = P_{max} T_b$$

$$\frac{E_{max}}{N_0} = \frac{P_{max} T_b}{N_0} = \frac{P_{max}}{N_0/T_b}$$

Probability of error in PCM :-

* Derive the expression for probability of error to estimate the performance of PCM System transmitted along channel - associated with AWGN

July 08 - 8M

* Consider a binary - encoded PCM wave $s(t)$ that uses NRZ unipolar format :

$$s(t) = \begin{cases} s_1(t) = \sqrt{\dfrac{E_{max}}{T_b}} , & 0 \leq t \leq T_b \text{ for Symbol } 1 \\ s_2(t) = 0 , & 0 \leq t \leq T_b \text{ for Symbol } 0 \end{cases} \longrightarrow (1)$$

Where T_b = bit duration

E_{max} = maximum or peak Signal energy.

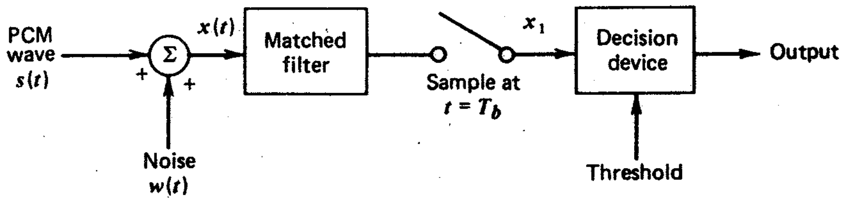


Figure Receiver for baseband transmission of binary-encoded PCM wave.

* The Channel noise $w(t)$ is modeled as additive white gaussian noise (AWGN) with mean and power Spectral density (PSD) $\frac{N_0}{2}$, then the received Signal

$$\boxed{x(t) = s(t) + w(t)} \qquad 0 \leq t \leq T_b$$

* To calculate the probability of error (P_e), we use the Signal Space approach. The basis function for eq (1) is

$$\phi_1(t) = \sqrt{\frac{1}{T_b}} , \quad 0 \leq t \leq T_b$$

then eq (1) becomes:

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$S(t) = \begin{cases} S_1(t) = \sqrt{E_{max}}\ \phi_1(t) \\ S_2(t) = 0 \end{cases}$$

* An ON-OFF PCM System is Characterized by having a Signal Space that is one-dimensional and with two message points

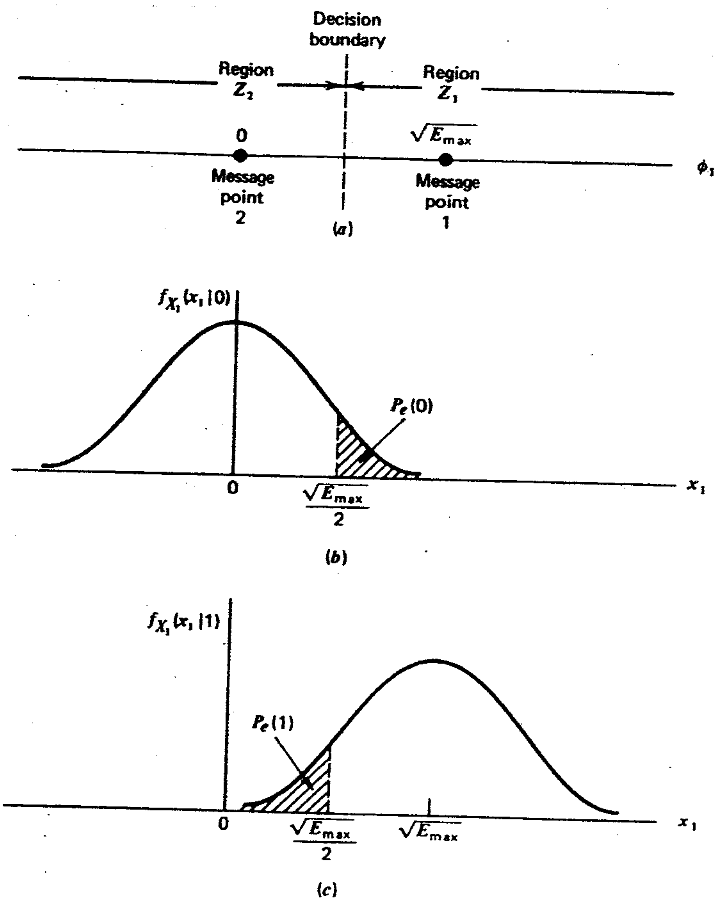


Figure (a) Signal space diagram for on–off PCM system. (b) Likelihood function, given that symbol 0 was sent. (c) Likelihood function, given that symbol 1 was sent.

The Co-ordinates of the two message points are:

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t) \cdot dt$$

$$\boxed{S_{11} = \sqrt{E_{max}}}$$

and

$$S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt$$

$$\boxed{S_{21} = 0}$$

for $S_1(t)$
$\mu = \sqrt{E_{max}}$ $\sigma^2 = \frac{N_0}{2}$
for $S_2(t)$
$\mu = 0$ $\sigma^2 = N_0/2$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* A threshold of $\frac{\sqrt{E_{max}}}{2}$ is used, which is half way point b/w two message points.

i.e. If $x(t) > \frac{\sqrt{E_{max}}}{2}$, then decision is taken in favour of symbol '1'

If $x(t) < \frac{\sqrt{E_{max}}}{2}$, then decision is taken in favour of symbol '0'

* The received signal point is calculated by sampling the matched filter o/p at time $t = T_b$

$$x_1 = \int_0^{T_b} x(t) \cdot \phi_1(t) \cdot dt$$

To calculate P_e of 1st Kind :-

Symbol '0' is sent but received as symbol '1' called as error of 1st kind.

* The decision region $Z_1 : \frac{\sqrt{E_{max}}}{2} < x_1 < \infty$

Since X_1 has gaussian distribution, it is defined by

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \exp\left[-\frac{(x-0)^2}{\cancel{2}\frac{N_0}{\cancel{2}}}\right]$$

$$\boxed{f_{X_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} \cdot e^{\left(\frac{-x^2}{N_0}\right)}} \longrightarrow ②$$

$\therefore$ Probability of symbol error, when symbol '0' is transmitted

$$P_e(0) = \int_{\frac{\sqrt{E_{max}}}{2}}^{\infty} f_{X_1}(x_1/0)\ dx_1 \longrightarrow ③$$

Substituting eq ② in eq ③, we get

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$P_e(0) = \int_{\frac{\sqrt{E_{max}}}{2}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\left(\frac{x_1}{\sqrt{N_0}}\right)^2} \cdot dx_1 \longrightarrow (4)$$

Let $Z = \dfrac{x_1}{\sqrt{N_0}}$

$x_1 = Z\sqrt{N_0}$

$dx_1 = dZ\sqrt{N_0}$

∴ Limits

When $x_1 = \frac{\sqrt{E_{max}}}{2}$	When $x_1 = \infty$
$Z\sqrt{N_0} = \frac{\sqrt{E_{max}}}{2}$	$Z\sqrt{N_0} = \infty$
$Z = \frac{\sqrt{E_{max}}}{2 N_0}$	$Z = \frac{\infty}{\sqrt{N_0}}$
$Z = \sqrt{\frac{E_{max}}{N_0}} \cdot \frac{1}{2}$	$Z = \infty$

∴ Equation (4) becomes

$$P_e(0) = \frac{1}{\sqrt{\pi \cancel{N_0}}} \int_{\sqrt{\frac{E_{max}}{N_0}} \times \frac{1}{2}}^{\infty} e^{-Z^2} dZ \sqrt{\cancel{N_0}}$$

$$\boxed{P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}}^{\infty} e^{-Z^2} dZ} \longrightarrow (5)$$

WKT

$$\boxed{\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-u^2} \cdot du} \longrightarrow (6)$$

From eq (6), we can write eq (5) as

$$P_e(0) = \frac{1}{\sqrt{\pi}} \frac{2}{2} \int_{\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}}^{\infty} e^{-Z^2}\, dZ$$

$$\boxed{P_e(0) = \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)}$$

Where $u = \frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}$

* Similarly P_e of 2nd kind is

$$\boxed{P_e(1) = \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)}$$

Average probability of error :-

Let probability of sending '0' is $P_0 = 1/2$

Let probability of sending '1' is $P_1 = 1/2$

* Then average probability of error

$$P_e = P_0 P_e(0) + P_1 P_e(1)$$

$$= \frac{1}{2}\left[\frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)\right]$$

$$\boxed{P_e = \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)}$$

* The ratio E_{max}/N_0 represents peak signal energy to noise spectral density ratio, it can be represented as peak signal to noise power ratio. WKT $E_{max} = P_{max} T_b$

$$\frac{E_{max}}{N_0} = \frac{P_{max} T_b}{N_0} = \frac{P_{max}}{N_0 / T_b}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* P_e decreases very rapidly as this ratio is increased.

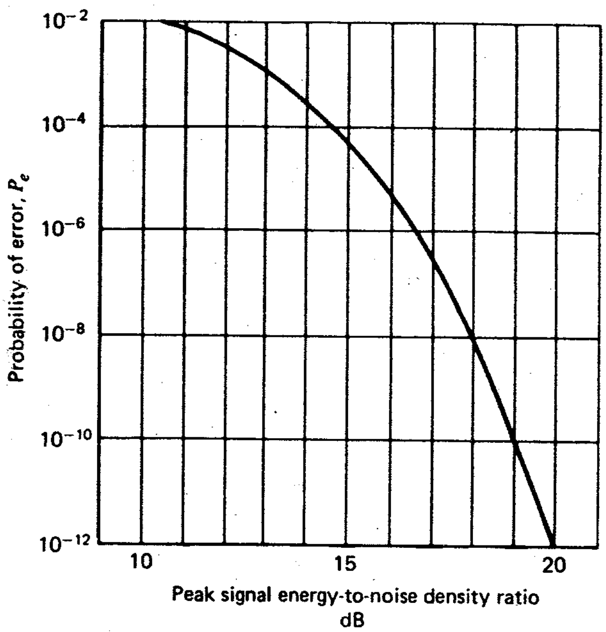


Figure Probability of error in a PCM receiver.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

ARUNKUMAR G M.Tech
Lecturer in E&CE Department
S.T.J.I.T.. Ranebennur

DIGITAL MODULATION TECHNIQUES

Introduction:

The modulation process involves Switching & Keying the amplitude, Frequency & phase of the carrier wave in accordance with the incoming data.

There are 3 - basic modulation techniques:

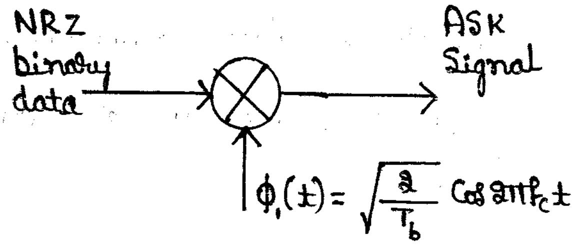
1) Amplitude Shift Keying (ASK)
2) Frequency Shift Keying (FSK)
3) Phase Shift Keying (PSK)

* The Choice of modulation techniques is based on the following goals:

1) Maximum data rate
2) Maximum resistance to interfering Signals
3) Minimum probability of Symbol error
4) Minimum transmitted power
5) Minimum channel bandwidth
6) Minimum Circuit Complexity.

Amplitude Shift Keying (ASK) :- or ON-OFF Keying :-

Generation of ASK :-



* In binary ASK System Symbol 1 & 0 are represented as follows:

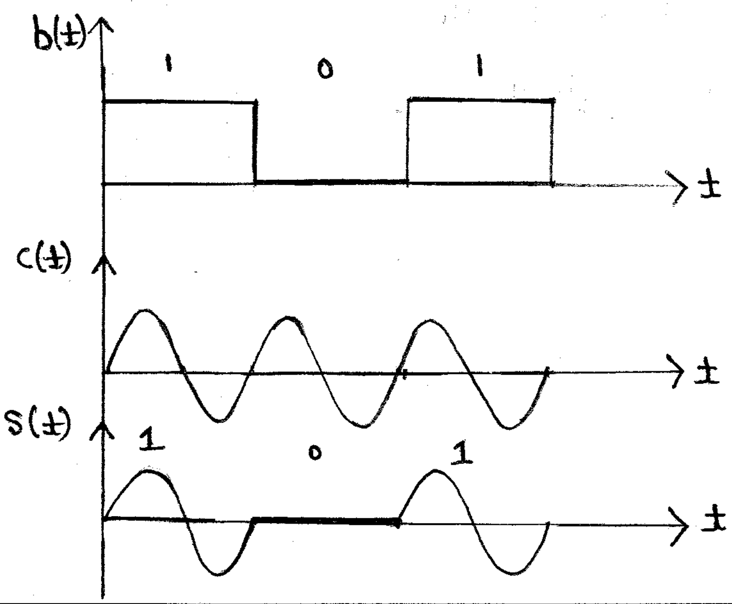
$$S(t) = \begin{cases} S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & 0 \leq t \leq T_b \quad \text{for Symbol 1} \\ S_2(t) = 0 & 0 \leq t \leq T_b \quad \text{for Symbol 0} \end{cases}$$

* Let us define a basis Function $\phi_1(t)$ which has unit energy

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

* Binary ASK can be written as:

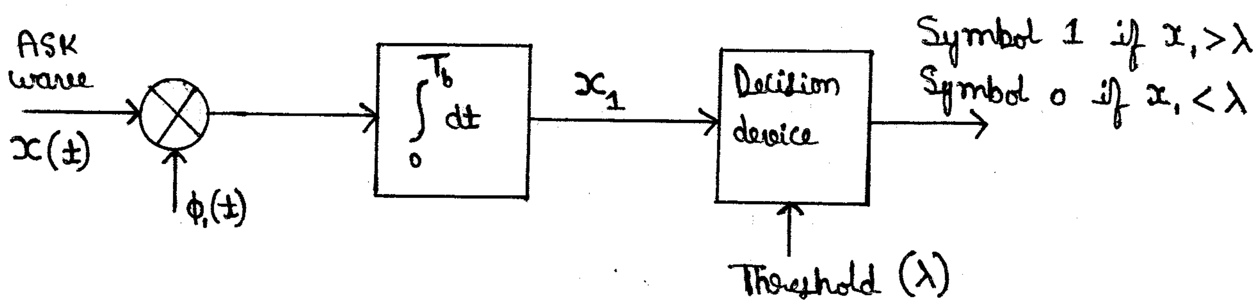
$$S(t) = \begin{cases} S_1(t) = \sqrt{E_b}\, \phi_1(t) & 0 \leq t \leq T_b \quad \text{for Symbol 1} \\ S_2(t) = 0 & 0 \leq t \leq T_b \quad \text{for Symbol 0} \end{cases}$$



ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* In ASK System, binary Symbol 1 is represented by transmitting a carrier $c(t) = A_c \cos 2\pi f_c t$ for a bit duration 'T_b' Seconds, where as Symbol 0 is represented by Switching off the carrier for 'T_b' Seconds.

Coherent (demodulation) detection of ASK Signal :-



* The received Signal is Cross correlated with local reference Signal $\phi_1(t)$. The o/p of the correlator 'x_1' is applied to the decision device.

* The decision device compares the o/p of correlator with preset threshold 'λ' i.e.

  If $x_1 > \lambda \rightarrow$ decides in favour of Symbol 1.

  If $x_1 < \lambda \rightarrow$ decides in favour of Symbol 0.

* In Coherent detection the o/p of local oscillator is in perfect Synchronization with the carrier used in the transmitter.

NOTE :- Usually phase and timing Synchronization is needed for the operation of Coherent or Synchronous detector.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

BASK

Probability error of ASK (or) BER Calculation of ASK :-

* Derive an expression for probability of error 'P_e' of a Coherent binary ASK. $\boxed{\text{Jan-10, 10M}}$

* In ASK System $S_1(t)$ & $S_2(t)$ are represented as:

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad \text{and}$$

$$S_2(t) = 0.$$

Where 'E_b' is the transmitted Signal energy per bit.

* The basis function is given by

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

* The transmitted ASK Signals are given by

$$S_1(t) = \sqrt{E_b}\, \phi_1(t) \quad \text{for Symbol 1}$$

$$S_2(t) = 0 \quad \text{for Symbol 0}$$

Hence, we have a one dimensional Signal space with two message points at $+\sqrt{E_b}$ and 0.

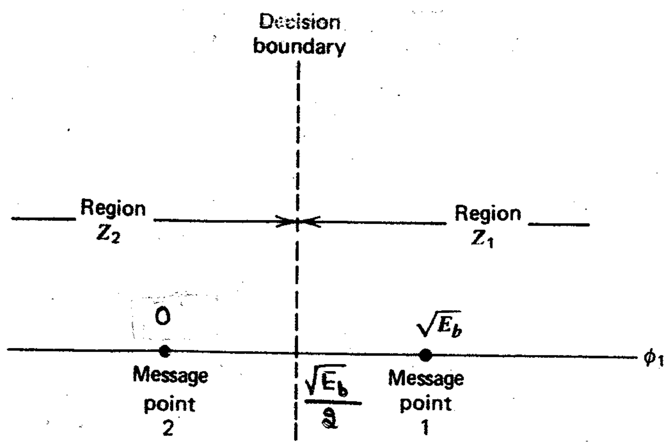
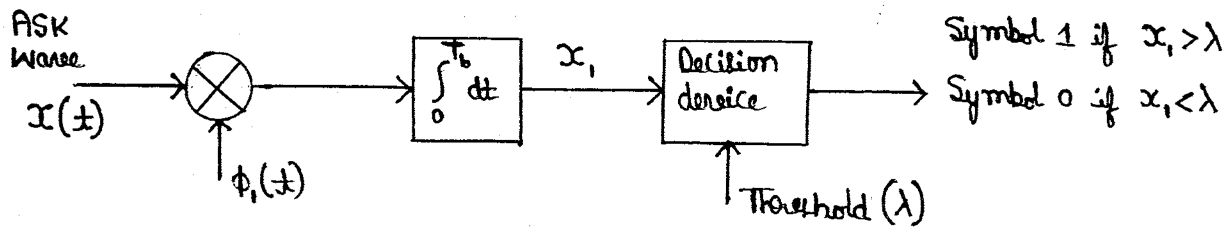


Figure Signal space diagram for coherent binary PSK system.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*



* Let $x(t)$ be the received Signal

$$\boxed{x(t) = s(t) + w(t)} \longrightarrow ①$$

Where $w(t)$ is the additive white gaussian noise (AWGN) with Zero mean $(\mu=0)$ and variance (σ) of $\frac{N_0}{2}$.

$$\therefore \quad x(t) = \begin{cases} s_1(t) + w(t) & , \text{ for Symbol 1} \\ 0 + w(t) & , \text{ for Symbol 0} \end{cases}$$

* Let us assume that Symbol '0' is transmitted. Then o/p of the Correlator is

$$x_1 = \int_0^{T_b} x(t) \cdot \phi_1(t) \cdot dt$$

$$= \int_0^{T_b} w(t)\, \phi_1(t) \cdot dt$$

$$\boxed{x_1 = W_1}$$

$$\therefore \quad E[x_1] = E[W_1] = 0 \quad \&$$

$$var[x_1] = \frac{N_0}{2}$$

$\therefore$

$\mu = 0$
$\sigma^2 = \frac{N_0}{2}$

Computing P_e of 1^{st} Kind :-

Decision region : $\sqrt{\frac{E_b}{2}} < x_1 < \infty$

(Since x_1 has gaussian distribution, it is defined by)

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(x_1-\mu)^2}{2\sigma^2}} \cdot dx_1$$

Conditional PDF when Symbol '0' is transmitted is given by:

ARUNKUMAR.G M.Tech., Lecturer in E&CE Dept, S.T.J.I.T, Ranebennur.

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \cdot e^{\frac{-(x_1-0)^2}{\cancel{2}\frac{N_0}{\cancel{2}}}}$$

$$\boxed{f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} e^{\left(\frac{-x_1^2}{N_0}\right)} \cdot dx_1} \longrightarrow (2)$$

$\therefore$ Probability of Symbol ~~error~~, when Symbol '0' is transmitted

$$P_e(0) = \int_{\frac{\sqrt{E_b}}{2}}^{\infty} f_{x_1}(x_1|0) \cdot dx_1 \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$P_e(0) = \int_{\sqrt{E_b/2}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_1}{\sqrt{N_0}}\right]^2} dx_1 \longrightarrow (4)$$

Let

$$Z = \frac{x_1}{\sqrt{N_0}}$$

$$Z\sqrt{N_0} = x_1$$

$$\boxed{dx_1 = dZ\sqrt{N_0}}$$

limits

When $x_1 = \frac{\sqrt{E_b}}{2}$	When $x_1 = \infty$
$Z\sqrt{N_0} = \frac{\sqrt{E_b}}{2}$ $Z = \frac{\sqrt{E_b}}{2} \cdot \frac{1}{\sqrt{N_0}}$ $Z = \sqrt{\frac{E_b}{N_0}} \times \frac{1}{2}$	$Z\sqrt{N_0} = \infty$ $Z = \frac{\infty}{\sqrt{N_0}}$ $Z = \infty$

$$P_e(0) = \int_{\sqrt{\frac{E_b}{N_0}} \times \frac{1}{2}}^{\infty} \frac{1}{\sqrt{\pi \cancel{N_0}}} e^{-Z^2} \cdot dZ \cancel{\sqrt{N_0}}$$

$$P_e(0) = \int_{\frac{1}{2}\sqrt{\frac{E_b}{2}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-Z^2} dZ \longrightarrow (5)$$

WKT Complementary error function

$$\boxed{erfc(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2}\, du} \longrightarrow (6)$$

From eq (6), we can write eq (5) as

$$P_e(0) = \frac{1}{\sqrt{\pi}} \frac{2}{2} \int_{\frac{1}{2}\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-Z^2}\, dZ$$

$$\boxed{P_e(0) = \frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)}$$

Where $u = \frac{1}{2}\sqrt{\frac{E_b}{N_0}}$

* Similarly we can calculate probability of error for 2nd Kind:

$$\boxed{P_e(1) = \frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)}$$

Let probability of Sending '0' is $p(0) = \frac{1}{2}$

Let probability of Sending '1' is $p(1) = \frac{1}{2}$

* Then average probability of error

$$P_e = P(0)\, P_e(0) + P(1)\, P_e(1)$$

$$P_e = \frac{1}{2}\left[\frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)\right]$$

$$\boxed{P_e = \frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)}$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Jan-07, 12M

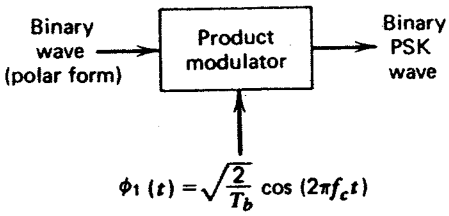
Binary phase Shift Keying (BPSK) :-

Jan-06, 8M

1) Explain the working of a BPSK transmitter and receiver. What are the drawbacks of BPSK System?

July-06, 6M

2) Explain a Coherent binary PSK transmitter & receiver



(a)

* In a PSK System a Sinusoidal Carrier wave of fixed amplitude & frequency 'f_c' is used to represent both Symbol '1' & '0' except that the carrier phase of each Symbol differs by a phase of 180°.

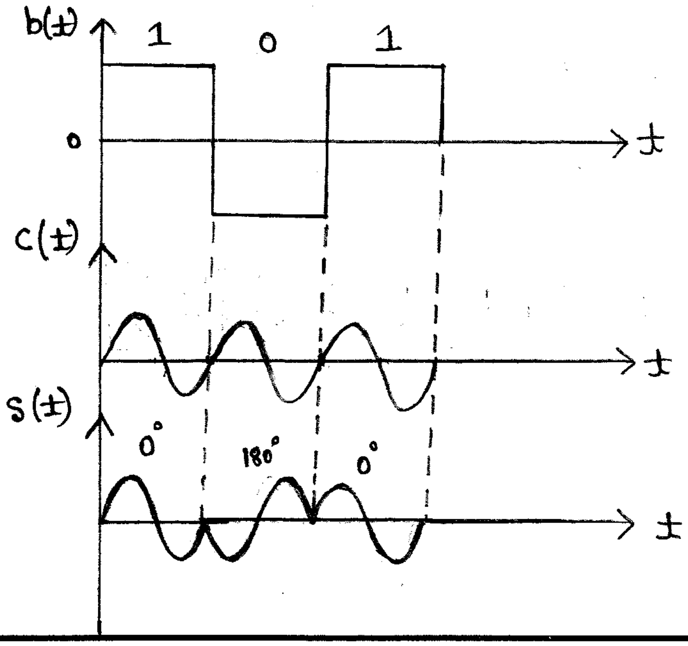
i.e.

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \qquad \text{for Symbol } 1$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) \qquad \text{for Symbol } 0$$

$$\therefore S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \qquad \text{for Symbol } 0.$$

$\rightarrow$ (1)



ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Eq ① can be represented by a single basis function

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \qquad 0 \leq t \leq T_b$$

* Then time domain - expression of the BPSK signal is given by

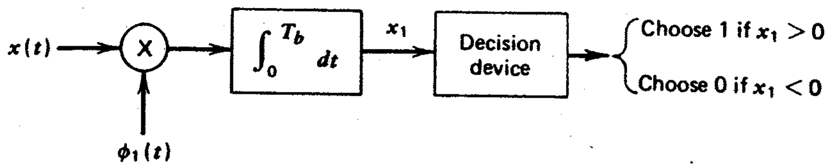
$$S(t) = \begin{cases} S_1(t) = \sqrt{E_b}\,\phi_1(t) & 0 \leq t \leq T_b, \text{ for Symbol 1} \\ S_2(t) = -\sqrt{E_b}\,\phi_1(t) & 0 \leq t \leq T_b, \text{ for Symbol 0} \end{cases}$$

* Thus in BPSK,

Symbol '1' is represented by transmitting a carrier wave with a 0° phase shift.

Symbol '0' is represented by transmitting a carrier wave with a 180° phase shift.

Coherent detection of BPSK :-



(b)

* Let $x(t)$ be the received signal which is the sum of AWGN noise and transmitted BPSK signal i.e. $x(t) = S(t) + w(t)$

* Let $\phi_1(t)$ be the local carrier signal which is synchronized interms of frequency & phase.

* If $x_1 > 0$, the decision is taken in favour of symbol '1'

If $x_1 < 0$, the decision is taken in favour of symbol '0'.

Downloaded from www.citystudentsgroup.blogspot.com

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Drawbacks of BPSK System :-

Probability error of BPSK :-

Jan-10, 10M

1) Explain coherent PSK receiver. obtain the expression for probability of error for PSK with coherent receiver

2) Assuming channel Noise to be additive white gaussian, obtain an expression for probability of error for PSK. July-06, 6M

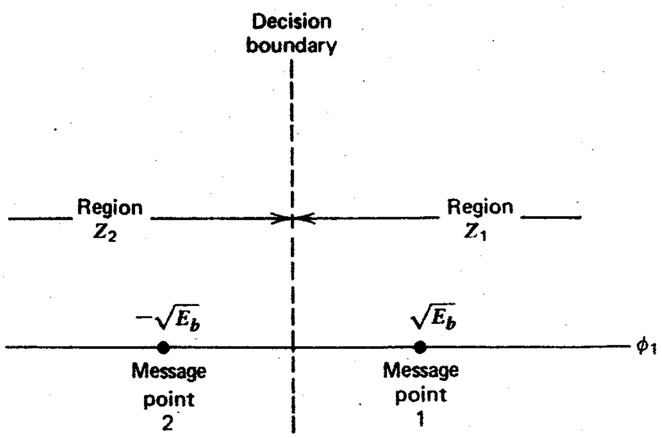
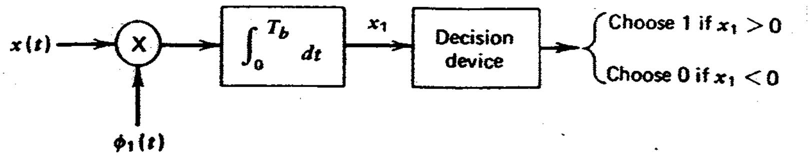


Figure Signal space diagram for coherent binary PSK system.

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.



(b)

* Let $x(t)$ be the received Signal

$$x(t) = s(t) + w(t) \longrightarrow (1)$$

$$x(t) = \begin{cases} s_1(t) = \sqrt{E_b}\,\phi_1(t) + w(t), & \text{for Symbol 1} \\ s_2(t) = -\sqrt{E_b}\,\phi_1(t) + w(t), & \text{for Symbol 0} \end{cases}$$

* Let us assume that the symbol 0 is transmitted. Then the o/p of the correlator is

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt = \int_0^{T_b} [s_1(t) + w(t)]\,\phi_1(t)\cdot dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\cdot dt$$

$$\boxed{x_1 = -\sqrt{E_b} + W_1}$$

* Mean of the random variable X_1 is

$$E[X_1] = E[\sqrt{E_b} + W_1] = E[\sqrt{E_b}] + E[W_1] = \sqrt{E_b} + 0$$

$$\boxed{E[X_1] = -\sqrt{E_b}}$$

* variance of X_1 is

$$var[X_1] = var[\sqrt{E_b} + W_1] = var[\sqrt{E_b}] + var[W_1] = 0 + N_0/2$$

$$\boxed{var[X_1] = N_0/2}$$

* Conditional pdf when symbol '0' is transmitted is given by:

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\left(\frac{x-\mu}{2\sigma^2}\right)} \longrightarrow (2)$$

Downloaded from www.citystudentsgroup.blogspot.com

$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \cdot e^{-\frac{(x_1+\sqrt{E_b})^2}{\cancel{2}\cdot\frac{N_0}{\cancel{2}}}} \qquad (e^x)$$

$$\boxed{f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \cdot e^{-\left[\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}\right]^2}} \longrightarrow (3)$$

* Let $P_e(0)$ denotes the Conditional probability of deciding in Favour of symbol '1' when '0' is transmitted.

Region Z_1 : $0 \leq x_1 \leq +\infty$

$$\therefore \quad P_e(0) = \int_0^\infty f_{X_1}(x_1/0)\, dx_1$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty e^{-\left[\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}\right]^2} \cdot dx_1$$

Let $Z = \frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}$

$dZ = \frac{dx_1}{\sqrt{N_0}} + 0$

$dx_1 = \sqrt{N_0}\, dZ$

Limits :-

When $x_1 = 0$	When $x_1 = \infty$
$Z = \frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}$ $Z = \frac{0+\sqrt{E_b}}{\sqrt{N_0}}$ $\boxed{Z = \sqrt{\frac{E_b}{N_0}}}$	$\boxed{Z = \infty}$

$$\therefore \quad P_e(0) = \int_{\sqrt{E_b/N_0}}^{\infty} \frac{1}{\sqrt{\pi \cancel{N_0}}}\, e^{-Z^2}\, dZ \sqrt{\cancel{N_0}} \longrightarrow (4)$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} e^{-Z^2} \cdot dZ$$

$$P_e(0) = \frac{1}{\sqrt{\pi}}\, \frac{2}{2} \int_{\sqrt{E_b/N_0}}^{\infty} e^{-Z^2}\, dZ \longrightarrow (5)$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

WKT Complementary error function

$$\boxed{\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-u^2}\, du} \longrightarrow (6)$$

From eq (6), we can write eq (5) as

$$\boxed{P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

Where $u = \sqrt{\frac{E_b}{N_0}}$

* Similarly we can calculate probability of error for 2nd kind:

$$\boxed{P_e(1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

Let probability of transmitting symbol '0' is $P(0) = \frac{1}{2}$

Let probability of transmitting symbol '1' is $P(1) = \frac{1}{2}$

* Then average probability of error

$$P_e = P(0)\, P_e(0) + P(1)\, P_e(1)$$

$$P_e = \frac{1}{2}\left[\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right]$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

P.T.O

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Binary Frequency Shift Keying (BFSK) :-

July-07, 8M

* Explain the operation of binary FSK transmitter and receiver with necessary block diagram

The binary FSK wave can be represented as follows:

$$S(t) = \begin{cases} S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t & 0 \le t \le T_b, \text{ for Symbol } 1 \\ S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t & 0 \le t \le T_b, \text{ for Symbol } 0 \end{cases}$$

* Let $\phi_1(t)$ & $\phi_2(t)$ are the basis function defined as

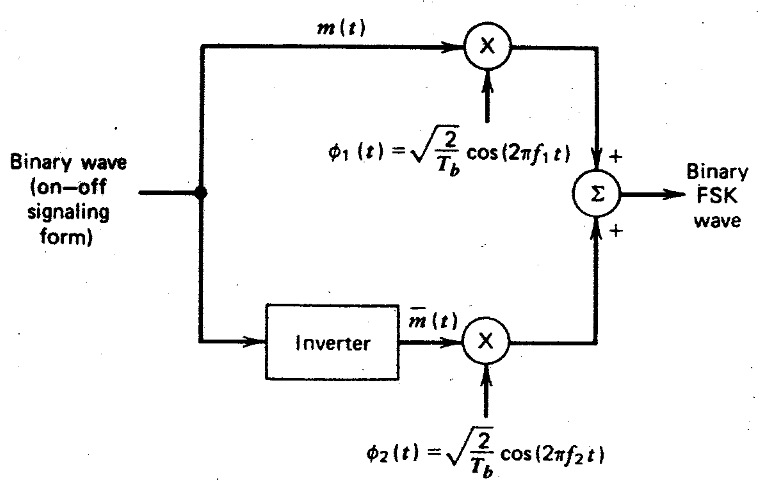
$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \quad \&$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t .$$

$\therefore$ We can write $S(t)$ as

$$S(t) = \begin{cases} S_1(t) = \sqrt{E_b}\, \phi_1(t) & \text{for Symbol '1'} \\ S_2(t) = \sqrt{E_b}\, \phi_2(t) & \text{for Symbol '0'} \end{cases}$$

BFSK Transmitter :-



(a)

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

Fig Shows the Scheme for generating BFSK Signal.

* I/p binary data is unipolar NRZ format applied to the multiplier 1 along with Carrier Signal $\phi_1(t)$

i.e $\boxed{S_1(t) = m(t) \cdot \phi_1(t)}$

* The I/p m(t) is applied to multiplier 2 through Inverter along with another Carrier Signal $\phi_2(t)$

i.e. $\boxed{S_2(t) = \overline{m}(t)\, \phi_2(t)}$

* When Symbol '1' is transmitted, o/p appears from upper path i.e. '$S_1(t)$' & no o/p from ~~the~~ lower path i.e. $S_2(t) = 0$.

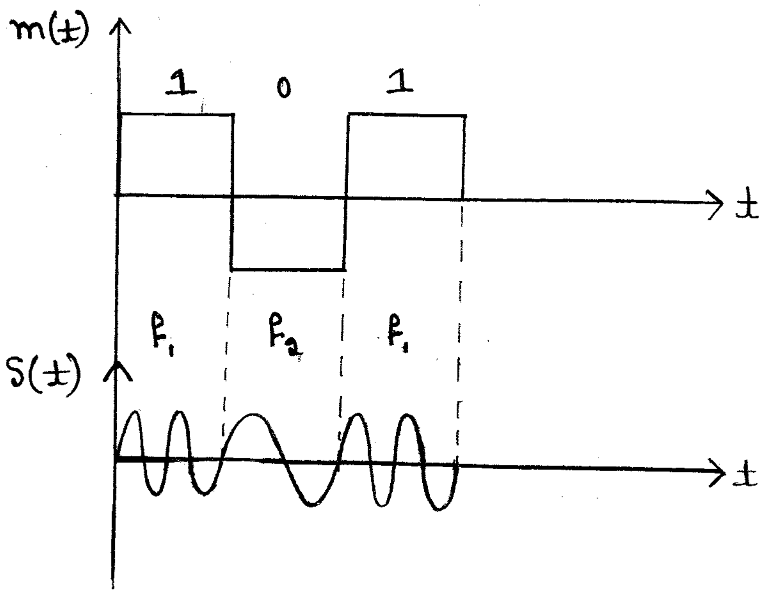
Thus frequency 'f_1' is transmitted for Symbol '1'.

* When Symbol '0' is transmitted, o/p appears from lower path i.e. '$S_2(t)$' & no o/p from the upper path i.e. $S_1(t) = 0$

Thus frequency 'f_2' is transmitted for Symbol '0'.

* Frequency $f_1 > f_2$ is chosen & the phase of the FSK remains continuous

∴ FSK is also known as Continuous phase FSK.

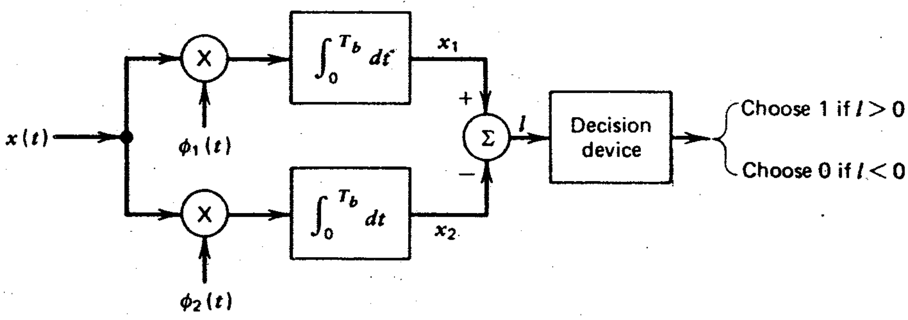


$f_1 > f_2$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Coherent detection of BFSK :-

Jan-08, 7M



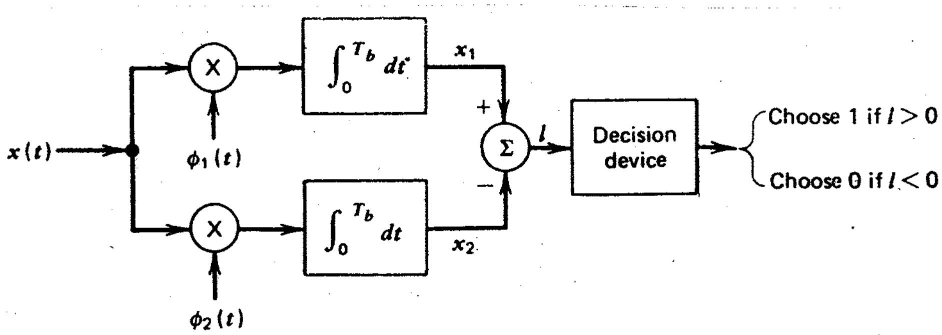
(b)

* The detector consists of two correlators that are individually tuned to two different frequencies.
* A correlator consists of a multiplier followed by an integrator (LPF).
* The o/p of top & bottom correlator be denoted by x_1 & x_2 respectively. The x_1 & x_2 are given to subtractor. The o/p of subtractor $l = x_1 - x_2$
* If $(x_1 - x_2) > 0$ i.e. $l > 0$, decision is taken in favour of symbol '1'.
* If $(x_1 - x_2) < 0$ i.e. $l < 0$, the decision is taken in favour of symbol '0'.
* If $(x_1 - x_2) = 0$ i.e. $l = 0$, the decision is arbitrary.

Probability of error in BFSK :-

July-08, 8M

* Derive an expression for probability of error in binary FSK generation & coherent detection.



⟶ Write Fig ③, page no → 20

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Let $x(t)$ be the received BPSK Signal given by:

$$\boxed{x(t) = S(t) + w(t)}$$

$$x = \begin{cases} S_1(t) + w(t) & \text{for Symbol '1'} \\ S_2(t) + w(t) & \text{for Symbol '0'} \end{cases}$$

Where $w(t)$ is AWGN noise having mean $(\mu) = 0$ & variance $(\sigma) = \frac{N_0}{2}$

* Let us consider the transmission of Symbol '0' then the received Signal is $\boxed{x(t) = S_2(t) + w(t)}$

* The o/p of the top Correlator is

$$x_1 = \int_0^{T_b} x(t)\, \phi_1(t) \cdot dt$$

$$x_1 = \int_0^{T_b} [S_2(t) + w(t)]\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} S_2(t)\, \phi_1(t)\, dt + \int_0^{T_b} w(t)\, \phi_1(t)\, dt$$

$$x_1 = S_{21} + W_1$$

$$\boxed{x_1 = 0 + W_1} \longrightarrow (1)$$

* Mean is given by

$$E[X_1] = E[0 + W_1] = 0 + 0$$

$$\boxed{E[X_1] = 0}$$

* variance of X_1 is

$$\text{var}[X_1] = \text{var}[0 + W_1] = 0 + \frac{N_0}{2}$$

$$\boxed{\text{var}[X_1] = \frac{N_0}{2}}$$

* The o/p of the bottom Correlator is

$$x_2 = \int_0^{T_b} x(t)\, \phi_2(t) \cdot dt$$

$$x_2 = \int_0^{T_b} [S_2(t) + w(t)]\, \phi_2(t)\, dt$$

$$x_2 = \int_0^{T_b} S_2(t)\,\phi_2(t)\, dt + \int_0^{T_b} w(t)\,\phi_2(t)\, dt$$

$$x_2 = S_{22} + W_2$$

$$\boxed{x_2 = \sqrt{E_b} + W_2} \longrightarrow ②$$

* The mean of X_2 is

$$\boxed{E[X_2] = \sqrt{E_b}}$$

* The variance of X_2 is

$$\boxed{\text{var}[X_2] = \frac{N_0}{2}}$$

* Let us find the mean & variance of random variable $l = x_1 - x_2$ is also gaussian.

Mean $E[L] = E[X_1] - E[X_2]$

$$E[L] = 0 - \sqrt{E_b}$$

$$\boxed{E[L] = -\sqrt{E_b}}$$

(* The variance of the random variable 'L' is independent of which binary Symbol was transmitted. Since the random variables X_1 & X_2 are Statistically independent each with a variance equal to $N_0/2$.)

* $\text{var}[L] = \text{var}[X_1] + \text{var}[X_2]$

$$= \frac{N_0}{2} + \frac{N_0}{2}$$

$$\boxed{\text{var}[L] = N_0}$$

* Conditional PDF when Symbol '0' transmitted is given by

$$\boxed{\mu = -\sqrt{E_b} \ \& \ \sigma^2 = N_0}$$

$$f_L(l/0) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(l-\mu)^2}{2\sigma^2}}$$

$$f_L(l/0) = \frac{1}{\sqrt{2\pi N_0}}\, e^{\frac{-(l+\sqrt{E_b})^2}{2N_0}}$$

$$\boxed{f_L(l/0) = \frac{1}{\sqrt{2\pi N_0}}\, e^{-\left[\frac{l+\sqrt{E_b}}{\sqrt{2N_0}}\right]^2}} \longrightarrow ③$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* Let $P_e(0)$ denotes the Conditional probability of deciding in favour of Symbol '1', when Symbol '0' is transmitted.

$$\therefore \quad \text{Region } Z_1 : \quad 0 \leq x \leq \infty$$

$$P_e(0) = \int_0^\infty f_L(l/0)\, dl = \int_0^\infty \frac{1}{\sqrt{2\pi N_0}}\, e^{-\left[\frac{l+\sqrt{E_b}}{\sqrt{2N_0}}\right]^2} dl$$

Let $Z = \dfrac{l+\sqrt{E_b}}{\sqrt{2N_0}}$

$$dZ = \frac{dl}{\sqrt{2N_0}} + 0$$

$$\boxed{dl = dZ\sqrt{2N_0}}$$

Limits

When $l = \infty$ then $Z = \infty$	When $l = 0$, $Z = \dfrac{l+\sqrt{E_b}}{\sqrt{2N_0}}$ $Z = 0 + \dfrac{\sqrt{E_b}}{\sqrt{2N_0}}$ $Z = \sqrt{\dfrac{E_b}{2N_0}}$

$$\therefore \quad P_e(0) = \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi \cancel{N_0}}}\, e^{-Z^2}\, dZ \sqrt{\cancel{2N_0}} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-Z^2} . dZ$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \frac{2}{2} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-Z^2} dZ \longrightarrow (4)$$

WKT Complementary error function

$$\boxed{\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-u} du} \longrightarrow (5)$$

From eq (5), we can write eq (4) as

$$\boxed{P_e(0) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]}$$

* Similarly we can calculate probability of error for 2nd kind :

$$\boxed{P_e(1) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]}$$

* If Symbol 0's & 1's are equiprobable then $p(0) = p(1) = \frac{1}{2}$

WKT $P_e(0) = P_e(1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

* Then average probability of error

$$P_e = P(0)\,P_e(0) + P(1)\,P_e(1)$$

$$P_e = \frac{1}{2}\left[\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)\right]$$

$$\boxed{P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

NOTE :-

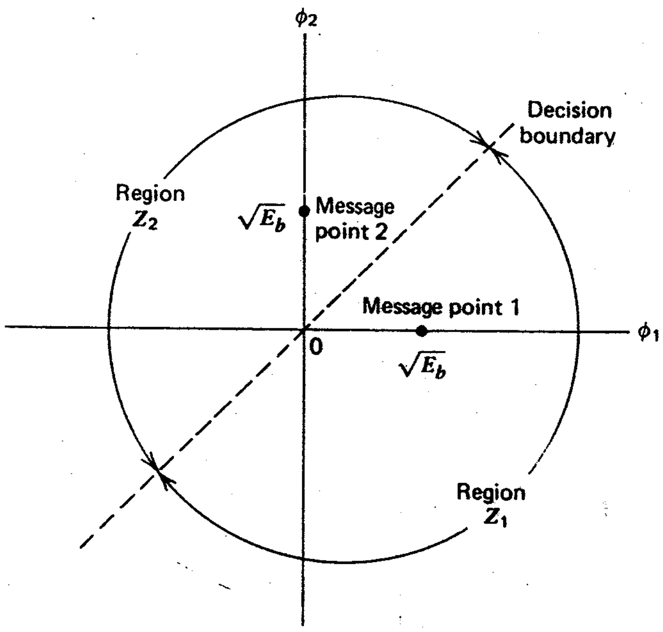


Figure 3 Signal space diagram for coherent binary FSK system.

Differential phase Shift Keying (DPSK) :-

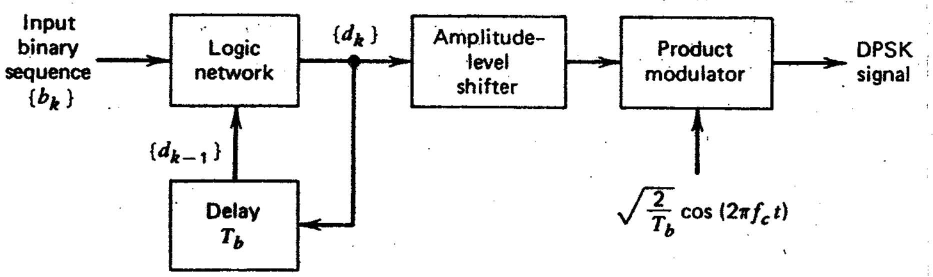
July-05, 4M
July-07, 8M
Jan-08, 4M

* DPSK eliminates the need for a Coherent reference signal at the receiver by Combining two basic operations at the transmitter :
  1) Differential encoding of the I/p binary wave &
  2) phase - Shift Keying

  hence the name DPSK.

* For Symbol '1', a carrier Signal with 0° phase Shift is transmitted.
* For Symbol '0', a Carrier Signal with 180° Phase Shift is transmitted.

DPSK Transmitter :-



(a)

* The differential encoding process at the transmitter Starts with an arbitrary bit, Serving as reference, & thereafter the differentially encode Sequence $\{d_K\}$ is generated by using the logic equation:

$$\boxed{d_K = b_K d_{K-1} + \bar{b}_K \bar{d}_{K-1}}$$

(XNOR gate)

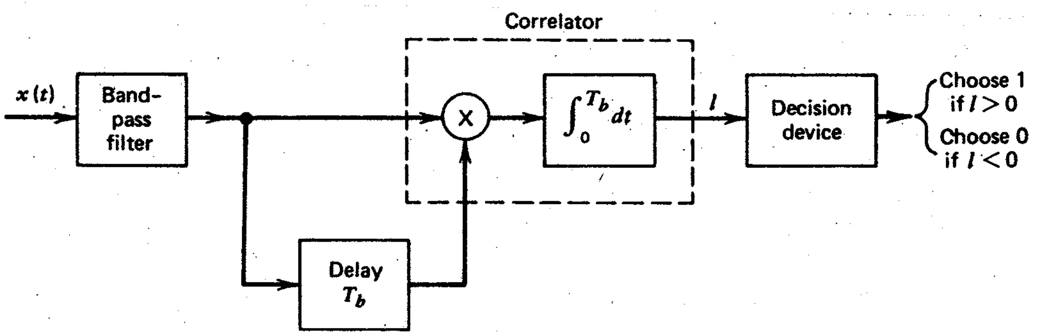
a	b	$y=\overline{a \oplus b}$
0	0	1
0	1	0
1	0	0
1	1	1

Where b_K is the I/p binary digit at time KT_b & d_{K-1} is the previous value of the differentially encoded digit.

Table Illustrating the Generation of DPSK Signal

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{\bar{b}_k\}$		0	1	1	0	1	1	0	0
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
$\{\bar{d}_{k-1}\}$		0	0	1	0	0	1	0	0
$\{b_k d_{k-1}\}$		1	0	0	1	0	0	1	1
$\{\bar{b}_k \bar{d}_{k-1}\}$		0	0	1	0	0	1	0	0
Differentially encoded sequence $\{d_k\}$	1	1	0	1	1	0	1	1	1
Transmitted phase (*radians*)	0	0	π	0	0	π	0	0	0

DPSK receiver :-



* At the receiver I/p, the received DPSK Signal plus noise is passed through a BPF Centered at the carrier frequency f_c.
* The BPF o/p & a delayed version of it (delay $= T_b$) are applied to the Correlator.
* The o/p of the Correlator 'l' is proportional to the Cosine of the difference between the carrier angles in the two Correlator I/p's.
* If $l > 0 \rightarrow$ receiver decides in favour of Symbol '1' & phase difference ranges inside $-\pi/2$ to $+\pi/2$.
* If $l < 0 \rightarrow$ receiver decides in favour of Symbol '0' & phase difference

ranges outside $-\pi/2$ to $+\pi/2$.

Quadrature phase Shift Keying (QPSK) :-

July-05, 4M	Jan-07, 10M

* With the help of a block diagrams, explain the operation of QPSK transmitter & receiver.

June-09, 6M	Jan-08, 4M	July-06, 8M

* In QPSK System, information is carried by 4 phase of the Sinusoidal carrier i.e $\pi/4$, $3\pi/4$, $5\pi/4$ & $7\pi/4$.

* A QPSK Signal can be represented in time domain as:

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\pi/4\right] & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \longrightarrow (1)$$

Where $i = 1, 2, 3, 4$ and

$E \rightarrow$ Signal energy per Symbol

$T \rightarrow$ Symbol duration.

* There are four message points & associated Signal vector are defined by

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \theta_i\right] & ; 0 \leq t \leq T \\ 0 & ; \text{otherwise} \end{cases} \longrightarrow (2)$$

Where $\theta_i = (2i-1)\pi/4$

i	$\theta_i = (2i-1)\pi/4$
1	$\pi/4$
2	$3\pi/4$
3	$5\pi/4$
4	$7\pi/4$

We can write eq (2) as

$S_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \pi/4)$; dibit 10

$S_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 3\pi/4)$; dibit 00

$S_3(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 5\pi/4)$; dibit 01

$S_4(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 7\pi/4)$; dibit 11

$$\boxed{\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B}$$

* Eq (1) can be written as

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \cdot \cos\left[(2i-1)\pi/4\right] - \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \cdot \sin\left[(2i-1)\pi/4\right] \; ; 0 \leq t \leq T \\ 0 \quad ; \text{otherwise} \end{cases} \longrightarrow (3)$$

* From eq (3), we observe that there are two orthonormal basis functions $\phi_1(t)$ & $\phi_2(t)$ defined by

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad ; 0 \leq t \leq T \longrightarrow (4)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad ; 0 \leq t \leq T$$

* The elements of the Signal vector, namely S_{i1} & S_{i2}.

i	$S_i(t)$	I/p dibit	Phase of QPSK Signal	Co-ordinates of message points	
				S_{i1}	S_{i2}
1	$S_1(t)$	10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
2	$S_2(t)$	00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
3	$S_3(t)$	01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
4	$S_4(t)$	11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

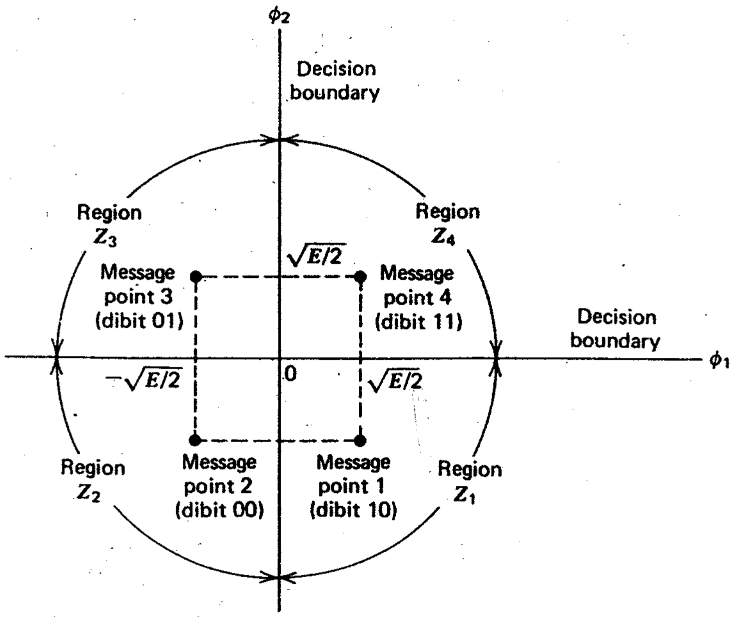
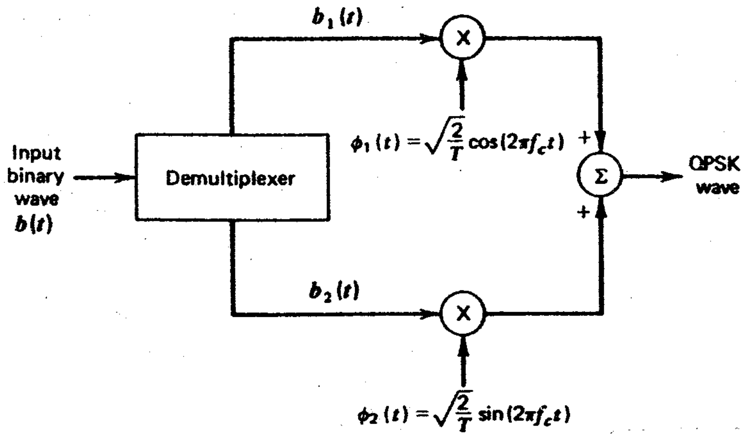


Figure Signal space diagram for coherent QPSK system.

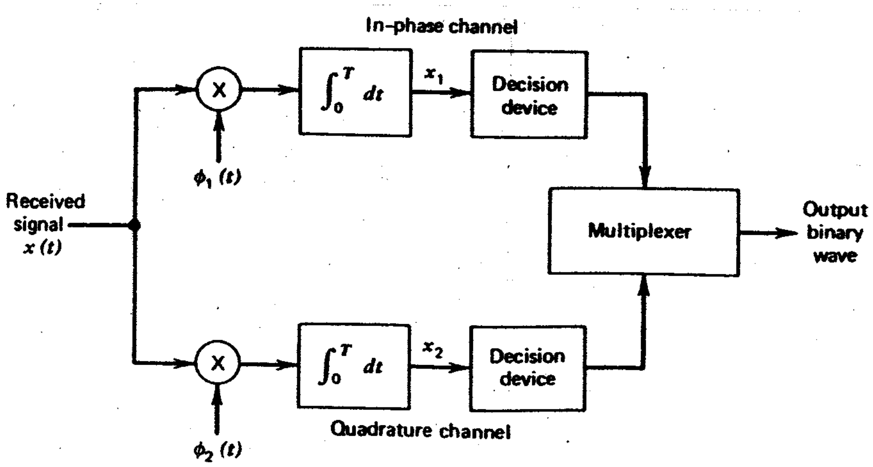
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

QPSK Transmitter :-



* The I/P binary Sequence $b(t)$ represented in polar form is divided into odd $[b_1(t)]$ & even $[b_2(t)]$ numbered bits by using demultiplexer. They are denoted as $b_1(t)$ & $b_2(t)$.
* These two Sequences, phase modulate two carrier Signal of Same frequency but quadrature in phase.
* Since each Symbol carries two bits, the Signalling rate decreased.
  ∴ BW required is half the bandwidth required compared to BPSK.

QPSK Receiver :-



(b)

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* The QPSK receiver consists of a pair of correlators with locally generated carrier signals $\phi_1(t)$ & $\phi_2(t)$.

* The o/p of the two correlators are x_1 & x_2 are compared with a threshold '0'.

If $x_1 > 0 \rightarrow$ decision is made in favour of symbol 1.

If $x_1 < 0 \rightarrow$ decision is made in favour of symbol 0.

If $x_2 > 0 \rightarrow$ decision is made in favour of symbol 1.

If $x_2 < 0 \rightarrow$ decision is made in favour of symbol 0.

* The two o/p's are combined in a multiplier to reproduce original binary sequence.

Probability of error in QPSK :-

June-09, 8M	Jan-09, 12M

* The signal points S_1, S_2, S_3 & S_4 are located symmetrically in two dimensional signal space diagram as shown in below figure.

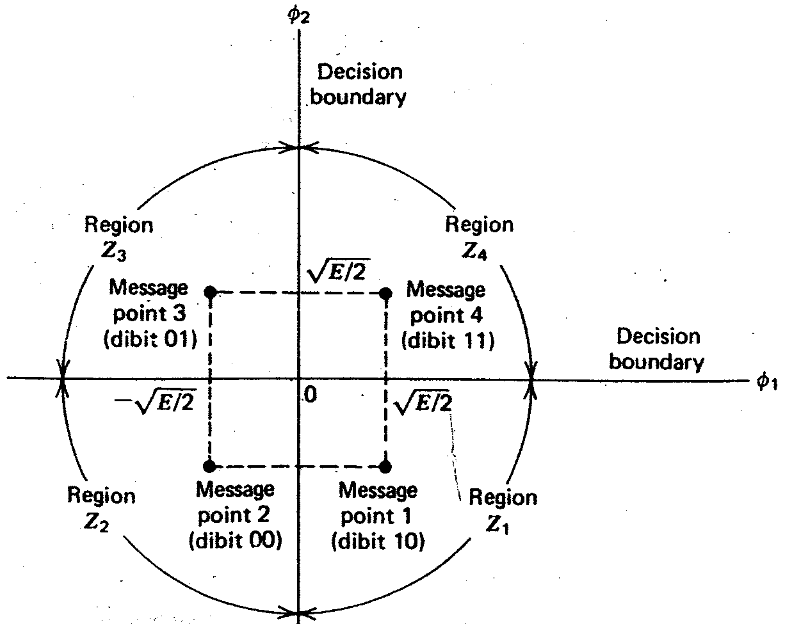


Figure Signal space diagram for coherent QPSK system.

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

∴ Computing probability for one message point, is remains same for other three points.

* Consider transmission of symbol $S_4(t)$ then received signal $x(t)$ will be

$$x(t) = S_4(t) + w(t) \qquad 0 \leq t \leq T$$

The samples x_1 & x_2 are computed as follows

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt = \int_0^T [S_4(t) + w(t)]\,\phi_1(t)\,dt$$

$$x_1 = \int_0^T S_4(t)\phi_1(t)\,dt + \int_0^T w(t)\,\phi_1(t)\,dt$$

$$x_1 = S_{41} + w_1$$

$$\boxed{x_1 = \sqrt{\frac{E}{2}} + W_1}$$

* Similarly

$$x_2 = \int_0^T x(t)\,\phi_2(t) = \int_0^T [S_4(t) + w(t)]\,\phi_2(t)\cdot dt$$

$$x_2 = \int_0^T S_4(t)\cdot\phi_2(t)\,dt + \int_0^T w(t)\,\phi_2(t)\cdot dt$$

$$x_2 = S_{42} + W_2$$

$$\boxed{x_2 = \sqrt{\frac{E}{2}} + W_2}$$

* X_1 & X_2 are gaussian random variables with mean 'μ' $= \sqrt{\frac{E}{2}}$ ~~variance~~ $\cancel{\sigma^2 = 0}$

* W_1 & W_2 are also gaussian random variables with variance $\sigma^2 = \frac{N_0}{2}$

* When signal $S_4(t)$ is transmitted, the received signal point -

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

lies in the decision region 'Z_4'.

If $x_1 > 0$ & $x_2 > 0$, leading to correct decision.
(1) (1)

* The conditional PDF is given by

$$f_{X_1}(x_1/S_4(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\left[\frac{(x_1-\mu)^2}{2\sigma^2}\right]} \quad \text{and}$$

$$f_{X_2}(x_2/S_4(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\left[\frac{(x_2-\mu)^2}{2\sigma^2}\right]}$$

$$f_{X_1}(x_1|S_4(t)) = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \cdot e^{-\left[\frac{\left(x_1-\sqrt{\frac{E}{2}}\right)^2}{\cancel{2}\frac{N_0}{\cancel{2}}}\right]} \quad \text{and}$$

$$f_{X_2}(x_2|S_4(t)) = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \cdot e^{-\left[\frac{\left(x_2-\sqrt{\frac{E}{2}}\right)^2}{\cancel{2}\frac{N_0}{\cancel{2}}}\right]}$$

$$\boxed{f_{X_1}(x_1|S_4(t)) = \frac{1}{\sqrt{\pi N_0}} \, e^{-\left[\frac{x_1-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2}} \longrightarrow (1) \quad \text{and}$$

$$\boxed{f_{X_2}(x_2|S_4(t)) = \frac{1}{\sqrt{\pi N_0}} \, e^{-\left[\frac{x_2-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2}} \longrightarrow (2)$$

* Let us assume $S_4(t)$ is transmitted. If the received signal 'x' should fall in region Z_4 i.e. both x_1 & x_2 should be +ve.

P.T.O

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

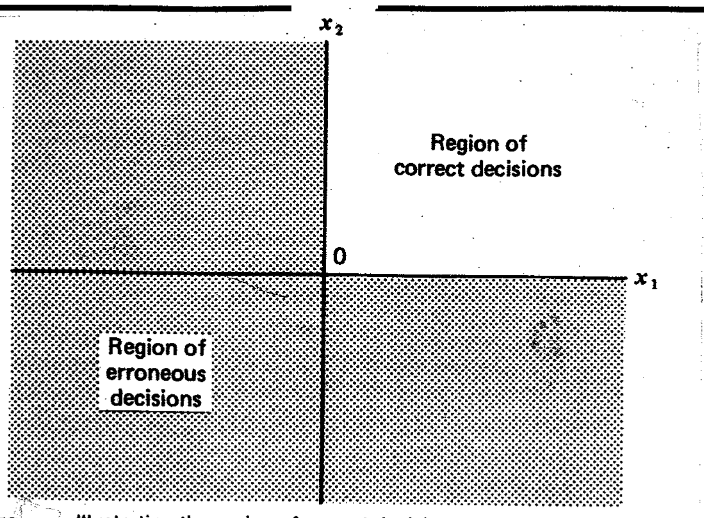


Figure Illustrating the region of correct decisions and the region of erroneous decisions, given that signal $s_4(t)$ was transmitted.

* Probability of correct decision 'P_c' is equal to the product of conditional probabilities of events $x_1 > 0$ & $x_2 > 0$, both given that $S_4(t)$ was transmitted.

Region Z_4 : $\boxed{\begin{array}{l} 0 \leqslant x_1 \leqslant \infty \\ 0 \leqslant x_2 \leqslant \infty \end{array}}$

Sub eq ① & ② in eq ③

$$P_c = \int_0^\infty f_{X1}(x_1/S_4(t))\, dx_1 \times \int_0^\infty f_{X2}(x_2/S_4(t)) \cdot dx_2 \longrightarrow ③$$

$$P_c = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2} dx_1 \cdot \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_2 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]} \cdot dx_2$$

Let

$Z = \dfrac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}$	$Z = \dfrac{x_2 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}$
$dZ = \dfrac{dx_1 - 0}{\sqrt{N_0}}$	$dZ = \dfrac{dx_2 - 0}{\sqrt{N_0}}$
$\boxed{dx_1 = \sqrt{N_0}\, dZ}$	$\boxed{dx_2 = \sqrt{N_0}\, dZ}$

Limits

$x_1 = \infty$	$x_2 = \infty$
then, $Z = \infty$	then, $Z = \infty$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$x_1=0,\quad Z=\dfrac{x_1-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}$ $Z=\dfrac{0-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}$ $\boxed{Z=-\sqrt{\dfrac{E}{2N_0}}}$	$x_2=0,\quad Z=\dfrac{x_2-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}$ $Z=\dfrac{0-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}$ $\boxed{Z=-\sqrt{\dfrac{E}{2N_0}}}$

$$P_c=\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty}\frac{1}{\sqrt{\pi \cancel{N_0}}}\cdot e^{-Z^2}\cdot\sqrt{\cancel{N_0}}\,dZ\ \cdot\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty}\frac{1}{\sqrt{\pi \cancel{N_0}}}\cdot e^{-Z^2}\sqrt{\cancel{N_0}}\,dZ$$

$$P_c=\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty}\frac{1}{\sqrt{\pi}}\cdot e^{-Z^2}dZ\cdot\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty}\frac{1}{\sqrt{\pi}}\cdot e^{-Z^2}.dZ$$

$$\boxed{P_c=\left[\frac{1}{\sqrt{\pi}}\int_{\sqrt{\frac{E}{2N_0}}}^{\infty}e^{-Z^2}.\,dZ\right]^2}\longrightarrow (4)$$

$\because\ \boxed{P_e=\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)}$

From the definition of the complementary error function we have

$$\frac{1}{\sqrt{\pi}}\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty}e^{-Z^2}dZ=1-\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\longrightarrow (5)$$

$\because\ \boxed{P_c=1-P_e}$

Substituting eq (5) in eq (4)

$$\therefore\ P_c=\left[1-\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2$$

WKT

$(a-b)^2=a^2+b^2-2ab$

$a=1,\ b=\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$

$$P_c=1+\frac{1}{4}\,\mathrm{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)-\cancel{2}(1)\cdot\frac{1}{\cancel{2}}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

Downloaded from www.citystudentsgroup.blogspot.com
ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{P_c = 1 + \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) - \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)}$$

Thus average probability of Symbol error

$$P_e = 1 - P_c$$

$$P_e = 1 - \left[1 + \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) - \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

$$= \cancel{1} - \cancel{1} - \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) + \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$\boxed{P_e = \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\,\text{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)}$$

* In the region $Z_4 : \left(\frac{E}{2N_0}\right) \gg 1$, hence we can ignore Second term

$$\boxed{P_e \approx \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)}$$

* In QPSK two bits are transmitted per Symbol

thus $\boxed{E = 2E_b}$

$$P_e \approx \text{erfc}\left(\sqrt{\frac{\cancel{2}E_b}{\cancel{2}N_0}}\right)$$

$$\boxed{P_e \approx \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

FORMULAE

	Coherent detector	Non Coherent detector
PSK	$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	
FSK	$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$	$P_e = \frac{1}{2}e^{\left(-E_b/2N_0\right)}$
ASK	$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]$	$P_e = \frac{1}{2}e^{\left(-E_b/4N_0\right)}$
DPSK	—	$P_e = \frac{1}{2}e^{\left(\frac{-E_b}{N_0}\right)}$
QPSK	$P_e = \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	—
MFSK	$P_e = \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	—

* $E_b = PT_b$
* $P = \dfrac{A_c^2}{2}$
* $T_b = \dfrac{1}{R_b}$
* $\mathrm{erf}(u) = 1 - \mathrm{erfc}(u)$
* Channel bandwidth $B_T = \dfrac{1}{T_b} = R_b$

Error Function Table :

Table Error Function

u	erf(u)	u	erf(u)
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

Jan - 10, 6M

1) A binary FSK System transmits data at a rate of 2Mbps over an AWGN Channel. The noise power Spectral density $\frac{N_0}{2} = 10^{-20}$ Watts/Hz. Determine the probability error 'P_e' for Coherent detection of FSK. Assume amplitude of the received Signal = 1 μV

Given :- $R_b = 2\text{Mbps}, \quad \frac{N_0}{2} = 10^{-20}\ \text{W/Hz}, \quad A_c = 1\times10^{-6}\ \text{V}$

$T_b = \frac{1}{R_b} = \frac{1}{2\times10^{6}}, \quad N_0 = 2\times10^{-20}\ \text{W/Hz}$

Sol :-

* $E_b = PT_b$
* $P = \frac{A_c^2}{2} = \frac{(1\times10^{-6})^2}{2} = 5\times10^{-13}\ \text{W}$
* $E_b = 5\times10^{-13} \times \frac{1}{2\times10^{6}} = 2.083\times10^{-19}$ Joules $\longrightarrow$ 2 - Marks

* WKT 'P_e' for FSK System is

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{2.083\times10^{-19}}{2\times2\times10^{-20}}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{5.2075}\right)$$

$$= \frac{1}{2}\,\text{erfc}\,(2.2819)$$

$$= \frac{1}{2}\left[1 - \text{erf}(2.2819)\right]$$

$$= \frac{1}{2}\left[1 - 0.99959\right]$$

$$= \frac{1}{2}\left(4.1\times10^{-4}\right)$$

$\because$ $\boxed{\text{erfc}(u) = 1 - \text{erf}(u)}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{P_e = 2.05 \times 10^{-4}} \longrightarrow \boxed{4\text{-Marks}}$$

2) A binary FSK System transmits data at a rate of 2Mbps over an AWGN Channel. The noise is Zero mean with PSD, $\frac{N_0}{2} = 10^{-20}$ W/HZ. The amplitude of received Signal in the absence of noise is 1 μv. Determine the average probability of error for Coherent detection of FSK. Take erfc $\sqrt{6.25} = 0.00041$.

July-09, 6M

Given :-

$$R_b = 2\text{Mbps}, \quad \frac{N_0}{2} = 10^{-20} \text{ W/HZ}$$

$$A_c = 1 \times 10^{-6} \text{ v}, \quad \text{erfc}\sqrt{6.25} = 0.00041$$

$$T_b = \frac{1}{R_b} = \frac{1}{2 \times 10^6} \qquad N_0 = 2 \times 10^{-20} \text{ W/HZ}.$$

Sol :- WKT 'P_e' for FSK System

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

* $E_b = PT_b$

* $P = \frac{A_c^2}{2} = \frac{(1 \times 10^{-6})^2}{2} = 0.5 \times 10^{-12}$ W

* $E_b = 0.5 \times 10^{-12} \times \frac{1}{2 \times 10^6} = \underline{0.25 \times 10^{-18}}$ Joules $\longrightarrow \boxed{2\text{-Marks}}$

* $P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{0.25 \times 10^{-18}}{2 \times 2 \times 10^{-20}}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{6.25}\right) = \frac{1}{2}\left[0.00041\right]$

$\because$ erfc$\left(\sqrt{6.25}\right) = 0.00041$

$$\boxed{P_e = 2.05 \times 10^{-4}} \longrightarrow \boxed{4\text{-Marks}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

3) A binary data is transmitted using ASK over AWGN channel at a rate of 2.4 Mbps. The carrier amplitude at the receiver is 1mV. Noise power spectral density $\frac{N_0}{2} = 10^{-15}$ W/Hz. Find the average probability of error if detector is coherent. Take $\text{erfc}(5) = 3 \times 10^{-6}$.

Jan-10, 6M

Given: $R_b = 2.4$ Mbps, $\frac{N_0}{2} = 10^{-15}$ W/Hz, $A_c = 1$ mV, $\text{erfc}(5) = 3\times10^{-6}$

$$T_b = \frac{1}{R_b} = \frac{1}{2.4\times10^{6}} \quad , \quad N_0 = 2\times10^{-15} \text{ W/Hz}$$

* WKT for ASK System: $P_e = \frac{1}{2}\,\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$

* $P = \dfrac{A_c^2}{2} = \dfrac{(1\times10^{-3})^2}{2} = 0.5\times10^{-6}$ $\longrightarrow$ 1-Mark

* $E_b = PT_b = 0.5\times10^{-6} \times \dfrac{1}{2.4\times10^{6}} = 2.083\times10^{-13}$ Joules $\rightarrow$ 2-Marks

* $$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{2.083\times10^{-13}}{4\times2\times10^{-15}}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{26.04}\right)$$

$$= \frac{1}{2}\,\text{erfc}(5.103)$$

$$= \frac{1}{2}\left(3\times10^{-6}\right)$$

$\boxed{P_e = 1.5\times10^{-6}}$ $\longrightarrow$ 3-Marks

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

4) A binary data is transmitted using ASK over a AWGN - Channel at a rate of 2.4 Mbps. The carrier amplitude at the receiver is 1 mv. The noise power Spectral density $\frac{N_0}{2} = 10^{-15}$ W/Hz Find the average probability of error if the detection is i) Coherent ii) Non-Coherent.

Jan -08, 6M

(Hint: Take $\operatorname{erfc}(5) = 3\times10^{-6}$)

Given: $R_b = 2.4$ Mbps , $T_b = \frac{1}{R_b} = \frac{1}{2.4\times10^{6}}$

$A_c = 1\text{mv}$, $\frac{N_0}{2} = 10^{-15}$ W/Hz, $N_0 = 2\times10^{-15}$ W/Hz

$\operatorname{erfc}(5) = 3\times10^{-6}$

Sol :-

$$* \quad P = \frac{A_c^2}{2} = \frac{(1\times10^{-3})^2}{2} = \underline{0.5\times10^{-6}\ \text{W}}$$

$$* \quad E_b = PT_b = 0.5\times10^{-6} \times \frac{1}{2.4\times10^{6}} = \underline{2.083\times10^{-13}} \text{ Joules.}$$

i) Coherent ASK:

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{0.208\times10^{-13}}{4\times2\times10^{-15}}}\right) = \frac{1}{2}\operatorname{erfc}(5.103)$$

$$= \frac{1}{2}(3\times10^{-6})$$

$$\boxed{P_e = 1.5\times10^{-6}}$$

ii) Non-Coherent :- WKT, $P_e = \frac{1}{2}\exp\left(-\frac{E_b}{4N_0}\right) = \frac{1}{2}\exp\left(\frac{-2.083\times10^{-13}}{4\times2\times10^{-15}}\right)$

$$= \frac{1}{2}\exp(-26.04125) = \frac{1}{2}(4.902\times10^{-12})$$

$$\boxed{P_e = 2.45\times10^{-12}}$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

5) Binary data is transmitted over AWGN channel using BPSK at a rate of 1 Mbps. It is desired to have average probability of error $P_e \leq 10^{-4}$. Noise PSD is $\frac{N_0}{2} = 10^{-12}$ W/Hz. Determine the average carrier power required at receiver I/p if the detector is of Coherent type. [Assume erfc(3.5) = 0.00025].

July-08, 6M

Given : $R_b = 1\text{Mbps}$, $T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$

$\frac{N_0}{2} = 10^{-12}$ W/Hz, $N_0 = 2\times10^{-12}$ W/Hz

$P_e \leq 10^{-4}$, $\text{erfc}(3.5) = 0.00025$.

Sol:- WKT for BPSK System

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$10^{-4} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = 2\times10^{-4}$$

$\text{erf}(u) = 1 - \text{erfc}(u)$

$$\text{erf}(u) = 1 - 2\times10^{-4}$$

$$\text{erf}(u) = 0.9998$$

from error function table $\boxed{u \approx 2.8 \text{ or } 2.9}$

$$\therefore \sqrt{\frac{E_b}{N_0}} = 2.8$$

$$\frac{E_b}{N_0} = (2.8)^2$$

$$\frac{E_b}{N_0} = 7.84$$

$$E_b = N_0 \times 7.84$$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$E_b = 2\times10^{-12} \times 7.8$

$PT_b = 1.568 \times 10^{-11}$

$$P = \frac{1.568\times10^{-11}}{T_b} = \frac{1.568\times10^{-11}}{1\times10^{-6}}$$

$$\boxed{P = 1.568\times10^{-5}\ W}$$

6) A binary FSK System transmits data at a rate of 2Mbps over a AWGN Channel. The noise is Zero mean with power Spectral density $\frac{N_0}{2} = 10^{-20}$ W/Hz. The amplitude of received Signal in the absence of noise is 1 μv. Determine the average probability of error for Coherent detection of FSK.

July-07, 6M

Given: $R_b = 2$ Mbps, $T_b = \frac{1}{R_b} = \frac{1}{2\times10^6}$

$\frac{N_0}{2} = 10^{-20}$ W/Hz, $N_0 = 2\times10^{-20}$ W/Hz, $A = 1$ μv.

Sol:

* $P = \frac{A^2}{2} = \frac{(1\times10^{-6})^2}{2} = 5\times10^{-13}$ W

* $E_b = PT_b = 5\times10^{-13} \cdot \frac{1}{2\times10^6} = 2.5\times10^{-19}$ Joules

WKT for FSK System: $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{2.5\times10^{-19}}{2\times2\times10^{-20}}}\right)$

$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{6.25}\right)$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$P_e = \frac{1}{2}\,\mathrm{erfc}(2.5)$$

$$= \frac{1}{2}\left(4.1\times10^{-4}\right)$$

$$\boxed{P_e = 2.05\times10^{-4}}$$

WKT

$$\mathrm{erfc}(2.5) = 1 - \mathrm{erf}(2.5)$$

From error function Table

$$\mathrm{erf}(2.5) = 0.99959$$

$$\mathrm{erfc}(2.5) = 1 - 0.99959$$

$$\boxed{\mathrm{erfc}(2.5) = 4.1\times10^{-4}}$$

---

6) Binary data are transmitted at a rate of 10^6 bits per second over a microwave link. Assuming channel noise is AWGN with zero mean & power spectral density at the receiver I/P is 10^{-10} W/Hz, find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent binary PSK. Determine the minimum channel bandwidth required.

Model - 6M

Given : $R_b = 1\times10^6$, $T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$

$\frac{N_0}{2} = 10^{-10}$ W/Hz, $N_0 = 2\times10^{-10}$ W/Hz , $P_e \leq 10^{-4}$

Sol :- WKT for PSK System: $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$10^{-4} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\boxed{\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = 2\times10^{-4}} \rightarrow (1)$$

WKT $\mathrm{erf}(u) = 1 - \mathrm{erfc}(u)$

ARUNKUMAR.G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.*

$$= 1 - 2\times10^{-4}$$

$$\boxed{erf(u) = 0.9998}$$

From error function table $u \approx 2.8$.

From eq ① $\sqrt{\frac{E_b}{N_0}} = 2.8$

$$\frac{E_b}{N_0} = (2.8)^2 = 7.84$$

$$E_b = 7.84 \times 2\times10^{-10}$$

$$\boxed{E_b = 1.568\times10^{-9} \text{ Joules}}$$

* WKT $E_b = PT_b$

$$P = \frac{E}{T_b} = \frac{1.568\times10^{-9}}{\frac{1}{1\times10^{6}}}$$

∴ Average carrier power

$$\boxed{P = 1.568\times10^{-3} \text{ W}}$$

* Channel bandwidth $B_T = \frac{1}{T_b} = R_b$

$$\boxed{B_T = 1\text{MHz}}$$

7) Binary data are transmitted at a rate of 10^6 bit per Second over a microwave link. Assuming Channel noise is AWGN with Zero mean & PSD at the receiver I/p is 10^{-6} W/Hz. Find the average Carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for Coherent binary FSK. Determine the minimum Channel bandwidth required.

Model-6M

Given: $R_b = 10^6$ bps , $T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$

$\frac{N_0}{2} = 10^{-6}$ W/Hz , $N_0 = 2\times10^{-6}$ W/Hz , $P_e = 10^{-4}$, $P=?$, $B_T=?$

Sol: WKT for FSK System:

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$10^{-4} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\boxed{\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) = 2\times10^{-4}}$$

WKT $\mathrm{erf}(u) = 1 - \mathrm{erfc}(u)$

$= 1 - 2\times10^{-4}$

$$\boxed{\mathrm{erf}(u) = 0.9998}$$

From error function table $\boxed{u \approx 2.8}$

From eq (1) $\sqrt{\frac{E_b}{2N_0}} = 2.8$

$$\frac{E_b}{2N_0} = (2.8)^2 = 7.84$$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$E_b = 2 \times 2 \times 10^{-6} \times 7.84$$

$$\boxed{E_b = 31.36 \times 10^{-6} \text{ Joules}}$$

* WKT $E_b = PT_b$

$$P = \frac{E_b}{T_b} = E_b R_b = 31.36 \times 10^{-6} \times 1 \times 10^{6}$$

$$\boxed{P = 31.36 \text{ W}}$$

* $B_T = R_b = \underline{1 \text{ MHz}}$

---

8) A binary data is transmitted using ASK over a AWGN Channel at a rate 2.5×10^6 bps. The carrier amplitude at the receiver is 1 microvolt (μV). Noise power spectral density 10^{-20} W/Hz. Find the average probability of error for coherent FSK detector.

Feb-03, 6M

Given: $R_b = 2.5 \times 10^6$ bps, $T_b = \frac{1}{R_b} = \frac{1}{2.5 \times 10^6}$

$\frac{N_0}{2} = 10^{-20}$ W/Hz, $N_0 = 2 \times 10^{-20}$ W/Hz, $A = 1 \times 10^{-6}$ V, $P_e = ?$

Sol:- WKT for FSK System:

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

* $P = \frac{A^2}{2} = \frac{(1 \times 10^{-6})^2}{2} = 0.5 \times 10^{-12}$ W

* $E_b = PT_b = 0.5 \times 10^{-12} \frac{1}{2.5 \times 10^6} = 2 \times 10^{-19}$ Joules

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{2\times10^{-19}}{2\times2\times10^{-20}}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{5}\right)$$

$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\,(2.236)}$$

WKT $\quad \text{erfc}(u) = 1 - \text{erf}(u)$

$$\text{erfc}(2.236) = 1 - \text{erf}(2.236)$$

From error function table $\boxed{\text{erf}(2.236) = 0.99959}$

$$\text{erfc}(2.236) = 1 - 0.99959 = 4.1\times10^{-4}$$

$$P_e = \frac{1}{2}\left[4.1\times10^{-4}\right]$$

$$\boxed{P_e = 0.205\times10^{-3}}$$

---

9) In PSK System binary 1 is represented by $S_1(t) = A\cos\omega_c t$ & binary 0 by $S_2(t) = -A\cos\omega_c t$ is transmitted over a AWGN channel at 0.2 msec. The carrier amplitude at the receiver is one millivolt. Noise PSD is 10^{-11} W/Hz. Find the average probability of error for Coherent PSK detector.

$\boxed{\text{Feb-04, 6M}}$

Given : $\quad S_1(t) = A\cos\omega_c t$ & $S_2(t) = -A\cos\omega_c t$.

$T_b = 0.2$ msec, $\quad \frac{N_0}{2} = 10^{-11}$ W/Hz, $\quad N_0 = 2\times10^{-11}$ W/Hz.

$P_e = ?\quad , \quad A = 1$ mV

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* $P = \frac{A^2}{2} = \frac{(1\times10^{-3})^2}{2} = 0.5\times10^{-6}$ W

* $E_b = PT_b = (0.5\times10^{-6})\times(0.2\times10^{-3}) = 1\times10^{-10}$ Joules

* $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{1\times10^{-10}}{2\times10^{-11}}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}(\sqrt{5})$$

$$= \frac{1}{2}\,\mathrm{erfc}(2.236)$$

$$= \frac{1}{2}(4.1\times10^{-4})$$

$$\boxed{P_e = 0.205\times10^{-3}}$$

WKT

$\mathrm{erfc}(u) = 1 - \mathrm{erf}(u)$

$\mathrm{erfc}(2.236) = 1 - \mathrm{erfc}(2.236)$

From error function table

$\mathrm{erf}(2.236) = 0.99959$

$= 1 - 0.99959$

$\boxed{\mathrm{erf}(2.236) = 4.1\times10^{-4}}$

10) A binary ASK System transmit data at a rate of 4.8 Mbps over an AWGN channel having bandwidth 10MHz. The noise is Zero mean with power spectral density 10^{-15} W/Hz. The amplitude of received signal is one mv. Determine the average probability of error for Coherent ASK detector.

Feb-04, 6M

Given: $R_b = 4.8$ Mbps, $T_b = \frac{1}{R_b} = \frac{1}{4.8\times10^6}$

$B_T = 10$ MHz, $A = 1\times10^{-3}$ V, $\frac{N_0}{2} = 10^{-15}$ W/Hz

$N_0 = 2\times10^{-15}$ W/Hz.

Sol:-

$* \; P = \frac{A^2}{2} = \frac{(1\times10^{-3})^2}{2} = 0.5\times10^{-6}\,W$

$* \; E_b = PT_b = 0.5\times10^{-6} \times \frac{1}{4.8\times10^{6}} = 0.10416\times10^{-12}$ Joules.

$* \; P_e = \frac{1}{2}\,\text{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_o}}\right] = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{4N_o}}\right)$

$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{0.10416\times10^{-12}}{4\times2\times10^{-15}}}\right)$

$= \frac{1}{2}\,\text{erfc}\,(3.608)$

$= \frac{1}{2}\left[1 - \text{erf}\,(3.608)\right]$

From error function table $\text{erf}\,(3.608) = 0.999998$

$= \frac{1}{2}\left[1 - 0.999998\right]$

$P_e = \frac{1}{2}(2\times10^{-6})$

$\boxed{P_e = 1\times10^{-6}}$

11) A binary FSK System transmits data at a rate of 10^6 bps over an AWGN Channel. Noise PSD is 10^{-10} W/Hz. Find the average Carrier power required to maintain an average Probability of error $P_e \leq 10^{-4}$ for Non Coherent binary FSK. Determine the channel bandwidth required.

Aug-04, 6M

Given : $R_b = 10^6$ bps , $T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$

$\frac{N_o}{2} = 10^{-10}$ W/Hz, $N_o = 2\times10^{-10}$ W/Hz.

$P_e \leq 10^{-4}$, $P = ?$, $B_T = ?$

Sol:- WKT for Non-Coherent BFSK System:

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

$$10^{-4} = \frac{1}{2} \exp\left(\frac{-E_b}{2N_0}\right)$$

$$(2\times10^{-4}) = \exp\left(-\frac{E_b}{2N_0}\right)$$

$$\log(2\times10^{-4}) = -\frac{E_b}{2N_0}$$

$$\cancel{-}\,3.698 = \cancel{-}\,\frac{E_b}{2N_0}$$

$$E_b = 2N_0 \times 3.698$$

$$= 2\times 2\times10^{-10} \times 3.698$$

$$\boxed{E_b = 1.4792\times10^{-9}}$$

* $E_b = PT_b$

$$P = \frac{E_b}{T_b} = E_b R_b = 1.4792\times10^{-9} \times 1\times10^{6}$$

$$\boxed{P = 1.4792\times10^{-3}}$$

* $\boxed{B_T = R_b = 1\text{MHz}}$

ARUNKUMAR.G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

12) Compare the average power requirements of binary non-Coherent DPSK, FSK, Coherent ASK & PSK Signalling Schemes operating at a data rate of 1000 bits/Sec over a bandpass Channel having a bandwidth of 3000Hz, $\eta/2 = 10^{-10}$ W/Hz, $P_e = 10^{-5}$.

Given : $R_b = 1000$ bps, $T_b = \frac{1}{1000}$, $B_T = 3000$Hz   July-05, 8M

$\frac{N_0}{2} = 10^{-10}$ W/Hz, $N_0 = 2\times10^{-10}$ W/Hz, $P_e = 10^{-5}$

i) Coherent ASK :-

$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

$$2\times10^{-5} = \text{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

WKT

$\text{erf}(u) = 1 - \text{erfc}(u)$

$\text{erf}(u) = 1 - 2\times10^{-5}$

$\text{erf}(u) = 0.99998$

From erf function table

$u = 3$

$$\therefore \sqrt{\frac{E_b}{4N_0}} = 3$$

$$\frac{E_b}{4N_0} = 9$$

$$E_b = 9\times4\times N_0$$

$$E_b = 36N_0$$

$$PT_b = 36N_0$$

$$P = \frac{36}{T_b}N_0$$

$$= \frac{36\times2\times10^{-10}}{10^{-3}}$$

$$\boxed{P = 7.2\times10^{-7}\ \text{W}}$$

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

Coherent PSK :-

WKT $P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$2\times10^{-5} = erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

WKT $erf(u) = 1 - erfc(u)$

$$erf\left(\sqrt{\frac{E_b}{N_0}}\right) = 1 - 2\times10^{-5}$$

$$erf\left(\sqrt{\frac{E_b}{N_0}}\right) = 0.99998$$

From error function table $erf(3) = 0.99998$

$$\sqrt{\frac{E_b}{N_0}} = 3$$

$$\frac{E_b}{N_0} = 9$$

$$E_b = 9N_0$$

$$E_b = 9\times2\times10^{-10}$$

$$PT_b = 18\times10^{-10}$$

$$P = \frac{18\times10^{-10}}{T_b} = 18\times10^{-10}\times R_b$$

$$\boxed{P = 1.8\times10^{-6}\,W}$$

Non - Coherent FSK :-

WKT $P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$

$$2\times10^{-5} = \exp\left(-\frac{E_b}{2N_0}\right)$$

$$\log\left(2\times10^{-5}\right) = -\frac{E_b}{2N_0}$$

$$\cancel{-}\,4.69 = \cancel{-}\,\frac{E_b}{2N_0}$$

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

$$E_b = 4.69 \times 2N_0$$

$$PT_b = 4.69 \times 2 \times 2 \times 10^{-10}$$

$$P = \frac{4.69 \times 2 \times 2 \times 10^{-10}}{T_b}$$

$$= 1.876 \times 10^{-9} \times R_b = 1.876 \times 10^{-9} \times 1 \times 10^{3}$$

$$\boxed{P = 1.876 \times 10^{-6}\, W}$$

Non Coherent DPSK:-

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$$

$$2 \times 10^{-5} = \exp\left(-\frac{E_b}{N_0}\right)$$

$$\log(2 \times 10^{-5}) = -\frac{E_b}{N_0}$$

$$\cancel{-}4.69 = \cancel{-}\frac{E_b}{N_0}$$

$$E_b = 4.69\, N_0$$

$$PT_b = 4.69 N_0$$

$$P = \frac{4.69 N_0}{T_b} = 4.69 N_0 \times R_b$$

$$P = 4.69 \times 2 \times 10^{-10} \times 1 \times 10^{3}$$

$$\boxed{P = 0.938 \times 10^{-6}\, W}$$

13) In an ASK System, Symbol 1 is transmitted by transmitting a Sinusoidal carrier of amplitude $\sqrt{2E_b/T_b}$, where E_b is the bit energy & T_b is Symbol duration and when Symbol '0', no signal is transmitted. If Symbols '0' & Symbol '1' occur with equal probability, Show that the average probability of error P_e is given by

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_{av}}{2N_0}}\right)$$

Assume Coherent detection & Channel noise is AWG with Zero mean & power Spectral density $N_0/2$. E_{av} is the average Signal energy.

Sol: WKT for ASK System:

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)} \longrightarrow (1)$$

$\because$

$$\boxed{\begin{aligned} P(1) &= \tfrac{1}{2} \\ P(0) &= \tfrac{1}{2} \end{aligned}}$$

* In ASK System,
  for Symbol '1' energy $= E_b$ &
  for Symbol '0' energy $= 0$.

$\therefore$ The average Signal energy is given by

$$E_{av} = P(1)\,E_b + P(0)\,0$$

$$= \frac{1}{2}(E_b) + \frac{1}{2}(0)$$

$$E_{av} = \frac{E_b}{2}$$

$$\boxed{E_b = 2E_{av}} \longrightarrow (2)$$

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

Substituting eq ② in eq ①, we get

$$P_e = \frac{1}{2}\, erfc\left(\frac{1}{2}\sqrt{\frac{2E_{av}}{N_0}}\right)$$

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{2E_{av}}{\cancel{2}4N_0}}\right)$$

$$\boxed{P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_{av}}{2N_0}}\right)}$$

14) Binary data are transmitted at a rate of 10^6 bps over a microwave link. Assuming channel noise is AWGN with Zero mean & power spectral density at the receiver I/P is 10^{-10} W/Hz. Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent BFSK. Determine the minimum channel bandwidth required.

Given : $R_b = 10^6$ bps , $T_b = \frac{1}{R_b} = \frac{1}{10^6}$, $\frac{N_0}{2} = 10^{-10}$ W/Hz

$N_0 = 2\times10^{-10}$ W/Hz, $P_e \leq 10^{-4}$, $P = ?$, $B_T = ?$

Sol :- WKT $P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$$2\times10^{-4} = erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

WKT $erf(u) = 1 - erfc(u)$

$$erf(u) = 1 - 2\times10^{-4}$$

$$\boxed{erf(u) = 0.9998}$$

From error function table $erf(3) = 0.99998$ & $erf(2.5) = 0.9959$.

Hence erf(2.7) will be approximately equal to 0.9998 i.e.

$$\sqrt{\frac{E_b}{2N_0}} = 2.7$$

$$\frac{E_b}{2N_0} = 7.29$$

$$E_b = 2 \times 2 \times 10^{-10} \times 7.29$$

$$\boxed{E_b = 2.916 \times 10^{-9} \text{ Joules}}$$

* $E_b = PT_b$

$$P = \frac{E_b}{T_b} = E_b R_b = 2.916 \times 10^{-9} \times 1 \times 10^{6}$$

$$\boxed{P = 2.916 \text{ mwatts}}$$

* $B_T = R_b = \underline{1\,MHz}$

---

15) An FSK System transmits binary data at a rate of 10^6 bps. Assuming channel noise is AWGN having PSD 2×10^{-20} W/Hz. Determine the average probability of error. Assume Coherent detection & amplitude of received sinusoidal Signal for both Symbol 1 & 0 to be 1.2 microvolt.

Given : $R_b = 10^6$ bps , $T_b = \frac{1}{R_b} = \frac{1}{1 \times 10^6}$, $\frac{N_0}{2} = 2 \times 10^{-20}$ W/Hz

$N_0 = 4 \times 10^{-20}$, $A = 1.2\ \mu V$, $P_e = ?$

Sol :-

* $P = \frac{A^2}{2} = \frac{(1.2 \times 10^{-6})^2}{2} = 0.72 \times 10^{-12}$

* $E_b = PT_b = 0.72 \times 10^{-12} \cdot \frac{1}{1 \times 10^6} = 0.72 \times 10^{-18}$ Joules

WKT $$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{0.72\times10^{-18}}{2\times4\times10^{-20}}}\right)$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}(3)} \longrightarrow (1)$$

From error function table $\mathrm{erf}(3) = 0.99998$

WKT $$\mathrm{erfc}(u) = 1 - \mathrm{erf}(u)$$

$$\mathrm{erfc}(3) = 1 - \mathrm{erf}(3)$$

$$= 1 - 0.99998$$

$$\boxed{\mathrm{erfc}(3) = 0.00002} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$P_e = \frac{1}{2} \times 0.00002$$

$$\boxed{P_e = 1\times10^{-5}}$$

ARUNKUMAR G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.*

1) A binary data Stream 101101100 is to be transmitted using DPSK. Determine the encoded & decoded o/p. Jan-10, 4M

I- Method :-

Binary data $\{b_K\}$		1	0	1	1	0	1	1	0	0
Differential encoded data $\{d_K\}$	0	0	1	1	1	0	0	0	1	0
phase of $\{d_K\}$	π	π	0	0	0	π	π	π	0	π
polarity of Integrator		+	-	+	+	-	+	+	-	-
Detected Sequence $\hat{b}_K$		1	0	1	1	0	1	1	0	0

II- Method :-

Binary data $\{b_K\}$		1	0	1	1	0	1	1	0	0
Differential encoded data $\{d_K\}$	1	1	0	0	0	1	1	1	0	1
phase of $\{d_K\}$	0	0	π	π	π	0	0	0	π	0
polarity of Integrator		+	-	+	+	-	+	+	-	-
Detected Sequence $\hat{b}_K$		1	0	1	1	0	1	1	0	0

2) A binary data stream $\{b_K\} = \{010010011\}$ is to be txed using DPSK. Choose $d_1 = 0$ & determine :

i) The differential encoded sequence $\{d_K\}$ & phase of the txed DPSK signal.

ii) Polarity of the integrator o/p at $t = T_b$ of the DPSK receiver

iii) Decision rule & detected binary sequence.

June-09, 6M

Binary data $\{b_k\}$		0	1	0	0	1	0	0	1	1
		same	same	different						
Differentially encoded data $\{d_k\}$	0	1	1	0	1	1	0	1	1	1
Phase of transmitted d_k	π	0	0	π	0	0	π	0	0	0
	Different	Same								
Polarity of output of integrator at $t = T_b$		$-$	$+$	$-$	$-$	$+$	$-$	$-$	$+$	$+$
Detected binary sequence, $\hat{b}_k$ (if $l < 0 \rightarrow 0$; $l > 0 \rightarrow 1$)		0	1	0	0	1	0	0	1	1

i) $$d_K = b_K d_{K-1} + \overline{b_K d_{K-1}}$$

$$d_K = \overline{b_K \oplus d_{K-1}}$$

Txed phase :

If $d_K = 1$, Txed phase $= 0$ radians

If $d_K = 0$, Txed phase $= \pi$ radians

ii) If $d_K = d_{K-1}$, then integrator o/p 'l' is +ve

If $d_K \neq d_{K-1}$, then integrator o/p 'l' is -ve

iii) Decision rule: If $l > 0$, then $\hat{b}_K = 1$

If $l < 0$, then $\hat{b}_K = 0$

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

DP8.11. A binary sequence 101101 is transmitted over a communication channel using DPSK transmitter shown in Figure DP8.11(a). The channel introduces a phase reversal of 180°.

(a) Sketch the transmitted DPSK waveform assuming an initial bit of 1. What is the effect of changing initial bit to 0?

(b) Assuming the channel is noise free, show that the DPSK detector in the receiver shown in Figure DP8.11(b) produces the original binary sequence, despite the 180° phase reversal in the channel. For demonstration, take DPSK waveform with $d_{k-1} = 1$.

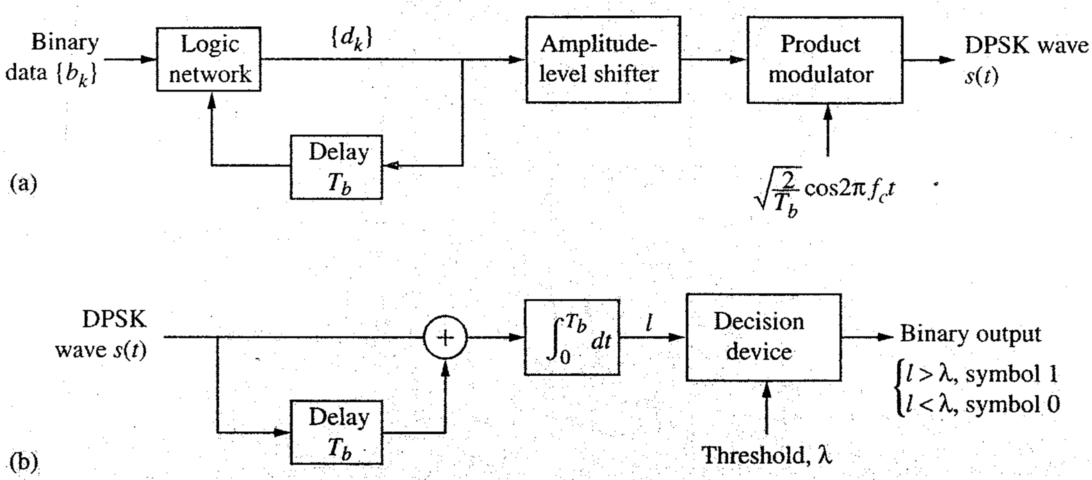


Figure DP8.11

Solution: (a) The logic used for generating DPSK sequence $\{d_k\}$ is

$$d_k = d_{k-1}b_k + \overline{d_{k-1}}\ \overline{b_k}$$

where the symbol + stands for logical OR operation.

(a) Initial bit, $d_{k-1} = 1$

$k \rightarrow$	−1	0	1	2	3	4	5
Binary data $\{b_k\}$		1	0	1	1	0	1
Differentially encoded data $\{d_k\}$	1	1	0	0	0	1	1
Phase of the DPSK wave (rad)	0	0	π	π	π	0	0

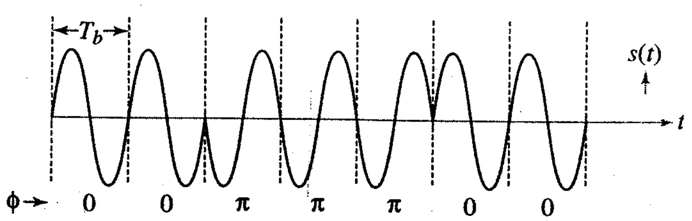


Figure DP8.11c

(b)

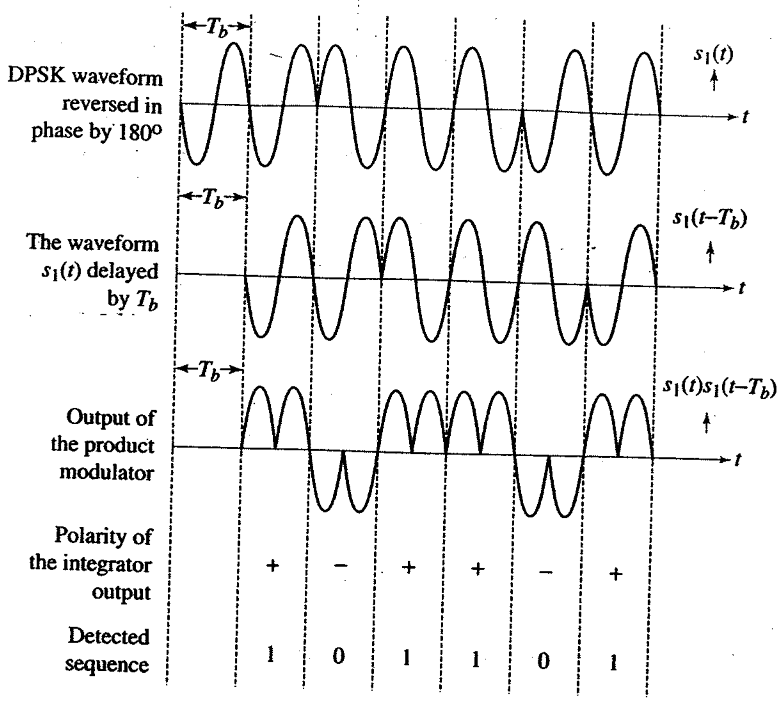


Figure DP8.11d

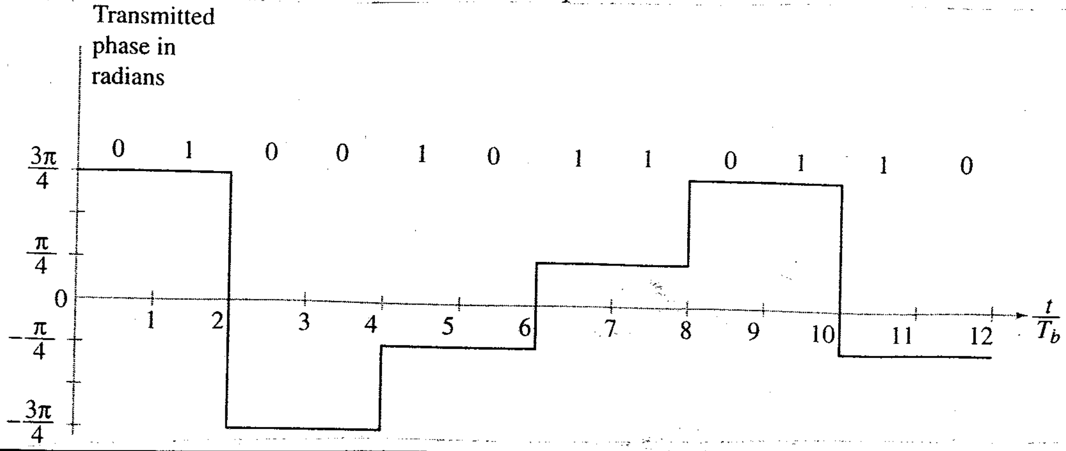
DP8.13. The input binary sequence to a QPSK modulator is

$$\{b_i\} = \{0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\}.$$

Sketch the transmitted phase of the carrier as a function of time.

Solution: The transmitted phase for each dibit is as follows.

Dibit	Transmitted phase
11	$\frac{\pi}{4}$ rad
01	$\frac{3\pi}{4}$ rad
00	$-\frac{3\pi}{4}$ rad
10	$-\frac{\pi}{4}$ rad



DP8.14. Sketch the inphase and quadrature components of a QPSK signal for the binary sequence 1 1 0 0 1 0 1 1 1. Assume carrier frequency f_c to be equal to $1/T_b$. Choose any convenient basis functions.

Solution: For the present context, $\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$ and $\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t$ are chosen as basis functions.

We have

$$\{bi\} = \{1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1\}$$
$$\{b_{ei}\} = \{1 \quad 0 \quad 1 \quad 1 \quad 1\}$$
$$\{b_{oi}\} = \{1 \quad 0 \quad 0 \quad 1\}$$

The sketches of $\{b_{ei}\}$, $\{b_{oi}\}$ and quadrature components of QPSK signal are shown in Figure DP8.14. Note that, we have taken, $f_c = 1/T_b$.

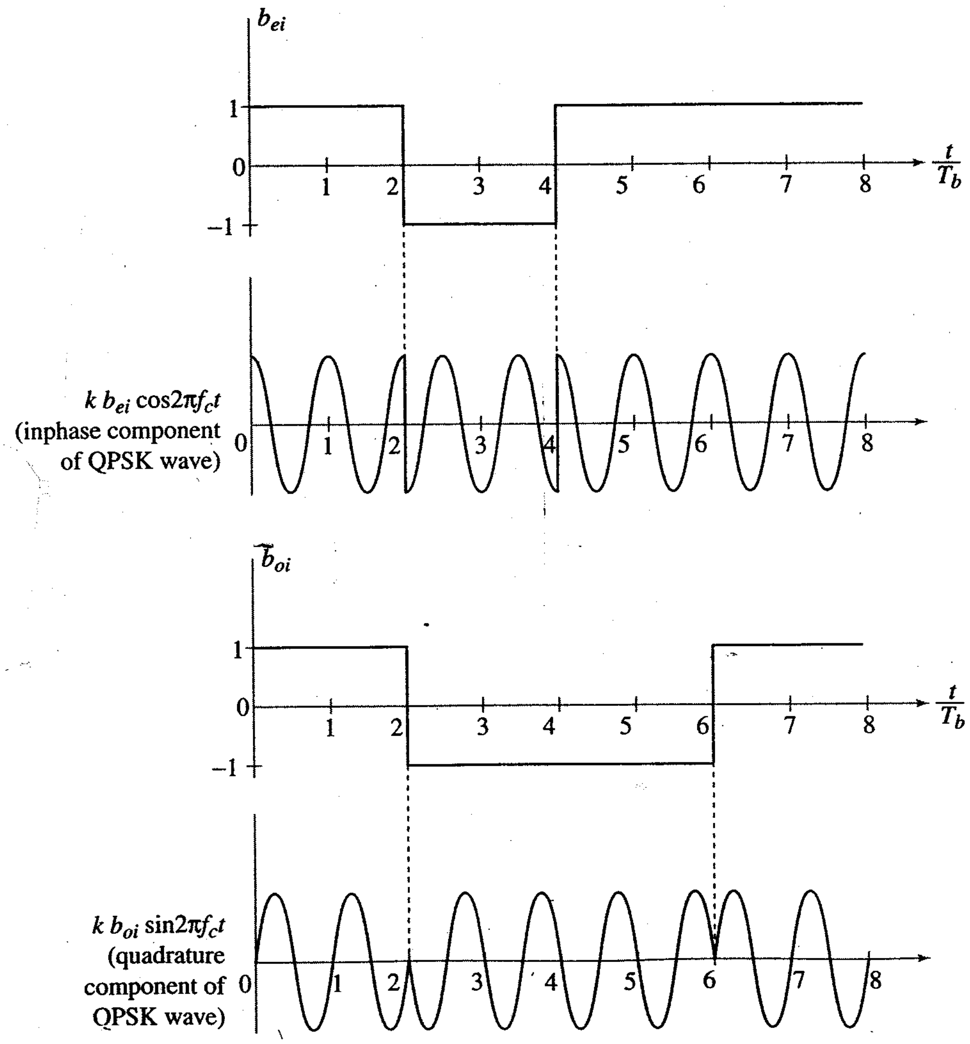
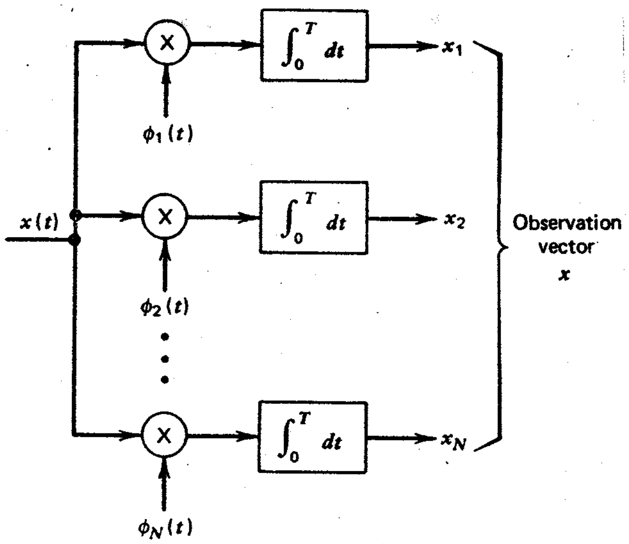


Figure DP8.14

Optimum Receiver or Maximum likelihood Receiver or Correlation receivers :-

Jan - 10, 5M
June - 09, 5M
Jan - 06, 5M

* For an AWGN Channel, when the transmitted signals $S_1(t), S_2(t), \cdots, S_M(t)$ are equally likely, the optimum receiver consists of two Subsystems as Shown in fig below.



(a)

1) The detector part of the receiver is as Shown in fig (a). It Consists of a bank of Correlators Supplied with a corresponding Set of orthonormal basis function $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ that are generated locally.
This bank of Correlators operate on the received Signal $x(t)$, to produce the observation vector $\mathbf{x}$.

P.T.O

ARUNKUMAR G M.Tech., Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

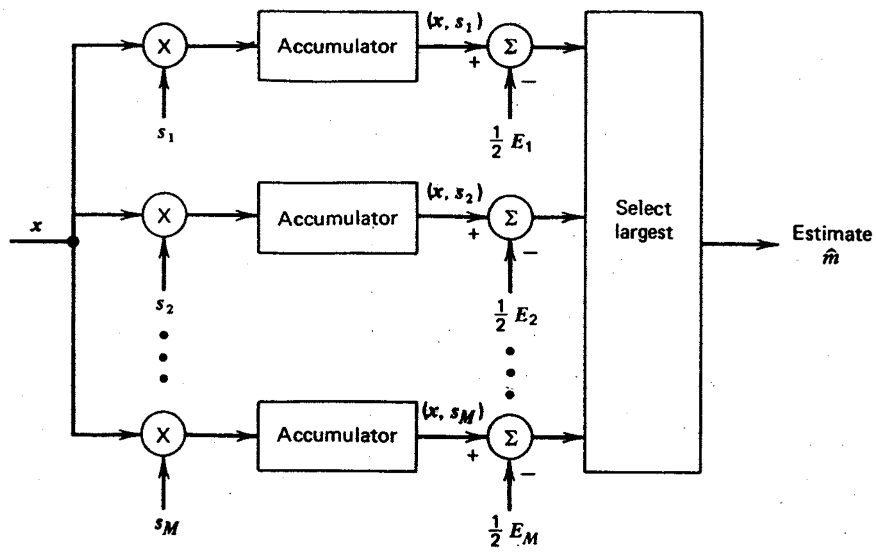


Figure (a) Detector. (b) Vector receiver.

2) The 2nd part of the receiver, namely the vector receiver as shown in fig (b).

The vector X is used to produce an estimate $\hat{m}$ of the transmitted symbol m_i, where $i = 1, 2, \ldots, M$ to minimize the average probability of symbol error.

* The observation vector X is multiplied by M signal vectors $S_1, S_2, \ldots, S_M$ & the resulting products are successively summed in accumulators.

Finally, the largest in the resulting set of numbers is selected & a corresponding decision is made on the transmitted signal.

* The optimum receiver is commonly referred to as a - correlation receiver.

Correlative Coding & Duobinary Signaling :-

By adding intersymbol interference to the transmitted signal in a controlled manner, it is possible to achieve a bit rate of $2B_0$ bits per second in a channel of bandwidth 'B_0' hertz. Such techniques are called correlative coding & partial-response signaling schemes.

Since intersymbol interference (ISI) introduced into the transmitted signal is known, its effect can be compensated at the receiver.

Thus correlative coding may be regarded as a practical means of achieving the theoretical maximum signaling rate of $2B_0$ bits per second in a bandwidth of B_0 Hz.

Duobinary Signaling :-

June-09, 5M	Jan-07, 5M

* Duo implies doubling of the transmission capacity of a system.
* Consider binary I/p sequence $\{b_k\}$ having duration T_b seconds, with symbol '1' represented by a pulse of amplitude +1V & symbol '0' by a pulse of amplitude -1V. i.e. $1 \rightarrow +1V$, $0 \rightarrow -1V$
* When this sequence is applied to a duobinary encoder, it is converted into a three-level o/p namely, -2V, 0V & +2V.
* To produce this transformation, the scheme is shown in Fig ①.

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

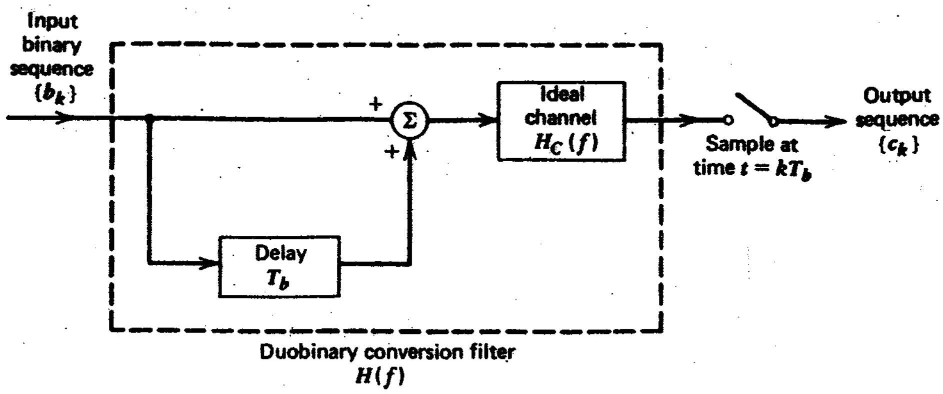


Figure Duobinary signaling scheme.

* The binary sequence $\{b_K\}$ is first passed through a simple filter involving a single delay element.

* The o/p of duobinary coder C_K is the sum of the present binary digit b_K & its previous value b_{K-1} i.e.

$$\boxed{C_K = b_K + b_{K-1}}$$

* Now, I/p sequence $\{b_K\}$ of uncorrelated digits are transformed into $\{C_K\}$ of correlated digits.

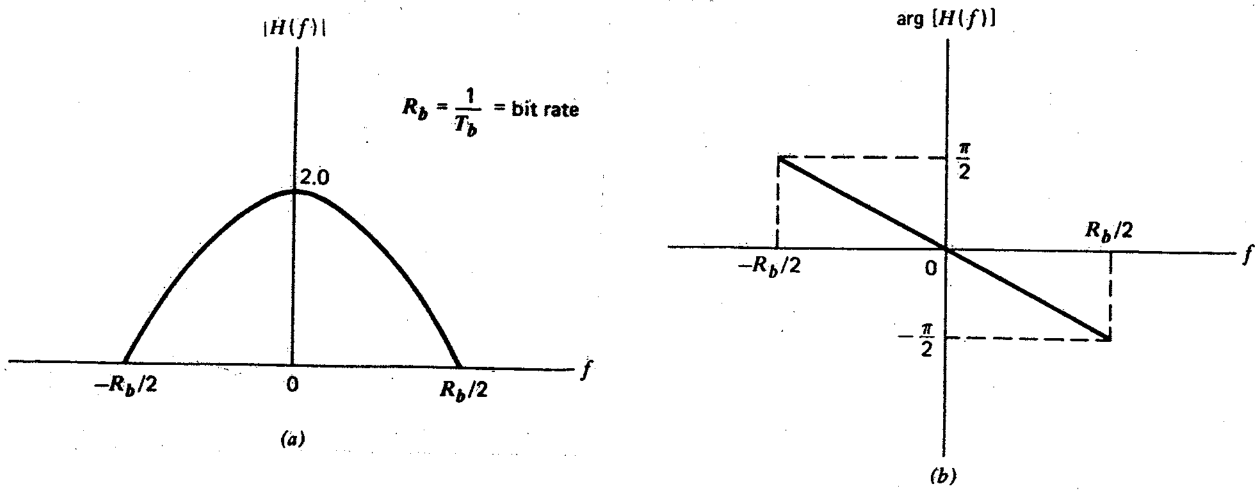


Figure Frequency response of duobinary conversion filter. (a) Amplitude response. (b) Phase response.

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

* The original data $\{b_k\}$ may be detected from the duobinary Sequence $\{c_k\}$ as shown below.

$$\boxed{\hat{b}_k = c_k - \hat{b}_{k-1}}$$

Drawback:- If error occurs, then they tend to propagate this error can avoided by use of precoder before the duobinary coding.

ex:- Demonstration of duobinary encoding & decoding:

Demonstration of duobinary encoding and decoding

1. Binary representation $\{b_k\}$	0	0	1	1	0	1	0	0	1
2. Polar representation $\{b_k\}$	−1	−1	+1	+1	−1	+1	−1	−1	+1
3. Duobinary encoder output $C_k = b_k + b_{k-1}$		−2	0	2	0	0	0	−2	0
4. $\hat{b}_k = C_k - \hat{b}_{k-1}$	−1	−1	1	1	−1	1	−1	−1	1
5. Output binary sequence	0	0	1	1	0	1	0	0	1

Modified duobinary Signalling or precoded duobinary Scheme:-

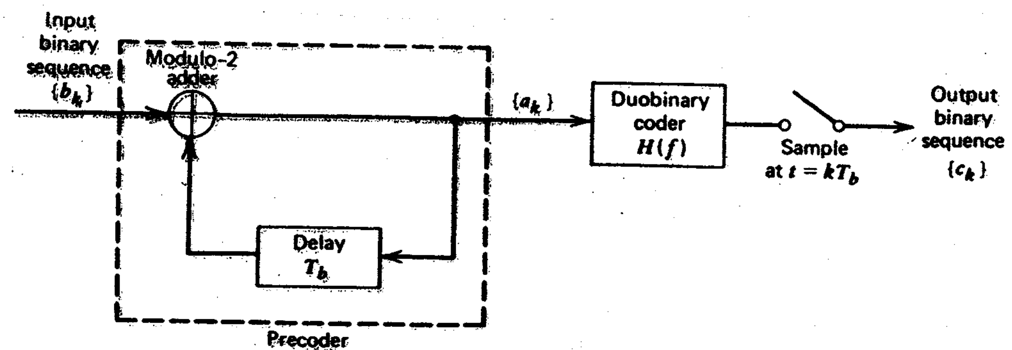


Figure ① A precoded duobinary scheme.

The block diagram of a duobinary encoder with a precoder is shown in fig ①.

* The precoded binary Sequence is $\boxed{a_k = b_k + a_{k-1}}$

ARUNKUMAR G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.*

* If the Symbol at the o/p of the precoder is represented in polar form, we find that

$$c_K = \begin{cases} \pm 2V, & \text{if } b_K = 0 \\ 0V & \text{if } b_K = 1 \end{cases}$$

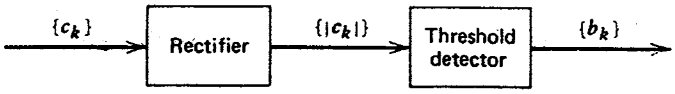


Figure Detector for recovering original binary sequence from the precoded duobinary coder output.

* The detector Consists of a rectifier, the o/p of which is compared to a threshold of 1V & the binary Sequence $\{b_K\}$ is thereby detected.

* The decision rule for detecting the original I/p binary Sequence $\{b_K\}$ from $\{c_K\}$:

$$b_k = \begin{cases} \text{symbol } 0 & \text{if } |c_k| > 1 \text{ volt} \\ \text{symbol } 1 & \text{if } |c_k| < 1 \text{ volt} \end{cases}$$

ex:

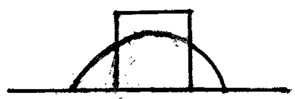
Demonstration of duobinary encoding and decoding with precoder

1. Binary sequence $\{b_k\}$:		0	0	1	1	0	1	0	0	1
	Assumed									
2. Precoded sequence $a_k = b_k \oplus a_{k-1}$	0	0	0	1	0	0	1	1	1	0
3. Polar representation of $\{a_k\}$:	−1	−1	−1	+1	−1	−1	+1	+1	+1	−1
4. Duobinary encoder output $C_k = a_k + a_{k-1}$:		−2	−2	0	0	−2	0	2	2	0
5. Recovered output:		0	0	1	1	0	1	0	0	1

NOTE :-

* For transmitting 0's & 1's on a communication channel, they must be converted into electrical waveforms compatible with the channel.

* When these waveforms are transmitted through the channel, they may tend to disperse as shown in below figure.



What is ISI?

* The transmitting pulses may tend to disperse to the neighbouring bits pulses as shown in fig ①. This phenomenon is known as ISI.

OR

* Signal overlapping may tend to give errors at the receiver. This phenomenon of pulse overlap & resulted - difficulty of discriminating symbols at the receiver is termed as ISI.

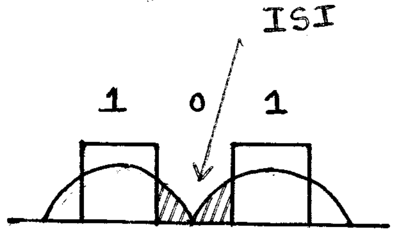


Fig ①

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.

Baseband transmission of binary data :-

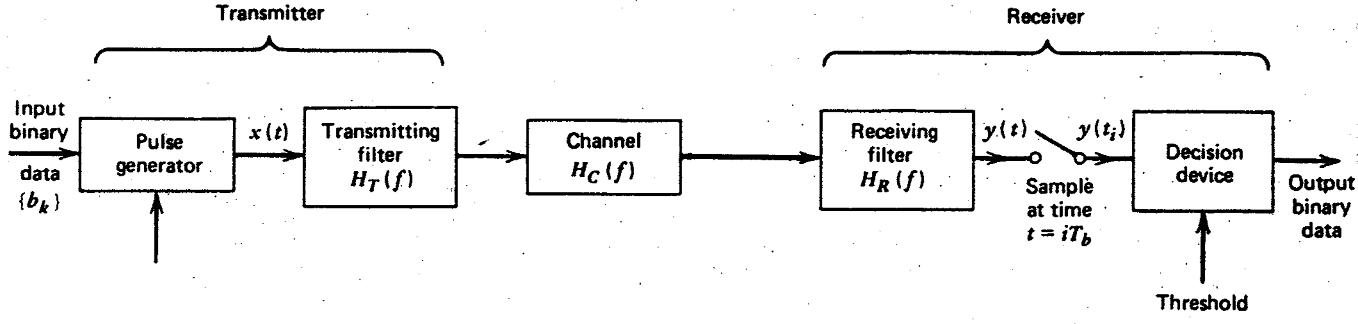


Figure Baseband binary data transmission system.

The basic elements of a baseband binary PAM System is Shown in fig ①.

* The I/p binary data Sequence $\{b_K\}$ with a bit duration of T_b Seconds. This Sequence is applied to a pulse generator, producing the discrete PAM Signal.

$$\boxed{x(t) = \sum_{K=-\infty}^{\infty} a_K v(t - KT_b)} \longrightarrow ①$$

Where, $v(t)$ denotes the basic pulse & normalized Such that $v(0)=1$ &

$$a_K = \begin{cases} a & ; \text{ Symbol } 1 \\ -a & ; \text{ Symbol } 0 \end{cases}$$

* The o/p of the pulse generator $x(t)$ is passed through a transmitting filter having the frequency response $H_T(f)$ & then through a channel of transfer function $H_c(f)$.
* The Channel may be co-axial cable or optical fiber. The major Source of System degradation is dispersion in channel.
* The Signal at the receiver is passed through a receiving filter of transfer function $H_R(f)$.

* The receiving filter o/p $y(t)$ is Sampled Synchronously with the transmitter at $t = iT_b$ i.e. $y(iT_b)$ & then applied to decision device.

* The decision device takes decision depending on the magnitude of $y(iT_b)$ as shown below:

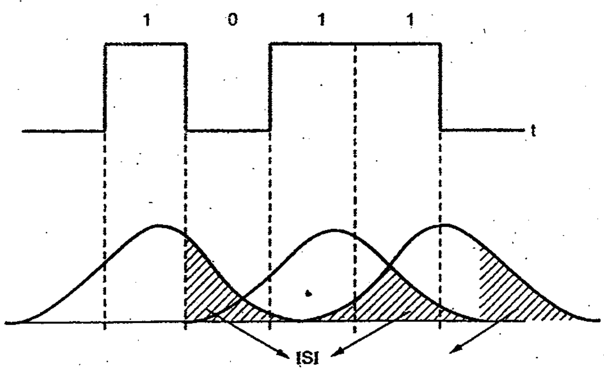
If $y(iT_b) > \lambda$, Selects Symbol '1'

If $y(iT_b) < \lambda$, Selects Symbol '0'

ISI :-

July-05, 5M	June-09, 5M

* The transmitting pulse dispersion or spreading causes overlap of pulses into adjacent time slots as shown in fig ①. The signal overlap may result in an error at the receiver. This phenomenon is known as ISI



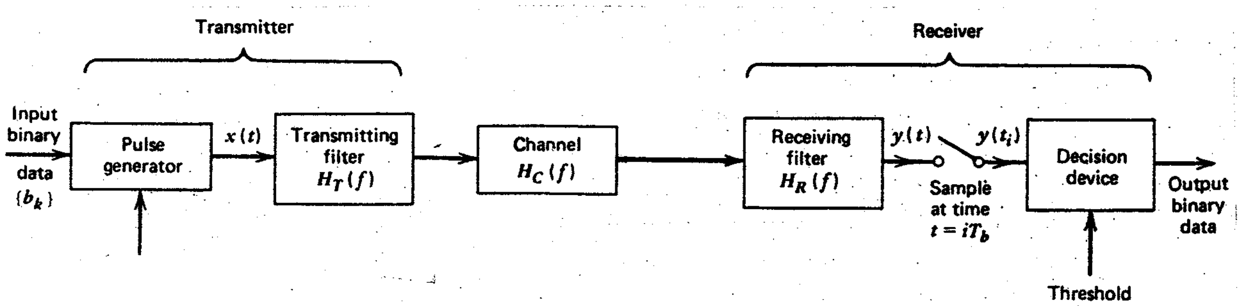


Figure Baseband binary data transmission system.

ARUNKUMAR G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.*

Assuming that the Channel is noise free. Mathematically, the o/p of the pulse generator can be described as:

$$\boxed{x(t) = \sum_{K=-\infty}^{\infty} a_K V(t-KT_b)} \longrightarrow (1)$$

Taking fourier transform on both Sides of eq (1), we get

$$\boxed{X(f) = \sum_{K=-\infty}^{\infty} a_K V(f) e^{-j2\pi f K T_b}} \longrightarrow (2)$$

* Let the o/p of the receiving filter be defined by

$$\boxed{y(t) = \sum_{K=-\infty}^{\infty} \mu a_K p(t-KT_b)} \longrightarrow (3)$$

Where, μ is Scaling factor &

$p(t)$ is pulse Shaping function of $y(t)$.

$p(t)$ is normalized Such that $\underline{p(0) = 1}$

* Taking fourier transform on both Sides of eq (3), we get

$$\boxed{Y(f) = \sum_{K=-\infty}^{\infty} \mu a_K P(f) e^{-j2\pi f K T_b}} \longrightarrow (4)$$

From fig (1), the o/p of the receiving filter in frequency domain is written as:

$$\underline{Y(f)} = \underline{X(f)}\ H_T(f) \cdot H_c(f)\ H_R(f) \longrightarrow (5)$$

Substituting eq (2) & eq (4) in eq (5), we get

$$\cancel{\sum_{K=-\infty}^{\infty}} \mu \cancel{a_K} P(f) \cancel{e^{-j2\pi f K T_b}} = \cancel{\sum_{K=-\infty}^{\infty}} \cancel{a_K} V(f) \cancel{e^{-j2\pi f K T_b}} \cdot H_T(f)\ H_c(f)\ H_R(f)$$

$$\mu P(f) = V(f)\ H_T(f)\ H_c(f)\ H_R(f)$$

$$P(f) = \frac{1}{\mu}\left[V(f)\, H_T(f)\, H_c(f)\, H_R(f) \right] \longrightarrow (6)$$

By taking IFT of eq (6), we get $p(t)$.

* $y(t)$ is sampled at time $\underline{t = iT_b}$ (eq (3)), we get

$$y(iT_b) = \sum_{K=-\infty}^{\infty} \mu a_K\, p(iT_b - KT_b)$$

$$y(iT_b) = \sum_{K=-\infty}^{\infty} \mu a_K\, p[T_b(i-K)]$$

Put $\underline{i = K}$

$$y(iT_b) = \mu a_i\, p(0) + \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} \mu a_K\, p[T_b(i-K)]$$

$$\boxed{y(iT_b) = \mu a_i + \underbrace{\sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} \mu a_K\, p[T_b(i-K)]}_{\text{ISI}}} \longrightarrow (7)$$

* In eq (7), 1st term '$\underline{\mu a_i}$' represents the contribution of i^{th} transmitted bit.

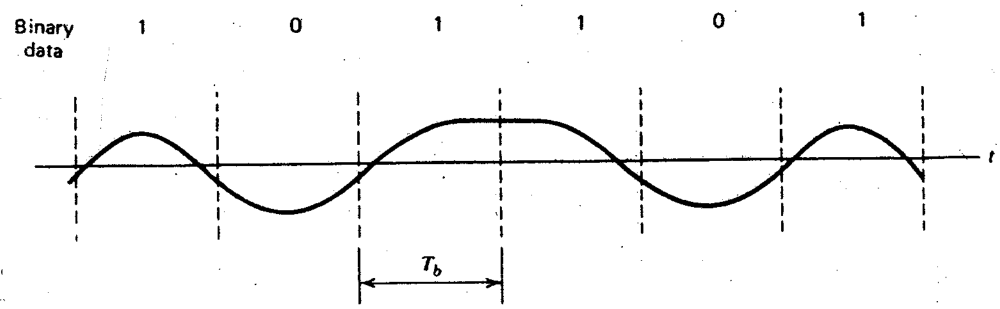
* The 2nd term represents $\underline{\text{ISI}}$.

ARUNKUMAR G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.*

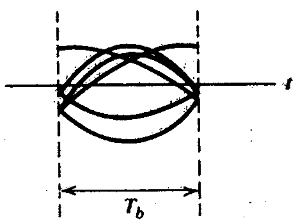
EYE pattern :-

June-09, 5M	June-08, 5M

* The eye pattern is used to study the effect of ISI in a PCM or data transmission system.
* Eye pattern can be obtained by applying the received wave to the vertical deflection plates of an oscilloscope & to apply a sawtooth wave to the horizontal deflection plates at a transmitted symbol rate $R = 1/T$.
* The waveforms in successive symbol intervals are thereby translated into one interval on the oscilloscope display as shown in fig 1 a & b.



(a)



(b)

Figure 1. (a) Distorted binary wave. (b) Eye pattern.

* The resulting display is called eye pattern because of its resemblance to the human eye for binary waves. The interior region of the eye pattern is called the eye opening.

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

* An eye pattern provides information about the performance of the System, as described in fig ②.

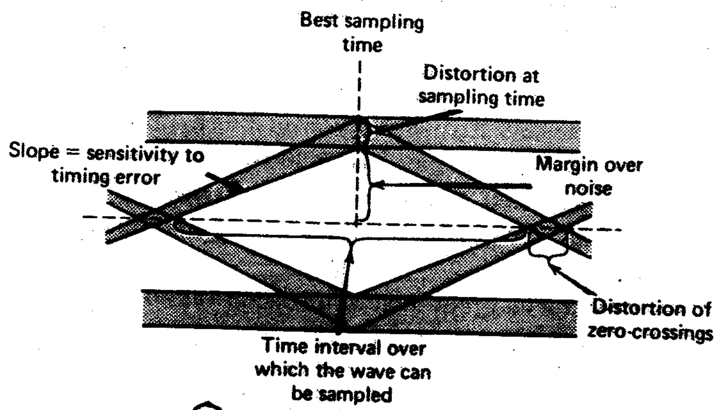


Figure ② Interpretation of eye pattern.

1) The width of the eye opening defines the time interval over which the received wave can be Sampled without error from ISI.

2) The Sensitivity of the System to timing error is determined by the rate of Closure of the eye as the Sampling time is varied.

3) The height of the eye opening, at a Specified Sampling time defines the margin over Noise.

Adaptive Equalization :- $\boxed{\text{July-08, 5M}}$

* The transmission characteristics of the Channel Keep on Changing. To Compensate this, adaptive equalization is used.

* In adaptive equalization, the Filters adapt themselves to the dispersive effects of the Channel. The co-efficients of the filters are Changed continuously according to the received data

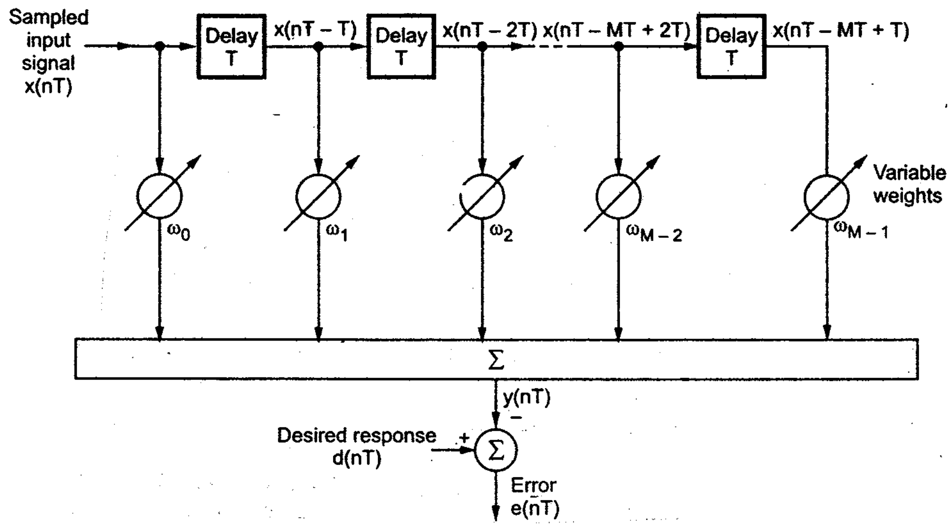
ARUNKUMAR G M.Tech, *Lecturer in E&CE Dept. S.T.J.I.T., Ranebennur.*

in Such a way that the distortion in the data is reduced.

* There are two types of equalization:
   1) pre-channel equalization
   2) post-channel equalization.

* pre-Channel equalization is done at the transmitting side. It requires feedback to know the amount of distortion in the received data.

* In post-channel equalization, feedback is not required. The equalizer is placed after the receiving filter in the receiver.



* Fig ① Shows the block diagram of an adaptive equalizing filter. It is adaptive in nature because it is capable of adjusting its co-efficients $W_0, W_1, \ldots, W_{M-1}$ by operating on the channel o/p in accordance with some algorithm.

* The adaptive equalizing filter consists of delay elements & adjustable filter co-efficients (Taps).

* The Sequence $x(nT)$ is applied to the I/p of the adaptive filter. The o/p $y(nT)$ of the adaptive filter will be:

ARUNKUMAR G M.Tech, Lecturer in E&CE Dept. S.T.J.I.T, Ranebennur.

$$\boxed{y(nT) = \sum_{i=0}^{M} w_i\, x(nT - iT)}$$

* A Known Sequence $\{d(nT)\}$ is transmitted 1st. This Sequence is Known to the receiver.

* An error Sequence is calculated i.e.

$$\boxed{e(nT) = d(nT) - y(nT)}$$

* If there is no distortion in the Channel, then $d(nT)$ & $y(nT)$ will be exactly Same producing Zero error Sequence.

* If there is distortion in the Channel, then $e(nT)$ exists. The weights of the filter i.e. w_i are Changed recursively Such that error $e(nT)$ is minimized.

* The algorithm used to change the weights of the adaptive filter is Least Mean Square Algorithm (LMS).

* The tap weights are adapted by this algorithm as follows:

$$\boxed{\hat{w}_i(nT+T) = \hat{w}_i(nT) + \mu\, e(nT)\, x(nT - iT)}$$

Where, $\hat{w}_i(nT)$ is the present estimate for Tap 'i' at time nT

$\hat{w}_i(nT+T)$ is the updated estimate for tap 'i' at time nT.

μ is the adaptation Constant.

$x(nT - iT)$ is the filter I/p &

$e(nT) \rightarrow$ error Signal.

Matched filter receiver :-

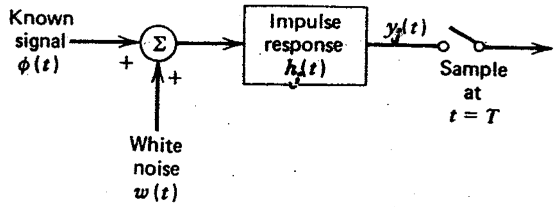


Figure (1) Illustrating the condition for derivation of the matched filter.

* Consider a linear filter with impulse response $h_j(t)$. With the received signal $x(t)$ used as the filter I/p, the resulting filter o/p $y_j(t)$ is defined as:

$$\boxed{y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, h_j(t-\tau)\, d\tau} \longrightarrow (1)$$

* Let us assume that the impulse response of the filter is

$$\boxed{h_j(t) = \phi_j(T-t)} \longrightarrow (2)$$

Where, T is the symbol duration &

(1) $\phi_j(T-t)$ is the time reversed & time delayed version of $\phi_j(t)$.

$$h_j(t-\tau) = \phi_j\left(T-[t-\tau]\right)$$

$$\boxed{h_j(t-\tau) = \phi_j(T-t+\tau)} \longrightarrow (3)$$

Substituting eq (3) in eq (1), we get

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T-t+\tau)\, d\tau \longrightarrow (3)$$

* The limits of integration are changed because $\phi_j(t)$ is Non-Zero

only in the interval (0,T).

Sampling this o/p at time $t=T$, we get

$$y_j(T) = \int_0^T x(\tau)\, \phi_j(\cancel{T}-\cancel{T}+\tau)\, d\tau$$

$$\boxed{y_j(T) = \int_0^T x(\tau)\, \phi_j(\tau)\, d\tau} \longrightarrow (4)$$

* A filter having an impulse response as suggested by eq (2) is known as Matched filter.

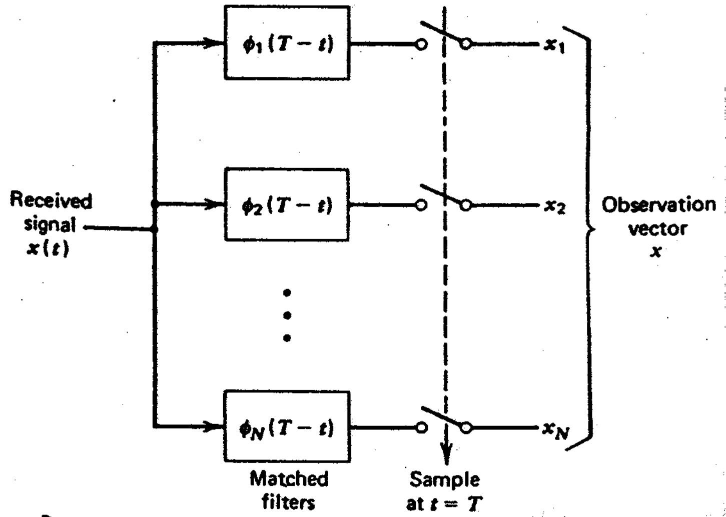


Figure (2) Detector part of matched filter receiver

Properties of Matched filter :-

Jan-09, 4M	July-08, 8M
July-07, 8M	July-06, 6M

Jan-06, 4M

{

NOTE :-

1) $h_{opt}(t) = \phi(T-t)$

2) $H_{opt}(f) = \phi^*(f)\, e^{-j2\pi fT}$

}

Property 1 :- The Spectrum of the o/p Signal of a matched filter with the matched Signal as I/p is, except for a time delay factor, proportional to the energy Spectral density of the I/p Signal.

* Let $\phi_o(f)$ denote the FT of the filter o/p $\phi_o(t)$, then

$$\phi_o(f) = \underline{H_{opt}}(f)\,\phi(f) \longrightarrow ①$$

$$= \phi^*(f)\, e^{-j2\pi fT}\, \phi(f)$$

$$= \phi^*(f)\,\phi(f)\, e^{-j2\pi fT}$$

$$\boxed{\phi_o(f) = |\phi(f)|^2\, e^{-j2\pi fT}}$$

Property 2 :- The o/p Signal of a matched filter is proportional to a Shifted version of the autocorrelation function of the I/p Signal to which the filter is matched.

3

WKT $$\boxed{\phi_o(f) = |\phi(f)|^2\, e^{-j2\pi fT}} \longrightarrow ①$$

Taking IFT of the eq ①, we get

$$\phi_o(f) = R_\phi(t-T)$$

Where $R_\phi(\tau)$ is the autocorrelation function of the I/p $\phi(t)$ for lag τ $\{$i.e. $R_\phi(\tau) = \phi(t) * \phi(t-T)\}$

We have $$\boxed{\phi_o(T) = R_\phi(0) = E}$$

Where 'E' is the Signal energy.

Property 3 :- The o/p of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter I/p.

WKT the average power of the o/p noise $n(t)$ is

$$E\left[n^2(t)\right] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$E\left[n^2(t)\right] = \frac{N_0}{2} \int_{-\infty}^{\infty} |\phi(f)|^2 \, df$$

$$\left(\because H(f) = \phi^*(f)\, e^{-j2\pi f T}\right)$$

From 2nd property, the maximum value of filter o/p at time $t=T$, is proportional to the signal energy 'E'.

$$\therefore \boxed{E\left[n^2(t)\right] = \frac{N_0}{2} E} \leftarrow \left(\times^{ly} \;\&\; \div \text{ by } E\right)$$

So, the o/p SNR will have the maximum value as

$$(SNR)_{o,max} = \frac{E^{\cancel{2}}}{\frac{N_0}{2} \times \cancel{E}} = \frac{2E}{N_0}$$

$$\boxed{(SNR)_{o,max} = \frac{2E}{N_0}}$$

* $\frac{2E}{N_0}$ is a dimensionless as E is in Joules & $\frac{N_0}{2}$ is in W/Hz.

* $\frac{E}{N_0}$ is termed as the signal energy to noise density ratio.

Property 4 :- The matched-filtering operation may be separated into two matching conditions; namely, spectral phase matching that produces the desired o/p peak at time 'T' & Spectral amplitude matching that gives this peak value its optimum Signal-to-Noise density ratio.

* In polar form,

$$\boxed{\phi(f) = |\phi(f)|\, e^{j\theta(f)}}$$

Where $|\phi(f)|$ is the amplitude Spectrum &

$\theta(f)$ is the phase Spectrum of the Signal.

i.e. $\boxed{\theta(f) = 2\pi f T}$

* For Spectral amplitude matching, the amplitude response $|H(f)|$ of the filter to Shape the o/p for best SNR at $t = T$

5 by using $\boxed{|H(f)| = |\phi(f)|}$

---

o/p SNR of a Matched filter :-

Jan-10, 10M	Jan-09, 12M

* Show that impulse response of a matched filter is a time reversed & delayed version of the I/p Signal. Jan-10, 8M

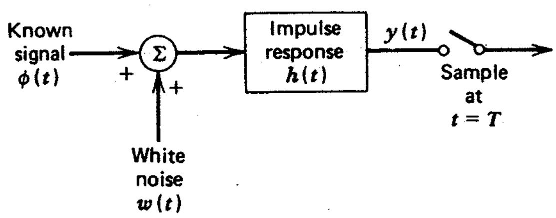


Figure 1 Illustrating the condition for derivation of the matched filter.

* From fig 1, we can write

$$\boxed{x(t) = \phi_1(t) + w(t)} \longrightarrow (1)$$

In eq ① $\phi(t)$ is the known I/p Signal & $W(t)$ AWGN noise.

* $\phi(t)$ is orthonormal function & $W(t)$ is AWGN, whose mean $\mu = 0$ & $PSD = \frac{N_0}{2}$.

* The o/p of the linear filter is

$$\boxed{y(t) = \phi_0(t) + n(t)} \longrightarrow ②$$

Where $\phi_0(t)$ & $n(t)$ are produced by the Signal & noise - Components of the I/p $x(t)$.

* Since, filter is both linear & time invariant, we can write:

$$\boxed{\phi_0(t) = \phi(t) * h(t)} \longrightarrow ③$$

Taking FT on bothside of eq ③, we get

⑥ $$\boxed{\phi_0(f) = \phi(f) \cdot H(f)} \longrightarrow ④$$

By definition

$$\phi_0(t) = \int_{-\infty}^{\infty} \underline{\phi_0(f)}\, e^{j2\pi f t}\, dt \longrightarrow ⑤$$

Substituting eq ④ in eq ⑤, we get

$$\phi_0(t) = \int_{-\infty}^{\infty} \phi(f)\, H(f)\, e^{j2\pi f t} \cdot dt$$

The filter o/p is Sampled at time $t = T$.

Normalized Signal power $= |\phi_0(T)|^2$

* Normalized Signal power

$$|\phi_0(T)|^2 = \left| \int_{-\infty}^{\infty} \phi(f)\, H(f)\, e^{j2\pi f T}\, df \right|^2$$

* PSD of I/p noise $= \frac{N_0}{2}$

* PSD of o/p noise $= \frac{N_0}{2} |H(f)|^2$

o/p noise power $= \int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 \cdot df$

$$\therefore \quad (SNR)_0 = \frac{\text{Signal power}}{\text{o/p Noise power}} = \frac{\left| \int_{-\infty}^{\infty} \phi(f)\, H(f)\, e^{j2\pi fT} \cdot df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df} \longrightarrow (6)$$

Applying Schwarz's inequality to the numerator of eq (6), we may write

$$\left| \int_{-\infty}^{\infty} H(f)\, \phi(f) \exp(j2\pi fT)\, df \right|^2 \leq \int_{-\infty}^{\infty} \left| H(f)\, e^{j2\pi fT} \right|^2 df \int_{-\infty}^{\infty} |\phi(f)|^2 \cdot df$$

$$\therefore \quad (SNR)_0 = \frac{\int_{-\infty}^{\infty} |H(f)\, e^{j2\pi fT}|^2\, df \int_{-\infty}^{\infty} |\phi(f)|^2\, df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df}$$

(7)

Since, $\left| e^{j2\pi fT} \right| = 1$, we get

$$(SNR)_0 = \frac{\cancel{\int_{-\infty}^{\infty} |H(f)|^2\, df} \int_{-\infty}^{\infty} |\phi(f)|^2\, df}{\frac{N_0}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2\, df}}$$

$$\boxed{(SNR)_0 = \frac{2}{N_0} \int_{-\infty}^{\infty} |\phi(f)|^2\, df} \longrightarrow (7)$$

* The RHS of eq (7) is defined by two quantities :

1) Signal energy given by

$$E = \int_{-\infty}^{\infty} |\phi(t)|^2\, dt = \int_{-\infty}^{\infty} |\phi(f)|^2\, df$$

2) The Noise PSD $N_0/2$.

* The o/p SNR will be maximum when $H(f)$ is chosen so that the equality holds i.e.

$$(SNR)_{o,max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |\phi(f)|^2 \cdot df$$

* For this condition, $H(f)$ assumes its optimum value denoted as $H_{opt}(f)$.

$$\boxed{H_{opt}(f) = \phi^*(f)\, e^{-j2\pi fT}} \longrightarrow (8)$$

Where $\phi^*(f)$ is the complex conjugate of the FT of the I/p signal $\phi(t)$.

* Taking IFT of $H_{opt}(f)$ of eq (8), we get

$$h_{opt}(t) = \int_{-\infty}^{\infty} H_{opt}(f)\, e^{j2\pi ft}\, df$$

(8)

$$h_{opt}(t) = \int_{-\infty}^{\infty} \phi^*(f)\, e^{-j2\pi fT}\, e^{j2\pi ft}\, df$$

Since for real valued signal $\underline{\phi^*(f) = \phi(-f)}$

$$h_{opt}(t) = \int_{-\infty}^{\infty} \phi(-f)\, e^{-j2\pi f[T-t]} \cdot df$$

$$\boxed{h_{opt}(t) = \phi(T-t)} \longrightarrow (9)$$

Eq (9) shows impulse response of optimum filter in a time reversed & delay version of I/p signal $\phi(t)$.