Contents

1 01.14.21

1.1 Automaton (automata)

- Self running machine requiring a continuous power source
  - Historically used power sources include water, steam, and electricity
- Course revolves around defining the mathematics powering machines

1.2 The Mathematics of Automata

1.2.1 Mathematicians & History

- Cantor defines sets as collections of objects
- Cantor also argues that infinites can be of different magnitudes - there are infinitely more real numbers than natural numbers
- Goedel eventually derives his incompleteness theorem
  - No logical system that contains the natural numbers can prove its own soundness
  - Every sound logical system containing the natural numbers contains valid statements that cannot be proved or disproved
- In 1936, Turing proves The Halting Problem is not decidable, it is impossible

  - The Halting Problem is an algorithm that can analyze any other algorithm and determine whether or not it goes into an infinite loop

- Turing creates the turing machine as an object consisting of sets and processes wherein the object can use any finite process to complete an action.

- Turing machine sets the basis for a computer, which leads to a series of important questions:

  - What can & can't a machine do?
  - What does it mean for a problem ot be harder than another?
  - What does it mean for a machine to be more powerfule than another?

1.2.2 Sequential Logic

- Sentential Logic- based on boolean results

  - Predicated on AND, OR, NOT
  - XOR, XAND, etc. can be derived using the above

1.3 Necessary Review

- Textbook Ch. 0
- Logic Statements
- Set Theory
- Functions

1.4 Functions

- Functions - something that maps objects from one set to another
- Given f: a $\rightarrow$ b;

  - Everything in a is mapped to something in b
    - For every x, such that x is an element of a, there exists a y, such that y is an element of b

- No one point in the domain can be mapped to two different points in the codomain
  - Logically, you can't have a function that takes in one input and returns two different outputs
  - If f maps x $\rightarrow$ y1 and $\rightarrow$ y2, y1 = y2

  -$\forall$ x $\in$ A $y_{1,y2} \in$ B [f(x)=y1$\wedge$ f(x)=y2 $\rightarrow$ y1 = y2]

1.5 TODO Types of Functions - Definition & Logical Statement

- Injective Functions
- Surjective Functions
- Proof by Induction ($\forall$)
- Proof by Contradiction ($\neg\exists$)

1.6 Finite Automaton (Finite State Machine)

- States are logical confirgurations
- States are generally based upon input
- Purpose of a state machine is to make a yes/no decision