①

NAME : SUDHARSAN SRINIVASAN

UTA ID : 1001755919

ASSIGNMENT - 3

Q1. Samples : 8.91, 11.85, 8.51, 10.82, 8.34, 8.99, 11.99, 8.83, 10.56, 9.83

a.

$$\text{Mean}_1 = \frac{98.63}{10} = 9.863$$

$$\text{Mean}_2 = \frac{(8.91 - 9.863)^2 + (11.85 - 9.863)^2 + \dots + (9.83 - 9.863)^2}{10}$$

$$\text{Mean}_2 = 1.784$$

$$\mu_1 = \frac{a+b}{2} \quad \text{as } \mu_1 = 9.863 \Rightarrow \frac{a+b}{2} = 9.863$$

$$a + b = 19.726 \quad — ①$$

$$\mu_2 = \frac{(b-a)^2}{12} \quad \text{as } \mu_2 = 1.784 \Rightarrow \frac{(b-a)^2}{12} = 1.784$$

$$(b-a)^2 = 21.36$$

$$b - a = 4.621$$

$$b = 4.621 + a \quad — ②$$

Substitute ② in ①, we get $2a + 4.621 = 19.726$

$$\therefore a = 7.552$$

Sub. (a) in equation ②

$$b = 4.621 + 7.552 = 12.1735$$

$\therefore$ Parameters of Uniform Distribution

$\boxed{a = 7.552}$ $\boxed{b = 12.1735}$

Q1. b. Negative Binomial Distribution:

Samples : 11, 10, 9, 11, 15, 10, 11, 10, 8, 13

Mean $\mu = \frac{108}{10} = 10.8$

Variance $\sigma^2 = \frac{(11-10.8)^2 + (10-10.8)^2 + (9-10.8)^2 + \dots + (13-10.8)^2}{10}$

$\sigma^2 = \frac{35.6}{10} = 3.56$

For Negative Binomial distribution $\mu = k/p \Rightarrow 10.8 = k/p$

$\sigma^2 = \frac{k(1-p)}{p^2}$ (Here $k = 10.8p$)

$= \frac{10.8\cancel{p}(1-p)}{p^{\cancel{2}}}$

$3.56p = 10.8 - 10.8p$

$14.36p = 10.8$

$\Rightarrow p = 0.752$

Substitute to get K =

$K = 10.8 \times 0.752 = 8.122$

$\therefore$ $\boxed{p = 0.752}$ $\boxed{k = 8.122}$

Q1. c. Gamma Distribution:

Samples: 1.05, 0.44, 0.74, 0.49, 0.84, 0.73, 0.70, 0.53, 0.50, 0.63

$$\mu_1 = \text{Mean} = \frac{6.65}{10} = 0.665$$

$$\mu_2 = \frac{(0.665-1.05)^2 + (0.665-0.44)^2 + \dots + (0.665-0.63)^2}{10}$$

$$(\sigma^2) = \frac{0.313}{10} = \boxed{0.0313}$$

$$\mu = \text{Mean} = \frac{\alpha}{\lambda}$$

$$\mu = \frac{\alpha}{\lambda}$$

$$0.665 = \alpha/\lambda$$

$$\boxed{\alpha = 0.665\lambda \quad \text{—— } 1}$$

Also, $\sigma^2 = \alpha/\lambda^2$

$$0.0313 = \frac{0.665\cancel{\lambda}}{\lambda^{\cancel{2}}} \rightarrow \text{from eq } 1$$

$$\lambda = \frac{0.665}{0.0313} = 21.24$$

Sub. λ in equation 1

$$\alpha = 0.665 \times 21.24$$
$$= 14.128$$

$\therefore \boxed{\alpha = 14.128}$, $\boxed{\lambda = 21.24}$

9.2 Geometric Distribution → 5 projects $(3, 7, 5, 3, 2)$

Find $p = ?$

(a) For Geometric distribution

$$E(x) = 1/p$$

Estimate p by Method of Moments

$$p = \frac{1}{\bar{X}}$$

$$= \frac{1}{\frac{1}{n}\sum_{i=1}^{n} X_i} = \frac{1}{\frac{1}{5}(3+7+5+3+2)} = \frac{1}{\frac{1}{5}(20)} = \frac{5}{20}$$

$$\boxed{p = 0.25}$$

(b) Method of Maximum Likelihood

$$\text{Likelihood function } (L) = p^n (1-p)^{\sum_{i=1}^{n} X_i}$$

$$\ln(L) = n \ln(p) + \left[\sum_{i=1}^{n} X_i - n\right] \ln(1-p)$$

$$\frac{dL}{dp} = \frac{n}{p} - \frac{\left[\sum_{i=1}^{n} X_i - n\right]}{(1-p)}$$

$$\Rightarrow \frac{n}{p} = \frac{n(\bar{x} - n)}{1-p}$$

$$n(1-p) = p(n\bar{x} - n)$$

$$n - np = np - n\bar{x}p$$

$$\Rightarrow \frac{n}{n} = p\bar{x} \Rightarrow p = \frac{1}{\bar{x}}$$

$$p = \frac{1}{\bar{x}} = \frac{1}{\frac{1}{5}(20)} = 0.25$$

$$\boxed{\therefore p = 0.25}$$

∴ Method of Maximum Likelihood is equal to Method of Moments

9.7 Number of users $n = 100$, average $\bar{x} = 37.7$

$S.D. = \sigma = 9.2$

Since S.D is known, go with Z-test

a. <u>90% C.I.</u>

$\therefore \quad 1 - \alpha = 0.90$

$\alpha = 0.10 \Rightarrow \alpha/2 = 0.05$

$Z_{\alpha/2} = Z_{0.05} = 1.645$

$$C.I. = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 37.7 \pm 1.645\left(\frac{9.2}{10}\right)$$

$$= 37.7 \pm 1.645(0.92)$$

$$= 37.7 \pm 1.5134$$

$$= (36.1866, 39.21)$$

Confidence Interval $= [36.19, 39.21]$

b. <u>Hypothesis Testing:</u> $\mu > 35 \Rightarrow$ to test

$H_0 \Rightarrow \mu = 35$ Significance Level $= 0.01$

$H_A \Rightarrow \mu > 35$ $\alpha = 0.01$

$Z_\alpha \Rightarrow Z_{0.01} = 2.326$

If $Z > Z_\alpha$, reject H_0, accept H_A

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{37.7 - 35}{9.2/10} = 2.935$$

Here $Z > Z_\alpha \Rightarrow$ accept H_A

$\therefore$ There is enough evidence to show that mean number of concurrent users is greater than 35

9.8 Number of users, $n = 64$, average $\bar{x} = 42$, S.D. $\sigma = 5$

a. <u>95% C.I:</u>

$$1-\alpha = 0.95$$
$$\alpha = 0.05$$
$$Z_{\alpha/2} \Rightarrow Z_{0.025} = 1.96$$

$$C.I. = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$= 42 \pm 1.96\left(\frac{5}{8}\right)$$
$$= 42 \pm 1.225$$
$$= (40.775, 43.225)$$

Confidence Interval (C.I) $= [40.775, 43.225]$

b. <u>Probability that Installation time be within the C.I.</u>

Installation time $X = 40$ minutes

$$P(40.775 < X < 43.225) = P\left(\frac{40.775-40}{5} < Z < \frac{43.225-40}{5}\right)$$
$$= P(0.151 < Z < 0.645)$$
$$= P(Z < 0.645) - P(Z < 0.151)$$
$$= 0.7405 - 0.5600$$
$$= 0.1805$$
$$= 0.181$$

$\therefore$ Probability is $\boxed{P = 0.181}$

9.9 Samples = 30, 50, 70

a. 90% CI: $1-\alpha = 0.90$

$\alpha = 0.10$

$Z_{\alpha/2} = Z_{0.05}$, Here $t_{0.05}$

Number of samples ⇒ $n = 3$

Mean $\bar{X} = \frac{150}{3} = 50$

$$S.D = \sqrt{\frac{(30-50)^2 + (50-50)^2 + (70-50)^2}{3-1}} = \sqrt{800/2}$$

$$= \sqrt{400} = 20$$

~~From~~ For the t- distribution, degrees of freedom ⇒ $n-1$

$(3-1) = \underline{\underline{2}}$

From the table A5 ⇒ $\alpha = 0.05$, $\nu = 2$

$t = 2.920$

$$C.I. = \bar{X} \pm t_{critical}\left(\frac{S}{\sqrt{n}}\right)$$

$$= 50 \pm 2.92\left(\frac{20}{\sqrt{3}}\right)$$

$$= 50 \pm 33.717$$

$$= 16.283, 83.717$$

$$\boxed{\text{Confidence Interval (CI)} = [16.283, 83.717]}$$

b. 10% significance

This is equal to 90% CI

∴ It provides enough evidence that at 10% of significance that the average salary of all entry-level computer engineers at CI [16.283, 83.717] is different from $80,000

c. 90% C.I:

$$1-\alpha = 0.90$$

$$\alpha = 0.10$$

$$\alpha/2 = 0.05$$

$$t_{\alpha/2} = t_{0.05} \quad \text{with } v = 2 \Rightarrow 5.991$$

$$[1 - t_{\alpha/2}] \Rightarrow [1-\alpha/2] = 0.95$$

$$\therefore 0.95 \text{ with } v = 2 \Rightarrow 0.103$$

$$90\%\ C.I \Rightarrow \left(\sqrt{\frac{(n-1)s^2}{\chi^2_{(0.05,9)}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{(0.95,9)}}} \right)$$

$$\Rightarrow \left(\sqrt{\frac{(3-1)20^2}{5.991}} \le \sigma \le \sqrt{\frac{(3-1)20^2}{0.103}} \right)$$

$$\Rightarrow \left(\sqrt{\frac{800}{5.991}} \le \sigma \le \sqrt{\frac{800}{0.103}} \right)$$

$$\Rightarrow (11.5556 \le \sigma \le 89.4427)$$

$\therefore$ We are 90% confident that the S.D. of entry-level salaries will be in the intervals of $[11.556 \le \sigma \le 89.4427]$

9.10 $\bar{x} = 24, \quad n = 200$

Sample proportion $\hat{p} = \frac{\bar{x}}{n}$

$$\hat{p} = \frac{24}{200} = 0.12$$

a. 96 % CI

$$1 - \alpha = 0.96$$
$$\alpha = 0.04$$
$$Z_{\alpha/2} = Z_{0.02} = 2.054$$

Confidence Interval (CI) $= \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$= 0.12 \pm 2.054 \sqrt{\frac{(0.12)(0.88)}{200}}$$
$$= 0.12 \pm 2.054\,(0.02297)$$
$$= 0.12 \pm 0.04719$$
$$= (0.073, 0.167)$$

$$\boxed{\text{Confidence Interval (CI)} = [0.073, 0.167]}$$

b. Hypothesis Testing: At most 10 % items are defective.

$H_0 = p \leq 0.1$ $\qquad \alpha = 0.04$

$H_A = p > 0.1$ $\qquad$ From Normal distribution table value is 1.751

If $Z > Z_\alpha \Rightarrow$ reject H_0, accept H_A

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.12 - 0.10}{\sqrt{\frac{(0.10)(0.90)}{200}}} = \frac{0.02}{0.0212}$$

$$\boxed{Z = 0.943}$$

Here $Z < Z_\alpha$

$\therefore$ Null hypothesis can't be rejected

Therefore, it can be concluded that atmost one in 10 items is defective.

For $\alpha = 0.15$ significance

$Z = 1.0364$

$Z < Z_\alpha \Rightarrow$ Null hypothesis can't be rejected.

$\therefore$ It can be concluded that atmost 1 in 10 items is defective.

9.11 Previous supplier: (9.10)

$x = 24$, $n = 200$

$\therefore$ Proportion $\hat{P}_1 = \frac{\bar{X}}{n} = \frac{24}{200} = 0.12$

New Supplier:

$\bar{X} = 13$, $n = 150$

$\therefore$ Proportion $\hat{P}_2 = \frac{\bar{X}}{n} = \frac{13}{150} = 0.087$

Population Proportion of Previous & Current Supplier

$$\hat{p} = \frac{\bar{X}_1 + X_2}{n_1 + n_2}$$

$$= \frac{24 + 13}{200 + 150}$$

$$= \frac{37}{350}$$

$$\hat{p} = 0.1057$$

Hypothesis Testing:

Null Hypothesis $H_0 = \hat{P}_1 \leq \hat{P}_2$

Alternate Hypothesis $H_A = \hat{P}_1 > \hat{P}_2$

Level of significance $\alpha = 0.05$

Test Statistic $Z = \dfrac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$= \frac{0.12 - 0.087}{\sqrt{(0.1057)(1-0.1057)\left(\frac{1}{200} + \frac{1}{150}\right)}}$$

$$= \frac{0.033}{\sqrt{(0.0945)(0.00116)}} = \frac{0.33}{\sqrt{0.0011025}}$$

$$= \frac{0.33}{0.33} = 1$$

$\boxed{Z = 1}$

P-value: P-value $= P(Z > Z_0)$

$= 1 - P(Z \leq 1)$

$= 1 - 0.841345$ (From standard normal table)

$\boxed{\text{p-value} = 0.158655}$

Here P-value > Level of significance (0.05). Therefore we cannot reject Null Hypothesis.

∴ There is no significant evidence to say that quantity of items by new supplier is higher than quantity of items

9.16 Lot A: $\bar{x} = 10$ defective items

Total $(n) = 250$

$$\hat{P}_A = \frac{\bar{x}}{n} = \frac{10}{250} = 0.04$$

Lot B: $\bar{x} = 18$ defective items

Total $(n) = 300$

$$\hat{P}_B = \frac{\bar{x}}{n} = \frac{18}{300} = 0.06$$

a. 98 % CI

$$1 - \alpha = 0.98$$

$$\alpha = 0.02 \quad , \quad \alpha/2 = 0.01$$

$$Z_{\alpha/2} = Z_{0.01} = 2.326$$

$$\text{Confidence Interval (CI)} = \left(\hat{P}_A - \hat{P}_B\right) \pm Z_{critical} \sqrt{\frac{\hat{P}_A(1-\hat{P}_A)}{n_A} + \frac{\hat{P}_B(1-\hat{P}_B)}{n_B}}$$

$$= (0.04 - 0.06) \pm 2.326 \sqrt{\frac{(0.04)(1-0.04)}{250} + \frac{0.0564}{300}}$$

$$= -0.02 \pm 2.326\ (0.01848)$$

$$= -0.02 \pm 0.043$$

$$\boxed{C.I. = [-0.063 , 0.023]}$$

b. Hypothesis Testing:

H_o = no significance between qualities of 2 lots.

H_A = significance between qualities of 2 lots.

Level of significance $\alpha = 0.02$

Z critical $= 2.326$

Absolute Test statistic $|z| = \dfrac{\hat{P}_1 - \hat{P}_2}{\sqrt{\dfrac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \dfrac{\hat{P}_2(1-\hat{P}_2)}{n_2}}}$

$$= \frac{0.04 - 0.06}{\sqrt{\dfrac{0.04(1-0.04)}{250} + \dfrac{(0.06(1-0.06))}{300}}}$$

$$= \frac{\cancel{-0.02}}{\sqrt{\cancel{(0.0001536)} + \cancel{(0.000128)}}}$$

$$= \frac{-0.02}{\sqrt{(0.0001536) + (0.000188)}} = \frac{-0.02}{\sqrt{0.0003416}}$$

$$= \frac{-0.02}{0.01848} = -1.08$$

$z = -1.06$

$\boxed{|z| = 1.06}$

Here $|z| < z_{critical}$

We cannot reject Null Hypothesis.

$\therefore$ There is no sufficient evidence that there is a significant difference between the quality of the 2 lots.

9.17 Find Margin of Error ($\hat{p}_1$) at 45%

$C.I = 95\%$

$1-\alpha = 0.95$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$\hat{p}_1 = 45\% \Rightarrow 0.45, \quad n = 900$

Margin of Error $ME = Z_{critical}\sqrt{\frac{p(1-p)}{n}}$

$= 1.96\sqrt{\frac{(0.45)(1-0.45)}{900}}$

$= 1.96(0.016583)$

$= 0.0325$

$\boxed{ME = 3.25\%}$

Margin of Error at ($\hat{p}_2$) 35%

$\hat{p}_2 = 35\% \Rightarrow 0.35, \quad n = 900$

$ME = Z_{critical}\sqrt{\frac{p(1-p)}{n}}$

$= 1.96\sqrt{\frac{(0.35)(1-0.35)}{900}}$

$= 1.96(0.01589)$

$= 0.0312$

$\boxed{ME = 3.12\%}$

Margin of Errors $(\hat{P}_1 - \hat{P}_2)$

$Z_{critical} = 1.96$

$ME = Z_{critical} SE(\hat{P}_1 - \hat{P}_2)$

$= 1.96 \sqrt{\frac{(0.45)(1-0.45)}{900} + \frac{(0.35)(1-0.35)}{900}}$

$= 1.96 (0.02297)$

$= 0.04502$

$\boxed{ME = 4.50\%}$

9.20 Sample Size $n = 40$, $S.D = 6.2$, $\alpha = 0.05$

Degree of freedom $\Rightarrow n-1 = 39$

To find Variability of population SD, use a test based on Chi-square distribution.

H_0 = Sample S.D. is not significantly different from the actual value

H_A = Sample S.D is significantly different from the actual value.

$\therefore H_0 \Rightarrow \sigma \neq 5$

$H_A \Rightarrow \sigma = 5$

Let alpha be 0.05

$\boxed{\alpha = 0.05}$

Test statistic is χ^2_{abs}

$$\chi^2_{abs} = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(40-1)(6.2)^2}{5^2}$$

$$= \frac{39 \times 38.44}{25} = \frac{1499.16}{25} = 59.9664$$

$$\boxed{\chi^2_{abs} = 59.9664}$$

For two tail test, $\chi^2_{\alpha/2} = \chi^2_{0.05/2}$

$\chi^2_{0.025}$ with $df = 39$

$\boxed{\chi^2_{0.025} = 58.1}$ (from Chi-square Table)

Here $\chi^2_{abs} > \chi^2_{\alpha/2}$

$\therefore$ Reject H_0, Accept H_A

$\therefore$ There is sufficient evidence that the sample standard deviation is significantly different from the assumed value. Hence, we agree with the Manager.