6. $\dfrac{a^2+b}{b^2-a}$, $\dfrac{b^2+a}{a^2-b}$ → integers.

$b^2-a \leq a^2+b$ $\qquad$ $b^2+a \geq a^2-b$

$b^2-a^2 \leq b+a$ $\qquad$ $b^2-a^2 \geq -a-b$

$b^2-a^2 \geq -(a+b)$

From these two conditions.

$(a+b) \geq b^2-a^2 \geq -(a+b)$ → 1.

$(b-a)(b+a) \leq (b+a)$ $\qquad$ $(b-a)(b+a) \geq -(a+b)$

~~If a+b ... a+b~~ > 0.

~~There~~

$(b+a)(b-a-1) \leq 0$. $\qquad$ $(b+a)(b-a+1) \geq 0$.

If $(b+a) > 0$.

$(b-a-1) \leq 0$. $(b-a+1) \geqslant 0$.

$b-a \leq 1$ $b-a \geqslant -1$.

$$\boxed{\begin{array}{l} b+a > 0 \\ b-a \leq 1 \\ b-a \geqslant -1 \end{array}}$$

If $b+a < 0$.

$(b-a-1) \geqslant 0$. $(b-a+1) \leq 0$.

$b-a \geqslant 1$ $(b-a) \leq -1$

not possible.

So, finally, $b+a > 0$

$b-a \leq 1$

$b-a \geqslant -1$

$\rightarrow$ $b+a > 0$, $b-a \in [-1,1]$.

$\Downarrow$

$(a-b) \in [-1,1]$.

$a^2+b = k(b^2-a)$ $\qquad$ $(b^2+a) = \ell(a^2-b)$.

$a^2 - kb^2 = -b - ka$ $\qquad$ $b^2 - \ell a^2 = -a - \ell b$.

$a(a+k) = b(bk-1)$ $\qquad$ $b(b+\ell) = a(a\ell - 1)$.

$$\frac{a}{b} = \frac{bk-1}{a+k} = \frac{b+\ell}{a\ell-1}.$$

$$(bk-1)(a\ell-1) = (b+\ell)(a+k).$$

$$abk\ell - bk - a\ell + 1 = ab + bk + a\ell + k\ell.$$

$$ab(k\ell-1) - 2(bk+a\ell) + (1-k\ell) = 0.$$

$$\underline{(ab-1)}\,\underline{(k\ell-1)} = \underline{2}\,\underline{(bk+a\ell)}.$$

$b+a>0$

$b-a \in [-1,1]$