Physical and Mathematical modelling of steelmaking processes (MSE629A)

Assignment-1

Steady State Laminar Flow in a cylindrical Pipe

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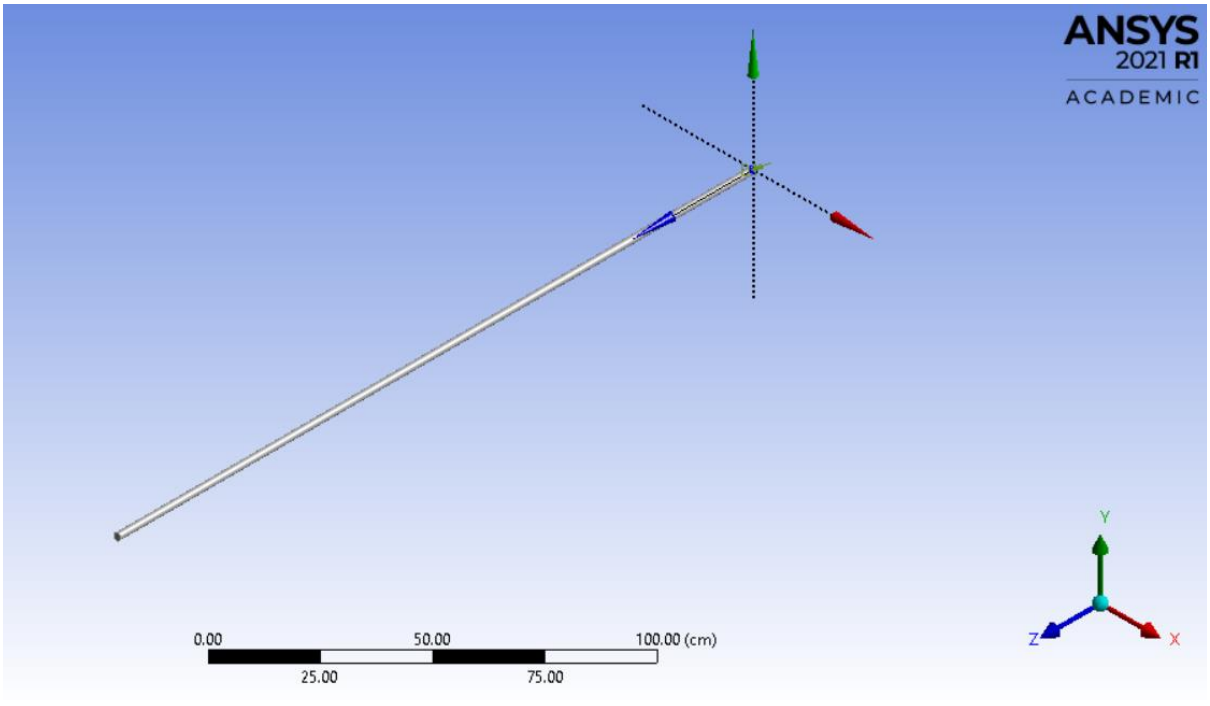
1. Objective:

1. To study the steady state laminar flow in a cylindrical pipe and generate the flow using 3D grid.
2. To determine the effect of grid size (6 , 8 and 12 lac number of elements) on the computational results.
3. The effect of convergence criteria on the computational results.
4. Comparison of computational and analytical results.



1.	Reynolds no.	100
2.	Density of fluid	1000 Kg/m^3
3.	Viscosity of fluid	0.001 Kg/m s
4.	Diameter of pipe (D)	0.02 m
5.	Length of pipe (H)	2 m
6.	V (Calculated Inlet velocity)	0.005 m/s

2. Governing equation:

Governing equations are given below for these calculations.

In this Assignment, the given conditions are steady state is not considered.

$$\frac{\partial(\rho)}{\partial t} = \frac{\partial(\rho u)}{\partial t} = \frac{\partial(pv)}{\partial t} = \frac{\partial(\rho w)}{\partial t} = 0, \rho g_x = \rho g_y = 0, \rho g_z \neq 0$$

The value of ρg_z is incorporated in the pressure force term.

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X component of the incompressible Navier stoke equation

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$

Y component of the incompressible Navier stoke equation

$$\frac{\partial(\rho vu)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)$$

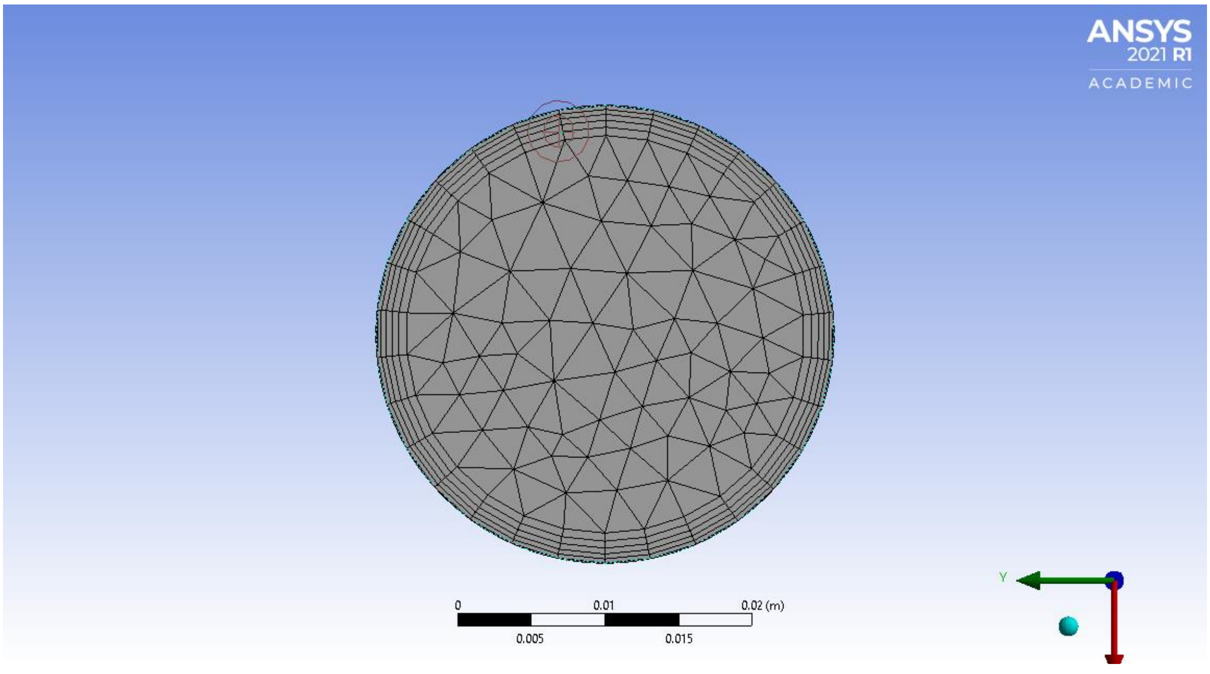
Z component of the incompressible Navier stoke equation

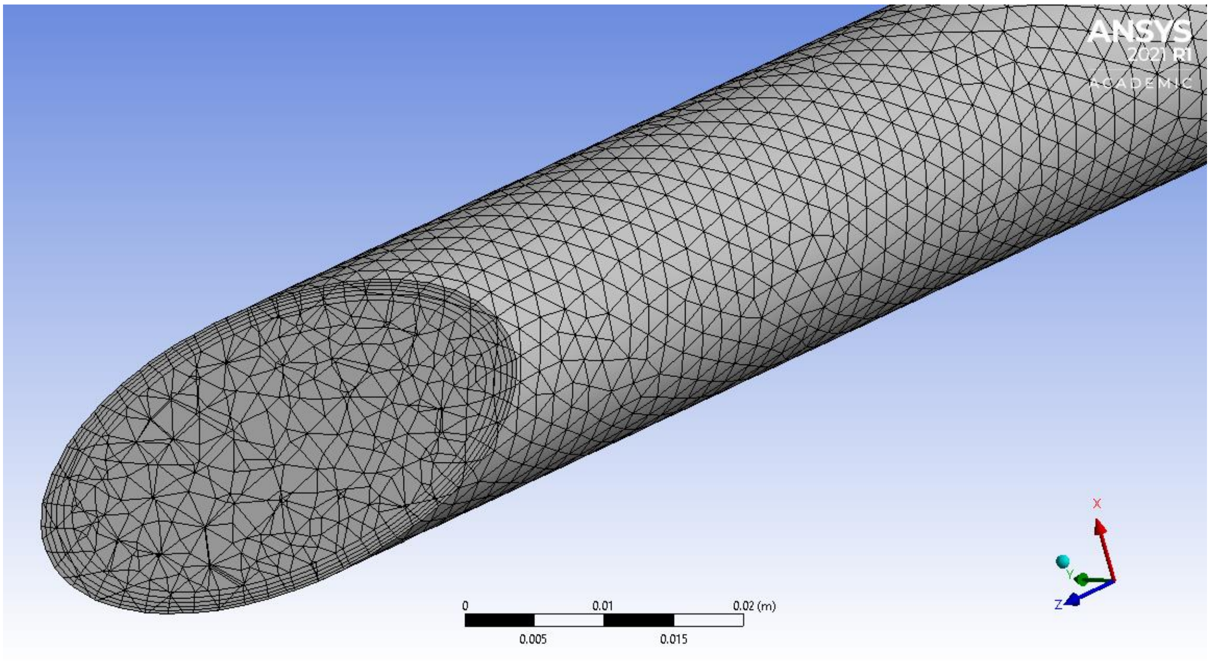
$$\frac{\partial(\rho wu)}{\partial x} + \frac{\partial(\rho wv)}{\partial y} + \frac{\partial(\rho ww)}{\partial z} = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right)$$

The solution was determined using **SIMPLE – (Semi implicit method for pressure correction)**

3. Meshing

Element size	2.1×10^{-3}
Inflation option	Smooth Transition
Transition ratio	0.272
Maximum Layer	5
Growth rate	1.2
Nodes	244203
Elements	661691





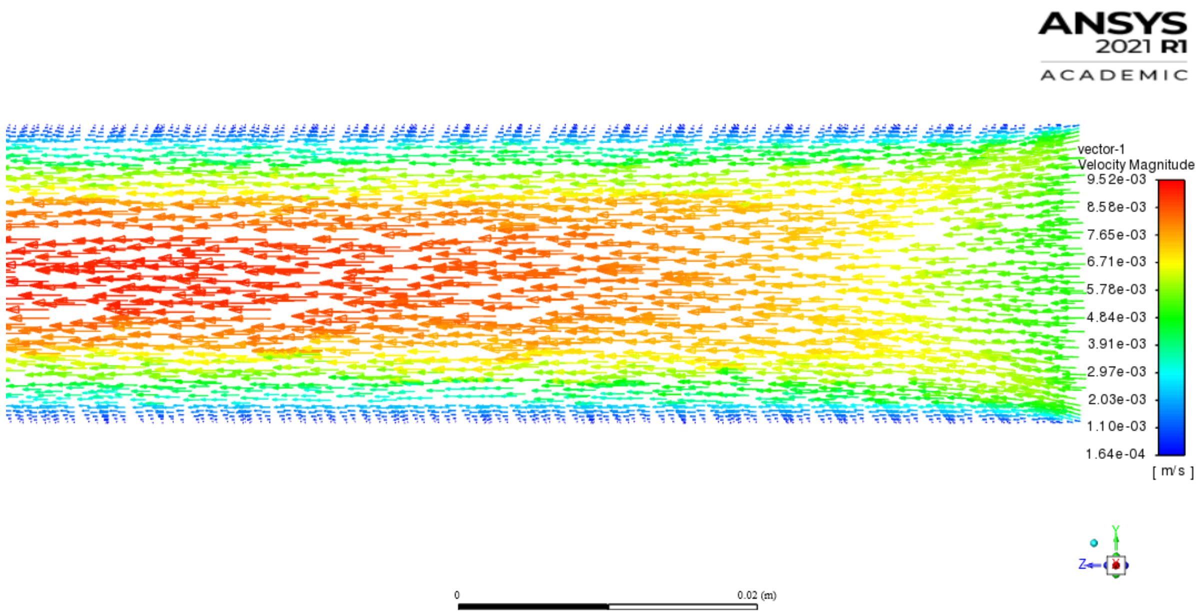
4. Fluent

Model	Viscous (Laminar)
Materials	
Fluid	Water liquid
Viscosity	0.001 Kg/m s
Density	998.2 Kg/m^3
Boundary conditions	
Inlet velocity	0.005 m/s
Wall	No slip
Outlet	Pressure = 0
Scheme	**SIMPLE**
Spatial Discretization	
Gradient	Least square cell based
Pressure	Second order
Momentum	Second order upwind
Monitor	Residual (default) = 10^{-3}
Standard Initialization	From Inlet
Run	Calculation
Iterations	500

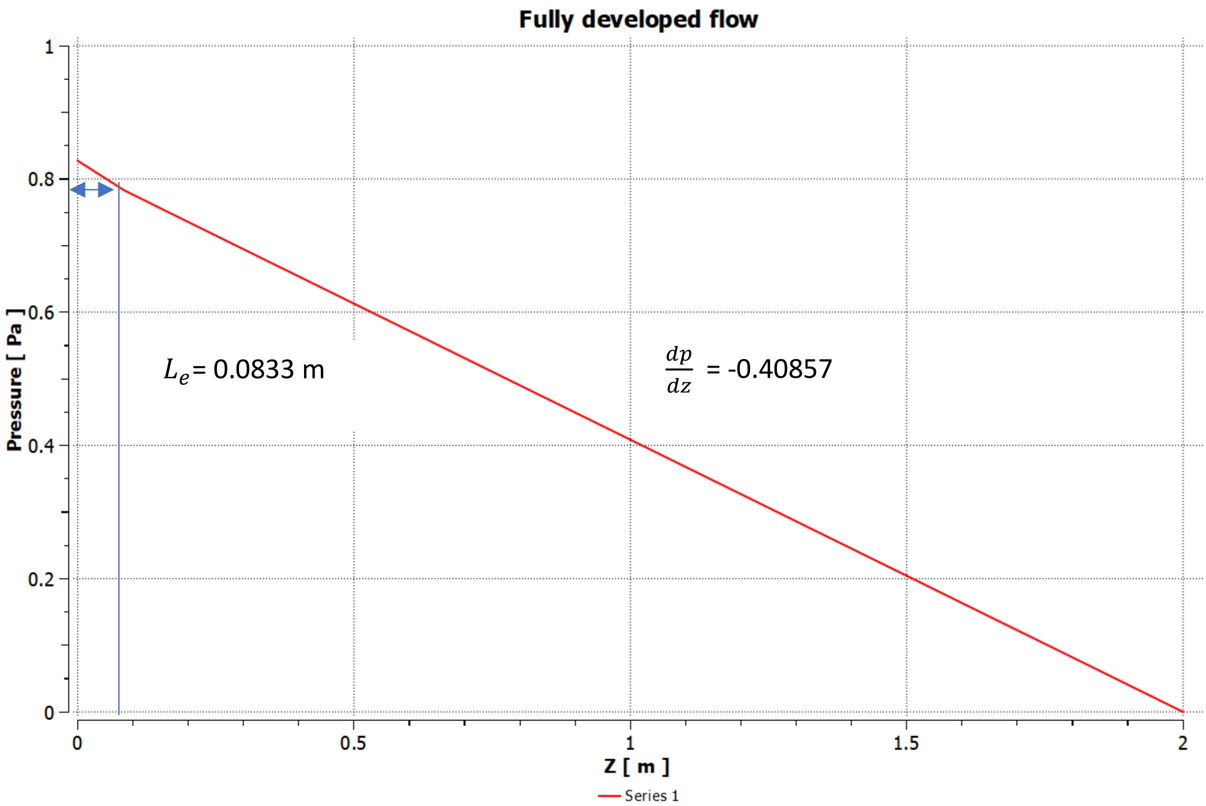
4.1 Results

(i) For elements 6 Lac

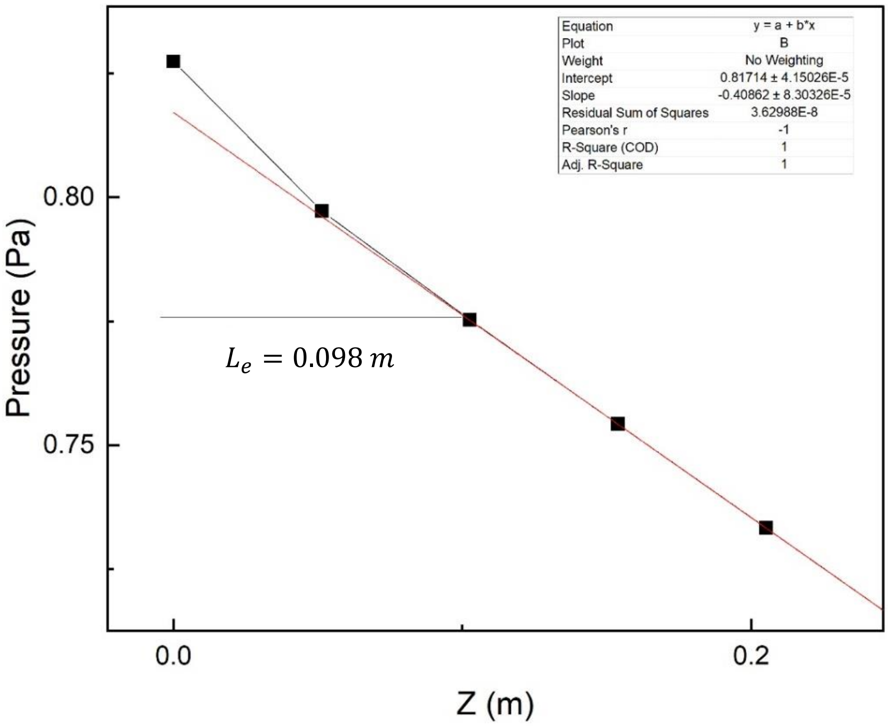
1. Velocity Vector plot



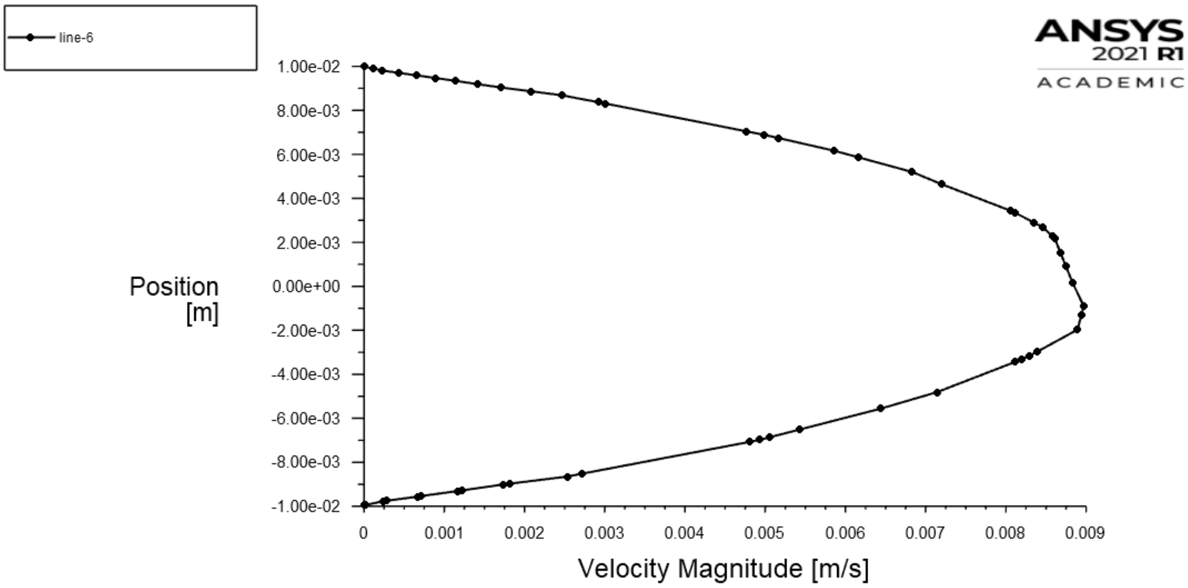
2. Pressure gradient plot



The part of this graph was taken and entrance length was determined using origin software.

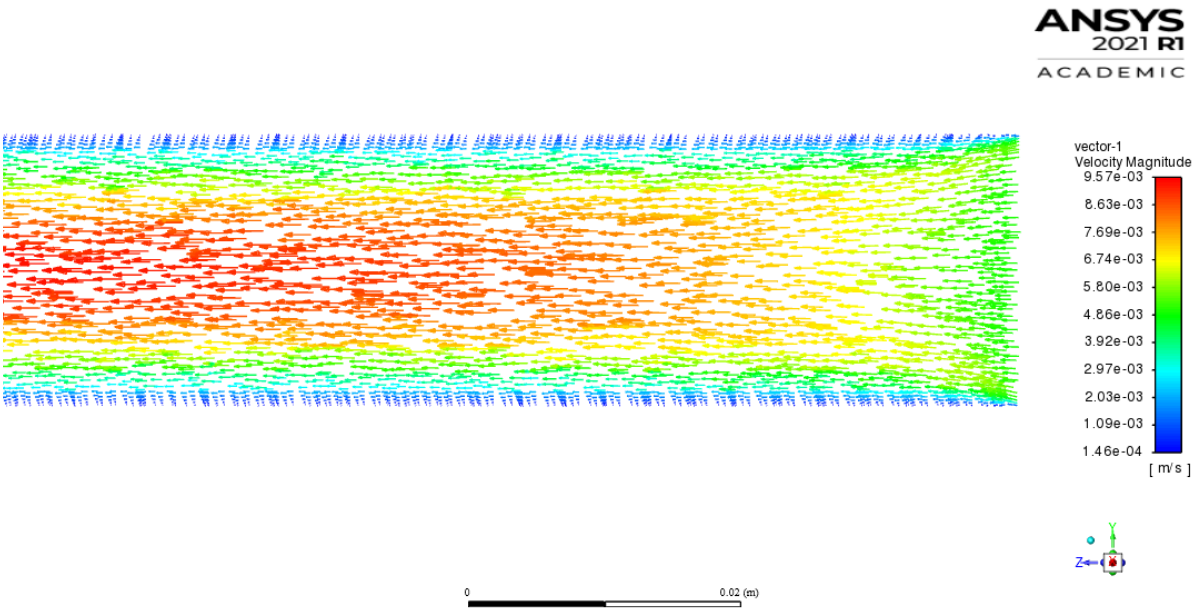


3. The axial velocity at z = 1.5 m

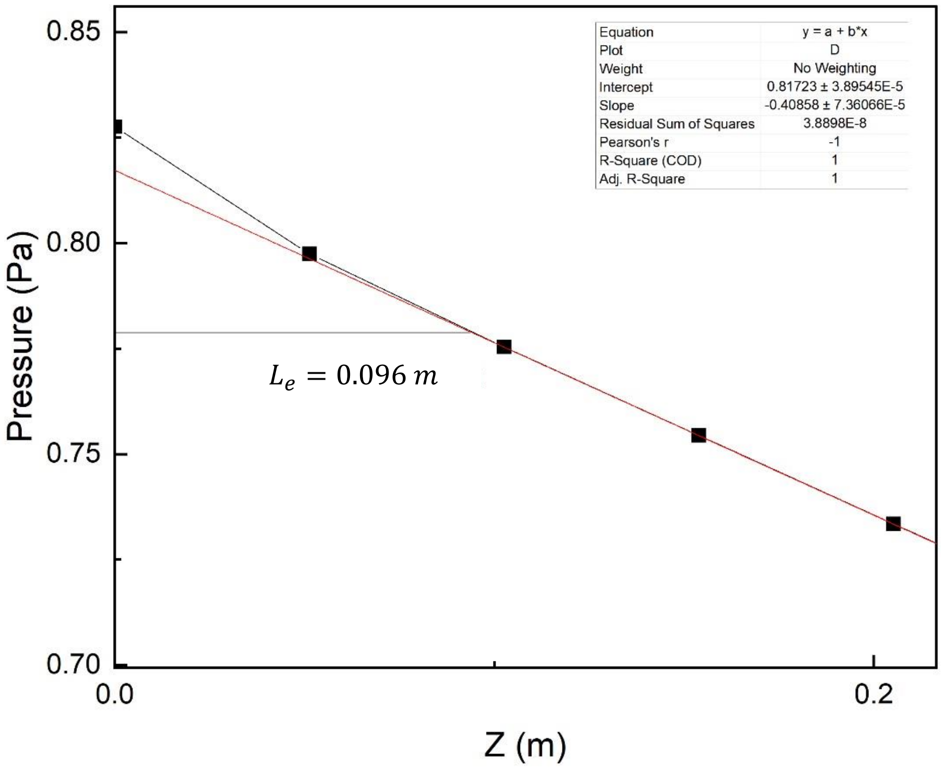


(ii) For elements 8 Lac

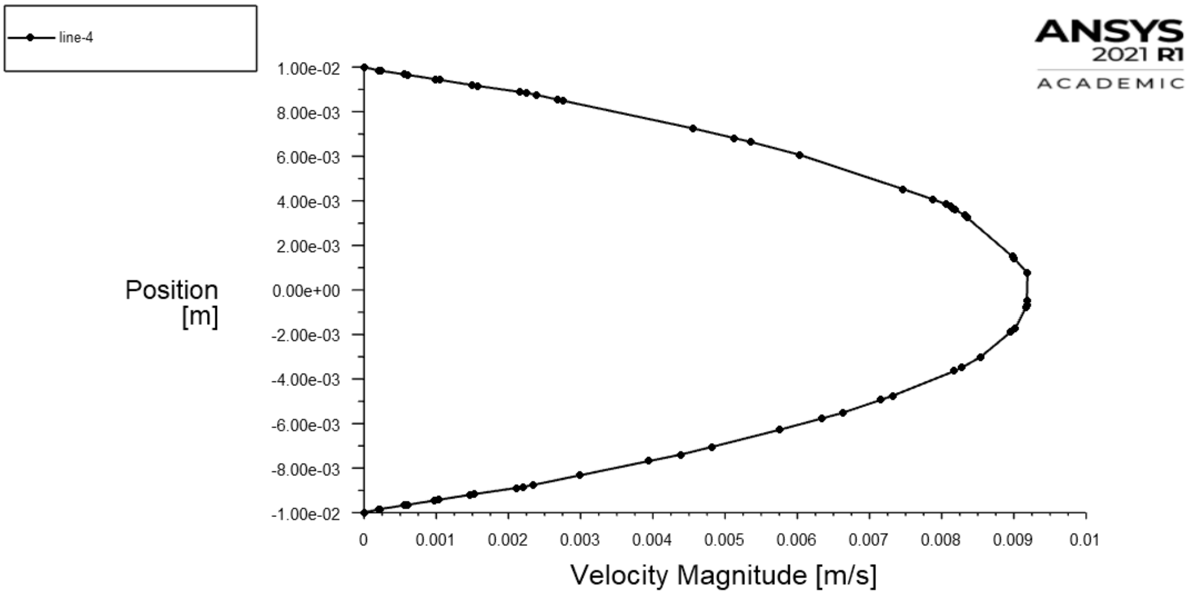
1. Velocity Vector plot



2. Pressure gradient plot

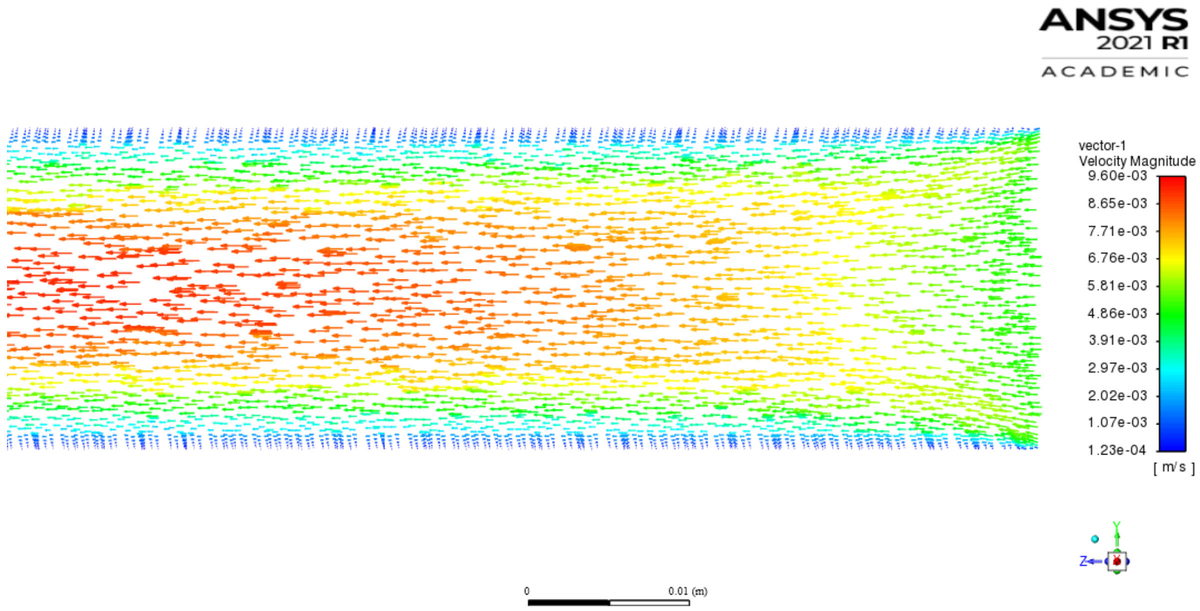


3. Axial velocity at z =1.5 m

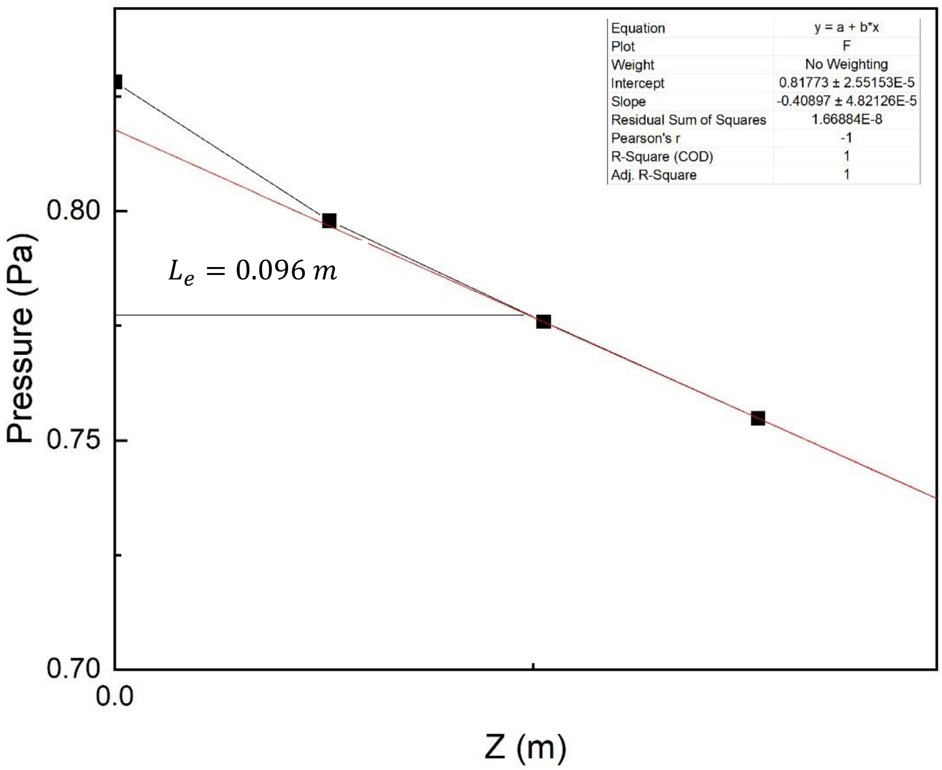


(iii) For elements 12 Lac

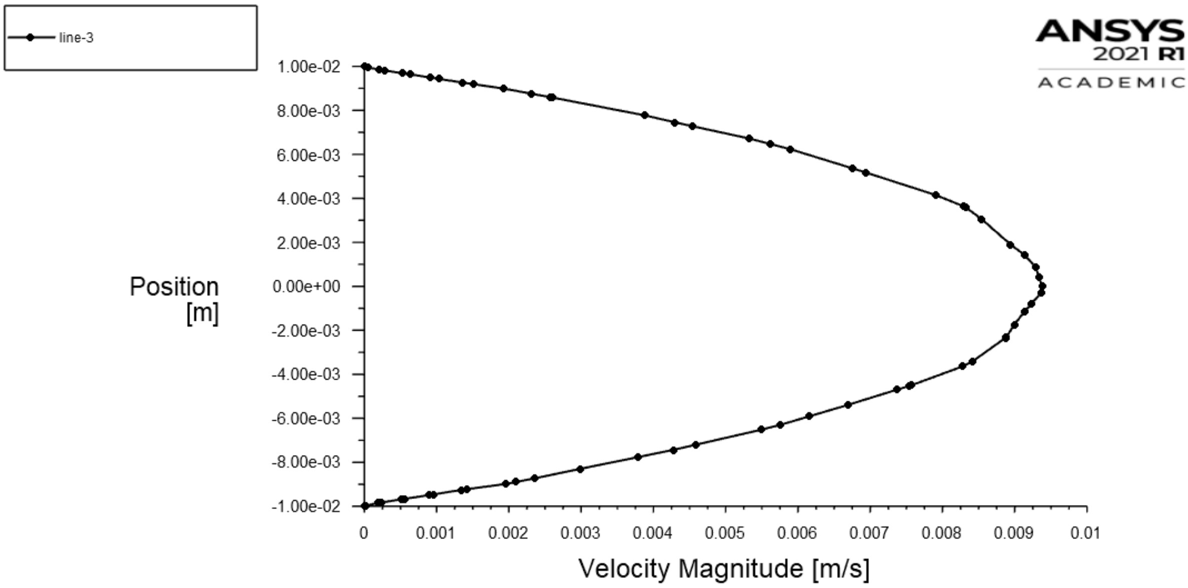
1. Velocity Vector plot



2. Pressure gradient plot

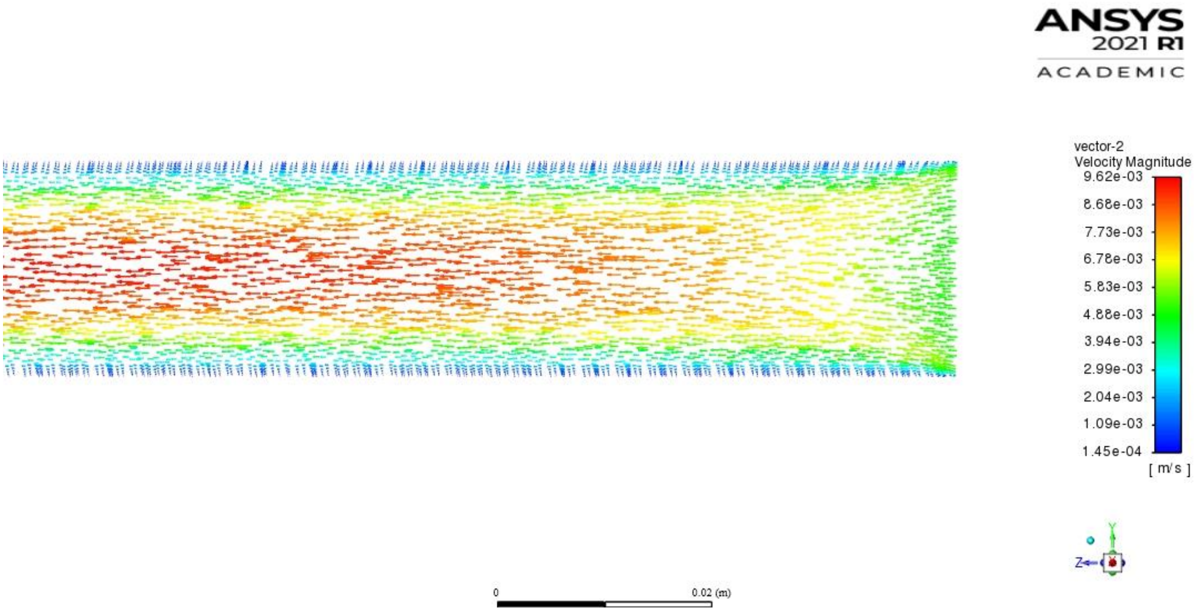


3. Axial velocity at z = 1.5 m

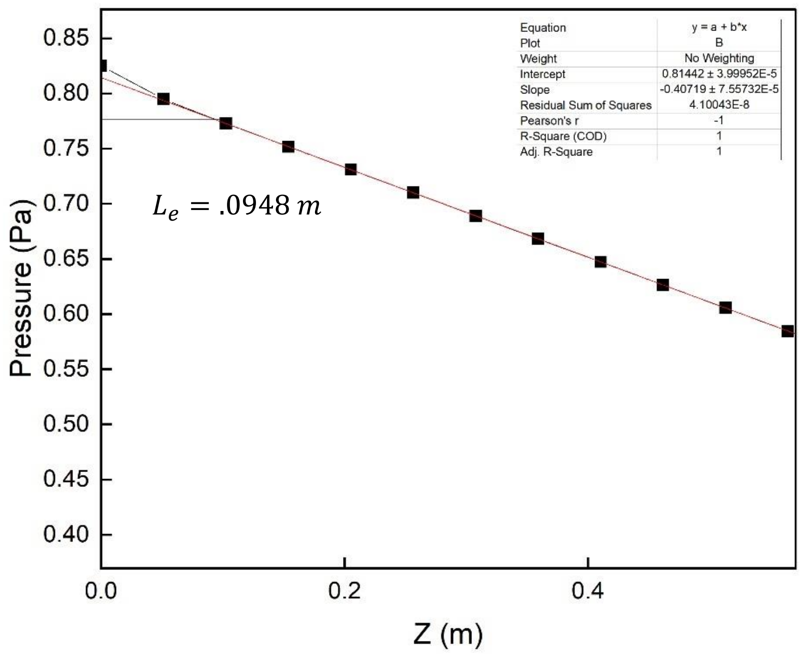


(iv) For Case 2 (R = 10^{-4}) of no elements 8 Lac

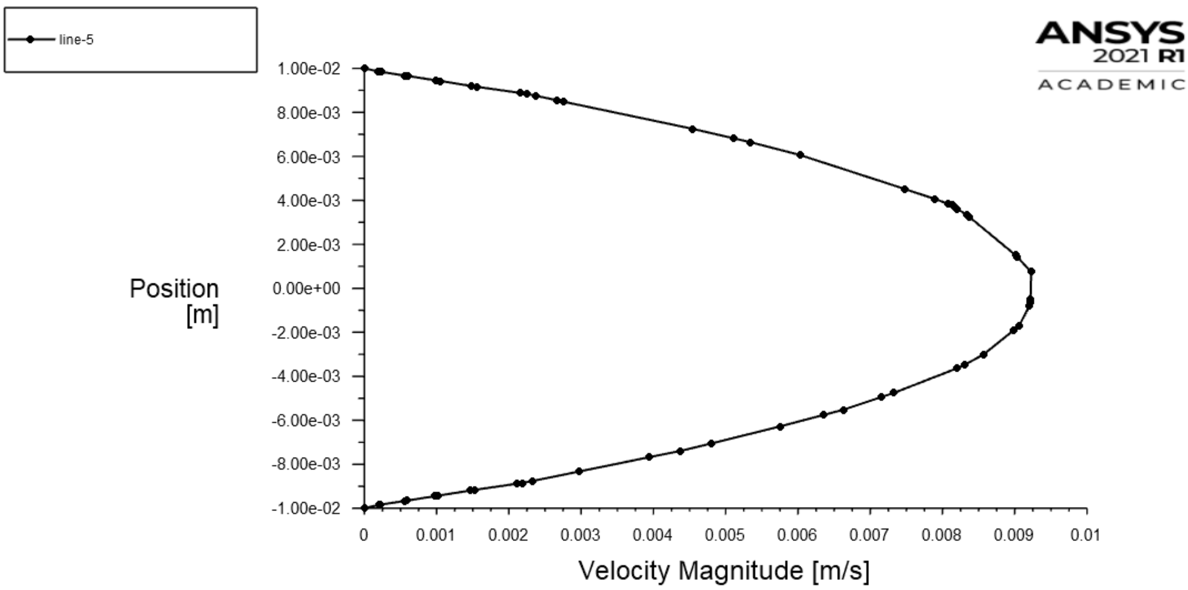
1. Velocity Vector plot



2. Pressure gradient plot



3. Axial velocity at z = 1.5 m



4.2 Calculation of analytical results

Analytical calculations for pressure gradient

Pressure drop $\Delta P = \frac{f\,L\,\rho\,V^2}{2\,D}$

f is friction factor (for laminar = $\frac{64}{Re}$ where Re is Reynolds number)

L is the length of pipe

D is the diameter of pipe

V is the average velocity of pipe

Re = 100 (Given)

Pressure drop = $\frac{64}{Re} \times \frac{200}{2} \times 998.5 \times \frac{0.005^2}{2}$ = 0.7988

Pressure gradient = $-\frac{0.7988}{2}$ = -0.3944

$U_{max} = -\frac{1}{4\mu}(\frac{\Delta P}{L})R^2 = 9.86\times10^{-3}\ m/s$

Entrance length calculation

For laminar flow the entrance length is given by

$$\frac{L_e}{D} = 0.06 \times Re$$

L_e = Length of entrance length

D = Diameter of the pipe = 2 cm

Re = Reynolds number

Re = 100

$$L_e = 6 \times 2 = 12\ cm$$

Analytical entrance length = 0.12 m.

The relative error is calculated using the formula $= \frac{|Measured\ value - Real\ value|}{Real\ value}$

The relative error was determined for entrance length and pressure gradient. The values of the error are given in the table below.

Table 1. For the residuals (R = 10^{-3})

Elements	Analytical L_e(m)	Computational value of L_e (m)	Relative Error of L_e	Analytical value of $\frac{dp}{dz}$	Computational Results $\frac{dp}{dz}$	Relative error $\frac{dp}{dz}$
661691	0.12	0.098	0.183	-0.3944	-0.4086	0.036
836043	0.12	0.096	0.2	-0.3944	-0.4085	0.0357
1245732	0.12	0.096	0.2	-0.3944	-0.4089	0.0357

Table 2. Comparison of residuals R = 10^{-3} vs R = 10^{-4} for no. of elements 836043

Convergence criteria	Analytical L_e(m)	Computational value of L_e (m)	Relative Error of L_e	Analytical value of $\frac{dp}{dz}$	Computational Results $\frac{dp}{dz}$	Relative error $\frac{dp}{dz}$
R = 10^{-3}	0.12	0.096	0.2	-0.3944	-0.408	0.034
R = 10^{-4}	0.12	0.0948	0.21	-0.3944	-0.407	0.0319

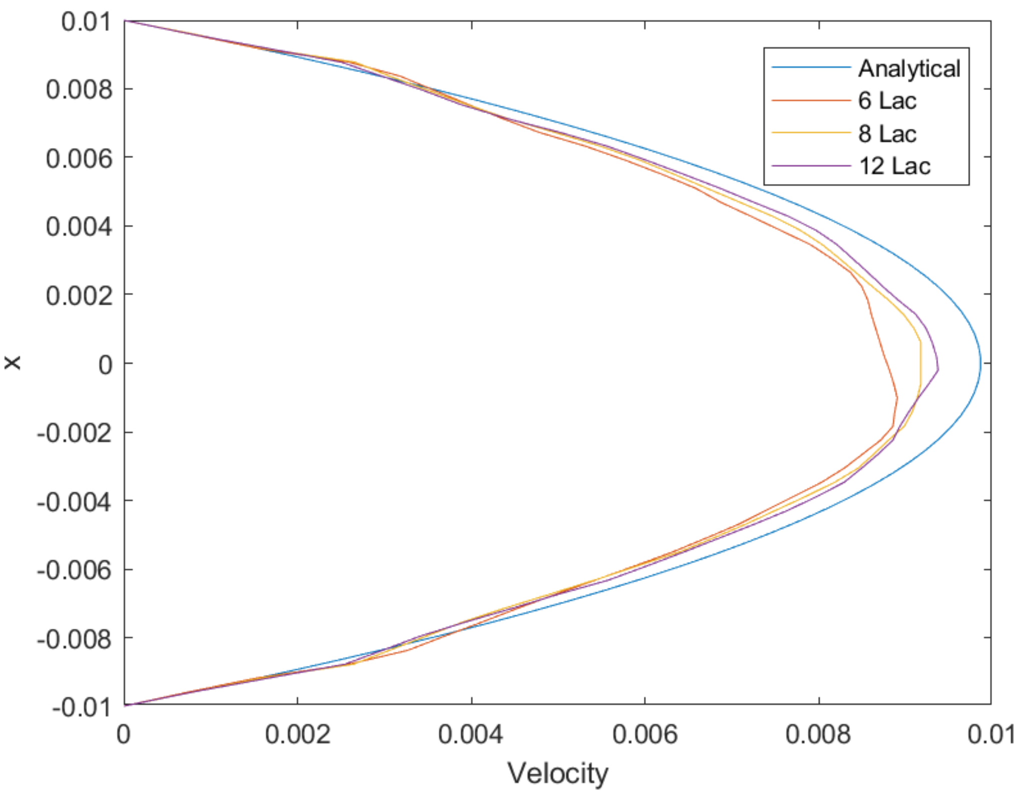
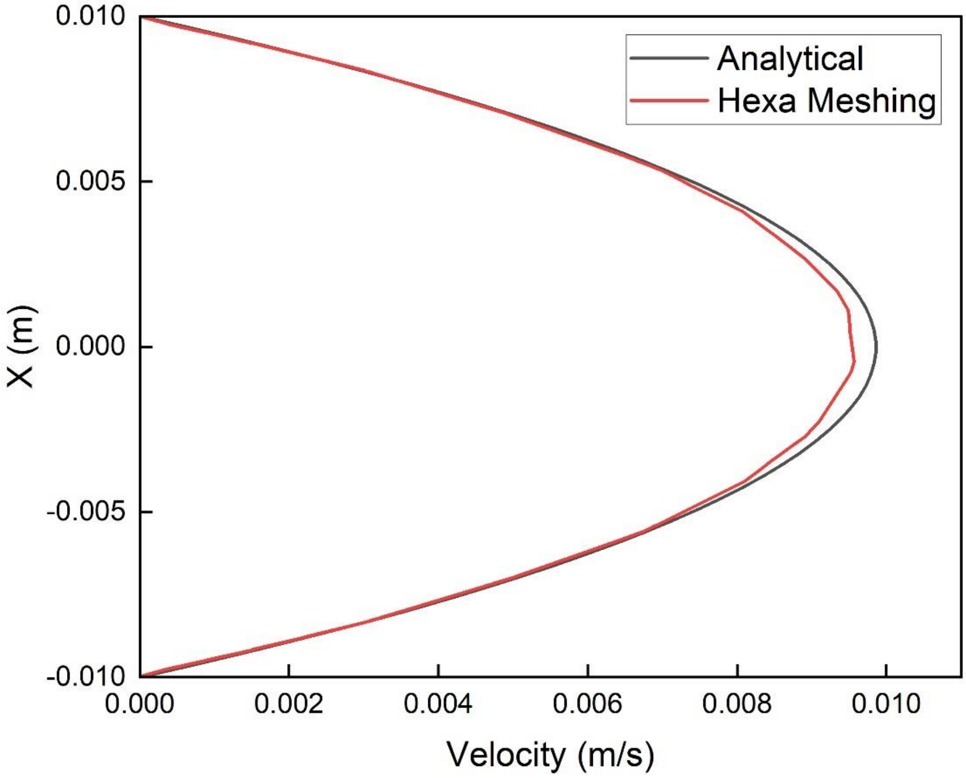
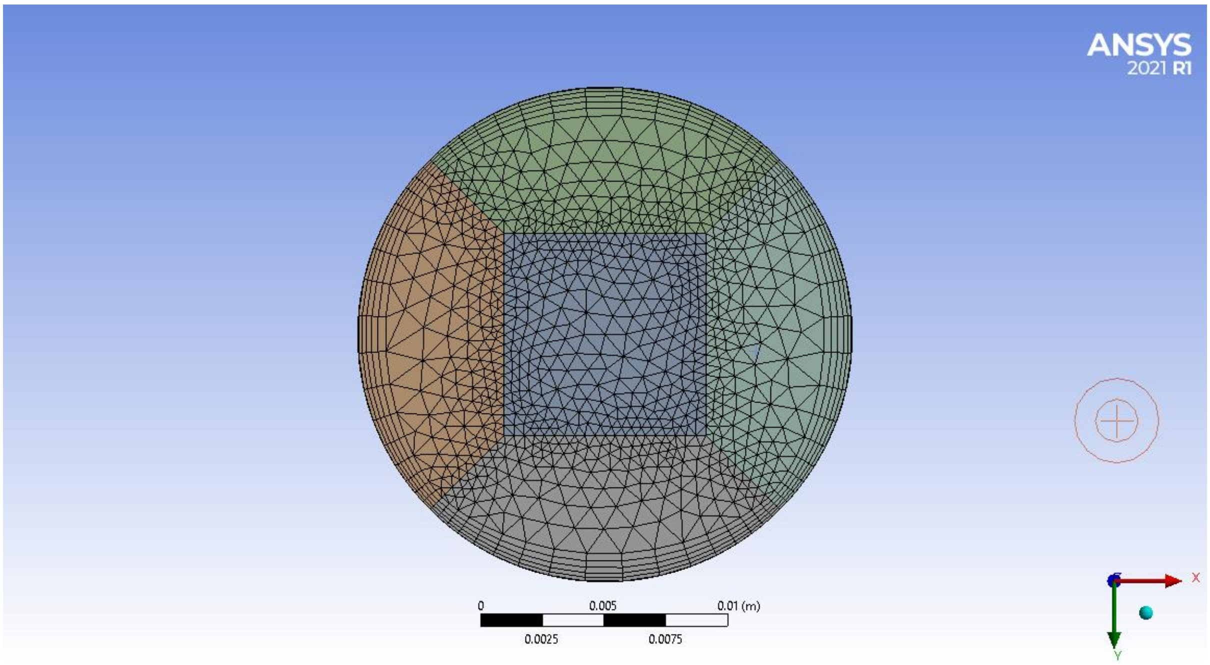


Figure: Comparison of Analytical and computational results



Analytical
Hexa Meshing
X (m)
0.010
0.005
0.000
-0.005
-0.010
0.000
0.002
0.004
0.006
0.008
0.010
Velocity (m/s)



ANSYS
2021 R1
0
0.005
0.01 (m)
0.0025
0.0075
X
Y

No of elements = 1309981

No. of Nodes = 420656

It clearly shows that when the number of elements in the center part of the pipe is increased. The axial velocity at the center of the pipe tends to approach the analytical velocity.

5. Summary

- Entrance length was calculated using analytical and computational methods. It was found that the analytical value was slightly more than computational results.
- The axial velocity was calculated at $z = 1.5$ m from $-0.01 \leq x \leq 0.01$ using Ansys and compared with analytical results. It was found that velocity magnitude is higher in the center part of the pipe than walls. Comparative studies show that the analytical values of velocity magnitude are similar to computational values at near wall region of the cylindrical pipe. However, the analytical velocity magnitude value is slightly higher in the center part of the pipe due to the unavailability of finer grid size at center part of the pipe.
- The grid independent test shows that when the mesh size was decreased. The property became independent of grid size. In this case the 8 lac number of elements gives consistent results.
- When the convergence criteria are increased from 10^{-3} to 10^{-4}. The value of entrance length and pressure gradient decreased further but the difference was not significant.

Appendix

Matlab code for comparison of axial velocity of computational and analytical

```
clc
clear all
close all
r=linspace(-0.01,0.01,60);
visocity = 0.001;
pressure_gradient = 0.3944;
R = 0.01;
Vmax = pressure_gradient*(R^2)/(4*visocity);
V = Vmax*(1-(r/R).^2);
plot(V,r)
hold on
Array3=xlsread('pipeflow12lac.csv');
col5 = Array3(:, 1);
col6 = Array3(:, 2);
plot(col5, col6)
hold on
Array2=xlsread('pipeflow8lac.csv');
col3 = Array2(:, 1);
col4 = Array2(:, 2);
plot(col3, col4)
hold on
Array=xlsread('pipeflow6lac.csv');
col1 = Array(:, 1);
col2 = Array(:, 2);
plot(col1, col2)
xlabel('Velocity')
ylabel('x')
```