Prajwal Shenoy KP
1DS16EC096 5-B

1) a) Linear, non-recursive, stable.

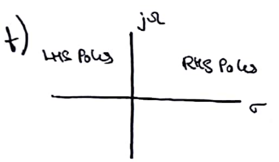
b) $y(n) = \sum_{k=0}^{N} b_k x(n-k)$

c) unit circle.

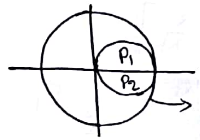
d) $H_n(s)\Big|_{s \to \frac{s^2+\Omega_0^2}{B_0 s}}$

e) $\omega = \Omega T$ or $z_i = e^{P_i T}$

f)



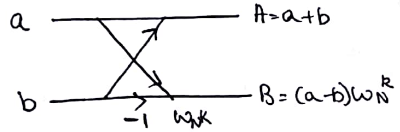
S Plane



Z-Plane.

g) In BLT, shrinking frequencies of frequencies near zero in lower frequencies is called frequency warping, introduces non linearity in mapping.

h)



a, b - inputs
A, B - outputs
W_N^k - Twiddle factor.

i) $N = 16$
memory required in the case of DIF is $2N = 32$

j) $N > \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ $K = \frac{\Omega_p}{\Omega_s}$ $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$

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⑨

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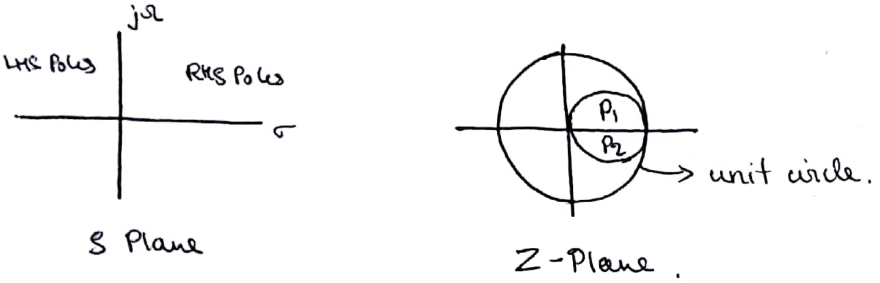

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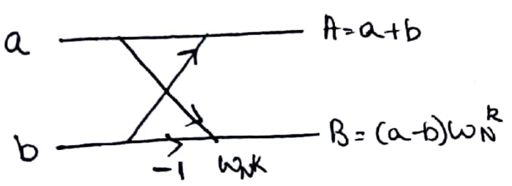

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2) Given $N = 5$ $\Delta p = 3dB$

$$K_p = 20\log(1-\Delta p) = -3dB$$

$$1-\Delta p = 0.707$$

$$\varepsilon = \sqrt{\frac{1}{(1-\Delta p)^2} - 1} = 0.9997 \simeq 1$$

Poles of chebyshev filter.

$$\Omega_k = b\cos\left[\frac{(2k-1)\pi}{2N}\right] \qquad \forall\ K = 0,1,2,\ldots N$$

$$\sigma_K = -a\sin\left[\frac{(2k-1)\pi}{2N}\right]$$

$$a = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/N} - \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-1/N}$$

$$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/N} + \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-1/N}$$

Let $\mu = \frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon} = \frac{1+\sqrt{1+1^2}}{1} = 2.4142.$

$$a = \frac{1}{2}(2.4142)^{1/5} - \frac{1}{2}(2.4142)^{-1/5} = 0.17188.$$

$$b = \frac{1}{2}(2.4142)^{1/5} + \frac{1}{2}(2.4142)^{-1/5} = 1.01557.$$

K	σ_K	Ω_K
1	-0.05311	0.96518
2	-0.1390	0.82161
3	-0.17188	0
4	-0.1390	-0.82161
5	-0.05311	-0.9658

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Poles of butterworth filter.

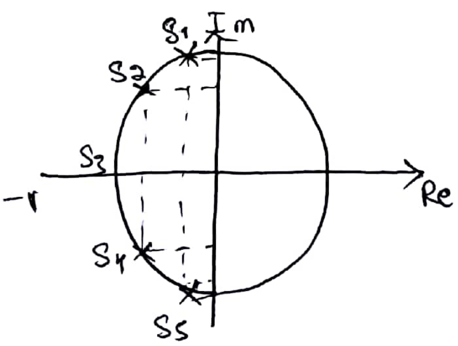
$\theta_K = \frac{(2K+1)\pi}{2N} + \frac{\pi}{2}$ K=0 to 4 $S_K = re^{j\theta_K}$

assume $r=1$

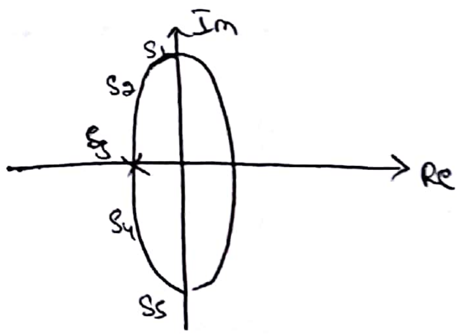
K	θ_K	S_K
0	105	$-0.3090+j0.951$
1	144	$-0.8090+j0.587785$
2	180	$-1+j0$
3	216	$-0.8090-0.58j$
4	252	$-0.3090-j0.951$

Plot of poles.

Butterworth.



Chebyshev



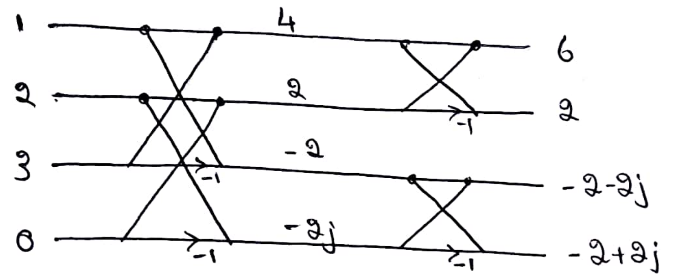
Poles of butterworth lie on unit circle where as poles of chebyshev are on ellipse.

3) $h(n) = \{1, 2, 3\}$ $\quad x(n) = \{1, 2\}$

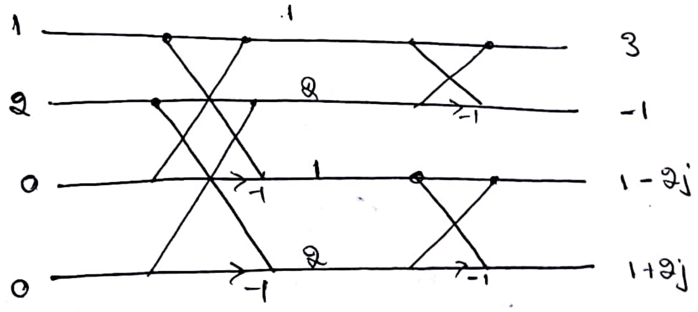
Response of a discrete time LTI system is linear convolution

∴ implement linear convolution using DIF

$h(n) = \{1, 2, 3, 0\}$ $\quad x(n) = \{1, 2, 0, 0\}$.

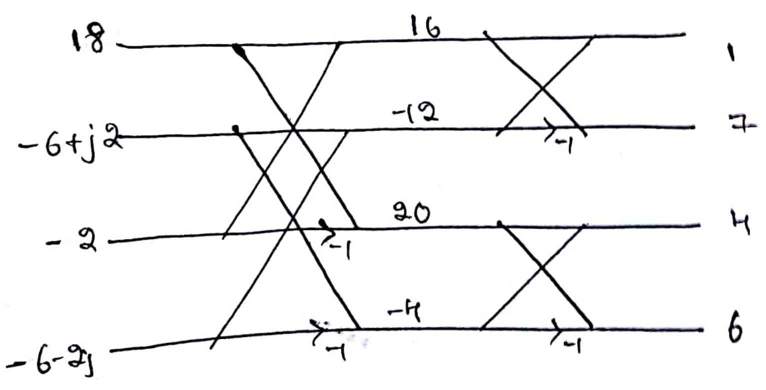


$\therefore H(k) = \{6, -2-2j, 2, -2+2j\}$.



$\therefore X(k) = \{3, 1-2j, -1, 1+2j\}$

$Y(k) = X(k)\,H(k) = \{18, -6+j2, -2, -6-2j\}$.



$\therefore y(n) = \{1, 4, 7, 6\}$

4) $0.6 \leq |H(e^{j\omega})| \leq 1 \;;\; 0 \leq \omega \leq 0.35\pi$

$|H(e^{j\omega})| \leq 0.1 \;;\; 0.7\pi \leq \omega \leq \pi$

given $\Delta p = 0.6 \qquad \Delta s = 0.1$

$\omega p = 0.35\pi \text{ r/s} \qquad \omega_s = 0.7\pi \text{ r/s}.$

$Ap = -20\log(\Delta p) = 4.436\,dB$

$As = -20\log(\Delta s) = 20\,dB.$

$\Omega p = \frac{2}{T}\tan\left(\frac{\omega p}{2}\right) = 1.0226 \text{ r/s}$

consider $T = 1$

$\Omega s = \frac{2}{T}\tan\left(\frac{\omega s}{2}\right) = 3.925 \text{ r/s}$

$$N \geq \frac{\log\left[\dfrac{10^{-0.1Ap} - 1}{10^{-0.1As} - 1}\right]}{2\log\left(\dfrac{\Omega p}{\Omega s}\right)} = 2$$

$\boxed{N = 2}$

$\Omega_c = \dfrac{\Omega p}{\left(10^{-0.1Ap} - 1\right)^{1/2N}} = 1.061 \text{ r/s}$

$\boxed{\Omega_c = 1.061 \text{ r/s}}$

$B_N(s) = s^2 + \sqrt{2}s + 1$

$H_N(S) = \dfrac{1}{s^2 + \sqrt{2}S + 1}$

$Ha(S) = H_N(S)\Big|_{s \to \frac{s}{\Omega u}}$

$\Omega u = \Omega p \cdot \Omega_c = 1.033 \text{ r/s}.$

$$H_a(s) = \frac{1}{\left(\frac{s}{1.83}\right)^2 + \sqrt{2}\left(\frac{s}{1.83}\right) + 1} = \frac{(1.361)^2}{s^2 + 1.83s + (1.30)^2}$$

$$H(z) = H_a(s)\Big|_{s \to \left(\frac{1+z^{-1}}{1-z^{-1}}\right)\frac{2}{T}} \quad \text{assume } T=1$$

$$= \frac{(1.36)^2 (1+z^{-1})^2}{4(1-z^{-1})^2 + 3.678(1+z^{-1})(1-z^{-1}) + 1.69(1+z^{-1})^2}$$

$$H(z) = \frac{1.69z^{-1} + 3.38z^{-1} + 1.69}{2.012z^{-2} - 4.62z^{-1} + 9.368}$$

5) $0.9 \leq |H(e^{j\omega})| \leq 1 \; ; \; 0 \leq \omega \leq 0.25\pi$

$|H(e^{j\omega})| \leq 0.24 \; ; \; 0.5\pi \leq \omega \leq \pi$

given $\Delta p = 0.9$ $\quad \Delta s = 0.24$

$A_p = 20\log(\Delta p) = 0.9151\,dB$ at $\omega_p = 0.25\pi$

$A_s = 20\log(\Delta s) = 12.3958\,dB$ at $\omega_s = 0.5\pi$

$\Omega_p = \frac{\omega_p}{T} = 0.7854$ r/s

assume $T=1$

$\Omega_s = \frac{\omega_s}{T} = 1.5708$ r/s

$$N \geq \frac{\cosh^{-1}\left(\sqrt{(10^{0.1A_s} - 1)/(10^{0.1A_p} - 1)}\right)}{\cosh^{-1}(\Omega_s/\Omega_p)} \geq 2.107.$$

$$\boxed{N = 3}$$

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$$\varepsilon = \sqrt{10^{-0.01 A_p \text{ in dB}} - 1} = \sqrt{10^{-0.1 \times -0.981} - 1}$$

$$\boxed{\varepsilon = 0.4843}$$

$Y_N = 0.5107$

$C_0 = Y_N = 0.5107$

$$H_N(S) = \frac{B_0}{S_N + C_0} \prod_{K=1}^{N-1/2} \frac{B_K}{S_N^2 + b_k S_n + C_0}$$

$b_1 = 0.5107$

$C_1 = 1.0108$

To evalate B_0 & B_K.

$$H_N(S)\Big|_{S_n \to 0} = 1$$

$$H_N(S) = \frac{0.5162}{S_n^3 + 1.0214 S_n^2 + 1.2716 S_n + 0.5162}.$$

$$H_a(S) = H_N(S)\Big|_{S \to S/\Omega_p}.$$

$$H_a(S) = \frac{0.2501}{S^3 + 0.8022 S^2 + 0.7844 S + 0.2506}.$$

Solving by partial fraction.

$A = 0.4011$ $B = -0.4011$ $C = 0$

$$H_a(S) = \frac{0.4011}{S + 0.4011} - \frac{0.4011 S}{S^2 + 0.4011 S + 0.6235}.$$

$$H(z) = \frac{0.4011}{1 - e^{-0.401} z^{-1}} - \frac{0.4011 (1)}{(1 - e^{0.2 + j0.7637} z^{-1})(1 - e^{0.2 - j0.7657} z^{-1})}.$$

$$H(z) = \frac{0.0906 z^{-1} + 0.0698 z^{-2}}{1 - 1.8516 z^{-1} + 1.4612 z^{-2} - 0.448 z^{-3}}$$

6)

Given

$A_p = -2dB$ at $\Omega_p = 400$ r/s.

$A_s = -20dB$ at $\Omega_s = 100$ r/s.

7)

$$N \geq \frac{\log\left[\frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} \geq \cancel{\log} \cancel{1}, 2$$

$$\boxed{N = 2}$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{-0.1A_p}-1\right)^{1/2N}} = 1.014 \text{ r/s}.$$

$$B_N(S) = S^2 + \sqrt{2}S + 1$$

$$H_N(S) = \frac{1}{S^2 + \sqrt{2}S + 1}$$

$$H_a(S) = H_N(S)\Big|_{S \to \frac{\Omega_u}{S}} \qquad \Omega_u = \frac{\Omega_p}{\Omega_c} = \frac{400}{1.014} = 350.8 \text{ r/s}.$$

$$H_a(S) = \frac{S^2}{S^2 + 496.106S + (350.8)^2}.$$

Verification.

$S \to j\Omega_p \to 400j$

$$H_a(S) = 20\log A_p = \frac{(400j)^2}{(400j)^2 + (496.166)(400j) + (350.8)^2}$$

$A_p = -2dB$

Hence Verified.

7)

Given

$A_p = -2.5\,dB$ at $\Omega_p = 20$ r/s.

$A_s = -30\,dB$ at $\Omega_s = 50$ r/s.

$$N \geq \frac{\cosh^{-1}\left(\sqrt{(10^{-0.1A_s}-1)/(10^{-0.1A_p}-1)}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}.$$

$$\boxed{N = 3}$$

$$\epsilon = \sqrt{10^{-0.1A_p}-1} = 0.8827$$

$b_1 = 0.3519$ $\quad c_1 = 0.8788$ $\quad y_N = 0.3519$.

$$H_N(s) = \frac{B_0}{s_N + C_0} \prod_{k=1}^{N-1/2} \frac{B_k}{s_N^2 + b_k s_N + C_k}.$$

To evaluate B_0 & B_K.

$$H_N(s)\Big|_{s_N \to 0} = 1$$

$$B_0 = B_1 = \sqrt{0.3074}.$$

$$H_N(s) = \frac{0.3074}{s_N^3 + 0.7038 s_N^2 + 0.99 s + 0.3074}.$$

$$H_a(s) = \frac{2.432.2}{s^3 + 14.076 s^2 + 396 s + 245.32},$$