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Assignment - 3

Aim: Implement Greedy search algorithm for any of the following application:
Prim's minimum spanning tree algorithm

Theory: The graph which does not have edges pointing to any direction in a graph is called an undirected graph and the graph always has a path from a vertex to any direction in a graph is called an undirected graph. and A spanning tree is a subgraph of the undirected connected graph where it include all the nodes of the graph with the minimum possible number of edges. The subgraph should contain each and every node of the original graph. If any node is missed out then it is not a spanning tree and also the spanning tree doesn't contain cycles. If graph has n number of nodes, then the total number of spanning trees created from a complete graph is equal to $n^{(n-2)}$. In a spanning tree, the edges may or may not have weights associated with them. Therefore, the spanning tree in which the sum of edges is minimum as possible then that spanning tree is called minimum spanning tree. One graph can have multiple spanning tree but it can have only one unique minimum spanning tree. There are two different ways to find the minimum spanning tree from the complete graph. i.e. kruskal's algorithm and Prim's algorithm.

Prim's algorithm:

It is minimum spanning tree algorithm which helps to find out the edges of the graph to form the tree including every node with the minimum sum of weights to form minimum spanning tree. It starts with the single source node and later explore all the adjacent nodes of the source node with all the connecting edge While we are exploring the graphs, we will choose the edges with the minimum weight an those which can't cause the cycle in graph.

Prim's algorithm for minimum spanning tree:

It basically follows the greedy algorithm approach to find the optimal solution. To find the minimum spanning tree using prim's algorithm, we will choose a source node and keep adding the edges with the lowest weight.

The algorithm is as given below:

- Initialize the algorithm by choosing the source vertex.
- Find the minimum weight edge connected to th source node and another node and add it to the tree.
- Keep repeating this process until we find the minimum spanning tree.

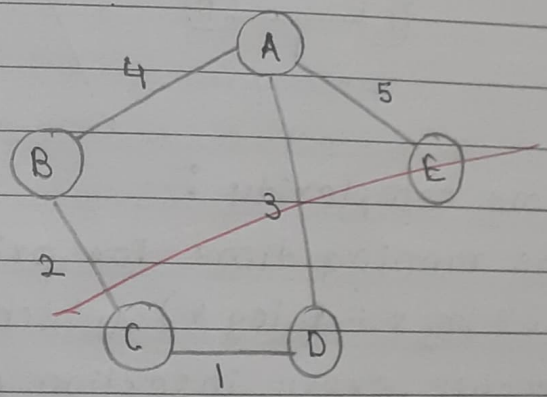
Pseudocode :

```
T = ϕ ;
M = {1} ;
while (M ≠ N)
    let (m,n) be the lowest cost edge such that
    m ∈ M and n ∈ N-M ;
```

Here we create two sets of nodes i.e. M and M-N. M set contains the list of nodes that will have been visited and the M-N set contains the nodes that haven't been visited. Later, we will move each node from M to M-N after each step by connecting the least weight edge.

Example -

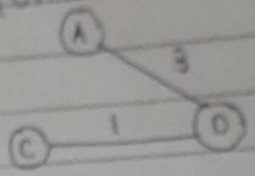
Let us consider the below-weighted graph.



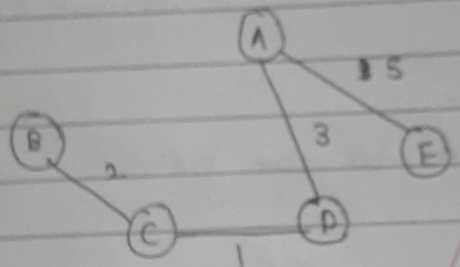
Later we will consider the source vertex to initialize the algorithm.

Now, we will choose the shortest edge from the source vertex and it to finding the spanning tree.

Then, choose the next nearest node connected with the minimum edge and add it to the solution. If there are multiple choices then choose anyone.



Continue the steps until all nodes are included and we find the minimum spanning tree.



Time complexity :

The running time for prim's algorithm is $O(V \log V + E \log V)$ which is equal to $O(E \log V)$ because every insertion of a node in the solution takes logarithmic time. Here, E is the number of edges & and V is the number of vertices/nodes. However, we can improve the running time complexity to $O(E + \log V)$ of prim's algorithm using Fibonacci Heaps.

Applications :

1. Prim's algorithm is used in network design.
2. It is used in network cycles and rail tracks connecting all the cities.
3. It is used in laying cables of electrical wiring.
4. It is used in irrigation channels and placing microwave towers.
5. It is used in cluster analysis.
6. Prim's algorithm is used in gaming development and cognitive science.
7. Path finding algorithm in artificial intelligence and travelling salesman problems make use of prim's algorithm.

Conclusion: As we studied, the minimum spanning tree has its own importance in the real world, it is important to learn the prim's algorithm which leads us to find the solution to many problems. When it comes to finding the minimum spanning tree for the dense graphs, prim's algorithm is the first choice.