Compound Interest

Example

Year end	Interest	Compound Amount	Simple Interest
0	—	100	100
1	15	115	115
2	17.25	132.25	130
3	19.84	152.09	145
4	22.81	174.9	160
5	26.24	201.13	175
6	30.17	231.31	190

$F = P(1+i)^n$ P = principal amt invested in time 0

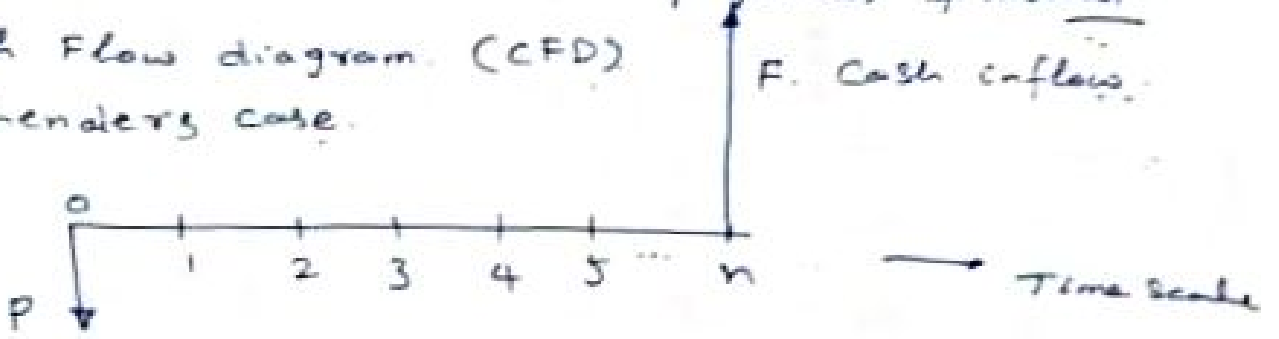
F = Future amount (compound amount)

i = Rate of interest compounded annually (Fraction).

n = no. of periods of interest.

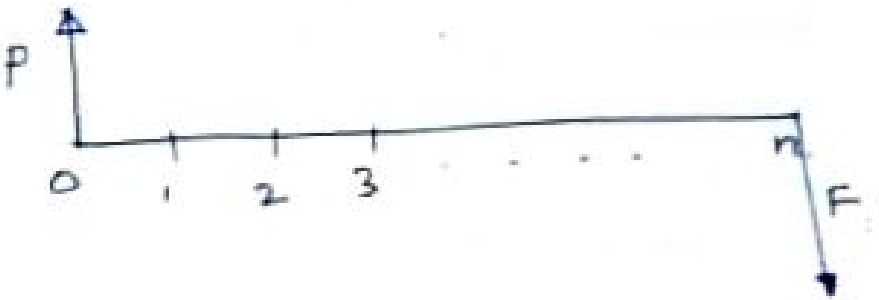
Cash Flow diagram (CFD)

Lenders case.



Cash out flow

Borrowers case.



Interest factors for discrete cash flow with end-of-period compounding

	Factor	To find	Given	Symbol
1)	Compound amount	Future worth F	Present amount P	(F/P, i%, N)
2)	Present worth	Present worth P	Future amount F	(P/F, i%, N)
3)	Sinking fund	Annuity amount A	Future amount F	(A/F, i, N)
4)	Series compound amount	Future worth F	Annuity amount A	(F/A, i, N)
5)	Capital recovery	Annuity amount A	Present amount A	(A/P, i, N)
6)	Series Present worth	Present worth P	annuity amt A	(P/A, i, N)
7)	Arithmetic gradient			
	Conversion	Annuity amt A	uniform change in amt G_1	(A/G_1) i, N)

To find the effective rate of interest

If the interest period is less than year, effective rate of interest (R). c = no. of interest periods/year. c = 4 quarterly payment, = 2 bi-annually, = 12 monthly interest

$$P(1+R) = P\left(1+\frac{i}{c}\right)^c$$

$$\therefore R = (1+i/c)^c - 1$$

(1) A person deposits a sum of Rs 10,000/ in a bank at an nominal rate of 12% for 10 years. Find the maturity amount of the deposit after 10 years, if the compounding is done quarterly.

Given $P = 10,000$, $i = 12$, quarterly interest $= \frac{12}{4} = 3\%$

no of interest period: $n = 4 \times 10 = 40$.

$$\therefore F = P\left(1+\frac{12}{4}\right)^{40} = 10,000\,(1+.03)^{40}$$

$$= 10,000\,(3.2620) = 32,620$$

Method 2

$$R = \left(1+\frac{.12}{4}\right)^4 - 1 = (1+.03)^4 - 1$$

$$= 1.1255 - 1 = .1255 = 12.55$$

$$\therefore F = P(1+R)^n = 10,000\,(1+.1255)^{10}$$

$$= 10,000\,(3.2620) = 32,620$$

(2) A person wishes to have a future sum of Rs 20 Lakhs for his son's engineering education 10 years from now. What is the single payment that he should deposit now so that he gets the desired amount after 10 years. (a) If the bank gives 12% rate of interest compounded annually b) If the rate of interest is 11% but compounded quarterly.

$(P/F, i, n)$ $\quad F = P(1+i)^n$

$\therefore P = \frac{F}{(1+i)^n}$ $\quad$ case 1 $= \frac{20,00000}{(1+.12)^{10}} = \frac{20,00000}{3.1058}$

$= 6,43,946.$

case 2 $F = P\left(1+\frac{.11}{4}\right)^{40} = P.$ $\quad$ (2.959873)

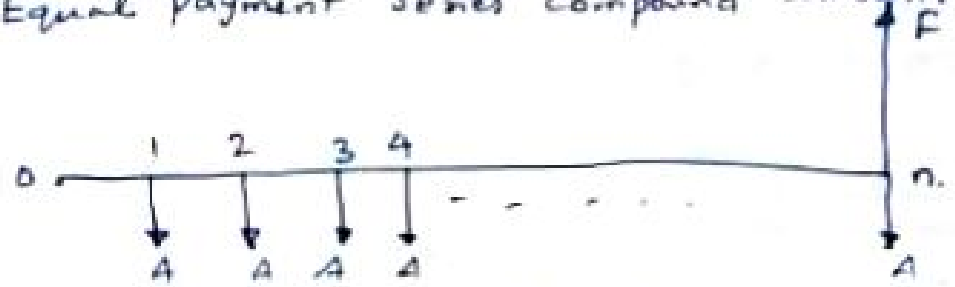
$\therefore P = \frac{20,00000}{2.959873.} = 6,75,704.$

Case 1 is better.

case 2: $R = \left(1+\frac{.11}{4}\right)^4 = 1.1146$ or 11.14%

$P = \frac{20,00000}{(1+.114)^{10}}$ $\quad \frac{20,00000}{(1+1.114)^{10}} = 6,75,704$

(3) Equal payment Series compound amount $A(F/A, i, N)$



$F = A\left[\frac{(1+i)^n - 1}{i}\right]$

(Q3) A 45 year old person is planning for his retired life. He plans to divert Rs 30,000 from his bonus as investment every year for the next 15 years. The bank gives 12% interest rate compounded annually. (1) Find the maturity value of his account when he is 60 year old (2) If he invests in Unit fund Rs 2500 every month but at 11% for the same period.

Case A $A = 30,000$, $n = 15$ years $i = \frac{12}{100}$ / $F = ?$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 30,000\left[\frac{(1+.12)^{15} - 1}{.12}\right]$$

$$= 11,18,391$$

$(1.12)^{15} = 5.4735$

$$\left[\frac{(1+.12)^{15} - 1}{.12}\right] = 37.279$$

Case B $A = 2500$/m , $i = 11$ compounded monthly $n = 15$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$A = 2,500$

$i = \frac{11}{12} = .916$ /

$= 0.00916$

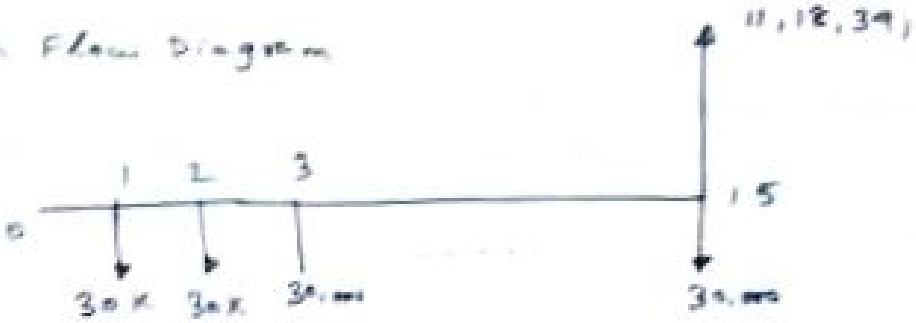
No of interest periods/year = 12

No of interest periods for 15 years, $n = 12 \times 15 = 180$

$$F = 2500\left[\frac{(1+.00916)^{180} - 1}{.00916}\right] = 11,35,875$$

Case B is better

Cash Flow Diagram



(4) Equal payment Series Sinking fund

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

A = equal amount deposited at the end of each interest period

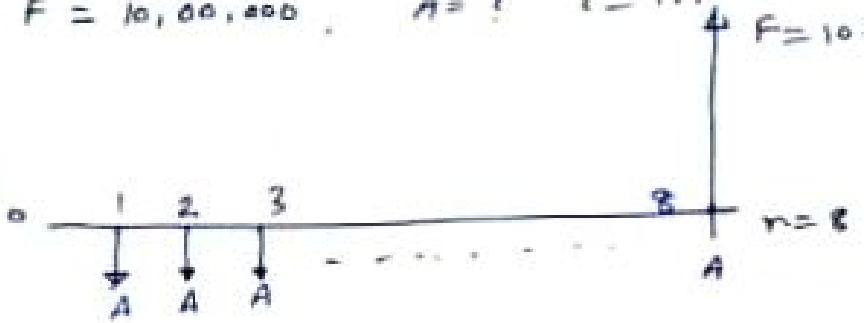
n = no. of interest periods.

i = rate of interest

F = Single Future amount at the end of nth period

(5) A person estimates an expenditure of Rs 10,00,000 for his daughters wedding about 8 years from now. He plans to deposit an equal amount at the end of every year for the next 8 years at a rate of interest of 11% compounded annually. Find the equivalent amount that must be deposited at the end every year for the next 8 years.

$F = 10,00,000$ $A = ?$ $i = 11\%$ (Compounded annually) $n = 8$ yrs



$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = 10,00,000\left[\frac{0.11}{(1+0.11)^8 - 1}\right]$$

$\therefore A = 84,321$

(6) If the same person had the choice of going to a private bank which assured him 10% rate of interest compounded monthly, should he go for it? His intention is to receive Rs 10 Lakhs after 8 years.

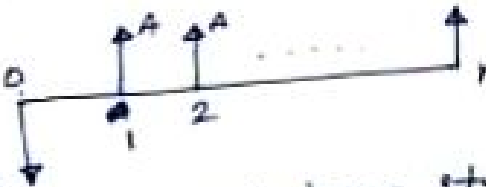
no. of interest per year $n = 12$, Total no. of interest periods $n = 8 \times 12$ - $n = 96$. $i = .10$, Rate of interest / month $\frac{0.10}{12}$

$$\therefore A = 10,00,000\left[\frac{.1/12}{(1+.1/12)^{96} - 1}\right] \quad \therefore A = 6,840/\text{month}$$

$\therefore$ per year he must pay $6,840 \times 12 = 82,089$ per year to realize 10 Lakhs 8 years. So this option is advantageous.

5 Equal payment series Present worth amount $A(P/A, i, n)$

$$\therefore P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$



(6) A person wants to give Scholarships to poor students in the tune of Rs 25,000/- every year, in memory of his late father. He wishes to deposit a lumpsum in the bank which would give him/receive the amount every year for the next 20 years. The reserve is assumed to grow annually at the rate of 9%. Find the single payment that must be made now as the reserve amount

(6) given $A = 25,000$ $i = 9\%$ $n = 20$ years $P = ?$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 25,000\left[\frac{(1+.09)^{20} - 1}{.09(1+.09)^{20}}\right]$$

$$P = 2,28,213$$

6(b) If the same person can deposit only Rs 2 lakh as reserve now for how many years will he be able to receive Rs 25,000 - 8 every year so as to give a very scholarship at the same rate of interest

$P = 2,00,000$, $A = 25,000$, $i = 0.09$. $n = ?$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad \frac{P}{A} = \frac{(1+i)^n - 1}{i(1+i)^n} \qquad \frac{2,00,000}{25,000} = \frac{(1+.09)^n - 1}{.09(1+i)^n}$$

$$.72(1+.09)^n = (1+.09)^n - 1 \qquad .72 = 1 - \frac{1}{(1+.09)^n}$$

$$\frac{1}{(1+.09)^n} = 1 - .72 = .28 \qquad \text{~~1+0.72~~}$$

$$(1+.09)^n = \frac{1}{0.28} = 3.571 \qquad 78 \text{ o } 28$$

$$n \log(1+.09) = \log 3.571 \qquad n = 14.77 \text{ years}$$

Equal payment Serial Capital recovery amount

$P(A/P, I, N)$ P is given, i & n given to find A

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad \therefore A = \frac{P(1+i)^n i}{(1+i)^n - 1}$$

.7x4

2.5

4.7

1.4

1.6

2.5

(7) ICICI bank is offering Rs 30 L home loan to a person to buy a new apartment at a interest rate of 7.5% compounded annually The amount should be repaid 15 years equal installments Find the annual installment amount the person has to pay to the bank

$$A = 30,00,000\frac{(1+.075)^{15}.075}{(1+0.075)^{15} - 1} \qquad A = 3,39,861 \text{ per annum}$$

If the bank decides to compound the rate of interest monthly rather than annually, how much money the person has to pay annually No of interest period in one year = 12

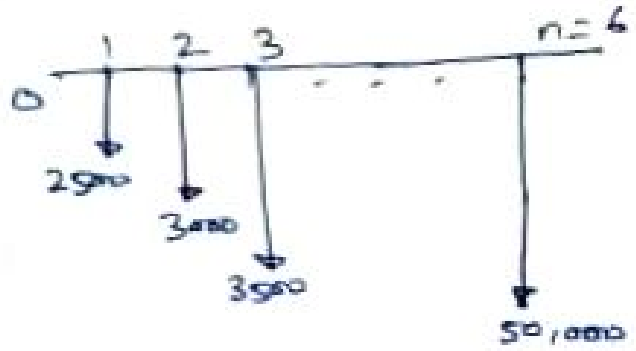
Total no of interest period $n = 12 \times 15 = 180$.

Solution to 7.1

Given $A1 = 25,000$, $G = 5,000$, $i = 11.5\%$, $n = 6$ years.

Annual equivalent $A = ?$ Maturity amount $F = ?$

CFD.



$\therefore$ Annual equivalent amount $A = A1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$

$\therefore A = 25,000 + 5000\left[\frac{(1+0.115)^6 - (0.115\times 6) - 1}{.115(1+.115)^6 - 0.115}\right]$

$A = 25,000 + 5,000\,(2.184)$.

$A = 35,924$.

$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right] = 35924\left[\frac{(1+0.115)^6 - 1}{0.115}\right]$

$F = 2,87,872$

In order to buy a car worth Rs 5 Lakhs he has to additionally put in 5,00,000 − Rs 2,87,872 = 2,12,127

7.2) A film star is at the height of his career. He wants to invest Rs 10 Lakhs from the end of this year and follow it up with 9 Lakhs, 8 Lakhs and so on for the next five years. Find the maturity amount 6 years later if the film producer agrees to pay him 15% rate of interest compounded annually.

Given $A1 = 10,00,000$, $G = 1,00,000$ $i = 15\%$, $n = 6$ years

$A = ?$ $F = ?$

Contd 7.2.

$$A = A_1 - G\left[\frac{(1+i)^n - in - i}{i(1+i)^n - i}\right]$$

-ve sign is used because of diminishing installments

$$A = 10,00,000 - 1,00,000\left[\frac{(1+.15)^6 - (0.15\times 6) - 1}{0.15(1+0.15)^6 - 0.15}\right]$$

$$A = 10,00,000 - 1,00,000\,[2.097]$$

$$A = 7,90,280$$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right] \quad \therefore F = 7,90,280\left[\frac{(1+.15)^6 - 1}{.15}\right]$$

$$F = \cancel{53,28,369}\ 69,17,912.404$$

(8) Geometric Series Present Amount.

A_1 = First year cost or revenue g = rate of increase every year

i = rate of interest n = no. of years, P_g = Present value

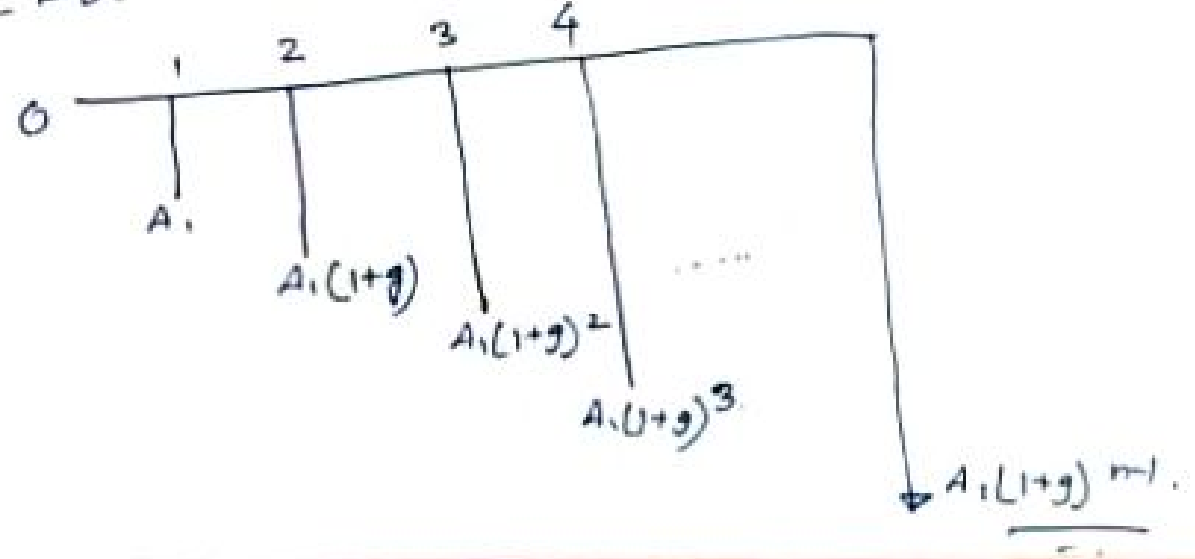
For these geometric series, the present amount is calculated for two cases i.e when $g \neq i$ & $g = i$

Case 1: when $g \neq i$

$$P_g = \frac{A_1}{(1+g)}\left[\frac{(1+g')^n - 1}{g'(1+g')^n}\right] \quad \text{where } g' = \frac{1+i}{1+g} - 1$$

Case 2. when $g = i$ $P_g = A_1\left[\frac{n}{1+g}\right]$

CFD.



(8.1) If an annual maintenance and operating costs of a plant are expected to be 4,00,000 for the first year and increases at a rate of 10% per year, determine the present value of all operating costs spent over a 30 year life. Assume rate of interest (compound) on 12%. If the rate of increase (g) per year increases to 12% instead of 10%, what is the difference in the answer

given $A_1 = 4,00,000$, $g = 10\%$, $i = 12\%$, $n = 30$ years

$g \neq i$

$$\therefore P_g = \left[\frac{A_1}{1+g}\right]\left[\frac{(1+g')^n - 1}{g'(1+g')^n}\right] \qquad g' = \frac{1+i}{1+g} - 1.$$

$$= \frac{1+.12}{1+.1} - 1 = 0.01818$$

$$P_g = \frac{4,00,000}{1+.1} \times \left[\frac{((1+0.01818)^{30} - 1)}{((0.01818)(1+0.01818)^{30})}\right]$$

$$= 83,51,505$$

If $i = g = 0.12$.

$$P_g = A_1\left[\frac{n}{1+g}\right] = 4,00,000\left[\frac{30}{1+0.12}\right]$$

$$= 107,14,285$$

~~Implicit function~~

(10) The rights to a patent have been sold under an agreement in which annual year end payments of Rs 1,00,000 are to be made in the next 10 years. What is the future sum of this annuity? What is the present worth of the annuity at an interest rate of 7%?

Given $A = 1,00,000$, $n = 10$ years, $i = 7\%$ $F = ?$ $P = ?$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A\left[\frac{(1+0.07)^{10} - 1}{.07}\right] = 1,00,000 \times 13.81645 = 13,81,645$$

$$P = F\left[\frac{1}{(1+i)^n}\right] \quad \text{or} \quad P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 13,81,645\left[\frac{1}{(1+.07)^{10}}\right] \qquad = 1,00,000\left[\frac{(1+.07)^{10} - 1}{.07(1+0.07)^n}\right]$$

$$= 7,02,360 \qquad = 7,02,360$$

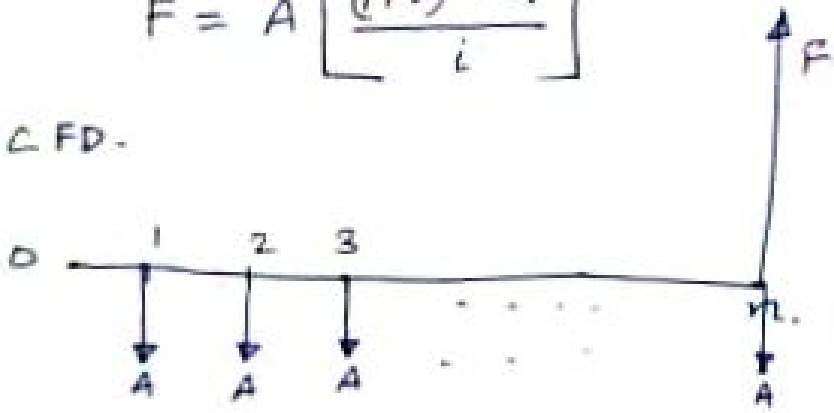
Prove.

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Equal payment

Compound amount.

CFD.



F_1 – compound amount of contribution at 1 year (A)

F_2 – – – – – – – – 2 " (A).

F_3 – – – – – – – – 3 " (A)

$$F_1 = A(1+i)^{n-1}$$

$$F_2 = A(1+i)^{n-2} \quad \ldots \text{etc}$$

$$F = F_1 + F_2 + F_3 \ldots\ldots F_{n-1}$$

$$= A(1+i)^{n-1} + A(1+i)^{n-2} + A(1+i)^{n-3} \ldots A(1+i) + A \quad \ldots (1)$$

multiply (1) by $(1+i)$.

$$F(1+i) = A(1+i)^n + A(1+i)^{n-1} + A(1+i)^{n-2} \ldots\ldots A(1+i) \quad \ldots (2)$$

$\therefore$ (2) − (1) gives:

$$F(1+i-1) = \left[A(1+i)^n + A(1+i)^{n-1} + A(1+i)^{n-2} \ldots A(1+i)\right] - \left[A(1+i)^{n-1} + A(1+i)^{n-2} \ldots A(1+i) + A\right].$$

$$F(i) = A\left[(1+i)^n - 1\right]$$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

III by Given compound Amount to find annual payment

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$