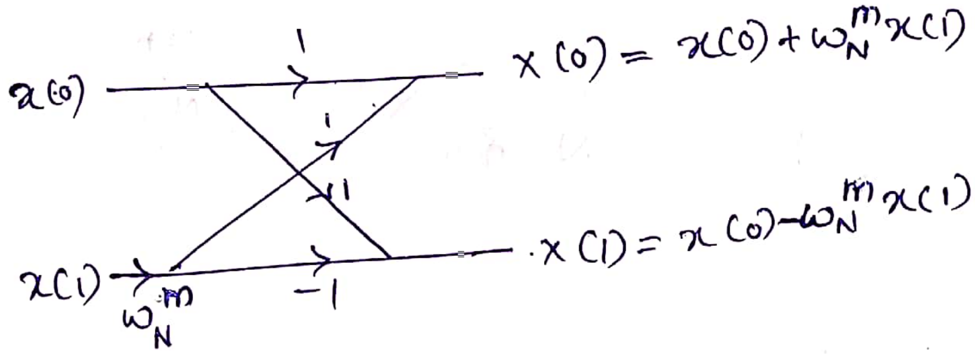
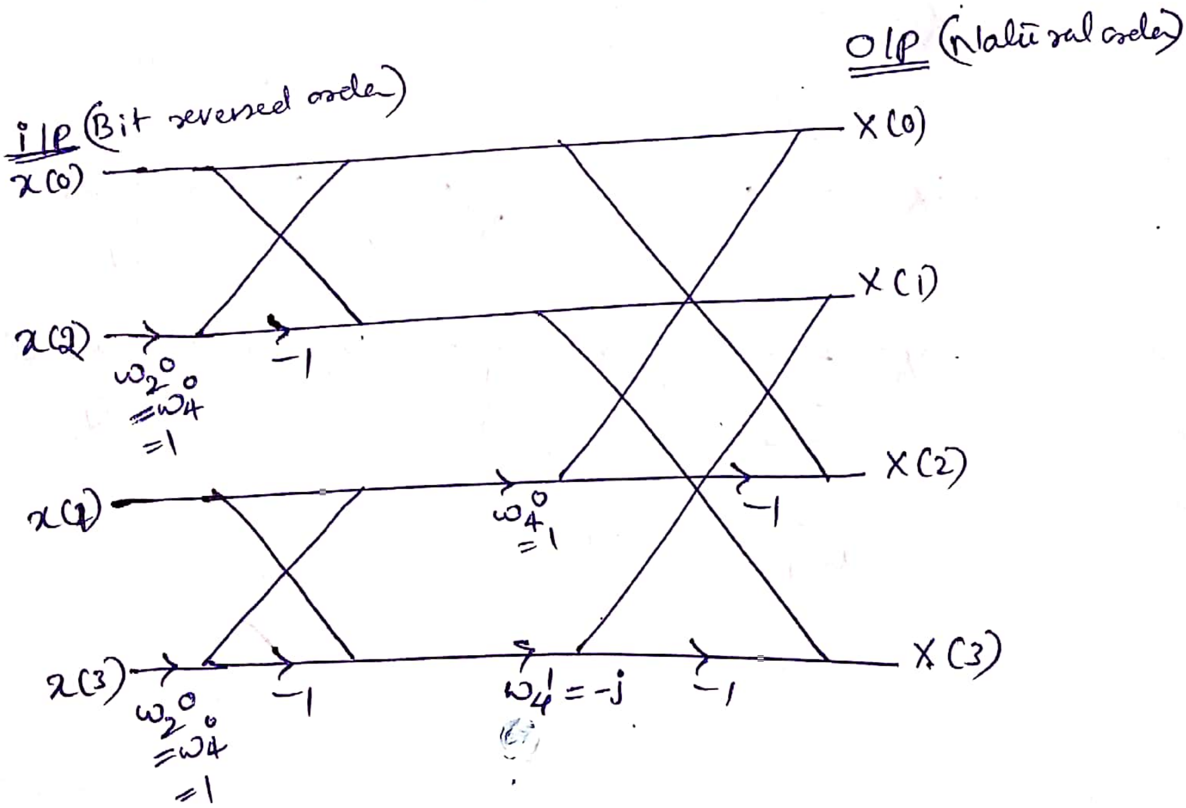
DIT-FFT algorithm Butterfly structures

For Two-Point computation



For 4-point computation



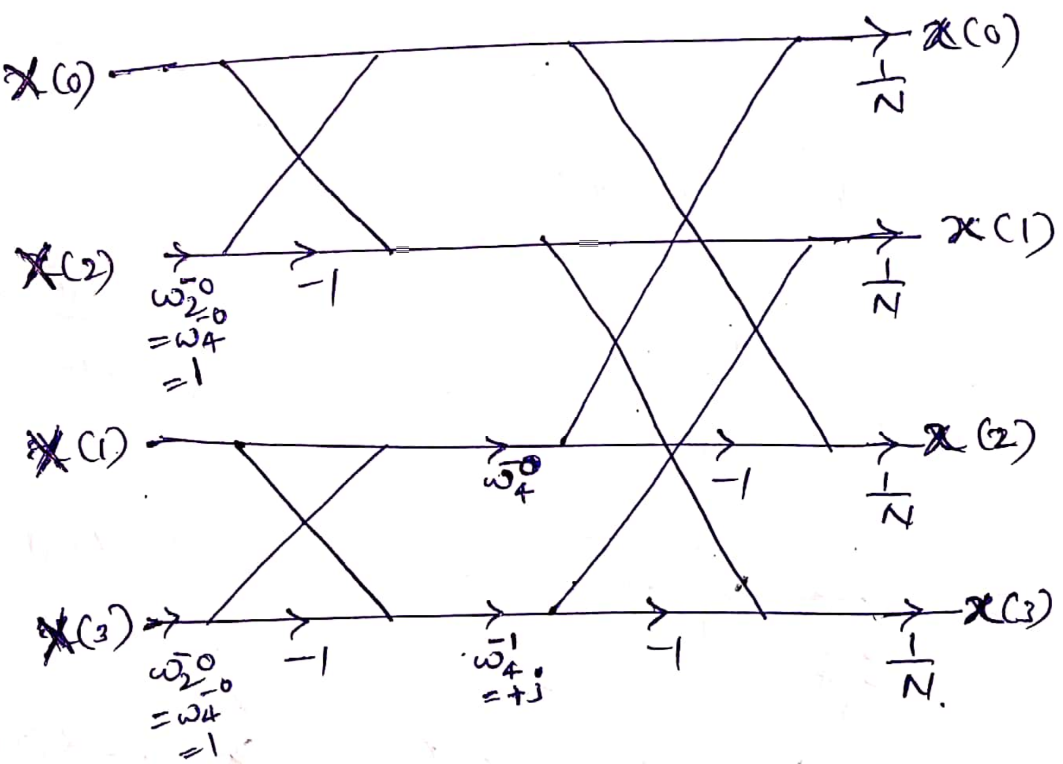
$W_4^0 = 1$

$W_4^1 = -j$

4. Point IDFT Computation

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, W_N^{-kn}$$

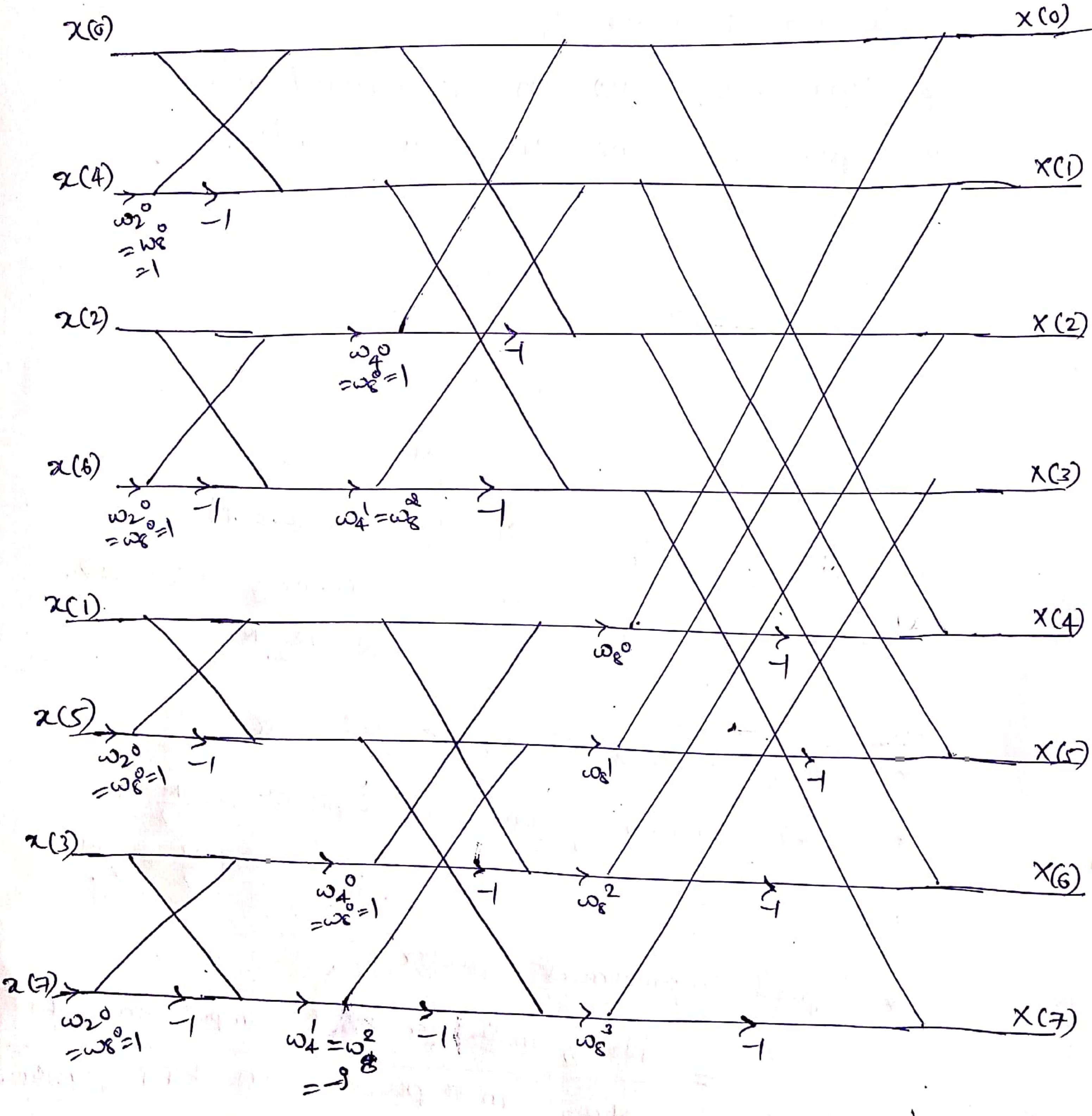
i/ps are X(K) o/ps are x(n)



$W_4^{-0} = W_4^0 = 1$

$W_4^{-1} = j$

8-Point computation

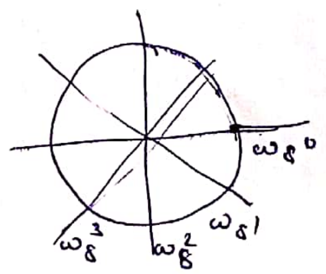


$$W_8^0 = 1$$

$$W_8^1 = +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} = 0.7071 - j0.7071$$

$$W_8^2 = -j$$

$$W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} = -0.7071 - j0.7071$$



8-point IDFT computation

* Replace all W_N^{m} by W_N^{-m} & multiply all o/p lines by $\frac{1}{N}$.
* i/ps are $X(K)$ in bitreversed order
* o/ps are $x(n)$ in natural order.

Number of computations

For N-point DFT.

* No of stages $= \nu = \log_2 N$
* No of complex additions in each stage $= N$
* Total no of complex additions $= N \cdot$ no of stages $= N\nu$.
$$= N \log_2 N$$

* No of complex multiplications in each stage $= \frac{N}{2}$
* Total number of complex multiplication $= \frac{N}{2}\nu$
$$= \frac{N}{2} \log_2 N.$$

* Speed improvement factor.

$$= \frac{\text{No of mults for Direct computation of DFT}}{\text{No of multiplications in FFT algoritm.}}$$

$$= \frac{N^2}{\frac{N}{2}\log_2 N.}$$

Speed improvement factor can also be expressed in terms of total number of computations as

Speed improvement factor =

$$\frac{\text{Total No of computations using DFT}}{\text{Total no of computations using FFT}}$$

$$= \frac{(\text{No of additions} + \text{No of multiplications})\text{ using DFT}}{(\text{No of additions} + \text{No of multiplications})\text{ using FFT}}$$

$$= \frac{N(N-1) + N^2}{N\log_2 N + \frac{N}{2}\log_2 N}$$

$$= \frac{N[2N-1]}{N\left[\log_2 N + \frac{1}{2}\log_2 N\right]}$$

$$= \frac{2N-1}{\frac{3}{2}\log_2 N}$$

$$= \frac{2(2N-1)}{3\log_2 N}.$$

To find no of real multiplications & additions in FFT algorithms & Direct computation.

W.K.T One complex multiplication needs
→ 4 real multiplications and
→ 3 real additions.

- One complex addition needs.
→ 2 real additions.

∴ Total number of additions & mults in computing N-DFT using direct method are

mults → $N^2 \times 4$.

additions → $2 \times N(N-1) + 3N^2 (\text{in mults}) = 5N^2 - 2N$

∴ Total computations $= 4N^2 + 5N^2 - 2N$

$= 9N^2 - 2N$.

III^ly using FFT algorithms

Mults $= 4 \times \frac{N}{2} \log_2 N = 2N \log_2 N$

adds $= 2 \times N \log_2 N + 3\frac{N}{2} \log_2 N = \frac{7N}{2} \log_2 N$

∴ speed improvement factor

$$= \frac{9N^2 - 2N}{\frac{7}{2} N \log_2 N}$$

$$= \frac{9N - 2}{\frac{7}{2} \log_2 N}$$ //.

1) Find 4 point DFTs of using DITFFT algorithm

i) $x_1(n) = \{1 \ -1 \ 2 \ -2\}$

ii) $x_2(n) = \{1/2, 1/3, 3/2, 1/5\}$.

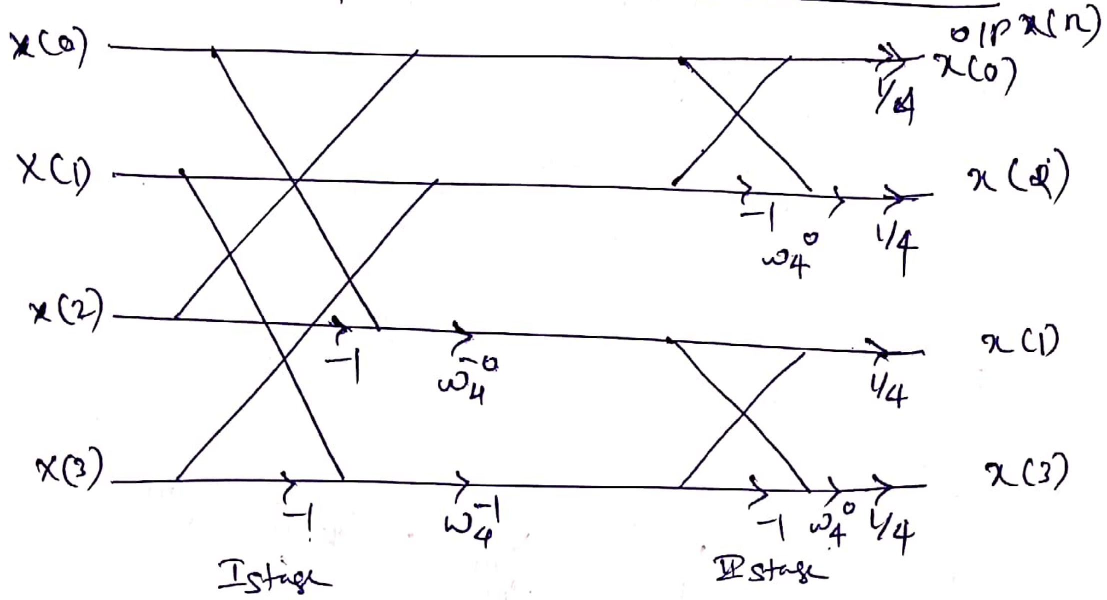
iii) $x_3(n) = \{1, -1\}$

2) Find 8-point DFT of $x(n) = \{1, 2, -1, 2, 3\}$ using DITFFT algorithm and verify by finding IDFT of the same using DIT-IFFT algorithm.

3) Find response of system whose impulse response is $h(n) = \{1 \ 2 \ 1\}$ for the i/p $x(n) = \{1, 2\}$ using DIT-FFT algorithm

i/p X(k)

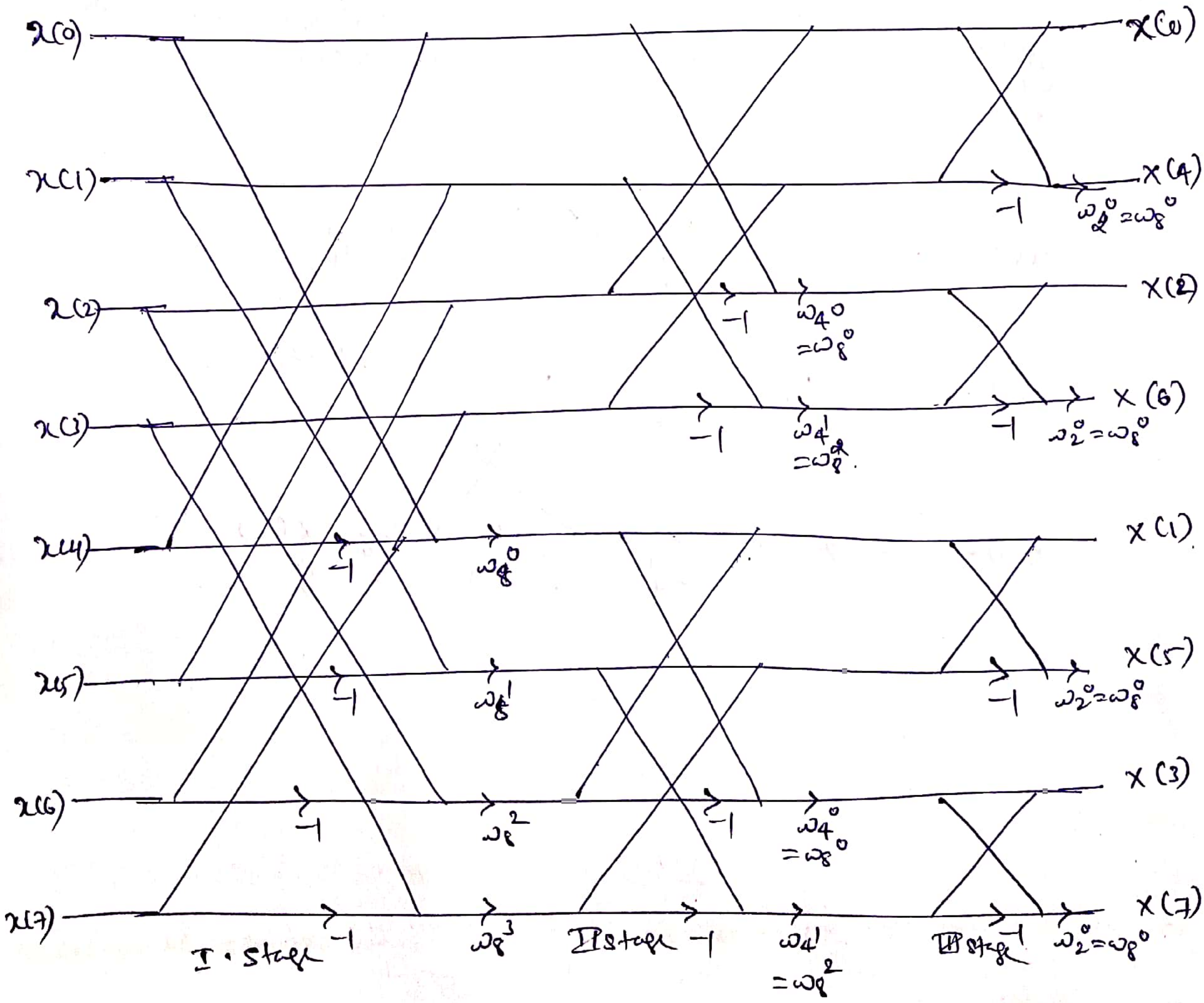
4-point IDFT computation



8-point computation

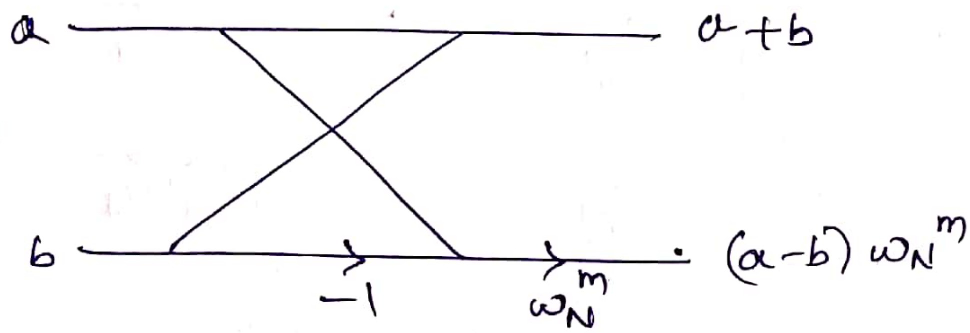
i/p x(n)

o/p X(k)



Radix-2 DIF-FFT Algorithms
Butterfly structures

2-point computation



4-point computation