MSE629
Physical and Mathematical modelling of steelmaking processes

ASSIGNMENT-3
Steady Laminar Flow and heat transfer in a circular cross section pipe

Under the instruction of Prof. Dipak Mazumdar,
Dept. of Materials Science and Engineering
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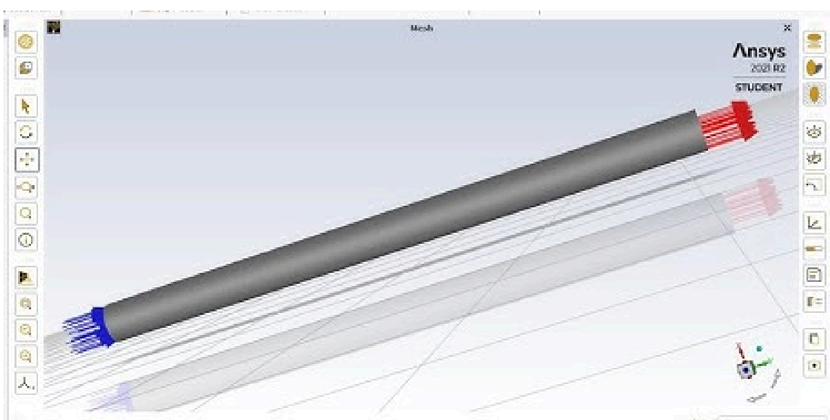
Objective:

1. To study a model of steady laminar flow and heat transfer in a circular pipe, specifying boundary conditions, dimensions, and coordinate systems.
2. Document the number of iterations and computational time required for the convergence of both flow and thermal equations.
3. Calculate the theoretical and numerical entrance lengths, illustrating the entrance region based on the numerically predicted flow field.
4. Compute and analyse the Nusselt number at 15 locations along the pipe wall, plotting its variation with length to validate theoretical expectations.
5. Evaluate the cross-sectional average temperature as a function of pipe length, and determine the exit temperature.

I. **Write down complete formulation of the problem with boundary conditions with the aid of neat sketch labelling dimensions, coordinate systems etc**

1. Geometry: The pipe is modelled as a straight cylindrical geometry with dimensions (length = 10m, diameter = 0.01m).



Boundary Conditions

- **Inlet Conditions**:
  - **Velocity**: Uniform inlet velocity of 0.1273 m/s .
  - **Temperature**: Inlet temperature of 25C.
- **Wall Conditions**:
  - **Temperature**: Constant wall temperature of 75C.
- **Fluid Domain**:
  - Newtonian, incompressible fluid with constant density and viscosity.The flow is assumed to be laminar and steady, so the velocity and pressure fields do not change with time.

Coordinate System:

- Use cylindrical coordinates (r, θ, z) for this problem, with:
  - **r**: Radial direction (from the centre of the cylinder outward),
  - **θ**: Tangential or angular direction (rotation of the fluid),
  - **z**: Axial direction (along the height of the cylinder)

Scheme	SIMPLE
Spatial Discretization	
Gradient	Least Squares Cell-Based
Pressure	Second Order
Momentum	Second Order Upwind
Energy	Second Order Upwind
Under-relaxation Factors	
Pressure	0.3
Density	0.4
Body Forces	0.5
Momentum	0.7
Energy	0.6
Monitor	Residual = 1e-06
Initialization Method	Standard Initialization
Compute from	inlet
Number of Iterations	500
Reporting interval	5

2. Governing equations:

Governing equations are given below for these calculations. In this assignment, we consider the system to be in a steady state.

$$\nabla \cdot \vec{v} + \frac{\partial t}{\partial \rho} = 0 \tag{1}$$

$$\rho g_x = \rho g_y = 0, \quad \rho g_z \neq 0$$

The value of ρg_z is incorporated in the pressure force term.

X Component of the Navier-Stokes Equation

$$\frac{\partial(\rho u u)}{\partial x} + \frac{\partial(\rho u v)}{\partial y} + \frac{\partial(\rho u w)}{\partial z} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \tag{2}$$

Y Component of the Navier-Stokes Equation

$$\frac{\partial(\rho v u)}{\partial x} + \frac{\partial(\rho v v)}{\partial y} + \frac{\partial(\rho v w)}{\partial z} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial v}{\partial z}\right) \tag{3}$$

Z Component of the Navier-Stokes Equation

$$\frac{\partial(\rho w u)}{\partial x} + \frac{\partial(\rho w v)}{\partial y} + \frac{\partial(\rho w w)}{\partial z} = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}\left(\mu \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial w}{\partial z}\right) \tag{4}$$

Heat Balance Equation / Energy Equation

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial r}\left(\frac{r^2 k \frac{\partial T}{\partial r}}{r^2}\right) + S_T \tag{5}$$

The solution was determined using SIMPLE (Semi-Implicit Method for Pressure Linked Equations). The solution was determined using SIMPLE – (Semi implicit method for pressure correction)

Coordinate System:

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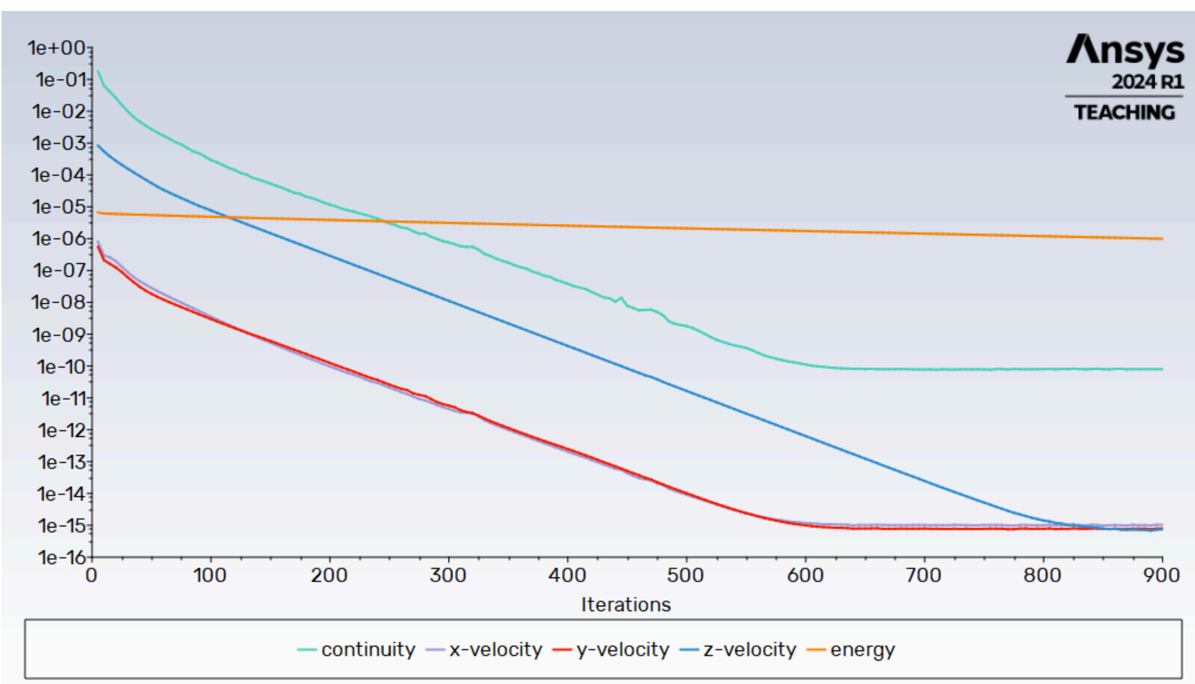
II.Note down the number of iterations and computational time for convergence of flow and thermal balance equation. Why does the flow equation take relatively more number of iterations to converge? Explain

From the residual vs. iteration plot, we can analyse the convergence history of your simulation

1. The total number of iterations required for the coupled flow and thermal balance equation convergence was **896**.
2. The flow equation alone requires fewer iterations because it only involves solving the momentum equations for velocity without considering the energy (temperature) field. When energy is included, the solver must account for both velocity and temperature fields simultaneously, coupling the momentum and energy equations.
3. This coupling introduces additional complexity, as temperature affects the flow due to variations in properties or additional boundary layers, requiring more iterative adjustments for an accurate, stable solution. The interaction between thermal gradients and flow patterns (especially near the pipe walls where temperature gradients are steepest) causes the convergence to take more steps.



(iii) Calculate Entrance length on the basis of the formula i.e., entrance length=0.05* Re* pipe diameter and compare with your numerical estimate. Illustrate your estimation of entrance length from the numerically predicted flow field.

Theoretical Entrance Length Calculation:

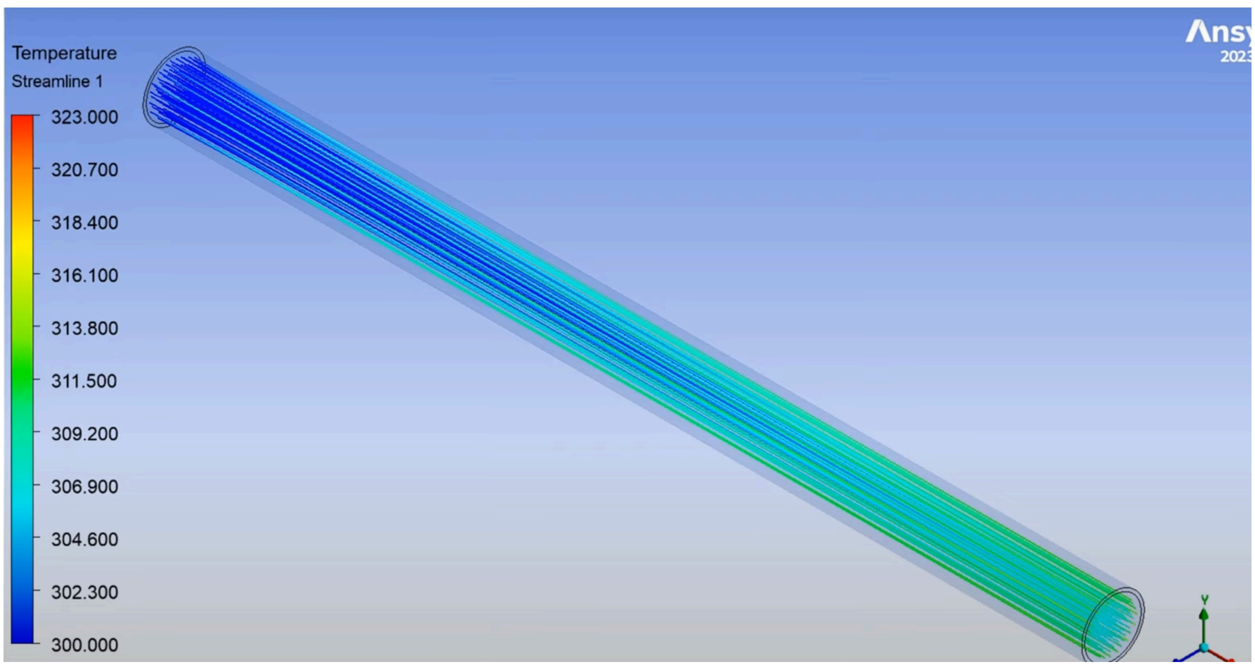
Using the formula: Entrance length=0.05×Re×pipe diameter

where:

- Reynolds number, Re=1273
- Pipe diameter = 0.05 m

Substituting the values: Entrance length=0.05×1273×0.05=3.1825 m

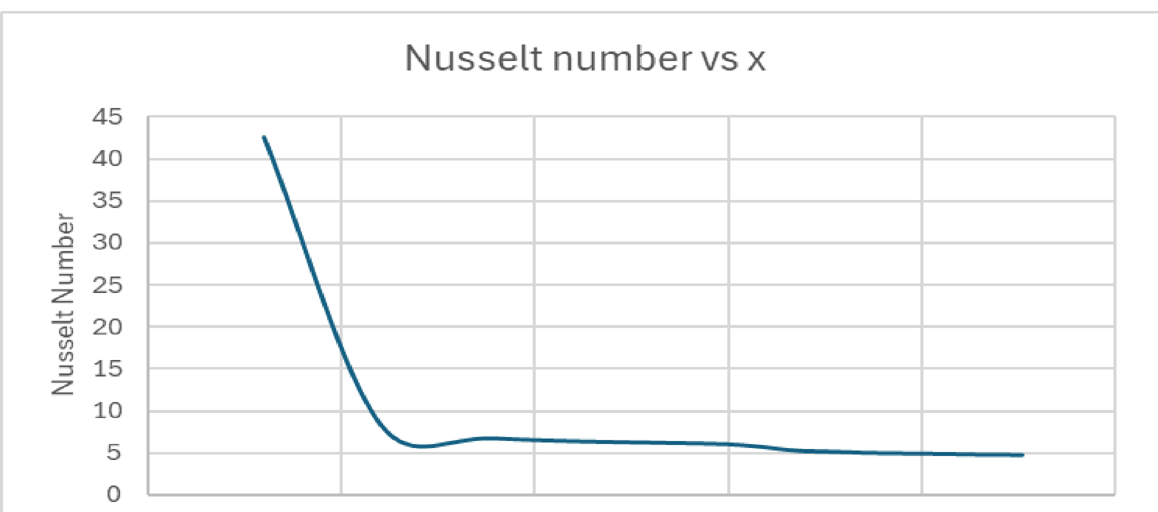
As model shows it is shown at Entrance length = 4.235m where the velocity profile at several locations along the pipe where the velocity profile becomes fully developed.



IV. Calculate Nusselt number at different locations (consider at least 15 locations) on the wall from x=0 to x=10m (equally spaced) and plot to show the variation as a function of length of the pipe. Does your value at any location come closer to 3.56? comment on your observation

$Nu = hD_h/k = qD_h/k(T_w-T_b)$

x	T_b	T_w	h	Nu
0.30	300.036	300.068	3106.217	42.564
0.60	300.454	303.679	620.076	8.496
1.90	300.930	304.972	494.871	6.781
2.50	301.408	305.914	443.824	6.181
2.80	311.885	312.700	415.374	6.091
3.25	312.362	319.392	397.310	5.944
3.70	315.840	327.035	384.979	5.795
4.50	319.317	326.634	376.171	5.794
5.35	320.795	329.205	369.678	5.765
5.95	324.272	329.755	364.787	4.698
6.05	325.749	332.289	361.014	4.547
6.90	328.227	338.812	358.075	4.406
7.15	335.704	341.326	355.735	4.374
7.20	335.182	341.833	353.900	4.249
8.26	336.659	342.334	352.406	4.123

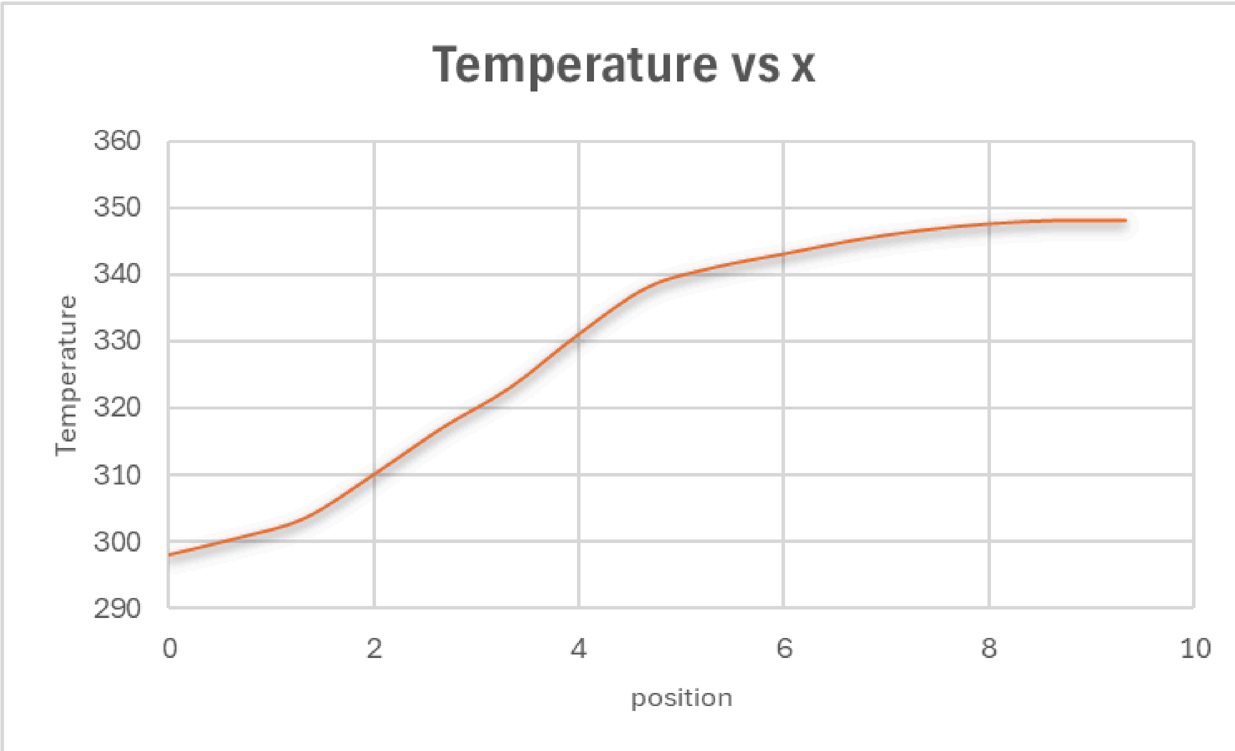


Observation:

1. The Nusselt number (Nu) shows a gradual decrease along the pipe length, indicating a reduction in convective heat transfer as the flow progresses and thermal equilibrium between the wall and fluid temperature is approached.
2. The NuNuNu value stabilises around 4.2, aligning closely with theoretical expectations for fully developed laminar flow in a circular pipe, validating the simulation accuracy.

V. Plot cross sectional average temperature as a function of length of the pipe and comment on thermal equilibrium between wall and liquid temperature. What is the exit temperature of fluid in your calculation? REPORT

Position along pipe(m)	Temperature
0	298
0.67	300.5
1.33	303.5
2.00	310
2.67	317
3.33	323
4.00	331
4.67	338
5.33	341
6.00	343
6.67	345
7.33	346.5
8.00	347.5
8.67	348
9.33	348
10.00	348



Comparison of Values:

As the fluid flows through the pipe, it absorbs heat from the wall, leading to a gradual increase in temperature until it approaches the wall temperature. Initially, there is a significant temperature gradient; however, as the fluid continues to travel, its average temperature asymptotically approaches the wall temperature. Ultimately, the exit temperature stabilises around 347 K.

(vi) Write a few comments on the agreement /disagreement between your computed results and published results.

Summary (what i understand and learn from this assignment)

I learned about the application of the heat energy balance equation and how it necessitates changes in boundary conditions, which significantly impacts the convergence of the solution. Analytical solutions for the Nusselt number exist for both constant wall heat flux (Nu = 4.3) and constant wall temperature (Nu = 3.56) under fully developed flow and heat transfer conditions. This assignment deepened my understanding of fluid flow and heat transfer principles, particularly in the context of numerical simulations and the significance of boundary conditions.