Subject Name:
DIGITAL COMMUNICATION SYSTEM
Subject code:
1DS5DCDCS

Module 1

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Class and Section : 5^{th} C

Purpose of communication system

- To transport an information-bearing signal from a source to a user destination via a communication channel.
- Basically, a communication system is of an analog or digital type.

Why growth of digital communications is large??

Sources and Signals

- A source of information generates a message
  - Human voice, television picture, teletype data, atmospheric temperature, pressure etc.
- The message signal can be of analog or digital.
- Analog to digital conversion is peformed by combining three basic operations;
  - Sampling
  - Quantizing
  - Encoding

Digital communication system

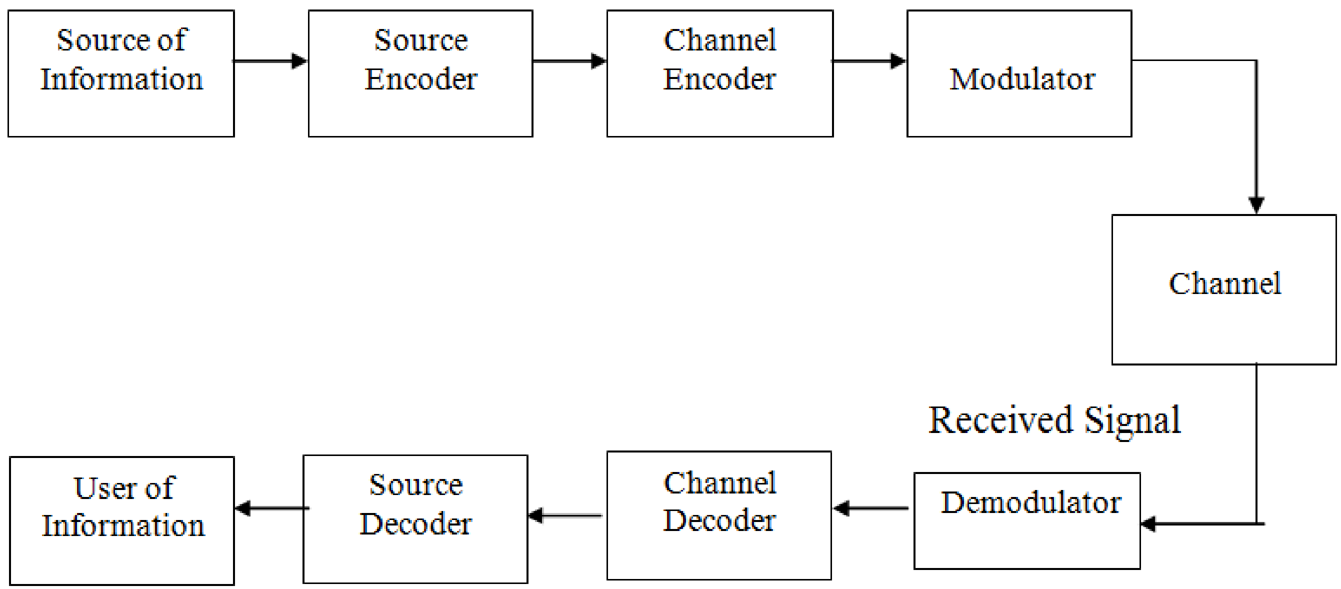


Fig 1: Block diagram

- Information Source:
  - Can be classified into Analog information source and discrete information sources.
  - Ex for analog: micophone actuated by speech.,
  - Ex for discrete: Teletype or numerical output of a computer consists of a sequence of discrete symbols or letters.
- Source Encoder :
  - Converting the output of whether analog or digital source into a sequence of binary digits.
- Channel encoder:
  - Introduce some redundancy in the binary information sequence. These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits.

- <u>Channel:</u>
  - Provides the electrical connection between the source and destination.Ex: Pair of wires, Coaxial cable, Optical fibre, Radio channel, Satellite channel or combination of any of these.
- <u>Modulator:</u>
  - It operates by keying shifts in the amplitude, frequency or phase of a sinusoidal carrier wave to the channel encoder output.
  - The digital modulation techniques are referred to as amplitude- shift keying, frequency- shift keying or phase-shift keying respectively.
  - Converts the input bit stream into an electrical waveform suitable for transmission over the communication channel.
- <u>Detector:</u>
  - The detector performs demodulation, thereby producing a signal the follows the time variations in the channel encoder output.

Review of probobality theory

- Deals with result of an experiment whose actual outcome is not known in advance.
- Mathematical approach used for studying the outcomes of such experiments is called " probabality theory"
- Ex: In the example of tossing of a coin, it is impossible to predict the outcome of the experiment with certainity.
- Hence 'chance' of getting an unexpected outcome may be defined as 'Probabality'

Information source

- The discrete information source consist of a discrete set of letters or symbol.
- a message emitted by a source consist of sequence of symbols
- Every message coming out of the source contains some information.
- However the information contains by the message varies from one another. Some message convey more information than others.
- In order to quantify the information contain in the message, it is necessary to define a measure of information.

Discrete Information source

- Discrete information source are characterized by

i) Source alphabet

Ex: a teletype having 26 letters of English alphabet plus several special characters such as full stop, comma etc, along with numericals.

ii) Symbol rate

- Rate at which the teletype produces characters. Ex- if the teletype operates at the speed of 10 characters/sec, then symbol rate is said to be 10 symbols/sec.

iii)Source alphabet probobalities; and **iv) probobalistic dependence of symbols in a sequence**

- Structural properties of symbol sequences can be characterized by probobality of occurance of individual symbols.

- In the block diagram of information system,
- Assume that discrete information source emitting discrete message symbols $s_{1,}$ $s2,$ $S_{3,\ldots\ldots}$ S_q with probobalities $p_{1,}$ $P2,\ldots\ldots$ P_q respectively.
- Then

$$P_1 + P_2 + P_3 + \ldots\ldots P_q = 1$$

$$\boxed{\sum_{i=1}^{q} p_i = 1}$$

Measure of information

- Let S_k be a symbol choosen for transmission at any instant of time with probobality equal to P_k .
- Then amount of information of message is given by
  - $$I_k = \log(1/P_k)$$
  - If base of logarithm is 2, units are Bits,
  - If base of logarithm is 10, units are Hartleys or Decits
  - If base of logarithm is e , units are NATS

Concept of amount of information

Example:

To know weather at chennai(peak winter time)

1. Sun will rise in east on the day of trip.
2. It will be a very cold day.
3. There will be snowfall on that particular day in chennai.

Conclusion: The message associated with an event least likely to occur contains more information.

Discrete memoryless source

- A source in which symbols emitted during successive signalling intervals are statistically independent.

Entropy

- Average information content per source symbol.
- Consider source alphabet $S = \{s_1, s_2, s_3, \ldots\ldots\ldots\ldots s_q\}$
- With probobalities $P = \{p_1, p_2, p_3, \ldots\ldots\ldots\ldots p_q\}$
- Consider long independent sequence of length L symbols. This long sequence then contains

p_1L number of messages of type s_1

p_2L number of messages of type s_2........................

pqL number of messages of type s_q

.

- We know that amount of information of $s_1 = \log(1/p_1)$ bits

∴ p_1L number of messages of type s_1 contain $p_1L \log(1/p_1)$ bits of information.

p_2L number of messages of type s_2 contain $p_2L \log(1/p_2)$ bits of information.

p_qL number of messages of type s_q contain $p_qL \log(1/p_q)$ bits of information.

∴ $\mathbf{I_{total} = p_1L \log(1/p_1) + p_2L \log(1/p_2) + \ldots\ldots + p_qL \log(1/p_q)}$ **bits**

$$= L\sum_{i=1}^{q} p_i \log\frac{1}{p_i}$$

$$\frac{I_{total}}{L}$$

- Average self-information H(S)

$$= \frac{I_{total}}{L}$$

$$= \sum_{i=1}^{q} p_i \log \frac{1}{p_i} \qquad \text{bits/message symbol}$$

Information Rate

- Let us suppose that the symbols are emitted by the source at a fixed time rate "$\mathbf{r_s}$"symbols/sec
- The average information rate $\mathbf{R_s}$ in bits/sec is defined as the product of the average information content per symbol(H(S)) and the message symbol rate $\mathbf{r_s}$

$\therefore$ $\mathbf{R_s = r_s\, H(S)}$ bits/sec or BPS

Numericals

- 1]Consider a binary source with source alphabet S= {s_1, s_2} with probobalities P={1/256, 255/256}.Find a)self-information of each message b)entropy.

Given : $S = \{S_1, S_2\}$

$P = \left\{\frac{1}{256}, \frac{255}{256}\right\}$

$\therefore$ entropy $H(S) = \sum_{i=1}^{2} p_i \log \frac{1}{p_i}$

$= \frac{1}{256} \log 256 + \frac{255}{256} \log \frac{256}{255}$

$= 0.037$ bits/message symbol

2]Consider a binary source with source alphabet $S^1= \{s_3, s_4\}$ with probobalities

$P^l=\{7/16, 9/16\}$.Find

a)self-information of each message

b)Entropy

3]Consider a binary source with source alphabet $S^{11}= \{s_5, s_6\}$ with probobalities

$P^{ll}=\{1/2, 1/2\}$.Find a)self-information of each message b)entropy

4]Consider a binary source with source alphabet $S= \{s_1, s_{2,} \ s_3\}$ with probobalities

$P=\{1/2, 1/4, 1/4\}$.Find a)self-information of each message b)entropy of source S

5] The collector voltage of certain circuit is to lie between -5 and -12 volts. The voltage can take on only these values -5,-6,-7,-9,-11,-12 volts with respective probobalities 1/6, 1/3,1/12,1/12,1/6,1/6. This voltage is recorded with a pen recorder. Determine the average self-information associated with the record in terms of bits/level.

6] A discrete source emits one of six symbols once every m-sec. The symbol probobalities are 1/2, ¼, 1/8, 1/16, 1/32 and 1/32 respectively. Find the source entropy and information rate.

7]Find relationship between Hartleys, Nats, and bits.

8] The output of an information source consists of **150** symbols, **32** of which occur with a probobality of **1/64** and the remaining **118** occur with a probobality of **1/236**. The source emits **2000** symbols/sec. Assuming that the symbols are choosen independently , find the average information rate of this source.

[note: The solution pdf is attached in google classroom, and also solved during online classes]

9] A code is composed of dots and dashes. Assuming that a dash is 3 times as long as a dot and has one-third the probobality of occurence. Calculate

(i) The information in a dot and dash

(ii) The entroy of dot-dash code

(iii) The average rate of information if a dot lasts for 10 m-sc and this time is allowed between symbols.

Solution:

Since only dots and dashes are present, we must have

$P_{dot} + P_{dash} = 1$

Given $P_{dash} = 1/3 P_{dot}$

$\therefore P_{dot} + \frac{1}{3} P_{dot} = 1 \quad \therefore P_{dot} = 3/4$

$P_{dash} = 1/4$

(i) Information in a dot $= I_{dot} = \log \frac{1}{P_{dot}} = \log 4/3 = \underline{0.415 \text{ bits}}$

Information in a dash $= I_{dash} = \log \frac{1}{P_{dash}} = \log 4 = \underline{\underline{2 \text{ bits}}}$

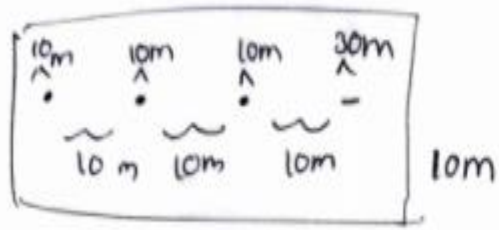
(ii) Entropy of dot dash code

$$H(S) = P_{dot} \log \frac{1}{P_{dot}} + P_{dash} \log \frac{1}{P_{dash}}$$

$$= 3/4 \log 4/3 + 1/4 \log 4$$

$$H(S) = \underline{0.8113} \text{ bits/message-symbol}$$

(iii)



Symbol rate $= r_s = 4$ symbols/100 m-sec

$= 40$ symbols/sec

$\therefore R_s = r_s H(S)$

$= (40$ symbols/sec$)(0.8113$ bits/symbol$)$

$= 32.452$ bits/sec

Source Encoding

- Source coding converts each symbols into codewords.
- Is the process by which the output of an information source is converted into a r-ray sequence.
- ‘r’ is the number of different symbols used in this transformation process.
- If r=2, then the output is a binary sequence,
- r=3, then the output is a ternary sequence,
- r=4, then the output is quaternary sequence

Fixed length code

- Consider size of source alphabet is 4;

$S=\{s_1, s_{2,}\ \ s_{3,}\ \ s_4\ \ \}$

Source symbols	Code
s_1	11
s_2	10
s_3	01
s_4	00

Its fixed length code.
Hence its uniquely decodable.

If we have 100100....
10- s_1
01- $s_{2,}$
00- s_3

$\mathbf{s_1 s_{2,} s_3}$

Variable length code(VLC code)

Source symbols	Code	No. of bits used
s_0	0	1-bit
s_1	1	1-bit
s_2	00	2-bit
s_3	11	2-bit

If we have 000....
May be correspond to 0 followed by 00 or 00 followed by 0. Hence can be decoded as

$\mathbf{S_0 s_2}$ **or** $\mathbf{s_{2,} s_0}$

Here, code is not uniquely decodable.

Variable length code(VLC code)

Source symbols	Code	No. of bits used
s_0	1	1-bit
s_1	01	2-bit
s_2	001	3-bit
s_3	000	3-bit

If we have 1001….

Can be decoded as

$\mathbf{S_0 s_2}$

Here, code is uniquely decodable

.

VLC code is prefix-free code
Also known as instantaneous code

Shannon's Encoding algorithm

Step1: List the source symbols in the order of non-increasing probobalities

Step2: Compute the sequences

$\alpha_1 = 0$

$\alpha_2 = p_1 = p_1 + \alpha_1$

$\alpha_3 = p_2 + p_1 = p_2 + \alpha_{12}$ ....

$\alpha_{q+1} = p_q + p_{q-1} + \ldots\ldots + p_2 + p_1 = p_q + \alpha_1$

Step3: Determine smallest integer value of l_i using the inequality

$$2^{l_i} \geq \frac{1}{p_i} \quad \text{for all } i=1,2,\ldots\ldots q$$

Step4: Expand the decimal number α_i in binary from upto l_i places neglecting expansion beyond l_i places

Step5: Remove the binary point to get the desired code.

1]Construct Shannon's binary code for the following message symbols $S=\{s_1, s_{2,}\ \ s_{3,}\ \ s_4\ \}$ with probobalities P=(0.4,0.3,0.2,0.1)

Solution:

Step1; 0.4>0.3>0.2>0.1

Step2: $\alpha_1\ =0$

$\alpha_2 = p_1 = 0.4$

$\alpha_3 = p_2 + \alpha_2\ = 0.3+0.4=0.7$

$\alpha_4\ = p_3 + \alpha_3\ = 0.2+0.7=0.9$

$\alpha_5 = p_4 + \alpha_4 = 0.1+0.9=1$

Step3: The smallest integer value of l_i is found using $2^{l_i} \geq 1/\ p_i$ for all i=1,2,...5

For i=1; $2^{l_1} \geq 1/o.4$

$2^{l_1} \geq 2.5$; hence $l_1 = 2$

For i=2; $2^{\ l_2} \geq 1/o.3$

≥ 3.333 ; hence $l_{2=}\ 2$

For i=3; $2^{l_3} \geq 1/o.2$

≥ 5 ; hence $l_3 = 3$

For i=4; $2^{l_4} \geq 1/o.1$

≥ 10 ; hence $l_4 = 4$

Step4: The decimal numbers α_i are expanded in binary form upto l_i places.

Step4: : α_1 $=0.\underline{00}00$ $(l_1 = 2)$

$\alpha_2 = 0.4 = 0.\underline{01}10$ $(l_2 = 2)$

$\alpha_3 = 0.\underline{101}10$ $(l_3 = 3)$

α_4 $= 0.\underline{1100....}$ $(l_4 = 4)$

0.4×2
$0.8 \times 2 \rightarrow 0$
$1.6 \rightarrow 1$
0.6×2
$1.2 \rightarrow 1$
0.2×2
$0.4 \rightarrow 0$
$\Rightarrow \underline{0.0110\ldots}$

α_2

0.7×2
$1.4 \rightarrow 1$
0.4×2
$0.8 \times 2 \rightarrow 0$
$1.6 \rightarrow 1$
$0.6 \times 2 \rightarrow 1$
1.2
0.2×2
$0.4 \times 2 \rightarrow 0$
$0.8 \rightarrow 0$
$\Rightarrow \underline{0.101100\ldots}$

α_3

0.9×2
$1.8 \rightarrow 1$
0.8×2
$1.6 \rightarrow 1$
$0.6 \times 2 \rightarrow 1$
1.2
$0.2 \times 2 \rightarrow 0$
0.4×2
$0.8 \times 2 \rightarrow 0$
$\Rightarrow \underline{0.11100\ldots}$

α_4

Source symbols	P_i	α_i	α_i in binary	l_i	Code
s_1	0.4	0	$(0.00000...)_2$	2	00
s_2	0.3	0.4	$(0.0110....)_2$	2	01
s_3	0.2	0.7	$(0.10110...)_2$	3	101
S_4	0.1	0.9	$(0..1101..)_2$	4	1100

Average length of this code is $L=\Sigma\, p_i\, l_i$ Bits/message symbol

L= 2x0.4+2x0.3+3x0.2+4x0.1= 2.4 binits/message symbol

Entropy ; H(S)=0.4log(1/0.4)+ 0.3log(1/0.3+ 0.2log(1/0.2)+ 0.1og(1/0.1)=1.84644 bits/message symbol

$$\%\eta_c = \frac{H(S)}{L} * 100 = \frac{1.8464}{2.4} * 100 = 76.93\%$$

2]Construct Shannon's binary code for the following message symbols S={s_1, $s_{2,}$ $s_{3,}$ s_4 } with probobalities P=(3/8,1/16, 3/16, ¼, 1/8). Obtain code efficiency and redundancy

step1; 3/8>1/4>3/16>1/8>1/16

Step2: : $\alpha_1 = 0$

$\alpha_2 = p_1 = 3/8 = 0.375$

$\alpha_3 = p_2 + \alpha_2 = (1/4) + (3/8) = 0.625$

$\alpha_4 = p_3 + \alpha_3 = (3/16) + 0.625 = 0.8125$

$\alpha_5 = p_4 + \alpha_4 = (1/8) + 0.8125 = 0.9375$

Step3: The smallest integer value of l_i is found using $2^{l_i} \geq 1/p_i$ for all i=1,2,...5

For i=1; $2^{l_1} \geq 1/(3/8)$

$2^{l_1} \geq 2.66$; hence $l_1 = 2$

For i=2; $2^{l_2} \geq 1/(1/4)$

$2^{l_1} \geq 4$; hence $l_1 = 2$

For i=3; $2^{l_3} \geq 1/(3/16)$

≥ 5.33 ; hence $l_{3=}$ 3

For i=4; $2^{l_4} \geq 1/(1/8)$

≥ 8 ; hence $l_{4=}$ 3

For i=5; $2^{l_5} \geq 1/(1/16)$

≥ 16 ; hence $l_{4=}$ 4

$\therefore$ l_i values are 2,2,3,3,4

Step4: **:** The decimal numbers α_i are expanded in binary form upto l_i places.

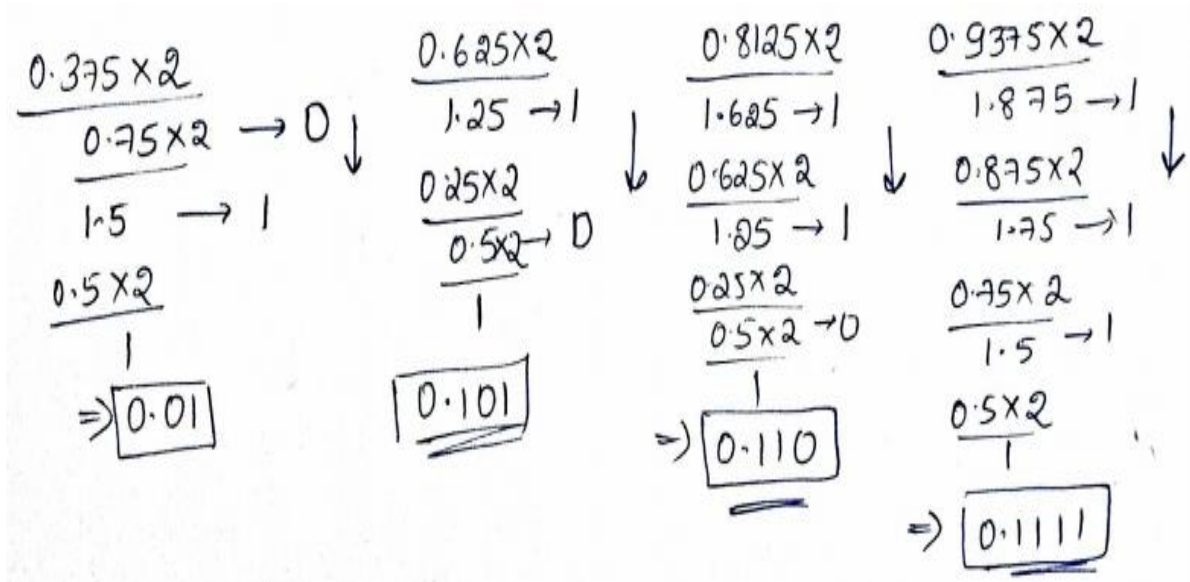
$\alpha_1 = 0.\underline{00}$

$\alpha_2 = 0.\underline{01}$

$\alpha_3 = 0.\underline{101}$

$\alpha_4 = 0.\underline{110}$

$\alpha_5 = 0.\underline{1111}$



S_i	P_i	α_i	l_i	binary	Code
S_1	3/8	0	2	0.00	00
S_2	1/4	0.375	2	0.01	01
S_3	3/16	0.625	3	0.101	101
S_4	1/8	0.8125	3	0.110	110
S_5	1/16	0.9375	4	0.1111	1111

- $H(S) = \frac{1}{4} log_2 4 + \frac{3}{8} log_2 \frac{8}{3} + \frac{1}{8} log_2 8 + \frac{3}{16} log_2 \frac{16}{3} + \frac{1}{4} log_2 4 + \frac{1}{16} log_2 16$
- $H(S) = 2.1085\ bits/symbol$
- $L = \sum_{i=1}^{q} P_i l_i = \frac{1}{4}(2) + \frac{3}{8}(2) + \frac{1}{8}(3) + \frac{3}{16}(3) + \frac{1}{16}(4)$
- $L = 2.4375\ bits$/symbol

- Redundancy?   Example
- Entropy ? significance

3] O.4, 0.25, 0.15, 0.12, 0.08

Solution:

Step1; 0.4>0.25>0.15>0.12>0.08

Step2: $\alpha_1 = 0$

$\alpha_2 = p_1 = 0.4$

$\alpha_3 = p_2 + \alpha_2 = 0.25+0.4=0.65$

$\alpha_4 = p_3 + \alpha_3 = 0.15+0.65=0.8$

$\alpha_5 = p_4 + \alpha_4 = 0.8+0.12=0.92$

$\alpha_6 = p_5 + \alpha_5 = 0.08+0.92=1$

Step3: The smallest integer value of l_i is found using $2^{l_i} \geq 1/p_i$ for all i=1,2,...5

For i=1; $2^{l_1} \geq 1/o.4$

$2^{l_1} \geq 2.5$; hence $l_1 = 2$

For i=2; $2^{l_2} \geq 1/o.25$

≥ 4 ; hence $l_2 = 2$

For i=3; $2^{l_3} \geq 1/o.15$

≥ 6.67 ; hence $l_3 = 3$

For i=4; $2^{l_4} \geq 1/0.12$

≥ 8.33 ; hence $l_4 = 4$

For i=5; $2^{l_5} \geq 1/0.08$

≥ 12.5 ; hence $l_5 = 4$

Step4: The decimal numbers α_i are expanded in binary form upto l_i places.

Step4: : $\alpha_1 = 0.\underline{00}00$ $(l_1 = 2)$

$\alpha_2 = 0.\underline{01}1$ $(l_2 = 2)$

$\alpha_3 = 0.\underline{101}$ $(l_3 = 3)$

$\alpha_4 = 0.\underline{1100}$ $(l_4 = 4)$

$\alpha_5 = 0.\underline{1100}$

Source symbols	p_i	Code
s_1	0.4	00
s_2	0.25	01
s_3	0.15	101
s_4	0.12	1100
s_5	0.08	1110

4]A source emits an independent sequence of symbols from an alphabet consisting of five symbols A, B,C,D and E with probobalities of ¼ , 1/8, 1/8, 3/16, and 5/16 respectively. Find the shanon code for each symbol and efficiency of the coding scheme.

4]Solution:

Step1;	p_1	p_2	p_3	p_4	p_5
	5/16	¼	3/16	1/8	1/8
	E	A	D	B	C

Step2: $\alpha_1 = 0$

$\alpha_2 = p_1 = 5/16 = 0.3125$

$\alpha_3 = p_2 + \alpha_2 = 0.5625$

$\alpha_4 = p_3 + \alpha_3 = 0.75$

$\alpha_5 = p_4 + \alpha_4 = 0.875$

$\alpha_6 = p_5 + \alpha_5 = 1$

Step3: The smallest integer value of l_i is found using $2^{l_i} \geq 1/\ p_i$ for all i=1,2,...6

For i=1; $2^{l_1} \geq 1/(5/16)$

$2^{l_1} \geq 3.2$; hence $l_1 = 2$

For i=2; $2^{l_2} \geq 1/(1/4)$

≥ 4 ; hence $l_2 = 2$

For i=3; $2^{l_3} \geq 1/(3/16)$

≥ 5.33 ; hence $l_3 = 3$

For i=4; $2^{l_4} \geq 1/(1/8)$

≥ 8 ; hence $l_4 = 3$

For i=5; $2^{l_5} \geq 1/(1/8)$

≥ 8 ; hence $l_5 = 3$

Step4: The decimal numbers α_i are expanded in binary form upto l_i places.

Step4: : α_1 =0.000 (l_1= 2)

α_2 =0.3125

0.3125 X 2= 0.625 with carry 0

0.625 x 2= 0.25 with carry 1

0.25x 2 = 0.5 with carry 0

$=(0.\mathbf{01}0)_2$

α_3 =0.100 (l_3= 3)

α_4 =0.11 (l_4= 3)

α_5 =0.111 (l_5=3)

Source symbols	Code	length
E	00	2
A	01	2
D	100	3
B	110	3
C	111	3

$L = \Sigma\, p_i\, l_i = (5/16)(2) + (1/4)(2) + (3/16)(3) + (1/8)3 + (1/8)3 = 2.4375$ binits/ message symbol

$H(S) = \Sigma\, p_i \log(1/p_i)$

$= 2.2272$ bits/message symbol

$n_c = H(S)/L \times 100\%$

91.37%

$Rnc = 1 - n_c = 8.628$

Homework problem

- 5]The source emits the messages consisting of two symbols each. These messages and their probobalities are given in table. Design source encoder using shanon encoding algorithm and also find

Message M_1	Probobality Pi
AA	9/32
AC	3/32
CC	1/16
CB	3/32
CA	3/32
BC	3/32
BB	9/32

Huffman coding

- The source symbols are listed in the decreasing order of probabilities.
- Check if $q = r + a(r-1)$ is satisfied and find the integer 'a', q is number of source symbols and r is number of symbols used in code alphabets.
- If 'a' is not integer, add dummy symbols of zero probability of occurrence.
- Combine the last 'r' symbols into a single composite symbol by adding their probability to get a reduced source.
- Repeat the above three steps, until in the final step exactly ***r***-symbols are left.

Cont...

- The last source with ‘r’ symbols are encoded with ‘r’ different codes 0,1,2,3,….r-1
- In binary coding the last source are encoded with 0 and 1
- As we pass from source to source working backward, decomposition of one code word each time is done in order to form 2 new code words.
- This procedure is repeated till we assign the code words to all the source symbols of alphabet of source ‘s’ discarding the dummy symbols.

1] Construct Huffman code for symbols having {1/2, 1/4 , 1/8, 1/8 }.

- $q=r+\alpha(r-1)$;

  $4=2+\alpha(1)$ => $\alpha=2$ € Z

CODE	Source symbols	P_i	1^{ST} reduction	2^{nd} reduction	
	s_1	0.5	0.5	0.5	
	s_2	0.25	0.25	0.5	
	s_3	0.125	0.25		
	s_4	0.125			

CODE	Source symbols	P_i	1^{ST} reduction	2^{nd} reduction
[0]	s_1	0.5	0.5	0.5
[10]	s_2	0.25	0.25	0.5
[110]	s_3	0.125	0.25	
[111]	s_4	0.125		

symbol	Probobality	code	Length l_i
s_1	1/2	0	1
s_2	1/4	10	1
s_3	1/8	110	3
s_4	1/8	111	3

2]Construct Huffman code for symbols having {0.4, 0.3 , 0.2, 0.1}. Find efficiency and redundancy

- $q = r + \alpha(r-1)$;

$4 = 2 + \alpha(1) \quad \Rightarrow \quad \alpha = 2 € Z$

CODE	Source symbols	P_i	1^{ST} reduction	2^{nd} reduction	
		0.4	0.4	0.6	
	S_2	0.3	0.3	0.4	
	S_3	0.2	0.3		
	S_4	0.1			

CODE			1^{ST} reduction	2^{nd} reduction
	s_1	0.4	0.4	0.6
	s_2	0.3	0.3	0.4
	s_3	0.2	0.3	
	s_4	0.1		

CODE			1^{ST} reduction	2^{nd} reduction
1	S_1	0.4	0.4	0.6 [0]
00	S_2	0.3	0.3 [00]	0.4 [1]
010	S_3	0.2 [010]	0.3 [01]	
011	S_4	0.1 [011]		

symbol	Probobality	code	length
s_1	0.4	1	1
s_2	0.3	00	2
s_3	0.2	010	3
s_4	0.1	011	3

$L=\Sigma\ p_i\ l_i\ = (0.4)\ (1)+(0.3)(2)+\ (0.2)(3)+\ (0.1)3$
$= 1.9$ binits/ message symbol

$H(S)= \Sigma\ p_i \log(1/p_i\ )$
$= 0.4 \log(1/0.4)+0.3 \log(1/0.3)+0.2 \log(1/0.2)+0.1 \log(1/0.1)$
$= 1.846$ bits/message symbol

$n_c = (H(S) / L)$
$=97.15\ \%$

Rnc $= \mathbf{1- n_c = 2.85\ \%}$

- 3] Given the messages x_1 , x_2 , x_3 , x_4 , $x_{5,}$ x_6 with respective probobalities of 0.4,0.2, 0.1, 0.07 and 0.03, construct a binary code by applying Huffman encoding procedure. Determine the efficiency and redundancy of the code so formed.

code	symbol	Probobaliti es	1^{st} reduction	2^{nd} reduction	3^{rd} reduction	4thre ductio n
1	x1	0.4	0.4	0.4	0.4	0.6
01	x2	0.2	0.2	0.2	0.4	0.4
000	x3	0.2	0.2	0.2	0.2	
0010	x4	0.1	0.1	0.2		
00110	x5	0.07	0.1			
00111	x6	0.03				

- Entropy= 2.209 bits/message-symbol
- Average length L = 2.3 binits/message-symbol
- Code efficiency n_c =96.04%
- Code redundancy R_{nc} = 2.96%

(steps with formula to be written)