7. Given, $\dfrac{x^2y+x+y}{xy^2+y+7} = k$ (k is an integer)

$xy^2+y+7 \overline{)\, x^2y+x+y \,(}$

Let $x = 7n^2$, $y = 7n$.

then, $n(xy^2+7+y) = x^2y+y+x$.

So, we got $\boxed{x=11,\ y=1.}$

giving $xy^2+7+y=19$, $x^2y+y+x = 7\cdot 19$

n

Another,

We have,

$$y(x^2y+x+y) - x(xy^2+y+7)$$
$$= y^2-7x.$$

so, xy^2+y+7 must divide y^2-7x.

One possibility $\boxed{y^2=7x}$. hence 7 divides y and also x.

so, we got the solutions,

$$y^2-7x < y^2 < xy^2+y+7$$

we must have $y^2 < 7x$

if xy^2+y+7 is a factor of $7x-y^2$.

so, $y = 1$ or 2

so, $y=1 \Rightarrow x = 11$

$y = 2 \Rightarrow$ ~~x~~

so, $\boxed{y=1,\ x=11}$ $\rightarrow$ only one pair is possible.