2.4 Given: 70% knows c, c++, 60% know Fortran, 50% both

Let c be c, c++, F be fortran $p(F \cap C)$

$p(C) = 0.7$, $P(F) = 0.6$ ~~$P(F \cup C)$~~ $= 0.5$

(a) does not know Fortran $\Rightarrow$ $p(F)' = 1 - p(F) = 1 - 0.6 = \boxed{0.4}$

(b) does not know Fortan and c/c++ $\Rightarrow$ $P(F' \cap C')$

$$P(F' \cap C') = 1 - P(F \cup C)$$
$$= 1 - (P(F) + P(C) - P(F \cap C))$$
$$= 1 - (0.6 + 0.7 - 0.5) = 1 - 0.8 = \boxed{0.2}$$

(c) knows c/c++ but not fortran $\Rightarrow$ $P(C \setminus F)$

$$P(C \setminus F) = P(C) - P(F \cap C) = 0.7 - 0.5 = \boxed{0.2}$$

(d) knows Fortran but not c/c++ $\Rightarrow$ $P(F \setminus C)$

$$P(F \setminus C) = P(F) - P(F \cap C) = 0.6 - 0.5 = \boxed{0.1}$$

(e) knows fortran, probability of knowing c/c++ $\Rightarrow$ $P(C|F)$

$$P(C|F) = \frac{P(C \cap F)}{P(F)} = \frac{0.5}{0.6} = \boxed{0.83}$$

(f) knows c/c++, probability of knowing fortran $\Rightarrow$ $P(F|C)$

$$P(F|C) = \frac{P(F \cap C)}{P(C)} = \frac{0.5}{0.7} = \boxed{0.7143}$$

2.5 Given: 3 independent tests

$P(A) = 0.2,\ P(B) = 0.3,\ P(C) = 0.5,$

Solution:

$P(A)' = 0.8,\ P(B)' = 0.7,\ P(C)' = 0.5$

Independent tests, A, B and C

$\therefore\ P(A' \cup B' \cup C') = P(A)' \cdot P(B)' \cdot P(C)'$

$= 0.8 \times 0.7 \times 0.5 = 0.28$

Found by atleast one test:

$P(\text{Found by } 1) = 1 - P(\text{Not found by any})$

$= 1 - (P(A') \cdot P(B)' \cdot P(C)')$

$= 1 - 0.28 = \boxed{0.72}$

2.14 Given: Spyware with 1 million tries

Solution:

(a) 6 different lower case letters $\Rightarrow {}_{26}P_6$

$26P_6 = \frac{26!}{(26-6)!} = \frac{26 \times 25 \times 24 \times 23 \times 22 \times 21 \times \cancel{20!}}{\cancel{20!}}$

$= 165765600$

$P(\text{correct password}) = \frac{\text{Number of tries (allowed)}}{\text{Total number of tries}}$

$= \frac{1,000,000}{165765600} = \boxed{0.006032}$

(b) 6 different letters and it is case sensitive (upper case)

$52P_6$

$P(\text{correct password}) = \frac{1,000,000}{52P_6} = \frac{1,000,000}{\left(\frac{52!}{46!}\right)}$

$= \frac{1,000,000}{52 \times 51 \times 50 \times 49 \times 48 \times 47} = \frac{1,000,000}{14,658,134,400} = \boxed{0.0000682}$

(c) any 6 letters and case sensitive $\Rightarrow 52^6$

$$P(\text{correct password}) = \frac{1,000,000}{52^6}$$

$$= \frac{1,000,000}{19770609664} = \boxed{0.00005058}$$

(d) any 6 characters including letters and digits $\Rightarrow 62^6$

$$P(\text{correct password}) = \frac{1,000,000}{62^6} = \frac{1,000,000}{56800235584}$$

$$= \boxed{0.00001761}$$

2.16 Given: 50% - S_1 , 20% - S_2 , 30% S_3

$P(S_1) = 0.5$, $P(S_2) = 0.2$, $P(S_3) = 0.3$

defect $\Rightarrow$ $P(D|S_1) = 0.05$

$P(D|S_2) = 0.03$

$P(D|S_3) = 0.06$

(a) All parts defective :-> Total $P(D)$

$$P(D) = P(D|S_1)\,P(S_1) + P(D|S_2)\,P(S_2) + P(D|S_3)\,P(S_3)$$

$$= (0.05 \times 0.5) + (0.03 \times 0.2) + (0.06 \times 0.3)$$

$$= 0.025 + 0.006 + 0.018 = \boxed{0.049}$$

(b) Probability of defect by S_1

$$P(S_1|D) = \frac{P(D|S_1)\,P(S_1)}{P(D)} = \frac{0.025}{0.049} = \boxed{0.510}$$

2.19 Given: P(Inspection) $P(I) = 0.2$, Inspected / no defect $= 0.95$

$\therefore \Rightarrow P(D|I) = (1 - 0.95) = 0.05$

Not inspected, no defect $= 0.7$

$\therefore P(D|I') = 1 - 0.7 = 0.3$

Find: Probability that customer received part has defect

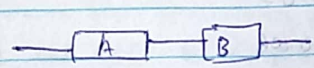
$$P(I|D) = \frac{P(D|I)\,P(I)}{P(D|I)\,P(I) + P(D|I')\,P(I')}$$

$$= \frac{0.05 \times 0.2}{(0.05 \times 0.2) + (0.7 \times 0.2)} = \cancel{\frac{0.01}{(0.01 + 0.14)}}$$

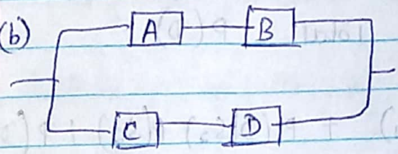
$$= \boxed{0.04}$$

2.23 Reliability of systems: A, B, C, D, E

Probabilities $\rightarrow$ 0.9, 0.8, 0.7, 0.6, 0.5

(a) 

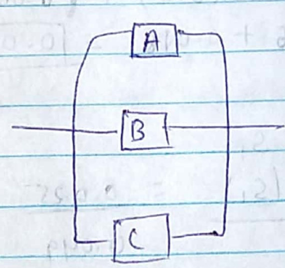
$\Rightarrow 0.9 \times 0.8 = \boxed{0.72}$

(b) 

$\Rightarrow 1 - [[1-(0.9)(0.8)][1-(0.7)(0.6)]]$

$= 1 - (1-0.72)(1-0.42)$

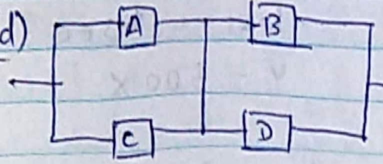
$= 1 - 0.1624 = \boxed{0.8376}$

(c) 

$\Rightarrow 1 - ((1-0.9)(1-0.8)(1-0.7))$

$= 1 - (0.1 \times 0.2 \times 0.3)$

$= 1 - 0.006 = \boxed{0.994}$

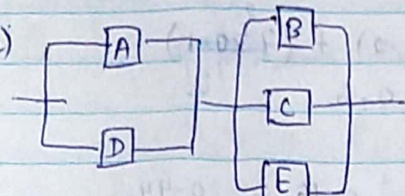
(d) 

$$\Rightarrow \left[1-(1-0.9)(1-0.7)\right]\left[1-(1-0.8)(1-0.6)\right]$$

$$= \left[1-(0.1\times 0.3)\right]\left[1-(0.2\times 0.4)\right]$$

$$= (1-0.03)(1-0.08)$$

$$= 0.97\times 0.92 = \boxed{0.8924}$$

(e) 

$$\Rightarrow \left[1-(1-0.9)(1-0.6)\right]\left[1-(1-0.8)(1-0.7)(1-0.5)\right]$$

$$= \left[1-(0.1\times 0.4)\right]\left[1-(0.2\times 0.3\times 0.5)\right]$$

$$= (1-0.04)(1-0.03)$$

$$= \cancel{0.96\times 0.97} =$$

$$= 0.96\times 0.97 = \boxed{0.9312}$$

2-26 2 out of 6 computers have hard drive problems.
3 are selected at random

Probability of choosing 3 from total 6 is $6C_3$ (order does not matter)

$$\therefore\ 6C_3 = \frac{6!}{3!(6-3)!} = \frac{6\times5\times4\times3!}{3\times2\times1\times3!} = \frac{120}{6} = 20$$

Probability of choosing 3 from 4 (the unaffected) $\Rightarrow 4C_3$

$$\therefore\ 4C_3 = \frac{4!}{3!\times1!} = \frac{4\times3\times2\times1}{3\times2\times1} = 4$$

$$P(\text{no hard drive affected}) = \frac{\text{choosing from 4}}{\text{choosing from total}} = \frac{4}{20} = \boxed{0.2}$$

3.2

x	0	1	2
$p(x)$	0.7	0.2	0.1

Loss = \$500

$Y = 500X$

<u>Find:</u> $E(x) = (0\times 0.7)+(1\times 0.2)+(2\times 0.1) = 0.4$

$Var(x) = E(x^2) - (E(x))^2 =$

$E(x^2) = (0^2\times 0.7) + (1^2\times 0.2) + (2^2\times 0.1)$

$= 0.2 + 0.4 = 0.6$

$(E(x))^2 = (0.4)^2 = 0.16$

$\therefore Var(x) = 0.6 - 0.16 = 0.44$

$\therefore E(Y) = 500\ E(x) = 500\times 0.4 = \boxed{\$200}$

$Var(Y) = 500^2\ Var(x) = 250000\times 0.44$

$= \boxed{\$110,000}$

3.7

x	0	1	2
$P(x)$	0.4	0.4	0.2

No. of games = 2

$Y = 2X$

$E(X) = (0\times 0.4) + (1\times 0.4) + (2\times 0.2) = 0.4 + 0.4 = 0.8$

$Var(x) = E(x^2) - (E(x))^2$

$E(x^2) = (0^2\times 0.4) + (1^2\times 0.4) + (2^2\times 0.2)$

$= 0 + 0.4 + 0.8 = 1.2$

$(E(x))^2 = (0.8)^2 = 0.64$

$Var(x) = 1.2 - 0.64 = 0.56$

$\therefore E(Y) = 2\ E(x) = 2\times 0.8 = \boxed{1.6}$

$Var(Y) = 2\ Var(x) = 2\times 0.56 = \boxed{1.12}$

3-18 PMF of X (Company A) PMF of Y (Company B)

X	-3	0	3
P(x)	0.3	0.2	0.5

Y	-3	0	3
P(y)	0.4	0	0.6

In terms of dollars, we can rewrite the above PMF as

A	\$-0.3	\$0	\$0.3
P_A	0.3	0.2	0.5

B	-\$1.5	\$0	\$1.5
P_B	0.4	0	0.6

3 Portfolios P_1, P_2, P_3

$P_1 = 100A$, $P_2 = 50A + 10B$, $P_3 = \cancel{40A + 12B}$ $20B$

(a) $E(A) = (-0.3 \times 0.3) + (0 \times 0.2) + (0.3 \times 0.5)$

$= -0.09 + 0 + 0.15 = 0.06$

$Var(A) = E(A^2) - (E(A))^2$

$E(A^2) = ((-0.3)^2 \times 0.3) + (0^2 \times 0.2) + [(0.3)^2 \times 0.5)]$

$= 0.27 + 0 + 0.45 = 0.72$

$(E(A))^2 = (0.06)^2 = 0.0036$

$Var(A) = 0.72 - 0.0036 = 0.0684$

$E(B) = (-1.5 \times 0.4) + (0 \times 0) + (1.5 \times 0.6)$

$= -0.6 + 0.9 = 0.3$

$Var(B) = E(B^2) - E(B)^2$

$E(B^2) = (-1.5^2 \times 0.4) + 0 + (1.5^2 \times 0.6)$

$= \cancel{(2.25 \times 0.4) + (2.25 \times 0.6)} = 2.25$

$= \cancel{29.25}$

$Var(B) = 2.25 - (0.3)^2 = 2.16$

$Var(P_1) = 100^2\ Var(A) = 100 \times 0.0684 = 684$

$Var(P_2) = 50^2\ Var(A) + 10^2\ Var(B)$

$= (2500 \times 0.0684) + 100 \times 2.16$

$= 387$

$Var(P_3) = 20^2\ Var(B)$

$= 400 \times 2.16 = 864$

(b) Since P_2 has lesser variance, it is least risky. when compared to P_1 & P_3.

(c) Since P_3 has highest variance, it is most risky.

3.25 10% do not close Windows Properly.

(a) On average, find users who do not close before someone closes it.

$p = 1 - 0.10 = 0.9$

$E(x) = 1/p = 1/0.9 = 1.111$

Excluding the last person (who closes).

$E(x-1) = E(x) - 1 = 1.111 - 1 = \boxed{0.111}$

(b) Exactly 8 out of 10 users close properly.

$P(x=8) = ?$

$P(x=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$x = 8,\ n = 10,\ p = 0.9$

$P(x=8) = \binom{10}{8} (0.9)^8 (0.1)^2$

$= {}^{10}C_8 \times 0.430467 \times 0.01$

$= 45 \times 0.430467 \times 0.01 = \boxed{0.194}$

3.26 Probability of damage $p = 0.2$

$n = 20$ files

(a) Probability atleast 5 out of 20 are damaged

$P(X \geq 5)$

PMF of $P(X = x) = \binom{n}{x} p^x q^{n-x}$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.6296$

$= \boxed{0.3704}$

(b) Probability atleast check 6 files to find 3 undamaged

$P(X = x) = \binom{x-1}{k-1} p^k q^{x-k}$

$x = 6, \; k = 3, \; p = 0.8$

$P(X = x) = \binom{x-1}{3-1} (0.8)^3 (0.2)^{x-3}$

$P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - P(X=3) - P(X=4) - P(X=5)$

$= 1 - \binom{3-1}{3-1} (0.8)^3 (0.2)^0 - \binom{4-1}{3-1} (0.8)^3 (0.2)^1$

$- \binom{5-1}{3-1} (0.8)^3 (0.2)^2$

$= 1 - 0.512 - 0.3072 - 0.12288$

$= \boxed{0.05792}$

3.27 $\lambda = 9$ messages per hour.

(a) Probability of receiving atleast 5 messages.

$$P(X=x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.055 \\ &= \boxed{0.945} \end{aligned}$$

(b) Probability of receiving exactly 5 messages.

$$P(X=5) = e^{-9} \frac{9^5}{5!} =$$

$$= \frac{0.000123 \times 81 \times 81 \times 9}{5 \times 4 \times 3 \times 2} = \boxed{0.006073}$$

3.28 Messages have Poisson Distribution, with λ.

To show $\Rightarrow$ Probability of receiving more than 4 does not exceed $1/9\lambda$

Chebyshev's Inequality : $P\{|X-\mu| > \varepsilon\} \leq \left(\frac{\sigma}{\varepsilon}\right)^2$

Here $\mu = \sigma^2 = \lambda$

$$P(X > 4\lambda) = P(X - \lambda > 3\lambda)$$

Substitute in Chebyshev's Inequality formula

$$P\{|X-\lambda| > 3\lambda\} \leq \left(\frac{\sigma}{3\lambda}\right)^2$$

$$P\{|X-\lambda| > 3\lambda\} \leq \left(\frac{\cancel{\lambda}}{\cancel{9\lambda}}\right)\left(\frac{\lambda}{9\lambda^2}\right)$$

$$\boxed{P\{|X-\lambda| > 3\lambda\} \leq 1/9\lambda}$$

4.2 $f(x) = \begin{cases} c(10-x)^2 & \text{if } 0 < x < 10 \\ 0 & \text{otherwise} \end{cases}$

(a) Compute c: General formula $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^{10} c(10-x)^2\,dx = 1$$

$$\Rightarrow c\int_0^{10}(10-x)^2\,dx = 1$$

$$\Rightarrow c\int_0^{10}(100+x^2-20x)\,dx = 1$$

$$\Rightarrow c\left[100x + \frac{x^3}{3} - 10x^2\right] = 1$$

$$\frac{1000c}{3} = 1 \Rightarrow c = 3/1000 = \boxed{0.003}$$

(b) Probability it takes 1 to 2 minutes to reboot the system

$$P(1 < X < 2) = \int_1^2 c(10-x)^2\,dx$$

$$= \int_1^2 0.003\,(100+x^2-20x)\,dx$$

$$= 0.003\left[\left[100(2)+\frac{2^2}{3}-10(2)^2\right]-\left[100(1)-\frac{1^2}{3}+10(1^2)\right]\right]$$

$$= 0.003\left[\left[200+\frac{4}{3}-40\right]-\left[100-\frac{1}{3}+10\right]\right]$$

$$= 0.003\left[\left[\frac{600+4-120}{3}\right]-\left[\frac{300-1+30}{3}\right]\right]$$

$$= 0.003\left[(484/3)-[329/3]\right]$$

$$= \cancel{0.003}\ \boxed{0.217}$$

4.4 $f(x) = \begin{cases} k - x/50 & \text{for } 0 < x < 10 \text{ years} \\ 0 & \text{for all other } x \end{cases}$

(a) Find k: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^{10} (k - x/50)\,dx = 1$$

$$\left(kx - \frac{x^2}{100}\right)_0^{10} = 1$$

$$\left[\left(k(10) - \frac{10^2}{100}\right) - 0\right] = 1$$

$$10k - 1 = 1$$

$$10k = 2$$

$$k = 2/10 = \boxed{0.2}$$

(b) Probability of failure within 1st 5 years

$$P(0 < X < 5) = \int_0^5 \left(k - \frac{x}{50}\right) dx$$

$$= \int_0^5 \left(0.2 - \frac{x}{50}\right) dx = \left(0.2x - \frac{x^2}{100}\right)_0^5$$

$$= \left[0.2(5) - \frac{1}{4}\right] - 0$$

$$= 1 - 0.25 = \boxed{0.75}$$

(c) Expectation of the lifetime $\Rightarrow E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

$$\int_0^{10} (0.2x - x^2/50)\,dx$$

$$= \left[\frac{0.2x^2}{2} - \frac{x^3}{150}\right]_0^{10} = \left[\frac{x^2}{10} - \frac{x^3}{150}\right]_0^{10} = \left[\left(\frac{10^2}{10} - \frac{10^3}{150}\right) - 0\right]$$

$$= 10 - 6.66667$$

$$= \boxed{3.333}$$

4.7 $\lambda = 1/12 \text{ sec}^{-1}$, $\alpha = 3$

$P(T \leq 60)$ follows Gamma-Poisson formula

$P(T \leq t) = P(X \geq \alpha)$

Probability that the job will be ready before 10:01

$$P(T < 60) = P(X \geq 3)$$
$$= 1 - P(X \leq 2)$$

From table A-3 for Poisson distribution

$$= 1 - 0.125$$
$$= \boxed{0.875}$$

4.30 Line 1 has Gamma Connection with $\alpha = 3$ $\lambda = 2 \text{ min}^{-1}$

Line 2 has Uniform Connection (20 sec, 50 sec)

$P(A) = 0.8$

$P(B) = 0.2$

$L_1 \to \text{Gamma}(3, 2)$

$L_2 \to \text{Uniform}(20, 50)$

Let X be the event it takes 30 seconds to connect

$$P(X) = P(A) \cdot P(X|A) + P(B) \cdot P(X|B)$$

Gamma - Poisson formula:

$P(X|A)$ $\quad P(t_1 > 0.5)$

$= P(x \le 3)$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 0.3679 + 0.3678 + 0.1839 + 0.0613$

(Table Lookup)

$= 0.9810$

Uniform Distribution:

$P(X|B) \rightarrow P(t_2 > 30)$

$$= \int_{30}^{50} \frac{1}{50-20} \, dt_2$$

$$= \frac{1}{30} (t_2)_{30}^{50}$$

$= 1/30 \, (20)$

$= 0.667$

$P(X) = P(A) \cdot P(X|A) + P(B) \cdot P(X|B)$

$= 0.8(0.981) + 0.2(0.667)$

$= 0.7848 + 0.1334$

$= \boxed{0.9182}$