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ASSIGNMENT - 3

TASK - 1

Two logical equivalents S1 & S2 are logically equivalent if (S1 ⟺ S2) is valid.

Pseudo code:

```
CHECK_EQUIVALENCE (KB1, KB2)
{
    if (TT_ENTAILS (KB1, KB2) &&
                 TT_ENTAILS (KB2, KB1))
            return TRUE
    else
            return FALSE
}
```

we use TT_ENTAILS.

TASK - 2:

Part a: Does KB entail S1 ?

KB entails S1, because as per the entailment rule, if KB is true for any state, S1 is true too and if false, S1 is also false. S1 is either true or false. Therefore KB entails S1.

Part b: Does NOT (KB) entail NOT (~~KB~~ S_1) ?

As per entailment rules, and the table given, there are 2 states where NOT (KB) is true and NOT (S_1) is false, which does not satisfy entailment.

∴ NOT (KB) does not entail NOT (S_1).

TASK - 3

Two cases where KB is false:

First case: A is True, B is false, C is True, D is True

Second case: A is false, B is false, C is True, D is false.

1st case: $\sim(A \land \sim B \land C \land D)$

2nd case: $\sim(\sim A \land \sim B \land C \land \sim D)$

Converting to CNF:

$\sim(A \land \sim B \land C \land D) \land \sim(\sim A \land \sim B \land C \land \sim D)$

$(\sim A \lor \sim(\sim B) \lor \sim C \lor \sim D) \land (\sim(\sim A) \lor \sim(\sim B) \lor \sim C \lor \sim(\sim D))$

$\boxed{(\sim A \lor B \lor \sim C \lor \sim D) \land (A \lor B \lor \sim C \lor D)}$

TASK - 4

Given, KB is

$A \Rightarrow B$

$B \Leftrightarrow C$

$D \Rightarrow A$

$E \Rightarrow D$

C AND $E \Rightarrow F$

E

→ Converting this, we get

$A \Rightarrow B$

$B \Rightarrow C$

$C \Rightarrow B$

$D \Rightarrow A$

$E \Rightarrow D$

$C \wedge E \Rightarrow F$

E

From KB, we can form the rules as follows:

1. $A \Rightarrow B$
2. $B \Rightarrow C$
3. $C \Rightarrow B$
4. $D \Rightarrow A$
5. $E \Rightarrow D$
6. $C \wedge E \Rightarrow F$
7. $E \Rightarrow$

(i) forward Chaining:

By applying modus ponens,

From 5 and 7, $E \Rightarrow D$, E $\Rightarrow$ D → 8

From 4 and 8, $D \Rightarrow A$, D $\Rightarrow$ A → 9

From 1 and 9, $A \Rightarrow B$, A $\Rightarrow$ B → 10

From 2 and 10, $B \Rightarrow C$, B $\Rightarrow$ C → 11

From 3 and 11, $C \Rightarrow B$, C $\Rightarrow$ B → already there

from 7 and 11, C, E $\Rightarrow$ $C \wedge E$ → 12 [conjunction]

From 12 and 6, $C \wedge E$, $C \wedge E \rightarrow F$ F

∴ The given KB entails F

(ii) Backward Chaining :

From KB

for F, we use rule 6 → C and E

for C, we use rule 2 → B and E

for B, we use rule 1 → A and E

for A, we use rule 4 → D and E

for D, we use rule 5 → E

∴ E is a fact, F is entailed by KB

From 5 and 7 $E \Rightarrow D$, E   D → 8

4, 8 $D \Rightarrow A$, D   A → 9

1, 9 $A \Rightarrow B$, A   B → 10

2, 10 $B \Rightarrow C$, B   C → 11

C and E are facts, so $\{C \wedge E,\ C \wedge E \Rightarrow F\} \Rightarrow F$

∴ Among the rules $C \wedge E \Rightarrow F$ is used for entailing F from KB.

iii) Resolution :

Rules: $A \Rightarrow B$

$B \Rightarrow C$

$C \Rightarrow B$

$D \Rightarrow A$

$E \Rightarrow D$

$C \wedge E \Rightarrow F$

1. Add $\neg F$, as we have to prove KB entails F.

$(A \Rightarrow B) \land (B \Rightarrow C) \land (C \Rightarrow B) \land (D \Rightarrow A) \land (E \Rightarrow D) \land$
$(C \land E \Rightarrow F) \land E \land \neg F$

2. Remove $\Rightarrow$

$(\neg A \lor B) \land (\neg B \lor C) \land (\neg C \lor B) \land (\neg D \lor A) \land (\neg E \lor D) \land$
$(\neg(C \land E) \lor F) \land E \land \neg F$

3. Move $\neg$ inwards,

$(\neg A \lor B) \land (\neg B \lor C) \land (\neg C \lor B) \land (\neg D \lor A) \land (\neg E \lor D) \land$
$(\neg C \lor \neg E \lor F) \land E \land \neg F$

$\therefore$ CNF for KB is

$(\neg A \lor B) \land (\neg B \lor C) \land (\neg C \lor B) \land (\neg D \lor A) \land (\neg E \lor D) \land$
$(\neg C \lor \neg E \lor F) \land E \land \neg F$

From the above CNF, we form new rules

1. $\neg A \lor B$
2. $\neg B \lor C$
3. $\neg C \lor B$
4. $\neg D \lor A$
5. $\neg E \lor D$
6. $\neg C \lor \neg E \lor \neg F$
7. E
8. $\neg F$

Applying Resolution for,

5, 7 $\Rightarrow$ $(\neg E \lor D)$, E $\Rightarrow$ D $\Rightarrow$ (9)

4, 9 $\Rightarrow$ $(\neg D \lor A)$, D $\Rightarrow$ A $\Rightarrow$ (10)

1, 10 $\Rightarrow$ $(\neg A \vee B)$, A $\Rightarrow$ B $\Rightarrow$ 11

2, 11 $\Rightarrow$ $(\neg B \vee C)$, B $\Rightarrow$ C $\Rightarrow$ 12

3, 12 $\Rightarrow$ $(\neg C \vee \neg B)$, C $\Rightarrow$ B (already present)

6, 12 $\Rightarrow$ $(\neg C \vee \neg E \vee F)$, C $\Rightarrow$ $\neg E \vee F$ $\Rightarrow$ 13

7, 13 $\Rightarrow$ E, $(\neg E \vee F)$ $\Rightarrow$ F $\rightarrow$ 14

8, 14 $\Rightarrow$ $\neg F$, F $\Rightarrow$ empty

$\therefore$ We get empty after resolution, so the KB entails F.

TASK – 5

Part – a: Let A : rains

B : gives checks

C : mows the lawn.

constants are May – x, John – j, Mary – m

The predicates in contract $\Rightarrow$ $A(x)$, $B(j, m)$, $C(m)$

First order logic from predicates and constants,

1. $A(x) \Rightarrow B(j, m)$
2. $B(j, m) \Rightarrow C(m)$

$\boxed{\therefore\ A(x) \Rightarrow B(j,m) \ \wedge\ B(j,m) \Rightarrow C(m)}$

Part b.

What truly happened when using above constants and predicates.

$$[A'(x) \wedge B(j,m) \wedge C(m)]$$

Part - c:

Symbols:

rains (may) : R , not rains (may) : $\neg R$
checks (John, Mary) : C , not check (John, Mary) : $\neg C$
checks (Mary, John) : D , not checks (Mary, John) : $\neg D$
Mows lawn (Mary) : M , not mows (Mary) : $\neg M$
Mows lawn (John) : J , not mows (John) : $\neg J$

Part - d:

contract :-

$$(R \Rightarrow C) \wedge (C \Rightarrow M)$$

what truly happened

$$\neg R \wedge C \wedge M$$

Part - e:

No, contract is not violated, as no individual part of the contract is violated.

TASK-6

- Taller (x, John) : Shorter (John, Mary)
- Taller (x, Mother(x)) : ~~Shorter~~ Taller (Bob, y)
- Shorter (Bob, Mother (Bob)) : Shorter (x, Mother (y))
- Shorter (Bob, x) : Shorter (John, Mary)
- Taller (x, y) : Taller (Mother (Bob), Bob)

Let us consider, Taller = A & Shorter A'
Rewriting the problem statement, we have

$$A(x, John) : A'(John, Mary)$$
$$A(x, Mother(x)) : A(Bob, y)$$
$$A'(Bob, Mother(Bob)) : A'(x, Mother(y))$$
$$A'(Bob, x) : A'(John, Mary)$$
$$A(x, y) : A(Mother(Bob), Bob))$$

We can infer from above, the following.

x = Mother (Bob)
y = Bob
x = Mary
x = Bob
Bob = John

From this we can conclude
x = John

Substitute this in 1st statement, we have

$$A(John, John) : A'(John, Mary)$$

x = John && x = Mary ⇒ John = Mary

$$\boxed{\therefore A(x, John) : A'(John, Mary)}$$

Hence Unified