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CSE 5311  
Test 1 - Closed Book  
Spring 2004

Name \_\_\_\_\_

Student ID # \_\_\_\_\_

Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. To reduce the probability of having any collisions to  $< 0.5$  when hashing  $n$  keys, the table should have at least this number of elements.
  - A.  $n$
  - B.  $n \ln n$
  - C.  $n^2$
  - D.  $n^3$
2. Path compression is used in which algorithm?
  - A. DECREASE-KEY (e.g. CASCADING-CUT) for Fibonacci heaps
  - B. FIND for disjoint sets
  - C. UNION for disjoint sets
  - D. UNION for Fibonacci heaps
3. When performing selection in worst-case linear time, roughly what fraction of the set of  $n$  keys is kept (in the worst case) for the next round?
  - A. 10%
  - B. 20%
  - C. 30%
  - D. 70%
4. Which Fibonacci heap operation has  $O(\log n)$  actual cost?
  - A. FIB-HEAP-DECREASE-KEY
  - B. FIB-HEAP-DELETE
  - C. FIB-HEAP-EXTRACT-MIN
  - D. FIB-HEAP-UNION
5. The main difference between MTF and OPT for self-organizing linear lists is:
  - A. MTF counts inversions

- B. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
- C. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
- D. OPT can do transpositions

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6. The minimum number of nodes in a tree in a Fibonacci heap where the root has 7 children is:
- A. 34  
B. 61  
C. 121  
D. 128
7. The maximum value of the potential function when comparing MTF and OPT on a list with  $n$  elements is:
- A.  $n$   
B.  $\frac{n^2-n}{2}$   
C.  $\frac{n^2+n}{2}$   
D.  $n^2-n$
8. In the worst case, the number of rotations for inserting a key in a treap with  $n$  keys is:
- A.  $\theta(1)$   
B.  $\theta(\log n)$   
C.  $\theta(n)$   
D.  $\theta(n \log n)$
9. Slow convergence toward the optimal fixed ordering is a property of which technique?
- A. Count  
B. Move-ahead-k  
C. Move-to-front  
D. Transpose
10. Sorting the edges is a property of which minimum spanning tree technique?
- A. Boruvka  
B. Kruskal  
C. Prim  
D. Path-based (Warshall)

Long Answer

1. Suppose there are 20 coupon types for the coupon collecting problem and you have already obtained 18 of the coupon types. How many boxes of cereal do you expect (mathematically) to open to get the remaining two coupon types? (5 points)
2. Suppose there are 20 coupon types for the coupon collecting problem. Each of the coupon types is

2. Suppose there are 20 coupon types for the coupon collecting problem. Each of the coupon types is identified by an integer in the range 1 to 20. If you are given two random cereal boxes, what is the probability (ignoring the order in which the coupons were obtained) that the two coupons have consecutive numbers, i.e. some  $i$  and  $i + 1$ ? (5 points)

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CSE 5311

Name \_\_\_\_\_

Test 1 - Open Book

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1. Suppose you wish to implement a verifier to test if a given subset (with  $V - 1$  edges) of the set of edges for a graph is a minimum spanning tree. Assuming you already have the code below for Kruskal's algorithm, explain how it should be modified to perform this verification. You may assume that each struct in edgeTab has a field subset that flags whether this edge is in the alleged MST. (15 points)

```
...
main()
{
    ...
qsort(edgeTab,numEdges,sizeof(edgeType),weightAscending);
for (i=0;i<numEdges;i++)
{
    root1=find(edgeTab[i].tail);
    root2=find(edgeTab[i].head);
    if (root1==root2)
        printf("%d %d %d discarded\n",edgeTab[i].tail,edgeTab[i].head,
               edgeTab[i].weight);
    else
    {
        printf("%d %d %d included\n",edgeTab[i].tail,edgeTab[i].head,
               edgeTab[i].weight);
        makeEquivalent(root1,root2);
    }
}
if (numTrees!=1)
    printf("MST does not exist\n");
}
```

2. Evaluate the following recurrences using the master method. (15 points)

a.  $T(n) = T(0.7n) + n$

b.  $T(n) = T(0.7n) + 1$

c.  $T(n) = 16T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

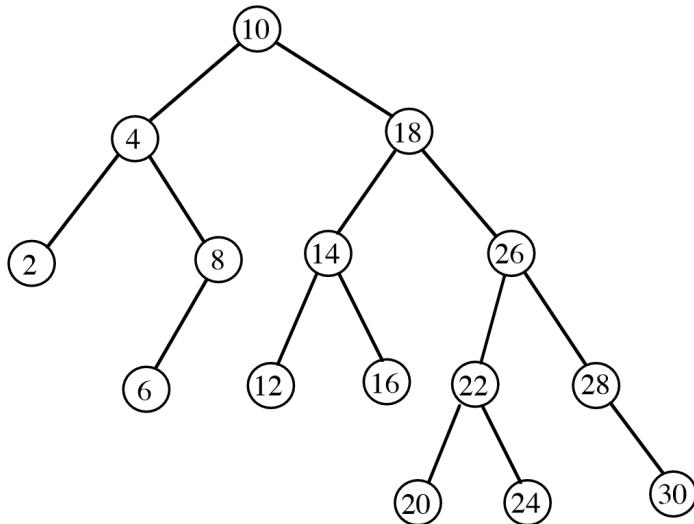
$n=6;$	$q[2]=0.03;$	$key[5]=50;$
$q[0]=0.01;$	$key[3]=30;$	$p[5]=0.02;$
$key[1]=10;$	$p[3]=0.2;$	$q[5]=0.04;$
$p[1]=0.19;$	$q[3]=0.04;$	$key[6]=60;$
$q[1]=0.02;$	$key[4]=40;$	$p[6]=0.12;$
$key[2]=20;$	$p[4]=0.2;$	$q[6]=0.03;$

p[2]=0.1; q[4]=0.0; w[0][0]=0.010000

2

```
w[0][1]=0.220000          w[6][6]=0.030000          c(6,6) cost 0.000000
w[0][2]=0.350000          Building c(0,2) using roots 1 thru 2   c(0,1) cost 0.220000 10
w[0][3]=0.590000          Building c(1,3) using roots 2 thru 3   c(1,2) cost 0.150000 20
w[0][4]=0.790000          Building c(2,4) using roots 3 thru 4   c(2,3) cost 0.270000 30
w[0][5]=0.850000          Building c(3,5) using roots 4 thru 5   c(3,4) cost 0.240000 40
w[0][6]=1.000000          Building c(4,6) using roots 5 thru 6   c(4,5) cost 0.060000 50
w[1][1]=0.020000          Building c(0,3) using roots 1 thru 3   c(5,6) cost 0.190000 60
w[1][2]=0.150000          Building c(1,4) using roots 3 thru 3   c(0,2) cost 0.500000 10(,20)
w[1][3]=0.390000          Building c(2,5) using roots 3 thru 4   c(1,3) cost 0.540000 30(20,,)
w[1][4]=0.590000          Building c(3,6) using roots 4 thru 6   c(2,4) cost 0.710000 30(,40)
w[1][5]=0.650000          Building c(0,4) using roots 2 thru 3   c(3,5) cost 0.360000 40(,50)
w[1][6]=0.800000          Building c(1,5) using roots 3 thru 4   c(4,6) cost 0.270000 60(50,,)
w[2][2]=0.030000          Building c(2,6) using roots 4 thru 4   c(0,3) cost 1.080000 20(10,30)
w[2][3]=0.270000          Building c(0,5) using roots 3 thru 3   c(1,4) cost 0.980000 30(20,40)
w[2][4]=0.470000          Building c(1,6) using roots 3 thru 4   c(2,5) cost 0.860000 40(30,50)
w[2][5]=0.530000          Building c(0,6) using roots ? thru ?   c(3,6) cost 0.720000 40(,60(50,,))
w[2][6]=0.680000          Counts - root trick 29 without root   c(0,4) cost 1.530000 30(10(,20),40)
w[3][3]=0.040000          trick 50                           c(1,5) cost 1.160000 30(20,40(,50))
w[3][4]=0.240000          Average probe length is ??????   c(2,6) cost 1.220000 40(30,60(50,,))
w[3][5]=0.300000          trees in parenthesized prefix   c(0,5) cost 1.710000 30(10(,20),40(,50))
w[3][6]=0.450000          c(0,0) cost 0.000000   c(1,6) cost 1.610000 40(30(20,,),60(50,,))
w[4][4]=0.000000          c(1,1) cost 0.000000   c(0,6) cost ?????????? ??????????????
w[4][5]=0.060000          c(2,2) cost 0.000000
w[4][6]=0.210000          c(3,3) cost 0.000000
w[5][5]=0.040000          c(4,4) cost 0.000000
w[5][6]=0.190000          c(5,5) cost 0.000000
```

4. Insert 23 into the following AVL tree to preserve the AVL properties. (10 points)



CSE 5311  
Test 2 - Closed Book  
Spring 2004

Name \_\_\_\_\_  
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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)
  - A. 3-satisfiability
  - B. Testing if a graph is 3-colorable
  - C. Testing if a table is in sorted order
  - D. Testing if the number of colors needed to edge color a graph is the degree of the graph
  
2. How many times will -1 occur in the style 1 fail link table for the pattern **abcabd**?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
  
3. How many times will -1 occur in the style 2 fail link table for the pattern **abcabd**?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
  
4. Suppose that you have devised a reduction from a known NP-complete problem A to a problem B in NP. Later you discover that the reduction takes exponential time. What conclusion may be drawn?
  - A. P  $\neq$  NP
  - B. Problem B is not NP-complete.
  - C. You need a reduction that takes linear time to prove that B is NP-complete.
  - D. You need a reduction that takes polynomial time to prove that B is NP-complete.
  
5. Which of the following is not part of the Gale-Shapley algorithm?
  - A. Men issue proposals
  - B. Finding rotations
  - C. Women break engagements to accept proposals from their more-preferred men
  - D. Women accept proposals if unengaged
  
6. Which of the following is not true regarding Strassen's algorithm?
  - A. It is not possible to have an asymptotically faster algorithm.
  - B. It requires more space than the everyday method.
  - C. It uses  $\Theta(n^{\lg 7})$  scalar additions when multiplying two n x n matrices.
  - D. It uses  $\Theta(n^{\lg 7})$  scalar multiplications when multiplying two n x n matrices.
  
7. Which algorithm is defined using the notions of left-turn and right-turn?
  - A. Closest points in 2-d space

- B. Graham scan  
 C. Jarvis march  
 D. Suffix array construction
8. Which of the following is not a condition for performing a push operation from u to v?
- the height at u is one more than the height at v
  - (u, v) has capacity > flow
  - u has excess > 0
  - v has a path to the sink in the residual graph
9. Which of the following does not have an approximation algorithm?
- Bin packing
  - Edge coloring
  - Traveling salesperson
  - Vertex covering
10. The length of a longest strictly increasing subsequence for 1 4 3 2 3 4 3 6 3 4 is:
- 4
  - 5
  - 6
  - 7
11. Which longest common subsequence method is potentially the most time-consuming?
- Compact version of dynamic programming
  - Method based on subsequence indices and longest strictly increasing subsequence
  - Ordinary dynamic programming
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a “restart”), it can be an articulation point only if
- there are no outgoing tree edges
  - there is one outgoing tree edge
  - there is more than one outgoing tree edge
  - there is no back edge returning to this vertex
13. Which of the following problems uses some edges with non-unit capacity when translated to a network flow problem?
- Bipartite matching
  - Edge connectivity
  - Minimum vertex cover for bipartite graph
  - Vertex connectivity
14. Which string search method is potentially the most time-consuming?
- A. Karp-Rabin

- B. KMP with fail 1 links
  - C. KMP with fail 2 links
  - D. Z table
15. What is the maximum number of times that an edge can become critical in the Edmonds-Karp method?
- A. Once
  - B. Twice
  - C.  $(V-2)/2$
  - D. VE
16. Suppose you have a set of points in Euclidean 2-d space. Give an algorithm for finding a  $\rho$ -approximation for the minimum traveling salesperson path. Be sure to give the value of  $\rho$ . (5 points)

CSE 5311  
 Test 2 - Open Book  
 Spring 2004

Name \_\_\_\_\_  
 Student ID # \_\_\_\_\_

1. Use the Gale-Shapley algorithm to determine the male-optimal solution for the following instance of the stable marriages problem. In addition, show the preference lists at termination. Note that the preference lists are given left-to-right. (15 points)

**male preference lists are:**

1: 1 2 3 4 5  
 2: 2 3 4 5 1  
 3: 3 4 2 5 1  
 4: 1 3 2 5 4  
 5: 3 5 4 2 1

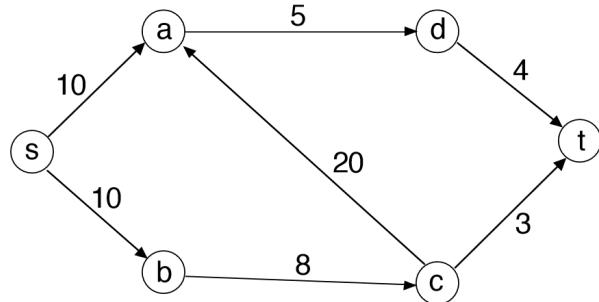
**female preference lists are:**

1: 4 3 5 2 1  
 2: 2 3 5 4 1  
 3: 1 2 5 4 3  
 4: 3 2 4 5 1  
 5: 2 4 3 1 5

2. The *hitting set* problem gives a collection C of subsets of a set S and a positive integer k. We would like to know if there is a subset  $S'$  of S with  $|S'| \leq k$  such that  $S'$  contains at least one element from each subset in C.

Give a proof that *hitting set* is NP-complete by using the fact that vertex cover is NP-complete. (15 points)

3. List the lift and push operations for the preflow-push algorithm on the following network. In addition, give a minimum cut. (20 points)



CSE 5311

Test 1 - Closed Book

Summer 2004

Name \_\_\_\_\_

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Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Which of the following random permutation techniques is not uniform?

- A. PERMUTE-BY-SORTING
- B. PERMUTE-WITH-ALL
- C. RANDOMIZE-IN-PLACE

- D. Randomly choose one of the  $n!$  ranks and then apply unranking
2. Fibonacci trees are used in the analysis of which technique?
    - A. AVL
    - B. red-black
    - C. splay
    - D. treap
  3. Which analysis of self-organizing linear search did not involve probabilities?
    - A. Count vs. optimal fixed order
    - B. Lab Assignment 1
    - C. Markov analysis of move-to-front for Zipf's distribution
    - D. Move-to-front (online) vs. optimal offline
  4. The median of a set of  $n$  numbers may be found optimally in time:
    - A.  $\Theta(\log n)$
    - B.  $\Theta(n)$
    - C.  $\Theta(n \log n)$
    - D.  $\Theta(n^2)$
  5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
    - A. the sum of all keys in the entire tree
    - B. the sum of all keys in the left subtree
    - C. the sum of all keys that are no larger than the stored key
    - D. the sum of all keys stored in the subtree rooted by this node
  6. When using Brent's rehash, the number of previously inserted keys that may move is:
    - A. 1
    - B. 2
    - C.  $\frac{1}{\alpha}$
    - D.  $H_m$ , where  $m$  is the number of stored keys
  7. Assuming that a random  $n$ -permutation is provided, the expected number of hires for the hiring problem is:
    - A. 2
    - B.  $H_n$
    - C.  $\frac{1}{n}$
    - D. n
  8. Which data structure is not used to implement a dictionary?
    - A. AVL tree
    - B. Red-black tree
    - C. Self-organizing list
    - D. Union-find

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9. Which binary search tree method stores the same information in each node as an unbalanced binary search tree, yet performs retrieval, insertion, and deletion in  $O(\log n)$  amortized time?
  - A. AVL
  - B. red-black
  - C. splay
  - D. treap
10. Which priority queue implementation generalizes binary heaps by increasing the branching?
  - A. Binomial heaps
  - B. d-heaps
  - C. Fibonacci heaps
  - D. Leftist heaps

11 Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected

11. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?

- A. 3
- B. 4
- C. 5
- D. 15

12. When is path compression used?

- A. After an insertion into any type of balanced binary search tree.
- B. After an insertion into a splay tree.
- C. After a FIND operation.
- D. After a UNION operation.

13. How many nodes does a  $B_5$  tree in a binomial heap have? (2 points)

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Test 1 - Open Book

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1. Give the range of possible heights for an AVL tree with 100 keys. (10 points)

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (15 points)

a.  $T(n) = 2T\left(\frac{n}{4}\right) + 1$

b.  $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

c.  $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (15 points)

n=6;	key[4]=40;	w[0][2]=0.250000
q[0]=0.01;	p[4]=0.2;	w[0][3]=0.490000
key[1]=10;	q[4]=0.0;	w[0][4]=0.690000
p[1]=0.09;	key[5]=50;	w[0][5]=0.850000
q[1]=0.02;	p[5]=0.12;	w[0][6]=1.000000
key[2]=20;	q[5]=0.04;	w[1][1]=0.020000
p[2]=0.1;	key[6]=60;	w[1][2]=0.150000
q[2]=0.03;	p[6]=0.12;	w[1][3]=0.390000
key[3]=30;	q[6]=0.03;	w[1][4]=0.590000
p[3]=0.2;	w[0][0]=0.010000	w[1][5]=0.750000
q[3]=0.04;	w[0][1]=0.120000	w[1][6]=0.900000

2

w[2][2]=0.030000	Building c(3,5) using roots 4 thru 5	c(1,2) cost 0.150000 20
w[2][3]=0.270000	Building c(4,6) using roots 5 thru 6	c(2,3) cost 0.270000 30
w[2][4]=0.470000	Building c(0,3) using roots 2 thru 3	c(3,4) cost 0.240000 40
w[2][5]=0.630000	Building c(1,4) using roots 3 thru 3	c(4,5) cost 0.160000 50
w[2][6]=0.780000	Building c(2,5) using roots 3 thru 4	c(5,6) cost 0.190000 60
w[3][3]=0.040000	Building c(3,6) using roots 4 thru 6	c(0,2) cost 0.370000 20(10,,)
w[3][4]=0.240000	Building c(0,4) using roots 3 thru 3	c(1,3) cost 0.540000 30(20,,)
w[3][5]=0.400000	Building c(1,5) using roots 3 thru 4	c(2,4) cost 0.710000 30(,40)
w[3][6]=0.550000	Building c(2,6) using roots 4 thru 5	c(3,5) cost 0.560000 40(,50)
w[4][4]=0.000000	Building c(0,5) using roots 3 thru 4	c(4,6) cost 0.470000 60(50,,)
w[4][5]=0.160000	Building c(1,6) using roots 4 thru 4	c(0,3) cost 0.860000 30(20(10,,),)
w[4][6]=0.310000	Building c(0,6) using roots ? thru ?	c(1,4) cost 0.980000 30(20,40)
w[5][5]=0.040000	Counts - root trick 28 without root	c(2,5) cost 1.060000 40(30,50)
w[5][6]=0.190000	trick 50	c(3,6) cost 0.980000 50(40,60)
w[6][6]=0.030000	Average probe length is ???	c(0,4) cost 1.300000 30(20(10,,),40)
Building c(0,2)	trees in parenthesized prefix	c(1,5) cost 1.450000 40(30(20,,),50)

```

using roots 1      c(0,0) cost 0.000000
thru 2            c(1,1) cost 0.000000
Building c(1,3)    c(2,2) cost 0.000000
    using roots 2  c(3,3) cost 0.000000
    thru 3          c(4,4) cost 0.000000
Building c(2,4)    c(5,5) cost 0.000000
    using roots 3  c(6,6) cost 0.000000
    thru 4          c(0,1) cost 0.120000 10
                                         c(2,6) cost 1.520000 40(30,60(50,))
                                         c(0,5) cost 1.780000 30(20(10,),40(.50))
                                         c(1,6) cost 1.910000 40(30(20,),60(50,))
                                         c(0,6) cost ??? ??????????

```

4. Suppose all  $2^k - 1$  nodes, along with the sentinel, in a red-black tree are colored black. Explain what will happen if any key is deleted. (10 points)

CSE 5311

Name \_\_\_\_\_

Test 2 - Closed Book

Summer 2004

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)
  - A. Testing if a graph is 3-colorable
  - B. Testing if an undirected graph has a Hamiltonian cycle
  - C. Testing if the number of colors needed to edge color a graph is the degree of the graph
  - D. 2-satisfiability
2. How many times will -1 occur in the style 1 fail link table for the pattern abaababaab?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
3. How many times will -1 occur in the style 2 fail link table for the pattern abaababaab?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
4. Which of the following do Kruskal's and Boruvka's algorithms have in common?
  - A. They can take advantage of Union-Find structures
  - B. They do not work correctly unless all edge weights are unique
  - C. They use min-heaps

- D. They use the edges in ascending weight order
5. Under what condition does an instance of stable marriages have only one solution?
  - A. Every male preference list is identical to some female preference list
  - B. No female appears at the beginning of multiple male preference lists
  - C. The male-optimal solution and female-optimal solution are the same
  - D. There is only one rotation
6. Which of the following algorithms does not preprocess using a sort?
  - A. Closest points in 2-d space
  - B. Graham scan
  - C. Jarvis march
  - D. Kruskal
7. Which algorithm is defined using the notions of left-turn and right-turn?
  - A. Closest points in 2-d space
  - B. Graham scan
  - C. Jarvis march
  - D. Longest common subsequence
8. Which of the following is not a condition for performing a lift operation at u?
  - A. all exiting saturated edges have heads with height  $\geq$  height of u
  - B. all exiting unsaturated edges have heads with height  $<$  height of u

- B. all existing unsaturated edges have heads with height  $\geq$  height of u  
 C. u has excess  $> 0$
9. Which of the following does not have a polynomial-time approximation algorithm?
- Bin packing
  - Edge coloring
  - Traveling salesperson with triangle inequality
  - Vertex coloring
10. The length of a longest monotone increasing subsequence for 1 4 3 2 3 4 3 6 3 4 is:
- 4
  - 5
  - 6
  - 7
11. Which longest common subsequence method is potentially the most space-consuming?
- Compact version of dynamic programming
  - Method based on subsequence indices and longest strictly increasing subsequence
  - Ordinary dynamic programming
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a “restart”), it can be an articulation point only if
- there are no incident tree edges
  - there is one incident tree edge
  - there is more than one incident tree edge
  - there is no back edge incident to this vertex
13. Which minimum spanning tree algorithm is the slowest?
- Boruvka
  - Kruskal
  - Prim
  - Warshall
14. Which string search method is potentially the most time-consuming?
- Karp-Rabin
  - KMP with fail 1 links
  - KMP with fail 2 links

5

- D. Z table
15. What is the minimum increase in the tail’s distance from the source between the first and second times that an edge becomes critical in the Edmonds-Karp method?
- 1
  - 2
  - $(V-2)/2$
  - VE
16. Explain how the Z Algorithm (“Fundamental String Preprocessing”) may be used to find all occurrences of string 1 within string 2. (5 points)

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1. Use the Gale-Shapley algorithm to determine the male-optimal solution for the following instance of the stable marriages problem. In addition, show the preference lists at termination. Note that the preference lists are given left-to-right. (15 points)

male preference lists are:

1: 1 2 3 4 5  
 2: 3 2 1 5 4  
 3: 3 4 2 5 1  
 4: 1 3 2 5 4  
 5: 2 3 4 5 1

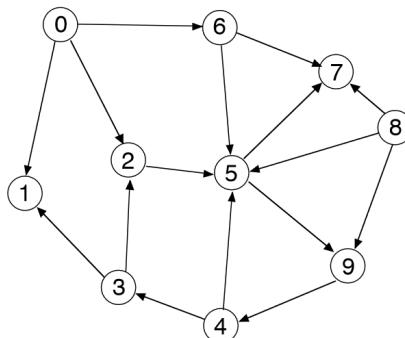
female preference lists are:  
 1: ~ ~ ~ ~ ~

```

1: 4 5 3 2 1
2: 1 2 3 4 5
3: 2 3 4 5 1
4: 3 2 1 5 4
5: 4 5 2 1 3

```

2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward). Assume that the adjacency lists are *ordered*. Write your answer in the tables below. 15 points

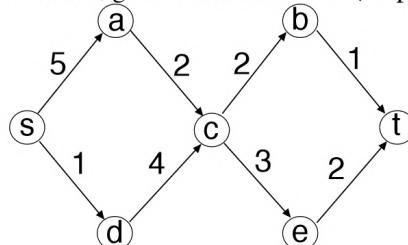


Vertex	Start	Finish	Edge	Type	Edge	Type
0	1	—	0 1	—	6 5	—
1	—	—	0 2	—	6 7	—
2	—	—	0 6	—	8 5	—
3	—	—	2 5	—	8 7	—
4	—	—	3 1	—	8 9	—
5	—	—	3 2	—	9 4	—
6	—	—	4 3	—		
7	—	—	4 5	—		

6

8	—	—	5 7	—
9	—	—	5 9	—

3. **Clearly** list the lift and push operations for the preflow-push algorithm on the following network. In addition, give a minimum cut. (20 points)



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Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Assuming that  $|E| > |V|$ , the time for Prim's algorithm using a Fibonacci heap is:
  - A.  $O(|E|)$
  - B.  $O(|E| \log |E|)$
  - C.  $O(|E| \log |V|)$
  - D.  $O(|V| \log |V| + |E|)$
2. What is the worst-case number of rotations when performing deletion on an AVL tree?
  - A.  $\Theta(1)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D. No rotations are ever used
3. The maximum value of the potential function when comparing MTF and OPT on a list with  $n$  elements is:
  - A.  $n$
  - B.  $\frac{n^2-n}{2}$
  - C.  $\frac{n^2+n}{2}$
  - D.  $n^2-n$
4. When performing selection in worst-case linear time, roughly what fraction of the set of  $n$  keys is kept (in the worst case) for the next round?
  - A. 10%
  - B. 20%
  - C. 30%
  - D. 70%
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
  - A. the sum of all keys in the entire tree
  - B. the sum of all keys in the left subtree
  - C. the sum of all keys that are no larger than the stored key

- D. the sum of all keys stored in the subtree rooted by this node
6. The worst-case placement of keys for an open-address method may be found using:
- Binary search on instances of unweighted bipartite matching
  - Brent's method
  - Perfect hashing
  - Weighted bipartite matching by placing weight  $jP_i$  at  $M_iS_{ij}$  for the probe sequence  $S_{i1}, S_{i2}, \dots, S_{im}$
7. Which of the following statements is true?
- A binary search tree may be assigned legal AVL balance factors if and only if it may be legally colored as a red-black tree.
  - If a binary search tree may be assigned legal AVL balance factors, then it may be legally colored as a red-black tree.
  - If a binary search tree may be legally colored as a red-black tree, then it may be assigned legal AVL balance factors.
  - No binary search tree may be assigned both legal AVL balance factors and be legally colored as a red-black tree.
8. Path compression is used in which algorithm?
- DECREASE-KEY (e.g. CASCADING-CUT) for Fibonacci heaps
  - FIND for disjoint sets
  - UNION for disjoint sets
  - UNION for Fibonacci heaps
9. Which priority queue is defined using the notion of null path length?
- Binary heap
  - Binomial heap
  - Fibonacci heap
  - Leftist heap
10. Which situation is true regarding a cascade cut that produces c trees for a Fibonacci heap?
- Both the actual and amortized costs are  $O(1)$ .

2

- B. The actual cost is  $O(c)$ . The amortized cost is  $O(1)$ .
- C. The actual cost is  $O(1)$ . The amortized cost is  $O(c)$ .
- D. The potential can become negative.
11. Which of the following is not true regarding the amortized analysis of binary tree traversals?
- INIT had an amortized cost of 0.
  - SUCC had an actual cost determined by the number of edges followed.
  - SUCC had an amortized cost of 2.
  - The potential was defined without regard for the type of traversal being performed.
12. Which of the following data structures offers similar capabilities and performance characteristics to skip lists?
- AVL trees
  - Splay trees
  - Treap
  - Union-find with path compression

13. Draw an  $S_5$  tree for a Fibonacci heap. (2 points)

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- "During Disney's biggest celebration, find one of 50 character Wobblers inside specially-marked **Kellogg's®** cereals!".
- Assuming all Wobblers are equally likely to be the one that occurs in a box, what is the expected number of boxes to obtain all 50 Wobblers? (3 points. You may leave your answer as an expression.)
- Under the same assumption as a., what is the expected number of boxes to obtain just 25 different Wobblers (of the available 50)? (7 points. You may leave your answer as an expression.)
- Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)
  - $T(n) = 8T\left(\frac{n}{2}\right) + n^4$
  - $T(n) = 8T\left(\frac{n}{2}\right) + n^2$
  - $T(n) = 8T\left(\frac{n}{2}\right) + n^3$
- Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

$n=6;$	$w[0][2]=0.560000$	$w[4][6]=0.100000$
$q[0]=0.12;$	$w[0][3]=0.700000$	$w[5][5]=0.040000$
$key[1]=10;$	$w[0][4]=0.900000$	$w[5][6]=0.090000$
$p[1]=0.12;$	$w[0][5]=0.950000$	$w[6][6]=0.020000$
$\dots$	$\dots$	$\dots$

```

q[1]=0.09;
key[2]=20;
p[2]=0.2;
q[2]=0.03;
key[3]=30;
p[3]=0.1;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.01;
q[5]=0.04;
key[6]=60;
p[6]=0.03;
q[6]=0.02;
w[0][0]=0.120000
w[0][1]=0.330000
w[1][0]=1.000000
w[1][1]=0.090000
w[1][2]=0.320000
w[1][3]=0.460000
w[1][4]=0.660000
w[1][5]=0.710000
w[1][6]=0.760000
w[2][2]=0.030000
w[2][3]=0.170000
w[2][4]=0.370000
w[2][5]=0.420000
w[2][6]=0.470000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.290000
w[3][6]=0.340000
w[4][4]=0.000000
w[4][5]=0.050000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 2
Building c(1,5) using roots 2 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 2 thru 2
Building c(1,6) using roots 2 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 31 without root trick
      50
Average probe length is ???

```

2

trees in	c(5,5) cost 0.000000	c(0,3) cost 1.200000 20(10,30)
parenthesized	c(6,6) cost 0.000000	c(1,4) cost 1.200000 20(,40(30,))
prefix	c(0,1) cost 0.330000 10	c(2,5) cost 0.640000 40(30,50)
c(0,0) cost	c(1,2) cost 0.320000 20	c(3,6) cost 0.490000 40(,60(50,))
0.000000	c(2,3) cost 0.170000 30	c(0,4) cost 1.770000 20(10,40(30,))
c(1,1) cost	c(3,4) cost 0.240000 40	c(1,5) cost 1.350000 20(,40(30,50))
0.000000	c(4,5) cost 0.050000 50	c(2,6) cost 0.790000 40(30,60(50,))
c(2,2) cost	c(5,6) cost 0.090000 60	c(0,5) cost 1.920000 20(10,40(30,50))
0.000000	c(0,2) cost 0.880000 10(,20)	c(1,6) cost 1.540000 40(20(,30),60(50,))
c(3,3) cost	c(1,3) cost 0.630000 20(,30)	c(0,6) cost ??? ??????????????????
0.000000	c(2,4) cost 0.540000 40(30,)	
c(4,4) cost	c(3,5) cost 0.340000 40(,50)	
0.000000	c(4,6) cost 0.150000 60(50,)	

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each key. Show the result from inserting 1300 using Brent's rehash when  $h_1(1300) = 5$  and  $h_2(1300) = 3$ . (10 points)

key	$h_1(key)$	$h_2(key)$
-----	------------	------------

0		
1	1000	6
2		2
3	1200	3
4	500	3
5	12	5
6	27	6

5. Give an instance of deletion from a red-black tree that will require the maximum number of rotations. Also, explain how the rotations will occur using the four cases. (10 points)

1. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)
  - A. 3-satisfiability
  - B. Testing if a bipartite graph has a vertex cover with no more than k vertices
  - C. Testing if a graph is 3-colorable
  - D. Testing if the number of colors needed to edge color a graph is the degree of the graph
2. How many times will -1 occur in the style 1 fail link table for the pattern acaababc?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
3. How many times will -1 occur in the style 2 fail link table for the pattern acaababc?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
4. Radix sort is useful for:
  - A. Constructing a longest common subsequence using O(m + n) space
  - B. Constructing a suffix array
  - C. Constructing the polynomial for Karp-Rabin string search
  - D. Constructing the Z table for a sequence
5. Which of the following is a deficiency of the maximum capacity path technique?
  - A. An augmenting path is blocked if it introduces a cycle of flow.
  - B. Augmenting paths will be discovered in descending incremental flow increase order.
  - C. Flow decomposition must be applied.
  - D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.
6. Which of the following algorithms does not preprocess using a sort?
  - A. Closest points in 2-d space - divide and conquer
  - B. Closest points in 2-d space - sweepline
  - C. Graham scan
  - D. Jarvis march
7. Which algorithm is defined using the notions of left-turn and right-turn?
  - A. Closest points in 2-d space
  - B. Graham scan
  - C. Jarvis march
  - D. TSP approximation when vertex distances observe the triangle inequality
8. Which of the following is not a condition for performing a push operation from u to v?
  - A. the height at u is one more than the height at v
  - B. (u, v) has capacity > flow
  - C. u has excess > 0
  - D. v has a path to the sink in the residual graph
9. Which of the following does not have a polynomial-time approximation algorithm?
  - A. Bin packing
  - B. Edge coloring
  - C. General traveling salesperson
  - D. Vertex cover
10. The length of a longest strictly increasing subsequence for 1 4 3 2 3 4 3 6 3 4 is:
  - A. 4
  - B. 5
  - C. 6
  - D. 7
11. Which of the following is not true regarding the reduction from 3-sat to 3-colorability?
  - A. The widget appears once for each literal.
  - B. The widget will use three colors when corresponding to a satisfied clause.
  - C. The widget will use four colors when corresponding to an unsatisfied clause.
  - D. There is a triangle of vertices for establishing the colors of TRUE, FALSE, and RED.
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a “restart”), it can be an articulation point only if
  - A. there are no incident tree edges
  - B. there is one incident tree edge
  - C. there is more than one incident tree edge
  - D. there is no back edge incident to this vertex
13. On an augmenting path, a critical edge is:
  - A. an edge from the source

- A. an edge from the source
  - B. an edge into the sink
  - C. an edge that used to be saturated
  - D. an edge with the minimum residual capacity

14. Which string search method computes signatures to avoid byte-to-byte comparisons?

  - A. Karp-Rabin
  - B. KMP with fail 1 links
  - C. KMP with fail 2 links
  - D. Z table

15. The most general approximation result that can be achieved for an NP-hard problem is:

  - A. Approximation Algorithm
  - B. Approximation Scheme
  - C. Fully Polynomial-time Approximation Scheme
  - D. Polynomial-time Approximation Scheme

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C. Fully Polynomial-time Approximation Scheme  
D. Polynomial-time Approximation Scheme

16. Explain how the Z Algorithm (“Fundamental String Preprocessing”) may be used to determine whether two strings are circularly equal. (5 points)

circularly  
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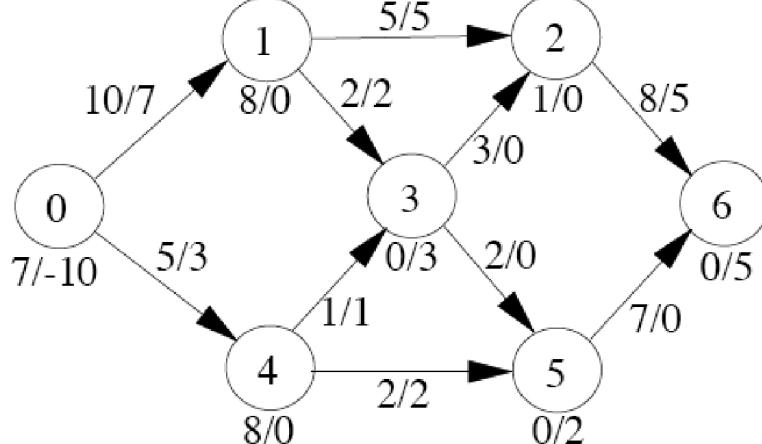
1. Solve the following instance of Longest Subsequence problem. (15 points)

0	1	2	3	4	5	6	7
a	b	d	c	d	e	f	g
b	c	d	a	e	d	g	f

2. Use dynamic programming, either with a table or lists, to determine a subset that sums to 20. DO NOT SOLVE BY INSPECTION! (15 points)

2    3    5    7    11    13    17

3. List the remaining operations to complete this instance of network flows by push-relabel. In addition, give a minimum cut. (20 points)



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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Sorting the set of input edges is a property of which minimum spanning tree technique?
  - A. Boruvka
  - B. Kruskal
  - C. Path-based (Warshall)
  - D. Prim
2. The balance factors in an AVL tree are computed as:
  - A.  $\text{height}_{\text{right}} - \text{height}_{\text{left}}$
  - B. same as the null path length in a leftist heap
  - C. the difference of the number of nodes in the left and right subtrees
  - D. the distance from a node to the root
3. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 16 you already have?
  - A. 3
  - B. 4
  - C. 5
  - D. 15
4. When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round?
  - A.  $\frac{n}{5}$
  - B.  $m$ , the median-of-medians
  - C.  $.7n$
  - D.  $W\left(\frac{n}{5}\right)$
5. To support computing the number of keys that are smaller than a query key, an augmented binary search tree stores the following at every node:
  - A. the count of the number of keys in the entire tree
  - B. the sum of all keys in the left subtree
  - C. the count of the number of keys stored in the subtree rooted by this node
  - D. the sum of all keys stored in the subtree rooted by this node
6. When using Brent's rehash during insertion, the number of previously inserted keys that may move is:
  - A. 1
  - B. 2
  - C.  $\frac{1}{\alpha}$
  - D.  $H_m$ , where  $m$  is the number of stored keys
7. Which of the following statements is true?
  - A. A binary search tree may be assigned legal AVL balance factors if and only if it may be legally colored as a red-black tree.
  - B. If a binary search tree may be legally colored as a red-black tree, then it may be assigned legal AVL balance factors.
  - C. If a binary search tree may be assigned legal AVL balance factors, then it may be legally colored as a red-black tree.

as a red-black tree.

- D. No binary search tree may be assigned both legal AVL balance factors and be legally colored as a red-black tree.

2

8. Which data structure is not used to implement a dictionary?
- A. AVL tree
  - B. Red-black tree
  - C. Self-organizing list
  - D. Union-find
9. The reason for marking nodes in a Fibonacci heap is:
- A. to allow computing the value of the potential function.
  - B. to assure that the structure is a Fibonacci heap rather than a binomial heap.
  - C. to improve the performance of CONSOLIDATE.
  - D. to indicate nodes that have lost a child since becoming a child themselves.
10. Which property does not hold for binomial heaps?
- A. DECREASE-KEY takes  $O(1)$  time.
  - B. MINIMUM takes  $O(\log n)$  time.
  - C. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.
  - D. The number of trees is based on the binary representation of the number of stored items.
11. When are Fibonacci trees used?
- A. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
  - B. Defining the potential function for Fibonacci heaps.
  - C. Demonstrating worst-case behaviors for AVL trees.
  - D. Demonstrating worst-case behaviors for red-black trees.
12. The perfect hashing method discussed in class depends on which fact?
- A.  $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$      $0 < x < 1$
  - B.  $\ln n < H_n < \ln n + 1$
  - C. The expected number of probes for a successful search in Brent's method is less than 2.5.
  - D. The probability of collisions among  $n$  keys stored in a hash table of size  $n^2$  is less than 0.5.
13. Dynamic optimality is a concept involving the comparison of
- A. a key-comparison based data structure to hashing.
  - B. amortized complexity to actual complexity.
  - C. an online data structure to a fixed, unchanging data structure.
  - D. an online data structure to an offline data structure.
14. Which of the following minimum spanning tree algorithms is theoretically slowest? Assume that the most efficient data structures are used.
- A. Boruvka
  - B. Kruskal
  - C. Path-based (Warshall)
  - D. Prim
15. The two parts of using a Markov model to analyze a self-adjusting data structure are:
- A. define the potential function, compare to the optimal offline (OPT) method.
  - B. determine the probability for each state, then compute the expected number of probes.
  - C. simulate the data structure long enough to ensure convergence, then compute the expected number of probes.
  - D. use Knuth's root trick to find the optimal subtrees in  $O(n^2)$  time, backtrace to print the tree.
16. Give the red-black (without sentinels) and AVL trees with four levels (maximum distance of a leaf from the root is three edges) with the fewest nodes. (5 points)

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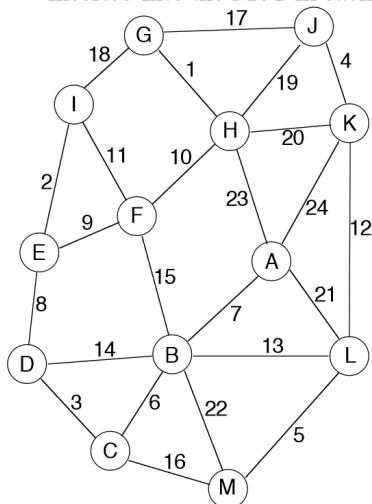
1. Suppose that you are given  $n$  values. Give a linear-time algorithm to build a binomial heap with the  $n$  values. 10 points.
2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)
  - a.  $T(n) = 4T\left(\frac{n}{2}\right) + n^3$
  - b.  $T(n) = 4T\left(\frac{n}{2}\right) + n^2$
  - c.  $T(n) = 4T\left(\frac{n}{2}\right) + 1$
3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=7;                      w[1][4]=0.490000          c(7,7) cost 0.000000
q[0]=0.06;                  w[1][5]=0.640000          c(0,1) cost 0.260000 1
key[1]=1;                   w[1][6]=0.710000          c(1,2) cost 0.180000 2
p[1]=0.14;                  w[1][7]=0.800000          c(2,3) cost 0.200000 3
q[1]=0.06;                  w[2][2]=0.060000          c(3,4) cost 0.230000 4
key[2]=2;                   w[2][3]=0.200000          c(4,5) cost 0.200000 5
p[2]=0.06;                  w[2][4]=0.370000          c(5,6) cost 0.120000 6
q[2]=0.06;                  w[2][5]=0.520000          c(6,7) cost 0.140000 7
key[3]=3;                   w[2][6]=0.590000          c(0,2) cost 0.560000 1(,2)
p[3]=0.08;                  w[2][7]=0.680000          c(1,3) cost 0.500000 3(2,)
q[3]=0.06;                  w[3][3]=0.060000          c(2,4) cost 0.570000 4(3,)
key[4]=4;                   w[3][4]=0.230000          c(3,5) cost 0.580000 4(,5)
p[4]=0.12;                  w[3][5]=0.380000          c(4,6) cost 0.390000 5(,6)
q[4]=0.05;                  w[3][6]=0.450000          c(5,7) cost 0.330000 7(6,)
key[5]=5;                   w[3][7]=0.540000          c(0,3) cost 0.980000 2(1,3)
p[5]=0.10;                  w[4][4]=0.050000          c(1,4) cost 0.900000 3(2,4)
q[5]=0.05;                  w[4][5]=0.200000          c(2,5) cost 0.920000 4(3,5)
key[6]=6;                   w[4][6]=0.270000          c(3,6) cost 0.800000 5(4,6)
p[6]=0.02;                  w[4][7]=0.360000          c(4,7) cost 0.690000 5(,7(6,))
q[6]=0.05;                  w[5][5]=0.050000          c(0,4) cost 1.480000 3(1(,2),4)
key[7]=7;                   w[5][6]=0.120000          c(1,5) cost 1.340000 4(3(2,),5)
p[7]=0.04;                  w[5][7]=0.210000          c(2,6) cost 1.180000 4(3,5(,6))
q[7]=0.05;                  w[6][6]=0.050000          c(3,7) cost 1.100000 5(4,7(6,))
w[0][0]=0.060000          w[6][7]=0.140000          c(0,5) cost 1.980000 3(1(,2),4(,5))
w[0][1]=0.260000          w[7][7]=0.050000          c(1,6) cost 1.600000 4(3(2,),5(,6))
w[0][2]=0.380000          Average probe length is ???
w[0][3]=0.520000          trees in parenthesized prefix
w[0][4]=0.690000          c(0,0) cost 0.000000
w[0][5]=0.840000          c(1,1) cost 0.000000
w[0][6]=0.910000          c(2,2) cost 0.000000
w[0][7]=1.000000          c(3,3) cost 0.000000
w[1][1]=0.060000          c(4,4) cost 0.000000
w[1][2]=0.180000          c(5,5) cost 0.000000
w[1][3]=0.320000          c(6,6) cost 0.000000
```

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each key. Show the result from inserting 9000 using Brent's rehash when  $h_1(9000) = 5$  and  $h_2(9000) = 4$ . (10 points)

$key$	$h_1(key)$	$h_2(key)$
0		
1		
2	5000	2
3	4000	3
4	3000	4
5	2000	5
6	1000	6

5. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)



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Test 2 - Closed Book

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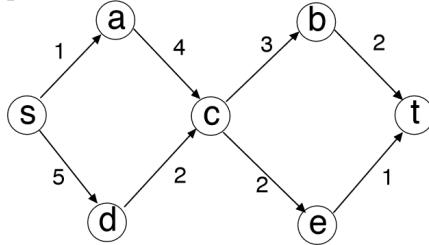
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. When performing bin packing using the First-Fit Decreasing technique, the total number of items placed in the bins past the optimal bins (1 .. OPT) is no more than:
  - A.  $1 + \epsilon$
  - B. 2
  - C.  $OPT - 1$
  - D.  $OPT$
2. How many times will -1 occur in the style 1 fail link table for the pattern abacabac?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
3. How many times will -1 occur in the style 2 fail link table for the pattern abacabac?
  - A. 1
  - B. 2
  - C. 3

- D. 4
4. Constructing a suffix array for a sequence with  $n$  symbols by using an optimal key-comparison sort has this worst-case time:
    - A.  $\Theta(n)$
    - B.  $\Theta(n \log n)$
    - C.  $\Theta(n^2)$
    - D.  $\Theta(n^2 \log n)$
  5. The algorithm for finding a maximum capacity path for network flows is most similar to which algorithm?
    - A. Breadth-first search
    - B. Decomposition of a flow into  $E$  augmenting paths
    - C. Dijkstra
    - D. Floyd-Warshall
  6. Which of the following algorithms may take time longer than  $O(n \log n)$ ?
    - A. Closest points in 2-d space - divide and conquer
    - B. Closest points in 2-d space - sweepline
    - C. Graham scan
    - D. Jarvis march
  7. When combining the left and right parts for the 2-d closest pair algorithm along the vertical dividing line, each left-side point “near” the line has its distance computed to no more than this number of right-side points.
    - A. 6
    - B. 8
    - C.  $\lg n$
    - D.  $n/2$
  8. Which of the following is not required before relabeling (“lifting”) a vertex to a new height?
    - A. Any eligible edges for the present height have been saturated.
    - B. Both breadth-first searches have been done.
    - C. The vertex is not the source or sink.
    - D. The vertex is overflowing.
  9. Which of the following does not have a polynomial-time approximation algorithm?
    - A. Bin packing
    - B. Edge coloring
    - C. Euclidean traveling salesperson
    - D. Vertex coloring
  10. The length of a longest increasing subsequence for 1 4 3 2 3 4 3 6 3 4 is:
    - A. 4
    - B. 5
    - C. 6
    - D. 7
  11. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?
    - A. Problem A is NP-complete.
    - B. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
    - C. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.
    - D. The reduction takes polynomial time.
- 4
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a “restart”), it can be an articulation point only if

- A. there are no incident tree edges  
 B. there is one incident tree edge  
 C. there is more than one incident tree edge  
 D. there is no back edge incident to this vertex
13. Which of the following problems uses some edges with non-unit capacity when translated to a network flow problem?  
 A. Bipartite matching  
 B. Edge connectivity  
 C. Minimum vertex cover for bipartite graph  
 D. Vertex connectivity
14. Which is not true regarding the Karp-Rabin method?  
 A. It always runs in  $\Theta(m + n)$  time.  
 B. It computes a signature for the pattern and each  $m$  contiguous text symbols.  
 C. The signature is computed using a polynomial.  
 D. When the signature for  $m$  contiguous text symbols matches the signature for the pattern, a `strcmp` must be performed.
15. The least general approximation result that can be achieved for an NP-hard problem is:  
 A. Approximation Algorithm  
 B. Approximation Scheme  
 C. Fully Polynomial-time Approximation Scheme  
 D. Polynomial-time Approximation Scheme
16. Explain how the Z Algorithm ("Fundamental String Preprocessing") may be used to perform a string search for all occurrences (possibly with overlap among occurrences) of a given pattern within a given text. Do not explain details of the Z algorithm. (5 points)
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- Explain how to use the Z algorithm to determine a sequence X for the largest possible value  $k$  such that for a given sequence  $W = X^k$ . Demonstrate your technique for  $ababab = (ab)^3$  and  $abcd = (abcd)^1$ . 15 points
  - Use dynamic programming, either with a table or lists, to determine a subset that sums to 39. DO NOT SOLVE BY INSPECTION! (15 points)  

$$2 \quad 4 \quad 5 \quad 11 \quad 12 \quad 13 \quad 17$$
  - List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (20 points)



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- Multiple Choice. Write your answer to the LEFT of each problem. 3 points each
- During which operation on a leftist heap may subtree swaps be needed?  
 A. DECREASE-KEY    B. EXTRACT-MIN

- C. UNION      D. All of A., B., and C.
2. What is the worst-case number of rotations when performing deletion on an AVL tree?
    - A.  $\Theta(1)$
    - B.  $\Theta(\log n)$
    - C.  $\Theta(n)$
    - D. No rotations are ever used
  3. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?
    - A. 3
    - B. 4
    - C. 5
    - D. 15
  4. When is path compression used?
    - A. After an insertion into any type of balanced binary search tree.
    - B. After an insertion into a splay tree.
    - C. With a FIND operation.
    - D. With a UNION operation.
  5. To support testing whether an interval intersects some interval in a set of intervals, an augmented binary search tree stores the following at every node:
    - A. the count of the number of intervals in the entire tree
    - B. the count of the number of intervals stored in the subtree rooted by this node
    - C. the maximum value appearing in any interval in the subtree rooted by this node
    - D. the sum of the lengths of all intervals in the left subtree
  6. Which Fibonacci heap operation has  $O(\log n)$  actual cost?
    - A. FIB-HEAP-DECREASE-KEY
    - B. FIB-HEAP-DELETE
    - C. FIB-HEAP-EXTRACT-MIN
    - D. FIB-HEAP-UNION
  7. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?
    - A. none of them
    - B. only the leaves
    - C. only the root
    - D. the leaves and the root
  8. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 1, 3?
    - A. 2
    - B. 3
    - C. 4
    - D. 5
  9. The reason for marking nodes in a Fibonacci heap is:
    - A. to allow computing the value of the potential function.
    - B. to assure that the structure is a Fibonacci heap rather than a binomial heap.
    - C. to improve the performance of CONSOLIDATE.
    - D. to indicate nodes that have lost a child since becoming a child themselves.
  10. Which property does not hold for binomial heaps?
    - A. DECREASE-KEY takes  $O(\log n)$  time.
    - B. MINIMUM takes  $O(1)$  time.
    - C. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.
    - D. The number of trees is based on the binary representation of the number of stored items.
  11. In the worst case, the number of rotations for inserting a key in a treap with  $n$  keys is:
    - A.  $\Theta(1)$
    - B.  $\Theta(\log n)$
    - C.  $\Theta(n)$
    - D.  $\Theta(n \log n)$
  12. Which priority queue is defined using the notion of null path length?
    - A. Binary heap
    - B. Binomial heap
    - C. Fibonacci heap
    - D. Leftist heap
  13. Which of the following is not a property of splay trees?
    - A. Splaying an accessed node to the root can cause the potential to decrease
    - B. The amortized cost of a splaying sequence is bounded logarithmically

- C. The zig rotation is only applied at the root
- D. They are a form of balanced tree
14. The main difference between MTF and OPT for self-organizing linear lists is:
  - A. OPT can do transpositions
  - B. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
  - C. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
  - D. MTF counts inversions

15. Assuming a random  $n$ -permutation is provided, the expected number of hires for the hiring problem is:

A. 2      B.  $H_n$       C.  $\frac{1}{n}$       D.  $n$

16. Give a tree of rank 5 that could appear in a Fibonacci heap and has the minimum possible number of nodes. Be sure to include the marks. (5 points)

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1. Describe the *format* of the system of equations that would be used to perform a Markov analysis on the *transpose* approach to self-adjusting lists for  $n = 4$ . (10 points)

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

- a.  $T(n) = T(0.7n) + n$
  - b.  $T(n) = T(0.7n) + 1$
  - c.  $T(n) = 16T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```

n=7; w[2][3]=0.160000 c(2,3) cost 0.160000 3
q[0]=0.06; w[2][4]=0.260000 c(3,4) cost 0.110000 4
p[1]=0.15; w[2][5]=0.330000 c(4,5) cost 0.120000 5
q[1]=0.03; w[2][6]=0.480000 c(5,6) cost 0.200000 6
p[2]=0.15; w[2][7]=0.610000 c(6,7) cost 0.180000 7
q[2]=0.05; w[3][3]=0.010000 c(0,2) cost 0.670000 1(,2)
p[3]=0.1; w[3][4]=0.110000 c(1,3) cost 0.500000 2(,3)
q[3]=0.01; w[3][5]=0.180000 c(2,4) cost 0.370000 3(,4)
p[4]=0.05; w[3][6]=0.330000 c(3,5) cost 0.290000 5(,)
q[4]=0.05; w[3][7]=0.460000 c(4,6) cost 0.390000 6(,5)

```

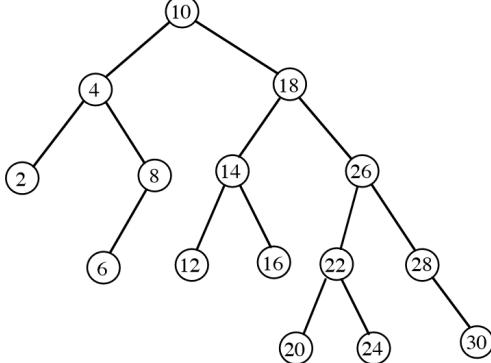
```

p[5]=0.02;           w[4][4]=0.050000          c(5,/) cost 0.510000 6(/,/)
q[5]=0.05;           w[4][5]=0.120000          c(0,3) cost 0.950000 2(1,3)
p[6]=0.1;            w[4][6]=0.270000          c(1,4) cost 0.780000 3(2,4)
q[6]=0.05;            w[4][7]=0.400000          c(2,5) cost 0.610000 4(3,5)
p[7]=0.08;            w[5][5]=0.050000          c(3,6) cost 0.620000 6(5(4,,))
q[7]=0.05;            w[5][6]=0.200000          c(4,7) cost 0.700000 6(5,7)
w[0][0]=0.060000      w[5][7]=0.330000          c(0,4) cost 1.260000 2(1,3(,4))
w[0][1]=0.240000      w[6][6]=0.050000          c(1,5) cost 1.030000 3(2,5(4,)) 
w[0][2]=0.440000      w[6][7]=0.180000          c(2,6) cost 1.030000 4(3,6(5,)) 
w[0][3]=0.550000      w[7][7]=0.050000          c(3,7) cost 0.930000 6(5(4,,),7)
w[0][4]=0.650000      Average probe length is ???      c(0,5) cost 1.570000 2(1,4(3,5))
w[0][5]=0.720000      trees in parenthesized prefix      c(1,6) cost 1.510000 3(2,6(5(4,,)))
w[0][6]=0.870000      c(0,0) cost 0.000000          c(2,7) cost 1.400000 6(4(3,5),7)
w[0][7]=1.000000      c(1,1) cost 0.000000          c(0,6) cost 2.140000 2(1,4(3,6(5,)))
w[1][1]=0.030000      c(2,2) cost 0.000000          c(1,7) cost 1.950000 3(2,6(5(4,,),7))
w[1][2]=0.230000      c(3,3) cost 0.000000          c(0,7) cost ??? ????????????????????
w[1][3]=0.340000      c(4,4) cost 0.000000
w[1][4]=0.440000      c(5,5) cost 0.000000
w[1][5]=0.510000      c(6,6) cost 0.000000
w[1][6]=0.660000      c(7,7) cost 0.000000
w[1][7]=0.790000      c(0,1) cost 0.240000 1
w[2][2]=0.050000      c(1,2) cost 0.230000 2

```

4. Give an example of a splay tree and an access (i.e. a search) such that the potential is increased by splaying the search key. (10 points)

5. Delete 2 from the following AVL tree (preserving AVL properties). (10 points)



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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

- When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round?
  - $\frac{n}{5}$
  - $m$ , the median-of-medians
  - $.7n$
  - $W\left(\frac{n}{5}\right)$
- What is the nature of the signature function for the Karp-Rabin method?
  - a polynomial of arbitrary precision implemented using a bignum package
  - it is similar to the KMP failure links
  - similar to a double hash function for string keys
  - the remainder by discarding the overflow for a polynomial
- Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?

- A. Convex hull  
 B. Delaunay triangulation  
 C. Euclidean minimum spanning tree  
 D. Voronoi diagram
4. While constructing a suffix array for a sequence with  $n$  symbols by using radix sorts, sequence symbols are used in which of the radix sorts?  
 A. All of them  
 B. None of them  
 C. The first one  
 D. The last one
5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:  
 A.  $f$   
 B.  $E$   
 C.  $V$   
 D.  $O(VE)$
6. Which algorithm is defined using the notions of left-turn and right-turn?  
 A. Closest points in 2-d space  
 B. Graham scan  
 C. Jarvis march  
 D. Suffix array construction
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?  
 A. BST of points with x-coordinates as the key  
 B. BST of points with y-coordinates as the key  
 C. Interval tree  
 D. Sorted array by ascending x-coordinates
8. The four russians' concept is to:  
 A. Implement longest common subsequences using linear space  
 B. Pack bits into an efficient storage unit  
 C. Trade-off between enumerating situations and referencing these situations  
 D. Trade-off between scalar additions and multiplications
9. When coloring the edges of a graph, a dc-path gets inverted because:  
 A. All edges in the fan will be colored with d or c.

3

- B. d is a free color for all fan vertices.  
 C. d is the free color for two fan vertices.  
 D. We are trying to minimize the number of colors used by the path.
10. Sorting the edges is a property of which minimum spanning tree technique?  
 A. Boruvka  
 B. Kruskal  
 C. Prim  
 D. Path-based (Warshall)
11. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:  
 A. Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.  
 B. The 3-sat instance is satisfiable.  
 C. The 3-sat instance is a tautology.  
 D. The 3-sat instance is unsatisfiable.
12. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)  
 A. 3-satisfiability  
 B. Testing if a graph is 3-colorable  
 C. Testing if a table is in sorted order  
 D. Testing if the number of colors needed to edge color a graph is the degree of the graph
13. Which of the following is a deficiency of the maximum capacity path technique?  
 A. An augmenting path is blocked if it introduces a cycle of flow.

- B. Augmenting paths will be discovered in descending incremental flow increase order.  
C. Flow decomposition must be applied.  
D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.

14. Which is not true regarding the first-fit decreasing method for bin packing?  
A. Each object is placed by going left-to-right until it fits in a bin or a new bin is allocated.  
B. Objects placed in bins beyond the optimal number have sizes no larger than  $\frac{1}{2}$ .  
C. The number of objects placed in bins beyond the optimal number is arbitrary.  
D. The number of objects with sizes larger than  $\frac{1}{2}$  is a lower bound on the optimal number of bins.

15. The most general approximation result that can be achieved for an NP-hard problem is:  
A. Approximation Algorithm  
B. Approximation Scheme  
C. Fully Polynomial-time Approximation Scheme  
D. Polynomial-time Approximation Scheme

16. Suppose you have a set of points in Euclidean 2-d space. Give an algorithm for finding a  $\rho$ -approximation for the minimum traveling salesperson path. Be sure to give the value of  $\rho$ . (5 points)

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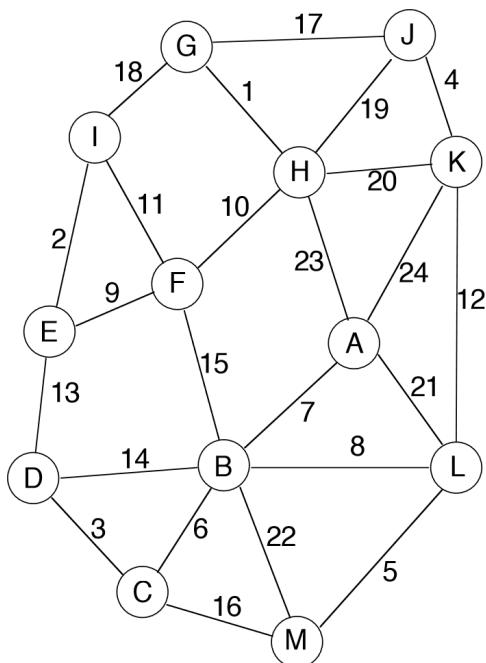
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1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (15 points)

4



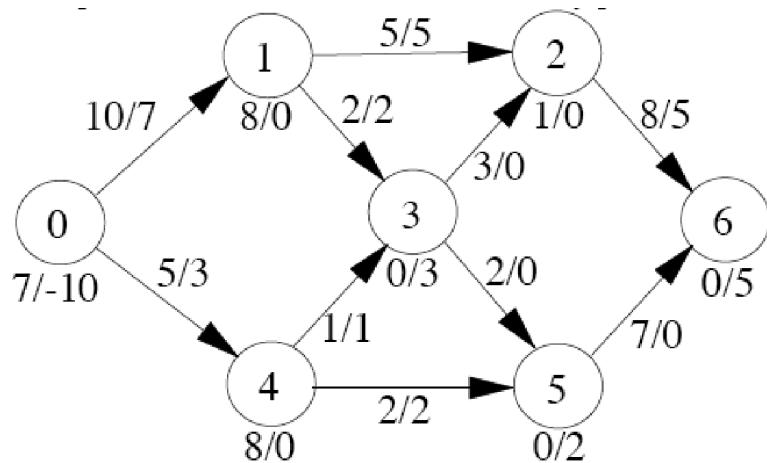
2. Give both types of KMP failure link tables and the table produced by the Z algorithm for the following string: (15 points)

0 a

```
-  
1 c  
2 a  
3 b  
4 a  
5 c  
6 a  
7 c  
8 a  
9 b  
10 a  
11 c  
12 a  
13 b  
14 a  
15 c  
16 a  
17 c  
18 a  
19 b  
20 a  
21 c  
22 a  
23 c  
24 a
```

3. List the remaining operations to complete this instance of network flows by push-relabel. In addition, give a minimum cut. (20 points)

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which priority queue is defined using the notion of null path length?  
A. Binary heap    B. Binomial heap    C. Fibonacci heap    D. Leftist heap
2. Dynamic optimality is a concept involving the comparison of  
A. a key-comparison based data structure to hashing.  
B. amortized complexity to actual complexity.  
C. an online data structure to a fixed, unchanging data structure.  
D. an online data structure to an offline data structure.
3. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 16 you already have?  
A. 3    B. 4    C. 5    D. 15
4. What is required when calling `union(i, j)` for maintaining disjoint subsets?  
A. `i` and `j` are in the same subset  
B. `i` and `j` are leaders for different subsets  
C. `i` and `j` are leaders for the same subset  
D. `i` is the ancestor of `j` in one of the trees
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:  
A. the sum of all keys in the entire tree  
B. the sum of all keys in the left subtree  
C. the sum of all keys that are no larger than the stored key  
D. the sum of all keys stored in the subtree rooted by this node
6. Which Fibonacci heap operation has  $\Theta(\log n)$  worst-case actual cost?  
A. FIB-HEAP-DECREASE-KEY    B. FIB-HEAP-DELETE  
C. FIB-HEAP-EXTRACT-MIN    D. FIB-HEAP-UNION
7. When are Fibonacci trees used?  
A. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.

- B. Defining the potential function for Fibonacci heaps.
  - C. Demonstrating worst-case behaviors for AVL trees.
  - D. Demonstrating worst-case behaviors for red-black trees.

8. To reduce the probability of having any collisions to  $< 0.5$  when hashing  $n$  keys, the table should have at least this number of elements.

  - A.  $n$
  - B.  $n \ln n$
  - C.  $n^2$
  - D.  $n^3$

9. The minimum number of nodes in a tree in a Fibonacci heap where the root has 7 children is:

  - A. 34
  - B. 61
  - C. 121
  - D. 128

10. Which property does not hold for binomial heaps?

  - A. DECREASE-KEY takes  $O(1)$  time.
  - B. MINIMUM takes  $O(\log n)$  time.
  - C. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.
  - D. The number of trees is based on the binary representation of the number of stored items.

11. Which of the following is not true regarding the amortized analysis of binary tree traversals?

  - A. INIT had an amortized cost of 0.
  - B. SUCC had an actual cost determined by the number of edges followed.
  - C. SUCC had an amortized cost of 2.
  - D. The potential was defined without regard for the type of traversal being performed.

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12. Which data structure is not used to implement a dictionary?  
A. AVL tree B. Red-black tree C. Self-organizing list D. Union-find

13. Which of the following is not a property of splay trees?  
A. Splaying an accessed node to the root can cause the potential to decrease  
B. The amortized cost of a splaying sequence is bounded logarithmically  
C. The zig rotation is only applied at the root  
D. They are a form of balanced tree

14. The two parts of using a Markov model to analyze a self-adjusting data structure are:  
A. define the potential function, compare to the optimal offline (OPT) method.  
B. determine the probability for each state, then compute the expected number of probes.  
C. simulate the data structure long enough to ensure convergence, then compute the expected number of probes.  
D. use Knuth's root trick to find the optimal subtrees in  $O(n^2)$  time, backtrace to print the tree.

15. Assuming a random  $n$ -permutation is provided, the expected number of hires for the hiring problem is:  
A. 2 B.  $H_n$  C.  $\sqrt{n}$  D.  $\ln \ln n$

16. Suppose  $f_1, f_2, \dots, f_m$  are functions from  $\{1, 2, \dots, n\}$  to  $\{1, 2, \dots, m\}$ . Then  $\sum_{i=1}^n \sum_{j=1}^m f_i(j) =$

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  - Suppose you are given  $n$  values. Give a linear-time algorithm to build a binomial heap with the  $n$  values. 10 points.
  - Evaluate the following recurrences using the master method. Indicate the case that is used for each.  
(10 points)
    - $T(n) = 4T\left(\frac{n}{2}\right) + n^3$
    - $T(n) = 4T\left(\frac{n}{2}\right) + n^2$
    - $T(n) = 4T\left(\frac{n}{2}\right) + 1$

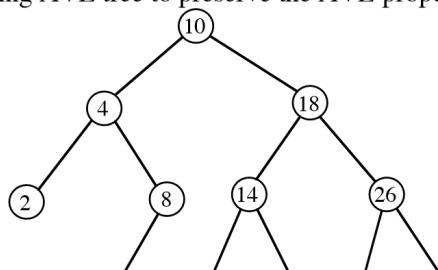
3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)
4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each key. Show the result from inserting 1300 using Brent's rehash when  $h_1(1300) = 5$  and  $h_2(1300) = 3$ . (10 points)

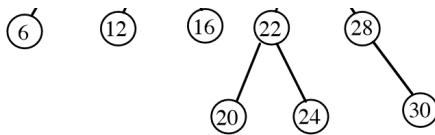
<i>key</i>	$h_1(key)$	$h_2(key)$
0		
1	1000	6
2		2
3	1200	3

			3
4	500	3	1
5	12	5	1
6	27	6	3

<i>key</i>
0
1
2
3
4
5
6

5. Insert 23 into the following AVL tree to preserve the AVL properties. (10 points)





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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems uses some edges with non-unit capacity when translated to a network flow problem?
  - A. Bipartite matching
  - B. Edge connectivity
  - C. Minimum vertex cover for bipartite graph
  - D. Vertex connectivity
2. Which is not true regarding the Karp-Rabin method?
  - A. It always runs in  $\Theta(m + n)$  time.
  - B. It computes a signature for the pattern and each  $m$  contiguous text symbols.
  - C. The signature is computed using a polynomial.
  - D. When the signature for  $m$  contiguous text symbols matches the signature for the pattern, a `strcmp` must be performed.
3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?
  - A. Convex hull
  - B. Delaunay triangulation
  - C. Euclidean minimum spanning tree
  - D. Voronoi diagram
4. Constructing a suffix array for a sequence with  $n$  symbols by using an optimal key-comparison sort has this worst-case time:
  - A.  $\Theta(n)$
  - B.  $\Theta(n \log n)$
  - C.  $\Theta(n^2)$
  - D.  $\Theta(n^2 \log n)$
5. The algorithm for finding a maximum capacity path for network flows is most similar to which algorithm?
  - A. Breadth-first search
  - B. Decomposition of a flow into  $E$  augmenting paths
  - C. Dijkstra
  - D. Floyd-Warshall
6. Which of the following algorithms may take time longer than  $O(n \log n)$ ?
  - A. Closest points in 2-d space - divide and conquer
  - B. Closest points in 2-d space - sweepline
  - C. Graham scan
  - D. Jarvis march
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
  - A. BST of points with x-coordinates as the key
  - B. BST of points with y-coordinates as the key
  - C. Interval tree
  - D. Sorted array by ascending x-coordinates
8. The length of a longest (monotone) increasing subsequence for 1 4 3 2 3 4 3 6 3 4 is:
  - A. 4
  - B. 5
  - C. 6
  - D. 7
9. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?
  - A. Problem A is NP-complete.
  - B. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
  - C. The reduction must be consistent for the decision results for each instance of problem A and and the corresponding instance of problem B

- D. The reduction takes polynomial time.
10. Which minimum spanning tree algorithm is the slowest?  
A. Boruvka      B. Kruskal      C. Prim      D. Warshall
11. When combining the left and right parts for the 2-d closest pair algorithm along the vertical dividing line, each left-side point “near” the line has its distance computed to no more than this number of right-side points.  
A. 6      B. 8      C.  $\lg n$       D.  $n/2$
12. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)

5

- A. Testing if a graph is bipartite  
B. Testing if an undirected graph has a Hamiltonian cycle  
C. Testing if the number of colors needed to edge color a graph is the degree of the graph  
D. 3-satisfiability
13. Which of the following is a deficiency of the maximum capacity path technique?  
A. An augmenting path is blocked if it introduces a cycle of flow.  
B. Augmenting paths will be discovered in descending incremental flow increase order.  
C. Flow decomposition must be applied.  
D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.
14. When performing bin packing using the First-Fit Decreasing technique, the total number of items placed in the bins past the optimal bins ( $1 \dots OPT$ ) is no more than:  
A.  $1 + \epsilon$       B. 2      C.  $OPT - 1$       D.  $OPT$
15. Which of the following does not have a polynomial-time approximation algorithm?  
A. Bin packing  
B. Edge coloring  
C. Traveling salesperson with triangle inequality  
D. Vertex coloring
16. Give the result of the Z algorithm for the following sequence. (5 points)

0 a  
1 c  
2 a  
3 b  
4 a  
5 c  
6 a  
7 c  
8 a  
9 b  
10 a  
11 c  
12 a  
13 b  
14 a  
15 c  
16 a  
17 c

18 a

19 b

20 a

6

21 c

22 a

23 c

24 a

25 b

CSE 5311

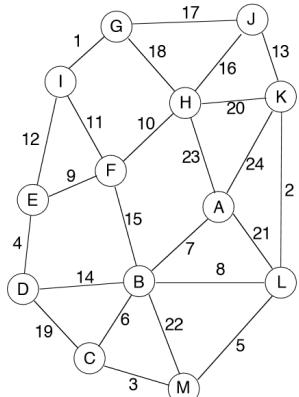
## Test 2 - Open Book

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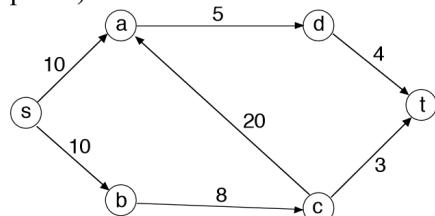
1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)



2. Solve the following instance of Longest Common Subsequence using the method based on the Longest Strictly Increasing Subsequence problem. (10 points)

0	1	2	3	4	5	6	7
a	b	d	c	d	e	f	g
b	c	d	f	e	d	g	f

3. List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (15 points)



4. Prove that the **SET PACKING** problem is NP-complete (15 points).

Hint 1: There is a straightforward reduction from **K-CLIQUE**.

Hint 1: There is a straightforward reduction from **K-CLIQUE**.  
Hint 2: It is often helpful to give an example of the reduction used.

**INSTANCE:** Collection  $C$  of finite sets, positive integer  $K \leq |C|$

**QUESTION:** Does  $C$  contain at least  $K$  mutually (i.e., pairwise)

QUESTION: Does  $C$  contain at least  $K$  mutually (i.e. pairwise) disjoint sets?

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following statements is true?
  - A. A binary search tree may be assigned legal AVL balance factors if and only if it may be legally colored as a red-black tree.
  - B. If a binary search tree may be assigned legal AVL balance factors, then it may be legally colored as a red-black tree.
  - C. If a binary search tree may be legally colored as a red-black tree, then it may be assigned legal AVL balance factors.
  - D. No binary search tree may be assigned both legal AVL balance factors and be legally colored as a red-black tree.
2. During which operation on a leftist heap may subtree swaps be needed?
  - A. DECREASE-KEY
  - B. EXTRACT-MIN
  - C. UNION
  - D. All of A., B., and C.
3. What is the worst-case number of rotations when performing deletion on an AVL tree?
  - A.  $\Theta(1)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D. No rotations are ever used
4. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?
  - A. 3
  - B. 4
  - C. 5
  - D. 15
5. Path compression is used in which algorithm?
  - A. DECREASE-KEY (e.g. CASCADING-CUT) for Fibonacci heaps
  - B. FIND for disjoint sets
  - C. UNION for disjoint sets
  - D. UNION for Fibonacci heaps
6. The balance factors in an AVL tree are computed as:
  - A.  $\text{height}_{\text{right}} - \text{height}_{\text{left}}$
  - B. same as the null path length in a leftist heap
  - C. the difference of the number of nodes in the left and right subtrees
  - D. the distance from a node to the root
7. Which situation is true regarding a cascade cut that produces c trees for a Fibonacci heap?
  - A. Both the actual and amortized costs are  $O(1)$ .
  - B. The actual cost is  $O(c)$ . The amortized cost is  $O(1)$ .

- C. The actual cost is  $O(1)$ . The amortized cost is  $O(c)$ .  
 D. The potential can become negative.
8. Which change to a binary search tree preserves the inorder traversal property?  
 A. rotation  
 B. zig-zag  
 C. zig-zig  
 D. all of the above
9. Which property does not hold for binomial heaps?  
 A. DECREASE-KEY takes  $O(1)$  time.  
 B. MINIMUM takes  $O(\log n)$  time.  
 C. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.  
 D. The number of trees is based on the binary representation of the number of stored items.
10. Which problem may be addressed using the assignment problem?  
 A. Optimal binary search tree  
 B. Optimal hashing  
 C. Perfect hashing  
 D. Primary clustering
11. Which of the following data structures offers similar capabilities and performance characteristics to skip lists?  
 A. AVL trees  
 B. Splay trees  
 C. Treap  
 D. Union-find with path compression
12. Slow convergence toward the optimal fixed ordering is a property of which technique?  
 A. Count  
 B. Move-ahead-k  
 C. Move-to-front  
 D. Transpose
13. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 1, 3?  
 A. 2      B. 3      C. 4      D. 5
14. The number of potential probe sequences when using double hashing with a table with  $m$  entries ( $m$  is prime) is:  
 A.  $O(\log m)$    B.  $m$       C.  $m(m - 1)$    D.  $m!$
15. When using Brent's rehash during insertion, the number of previously inserted keys that may move is:

- A. 1
- B. 2
- C.  $\frac{1}{\alpha}$
- D.  $H_m$ , where  $m$  is the number of stored keys

16. Suppose there are 20 coupon types for the coupon collecting problem and you have already obtained 17 of the coupon types. How many boxes of cereal do you expect (mathematically) to open to get the remaining three coupon types? (5 points)

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1. Give the range of possible heights for an AVL tree with 200 keys. (10 points)
2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)
  - a.  $T(n) = T(0.7n) + n$
  - b.  $T(n) = T(0.7n) + 1$
  - c.  $T(n) = 16T\left(\frac{n}{2}\right) + n^3$
3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;
q[0]=0.12;
key[1]=10;
p[1]=0.12;
q[1]=0.09;
key[2]=20;
p[2]=0.2;
q[2]=0.03;
key[3]=30;
p[3]=0.1;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.01;
q[5]=0.04;
```

```

key[6]=60;
p[6]=0.03;
q[6]=0.02;
w[0][0]=0.120000
w[0][1]=0.330000
w[0][2]=0.560000
w[0][3]=0.700000
w[0][4]=0.900000
w[0][5]=0.950000
w[0][6]=1.000000
w[1][1]=0.090000
w[1][2]=0.320000
w[1][3]=0.460000
w[1][4]=0.660000
w[1][5]=0.710000
w[1][6]=0.760000
w[2][2]=0.030000
w[2][3]=0.170000
w[2][4]=0.370000
w[2][5]=0.420000
w[2][6]=0.470000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.290000
w[3][6]=0.340000
w[4][4]=0.000000
w[4][5]=0.050000
w[4][6]=0.100000
w[5][5]=0.040000
w[5][6]=0.090000
w[6][6]=0.020000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 2
Building c(1,5) using roots 2 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 2 thru 2
Building c(1,6) using roots 2 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 31 without root trick 50
Average probe length is ???
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000

```

```

c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000

```

```

c(0,1) cost 0.330000 10
c(1,2) cost 0.320000 20
c(2,3) cost 0.170000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.050000 50
c(5,6) cost 0.090000 60
c(0,2) cost 0.880000 10(,20)
c(1,3) cost 0.630000 20(,30)
c(2,4) cost 0.540000 40(30,)
c(3,5) cost 0.340000 40(,50)
c(4,6) cost 0.150000 60(50,)
c(0,3) cost 1.200000 20(10,30)
c(1,4) cost 1.200000 20(,40(30,))
c(2,5) cost 0.640000 40(30,50)
c(3,6) cost 0.490000 40(,60(50,))
c(0,4) cost 1.770000 20(10,40(30,))
c(1,5) cost 1.350000 20(,40(30,50))
c(2,6) cost 0.790000 40(30,60(50,))
c(0,5) cost 1.920000 20(10,40(30,50))
c(1,6) cost 1.540000 40(20(,30),60(50,))
c(0,6) cost ??? ??????????????????

```

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each  $key$ . Show the result from inserting 9000 using Brent's rehash when  $h_1(9000) = 5$  and  $h_2(9000) = 4$ . (10 points)

$key$	$h_1(key)$	$h_2(key)$
-------	------------	------------

0		
1		
2	5000	2
3	4000	3
4	3000	4
5	2000	5
6	1000	6

$key$

0	
1	

2	
3	
4	

5. Suppose all  $2^k - 1$  nodes, along with the sentinel, in a red-black tree are colored black. Explain what will happen if any key is deleted. (10 points) CSE 5311

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Test 2 - Closed Book

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round?

A.  $W\left(\frac{n}{5}\right)$       B.  $\frac{n}{5}$       C.  $m$ , the median-of-medians      D.  $0.7n$

2. What data structure is used for the sweep-line status when detecting intersections among rectilinear rectangles?

- A. BST of points with x-coordinates as the key
- B. BST of points with y-coordinates as the key
- C. Interval tree
- D. Sorted array by ascending x-coordinates

3. While constructing a suffix array for a sequence with  $n$  symbols by using radix sorts, sequence symbols are used in which of the radix sorts?

- A. All of them
- B. None of them
- C. The last one
- D. The first one

4. Constructing a suffix array for a sequence with  $n$  symbols by using the original Manber-Myers radix sort construction has this worst-case time:

A.  $\Theta(n)$       B.  $\Theta(n \log n)$       C.  $\Theta(n^2)$       D.  $\Theta(n^2 \log n)$

5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:

- A.  $V$
- B.  $O(VE)$
- C.  $f$
- D.  $E$

6. Which of the following is not true regarding Strassen's algorithm?

- A. It is not possible to have an asymptotically faster algorithm.
- B. It requires more space than the everyday method.
- C. It uses  $\Theta(n^{\lg 7})$  scalar additions when multiplying two  $n \times n$  matrices.
- D. It uses  $\Theta(n^{\lg 7})$  scalar multiplications when multiplying two  $n \times n$  matrices.

7. What is the maximum number of times that an edge can become critical in the Edmonds-Karp method?

- A. Once      B. Twice      C.  $(V-2)/2$       D. VE
8. The worst-case time to determine a longest (monotone) increasing subsequence for a sequence of  $n$  numbers is:
- A.  $\Theta(m+n)$       B.  $\Theta(n)$       C.  $\Theta(n \log n)$       D.  $\Theta(n^2)$
9. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?
- A. The reduction has an inverse that takes each instance of problem B to an instance of problem A.  
B. The reduction takes polynomial time.  
C. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.  
D. Problem A is NP-complete.
10. On an augmenting path, a critical edge is:
- A. an edge with the minimum residual capacity  
B. an edge from the source  
C. an edge into the sink  
D. an edge that used to be saturated
11. Which algorithm is defined using the notions of left-turn and right-turn?
- A. Closest points in 2-d space      B.      Graham scan  
C. Jarvis march      D.      Suffix array construction
12. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:
- A. The 3-sat instance is satisfiable.  
B. Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.  
C. The 3-sat instance is unsatisfiable.  
D. The 3-sat instance is a tautology.
13. Which of the following is not required before relabeling (“lifting”) a vertex to a new height?
- A. Both breadth-first searches have been done.  
B. The vertex is overflowing.  
C. Any eligible edges for the present height have been saturated.  
D. The vertex is not the source or sink.
14. When coloring the edges of a graph, a dc-path gets inverted because:
- A. d is a free color for all fan vertices.  
B. We are trying to minimize the number of colors used by the path.  
C. All edges in the fan will be colored with d or c.  
D. d is the free color for two fan vertices.
15. The least general approximation result that can be achieved for an NP-hard problem is:

- A. Approximation Scheme
- B. Approximation Algorithm
- C. Fully Polynomial-time Approximation Scheme
- D. Polynomial-time Approximation Scheme

16. Give the style 2 KMP failure links for this sequence. (5 points)

0 a  
1 c  
2 a  
3 b  
4 a  
5 c  
6 a  
7 c  
8 a  
9 b  
10 a  
11 c  
12 a  
13 b  
14 a  
15 c  
16 a  
17 c  
18 a  
19 b

9

20 a  
21 c  
22 a  
23 c  
24 a  
25 b

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Summer 2009

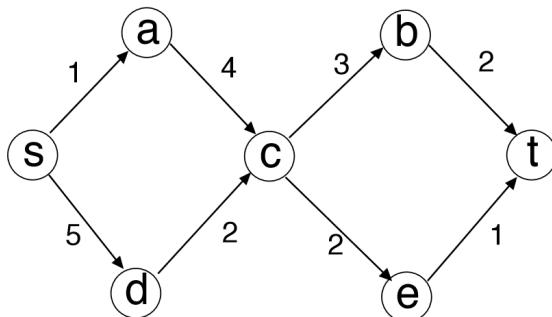
Name \_\_\_\_\_  
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1. Explain how to use the Z algorithm to determine the maximum overlap for a suffix of string X and a prefix of string Y. For example, if X=“dogcatcat” and Y=“catcatdog”, then the maximum overlap would be “catcat”. Do not explain details of the Z algorithm. 10 points

2. Solve the following instance of Longest Common Subsequence using the method based on the Longest Strictly Increasing Subsequence problem. (10 points)

0	1	2	3	4	5	6	7
a	b	d	c	d	e	b	a
b	c	d	c	e	d	b	a

3. List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (15 points)



4. Show that deciding whether an undirected graph is 5-colorable is NP-complete by a simple reduction from the 3-colorability problem. In addition to your proof, give an example of your reduction on a 3-colorable graph. (15 points)

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. During which operation on a leftist heap may subtree swaps be needed?
  - A. DECREASE-KEY
  - B. EXTRACT-MIN
  - C. UNION
  - D. All of A., B., and C.
2. What is the worst-case number of rotations when performing deletion on an AVL tree?
  - A.  $\Theta(1)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D. No rotations are ever used
3. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?
  - A. 3
  - B. 4
  - C. 5
  - D. 15
4. When is path compression used?
  - A. After an insertion into a splay tree.
  - B. With a FIND operation.
  - C. After an insertion into any type of balanced binary search tree.
  - D. With a UNION operation.
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
  - A. the sum of all keys in the entire tree
  - B. the sum of all keys in the left subtree
  - C. the sum of all keys that are no larger than the stored key
  - D. the sum of all keys stored in the subtree rooted by this node
6. To reduce the probability of having any collisions to  $< 0.5$  when hashing  $n$  keys, the table should have at least this number of elements.

- A.  $n$       B.  $n \ln nC$ .  $n^2$       D.  $n^3$

  7. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?
    - A. none of them
    - B. only the leaves
    - C. only the root
    - D. the leaves and the root
  8. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 1, 3?
    - A. 2
    - B. 3
    - C. 4
    - D. 5
  9. The reason for marking nodes in a Fibonacci heap is:
    - A. to assure that the structure is a Fibonacci heap rather than a binomial heap.
    - B. to allow computing the value of the potential function.
    - C. to indicate nodes that have lost a child since becoming a child themselves.
    - D. to improve the performance of CONSOLIDATE.
  10. Which property does not hold for binomial heaps?
    - A. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.
    - B. The number of trees is based on the binary representation of the number of stored items.
    - C. DECREASE-KEY takes  $O(\log n)$  time.
    - D. MINIMUM takes  $O(1)$  time.
  11. In the worst case, the number of rotations for inserting a key in a treap with  $n$  keys is:
    - A.  $\theta(1)$
    - B.  $\theta(\log n)$
    - C.  $\theta(n)$
    - D.  $\theta(n \log n)$
  12. Which priority queue is defined using the notion of null path length?
    - A. Leftist heap
    - B. Binomial heap
    - C. Fibonacci heap
    - D. Binary heap
  13. When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round?
    - A.  $\frac{n}{5}$
    - B.  $m$ , the median-of-medians
    - C.  $0.7n$
    - D.  $W\left(\frac{n}{5}\right)$
  14. The main difference between MTF and OPT for self-organizing linear lists is:
    - A. OPT can do transpositions
    - B. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
    - C. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
    - D. MTF counts inversions
  15. Assuming a random  $n$ -permutation is provided, the expected number of hires for the hiring problem is:
    - A.  $H_n$
    - B. 2
    - C.  $\sqrt{n}$
    - D.  $\ln \ln n$

2

16. Suppose there are 20 coupon types for the coupon collecting problem. Each of the coupon types is identified by an integer in the range 1 to 20. If you are given two random cereal boxes, what is the probability (ignoring the order in which the coupons were obtained) that the two coupons have consecutive numbers, i.e. some  $i$  and  $i + 1$ ? (5 points)

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## Test 1 - Open Book

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1. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each  $key$ . Show the result from inserting 1300 using Brent's rehash when

$$h_1(1300) = 5 \text{ and } h_2(1300) = 3. \quad (10 \text{ points})$$

	<i>key</i>	$h_1(key)$	$h_2(k)$
0			
1	1000	6	2
2			
3	1200	3	3
4	500	3	1
5	12	5	1
6	27	6	3
	<i>key</i>		

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

$$T(n) = 4T\left(\frac{n}{2}\right) + n^3$$

.. \* \\" - .. \2) ..

b.  $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

c.  $T(n) = 4T\left(\frac{n}{2}\right) + 1$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```

n=7;           w[0][3]=0.550000          w[3][4]=0.110000
q[0]=0.06;     w[0][4]=0.650000          w[3][5]=0.180000
p[1]=0.15;     w[0][5]=0.720000          w[3][6]=0.330000
q[1]=0.03;     w[0][6]=0.870000          w[3][7]=0.460000
p[2]=0.15;     w[0][7]=1.000000          w[4][4]=0.050000
q[2]=0.05;     w[1][1]=0.030000          w[4][5]=0.120000
p[3]=0.1;      w[1][2]=0.230000          w[4][6]=0.270000
q[3]=0.01;      w[1][3]=0.340000          w[4][7]=0.400000
p[4]=0.05;      w[1][4]=0.440000          w[5][5]=0.050000
q[4]=0.05;      w[1][5]=0.510000          w[5][6]=0.200000
p[5]=0.02;      w[1][6]=0.660000          w[5][7]=0.330000
q[5]=0.05;      w[1][7]=0.790000          w[6][6]=0.050000
p[6]=0.1;       w[2][2]=0.050000          w[6][7]=0.180000
q[6]=0.05;       w[2][3]=0.160000          w[7][7]=0.050000
p[7]=0.08;       w[2][4]=0.260000          Average probe length is ???
q[7]=0.05;       w[2][5]=0.330000          trees in parenthesized prefix
w[0][0]=0.060000 w[2][6]=0.480000          c(0,0) cost 0.000000
w[0][1]=0.240000 w[2][7]=0.610000          c(1,1) cost 0.000000
w[0][2]=0.440000 w[3][3]=0.010000          c(2,2) cost 0.000000

```

2

c(3,3) cost 0.000000	c(2,3) cost 0.160000 3 c(3,4) cost 0.110000 4	c(4,7) cost 0.700000 6(5,7) c(0,4) cost 1.260000 2(1,3(,4))
c(4,4) cost 0.000000	c(4,5) cost 0.120000 5 c(5,6) cost 0.200000 6	c(1,5) cost 1.030000 3(2,5(4,)) c(2,6) cost 1.030000 4(3,6(5,))
c(5,5) cost 0.000000	c(6,7) cost 0.180000 7 c(0,2) cost 0.670000 1(,2)	c(3,7) cost 0.930000 6(5(4,),7) c(0,5) cost 1.570000 2(1,4(3,5))
c(6,6) cost 0.000000	c(1,3) cost 0.500000 2(,3) c(2,4) cost 0.370000 3(,4)	c(1,6) cost 1.510000 3(2,6(5(4,),)) c(2,7) cost 1.400000 6(4(3,5),7)
c(7,7) cost 0.000000	c(3,5) cost 0.290000 5(4,) c(4,6) cost 0.390000 6(5,)	c(0,6) cost 2.140000 2(1,4(3,6(5,))) c(1,7) cost 1.950000 3(2,6(5(4,),7))
c(0,1) cost 0.240000 1	c(5,7) cost 0.510000 6(,7) c(0,3) cost 0.950000 2(1,3)	c(0,7) cost ??? ??????????????????
c(1,2) cost 0.230000 2	c(1,4) cost 0.780000 3(2,4) c(2,5) cost 0.610000 4(3,5)	
	c(3,6) cost 0.620000 6(5(4,),)	

4. Fill in the min and max blanks for the following instance of a van Emde Boas tree for the set {0, 1, 8, 10, 11, 12, 13}. You should give these as values in the local universe (0 .. u-1). Instead of using the symbol “/” for NIL, use the symbol “Ø”. (10 points)

```

root (base 0) u 16 min _____ max _____
summary (base 0) u 4 min _____ max _____
    summary (base 0) u 2 min _____ max _____
        cluster[0] (base 0) u 2 min _____ max _____
        cluster[1] (base 2) u 2 min _____ max _____
cluster[0] (base 0) u 4 min _____ max _____
    summary (base 0) u 2 min _____ max _____
        cluster[0] (base 0) u 2 min _____ max _____
        cluster[1] (base 2) u 2 min _____ max _____
cluster[1] (base 4) u 4 min _____ max _____
    summary (base 0) u 2 min _____ max _____
        cluster[0] (base 4) u 2 min _____ max _____
        cluster[1] (base 6) u 2 min _____ max _____

```

```

cluster[2] (base 8) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 8) u 2 min _____ max _____
cluster[1] (base 10) u 2 min _____ max _____
cluster[3] (base 12) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 12) u 2 min _____ max _____
cluster[1] (base 14) u 2 min _____ max _____

```

5. Give the range of possible heights for an AVL tree with 500 keys. Your answer should be two natural numbers giving the minimum and maximum heights. (A tree with one node has height 0.) Show your work! (10 points)

CSE 5311

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Test 2 - Closed Book

Summer 2010

Student ID # \_\_\_\_\_

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Constructing a suffix array for a sequence with  $n$  symbols by using the original Manber-Myers radix sort construction has this worst-case time:

$$\begin{array}{llll} \text{A. } \Theta(n) & \text{B. } \Theta(n \log n) & \text{C. } \Theta(n^2) & \text{D. } \Theta(n^2 \log n) \end{array}$$

2. What is the nature of the signature function for the Karp-Rabin method?

- A. a polynomial of arbitrary precision implemented using a bignum package
- B. similar to a double hash function for string keys
- C. it is similar to the KMP failure links
- D. the remainder by discarding the overflow for a polynomial

3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?

- A. Euclidean minimum spanning tree

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- B. Voronoi diagram
  - C. Convex hull
  - D. Delaunay triangulation
4. While constructing a suffix array for a sequence with  $n$  symbols by using radix sorts, sequence symbols are used in which of the radix sorts?
- A. The last one
  - B. The first one
  - C. None of them
  - D. All of them
5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:
- A.  $f$
  - B.  $E$
  - C.  $V$
  - D.  $O(VE)$
6. Which algorithm is defined using the notions of left-turn and right-turn?
- A. Graham scan
  - B. Closest points in 2-d space
  - C. Jarvis march
  - D. Suffix array construction
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
- A. BST of points with x-coordinates as the key
  - B. Sorted array by ascending x-coordinates
  - C. BST of points with y-coordinates as the key
  - D. Interval tree
8. The four russians' concept is to:
- A. Trade-off between enumerating situations and referencing these situations
  - B. Trade-off between scalar additions and multiplications
  - C. Implement longest common subsequences using linear space
  - D. Pack bits into an efficient storage unit
9. When coloring the edges of a graph, a dc-path gets inverted because:
- A. All edges in the fan will be colored with d or c.
  - B. d is a free color for all fan vertices.
  - C. d is the free color for two fan vertices.
  - D. We are trying to minimize the number of colors used by the path.
10. Which longest common subsequence method is potentially the most time-consuming?
- A. Compact version of dynamic programming based on divide-and-conquer
  - B. Method based on subsequence indices and longest strictly increasing subsequence
  - C. Ordinary dynamic programming using full matrix

- D. Four russians' implementation of C.
11. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:
- The 3-sat instance is satisfiable.
  - The 3-sat instance is a tautology.
  - The 3-sat instance is unsatisfiable.
  - Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.
12. Which of the following problems is not NP-complete? (Assume P  $\neq$  NP)
- Testing if a table is in sorted order
  - 3-satisfiability
  - Testing if a graph is 3-colorable
  - Testing if the number of colors needed to edge color a graph is the degree of the graph
13. Which of the following is a deficiency of the maximum capacity path technique?
- An augmenting path is blocked if it introduces a cycle of flow.
  - The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.
  - Augmenting paths will be discovered in descending incremental flow increase order.
  - Flow decomposition must be applied.
14. Which is not true regarding the first-fit decreasing method for bin packing?
- Each object is placed by going left-to-right until it fits in a bin or a new bin is allocated.
  - The number of objects placed in bins beyond the optimal number is arbitrary.
  - The number of objects with sizes larger than  $\frac{1}{2}$  is a lower bound on the optimal number of bins.

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- D. Objects placed in bins beyond the optimal number have sizes no larger than  $\frac{1}{2}$ .
15. The most general approximation result that can be achieved for an NP-hard problem is:
- Approximation Algorithm
  - Approximation Scheme
  - Polynomial-time Approximation Scheme
  - Fully Polynomial-time Approximation Scheme
16. What is the Steiner tree problem? (5 points)

CSE 5311

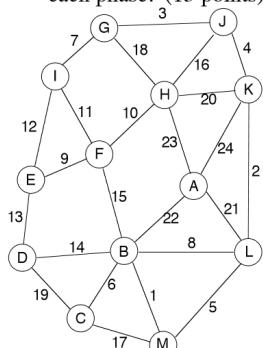
Name \_\_\_\_\_

Test 2 - Open Book

Summer 2010

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1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (15 points)



2. Give both types of KMP failure link tables and the table produced by the Z algorithm for the following string: (20 points)

```

0 a
1 c
2 a
3 b
4 a
5 c
6 a
7 c
8 a
9 b
10 a
11 c
12 a
13 b
14 a
15 c
16 a
17 c
18 a
19 b
20 a

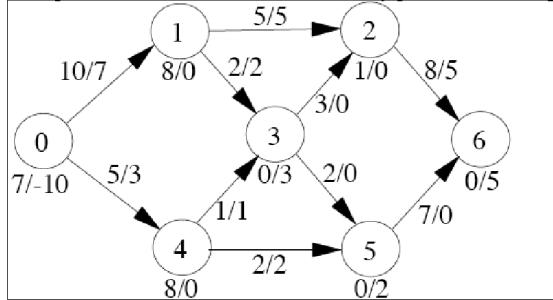
```

```

21 c
22 a
23 c
24 a

```

3. List the remaining operations to complete this instance of network flows by push-relabel. In addition, give a minimum cut. (15 points)



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Name \_\_\_\_\_

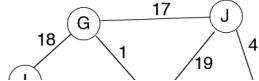
Test 1 - Closed Book

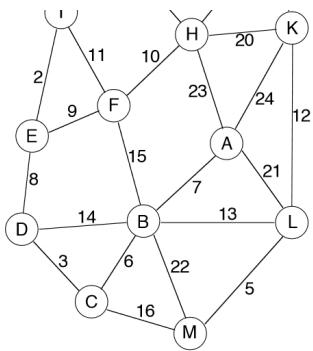
Summer 2011

Student ID # \_\_\_\_\_

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The number of potential probe sequences when using double hashing with a table with  $m$  entries ( $m$  is prime) is:
  - A.  $O(\log m)$
  - B.  $m$
  - C.  $m(m-1)$
  - D.  $m!$
2. What is the worst-case number of rotations when performing insertion on an AVL tree?
  - A.  $\Theta(1)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D. No rotations are ever used
3. Suppose you already have 10 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 10 you already have?
  - A. 2
  - B. 3
  - C. 4
  - D. 5
4. Which of the following is not true regarding Bloom filters?
  - A. An indication that a candidate element is not in the set is always correct.
  - B. They are an especially compact dictionary representation.
  - C. Several hash functions are used.
  - D. The optimal bit array size depends on the number of items and the false positive probability.
5. If the universe size at the root of a van Emde Boas tree is  $u$ , then the number of children is:
  - A. 2
  - B.  $\log u$
  - C.  $\log \log u$
  - D.  $\sqrt{u}$
6. The summary structures are critical for implementing which operation efficiently for van Emde Boas trees?
  - A. Member
  - B. Minimum
  - C. Successor
  - D. all of these
7. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 3, 1?
  - A. 2
  - B. 3
  - C. 4
  - D. 5
8. Suppose there are  $n$  entries in a Fibonacci heap. The maximum number of trees is:
  - A.  $\Theta(\log \log n)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D.  $\Theta(n \log n)$
9. Which property does not hold for binomial heaps?
  - A. DECREASE-KEY takes  $O(1)$  time.
  - B. MINIMUM takes  $O(\log n)$  time.
  - C. Performing  $n$  INSERT operations into an empty heap will take  $O(n)$  time.
  - D. The number of trees is based on the binary representation of the number of stored items.
10. What is the main contribution of leftist heaps?
  - A. The minimum is found in  $O(1)$  time.
  - B. The amortized complexity of DECREASE-KEY is  $O(1)$ .
  - C. The height of the tree is  $O(\log n)$ .
  - D. The UNION is computed in  $O(\log n)$  time.
11. Dynamic optimality is a concept involving the comparison of
  - A. a key-comparison based data structure to hashing.
  - B. amortized complexity to actual complexity.
  - C. an online data structure to a fixed, unchanging data structure.
  - D. an online data structure to an offline data structure.
12. Give the total weight of the minimum spanning tree for this graph. (5 points)





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13. Give an example of a binary search tree that can be colored as a legal red-black tree, but does not satisfy the AVL balance conditions. (6 points)

14. Give the binomial (min) heap that results from inserting 1, 2, 3, 4, 5, 6, 7, 8 (in that order) into an empty heap. (6 points)

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Test 1 - Open Book

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1. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each key. Show the result from inserting 7095 using Brent's rehash when

$$h_1(7095) = 4 \text{ and } h_2(7095) = 4. \quad (10 \text{ points})$$

	key	$h_1(key)$	$h_2(key)$
0	7000	0	5
1	7081	4	2
2	7002	2	1
3			
4	7004	4	3
5			
6	7006	6	5

	key
0	
1	
2	
3	
4	
5	
6	

2. Evaluate the following recurrences using the master method. (10 points)

a.  $T(n) = T(0.7n) + n$

b.  $T(n) = T(0.7n) + 1$

c.  $T(n) = 16T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```

n=6;           w[0][0]=0.010000          w[3][5]=0.300000
q[0]=0.01;     w[0][1]=0.220000          w[3][6]=0.450000
key[1]=10;     w[0][2]=0.350000          w[4][4]=0.000000
p[1]=0.19;     w[0][3]=0.590000          w[4][5]=0.060000
q[1]=0.02;     w[0][4]=0.790000          w[4][6]=0.210000
key[2]=20;     w[0][5]=0.850000          w[5][5]=0.040000
p[2]=0.1;      w[0][6]=1.000000          w[5][6]=0.190000
q[2]=0.03;     w[1][1]=0.020000          w[6][6]=0.030000
key[3]=30;     w[1][2]=0.150000          Building c(0,2) using roots 1
p[3]=0.2;      w[1][3]=0.390000          thru 2
q[3]=0.04;     w[1][4]=0.590000          Building c(1,3) using roots 2
key[4]=40;     w[1][5]=0.650000          thru 3

```

```

p[4]=0.2;           w[1][6]=0.800000      Building c(2,4) using roots 3
q[4]=0.0;           w[2][2]=0.030000      thru 4
key[5]=50;          w[2][3]=0.270000      Building c(3,5) using roots 4
p[5]=0.02;          w[2][4]=0.470000      thru 5
q[5]=0.04;          w[2][5]=0.530000      Building c(4,6) using roots 5
key[6]=60;          w[2][6]=0.680000      thru 6
p[6]=0.12;          w[3][3]=0.040000      Building c(0,3) using roots 1
q[6]=0.03;          w[3][4]=0.240000      thru 3

```

2

Building c(1,4)	Building c(1,6) using roots 3	c(1,3) cost 0.540000 30(20,)
using roots	thru 4	c(2,4) cost 0.710000 30(,40)
3 thru 3	Building c(0,6) using roots ?	c(3,5) cost 0.360000 40(,50)
Building c(2,5)	thru ?	c(4,6) cost 0.270000 60(50,)
using roots	Counts - root trick 29 without	c(0,3) cost 1.080000 20(10,30)
3 thru 4	root trick 50	c(1,4) cost 0.980000 30(20,40)
Building c(3,6)	Average probe length is ????	c(2,5) cost 0.860000 40(30,50)
using roots	trees in parenthesized prefix	c(3,6) cost 0.720000 40(,60(50,))
4 thru 6	c(0,0) cost 0.000000	c(0,4) cost 1.530000
Building c(0,4)	c(1,1) cost 0.000000	30(10(,20),40)
using roots	c(2,2) cost 0.000000	c(1,5) cost 1.160000
2 thru 3	c(3,3) cost 0.000000	30(20,40(,50))
Building c(1,5)	c(4,4) cost 0.000000	c(2,6) cost 1.220000
using roots	c(5,5) cost 0.000000	40(30,60(50,))
3 thru 4	c(6,6) cost 0.000000	c(0,5) cost 1.710000
Building c(2,6)	c(0,1) cost 0.220000 10	30(10(,20),40(,50))
using roots	c(1,2) cost 0.150000 20	c(1,6) cost 1.610000
4 thru 4	c(2,3) cost 0.270000 30	40(30(20,),60(50,))
Building c(0,5)	c(3,4) cost 0.240000 40	c(0,6) cost ??????????
using roots	c(4,5) cost 0.060000 50	????????????????
3 thru 3	c(5,6) cost 0.190000 60	
	c(0,2) cost 0.500000 10(,20)	

4. Fill in the min and max blanks for the following instance of a van Emde Boas tree for the set {1, 8, 9, 10, 12, 13, 14}. You should give these as values in the local universe (0..u-1). Instead of using the symbol “/” for NIL, use the symbol “Ø”. (10 points)

```

root (base 0) u 16 min _____ max _____
summary (base 0) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 0) u 2 min _____ max _____
cluster[1] (base 2) u 2 min _____ max _____
cluster[0] (base 0) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 0) u 2 min _____ max _____
cluster[1] (base 2) u 2 min _____ max _____
cluster[1] (base 4) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 4) u 2 min _____ max _____
cluster[1] (base 6) u 2 min _____ max _____
cluster[2] (base 8) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 8) u 2 min _____ max _____
cluster[1] (base 10) u 2 min _____ max _____
cluster[3] (base 12) u 4 min _____ max _____
summary (base 0) u 2 min _____ max _____
cluster[0] (base 12) u 2 min _____ max _____
cluster[1] (base 14) u 2 min _____ max _____

```

Give the range of possible heights for a red-black tree with 200 keys. Your answer should be two natural numbers giving the minimum and maximum heights. (A tree with one node has height 0.) Show your work! (10 points)

CSE 5311

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Test 2 - Closed Book

Summer 2011

Last 4 Digits of Student ID # \_\_\_\_\_

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round? \_\_\_\_\_

the first round?

- A.  $W\left(\frac{n}{5}\right)$
  - B.  $m$ , the median-of-medians
  - C.  $.7n$
  - D.  $\frac{n}{5}$
2. What is the nature of the signature function for the Karp-Rabin method?
- A. the remainder by discarding the overflow for a polynomial
  - B. it is similar to the KMP failure links
  - C. similar to a double hash function for string keys
  - D. a polynomial of arbitrary precision implemented using a bignum package
3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?
- A. Euclidean minimum spanning tree
  - B. Delaunay triangulation
  - C. Convex hull
  - D. Voronoi diagram
4. Which of the following is not a goal for suffix array construction methods?
- A. Fast in practice
  - B. Based on radix sort
  - C. Lightweight
  - D. Linear-time in worse case
5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:
- A.  $V$
  - B.  $O(VE)$
  - C.  $f$
  - D.  $E$
6. Which algorithm is defined using the notions of left-turn and right-turn?
- A. Graham scan
  - B. Closest points in 2-d space
  - C. Suffix array construction
  - D. Jarvis march
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
- A. BST of points with y-coordinates as the key
  - B. Interval tree
  - C. BST of points with x-coordinates as the key
  - D. Sorted array by ascending x-coordinates
8. The four russians' concept is to:
- A. Pack bits into an efficient storage unit
  - B. Trade-off between enumerating situations and referencing these situations
  - C. Implement longest common subsequences using linear space
  - D. Trade-off between scalar additions and multiplications
9. When coloring the edges of a graph, a dc-path gets inverted because:
- A. All edges in the fan will be colored with d or c.
  - B. d is a free color for all fan vertices.
  - C. d is the free color for two fan vertices.
  - D. We are trying to minimize the number of colors used by the path.
10. How many times will  $-1$  occur in the style 2 fail link table for the pattern abacabac?
- A. 1
  - B. 2
  - C. 3
  - D. 4
11. How many times will  $-1$  occur in the style 1 fail link table for the pattern abacabac?
- A. 1
  - B. 2
  - C. 3
  - D. 4
12. Which of the following problems is NP-complete? (Assume P  $\neq$  NP)
- A. 3-satisfiability
  - B. Testing if a graph is 2-colorable
  - C. Testing if a table is in sorted order
  - D. Verifying a solution to traveling salesperson
13. Which of the following is a deficiency of the maximum capacity path technique?
- A. Augmenting paths will be discovered in descending incremental flow increase order

- B. Flow decomposition must be applied.  
 C. An augmenting path is blocked if it introduces a cycle of flow.  
 D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.

14. Constructing a suffix array for a sequence with  $n$  symbols by using an optimal key-comparison sort has this worst-case time:

A.  $\Theta(n)$       B.  $\Theta(n \log n)$       C.  $\Theta(n^2)$       D.  $\Theta(n^2 \log n)$

15. Which of the following does not have a polynomial-time approximation algorithm?

13. Which of the following does not have a polynomial-time approximation algorithm?

- A. Bin packing
  - B. Edge coloring
  - C. Traveling salesperson with triangle inequality
  - D. Vertex coloring

16. Determine a monotone longest increasing subsequence for the sequence below. (5 points)

1

3 4  
N

CSE 5311

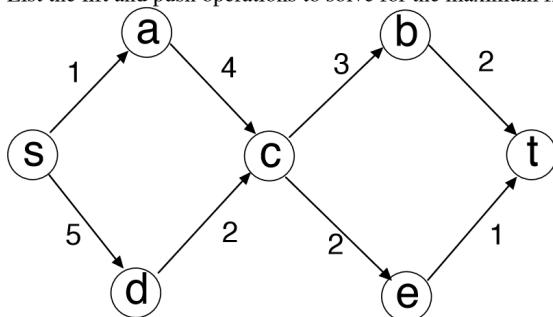
Text 3 - Open Book

Test 2 - Open  
S... 2011

Summer 2011 Last 4 Digits of Student ID # \_\_\_\_\_  
1. Fill in the blanks in the following instance of a suffix array with lcp values and ranks. As usual,  $s[21]$  is NULL ('\0'). (15 points)

i	sa	suffix	lcp	s	rank	lcp[rank]
0	21		-1	0	—	—
1	20	0	0	1	—	—
2	7	00100101001010	1	0	—	—
3	15	001010	4	0	—	—
4	2	0010100100101001010	6	1	20	4
5	10	00101001010	9	0	7	3
6	18	010	1	1	15	2
7	5	0100100101001010	3	0	2	—
8	13	01001010	6	0	10	—
9	0	010010100100101001010	8	1	18	—
10		0100101001010	11	0	5	9
11	16	01010	3	0	13	8
12	3	010100100101001010	5	1	21	7
13	11	—101001010	8	0	8	6
14	19	—0	0	1	16	5
15	6	100100101001010	2	0	3	4
16	14	1001010	5	0	11	3
17	1	10010100100101001010	7	1	19	2
18	9	100101001010	10	0	6	1
19		1010	2	1	14	0
20	4	10100100101001010	4	0	1	0
21	12	10100101010	7	0	—	-1

2. List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (15 points)



3. Give the result of the Z algorithm and both KMP methods for the following sequence. (20 points)

9 a

1 c

2 a

3 b

4 a

5 c

6 a

7 c

8 a

9 b

10 a

11 c

12 a

13 b

14 a

15 c

16 a

17 c

18 a

19 b

20 a

21 c

22 a

23 c

24 a

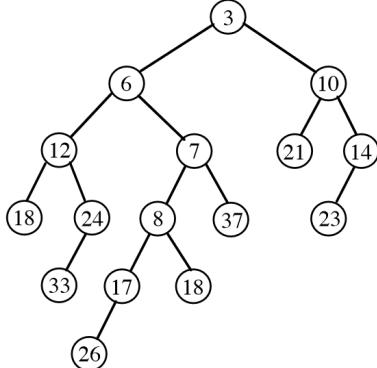
25 b

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

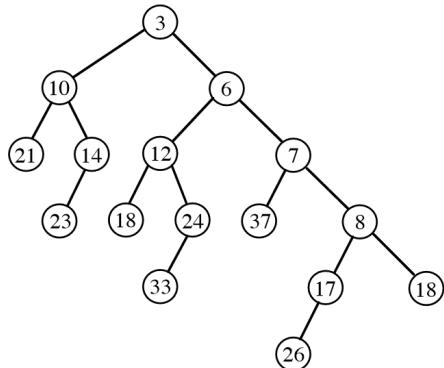
1. When are Fibonacci trees used?

- A. Defining the potential function for Fibonacci heaps.
- B. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
- C. Demonstrating worst-case behaviors for red-black trees.
- D. Demonstrating worst-case behaviors for AVL trees.

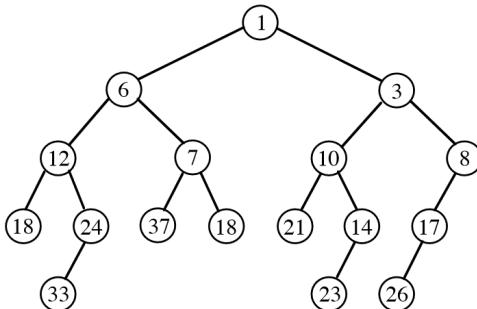
2. Which of the following is not a legal leftist heap?



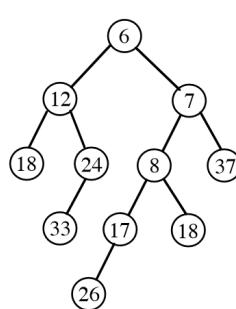
A.



B.



C.



D.

3. Dynamic optimality is a concept involving the comparison of
- A. a key-comparison based data structure to hashing.
  - B. amortized complexity to actual complexity.
  - C. an online data structure to an offline data structure.
  - D. an online data structure to a fixed, unchanging data structure.
4. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 16 you already have?
- A. 4
  - B. 5
  - C. 6
  - D. 10
5. If the universe size at the root of a van Emde Boas tree is  $u$ , then the number of children is:
- A.  $\lg u$
  - B.  $\log u$
  - C.  $\log \log u$
  - D.  $\sqrt{u}$
6. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
- A. the sum of all keys in the entire tree
  - B. the sum of all keys in the left subtree
  - C. the sum of all keys that are no larger than the stored key
  - D. the sum of all keys stored in the subtree rooted by this node
7. Which Fibonacci heap operation has  $\Theta(\log n)$  worst-case actual cost?
- A. FIB-HEAP-EXTRACT-MIN
  - B. FIB-HEAP-UNION
  - C. FIB-HEAP-DECREASE-KEY
  - D. FIB-HEAP-DELETE
8. What is the main contribution of leftist heaps?
- A. The minimum is found in  $O(1)$  time.
  - B. The amortized complexity of DECREASE-KEY is  $O(1)$ .
  - C. The height of the tree is  $O(\log n)$ .
  - D. The UNION is computed in  $O(\log n)$  time.
9. The number of potential probe sequences when using linear probing with a table with  $m$  entries ( $m$  is prime) is:

10. Which of the following is not true regarding the amortized analysis of binary tree traversals?
- INIT had an amortized cost of 1.
  - SUCC had an actual cost determined by the number of edges followed.
  - SUCC had an amortized cost of 2.
  - The potential was defined with regard to the type of traversal being performed.
11. Which data structure is not used to implement a dictionary?
- vEB tree
  - Red-black tree
  - Self-organizing list
  - Splay tree
12. The reason for marking nodes in a Fibonacci heap is:
- to indicate nodes that have lost a child since becoming a child themselves.
  - to allow computing the value of the potential function.
  - to assure that the structure is a Fibonacci heap rather than a binomial heap.
  - to improve the performance of CONSOLIDATE.
13. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?
- none of them
  - only the root
  - only the leaves
  - the leaves and the root
14. The main difference between MTF and OPT for self-organizing linear lists is:
- MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
  - OPT counts inversions
  - OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
  - MTF can do transpositions
15. To reduce the probability of having any collisions to  $< 0.5$  when hashing  $n$  keys, the table should have at least this number of elements.
- $n$
  - $n \ln n$
  - $n^2$
  - $n^3$
16. Suppose you roll three standard six-sided dice. What is the probability that the sum of the three values rolled does not exceed 11? Show your work. (5 points)

CSE 5311

Name \_\_\_\_\_

Test 1 - Open Book

Summer 2012

Last 4 Digits of Student ID # \_\_\_\_\_

1. Fill in the min and max blanks for the following instance of a van Emde Boas tree for the set  $\{1, 2, 3, 8, 9, 10, 14, 15\}$ . You should give these as values in the local universe ( $0 \dots u-1$ ). Instead of using the symbol “/” for NIL, use the symbol “Ø”. (10 points)

```

root (base 0) u 16 min _____ max _____
    summary (base 0) u 4 min _____ max _____
        summary (base 0) u 2 min _____ max _____
            cluster[0] (base 0) u 2 min _____ max _____
            cluster[1] (base 2) u 2 min _____ max _____
        cluster[0] (base 0) u 4 min _____ max _____
            summary (base 0) u 2 min _____ max _____
            cluster[0] (base 0) u 2 min _____ max _____
            cluster[1] (base 2) u 2 min _____ max _____
        cluster[1] (base 4) u 4 min _____ max _____
            summary (base 0) u 2 min _____ max _____
            cluster[0] (base 4) u 2 min _____ max _____
            cluster[1] (base 6) u 2 min _____ max _____
        cluster[2] (base 8) u 4 min _____ max _____
            summary (base 0) u 2 min _____ max _____
            cluster[0] (base 8) u 2 min _____ max _____
            cluster[1] (base 10) u 2 min _____ max _____
        cluster[3] (base 12) u 4 min _____ max _____
            summary (base 0) u 2 min _____ max _____
            cluster[0] (base 12) u 2 min _____ max _____
            cluster[1] (base 14) u 2 min _____ max _____

```

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

- $T(n) = 4T\left(\frac{n}{2}\right) + n^3$
- $T(n) = 4T\left(\frac{n}{2}\right) + n^2$
- $T(n) = 4T\left(\frac{n}{2}\right) + 1$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```

n=6;
q[0]=0.01;
key[1]=10;
w[1][5]=0.650000
w[1][6]=0.800000
w[2][2]=0.030000

```

```

Building c(1,6) using roots 3
      thru 4
Building c(0,6) using roots ?

```

```

p[1]=0.19; w[2][3]=0.270000 thru ?
q[1]=0.02; w[2][4]=0.470000 Counts - root trick 29 without
key[2]=20; w[2][5]=0.530000 root trick 50
p[2]=0.1; w[2][6]=0.680000 Average probe length is *****
q[2]=0.03; w[3][3]=0.040000 trees in parenthesized prefix
key[3]=30; w[3][4]=0.240000 c(0,0) cost 0.000000
p[3]=0.2; w[3][5]=0.300000 c(1,1) cost 0.000000
q[3]=0.04; w[3][6]=0.450000 c(2,2) cost 0.000000
key[4]=40; w[4][4]=0.000000 c(3,3) cost 0.000000
p[4]=0.2; w[4][5]=0.060000 c(4,4) cost 0.000000
q[4]=0.0; w[4][6]=0.210000 c(5,5) cost 0.000000
key[5]=50; w[5][5]=0.040000 c(6,6) cost 0.000000
p[5]=0.02; w[5][6]=0.190000 c(0,1) cost 0.220000 10
q[5]=0.04; w[6][6]=0.030000 c(1,2) cost 0.150000 20
key[6]=60; Building c(0,2) using roots 1 c(2,3) cost 0.270000 30
p[6]=0.12; thru 2 c(3,4) cost 0.240000 40
q[6]=0.03; Building c(1,3) using roots 2 c(4,5) cost 0.060000 50
w[0][0]=0.01000 thru 3 c(5,6) cost 0.190000 60
0 Building c(2,4) using roots 3 c(0,2) cost 0.500000 10(,20)
w[0][1]=0.22000 thru 4 c(1,3) cost 0.540000 30(20, )
0 Building c(3,5) using roots 4 c(2,4) cost 0.710000 30(,40)
w[0][2]=0.35000 thru 5 c(3,5) cost 0.360000 40(,50)
0 Building c(4,6) using roots 5 c(4,6) cost 0.270000 60(50, )
w[0][3]=0.59000 thru 6 c(0,3) cost 1.080000 20(10,30)
0 Building c(0,3) using roots 1 c(1,4) cost 0.980000 30(20,40)
w[0][4]=0.79000 thru 3 Building c(1,4) using roots 3 c(2,5) cost 0.860000 40(30,50)
0 Building c(1,4) using roots 3 thru 3 c(3,6) cost 0.720000 40(,60(50, ))
w[0][5]=0.85000 Building c(2,5) using roots 3 c(0,4) cost 1.530000
0 thru 4 Building c(2,5) using roots 3 30(10(,20),40)
w[0][6]=1.00000 Building c(3,6) using roots 4 c(1,5) cost 1.160000
0 thru 6 Building c(3,6) using roots 4 30(20,40(,50))
w[1][1]=0.02000 Building c(0,4) using roots 2 c(2,6) cost 1.220000
0 thru 3 Building c(0,4) using roots 2 40(30,60(50, ))
w[1][2]=0.15000 Building c(1,5) using roots 3 c(0,5) cost 1.710000
0 thru 4 Building c(1,5) using roots 3 30(10(,20),40(,50))
w[1][3]=0.39000 Building c(2,6) using roots 4 c(1,6) cost 1.610000
0 thru 4 Building c(2,6) using roots 4 40(30(20, ),60(50, ))
w[1][4]=0.59000 Building c(0,5) using roots 3 c(0,6) cost ??????????
0 thru 3 Building c(0,5) using roots 3 ???????????????

```

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each  $key$ . Show the result from inserting 5559 using Brent's rehash when

$$h_1(5559) = 1 \text{ and } h_2(5559) = 4. \quad (10 \text{ points})$$

	<i>key</i>	$h_1(key)$	$h_2(key)$
0			
1	2199	1	4
2	3299	2	6
3	5500	5	5
4	4400	4	3
5	1111	5	2

key

0  
1  
2  
3  
4  
5  
6

5. Give the precise range of possible heights for an AVL tree with 300 keys. (10 points)

CSE 5311

Name \_\_\_\_\_

Test 2 - Closed Book

Summer 2012

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Constructing a suffix array for a sequence with  $n$  symbols by using the original Manber-Myers radix sort construction has this worst-case time:

$$\text{A. } \Theta(n) \quad \text{B. } \Theta(n \log n) \quad \text{C. } \Theta(n^2) \quad \text{D. } \Theta(n^2 \log n)$$

2. What is the nature of the signature function for the Karp-Rabin method?

- A. it is similar to the KMP failure links
- B. the remainder by discarding the overflow for a polynomial
- C. a polynomial of arbitrary precision implemented using a bignum package
- D. similar to a double hash function for string keys

3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?

- A. Voronoi diagram
- B. Convex hull
- C. Delaunay triangulation
- D. Euclidean minimum spanning tree

4. While constructing a suffix array for a sequence with  $n$  symbols by using radix sorts, sequence symbols are used in which of the radix sorts?

- A. The last one
- B. The first one
- C. None of them
- D. All of them

5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:

- A. the number of edges in the graph
- B. the number of saturated edges
- C. the number of unsaturated edges
- D. the number of vertices

6. Which algorithm is defined using the notions of left-turn and right-turn?

- A. Suffix array construction
- B. Graham scan
- C. Closest points in 2-d space
- D. Jarvis march

7. What data structure is used for the sweep-line status when computing the 2-d closest pair?

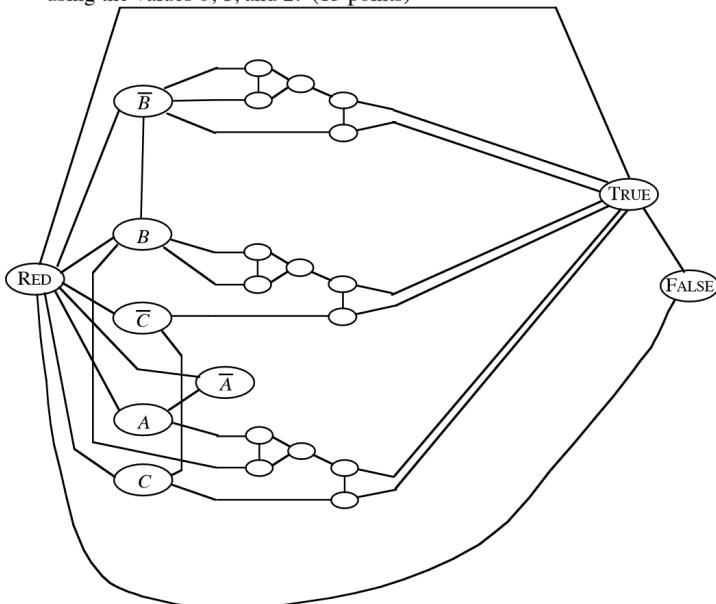
- A. BST of points with y-coordinates as the key
- B. Sorted array by ascending x-coordinates
- C. BST of points with x-coordinates as the key
- D. Interval tree

8. The four russians' concept is to:

- A. Trade-off between generating situations and referencing these situations
- B. Trade-off between scalar additions and multiplications
- C. Implement longest common subsequences using linear space

- D. Pack bits into an efficient storage unit
9. When coloring the edges of a graph, a dc-path gets inverted because:
- A. d is the free color for two fan vertices.
  - B. We are trying to minimize the number of colors used by the path.
  - C. All edges in the fan will be colored with d or c.
  - D. d is a free color for all fan vertices.
10. What is the nature of the linear-space method for the longest common subsequence problem?
- A. Build a suffix array and lcp array for the concatenated input sequences
  - B. Recursive divide-and-conquer
  - C. Radix sort
  - D. Use a polynomial for the signature function
11. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:
- A. Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.
  - B. The 3-sat instance is unsatisfiable.
  - C. The 3-sat instance is satisfiable.
  - D. The 3-sat instance is a tautology.
12. How many times will 1 occur in the style 1 fail link table for the pattern abcaabbd?

12. How many times will -1 occur in the style 1 fail link table for the pattern abcabd?
- 1
  - 2
  - 3
  - 4
13. How many times will -1 occur in the style 2 fail link table for the pattern abcabd?
- 1
  - 2
  - 3
  - 4
14. Vizing's theorem is the basis for:
- Approximate bin packing.
  - Traveling salesperson approximation when the triangle inequality applies.
  - Approximate edge coloring.
  - Approximate a vertex coloring.
15. The technique for approximating a subset cover proceeds by:
- Choosing the subset with the smallest fraction of its elements uncovered
  - Choosing the subset with the smallest number of uncovered elements
  - Choosing the subset with the largest fraction of its elements uncovered
  - Choosing the subset with the largest number of uncovered elements
16. Describe the initialization for push/relabel techniques. You may give an example. (5 points)
- CSE 5311 Name \_\_\_\_\_
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- Summer 2012 Student ID # \_\_\_\_\_
1. Give the set of clauses for the instance below of the reduction from 3-sat to 3-colorability and an appropriate coloring using the values 0, 1, and 2. (15 points)



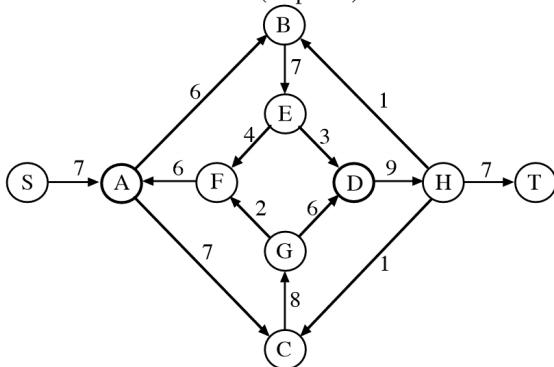
6

2. Give both types of KMP failure link tables for the following string: (10 points)

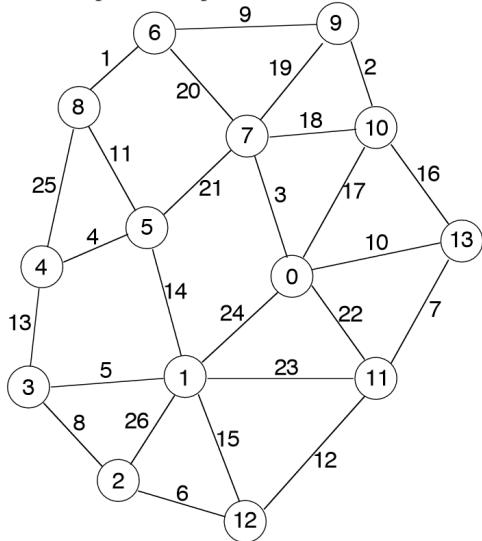
0	C
1	A
2	B
3	A
4	B
5	C
6	C
7	A
8	B
9	C
10	A
11	B
12	*

12 A  
13 B  
14 C

3. Give a flow decomposition for the network flow below. (10 points)



4. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (15 points)



CSE 5311

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Test 1 - Closed Book

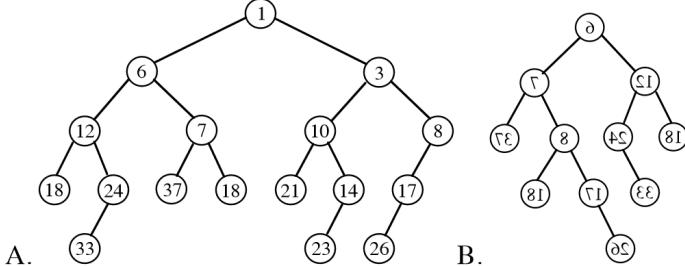
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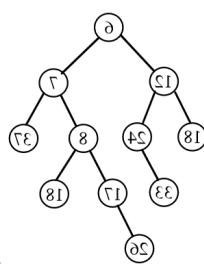
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. During which operation on a leftist heap may subtree swaps be needed?
  - A. DECREASE-KEY
  - B. EXTRACT-MIN
  - C. UNION
  - D. All of A., B., and C.
2. Which of the following is not true regarding the amortized analysis of binary tree traversals?
  - A. INIT had an amortized cost of 1.
  - B. SUCC had an actual cost determined by the number of edges followed.
  - C. SUCC had an amortized cost of 2.
  - D. The potential was defined with regard to the type of traversal being performed.
3. Suppose you already have 10 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 10 you already have?
  - A. 2
  - B. 3
  - C. 4
  - D. 5
4. When is path compression used?
  - A. After an insertion into a splay tree.
  - B. With a FIND operation.
  - C. After an insertion into any type of balanced binary search tree.

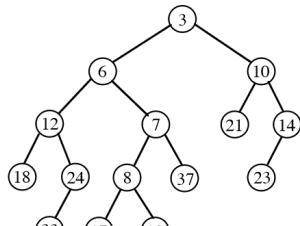
- D. With a UNION operation.
5. Which of the following data structures offers similar capabilities and performance characteristics to skip lists?  
 A. AVL trees    B. Splay trees    C. Treap    D. Union-find with path compression
6. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?  
 A. none of them    B. only the leaves  
 C. only the root    D. the leaves and the root
7. How many inversions are there for the lists 1, 2, 5, 4, 3 and 1, 2, 3, 4, 5?  
 A. 2    B. 3    C. 4    D. 5
8. Which of the following is not a legal leftist heap?



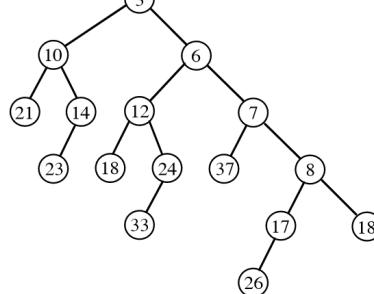
A.



B.



C.



D.

9. Which situation is true regarding a cascading cut that produces c trees for a Fibonacci heap?

2

- A. Both the actual and amortized costs are O(1).  
 B. The actual cost is O(c). The amortized cost is O(1).  
 C. The actual cost is O(1). The amortized cost is O(c).  
 D. The potential can become negative.
10. Which property does not hold for binomial heaps?  
 A. MINIMUM takes O(1) time.  
 B. Performing  $n$  INSERT operations into an empty heap will take O( $n$ ) time.  
 C. The number of trees is based on the binary representation of the number of stored items.  
 D. DECREASE-KEY takes O(log  $n$ ) time.
11. When using Brent's rehash, the number of previously inserted keys that may move is:  
 A. 1    B. 2    C.  $\frac{1}{\alpha}$     D.  $H_m$ , where  $m$  is the number of stored keys
12. Which priority queue implementation generalizes binary heaps by increasing the branching?  
 A. Binomial heaps    B. d-heaps    C. Fibonacci heaps    D. Leftist heaps
13. What is minimized in the dynamic programming solution to the subset sum problem?  
 A. The number of input values used to sum to each  $C(i)$     B.  $S_j$   
 C.  $m$     D. The index stored for each  $C(i)$
14. What is the worst-case number of rotations when performing deletion on an AVL tree?  
 A.  $\Theta(1)$     B.  $\Theta(\log n)$     C.  $\Theta(n)$     D. No rotations are ever used
15. If the universe size at the root of a van Emde Boas tree is  $u$ , then the number of children is:  
 A.  $\lg u$     B.  $\log u$     C.  $\log \log u$     D.  $\sqrt{u}$
16. Give the binomial min-heap that results from inserting 1, 2, 3, 4, 5, 6, 7, 8 (in that order) into an

empty heap. (5 points)

CSE 5311

Test 1 - Open Book

Summer 2014

Name \_\_\_\_\_

Student ID # \_\_\_\_\_

1. "During Disney's biggest celebration, find one of 50 character Wobblers inside specially-marked

**Kellogg's® cereals!".**

- a. Assuming all Wobblers are equally likely to be the one that occurs in a box, what is the expected number of boxes to obtain all 50 Wobblers? (3 points. You may leave your answer as an expression.)
- b. Under the same assumption as a., what is the expected number of boxes to obtain just 25 different Wobblers (of the available 50)? (7 points. You may leave your answer as an expression.)

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

a.  $T(n) = 8T\left(\frac{n}{2}\right) + n^4$

b.  $T(n) = 8T\left(\frac{n}{2}\right) + n^2$

c.  $T(n) = 8T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

n=6;	p[2]=0.2;	p[4]=0.2;
q[0]=0.12;	q[2]=0.03;	q[4]=0.0;
key[1]=10;	key[3]=30;	key[5]=50;
p[1]=0.12;	p[3]=0.1;	p[5]=0.01;
q[1]=0.09;	q[3]=0.04;	q[5]=0.04;
key[2]=20;	key[4]=40;	key[6]=60;

2

p[6]=0.03;	w[5][5]=0.040000	c(5,5) cost 0.000000
q[6]=0.02;	w[5][6]=0.090000	c(6,6) cost 0.000000
w[0][0]=0.120000	w[6][6]=0.020000	c(0,1) cost 0.330000 10
w[0][1]=0.330000	Building c(0,2) using roots 1 thru 2	c(1,2) cost 0.320000 20
w[0][2]=0.560000	Building c(1,3) using roots 2 thru 3	c(2,3) cost 0.170000 30
w[0][3]=0.700000	Building c(2,4) using roots 3 thru 4	c(3,4) cost 0.240000 40
w[0][4]=0.900000	Building c(3,5) using roots 4 thru 5	c(4,5) cost 0.050000 50
w[0][5]=0.950000	Building c(4,6) using roots 5 thru 6	c(5,6) cost 0.090000 60
w[0][6]=1.000000	Building c(0,3) using roots 1 thru 2	c(0,2) cost 0.880000 10(,20)
w[1][1]=0.090000	Building c(1,4) using roots 2 thru 4	c(1,3) cost 0.630000 20(,30)
w[1][2]=0.320000	Building c(2,5) using roots 4 thru 4	c(2,4) cost 0.540000 40(30,)
w[1][3]=0.460000	Building c(3,6) using roots 4 thru 6	c(3,5) cost 0.340000 40(,50)
w[1][4]=0.660000	Building c(0,4) using roots 2 thru 2	c(4,6) cost 0.150000 60(50,)
w[1][5]=0.710000	Building c(1,5) using roots 2 thru 4	c(0,3) cost 1.200000 20(10,30)
w[1][6]=0.760000	Building c(2,6) using roots 4 thru 4	c(1,4) cost 1.200000 20(,40(30,))
w[2][2]=0.030000	Building c(0,5) using roots 2 thru 2	c(2,5) cost 0.640000 40(30,50)
w[2][3]=0.170000	Building c(1,6) using roots 2 thru 4	c(3,6) cost 0.490000 40(,60(50,))
w[2][4]=0.370000	Building c(0,6) using roots ? thru ?	c(0,4) cost 1.770000 20(10,40(30,))
w[2][5]=0.420000	Counts - root trick 31 without root	c(1,5) cost 1.350000 20(,40(30,50))
w[2][6]=0.470000	trick 50	c(2,6) cost 0.790000 40(30,60(50,))
w[3][3]=0.040000	Average probe length is ???	c(0,5) cost 1.920000 20(10,40(30,50))
w[3][4]=0.240000	trees in parenthesized prefix	c(1,6) cost 1.540000 40(20(,30),60(50,))
w[3][5]=0.290000	c(0,0) cost 0.000000	c(0,6) cost ??? ????????????????????
w[3][6]=0.340000	c(1,1) cost 0.000000	
w[4][4]=0.000000	c(2,2) cost 0.000000	
w[4][5]=0.050000	c(3,3) cost 0.000000	
w[4][6]=0.100000	c(4,4) cost 0.000000	

4. Fill in the **min** and **max** blanks for the following instance of a van Emde Boas tree for the set  $\{1, 3, 5, 6, 7, 13, 14, 15\}$ . You should give these as values in the local universe ( $0 \dots u-1$ ). Instead of using the symbol “/” for NIL, use the symbol “ $\emptyset$ ”. (10 points)

```

root (base 0) u 16 min _____ max _____
  summary (base 0) u 4 min _____ max _____
    summary (base 0) u 2 min _____ max _____
      cluster[0] (base 0) u 2 min _____ max _____
      cluster[1] (base 2) u 2 min _____ max _____
    cluster[0] (base 0) u 4 min _____ max _____
      summary (base 0) u 2 min _____ max _____
      cluster[0] (base 0) u 2 min _____ max _____
      cluster[1] (base 2) u 2 min _____ max _____
    cluster[1] (base 4) u 4 min _____ max _____
      summary (base 0) u 2 min _____ max _____
      cluster[0] (base 4) u 2 min _____ max _____
      cluster[1] (base 6) u 2 min _____ max _____
    cluster[2] (base 8) u 4 min _____ max _____
      summary (base 0) u 2 min _____ max _____
      cluster[0] (base 8) u 2 min _____ max _____
      cluster[1] (base 10) u 2 min _____ max _____
    cluster[3] (base 12) u 4 min _____ max _____
      summary (base 0) u 2 min _____ max _____
      cluster[0] (base 12) u 2 min _____ max _____
      cluster[1] (base 14) u 2 min _____ max _____

```

2

5. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment ( $h_2(key)$ ) are given for each  $key$ . Show the result from inserting 1300 using Brent's rehash when  $h_1(1300) = 5$  and  $h_2(1300) = 3$ . (10 points)

	$key$	$h_1(key)$	$h_2(key)$
0			
1	1000	6	2
2			
3	1200	3	3
4	500	3	1
5	12	5	1
6	27	6	3

	$key$
0	
1	
2	
3	
4	
5	
6	

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Test 2 - Closed Book

Summer 2014

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. What is the nature of the signature function for the Karp-Rabin method?

- A. it is similar to the KMP failure links
- B. the remainder by discarding the overflow for a polynomial
- C. a polynomial of arbitrary precision implemented using a bignum package
- D. similar to a double hash function for string keys

2. When performing selection in worst-case linear time for  $n$  numbers, roughly how many column medians are computed in the first round?
  - A.  $W\left(\frac{n}{5}\right)$
  - B.  $m$ , the median-of-medians
  - C.  $0.7n$
  - D.  $\frac{n}{5}$
3. While constructing a suffix array for a sequence with  $n$  symbols by using radix sorts, sequence symbols are used in which of the radix sorts?
  - A. All of them
  - B. None of them
  - C. The last one
  - D. The first one
4. Which of the following is a deficiency of the maximum capacity path technique?
  - A. Augmenting paths will be discovered in descending incremental flow increase order.
  - B. Flow decomposition must be applied.
  - C. An augmenting path is blocked if it introduces a cycle of flow.
  - D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.
5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:
  - A.  $V$
  - B.  $O(VE)$
  - C.  $f$
  - D.  $E$
6. Which of the following is not true regarding Strassen's algorithm?
  - A. It is not possible to have an asymptotically faster algorithm.
  - B. It requires more space than the everyday method.

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- C. It uses  $\Theta(n^{\lg 7})$  scalar additions when multiplying two  $n \times n$  matrices.
- D. It uses  $\Theta(n^{\lg 7})$  scalar multiplications when multiplying two  $n \times n$  matrices.
7. The four Russians' concept is to:
  - A. Implement longest common subsequences using linear space
  - B. Pack bits into an efficient storage unit
  - C. Trade-off between enumerating situations and referencing these situations
  - D. Trade-off between scalar additions and multiplications
8. Which is not true regarding the first-fit decreasing method for bin packing?
  - A. Each object is placed by going left-to-right until it fits in a bin or a new bin is allocated.
  - B. Objects placed in bins beyond the optimal number have sizes no larger than  $\frac{1}{2}$ .
  - C. The number of objects placed in bins beyond the optimal number is arbitrary.
  - D. The number of objects with sizes larger than  $\frac{1}{2}$  is a lower bound on the optimal number of bins.
9. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?
  - A. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
  - B. The reduction takes polynomial time.
  - C. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.
  - D. Problem A is NP-complete.
10. The algorithm for finding a maximum capacity path for network flows is most similar to which algorithm?
  - A. Breadth-first search
  - B. Decomposition of a flow into  $E$  augmenting paths
  - C. Dijkstra
  - D. Floyd-Warshall
11. Which minimum spanning tree algorithm is the slowest?
  - A. Boruvka
  - B. Kruskal
  - C. Prim
  - D. Warshall
12. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:
  - A. Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.
  - B. The 3-sat instance is unsatisfiable.
  - C. The 3-sat instance is satisfiable.

- D. The 3-sat instance is a tautology.
13. Which of the following is not required before relabeling (“lifting”) a vertex to a new height?
- Both breadth-first searches have been done.
  - The vertex is overflowing.
  - Any eligible edges for the present height have been saturated.
  - The vertex is not the source or sink.
14. Which of the following does not have a polynomial-time approximation algorithm?
- Bin packing
  - Edge coloring
  - Traveling salesperson with triangle inequality
  - Vertex coloring
15. How many times will  $-1$  occur in the style 2 fail link table for the pattern **acaababc**?
- 1
  - 2
  - 3
  - 4
16. Suppose you have a set of points in Euclidean 2-d space. Give an algorithm for finding a  $\rho$ -approximation for the minimum traveling salesperson path. Be sure to give the value of  $\rho$ . (5 points)

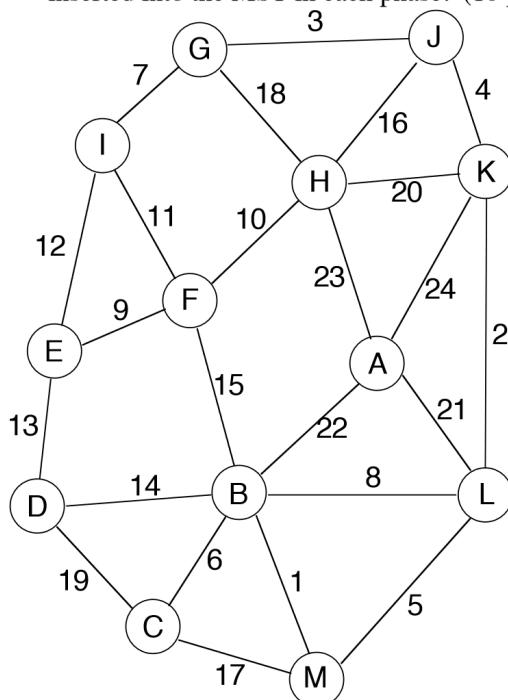
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Test 2 - Open Book  
Summer 2014

1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)



2. Give both types of KMP failure link tables and the table produced by the Z algorithm for the following string: (15 points)

0	a
1	c
2	a
3	b
4	a
5	c
6	a
7	c
8	a

```

9   b
10  a
11  c
12  a
13  b
14  a
15  c
16  a
17  c
18  a
19  b
20  a
21  c
22  a

```

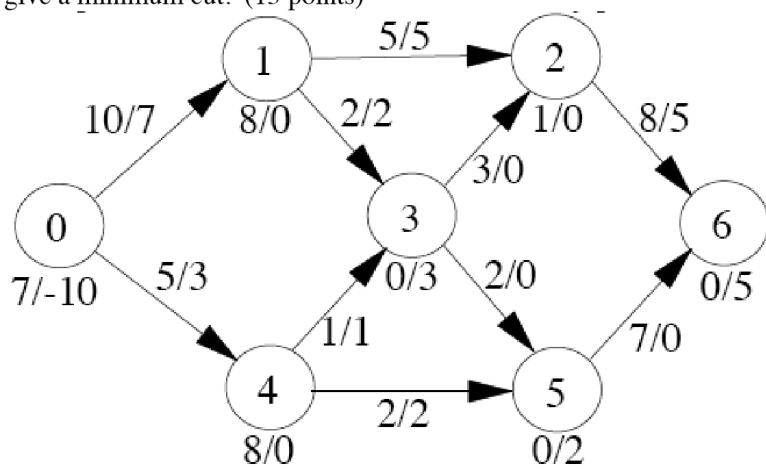
5

```

23  c
24  a

```

3. List the remaining operations to complete this instance of network flows by push-relabel. In addition, give a minimum cut. (15 points)



4. Use dynamic programming, either with a table or lists, to determine a subset that sums to 20. DO NOT SOLVE BY INSPECTION! (10 points)

2    3    5    7    11    13    17

