$$= 2.666 - 2.066$$

$$\mathbf{H(B/A) = 0.6 \text{ bits/message-symbol}}$$

$$H(A/B) = H(A, B) - H(B)$$
$$= 2.666 - 1.857$$
$$\mathbf{H(A/B) = 0.809 \text{ bits/message-symbol}}$$

$$I(A, B) = H(A) - H(A/B)$$
$$= 2.066 - 0.809$$
$$\mathbf{I(A, B) = 1.257 \text{ bits/message-symbol}}$$

$$I(A, B) = H(B) - H(B/A)$$
$$= 1.857 - 0.6$$
$$\mathbf{I(A, B) = 1.257 \text{ bits/message-symbol}}$$

Ex.2: A transmitter transmits 5 symbols with probabilities 0.2, 0.3, 0.2, 0.1 and 0.2. Given the channel matrix $P(B/A)$, Calculate H(B) and H(A,B)

$$P(B/A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$I(A, B) = 1.257$ bits/message-symbol

Ex.2: A transmitter transmits 5 symbols with probabilities 0.2, 0.3, 0.2, 0.1 and 0.2. Given the channel matrix $P\left({}^{B}/_{A}\right)$, Calculate H(B) and H(A,B)

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Solution:

We know that, $P(a_i, b_j) = P(a_i)P(b_j/a_i)$

The JPM may now be constructed by multiplying 1st row elements by $P(a_1) = 0.2 = \frac{1}{5}$, 2nd row by $P(a_2) = 0.3 = \frac{3}{10}$, 3rd row by $P(a_3) = 0.2 = \frac{1}{5}$, 4th row by $P(a_4) = 0.1 = \frac{1}{10}$ and 5th row by $P(a_5) = 0.2 = \frac{1}{5}$.

$$P(A, B) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 1/5 & 0 & 0 & 0 \\ 3/40 & 9/40 & 0 & 0 \\ 0 & 1/15 & 2/15 & 0 \\ 0 & 0 & 1/30 & 1/15 \\ 0 & 0 & 1/5 & 0 \end{bmatrix} \end{array}$$

Adding the element pf each column, we get

$$P(b_1) = \frac{1}{5} + \frac{3}{40} = \frac{11}{40}$$

$$P(b_2) = \frac{9}{40} + \frac{1}{15} = \frac{7}{24}$$

$$P(b_3) = \frac{2}{15} + \frac{1}{30} + \frac{1}{5} = \frac{11}{30}$$

$$P(b_4) = \frac{1}{15}$$

$$H(B) = \sum_{j=1}^{4} P(b_j) \log \frac{1}{P(b_j)}$$

$$= \frac{11}{40} \log \frac{40}{11} + \frac{7}{24} \log \frac{24}{7} + \frac{11}{30} \log \frac{30}{11} + \frac{1}{15} \log 15$$

H(B) = 1.822 bits/message-symbol

$$H(A, B) = \sum_{i=1}^{5} \sum_{j=1}^{4} P(a_i, b_j) \log \frac{1}{P(a_i, b_j)}$$

$$= \frac{1}{5} \log 5 + \frac{3}{40} \log \frac{40}{3} + \frac{9}{40} \log \frac{40}{9} + \frac{1}{15} \log 15 + \frac{2}{15} \log \frac{15}{2}$$

$$+ \frac{1}{30} \log 30 + \frac{1}{5} \log 5 + \frac{1}{15} \log 15$$

H(A, B) = 2.7653 bits/message-symbol

$$P(X/Y) = \begin{bmatrix} \cdots & & & \\ 0 & \frac{0.10}{0.15} & \frac{0.1}{0.5} & 0 \\ 0 & 0 & \frac{0.2}{0.5} & \frac{0.10}{0.25} \\ \frac{0.05}{0.10} & \frac{0.05}{0.15} & 0 & \frac{0.10}{0.25} \end{bmatrix}$$

$$\therefore \quad P(X/Y) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 1/2 & 0 & 2/5 & 1/5 \\ 0 & 2/3 & 1/5 & 0 \\ 0 & 0 & 2/5 & 2/5 \\ 1/2 & 1/3 & 0 & 2/5 \end{bmatrix} \end{array}$$

$$\therefore \quad H(X/Y) = 0.05 \log 2 + 0.05 \log 2 + 0.1 \log 3/2 + 0.05 \log 3 + 0.2 \log 5/2$$
$$+ 0.1 \log 5 + 0.2 \log 5/2 + 0.05 \log 5 + 0.1 \log 5/2 + 0.1 \log 5/2$$

$$\therefore \quad \mathbf{H(X/Y) = 1.379\ bits/message\ symbol}$$

$$H(Y/X) = \sum_{i=1}^{4}\sum_{j=1}^{4} P(x_i, y_j) \log \frac{1}{P(y_j/x_i)} \text{ bits/message-symbol.}$$

Using the relationship $P(y_j/x_i) = \dfrac{P(x_i, y_j)}{P(x_i)}$, the channel matrix $P(Y/X)$ is constructed below:

$$P(Y/X) = \begin{bmatrix} \frac{0.05}{0.30} & 0 & \frac{0.20}{0.30} & \frac{0.05}{0.30} \\ 0 & \frac{0.10}{0.20} & \frac{0.10}{0.20} & 0 \\ 0 & 0 & \frac{0.20}{0.30} & \frac{0.10}{0.30} \\ \frac{0.05}{0.20} & \frac{0.05}{0.20} & 0 & \frac{0.10}{0.20} \end{bmatrix}$$

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$$P(Y/X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 1/6 & 0 & 2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix} \end{array}$$

$$\begin{aligned} H(Y/X) &= 0.05 \log 6 + 0.2 \log 3/2 + 0.05 \log 6 + 0.1 \log 2 + 0.1 \log 2 \\ &\quad + 0.2 \log 3/2 + 0.1 \log 3 + 0.05 \log 4 + 0.05 \log 4 + 0.1 \log 2 \\ &= \textbf{1.151 bits/message-symbol} \end{aligned}$$

Stop sharing Hide

Verification:

$P(x_i)$

below:

$$P(Y/X) = \begin{bmatrix} \frac{0.05}{0.30} & 0 & \frac{0.20}{0.30} & \frac{0.05}{0.30} \\ 0 & \frac{0.10}{0.20} & \frac{0.10}{0.20} & 0 \\ 0 & 0 & \frac{0.20}{0.30} & \frac{0.10}{0.30} \\ \frac{0.05}{0.20} & \frac{0.05}{0.20} & 0 & \frac{0.10}{0.20} \end{bmatrix}$$

$$P(Y/X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 1/6 & 0 & 2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix} \end{array}$$

$$\begin{aligned} H(Y/X) = {} & 0.05 \log 6 + 0.2 \log 3/2 + 0.05 \log 6 + 0.1 \log 2 + 0.1 \log 2 \\ & + 0.2 \log 3/2 + 0.1 \log 3 + 0.05 \log 4 + 0.05 \log 4 + 0.1 \log 2 \end{aligned}$$

$$\mathbf{H(Y/X) = 1.151 \text{ bits/message-symbol}}$$

Verification:

$$\begin{aligned} &= H(X, Y) - H(X) \\ &= 3.122 - 1.971 \end{aligned}$$

$$\mathbf{H(Y/X) = 1.151 \text{ bits/message-symbol as before}}$$

$$H(X/Y) = H(X, Y) - H(Y)$$
$$= 3.122 - 1.743$$
$$\mathbf{H(Y/X) = 1.379 \text{ bits/message-symbol as before}}$$

$$I(X, Y) = H(X) - H(X/Y)$$
$$= 1.971 - 1.379$$
$$\mathbf{I(X, Y) = 0.592 \text{ bits/message-symbol}}$$

$$I(X, Y) = H(Y) - H(Y/X)$$
$$= 1.743 - 1.151$$
$$\mathbf{I(X, Y) = 0.592 \text{ bits/message-symbol}} \quad - \text{verified.}$$

1.7 Binary Symmetric Channel:

ry symmetric channel is one of the most commonly and widely used channel whose channel diagram is given below

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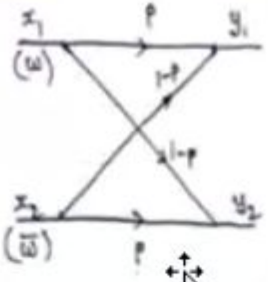
The binary symmetric channel is one of the most commonly and widely used channel whose channel diagram is given below



From the above diagram, channel matrix can be written as

$$P(X/_Y) = \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix} = \begin{bmatrix} P & \bar{P} \\ \bar{P} & P \end{bmatrix}$$

The matrix is a symmetric matrix. Hence the channel is binary symmetric channel.

nel Capacity

It is known that $C = \text{Max}\{[H(Y) - H(Y/_X)]\, r_s\}$.

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$$P\left({}^{X}/_{Y}\right) = \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix} = \begin{bmatrix} P & \bar{P} \\ \bar{P} & P \end{bmatrix}$$

The matrix is a symmetric matrix. Hence the channel is binary symmetric channel.

1.8 Channel Capacity

It is known that $C = \text{Max}\{[H(Y) - H\left({}^{Y}/_{X}\right)]\, r_s\}$.

For symmetry channel, $H\left({}^{Y}/_{X}\right) = h = P\log\frac{1}{P} + \bar{P}\log\frac{1}{\bar{P}}$

Since it is a binary symmetric channel, $H(Y)_{max} = log_2 s = log_2 2 = 1$

$\therefore C = 1 - h$ bits/sec.

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Since it is a binary symmetric channel, $H(Y)_{max} = log_2 s = log_2 2 = 1$

$\therefore C = 1 - h$ bits/sec.

Ex.1: A binary symmetric channel has the following noise matrix with source probabilities of $P(x_1)$=2/3 and $P(x_2)$=1/3. $P\left({}^{Y}/_{X}\right) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$. Determine H(X), H(Y), H(X,Y), H(Y/X), H(X/Y), I(X,Y), Channel Capacity, Channel efficiency and redundancy.

Solution: