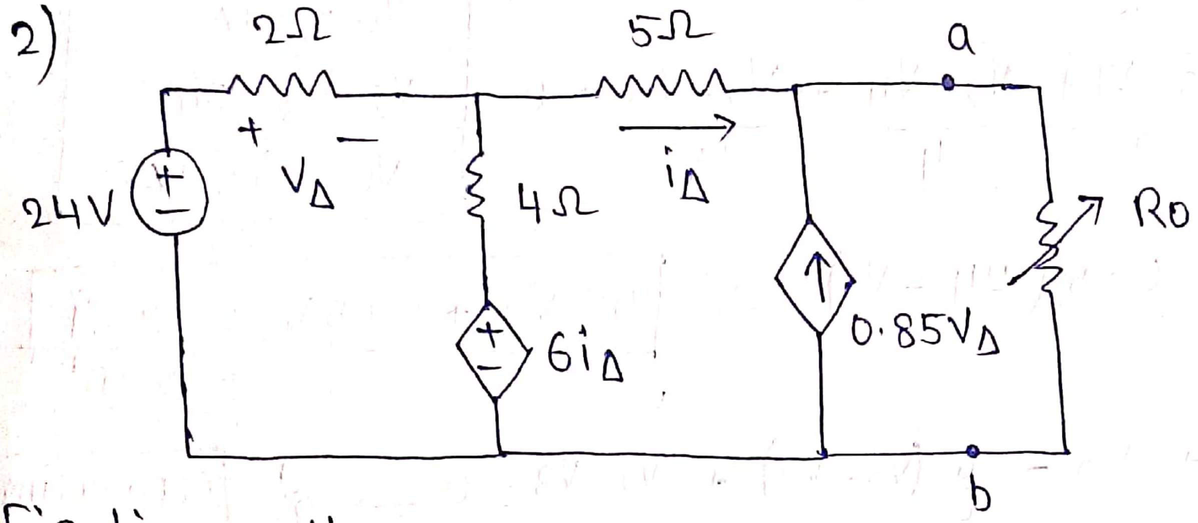
Network Theory

Home-work-2

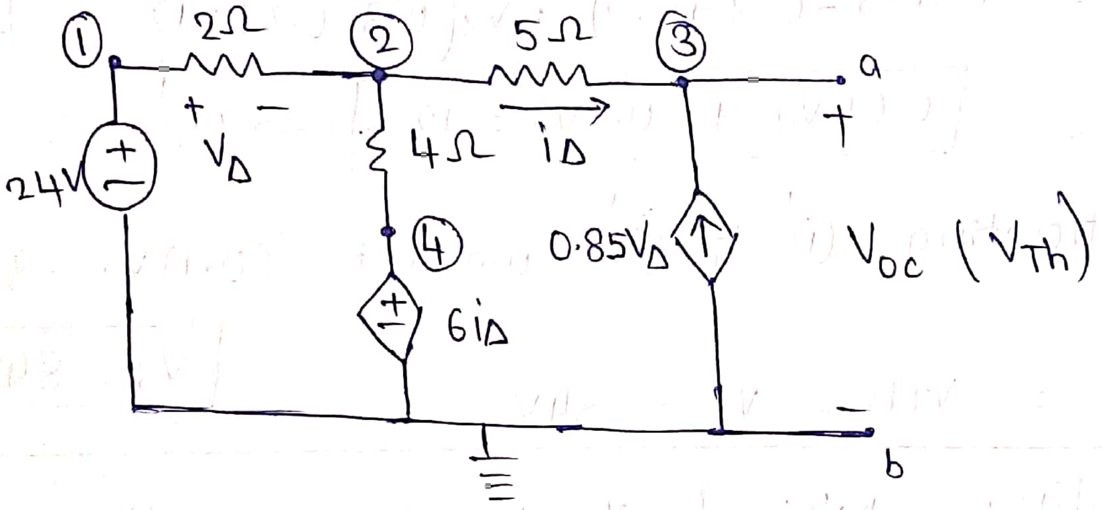
A. Sai Naga Manu
19EE01065

2)



Finding the thevenin's equivalent between the terminals a & b:

Voc:



At node-3:
$$\frac{V_3 - V_2}{5} + (-0.85V_\Delta) = 0 \quad — (1)$$

At node-2:
$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_4}{4} + \frac{V_2 - V_3}{5} = 0 \quad — (2)$$

Extra information:

$V_4 - 6i_\Delta = 0 \Rightarrow \boxed{V_4 = 6i_\Delta}$ — (3)

$V_1 - 24 = 0 \Rightarrow \boxed{V_1 = 24}$ — (4)

$V_2 - V_3 = 5i_\Delta$, — (5)

$V_1 - V_2 = V_\Delta$ — (6)

6- eq's & 6 unknowns ($V_1, V_2, V_3, V_4, i_\Delta, V_\Delta$)

1) $\Rightarrow \dfrac{V_3 - V_2}{5} = 0.85\, V_\Delta$

2) $\Rightarrow \dfrac{V_2 - V_1}{2} + \dfrac{V_2 - V_4}{4} + \dfrac{V_2 - V_3}{5} = 0$

$V_1 = 24$
$V_4 = 6 i_\Delta$
$V_2 - V_3 = 5 i_\Delta$
$V_1 - V_2 = V_\Delta$

$\dfrac{V_3 - V_2}{5} = 0.85(24 - V_2) \Rightarrow \boxed{0.2 V_3 + 0.65 V_2 = 20.4} \longrightarrow ①'$

$\dfrac{V_2 - 24}{2} + \dfrac{V_2 - \frac{6}{5}(V_2 - V_3)}{4} + \dfrac{V_2 - V_3}{5} = 0 \longrightarrow ②'$

converting everything interms of V_2 & V_3

$V_2(0.5 + 0.25 - 0.3 + 0.2) + V_3(0.3 - 0.2) = 12$

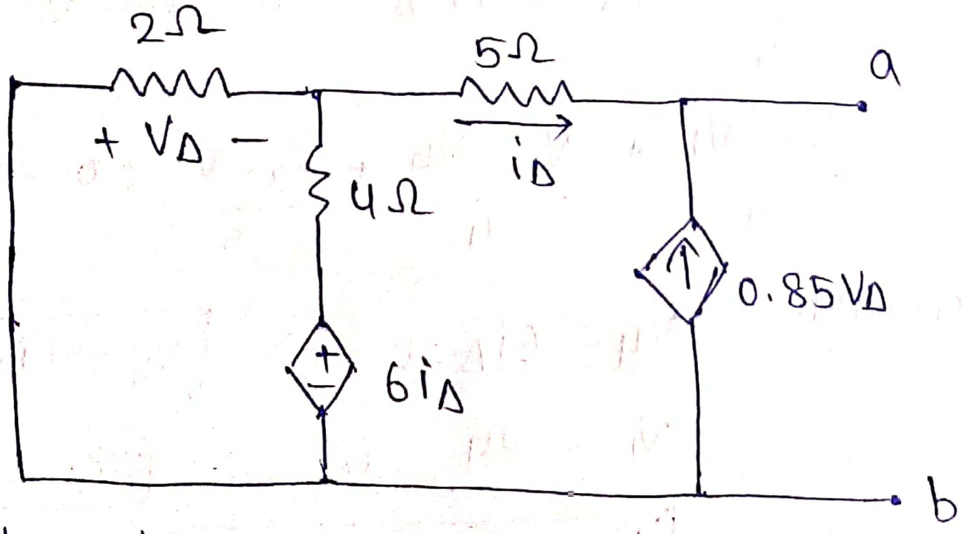
$\boxed{0.65 V_2 + 0.1 V_3 = 12} \longrightarrow ②'$

Subtracting ①' & ②' we get $(0.1) V_3 = 8.4$

$\boxed{V_3 = 84V}$

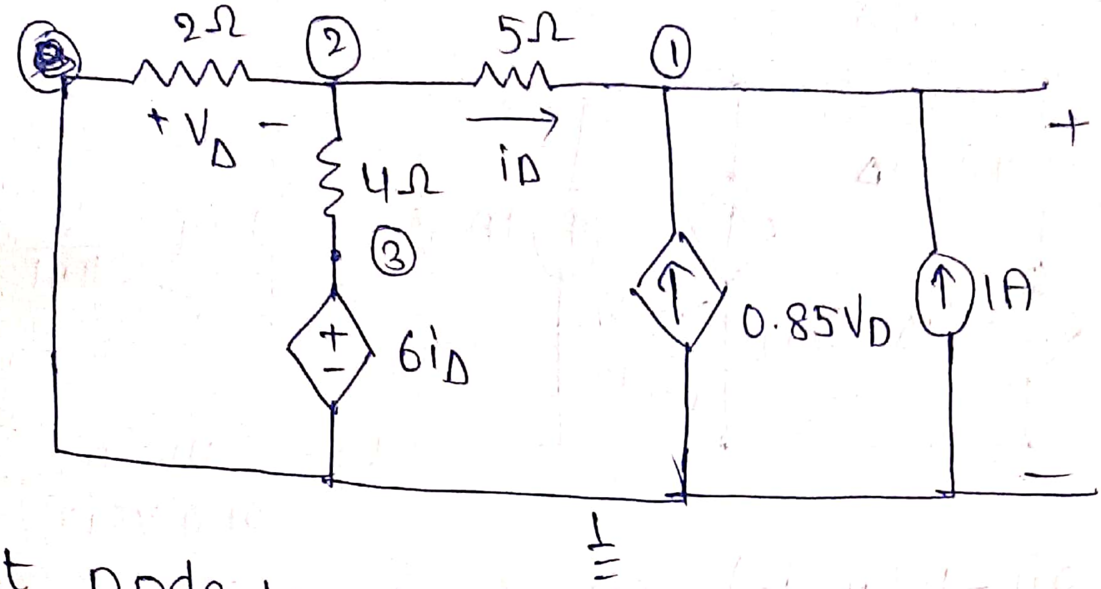
$V_{oc} = V_{Th} = V_3 = 84V$

Finding R_{Th}: (short - circuting the 24 V Source)



(No independent Source is present)

Connecting a test - Source of 1A:



at node-1: $i_\Delta + 0.85 V_\Delta + 1 = 0$

at node-2 $i_\Delta + \frac{V_2 - V_3}{4} + \frac{V_2}{2} = 0$ — eq ②

extra information $V_3 = 6 i_\Delta$, $V_2 = -V_\Delta$

conventing eq-② we get,

$$i_\Delta + \frac{-V_\Delta - 6 i_\Delta}{4} - \frac{V_\Delta}{2} = 0$$

$i_\Delta - 0.75 V_\Delta - 1.5 i_\Delta = 0 \Rightarrow 0.75 V_\Delta = -0.5 i_\Delta$

$$\boxed{i_\Delta = (V_\Delta)(-1.5)}$$

We get $i_\Delta + 0.85 V_\Delta + 1 = 0 \Rightarrow -1.5 V_\Delta + 0.85 V_\Delta + 1 = 0$

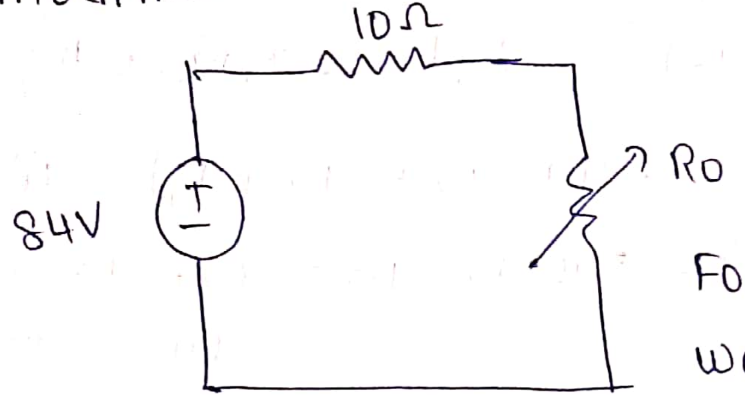
$$V_\Delta = \frac{1}{1.5 - 0.85} = \boxed{1.538 V}$$

$V_1 = V_2 - 5 i_\Delta = -V_\Delta + 7.5 V_\Delta = 6.5 V_\Delta = 10V$

$$\boxed{R_{Th} = \frac{V_1}{1A} = 10\Omega}$$

Scanned with CamScanner

Our modified circuit is

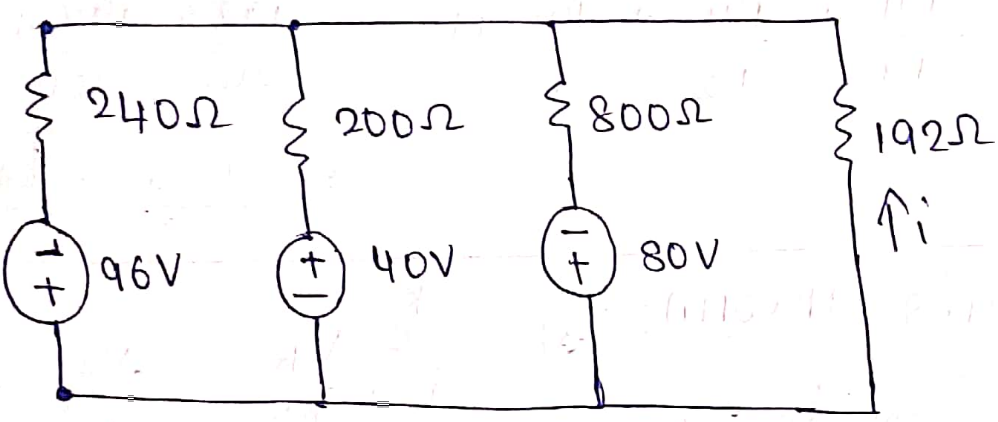


For $R_o = R_{Th} = 10\Omega$ we will get max power

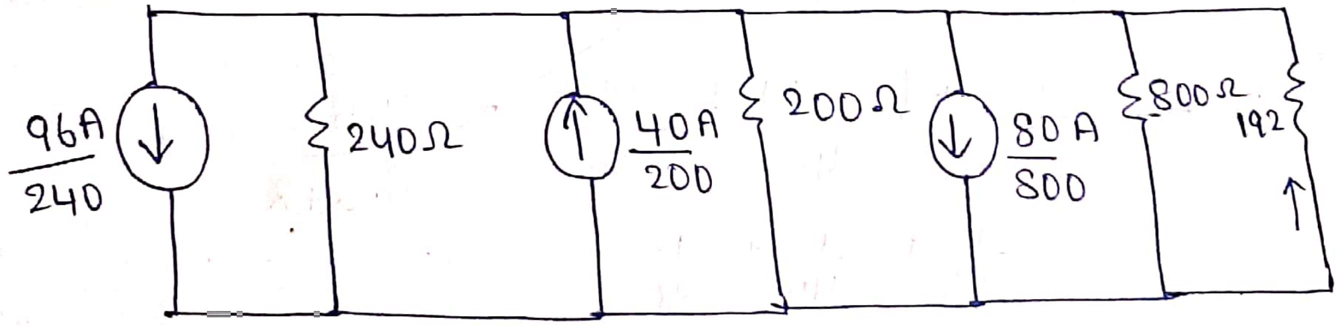
max power = $\dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{84\times 84}{4\times 10} = \boxed{176.4W}$

---

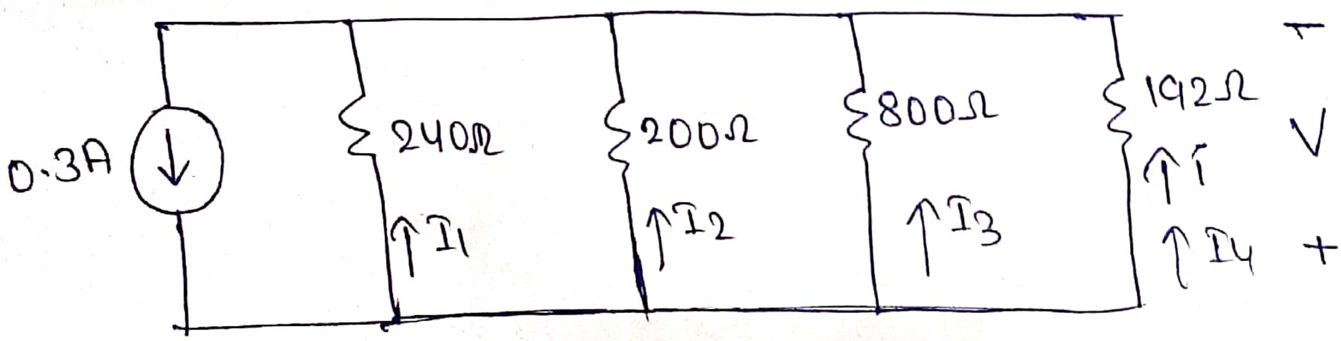
(4)



a) By source transformation:



Current sources can be grouped together!



as they are connected in parellel V is same $I_1 R_1 = I_2 R_2 = I_3 R_3 = I_4 R_4 = k$

By KCL, $0.3 = (I_1 + I_2 + I_3 + I_4)$ where

$R_1 = 240\,\Omega$, $R_2 = 200\,\Omega$, $R_3 = 800\,\Omega$,

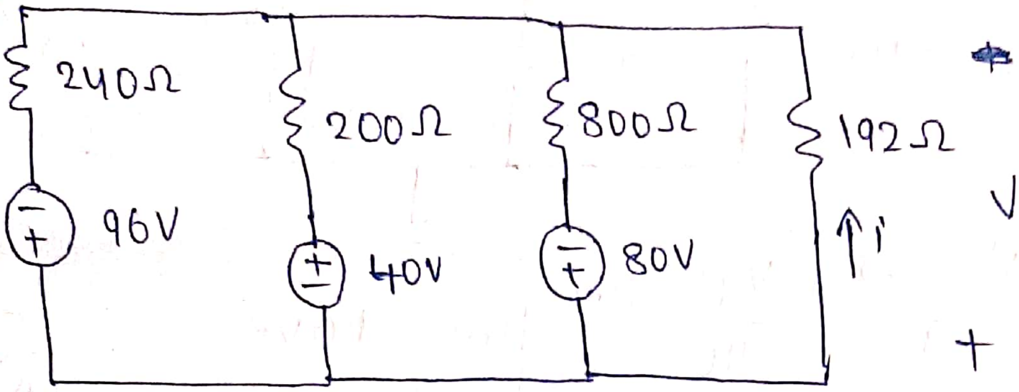
$R_4 = 192\,\Omega$

$$0.3 = I_4 \frac{R_4}{R_1} + I_4 \frac{R_4}{R_2} + I_4 \frac{R_4}{R_3} + I_4$$

$$I_4 = i = \frac{0.3}{\frac{R_4}{R_1} + \frac{R_4}{R_2} + \frac{R_4}{R_3} + 1} = \frac{0.3}{\frac{192}{240} + \frac{192}{200} + \frac{192}{800} + 1}$$

$$= \frac{0.3}{3} = \boxed{0.10A}$$

By millman's theorm:



(polarity is properly considered)

$$V = \frac{\sum_{i=1}^{3} \frac{V_i}{R_i}}{\sum_{i=1}^{3} \frac{1}{R_i}} \text{, then } \boxed{i = \frac{V}{192}\ A}$$

[considered as '0' voltage source]

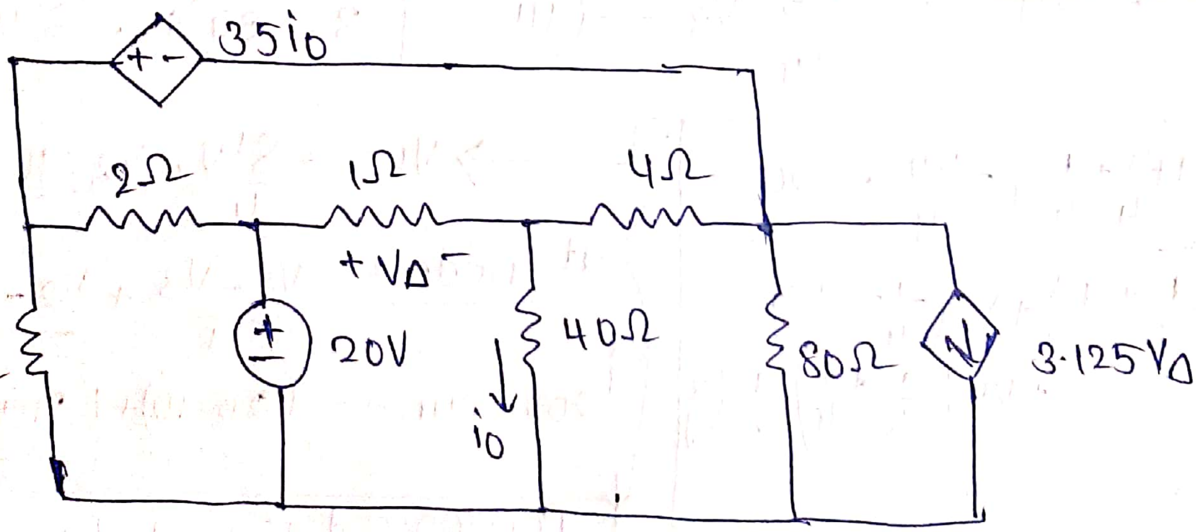
$$\Rightarrow V = \frac{\frac{96}{240} - \frac{40}{200} + \frac{80}{800} + \frac{0}{192}}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800} + \frac{1}{192}} = \frac{0.3}{0.015625}$$

$$\boxed{V = 19.2V} \Rightarrow i = \frac{19.2}{192} = \boxed{0.10A}$$

By both Source transformation and millman's theorm i through 192Ω came out to be <u>0.1A</u>

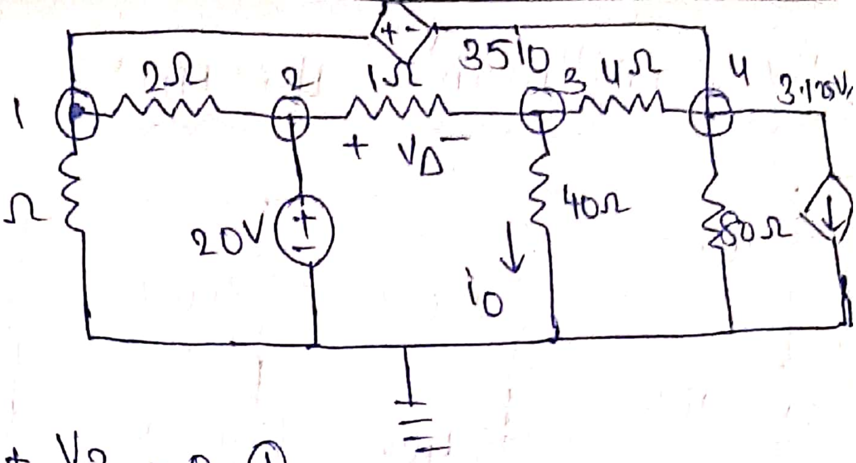
---

6)



Power developed by 20V Source = ?

Using nodal analysis:



At node 3:

$$\frac{V_3-V_4}{4}+\frac{V_3-V_2}{1}+\frac{V_3}{40}=0 \quad (1)$$

We have

$V_2 = 20V$, $i_o = \frac{V_3}{40}$ A

considering (1) & (4) as super node we get

$$\frac{V_1}{20}+\frac{V_1-V_2}{2}+\frac{V_4}{80}+3.125V_\Delta+\frac{V_4-V_3}{4}=0 \quad (2)$$

$$V_1 - V_4 = 35\, i_o$$

$$\boxed{V_1 - V_4 = \frac{35\,V_3}{40}} \quad (3)$$

$$\boxed{V_\Delta = V_2 - V_3 = 20 - V_3}$$

From (1)

$$\frac{V_3-V_4}{4}+V_3-20+\frac{V_3}{40}=0 \quad (\mathrm{I})$$

$$\frac{V_1}{20}+\frac{V_1-20}{2}+\frac{V_4}{80}+3.125(20-V_3)+\frac{V_4-V_3}{4}=0 \quad (\mathrm{II})$$

$$V_1 - V_4 = \frac{35}{40}V_3 \quad (\mathrm{III})$$

[3 − eq's, 3 unknowns]

$$V_3\left(1+\frac{1}{4}+\frac{1}{40}\right)-\frac{V_4}{4}=20$$

$$V_1\left(\frac{1}{20}+\frac{1}{2}\right)+V_3\left(-\frac{1}{4}-3.125\right)+V_4\left(\frac{1}{80}+\frac{1}{4}\right)=10-(20\times 3.125)$$

$$V_1 - V_4 - \frac{35\,V_3}{40}=0$$

→ $V_1 = -\frac{81}{4}V$, $V_3 = 10V$, $V_4 = 29$

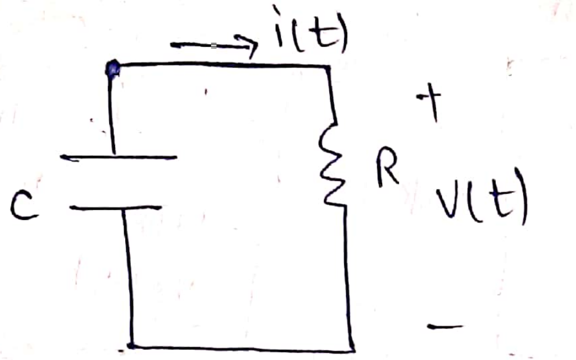
at node-2 $\frac{V_2-V_3}{1}+\frac{V_2-V_1}{2}-i=0$

$i_{20V\ source} = (20-10)+\left(\frac{20+\frac{81}{4}}{2}\right) = 30.125A$

Power delievered by 20V source

$= 20 \times 30.125 = 602.5\,W$

8)



Given $v(t) = 72e^{-500t}$ V $(t \geq 0)$

$i(t) = 9e^{-500t}$ mA $(t \geq 0^+)$

Differential equation: $C\frac{dv}{dt} + \frac{v}{R} = 0$

$$\frac{dv}{dt} = \frac{-v}{RC}$$

$$\boxed{V = V_0 e^{-t/RC}} \quad t \geq 0$$

Comparing with given data, we get $\boxed{\gamma = RC = \frac{1}{500}}$

$V_0 = 72V$ & $RC = \frac{1}{500}$

We can also say that $\boxed{V_c(0^-) = V_c(0^+) = 72V}$

at $t = 0$, $i(0) = 9$ mA

$V(0) = V_0 = 72V$

∴ Just after C is connected to R it behaves as a dc source and then decays exponentially

$V(0) = i(0) R$

$72 = 9 \, 10^{-3} R \Rightarrow \boxed{R = 8k\Omega}$

✓ We also have $RC = \frac{1}{500}$ $\Rightarrow C = \frac{1}{500 \times 8 \times 10^3}$

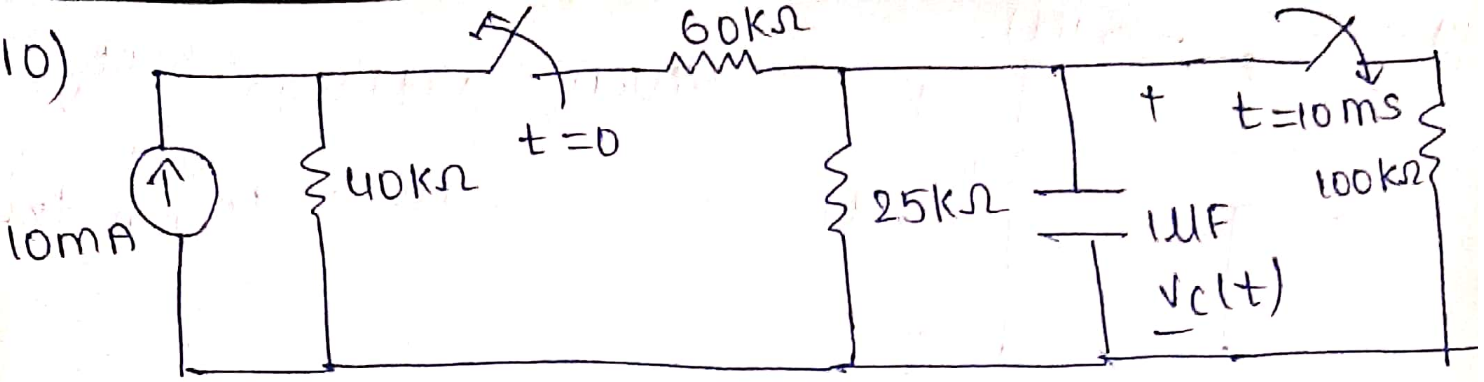
(Time constant = 1/500) $\boxed{C = 250 \text{ nF}}$

✓ Initial energy stored in the capacitor

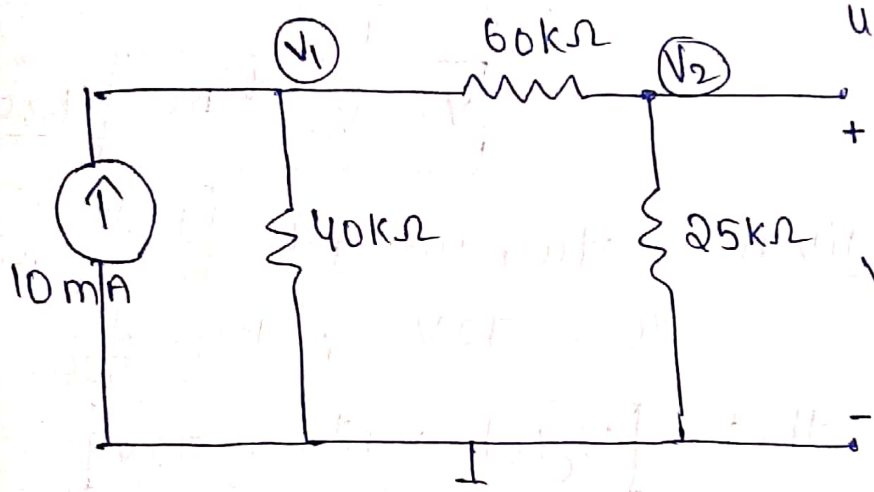
$= \frac{1}{2} C V_0^2 = \frac{1}{2} \times 250 \times 10^{-9} \times 72 \times 72$

$= \boxed{648 \, 10^{-6} \text{ J}}$ (648 μJ)

10)



For $t \leq \bar{0}$: (1μF capacitor will be considered as open circuit as it is under dc source for long time)



node-1 $\qquad \dfrac{V_1}{40} + \dfrac{V_1 - V_2}{60} - 10 = 0$

node-2 $\qquad \dfrac{V_2}{25} + \dfrac{V_2 - V_1}{60} = 0 \implies V_2\left(\dfrac{1}{25} + \dfrac{1}{60}\right) = \dfrac{V_1}{60}$

$$V_2\left(1 + \frac{60}{25}\right) = V_1$$

$$\boxed{V_1 = V_2 \frac{17}{5}}$$

$$V_1\left(\frac{1}{40} + \frac{1}{60}\right) - \frac{V_2}{60} = 10$$

$$\frac{17V_2}{\cancel{5}}\left(\frac{\cancel{3+2}}{120}\right) - \frac{V_2}{60} = 10$$

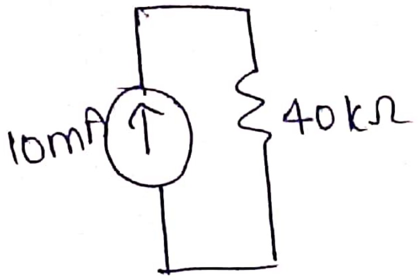
$$\frac{17V_2}{120} - \frac{2V_2}{120} = 10$$

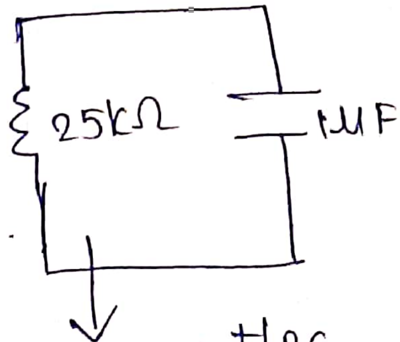
$$\frac{\cancel{15}V_2}{\cancel{120}\,8} = 10$$

$$\boxed{V_2 = 80V}$$

$$\implies \boxed{V_c(\bar{0}) = 80V}$$

For $0 \le t \le 10$ ms





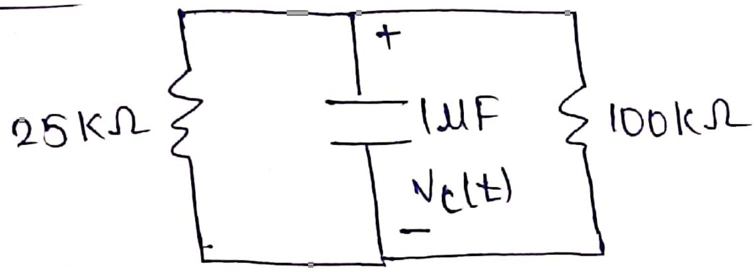
$$V_c = V_0 e^{-t/RC}$$

$$V_0 = V_c(0^+) = V_c(0^-) = 80$$

$$RC = 25\ 10^{3}\ 10^{-6} = 25 \times 10^{-3} \Rightarrow V_c(t) = 80\ e^{-\frac{t\ 10^3}{25}}$$

$$\boxed{V_c(t) = 80\ e^{-40t}\ V}$$

For $t \ge 10$ms



$$Req = \frac{\cancel{25} \times 100}{\cancel{125}\ 5} = 20k\Omega \qquad \tau = RC = 20 \times 10^{3} \times 10^{-6}$$

$$C = 1\mu F \qquad = \boxed{20 \times 10^{-3}}$$

$$V_c(10ms^+) = V_c(10ms^-) = 80 \times e^{-40 \times 10 \times 10^{-3}}$$

$$\boxed{V_c(10^-) = 53.62V}$$

For $t \ge 10$ms $\quad V_c(t) = 53.62\ e^{-\frac{(t - 10 \times 10^{-3})\ 10^3}{20}}$

$$= 53.62\ e^{-50(t-0.01)}V$$

$$\boxed{V_c(t) = \begin{cases} 80 & t \le 0^- \\ 80\ e^{-40t} & 0 \le t \le 10\text{ms} \\ 53.62\ e^{-50(t-0.01)} & 10\text{ms} \le t < \infty \end{cases}}$$