5.2 Geometric Representation of Signals

The essence of geometric representation of signals is to represent any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \le M$. That is to say, given a set of real-valued energy signals $s_1(t), s_2(t), \ldots, s_M(t)$, each of duration T seconds, we write

$$s_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t), \quad \begin{cases} 0 \le t \le T \\ i=1,2,\ldots,M \end{cases}$$

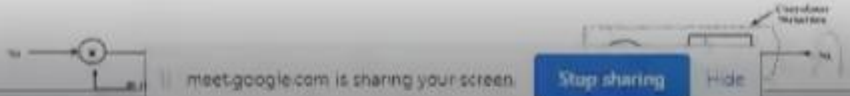
Where the coefficients of the expansion are defined by:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt \quad \begin{cases} i=1,2,\ldots,M \\ j=1,2,\ldots,N \end{cases}$$

The real-valued basis functions are orthonormal which means

$$s_{ij} = \int_0^T \phi_i(t)\phi_j(t)\,dt \quad \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\ne j \end{cases}$$

The set of coefficients may naturally be viewed as an N-dimensional vector, denoted by s_i. The important point to note here is that the vector S_i bears a one-to-one relationship with the Transmitted signal



received. So, an optimum decision rule may heuristically be framed as:

Set $m^{*} = m_i$ ***if*** $P(m_i \text{ sent} | x) \geq P(m_k \text{ sent} | x)$***, for all*** $k \neq i$

This decision rule is known as ***maximum a posteriori probability*** rule. This rule requires the receiver to determine the probability of transmission of a message from the received vector. Now, for practical convenience, we invoke Bayes' rule to obtain an equivalent statement of optimum decision rule in terms of a priori probability:

$$\Pr\left(m_i \middle| \bar{r}\right) \quad \Pr\left(\bar{r}\right) \quad = \Pr\left(\bar{r} \middle| m_i\right) \Pr\left(m_i\right)$$

we see that determination of maximum a posteriori probability is equivalent to determination of maximum a priori probability $p(r^{*} \mid m_i)$. This a priori probability is also known as the '***likelihood function***'.

So the decision rule can equivalently be stated as:

Set $m^{*} = m_i$ if $pr(r^{*} \mid m_i)$ is maximum for k = i

For an AWGN channel, the following statement is equivalent to ML decision:

The received vector r lies in decision region Z_i if,

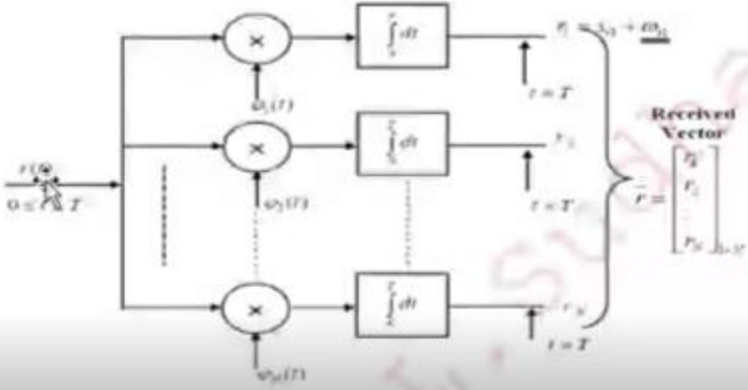
$$\left(\sum_{j=1}^{N} r_j s_{kj} - \frac{1}{2} E_k \right)$$ is maximum for k = i

In the correlation detector, multiply received signal with different basis functions and integrate the product to identify the correlation between the received signal and individual basis function and form the received vector.

The vector receiver is constructed based on ML detection as per the equation

$$\left(\sum_{j=1}^{N} r_j s_{kj} - \frac{1}{2} E_k\right)$$

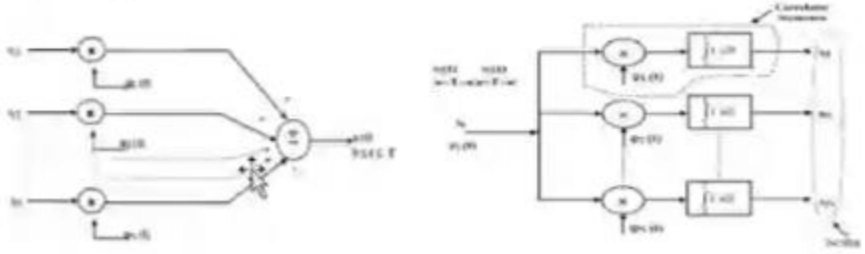
Decision is made by selecting the signal with maximum likelihood value.



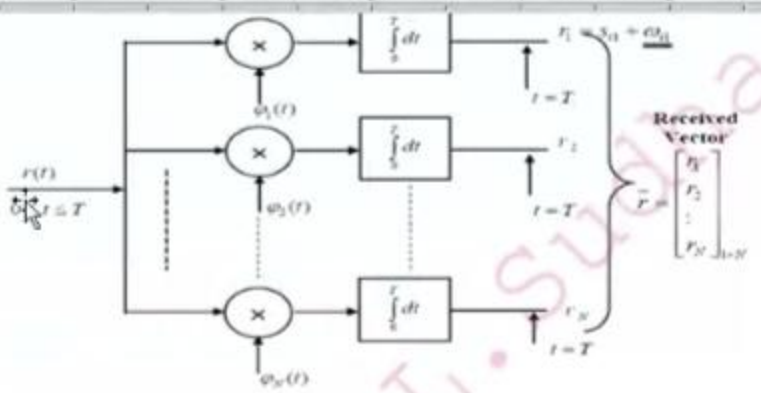
Fig(5) The structu

$$s_{ij} = \int_0 \phi_i(t)\phi_j(t)dt \qquad \begin{cases} 0 & \text{if } \neq j \end{cases}$$

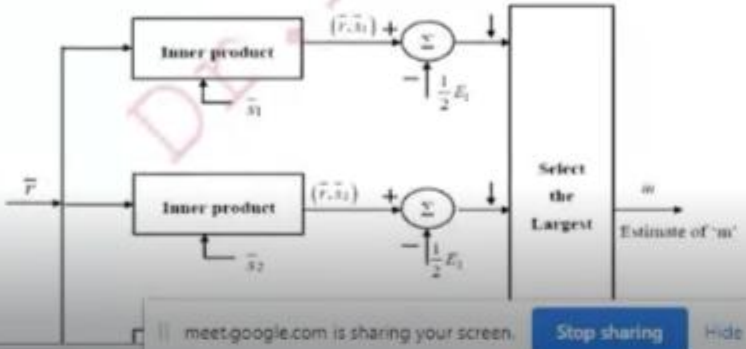
The set of coefficients may naturally be viewed as an N dimensional vector, denoted by $\mathbf{s_j}$. The important point to note here is that the vector $\mathbf{s_i}$ bears a one to one relationship with the Transmitted signal

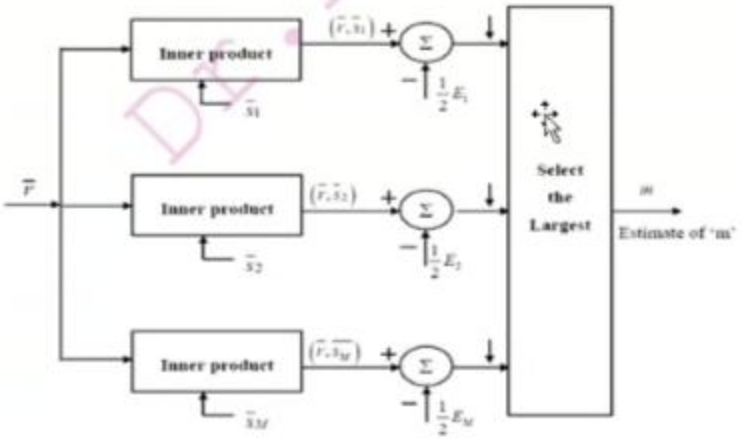


Fig(2) (a)Construction of signal from basis functions. (b) Getting coefficients from Basis functions



Fig(5) The structure of a Correlation Detector





Fig(6) Block schematic diagram for the Vector Receiver

5.8 Matched Filter Receiver:

Certain structural modification and simplifications of the correlation receiver are possible by observing that,

(a) All orthonormal basis functions ϕ_j are defined between $0 \le t \le T_b$ and they are zero outside this range.

(b) Analog multiplication, which is not always very simple and accurate to implement, of the received signal r(t) with time limited basis functions may be replaced by some filtering operation. Let, $h_j(t)$ represent the impulse response of a linear filter to which r(t) is applied.

Then, the filter output $Y_j(t)$ may be expressed as:

$$y_j(t) = \int_{\infty}^{\infty} r(\tau)\, h_j(t-\tau)\, d\tau$$

Now, let $h_j(t) = \varphi_j(T-t)$.

$$y_j(t) = \int_{-\infty}^{\infty} r(\tau).\varphi_j[T-(t-\tau)]d\tau$$

$$= \int^{\infty} r(\tau).\varphi_j(T+\tau-t)d\tau$$

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If we sample this output at t = T, and recalling that $\phi_j(t)$ is zero outside the interval $0 \le t \le T$

Using this, the above equation may be expressed as,

$$y_j(T) = \int_0^T r(\tau)\varphi_j(\tau)\, d\tau$$

From our discursion on correlation receiver, we recognize that,

$$r_j = \int_0^T r(\tau)\varphi_j(\tau)\, d\tau = y_j(\tau)$$

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The important expression of above equation is that it tells us that the j^{th} correlation output can equivalently be obtained by using a filter with $h_j(t)=\phi_j(T-t)$ and sampling its output at t = T.

The filter is said to be matched to the orthonormal basis function $\phi_j(t)$ and the alternate receiver structure is known as a matched filter receiver. The detector part of the matched filter receiver is shown in fig below.

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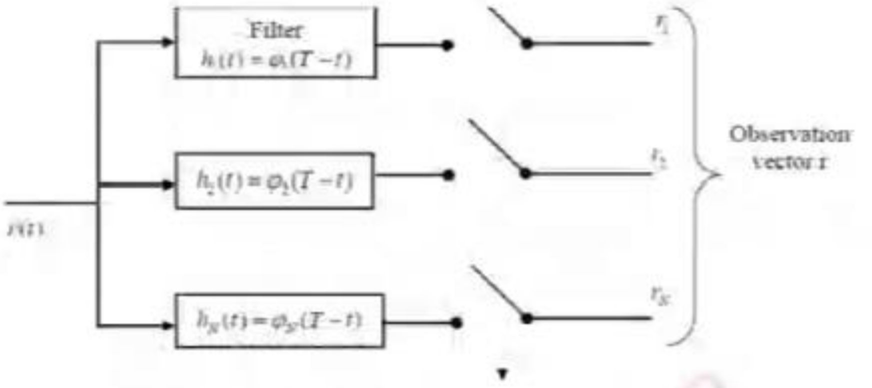

Fig(7)The block diagram of a matched filter bank that is equivalent to a Correlation Detector

A physically realizable matched filter is to be causal i.e $h_j(t)=0$ for $t<0$.

Properties of a Matched Filter

We note that a filter which is matched to a known signal $\phi(t), 0 \le t \le T$ is characterized by an impulse response h(t) which is a time reversed and delayed version of $\phi(t)$ i.e.

$h_j(t)=\phi_j(T-t)$

In the frequency domain, the matched filter is characterized by a transfer function, which is, except for a delay factor, the complex conjugate of the F.T. of $\phi(t)$, i.e.

$$H(f) = \Phi^*(f)\exp(-j2\pi fT)$$

Property (1) : The spectrum of the output signal of a matched filter with the matched signal as input is, except for a time delay factor, proportional to the energy spectral density of the input signal.

Let, $\Phi_0(f)$ denote the F.T. of the filter of output $\Phi_0(t)$. Then,

$$\begin{aligned}\Phi_0(f) &= H(f)\Phi(f)\\ &= \Phi^*(f)\Phi(f)\exp^{(-j2\pi fT)}\\ &= \underbrace{|\Phi(f)|^2}_{\substack{\text{Energy spectral}\\ \text{density of } \varphi(t)}} \exp^{(-j2\pi fT)}\end{aligned}$$

Property (2): The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.