(1T)

Graphical solution of LP Problem

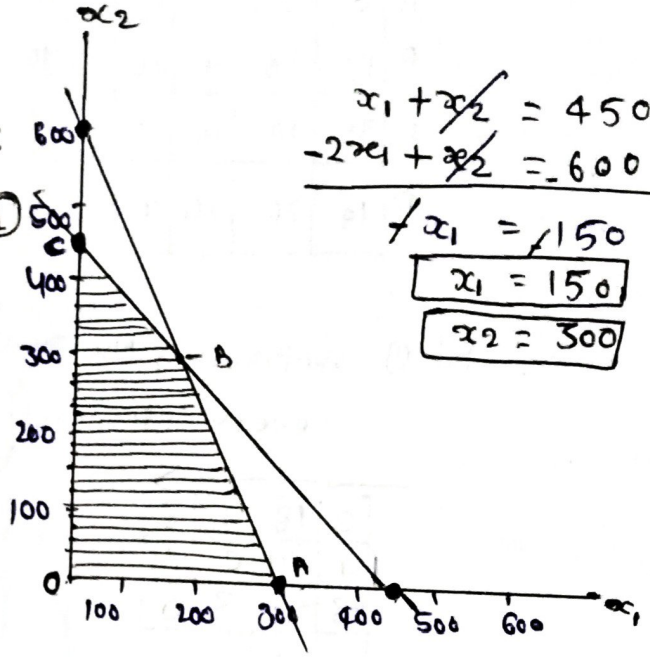
8. max $Z = 3x_1 + 4x_2$

Subject to constraints

$x_1 + x_2 \leq 450$ — (1)

$2x_1 + x_2 \leq 600$ — (2)

And $x_1, x_2 \geq 0$



$x_1 + x_2 = 450$

$-2x_1 + x_2 = -600$

$x_1 = 150$

$x_1 = 150$

$x_2 = 300$

Sol ⇒ From constraints (1)

$x_1 + x_2 = 450$

Put $x_1 = 0 \Rightarrow 0 + x_2 = 450$

$\Rightarrow x_2 = 450$

Put $x_2 = 0 \Rightarrow x_1 + 0 = 450$

$x_1 \Rightarrow 450$

From constraints (2)

$2x_1 + x_2 = 600$

Put $x_1 \Rightarrow 0 \Rightarrow 0 + x_2 = 600$

$x_2 = 600$

Put $x_2 = 0 \Rightarrow 2x_1 + 0 = 600$

$2x_1 = 600$

$x_1 = 300$

Constraints (1)

x_1	x_2
0	450
450	0

Constraints (2)

x_1	x_2
0	600
300	0

Point	Coordinate	$Z = 3x_1 + 4x_2$
O	(0, 0)	(0)
A	(300, 0)	(900, 0)
B	(150, 300)	(1650)
C	(0, 450)	(1800)

Optimal Solution max Z = (1800)

Optimal Solution max min = (900)

* Assignment problems

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How the tasks should be allocated to each person so as to minimize the total man-hours?

Sol-① subtracting the smallest element in each row from every element of that row.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Solve② Subtracting the smallest element in each colom from every element of that colom.

	I	II	III	IV
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~8~~
D	9	12	14	[0]

Colom Reduction (☐) assiment 0

0 = ④

Optimal Assiment A-I B-III C-II D-IV

8 4 19 10 = (41 hours)

Ans

Finding IBFS

* following method —

① North west corner (NWC)
② Least cost method (LCM)
③ Vogel's approximation method (VAM)

① North west Corner (NWC) ⇒

Q) obtain the initial basic feasible solution of a transportation problem whose cost and ~~rim~~ requirment table is given bl.

origin / Destination	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	34

S = 34
D = 34

* NWC method को सबसे North West वाली value से शुरू करेंगे

Solution ⇒ The problem is balanced TP as the total supply to the total demand.

Destination	D_1	D_2	D_3	Supply
O_1	2 [5]	7	4	~~5~~ 0
O_2	3 [2]	3 [6]	1	~~8~~ ~~6~~ 0
O_3	5	4 [3]	7 [4]	~~7~~ ~~4~~ 0
O_4	1	6	2 [14]	~~14~~ 0
Demand	~~7~~ 2 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	

⇒ The minimum transportation cost given by

⇒ 5×2 + 3×2 + 6×3 + 3×4 + 4×7 + 14×2

⇒ 10 + 6 + 18 + 12 + 28 + 28

⇒ Rs = **102**

② Least cost or matrix (LCM)

Q) Solve the TP using Least cost method.

	D_1	D_2	D_3	D_4	Suppy
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	35

T.S = 14+16+5 = 35

T.D = 6+10+15+4 = 35

* Total Suppply is equal to total demand.

	D_1	D_2	D_3	D_4	Suppy
O_1	6	4	1 [14]	5	14 ~~0~~
O_2	8 [6]	9 [9]	2 [1]	7	16 ~~15~~ ~~9~~ 0
O_3	4	3 [1]	6	2 [4]	~~5~~ ~~1~~ 0
Demand	~~6~~ 0	~~10~~ ~~9~~ 0	~~15~~ ~~1~~ 0	~~4~~ 0	

* LCm method को वहां से शुरू करेंगे ऊपर जहां सबसे छोटी value हो

* Demand और Suppy में जो सबसे छोटी value होगी उसी को हमेशा लेंगे

* The total minimum cost is given by,

$1\times14 + 8\times6 + 9\times9 + 2\times1 + 2\times4$

$14 + 48 + 81 + 2 + 8$

$= \boxed{RS = 156}$

* last में Suppy व Demand की value 0 आनी चाहिए।

* जिस row या col में सबसे छोटी value होगी उसी को लेंगे।

(2) Vogel's Approximation method (VAM)

(Q) Find the initial basic feasible saluation for the TP by VAM.

	D_1	D_2	D_3	D_4	Suppy
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

Solution = Total Supply = 250+300+400 = 950

Total demand = 200+250+275+250 = 950

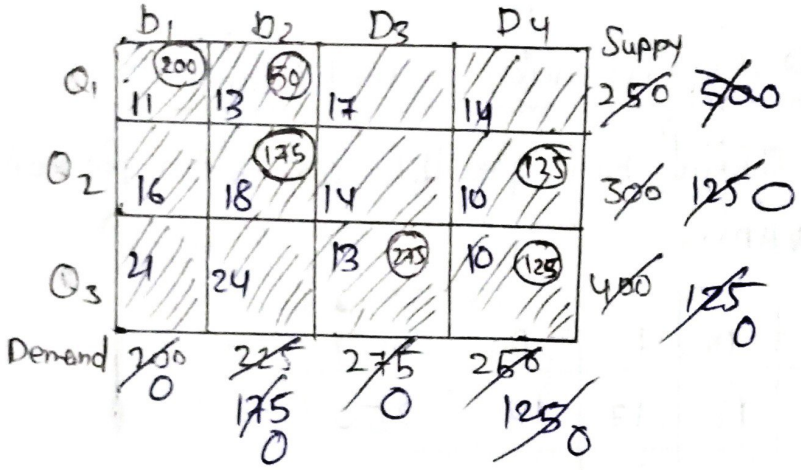
* Total Suppy is equal to total demand.

* find the penalty cost, namely the different b/w the Smallest and next smallest cost in next row and colom.

* Choose the maximum penalty.

Row P_1	P_2	P_3	P_4	P_5
2	1	–	–	–
4	4	4	4	–
3	3	3	3	3

Colom				
P_1	5↑	5	1	0
P_2		5↑	1	0
P_3		6↑	1	0
P_4			1	0
P_5	–	–	13↑	10



* हमेशा minimum Value से Solve करेंगे

	D_1	D_2	D_3	D_4	Supply
O_1	11 (200)	13 (50)	17	14	250
O_2	16	18 (175)	14	10 (125)	300
O_3	21	24	13 (275)	10 (125)	400
Demand	200	225	275	250	

No. of allocate cell = $m+n-1 = 6$

$= 11\times200 + 13\times50 + 18\times175 + 10\times125 + 13\times275 + 10\times125$

$= 2200 + 650 + 3150 + 1250 + 3575 + 1250$

$= \boxed{\text{Rs } 12075}$

Sequencing Problem

Processing n job through two machine

Q.) Find the Sequence that minimizes the total elapsed time (in hours) required to complete the following tasks on two machine.

Task	A	B	C	D	E	F	G	H	I
machine I	2	5	4	9	6	8	7	5	4
machine II	6	8	7	4	3	9	3	8	11

* जो दोनों machine में minimum value हो उसे लेंगे पहले

Solution ⇒ The optimal sequencing for the task is —

A	C	I	D	H	F	D	G	E

machine I — machine II

* machine I की value को first में लिखेंगे और II की last में

* The total elapsed time and Idle time for each machine can be obtained as follow.

Jobs	machine I		machine II	
	IN	OUT	IN	OUT
A	0	2	2	8
C	2	6	8	15
I	6	10	15	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
G	37	44	55	58
E	44	50	58	61

machine II IN 2+6 = 8

* out में जो सबसे बड़ी value होगी उसे ही machine II के IN में लिखेंगे।

Total elapsed time = (61 hours)

Idle time for machine-I = 61 - 50 = (11 hours)

Idle time for machin II = (61 - 61) + 2 = (2 hours)

* Problems with n Job Throgn 3 machine

Q.) Determine the sequence that will minimize the total elapsed time. also find Idle time of all machine.

Job	1	2	3	4	5	6
machin A	3	12	5	2	9	11
Machine B	8	6	4	6	3	1
machine C	13	14	9	12	8	13

* Solution ⇒ I ⇒ machine (A) और machin (C) मेंजो minimum होगा उसे लेंगे

II ⇒ machine (B) मे जो maximum value होगी उसे लेंगे

III ⇒ machin (A) ≥ machin (B) और machine (C) ≥ machin (B) अगर दोनो condition fobe हो जाते है तो हमारा equation Solve नही होगा

⟹ machine (A) = 2 machine (B) = 8.

machine (C) = 8

* we define two machine G and H such that

machG = A + B

machH = B + C

Job	1	2	3	4	5	6
G = A+B	11	18	9	8.	12	12
H = B+C	21	20	13	18	11	14

* Optimal Sequence for the Job.

4	3	1	6	2	5

mach- G machine H

- To find the min. total elapsed time and Idle time for machine A, B, C.

Job	machine A		machin B		machine C	
	IN	OUT	IN	OUT	IN	OUT
4	0	2	2	8	8	20
3	2	7	8	12	20	29
1	7	19	12	20	29	42
6	10	21	21	22	42	55
2	21	33	33	39	55	69
5	33	42	42	45	69	77

elapsed time

Total elapsed time = 77 Unit

Idle time for machine (A) = 77 - 42 = (35) Unit

Idle time for machine (B) = (77 - 45) + 2 + 1 + 11 + 3 = (49)

Idle time for machine (C) (77 - 77) + 8 = (8) unit

* Processing n job through m machine

Q.) four jobs 1, 2, 3, 4 and 4 are to be processed on each of the 5 machine A, B, C, D, and E in the order ABCDE find the total minimum elapsed time also find the idle time for each machine.

Job / machine	i	1	2	3	4
	A	7	6	5	8
	B	5	6	4	3
	C	2	4	5	3
	D	3	5	6	2
	E	9	10	8	6

Solve

I ⇒ find minimum(Ai) = 5 , min (Ei) = 6 , maximum(B, C, D) = 6

II ⇒ check min (A) ≥ max (B, C D) , min (E) ≥ max (B, C, D)

5 ≥ 6 (false) , 6 ≥ 6 (True)

III ⇒ convert 5 machine into two machine

Job		1	2	3	4
machine	G = (A+B+C+D)	17	21	20	16
	H = (B+C+D+E)	19	25	23	14
		X	X	X	X

IV ⇒

1	3	2	4

Job Sequence	Machine A		B		C		D		E	
	Time IN	Time Out	Time IN	Time OUT	Time IN	Time OUT	Time IN	Time OUT	Time IN	Time OUT
1	0	7	7	12	12	14	14	17	17	26
3	7	12	12	16	16	21	21	27	27	35
2	12	18	18	24	24	28	28	33	33	43
4	18	26	26	29	29	32	33	35	45	51

Travelling Salesman problem

Q.) find the Solution of travilling Salseman problem.

from city \ To city	A	B	C	D
A	∞	4	9	5
B	6	∞	4	8
C	9	4	∞	9
D	5	8	9	∞

Solve ⇒ * The number of row and colom are equal, So the matrise is balansed.

* Subtract the minimum element in each row from all the element in its row. (जो minivalue होगा उसी को सब value में से घटायेंगे row - row)

	A	B	C	D
A	∞	0	5	1
B	2	∞	0	4
C	5	0	∞	5
D	0	3	4	∞

* Subtract the minimum element in each colom from all the element in its colom (जो mini value होगा उसी को सब value में से घटायेंगे colom - coulom)

	A	B	C	D
A	∞	0	1	0
B	2	∞	0	3
C	5	0	∞	4
D	0	3	4	∞

* जिसमे 0 और ∞ होगा उसे cross कर देंगे

Total elapsed time = 51 hours

Idle time of machine:- A = (51-26) = 25 hours

B = 7+2+2+22 = 33 hours

C = 12+2+3+1+19 = 37 hours

D = 14+4+1+16 = 35 hours

E = 17+1 = 18 hour