BILINEAR TRANSFORMATION

Infinite Impulse Response (IIR) Filters

Recursive Filters:

$$y(n) + a_1 y(n-1) + \ldots + a_N y(n-N) = b_0 x(n) + \ldots + b_N x(n-N)$$

with $a_i, \ldots, b_j$ *constant coefficients.*

Advantages: *very selective filters with a few parameters;*

Disadvantages: a) *in general nonlinear phase*

b) *can be unstable.*

$$\frac{nT - nT + T}{2}$$

$$\frac{dy(t)}{dt} = x(t) \Rightarrow sY(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s}$$

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt}\,dt = \int_{(n-1)T}^{nT} x(t)\,dt$$

$$\Rightarrow y(nT) - y[(n-1)T] = \int_{(n-1)T}^{nT} x(t)\,dt$$

$$y(nT) - y[(n-1)T] = \frac{T}{2}\left[x(n-1) + x(n)\right]$$

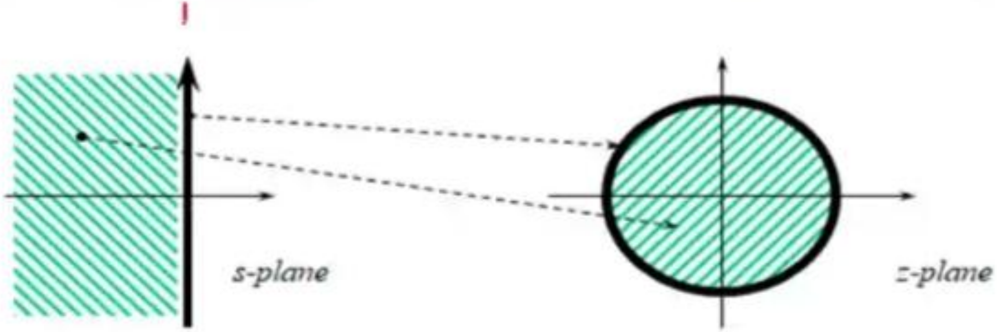
$Z - T$

$$Y(z) - Y(z)z^{-1} = \frac{T}{2}\left[X(z)z^{-1} + X(z)\right]$$

$$Y(z)\left[1 - z^{-1}\right] = \frac{T}{2} X(z)\left[1 + z^{-1}\right]$$

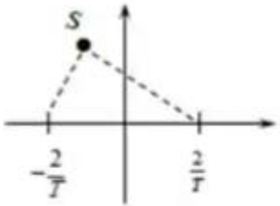
$$H(z) = \frac{Y(z)}{X(z)} = \frac{T}{2}\left(\frac{1 + z^{-1}}{1 - z^{-1}}\right)$$

Main Property of the Bilinear Transformation: *it preserves the stability regions.*



since:

$$|z| = \frac{\left| s + \frac{2}{T} \right|}{\left| s - \frac{2}{T} \right|} \le 1$$



THENMOZHI S

$$SY(s) = X(s) \quad \text{--- (1)}$$

$$\boxed{S \to \left[\frac{2}{T}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)\right]}$$

$$\frac{T}{2}S = \frac{Z-1}{Z+1}$$

$$\frac{T}{2}s(Z+1) = Z-1$$

$$\frac{T}{2}sZ + \frac{T}{2}S = Z-1$$

$$\left(\frac{T}{2}s-1\right)Z = -\left(1+\frac{T}{2}S\right)$$

$$\cancel{-}Z\left(1-\frac{T}{2}S\right) = \cancel{-}\left(1+\frac{T}{2}S\right)$$

$$Z = \frac{1+\frac{T}{2}S}{1-\frac{T}{2}S}$$

$$(1-Z^{-1})Y(Z) = \frac{T}{2}(1+Z^{-1})X(Z)$$

$$\left[\frac{2}{T}\frac{(1-Z^{-1})}{1+Z^{-1}}\right]Y(Z) = X(Z) \quad \text{--- (2)}$$

$$S = \sigma + j\Omega \qquad Z = re^{j\omega}$$

$$re^{j\omega} = \frac{1+\frac{T}{2}(\sigma+j\Omega)}{1-\frac{T}{2}(\sigma+j\Omega)}$$

$$|Z| = \frac{\sqrt{\left(1+\frac{T}{2}\sigma\right)^2 + \left(\frac{T}{2}\Omega\right)^2}}{\sqrt{\left(1-\frac{T}{2}\sigma\right)^2 + \left(\frac{T}{2}\Omega\right)^2}} = 1$$

THENMOZHI S