An introduction to numerical ocean modelling

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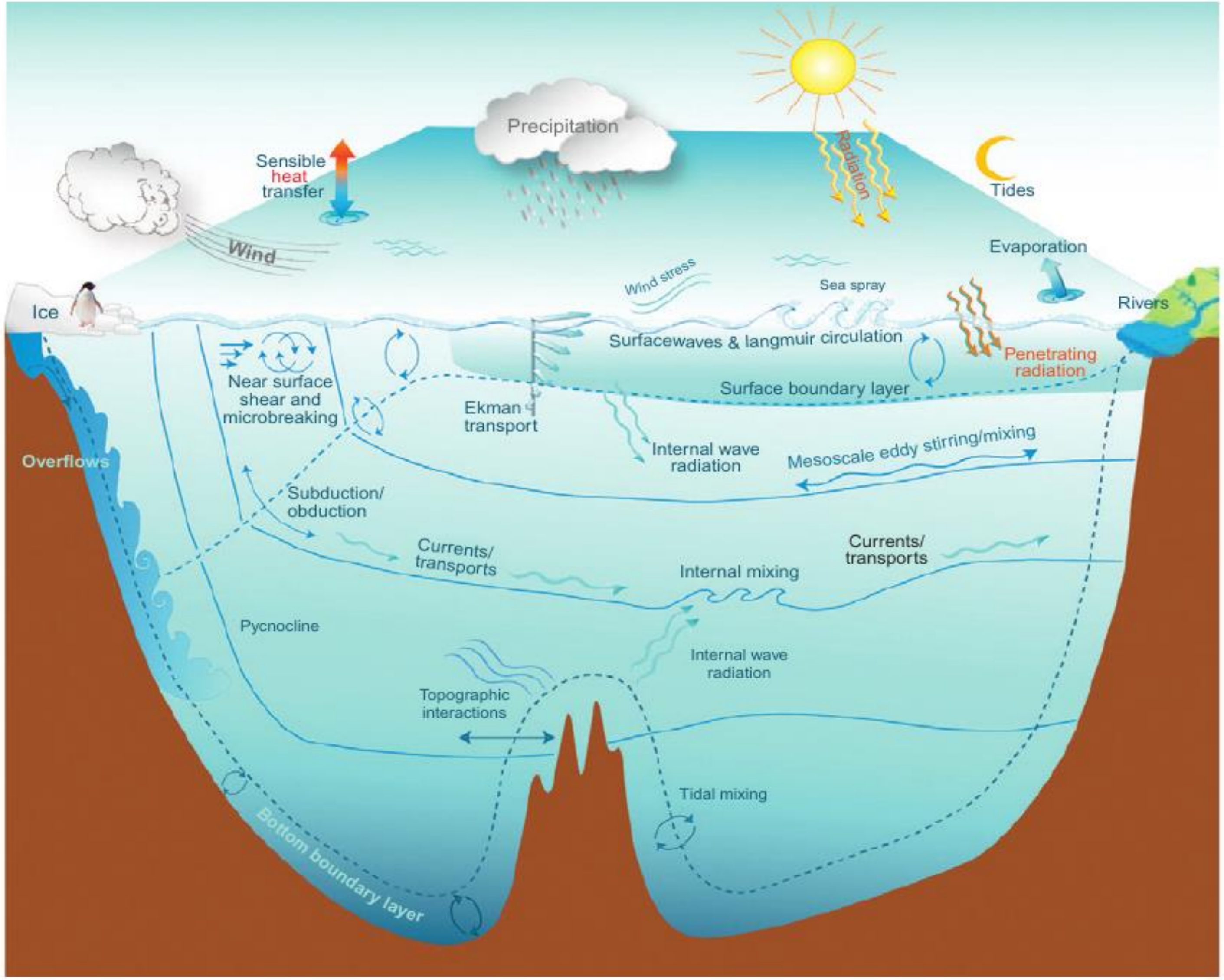
22-09-2020

Oceans of the planet Earth

"Our planet is misnamed 'Earth' because we live on the land portion of the planet. If we were marine animals, our planet would probably be called 'Ocean', 'Water', 'Hydro', 'Aqua', or even 'Oceanus' to indicate the prominence of Earth's oceans." (Thurman, H. V. & Trujillo, A. P., 2016)

- 70.8% of the Earth's surface is covered by oceans.
- Oceans contain 97.2% of all the water on or near the Earth's surface.
- Ocean influence the weather and climate of the Earth.
- Ocean plays an important role in rainfall distribution, drought, floods, cyclone-formation, regional and global climate system etc.

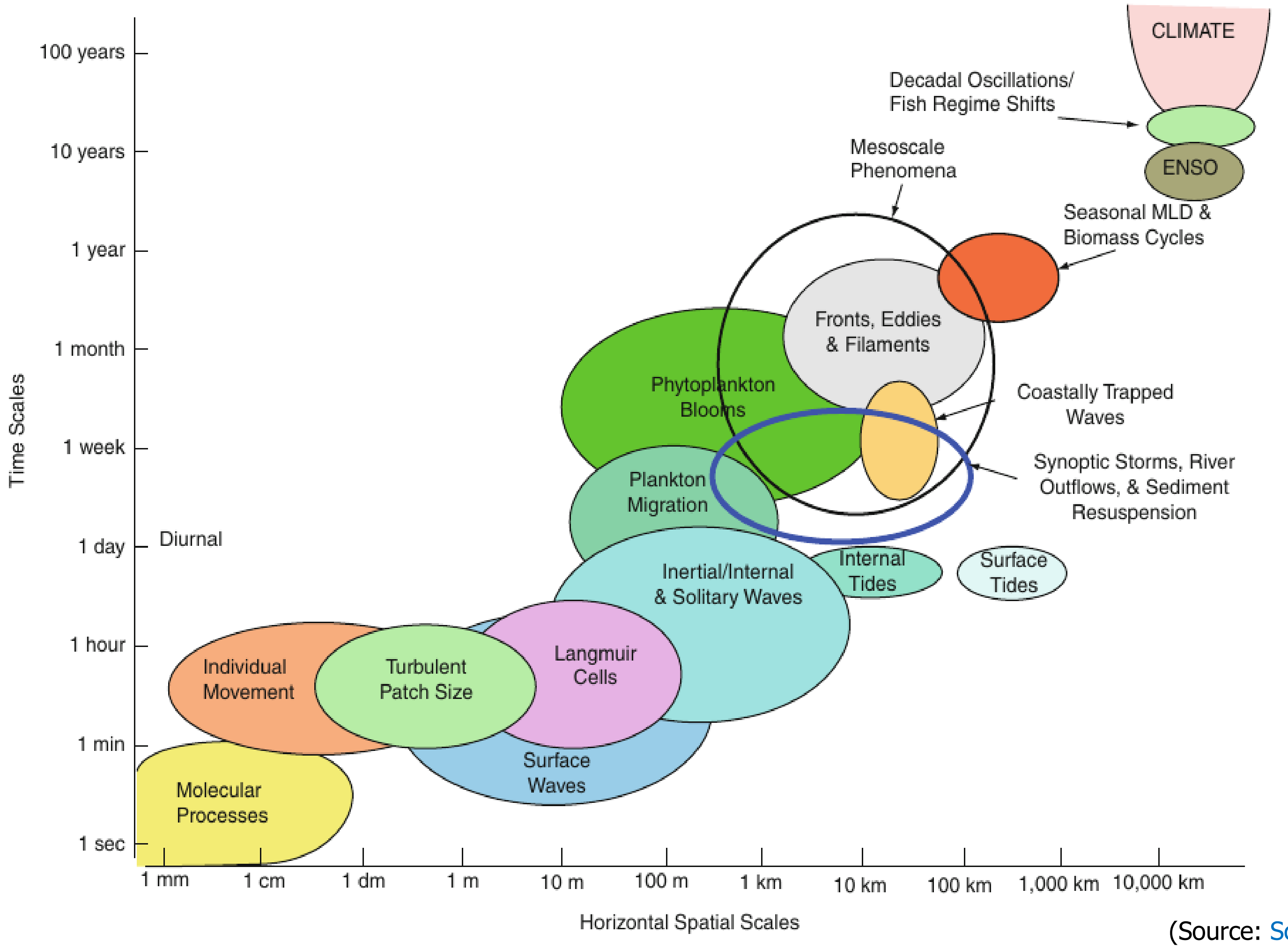


- **Surface ocean:** Exchange mass, energy, momentum
- Mixed layer and Ekman layer formation in response to surface wind forcing.
- **Internal waves:** Interaction of tidal forcing with bottom topography
- **Currents:** transport
- **Rivers:** bring freshwater, nutrients, sediments
- **Sea-ice**

(Source: Griffies, S. M., & Treguier, A. M., 2013)

Ocean plays an important role in global climate system through various physical and dynamical processes.

Temporal and spatial scales of various oceanic processes

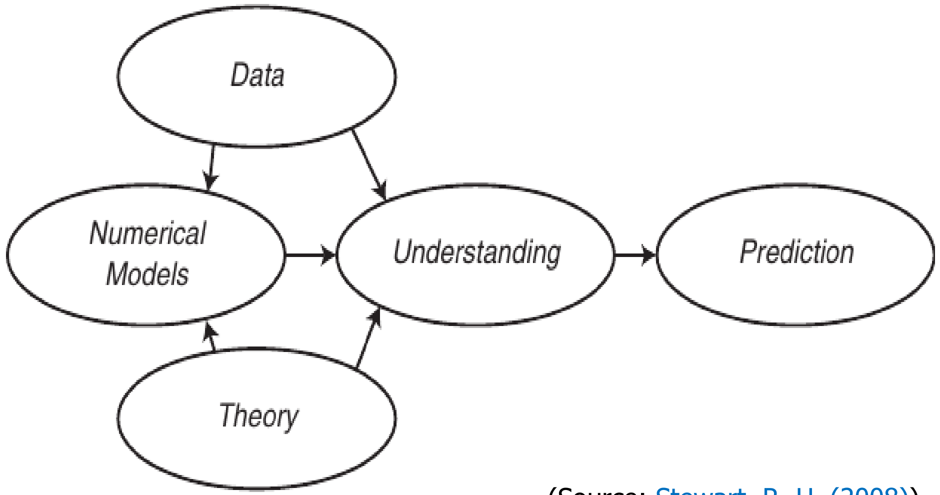


(Source: Schofileld et al., 2013)

Oceanic processes are characterized by a broad range of temporal and spatial scales.

How do we study/understand the ocean and oceanic processes? Can we predict the possible impacts of these processes?

- Theories, observations and models are the most important components of studying and understanding the oceanic processes (None is sufficient by itself).
- Theories: based on fundamental physical laws to explain various processes.
- Observations (*in-situ* and satellite): provide valid information but sparse both temporally and spatially.
- Models (physical and numerical): to simulate the 3-dimensional oceanic processes.



(Source: Stewart, R. H. (2008))

Observations, theory and models are all necessary to understand and predict the future states of the system.

What is a numerical ocean model?

- Numerical ocean models are tools used to simulate the physical and dynamical processes in the ocean having a broad range of space-time scales.

- Oceanic processes in the model are represented using a set of equations based on the fundamental physical laws.

- The equations are solved using numerical methods transforming into a computational tool.

Conservation laws governing the basic equations of fluid motion

Conservation law	Equation of fluid motion
Conservation of mass	Equation of continuity
Conservation of energy	a) Heat budget equation from conservation of heat b) Wave equation from conservation of mechanical energy
Conservation of momentum	Momentum equations (Navier-Stokes equations)
Conservation of Angular momentum	Vorticity equation

Why use ocean models?

- Simulate the state of the ocean.
- Predict how ocean will change.
- Provide the data of desired spatial/temporal resolution
    - Satellite provides only surface data
    - In-situ measurements are limited spatial coverage
- Understand the three-dimensional ocean dynamics.

Types of ocean models

Ocean models can be classified in many different ways and the most common criteria is based on the process to be studied. Some general classifications are;

- Global or Regional
- Deep basin or shallow coastal
- Rigid lid (fixed surface) or free surface (surface to move with the flow)
- Hydrostatic or non-hydrostatic
- Barotropic (depth-independent) or baroclinic (depth dependent)
- Stand-alone or coupled
- Purely physical or Physical-Chemical-Biological
- Short term simulations or long term climate studies

Primitive equations for ocean dynamics

Horizontal momentum equations:

$$\frac{du}{dt} \quad - \quad (2\Omega sin\Phi)v \; = \; -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad + \quad Fu$$

(acceleration) (Coriolis) (Pressure gradient) (Friction)

$$\frac{dv}{dt} + (2\Omega sin\Phi)u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + Fv$$

Vertical momentum equation:

$$\frac{dw}{dt} - (2\Omega cos\Phi)u = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\rho}{\rho_0}g + F_w$$

Hydrostatic

Geostrophic

Arakawa grid

Primitive equations for ocean dynamics (Cont.)

Continuity equation:

$$\frac{1}{\rho}\frac{d\rho}{dt}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

For incompressible flow,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Elimination of sound waves, which depends on compressibility for their propagation.

Primitive equations for ocean dynamics (Cont.)

Tracer equations (Advection-Diffusion equation):

$$\frac{dT}{dt} = k_T \nabla^2 T + F_T$$

$$\frac{dS}{dt} = ks\nabla^2 S + F_s$$

Where, $\frac{d}{dx} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

(local) (advection)

Primitive equations for ocean dynamics (Cont.)

Equation of state (linear):

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

Where, ρ_0, T_0, S_0 are reference values of density, temperature and salinity,

α is the coefficient of thermal expansion,

β is the coefficient of saline contraction.

For typical seawater, ρ_0 = 1028 kg m^{-3}

T_0 = 10°C

S_0 = 35 psu

α = 1.7 × 10^{-4} K^{-1}

β = 7.6 × 10^{-4}

Equation of state (Nonlinear):

$$\rho = \rho(T, S, P)$$

(Empirical relation)

Primitive equations for ocean dynamics (Cont.)

7 equations and 7 unknown variables

(u, v, w) – 3 velocity components

T – Temperature

S – Salinity

ρ – Density

P – Pressure

Approximations

Boussinesq Approximation:

- Since density variations are small, their effect on the mass of the fluid can be neglected, but not their effect on the weight (mass times gravity) of the fluid parcel.
- This approximation is applicable in the horizontal momentum equations and not in the vertical.
- Cannot represent sound waves traveling through the water.

Approximations (Cont.)

Hydrostatic Approximation:

- When vertical accelerations are small compared to the gravitational acceleration, the pressure gradient force is balanced by the gravity.

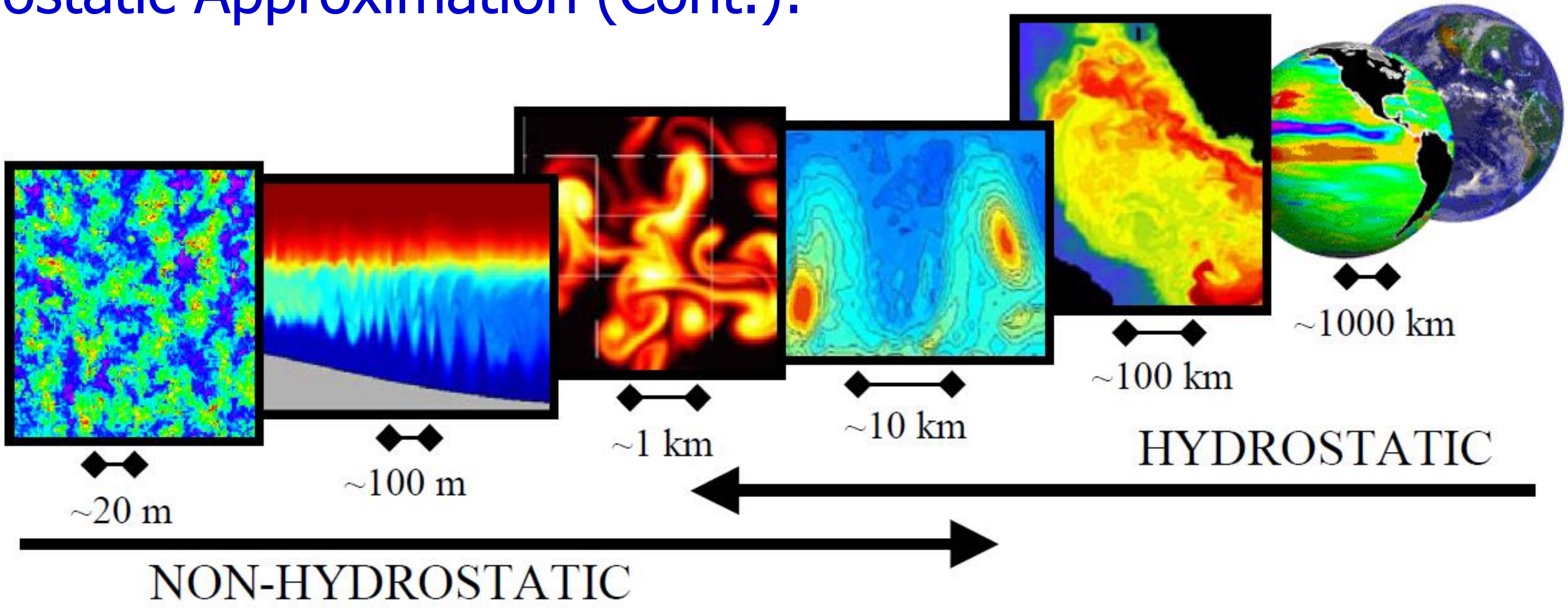
$$\frac{\partial p}{\partial z} = -\rho g$$

- If the density is constant with respect to depth, then the hydrostatic pressure is,

$$p = \rho g z$$

Equations

Hydrostatic Approximation (Cont.):



(Source: Adcroft et al., 2004)

The non-hydrostatic capability allows the model to simulate overturning and mixing processes.

Approximations (Cont.)

Geostrophic Approximation:

- Represents a steady-state balance between the Coriolis force and the pressure gradient force.

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y};\ fv = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Geostrophic currents can be computed from these equations using the pressure gradient obtained by integrating the hydrostatic equation.
- Applicable for large spatial scales.

Equations

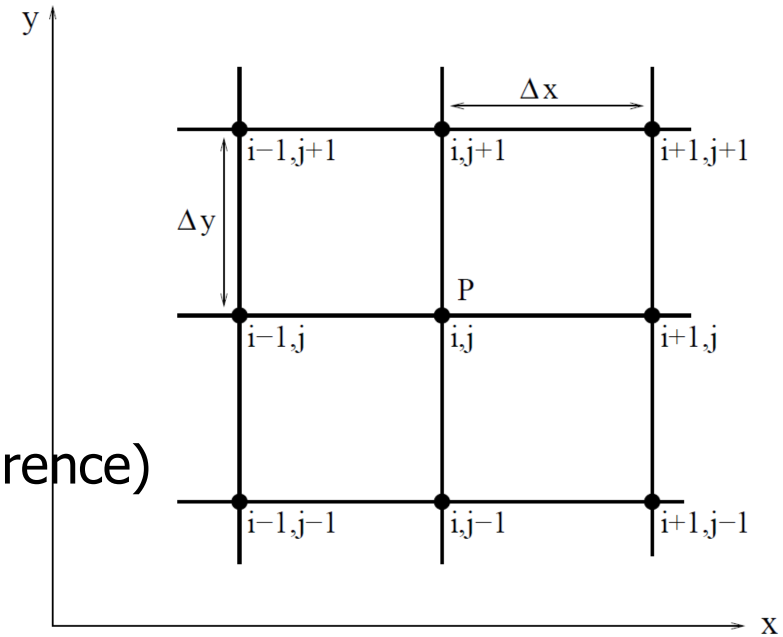
Discretize equations

To convert a continuous equation into a discrete counterpart that can be solved by using numerical methods.

- Finite difference method
- Finite element method
- Finite volume method

Discretize equations (Cont.)

The computational domain is usually divided into grid cells and the solution can be obtained at each point.



Taylor series expansion:

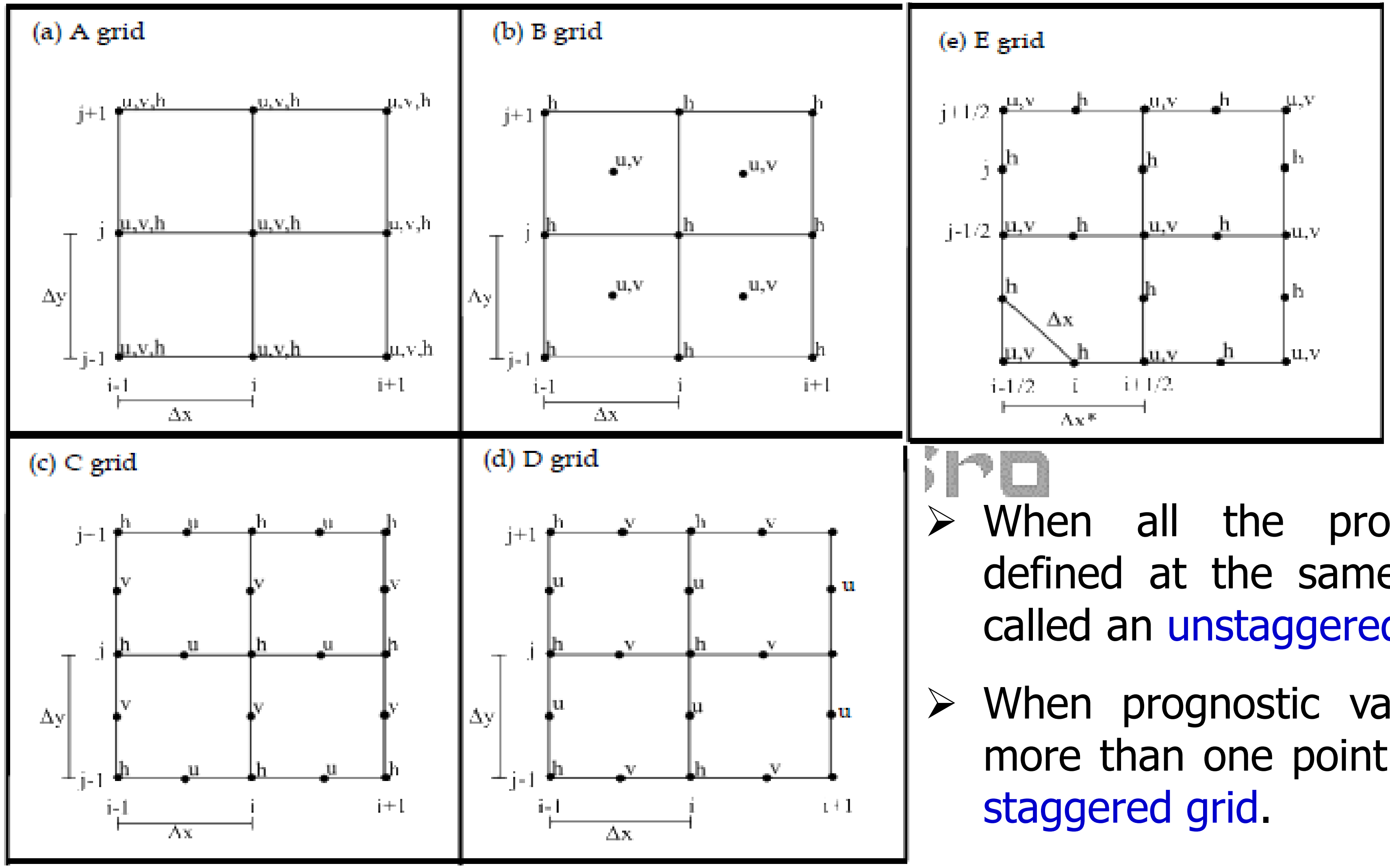
$$u_{i+1,j} = u_{i,j} + \left(\frac{\partial u}{\partial x}\right)_{i,j} \Delta x + \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \frac{(\Delta x)^2}{2} + - - - - -$$

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} + \mathcal{O}(\Delta x)$$ (First order forward difference)

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \mathcal{O}(\Delta x)$$ (First order backward difference)

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \mathcal{O}(\Delta x)^2$$ (Second order central difference)

Discretize equations (Cont.)



(Source: Arakawa and Lamb, 1977)

Equations

- When all the prognostic variables are defined at the same point in a grid, it is called an unstaggered grid.
- When prognostic variables are defined at more than one point in a grid, it is called a staggered grid.

Horizontal grid

Number of discrete points covering the surface of Earth; dividing it up into a lot of small boxes. Most commonly used horizontal grid types are regular, tripolar and irregular.

1. Regular grid:

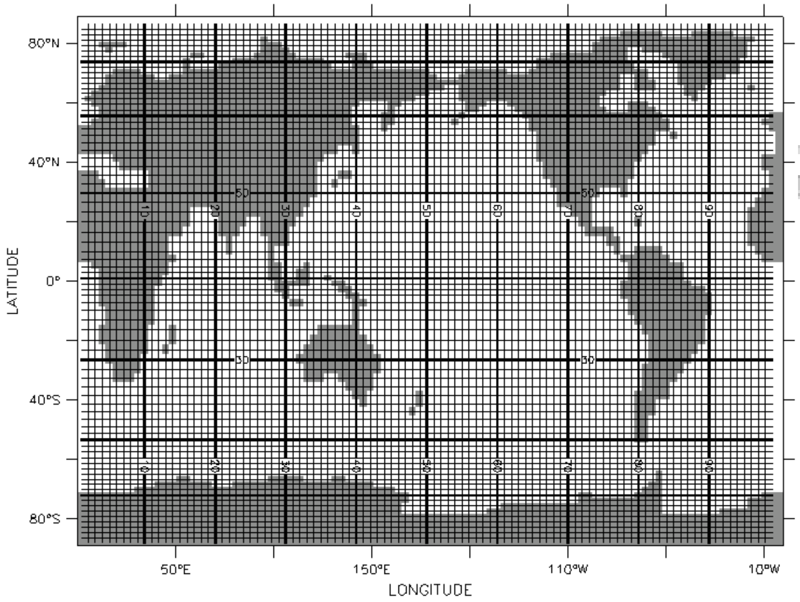
- Contains equally spaced lines (applicable for regional domain).
- As the Earth is spherical in shape, we cannot apply uniform spacing across the grids and keep the lines straight.
- Thus, the lines tend to be curvilinear and their internal spacing tends to vary.
- Regular grids have a problem at the poles where the grid lines converge, resulting in shrinking grid cells.

Horizontal grid (Cont.)

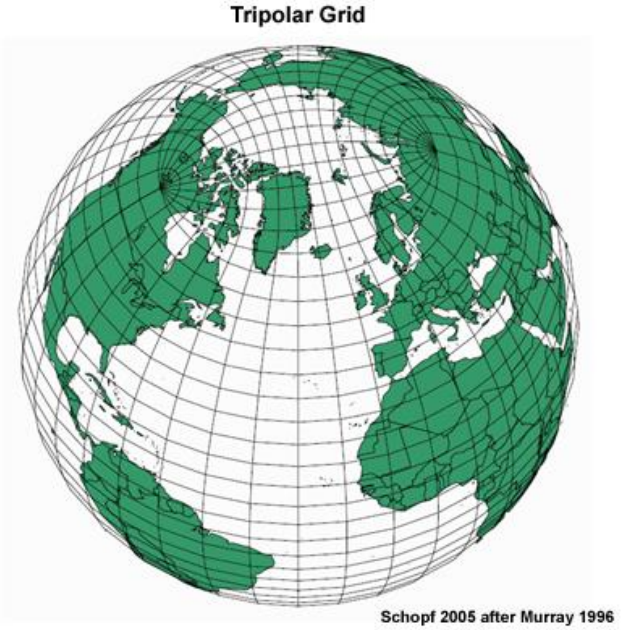
2. **Tripolar grid:** Circular grid laid over Arctic region with poles on land.

3. **Irregular grid:** Gives more freedom to vary the spatial resolution.

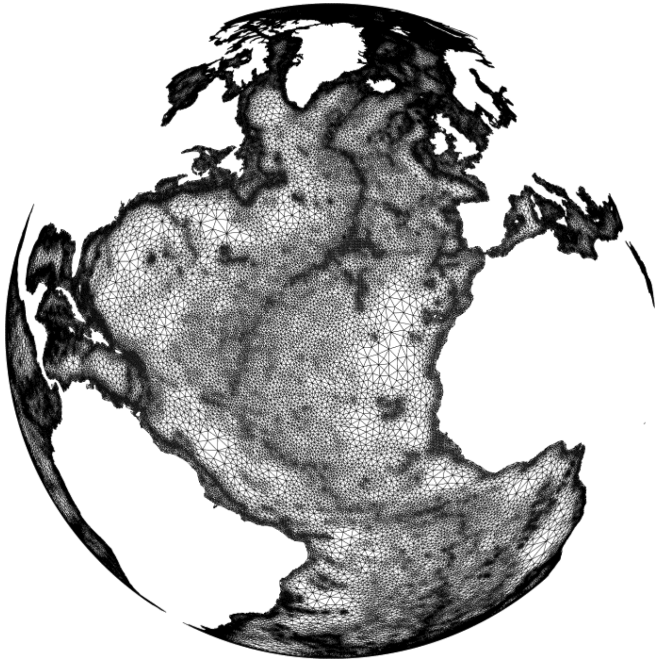
Regular grid



Tripolar grid



Irregular grid



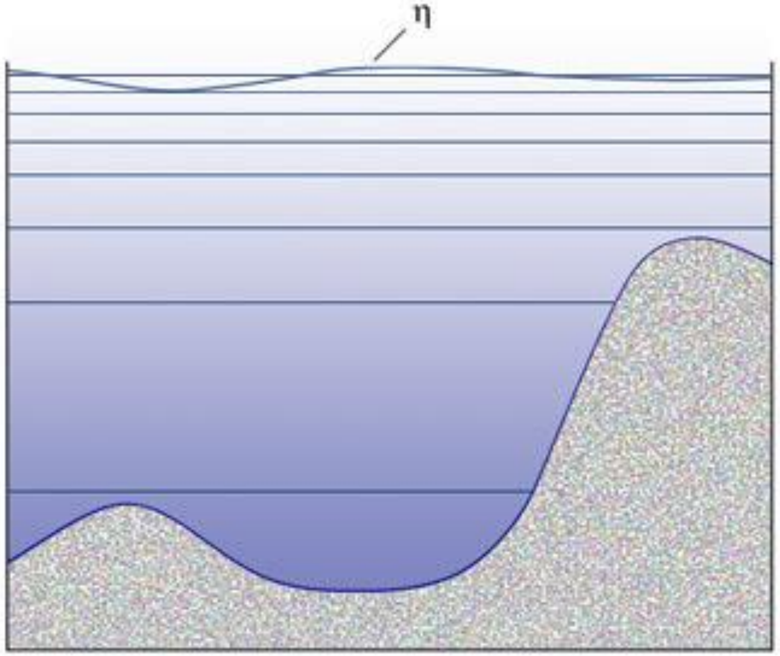
(Source: Gorman et al., 2006)

Vertical grid

There are four main vertical coordinate systems in use, i.e., Depth, sigma, isopycnal and hybrid coordinates.

1. Absolute depth/z-coordinate:

- Simple to set up.
- Common to increase the vertical resolution in the upper ocean relative to the deep.
- Good in simulation of turbulent processes in mixed layer.
- Disadvantage is that, in regions of sloping topography; unable to resolve the actual bottom topography; results unrealistic vertical velocities near the bottom.

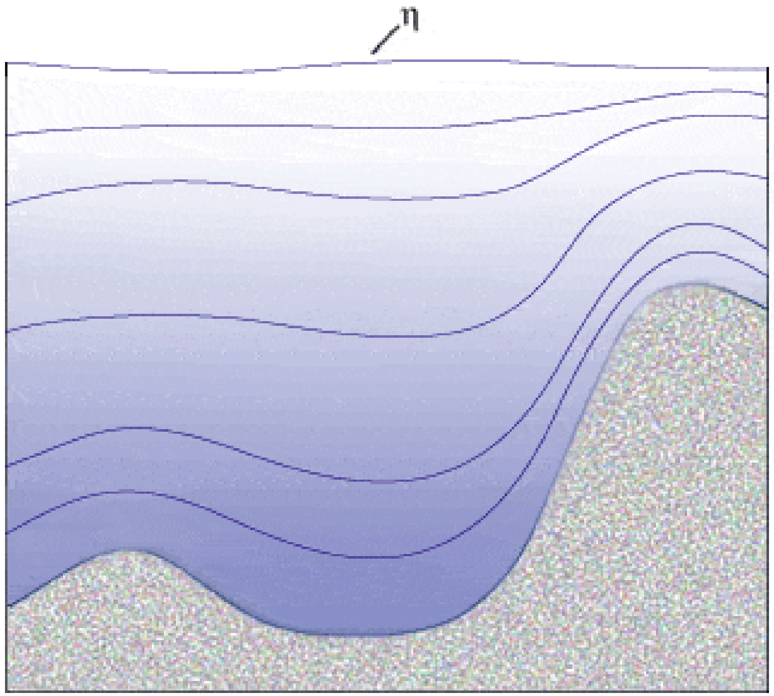


(Source: https://www.oc.nps.edu/)

Vertical grid (Cont.)

2. Terrain following/σ-coordinate:

- Follows the bathymetry keeping same number of vertical grid points everywhere in the domain.
- Closely spaced near the surface and/or bottom than in the interior, thus allowing the boundary layers to be better resolved.
- Appropriate for continental shelf and coastal regions.
- Difficulty in handling sharp topographic changes.

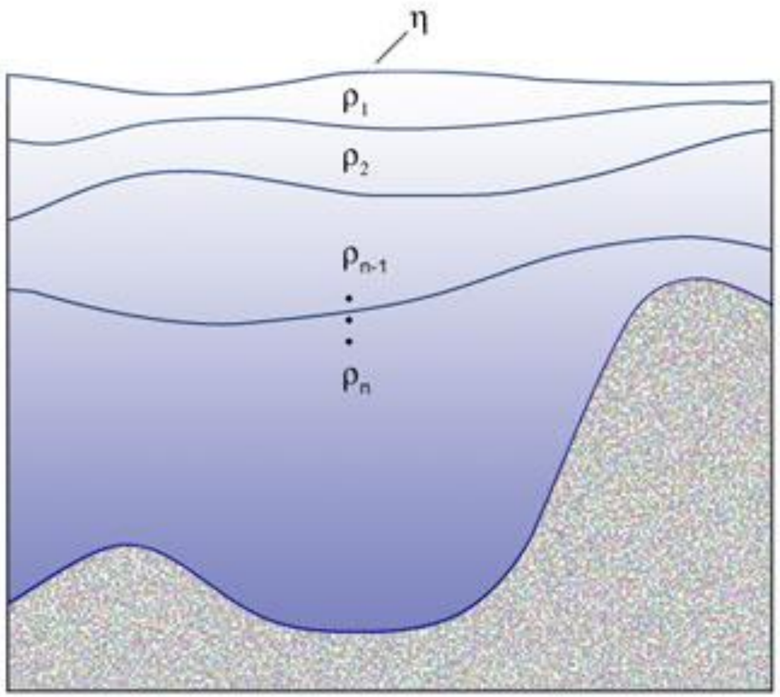


(Source: https://www.oc.nps.edu/)

Vertical grid (Cont.)

3. Density/isopycnal coordinate:

- Vertical grid is defined by the density surfaces.
- Divides the water column into distinct homogeneous layers, whose thicknesses can vary from place to place and from one time step to the next.
- This coordinate works well for modeling tracer transport.
- Not applicable to the surface mixed layer region where thermodynamic processes change the density.

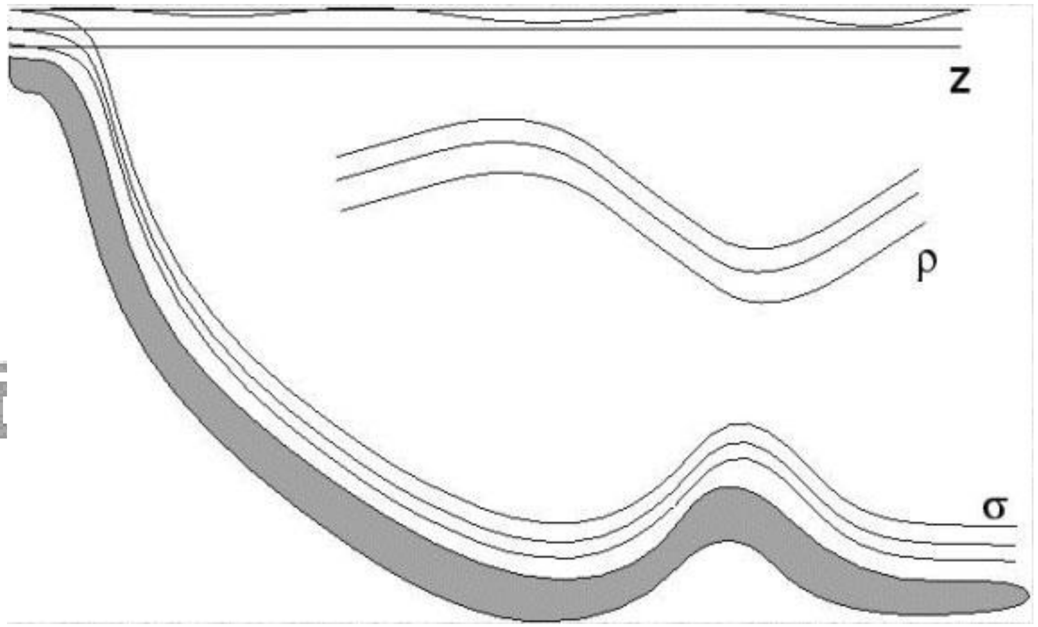


(Source: https://www.oc.nps.edu/)

Vertical grid (Cont.)

4. Hybrid coordinate:

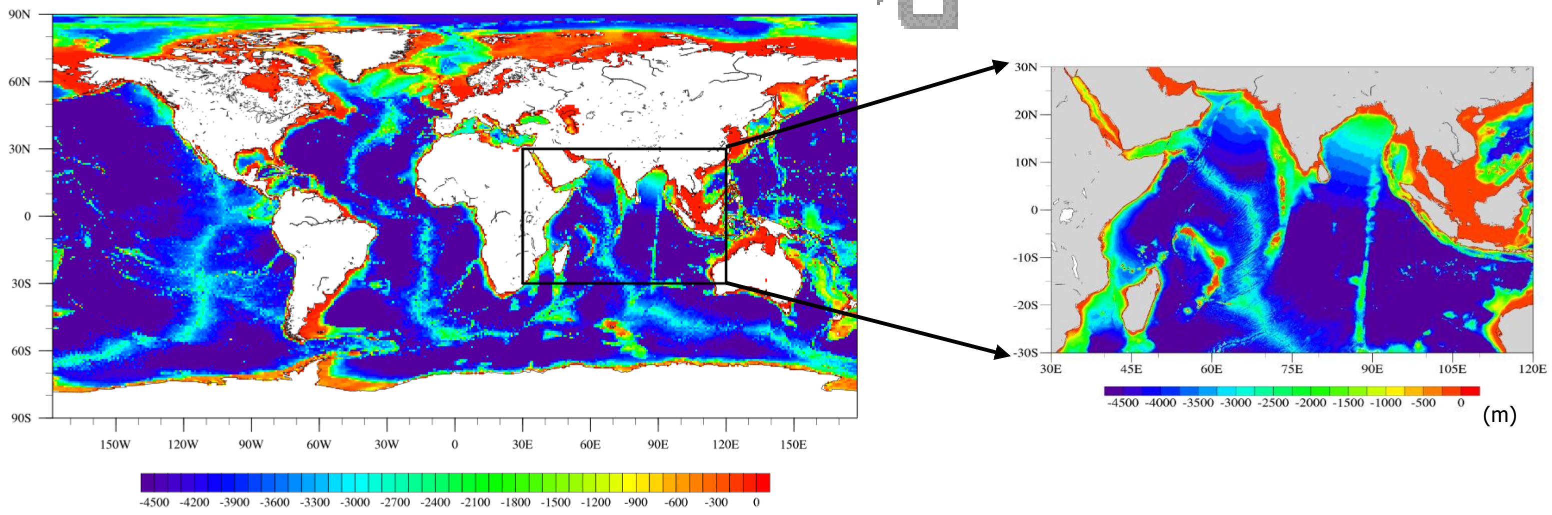
- Combined the best suited systems in different regions based on the dominant processes.
- Z coordinate in the surface mixed layer, density coordinate near the stratified ocean and sigma co-ordinate to follow the bottom topography.
- Dynamically optimized coordinate system gives improved results.
- High computational cost.



(Source: Courtesy of Eric Chassignet (U. Miami))

Model domain

- What are the limits of the region or domain of interest over which the model to be configured?
- The model domain can range from a very localized patch of ocean to the entire global ocean.
- When choosing the domain, one must consider the processes to be studied, the grid resolution, how long it will take to complete the model run depending on the availability of computer resources.

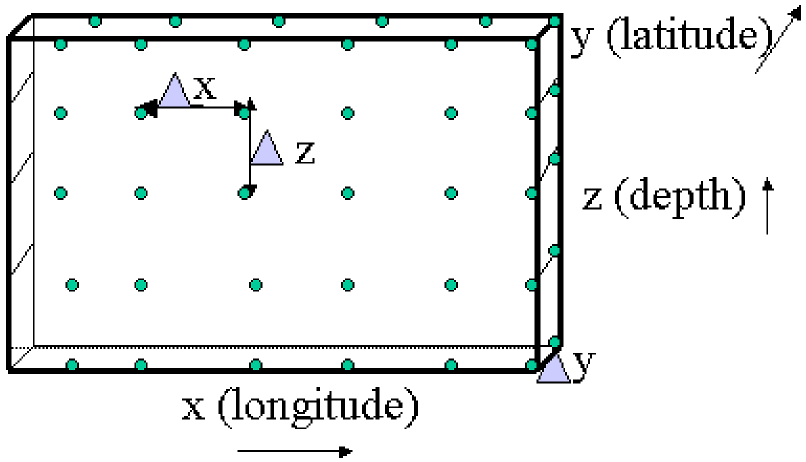


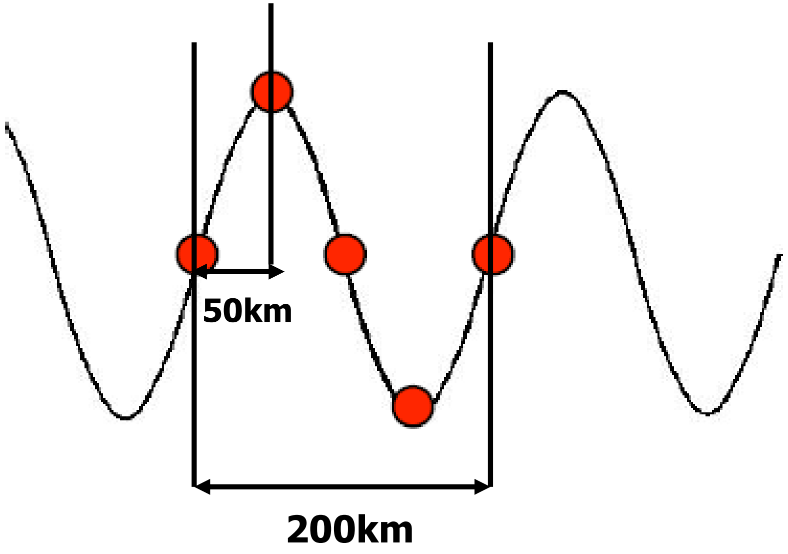
Model resolution

- The phenomena of interest need to be resolved.
- Availability of computer resources.
- The time step must be small enough for the computational stability of the numerical schemes which are used to solve the equations.

Courant-Friedrich-Lewy (CFL) criteria:

$$\Delta t \leq \Delta x / c$$





Initial conditions

The necessary initial fields to begin a model simulation are temperature, salinity, velocity and sea surface height.

1. Climatology: Climatological values from existing database.

2. Output of previous model run: Another common way to initialize a model is with fields from a previous model run.

Boundary conditions

1. Lateral Boundary
2. Ocean surface and bottom boundary

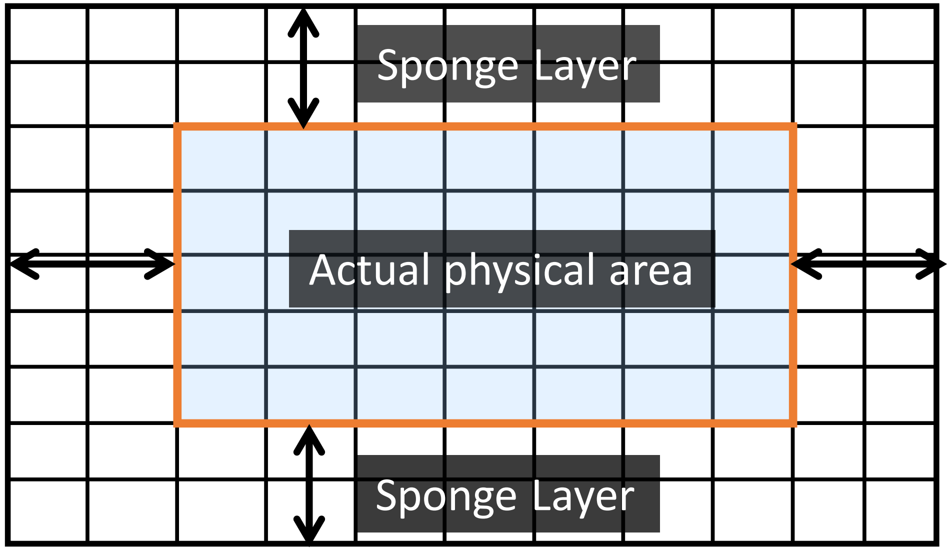
Lateral boundary conditions

- Closed boundary
  (No flows across the boundary)
  - No slip
    (No flow along and across the boundary)
  - Free slip
    (can be flow along the boundary but not perpendicular or normal to it)
- Open boundary
  (not bounded by land)

Open boundary: to allow waves and disturbances originating within the model domain to leave the domain without affecting the interior solution and to pass the information from the ocean into the domain through the boundary.

(i) **Specified boundary condition:** Boundaries can be set to climatological values or prescribed real time values from any global model output.

(ii) **Radiative/Sponge boundary condition:** to absorb waves reaching the boundary rather than having it reflect back into the model domain.



Model forcing fields

- **Wind:** from satellite or reanalysis data
- **Heat flux:** The components of surface heat flux are;

  Short-wave radiation: Rate of inflow of solar energy at the sea surface

  Long-wave radiation: Net rate of heat loss by the ocean to atmosphere and space by back radiation

  Latent heat flux: Rate of heat loss/gain by evaporation/condensation

  Sensible heat flux: Rate of heat loss/gain through the sea surface by conduction

Model forcing fields (Cont.)

- **Salinity flux:** Evaporation

  Precipitation

  River runoff

- **Tide:** The rise and fall of sea levels caused by the combined effects of the gravitational forces exerted by the Moon and the Sun, and the rotation of the Earth.

  (i) Sea surface height data from satellite altimeter

  (ii) Amplitude and phase of different tidal constituents from global tidal model

Model validation

- Model simulated parameters should be validated prior to the interpretation of the results.
- *In-situ* and satellite data can be used for the model validation.
- Different statistical measures have to be used for quantifying the difference between the model simulation and observations.
- The possible reasons for the deviation of model results from the observations should be identified.

Errors in numerical modelling

- Mathematical error is caused by the imperfections of the mathematical model in representing the physical system.
- Propagating errors arise from the errors in the forcing fields.
- Discretization error is introduced when the original equations are approximated to transform them into a computer code.
- Rounding errors are due to the fact that only a finite number of digits are used in the computer to represent real numbers.

Challenges in ocean modeling

- Variable spatial/temporal scales of various oceanic processes
- Coupling of other earth components (atmosphere, ice and land etc. to the ocean model)
- Complex topography and lateral boundaries
- Availability of a few observational measurements for validation
- Requirement of large computational power

Ocean models

- ROMS (Regional Ocean Modeling System; www.myroms.org)
- MOM (Modular Ocean Model; https://www.gfdl.noaa.gov/mom-ocean-model/)
- MITgcm (MIT general circulation model; http://mitgcm.org/)
- HYCOM (Hybrid Coordinate Ocean Model; https://hycom.org/)
- NEMO (Nucleus for European Modelling of the Ocean; https://www.nemo-ocean.eu/)
- ADCIRC (ADvanced CIRCulation model; http://adcirc.org/)
- SUNTANS (Stanford Unstructured Non-hydrostatic Terrain-following Adaptive Navier-Stokes Simulator)

Important steps of regional ocean modeling

Choose a region (area of interest)

↓

Prepare model domain with necessary horizontal resolution

↓

Build bathymetry

↓

Choose vertical resolution

↓

Interpolate the initial condition fields to the model domain

↓

Prepare forcing fields and boundary conditions

↓

Run the model

↓

Validate and analyze the outputs

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Thank You

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