Design a Butterworth HP filter to meet following specification

i) max passband attenuation = 2dB

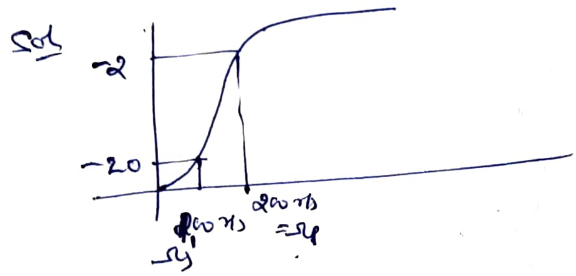
ii) pass band edge freq = 200 r/s.

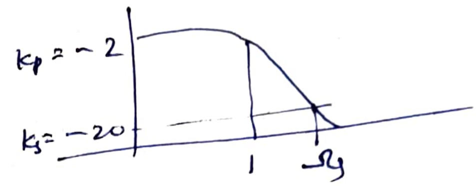
iii) min stopband atten = 20dB

iv) stop band edge freq = 100 r/s.

Prototype

Sol





$$\Omega_s = \frac{\Omega_p}{\Omega_s'} = 2$$

Now $N = \log\left[\frac{10^{-0.1k_p} - 1}{10^{-0.1k_s} - 1}\right] \div 2\log(1/2) = 3.7 = 4$

→ Normalized filter

$$\therefore\ H_4(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$

To find cut off freq (Normalized filter)

$$\Omega_c = \frac{\Omega_p}{\left(10^{-0.1k_p} - 1\right)^{1/2N}} = \frac{1}{\left(10^{-0.1\times -2} - 1\right)^{\frac{1}{8}}} = 1.0693$$

∴ The prototype LPF is obtained by performing LP to LP Tx

$$\text{ie}\quad H_p(s) = H_4(s)\Big|_{s=\frac{s}{\Omega_c}} = \frac{s}{1.06193}$$

⇒ This transformation is done to meet exact specification of required filter ie exact -2dB at $\Omega_p = 200$ r/s.

$$H_p(s) = H_4(s)\Big|_{s=\frac{s}{\Omega_c}}$$

To find HP Filter transform LP proto type to Hp filter of Ω_u ie upper cutoff freq of given desired freq Ω_{c1} = HPF ie $\Omega_u = 200$ r/s.

ie $H'(s) = H_p(s)\Big|_{s=\frac{\Omega_u}{\Omega_c s}}$

$= H_{LP}(s)\Big|_{s=\frac{\Omega_u}{\Omega_c s}} = \frac{200}{1.06935} = \frac{187.031}{s}.$

$$\therefore H'(s) = \frac{s^4}{(s^2+143.146s+34980.7521)(s^2+345.5592s+34980.7521)}$$

Verification

Sub $s = j\Omega$ in $H'(s)$

$$H'(j\Omega) = \frac{\Omega^4}{\left[(34980.7521-\Omega^2)+j143.1464\Omega\right]\left[(34980.7521-\Omega^2)+j(345.5592\Omega)\right]}$$

$H'(j\Omega)\Big|_{\Omega=200} = \cancel{0.7920}\; 0.79433.$

$20\log|H(j\Omega)|_{\Omega=200} = -2\,dB$

$20\log|H(j\Omega)|_{\Omega=100\, r/s} = 20\log 0.081 = -21.83.$

IIly

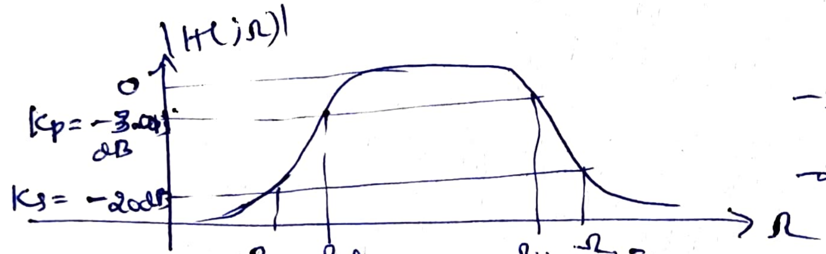
Design an analog BPF to meet given specifications

i) -3.0103 dB upper & lower cutoff freq of 50 Hz & 20 kHz

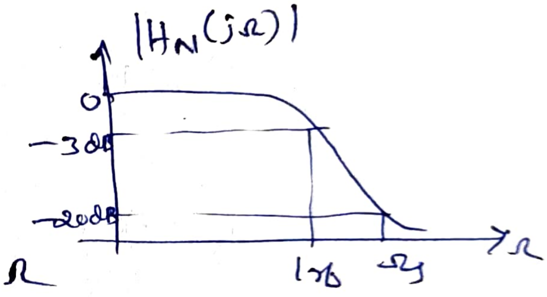
ii) Stop band attenuation of at least 20 dB at 20 Hz & 45 kHz

iii) monotonic freq response.

Sol



$\Omega_1 = 20\text{ Hz} = 40\pi$ r/s, $\Omega_l = 50\text{ Hz} = 100\pi$, $\Omega_u = 20\text{k Hz} = 40\pi \times 10^3$, $\Omega_2 = 45 \times 2\pi = 90\pi$ r/s $\times 10^3$



$\Omega_s = \min(|A|, |B|)$

given $\Omega_1 = 2\pi f_1 = 40\pi$ r/s

$\Omega_l = 2\pi f_l = 100\pi$ r/s

$\Omega_u = 2\pi f_u = 40 \times 10^3 \pi$ r/s

$\Omega_2 = 2\pi f_2 = 90 \times 10^3 \pi$ r/s

$K_p = -3.01$ dB, $K_s = -20$ dB

$\Omega_0^2 = \Omega_l \Omega_u$

$= 4 \times \pi^2 \times 10^6 = 39.478 \times 10^6$

$B_0 = \Omega_u - \Omega_l$

$= 39900\pi$

To find Ω_s

$\Omega_s = \min(|A|, |B|)$

$$A = \frac{-\Omega_1^2 + \Omega_0^2}{\Omega_1 B_0} = \frac{-\pi^2((40)^2 - (4 \times 10^6)^2)}{40\pi \times 39900\pi} = 2.505$$

$$B = \frac{-(\Omega_2^2 + \Omega_0^2)}{\Omega_2 B_0} = \frac{\Omega_2^2 - \Omega_0^2}{\Omega_2 B_0} = \cancel{2.505} / A = 2.25$$

$\therefore \Omega_s = m(|A|, |B|) = 2.25$ r/s

To find order of proto type filter

$$N = \log\left[\frac{10^{-0.1K_p} - 1}{10^{-0.1K_s} - 1}\right] \div 2\log\left(\frac{1}{\Omega_s}\right)$$

$$= 2.83$$

$$\boxed{N = 3}$$

TF of proto type analog filter is

$$H_3(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+2s^2+2s+1}$$

To find analog BPF apply LP to BP transform

ie $H_3'(s) = H_3'(s)\Big|_{s=\frac{s^2+\Omega_0^2}{s(B_0)}} = \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}$

$$H_3'(s) = \frac{1.9695\times10^{15}\,s^3}{s^6+2.571\times10^5 s^5+3.154\times10^{10} s^4+1.989\times10^{15} s^3 + 1.2453\times10^{18} s^2+3.9073\times10^{20} s+6.1529\times10^{22}}$$

IIR filter design using Bilinear Transformation

Design Steps

1) Given the specifications identify the spec.s ie if given freq. are analog convert them to digital freq. using $\omega = \Omega T = \frac{\Omega}{f_s} = \frac{2\pi f}{f_{samp}}$
where f_{samp} is sampling freq & f is specifications f_s & f_p in Hz.

2) Prewarp ω_p & ω_s $\left(\omega_p = \frac{2\pi f_p}{f_{samp}}, \omega_s = \frac{2\pi f_s}{f_{samp}} \right)$
using $\Omega = \frac{2}{T} \tan(\omega/2)$, $T = 1$ sec

3) For given K_p, K_s (A_p, A_s), Ω_p & Ω_s design normalized filter by finding N & Ω_{cn} (based on type of filter)

4) Find desired freq resp. $H_a(s)$ of analog filter.

5) Transform $H_a(s)$ to $H(z)$ using transformation
$$H(z) = H_a(s) \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$
assume $T = 1s$.

Summary

step 1: Identify specifications convert to digital if analog

2: Prewarp

3: Design analog filter $H_a(s)$

4: Transform using bilinear transformation.

IIR filter design using IIT

1) Identify specifications & get k_p, k_s.

If ω_p & ω_s are given convert them to Ω_p & Ω_s using $\Omega = \frac{\omega}{T}$ where T is sampling time. [$T = 1/f_{samp}$.]

(or) If Ω_p & Ω_s are given design analog filter and get TF $H_a(s)$

2) express $H_a(s)$ in the form of

$$H_a(s) = \sum_{i=1}^{N} \frac{c_i}{(s-s_i)}$$ using partial fraction expansion.

3) Replace $\frac{1}{s-s_i}$ by $\frac{1}{1-e^{s_iT}z^{-1}}$

4) Simplify to get $H(z)$.

Design a digital LPF using impulse invariant transform for the following specifications.

$$20\log|H(\omega)|\Big|_{\omega=0.2\pi} \geq -1.9328\,dB$$

$$20\log|H(\omega)|\Big|_{\omega=0.6\pi} \leq -13.9794\,dB.$$

The filter must have maximally flat freq response.

Sol: Given

$\omega_p = 0.2\pi$ rad, $\quad K_p = -1.9328\,dB$

$\omega_s = 0.6\pi$ rad $\quad K_s = -13.9794\,dB.$

Step 1 To design using impulse invariant transform convert given dig edge freq in to analog.

using $\quad \omega = \Omega T_s$

T_s is not given $\therefore$ Take $T_s = 1s.$

$\therefore \quad \omega = \Omega \Rightarrow \Omega_p = \omega_p \quad \& \quad \Omega_s = \omega_s.$

$\therefore$ Analog specifications are

$\Omega_p = 0.2\pi$ r/s $\quad K_p = -1.9328\,dB$

$\Omega_s = 0.6\pi$ r/s $\quad K_s = -13.9794\,dB.$

For monotonic resp design using Butterworth approximation.

step 2: To design $H_a(s)$ for Butterworth approx:

$$N = \log\left(\frac{10^{-0.1K_p}-1}{10^{-0.1K_s}-1}\right) \div \log\left(\frac{\Omega_p}{\Omega_s}\right)$$

$= 1.7$

$\boxed{N = 2}$

$$\theta_k = \frac{\pi}{2} + (2k+1)\frac{\pi}{2N} \cdot \quad k = 0, 1.$$

$$\theta_0 = \frac{3\pi}{4} \qquad \theta_1 = \frac{5}{4}\pi.$$

$$S_0 = (0.707 + j0.707) \quad , S_1 = (-0.707 \mp j0.707).$$

$$H_N(s) = \frac{1}{\prod(s - s_k)} = \frac{1}{(s + 0.707 - j0.707)(s + 0.707 + 0.707j)}$$

Transform $H_N(s)$ to $H_a(s)$ by replacing

$$s = \frac{s}{\Omega_{c1}} \quad , \quad \Omega_{c1} = \frac{\Omega_p}{(10^{0.1 k_p} - 1)^{\frac{1}{2N}}} = 0.7255$$

$$\therefore H_a(s) = \frac{(0.7255)^2}{(s + 0.5129 - j0.5129)(s + 0.5129 + 0.5129j)}$$

Transform to $\sum_{i=1}^{N-1} C_i \cdot \frac{1}{s - s_i}$ using partial fraction expansion.

$$H_a(s) = \frac{A}{(s + 0.5129 - j0.5129)} + \frac{B}{(s + 0.5129 + 0.5129j)}$$

$$A(s + 0.5129 + j0.5129) + B(s + 0.5129 - 0.5129j) = 0.5263$$

at $s = -0.5129 - j0.5129$; $B(1.0258) = -0.5263.$

$$\boxed{B = -0.5130} \qquad \text{III}^{ly} \quad \boxed{A = 0.5130}$$

$$\therefore H_a(s) = \frac{0.5130}{(s + 0.5129 - 0.5129j)} - \frac{0.5130}{(s + 0.5129 + j0.5129)}$$

replacing $\frac{1}{s-s_k}$ by $\frac{1}{1-e^{s_k T_s} z^{-1}}$

$$H_a(z) = \frac{0.5130}{1-e^{(-0.5129+0.5129j)T} z^{-1}} - \frac{0.5130}{1-e^{(-0.5129-0.5129j)} z^{-1}}$$

$$= \frac{0.5130z}{\underset{(z-e^{s_0})}{z-e^{(-0.5129+0.5129j)T_s}}} - \frac{0.5130z}{\underset{(z-e^{s_1})}{z-e^{T_s(-0.5129-j0.5129)}}}$$

$$= \frac{(0.5130z)(z-e^{s_0}) - 0.5130z(z-e^{s_1})}{z^2 - ze^{s_0T_s} - ze^{s_1T_s} + e^{s_0T_s}e^{s_1T_s}}$$

$$= \frac{0.5130z(-e^{s_0}+e^{+s_1})}{z^2 - z(e^{s_0}+e^{s_1}) + e^{(s_0+s_1)T_s}}$$

$$= \frac{0.5130z\left(e^{j(0.5129)T_s} - e^{-j0.5129T_s}\right)e^{-0.5129T_s}}{z^2 - z\left(e^{j0.5129T_s} + e^{-1.0258T_s}\right)e^{-0.5129T_s} + e^{\cdots}}$$

$$= \frac{j0.5130 \times 2ze^{-0.5129T_s}\sin(0.5129T_s)}{z^2 - 2z\cos(0.5129T_s)e^{0.5129T_s} + e^{-1.0258T_s}}$$

$$= \frac{j1.0258e^{-0.5129T_s}\sin(0.5129T_s)}{z^2 - 2ze^{-0.5129T_s}\cos(0.5129T_s) + e^{-1.0258T_s}}$$

with $T_s=1$

$$\boxed{H(z) = \frac{0.3020z}{z^2 - 1.0434z + 0.3585}}$$

(a) using direct Transformation.

ie $$\frac{b}{(s+a)^2+b^2} \overset{IIT}{\longleftrightarrow} \frac{e^{-aT} \sin bT_s z^{-1}}{\left(1-e^{-aT}\cos bT z^{-1} + e^{-2aT} z^{-2}\right)}$$

$$H_N(s) = \frac{1}{(s-s_0)(s-s_1)}$$

Verify Pg 330
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$$s_0 = s_1^*$$

Verification

Let $z = e^{j\omega}$ in $H(z)$ and find magnitude &

$$|H(z)|\Big|_{z=e^{j\omega}=e^{j\omega_p}}$$

$$20\log |H(\omega)|\Big|_{\omega=\omega_p} = -2\,dB$$

$$20\log |H(\omega)|\Big|_{\omega=\omega_s} = -14.4\,dB.$$

Design an IIR digital filter using bilinear trans-formation used in pre filter A/D – H(z) – D/A structure for following specifications.

a) LP filter with –2dB cutoff at 100Hz

b) Stop band attenuation of 20dB or more at 500Hz &

c) Sampling rate of 4000 ~~B/s~~ samples/sec.

Sol: i) Given specifications are

Chebyshev approximation.

$K_p = -2dB$, $f_p = 100Hz$

$K_s = -20dB$, $f_s = 500Hz$

given freq are in Hz $\therefore$ $\Omega_p = 2\pi \times 100 = 200\pi$ r/s

$\Omega_s = 2\pi \times 500 = 1000\pi$ r/s.

step ii) Convert analog specifications in to digital

using $\omega = \frac{\Omega}{f_s} = \Omega T$

f_s is given $= 4000$ s/s. [Note: if f_s is not given

choose $f_s \geq 2f_{max}$ ie $f_s = 2f_{max}$ where f_{max} is

highest freq component in i/p signal]

$$\omega_p = \frac{\Omega_p}{f_s} = \frac{200\pi}{4000} = 0.05\pi \text{ r}$$

$$\omega_s = \frac{\Omega_s}{f_s} = \frac{1000\pi}{4000} = 0.25\pi \text{ r.}$$

step 3: Pre warp the freq. (with T = 1 sec)

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 0.1574 \text{ r/s} \qquad K_p = -2dB$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 0.8284 \text{ r/s} \qquad K_s = -20dB$$

Ex Design filter using bilinear transformation that has following specifications. $f_p = 1000\,Hz$, $f_s = 350\,Hz$, $f_{samp} = 5\,KHz$ monotonic freq resp., $A_s = 10\,dB$, $A_p = 3\,dB$.

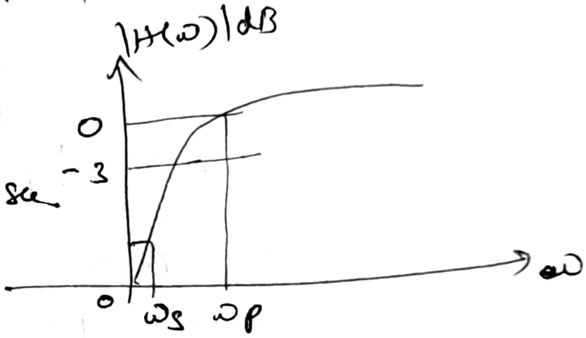
Sol. $f_p = 1000\,Hz$, $f_s = 350\,Hz \Rightarrow$ HPF.

Step 1) Given spec. are in analog domain.

$\therefore$ Converting freq to digital $\omega_p = \frac{2\pi f_p}{f_{samp}}$.

$\omega_p = \frac{2000\pi}{5000}$ |||ly $\omega_s = \frac{2\pi f_s}{f_{samp}} = \frac{700\pi}{5000}$

$\omega_p = 0.4\pi$ rad, $\omega_s = 0.14\pi$ rad



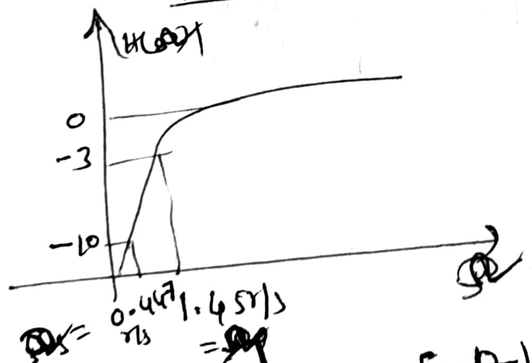
Step 2: Prewarping with T = 1 sec

$\Omega_p = \frac{2}{T} \tan\left(\omega_p/2\right)$

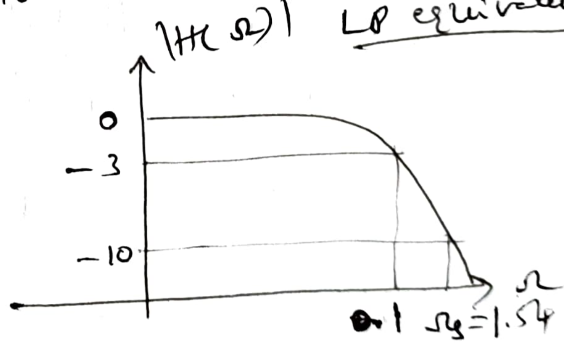
$\Omega_p = 1.453\,r/s$ |||ly $\Omega_s = \cancel{0.4470}\ 0.9411\ r/s$

Step 3: Design analog filter

given filter



|H(Ω)| LP equivalent



$$N = \log\left[\frac{10^{0.1A_p} - 1}{10^{0.1A_s} - 1}\right] \div 2\log\left(\frac{1}{\Omega_s}\right)$$

$= 2.49 \cong 3$

$\boxed{N = 3}$