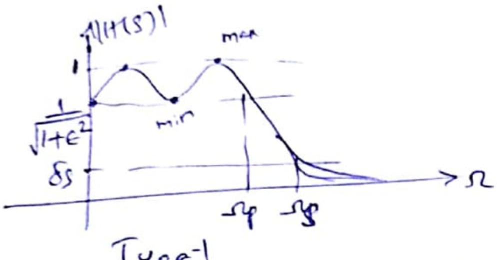
Chebyshev Filters

→ Chebyshev filters are characterized by chebyshev polynomial.

→ chebyshev filters are also known as equiripple filters as they have ripples in pass band & stop band

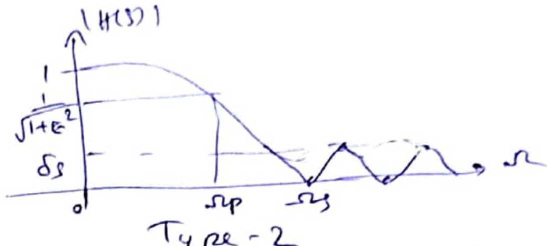
→ Based on ripple they are classified as



Type-1

→ Ripple in pass band
Monotonic stop band

→ All pole filters



Type - 2

→ Monotonic pass band
Ripple in stop band

→ Pole zero filters.

Chebyshev polynomials

→ It is given by $T_N(\Omega)$ or $C_N(\Omega)$

$$T_N(\Omega) = \begin{cases} \cos(N\cos^{-1}\Omega), & |\Omega| \leq 1 \\ \cosh(N\cosh^{-1}\Omega), & |\Omega| > 1 \end{cases}$$

→ for $N = 0$, $T_N(0) = 1$ & $N=1$, $T_N(1) = \Omega$

→ Chebyshev polynomial can be generated using the recursive formula for $N \geq 2$.

ie $\cdot$ $T_N(\Omega) = 2\Omega\, T_{N-1}(\Omega) - T_{N-2}(\Omega), N \geq 2.$

To find chebyshev polynomial

W.K.T

$$T_N(\Omega) = \begin{cases} \cos(N\cos^{-1}\Omega) & \text{for } |\Omega| \leq 1 \\ \cosh(N\cosh\Omega) & |\Omega| > 1 \end{cases}$$

for $N=0$, $T_N(\Omega) = 1 = T_0(\Omega)$

$N=1$, $T_N(\Omega) = \Omega = T_1(\Omega)$

$N=2$, $T_N(\Omega) = T_2(\Omega) = 2\Omega T_{N-1}(\Omega) - T_{N-2}(\Omega)$

ie $T_2(\Omega) = 2\Omega\, T_1(\Omega) - T_0(\Omega)$

$= 2\Omega \cdot \Omega - 1 = 2\Omega^2 - 1$

$N=3, \quad T_3(\Omega) = 2\Omega T_2(\Omega) - T_1(\Omega)$

$$= 2\Omega(2\Omega^2-1) - \Omega$$
$$= 4\Omega^2 - 2\Omega - \Omega$$
$$= 4\Omega^2 - 3\Omega$$

ie

N	$T_N(\Omega)$
0	1
1	Ω
2	$2\Omega^2-1$
3	$4\Omega^2-3\Omega$.
4	$8\Omega^4-8\Omega^2+1$
5	$16\Omega^5-20\Omega^3+5\Omega$
⋮	

Properties of Chebyshev Polynomials

→ For $|\Omega| \leq 1$ (or), $T_N(\Omega) \leq 1$ & it oscillates b/w +1 & −1 & no of oscillations is proportional to N. $[T_N(\Omega) = \cos(N\cos^{-1}\Omega)]$

→ for $|\Omega| > 1$, $|T_N(\Omega)| \geq 1$ & it is monotonically increasing in $|\Omega|$. $[T_N(\Omega) = \cosh(N\cosh^{-1}\Omega)]$

→ Cheby polynomials ~~for~~ odd value of N are odd functions of Ω, & for N even, are even functions of Ω.

→ Also polynomials for odd N contain only odd powers of Ω & for even N, even powers of Ω.

→ $T_N(0) = \pm 1$ for even N
$T_N(0) = 0$ for odd N

→ $T_N(-\Omega) = (-1)^N T_N(\Omega)$

→ $T_N(\pm 1) = 1$ for all N.

→ The equiripple prop of Cheby-(filters) polynomials when used in cheby- filters leads to narrower transition band in cheby filters compared to Butterworth filters.

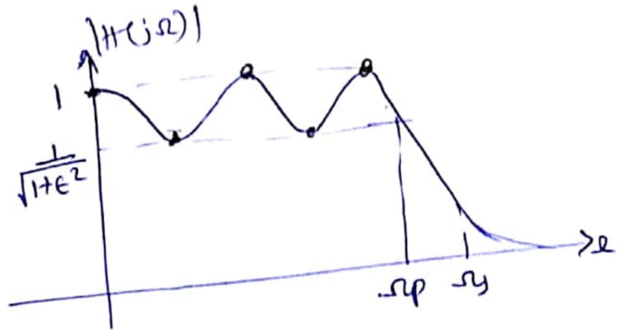
→ Hence the order of cheby filter is lower than the Butterworth filters for the given specifications.

Magnitude response of Chebyshev - I filter

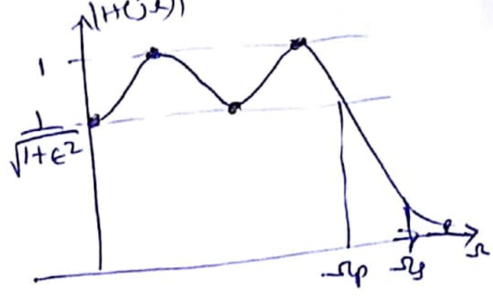
It is given by

$$|H(j\Omega)| = \frac{1}{\left[1+\epsilon^2 T_N^2\left(\frac{\Omega}{\Omega_p}\right)\right]^{1/2}} \quad -(1)$$

where ϵ is a parameter of filter related to ripple in pass band, Ω_p is passband edge freq & $T_N(x)$ is N^{th} order chebyshev polynomial.



N-odd



N-even

Properties of Cheby-I filters

→ $H(j0) = 1$ for N odd
$= \frac{1}{\sqrt{1+\epsilon^2}}$ for N even. } → sub $\Omega = 0$ in (1)

→ Response has uniform ripple in PB & monotonic in stop band.

→ Sum of maxim & min in pass band is equal to order N of the filter.

To find poles of chebyshev filters

$$H_N(j\Omega) = \frac{1}{\sqrt{1+\epsilon^2 T_N^2(\Omega)}} \quad -① $$

$$|H_N(j\Omega)|^2 = \frac{1}{1+\epsilon^2 T_N^2(\Omega)} \quad -② $$

$s = j\Omega \Rightarrow \Omega = s/j$

$$\therefore |H_N(s)|^2 = \frac{1}{1+\epsilon^2 T_N^2(s/j)}$$

$$H_N(s)\, H_N(-s) = \frac{1}{1+\epsilon^2 T_N^2(s/j)} \quad -③ $$

To find poles equate Dr of ③ to Zero

ie $1+\epsilon^2 T_N^2(s/j) = 0$

$\epsilon^2 T_N(s/j) = -1$

Soln of above eqn is given by

$s = \sigma_k + j\Omega_k$

$\sigma_k = -a \sin\left((2k-1)\frac{\pi}{2N}\right)$

& $\Omega_k = b\cos\left((2k-1)\frac{\pi}{2N}\right)$, $k = 1, 2, \ldots, 2N$

where $a = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{1/N} - \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{-1/N}$

& $b = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{1/N} + \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{-1/N}$

$N = 1, 2, \ldots, 2N$

$$\boxed{S_k = \sigma_k + j\Omega_k}$$

The poles of chebyshev filter lies on ellipse.

The normalized Chebyshev Transfer fun using only poles to the left of s-plane we can write $H_N(s)$ as

$$H_N(s) = \frac{K_N}{V_N(s)} = \frac{K_N}{\prod_{k=0}^{N-1} (S - S_k)}$$

also $V_N(S) = S^N + b_{N-1} S^{N-1} + b_{N-2} S^{N-2} + \dots b_0.$

& K_N is normalizing factor. that ensures.

$|H_N(j0)| = 1$ for N-odd &

$= \frac{1}{\sqrt{1+\epsilon^2}}$ for N-even

$$\therefore \quad K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N\text{-odd} \end{cases}$$

<u>To find N order of chebyshev filter</u>

WKT $|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 T_N^2(\Omega/\Omega_p)}$ for cheby-I filter

$$T_N(\Omega) = \begin{cases} \cos(N \cos^{-1}\Omega) & x \leq 1 \\ \cosh(N \cosh^{-1}\Omega) & x > 1 \end{cases}$$

Over the pass band $\Omega < \Omega_p \therefore \Omega/\Omega_p \leq 1$.

& magnitude of $H(j\Omega)$ oscillates b/w. 1 & $\frac{1}{\sqrt{1+\epsilon^2}}$.

$\therefore$ The passband ripple amplitude is .

$$\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}}.$$

ie $\epsilon^2 = (1-\delta_p)^{-2} - 1$

Also at $\Omega = \Omega_s$

$$|H(\Omega_s)|^2 \leq \delta_s^2.$$

$$\frac{1}{1+\epsilon^2 T_N^2(\Omega_s/\Omega_p)} \leq \delta_s^2$$

$$T_N^2(\Omega_s/\Omega_p) \geq \frac{\delta_s^{-2}-1}{\epsilon^2}.$$

ie $$T_N^2\left(\frac{\Omega_s}{\Omega_p}\right) \geq \frac{\delta_s^{-2}-1}{(1-\delta_p)^{-2}-1}.$$

$$T_N\left(\frac{\Omega_s}{\Omega_p}\right) = \frac{1}{d}. \quad -(A) \qquad d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

$\because \frac{\delta_s}{\delta_p} > 1$ after Ω_s.

$$T_N(\Omega_s/\Omega_p) = \cosh\left[N \cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right]$$

(A) becomes

$$\cosh\left[N \cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right] \geq \frac{1}{d}$$

(a) $$N \cdot \cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right) = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{1}$$

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} \qquad \text{Let } K = \frac{\Omega_p}{\Omega_s}.$$

$$\therefore \boxed{N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)}}$$

where $d \to$ discrimination factor & $K \to$ selectivity factor.
N is approximated to next larger integer.

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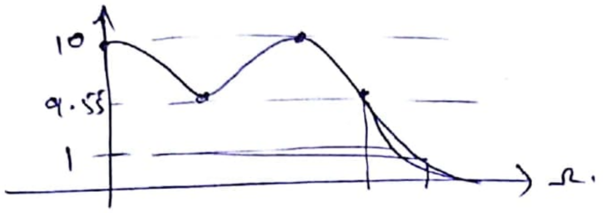
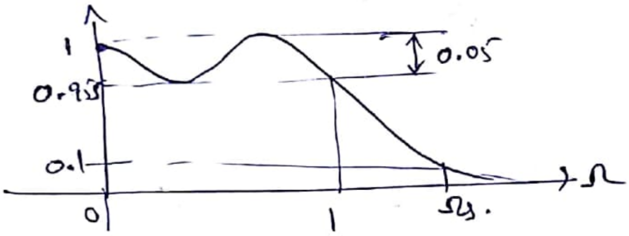
Problem

(1) Find order N, the value of ripple factor ϵ & normalized passband attenuation of cheb-I LP filter for following specification



Sol. The normalized response can be found by ÷ y-axis by 10



From fig

$\delta_p = 0.045$, $\delta_s = 0.1$

$K =$. $\quad d = \sqrt{\dfrac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$

$$\frac{1}{\sqrt{1+\epsilon^2}} = 0.955 \Rightarrow \epsilon = 0.3016$$

$$K_p = -A_p = -20 \log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right) = +0.399\,dB. = +0.4\,dB$$

$$K_s = -A_s = -20 \log(0.1) = +20\,dB.$$

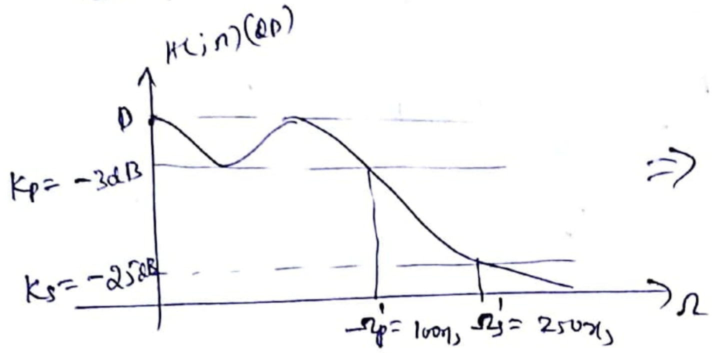
To find order

From fig total no of maxima & minima = 3

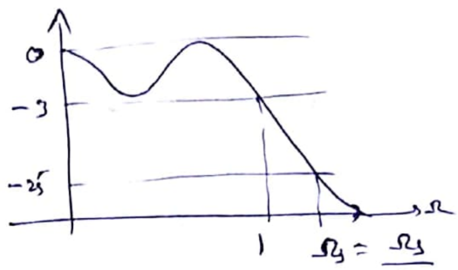
$\therefore \boxed{N = 3}$

(5)

Design a chebyshev analog filter (LP) that has −3dB cutoff freq of 100 r/s & stop band attenuation of 25 dB or greater for all freq above 250 r/s. verify the design.



⟹



$\Omega_s = \frac{\Omega_s}{\Omega_p} = 2.5 \text{ r/s}$

To find order

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)}$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$1/k = \frac{\Omega_s}{\Omega_p} = 2.5$ ie $k = \frac{\Omega_p}{\Omega_s} = 0.4$

$20\log(1-\delta_p) = -3 \Rightarrow (1-\delta_p) = 0.707945 \Rightarrow \delta_p = 0.29205.$

$\frac{1}{\sqrt{1+\epsilon^2}} = 1-\delta_p = 0.707 \Rightarrow \epsilon = \sqrt{(1-\delta_p)^{-2} - 1} = 0.997628$

$\therefore 1/d = 17.8444.$ $d = \sqrt{\frac{0.99529}{(0.05623)^{-2} - 1}} = 0.056$

$$N = \frac{\cosh^{-1}\left(\frac{1}{0.056}\right)}{\cosh^{-1}\left(\frac{1}{2.5}\right)} = \frac{3.5747}{1.5667} = 2.28$$

$\boxed{N = 3}$

To find $H_N(s)$

$$H_N(s) = \frac{1}{\prod_{k=0}^{3}(s - s_k)}$$ $k = 0 \text{ to } 3$

To find S_K

$$S_k = \sigma_k + j\Omega_k. \qquad \sigma_k = -a\sin\left[(2k-1)\frac{\pi}{2N}\right]$$

$$\text{ie } \sigma_k = -a\sin\left[(2k-1)\frac{\pi}{6}\right] \qquad \Omega_k = b\cos\left[(2k-1)\frac{\pi}{6}\right]$$

where $a = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{1/N} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-\frac{1}{N}}.$

$$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{1/N} + \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{-1/N}.$$

$$\frac{1+\sqrt{1+\epsilon^2}}{\epsilon} = 2.41827$$

$\therefore a = 0.2986202 \qquad b = 1.043635.$

K	σ_K	Ω_k
1	-0.14931	0.903814
2	-0.2986	0
3	-0.14931	-0.90381

$$H_3(s) = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)} = \frac{K_N}{s^3 + 0.59724045^2 + 0.928348s + 0.2505943.}$$

To find K_N ie K_3

Here $N=3$ is odd $\therefore K_N = K_3 = b_0 = 0.2505943.$

$$\therefore H_3(s) = \frac{0.2505943}{(\qquad\qquad)}$$

Required LPF is obtained by applying LP to LP transform

ie $H_a(s) = H_N(s)\Big|_{s=\frac{s}{\Omega_u}}.$

ie $H_a(s) = H_{a1}(s)\Big|_{s=\frac{s}{100}}$

$$\boxed{H_a(s) = \frac{250594.3}{s^3 + 59.72404s^2 + 9283.48s + 250594.3}}$$

Verify

find $20\log |H_a(s)|_{s=100} = -3\,dB$

$20\log |H_a(s)|_{s=250} \geq -35\,dB$.

③ Find the order of BW & chebyshev-1 filters for following specifications & comment.

$\delta_p = \delta_s = 0.01$, $\Omega_p = 0.6682$ r/s

$\Omega_s = 1$ r/s.

Sol: N- for BW filter.

$$N = \log\left[\frac{10^{-0.1K_p} - 1}{10^{-0.1K_s} - 1}\right] \div 2\log\left(\frac{\Omega_p}{\Omega_s}\right)$$

$K_p = A_p(\text{in dB}) = 20\log(1-\delta_p) = -0.087296\ dB$

$K_s = A_s(\text{in dB}) = 20\log\delta_s = -40.\ dB$

$$\therefore N = \log\left[\frac{0.020304}{9999}\right] \div 2\log(0.6682)$$

$N = 16.25$

$\boxed{N = 17}$

Order of chebyshev-filter

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)}$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}} = 1.0424994 \times 10^{-3}$$

$$K = \frac{\Omega_p}{\Omega_s} = 0.6682$$

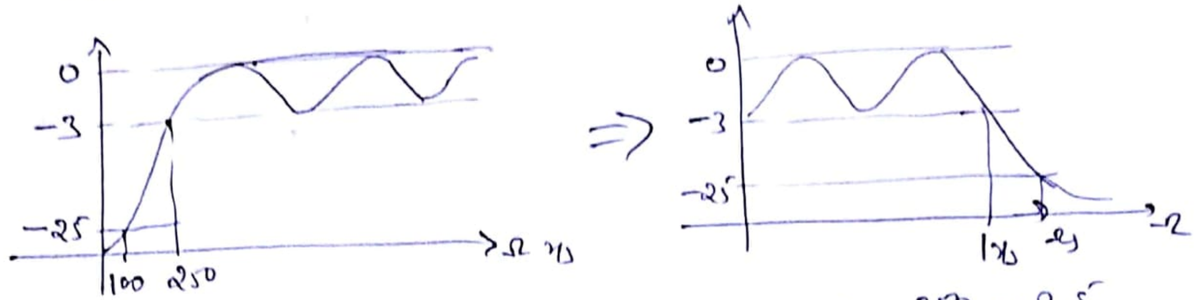
$$N = \frac{7.24666}{0.95933} = 7.5538 = 8$$

$$\boxed{N = 8}$$

∴ For given specifications order of Cheby-I filter is less than order of Butterworth filter.

Assignment

10) Given



$\Omega_s = \frac{250}{100} = 2.5$

$$N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$$

$(1-\delta_p) = \text{Antilog}\left(\frac{-3}{20}\right)$

$= 0.7079$

$\delta_s = \text{Antilog}\left(-\frac{25}{20}\right)$

$= 0.056234$

$K = \frac{\Omega p}{\Omega s} = \frac{1}{2.5} = 0.4$

$$d = \sqrt{\frac{(1-\delta p)^{-2} - 1}{\delta_s^{-2} - 1}} = \sqrt{\frac{(0.7079)^{-2} - 1}{(0.0562)^{-2} - 1}}$$

$d = 0.05619.$

Also $1 - \delta_p = 0.7079$

$1 - \delta_p = \frac{1}{\sqrt{1+\epsilon^2}} = 0.7079$

~~$\delta_p = \frac{1}{\sqrt{1+\epsilon^2}} = 0.2921$~~

~~$\epsilon = \sqrt{\frac{1}{\delta_p^2} - 1} = 3.2741$~~

$N = 2.2793 \cong 3.$

$\boxed{N = 3}$

$\frac{1}{\sqrt{1+\epsilon^2}} = 0.7079$

$\epsilon = \sqrt{\frac{1}{0.7079^2} - 1}$

$\boxed{\epsilon = 0.9977}$

To find $H_N(s)$

$$H_N(s) = \frac{K_N}{\prod_{k=1}^{3}(s - s_k)}$$

$s_k = \sigma_k + j\Omega_k$

$\sigma_k = -a \sin\left((2k-1)\frac{\pi}{2N}\right) = -a \sin\left[(2k-1)\frac{\pi}{6}\right]$

$\Omega_k = b \cos\left((2k-1)\frac{\pi}{2N}\right) = b \cos\left[(2k-1)\frac{\pi}{6}\right]$

K	σ_K	Ω_K
1		
2		
3		

$$a = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} + \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{-1/N}$$

$$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} - \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{-\frac{1}{N}}$$

$a = 0.6711 + 0.3725$

$= 1.0436$

$\cancel{b=}$

$b = 0.2986.$

(7) Determination of order of filter

→ squared magnitude response of N order analog LP type-I Chebyshev filter is given by

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 T_N^2(\Omega/\Omega_p)}$$

$$= \frac{1}{\sqrt{1+\epsilon^2 T_N^2(\Omega/\Omega_p)}}$$

$$20\log|H(j\Omega)| = -20\log\left(1-\epsilon^2 T_N^2(\Omega/\Omega_p)\right)^{1/2}$$

$$= -10\log\left(1-\epsilon^2 T_N^2(\Omega/\Omega_p)\right)$$

(i) at $\Omega = \Omega p$

$$A_p \text{ in dB} = -10\log\left(1-\epsilon^2 T_N^2(\Omega/\Omega_p)\right)$$

$$= -10\log(1+\epsilon^2)$$

$$\epsilon^2 = 10^{-0.1 A_p \text{ in dB}} - 1$$

$$\epsilon = \sqrt{10^{-0.1 A_p \text{ in dB}} - 1}$$

$$\epsilon = \left(10^{-0.1 A_p \text{ in dB}} - 1\right)^{1/2} \longrightarrow (1)$$

(ii) at $\Omega = \Omega_s$

$$A_s \text{ in dB} = -10\log\left(1-\epsilon^2 T_N^2\left(\frac{\Omega_s}{\Omega_p}\right)\right)$$

Here $\frac{\Omega_s}{\Omega_p} > 1 \Rightarrow C_N(\Omega) = \cosh(N\cosh^{-1}\Omega)$

$$\therefore A_s \text{ in dB} = -10\log\left(1+\epsilon^2[\cosh(N\cosh^{-1}(\Omega)]^2\right)$$

Substitute : ϵ & solve for N

$$N \geq \frac{\cosh^{-1}\left[\frac{10^{-0.1 A_s \text{ in dB}} - 1}{10^{-0.1 A_p} - 1}\right]}{\cosh^{-1}(\Omega_s/\Omega_p)}$$

(4) Find the half-power frequency of a fifth-order Chebyshev I LPF with a 2dB PB edge at 1KHz.

Sol^n

$$-2\,dB = 20 \log \left[\frac{1}{\sqrt{1+\epsilon^2}}\right]$$

$$\epsilon = 0.7648$$

$$\Omega_p = 2\pi f_p = 2\pi \times 1 \times 10^3 = 2\pi \times 10^3 \ r/s.$$

$$|H(j\Omega)| = \frac{1}{\left[1+\epsilon^2 T_5^2\left(\frac{\Omega}{2\pi\times10^3}\right)\right]^{1/2}} \qquad \Omega \to \Omega/\Omega_p.$$

$$N=5 \Rightarrow T_5(\Omega) = 16\Omega^5 - 20\Omega^3 + 5\Omega.$$

$$= 16\left(\frac{\Omega}{2\pi\times10^3}\right)^5 - 20\left(\frac{\Omega}{2\pi\times10^3}\right)^3 + 5\left(\frac{\Omega}{2\pi\times10^3}\right)$$

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2\left[16\left(\frac{\Omega}{2\pi\times10^3}\right)^5 - 20\left(\frac{\Omega}{2\pi\times10^3}\right)^3 + 5\left(\frac{\Omega}{2\pi\times10^3}\right)\right]}$$

let $\Omega = \Omega_c$ at $|H(j\Omega)|^2 = 1/2$

$$\frac{1}{2} = \frac{1}{1+(0.7648)^2\left[16\left(\frac{\Omega_c}{2\pi\times10^3}\right)^5 - 20\left(\frac{\Omega_c}{2\pi\times10^3}\right)^3 + 5\left(\frac{\Omega_c}{2\pi\times10^3}\right)\right]}$$

$$\Omega_c = 6552.157 \ r/s.$$

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Design Procedure for IIR filters

① → Find order N & 3dB cut off freq Ω_{cn} using given of specifications by using normalized filter.
(Let given specifications be Ω_p' & Ω_s', K_p & K_s)
& normalized filter parameters be $\Omega_p = 1$ r/s, $\Omega_s = \frac{\Omega_s'}{\Omega_p'}$, K_p, K_s)

② →

$$N = \log\left[\frac{10^{-0.1 K_p} - 1}{10^{-0.1 K_s} - 1}\right] \div 2\log\left(\frac{\Omega_p}{\Omega_s}\right)$$

$$\Omega_{cn} = \frac{\Omega_p}{\left(10^{-0.1 K_p} - 1\right)^{\frac{1}{2N}}}$$

③ → Find $H_N(s) = \frac{1}{B_N(s)}$ ie TF of normalized filter.

④ → Perform analog transformation

$$H_p(s) = H_N(s)\Big|_{s = \frac{s}{\Omega_{cn}}}$$ → Normalized prototype filter.

⑤ → Perform $H_a(s) = H_p(s)\Big|_{s = \frac{s}{\Omega_p'}}$ for LP filter

similar to $s \to \frac{s}{\Omega_{cn}\Omega_p'}$ in $H_N(s)$

$H_a(s) = H_p(s)\Big|_{s = \frac{\Omega_p'}{s}}$ for HP filter.

similar to $s \to \Omega_p'/\Omega_{cn}$ in $H_N(s)$

$H_a(s)$ is TF for given specifications.

For BP & BS filters.

① → Find normalized filter by using transformation of specifications.

$\Omega_p = 1$ r/s & $\Omega_s = \min(|A|, |B|)$

② → Find order N using normalized filter.

③ → Find T.F. $H_N(s)$

④ → Transform $H_N(s)$ to $H_a(s)$ by using following Table.

To find Ω_s use:

For BPF $\Rightarrow$ $A = \dfrac{-\Omega_1^2 + \Omega_0^2}{\Omega_1 (B_0)}$ & $B = \dfrac{+\Omega_2^2 - \Omega_0^2}{\Omega_2 (B_0)}$

$\Omega_0^2 = \Omega_u \Omega_l$

$B_0 = (\Omega_u - \Omega_l)$

$\Omega_s = \min(|A|, |B|)$

For BSF or BEF $A = \dfrac{\Omega_1 B_0}{-\Omega_1^2 + \Omega_0^2}$, $B = \dfrac{\Omega_2 B_0}{-\Omega_2^2 + \Omega_0^2}$

$\Omega_s = \min(|A|, |B|)$

Transformations

$\Omega_0^2 = \Omega_u \Omega_l$.

$B_0 = \Omega_u - \Omega_l$

LP to BP $\Rightarrow$ $s \rightarrow \dfrac{s^2 + \Omega_0^2}{s B_0}$;

LP to BS $\Rightarrow$ $s \rightarrow \dfrac{s B_0}{s^2 + \Omega_0^2}$

In LPF design

steps (4) & (5) can be replaced by single Transformation

$H_a(s) = H_N(s) \Big|_{s = \frac{s}{\Omega_{c1}}}$

where $\Omega_{c1} = \dfrac{\Omega_p'}{\left(10^{-0.1 k_p} - 1\right)^{\frac{1}{2N}}}$

$\therefore$ LPF $\rightarrow$ $H_N(s) \xrightarrow{s = \frac{s}{\Omega_{cn}}} H_p(s) \rightarrow H_a(s)$ $\quad s = \frac{s}{\Omega_p'} \Rightarrow s = \frac{s}{\Omega_p' \Omega_{cn}}$

or $H_N(s) \rightarrow H_a(s) \big|_{s = s/\Omega_{c1}}$ $\Rightarrow$

HPF $\rightarrow$ $H_N(s) \xrightarrow{s = s/\Omega_{cn}} H_p(s) \rightarrow H_a(s)$ $\quad s = \frac{\Omega_p'}{s} \Rightarrow s = \frac{\Omega_p'}{\Omega_{cn} s}$

BPF $\rightarrow$ $H_N(s) \rightarrow H_a(s) \Big|_{s = \frac{s^2 + \Omega_0^2}{s (B_0)}}$

BEF $\rightarrow$ $H_N(s) \rightarrow H_a(s) \Big|_{s = \frac{s B_0}{s^2 + \Omega_0^2}}$.