(1) Properties of Maximum-length Sequences

Maximum-length sequences* have many of the properties possessed by a truly *random binary sequence*. A random binary sequence is a sequence in which the presence of a binary symbol 1 or 0 is equally probable. Some properties of maximum-length sequences are listed below:

PROPERTY 1

In each period of a maximum-length sequence, the number of 1s is always one more than the number of 0s. This property is called the *balance property*.

PROPERTY 2

Among the runs of 1s and of 0s in each period of a maximum-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs. This property is called the *run property*.

By a "run" we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence. The length of this subsequence is the length of the run. For a maximum-length sequence generated by a feedback shift register of length m, the total number of runs is $(m + 1)/2$.

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$\Rightarrow 3 < 4$ ✓

$\Rightarrow \underbrace{00}_{1}\underbrace{111}_{2}\underbrace{0}_{3}\underbrace{1}_{4}$ ✓ $\Rightarrow$ $\frac{N+1}{2}$

$n=1$

$\frac{1}{2^n}(\text{Runs})$

$\Rightarrow \frac{1}{2}(4) = 2 \Rightarrow$ of length 1

PROPERTY 3

The autocorrelation function of a maximum-length sequence is periodic and binary-valued. This property is called the *correlation property*.

Let binary symbols 0 and 1 be represented by -1 volt and $+1$ volt, respectively. By definition, the *autocorrelation sequence* of a binary sequence $\{c_n\}$, so represented, equals

$$R_c(k) = \frac{1}{N}\sum_{n=1}^{N} c_n c_{n-k}$$

where N is the *length* or *period* of the sequence and k is the *lag* of the autocorrelation sequence. For a maximum-length sequence of length N, the autocorrelation sequence is periodic with period N and two-valued, as shown by

$$R_c(k) = \begin{cases} 1 & k = lN \\ -\dfrac{1}{N} & k \neq lN \end{cases}$$

where l is any integer. When the length N is infinitely large, the autocorrelation sequence $R_c(k)$ approaches that of a completely random binary sequence.

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$$\begin{array}{lccccccc} & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ c(n) & -1 & -1 & 1 & 1 & 1 & -1 & 1 \\ & -1 & -1 & 1 & 1 & 1 & -1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$$R_c(0) = \frac{1}{7}\sum_{n=1}^{7} C_n C_n = \frac{1}{7}(7) = 1$$

$k = 1$

$$R_c(1) = \frac{1}{7}\sum_{n=1}^{7} C_n C_{n-1} =$$

$$= \frac{1}{7}(-1) = -\frac{1}{7}$$

$$\begin{array}{ccccccc} -1 & -1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & 1 & -1 \\ \hline -1 & 1 & -1 & 1 & 1 & -1 & -1 \end{array}$$

$0 \quad 0 \quad 0 \quad -1$

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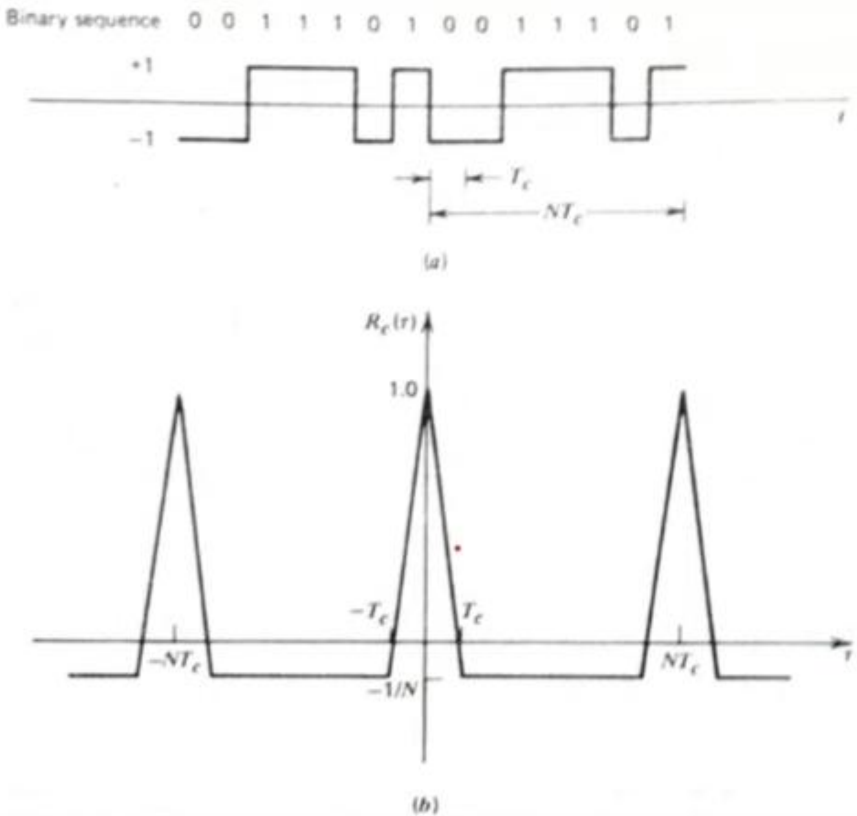

Figure 9.2 (*a*) Waveform of maximum-length sequence. (*b*) Autocorrelation of maximum-length sequence. Both parts refer to the output of the feedback shift register of Fig. 9.1.

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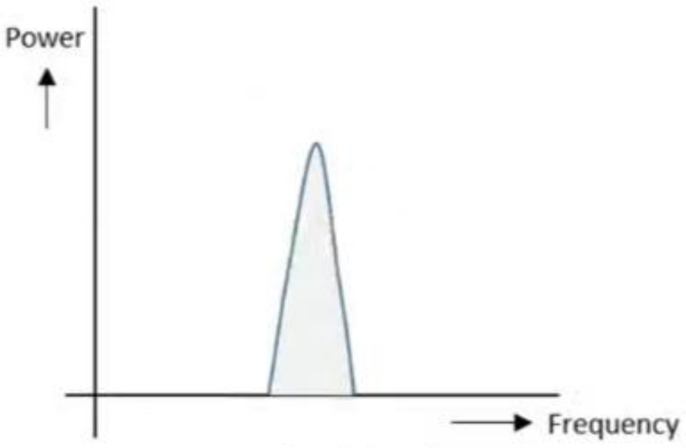
Narrow-band Signals

The Narrow-band signals have the signal strength concentrated as shown in the following frequency spectrum figure.



Narrow-band Signal

Following are some of its features −

- Band of signals occupy a narrow range of frequencies.
- Power density is high.
- Spread of energy is low and concentrated.

Though the features are good, these signals are prone to interference.

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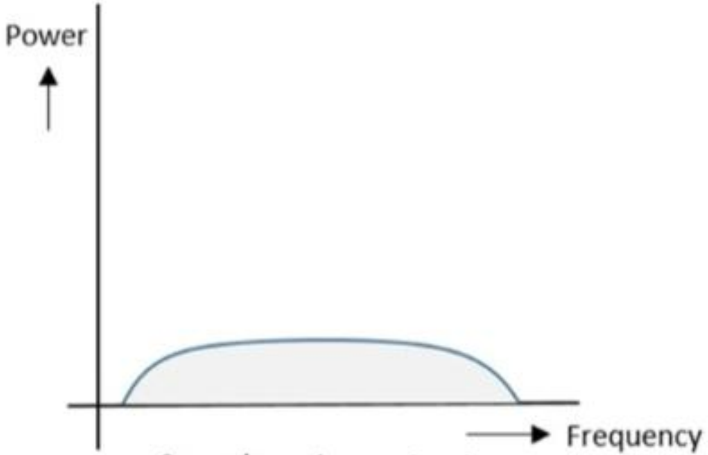
Spread Spectrum Signals

The spread spectrum signals have the signal strength distributed as shown in the following frequency spectrum figure.



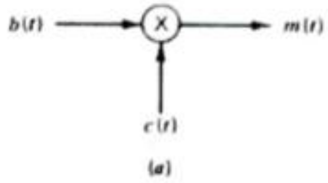
Spread spectrum signal

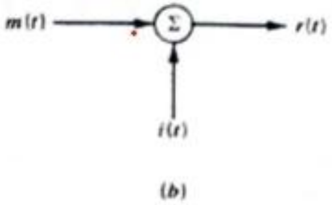
Following are some of its features −

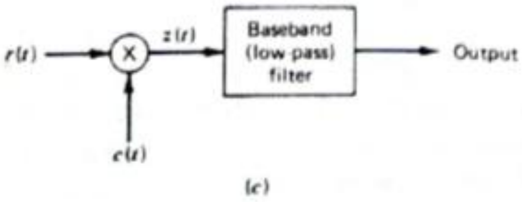
- Band of signals occupy a wide range of frequencies.
- Power density is very low.
- Energy is wide spread.

With these features, the spread spectrum signals are highly resistant to interference or jamming. Since multiple users can share the same spread spectrum bandwidth without interfering with one another, these can be called as **multiple access techniques**.

A Notion of Spread Spectrum







$$m(t) = c(t)b(t)$$

$$r(t) = m(t) + i(t) = c(t)b(t) + i(t)$$

$$z(t) = c(t)r(t) = c^2(t)b(t) + c(t)i(t)$$

$$c^2(t) = 1 \qquad \text{for all } t$$

$$z(t) = b(t) + c(t)i(t)$$

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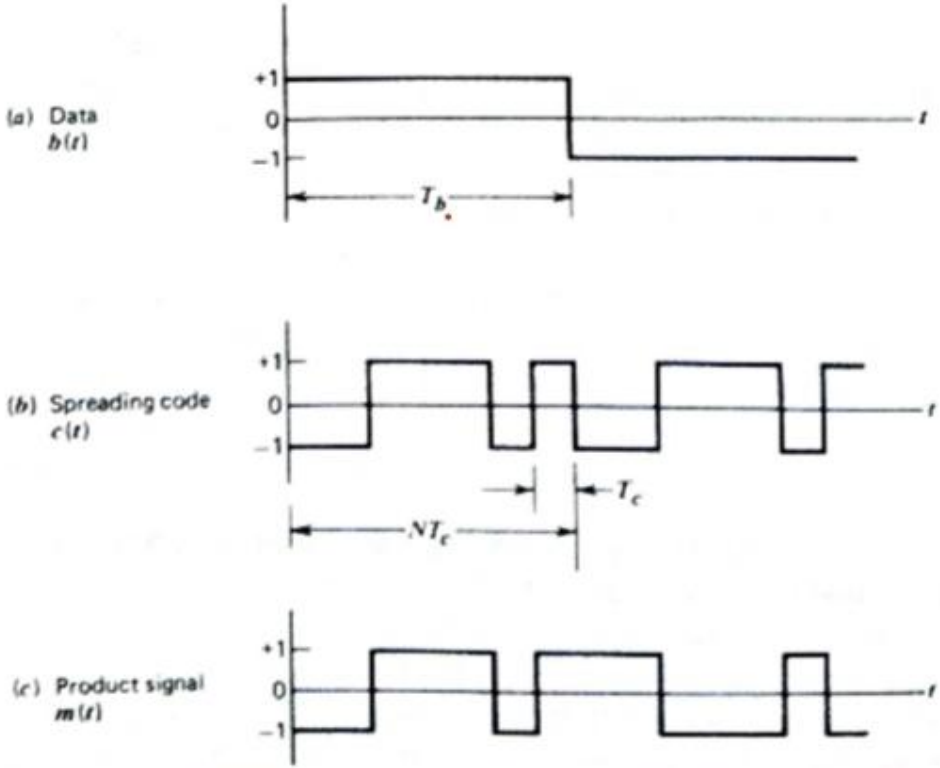

Figure 9.4 Illustrating the waveforms in the transmitter of Fig. 9.3*a*.

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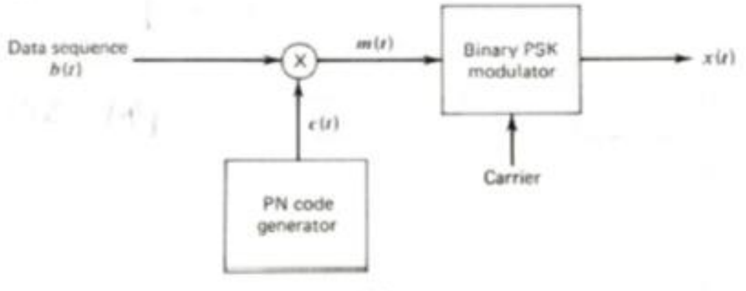
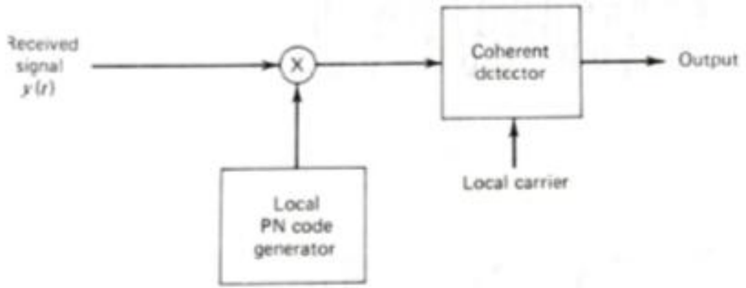
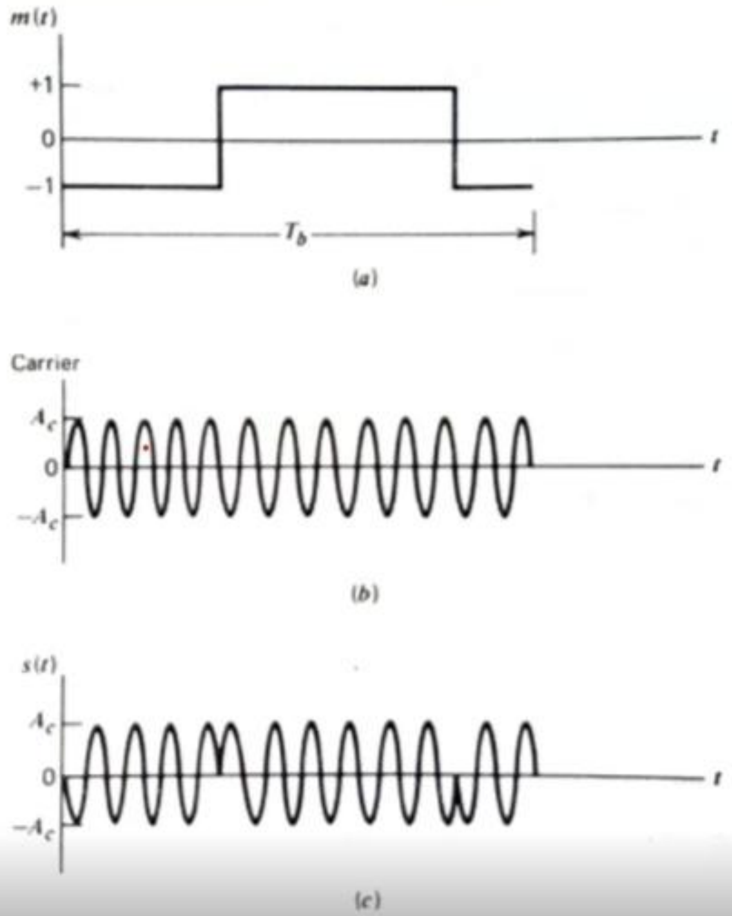
Direct sequence spread spectrum



(a)



(b)



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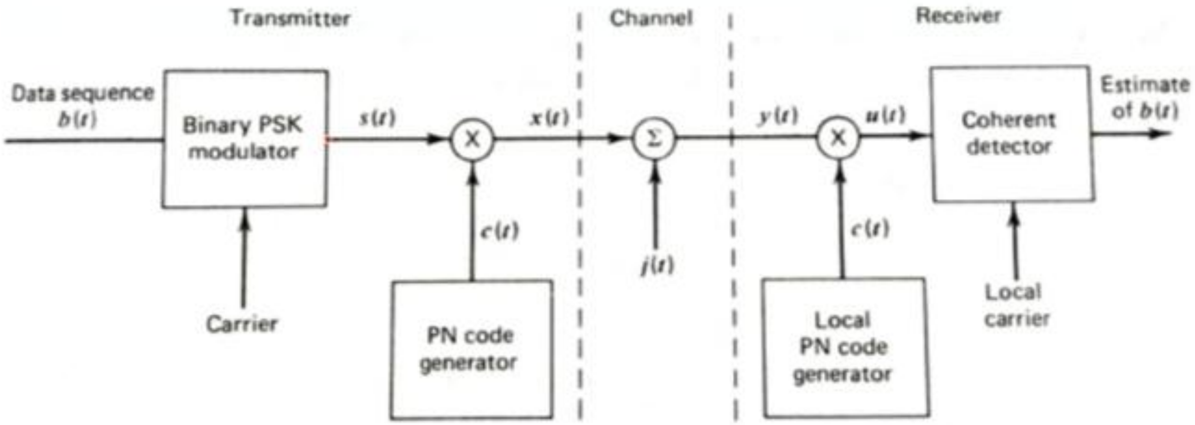

Model of direct-sequence spread binary PSK system.

$$
\begin{aligned}
y(t) &= x(t) + j(t) \\
&= c(t)s(t) + j(t)
\end{aligned}
$$

$$
\begin{aligned}
u(t) &= c(t)\, y(t) \\
&= c^2(t)\, s(t) + c(t)\, j(t) \\
&= s(t) + c(t)\, j(t),
\end{aligned}
$$

$$c^2(t) = 1 \qquad \text{for all } t$$

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Signal space dimensionality and processing gain

Signal space representation of the transmission and interfering signal

Orthogonal basis functions

$$\phi_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \cos(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \sin(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$k = 0, 1, \ldots, N-1$$

Tc= Chip duration, N=Number of chips per bit

The transmitted signal X(t) is

$$\begin{aligned} x(t) &= c(t)s(t) \\ &= \pm \sqrt{\frac{2E_b}{T_b}}\, c(t) \cos(2\pi f_c t) \\ &= \pm \sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \phi_k(t) \qquad 0 \leq t \leq T_b \end{aligned}$$

$$j(t) = \sum_{k=0}^{N-1} j_k \phi_k(t) + \sum_{k=0}^{N-1} \tilde{j}_k \tilde{\phi}_k(t) \qquad 0 \leq t \leq T_b$$

$$(\text{SNR})_I = \frac{E_b/T_b}{J} \qquad (\text{SNR})_O = \frac{2T_b}{T_c}(\text{SNR})_I \tag{9.36}$$

It is customary practice to express signal-to-noise ratios in decibels. We may thus write Eq. 9.36 in the equivalent form

$$10\log_{10}(\text{SNR})_O = 10\log_{10}(\text{SNR})_I + 3 + 10\log_{10}(PG),\ \text{dB} \tag{9.37}$$

where

$$PG = \frac{T_b}{T_c} \tag{9.38}$$

$$\textit{spread factor}\ N = T_b/T_c$$

Average Interference power of j(t) $\quad J = \frac{1}{T_b}\int_0^{T_b} j^2(t)\,dt$

The 3-dB term on the right side of Eq. 9.37 accounts for the gain in SNR that is obtained through the use of coherent detection (which presumes exact knowledge of the signal phase by the receiver). This gain in SNR has nothing to do with the use of spread spectrum. Rather, it is the last term, $10\log_{10}(PG)$, that accounts for the *gain in SNR obtained by the use of spread spectrum*. The ratio PG, defined in Eq. 9.38, is therefore referred to as the *processing gain*. Specifically, it represents the gain achieved by processing a spread-spectrum signal over an unspread signal. Note that both the processing gain PG and the spread factor N (i.e., PN sequence length) equal the ratio T_b/T_c. Thus the longer we make the PN sequence (or, correspondingly, the smaller the chip time T_c is), the larger will the pr[illegible]

We may define the processing gain in another way by making two observations:

1. The *bit rate* of the binary data entering the transmitter input is given by

$$R_b = \frac{1}{T_b}$$

2. The bandwidth of the PN sequence $c(t)$, defined in terms of the main lobe of its spectrum, is given by

$$W_c = \frac{1}{T_c}$$

Note that both R_b and W_c are baseband parameters. Hence, we may reformulate the processing gain of Eq. 9.38 as

$$PG = \frac{W_c}{R_b}$$

Probability of Error

BPSK

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e \simeq \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

Antijam characteristics

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

$$E_b = PT_b$$

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right)$$

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

$$(\text{Jamming margin})_{\text{dB}} = (\text{Processing gain})_{\text{dB}} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{\min}$$

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Probability of Error

BPSK

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e \simeq \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

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$$\frac{N_0}{2} = \frac{JT_c}{2}$$

$$E_b = PT_b$$

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right)$$

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

$$(\text{Jamming margin})_{\text{dB}} = (\text{Processing gain})_{\text{dB}} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{\text{min}}$$