

6.

$$\frac{a^2+b}{b^2-a}$$

$$\frac{b^2+a}{a^2-b} \rightarrow \text{integers.}$$

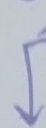
$$b^2-a \leq a^2+b$$

$$b^2+a \geq a^2-b$$

$$b^2-a^2 \leq b+a$$

$$b^2-a^2 \geq -a-b$$

$$b^2-a^2 \geq -(a+b)$$



From these two conditions.

$$(a+b) \geq b^2-a^2 \geq -(a+b) \rightarrow \textcircled{1}$$

$$(b-a)(b+a) \leq (b+a)$$

$$(b-a)(b+a) \geq -(a+b)$$

~~$$\text{If } a+b \leq 0$$~~

Then:

~~$$b-a \leq 1$$~~

~~$$b-a \geq -1$$~~

$$(b+a)(b-a-1) \leq 0$$

$$(b+a)(b-a+1) \geq 0$$

If $(b+a) > 0$.

$$(b-a-1) \leq 0$$

$$(b-a+1) \geq 0$$

$$b-a \leq 1$$

$$b-a \geq -1$$

$$\begin{aligned} b+a &> 0 \\ b-a &\leq 1 \\ b-a &\geq -1 \end{aligned}$$

If $b+a < 0$.

$$(b-a-1) \geq 0$$

$$(b-a+1) \leq 0$$

$$b-a \geq 1$$

$$(b-a) \leq -1$$

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 no +
 possible.

So, finally,

$$\begin{aligned} b+a &> 0 \\ b-a &\leq 1 \\ b-a &\geq -1 \end{aligned}$$

$\rightarrow b+a > 0$, $\bullet b-a \in [-1, 1]$.

\Downarrow

$(a-b) \in [-1, 1]$.

$$a^2 + b = k(b^2 - a)$$

$$(b^2 + a) = l(a^2 - b)$$

$$a^2 - kb^2 = -b - ka$$

$$b^2 - la^2 = -a - lb$$

$$a(a+k) = b(bk-1)$$

$$b(b+l) = a(al-1)$$

$$\frac{a}{b} = \frac{bk-1}{a+k} = \frac{b+l}{al-1}$$

$$(bk-1)(al-1) = (b+l)(a+k)$$

$$abkl - bk - al + 1 = ab + bk + al + kl$$

$$ab(kl-1) - 2(bk+al) + (1-kl) = 0$$

$$\underline{(ab-1)} \underline{(kl-1)} = \underline{2(bk+al)}$$

~~$$a+b > 0$$~~

$$b+a > 0$$

$$b-a \in [-1, 1]$$