CBNST

Computer Base numerical and Statistical Technique

* Error ⇒

The difference b/w actual value, True Value, Real Value and approximate value is called the error.

Error = |actual value − approximate val|

* Numerical *

Q⇒ The actual value $= \frac{1}{3}$, and approximate value = 0.30, 0.34, 0.33 find the lowest error?

Sol⇒

Error = |actual value − approximate v|

Erro 1 = |actual value$_1$ − approx.value$_1$|

$= \left| \frac{1}{3} - 0.30 \right|$

$= |0.333 - 0.30|$

$= |0.033|$

Error$_1$ = 0.033

$$Error_2 = |actual\ value_2 - Approx.\ value|$$

$$= |0.333 - 0.33|$$

$$= |0.003|$$

$$Error_3 = |actual\ value_3 - Approx.\ value|$$

$$= |0.333 - 0.34|$$

$$= |-0.007|$$

$$= 0.007$$

So, least error comes in <u>0.33</u>

* <u>Types of error :⇒</u>

There are three type of error.

① <u>Absolute Error :⇒</u>

Difference b/w the true value and approx. value is know as absolute error.

Formula =

$$\boxed{E_a = |x_t - x_a|}$$

Q. $x_a = 22/7 \Rightarrow 3.1428571$ and it's true value $x_t = 3.1415926$ find the absolate error?

Sol ⇒ Given that,

$$x_a = \frac{22}{7} \Rightarrow 3.1428571$$

$$x_t = 3.1415926$$

$$E_a = ?$$

So, we know that,

$$E_a |x_a - x_t|$$

$$= |3.1428571 - 3.1415926|$$

$$= |0.0012645|$$

$$E_a = 0.0012645$$

✳ **Relative Error:**

$$\left[E_R = \frac{E_A}{E_T} = \frac{\text{absolute Error}}{\text{true value}} \right]$$

where,

E_A = Absolute error.

Ex = Find relative error if $\frac{2}{3}$ is approximated to 0.667.

Sol ⇒ $\frac{2}{3} = 0.667$ = True Value = 0.666

Approximate Value = 0.667

absolute error = ?

Relative error = ?

$$\left[E_R = \frac{E_A}{E_T} \right] \text{where, } E_A \Rightarrow \left[E_T - E_{App} \right]$$

So, we get

$$E_R = \frac{|E_{True} - E_{approximate}|}{E_{True}}$$

$$= \frac{|0.666 - 0.667|}{0.666}$$

$$= \left| \frac{-0.001}{0.666} \right|$$

$$= \frac{0.001}{0.666}$$

$$= \frac{1}{666}$$

$$E_R = .0001$$

Percentage Error:

Percentage error is defined as $[E_p = 100 \times E_R]$

formula:

$$\left[E_p = 100 \times \left|\frac{E_T - E_{App}}{E_T}\right|\right]$$

Ex = If 0.333 is the approximate value of $\frac{1}{3}$ find absolute relative and Percentage error.

Sol ⇒ True value $(x) = \frac{1}{3}$

Approximate Value $(x') = 0.333$

1) Absolute Error:

$$E_A = |x - x'|$$

$$= \left|\frac{1}{3} - 0.333\right|$$

$$= 0.000333$$

2) Relative Error: $\left[E_R = \frac{E_A}{x}\right]$

$$= \frac{0.000333}{0.333333}$$

$$= 0.000999$$

3) Percentage Error:

$$E_p = 100 \times E_R$$

$$= 100 \times 0.000999$$

$$= .0999\%.$$

Q: Find the percentage error if 625.483 is approximated to three significant digits.

Sol: $E_a = |625.483 - 625| = 0.483$

$$E_r = \frac{0.483}{625.483} = 0.000772$$

$$E_p = 100 \times 0.000772$$

$$= .0772\%$$

p

Representation of floating point Number =)

* IEEE-754 floating point Standard:

There are 2 type of format =

1) Single precision format (32 bits)

83 cart 2) Double precision format (64 bits)

*1 Single precision format (32 bit) =)

Sign 1 bits	exponent 8 bits	mantissa 23 bits

*2 Double Precision format =)

sign 1 bits	exponent 11 bits	mantissa 52 bits

* Exponent formula (Biased exponent):

Single precision formet	Double precisionformet
$\boxed{E = E' + 127}$	$\boxed{E = E' + 1023}$

Q. Represent $(1259.125)_{10}$ is single precision format.

Sol⇒ 1) Convert to Binary number:

$(1259)_{10} = (10011101011)_2$

$(0.125)_{10} = (.001)_2$

$(1259.125)_{10} = (10011101011.001)_2$

2) Normalize the number.

$(10011101011.001)_2$

Now normalize it

$1.0011101011001 \times 2^{10}$

3) Using formula

$(1.N) \times 2^{(E+127)}$ → Single

$(1.N) \times 2^{E+1023}$ — Double

Where —

N = Number after Decimal point and
E = power of 2 after normalize

sion

$E' + 127 \Rightarrow 10 + 127 \Rightarrow (137)_{10}$

Sign	Exponent	mantissa bit fracti
0	10001001	00111010 11001.

$(137)_{10}$ will convert into Binary

$(137)_{10} = (10001001)_2$ This is Exponent

Q= Now for Double precision:=

we know that what is Binary of the number.

$(1259.125)_{10} = (10011101011.001)_2$

* Now it's turn to. Normalized it in form of $(1.N) \times 2^{E'}$ where N = number after decimal and E' = Power of 2

$1.0011101011001 \times 2^{10}$

Now, using formula

$[E = E' + 1023]$

d

$E = 10 + 1023$
$= 1023$

Now change the value of Exponent into Binary.

$(1033)_{10} - (10000001001)_2$

Sign	Exponent	mantissa / fraction
0	10000001001	001110101001..0

Important

Q. The value of a float type variable is represented using the single precision 32 bit floating point format IEEE-754 Standard that uses 1 bit for sign, 8 bit for biased Exponent and 23 bits for mantissa A float type variable x is assigned the decimal value of -14.25 The representation of x in hexadecimal notation is

A) C1640000H
B) 416C0000H
C) 41640000H
D) C16C0000H

Sol: Since, number is negative so *sign bit* will be 1 now, convert 14.25 in to Binary

$(14.25)_{10} = (1110.01)_2$

Now,

Normalize it

$1110.01 = 1.11001 \times 2^3$

Biased Exponent add 127;

$E = 3 + 127$ $(E = E' + 127)$

$= 130$

Now convert it into Binory

$(130)_{10} = (10000010)$

mantissa : 110010.....0 (Total 23 bits)

Number Represent in IEE-754 Single precision

1 10000010 11001000000000000

In Hex (Group of 4 bits)

1100 0001 0110 0100 0000 000 0000

and the num becomes : C1640000

* Bisection Method :=>

This method is based on the repeated application of intermediate value property

let, $f(x)$ be continous b/w a & b and let, *$f(a)$ be +ve* & *$f(b)$ be −ve*

Then the first approximation of the root is

$$\boxed{x_1 = \frac{a+b}{2}}$$

Then x_1 is the root of $f(x)=0$ Otherwise, the root lines between $(a \,\&\, x_1)$ or $(x_1 \,\&\, b)$ according as $f(x_1)$ is positive or negetive

Then we bisect the interval as before and contince the process until the root is found accuracy.

Q-1 $x^3 - x - 4$ find the root.

I $f(0) = 0 - 0 - 4$

$f(1) = 1^3 - 1 - 4 = -4$

$f(2) = 2^3 - 2 - 4 = 2$

root b/w $a = 1$, $b = 2$

II formula $\Rightarrow x_1 = \dfrac{a_0 + b_0}{2}$

$a = 1$, $b = 2$ $\quad \dfrac{1+2}{2} = \dfrac{3}{2} = (1.5)$

III (1.5) put in equation $f(x) = x^3 - x - 4$

$f(1.5) = (1.5)^3 - 1.5 - 4$

$= -2.125$

iv Sign (−ve) & we know $a = (-ve)$

we put in $a_0 = (1.5)$,

in formula

$\dfrac{a_1 + b_0}{2} = \dfrac{1.5 + 2}{2} = 1.75$

(1.75) put in equation $f(x) = x^3 - x - 4$

$f(x) = f(1.75) = (1.75)^3 - 1.75 - 4 = -0.3906$

Sign is (−ve) & we know $a = (-ve)$

$= a_0 = (1.75)$, $b = 2$

DATE :

$$\frac{a+b}{2} = \frac{1.75+2}{2} = 1.875$$

(1.875) put in equation $f(x) = x^3 - x - 4$

$f(1.875) = (1.875)^3 - 1.875 - 4 = +0.716$

{ In the equation sign (+ve) & a = (−ve)
b = (+ve) in formula so = b = 1.875, a = 1
& other condition | process will be same }

iv)

$\frac{1+1.875}{2} = [1.4375]$ — Put in eq =

$f(x) = x^3 - x - 4$

$f(1.437) = (1.437)^3 - (1.437) - 4$

$= -2.469$

{ Sign is (−ve)
$a = 1.437$, $b = 2$ = in formula

v)

$$\frac{1.4375+2}{2} = 1.718$$

(1.718) put in equation $f(x) = x^3 - x - 4$

$f(1.718) = (1.718)^3 - (1.718) - 4 = -0.647$

(Siogn (−ve) & a = 1.718, b = 2 in for)

$= \frac{1.718+2}{2} = 1.859$

$\Rightarrow$ $I^{st} = (1.5)$
$II^{nd} = (1.75)$
$III^{rd} = (1.875)$
$IV = (1.4375)$
$V = (1.71875)$
bisection method iv sarting

Q2 $\Rightarrow$ $x^3 - 4x - 9$ Find the real root

$$\left\{ \begin{array}{l} f(2) = 8 - 4(2) - 9 = -9 \\ f(3) = 27 - 4(3) - 9 = 6 \end{array} \right\}$$

root line consist b/w $(2, 3)$

2	3
$(-ve)$	$(+ve)$
a	b

formula $= x_1 = \frac{a+b}{2}$

I^{st} $\frac{2+3}{3} = 2.5$

(2.5) in equation $= x^3 - 4x - 9$
$(2.5)^3 - 4(2.5) - 9 = -3.375$

Sign $(-v)$ $a = 2.5$, $b = 3$

IInd $\frac{2.5+3}{2} = 2.75 \Rightarrow$

(2.75) in equation $= x^3 - 4x - 9$

$(2.75)^3 - 4(2.75) - 9 = 0.796$

sign $(+ve)$ $a = 2$, $b = 2.75$

IIIrd $\frac{2+2.75}{2} = 2.375$

(2.375) in equation $= x^3 - 4x - 9$

$(2.375)^3 - 4(2.375) - 9 = 2.296$

$(+ve)$ sign $= 2.296$

$a = 2$, $b = 2.375$

IVth $\frac{2+2.375}{2} = 2.1875$

(2.1875) in equation $= x^3 - 4x - 9$

$(2.1875)^3 - 4(2.1875) - 9 = -7.284$

sign$(-ve)$ $a = 2.1875$ $b = 3$

Regula falsi method

Q.1⇒ Find a real root $x^3 - 2x - 5 = 0$ using the method false position upto 4 iteration

Solve ⇒ $f(2) = 8 - 4 - 5 = -1$

$f(3) = 27 - 6 - 5 = 16$

$a = 2$ $\quad$ $b = 3$

$f(a) = -1$ $\quad$ $f(b)\ 16$

Formula $\boxed{x = \dfrac{af(b) - bf(a)}{f(b) - f(a)}}$

$$x = \frac{2 * 16 - 3(-1)}{16 - (-1)} = \frac{32+3}{16+1} = \frac{35}{17} = 2.058$$

(2.0588) put in equation ⇒ $x^3 - 2x - 5$

$f = (2.0588)^3 - 2(2.0588) - 5 = -0.3908$

Sign is negetive $(-)$

$a = 2.0588$ $\quad$ $b = 3$

$f(a) = -0.3908$ $\quad$ $f(b) = 16$

$$\frac{2.0588(16) - 3(0.3908)}{16 + 0.3908} = 2.0812$$

(2.0812) put in equation $= x^3 - 2x - 5$

$(2.0812)^3 - 2(2.0812) - 5 = -0.1479$

$a = 2.0812 \qquad b = 3$

$f(a) = -0.1479 \qquad f(b) = 16$

$$x = \frac{16(2.0812) + 3(0.1479)}{16 + 0.1479}$$

$= 2.0896$

(2.0896) put on equation $x^3 - 2x - 5$

$f(x) = (2.0896)^3 - 2(2.0896) - 5 = -0.0551$

$a = 2.0896 \qquad b = 3$

$f(a) = -0.0551 \qquad f(b) = 16$

$$x = \frac{16(2.0896) + 3(0.3551)}{16 + 0.0551}$$

$= \boxed{2.0927}$ ans

Q.2 Find a root of an equation $f(x) = x \log_{10} x = 1.2$

Sol => Here $x \log_{10} x = 1.2$

$$x \log(10x) - 1.2 = 0$$

let $f(x) = x \log(10x) - 1.2$

x	0	1	2
$f(x)$		-0.2	1.4021

1

Ist $f(1) = 0.2 < 0$ & $f(2) = 1.4021 > 0$

now root line b/w $x_0 = 1$ & $x_1 = 2$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = = 1 - (-0.2) \cdot \frac{2 - 1}{1.4021 - 1 - 0.2} = 1.1248$$

$f(x) = f(1.1248) = 1.1248 \log 1.1248 - 1.2 = 0.0177 < 0$

IInd Here $f(1.1248) = -0.0177 < 0$ & $f(2) = 1.4021 > 0$

root line b/w $x_0 = 1.1248$ & $x_1 = 2$

$$x_3 = 1.1248 - (-0.0177) \frac{2 - 1.1248}{1.4021 - (0.0177} = 1.1357$$

$f(x_3) = f(1.1357) = 1.1357 \log 1.1357 - 1.2 = -0.00152 < 0$

IIIrd

$f(1.1357) -0.0015 < 0$ & $f(2) = 1.402 > 0$

Root lie b/w $x_0 = 1.1357$ & $x_1 = 2$

$$x_4 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_4 = 1.1357 - (-0.0015) \cdot \frac{2 - 1.1357}{1.4021 - (-0.0015)}$$

$= 1.1367$

$f(x_4) = (1.1367) = 1.1367 \log 1.1367 - 1.2 = -0.0011 < 0$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
1	-0.2	2	1.4021	1.1242	-0.0177
1.1298	-0.0177	2	1.4021	1.1357	-0.0015
1.1357	-0.0015	2	1.4021	1.1367	-0.0001

Newton Raphson method

Formula $\left[x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \right]$

Q=1 ⇒ $f(x) = 2x^3 - 3x - 6$ $\quad f'(x) = 6x^2 - 3$ by $(x^n\ n-1)$

$$\cancel{f(1) = 2(1)^3 - 3(1) - 6 = -7}$$

$$f(2) = 2(2)^3 - 3(2) - 6 = 4$$

root line b/w = $(1, 2)$

$$f'(x) = \frac{d}{dx} = (2(3x^2) - 3$$

$$= 6x^2 - 3$$

$$x_{n+1} = x_n - \frac{f(x_0)}{f'(x_n)}$$

$$\cancel{x_{n+1} = x_0 - \frac{2x_0^3 - 3x_0 - 6}{6x_0^2 - 3}}$$

$$\frac{6x^3 - 6 - 3x_0 - 2x_0^3 + 3x_0 + 6}{6x_0^2 - 3}$$

formula $x^n = nx^{n-1}$

$$x_1 = \frac{4x^3 + 6}{6x_0^2 - 3}$$

$$\frac{4\times(2)^3+6}{6(2)^2-3} = \frac{38}{21} = 1.8095$$

$$x_2 = \frac{4(x_1)^3+6}{6x_1^2-3}$$

$$= \frac{4(1.8.09524)^3+6}{6(1.809524)^2-3}$$

$$x_2 = 1.784200$$
$$x_3 = 1.783769$$
$$x_n = 1.783769$$

11/12/21.

Q.2⇒ find a root of on equation (f(x) 2* cos(x) − x

Sol ⇒ Here $2 \cdot \cos(x) - x = 0$

let $f(x) = 2 \cdot \cos(x) - x$

$f'(x) = -2\sin x$

x	0	1	2
$f(x)$	2	0.0806	−2.8323

Here $f(1) = 0.0806 > 0$ & $f(2) = -2.8323 < 0$

root line b/w (1 & 2)

$$x_0 = \frac{1+2}{2} = 1.5$$

$$\boxed{x_0 = 1.5}$$

1st $f(x_0) = f(1.5) = 2 \cdot \cos(1.5) - 1.5 = -1.3585$

$f'(x)\ f'(1.5) = -2\sin(1.5) = -1995$

$$\boxed{x_1 = x_0 = \frac{f(x)}{f'(x)}}$$

$$x = 1.5 - \frac{-1.3585}{-1.995}$$

$$\boxed{x_1 = 0.819}$$

$\underline{\underline{II^{nd}}}$ $f(x_1) = f(0.819) = 2\cos(0.819) - 0.819 = 0.5468$

$f'(x_1) = f(0.819) = -2\sin(0.819) = -1.461$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

$$x_2 = 0.819 - \frac{0.5468}{-1.461} = 1.1933$$

$$\boxed{x_2 = 1.1933}$$

$\underline{\underline{III^{rd}}}$ $f(x_2) = f(1.1933) = 2\cos(1.933) - 1.933 = -0.4562$

$f'(x_2) = f(1.1933) = -2\sin(1.933) = -1.8592$

$$\boxed{x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}}$$

$$x_3 = 1.933 - \frac{-0.4562}{-1.8592} = 0.948$$

$$\boxed{x_3 = 0.948}$$

$\underline{\underline{IV^{th}}}$ $f(x_3) = f(0.948) = 2\cos(0.948) - 0.948 = 0.2187$

$f'(x_3) = f'(0.948) = -2\sin(0.948) = -1.6245$

$$x_4 = x_3 - \frac{f(x}{f'(x)}$$

$$x_4 = 0.948 - \frac{0.2187}{-1.6245}$$

$$\boxed{x_4 = 1.0826}$$

5th $\boxed{x_5 = 1.0008}$

6th $\boxed{x_6 = 1.0474}$

7th $\boxed{x_7 = 1.0198}$

x	$f(x_0)$	$f'(x_0)$	x_1
1.5	-1.3585	-1.995	0.819
0.819	0.5468	-1.461	1.1933
1.1933	-0.4562	-1.8592	0.948
0.948	0.2187	-1.6245	1.0826
1.0826	-0.1445	-1.7664	1.008
1.008	0.0785	-1.6838	1.0474
1.0474	-0.0478	-1.7323	1.0198
1.0198	0.0272	-1.704	1.0358

Gaass elimination Direct method

Guass seidel method

Q.1 ⇒ $2x + y + z = 5$

$3x + 5y + 2z = 15$

$2x + y + 4z = 8$

$|2| > |1| + |1|$

$|5| > |3| + |2|$

$|4| > |2| + |1|$

from the equation

$2x = |5 - y - z|$

$\Rightarrow x = \frac{1}{2} |5 - y - z| \quad — ①$

$5y = |15 - 3x - 2z|$

$y = \frac{1}{5} |15 - 3x - 2z| \quad — ②$

$4z = |8 - 2x - y| \quad — ③$

$z = \frac{1}{4} |8 - 2x - y|$

initial guass $(x = 0, y = 0)$

1st approximation

$x_1 = \frac{1}{2}|5 - y - z|$

$\frac{1}{2}|5 - 0 - 0|$

$\left|\frac{5}{2} = 2.5\right|$

$\boxed{x_1 = 2.5}$

$y_1 = \frac{1}{5}|15 - 3x - 2z|$

$\frac{1}{5}|15 - 3(2.5) - 2(0)|$

$\left|\frac{7.5}{5} = 1.5\right|$

$\boxed{y_1 = 1.5}$

$z_1 = \frac{1}{4}|8 - 2x - y|$

$\frac{1}{4}|8 - 2(2.5) - (1.5)|$

$\left|\frac{1.5}{4} = 0.375\right|$

$\boxed{z_1 = 0.375}$

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IInd $x_2 = \frac{1}{2}\left| 5 - 1.5 - 0.375 \right|$

$\boxed{x_2 = 1.562}$

$y_2 = \frac{1}{5}\left| 15 - 3.(1.562) - 2(0.375) \right|$

$\boxed{y_2 = 1.912}$

$z_2 = \frac{1}{4}\left| 8 - 2(1.562) - (1.912) \right|$

$\boxed{z_2 = 0.741}$

IIIrd $x_3 = \frac{1}{2}\left| 5 - (1.912) - (0.741) \right|$

$\boxed{x_3 = 1.173}$

$y_3 = \frac{1}{5}\left| 15 - 3(1.173) - 2(0.741) \right|$

$\boxed{y_3 = 1.999}$

$z_3 = \frac{1}{4}\left| 8 - 2(1.173) - 1.999) \right|$

$\boxed{z_3 = 0.913}$

4th

$$x_4 = \frac{1}{2}\left| 5-(1.999)-(0.913) \right|$$

$$\boxed{x_4 = 1.044}$$

$$y_4 = \frac{1}{5}\left| 15-3(1.044)-2(0.913) \right|$$

$$\boxed{y_4 = 2.008}$$

$$z_4 = \frac{1}{4}\left| 8-2(1.044)-(2.008) \right|$$

$$\boxed{z_4 = 0.0976}$$

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Matrix inversion method

Q.1 Solve equation $2x + y = 8$, $x + 2y = 1$ using inversion matrix method.

$2x + y = 8$
$x + 2y = 1$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, b = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$AX = B$
$X = A^{-1} B$

$$|A| = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$= 2 \times 2 - 1 \times 1$
$= 4 - 1 = 3$

Here, $|A| = 3 \neq 0$
A^{-1} is posible

Cofactor $\Rightarrow$ $Adj(A) = Adj \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} +(2) & -(1) \\ -(1) & +(2) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

now $A^{-1} = \frac{1}{|A|} \times adj\ (A)$

Here $x = A^{-1} \times B$

$x = \frac{1}{|A|} + Adj\ (A) * B$

$$= \frac{1}{3} \times \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 16 + (-1) \\ -8 + 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 15 \\ -6 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$(x = 5) \ (y = -2)$

Q.2 → Solve equation $2x + y + z = 5$,
$3x + 5y + 2z = 15$

$2x + 7y + 4z = 8$ using inverse m.m

~~$3x + 5y + 2z = 15$~~

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 15 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 1 & 1 & 4 \end{vmatrix}$$

$$= 2\begin{vmatrix} 5 & 2 \\ 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} + 1\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 2(20-2) - 1(12-4) + 1(3-10)$

$= 2(18) - 1(8) + 1(-7)$

$= 36 - 8 - 7$

$= \underline{21}$

cofactor $\Rightarrow$ $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} & - & \\ - & & - \\ & - & \end{bmatrix}$

$$\begin{bmatrix} (20-2) & -(12-4) & (3-10) \\ -(4-1) & (8-2) & -(2-2) \\ (2-5) & -(4-3) & (10-3) \end{bmatrix} = \begin{bmatrix} 18 & -8 & 7 \\ -3 & 6 & 0 \\ -3 & -1 & 7 \end{bmatrix}$$

$\mathrm{adj}(A) = \begin{bmatrix} 18 & -3 & -3 \\ -8 & 6 & -1 \\ -7 & 0 & 7 \end{bmatrix}$ now, $A^{-1} = \frac{1}{|A|} \times \mathrm{Adj}(A)$

Here $x = A^{-1} \times B$

$\left(x = \frac{1}{|A|} \times \mathrm{Adj}(A) \times B \right)$

$$= \frac{1}{21} \begin{bmatrix} 18 & -3 & -3 \\ -8 & 6 & -1 \\ -7 & 0 & 7 \end{bmatrix} \begin{bmatrix} 5 \\ 15 \\ 8 \end{bmatrix}$$

$$= \frac{1}{21} \begin{bmatrix} 18 \times 5 - 3 \times 15 - 3 \times 8 \\ -8 \times 5 + 6 \times 15 - 1 \times 8 \\ -7 \times 5 + 0 \times 15 + 7 \times 8 \end{bmatrix}$$

$$= \frac{1}{21} \begin{bmatrix} 21 \\ 42 \\ 21 \end{bmatrix}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix} \quad x = 1,\ y = 2,\ z = 1$$

(Gauss Elimanation block Substitution)

Solve the sum by Guass elimination method

$$x - y + 2z = 3$$
$$x + 2y + 3z = 5$$
$$3x - 4y - 5z = -13$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -13 \end{bmatrix}$$

$$AX = B$$

$$(A:B) = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 3 & -4 & -5 & -13 \end{array}\right] \quad R_2 - R_2 - R_1$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 6 & 2 & 3 & 2 \\ 3 & -4 & -5 & -13 \end{array}\right] \quad R_3 - R_3 - R_1$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 2 & 3 & 2 \\ 0 & -1 & -11 & -22 \end{array}\right] \quad R_3 - 3R_3 + R_2$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -32 & -64 \end{array}\right]$$

$x - y + 2z = 3$ — ①

$3y + z = 2$ — ②

$-3z + z = 64$ — ③

$= z = \frac{64}{32} = ② - z = 2$

$\boxed{z = 2}$ Put on eq (ii)

$$\left\{ \begin{aligned} 3y + z &= 2 \\ y &= 0 \end{aligned} \right\}$$

Put $= y = 0 , z = 2$ in (i)

$x - 0 + 2(2) = 3$ $\quad x - 4 = 3 = 3 - 4$

(-1)

$\boxed{x = -1 , y = 0 , z = 2}$

DATE :

Q2) $2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

convert into matrix form

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix} \quad \text{or} \quad \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_2 - R_2 - 1.5 \times R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & 0.5 & 1.5 & 3 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_3 - R_3 - \frac{2}{0.5} \times R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & 0.5 & 1.5 & 3 \\ 0 & 3.5 & 8.5 & 11 \end{array}\right]$$

$R_3 - R_3 - 7 \times R_2$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & 0.5 & 1.5 & 3 \\ 0 & 0 & -2 & -10 \end{array}\right]$$

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(i) $2x + y + z = 10$

$0.5y + 1.5z = 3$

$-2z = -10$

(iii) $-2z = -10$

$z = \dfrac{-10}{-2} = 5$

(ii) $0.5y + 1.5z = 3$

$0.5y + 1.5(5) = 3$

$0.5y + 7.5 = 3$

$0.5y \pm 3 - 7.5$

$\cancel{0.5y = -4.5}$

$\cancel{y = -4.5 \times 2}$

17/12/21

i) Forword table ⇒

constant the forword different table given that.

x	y	Δ	Δ^2	Δ^3	Δ^4
0	1				
1	1	0	8		
2	9	8	14	6	0
3	31	22	20	6	0
4	73	42	26	6	0
5	141	68	32	6	
6	241	100			

ii) Backword table ⇒

constant the backword different table given that.

x	y	∇	∇^2	∇^3	∇^4	∇^5
20	354	-22				
25	332		-19	29		
30	241	-41	10		-37	-37
35	260	-31	2	-8	0	
40	231	-29	2	0		
45	204	-27				

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Q) The population of the town was as given below stimate the population of the year 1896

Sol ⇒

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1893	48				
		21			
1903	69		-7		
		14		1	
1913	83		-6		4
		8		5	
1923	91		-1		
		7			
1933	98				

$$u = \frac{x - x_0}{h} = \frac{1896 - 1893}{10} = \frac{3}{10} = 0.3$$

$$y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y_0}{2!} + \frac{u(u-1)(u-2)\Delta^3 y_0}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 y_0}{4!}$$

$$48(0.3(21)) + \left(\frac{0.3(0.3-1)-7}{2!}\right) + \left(\frac{0.3(0.3-1)(0.3-2)(9)}{3!}\right)\left(\frac{0.3(0.3-1)(0.3-2)(0.3-3)4}{4!}\right)$$

$$48[6.3] + [-3.605] + [0.6595] + [-0.0462]$$

[50.~~31288~~] Ans

50.5383

Q2) from the data given below find the no of student who secured mark b/w 45

mark x	Student y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	35				
		49			
50	84		23		
		72		+54	
60	156		-31		67
		41		-13	
70	197		-18		
		23			
80	220				

$$u = \frac{x - x_0}{h} = \frac{45-40}{10} = \frac{5}{10} = 0.5$$

$$y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y}{2!} + \frac{u(u-1)(u-2)\Delta^3 y}{3!} + \frac{u(u-1)(u-2)(u-3)}{4!}$$

$$35 + \left[0.5(49)\right] + \left[\frac{0.5(0.5-1)\,23}{2!}\right] + \left[\frac{0.5(0.5-1)(0.5-2)}{3!} +54\right] + \left[\frac{0.5(0-5-1)(0.5-2)(0.5-3)\,67}{4!}\right]$$

$$35 + [24.5] + [-2.875] + [\cancel{-8.9375}\ 1.125] + [-2.6171875]$$

$$[55.9328] \text{ Ans}$$

$$[\cancel{55.13282}] \text{ Ans}$$

$$\boxed{57.3828}$$

8) The following were taken an a day find 3am

Time	2am	6am	10am	14am	
Tyam	40.2°C	42.4°C	51°C	72.4°C	

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
2	40.2°C			
		2.2		
6	42.4°C		6.4	
		8.6		6.4
10	51°C		12.8	
		21.4		
14	72.4°C			
~~14~~				

$$u = \frac{x - x_0}{h} = \frac{3-2}{4} = \frac{1}{4} = 0.25$$

$$40.2 + 0.25(2.2) + \left(\frac{0.25(0.25-1)6.4}{2!}\right) + \left(\frac{0.25(0.25-1)(0.25-2)6.4}{3!}\right)$$

$$= 40.2 + (0.55) + (-0.6) + (0.35)$$

$$= (40.5) \text{ Am}$$

57.3828

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Q) find a number of men getting wages b/w rupees 20 and 25 from following table ?

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
20	41				
		62			
40	103		3		
		65		-18	
60	168		-15		+ ~~36~~ 0
		50		-18	
80	218		-33		
		17			
100	235				

$$u = \frac{x - x_0}{h} = \frac{25-20}{20} = \frac{5}{20} = 0.25$$

$$41 + 0.25(62) + \left(\frac{0.25(0.25-1)3}{2!}\right) + \left(\frac{0.25(0.25-1)(0.25-2)(-18)}{3!}\right) + \left(\frac{0.25(0.25-1)(0.25-2)(0.25-3)(-0)}{4!}\right)$$

$$41 + (15.5) + (-0.0703125) + (0.984375) + (\cancel{-1.587597}\ 0)$$

$$= (53.876455)$$

53.2734

Newton Backword Interpolution ⇒

formula = $y_n + u\nabla y_n + \left(\frac{u(u+1)\nabla^2 y}{2!}\right) + \left(\frac{u(u+1)(u+2)\nabla^3 y}{3!}\right)$
$+ \left(\frac{u(u+1)(u+2)(u+3)\nabla^4 y}{4!}\right)$

Q) from the following table of half year primeum for policy maturing and different find the primeum for policy 64

Age	46	51	56	61	66
Primeum	116	97	83	74	68

Age x	Primuem y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
46	116				
		−19			
51	97		5		
		−14		0	
56	83		5		−2
		−9		−2	
61	74		3		
		−6			
66	68				

$$u = \frac{x - x_0}{h} = \frac{64 - 66}{5} = \frac{-2}{5} = (-0.4)$$

$$68 + \left(-0.4(+6)\right) + \left(\frac{-0.4(-0.4+1)}{2!}\,3\right) +$$

$$\left(\frac{-0.4(-0.4+1)(-0.4+2)}{6}(-2)\right) + \Big((1-0.4)$$

$$\left.\frac{(-0.4+1)(-0.4+2)(-0.4+3)}{24} - 2\right)$$

$$= 68 + (2.4) + (0.128) + (-0.124933)$$

$$= (70.403067)$$

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Q) find y at $x = 42$ from the following table =

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	354					
		-22				
25	332		-19			
		-41		29		
30	291		10		-37	
		-31		-8		29
35	260		2		-8	
		-29		0		
40	231		2			
		-27				
45	204					

$$u = \frac{x - x_0}{h} = \frac{42 - 45}{5} = \frac{-3}{5} = -0.6$$

$$204 + \left[-0.6(-27)\right] + \left[\frac{-0.6(-0.6+1)2}{2}\right] + \left[\frac{0.6(-0.6+1)(-0.6+2)0}{6}\right] + \left[\frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)-8}{24}\right] + \left[\frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)29}{120}\right]$$

$$= 204 + [16.2] + [-0.24] + [0] + [-0.374933] + [-1.302912]$$

$$= [218.282155] \text{ Ans}$$

9) The population of the town in the Secunce is given below stimate is the increge in the populatfon deruring peroid 1896 to 1916

Year	Population	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Forword table =>

$$\frac{1896 - 1891}{10} = \frac{5}{10} = 0.5$$

$$46 + \left[0.5(20)\right] + \left[\frac{0.5(0.5-1) - 5}{2}\right] + \left[\frac{0.5(0.5-1)(0.5-2)\,2}{6}\right] + \left[\frac{0.5(0.5-1)(0.5-2)(0.5-3) - 3}{24}\right]$$

$$46 + [10] + [-2.625] + [0.125] + [-0.1640625]$$

$$= \left[53.335937\right]$$

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Backwoord table =)

$$\frac{1916-1931}{10} = \frac{-15}{10} = -1.5$$

$$101 + \left[-1.5(8)\right] + \left[-1.5\frac{(-1.5+1)-4}{2}\right] + \left[-1.5\frac{(-1.5+1)(-1.5+2)-2}{6}\right] + \left[-1.5\frac{(-1.5+1)(-1.5+2)(-1.5+3)-3}{24}\right]$$

$$= 101 + [-12] + [-1.625] + [-0.104166] + [-0.0101]$$

$$= \begin{array}{r} 87.377604 \\ -\ 53.335937 \\ \hline 34.041667 \end{array} \text{ Ans}$$

Q) Given $\sin 45^\circ = 0.7071$
$\sin = 50^\circ = 0.7660$
$\sin = 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$
$\sin = 65^\circ = 0.9063$
find $\sin = 57^\circ c$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
45	0.7071				
		0.0589			
50	0.7660		-0.057		
		0.0532		-0.007	
55	0.8192		-0.064		0.006
		0.0468		-0.001	
60	0.8660		-0.0065		
		0.0403			
65	0.9063				

$$u = \frac{x - x_0}{h} = \frac{57 - 65}{5} = \frac{-8}{5} = -1.6$$

$$(0.9063) + \left[(-1.6(0.0403)\right] + \left[\frac{(-1.6(-1.6+1)}{2} - 0.0065)\right] + \left[\frac{-1.6(-1.6+1)(-1.6+2) - 0.001}{6}\right]$$

$$\left[\frac{-1.6(-1.6+1)(-1.6+2)(-1.6+3)\,0.006}{24}\right]$$

$$(0.9063) + (-0.06448) + (-0.00312) + (-0.0064)$$
$$+ (0.0001344)$$
$$= (0.83870) \text{ Ans}$$

(1) Gauss forword Interpoluation =

The various central difference interpolution formula

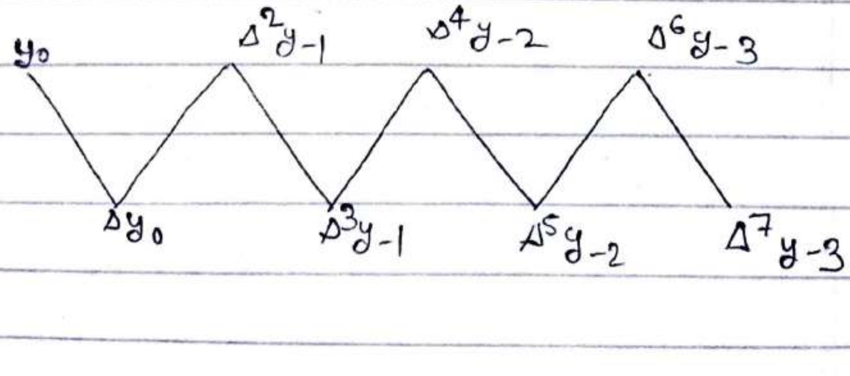
(i) Gauss forword
(ii) Gauss Backword
(3) central defference striling

(I) Gauss forword Interpoluation =

formula

$$y_n(x) = y_0 + u\Delta y_0 + \left[\frac{u(u-1)\Delta^2 y_{-1}}{2!}\right] + \left[\frac{(u+1)u(u-1)\Delta^3 y_{-1}}{3!}\right] + \left[\frac{(u+1)u(u-1)(u-2)\Delta^4 y_{-1}}{4!}\right] + \left[\frac{(u+2)(u+1)u(u-1)(u-2)\Delta^5 y_{-2}}{5!}\right]$$

centralline ⇒



Q) Given $\sqrt{100} = 10$, $\sqrt{110} = 10.489$, $\sqrt{120} = 10.954$, $\sqrt{130} = 11.402$, $\sqrt{140} = 11.832$, $\sqrt{122} = ?$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
100	10				
		0.489			
110	10.489		-0.024		
		0.465		0.007	
120	10.954		-0.017		-0.008
		0.448		-0.001	
130	11.402		-0.0018		
		0.430			
140	11.832				

$$y_0 + u\Delta y_0 + \frac{u(u-1)\Delta^2 y_{-1}}{2} + \frac{(u+1)u(u-1)\Delta^3 y_{-1}}{3!} +$$

$$\frac{(u+1)u(u-1)(u-2)\Delta^4 y_{-2}}{4!} + \frac{(u+2)(u+1)u(u-1)(u-2)\Delta^5 y_{-2}}{5!}$$

$$u = \frac{x - x_0}{h} = \frac{122 - 120}{10} = \frac{2}{10} = 0.2$$

$$(10.954) + (0.2(0.448)) + \left(\frac{0.2(0.2-1) - 0.017)}{2}\right) +$$

$$\left(\frac{(0.2+1)0.2(0.2-1) - 0.001)}{3}\right) + \left(\frac{(0.2+1)(0.2)(0.2-1)(0.2-2) - 0.008}{24}\right) \Rightarrow (10.954) + (0.0896) +$$

$$(-0.0885) + (0.0065) + (0.01406667) \Rightarrow 11.10226667$$

Q) from the following table find the value of $e^{1.17}$ using gauss forword

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.00	2.7183						
		0.1394					
1.05	2.8577		0.0071				
		0.1465		0.0004			
1.10	3.0042		0.0075		0.0000		
		0.1540		0.0004		0	
1.15	3.1582		0.0079		0.0000		0.0001
		0.1619		0.0004		0.0001	
1.20	3.3201		0.0083		0.0001		
		0.1702		0.0005			
1.25	3.4903		0.0088				
		0.1790					
1.30	3.6093						

$$u = \frac{x - x_0}{h} = \frac{1.17 - 1.15}{0.5} = 0.4$$

$$3.1582 + \left[0.4(0.1619)\right] + \left[\frac{0.4(0.4-1)0.0079}{2}\right]$$

$$+ \left[\frac{(0.4+1)(0.4)(0.4-1)0.0004}{6}\right] + \left[\frac{(0.4+1)(0.4)(0.4-1)(0.4-2)0}{24}\right] + \left[\frac{(0.4+2)(0.4+1)(0.4)(0.4-1)(0.4-2)0.0001}{120}\right]$$

$$+ \left[\frac{(0.4+3)(0.4+2)(0.4+1)(0.4)(0.4-1)(0.4-2)(0.4-3)0001}{720}\right]$$

$$= [3.1582] + [0.06476] + [-0.000948] + [-0.0000224] + [0] + [0.000010752] + [-0.00000158413]$$

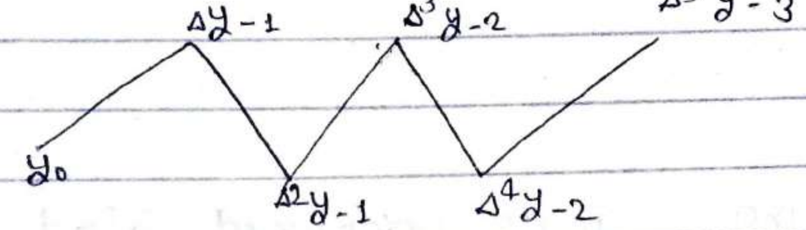
$$= [3.221989091] \text{ Ans}$$

(2) Gauss Backword Interpaluation ⇒

formula

$$y(x) = \left[y_0\right] + \left[u\Delta y_{-1}\right] + \left[\frac{(u+1)u\Delta^2 y}{2!}\right] + \left[\frac{(u+1)u(u-1)\Delta^3 y_{-2}}{3!}\right] + \left[\frac{(u+2)(u+1)u(u-1)\Delta^4 y_{-2}}{4!}\right]$$

central line



Q) Using gauss Backword farmula to find Can 51'42?

x	y tan	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
50	1.1918				
		0.0431			
51	1.2349		0.0019		
		0.0450		0.0002	
52	1.2799		0.0021		0
		0.0471		0.0002	
53	1.3270		0.0023		
		0.0494			
54	1.3364				

$$u = \frac{x - x_0}{h} = \frac{51^{\circ}42' - 52^{\circ}}{60'} = \frac{-18'}{60'} = \frac{-3'}{10}$$

$$= -0.3$$

$$\left[1.2799\right] + \left[-0.3(0.0450)\right] + \left[\frac{(-0.3+1)\,0.0021}{2!}\right] + \left[\frac{(-0.3+1)(-0.3)(-0.3-1)0.0002}{3!}\right]$$

$$= (1.2799) + (-0.0135) + (-0.0002205)$$
$$(0.000091)$$

$$= (1.2661886) \text{ An}$$

Q) using gauss backword find the population 1936 ?

year x	population y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1901	12					
		3				
1911	15		2			
		5		0		
1921	20		2		3	~~7~~
		7		3		-10
1931	27		5		-7	
		12		-4		
1941	39		1			
		13				
1951	52					

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$$u = \frac{x - x_0}{h} = \frac{1936 - 1941}{10} = \frac{-5}{10} = -0.5$$

$$39 + \left[(-0.5) \times 12\right] + \left[\frac{-0.5(-0.5+1)\,1}{2}\right] +$$

$$\left[\frac{(-0.5+1) - 0.5(-0.5-1) - 4}{6}\right] + \left[\frac{(-0.5+2)(-0.5+1)(-0.5)(-0.5-1) - 7}{24}\right]$$

$$= 39(-6) + (-0.725) + (-0.60416)$$

$$= (32.270884)$$

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③ Central defference Striling ⇒

$$y = y_0 + \frac{u}{1!}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2!}\right] + \left[\frac{u^2}{2!}\Delta^2 y_{-1}\right] + \left[\frac{u(u^2-1^2)}{3!}\right]$$

$$\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-1}}{2}\right] + \left[\frac{u^2(u^2-1^2)\,\Delta^4 y_{-2}}{4!}\right] =$$

$$(u^2-1^2)\ (u-1)\ (u+1)$$

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Q) using striling formula find $y(28)$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	20	49225				
			-909			
-1	25	48316		-171		
			-1080 I		-59 I	
0	30	47236		-230		-21
			-1310 II		-80 II	
1	35	45926		-310		
			-1620			
2	40	44306				

$\cancel{47236} +$ $\quad u = \dfrac{x - x_0}{h} = \dfrac{28-30}{5} = \dfrac{-2}{5} = -0.4$

$$47236 + (-0.4)\left[\frac{(-1310 - 1080)}{2}\right] + \frac{(0.16)}{2}$$

$$(-2.30) + \left[\frac{(-0.4)(0.16-1)}{6}\right] + \left[\frac{-80-59}{2}\right] +$$

$$\frac{(0.16)(0.16-1)}{24}(-21)$$

$$47236 + 478 - 18.4 - 3.892 + 0.1176$$

$$= 47691.8256$$

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Find soluation using striling farmula f(12)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	54.14			
		6		
10	60.14		1.58	
		7.58		-1
15	67.72		0.58	
		8.16		
20	75.88			

$$u = \frac{x - x_0}{h} = \frac{12 - 15}{5} = -0.6$$

$$67.72 + (-0.6)\left[\frac{(8.16 + 7.58)}{2}\right] + \frac{(0.36)}{2} +$$

$$(0.58) + \left[\frac{(-0.6)(0.36-1)}{6}\right]\left(\frac{-0.61}{2}\right)$$

$$= 67.72 - 4.722 + 0.1044 - 0.032$$

$$= 63.0704 \ \underline{\text{Ans}}$$

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Q) Find saluation using Striling formula

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
40	51.08			
		12.16		
44	63.24		−4.52	
		7.64		5.84
48	70.88		1.32	
		8.96		
52	79.84			

$$70.88 + (-8)\left[\frac{(8.96+7.64)}{2}\right] + \frac{(64)}{2}(1.32)$$

$$+\left[\frac{(-8)(64-1)}{6}\right] + \left[\frac{10+5.84}{2}\right]$$

$$70.88 - 66.4 + 42.24 - 245.28$$

-198.56 Ans

(4) For unequal interval lagranges formula ⇒

$$y = f(x) = \left[\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0)\right] +$$

$$\left[\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)\right] + \left[\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2)\right] + \left[\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)\right]$$

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Q) find the saluation using lagrages farmula =)

	x	fx
x_0	300	2.4771
x_1	304	2.4829
x_2	305	2.4843
x_3	307	2.4871

find the value of $f(x = 301)$

$$(301) = \left[\frac{(301-304)(301-305)(301-307)}{(300-304)(300-305)(300-307)} \times 2.4771\right] + \left[\frac{(301-300)(301-305)(301-307)}{(304-300)(304-305)(304-307)} \times 2.4829\right]$$

$$+ \left[\frac{(301-300)(301-304)(301-307)}{(305-300)(305-304)(305-307)} \times 2.4843\right] +$$

$$\left[\frac{(301-300)(301-304)(301-305)}{(307-300)(307-304)(307-305)} \times 2.4871\right]$$

$$y(301) = \left[\frac{(-3)(-4)(-6)}{(-4)(-5)(-7)} \times 2.4771\right] + \left[\frac{(1)(-4)(-6)}{(4)(-1)(-3)} \times\right.$$

$$\left. 2.4829\right] + \left[\frac{(-1)(-3)(-6)}{(5)(1)(-2)} \times 2.4843\right] + \left[\frac{(1)(-3)(-4)}{(1)(3)(2)} \times 2.4871\right]$$

$(301) = 0.5143 * 2.4111 * 2.4829 + (-1.8) + 2.4843 + 0.2951 + 2.4871$

$= \underline{2.4786}$ Ans

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Q) find saluation using lagranges interpolation

$\underline{\underline{x = 2.7}}$

x	y
2	0.69315
2.5	0.91629
2	1.09861

$$(2.7) = \left[\frac{(2.7-2.5)(2.7-3)}{(2-2.5)(2-3)} \times 0.69315\right] + \left[\frac{(2.7-2)}{(2.5-2)}\frac{(2.7-3)}{(2.5-3)} \times 0.9163\right] + \left[\frac{(2.7-2)(2.7-2.5)}{(3-2)(3-2.5)} \times 1.0986\right]$$

$$= \left[\frac{(0.2)(-0.3)}{(0.5)(-1)} \times 0.6932\right] + \left[\frac{(0.7)(-0.3)}{(0.5)(-0.5)} \times 0.9163\right] + \left[\frac{(0.7)(0.2)}{(1)(0.5)} \times 1.0986\right]$$

$$= (-0.12)+(0.6932) + (0.84) + (0.9163) + (0.28) + (1.0986)$$

$$= (0.9941) \text{ Ans}$$

⑤ Newton's divided difference interpolation ⇒

it is using for unequal interval

formula ⇒ $f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)f(x_0,x_1,x_2,x_3,x_4) + \cdots$

Q) find solucation using netwon's divided-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$
5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$
7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$
10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$	
11	1210	$\frac{2028-1210}{13-11} = 409$		
13	2028			

$= 48 + (15-4) + (52) + (15-4)$

Corelation

* Significance study of corelation ⇒

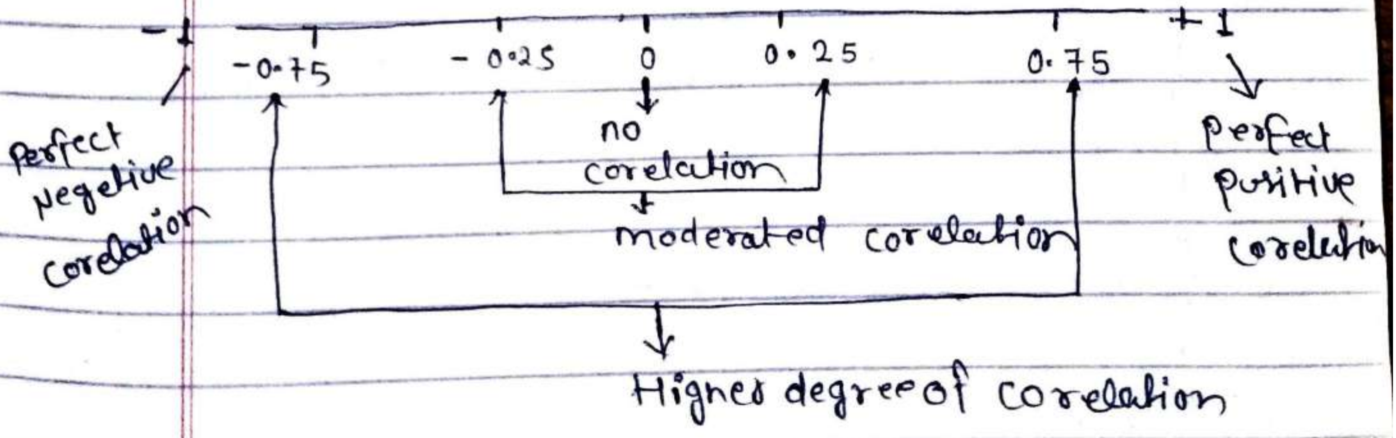
① Corelation is a statical tool that help major and analyzation the degree of b/w relationship two variable.

② corelation analysis deal with the association b/w two are more variable.

③ The value of corelation lize b/w (−1 to +1)

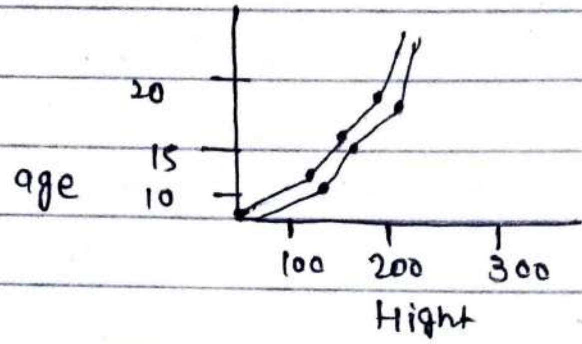
Types of corelatioon ⇒

① Positive corelation
② Negetive corelation
③ Partial corelation
④ multiple corelation
⑤ linear corelation
⑥ non linear corelation



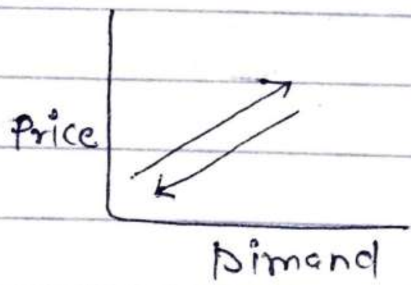
① Positive corelatione ⇒

The corelation is said to be positive if the value of two variable changing with the same direction.



② Negetive corelation ⇒

The corelation said to be negetive corelation when the value of variable change with the the upsite direction.



③ Partial corrlation ⇒

we have more than two variable be study only two variable at a time.

④ Multiple correlation ⇒

we deals atleast 3 variable simultaneously.

⑤ linear corelation ⇒

In which any change in one variable leads to change in other variable in same retio.

x	y	$\frac{x}{y}$
1	10	$\frac{1}{10}, \frac{1}{10}$
2	20	$\frac{2}{20}, \frac{1}{10}$
3	30	$\frac{3}{30}, \frac{1}{10}$
4	40	$\frac{4}{40}, \frac{1}{10}$
5	50	$\frac{5}{50}, \frac{1}{10}$

⑥ non leniar corelation ⇒

It is a setuation in which any change in one vaoiable leads to change to onother variable in difference retio.

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x	y	$\frac{x}{y}$
1	10	$\frac{1}{10}, \frac{1}{10}$
2	18	$\frac{2}{18}, \frac{1}{9}$
3	24	$\frac{3}{24}, \frac{1}{8}$
4	28	$\frac{4}{28}, \frac{1}{7}$
5	30	$\frac{5}{30}, \frac{1}{6}$

(7) simple corelation $\Rightarrow$

The relationship b/w two variable only (x, y)

(1) method of measureuct.

- Karl-Pearson's method
  - Actual means method
  - Assumed means method
- Spearman's Rank method
  - Rank are given
  - Rank are not given
  - Rank are equal

① Spearman's method =

(i) Rank are given =

$$\boxed{r_K = 1 - \frac{6\Sigma D^2}{N^3 - N} = D = R_1 - R_2}$$

Q)

R_1	R_2	$D = R_1 - R_2$	D^2
1	3	-2	4
3	4	-1	1
4	5	1	1
2	1	-1	1
5	2	3	9

$$1 - \frac{6 \times 16}{(5)^3 - 5}$$

$$\Sigma D^2 = \frac{96}{120}$$

$$= \frac{120 - 96}{120} = \frac{24}{120} = \frac{1}{5} = (0.2)$$

(ii) Rank are not given $\Rightarrow$

$$\boxed{r_K = 1 - \frac{6\Sigma D^2}{N^3 - N}}$$

x	y	R_1	R_2	$D = R_1 - R_2$	D^2
6	8	3	4	-1	1
8	9	2	3	-1	1
11	11	1	1	0	0
3	3	5	5	0	0
4	10	4	2	2	4
					$\Sigma D^2 = 6$

$$= 1 - \frac{6 \times 6}{(5)^3 - 5}$$

$$= 1 - \frac{36}{120} = \frac{120 - 36}{120} = \frac{84}{120} = (0.7)$$

(iii) Ranks are equal ⇒

x	y	R_1	R_2	$D = R_1 - R_2$	D^2
2	8	5.5	2	3.5	12.5
3	3	4	6	−2	4
5	4	3	4.5	−1.5	2.25
6	8	2	2	0	0
2	4	5.5	4.5	1	1
9	8	1	2	−1	1
					$\Sigma D^2 = 20.50$

Repeted 2 time

$2 = 2 \quad \frac{5+6}{2} = 5.5$

8 R. 3 time

$1+2+3 = \frac{6}{3} = 2$

$\frac{4+5}{2} = 4.5$

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formula $1-6\left[\sum D^2+\frac{1}{12}(m_1^3-m_1)+\frac{1}{12}(m_2^3-m_2)+\frac{1}{12}(m_3^3-m_3)\right]$

$$= 1-6\left[\frac{20.50+\frac{1}{12}(2^3-2)+\frac{1}{12}(8^3-8)+\frac{1}{12}(4^3-4)}{216-6}\right]$$

$$= 1-\left[\frac{20.50+0.5+42+5}{210}\right].$$

$$= 1-0.3238$$

$$= \underline{0.6762}\ \text{Ans}$$

(2) Karl - Pearson's coefficient corelation ⇒

(i) Actual means method ⇒

$$\boxed{r=\frac{\sum xy}{\sqrt{\sum x^2\cdot\sum y^2}}}$$

x	y	$x=(x-\bar{x})$	x^2	$y=(y-\bar{y})$	y^2	$x\cdot y$
10	8	-20	400	2	4	-40
30	6	0	0	0	0	0
20	4	-10	100	-2	4	20

40	2	10	100	−4	16	−40
50	10	20	400	4	16	80
$\Sigma x = \frac{150}{5} = 30$	$\Sigma y = \frac{30}{5} = 6$		$\Sigma x^2 = 1000$		$\Sigma y^2 = 40$	$\Sigma xy = 20$

$$r = \frac{\Sigma + y}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}} = \frac{20}{\sqrt{1000 \times 40}}$$

$$= \frac{20}{\sqrt{10 \times 10 \times 10 \times 10 \times 2 \times 2}} = \frac{\cancel{20}}{\cancel{10} \times 10 \times \cancel{2}} = \frac{1}{10} = \underline{\underline{0.1}}$$