5.22 Consider the periodic sequence

$$x_p(n) = \cos \frac{2\pi}{10} n \qquad -\infty < n < \infty$$

with frequency $f_0 = \frac{1}{10}$ and fundamental period $N = 10$. Determine the 10-point DFT of the sequence $x(n) = x_p(n)$, $0 \le n \le N-1$.

5.23 Compute the N-point DFTs of the signals

(a) $x(n) = \delta(n)$

(b) $x(n) = \delta(n - n_0) \quad 0 < n_0 < N$

(c) $x(n) = a^n \quad 0 \le n \le N-1$

(d) $x(n) = \begin{cases} 1, & 0 \le n \le N/2 - 1 (N \text{ even}) \\ 0, & N/2 \le n \le N-1 \end{cases}$

(e) $x(n) = e^{j(2\pi/N)k_0} \quad 0 \le n \le N-1$

(f) $x(n) = \cos \frac{2\pi}{N} k_0 n \quad 0 \le n \le N-1$

(g) $x(n) = \sin \frac{2\pi}{N} k_0 n \quad 0 \le n \le N-1$

(h) $x(n) = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \quad 0 \le n \le N-1 \end{cases}$

5.24 Consider the finite-duration signal

$$x(n) = \{1, 2, 3, 1\}$$

(a) Compute its four-point DFT by solving explicitly the 4-by-4 system of linear equations defined by the inverse DFT formula.

(b) Check the answer in part (a) by computing the four-point DFT, using its definition.

5.25 **(a)** Determine the Fourier transform $X(\omega)$ of the signal

$$x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1, 0\}$$

(b) Compute the 6-point DFT $V(k)$ of the signal

$$v(n) = \{3, 2, 1, 0, 1, 2\}$$

(c) Is there any relation between $X(\omega)$ and $V(k)$? Explain.

5.26 Prove the identity

$$\sum_{l=-\infty}^{\infty} \delta(n + lN) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j(2\pi/N)kn}$$

(*Hint:* Find the DFT of the periodic signal in the left-hand side.)

5.27 *Computation of the even and odd harmonics using the DFT* Let $x(n)$ be an N-point sequence with an N-point DFT $X(k)$ (N even)

(a) Consider the time-aliased sequence

$$y(n) = \begin{cases} \displaystyle\sum_{l=-\infty}^{\infty} x(n + lM), & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases}$$

What is the relationship between the M-point DFT $Y(k)$ of $y(n)$ and the Fourier transform $X(\omega)$ of $x(n)$?

(b) Let

$$y(n) = \begin{cases} x(n) + x\left(n + \frac{N}{2}\right), & 0 \le n \le N-1 \\ 0, & \text{elsewhere} \end{cases}$$

and

$$y(n) \underset{N/2}{\overset{\text{DFT}}{\longleftrightarrow}} Y(k)$$

Show that $X(k) = Y(k/2)$, $k = 2, 4, \ldots, N-2$.

(c) Use the results in parts (a) and (b) to develop a procedure that computes the odd harmonics of $X(k)$ using an $N/2$-point DFT.

5.28* *Frequency-domain sampling* Consider the following discrete-time signal

$$x(n) = \begin{cases} a^{|n|}, & |n| \le L \\ 0, & |n| > L \end{cases}$$

where $a = 0.95$ and $L = 10$

(a) Compute and plot the signal $x(n)$.

(b) Show that

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = x(0) + 2\sum_{n=1}^{L} x(n)\cos\omega n$$

Plot $X(\omega)$ by computing it at $\omega = \pi k/100$, $k = 0, 1, \ldots, 100$.

(c) Compute

$$c_k = \frac{1}{N} X\left(\frac{2\pi}{N}K\right) \qquad k = 0, 1, \ldots, N-1$$

for $N = 30$.

(d) Determine and plot the signal

$$\tilde{x}(n) = \sum_{k=0}^{N-1} c_k e^{j(2\pi/N)kn}$$

What is the relation between the signals $x(n)$ and $\tilde{x}(n)$? Explain.

(e) Compute and plot the signal $\tilde{x}_1(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$, $-L \le n \le L$ for $N = 30$. Compare the signals $\tilde{x}(n)$ and $\tilde{x}_1(n)$.

(f) Repeat parts (c) to (e) for $N = 15$.

5.29* *Frequency-domain sampling* The signal $x(n) = a^{|n|}$, $-1 < a < 1$ has a Fourier transform

$$X(\omega) = \frac{1 - a^2}{1 - 2a\cos\omega + a^2}$$

(a) Plot $X(\omega)$ for $0 \le \omega \le 2\pi$, $a = 0.8$.

Reconstruct and plot $X(\omega)$ from its samples $X(2\pi k/N)$, $0 \le k \le N-1$ for:

(b) $N = 20$

(c) $N = 100$

(d) Compare the spectra obtained in parts (b) and (c) with the original spectrum $X(\omega)$ and explain the differences.

(e) Illustrate the time-domain aliasing when $N = 20$.