Lattice structure for FIR filters

We have the transfer fun of an FIR filter given by

$$\frac{Y(z)}{X(z)} = B(z) = 1 + \sum_{k=1}^{M} a_M(k) z^{-k}.$$

where M is the order of the FIR filter.

if M=1 ie a first order system TF becomes

$$\frac{Y(z)}{X(z)} = 1 + a_1(1) z^{-1} \quad - \text{(I)}$$

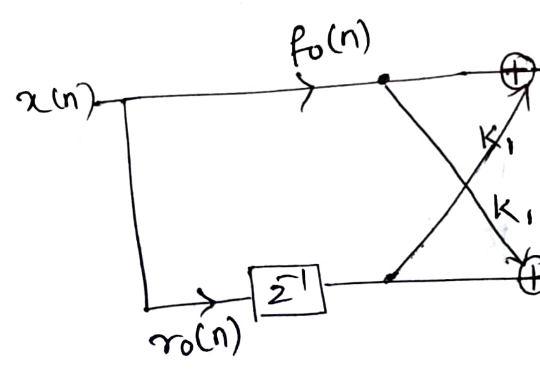
ie $Y(z) = X(z)(1 + a_1(1) z^{-1})$

$= X(z) + a_1(1) z^{-1} X(z)$

∴ Taking IZT

$$y(n) = x(n) + a_1(1)\, x(n-1) \quad - \text{(II)}$$

The (II) can be written in lattice structure form as.



$y(n) = X(z) + k z^{-1} X(z)$ (shift)

$= x(n) + k_1 x(n-1)$

$= f_0(n) + k_1 r_0(n-1)$

$r_1(n) = k_1 f_0(n) + r_0(n-1)$

This is the first order lattice structure.

→ I/p is $x(n)$ and two outputs are

Upper channel o/p $f_1(n) = y(n) = f_0(n) + k_1 r_0(n-1) = x(n) + k_1 x(n-1)$ — (III)

Lower channel o/p $r_1(n) = r_1(n) = k_1 f_0(n) + r_0(n-1) = k_1 x(n) + x(n-1)$ — (IV)

In eqn (III) if $k_1 = a_1(1)$ then III is equal to (II)

Now let $M=2$.

then $H(z) = \frac{Y(z)}{X(z)} = B(z) = \sum_{k=0}^{2} a_M(k) z^{-k}$

$Y(z) = [a_2(0) z^{0} + a_2(1) z^{-1}] X(z) + a_2(2) z^{-2} X(z)$

ie $Y(z) = z^{0} a_2(0) X(z) + a_2(1) z^{-1} X(z) + a_2(2) z^{-2} X(z)$

$\therefore y(n) = a_2(0) x(n) + a_2(1) x(n-1) + a_2(2) x(n-2)$ — (V)

Let $y(n) = f_1(n) + K_2 r_1(n-1)$ — (VI)

Sub (III) & (IV) in (VI)

$y(n) = f_0(n) + K_1 r_0(n-1) + K_2 [K_1 f_0(n) + r_0(n-2)]$

$= f_0(n) + K_1 r_0(n-1) + K_1 K_2 f_0(n) + K_2 r_0(n-2)$.

$\because f_0(n) = r_0(n) = x(n)$ $y(n)$ becomes

$y(n) = x(n) + K_1 x(n-1) + K_1 K_2 x(n-1) + K_2 x(n-2)$.

$y(n) = x(n) + x(n-1)(K_1 + K_1 K_2) + K_2 x(n-2)$ — (VII)

Comparing (VII) with (V) & (V)

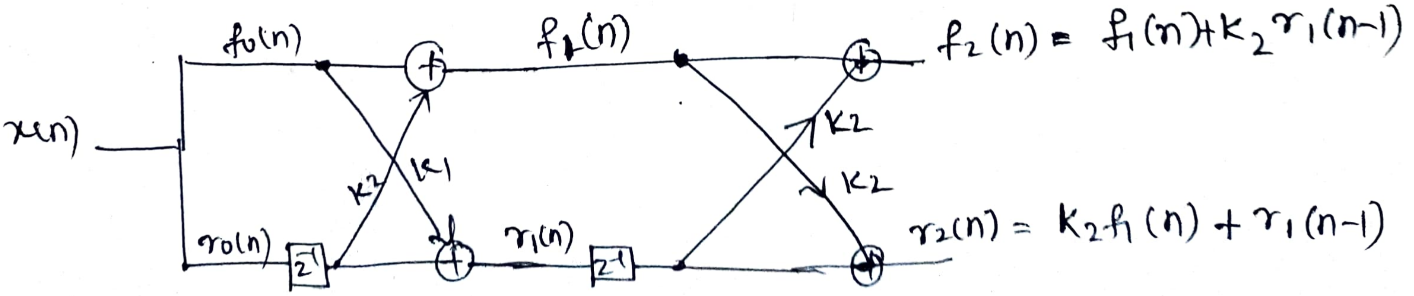
we have $a_2(0) = 1$, $a_2(1) = (K_1 + K_1 K_2)$, $a_2(2) = K_2$.

Solving for K_2 & K_1.

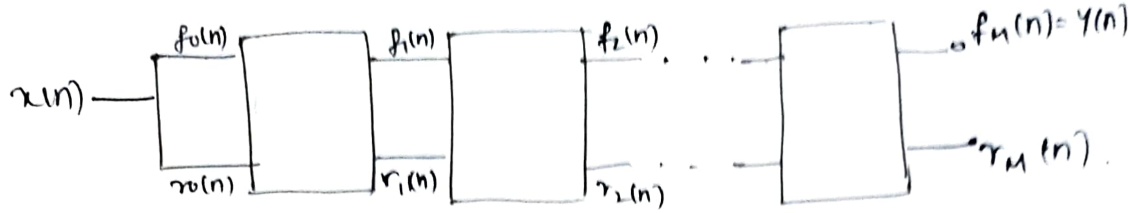
$a_2(2) = K_2$ & $a_2(1) = K_1 + K_1 K_2 \Rightarrow K_1(1+K_2) = a_2(1)$

$K_1 = \frac{a_2(1)}{1 + a_2(2)}$.

Eq (VII) is lattice structure of 2nd order & can be written as.



IIIly an M^{th} order FIR filter can be represented by M stage lattice structure.



$\Rightarrow$ To convert Direct form I to Lattice structure

for $m = M, M-1, \ldots 1$ do

$$K_m = a_m(m) \quad \text{— (A)}$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\,a_m(m-i)}{1 - K_m^2} \quad \text{— (B)} \quad i = 1 \text{ to } (m-1)$$

$\Rightarrow$ if $K_m = 1$ above eqn fails ie $a_{m-1}(i) = \infty$ which means there is a zero on unit circle then we need to factor out the the root from $B(Z)$ & above recursive formula can be used.

$\Rightarrow$ eqn (A) & (B) can be used to find to find lattice structure coefficients.

$\Rightarrow$ For $M = 1$

$$K_1 = a_1(1)$$

$M = 2$

$$K_2 = a_2(2)$$

$$a_1(1) = \frac{a_2(1) - a_2(2)\,a_2(1)}{1 - K_2^2}$$

$$\text{ie } K_1 = \frac{a_2(1)(1 - a_2(2))}{1 - a_2^2(2)}$$

$$= \frac{a_2(1) \cdot \cancel{[1 - a_2(2)]}}{\cancel{(1 - a_2(2))}(1 + a_2(2))}$$

$$\boxed{K_1 = \frac{a_2(1)}{1 + a_2(2)}}$$

For 3^{rd} order lattice $m = 3 \; 2 \; 1$

for $\underline{\underline{M=3}}$ $\quad K_1, K_2, K_3$

$K_3 = a_3(3).$

for $i = 1$ to 2 &

$$a_2(2) = \frac{a_3(2) - a_3(3) \cdot a_3(1)}{1 - K_3^2} =$$

$$a_2(1) = \frac{a_3(1) - a_3(3) \cdot a_3(2)}{1 - K_3^2}$$

$\underline{\underline{M=2}}.$

⋮

Continue to get $a_M(k)$ coefficients

Problem

Determine the coefficients K_m for the lattice filter corresponding to FIR filter described by

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}.$$

and draw the structure in both Lattice form & Direct form-I

Sol given $a_2(0) = 1$, $a_2(1) = 2$, $M = 2$.

$a_2(2) = \frac{1}{3}$.

use $m = M, M-1 \ldots 1$ do

$$K_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\,a_m(m-i)}{1 - K_m^2}, \quad i = 1, 2 \ldots M-1$$

ie for $m = 2, 1$ do above eqns $i = 1$ to $(m-1)$ each time.

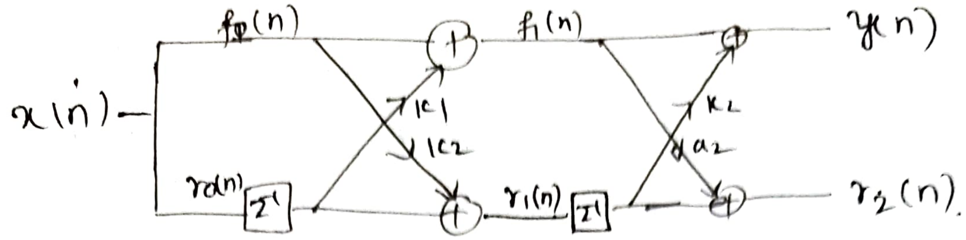
ie for $\underline{m = 2}$, $i = 1$

$$K_2 = a_2(2) = \frac{1}{3}$$

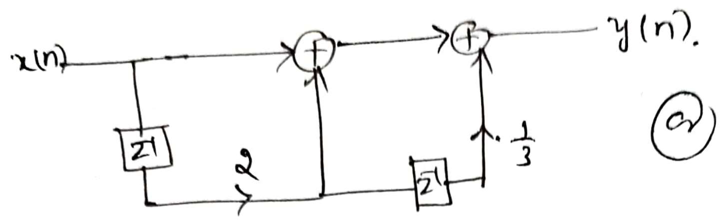
$$a_1(1) = \frac{a_2(1) - a_2(2)\,a_2(1)}{1 - K_2^2} = \frac{2 - \frac{1}{3} \times 2}{1 - \frac{1}{9}}$$

$a_1(1) = 1.5$. for $m = 1$, $K_1 = a_1(1) = 1.5$..

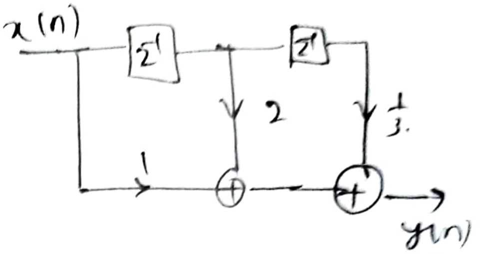
∴ Lattice structure is



Direct form structure



(or)

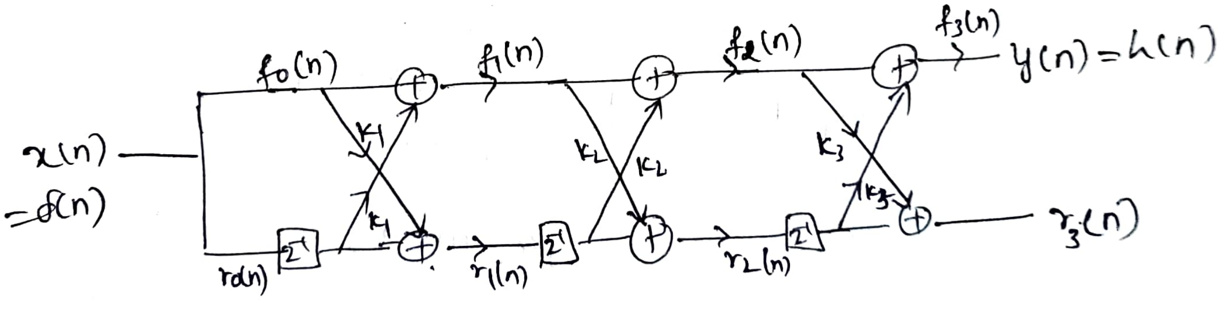


(2). Consider 3stage FIR structure having

$K_1 = 0.65$, $K_2 = -0.34$ & $K_3 = 0.8$

Evaluate impulse response by tracing a unit sample i/p $\delta(n)$ at its i/p through lattice structure.

Sol: The 3-stage lattice structure is



$f_1(n) = \delta(n) + K_1\delta(n-1) = \cancel{1}$

$r_1(n) = K_1\delta(n) + \delta(n-1) = \cancel{K_1}$

$f_2(n) = K_2 r_1(n-1) + f_1(n) = \delta(n) + K_1\delta(n-1) + K_2(K_1\delta(n-1) + \delta(n-2))$

$f_2(n) = \delta(n) + K_1\delta(n-1) + K_1K_2\delta(n-1) + K_2\delta(n-2)$

$= \cancel{\delta(n)(1+K_1K_2)} + \cancel{\delta(n-1)(K_1+K_2)}$

$= \delta(n) + (K_1 + K_1K_2)\delta(n-1) + K_2\delta(n-2)$

$r_2(n) = K_2 f_1(n) + r_1(n-1) = K_2(\delta(n) + K_1\delta(n-1)) + K_1\delta(n-1) + \delta(n-2)$

ie $r_2(n) = K_2\delta(n) + \delta(n-1)(K_1 + K_1K_2) + \delta(n-2)$.

Now: $f_3(n) = f_2(n) + K_3\, r_2(n-1)$.

$$= \delta(n) + (K_1 + K_1K_2)\,\delta(n-1) + K_2\,\delta(n-2) + K_3\left[K_2\,\delta(n-1) + \delta(n-2)(K_1 + K_1K_2) + \delta(n-3)\right]$$

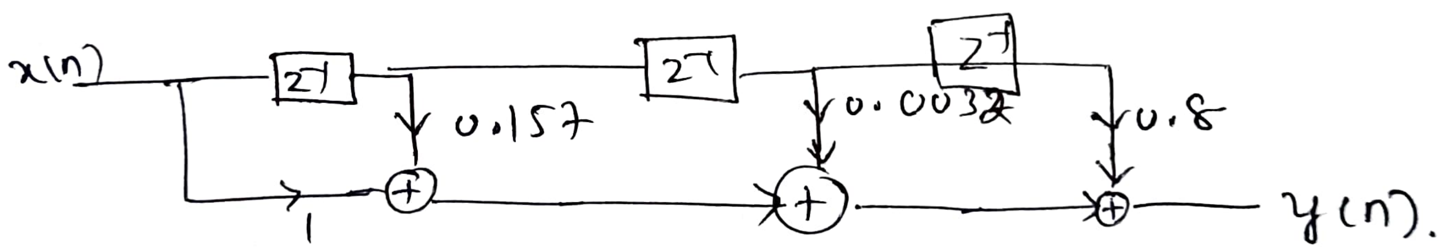
$$= \delta(n) + \delta(n-1)\left[K_1 + K_1K_2 + K_2K_3\right] + \delta(n-2)\left[(K_1 + K_1K_2)K_3 + K_2\right] + K_3\,\delta(n-3)$$

$$f_3(n) = \delta(n) + (K_1 + K_1K_2 + K_2K_3)\,\delta(n-1) + \left[K_2K_3 + K_1K_2K_3\right]\delta(n-2) + K_3\,\delta(n-3)$$

$y(n) = h(n)$ if $x(n) = \delta(n)$

ie $h(n) = f_3(n) = \delta(n) + 0.157\,\delta(n-1) + \cancel{(0.4488)}\ 0.0032\,\delta(n-2) + 0.8\,\delta(n-3)$

$\therefore\ H(z) = 1 + 0.157z^{-1} + 0.0032z^{-2} + 0.8z^{-3}$



3rd order Lattice structure

Given FIR filter with diff eqn

$y(n) = x(n) + 3.1x(n-1) + 5.5x(n-2) + 4.2x(n-3) + 2.3x(n-4)$

Sketch Lattice realization.

Sol $\underline{M=4}$ $\therefore$ find K_4, K_3, K_2 & K_1. given $a_0 = 1$

$a_4(1) = 3.1$

$a_4(2) = 5.5$

$a_4(3) = 4.2$

$a_4(4) = 2.3$

$i = 1$ to $(4-1) = 3$

For $m = 4, 3, 2, 1$

$K_m = a_m(m)$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

For $\underline{m=4}$, $i = 3, 2, 1$

$$K_4 = a_4(4) = 2.3, \quad a_3(3) = \frac{a_4(3) - a_4(4)\, a_4(1)}{1 - K_4^2} = 0.683$$

$$a_3(2) = \frac{a_4(2) - a_4(4)\, a_4(2)}{1 - K_4^2} = 1.667$$

$$a_3(1) = \frac{a_4(1) - a_4(4)\, a_4(3)}{1 - K_4^2} = 1.529$$

For $\underline{m=3}$, $i = 2, 1$

$K_3 = a_3(3) = 0.683$

$$a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - K_3^2} = 1.167$$

$$a_2(1) = \frac{a_3(1) - a_3(3) \cdot a_3(2)}{1 - K_3^2} = 0.7318$$

$\underline{\text{For } m=2}$, $a_2(2) = K_2 = 1.167$, $i = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - K_2^2} = 0.13548$$

For $m = 1$,

$K_1 = a_1(1) = 0.13548$

$\therefore K_1 = 0.13548$, $K_2 = 1.167$, $K_3 = 0.683$, $K_4 = 2.3$