

Theory of Computing

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1.1 Deterministic Finite Automata

- We know that $\delta = Q \times \Sigma \rightarrow Q_2$
- We want a function that takes a starting state and a string, then returns the state after the machine has read that string
- Let's define $\delta^* = Q \times \Sigma^* \rightarrow Q$
 - δ^* takes a state and a string
 - δ takes a state and a symbol
- Now, we need a recursive definition
 - Base case:
 - * Let $q_i \in Q$
 - * $\delta^*(q_i, \epsilon) = q_i$
 - Recursive step:
 - * If $q_i \in Q$, $w \in \Sigma^*$, and $c \in \Sigma$
 - * then $\delta^*(q, w \cdot c) = \delta(\delta^*(q_i, w), c)$

1.2 Aside: On Σ and Σ^*

- Σ^* is the universe of all strings over Σ
 - $\Sigma = \{0, 1\}$
 - $\Sigma^* = \{\epsilon, 0, 1, 00, 01, 11, 10, 000, \dots\}$
- We can see this recursively
 - Base step: $\epsilon \in \Sigma^*$
 - Recursive step:
 - Let $w \in \Sigma^*$, let $c \in \Sigma$
 - Then $w \times c \in \Sigma^*$
- We can see this recursion graphically

w	c	w · c	step
ϵ		-	base
ϵ	1	1	recursive
1	0	10	recursive
10	1	101	recursive

1.3 Recursively Testing 101

- Solve $\delta^*(q_1, 101)$
- $\delta(\delta^*(q_1, 10), 1)$
- $\delta^*(q_1, 10)$
- $\delta(\delta^*(q_1, 1), 0)$

1.3.1 TODO Complete Recursion Sequence

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2.1 Tuples & DFAs

- Tuples are sequences which are always finite in length
- The deterministic finite automaton shown is a 5-tuple:
 1. Q : finite nonempty set of states
 - state: configuration of logic of a machine
 2. Σ (Sigma) - input alphabet
 - alphabet: a finite, nonempty set of symbols where symbols are an object of length 1
 3. δ (Delta) - transition function
 4. $Q_0 \in Q$ - starting state
 5. $F \subset Q$ - set of final states
- For this deterministic finite automaton,
 - $\delta: Q \times \Sigma \rightarrow Q_2$

Represented as a table,

Step	State	Input	Transition
1	Q_1	1	$Q_1 \rightarrow Q_2$
2	Q_2	0	$Q_2 \rightarrow Q_1$
3	Q_1	1	$Q_1 \rightarrow Q_2$
4	Q_2	1	$Q_2 \rightarrow Q_2$

2.2 Domains & Codomains

- Domain: set of all possible function inputs
- Codomain: set of all possible outputs

2.3 Strings

- In computer science, strings are character arrays
- In mathematics, strings are sequences of symbols
- Specifically a string over an alphabet, Σ , is a sequence of symbols belonging to Σ
- ϵ is the empty string
- Concatenation: If $w_1, w_2 \in \Sigma$, $w_1 \cdot w_2 = w_1w_2$
- If $c \in \Sigma$, then $\epsilon \cdot c = c \cdot \epsilon = c$

2.4 TODO Review Recursive Definitions

- Base step: a step that can not be broken down any further, a fact that is always true regardless of the input
- Recursive step:
- Defining the length of a string over Σ
 - Base: $|\epsilon| = 0$
 - Recursive:
 - * let w be a string over Σ , and $c \in \Sigma$
 - * then $|w \cdot c| = |w| + 1$
- Using this to define $|1011|$,
 1. $|1011| = |101 \cdot 1| = |101| + 1 =$
 2. $|10 \cdot 1| + 1 = |10| + 1 + 1 =$
 3. $|1 \cdot 0| + 1 + 1 = |1| + 1 + 1 + 1 =$
 4. $|\epsilon \cdot 1| + 1 + 1 + 1 =$
 5. $|\epsilon| + 1 + 1 + 1 + 1 =$
 6. $0 + 1 + 1 + 1 + 1 = 4$

2.5 Languages

- Languages over Σ - a set of finite strings over Σ
- Languages recognized by an automaton, M , $L(M)$ is the language accepted by M
- \emptyset is the empty language
- $\epsilon \neq \emptyset$
- $\epsilon \neq \{\epsilon\}$
- ϵ is not a symbol in any alphabet

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3.1 Automaton (automata)

- Self running machine requiring a continuous power source
 - Historically used power sources include water, steam, and electricity
- Course revolves around defining the mathematics powering machines

3.2 The Mathematics of Automata

3.2.1 Mathematicians & History

- Cantor defines sets as collections of objects
- Cantor also argues that infinities can be of different magnitudes - there are infinitely more real numbers than natural numbers
- Goedel eventually derives his incompleteness theorem
 - No logical system that contains the natural numbers can prove its own soundness
 - Every sound logical system containing the natural numbers contains valid statements that cannot be proved or disproved
- In 1936, Turing proves The Halting Problem is not decidable, it is impossible

- The Halting Problem is an algorithm that can analyze any other algorithm and determine whether or not it goes into an infinite loop
- Turing creates the turing machine as an object consisting of sets and processes wherein the object can use any finite process to complete an action.
- Turing machine sets the basis for a computer, which leads to a series of important questions:
 - What can & can't a machine do?
 - What does it mean for a problem to be harder than another?
 - What does it mean for a machine to be more powerful than another?

3.2.2 Sequential Logic

- Sentential Logic- based on boolean results
 - Predicated on AND, OR, NOT
 - XOR, XAND, etc. can be derived using the above

3.3 Necessary Review

- Textbook Ch. 0
- Logic Statements
- Set Theory
- Functions

3.4 Functions

- Functions - something that maps objects from one set to another
- Given $f: a \rightarrow b$;
 - Everything in a is mapped to something in b
 - * For every x , such that x is an element of a , there exists a y , such that y is an element of b

- No one point in the domain can be mapped to two different points in the codomain
 - * Logically, you can't have a function that takes in one input and returns two different outputs
 - * If f maps $x \rightarrow y_1$ and $x \rightarrow y_2$, $y_1 = y_2$
- $$\forall x \in A \ y_1, y_2 \in B \ [f(x)=y_1 \wedge f(x)=y_2 \rightarrow y_1 = y_2]$$

3.5 TODO Types of Functions - Definition & Logical Statement

- Injective Functions
- Surjective Functions
- Proof by Induction (\forall)
- Proof by Contradiction ($\neg\exists$)

3.6 Finite Automaton (Finite State Machine)

- States are logical configurations
- States are generally based upon input
- Purpose of a state machine is to make a yes/no decision