

## Drain to Source Current $I_{ds}$ versus Voltage $V_{ds}$ Relationships

$$I_{ds} = -I_{sd} = \frac{\text{Charge induced in channel}(Q)}{\text{Electron Transit Time}(\tau)}$$

$$\tau_{sd} = \frac{\text{Length of channel}(L)}{\text{Velocity}(v)}$$

$$v = \mu E_{ds}$$

$\mu$  = electron or hole mobility(surface)

$E_{ds}$  = electric field(drain to source)

$$E_{ds} = \frac{V_{ds}}{L} \quad v = \frac{\mu V_{ds}}{L}$$

Thus, Transit Time is

$$\tau_{sd} = \frac{L^2}{\mu V_{ds}} \quad (2)$$

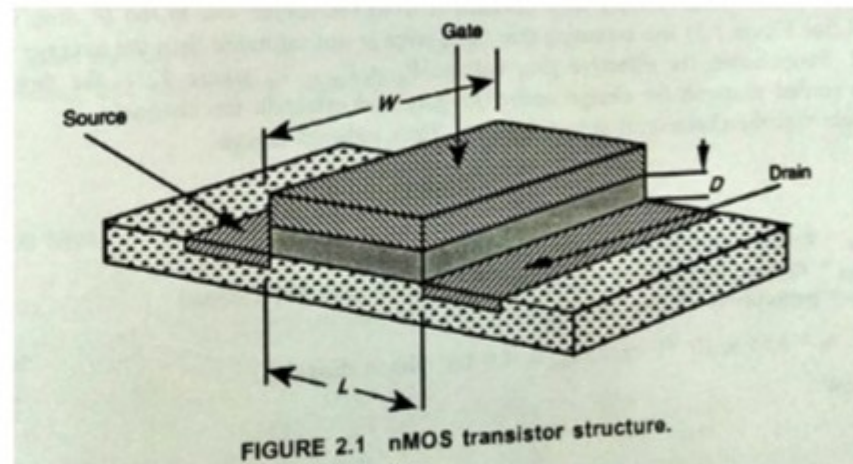


FIGURE 2.1 nMOS transistor structure.

Typical Values of  $\mu$  at room temperature

$$\mu_n = 650 \text{ cm}^2 / \text{V sec (surface)}$$

$$\mu_p = 240 \text{ cm}^2 / \text{V sec (surface)}$$

$$\tau_{sd} = \frac{L}{\mu V_{ds}} = \frac{L^2}{\mu V_{ds}}$$



### The Non Saturated region

- Charge induced in channel due to gate voltage is due to the voltage difference between the gate and channel
- Voltage along the channel varies linearly with distance  $x$  from the source due to the IR drop in channel
- Device in non saturated then the average value is  $V_{ds}/2$
- The effective gate voltage  $V_b = V_{gs} - V_t$
- $V_t$  is the threshold voltage needed to invert the charge under the gate and establish the channel

$$\text{Charge/unit area} = E_g \epsilon_{ins} \epsilon_0$$

Induced Charge is

$$Q_c = \frac{E_g}{L} \epsilon_{ins} \epsilon_0 WL$$

*Handwritten notes in pink:  $E_g = \frac{V_{gs} - V_t}{L}$*

$$\epsilon_0 = 8.85 \times 10^{-14} \text{ Fcm}^{-1}$$

$$\epsilon_{ins} = 4.0 \text{ for Silicon dioxide}$$

$E_g$  = average electric field gate to channel

$\epsilon_{ins}$  = relative permittivity of insulation between gate and channel

$\epsilon_0$  = permittivity of free space

$$E_g = \frac{\left( (V_{gs} - V_t) - \frac{V_{ds}}{2} \right)}{D}$$

D- oxide thickness



$$Q_c = \frac{WL \epsilon_{ins} \epsilon_0}{D} \left( (V_{gs} - V_t) - \frac{V_{ds}}{2} \right) \quad (3)$$

Combining equations (2) and (3) in (1), we have

$$I_{ds} = \frac{\epsilon_{ins} \epsilon_0 \mu}{D} \frac{W}{L} \left( (V_{gs} - V_t) - \frac{V_{ds}}{2} \right) V_{ds}$$

$$I_{ds} = K \frac{W}{L} \left( (V_{gs} - V_t) V_{ds} - \frac{V_{ds}^2}{2} \right) \quad (4)$$

In the non-saturated or resistive region where  $V_{ds} < V_{gs} - V_t$

$$K = \frac{\epsilon_{ins} \epsilon_0 \mu}{D}$$

The factor W/L is of course, contributed by the geometry and it is common practice to write

$$\beta = K \frac{W}{L}$$

$$I_{ds} = \beta \left( (V_{gs} - V_t) V_{ds} - \frac{V_{ds}^2}{2} \right) \quad (4a)$$

## The Saturated Region

- Saturation begins when  $V_{ds} = V_{gs} - V_t$  since at this point the IR drop in the channel equals the effective gate to channel voltage at the drain.
- The current remains constant as  $V_{ds}$  increases

$$I_{ds} = K \frac{W}{L} \frac{(V_{gs} - V_t)^2}{2} \quad (5)$$

Or we may write

$$I_{ds} = \frac{\beta}{2} (V_{gs} - V_t)^2 \quad (5a)$$

$$I_{ds} = K \frac{C_g \mu}{2L^2} (V_{gs} - V_t)^2 \quad (5b)$$

$$\begin{aligned} I_{ds} &= K \frac{W}{L} \left[ (V_{gs} - V_t)(V_{gs} - V_t) - \frac{(V_{gs} - V_t)^2}{2} \right] \\ &= K \frac{W}{L} \left[ \frac{(V_{gs} - V_t)^2}{2} \right] \end{aligned}$$

we may also write

$$I_{ds} = C_0 \mu \frac{W}{2L} (V_{gs} - V_t)^2 \quad (5c)$$

Threshold voltage for nMOS depletion mode device denoted as  $V_{td}$

## Aspects of MOS Transistor Threshold Voltage $V_t$

The threshold voltage  $V_t$  may be expressed as

$$V_t = \phi_{ms} \frac{Q_B - Q_{ss}}{C_0} + 2\phi_{fN} \quad (6)$$

Where

$Q_B$  = the charge per unit area in the depletion layer beneath the oxide

$Q_{ss}$  = Charge density at Si:SiO<sub>2</sub> interface

$C_0$  = capacitance per unit gate area

$\phi_{ms}$  = Workfunction difference between gate and Si

$\phi_{fN}$  = Fermi level potential between inverted surface and bulk Si

$$\left. \begin{array}{l} V_{SB} = 0 \text{ V}; V_t = 0.2V_{DD} (= +1 \text{ V for } V_{DD} = +5 \text{ V}) \\ V_{SB} = 5 \text{ V}; V_t = 0.3V_{DD} (= +1.5 \text{ V for } V_{DD} = +5 \text{ V}) \end{array} \right\} \begin{array}{l} \text{Similar but} \\ \text{negative values} \\ \text{for pMOS} \end{array}$$

For nMOS depletion mode transistors:

$$\left. \begin{array}{l} V_{SB} = 0 \text{ V}; V_{td} = -0.7V_{DD} (= -3.5 \text{ V for } V_{DD} = +5 \text{ V}) \\ V_{SB} = 5 \text{ V}; V_{td} = -0.6V_{DD} (= -3.0 \text{ V for } V_{DD} = +5 \text{ V}) \end{array} \right\}$$

## MOS TRANSISTOR TRANSCONDUCTANCE $g_m$ , AND OUTPUT CONDUCTANCE $g_{ds}$

Transconductance expresses the relationship between output current and the input voltage is defined as

$$g_m = \frac{\delta I_{ds}}{\delta V_{gs}} \bigg|_{V_{ds} = \text{constant}}$$

To find an expression for  $g_m$  in terms of circuit and transistor parameters, consider the charge in channel  $Q_c$  is such that

$$\frac{Q_c}{I_{ds}} = \tau$$

Where  $\tau$  is transit time. Thus change in current

$$\delta I_{ds} = \frac{\delta Q_c}{\tau_{ds}}$$

$$\tau_{ds} = \frac{L^2}{\mu V_{ds}}$$

$$\delta I_{ds} = \frac{\delta Q_c V_{ds} \mu}{L^2}$$

but change in charge

$$\delta Q_c = C_g \delta V_{gs}$$

so that

$$\delta I_{ds} = \frac{C_g \delta V_{gs} \mu V_{ds}}{L^2}$$

Now

$$g_m = \frac{\delta I_{ds}}{\delta V_{gs}} = \frac{C_g \mu V_{ds}}{L^2}$$

In saturation

$$V_{ds} = V_{gs} - V_t$$

$$g_m = \frac{C_g \mu}{L^2} (V_{gs} - V_t) \quad (2.7)$$

and substituting for  $C_g = \frac{\epsilon_{ins} \epsilon_0 WL}{D}$

$$g_m = \frac{\mu \epsilon_{ins} \epsilon_0}{D} \frac{W}{L} (V_{gs} - V_t) \quad (2.7a)$$

Alternatively,

$$g_m = \beta (V_{gs} - V_t)$$