[Special Topics] Homework 1 CS 4301.001 Sudarshana Jagadeeshi

Question 1-a:

Proof convex:

We want the second derivative to be greater or equal to 0. Differentiating w.r.t to w we get, $[\max(0, 1 - y_m w x_m)^2]$ " = $[\max(0, 2 - 2y_m w x_m)]$ * $(-y_m x_m)$] = $[\max(0, -2y_m x_m)]$. This is positive everywhere, because the minimum value the function can take is 0.

Compute Gradient/Subgradient:

The ith entry of the gradient is: $max(0, 2 - 2y_m w_i x_m) * (-y_m x_m)$). This is just the first derivative I put above, but w_i instead of w_i because all other dimensions should differentiate out to 0.

Updates for Gradient w/ Fixed step size:

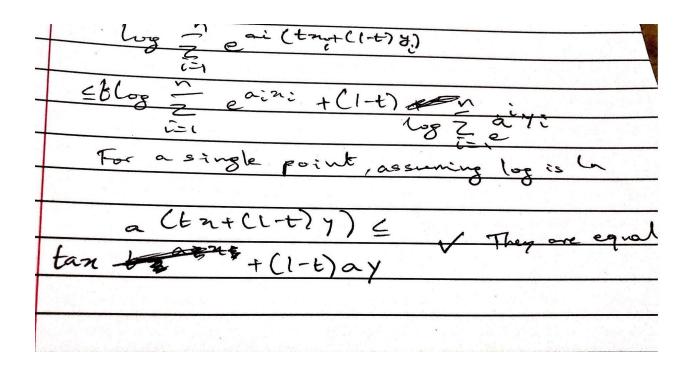
$$(w_i)_t+1 = (w_i)_t - gamma * gradient of (w_i)_t$$

 $\Rightarrow (w_i)_t+1 = (w_i)_t - gamma * max(0, 2 - 2y_m w_i x_m) * (-y_m x_m)).$

Question 1-b:

Proof convex:

If we choose an x and y, we want to show $f(tx + (1-t)y) \le t f(x) + (1-t)f(y)$



Compute Gradient/Subgradient:

x is a vector(i.e. $w_1 x_1 + w_2 x_2 + ... w_n x_n$) The ith entry of the gradient is $aw_i^* exp(a^i x^i)$. The exponent stays the same, but we differentiate the vector x and get only a w_i .

Updates for Gradient w/ Fixed step size: (w_i)_t+1 = (w_i)_t - gamma * gradient of (w_i)_t ⇒ (w i) t+1 = (w i) t - gamma * aw i* exp(a^i x^i)

Question 1-c:

Proof convex:

Compute Gradient/Subgradient:

Updates for Gradient w/ Fixed step size:

Question 2-1-a:

Refer to gd_userinput.py. I've hosted the linreg.data file via url. Input for X and y must be separated by spaces.

Question 2-1-b:

Refer to problem2-1b.xlsx. The generating code is in gd_final.py.

Question 2-2-a

Refer to file subgradient_final.py

Question 2-2b

Refer to file problem2-2b.xlsx

Question 3-1-a:

Yes. We know that the infimum preserves convexity, so we want to just know if the inside is a convex function. Deriving once we get: $[e^a (\ln(e^a + 1) + \ln(e^a + 1))]/(e^a + 1)^2$.

Deriving again we get,

$$-rac{\mathrm{e}^a\left(\left(\mathrm{e}^a-1
ight)\ln\!\left(rac{\mathrm{e}^a}{\mathrm{e}^a+1}
ight)+\left(\mathrm{e}^a-1
ight)\ln\!\left(\mathrm{e}^a+1
ight)-\mathrm{e}^a-1
ight)}{\left(\mathrm{e}^a+1
ight)^3}$$

Which should be positive everywhere. Something went wrong with my math, but the original function is convex.

Question 3-1-b:

Question 3-1-c:

Question 3-2-a:

Refer to file cvxnum.py min of -132.66535571301728 @ approximately x=-4.75