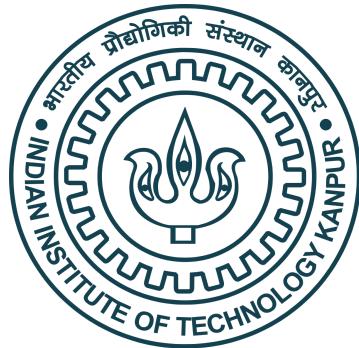


MSE629

# Physical and Mathematical modelling of steelmaking processes



## ASSIGNMENT-2

### Steady Laminar Flow in an Rotating Viscometer With Inner Cylinder Rotating

Under the instruction of Prof. Dipak Mazumdar,  
Dept. of Materials Science and Engineering  
Made By:  
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## **Objective:**

1. To study the steady, laminar velocity field in a rotating viscometer using CFD techniques with an inner rotating cylinder.
2. To investigate the impact of a moving wall on the flow characteristics in a rotating viscometer, specifically focusing on the overall flow structure at a viscosity = 2.26 kg/(ms)
3. To create flow visualisations such as vector plots on the central horizontal plane to observe flow patterns
4. To compute the wall shear stress at different rotational speeds (2, 4, and 10 rpm) and compare these values with published data.
5. To analyse the convergence behaviour, tracking the residuals and documenting the number of iterations and computation time required for convergence.

### **I. Write down complete formulation of the problem with boundary conditions with the aid of neat sketch labelling dimensions, coordinate systems etc**

1. Geometry: A cylinder rotating viscometer with dimensions (height = 0.04215m, diameter = 0.055m).
2. Inner rotating cylinder Height = 0.025m, diameter: 11.76 mm
3. Outer stationary cylinder diameter: 0.055 m
4. Diameter of shaft : 3 mm
5. Height of the cylinder shaft: 0.1 m
6. Distance from bottom to top boundary (shaft height): 0.04215 m

## **Boundary Conditions:**

1. **Inner Cylinder (Rotating) Boundary:**
  - Rotational velocity is specified on the surface of the inner cylinder, rotating at a constant RPM (given in the problem as 2, 4, and 10 RPM).
  - No-slip condition.
2. **Outer Cylinder (Stationary) Boundary:**
  - The outer cylinder is fixed.
  - No-slip condition.
3. **Top and Bottom Boundaries:**
  - Both top and bottom boundaries are stationary with no velocity components.
4. **Shaft (along the centerline):**
  - The shaft attached to the inner cylinder rotates with it, with the same RPM
5. **Fluid Domain:**
  - Newtonian, incompressible fluid with constant density and viscosity.
  - The flow is assumed to be laminar and steady, so the velocity and pressure fields do not change with time.

## Coordinate System:

- Use cylindrical coordinates ( $r, \theta, z$ ) for this problem, with:
  - $r$ : Radial direction (from the centre of the cylinder outward),
  - $\theta$ : Tangential or angular direction (rotation of the fluid),
  - $z$ : Axial direction (along the height of the cylinder).

Equation & Formulas

$$\frac{\partial(\rho)}{\partial t} = \frac{\partial(\rho u)}{\partial t} = \frac{\partial(\rho v)}{\partial t} = \frac{\partial(\rho w)}{\partial t} = 0,$$

Pressure terms

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X component of the incompressible Navier-Stokes equation

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\mu \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\mu \frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(\mu \frac{\partial u}{\partial z})$$

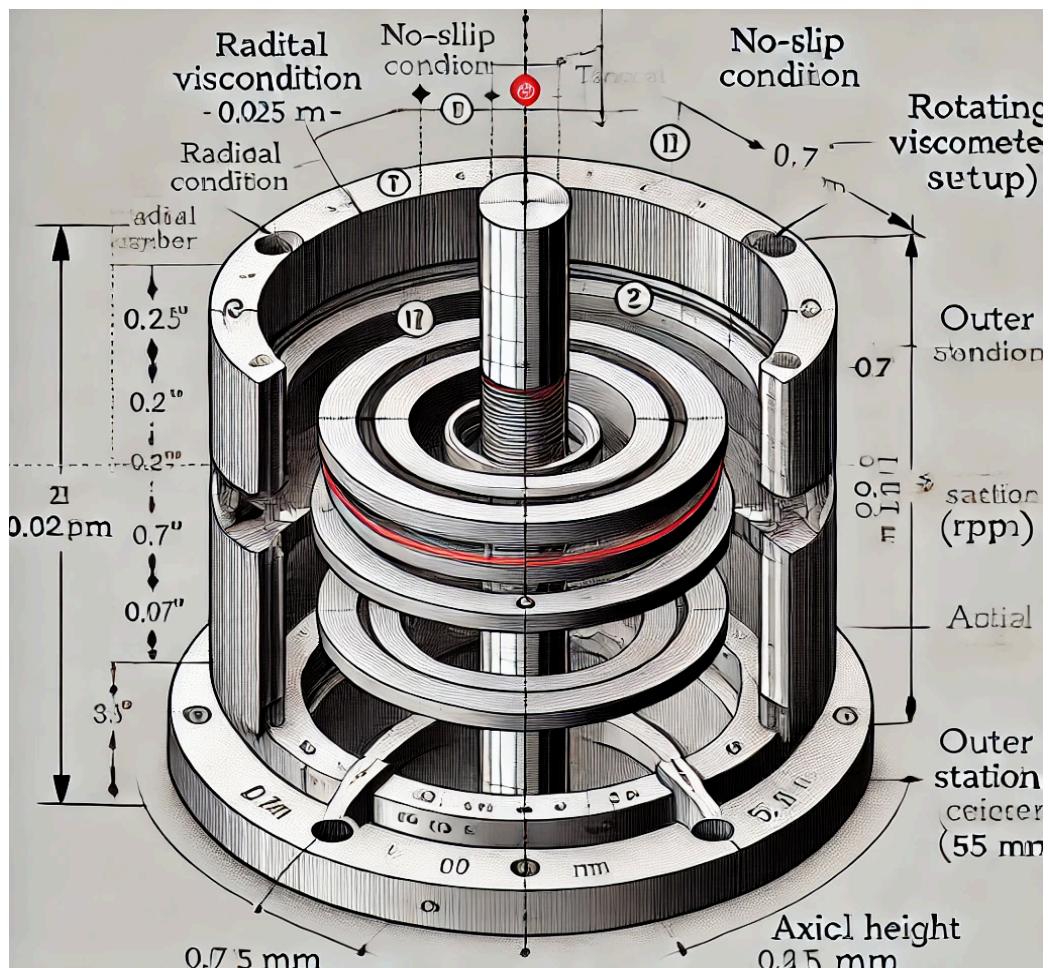
Y component of the incompressible Navier-Stokes equation

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}(\mu \frac{\partial v}{\partial x}) + \frac{\partial}{\partial z}(\mu \frac{\partial v}{\partial z})$$

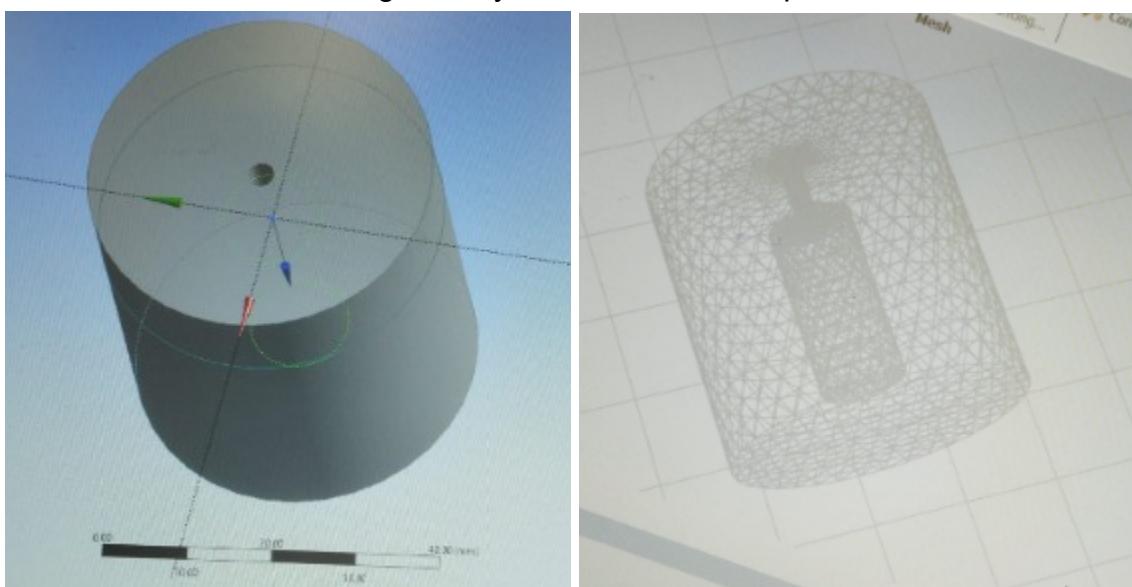
Z component of the incompressible Navier-Stokes equation

$$\frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho ww)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}(\mu \frac{\partial w}{\partial x}) + \frac{\partial}{\partial y}(\mu \frac{\partial w}{\partial y}) + \frac{\partial}{\partial z}(\mu \frac{\partial w}{\partial z})$$

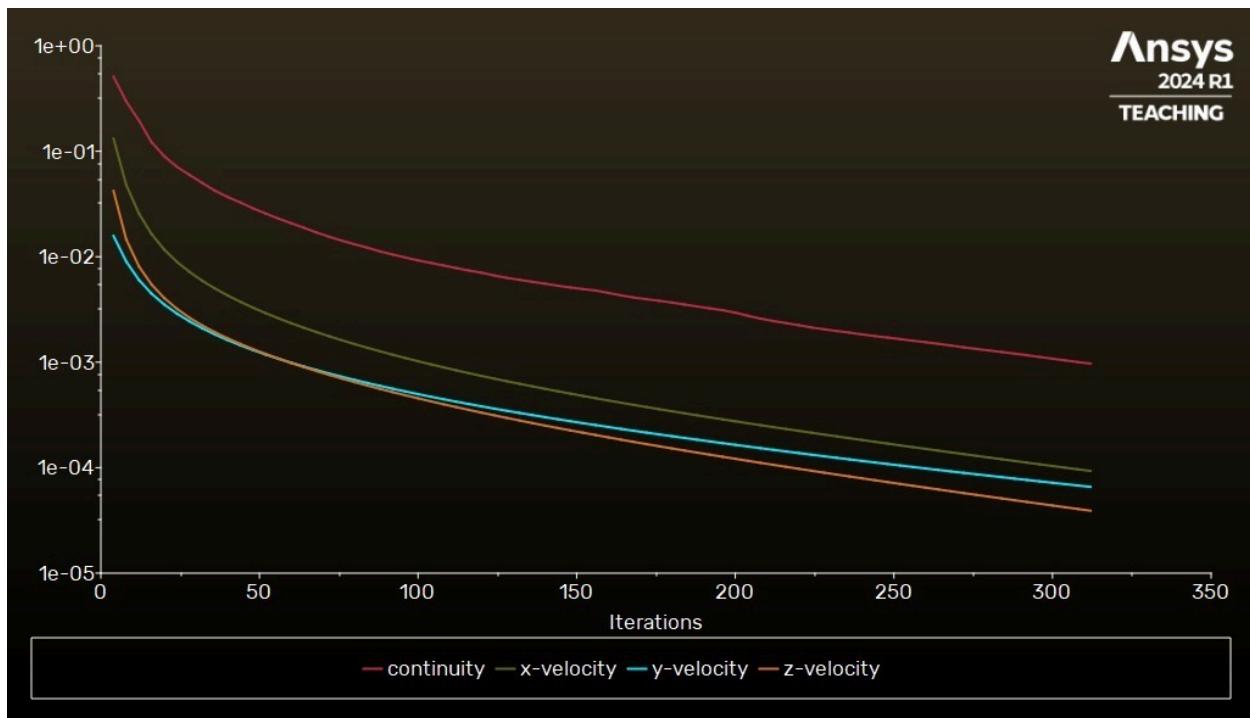
The solution was determined using SIMPLE  
Crank-Nicolson method for pressure correction



AI Generated Figure only for visualisation of process



**II. Note down the number of iterations and computational time to convergence. Also study the convergence history from the residual vs. iteration plot and write a few comments.**



From the residual vs. iteration plot, we can analyse the convergence history of your simulation

1. The plot shows that around 323 iterations were carried out.
2. The residuals have reduced to values below  $10^{-3}$  for continuity and well below  $10^{-4}$  for the velocity components.
3. The graph shows that the solution is converging steadily, with residuals decreasing throughout the iterations. There are no spikes, indicating a stable solution process. The residuals are approaching standard stopping criteria, confirming the accuracy and stability of the flow solution.

**IV. Calculate wall shear stress ( in SI units using FLUENT; on shaft + rotating cylinder, i.e., the rotating assembly) at rotational speed of 2, 4 and 10 rpm**

Rotational Speed (RPM)	Wall Shear Stress (Pa) - Shaft	Wall Shear Stress (Pa) - Spindle	Net Wall Shear Stress (Pa)
2 RPM	0.4870	2.7657	2.5376
4 RPM	0.9781	3.1368	2.9207
10 RPM	2.3938	4.3030	4.1119

**Observation:**

1. The data shows that as the rotational speed of the inner cylinder (spindle) increases, the wall shear stress increases both on the shaft and the spindle.
2. At 2 RPM, the wall shear stress on the shaft is 0.48 Pa, while it reaches 2.393 Pa at 10 RPM, showing a direct relationship between speed and shear stress.
3. Similarly, the wall shear stress on the spindle rises from 2.7657 Pa at 2 RPM to 4.3030 Pa at 10 RPM.
4. The net wall shear stress (which combines the effects on the shaft and spindle) also increases with RPM. It rises from 2.5376 Pa at 2 RPM to 4.1 Pa at 10 RPM.
5. This implies that the overall resistance encountered by the rotating assembly increases with the speed, which is a common trend in rotating systems. The fluid offers more resistance to motion as the speed increases, causing higher shear forces along the surfaces.
6. It's important to observe that the increase in shear stress is **non-linear**. This suggests that as the system operates at higher speeds, the rate at which shear stress increases accelerates. This could be due to the increased turbulence or complexity of fluid dynamics at higher rotational speeds, even though the flow is laminar.
7. At every RPM, the wall shear stress on the **spindle** is higher than on the **shaft**. This is consistent with the fact that the spindle is the primary rotating component and faces more fluid drag compared to the shaft

**V. From Figure 2.2 , READ The values of wall shear stress and tabulate these at 2,4 and 10 rpm.**

Rotational Speed (RPM)	Wall Shear Stress (Pa)
2 RPM	1
4 RPM	1.9
10 RPM	4.4

In both graphs, the results are approximately similar, demonstrating good agreement between the numerical simulation and the benchmark solution.

#### **Comparison of Values:**

- The wall shear stresses at the measured speeds are lower than those obtained in my analysis from Fluent:
  - At 2 RPM, the Fluent data showed approximately 0.4860 Pa on the shaft, which is significantly lower than the 1.0 Pa obtained from Figure 2.2.
  - At 4 RPM, the Fluent measurement of 0.97 Pa is also lower than 1.9 Pa.
  - At 10 RPM, the Fluent value of 2.3938 Pa is again lower than 4.4 Pa.
- These differences may arise from variations in the experimental setup, measurement techniques, or assumptions made during simulations.

This comparison confirms the accuracy of the numerical calculations relative to the benchmark data.

**(vi) Make a table to illustrate a comparison of Wall shear stress as per your computation ( as in (iii)) and Reading ( as in (iv) ) above**

Rotational Speed (RPM)	Wall Shear Stress (Pa) - Computation	Wall Shear Stress (Pa) - Reading	Difference
2 RPM	2.5476	1.0	1.5376
4 RPM	2.9207	1.9	1.0207
6 RPM	4.1119	4.9	0.78

1. The wall shear stress values obtained from Fluent simulations are significantly lower than those reported in Figure 2 across all rotational speeds.
2. Both datasets show a consistent trend of increasing wall shear stress with higher rotational speeds. This reinforces the fundamental understanding that as rotational speeds increase, the resistance to motion and shear forces acting on surfaces also increase

**(vii) Write a few comments on the agreement /disagreement between your computed results and published results.**

1. Overall Trend Consistency: Both the computed results from Fluent and the published results from Figure 2.2 show a similar trend where wall shear stress increases with rotational speed. This is a positive indication that our understanding of fluid dynamics in a rotating viscometer is aligned with our knowledge.
2. There are noticeable differences in the actual values of wall shear stress. The Fluent results are consistently lower than those reported in the published data. This could suggest that our CFD model may not fully capture the complexities of fluid behaviour under the given conditions.
3. These experiments highlight the importance of validating CFD models with experimental data. It's essential to conduct further experiments or refine the simulation parameters to achieve results that are closer to the published values. This would enhance the reliability of the computational methods used for analysing fluid behaviour in rotating systems.

### **Summary ( what i understand and learn from this assignment)**

I learned how to set up and analyse the steady, laminar flow between concentric cylinders, focusing on key parameters like wall shear stress at varying rotational speeds. By comparing computed results from CFD simulations with published data, I understood the significance of validating numerical models against experimental findings. Overall, this assignment deepened my understanding of fluid mechanics principles and their practical applications in engineering, showcasing the essential role of CFD in designing and optimising rotating systems. This knowledge will be instrumental as I continue to explore advanced topics in fluid dynamics and computational methods.

I observed that the flow behaviour was influenced by the moving wall. The velocity profiles near the walls demonstrate a gradual decrease, with zero velocity at stationary walls and maximum velocity near the moving wall. Grid refinement showed improved accuracy, especially in capturing flow details near the boundaries.