Normalized/prototype Low pass analog filter

1.) N-order

$$N \geq \frac{\log\left[\frac{(10^{-0.1 A_p} - 1)}{(10^{-0.1 A_s} - 1)}\right]}{2 \log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

2.) Ω_c → 3dB cutoff freq in rad/sec.

3.) Determine poles S_k

$$\Omega_c e^{j\left(\frac{\pi}{2} + \frac{(2k+1)}{2N}\pi\right)} \qquad k = 0 \cdots 2N-1$$

4.) Transfer function

$$\boxed{H_n(s) = \frac{1}{\prod_{\substack{\text{LHS poles} \\ k=0}}^{N-1} (s - s_k)}}$$

Design an analog butterworth LPF to satisfy the following specifications

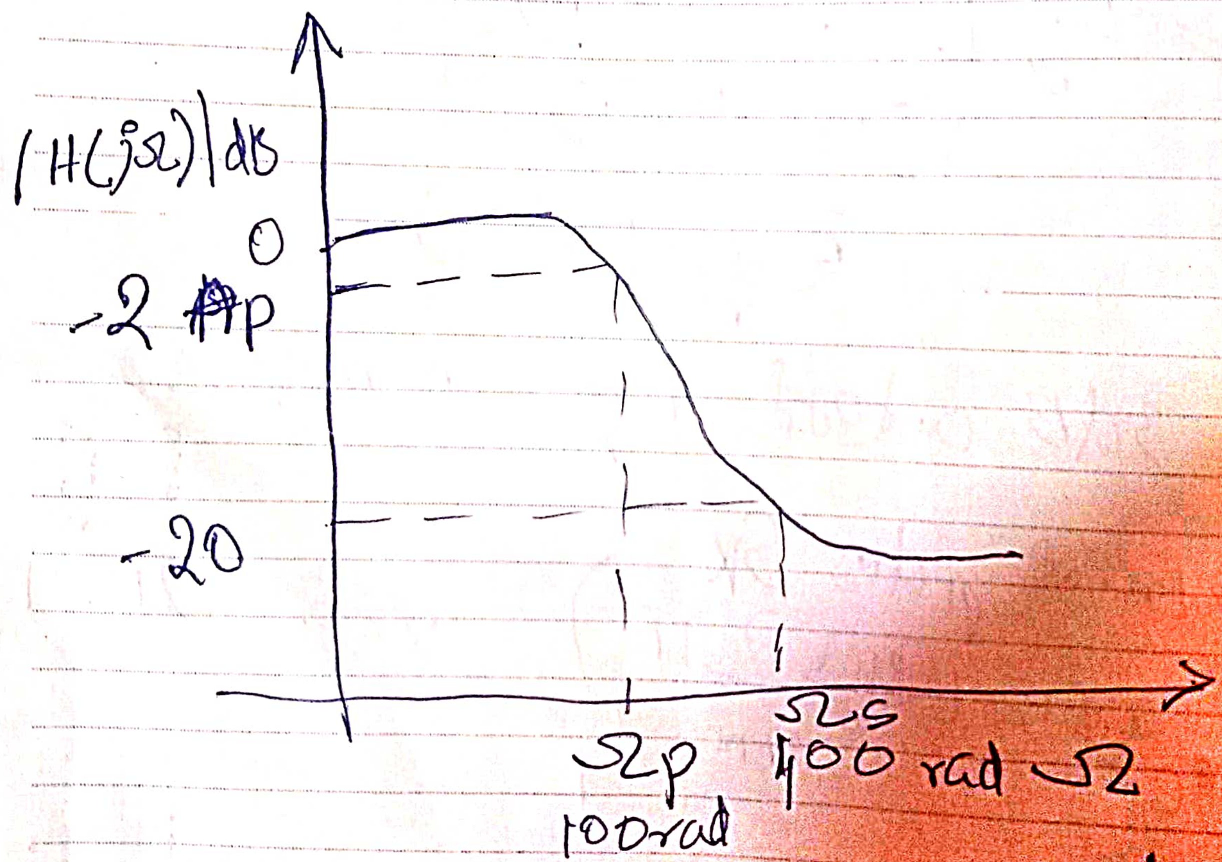
i) monotonic response :

ii) $A_p \leq 2$ dB at $\Omega_p = 100$ rad/sec

monotonic response $A_s = -20$ dB

$\Omega_s = 400$ rad/sec

<u>Design</u>

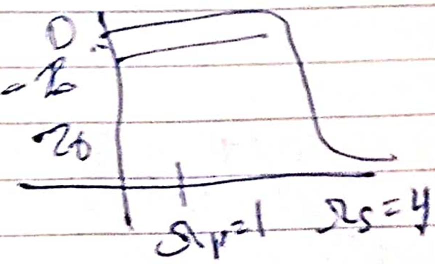


Transformation of un normalized filter to normalized low pass filter

① $\frac{\Omega_s}{\cancel{\quad}} = \frac{400}{100} = 4$ rad/s

② order of the filter

N



② $$N \geq \frac{\log\left[\dfrac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}\right]}{2\log\left(\dfrac{\Omega_p'}{\Omega_s'}\right)}$$

$$\frac{\log\left[\dfrac{10^{-0.1(-2)}-1}{10^{-0.1(-20)}-1}\right]}{2\log\left(\frac{1}{4}\right)} = 1.85 \approx 2$$

③ $$\boxed{\Omega_c = \frac{\Omega_p'}{\left(10^{-0.1(-2)}-1\right)^{1/2N}} = \frac{1}{\left(10^{0.2}-1\right)^{1/4}} = 1.14 \text{ rad/s}}$$

$$S_k = 1 \cdot e^{j\left(\frac{\pi}{2} + \frac{(2k \pm 1)\pi}{2 \times N}\right)}$$

$k = 0 \ldots 2N-1$

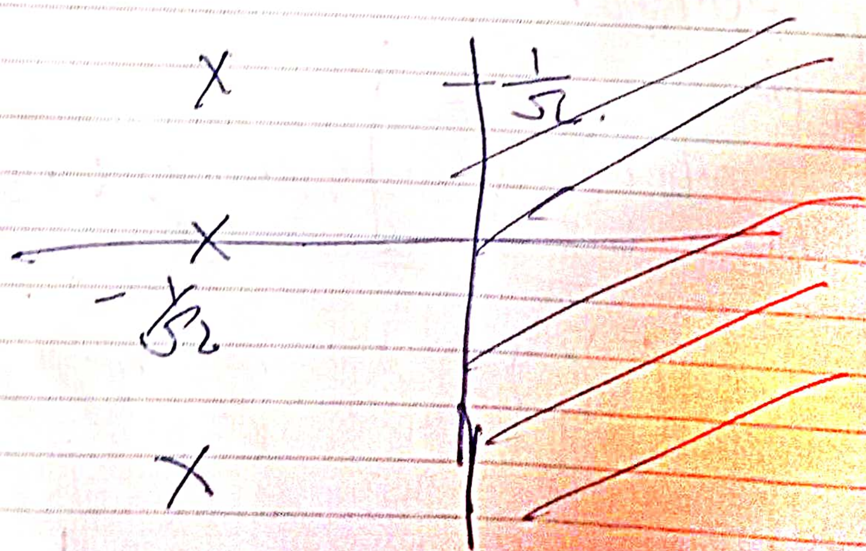
$= 0, 1, 2, 3$

$$S_0 = e^{j\left(\frac{\pi}{2} + \frac{\pi}{4}\right)} = e^{j\frac{3\pi}{4}}$$

$$= -0.707 + j0.707$$

$$S_1 = e^{j\left(\frac{\pi}{2} + \frac{3\pi}{4}\right)} = e^{j\frac{5\pi}{4}}$$

$$= -0.707 - j0.707$$



(4) $$H_n(S) = \frac{1}{\prod_{k=0}^{N-1} (S - S_k)} = \frac{1}{(S + 0.707 - j0.707)(S + 0.707 + j0.707)}$$

$$= \frac{1}{S_n^2 + 1.414 S_n + 1}$$

$$= \cancel{\frac{1}{\left(\frac{S_n}{141.4}\right)^2 + 1.414\left(\frac{S_n}{\frac{141.6}{100}}\right) + 1}}$$

$$= \frac{1}{\frac{S_n^2}{12996} + 1.414\left(\frac{S}{114}\right) + 1}$$

$$= \frac{12996}{S^2 + 161.196(S) + 12996}$$