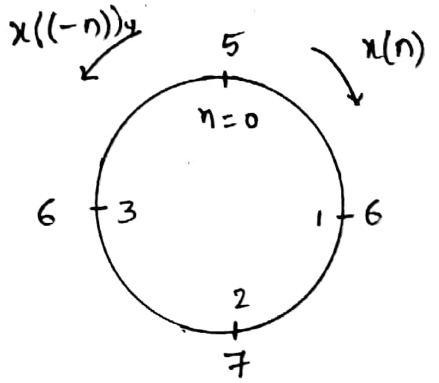
Note: (i) A sequence $x(n)$ is said to be even sequence if it satisfies the condition

$$x((-n))_N = x(n)$$

Example: $x(n) = (\underset{\uparrow}{5}, 6, 7, 6)$



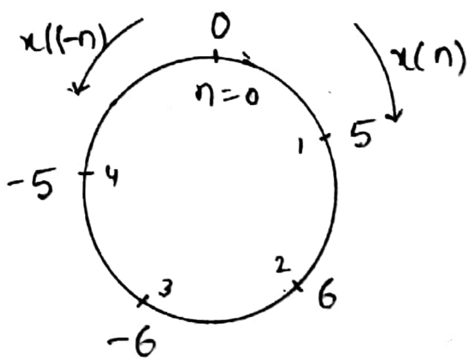
From figure $x(n) = x((-n))_4$ $\therefore$ the sequence $x(n)$ is even.

(ii) A sequence $x(n)$ is said to be odd sequence if it satisfies the condition.

$$x(n) = -x((-n))_N \quad \text{(or)} \quad x((-n))_N = -x(n)$$

Example: $x(n) = (0, 5, 6, -6, -5)$

$-x(n) = (0, -5, -6, 6, 5)$



From figure $x((-n))_5 = (0, -5, -6, 6, 5)$

$$x((-n))_5 = -x(n)$$

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(iii) For a Real sequence $x(n)$ W.K.T

$$X(k) = X^*(N-k)$$

Replace k by $-k$

$$X(-k) = X^*(N+k)$$

Since folding is circular $X(-k) \rightarrow X((-k))_N$

$$X((-k))_N = X^*(N+k)$$

Since $X(k)$ & $X^*(k)$ are periodic with period N.

$$X(k) = X(N+k), \quad X^*(k) = X^*(N+k)$$

$$\therefore \boxed{X((-k))_N = X^*(k)}$$

(iv) It can be proved that

$$x(n) \xleftrightarrow{DFT} X(k)$$

$$x((-n))_N \xleftrightarrow{DFT} X((-k))_N.$$

Prove that for a real & even sequence the DFT is purely real & for a real & odd sequence the DFT is purely imaginary.

Proof:

Consider a real sequence $x(n)$

$$x(n) = x_e(n) + x_o(n)$$

Ex: $x(n) = \cos\pi n + \sin\pi n$

$$x_e(n) = \frac{1}{2}\left[x(n) + x((-n))_N\right] \longrightarrow (1)$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x((-n))_N\right] \longrightarrow (2)$$

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Let $X(k)$ be the DFT of $x(n)$, which could be a complex, written as.

$$X(k) = A + jB \longrightarrow (3)$$

$$X^*(k) = A - jB \longrightarrow (4)$$

(3) + (4)

$$A = \frac{1}{2}\left[X(k) + X^*(k)\right]$$

$$A = \frac{1}{2}\left[X(k) + X((-k))_N\right] \longrightarrow (5)$$

(3) − (4)

$$jB = \frac{1}{2}\left[X(k) - X^*(k)\right]$$

$$jB = \frac{1}{2}\left[X(k) - X((-k))_N\right] \longrightarrow (6)$$

Taking DFT of Eqn (1)

$$X_e(k) = \frac{1}{2}\left[X(k) + X((-k))_N\right]$$

$$= A = \text{Real part of } X(k)$$

∴ DFT of Real & Even sequence is purely real.

Taking DFT of Eqn (2)

$$X_o(k) = \frac{1}{2}\left[X(k) - X((-k))_N\right]$$

$$X_o(k) = jB$$

∴ DFT of Real & odd sequence is purely imaginary.

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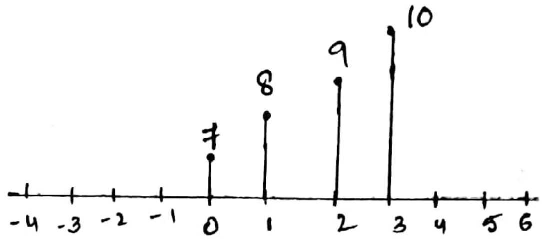
Note: Difference b/w Linear & Circular i) Shifting ii) Foldings

Consider a sequence $x(n) = (\underset{\uparrow}{7}, 8, 9, 10)$

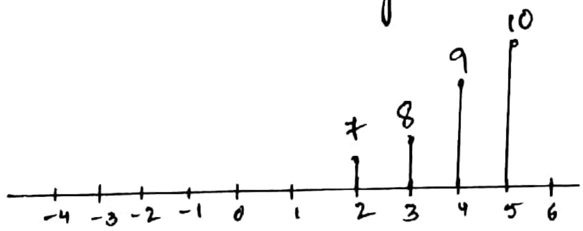
Sketch (i) $x(n)$ (ii) $x(n-2)$ (iii) $x(n+2)$ (iv) $x(-n)$ (v) $x(2-n)$ (vi) $x(-2-n)$ (vii) $x((n-1))_4$ (viii) $x((n+1))_4$ (ix) $x((-n))_4$ (x) $x((1-n))_4$ (xi) $x((-1-n))_4$

LINEAR

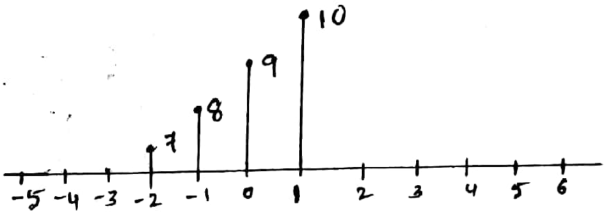
(i) $x(n)$



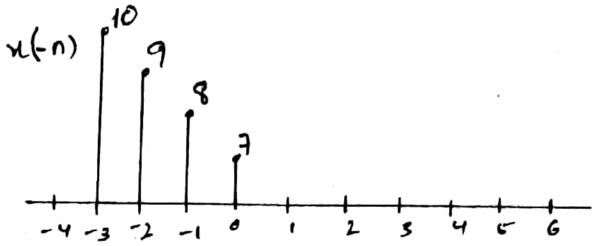
(ii) $x(n-2)$
$= x(n-n_0)$ $n_0 = 2$ +ve, shift $x(n)$ right 2 unit



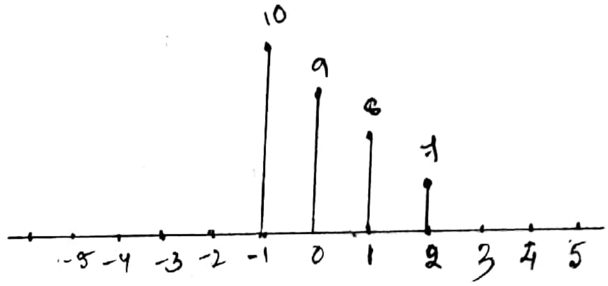
(iii) $x(n+2)$
$= x(n-n_0)$ $n_0 = -2$ -ve, shift $x(n)$ left 2 unit



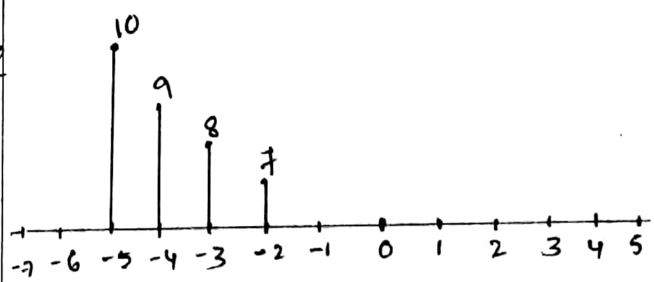
(iv) $x(-n)$
Taking Folding / Reflection of $x(n)$



(v) $x(2-n)$
2 +ve, shift $x(-n)$ right by 2 unit



(vi) $x(-2-n)$
-2 -ve shift $x(-n)$ left by 2 units



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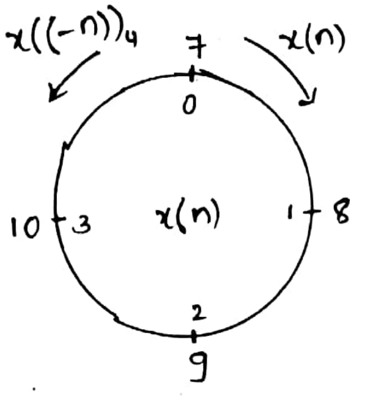
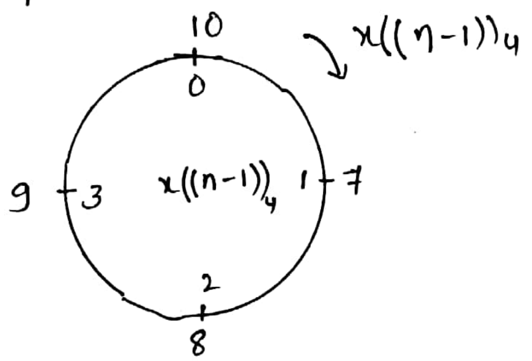
CIRCULAR



$x(n) = (\underset{\uparrow}{7}, 8, 9, 10)$

(vii) $x((n-1))_4$

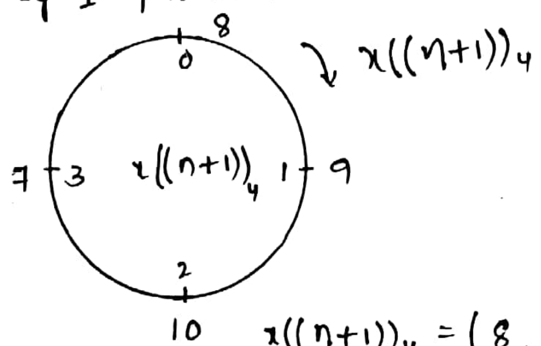
$x((n-1))_4$ is obtained by shifting (rotating) the values of $x(n)$ towards right (clockwise) by 1 position.



$x((n-1))_4 = (\underset{\uparrow}{10}, 7, 8, 9)$

(viii) $x((n+1))_4$

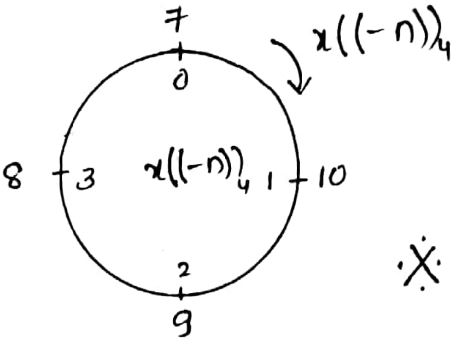
$x((n+1))_4$ is obtained by shifting (rotating) the values of $x(n)$ towards left (anti-clockwise) by 1 position.



$x((n+1))_4 = (\underset{\uparrow}{8}, 9, 10, 7)$

(ix) $x((-n))_4$ **IMPORTANT**

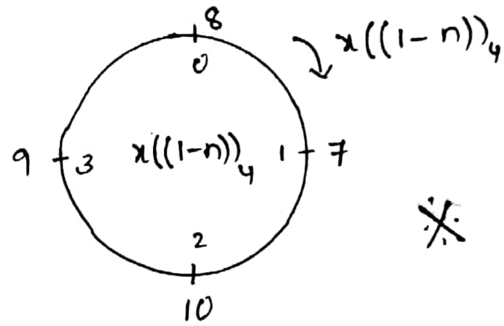
$x((-n))_4$ is obtained by reading $x(n)$ in anticlockwise direction & sketched as below.



$x((-n)) = (\underset{\uparrow}{7}, 10, 9, 8)$

(x) $x((1-n))_4$ **IMPORTANT**

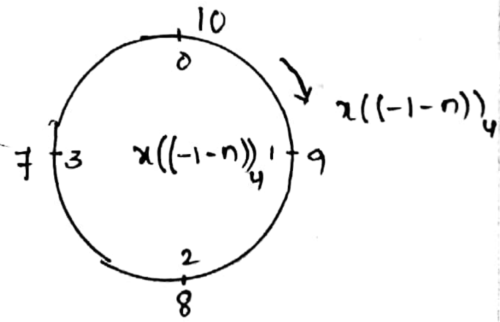
Shift $x((-n))_4$ right by 1



$x((1-n))_4 = (\underset{\uparrow}{8}, 7, 10, 9)$

(xi) $x((-1-n))_4$

shift $x((-n))_4$ left by 1



$x((-1-n))_4 = (\underset{\uparrow}{10}, 9, 8, 7)$

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Qn: Perform the circular convolution of the two sequences given using graphical method.

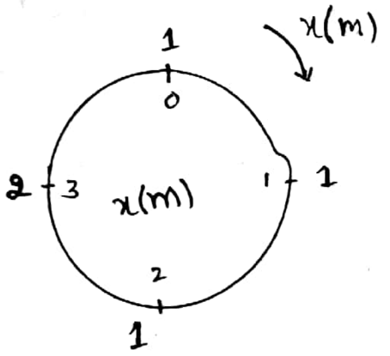
$x(n) = (\underset{\uparrow}{1}, 1, 1, 2)$. $h(n) = (4, 6, 2, 1)$

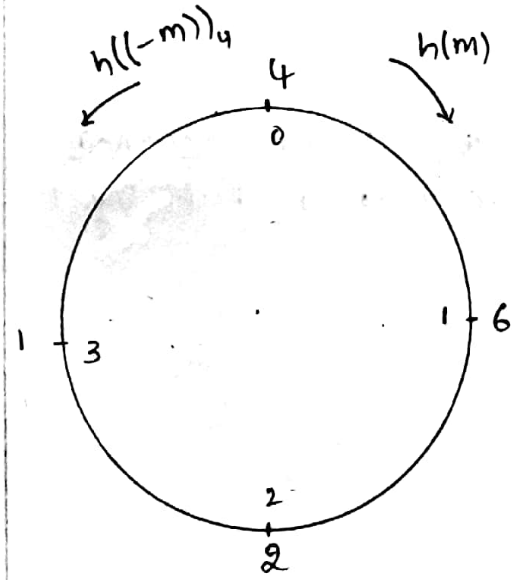
Soln. $N = 4$

$$y_c(n) = x(n) \circledast_N h(n)$$

$$y_c(n) = \sum_{m=0}^{N-1} x(m)\, h((n-m))_N$$

$$y_c(n) = \sum_{m=0}^{3} x(m)\, h((n-m))_4 \longrightarrow (1)$$

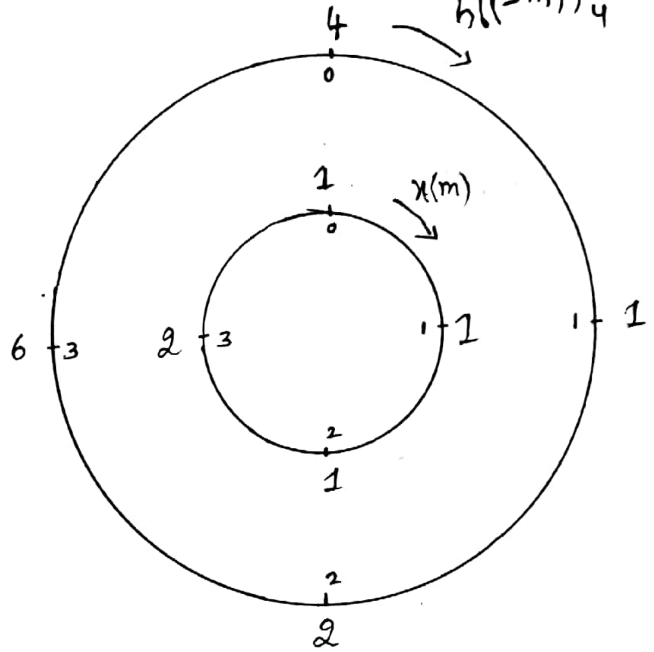




$y(0) = ?$

put $n = 0$ in Eqn (1)

$$y(0) = \sum_{m=0}^{3} x(m)\, h((-m))_4$$



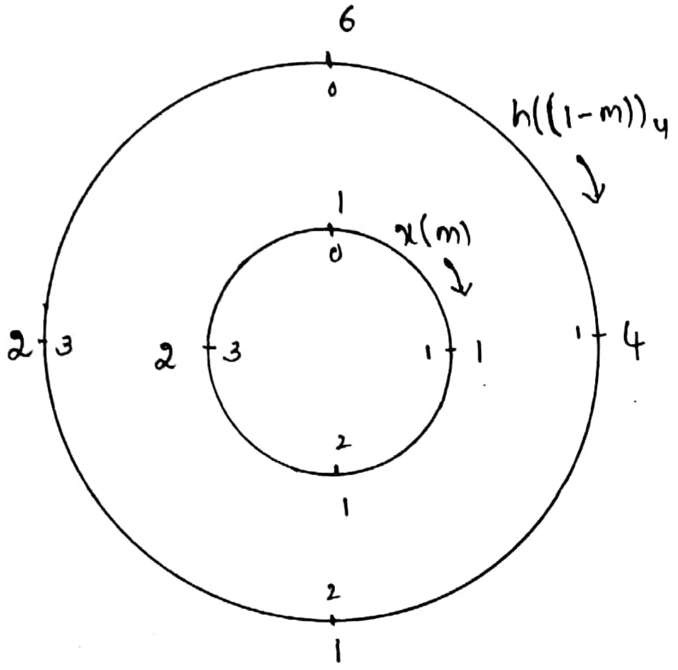
$$y(0) = 1 \times 4 + 1 \times 1 + 2 \times 1 + 2 \times 6$$
$$= 4 + 1 + 2 + 12$$

$$\underline{\underline{y(0) = 19}}$$

$y(1) = ?$

put $n = 1$ in Eqn (1)

$$y(1) = \sum_{m=0}^{3} x(m)\, h((1-m))_4$$

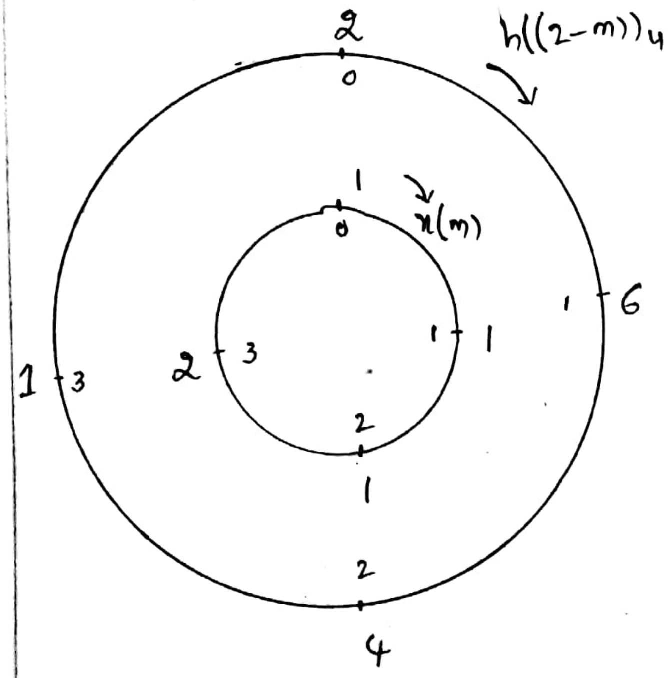


$y(1) = 1\times6 + 1\times4 + 1\times1$
$\quad + 2\times2$
$\quad = 6 + 4 + 1 + 4$

$\underline{\underline{y(1) = 15}}$

$y(2) = ?$

$$y(2) = \sum_{m=0}^{3} x(m)\, h((2-m))_4$$

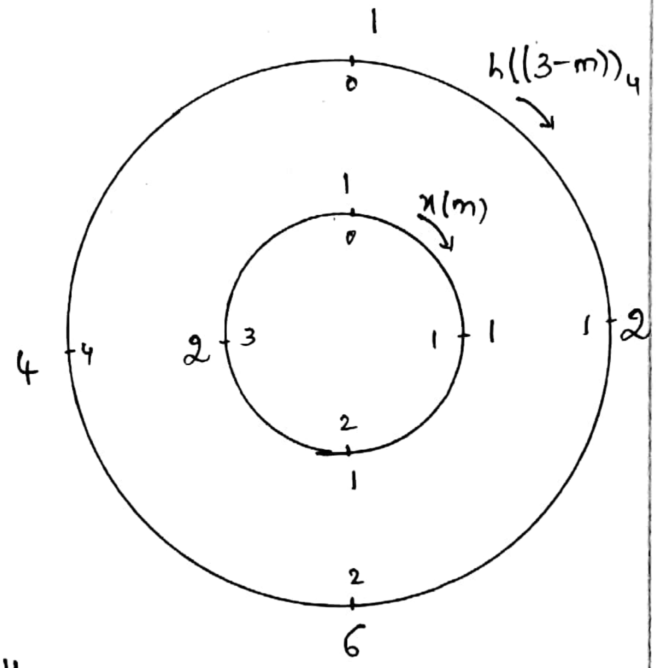


$y(2) = 1\times2 + 1\times6 + 1\times4$
$\quad + 2\times1$
$\quad = 2 + 6 + 4 + 2$

$\underline{\underline{y(2) = 14}}$

$y(3) = ?$

$$y(3) = \sum_{m=0}^{3} x(m)\, h((3-m))_4$$



$y(3) = 1\times1 + 1\times2 + 1\times6$
$\quad + 2\times4$

$y(3) = 1 + 2 + 6 + 8$

$\underline{\underline{y(3) = 17}}$

$y(n) = (\underset{\uparrow}{19}, 15, 14, 17)$

21) Consider a length - 12 sequence $x(n)$ whose 12 - point DFT is given by - $X(K)$. Sequence -

$$x(n) = (\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11).$$

Evaluate the following functions of $X(K)$ without computing DFT.

a) $X(0)$ b) $X(6)$ c) $\sum_{K=0}^{11} X(K)$

d) $\sum_{K=0}^{11} e^{-j\frac{4\pi}{6}K}$ e) $\sum_{K=0}^{11} |X(K)|^2$

Soln : Part (a) : WKT, $X(K) = \sum_{n=0}^{11} x(n) W_{12}^{kn}$ —— ①

Letting $k=0$, we get,

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$$X(0) = \sum_{n=0}^{11} x(n)$$

$$= 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11.$$

$$= \underline{\underline{66}}$$

Part (b) : Letting $k=6$ in eq. ①, we get -

$$X(6) = \sum_{n=0}^{11} x(n) \cdot W_{12}^{6n}$$

$$= \sum_{n=0}^{11} x(n) \cdot e^{-j\frac{2\pi}{12}(6n)} = \sum_{n=0}^{11} x(n) \cdot e^{-j\pi n}$$

$$= \sum_{n=0}^{11} (-1)^n \cdot x(n).$$

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$= 0-1+2-3+4-5+6-7+8-9+10-11$

$X(6) = \underline{\underline{-6}}$

Part (c) : $x(n) \triangleq \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, W_{12}^{-Kn}$

Letting $n=0$, we get,

$$x(0) = \frac{1}{12}\sum_{K=0}^{11} X(K)$$

$$\Rightarrow \sum_{K=0}^{11} X(K) = 12\,x(0) = 12(0) = 12(0) = \underline{\underline{0}}.$$

Part (d) : $\text{IDFT}\left\{e^{-j\frac{4\pi K}{6}} X(K)\right\} = \text{IDFT}\left\{e^{-j\frac{2\pi}{12}\times 4K} X(K)\right\}$

$$= \text{IDFT}\left\{W_{12}^{4K} X(K)\right\}$$

Recall : $\text{DFT}\left\{x((n-m))_N\right\} = W_N^{Km} X(K)$

$$\text{IDFT}\left\{e^{-j\frac{4\pi K}{6}} X(K)\right\} = x((n-4))_{12} \quad \text{—— (1)}$$

By definition,

$$\text{IDFT}\left\{e^{-j\frac{4\pi K}{6}} X(K)\right\} = \frac{1}{12}\sum_{K=0}^{11} e^{-j\frac{4\pi K}{6}} X(K).\, W_{12}^{-Kn} \quad \text{—— (2)}$$

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Equating (1) and (2), we get -

$$\frac{1}{12}\sum_{K=0}^{11} e^{-j\frac{4\pi K}{6}} X(K) \cdot W_{12}^{-Kn} = x((n-4))_{12}$$

Letting $n=0$, we get

$$\frac{1}{12}\sum_{K=0}^{11} e^{-j\frac{4\pi K}{6}} \cdot X(K) = x((-4))_{12} = x(-4+12) = x(8).$$

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$$\Rightarrow \sum_{k=0}^{n} e^{-j\frac{4\pi k}{6}} x(k) = 12\, x(8) = 12(8) = \underline{96}$$

Part (e) Parseval's Theorem:

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

Proof : $$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) \cdot x^*(n) \quad \text{--- (1)}$$

WKT, $$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot W_N^{-kn}$$

$$\Rightarrow x^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) W_N^{kn} \quad \text{--- (2)}$$

Substituting eqn. (2) in eqn. (1), we get -

$$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) \cdot \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) W_N^{kn}$$

$$\Rightarrow \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) X(k)$$

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$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

$$\Rightarrow \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

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$$\Rightarrow \sum_{K=0}^{N-1} |X(K)|^2 = N \sum_{n=0}^{N-1} |x(n)|^2$$

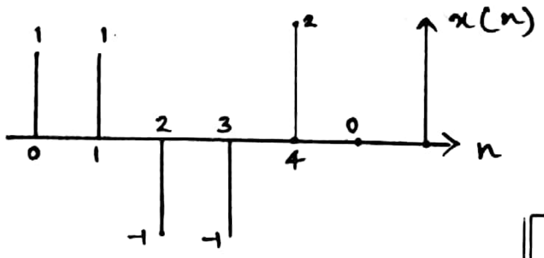
Here, $N = 12$. We get -

$$\sum_{K=0}^{11} |X(K)|^2 = 12 \sum_{n=0}^{11} |x(n)|^2$$

$$= 12 \{0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2\}$$

$$= \underline{\underline{6072}}$$

22) Let $X(K)$ denote a 6-point DFT of a length-6 sequence $x(n)$ shown in figure below. Without computing IDFT, determine the length-6 sequence $g(n)$ whose 6-point DFT is given by $G(K) = W_3^{2K} X(K)$.



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Sol^n: Given, $G(K) = W_3^{2K} X(K)$

$$= e^{-j\frac{2\pi}{3} \times 2K} X(K)$$

$$= e^{-j\frac{2\pi}{6} \times 4K} X(K).$$

$$= W_6^{4K} X(K)$$

$$\therefore g(n) = x((n-4))_6.$$

Here, $x(n) = (\underset{\uparrow}{1}, 1, -1, -1, 2, 0)$

$\therefore g(n) = (\underset{\uparrow}{-1}, -1, 2, 0, 1, 1)$

23) Let $x(n)$ be a real sequence of length - N having an N-point DFT given by $X(K)$.

a) ST $X(N-K) = X^*(K)$

b) ST. $X(0)$ is real.

c) ST. $X\left(\frac{N}{2}\right)$ is real.

d) Check the results of part a, b, c by computing 4-point DFT of a real sequence $x(n) = 2^n$. $0 \le n \le 3$.

Soln : Part (a) : $X(K) \triangleq \sum_{n=0}^{N-1} x(n) W_N^{Kn}$ — ①

$$\Rightarrow X(N-K) = \sum_{n=0}^{N-1} x(n) W_N^{(N-K)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{Nn} \cdot W_N^{-Kn}.$$

Since, $W_N^{Nn} = e^{-j\frac{2\pi}{N} \cdot Nn} = 1$, we get -

$$X(N-K) = \sum_{n=0}^{N-1} x(n) W_N^{-kn}$$

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Since $x(n)$ is a real sequence, we have,

$$x(n) = x^*(n)$$

$$\therefore X(N-K) = \sum_{n=0}^{\text{11}} x^*(n) W_N^{-kn}$$

$$= \left[\sum_{n=0}^{N-1} x(n) W_N^{Kn} \right]^* = X^*(K)$$

Part (b) :

Letting $K=0$ in eqn. ①, we get -

$$X(0) = \sum_{n=0}^{N-1} x(n).$$

Since $x(n)$ is a real sequence, the above summation is real. This means that $X(0)$ is real.

Part (c) : Letting $K = \frac{N}{2}$ in eqn. ①, we get -

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n) W_N^{\frac{N}{2}n}.$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} \cdot \frac{N}{2} \times n}$$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n) (-1)^n$$

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$\because$ $x(n)$ is real, the summation of $x(n)$ with alternating signs is also real. Hence, $X\left(\frac{N}{2}\right)$ is also real.

Part (d) : Given, $x(n) = 2^n$, $0 \le n \le 3$.

$$\Rightarrow x(n) = (\underset{\uparrow}{1}, 2, 4, 8)$$

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$$X(K) \triangleq \sum_{n=0}^{3} x(n)\, W_4^{Kn}$$

$$= 1 + 2W_4^{K} + 4W_4^{2K} + 8W_4^{3K} \qquad K = 0,1,2,3.$$

$$X(0) = 1+2+4+8 = 15.$$

$$X(1) = 1 + 2W_4^1 + 4W_4^2 + 8W_4^3$$

$$= 1 + 2(-j) + 4(-1) + 8(j) = 1-2j-4+8j$$

$$= -3+6j.$$

$$X(2) = 1 + 2W_4^2 + 4W_4^0 + 8W_4^2$$

$$= 1 + 2(-1) + 4(1) + 8(-1) = 1-2+4-8 = -5.$$

$$X(3) = 1 + 2W_4^3 + 4W_4^2 + 8W_4^1 = -3-j6.$$

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K	X(K)	
0	15	$\Rightarrow$ $X(0)$ is real.
1	$-3+6j$	$\Rightarrow$ $X^*(K) = X(N-K)$
2	-5	$\Rightarrow$ $X(2) = X(\frac{N}{2})$ is real.
3	$-3-j6$	$\Rightarrow$ $X^*(K) = X(N-K)$.

$FI = \frac{N}{2} = 2$

24) Consider the following length -8 sequences defined for $n = 0, 1, 2, \ldots 7$.

$x_1(n) = (1, 1, 1, 0, 0, 0, 1, 1)$

$x_2(n) = (1, 1, 0, 0, 0, 0, -1, -1)$

$x_3(n) = (0, 1, 1, 0, 0, 0, -1, -1)$

$x_4(n) = (0, 1, 1, 0, 0, 0, 1, 1)$

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Qn: If $x(n)$ is a 6 point sequence with $X(k)$ as its DFT, without computing IDFT find sequence $y(n)$ whose 6 point DFT is given by $Y(k) = W_6^{5k} X(k)$

Soln

Given: $x(n) \longleftrightarrow X(k)$

$$Y(k) = W_6^{5k} X(k) \longrightarrow (1)$$

Recall: $\mathrm{DFT}\{x((n-m))_N\} = W_N^{km} X(k)$

$$x((n-m))_N = \mathrm{IDFT}\{W_N^{km} X(k)\}$$

with $N = 6,\ m = 5,\ x((n-5))_6 = \mathrm{IDFT}\{W_6^{5k} X(k)\} \longrightarrow (2)$

Taking IDFT of Eqn (1)

$$\mathrm{IDFT}\{Y(k)\} \quad \mathrm{IDFT}\{W_6^{5k} X(k)\} \longrightarrow (3)$$

on comparing eqn (2) & (3)

$$y(n) = x((n-5))_6$$

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$$\Rightarrow \sum_{K=0}^{N-1} |X(K)|^2 = N \sum_{n=0}^{N-1} |x(n)|^2$$

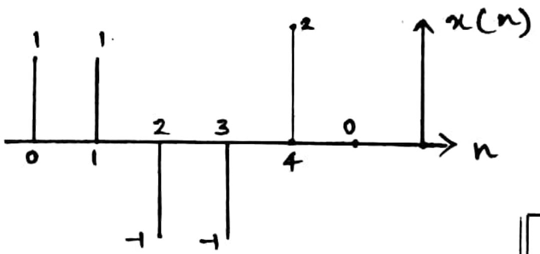
Here, $N = 12$. We get -

$$\sum_{K=0}^{11} |X(K)|^2 = 12 \sum_{n=0}^{11} |x(n)|^2$$

$$= 12 \{0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2\}$$

$$= \underline{\underline{6072}}$$

22) Let $X(K)$ denote a 6-point DFT of a length-6 sequence $x(n)$ shown in figure below. Without computing IDFT, determine the length-6 sequence $g(n)$ whose 6-point DFT is given by $G(K) = W_3^{2K} X(K)$.



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Sol^n : Given, $G(K) = W_3^{2K} X(K)$

$$= e^{-j\frac{2\pi}{3} \times 2K} X(K)$$

$$= e^{-j\frac{2\pi}{6} \times 4K} X(K).$$

$$= W_6^{4K} X(K)$$

$$\therefore g(n) = x((n-4))_6$$

Here, $x(n) = (\underset{\uparrow}{1}, 1, -1, -1, 2, 0)$

$\therefore g(n) = (\underset{\uparrow}{-1}, -1, 2, 0, 1, 1)$

23) Let $x(n)$ be a real sequence of length - N having an N - point DFT given by $X(K)$.

a) ST $X(N-K) = X^*(K)$

b) ST. $X(0)$ is real.

c) ST. $X\left(\frac{N}{2}\right)$ is real.

d) Check the results of part a, b, c by computing 4-point DFT of a real sequence $x(n) = 2^n$. $0 \le n \le 3$.

Soln : Part (a) : $X(K) \triangleq \sum_{n=0}^{N-1} x(n) W_N^{Kn}$ — ①

$$\Rightarrow X(N-K) = \sum_{n=0}^{N-1} x(n) W_N^{(N-K)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{Nn} \cdot W_N^{-Kn}.$$

Since, $W_N^{Nn} = e^{-j\frac{2\pi}{N} \cdot Nn} = 1$, we get -

$$X(N-K) = \sum_{n=0}^{N-1} x(n) W_N^{-Kn}$$

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Since $x(n)$ is a real sequence, we have,

$$x(n) = x^*(n)$$

$$\therefore X(N-K) = \sum_{n=0}^{11} x^*(n) W_N^{-Kn}$$