Question 2:

```
i ← n;
while(i > 1){
 j ← i;
 while (j < n) {
  k ← 0;
  while (k < n) {
   k ← k + 2;
  }
  j ← j * 2;
 }
 i ← i / 2;
}
```

Because k increases by 2 each iteration, this k increases linearly making this O(n)

Here, J is multiplied by 2 each iteration. Because of this exponential increase and the while loop running while J < N, this is O(log(n))

Here, I decreases by half of its original value each iteration. Given this exponential decrease, it will be in O(log(n))

Using the above steps, we see that this function is composed of $O(n), O(log(n))$, and $O(log(n))$. We may multiply these together to find the final Big-O of this function, as follows:

$$O(n) = O(n) \cdot O(log(n)) \cdot O(log(n))$$
$$O(n) = n(log^2(n))$$

Question 3:

Given $a_n = a_{n-1} + 2^n$, we can show that $a_n = n2^n$ is a solution to this recurrence relationship.

Firstly, we recognize that if $a_n = n2^n$, then $a_{n-1} = (n-1)2^{n-1}$.

We can accordingly rewrite $a_{n-1} + 2^n$ as $2((n-1)2^{n-1}) + 2^n$.

From here,

$$\begin{aligned} 2((n-1)2^{n-1}) + 2^n &= 2^n(n-1) + 2^n \\ &= n2^n - 2^n + 2^n \\ &= n2^n \end{aligned}$$

$\therefore a_n = n2^n$ is a solution to $a_n = a_{n-1} + 2^n$

Question 4:

The provided Fibonacci relationship follows the below relationship:

$$T(n) = T(n-1) + T(n-2) + 1$$

Following this recurrence relationship, we can solve for the Big-O.
We start by substituting:

$$\begin{aligned} T(n) &= T(n-1) + T(n-2) + 1 \\ T(n) &= 2T(n-1) + 1 \\ T(n-1) &= 2T(n-2) + 1 \\ T(n) &= 2(2T(n-2) + 1) + 1) \\ T(n-2) &= 2T(n-3) + 1 \\ T(n) &= 2(2(2T(n-3) + 1) + 1) + 1 \end{aligned}$$

This allows us to generalize $T(n)$ as follows:

$$\begin{aligned} T(n) &= 2^n \cdot T(n-n) + 2^n - 1 \\ &= 2^n \cdot T(0) + 2^n - 1 \\ &= 2^n + 2^n - 1 \end{aligned}$$

This generalization allows us to determine that this relationship is $O(2^n)$.

Question 5:

Given the master theorem,

$$T(n) = aT(\frac{n}{b}) + f(n)$$

Wherein

$$f(n) = \Theta(n^k log^p n)$$

And given this problem's recurrence relationship,

$$T(n) = T(\frac{n}{2}) + 1$$

We know that the master theorem value correspond as follows:

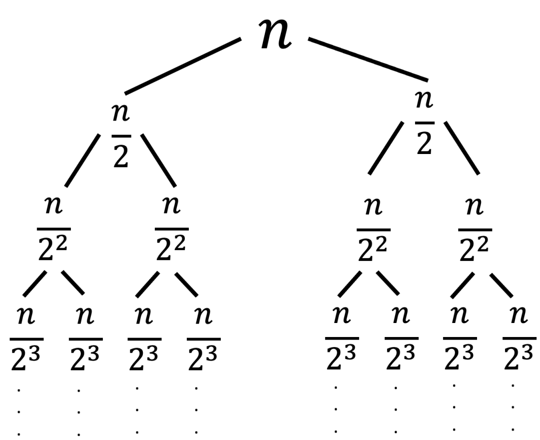
$$\begin{aligned} a &= 1 \\ b &= 2 \\ k &= 0 \\ p &= 0 \end{aligned}$$

Because $a = b^k(1 = 2^0)$, and $p > -1(0 > -1)$,

$$\begin{aligned} T(n) &= \Theta(n^{log_b^a} \cdot log^{p+1}(n)) \\ &= \Theta(n^{log_2^1} \cdot log^{0+1}(n)) \\ &= \Theta(n^0 \cdot log^1(n)) \\ &= \Theta(1 \cdot log(n)) \\ &= \Theta(log(n)) \end{aligned}$$

$\therefore$ This recurrence relation is $O(log(n))$.

Question 6:



From the graph above, we can see that there exist n levels. We can calculate the total cost as follows:

$$\begin{aligned} TotalCost &= \sum_{i=0}^{log(n-1)} \frac{n}{2^i} + n(1) \\ &= n \sum_{i=0}^{log(n-1)} \frac{1}{2^i} + n(T(1)) \\ &= n \cdot 2 + n \end{aligned}$$

$\therefore$ the Big-O of this function is O(n).