* ALGEBRAIC STRUCTURE OF CYCLIC CODES :-

"A (n, K) linear block code is said to be a cyclic code if every cyclic shift of the code is also a code vector."

<u>Ex:</u> If

$C_1 = 0111110$

$C_2 = 0011111$

$C_3 = 1001111$

$C_4 = 1100111$

$\vdots \quad \vdots$

If $C_1, C_2, C_3 \ldots$ are also code vectors belonging to the same code, then the code is called CYCLIC CODE.

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In general, let the n-bit vector be represented as

$$V = (V_0\, V_1\, V_2 \cdots\cdots V_{n-1})$$

$$V(1) = (V_{n-1}\, V_0\, V_1\, V_2 \cdots\cdots V_{n-2})$$

$$V(2) = (V_{n-2}\, V_{n-1}\, V_0\, V_1 \cdots\cdots V_{n-3})$$

$$\vdots \qquad \vdots$$

$$V(i) = (V_{n-i}\, V_{n-i+1} \cdots\cdots V_0\, V_1\, V_2 \cdots\cdots V_{n-i-1})$$

These equations which are obtained by shifting the 'V' vector cyclically successively are also the

$$V(i) = (V_{n-i}\ V_{n-i+1} \cdots\cdots V_0\ V_1\ V_2 \cdots\cdots V_{n-i-1})$$

These equations which are obtained by shifting the 'V' vector cyclically successively are also the code vectors 'C'. This property of cyclic codes also allows to treat the elements of each code vector as a co-efficients of polynomial of degree ~~(n-1)~~. n

$\therefore$ The equation will be

$$V(x) = V_0 + V_1 x + V_2 x^2 + V_3 x^3 + \cdots\cdots + V_{n-1} x^{n-1}$$

$$V'(x) = V_{n-1} + V_0 x + \cdots V_{n-2} x^{n-1}$$

as a the co-efficients of polynomial of degree $(\cancel{n-1})\ n$

$\therefore$ The equation will be

$$V(x) = V_0 + V_1 x + V_2 x^2 + V_3 x^3 + \cdots + V_{n-1} x^{n-1}$$

$$V^1(x) = V_{n-1} + V_0 x + V_1 x^2 + V_2 x^3 + \cdots + V_{n-2} x^{n-1}$$

$$V^2(x) = V_{n-2} + V_{n-1} x + V_0 x^2 + V_1 x^3 + \cdots + V_{n-3} x^{n-1}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$V^i(x) = V_{n-i} + V_{n-i+1} x + V_{n-i+2} x^2 + V_{n-i+3} x^3 + \cdots + V_{n-i-1} x^{n-1}$$

* MODULO-2 ALGEBRA :-

P1) Find the product of polynomials $f_1(x) = x+1$ & $f_2(x) = x^3+x+1$ using modulo-2 algebra.

$$f_1(x) \cdot f_2(x) = (x+1)(x^3+x+1)$$
$$= x^4 + x^2 + x + x^3 + x + 1$$
$$= x^4 + x^2 + x^3 + x(1 \overset{0}{\cancel{+}} 1) + 1$$
$$= \underline{x^4 + x^3 + x^2 + 1}$$

P2) Multiply $f_1(x) = 1+x+x^3$ and $f_2(x) = 1+x+x^2+x^4$

$$f_1(x) \cdot f_2(x) = (1+x+x^3)(1+x+x^2+x^4)$$
$$= 1+x+x^2+x^4 + x+x^2+x^3+x^5 + x^3+x^4+x^5+ \ldots$$
$$= 1 + x(1 \overset{0}{\cancel{+}} 1) + x^2(1 \overset{0}{\cancel{+}} 1) + x^3(1 \overset{0}{\cancel{+}} 1) + x^4(1 \overset{0}{\cancel{+}} 1) + x^5(\ldots$$
$$= \underline{1+x^7}$$

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ii) The generator polynomial $g(x)$ of a (n,k) cyclic code is a factor of x^n+1

i.e., $x^n + 1 = g(x)\,h(x)$

where, $h(x)$ is another polynomial of degree 'k' called PARITY-CHECK Polynomial.

iii) If $g(x)$ is a polynomial of degree $(n-k)$ & is a factor of x^n+1, then it generates the (n,k) cyclic code.

iv) The code vector polynomial can be found using

$$V(x) = D(x) \cdot g(x)$$

where $D(x) = d_0 + d_1 x + d_2 x^2 + \ldots\ldots + d_{k-1} x^{k-1}$ ... of degree 'k'

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where $D(x) = d_0 + d_1 x + d_2 x^2 + \ldots\ldots + d_{k-1} x^{k-1}$ of degree 'k'. is the message vector polynomial.

This method generates NON-SYSTEMATIC CYCLIC codes.

v) To generate a systematic cyclic code, the remainder polynomial $R(x)$ is obtained from the division of

$$\frac{x^{n-k} D(x)}{g(x)} = R(x)$$

... $R(x)$ are placed in beginning

v) To generate a systematic cyclic code, the [illegible] [illegible] R(x) is obtained from the division of

$$\frac{x^{n-k} D(x)}{g(x)} = R(x)$$

The co-efficients of R(x) are placed in beginning of code vector followed by co-efficients of message polynomial D(x) to get the code vector.

n-bit code vector	
co-efficients of R(x)	Co-efficients of D(x)

P) For (7,4) single error correcting cyclic code, $D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$ and $x^n + 1 = x^7 + 1 =$